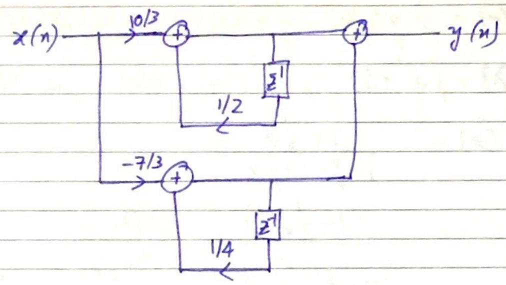


Q-7) $H[z] = \frac{3(2z^2+5z+4)}{(2z+1)(z+2)}$

$$\frac{H[z]}{3z} = \frac{(2z^2+5z+4)}{z(2z+1)(z+2)} = \frac{A}{z} + \frac{B}{(2z+1)} + \frac{C}{(z+2)}$$

$$2z^2+5z+4 = A(z+2)(2z+1) + B(z+2)z + C\,z(2z+1)$$

$z=0$ $\quad z=-\frac{1}{2}$ $\quad z=-2$

$A = 4/2 = 2$

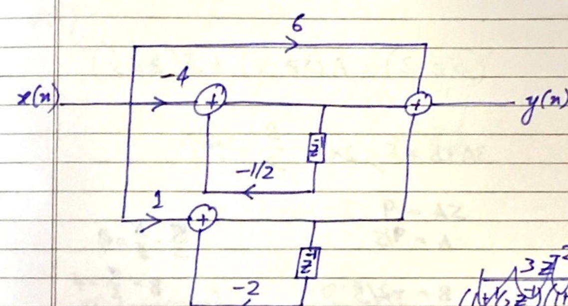
$B = -8/3$

$C = 1/3$

$$H[z] = 3z \cdot \left[\frac{2}{z} - \frac{4}{3(2z+1)} + \frac{1}{3(z+2)} \right]$$

$$H[z] = 6 - \frac{4z}{2z+1} + \frac{z}{z+2}$$

$$H[z] = 6 - \frac{2}{1+\frac{1}{2}z^{-1}} + \frac{1}{1+2z^{-1}}$$



$$\frac{3z^{2}}{(1+\frac{1}{2}z^{-1})(1+2z^{-1})}$$

$$\frac{A}{(1+\frac{1}{2}z^{-1})} + \frac{B}{(1+2z^{-1})} = \vdots$$

A+B, A0,

~~Q-8) $H(z) = \frac{3z(5z-2)}{(z+1/2)(3z-1)}$~~

$\left(6 - \frac{4}{1+\frac{1}{2}} + \frac{1}{1+2}\right)$

$= 6 - \frac{8}{3} + \frac{1}{3}$

11/3 $\quad 3\left(\frac{4}{\ldots} - \frac{2}{1+2}\right) + \frac{2}{3}$

11/3

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Q-8) $H(z) = \dfrac{3z(5z-2)}{(z+\frac{1}{2})(3z-1)}$

$$\frac{H(z)}{3z} = \frac{5z-2}{(z+1/2)(3z-1)} = \frac{A}{(z+1/2)} + \frac{B}{(3z-1)}$$

$$(5z-2) = A(3z-1) + B(z+\tfrac{1}{2})$$

$$3A+B=5 \;;\; 2\times\left(A-\frac{B}{2}=2\right)$$

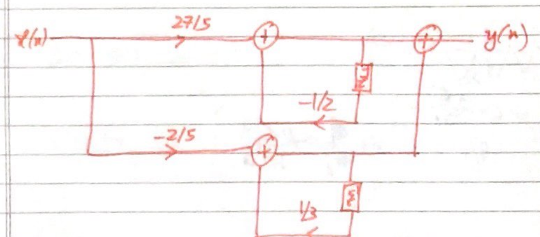
$$5A = 9$$
$$A = 9/5$$

$$\frac{18}{5} - B = 4$$
$$B = \frac{18}{5} - 4$$

$$B = -2/5$$

$$H(z) = \frac{3z \cdot 9/5}{(z+1/2)} - \frac{3z \cdot 2/5}{(3z-1)}$$

$$= \frac{27/5}{(1+\frac{1}{2}z^{-1})} - \frac{2/5}{(1-\frac{1}{3}z^{-1})}$$



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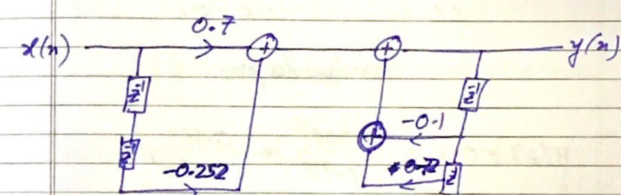
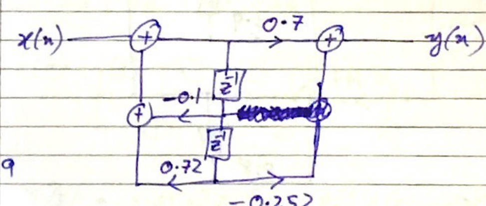
Q-9) $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

$$H(z) = \frac{0.7 - 0.252z^{-2}}{1+0.1z^{-1} - 0.72z^{-2}}$$



Dir. Form-I



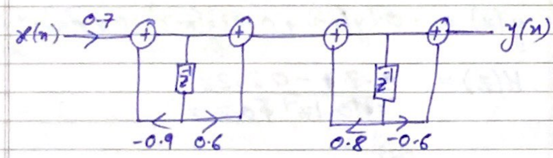
$0.72 = 0.8 \times 0.9$

Dir Form-II (Canonical)

