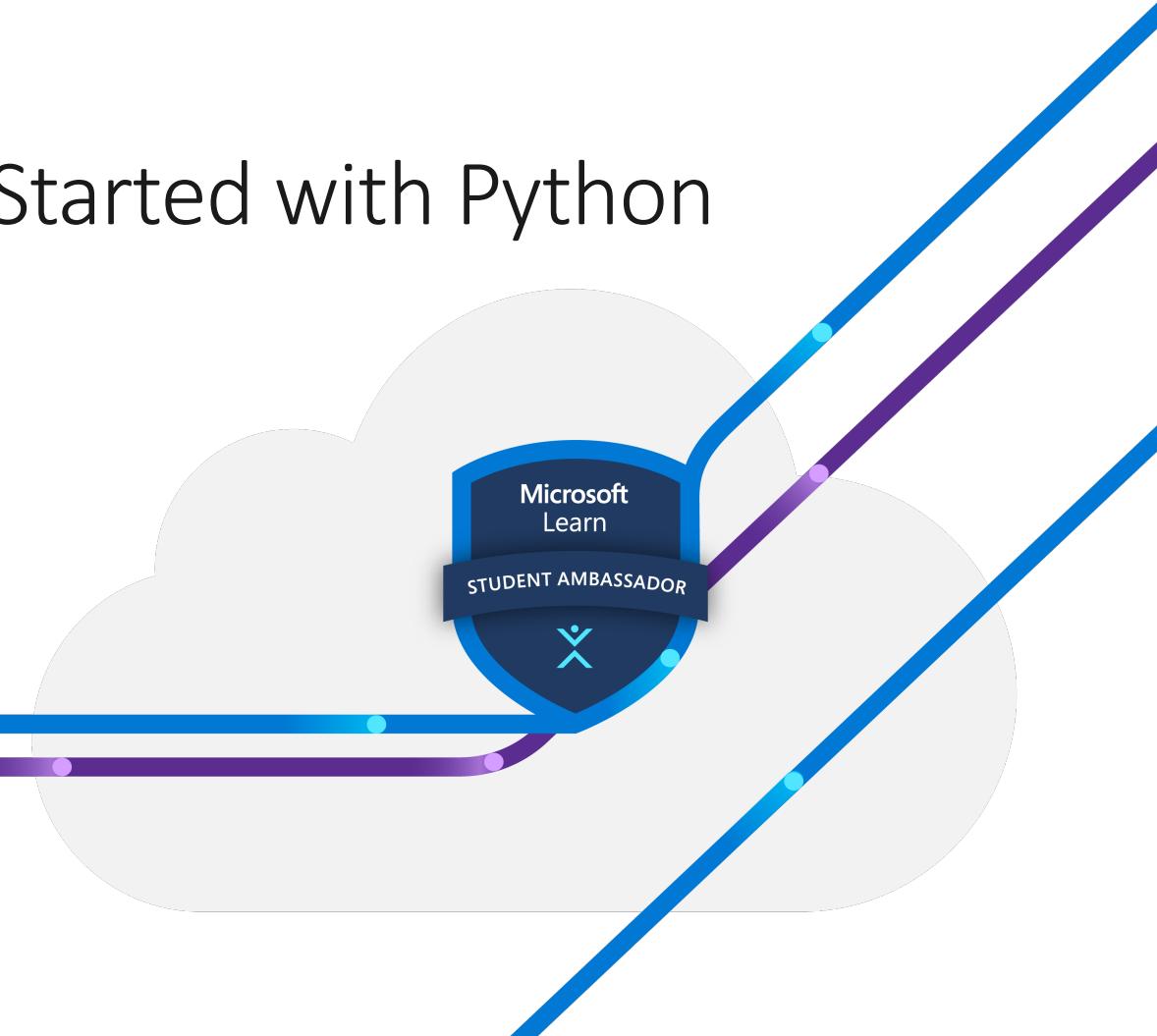


# ML Bootcamp Session 2 : Getting Started with Python and Linear Algebra





# Python Setup (Local)

To setup python on your local device:

1. Download python from [python.org](https://www.python.org)
2. Use the packaged Python Idle as Integrated development environment.

OR

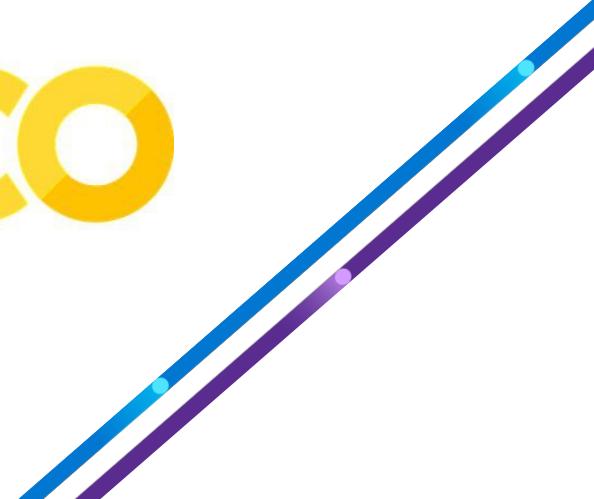
1. Download [Anaconda](https://www.anaconda.com/distribution/) Installer / Miniconda
2. Choose your IDE:
  - i. VS CODE
  - ii. Jupyter Notebook([Anaconda](https://jupyter.org))
  - iii. Spyder
  - iv. JupyterLab



