My Bayesian OLS python package

I don't really code

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Abstract

Suppose you want to do OLS regressions to fit data from snap-shots captured at some frequency. Gauthier and Simonato (2012) show there can be several sets of parameters which produce very similar fits. A Bayesian approach is used to regularize the regressions and stabilize the betas. The regression packaged here is directly from François et. al. (2022).

1 Bayesian Generalized Least-Squares Regression: BGLS

The objective is to perform a regular OLS regression, as $Y = X\beta + \epsilon$, but with priors on β. We specify

$$\begin{bmatrix} Y \\ \beta_{prior} \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix} \beta + \begin{bmatrix} \epsilon \\ \delta \end{bmatrix}, \tag{1}$$

where Y is the vector of observations, β_{prior} the prior's expectation, X the factors, R the matrix linking β to the prior, and (ϵ, δ) the vector of errors which follows a multivariate normal distribution with diagonal covariance matrix

$$\Omega = \begin{bmatrix} \Sigma_\epsilon & 0 \\ 0 & \Sigma_\delta. \end{bmatrix} \tag{2}$$

The generalized least quares estimator of β with prior information is

$$\hat{\beta} = \left(\begin{bmatrix} X \\ R \end{bmatrix}^T \Omega^{-1} \begin{bmatrix} X \\ R \end{bmatrix} \right)^{-1} \begin{bmatrix} X \\ R \end{bmatrix}^T \Omega^{-1} \begin{bmatrix} Y \\ \beta_{prior} \end{bmatrix} \tag{3}$$

The value of Σ_ϵ can be estimated using OLS without priors and the value of Σ_δ is a hyperparameter that controls the prior distribution. The smaller the values on the diagonal of Σ_δ are, the closer $\hat{\beta}$ will be to β_{prior}

2 Use case

As an example, consider fitting a simple model to an Implied Volatility surface (IV) for a specific moment in time. Then repeat the fit at a certain frequency to obtain times series of $\hat{\beta}_t$, with the objective of fitting a time series model on $\hat{\beta}_t$.

Having well-behaved $\hat{\beta}$ will be instrumental in being successful.

3 Reference

François, P., Galarneau-Vincent, R., Gauthier, G., and Godin, F. (2022). Venturing into uncharted territory: An extensible implied volatility surface model. Journal of Futures Markets, 42(10), 1912-1940.

Gauthier, G. and Simonato, J.-G. (2012). Linearized Nelson-Siegel and Svensson models for the estimation of spot interest rates. European Journal of Operational Research, 219(2):442–451.