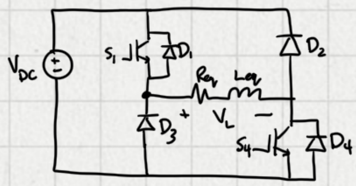
Given the ckt

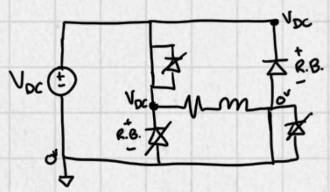


Only 2 possible states

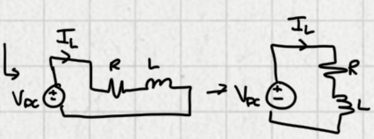
(I) S_1 & S_4 are High

(II) S_1 & S_4 are Low

In State (I)



↳



Assuming some t_0 and an initial condition @ t_0, $I_L(t_0^-) = I_0$

KVL

$V_{DC} = V_R + V_L$

Ohm's Law

$V_R = R \cdot I_R$

Inductor Equation

$V_L = L I_L'$

KCL

$I_L = I_R$

no current information can be found from a voltage source.

$\therefore$ $V_R = R \cdot I_L$ & $V_{DC} = R \cdot I_L + L I_L'$

$$I_L' + \frac{R}{L} I_L = \frac{V_{DC}}{L}$$

Homogeneous Solution

$$I_{LH} = C_1 e^{s(t-t_0)}$$

where the characteristic equation is:

$$s + \frac{R}{L} = 0 \quad \therefore \quad \rightarrow \quad s = -\frac{R}{L}$$

↳ $I_{LH} = C_1 e^{-\frac{R}{L}(t-t_0)}$

Particular Solution

$I_{LP} = A$

$I_{LP}' = 0$ $\quad \therefore \quad \frac{R}{L} \cdot A = \frac{V_{DC}}{L} \rightarrow A = \frac{V_{DC}}{R}$

$\therefore$

$$I_{LP} = \frac{V_{DC}}{R}$$

Therefore the total general solution becomes:

$$I_L = \underbrace{C_1 e^{-\frac{R}{L} \cdot t}}_{\text{transient solution}} + \underbrace{\frac{V_{DC}}{R}}_{\text{steady state solution}}$$

using initial condition

$$I_L(t_0^-) = I_0 = C_1 e^{-\frac{R}{L}\overbrace{(t_0 - t_0)}^{0}} + \frac{V_{DC}}{R}$$

$$\therefore \quad I_0 = C_1 + \frac{V_{DC}}{R}$$

$$↳ \; C_1 = I_0 - \frac{V_{DC}}{R}$$

Therefore, the solution becomes

$$\boxed{I_{L,I} = \left(I_0 - \frac{V_{DC}}{R}\right) e^{\frac{-(t-t_0)}{\tau}} + \frac{V_{DC}}{R}}$$

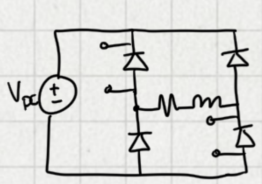
This is a solution that can be used for any simple R-L ckt that has a known starting current.

It can be also written as:

$$\boxed{\begin{gathered} i_L(t) = (i_0 - i_\infty) e^{\frac{-(t-t_0)}{\tau_{RL}}} + i_\infty \\ \text{where } \tau_{RL} = \frac{L}{R} \end{gathered}}$$

State II we find

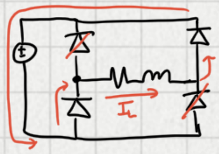
S_1 & S_4 are off:



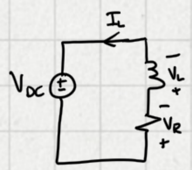
if we start @ this state, we see none of the diodes are on and no current flows any where.