0.36

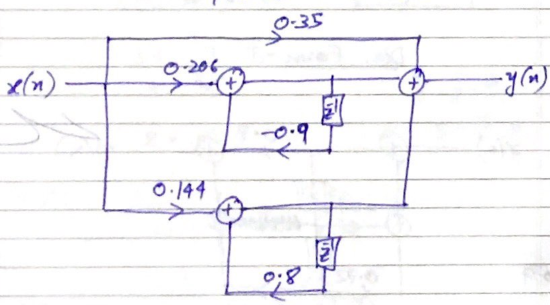
$$H(z) = \frac{(0.7 - 0.252z^{-2})}{(1+0.9z^{-1})(1-0.8z^{-1})}$$

$$= \frac{0.7(1+0.6z^{-1})(1-0.6z^{-1})}{(1+0.9z^{-1})(1-0.8z^{-1})}$$



Parr Cascade form

$$H(z) = 0\cdot35 + \frac{0\cdot206}{1+0\cdot9z^{-1}} + \frac{0\cdot144}{1-0\cdot8z^{-1}}$$



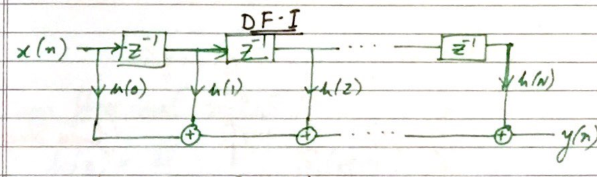
✱ FIR Filter Structure (rep. by zeroes only)

$$H(z) = \sum_{n=0}^{N} h(n)\, z^{-n}$$

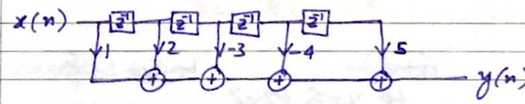
$$y(n) = \sum_{k=0}^{N} h(k)\, x(n-k)$$

$$= h(0)\,x(n) + h(1)\,x(n-1) + \cdots + h(N)\,x(n-N)$$

DF-I



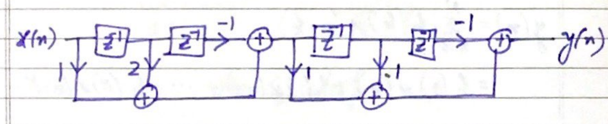
Q-1 $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

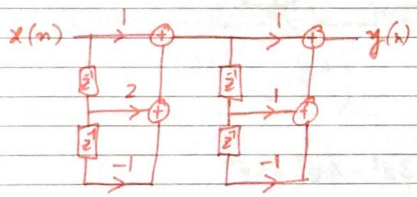


$$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$$

Cascade form

Q-2 $H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$





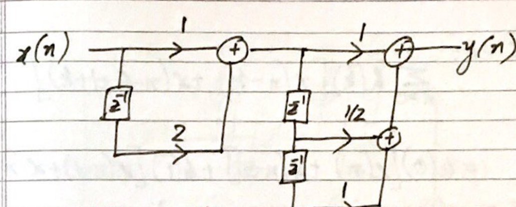
(fancier design LOL :P)

Q-3 $H(z) = 1+\frac{5}{2}z^{-1}+2z^{-2}+2z^{-3}$

$H(z) = (1+2z^{-1})(\quad)$

$H(z) = (1+2z^{-1}) + \left(\frac{1}{2}z^{-1}+z^{-2}\right) + \left(z^{-2}+2z^{-3}\right)$

$= (1+2z^{-1})\left(1+\frac{1}{2}z^{-1}+z^{-2}\right)$



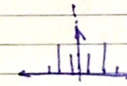
Linear phase system → no-phase-change → FIR (stable)

non-lin. phase system → phase-change → IIR (unstable)

easier to realise

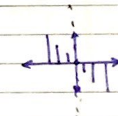
→ Linear phase cond^n for FIR

$h(n) = h(n-N-1)$ (symm)



symmetric sample

$h(n) = -h(n-N-1)$ (antisymm)



N even

$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$

$= \sum_{k=0}^{(N/2-1)} h(k)x(n-k) + \sum_{k=N/2}^{N-1} h(k)x(n-k)$

$k = N-1-l$ symm.

$= \sum_{k=0}^{(N/2-1)} h(k)x(n-k) + \sum_{l=0}^{(N/2-1)} h(N-1-l)\,x(n-N+1+l)$

$= \sum_{k=0}^{(N/2-1)} h(k)\left\{x(n-k)+x(n-N+1+l)\right\}$

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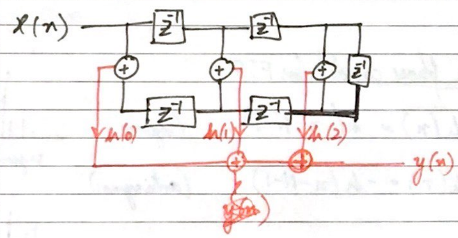
Let N=6; $\sum_{n=0}^{2} h(k)\left[x(n-k) + x(n-6+1+k)\right]$

$$= h(0)\left[x(n) + x(n-5)\right] + h(1)\left[x(n-1) + x(n-4)\right]$$
$$+ h(2)\left[x(n-2) + x(n-3)\right]$$



Multipliers are reduced

If N odd; $y(k) = \sum_{k=0}^{(N-1/2-1)} h(k)\left[x(n-k) + x(n-N+1+k)\right]$
$$+ h\left(\frac{N-1}{2}\right)x\left(n-\frac{N-1}{2}\right) \quad \cancel{+ h\left(\frac{N-1}{2}\right)x\left(n-\frac{N-1}{2}\right)}$$

N=7 $\quad h(0)\left[x(n) + x(n-6)\right] + h(1)\left[x(n-1) + x(n-5)\right]$
$$+ h(2)\left[x(n-2) + x(n-4)\right] + h(3)\,x(n-3)$$

