You should solve five out of six problems at your choice.

1. Find the expected value of $\mathrm{E}(\exp(aW_t))$, $\mathrm{E}(1_{W_t \le b})$, $\mathrm{E}(\exp(aW_t) \cdot 1_{W_t \le b})$ and $\mathrm{E}(W_t 1_{W_t \le b})$. Naturally, you may use the standard normal cumulative distribution function F in your answer

2. It is known that $\mathrm{E}(Y|X) = 0$. Which of the following quantities must be zero: $\mathrm{E}(Y)$? $\mathrm{E}(X)$? $\mathrm{Cov}(X,Y)$? $\mathrm{Cov}(X^2,Y)$? $\mathrm{Cov}(X,Y^2)$? Prove or provide a counter-example.

3. Alisa and Bob throw a fair coin until either the sequence THTH or the sequence HTHH appears. Alisa wins if the sequence THTH appears first and Bob wins if the sequence HTHH appears first.

   (a) What is the probability that Alisa will win?

   (b) What is the expected duration of the game in tosses?

   Hint: you may introduce a martingale from lecture or solve this problem without martingales at all

4. Consider the process $Y_t = \exp(2W_t - 2t)$.

   (a) Find dY_t

   (b) Find $\int_0^t Y_u \, dW_u$

   (c) Find $\mathrm{E}(Y_t)$ and $\mathrm{Var}(Y_t)$

5. Consider stochastic differential equation

$$dX_t = (a - bX_t)dt + c dW_t, \; X_0 = x_0$$

   (a) Solve this differential equation[1].

   (b) Find $\mathrm{E}(X_t)$ and $\mathrm{Var}(X_t)$

6. In the framework of Black and Scholes model find the price at $t = 0$ of the classic European call option by calculating corresponding expected value.

   European call option with strike price K is the right to buy at time t one share at price K. So, at time t it pays you $S_t - K$ if $S_t > K$ and zero otherwise.

You may decide not to solve any number of problems from this hometask and send me a solution of corresponding number of problems from old exams collection instead. In this case you should use LaTeX or markdown formats. Please, be polite and avoid sending solutions of already solved problems or problems taken by someone else. You may choose problems and fill in your choice at `http://goo.gl/8TFhWE`.

The due date for this hometask — 11.01.2016. If you decide to solve problems from an old exam you should send the solution of these problems by e-mail to boris.demeshev@gmail.com before 09.01.2016. I will check your solutions and add them into the exams collection so they will be available for everyone. You will receive extra bonus points in this case.

[1]The answer may contain an Ito integral that cannot be simplified.