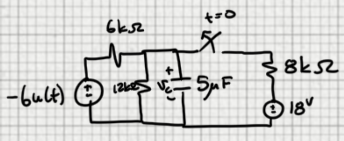
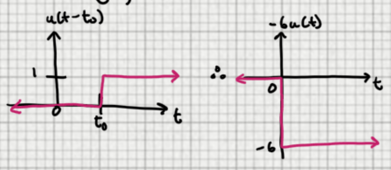
Given the ckt:



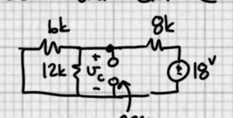

The unit step function (Heaviside Function) is a piecewise function given by:

$$u(t-t_0) = \begin{cases} 1, & \arg() \geq 0 \therefore t-t_0 \geq 0 \rightarrow t \geq t_0 \\ 0, & \text{otherwise} \end{cases}$$

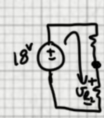



Assuming the switch is closed for a long time what is $v_c(t)$ when $t=0^+$.

Well the ckt @ $t=0^-$ is:




or if further simplified




$(6k \| 12k) \rightarrow \frac{6 \cdot 12\, k^2}{(6+12)\, k} = \frac{72k}{18} = 4k$

Since 8k & 4k share the same current we can use the voltage divider equation.

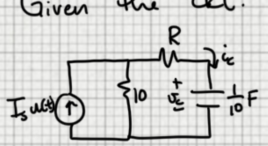
$$v_{R_2} = \left(\frac{4k}{4k+8k}\right) 18^V = 6^V$$

we see that given the configuration $v_{R_2}$ is in parallel with $v_c$. Therefore $v_c(t=0^-) = 6^V$. Since we assume voltage cannot change instantaneously across a capacitor then

$$\boxed{v_c(t=0^+) = v_c(t=0^-) = 6^V}$$

---

Given the ckt:

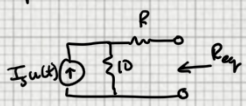



$$\left.\begin{aligned} v_c(t) &= 20 - 10e^{-0.4t} \\ i_c(t) &= 0.4\, e^{-0.4t} \end{aligned}\right\} \text{for } t \geq 0$$

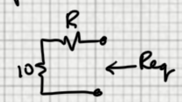
Find the value of R.

This problem is doing nothing more than finding equivalent resistance felt by the capacitor. Recall that any simple RC ckt has a $\tau = RC$.

We know the Capacitance, we just need to find the equivalent resistance seen by capacitor. In other words:




When you have access to the sources, zero them out. Recall a current source with $0^A$ is like an open. So the ckt reduces to:




$\therefore R_{eq} = R + 10 \therefore \tau = (R+10) \cdot \frac{1}{10}$

We can see from the general equation describing the voltage of a capacitor for a simple RC network:

$$v_c(t) = (v_0 - v_\infty)e^{\frac{-(t-t_0)}{\tau}} + V_\infty$$

we see that $\frac{1}{\tau} = 0.4 = \frac{4}{10}$

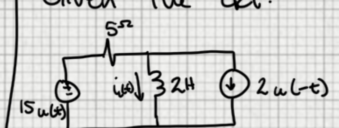
$\therefore \tau = \frac{10}{4}$ so we equate them.

$$(R+10) \cdot \frac{1}{10} = \frac{10}{4} \rightarrow 4R + 40 = 100$$

$$\hookrightarrow 4R = 60 \rightarrow \boxed{R = 15^\Omega}$$

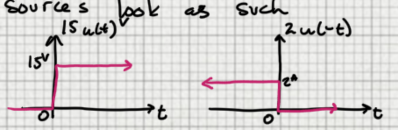
---

Given the ckt:

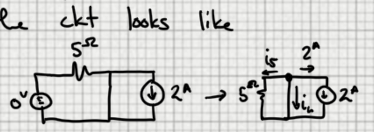



find the equation of $i_L(t)$ for $t \geq 0$.

The equation for the voltage & current sources look as such




we can now assume any initial states. Assuming the ckt is steady state @ negative infinity the ckt looks like




recall ohms law for resistors.