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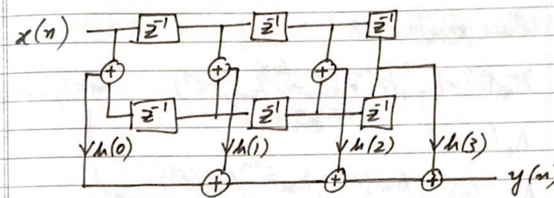


Antisymm

N is even $\quad y(n) = \sum_{k=0}^{\frac{N}{2}-1} h(k)\left[x(n-k) - x(n-N+1+k)\right]$

N is odd $\quad y(n) = \sum_{k=0}^{\left(\frac{N-1}{2}-1\right)} h(k)\left[x(n-k) - x(n-N+1+k)\right]$
$$+ h\left(\frac{N-1}{2}\right)x\left(n-\frac{N-1}{2}\right)$$

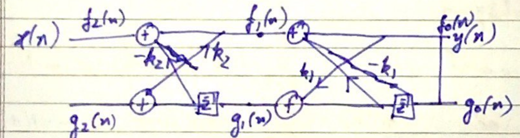
Lattice structure IIR

$\cancel{A_m(z) = \frac{A_{m+1}(z) + k_{m+1} z^{-(m+1)} A_{m+1}(z^{-1})}{1-k_{m+1}^2}}$

$A_0(z) = 1$

$$A_{m-1}(z) = \frac{A_m(z) - k_m z^{-m} A_m(z^{-1})}{1-k_m^2} \qquad m = 1, \ldots, N$$

$y(n) = x(n) - a_2(1)\, y(n-1) - a_2(2)\, y(n-2)$



Q-1) $H(z) = \frac{1}{1-0.2z^{-1}+0.4z^{-2}+0.6z^{-3}} = A_3^{-1}(z)$

Compare latt. struc. to direct struc. realisation of $H(z)$ in terms of z^{-1}, ⊕ & × (mult.)

$\cancel{A_3(z) = A_2(z) + k_3 z^{-3} A_2(z^{-1})}$

$k_3 = 0.6$

$$A_2(z) = \frac{A_3(z) - k_3 z^{-3} A_3(z^{-1})}{1-k_3^2}$$

$0.4 + 0.12$

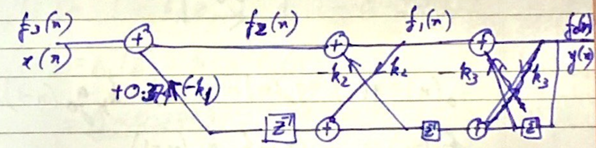
$$= \frac{(1-0.2z^{-1}+0.4z^{-2}+0.6z^{-3}) - 0.6z^{-3}(1-0.2z+0.4z^{2}+0.6z^{3})}{1-(0.6)^2}$$

$$= \frac{0.64 - 0.44z^{-1} + 0.52z^{-2}}{0.64}$$

$$A_2(z) = 1 - \left(\frac{0.44}{0.64}\right)z^{-1} + \left(\frac{0.52}{0.64}\right)z^{-2}$$

$$= 1 - 0.6875z^{-1} + 0.8125z^{-2}$$

$k_2 = 0.8125$



3 delays, 5 add, 5 mult.

$$A_1(z) = \frac{A_2(z) - k_2 z^{-2} A_2(z^{-1})}{1-k_2^2}$$

$$= \frac{(1-0.6875z^{-1}+0.8125z^{-2}) - 0.8125z^{-2}(1-0.6875z+0.8125z^{2})}{1-(0.8125)^2}$$

$$= \frac{1-0.6875z^{-1}+0.8125z^{-2}-0.8125z^{-2}-0.66+0.56z^{-1}}{\cancel{1-0.66}\ 0.34}$$

$$= \frac{0.34 - 0.127z^{-1}}{0.34} = 1 - 0.374z^{-1} \qquad k_1 = 0.374$$

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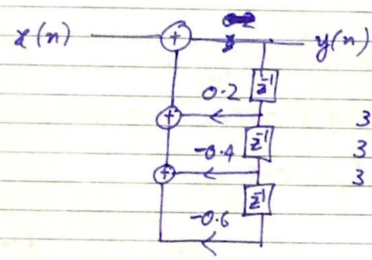
$X(z) = Y(z)\left(1 - 0.2z^{-1} + 0.4z^{-2} + 0.6z^{-3}\right)$

$Y(z) = X(z) + 0.2z^{-1}Y(z) - 0.4z^{-2}Y(z) - 0.6z^{-3}$



3 mult
3 add
3 delay

$f_0(n) = y(n) = g_0(n) = f_1(n) - k_1 g_0(n-1)$

$f_1(n) = f_2(n) - k_2 g_1(n-1)$

$g_1(n) = k_3 f_0(n) + g_0(n-1)$

$f_2(n) = f_3(n) - k_3 g_2(n-1)$

$\cancel{g_2(n) = k_2 f}$

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Q-2) An all-pole IIR filter has refl^{n} coef. $k_1 = -0.3793$
$k_2 = 0.8125$, $k_3 = 0.6$

