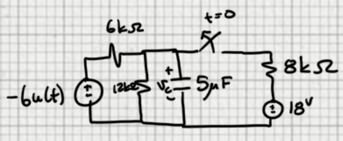
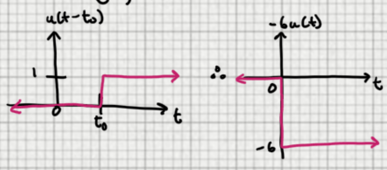
Given the ckt:



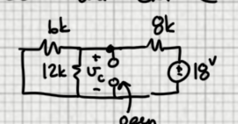
The unit step function (Heaviside Function) is a piecewise function given by:

$$u(t-t_0) = \begin{cases} 1, & \arg() \geq 0 \therefore t-t_0 \geq 0 \rightarrow t \geq t_0 \\ 0, & \text{otherwise} \end{cases}$$

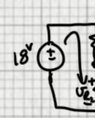


Assuming the switch is closed for a long time what is $v_c(t)$ when $t=0^+$.

Well the ckt @ $t=0^-$ is:



or if further simplified



$(6k \| 12k) \rightarrow \frac{6 \cdot 12\, k^2}{(6+12)\, k} = \frac{72k}{18} = 4k$

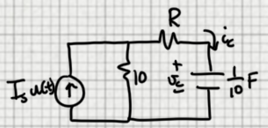
Since 8k & 4k share the same current we can use the voltage divider equation.

$$v_{R_2} = \left(\frac{4k}{4k+8k}\right) 18^V = 6^V$$

we see that given the configuration v_{R_2} is in parallel with v_c. Therefore $v_c(t=0^-) = 6^V$. Since we assume voltage cannot change instantaneously across a capacitor then

$$\boxed{v_c(t=0^+) = v_c(t=0^-) = 6^V}$$

Given the ckt:

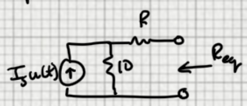


$$\left.\begin{aligned} v_c(t) &= 20 - 10e^{-0.4t} \\ i_c(t) &= 0.4e^{-0.4t} \end{aligned}\right\} \text{for } t \geq 0$$

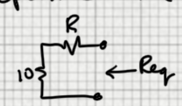
Find the value of R.

This problem is doing nothing more than finding equivalent resistance felt by the capacitor. Recall that any simple RC ckt has a $\tau = RC$.

We know the capacitance, we just need to find the equivalent resistance seen by capacitor. In other words:



When you have access to the sources, zero them out. Recall a current source with 0^A is like an open. So the ckt reduces to:



$\therefore R_{eq} = R + 10 \therefore \tau = (R+10) \cdot \frac{1}{10}$

We can see from the general equation describing the voltage of a capacitor for a simple RC network:

$$v_c(t) = (v_0 - v_\infty)e^{\frac{-(t-t_0)}{\tau}} + V_\infty$$

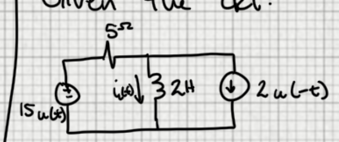
we see that $\frac{1}{\tau} = 0.4 = \frac{4}{10}$

$\therefore \tau = \frac{10}{4}$ so we equate them.

$$(R+10) \cdot \frac{1}{10} = \frac{10}{4} \rightarrow 4R + 40 = 100$$

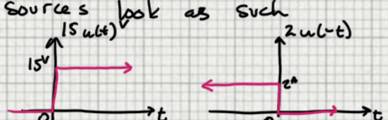
$\hookrightarrow 4R = 60 \rightarrow \boxed{R = 15^\Omega}$

Given the ckt:

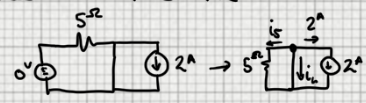


find the equation of $i_L(t)$ for $t \geq 0$.

The equation for the voltage & current sources look as such



we can now assume any initial states. Assuming the ckt is steady state @ negative infinity the ckt looks like



recall ohms law for resistors.

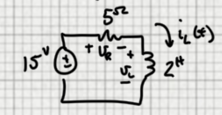
$i_R = \frac{\Delta V_R}{R}$, and since a wire drops/gains 0^V $\Delta V_R = 0^V$. Due to Ohm's law 0^A can flow through the 5^Ω. So using KCL

$\cancel{i_5}^{0} + i_L + 2^A = 0^A$, we see $i_L = -2^A$.

or $i_0 = -2^A$. $i_L(t=0^-) = -2^A = i_L(t=0^+)$

Since we assume current in an inductor cannot change instantaneously.

Given the initial condition we can now look @ the ckt for $t \geq 0$. Which looks like:



KVL	KCL	Component eqn's
$15^V = V_R + V_L$	$I_R = I_L$	$\Delta V_R = R \cdot I_R$
		$\Delta V_L = L\frac{dI_L}{dt}$

given the KCL:

$\Delta V_R = R \cdot I_L$ & plugging it all into the KVL

$15 = R \cdot I_L + L I_L'$

or

General 1st order O.D.E.

$$I_L' + \frac{R}{L} I_L = \frac{15}{L} \rightarrow \boxed{I_L' + 2.5 I_L = 7.5}$$

Homogeneous Solution, where the characteristic eqn:

$I_{L,H}(t) = C_1 e^{st}$ $\qquad$ $s + 2.5 = 0 \rightarrow s = -2.5$

$\therefore I_{L,H}(t) = C_1 e^{-2.5t}$

Particular Solution

$I_{L,P} = A$ & $I_{L,P}' = 0$

Since the driving function is a constant we assume the particular solution is a constant.

$\therefore 0 + 2.5 \cdot A = 7.5$

$\therefore A = \frac{7.5}{2.5} = 3$

The general solution becomes:

$$I_L(t) = I_{L,H}(t) + I_{L,P}(t) = C_1 e^{-2.5t} + 3$$

Given the initial condition $I_L(t=0^+)$ we can solve for C_1.

$I_L(0^+) = -2 = C_1 \cancel{e}^{1} + 3 \therefore C_1 = -5$

and the solution becomes:

$$\boxed{I_L(t) = -5e^{-2.5t} + 3}$$

if we use the general current equation for an inductor in a simple LR ckt we know

$\tau = \frac{L}{R} = \frac{2}{5}$ & $\frac{1}{\tau} = 2.5$

we know $t_0 = 0$

we know $I_L(0^-) = -2^A$ $\quad I_L(\infty) = \frac{V}{R} = \frac{15^V}{5^\Omega} = 3^A$

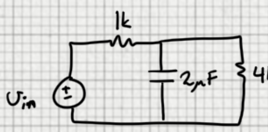
Plugging these values into the general equation:

$$I_L(t) = (I_{L_0} - I_{L_\infty})e^{\frac{-(t-t_0)}{\tau}} + I_{L_\infty}$$

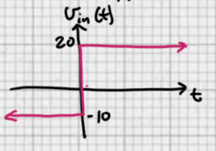
yields the same answer but way faster.

$$I_L(t) = (-2-3)e^{-2.5t} + 3$$

Given the ckt:



where $v_{in}(t) = -10u(-t) + 20u(t)$



Find the zero input response and the zero state response for $t \geq 0$.

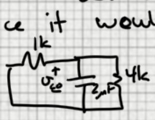
All this is asking is,

① How does this ckt respond if the capacitor has only initial conditions and the source is zeroed out? (zero input response) ← similar to homogeneous solution

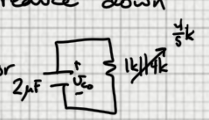
② How does this ckt respond if the capacitor has no initial conditions and the source is the only thing exciting it? (zero state response) ← kind of like looking @ a particular solution

Let's start w/ ①

if this ckt had a zeroed out source it would reduce down to



or



we need to find $V_c(t=0^-)$ to use as an initial condition.