But if this state follows state one, then we know that there is energy stored in the inductors' magnetic field and current must flow

Therefore, the resulting ckt assumes the same current flows at t_0^+ as it did at t_0^-



Therefore, the resulting ckt is



KVL

$V_{DC} = -V_L - V_R$

KCL

$I_L = I_R$

Ohm's Law

$V_R = R \cdot I_R$

Inductor Equation

$V_L = L I_L'$

$$-V_{DC} = L I_L' + R I_L \rightarrow I_L' + \frac{R}{L} I_L = -\frac{V_{DC}}{L}$$

$\therefore$ Homogeneous Solution is the same:

$$I_{LH} = C_1 e^{-\frac{R}{L} \cdot (t-t_0)}$$

Particular Solution is:

$I_{LP} = A \quad I_{LP}' = 0 \quad \therefore$

$$\frac{R}{L} A = \frac{-V_{DC}}{L} \rightarrow A = \frac{-V_{DC}}{R}$$

$$\therefore \; I_{LP} = \frac{-V_{DC}}{R}$$

$$I_L = C_1 e^{-\frac{R}{L}(t-t_0)} - \frac{V_{DC}}{R}$$

using i.c. $I_0 = I_L(t_0^-)$

$$I_L(t_0) = I_0 = C_1 - \frac{V_{DC}}{R} \rightarrow C_1 = I_0 + \frac{V_{DC}}{R}$$

$\therefore$

$$\boxed{I_{L,II} = \left(I_0 + \frac{V_{DC}}{R}\right) e^{\frac{-(t-t_0)}{\tau}} - \frac{V_{DC}}{R}}$$

notice $I_{L,II}$ cannot go negative. This is due to diodes only allowing current flow in one direction. Once $I_{L,II}$ reaches 0^+ the diodes become an open again.

First problem asks to make a ckt w/ opamps, dc voltage sources & resistors to make a -100^{A} to 100^{A} input swing an output voltage from 0^{V} to 3^{V}. In other words, when the I/P input is -100^{A} the O/P output needs to $\sim 0^{V}$ and when I/P is 0^{A} the O/P needs to be $\sim 1^{V}$ and when I/P is 100^{A} the O/P is $\sim 3^{V}$

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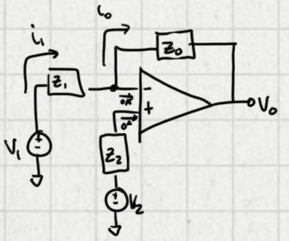
looking @ Op-Amps in a negative feedback operation.



Rules of ideal op-amp

1) when no feedback is attached then $V_o = A(V_+ - V_-)$ where $A = \infty$
2) when there is feedback and V_+ & V_- are not directly controlled then $V_+ = V_-$
3) $i_- = 0^{A}$ & $i_+ = 0^{A}$ * this is the rule most commonly forgotten.

Given this new set of rules and the laws & rules we already know lets analyze this ckt.

We 1st see that Z_2 is useless due to no current can pass through it because $i_+ = 0$. Therefore, $V_+ = V_2$ and since $V_+ = V_-$, then $V_- = V_2$ as well. Now we use Ohm's Law & KCL to solve the rest.

KCL shows: $i_1 = i_o$

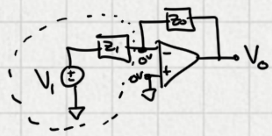
Ohms law shows: $i_1 = \frac{V_1 - V_2}{Z_1}$ & $i_o = \frac{V_2 - V_o}{Z_o}$

$$\therefore \frac{V_1 - V_2}{Z_1} = \frac{V_2 - V_o}{Z_o}$$

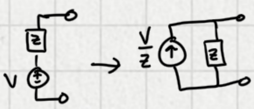
Rearranging, we get

$$V_o = \left(1 + \frac{Z_o}{Z_1}\right) V_2 - \frac{Z_o}{Z_1} \cdot V_1$$

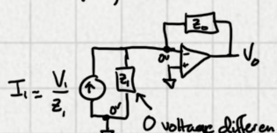
Another thing to notice is how this configuration is a voltage output based on the current input. To simplify let's zero out V_2 and do a transformation.



Recall:

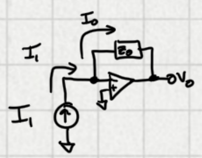


Therefore,



0 voltage difference across therefore no current can pass just like an open

↳



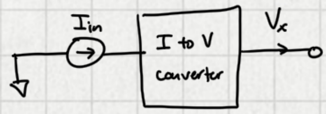
We see that this becomes easier to see that this op-Amp configuration is a current to voltage transformer.

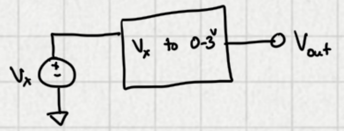
That V_o must change in order to satisfy KCL.