Determine $H(z)$

$A_0(z) = 1$

$A_m(z) = A_{m-1}(z) + k_m z^{-m} A_m(z^{-1})$

$A_1(z) = 1 + k_1 z^{-1} \cdot 1 = 1 - 0.3793z^{-1}$

$A_2(z) = A_1(z) + k_2 z^{-2} A_1(z^{-1})$

$= \cancel{(1 - 0.3793z^{-1}) + (0.3793z^{-1} - (0.3793)^2)}$

$= \cancel{0.8561 - 0.7586z^{-1}}$

$= \cancel{1 - 0.7586z^{-1} + 0.1439z^{-2}}$

$\cancel{1 - 0.8561}$ $= \cancel{1 - 0.3793}$
0.1439

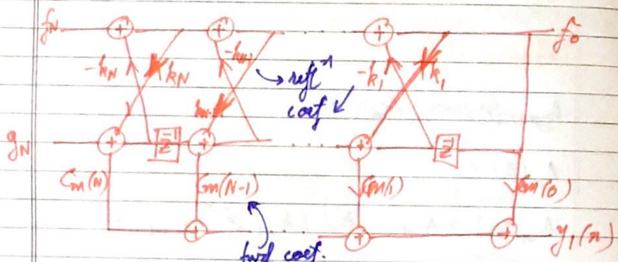
$= (1 - 0.37z^{-1}) + \overset{0.8125}{\cancel{0.37}} z^{-2}(1 - 0.37z)$

$= 1 - 0.37z^{-1} - \overset{(0.8125)(0.37)}{\cancel{(0.37)}} z^{-1} + \overset{0.8125}{\cancel{0.37}} z^{-2}$

$\cancel{= 1 - (1 - 0.37)0.37z^{-1} - 0.37z^{-2}}$

$\cancel{= 1} - (0.1875)(0.37)z^{-1} - 0.8125z^{-2}$

★ Lattice Ladder n/w (IIR Network)



To find refl^n coef. values, use step down recursion

$$H(z) = \frac{1 + \sum_{k=1}^{M} b_M(k) z^{-k}}{1 + \sum_{k=1}^{N} a_N(k) z^{-k}}$$

$$A_{m-1}(z) = \frac{A_m(z) - k_m z^{-m} A_m(z^{-1})}{1 - k_m^2}$$

$k_m = a_m(m)$; $m = 1, 2, \ldots, N$

$$c_m(k) = b_m(k) - \sum_{j=k+1}^{m} c_m(j) a_j(j-k)$$

$C_m(m) = b_m(m)$ $\qquad k = m-1, m-2, \ldots, 0$

If k_m & C_m are given, use step up recursion

$$A_m(z) = A_{m-1}(z) + k_m z^{-m} A_{m-1}(z^{-1}) \qquad m = 1, 2, \ldots, N$$

$A_0(z) = 1$

$$b_m(k) = \sum_{j=k}^{M} c_m(j) a_j(j-k) \qquad k = m-1, m-2, \ldots, 0$$

Q-1) $H(z) = \dfrac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 - 0.8z^{-1} + 0.49z^{-2} - 0.57z^{-3}}$

Find k_3, k_2, k_1 & $C_m(m)$s

$A_3(z) = 1 - 0.8z^{-1} + 0.49z^{-2} - 0.57z^{-3}$ $\quad \{k_3 = 0.57\}$

$$A_2(z) = \frac{(1 - 0.8z^{-1} + 0.49z^{-2} - 0.57z^{-3}) + 0.57z^{-3}(1 - 0.8z + 0.49z^{2} - 0.57z^{3})}{1 - (0.57)^2}$$

$$= \frac{\{1 - (0.57)^2\} - (0.8 - 0.57 \times 0.49) z^{-1} + (0.49 - 0.57 \times 0.8) z^{-2}}{1 - (0.57)^2}$$

$= \cancel{1 - 0.325}$

$$= \frac{0.675 - (0.521) z^{-1} + (0.034) z^{-2}}{0.675}$$

$$= 1 - 0.771 z^{-1} + 0.05 z^{-2}$$

$\{k_2 = 0.05\}$

$a_2(0) = 1$
$a_2(1) = -0.771$
$a_2(2) = 0.05$

$$A_1(z) = \frac{(1-0.771z^{-1}+0.05z^{-2})-0.05z^{-2}(1-0.771z+0.05z^{2})}{1-(0.05)^2}$$

$$= \frac{\{1-(0.05)^2\}-(1-0.05)0.771z^{-1}}{\{1-(0.05)^2\}}$$

$$= \frac{0.9975-(0.7325)z^{-1}}{0.9975}$$

$$= 1-0.73z^{-1}$$

$a_1(0)=1$
$a_1(1)=-0.73$

$\{k_1=-0.73\}$

$\cancel{C_m(k)=b_m(k)-\sum_{j\geq k}}$

$b_3(0) = 1$
$b_3(1) = 2$
$b_3(2) = 2$
$b_3(3) = 1$

$B_3(z)=1+2z^{-1}+2z^{-2}+z^{-3}$

$a_3(0) = 1$
$a_3(1) = -0.8$
$a_3(2) = 0.49$
$a_3(3) = -0.57$

$A_3(z)=1-0.8z^{-1}+0.49z^{-2}-0.57z^{-3}$

$\boxed{C_3(3)=1}$

$$C_3(2) = b_3(2) - \sum_{j=3}^{3} C_3(j)\,a_j(j-k)$$

$$= b_3(2) - C_3(3)a_3(1)$$

$$= 2 - 1\times(-0.8)$$

$\boxed{C_3(2)=2.8}$

$$C_3(1) = b_3(1) - \sum_{j=2}^{3} C_3(j)\,a_j(j-1)$$

$$= 2-\{C_3(2)a_2(1)+C_3(3)a_3(2)\}$$

$$= 2-\{(2.8)(-0.771)+1\times0.49\}$$

$\boxed{C_3(1) = 3.66}$

$$C_3(0) = b_3(0) - \sum_{j=1}^{3} C_3(j)\,a_j(j)$$

$$= 1-\{C_3(1)a_1(1)+C_3(2)a_2(2)+C_3(3)a_3(3)\}$$

$$= 1-\{3.66\times(-0.73)+2.8\times(0.05)+(-0.57)\}$$

$$= 1-\{-2.672+0.14-0.57\}$$

$\boxed{C_3(0)=4.11}$

W

☆ Lattice n/w for FIR filters

$H_m(z) = A_m(z) \qquad m = 0,1,2,\dots,m-1$

$$A_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k}$$

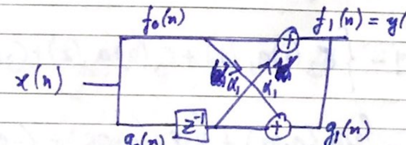
$$A_0(z) = 1$$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=0}^{m} \alpha_m(k)\, x(n-k)$$