@ $t=-\infty$ the capacitor acts as an open and $V_c = V_{4k}$. Using the voltage we see $V_{4k} = \left(\frac{4k}{5k}\right)\cdot(-10^v)$

$\therefore V_c(t=0^-) = -8^v \Rightarrow V_{c_0} = -8^v$

with a potential of -8^v we see the RC network will decay to 0^v with nothing driving it. Using the general equation

$$V_c(t) = (V_{c_0} - V_{c_\infty})e^{-\frac{t-t_0}{\tau}} + V_{c_\infty}$$

we know for this configuration

$\tau = \left(\frac{4}{5}k\Omega\right)(2\times10^{-6}F) = \frac{8}{5}\times10^{-3}$

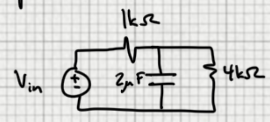
& $\frac{1}{\tau} = \frac{5}{8}\times10^3 = 625$

$t_0 = 0 \qquad V_{c_0} = -8^v \qquad V_{c_\infty} = 0^v$

$\therefore$ The zero input response is

$$\boxed{V_{c,ZIR} = -8^v e^{-625t}}$$

The zero state response looks at what happens when $V_{c_0} = 0^v$ and $V_{c_\infty} = V_{in}(t=\infty)$. The ckt for $t \geq 0$ w/ the input becomes



τ & $\frac{1}{\tau}$ is the same.

t_0 is the same

$V_{c_0} = 0^v$ & $V_{c_\infty} = 20^v$

Using the general voltage equation for a simple RC network

$$V_c(t) = (V_{c_0} - V_{c_\infty})e^{-\frac{(t-t_0)}{\tau}} + V_{c_\infty}$$

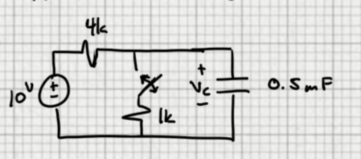
$\therefore$

$$V_{c,ZSR}(t) = (0-20)e^{-625t} + 20$$

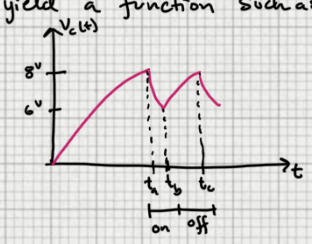
or

$$\boxed{V_{c,ZSR}(t) = 20(1-e^{-625t})}$$

Given the ckt:



Assume, $V_c(t=0) = 0^v$. Also that the switch closes when $V_c = 8^v$ and opens when $V_c = 6^v$, to yield a function such as:



the function is given by the piecewise:

$$V_c(t) = \begin{cases} 10(1-e^{-0.5t}), & 0<t<t_a \quad \text{off} \\ 2+6e^{-2.5(t-t_a)}, & t_a<t<t_b \quad \text{on} \\ 10-4e^{-0.5(t-t_b)}, & t_b<t<t_c \quad \text{off} \\ \vdots \end{cases}$$

Find $t_b - t_a$.

You could go through multiple steps to find $t_b - t_a$ or you can literally find the time it takes for voltage to fall from 8^v to 6^v using the second equation in the piecewise.

Using $V_{c,on}(t) = 2+6e^{-2.5(t-t_a)}$ for $t_a<t<t_b$ we already know the bounds. At $t=t_a$ the $V_c = 8^v$ & @ $t=t_b$ the $V_c = 6^v$ we can check the first assumption easy by plugging in $t=t_a$ to the equation

$V_c(t_a) = 2+6e^{-2.5(t_a-t_a)}$

$V_c(t_a) = 2+6\cdot 1 = 8^v$

this checks out.

So if we plug in $t=t_b$ we get the form of what is needed to be found.

$\therefore$

$6^v = 2+6e^{-2.5(t_b-t_a)}$

↳ $4 = 6e^{-2.5(t_b-t_a)}$

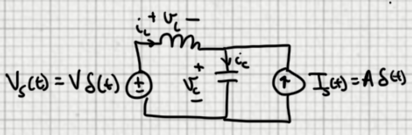
↳ $\frac{2}{3} = e^{-2.5(t_b-t_a)}$

↳ $\ln\left(\frac{2}{3}\right) = \ln\left(e^{-2.5(t_b-t_a)}\right) = -2.5(t_b-t_a)\cancel{\ln(e)}$

$\therefore (t_b-t_a) = \frac{\ln\left(\frac{2}{3}\right)}{-2.5}$

$$\therefore \boxed{(t_b-t_a) \approx 0.162\,s \text{ or } 162\,ms}$$

Given the ckt:



$i_L(0) = 0^A$ & $v_c(0^-) = 0^V$

Find $i_L(0^+)$ & $v_c(0^+)$

KCL	KVL	Component eqn
$i_L + I_s = i_c$	$V_s = v_L + v_c$	$i_c = Cv_c'$
$i_L = i_c - A\delta(t)$		$i_c' = Cv_c''$
$i_L' = i_c' - A\delta'(t)$		$v_L = Li_L'$

$$V\delta(t) = Li_L' + v_c = L[Cv_c'' - A\delta'(t)] + v_c$$

$$\therefore v_c'' + \frac{v_c}{LC} = \frac{V\delta(t)}{LC} + \frac{A}{C}\delta'(t)$$

Because I don't know how to handle $\delta'(t)$ in time domain, I am going to use the Laplace Transform

Recall

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\therefore \mathcal{L}\left\{v_c'' + \frac{v_c}{LC}\right\} = s^2V_c(s) - sv_c(0) - v_c'(0) + \frac{V_c(s)}{LC}$$

where $v_c(0) = 0$

and $i_c = i_L + A\delta(t) = Cv_c'$

$$\therefore v_c'(0) = \frac{\cancel{i_L(0)}^{0}}{C} + \frac{A}{C}\delta(0)'$$

$$\therefore v_c'(0) = \frac{A}{C}$$

$$\therefore s^2V_c(s) - \cancel{s}^{0}\,0 - \frac{A}{C} + \frac{V_c(s)}{LC} = V_c(s)\left[s^2 + \frac{1}{LC}\right] - \frac{A}{C}$$

$$\& \ \mathcal{L}\left\{\frac{V}{LC}\delta(t) + \frac{A}{C}\delta'(t)\right\} = \frac{V}{LC} + s\frac{A}{C}$$

so,

$$V_c(s)\left[s^2 + \frac{1}{LC}\right] - \frac{A}{C} = \frac{A}{C}s + \frac{V}{LC}$$

$$\therefore V_c(s) = \frac{\frac{A}{C}s + \frac{V+AL}{LC}}{s^2 + \frac{1}{LC}} = \frac{A}{C}\cdot\frac{s}{s^2+\frac{1}{LC}} + \frac{V+AL}{\sqrt{LC}}\,\frac{\sqrt{\frac{1}{LC}}}{s^2+\frac{1}{LC}}$$