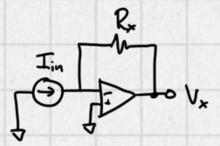
$$I_1 = I_o = \frac{0 - V_o}{Z_o} \therefore \boxed{V_o = -Z_o \cdot I_1}$$

So to complete this problem we need to take the idea that we can transform current to voltage and the make that voltage be stepped up or down to O/P a certain range.

So let's draw a diagram of what we need to do.

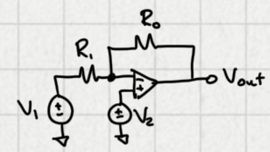






$$\therefore V_x = -I_{in} \cdot R_x$$

now we need to see about taking this V_x and offsetting it.



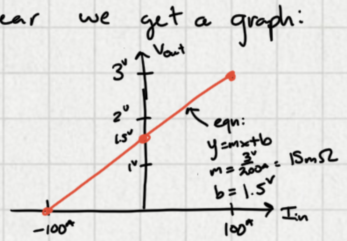
$V_{out} = \left(1 + \frac{R_o}{R_1}\right) V_2 - \frac{R_o}{R_1} V_1$

Assuming $V_1 = V_x = -R_x \cdot I_{in}$

Rearranging and simplifying we get:

$$V_{out} = \frac{R_o}{R_1} \cdot R_x \cdot I_{in} + \left(1 + \frac{R_o}{R_1}\right) \cdot V_2$$

Using the parameters given and assuming the current to voltage relationship is linear we get a graph:



looking @ our general eqn and matching to this line we get that:

$$\frac{R_o}{R_1} \cdot R_x = 0.015^{\Omega} \quad \& \quad \left(1 + \frac{R_o}{R_1}\right) V_2 = 1.5^{V}$$

This is the best you can get. You may try as hard as you want, but you will not get any more new information. Therefore, there are infinite solutions to this problem.

To find your values, all you need to due is to create a $\frac{R_o}{R_1}$ relationship.

$\therefore \frac{R_o}{R_1} = k$ or $R_o = k \cdot R_1$

so you choose a k & R_1 and the rest solves itself.

Ex: $k = 2$ $R_1 = 10\,M\Omega$

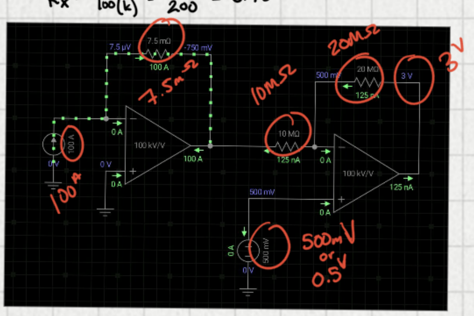
$\therefore R_o = 20 M\Omega$ ← MegaOhms

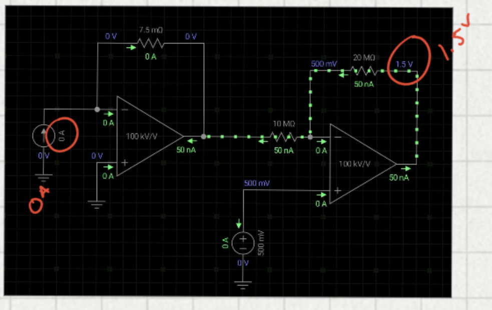
then

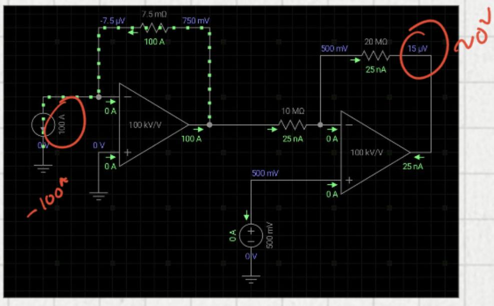
$\left(1+\frac{R_o}{R_1}\right)V_2 = 1.5^V$ or

$V_2 = \frac{1.5^V}{(1+k)} = \frac{1.5^V}{(1+2)} = 0.5^V$

$R_x = \frac{1.5}{100(k)} = \frac{1.5}{200} = 0.75^{\Omega}$







Now that you have seen some ways to solve the problem using OpAmp stages let's see if it can be done with one OpAmp.