# Python Setup (Online)



Using [colab.research.google.com](https://colab.research.google.com)

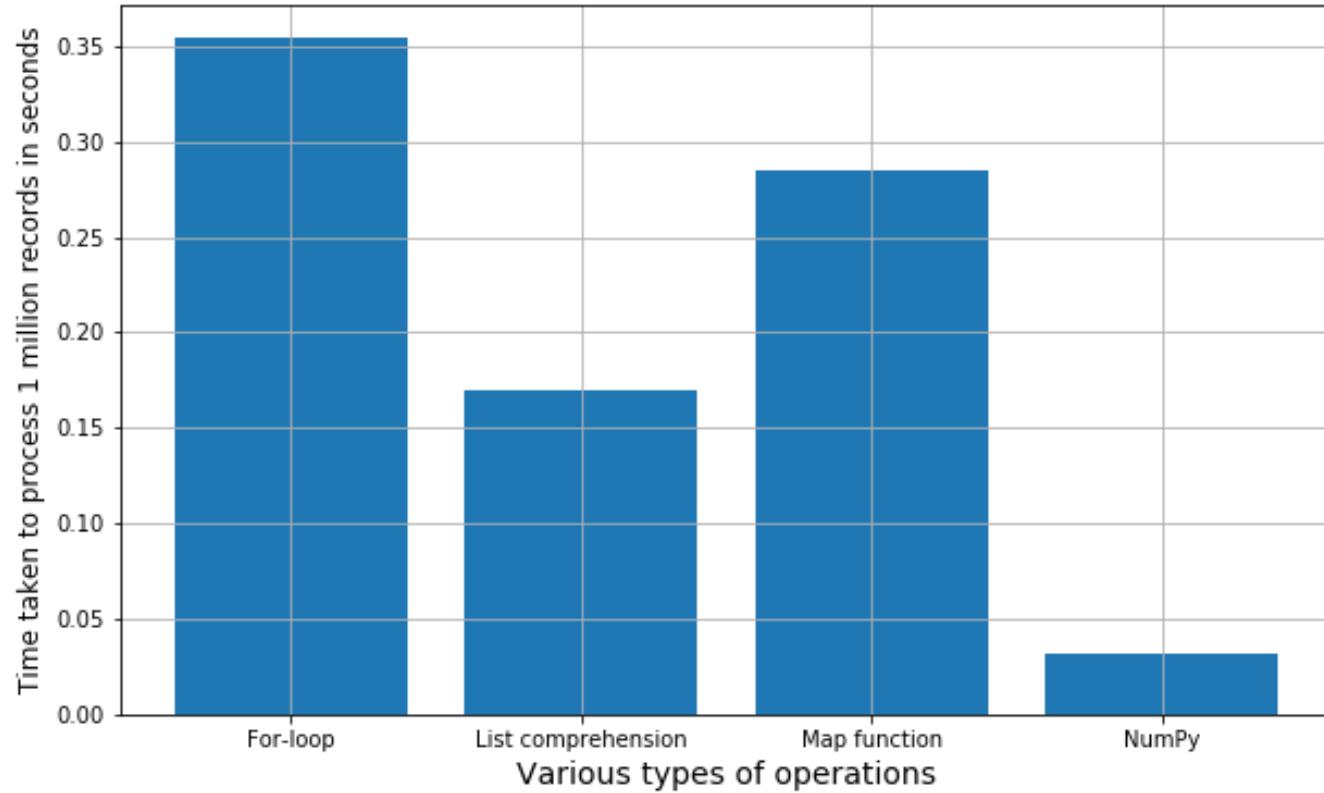


# LINEAR ALGEBRA



# Why Vectors?

SPEED



# Why Vectors?

SPEED

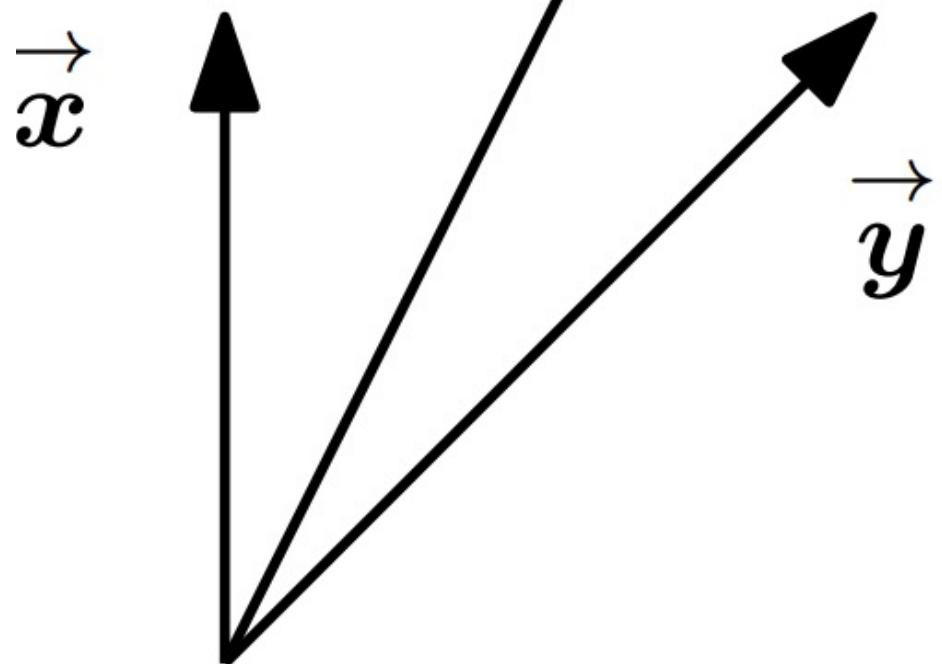
Organized Data

