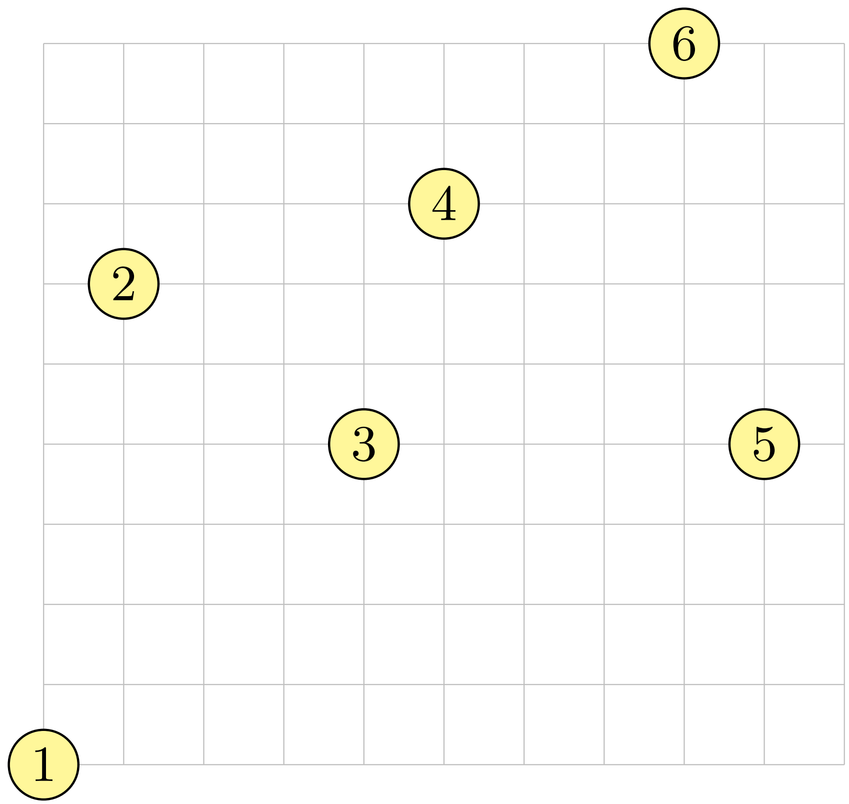
Heuristics for the TSP[2]

- Phase 1 – Tour construction (nearest neighbor)
- Phase 2 – Improvement (subtour reversal)



[2]This example comes from Goetschalckx, p. 223

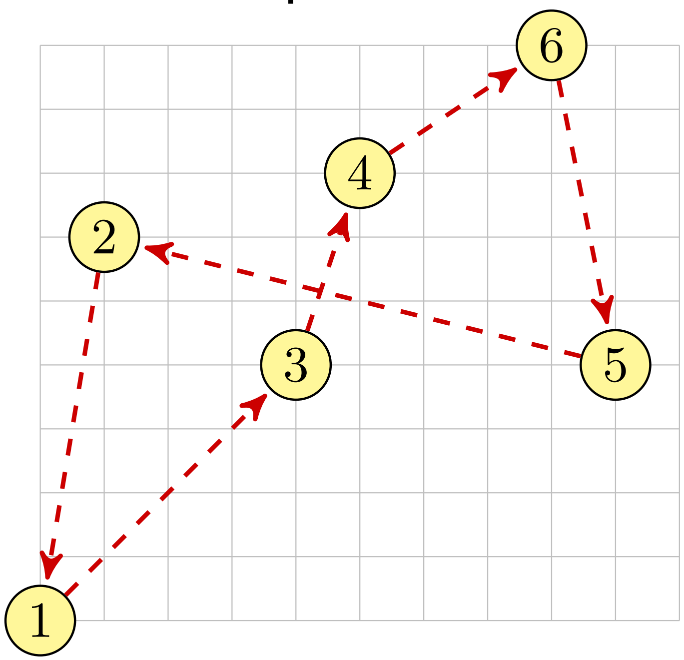
Euclidean (and symmetric) Distances

	1	2	3	4	5	6
1	-	608	566	860	985	1,204
2	608	-	361	412	825	762
3	566	361	-	316	500	640
4	860	412	316	-	500	361
5	985	825	500	500	-	510
6	1,204	762	640	361	510	-