Using a Laplace Transform Table

$f(t)$		$F(s)$
$\delta(t)$	$\leftrightarrow$	1
$u(t)$	$\leftrightarrow$	$\frac{1}{s}$
$e^{-\alpha t}u(t)$	$\leftrightarrow$	$\frac{1}{s+\alpha}$
$\sin(\omega t)u(t)$	$\leftrightarrow$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)u(t)$	$\leftrightarrow$	$\frac{s}{s^2+\omega^2}$

$$\mathcal{L}^{-1}\left\{\frac{A}{C}\,\frac{s}{s^2+\frac{1}{LC}}\right\} \rightarrow \frac{A}{C}\mathcal{L}^{-1}\left\{\frac{s}{s^2+\frac{1}{LC}}\right\} = \frac{A}{C}\cos\left(\frac{t}{\sqrt{LC}}\right)u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{V+AL}{\sqrt{LC}}\cdot\frac{\sqrt{LC}}{s^2+\frac{1}{LC}}\right\} = \frac{V+AL}{\sqrt{LC}}\cdot\sin\left(\frac{t}{\sqrt{LC}}\right)u(t)$$

$$\mathcal{L}^{-1}\{V_c(s)\} = v_c(t)u(t)$$

Therefore, for $t \geq 0$

$$v_c(t) = \frac{A}{C}\cos\left(\frac{t}{\sqrt{LC}}\right) + \frac{V+AL}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$\therefore \boxed{v_c(0) = \frac{A}{C}}$$

$$i_L(t) = Cv_c'(t) - A\delta(t)$$

where

$$v_c'(t) = \frac{-A}{C\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right) + \frac{V+AL}{LC}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$v_c'(0) = \frac{V+AL}{LC}$$

$\therefore$

$$i_L(0) = Cv_c'(0) - A\delta(0)$$

$$\therefore i_L(0) = \cancel{C}\left(\frac{V+AL}{L\cancel{C}}\right) - A$$

$$\boxed{i_L(0) = \frac{V}{L}}$$

The frequency of this ckt comes from inside the trig.

$$\cos(\omega t) = \cos\left(\frac{1}{\sqrt{LC}}t\right)$$

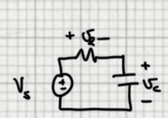
$$\omega = \frac{1}{\sqrt{LC}} \quad \text{where } \omega = 2\pi f$$

$$\therefore 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Delta functions are a funny function. They set i.c. conditions.

Let's look @ a simple RC network. for $t \geq 0$



KVL	KCL
$V_s = v_R + v_c$	$i_R = i_c$

Component eqn

$v_c(0^-) = 0^V$

$$v_R = R\cdot i_R = Ri_c = RCv_c'$$

$$i_c = C\cdot v_c'$$

$$\therefore V_s = RCv_c' + v_c \quad \text{where } V_s = V\delta(t)$$

$$\hookrightarrow v_c' + \frac{v_c}{RC} = \frac{V_s}{RC} = \frac{V\delta(t)}{RC}$$

$$\mathcal{L}\left\{v_c' + \frac{v_c}{RC}\right\} = sV_c(s) - v_c(0) + \frac{V_c(s)}{RC}$$

$$\mathcal{L}\left\{\frac{V}{RC}\delta(t)\right\} = \frac{V}{RC}$$

$$\hookrightarrow V_c(s) = \frac{V}{RC}\left(\frac{1}{s+\frac{1}{RC}}\right)$$

Recall:

$f(t)$	$F(s)$
$e^{-\alpha t}u(t) \leftrightarrow$	$\frac{1}{s+\alpha}$

$$\therefore v_c(t) = \frac{V}{RC}e^{-\frac{t}{RC}}u(t)$$

and for $t \geq 0$

$$v_c(t) = \frac{V}{RC}e^{-\frac{t}{RC}}$$

Another way to look @ an impulse (delta) function as the driving function is to analyze a step response and take the derivative of that.

Recall: $v_c' + \frac{v_c}{RC} = \underset{\text{step function}}{u(t)}$ where $v_c(0) = 0^V$

$$\therefore \mathcal{L}\{\} \rightarrow sV_c(s) + \frac{V_c(s)}{RC} = \frac{1}{sRC}$$

$$\hookrightarrow V_c(s)\left[s + \frac{1}{RC}\right] = \frac{1}{sRC} \qquad V_c(s) = \frac{\frac{1}{RC}}{s\left[s+\frac{1}{RC}\right]}$$

$$\hookrightarrow V_c(s) = \frac{1}{RC}\left[\frac{RC}{s} - \frac{RC}{s+\frac{1}{RC}}\right] = \frac{1}{s} - \frac{1}{s+\frac{1}{RC}}$$

$$\therefore \mathcal{L}^{-1}\{\} \rightarrow v_c(t) = u(t) - e^{-\frac{t}{RC}}u(t)$$

& for $t \geq 0$

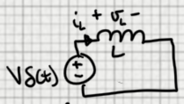
$$v_c(t) = 1 - e^{-\frac{t}{RC}} \leftarrow \text{step response}$$

to find impulse response just take a derivative of the step response.

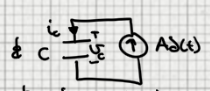
$$v_c(t) = \frac{1}{RC}e^{-\frac{t}{RC}}$$

what's happening?

well the current source is charging the capacitor and the voltage source is charging the inductor.



&



$\therefore v_L(t) = V\delta(t)$ & $i_c(t) = A\delta(t)$

$i_L(0^-) = 0$ $\qquad$ $v_c(0^-) = 0$

Given:

$$v_L = Li_L' \quad \text{or} \quad i_L = \int_0^t \frac{v_L(t')}{L}dt' + i_L(0^-)$$

&

$$i_c = Cv_c' \quad \text{or} \quad v_c = \int_0^t \frac{i_c(t')}{C}dt' + v_c(0^-)$$

$$\therefore i_L(0^+) = \int_0^{0^+}\frac{V}{L}\delta(t)\,dt + \cancel{i_L(0)}^{0^A}$$

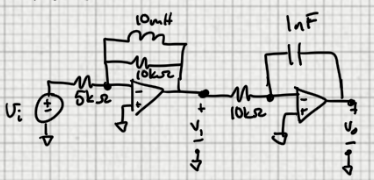
$$i_L(t) = \frac{V}{L}$$

if you don't understand $\delta(t)$ it is defined as having an area of 1 a $t = 0$. Therefore, the current is following a voltage source that has a zero rise & zero fall time. It is this energy that is setting the of the current. Just like the current source is setting the voltage on the capacitor.

$$v_c(0^+) = \int_0^{0^+}\frac{A}{C}\delta(t)\,dt + \cancel{v_c(0)}^{0^V}$$

$$v_c(0^+) = \frac{A}{C}$$

Given the ckt:

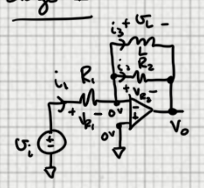


assume $i_L(0^-)=0^A$ $\quad v_C(0^-)=0^V$

and $v_i = 2u(t)$

and ideal op-amps. find v_1 & v_o t>0

Stage 1



we must solve the ODE before being able to find v_0

KCL	KVL	Component Eqn
$i_1=i_2+i_3$	$v_i=v_{R_1}$	$v_{R_x}=R_x\cdot i_{R_x}$
$i_1=i_{R_1}$	$v_{R_2}=v_L=-v_0$	$v_L=L\cdot i_L'$
$i_2=i_{R_2}$		
$i_3=i_L$		

∴ $\frac{v_{R_1}}{R_1}=\frac{v_{R_2}}{R_2}+i_L$, where $v_{R_2}=v_L=Li_L'$

↳ $\frac{v_{R_1}}{R_1}=\frac{L}{R_2}i_L'+i_L \to i_L'+\frac{R_2}{L}i_L=\frac{R_2}{R_1L}\cdot \cancel{v_{R_1}}\, v_i$

$\mathcal{L}\{i_L'+\frac{R_2}{L}i_L\}=sI_L(s)-\cancel{i_L(0)}^{\,0}+\frac{R_2}{L}I_L(s)$

$\mathcal{L}\{\frac{R_2}{R_1L}\cdot 2u(t)\}=\frac{2R_2}{R_1L}\cdot\frac{1}{s}$

∴ $I_L(s)[s+\frac{R_2}{L}]=\frac{2R_2}{R_1L}\cdot\frac{1}{s}$

∴ $I_L(s)=\frac{2R_2}{R_1L}\cdot\frac{1}{s[s+\frac{R_2}{L}]}=\frac{2R_2}{R_1L}\left(\frac{\frac{L}{R_2}}{s}-\frac{\frac{L}{R_2}}{s+\frac{R_2}{L}}\right)$

used partial fraction decomp.