Mathematically Efficient

Supports GPU

(And is building block for DL)

$$\vec{x} + \vec{y}$$



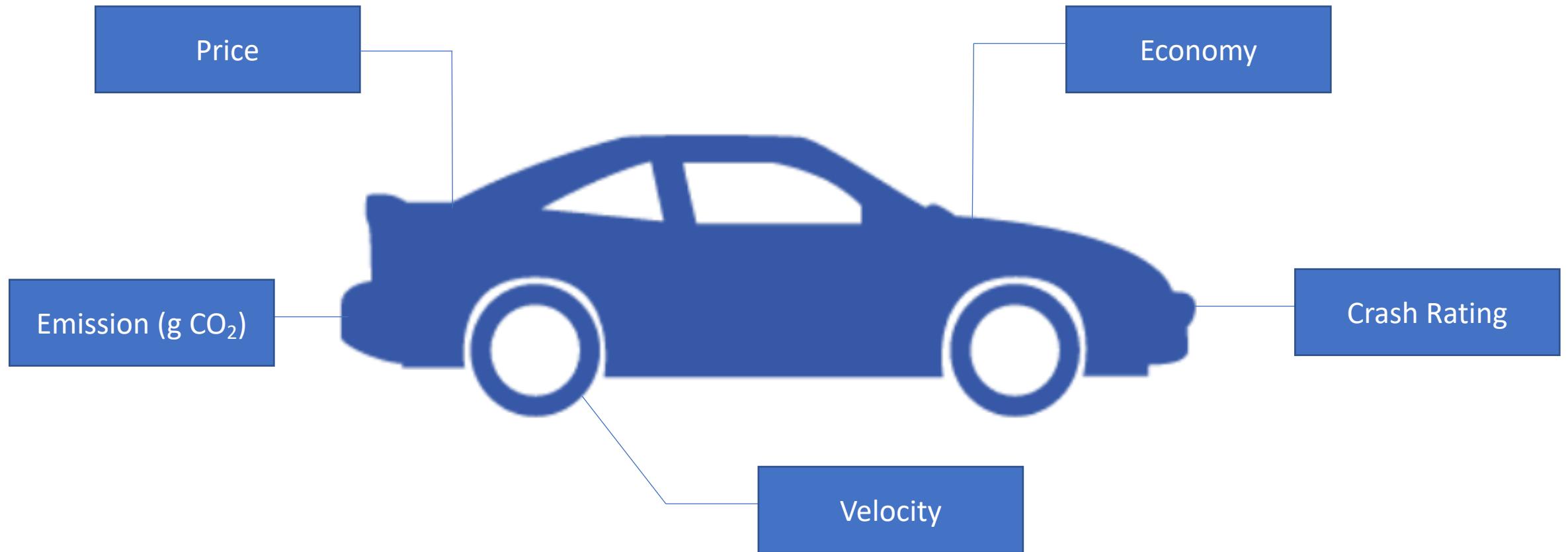
## Vector

---

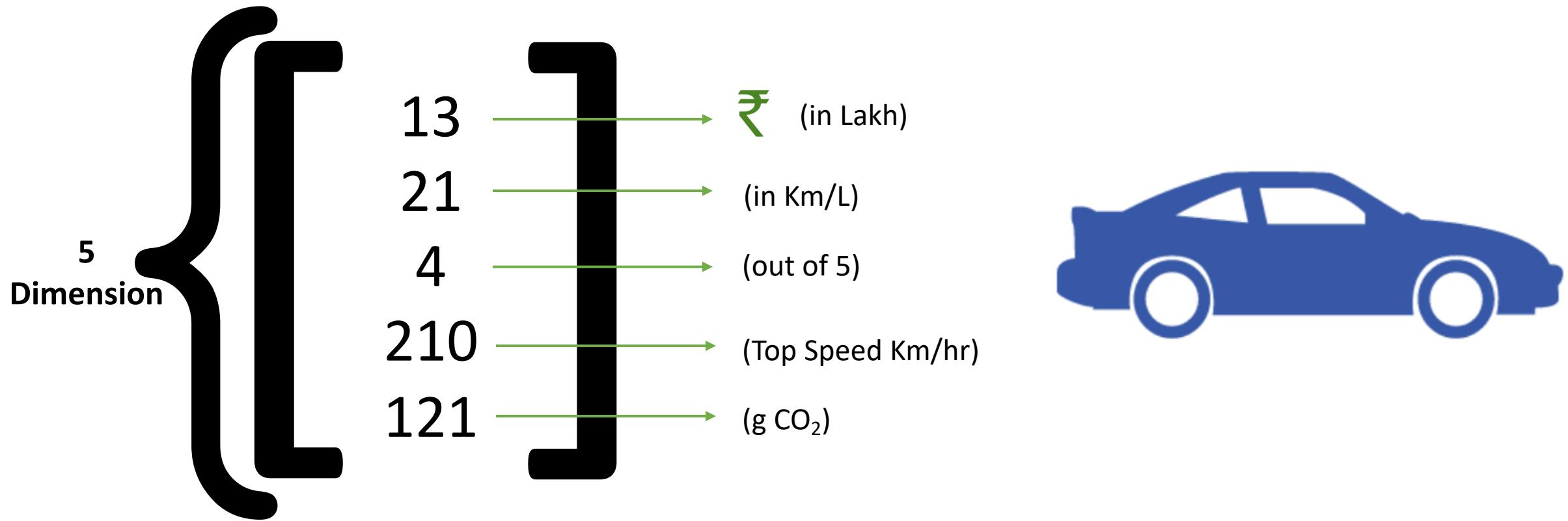
- can be added, such that
- $\vec{x} + \vec{y} = \vec{z}$