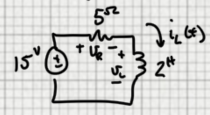
$i_R = \frac{\Delta V_R}{R}$, and since a wire drops/gains $0^V$ $\Delta V_R = 0^V$. Due to Ohm's law $0^A$ can flow through the $5^\Omega$. So using KCL

$\cancel{i_5}^{\,0} + i_L + 2^A = 0^A$, we see $i_L = -2^A$.

Or $i_0 = -2^A$. $i_L(t=0^-) = -2^A = i_L(t=0^+)$

Since we assume current in an inductor cannot change instantaneously.

Given the initial condition we can now look @ the ckt for $t \geq 0$. Which looks like:




| KVL | KCL | Component eqn's |
|---|---|---|
| $15^V = V_R + V_L$ | $I_R = I_L$ | $\Delta V_R = R \cdot I_R$ |
| | | $\Delta V_L = L \frac{dI_L}{dt}$ |

given the KCL:

$\Delta V_R = R \cdot I_L$ & plugging it all into the KVL

$$15 = R \cdot I_L + L I_L'$$

or

$$I_L' + \frac{R}{L} I_L = \frac{15}{L} \rightarrow \boxed{I_L' + 2.5 I_L = 7.5}$$

General 1st order O.D.E.

Homogeneous Solution, where the characteristic eqn:

$I_{L,H}(t) = C_1 e^{st}$ $\qquad s + 2.5 = 0 \rightarrow s = -2.5$

$\therefore I_{L,H}(t) = C_1 e^{-2.5t}$

Particular Solution

$I_{L,P} = A$ & $I_{L,P}' = 0$

Since the driving function is a constant we assume the particular solution is a constant.

$\therefore 0 + 2.5 \cdot A = 7.5$

$\therefore A = \frac{7.5}{2.5} = 3$

The general solution becomes:

$$I_L(t) = I_{L,H}(t) + I_{L,P}(t) = C_1 e^{-2.5t} + 3$$

Given the initial condition $I_L(t=0^+)$ we can solve for $C_1$.

$$I_L(0^+) = -2 = C_1 \cancel{e}^{\,1} + 3 \therefore C_1 = -5$$

and the solution becomes:

$$\boxed{I_L(t) = -5e^{-2.5t} + 3}$$

if we use the general current equation for an inductor in a simple LR ckt we know

$\tau = \frac{L}{R} = \frac{2}{5}$ & $\frac{1}{\tau} = 2.5$

we know $t_0 = 0$

we know $I_L(0^-) = -2^A$ $\quad I_L(\infty) = \frac{V}{R} = \frac{15^V}{5^\Omega} = 3^A$

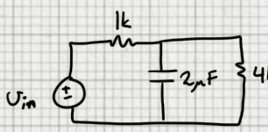
Plugging these values into the general equation:

$$I_L(t) = (I_{L_0} - I_{L_\infty})e^{-\frac{(t-t_0)}{\tau}} + I_{L_\infty}$$

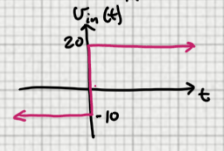
yields the same answer but way faster.

$$I_L(t) = (-2-3)e^{-2.5t} + 3$$

Given the ckt:




where $v_{in}(t) = -10u(-t) + 20u(t)$




Find the zero input response and the zero state response for $t \geq 0$.

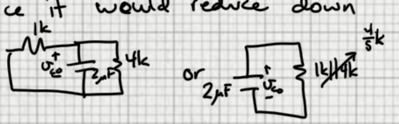
All this is asking is,

① How does this ckt respond if the capacitor has only initial conditions and the source is zeroed out? (zero input response) ← similar to homogeneous solution

② How does this ckt respond if the capacitor has no initial conditions and the source is the only thing exciting it? (zero state response) ← kind of like looking @ a particular solution

Let's start w/ ①

if this ckt had a zeroed out source it would reduce down to




we need to find $V_c(t=0^-)$ to use as an initial condition. @ $t=-\infty$ the capacitor acts as an open and $V_c = V_{4k}$. Using the voltage we see $V_{4k} = \left(\frac{4k}{5k}\right)\cdot(-10^V)$