$\mathcal{L}^{-1}\left\{I_L(s)=\frac{2R_2}{R_1L}\left[\frac{L}{R_2}\cdot\frac{1}{s}-\frac{L}{R_2}\frac{1}{s+\frac{R_2}{L}}\right]\right\}$

using Laplace Tables

$f(t)\leftrightarrow F(s)$

$u(t)\leftrightarrow \frac{1}{s}$

$e^{at}u(t)\leftrightarrow\frac{1}{s-a}$

∴ $i_L(t)=\frac{2R_2}{R_1L}\left(\frac{L}{R_2}\cdot u(t)-\frac{L}{R_2}e^{-\frac{R_2}{L}t}u(t)\right)$

for t>0

$i_L(t)=\frac{2}{R_1}-\frac{2}{R_1}e^{-\frac{R_2}{L}t}$ & $i_L'(t)=\left(-\frac{2}{R_1}\right)\left(-\frac{R_2}{L}\right)e^{-\frac{R_2}{L}t}$

$i_L'(t)=\frac{2R_2}{R_1L}e^{-\frac{R_2}{L}t}$

∴ $v_L(t)=L\cdot i_L'(t)=L\left(\frac{2R_2}{R_1L}\right)e^{-\frac{R_2}{L}t}$

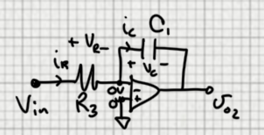
$v_L(t)=\frac{2R_2}{R_1}e^{-\frac{R_2}{L}t}$ & $v_0=-v_L$

∴ $\boxed{v_o(t)=-\frac{2R_2}{R_1}e^{-\frac{R_2}{L}t}}$ Given Values $v_o(t)=-4e^{-10^6t}$

Now to Solve Stage 2

we use the same approach but apply the capacitor eqn:

$i_c=Cv_c'$



KCL	KVL	Component Eqn
$i_{R_3}=i_{C_1}$	$v_{R_3}=v_{in}$	$v_R=R\cdot i_R$
	$v_{C_1}=-v_{o_2}$	$i_c=Cv_c'$

∴ $\frac{v_{R_3}}{R_3}=C_1v_{C_1}' \to \frac{v_{in}}{R_3}=C_1v_{C_1}'$

$v_{C_1}'=\frac{v_{in}}{RC_1}$ → Homogeneous $v_{c_H}=K_1e^{st}=K_1$

characteristic eqn $s=0$

↳ Particular

need to know the input.

$v_i=-\frac{2R_2}{R_1}e^{-\frac{R_2}{L}t}$ ∴ $v_{c_{1p}}=Ae^{-\frac{R_2}{L}t}$

$v_{c_{1p}}'=-\frac{AR_2}{L}e^{-\frac{R_2}{L}t}$

∴ $-\frac{AR_2}{L}\cancel{e^{-\frac{R_2}{L}t}}=-\frac{2R_2}{R_1R_3C_1}\cancel{e^{-\frac{R_2}{L}t}}$

↳ $-\frac{AR_2}{L}=-\frac{2R_2}{R_1R_3C_1} \to A=\frac{2L}{R_1R_3C_1}$

∴ $v_{C_1}=v_{c_{1H}}+v_{c_{1p}}=K_1+\frac{2L}{R_1R_3C_1}e^{-\frac{R_2}{L}t}$

where $v_{C_1}(0)=0$ ∴

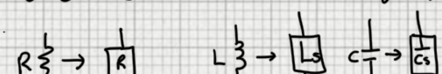
$0=K_1+\frac{2L}{R_1R_3C_1}\cancel{e^0}^{\,1} \to K_1=\frac{-2L}{R_1R_3C_1}$

∴ $v_{C_1}(t)=\frac{2L}{R_1R_3C_1}\left(e^{-\frac{R_2}{L}t}-1\right)$

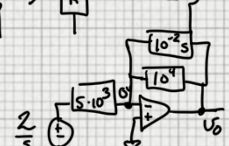
& $\boxed{v_{o_2}=-v_{C_1}=\frac{2L}{R_1R_3C_1}\left(1-e^{-\frac{R_2}{L}t}\right)}$

given the values → $v_{o_2}=\frac{2\cdot10^{-2}}{5\cdot10^3\cdot10^{-9}}(1-e^{-10^6t})=\frac{4}{10}(1-e^{-10^6t})$

If you know the Laplace Transform for components let's analyze this: (note: w/out i.c. transforms)



∴



$10^{-2}s\,||\,10^4=\frac{10^2s}{10^{-2}(s+10^6)}=\frac{10^4s}{s+10^6}$

Using the negative feedback general equation

$v_o=\left(1+\frac{Z_o}{Z_i}\right)v_2-\left(\frac{Z_o}{Z_i}\right)v_1$

where $v_2=0$ $\quad Z_o=\frac{10^4s}{s+10^6}$ $\quad Z_i=5\cdot10^3$

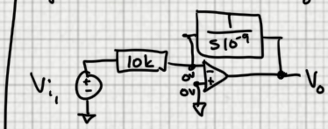
$v_1=\frac{2}{s}$

$V_o(s)=-\frac{10^4s}{5\cdot10^3(s+10^6)}\cdot\frac{2}{s}=\frac{-4\cdot\cancel{10^4s}}{\cancel{10^4s}(s+10^6)}$

$V_o(s)=\frac{-4}{(s+10^6)}\xrightarrow{\mathcal{L}^{-1}\{\}} v_o(t)=-4e^{-10^6t}u(t)$

for t≥0 $v_o(t)=-4e^{-10^6t}$

for the second stage



using the same eqn we get:

$V_2=0$ $\quad Z_o=\frac{10^9}{s}$

$V_1=\frac{-4}{s+10^6}$ $\quad Z_i=10^4$

∴ $V_o(s)=\frac{-10^9}{s10^4}\left(\frac{-4}{s+10^6}\right)$

↳ $V_o(s)=\frac{4\cdot10^5}{s(s+10^6)}=\frac{A}{s}+\frac{B}{s+10^6}$

$A=4/10$ $\quad B=-4/10$

∴ $V_o(s)=\frac{4}{10}\cdot\frac{1}{s}-\frac{4}{10}\cdot\frac{1}{s+10^6}$

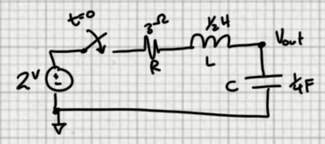
$\mathcal{L}^{-1}\{\}$ ↳ $v_o(t)=\frac{4}{10}u(t)-\frac{4}{10}e^{-10^6t}u(t)$

for t≥0

$v_o(t)=\frac{4}{10}-\frac{4}{10}e^{-10^6t}=\frac{4}{10}(1-e^{-10^6t})$

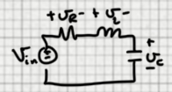
We see how powerful the Laplace technique is and how quickly a solution can be found for basic driving functions.

Given the ckt:



no initial conditions

Find $v_{out}(t)$ for $t \geq 0$.