multiplication by a scalar  
 $\lambda\vec{x}$

# Vector (Intuition)



# Vector (Intuition)



# Vector (Definition)

```
[1] # Import NumPy: Numerical Python Library
    import numpy as np
```

```
    # Make a vector from list
    v = np.array([1, 2, 3, 4])
    print(v)
```

```
[1 2 3 4]
```

```
[2] # Make same from range function
    v = np.array(range(1, 5))
    print(v)
```

```
[1 2 3 4]
```

# Vector Arithmetic

## Addition

$$\begin{bmatrix} 1 \\ 10 \\ 4 \\ 76 \\ -5 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 7 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 1 \\ 83 \\ -10 \end{bmatrix}$$

# Vector Arithmetic

## Subtraction

$$\begin{bmatrix} 1 \\ 10 \\ 4 \\ 76 \\ -5 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ -3 \\ 7 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 0 \\ -3 \\ 7 \\ -5 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \\ 7 \\ 69 \\ 0 \end{bmatrix}$$

# Vector Arithmetic

## Subtraction

$$\begin{bmatrix} 1 & - & 4 \\ 10 & - & 0 \\ 4 & - & -3 \\ 76 & - & 7 \\ -5 & - & -5 \end{bmatrix} - \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$

```
[5] # vector subtraction
from numpy import array
# define first vector
a = array([1, 10, 4, 76, -5])
print(a)
# define second vector
b = array([4, 0, -3, 7, -5])
print(b)
# subtract vectors
c = a - b
print(c)
```

[ 1 10 4 76 -5]  
[ 4 0 -3 7 -5]  
[-3 10 7 69 0]

# Vector Arithmetic

## Multiplication by a scalar

```
[6] # Multiplication by a scalar
    import numpy as np
    # define a vector
    v = np.array([1, 10, 4, 76, -5])
    # multiply with scalar
    print(2*v)
```

```
[ 2 20  8 152 -10]
```

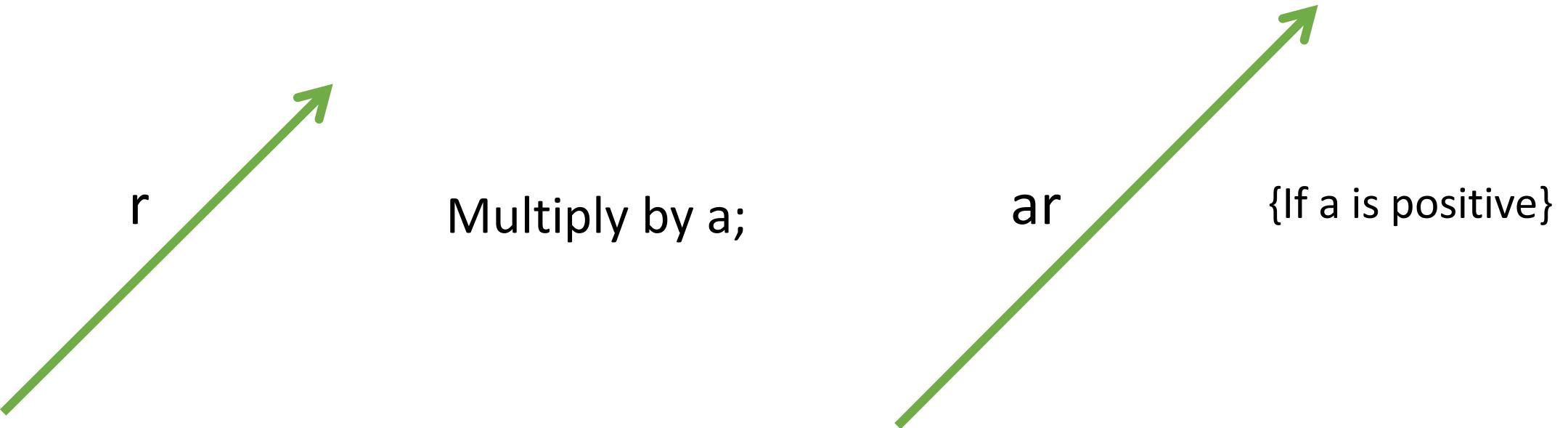
```
[7] # multiplying with a vector of same length as v
    # and having same value of all elements
    scalar_vec = np.array([2, 2, 2, 2, 2])
```

```
print(scalar_vec*v)
```

```
[ 2 20  8 152 -10]
```