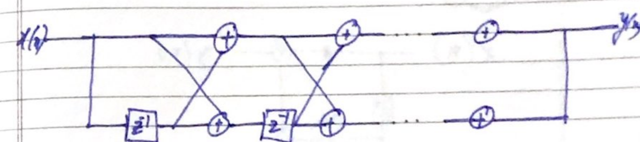
$$h_k(z) = \begin{cases} 1 & k = 0 \\ \alpha_m & k = 1,2,3,\dots m \end{cases}$$

$y(n) = x(n) + \alpha_1(1)\, x(n-1)$ ~~$+ \alpha_2 x(n-2)$~~



$y(n) = x(n) + \alpha_2(1)\, x(n-1) + \alpha_2(2)\, x(n-2)$

$y(n) = x(n) + \alpha_N(1)\, x(n-1) + \dots + \alpha_N(N)\, x(n-N)$



Q-1) $k_1 = 1/4,\ k_2 = 1/2,\ k_3 = 1/3$

Determine FIR coef. for direct form

$A_1(z) = A_0(z) + k_1 z^{-1} A_0(z^{-1})$

$= 1 + \frac{z^{-1}}{4} \times 1 = 1 + 0.25 z^{-1}$

$A_2(z) = A_1(z) + k_2 z^{-2} A_1(z^{-1})$

$= (1 + 0.25 z^{-1}) + \frac{z^{-2}}{2}(1 + 0.25 z)$

$= (1 + 0.375 z^{-1} + 0.5 z^{-2})$

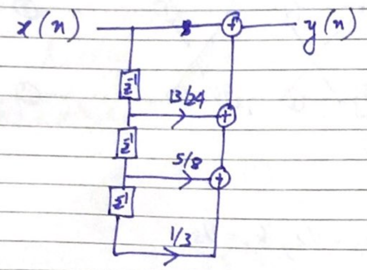
$A_3(z) = A_2(z) + k_3 z^{-3} A_2(z^{-1})$

$= \left(1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2}\right) + \frac{z^{-3}}{3}\left(\frac{1}{3} + \frac{3}{8} z + \frac{1}{2} z^{2}\right)$

$= 1 + \left(\frac{3}{8} + \frac{1}{6}\right) z^{-1} + \left(\frac{1}{2} + \frac{1}{8}\right) z^{-2} + \frac{z^{-3}}{3}$

$$A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$

$$\frac{Y(z)}{\cancel{X(z)}} = H(z)\,X(z)$$



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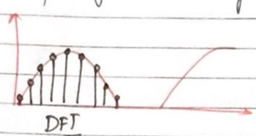
$1+e^{-j3\pi/2}$

$= \cancel{1+(\cos\frac{3\pi}{2}+j)}$

★ DFT

↳ Samples of ~~ont~~ DTFT for 1 T.P.



DTFT → periodic & continuous.

$$X(k)=\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}}$$

eg. 1 Find the DFT of a sequence : $x(n)=\{1, 1, 0, 0\}$

Find IDFT of : $Y(k)=\{1, 0, 1, 0\}$

$$X(k)=\sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{4}}$$

$$= (1+e^{-j\frac{k\pi}{2}})$$

$X(0)=2$
$X(1)=1-j$
$X(2)=0$
$X(3)=1+j$

$$y(n)=\frac{1}{4}\sum_{k=0}^{3} Y(k)\, e^{j\frac{2\pi kn}{4}}$$

$$=\frac{1}{4}(1+e^{j\pi n})$$

$y(0)=1/2$
$y(1)=0$
$y(3)=1/2$
$y(4)=0$

$X(k)=\{2, 1-j, 0, 1+j\}$

$y(n)=\{1/2, 0, 1/2, 0\}$

$$e^{-j\frac{2\pi}{N}\cdot nk} = W_N^{nk}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

Twiddle Factor → Periodicity, Symmetricity

$W_N^{k} = W_N^{(k+iN)}$ → integer

$$W_N^{k} = -W_N^{k\pm N/2}$$

$$e^{-j\frac{2\pi kn}{N}} = -e^{-j\frac{2\pi(kn\pm N/2)}{N}}$$

$$= -e^{-j\frac{2\pi kn}{N}}\, e^{-j\pi}$$

$$= -e^{-j\frac{2\pi kn}{N}}(\cos\pi - j\sin\pi)$$

$$= -e^{-j\frac{2\pi kn}{N}}(-1-0)$$

$$= e^{-j\frac{2\pi kn}{N}}$$

$$x_N = \begin{bmatrix} x(0)\\ x(1)\\ \vdots\\ x(N-1)\end{bmatrix} \qquad X_N = \begin{bmatrix} X(0)\\ X(1)\\ \vdots\\ X(N-1)\end{bmatrix}$$

$$W_N = \begin{array}{c|c} & n=0 \quad \cdots \quad n=N-1 \\ \hline \begin{matrix} k=0\\ \vdots\\ \vdots\\ \vdots\\ k=N-1\end{matrix} & \begin{bmatrix} 1 & 1 & \cdots & 1\\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1}\\ 1 & W_N^2 & W_N^4 & \cdots & \vdots\\ \vdots & \vdots & \vdots & & \vdots\\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)}\end{bmatrix}_{N\times N}\end{array}$$

$$X_N = W_N x_N$$

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same

$$N=4; \quad W_4 = \begin{array}{c} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

