Graded hometask on stochastic calculus. Icef. 2014.

1. Researcher Veniamin throws a fair dice until 6 appears. Let denote by T the total number of throws and by N the number of throws when 5 appeared. Find $\mathrm{E}(N|T)$, $\mathrm{Var}(N|T)$, $\mathrm{E}(N)$, $\mathrm{Var}(N)$ and $\mathrm{E}(T|N)$.

2. It is $25°C$ in Australia today. Each day the temperature goes one degree up or down with equal probability. Each day I will put the sum equal to the temperature in my piggy bank. So on the first day I will put 25 roubles. I will stop my investment strategy when the temperature will reach $15°C$ or $30°C$ for the first time.

   What will be the expected value of my account in my piggy bank?

3. Veniamin and Varvara are managers of two gladiators-vampires teams. Veniamin has 4 gladiators-vampires with initial strengths 1, 2, 3 and 4. Varvara has 3 glagiators-vampires with initial strengths 1, 3 and 5.

   The competition between two teams is organised as a sequence of rounds. In each round two gladiators-vampires (one from each team) will meet and fight to the death. Varvara always selects the best gladiator from her team to fight in the next round.

   When the gladiators of strengths a and b meet the first will win with probability $a/(a+b)$, the second — with probability $b/(a+b)$. The gladiators are vampires, so the strength of the winner will become $a+b$.

   What is best strategy for Veniamin? What is the probability that Veniamin's team will win?

4. Consider the processes $X_t = \int_0^t (sW_s)^3\, dW_s$ and $Y_t = \int_0^t W_s\, dW_s$. Find $\mathrm{E}(X_t)$, $\mathrm{Var}(X_t)$, $\mathrm{Cov}(X_t, Y_t)$

5. The process X_t is given by

$$dX_t = tW_t e^{W_t}\, dt + \sin(tW_t)\, dW_t,\ X_0 = 1$$

   Using Ito-Doeblin lemma find Z_0 and dZ_t if $Z_t = X_t^2 + t\cos(X_t)$

6. Solve the stochastic differential equation:

$$dX_t = X_t(1 - X_t)\, dt + X_t\, dW_t,\ X_0 = 1$$

   Some hints: you may find the substitution $Y_t = \ln X_t$ useful.

7. In the framework of Black and Scholes model find the price at $t = 0$ of the asset, which pays you 1\$ if $S_{T-1} > 1$ and 0 otherwise. Here S_t denotes the price of a share at time t. The payment is made at the fixed moment of time $T > 1$.

Note: you may decide not to solve one problem of your choice from this hometask and send me a solution of one problem from old exams collection instead. In this case you should use LaTeX or markdown or (ugly) Word. Please, be polite and avoid sending solutions of already solved problems or problems taken by someone else. You may choose a problem and fill in your choice at `http://goo.gl/8TFhWE`.

The due date for this hometask — 27.12.2014. If you decide to solve one problem from an old exam you should sent the solution of this problem by e-mail to boris.demeshev@gmail.com before 20.12.2014.