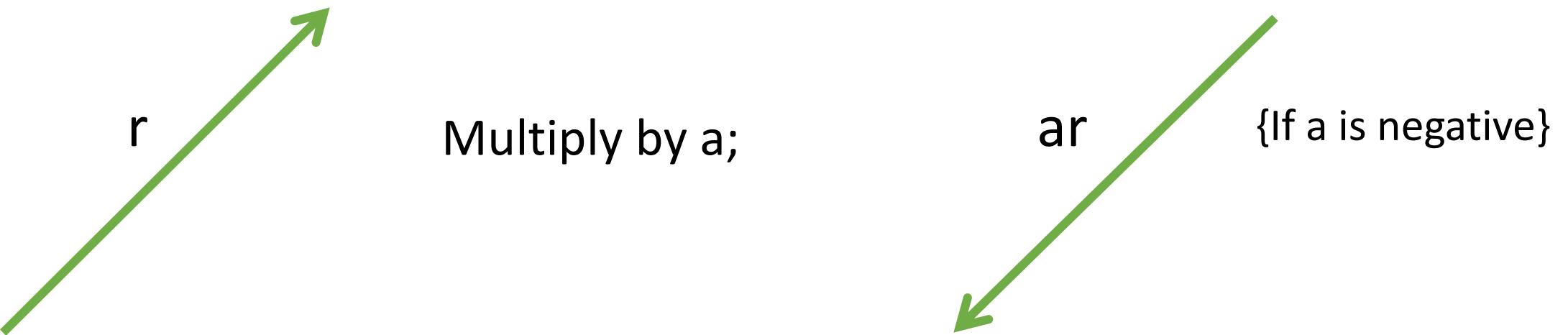
# Vector Arithmetic

## Multiplication by a scalar



# Vector Arithmetic

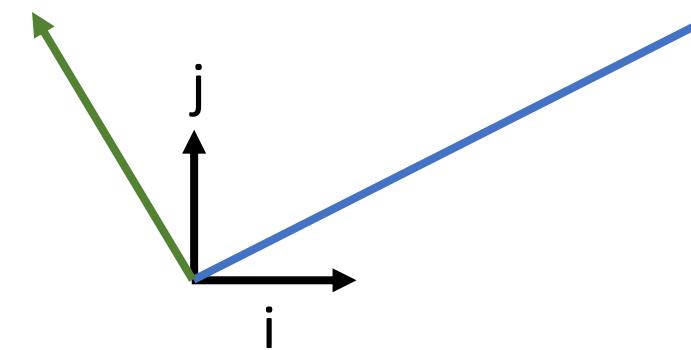
## Multiplication by a scalar



# Vector Arithmetic

## Modulus and Inner Product

$$\begin{bmatrix} s_i \\ s_j \end{bmatrix} \leftarrow s = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} r_i \\ r_j \end{bmatrix}$$



Dot Product:

$$\begin{aligned} r \cdot s &= r_i s_i + r_j s_j \\ &= 3 \times (-1) + 2 \times 2 \\ &= 1 = s \cdot r \end{aligned}$$

**Commutative**

# Vector Arithmetic

## Transpose

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, x \in \mathbb{R}^n \quad x^T = [x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_n], x^T \in \mathbb{R}^{1 \times n}$$

# Vector Arithmetic

## Transpose

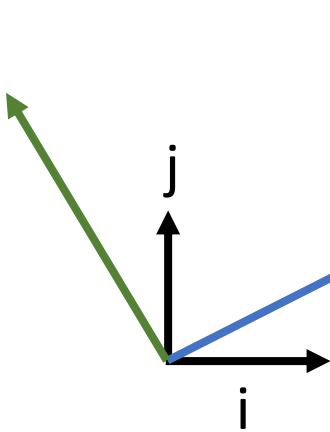
```
[16] import numpy as np
# Make a random vector
v = np.random.rand(5).reshape(-1, 1)
# print vector and its shape
print(v, v.shape)
# transpose with T
print(v.T, v.T.shape)
# transpose with transpose method
print(v.transpose(), v.transpose().shape)
# also same as
print(np.transpose(v), np.transpose(v).shape)
```

```
[[0.17909345]
 [0.35013963]
 [0.29005948]
 [0.56221015]
 [0.61924807]] (5, 1)
[[0.17909345 0.35013963 0.29005948 0.56221015 0.61924807]] (1, 5)
[[0.17909345 0.35013963 0.29005948 0.56221015 0.61924807]] (1, 5)
[[0.17909345 0.35013963 0.29005948 0.56221015 0.61924807]] (1, 5)
```