-ve

symmetricity

MATH

eg. $x_N = [0 \quad 1 \quad 2 \quad 3]$

$$X_N = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$X_N = W_N x_N$$

$$x_N = W_N^{-1} X_N = \frac{1}{N} W_N^{*} X_N$$

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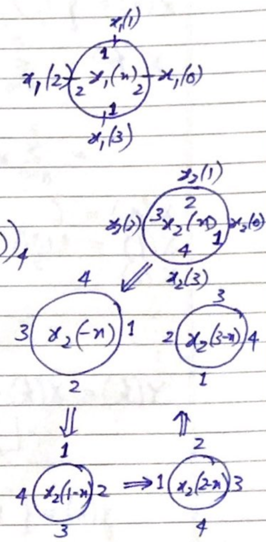
eg. 1 $x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N$

$x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$



$N = 4$

$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2((m-n))_4$

$x_3(0) = 2 + 4 + 6 + 2 = 14$

$x_3(1) = 4 + 1 + 8 + 3 = 16$

$x_3(2) = 6 + 2 + 2 + 4 = 14$

$x_3(3) = 8 + 3 + 4 + 1 = 16$

$x_3(m) = \{14, 16, 14, 16\}$

eg. 2 $h(n) = \{1, 2, 3\} \longrightarrow m = 3$ pad w/ $l-1$

$x(n) = \{1, 2, 2, 1\} \longrightarrow l = 4$ pad w/ $m-1$

$y(n) = x(n) * h(n) \longrightarrow n = 3 + 4 - 1 = 6$

↳ assume 8 ($\to 2^n$)

$X(k) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi \frac{kn}{N}}$

$x(n) = \{1, 2, 2, 1, \underbrace{0, 0, 0, 0}\}$

zero-padding does not affect freq. spectrum

$$x(n) = \{1, 2, 2, 1, 0, 0, 0, 0\}$$

$$X(k) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi kn/8}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j\pi kn/4}$$

$X(0) = 6$ $\qquad H(0) =$

$X(1) = \frac{4+3\sqrt{2}}{2}$ $\qquad H(1) =$

$X(2)$

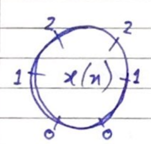
$\vdots$

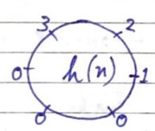
$$Y(k) = X(k) \cdot H(k)$$

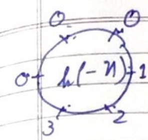
$\quad \rightarrow Y(0) \cdots Y(7)$

$$y(n) = \{1, 4, 9, 11, 8, 3, 0, 0\}$$

$$x(n) = \{1, 2, 2, 1, 0, 0\}$$
$$h(n) = \{1, 2, 3, 0, 0, 0\}$$







$y(0) = 1$

$y(1) = 2 + 2 = 4$

$y(2) = 2 + 4 + 3 = 9$

$y(3) = 1 + 4 + 6 = 11$

$y(4) = 2 + 6 = 8$

$y(5) = 3$

Circular Conv. = Linear for $l' = m' = N$

→ If $x(n)$ seq is complex, then $X(k)$ is also complex.

$$x(n) = x_R(n) + x_I(n) \qquad 0 \le n \le N-1$$

$$X(k) = X_R(k) + X_I(k)$$

$$X_R(k) = \sum_{n=0}^{N-1} \left[x_R(n) \cos\left(\frac{2\pi kn}{N}\right) + x_I(n) \sin\left(\frac{2\pi kn}{N}\right) \right]$$

$$X_I(k) = \sum_{n=0}^{N-1} \left[x_R(n) \sin\left(\frac{2\pi kn}{N}\right) + x_I(n) \cos\left(\frac{2\pi kn}{N}\right) \right]$$

$$x_R(n) = \sum_{k=0}^{N-1} \left[X_R(k) \cos\left(\frac{2\pi kn}{N}\right) - X_I(k) \sin\left(\frac{2\pi kn}{N}\right) \right]$$

$$x_I(n) = \sum_{k=0}^{N-1} \left[X_R(k) \sin\left(\frac{2\pi kn}{N}\right) + X_I(k) \cos\left(\frac{2\pi kn}{N}\right) \right]$$

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→ for real sequence $x(n)$

$$X(N-k) = X^*(k) = X(-k)$$

$$|X(N-k)| = |X(k)|$$

$$\angle X(N-k) = -\angle X(k)$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$X(N-k) = \sum_{n=0}^{N-1} x(n)\, e^{+j\frac{2\pi kn}{N}}\, e^{-j2\pi k}$$

$$= \left[\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}\right]^* \qquad (x^*(n) = x(n))$$

$$X(N-k) = X^*(k)$$

→ for real & even $x(n)$

→ for real & odd $x(n)$

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★ Goertzel Algorithm

$$W_N^{k} = e^{-j\frac{2\pi k}{N}}$$

$$W_N^{-kN} = e^{+j\frac{2\pi kN}{N}} = 1$$

$$X(k) = \sum_{m=0}^{N-1} x(m)\, W_N^{km}$$

$$= W_N^{-kN} \sum_{m=0}^{N-1} x(m)\, W_N^{km}$$

$$= \sum_{m=0}^{N-1} x(m)\, W_N^{-k(N-m)}$$

$$y(n) = x(n) * h(n) = \sum_{k=0}^{N-1} x(k)\, h(n-k)$$

$$y_k(n) = x(n) * W_N^{-kn}\, u(n)$$

$$y_k(n) = \sum_{m=0}^{N-1} x(m)\, W_N^{-k(n-m)}$$

$$h_k(n) = W_N^{-kn}\, u(n) \xleftrightarrow{Z} H_k(Z)$$

$$y_k(n) = \Big|_{n=N} = \sum_{n=0}^{N-1} x(m)\, W_N^{-k(N-m)} = X(k)$$

$$H_k(Z) = \sum_{n=0}^{\infty} W_N^{-kn} Z^{-n} = \frac{1}{1 - W_N^{-k} Z^{-1}} = \frac{Y_k(Z)}{X_k(Z)}$$