Recall the equation of the line was:

$y = (0.015)x + 1.5$

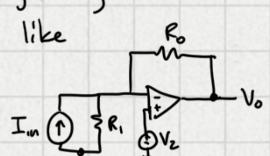
if we recall the general equation for any opamp with negative feed back

$V_o = \left(1+\frac{R_o}{R_1}\right)V_2 - \frac{R_o}{R_1}\cdot V_1$

doing some rearranging we see:

$V_o = (R_o+R_1)\frac{V_2}{R_1} - R_o\frac{V_1}{R_1}$

if we replace $\frac{V_1}{R_1}$ with I_{in} yielding the ckt to look like



then $I_{in} = \frac{V_1}{R_1}$ and the eqn becomes

$V_o = -R_o \cdot I_{in} + \frac{R_o+R_1}{R_1}\cdot V_2$

we have a problem.

$-R_o = 15 m\Omega$

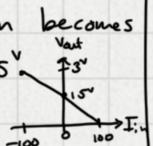
since resistors cannot have a negative value we can't get this to work.

The problem never states though that -100^A has to map to 0^V & 100^A to 3^V.

What if we swap them -100^A to 3^V and 100^A to 0^V

the equation then becomes

$y = -15m\Omega\, x + 1.5^V$



this will work

$-R_o = -15 m\Omega$

& $\frac{R_o+R_1}{R_1}\cdot V_2 = 1.5^V$

Therefore we have to come up w/ V_2 or R_1.

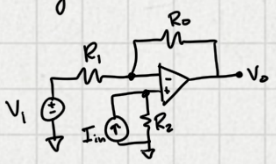
Given $V_2 = \frac{1.5\cdot R_1}{(R_o+R_1)}$ let's say

$R_1 = R_o$ to make the math easy, then

$V_2 = \frac{1.5\cancel{R_o}}{2\cancel{R_o}} = 0.75^V$

And then check it out.

One last design and let's see if we can get it.



We see $V_+ = R_2 \cdot I_{in}$ and V_+ was where V_2 was so going back to the general equation:

$V_{out} = \left(1+\frac{R_o}{R_1}\right)V_2 - \frac{R_o}{R_1}\cdot V_1$

we get

$V_{out} = \overbrace{\left(\left(1+\frac{R_o}{R_1}\right)\cdot R_2\right)}^{m} I_{in} + \overbrace{\left(\frac{R_o}{R_1}V_1\right)}^{b}$

$y = (0.015)\cdot x + b$

$\therefore R_2\left(1+\frac{R_o}{R_1}\right) = 0.015$

& $-\frac{R_o}{R_1}V_1 = 1.5$

so let's do this again:

$\frac{R_o}{R_1} = k \rightarrow R_o = k\cdot R_1$

let's choose $k = 2$ & $R_1 = 1k\Omega$

then $R_o = 2k\Omega$

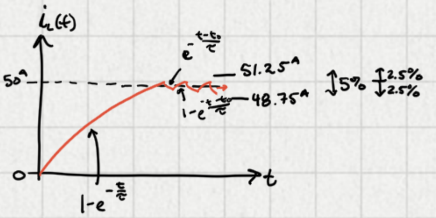
$V_1 = \frac{-1.5}{2} = -0.75^V$

$R_2 = \frac{0.015}{1+2} = 0.005^{\Omega}$

Eureka!

We can do it w/ 1 opamp.

Assuming that the current controller must have a hysteresis band of 5%, means the error from the tracked current isn't more than ±2.5%



so we notice that S_1/S_4 need to be on long enough so that $i_L(t) = 50^A(1.025) = 51.25^A$

Therefore, we use the RL ckt current equation

$$i_L(t) = (i_0 - i_\infty)e^{-\frac{t}{\tau}} + i_\infty$$

Assuming the diodes take no turn on voltage. $V_{in} = 200^V$ $R = 100m\Omega$ $L = 1mH$.

