Here W_t denotes the standard Wiener process.

1. Find the following stochastic integral:

$$I_t = \int_0^t 3W_u - 5\,dW_u$$

   Find $\mathbb{E}(I_t)$ and $\mathrm{Var}(I_t)$

2. For the standard Wiener process W_t find
   (a) $\mathbb{E}(W_6)$, $\mathbb{E}(W_3^7)$, $\mathbb{E}(W_5 \cdot W_6)$
   (b) $\mathrm{Var}(W_6)$, $\mathrm{Var}(W_6 + W_5)$
   (c) $\mathrm{Cov}(2W_3 + W_4, W_1)$

3. Consider the process $X_t = W_t^3 - 4tW_t$.
   (a) Use Ito's lemma to find dX_t
   (b) Write the corresponding full form
   (c) Is X_t a martingale?

4. Let τ be a stopping time, the first moment of time when W_t hits 2 or -1.
   (a) Apply Doob's theorem for the martingale W_t and find $\mathbb{P}(W_\tau = 2)$
   (b) Apply Doob's theorem for the martingale $Y_t = W_t^2 - t$ and find $\mathbb{E}(\tau)$

5. In the framework of Black and Scholes model find the price of an asset which pays you $X_2 = \ln(S_1)$ at the moment $T = 2$. Here S_1 denotes the price of the share at time $t = 1$. Negative value of X_2 means that you should pay the corresponding amount. The parameters are: starting share price, $S_0 = 100$, risk-free rate, $r = 0.1$, volatility, $\sigma = 0.2$.