$$y_k(n) - W_N^{-k}\, y_k(n-1) = x_k(n)$$

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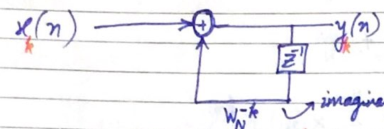
$$y_k(n) = x_k(n) + W_N^{-k}\, y(n-1)$$



imaginary + real = 2x mult. $\therefore$ 2N mult. in total

Put filter in parallel w/ $k = 1, 2, \dots, N$
Add o/p to get DFT.

$$H_k(z) = \frac{1}{1 - e^{j\frac{2\pi k}{N}} z^{-1}} \times \frac{1 - e^{-j\frac{2\pi k}{N}} z^{-1}}{1 - e^{-j\frac{2\pi k}{N}} z^{-1}}$$

$$\frac{Y_k(z)}{X_k(z)} = \frac{1 - W_N^{-k} z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}}$$

$$\frac{Y_k(z)}{V_k(z)} \cdot \frac{V_k(z)}{X_k(z)} = \frac{(1 - W_N^{-k} z^{-1})}{(1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2})}$$

$$\frac{Y_k(z)}{V_k(z)} = (1 - W_N^{-k} z^{-1})$$

$$y_k(n) = v_k(n) - W_N^{-k} v_k(n-1)$$

$$\frac{V_k(z)}{X_k(z)} = \frac{1}{(1 - 2\cos\frac{2\pi k}{N} z^{-1} + z^{-2})}$$

$$x_k(n) = v_k(n) - 2\cos\frac{2\pi k}{N} v_k(n-1) + v_k(n-2)$$

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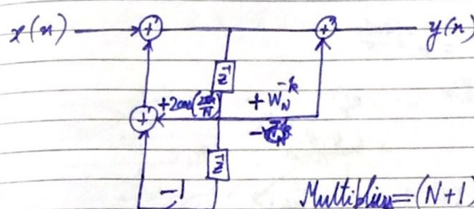
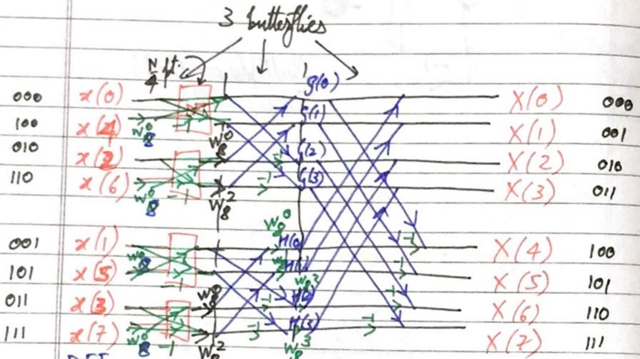


Multipliers = (N+1)
Adders = 3

suitable for $M < \log_2 N$
samples of filter response.

$$W_N^{kn} = e^{-j\frac{2\pi kn}{N}}$$

$$W_N^{(k+\frac{N}{2})n} = -W_N^{kn}$$



DFT

$$X[k] = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad k = 0, 1, \dots, N-1$$

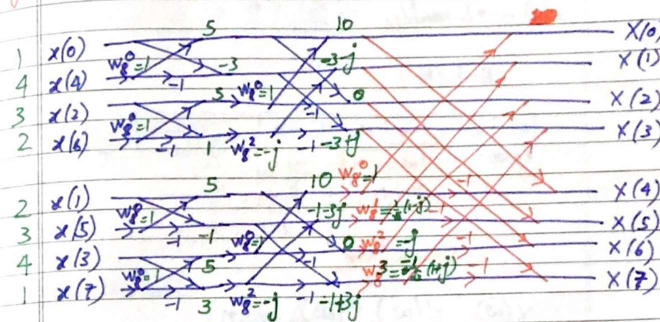
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x(2r) W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x(2r+1) W_N^{(2r+1)k}$$

$$X[k] = \boxed{G(k) + W_N^k H(k)}$$

$$G(k) = \sum_{r=0}^{\frac{N}{2}-1} g(r) W_{N/2}^{rk}$$

$$= \sum_{l=0}^{\frac{N}{4}-1} g(2l) W_{N/2}^{2lk} + \sum_{l=0}^{\frac{N}{4}-1} g(2l+1) W_{N/2}^{(2l+1)k}$$

eg. $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$



$$W_8^0 = 1;\ W_8^1 = e^{-j\frac{2\pi}{8}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}};$$

$$W_8^2 = -j;\ W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$X(0) = 20$

$X(1) = \cancel{-(3+\sqrt{2})-(1+\sqrt{2})j}\ -5.828 - j2.414$

$X(2) = 0$

$X(3) = \cancel{-(3+\sqrt{2})}\ 0.172 - j0.414$

$X(4) = 0$

$X(5) = 0.172 + j0.414$

$X(6) = 0$

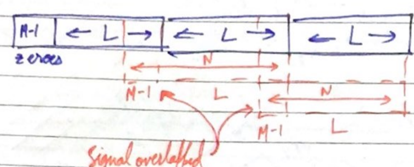
$X(7) = -5.828 + j2.414$

$X(N-k) = X^*(k)$

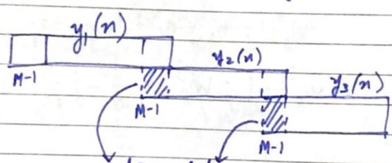
☆ Overlap Save Method (for filtering of long seq.)