KCL	KVL	Component Eqn
$i_R = i_L = i_C$	$v_{in} = v_R + v_L + v_C$	$v_R = R \cdot i_R$
	$v_C = v_{out}$	$v_L = L \cdot i_L'$
		$i_C = C v_C'$
		$i_C' = C v_C''$

∴ $v_{in} = R \cdot \overset{i_C}{\cancel{i_R}} + L \overset{i_C'}{\cancel{i_L'}} + v_C$

↳ $v_{in} = RCv_C' + LCv_C'' + v_C$

∴ $v_C'' + \frac{R}{L}v_C' + \frac{1}{LC}v_C = \frac{v_{in}}{LC}$

Homogeneous Sol'n

$v_{C,H} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

characteristic eqn

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$s_{1,2} = -\frac{b}{2} \pm \sqrt{\left(\frac{-b}{2}\right)^2 - c}$

$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{-R}{2L}\right)^2 - \frac{1}{LC}}$ R=3, L=1/2, C=1/4

$s_{1,2} = \frac{-3}{2(1/2)} \pm \sqrt{(-3)^2 - \frac{1}{(1/2)(1/4)}} = -3 \pm \sqrt{9-8}$

$s_{1,2} = -3 \pm 1$ $s_1 = -2$ $s_2 = -4$

∴ $v_{C,H} = K_1 e^{-2t} + K_2 e^{-4t}$

Particular Sol'n

need to know v_{in}

$v_{in} = 2$ ∴ $v_{C,P} = A$, $v_{C,P}' = 0$, $v_{C,P}'' = 0$

∴ $\frac{A}{LC} = \frac{2}{LC} \rightarrow A = 2$

∴ $v_{C,P} = 2$

General Sol'n

∴ $\boxed{v_C = v_{C,H} + v_{C,P} = K_1 e^{-2t} + K_2 e^{-4t} + 2}$

given $v_C(0) = 0^V$

$0 = K_1 + K_2 + 2 \rightarrow K_1 + K_2 = -2$ (1)

to get the 2nd eqn we use the 2nd i.c. $i_L(0) = 0^A$

recall $i_L = i_C = Cv_C'$

∴ $v_C' = -2K_1 e^{-2t} - 4K_2 e^{-4t}$

∴ $i_L = \overset{1/4}{C}(-2K_1 e^{-2t} - 4K_2 e^{-4t}) = -\frac{1}{2}K_1 e^{-2t} - K_2 e^{-4t}$

∴ $0 = -\frac{1}{2}K_1 - K_2 \rightarrow K_1 + 2K_2 = 0$ (2)

solving for K_1 & K_2

$$\begin{array}{c c} & \begin{array}{ccc} K_1 & K_2 & \text{Known} \end{array} \\ \begin{array}{c} (1) \\ (2) \end{array} & \begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

∴ $\text{rref}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \end{bmatrix}$

∴ $K_1 = -4$, $K_2 = 2$

∴ $v_C(t) = -4e^{-2t} + 2e^{-4t} + 2$

Recall

$v_{out} = v_C$

∴ $\boxed{v_{out}(t) = -4e^{-2t} + 2e^{-4t} + 2}$

Another way to approach: using unilateral Laplace Transform

Using $v_C'' + \overset{6}{\cancel{\frac{R}{L}}}v_C' + \overset{8}{\cancel{\frac{1}{LC}}}v_C = \frac{v_{in}}{\cancel{LC}} \; 8 \cdot 2u(t)$

where

$v_{in} = 2u(t)$

$R = 3$ $L = 1/2$ $C = 1/4$

$v_C(0) = 0$

$v_C'(0) = \frac{i_C(0)}{C} = \frac{\overset{0}{\cancel{i_L(0)}}}{C} = 0$

Recall:

∴ $\mathcal{L}\{\alpha v_C''(t)\} = \alpha(s^2 V_C(s) - s\overset{0}{\cancel{v_C(0)}} - \overset{0}{\cancel{v_C'(0)}})$

$\mathcal{L}\{\beta v_C'(t)\} = \beta(sV_C(s) - \overset{0}{\cancel{v_C(0)}})$

$\mathcal{L}\{\Delta v_C(t)\} = \Delta V_C(s)$

$\mathcal{L}\{\gamma u(t)\} = \gamma\left(\frac{1}{s}\right)$

∴

$s^2 V_C(s) + s6V_C(s) + 8V_C(s) = 16\left(\frac{1}{s}\right)$

↳ $V_C(s)(s^2 + 6s + 8) = \frac{16}{s}$

↳ $V_C(s) = \frac{16}{s(s^2+6s+8)} = \frac{16}{s(s+2)(s+4)}$

∴ Partial fract decomp

$\frac{1}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$

$A = \frac{1}{(s+2)(s+4)}\Big|_{s=0} = \frac{1}{8}$

$B = \frac{1}{s(s+4)}\Big|_{s=-2} = \frac{-1}{4} = \frac{-2}{8}$

$C = \frac{1}{s(s+2)}\Big|_{s=-4} = \frac{1}{8}$

∴ $V_C(s) = 16\left(\frac{1}{8}\left(\frac{1}{s}\right) - \frac{2}{8}\left(\frac{1}{s+2}\right) + \frac{1}{8}\left(\frac{1}{s+4}\right)\right)$

↳ $V_C(s) = 2\frac{1}{s} - 4\left(\frac{1}{s+2}\right) + 2\left(\frac{1}{s+4}\right)$

Using Laplace Transform Table (unilateral) $t>0$

$f(t) \rightleftharpoons F(s)$

$Au(t) \rightleftharpoons A\frac{1}{s}$

$Ae^{-at}u(t) \rightleftharpoons A\frac{1}{s+a}$

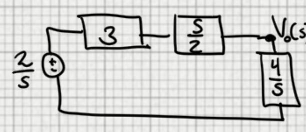
∴ $\mathcal{L}^{-1}\left\{2\frac{1}{s}\right\} = 2u(t)$

$\mathcal{L}^{-1}\left\{-4\frac{1}{s+2}\right\} = -4e^{-2t}u(t)$

$\mathcal{L}^{-1}\left\{2\frac{1}{s+4}\right\} = 2e^{-4t}u(t)$

∴ $\boxed{t>0 \quad v_C(t) = -4e^{-2t} + 2e^{-4t} + 2}$

or using the Laplace Transform from the beginning



using the voltage divider equation because all the components share the same current

$V_o(s) = \frac{\overbrace{\left(\frac{4}{s}\right)}^{Z_C}}{\underbrace{\left(3 + \frac{s}{2} + \frac{4}{s}\right)}_{Z_{tot}}}\overbrace{\left(\frac{2}{s}\right)}^{V_{in}}$

$V_o(s) = \frac{8}{s^2\left(\frac{6s+s^2+8}{2s}\right)} = \frac{16}{s(s+2)(s+4)}$

$V_o(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$

$A = 2$ $B = -4$ $C = 2$

$V_o(s) = \frac{2}{s} - \frac{4}{s+2} + \frac{2}{s+4}$

$\mathcal{L}^{-1}\{\}$

↳ $v_o(t) = [2 - 4e^{-2t} + 2e^{-4t}]u(t)$

for $t \geq 0$

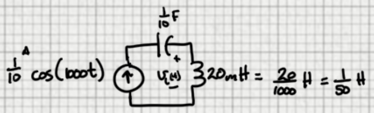
$v_o(t) = 2 - 4e^{-2t} + 2e^{-4t}$

The reason we like to transform using the Laplace or Phasor technique is because KVL, KCL and the concept of Ohm's Law hold's.

$V_z = Z \cdot I_z$

This also means Voltage & current dividers work as well as impedance reduction techniques.

Given the ckt:



$v_C(0) = 0^V$
Determine $v_L(t)$.

Multiple ways can be used to solve this problem.