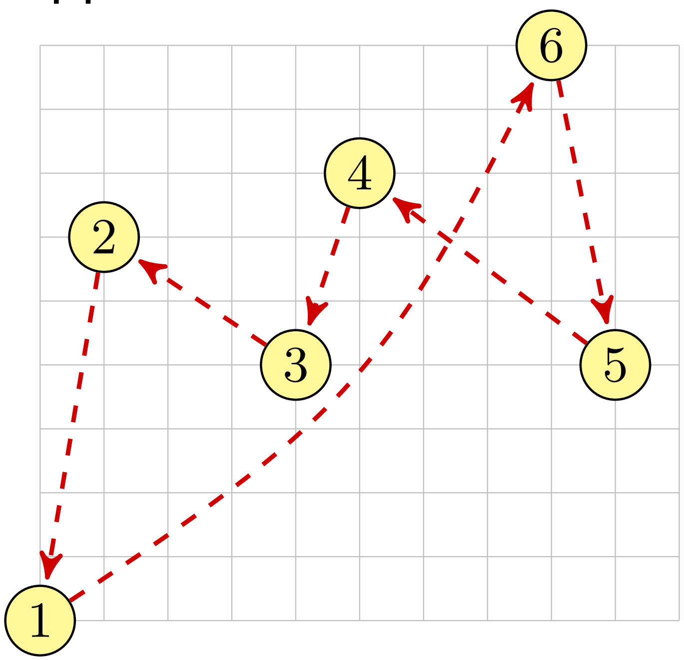
Nearest Neighbor

1) Choose a starting node, i. $N = \{1, \ldots, n\} \setminus i$.
2) Choose the nearest node to i. Call this node j.
   $N \leftarrow N \setminus j$ (remove j from the list of candidate nodes)
   $i \leftarrow j$ (The current node is now j)
3) Repeat Step 2 until $N = \varnothing$.
   Connect the first and last nodes.

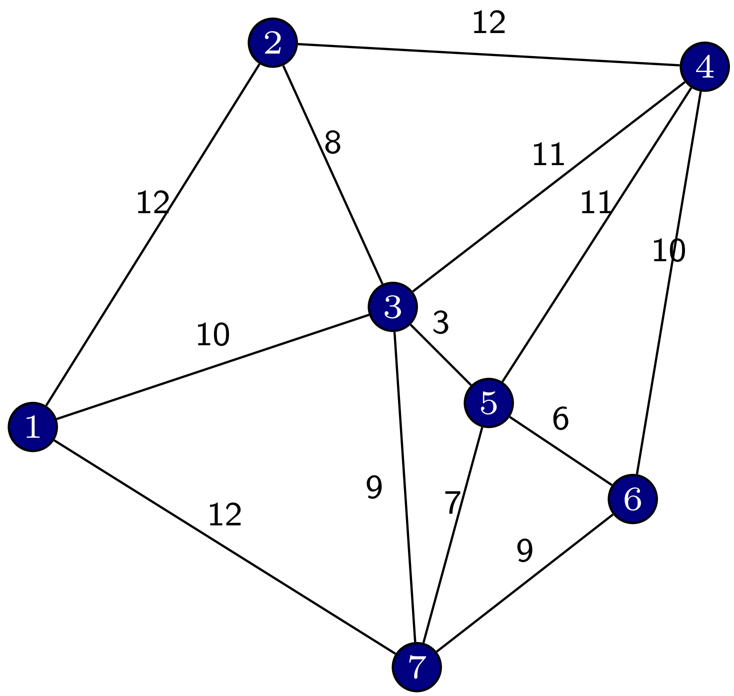
Textbook example starts in node 3:



Suppose we start in node 5:

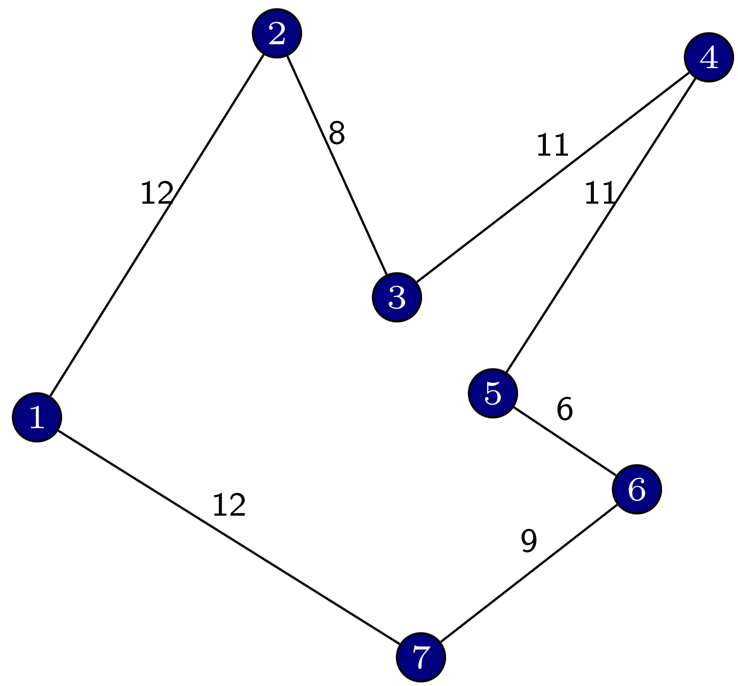


Another TSP Instance (from H&L)



A Subtour Reversal Heuristic for TSP (from H&L)

- **Step 1** – Start with any feasible tour.
  - We'll try: 1-2-3-4-5-6-7-1



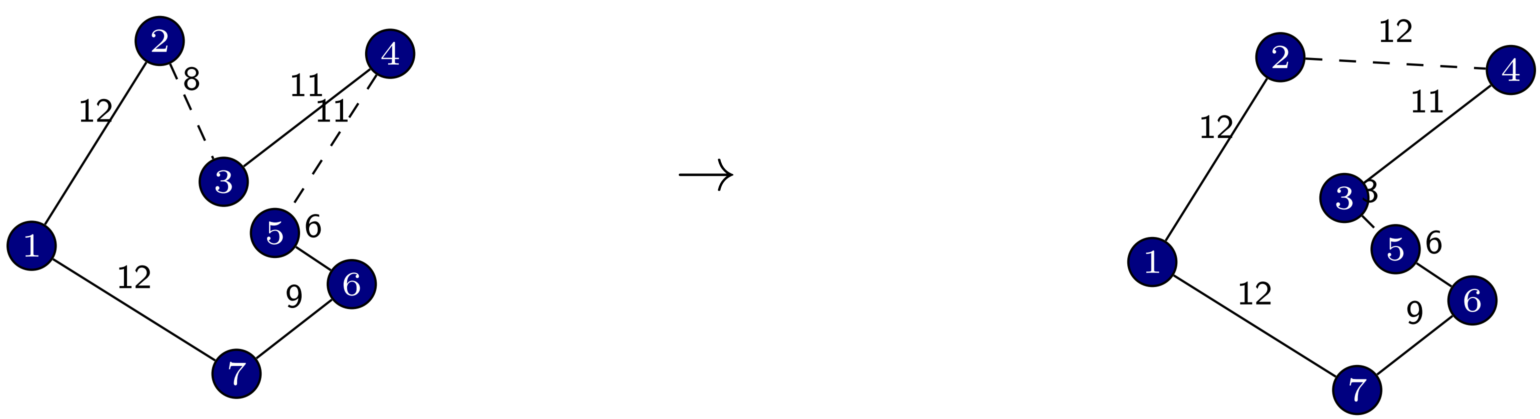
Total Distance = 69.