For the turn on $t \geq 0$ in the 1st section. $t_0 = 0_s$

$i_0 = 0^A$ $i_\infty = \frac{V_{DC}}{R} = \frac{200^V}{100m\Omega} = 2kA$

$\frac{L}{R} = \frac{1mH}{100m\Omega} = 10ms$

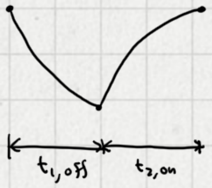
$\therefore i_{L_1}(t) = 2kA(1 - e^{-\frac{t}{10ms}})$

How long does it take to reach 51.25? Do some algebra & rearrange to get:

$$t = -10ms \cdot \ln\left(1 - \frac{51.25}{2000}\right) \approx 259\mu s$$

when plugging this in you will get $i_L(259\mu s) \approx 51.13^A$ ✓

Now that the current is around the 50^A lets find the period and switching frequency.



$T = t_1 + t_2$

$f = \frac{1}{T}$

So we need to find the time it takes for the circuit to fall from 51.25^A to 48.75^A & also the time it takes for it to rise from 48.75^A to 51.25^A.

Calculating the fall time.

$i_0 = 51.13^A$ $\frac{L}{R} = 10ms$

$i_\infty = \frac{-V_{DC}}{R} = -2^{kA}$

$$i_L(t) = (51.13^A + 2^{kA})e^{-\frac{t}{10ms}} - 2^{kA}$$

what is the time for it to decay to 48.75^A

Doing some algebra we get

$$t = -10ms \ln\left(\frac{48.75+2000}{51.13+2000}\right) \approx 12.195\mu s$$

$t \approx 12.195\mu s$ to rise from 48.75^A to 51.25^A

Now to calculate the rise time

$i_0 = 48.75^A$ $i_\infty = 2000^A$

$\frac{L}{R} = 10ms$

$$i_L(t) = (48.75 - 2000)e^{-\frac{t}{10ms}} + 2000$$

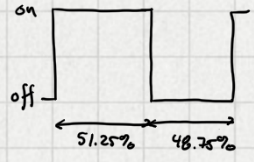
How long does it take to reach 51.25^A?

$$t = -10ms \cdot \ln\left(\frac{51.25-2000}{48.75-2000}\right) \approx 12.82\mu s$$

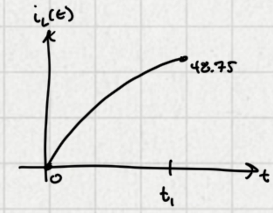
So the square wave generation needs to have a period of

$T = 12.82\mu s + 12.195\mu s = 25.015\mu s$

w/ a Duty cycle of on for $\frac{12.82\mu s}{25.015\mu s} \approx 51.25\%$



$\boxed{f = 39{,}976\ Hz}$

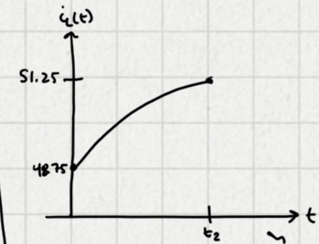


$i_0 = 0^A$ $\frac{L}{R} = 10ms$

$i_\infty = 2000^A$

$$i_L(t) = (1 - e^{-\frac{t}{10ms}})2000$$

$t = 246.77\mu s$ to reach 48.75^A



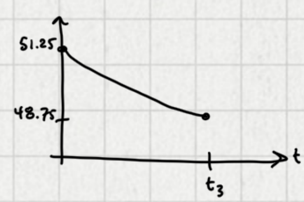
$i_0 = 48.75$ $\frac{L}{R} = 10ms$

$i_\infty = 2000$

$$i_L(t) = (48.75 - 2000)e^{-\frac{t}{10ms}} + 2000$$

$t \approx 12.82\mu s$

to reach 51.25^A from 48.75^A

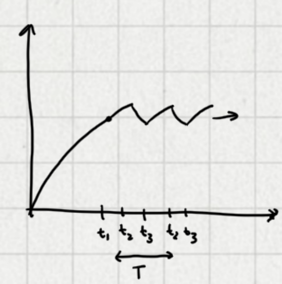


$i_0 = 51.25$ $\frac{L}{R} = 10ms$

$i_\infty = -2000$

$$i_L(t) = (51.25 + 2000)e^{-\frac{t}{10ms}} - 2000$$

$t \approx 12.195\mu s$ to fall to 48.75^A from 51.25^A



$T = 25.015\mu s$ $f = 39{,}976\ Hz$ after a $246.77\mu s$ delay