① Time domain

$v_L(t) = L\, i_L'(t)$ where KCL $i_L = i_C = \frac{1}{10}\cos(1000t)$

$\therefore v_L(t) = \frac{1}{50}\frac{d}{dt}\left(\frac{1}{10}\cos(1000t)\right) = \frac{1}{500}\cdot(-\sin(1000t)\cdot 1000)$

$v_L(t) = -\frac{1000}{500}\cdot\sin(1000t) = -2\sin(1000t)$

$\therefore \boxed{v_L(t) = -2\sin(1000t)}$

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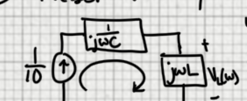
② Phasor technique



where $\omega = 1000$

KCL still shows all currents to be the same. Therefore,

$V_L(\omega) = Z_L(\omega)\cdot I_L(\omega) = j(1000)\left(\frac{1}{50}\right)\cdot\frac{1}{10}$

$V_L(\omega) = j\frac{1000}{500} = j2$

$|V_L(\omega)| = 2 \qquad \angle V_L(\omega) = \frac{\pi}{2}$

$\therefore v_L(t) = |V_L(\omega)|\cdot\cos(1000t + \angle V_L(\omega))$

$v_L(t) = 2\cos\left(1000t + \frac{\pi}{2}\right)$

we know from Trig that $\cos\left(\omega t + \frac{\pi}{2}\right) = -\sin(\omega t)$

$\therefore \boxed{v_L(t) = -2\sin(1000t)}$ ✓

note: Laplace Tranform and Phasor technique are the same in this case:
- sinusoidal input
- no initial conditions
- no transient information (for this case that current source has been on for all time)

Given the ckt:

swt open for a very long time and closes @ t=0. find $i_R(0^+)$

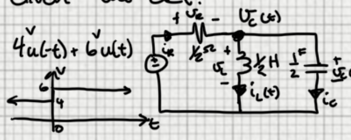
Analysis of the inductor shows $i_L(0^-) = 0^A$ due to no path for current.

$\therefore i_L(0^+) = 0^A$. The KCL shows

$0.03^A = i_L(t) + i_R(t) = \cancel{i_L(0^+)}^{0A} + i_R(0^+)$

$\therefore \boxed{i_R(0^+) = 0.03^A}$

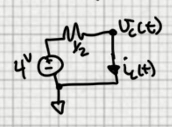
Given the ckt:



Find $i_L(t)$ for $t>0$.

we see the source was 4^V for all time before zero. Assuming the states for inductor and capacitor; (the inductor = short capacitor = open.)

$\therefore$ ckt for $t<0$ becomes



we see:

$v_C(0^-) = 0^V$

$i_L(0^-) = \frac{4^V}{\frac{1}{2}\Omega} = 8^A$

Now we know the initial conditions (i.c.) let's solve this ckt:

① Using Diff Eq.

KVL	KCL	Component eqns
$V_{in} = v_R + v_L$	$i_R = i_L + i_C$	$v_R = R\cdot i_R$
$v_C = v_L$		$v_L = L\, i_L'$
		$i_C = C v_C'$

we see $v_R = R\cdot i_R = R(\overbrace{i_L + i_C}^{KCL})$

$\therefore V_{in} = R\cdot i_L + R\overbrace{\cancel{i_C}}^{Cv_C'} + \overbrace{\cancel{v_L}}^{Li_L'}$

$\hookrightarrow V_{in} = R\cdot i_L + RC\underbrace{\cancel{v_C'}}_{v_L'} + L i_L'$

$\hookrightarrow V_{in} = R i_L + RCL i_L'' + L i_L'$

$\hookrightarrow \frac{V_{in}}{RLC} = i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L$

$48 = i_L'' + 4 i_L' + 4 i_L \leftarrow R=L=C=\frac{1}{2}$

Homogeneous Soln

characteristic eqn

$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \rightarrow s^2 + 4s + 4 = 0$

$\therefore (s+2)(s+2) = 0 \qquad S_{1,2} = 2$

repeated roots needs special handling

$\therefore \boxed{i_{L,H} = K_1 e^{-2t} + K_2 t e^{-2t}}$

Particular Solution

Given a constant driving function

$i_{L,P} = A \quad \therefore i_{L,P}' = i_{L,P}'' = 0$

plugging back in and solving for A:

$48 = \cancel{0}^0 + \cancel{4\cdot 0}^0 + 4\cdot A$

$\therefore A = 12$

General Soln:

$i_L(t) = K_1 e^{-2t} + K_2 t e^{-2t} + 12^A$

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using i.c.

$i_L(0) = 8^A = K_1 \cancel{e}^1 + \cancel{K_2(0)e}^1 + 12^A$

$\therefore K_1 = -4^A$

To use the second i.c. we need to take equation over to $v_C(t)$. Recall:

$v_C(t) = v_L(t) = L\cdot i_L'(t)$

$\therefore v_C(t) = \frac{1}{2}\frac{d}{dt}\left(K_1 e^{-2t} + K_2 t e^{-2t} + 12^A\right)$

$\hookrightarrow v_C(t) = \frac{1}{2}\left(-2K_1 e^{-2t} + K_2\left(e^{-2t} - 2te^{-2t}\right)\right)$

given $v_C(0) = 0 = \frac{1}{2}(-2K_1 + K_2)$

$\therefore K_2 = 2K_1 \Rightarrow 2(-4) = -8$

and now plugging back in coeffecients we find:

$\boxed{i_L(t) = -4e^{-2t} - 8te^{-2t} + 12 \;\;\&\;\; v_C(t) = 8te^{-2t}}$

Because of initial conditions and no frequency input Phasor technique cannot be used. Also Phasor technique only yields steady state response & does not have transient info.

② Laplace Technique

Before jumping straight into it, let's go over some Laplace info.

You are using the unilateral Laplace transform. This means the transforms are for functions multiplied by u(t). Here is a quick table:

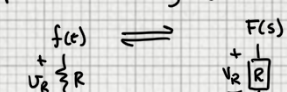
	f(t)	⇌	F(s)
	$\delta(t)$		1
this is u(t) →	1		$1/s$
	$e^{-\alpha t}$		$\frac{1}{(s+\alpha)}$
	$\sin(\omega t)$		$\frac{\omega}{(s^2+\omega^2)}$
	$\cos(\omega t)$		$\frac{s}{(s^2+\omega^2)}$
not common → but needed for repeated roots	$t^n e^{-\alpha t}$ where $n = 1,2,3\ldots$		$\frac{n!}{(s+\alpha)^{n+1}}$

we like laplace because it can handle initial conditions and aperiodic driving functions as well.

f(t) ⇌ F(s)

$\mathcal{L}\{v_R = R\cdot i_R\}$

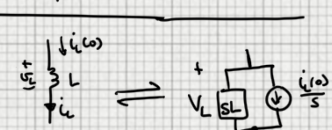
$\hookrightarrow V_R = R\cdot I_R$



$\mathcal{L}\{v_L = L i_L'\}$

$\hookrightarrow V_L = L(sI_L - i_L(0))$

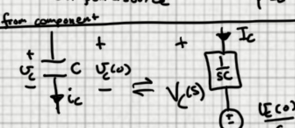
$\hookrightarrow I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s}$ ← current from a source; current from component



$\mathcal{L}\{i_C = C v_C'\}$

$\hookrightarrow \frac{I_C}{C} = sV_C - v_C(0)$

$\hookrightarrow V_C = \frac{I_C}{Cs} + \frac{v_C(0)}{s}$ ← voltage from a source; voltage from component



Given the time domain eqn:

$i_L'' + 4i_L' + 4i_L = 48$ when $i_L(0) = 8^A$

$i_L'(0) = L \cdot v_L(0)$

$i_L'(0) = L \cdot \cancel{v_L(0)}^{0} = 0$

taking the Laplace of this eqn we get

$$\left[s^2 I_L(s) - s\cancel{i_L(0)}^{8} - \cancel{i_L'(0)}^{0}\right] + 4\left[sI_L(s) - \cancel{i_L(0)}^{8}\right] + 4I_L(s) = \frac{48}{s}$$

∴

$$I_L(s)\left[s^2 + 4s + 4\right] - 8s - 32 = \frac{48}{s}$$

↳ $I_L(s) = \dfrac{\frac{48}{s} + 8s + 32}{(s+2)^2} = \dfrac{48 + 8s^2 + 32s}{s(s+2)^2}$