$\therefore V_c(t=0^-) = -8^V \Rightarrow V_{c_0} = -8^V$

with a potential of $-8^V$ we see the RC network will decay to $0^V$ with nothing driving it. Using the general equation

$$V_c(t) = (V_{c_0} - V_{c_\infty})e^{-\frac{t-t_0}{\tau}} + V_{c_\infty}$$

we know for this configuration

$\tau = (\frac{4}{5}k\Omega)(2\times10^{-6}F) = \frac{8}{5}\times10^{-3}$

& $\frac{1}{\tau} = \frac{5}{8}\times10^{3} = 625$

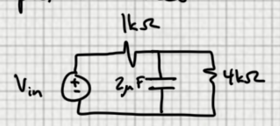
$t_0 = 0$ $V_{c_0} = -8^V$ $V_{c_\infty} = 0^V$

∴ The zero input response is

$$\boxed{V_{C,ZIR} = -8^V e^{-625t}}$$

The zero state response looks at what happens when $V_{c_0} = 0^V$ and $V_{c_\infty} = V_{in}(t=\infty)$

The ckt for $t \geq 0$ w/ the input becomes




$\tau$ & $\frac{1}{\tau}$ is the same.

$t_0$ is the same

$V_{c_0} = 0^V$ & $V_{c_\infty} = 20^V$

Using the general voltage equation for a simple RC network

$$V_c(t) = (V_{c_0} - V_{c_\infty})e^{-\frac{(t-t_0)}{\tau}} + V_{c_\infty}$$

∴

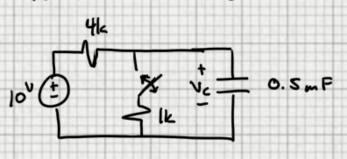
$$V_{c,ZSR}(t) = (0-20)e^{-625t} + 20$$

or

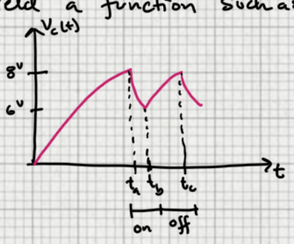
$$\boxed{V_{c,ZSR}(t) = 20(1-e^{-625t})}$$

---

Given the ckt:




Assume, $V_c(t=0) = 0^V$. Also that the switch closes when $V_c = 8^V$ and opens when $V_c = 6^V$, to yield a function such as:




the function is given by the piecewise:

$$V_c(t) = \begin{cases} 10(1-e^{-0.5t}), & 0<t<t_a \quad \text{off} \\ 2+6e^{-2.5(t-t_a)}, & t_a<t<t_b \quad \text{on} \\ 10-4e^{-0.5(t-t_b)}, & t_b<t<t_c \quad \text{off} \\ \vdots & \end{cases}$$

Find $t_b - t_a$.

You could go through multiple steps to find $t_b - t_a$ or you can literally find the time it takes for voltage to fall from $8^V$ to $6^V$ using the second equation in the piecewise.

Using $V_{c,on}(t) = 2+6e^{-2.5(t-t_a)}$ for $t_a<t<t_b$ we already know the bounds. At $t=t_a$ the $V_c = 8^V$ & @ $t=t_b$ the $V_c = 6^V$ we can check the first assumption easy by plugging in $t=t_a$ to the equation

$V_c(t_a) = 2+6e^{-2.5(\cancel{t_a-t_a})^0}$

$V_c(t_a) = 2+6\cancel{e}^{1} = 8^V$

this checks out.

So if we plug in $t=t_b$ we get the form of what is needed to be found.

∴ $6^V = 2+6e^{-2.5(t_b-t_a)}$

↳ $4 = 6e^{-2.5(t_b-t_a)}$

↳ $\frac{2}{3} = e^{-2.5(t_b-t_a)}$

↳ $\ln\left(\frac{2}{3}\right) = \ln\left(e^{-2.5(t_b-t_a)}\right) = -2.5(t_b-t_a)\cancel{\ln(e)}^{1}$

∴ $(t_b-t_a) = \frac{\ln\left(\frac{2}{3}\right)}{-2.5}$

∴ $\boxed{(t_b-t_a) \approx 0.162\,s \text{ or } 162\,ms}$