A Subtour Reversal Heuristic for TSP

- **Step 2** – For the current solution, consider all ways of reversing the order of a subset of cities. Select the one that provides the largest decrease in the travel distance. Break ties arbitrarily.

	1-2-3-4-5-6-7-1	Distance = 69
Reverse 2-3:	1-3-2-4-5-6-7-1	Distance = 68
Reverse 2-3-4:	1-4-3-2-5-6-7-1	Infeasible (1-4 is invalid)
Reverse 3-4:	1-2-4-3-5-6-7-1	Distance = 65
Reverse 4-5:	1-2-3-5-4-6-7-1	Distance = 65
Reverse 5-6:	1-2-3-4-6-5-7-1	Distance = 66

We'll choose 1-2-4-3-5-6-7-1.

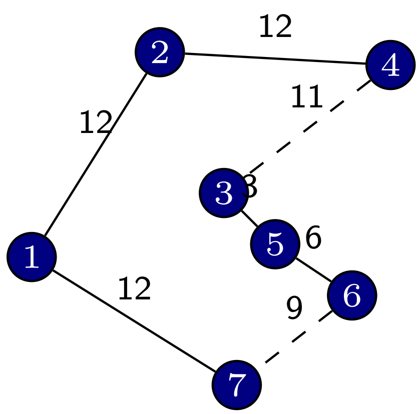


Repeat Step 2 until no subtour reversal will improve the current solution.

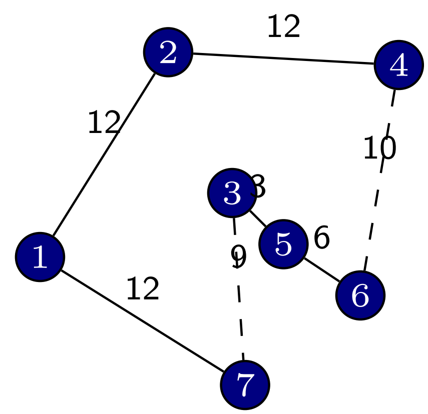
A Subtour Reversal Heuristic for TSP

- **Step 2** (again)

	1-2-4-3-5-6-7-1	Distance = 65
Reverse 3-5-6:	1-2-4-6-5-3-7-1	Distance = 64



$\rightarrow$



- **Step 2** (for the 3rd time): There are no subtour reversals that give us a distance of less than 64. The heuristic stops.

The optimal solution is 1-2-4-6-7-5-3-1 (distance of 63). However, there was no way to get to this solution with subtour reversal moves.

Local vs. Global Optimum

- The subtour reversal heuristic for the TSP is known as a **local improvement procedure**.
- One drawback is that local improvement procedures (or local search procedures) can lead to **local optima**.

Metaheuristics

- Traditionally, new heuristics were created to solve each new problem type.
- **Metaheuristics** are more general solution methods that can be applied to a variety of different problems.
  - "A metaheuristic is a general kind of solution method that orchestrates the interaction between local improvement procedures and higher level strategies to create a process that is capable of escaping from local optima and performing a robust search of a feasible region." (H&L)
  - Metaheuristics provide a general framework and some basic rules to follow.
  - Many popular metaheuristics are inspired by natural phenomena.

Metaheuristics, continued

- Examples:
  - Simulated Annealing (SA)
  - Tabu Search (TS)
  - Genetic Algorithms (GA)
  - Ant Colony Optimization (ACO)
  - Particle Swarm Optimization (PSO)
- These are iterative procedures.

