

1. [10 points] In the framework of Black and Scholes model find the price of the asset, which pays you S_2/S_1 at the fixed moment of time $T = 2$
2. [10 points] Let τ be a stopping time, the first moment when W_t hits 2 or -1 .
 - (a) Find the probability $\mathbb{P}(W_\tau = 2)$
 - (b) Find a martingale of the form $X_t = W_t^2 + f(t)$.
 - (c) Find $E(\tau)$
3. [10 points] Let $Y_t = W_t^3 - tW_t^4$. Find $E(Y_t)$ and $\text{Var}(Y_t)$.
4. [20 points] Solve the stochastic differential equation

$$dX_t = (X_t/t + t)dt + 2\sqrt{tX_t}dW_t$$

Hint: You may suppose without a proof that the solution has the form $X_t = f(t)g(W_t)$.