# Vector Arithmetic

## Dot Product and Transpose

$$s = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$



$$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

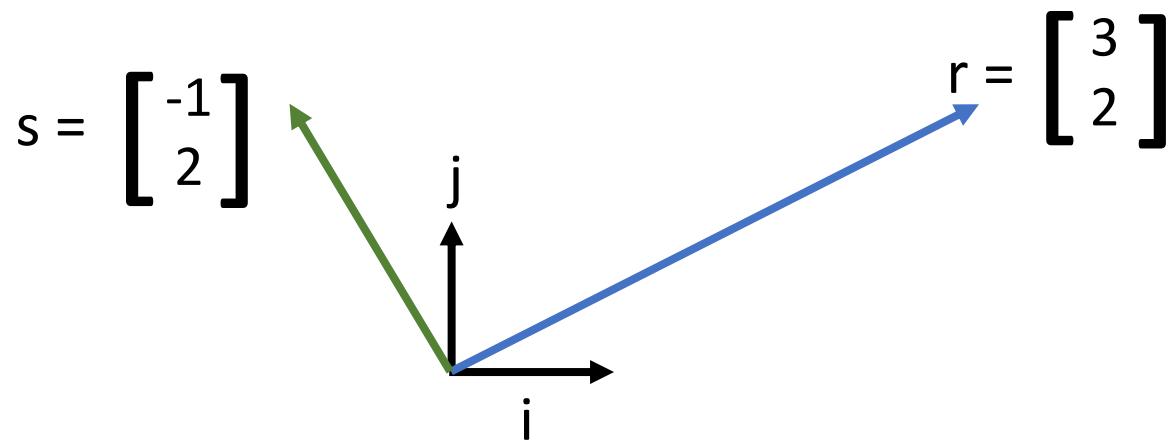
$$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, r \in \mathbb{R}^2$$

$$r^T = [3 \ 2], r^T \in \mathbb{R}^{1 \times 2}$$

$$\begin{aligned} r^T s &= [3 \ 2] \begin{bmatrix} -1 \\ 2 \end{bmatrix} \\ &= (3 \times -1) + (2 \times 2) = 1 \\ &= r \cdot s \end{aligned}$$

# Vector Arithmetic

## Dot Product and Transpose



```
Vector s: [-1 2]
Vector r: [3 2]
Dot product: 1
Dot product: 1
```

```
[34] # vector dot product
    from numpy import array
    # define first vector
    s = array([-1, 2])
    print('Vector s: ', s)
    # define second vector
    r = array([3, 2])
    print('Vector r: ', r)
    # multiply vectors
    c = r.dot(s)
    print('Dot product: ', c)
    # Dot product by multiplication and transpose
    c = np.matmul(r.T, s)
    print('Dot product: ', c)
```

# Vector Arithmetic

## Modulus and Inner Product

Associative over scalar multiplication

$$r \cdot (as) = a(r \cdot s)$$

# Vector Arithmetic

## Modulus and Inner Product

Dot Product with vector itself

$$\begin{aligned} \mathbf{r} \cdot \mathbf{r} &= r_1 r_1 + r_2 r_2 \\ &= r_1^2 + r_2^2 \\ &= \left( \sqrt{r_1^2 + r_2^2} \right)^2 \\ &= |\mathbf{r}|^2 \end{aligned}$$

Link between the dot product and the length or modulus of a vector

# Vector Norm

- A function that ***operates on a vector (matrix)*** and returns a **scalar element**.
- A norm is denoted by  $\mathcal{L}^p$  in which  $p$  ***shows the order of the norm*** and  $p \geq 1, p \in \mathbb{R}$ .
- The order a vector (matrix) is ***always a non-negative value***.
- The intuition behind the norm is to ***measure a kind of distance***.

# Vector Norm

The  $\ell_p$  norm

$$\ell_p = \left( \sum_{i=1}^n |x_i|^p \right)^{\frac{1}{p}}, \text{ for } p \geq 1$$

For  $p = 1$ ,  $\ell_1 = |x_1| + |x_2| + \dots + |x_n|$

For  $p = 2$ ,  $\ell_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

For  $p \rightarrow \infty$ ,  $\ell_\infty = \max_i(x_1, x_2, \dots, x_n)$

# Vector Norm

The  $\ell_p$  norm

For  $p = 1$ ,  $\ell_1 = |x_1| + |x_2| + \dots + |x_n|$

```
[40] # vector L1 norm
    import numpy as np
    from numpy.linalg import norm
    # define vector
    a = np.arange(1, 6)
    print(a)
    # calculate norm
    l1 = norm(a, 1)
    print(l1)
```

```
[1 2 3 4 5]
15.0
```

# Vector Norm

The  $\ell_p$  norm

$$\text{For } p = 2, \quad \ell_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

```
[41] # vector L2 norm
    import numpy as np
    from numpy.linalg import norm
    # define vector
    a = np.arange(1, 6)
    print(a)
    # calculate norm
    l2 = norm(a)      # or norm(a, 2)
    print(l2)
```

```
[1 2 3 4 5]
7.416198487095663
```