↳ $I_L(s) = \dfrac{8(s^2+4s+6)}{s(s+2)^2} = \dfrac{A}{s} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+2}$

where $A = \dfrac{8(s^2+4s+6)}{(s+2)^2}\Big|_{s=0} = \dfrac{48}{4} = 12$

$B = \dfrac{8(s^2+4s+6)}{s}\Big|_{s=-2} = \dfrac{8(2)}{-2} = -8$

&

$C = \left[\dfrac{8(s^2+4s+6)}{s(s+2)} - \dfrac{\cancel{A}^{12}(s+2)}{s} - \dfrac{\cancel{B}^{-8}}{(s+2)}\right]_{s=1}$

choose any value not 0 or -2

↳ $C = \left[\dfrac{8(11)}{3} - 12(3) + \dfrac{8}{3}\right] = \dfrac{\cancel{96}^{32}}{\cancel{3}} - 36$

$C = -4$

∴ $I_L(s) = \dfrac{12}{s} - \dfrac{8}{(s+2)^2} - \dfrac{4}{s+2}$

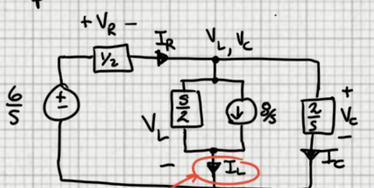
$\mathcal{L}^{-1}\{\}$

↳ $i_L(t) = 12u(t) - 4e^{-2t}u(t) - 8te^{-2t}u(t)$

for $t \geq 0$

$$\boxed{i_L(t) = 12 - 4e^{-2t} - 8te^{-2t}}$$ ✓

To save time:
Let's build the ckt from scratch straight into the Laplace domain.



remember $I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s}$

KCL	KVL	component eqns
$I_R = I_L + I_C$	$\frac{6}{s} = V_R + V_L$	$V_R = R \cdot I_R$
	$V_L = V_C$	$I_L = \frac{V_L}{Ls} + \frac{\cancel{i_L(0)}^{8}}{s}$
		$V_C = \frac{I_C}{Cs} + \frac{\cancel{v_C(0)}^{0}}{s}$

Therefore,

$V_R = \frac{6}{s} - V_L$ & $I_R = \dfrac{\frac{6}{s} - V_L}{\cancel{R}\ 1/2}$

↳ $I_R = \frac{12}{s} - 2V_L$

$I_C = CsV_C = CsV_L = \frac{s}{2}V_L$

$I_L = \frac{V_L}{Ls} + \frac{8}{s} = \frac{2}{s}V_L + \frac{8}{s}$

∴ plugging all this into KCL:

$$\underbrace{\frac{12}{s} - 2V_L}_{I_R} = \underbrace{\frac{2}{s}V_L + \frac{8}{s}}_{I_L} + \underbrace{\frac{s}{2}V_L}_{I_C}$$

$V_L\left(2 + \frac{2}{s} + \frac{s}{2}\right) = \frac{12}{s} - \frac{8}{s} = \frac{4}{s}$

$V_L\left(\frac{s^2+4s+4}{2\cancel{s}}\right) = \frac{4}{\cancel{s}}$

$V_L = \frac{8}{(s+2)^2} = V_C$

$\mathcal{L}^{-1}\left\{\frac{8}{(s+2)^2}\right\} = 8te^{-2t}u(t)$ ✓

Recall

$I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s} = \frac{8}{(s+2)^2} \cdot \frac{2}{s} + \frac{8}{s}$

$I_L = \frac{16}{s(s+2)^2} + \frac{8}{s} = \frac{16 + 8(s^2+4s+4)}{s(s+2)^2}$

$I_L = \frac{8s^2 + 32s + 48}{s(s+2)^2} = \frac{8(s^2+4s+6)}{s(s+2)^2}$

∴ $I_L = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{s}$

$B = \frac{8(s^2+4s+6)}{s}\Big|_{s=-2} = \frac{8(4-8+6)}{-2} = -8$

$C = \frac{8(s^2+4s+6)}{(s+2)^2}\Big|_{s=0} = \frac{8(6)}{4} = 12$

$A = (s+2)\left[\frac{8(s^2+4s+6)}{s(s+2)^2} - \frac{-8}{(s+2)^2} - \frac{12}{s}\right]_{s=1}$

$A = 3\left[\frac{8(1+4+6)}{9} + \frac{8}{9} - 12\right]$

$A = \frac{88}{3} + \frac{8}{3} - 36 = \frac{\cancel{96}^{32}}{\cancel{3}} - 36 = -4$

∴ $I_L = \frac{-4}{(s+2)} - \frac{8}{(s+2)^2} + \frac{12}{s}$

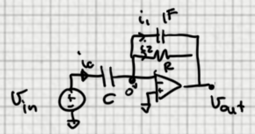
$\mathcal{L}^{-1}\{\}$

$i_L(t) = -4e^{-2t}u(t) - 8te^{-2t}u(t) + 12u(t)$

for $t \geq 0$

$$\boxed{i_L(t) = -4e^{-2t} - 8te^{-2t} + 12}$$ ✓

Given the ckt:



where $v_{in} = u(t)$

$v_{out} = -\frac{1}{2}e^{-4t}u(t)$

find RC to give the step response

From the op-amp rules

where

$i_- = i_+ = 0^A \qquad V_+ = V_-$

we see

$v_C = \overset{u(t)}{\cancel{v_{in}}} - 0 = u(t)$

$i_C = Cv_C' = C\frac{d}{dt}(u(t)) = C\delta(t)$

The KCL shows

$i_C = i_1 + i_2$

where $i_1 = (1F)\frac{d}{dt}(0 - v_{out})$

$i_2 = \frac{0 - v_{out}}{R}$

$\therefore C\delta(t) = -\frac{d}{dt}(v_{out}) - \frac{v_{out}}{R}$

↳ $v_{out}' + \frac{v_{out}}{R} = -C\delta(t)$

Knowing that:

$v_{C_2} = -v_{out}$ & $v_{C_2}(0) = 0^V$

then $v_{out}(0) = 0^V$

$\mathcal{L}\{\}$ ↳ $sV_{out}(s) - \overset{0}{\cancel{v_{out}(0)}} + \frac{V_{out}(s)}{R} = -C$

$V_{out}(s)\left[s + \frac{1}{R}\right] = -C$

$V_{out}(s) = \frac{-C}{\left[s + \frac{1}{R}\right]}$

$\mathcal{L}^{-1}\{\}$ ↳ $v_{out}(t) = -Ce^{-\frac{t}{R}}u(t)$

given

$v_{out} = -\frac{1}{2}e^{-4t}u(t)$

then $C = 1/2$

$R = 1/4$

---

without solving using Laplace a trick must be invoked. Since no initial conditions the $\delta(t)$ will set one and there will be no particular solution only a homogeneous solution.

Recall:

$v_{out}' + \frac{v_{out}}{R} = -C\delta(t)$

$\therefore v_{out}(0) = -C$

& the O.D.E. becomes

$v_{out}' + \frac{v_{out}}{R} = 0$

$v_{out,H} = K_1 e^{st}$

characteristic eqn:

$s + \frac{1}{R} = 0$

$s = -\frac{1}{R}$

$\therefore v_{out,H} = K_1 e^{-\frac{t}{R}}$

given that now no particular,

$v_{out} = K_1 e^{-\frac{t}{R}}$

using new i.c.