$x(n) \longrightarrow L$ samples.

$N = L + M - 1$



$X(\omega), H(\omega), Y(\omega), y(n)$



divided into blocks of 4 $(L=4)$

eg. $x(n) = \{1, 2, 3, 2, 3, 4, 1, 2, 0, 1, 2, 2\}$

$h(n) = \{1, 3, 4\} \longrightarrow m = 3$

$N = 4 + 3 - 1 = 6$

$x(n)$ 0 0 1 2 3 2 Block1

$h(n)$ 1 3 4

$x(n) * h(n)$ 0, 0, 1, 5, 13, 19, 18, 8

4 3 1 discard first M-1 samples; only 6 samples to be kept

$x(n)$ 3 2 3 4 1 2 Block2

$h(n)$ 1 3 4

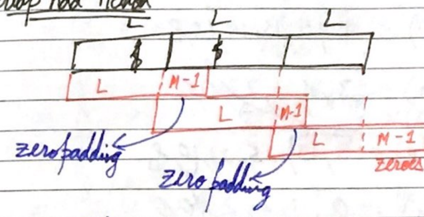
$x(n) * h(n)$ 3, 11, 21, 21, 25, 21, 10, 8

Block3 1 2 0 1 2 2

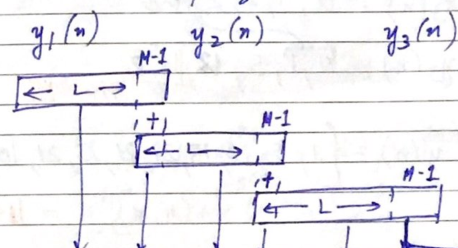
1, 5, 10, 9, 5, 12, 14, 8

$y(n) = \{1, 5, 13, 19, 21, 21, 25, 21, 10, 9, 5, 12, 14, 8\}$

☆ Overlap Add Method



$\{L + M - 1 = N\}$



$y(n)$ As is; add; As is; add; As is; ~~As is~~ ~~Ignore~~ ~~Discard~~ As is

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eg. $x(n) = \{1, 2, 3, 2, 3, 4, 1, 2, 0, 1, 2, 2\}$

$h(n) = \{1, 3, 4\}$

$x_1(n) = 1, 2, 3, 2, 0, 0$ 431

$y_1(n) = 1, 5, 13, 19, \underbrace{18, 8}$

$x_2(n) = 3, 4, 1, 2, 0, 0$

$y_2(n) = \underbrace{3, 13}, 25, 21, \underbrace{10, 8}$

$x_3(n) = 0, 1, 2, 2, 0, 0$

$y_3(n) = \underbrace{0, 1}, 5, 12, \boxed{14, 8}$ Keep

$y(n) = \{1, 5, 13, 19, 21, 21, 25, 21, 10, 9, 5, 12, 14, 8\}$

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☆ DIF (Decimation in Frequency)

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$X(k) = \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=N/2}^{N-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{\left(n+\frac{N}{2}\right)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + W_N^{\frac{N}{2}k} \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{nk}$$

$$\hookrightarrow e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot k} = e^{-j\pi k} = (-1)^k$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left\{ x(n) + (-1)^k x\left(n+\frac{N}{2}\right) \right\} W_N^{nk}$$

$k \to$ even → add

$k \to$ odd → subtract.

$$X(2r) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2r} x(n+N/2)\right] W_N^{2rn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[\underbrace{x(n) + x(n+N/2)}_{g(n)}\right] W_{N/2}^{rn}$$

$$X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1} \left[\underbrace{x(n) - x(n+N/2)}_{h(n)}\right] W_N^{n} \cdot W_{N/2}^{rn}$$

$$g(n) = x(n) + x\left(n+\frac{N}{2}\right) \qquad (N=8)$$

$g(0) = x(0) + x(4)$

$g(1) = x(1) + x(5)$

~~$g(2) = x(2) + x(6)$~~

$g(2) = x(2) + x(6)$

$g(3) = x(3) + x(7)$

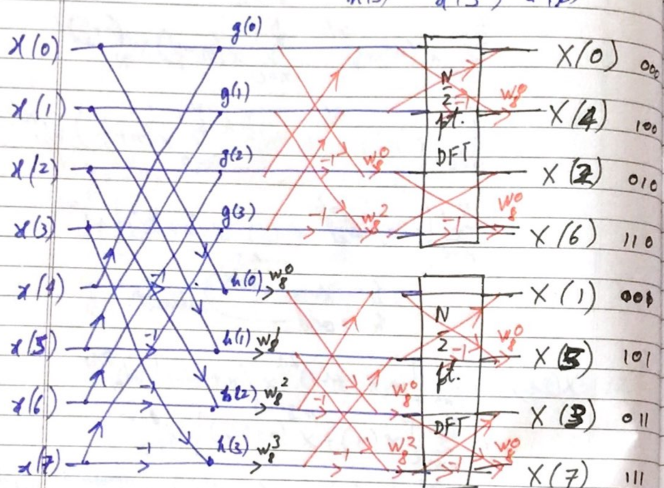
$h(n) = x(n) - x(n + N/2)$

$h(0) = x(0) - x(4)$

$h(2) = x(2) - x(6)$

$h(1) = x(1) - x(5)$

$h(3) = x(3) - x(7)$



8-point DFT

For IDFT, $W_8^k \Rightarrow -W_8^k$