# Vector Norm

The  $\ell_p$  norm

For  $p \rightarrow \infty$ ,  $\ell_\infty = \max_i(x_1, x_2, \dots, x_n)$

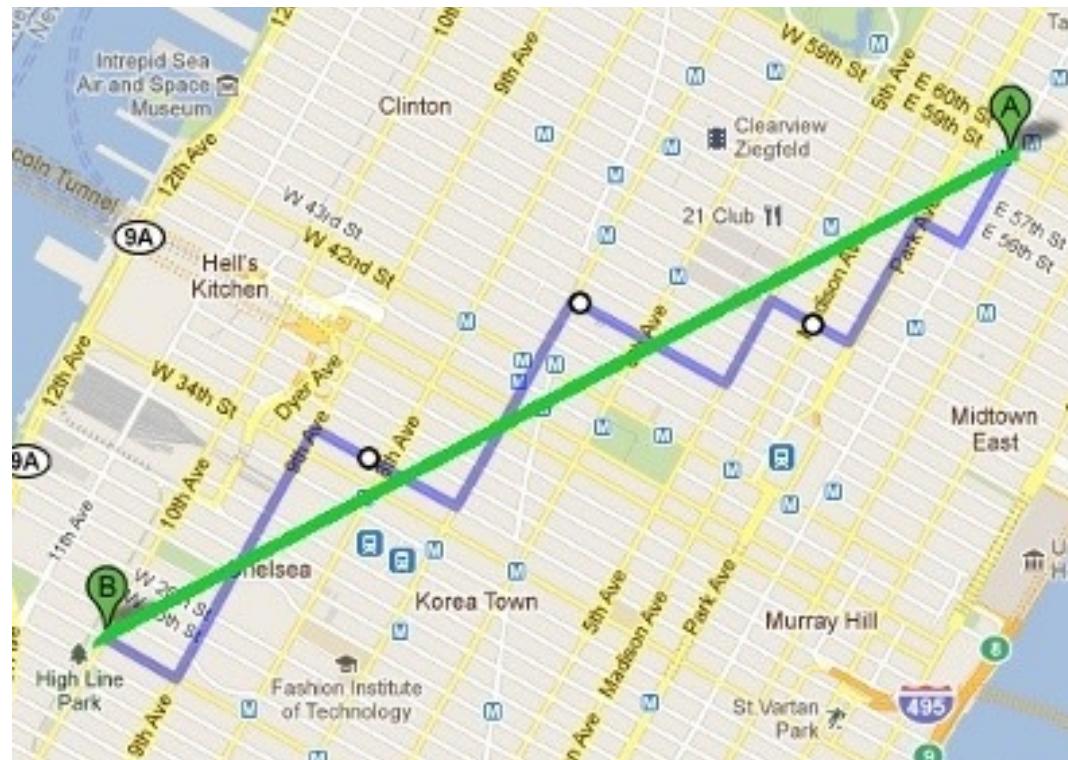
```
[42] # vector max norm
      from math import inf
      from numpy import array
      from numpy.linalg import norm
      # define vector
      a = np.arange(1, 6)
      print(a)
      # calculate norm
      maxnorm = norm(a, inf)
      print(maxnorm)
```

```
[1 2 3 4 5]
5.0
```

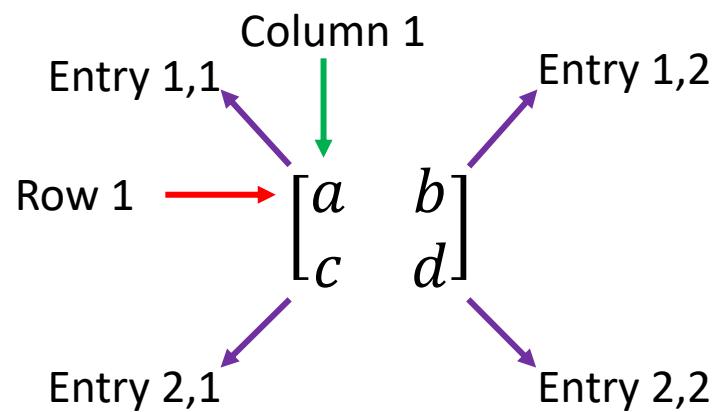
# Vector Norm

$\ell_1$  norm vs  $\ell_2$  norm

$$\ell_1 = |x_1| + |x_2| + \dots + |x_n| , \quad \ell_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$



# Matrices Intuition



$$A \in \mathbb{R}^{m \times n}$$
$$A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \ddots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{n \text{ columns}} \}_{m \text{ rows}}$$

# Matrices Notation

$$A = \begin{bmatrix} | & | & & | \\ a^1 & a^2 & \dots & a^n \\ | & | & & | \end{bmatrix}$$

*j*th column of A by  $a^j$  or  $A_{:,j}$

$$A = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ \vdots & & \\ - & a_n^T & - \end{bmatrix}$$

*i*th row of A by  $a^T$  or  $A_{i,:}$

# Matrices Structure

$$A = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ \vdots & & \\ - & a_n^T & - \end{bmatrix}$$

|         | Living area (feet <sup>2</sup> ) | Price (1 Lakh) |
|---------|----------------------------------|----------------|
| $a_1^T$ | 2104                             | 62             |
| $a_2^T$ | 1600                             | 45             |
| :       | 2400                             | 76             |
|         | 1416                             | 80             |
| $a_5^T$ | 2900                             | 103            |