Simulated Annealing

- Physical Annealing – Forming Metal
  - Heat it, form it, cool it (according to a schedule), repeat
- Simulated Annealing – Metaheuristic
  1) Start with a feasible trial solution. Choose a **starting temperature** and a **temperature schedule**.
  2) Use a **local search procedure** to find a **neighboring solution**.
  3) When the temperature is hot, we have a high probability of accepting "bad" neighbors. As the temperature cools, we're more selective.
  4) Repeat Steps 2 and 3 until
     - We can't find an acceptable neighbor, or
     - Our temperature has cooled to its minimum value, or
     - We run out of time.

Simulated Annealing Procedure - Phase I

- Initialize Parameters:
  - Pick T_0 (initial temperature)
  - Pick I (number of iterations per temperature)
  - Pick δ (cooling schedule temperature reduction)
  - Pick T_{final} (minimum allowable temperature)
  - Pick `cutoffTime` (maximum allowable runtime)
- Start the Procedure:
  - Find X_O (an initial solution)
  - Set $X_{\text{cur}} = X_0$ (current solution)
  - Set $Z_{\text{cur}} = Z(X_{\text{cur}})$ (current OFV)
  - Set $T_{\text{cur}} = T_0$ (current temperature)
  - Set $X_{\text{best}} = X_{\text{cur}}$ (best known solution)
  - Set $Z_{\text{best}} = Z_{\text{cur}}$ (best known OFV)

Simulated Annealing Procedure - Phase II

The code below assumes we are solving a minimization problem.

for count = 1 to I:
  Generate a neighbor solution, $X_{\texttt{count}}$
  if $(Z(X_{\texttt{count}}) < Z_{\texttt{cur}})$:
    $X_{\texttt{cur}} = X_{\texttt{count}}$
    $Z_{\texttt{cur}} = Z(X_{\texttt{count}})$
  else:
    $\Delta C = Z(X_{\texttt{count}}) - Z_{\texttt{cur}}$
    if $(\texttt{rand()} \leq e^{-\Delta C / T_{\texttt{cur}}})$:
      $X_{\texttt{cur}} = X_{\texttt{count}}$
      $Z_{\texttt{cur}} = Z(X_{\texttt{count}})$
  if $(Z(X_{\texttt{cur}}) < Z_{\texttt{best}})$:
    $Z_{\texttt{best}} = Z(X_{\texttt{cur}})$
    $X_{\texttt{best}} = X_{\texttt{cur}}$

The exponent should be NEGATIVE. See the WebEx video from May 7, 2020 for an explanation.

Simulated Annealing Procedure - Phase III

$T_{\texttt{cur}} = T_{\texttt{cur}} - \delta$

if $((T_{\texttt{cur}} < T_{\texttt{final}})$ or $(\texttt{runTime} \geq \texttt{cutoffTime}))$:

  STOP

else:

  Return to Phase II

Law of Thermodynamics

- At temperature t, the probability of an increase in energy of magnitude ΔE is given by

$$P\{\Delta E\} = e^{\frac{-\Delta E}{kt}},$$

where k is a constant known as Boltzmann's constant.

Improving SA Performance

- (Some) Things you can change:
  - initial solution
  - local search procedure / neighborhood structure
  - starting temperature
  - cooling schedule (cool at a different rate)
  - number of iterations per temperature
  - do a re-start with a new/random solution

Simulated Annealing Terminology

- Local Search Procedure – An approach to find new candidate solutions in the neighborhood of the current solution (e.g., subtour reversal).

- Neighborhood – All solutions that may be found by applying a local search procedure to the current solution.

- Move Selection Rule – A procedure to select one of the candidate/neighboring solutions.
  - $P\{accept\} = e^x$ where $x = (Z_{\text{count}} - Z_{\text{cur}})/T_{\text{cur}}$

- Temperature Schedule – A procedure for determining the starting temperature and the rate at which the temperature cools over time.