# Matrices Structure

$$X = \begin{bmatrix} - & x_1^T & - \\ - & x_2^T & - \\ \vdots & & \\ - & x_n^T & - \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

|          | Living area (feet <sup>2</sup> ) | # Bedrooms | # Parks in locality | Price (1 Lakh) |          |
|----------|----------------------------------|------------|---------------------|----------------|----------|
| $x_1^T$  | 2104                             | 2          | 1                   | 62             | $y_1$    |
| $x_2^T$  | 1600                             | 1          | 1                   | 45             | $y_2$    |
| $\vdots$ | 2400                             | 2          | 0                   | 76             | $\vdots$ |
| $x_4^T$  | 1416                             | 1          | 2                   | 80             |          |
| $x_5^T$  | 2900                             | 3          | 1                   | 103            | $y_5$    |

# Matrices Structure

```
[31] # Import NumPy: Numerical Python Library
     import numpy as np

     # Defining a matrix
     mat = np.array([[1, 2, 3],
                    [4, 5, 6],
                    [7, 8, 9]])

     print("Matrix: ")
     print(mat, mat.shape)
```

```
Matrix:
[[1 2 3]
 [4 5 6]
 [7 8 9]] (3, 3)
```

# Matrices Arithmetic

## Multiplication

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 0 \\ 3 & 4 & 7 \end{bmatrix} \times \begin{bmatrix} 0 & 2 & 3 \\ 4 & 7 & 0 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 17 & 28 & 21 \\ 32 & 64 & 12 \\ 37 & 62 & 51 \end{bmatrix}$$

# Matrices Arithmetic

## Multiplication

```
[36] # Import NumPy: Numerical Python Library
      import numpy as np

      # Defining matrices
      mat1 = np.array([[1, 2, 3],
                      [4, 8, 0],
                      [3, 4, 7]])
      print("Matrix 1: ")
      print(mat1, mat1.shape)

      mat2 = np.array([[0, 2, 3],
                      [4, 7, 0],
                      [3, 4, 6]])
      print("Matrix 2: ")
      print(mat2, mat2.shape)

      # Multiplication of two matrices
      mat3 = np.matmul(mat1, mat2)
      print("Resultant Matrix: ")
      print(mat3, mat3.shape)
```

Matrix 1:  
[[1 2 3]  
 [4 8 0]  
 [3 4 7]] (3, 3)

Matrix 2:  
[[0 2 3]  
 [4 7 0]  
 [3 4 6]] (3, 3)

Resultant Matrix:  
[[17 28 21]  
 [32 64 12]  
 [37 62 51]] (3, 3)

# Matrices Arithmetic

## Multiplication by a vector

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 2 & 3 \\ 4 & 7 & 0 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

1x3                          3x3                          1x3

$$(1 \times 0) + (0 \times 4) + (1 \times 3) = 0 + 0 + 3$$

# Matrices Arithmetic

## Multiplication by a vector

$$\begin{array}{c} \xrightarrow{\hspace{1cm}} \\ [1 \quad 0 \quad 1] \end{array} \times \left[ \begin{array}{cc|c} 0 & 2 & 3 \\ 4 & 7 & 0 \\ 3 & 4 & 6 \end{array} \right] = [3 \quad 6 \quad 9]$$

$$(1 \times 3) + (0 \times 0) + (1 \times 6) = 3 + 0 + 6$$

# Matrices Arithmetic

## Multiplication by a vector

```
[34] # Import NumPy: Numerical Python Library
      import numpy as np

      # Defining vector
      vec = np.array([[1, 0, 1]])
      print("Vector: ")
      print(vec, vec.shape)

      # Defining matrix
      mat = np.array([[0, 2, 3],
                     [4, 7, 0],
                     [3, 4, 6]])
      print("Matrix: ")
      print(mat, mat.shape)

      # Multiplying a row vector with matrix
      print("Resultant Matrix: ")
      print(np.matmul(vec, mat))
```

Vector:  
[[1 0 1]] (1, 3)  
Matrix:  
[[0 2 3]  
 [4 7 0]  
 [3 4 6]] (3, 3)  
Resultant Matrix:  
[[3 6 9]]

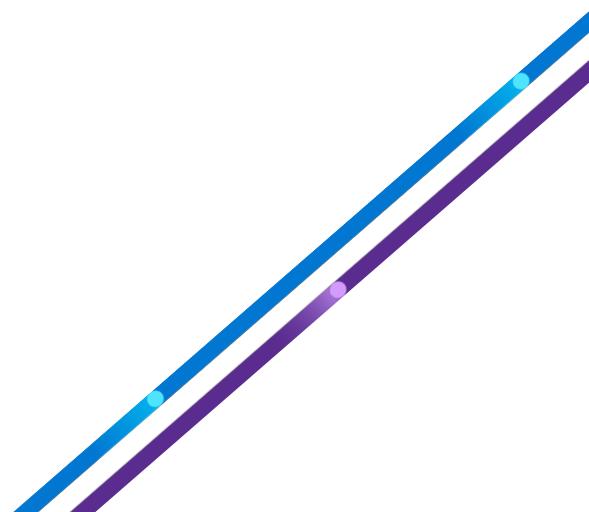
# Resources to learn Python

[Python documentation](#)

[W3 schools](#) (Our favorite)

CodeAcademy

Datacamp



# Quiz Time

130 °C

50



2

1

0

10:25  
5.08.2017