$v_{out}(0) = -C = K_1 \overset{1}{\cancel{e^{0}}}$

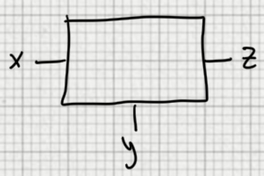
$\therefore K_1 = -C$

& $v_{out} = -Ce^{-\frac{t}{R}}$

just like the Laplace transform

$\therefore \quad C = \frac{1}{2} \qquad R = \frac{1}{4}$

Given a Black Box



we know an RLC ckt is inside; one of each component.

Given $C = 1\mu F$

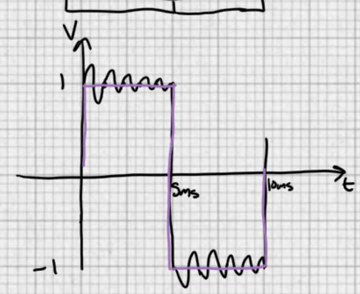
Ohm reading are as follows:

$\Omega_{xy} = \infty$

$\Omega_{xz} = 40\Omega$

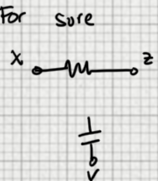
$\Omega_{yz} = \infty$

Applying a square wave @ x-y yields a voltage reading of z-y



so what do we know

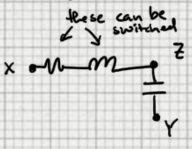
For sure



we also know the ckt resonates so all 3 components must be connected

Therefore

these can be switched



we also see the o/p of $V_{z\text{-}y}$ follows the I/P therefore $V_{z\text{-}y}$ must be the capacitor voltage.

we find the R to be equal to 40Ω because of the Ohm meter reading.

we also see the O/P voltage resonates. Therefore this must be an under damped system.

We know the RLC series ckt is of the form:

$$v_c'' + \underbrace{\frac{R}{L}}_{2\sigma} v_c' + \overbrace{\frac{1}{LC}}^{\omega_0^2} v_c = v_{in}$$

and if under damped then

$$\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$$

and

$$v_c(t) = e^{-\sigma t}\left(K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t)\right)$$

where

$$\omega_0^2 = \frac{1}{LC} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\sigma = \frac{R}{L} \rightarrow \sigma = \frac{R}{2L}$$

$$\therefore \; v_c(t) = e^{-\frac{R}{2L}t}\left(K_1 \cos\left(\frac{t}{\sqrt{LC}}\right) + K_2 \sin\left(\frac{t}{\sqrt{LC}}\right)\right)$$

2 ways to solve for L.

1) if we assume $5ms \approx 4\tau$

then

$$4\left(\frac{2L}{R}\right) \approx 5\times10^{-3}$$

$$\frac{8L}{40} \approx 5\times10^{-3} \rightarrow L \approx 25mH$$

or

2) Assume there are 5 periods in 5 ms therefore $T = 1ms$

from the sinusoids we get

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f = \frac{2\pi}{T}$$

$$\therefore \; \frac{1}{\sqrt{L\cdot10^{-6}}} \approx \frac{2\pi}{10^{-3}}$$

$$\frac{1}{L\cdot10^{-6}} \approx \frac{4\overbrace{\pi^2}^{\sim 9.87}}{10^{-6}}$$

$$L \approx \frac{\cancel{10^{-6}}}{\cancel{10^{-6}}} \cdot \frac{1}{\underbrace{4(9.87)}_{39.5}}$$

$$L = \frac{1}{39.5} \approx 25mH$$

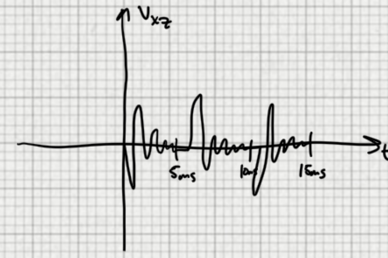
What does V_{xz} look like w/ the same I/P?

Well

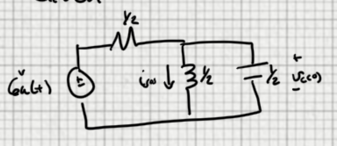
$$V_{xy} = V_{xz} + V_{zy}$$

$$\therefore \; V_{xz} = V_{xy} - V_{zy}$$

so you just need to substract the 2 signals & you will get something like



Given



Given
$i_L(0) = 9^A$ solve for
$v_C(0) = 1^V$ $i_L(t)$ & $v_C(t)$

The O.D.E. is:

$i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L = \frac{V_{in}}{R} \cdot \frac{1}{LC}$

or for these values

$i_L'' + 4i_L' + 4i_L = 48$

Therefore,

$i_{L_H} = K_1 e^{-2t} + K_2 t e^{-2t}$

$i_{L_P} = 12$

$i_L(t) = K_1 e^{-2t} + K_2 t e^{-2t} + 12$

$v_C(t) = \left(\frac{K_2}{2} - K_1\right) e^{-2t} - K_2 t e^{-2t}$

using i.c.

$i_L(0) = 9 = K_1 + 12 \quad \therefore K_1 = -3$

$v_C(0) = 1 = \frac{K_2}{2} - \cancel{K_1}^{-3} \quad \therefore K_2 = -4$

Thus,

$i_L(t) = -3e^{-2t} - 4te^{-2t} + 12$

$v_C(t) = e^{-2t} + 4te^{-2t}$

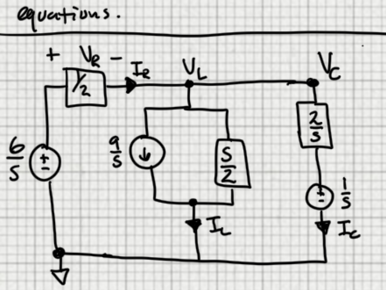
Laplace Transform is:

$I_L(s) = \frac{-3}{s+2} - \frac{4}{(s+2)^2} + \frac{12}{s} = \frac{9s^2+38s+48}{s(s+2)^2}$

or $-\frac{3s-10}{(s+2)^2} + \frac{12}{s}$

$V_C(s) = \frac{1}{s+2} + \frac{4}{(s+2)^2} = \frac{s+6}{(s+2)^2}$

when doing the transform if it doesn't yield these equations it will not yield the correct equations.



KCL
$I_R = I_L + I_C$

KVL
$\frac{6}{s} = V_R + V_L$
$V_L = V_C$

component eqn
$V_R = R \cdot I_R$
$I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s}$
$V_C = \frac{I_C}{Cs} + \frac{v_C(0)}{s}$

$\underbrace{\frac{V_R}{R}}_{I_R} = \underbrace{\left(\frac{V_L}{Ls} + \frac{i_L(0)}{s}\right)}_{I_L} + \underbrace{\left(CsV_C + Cv_C(0)\right)}_{I_C}$

$\frac{\frac{6}{s} - V_C}{R} = \frac{V_C}{Ls} + \frac{i_L(0)}{s} + CsV_C + Cv_C(0)$

$\frac{12}{s} - 2V_C = \frac{2V_C}{s} + \frac{9}{s} + \frac{sV_C}{2} - \frac{1}{2}$

Therefore,

$\frac{24}{2s} - \frac{18}{2s} + \frac{s}{2s}$

$V_C\left(\frac{s}{2} + \frac{2}{s} + 2\right) = \cancel{\frac{12}{s}} - \cancel{\frac{9}{s}} + \frac{1}{2}$

$V_C\left(\frac{s^2+4s+4}{2s}\right) = \frac{s+6}{2s}$

$\boxed{V_C = \frac{s+6}{(s+2)^2}}$ ✓

we know from earlier that

$\mathcal{L}\{e^{-2t} + 4te^{-2t}\} = \frac{s+6}{(s+2)^2}$

For I_L we know

$V_L = V_C = \frac{s+6}{(s+2)^2}$

& $I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s}$

∴ $I_L = \frac{2}{s}\left(\frac{s+6}{(s+2)^2}\right) + \frac{9}{s}$

↳ $= \frac{2s+12+9s^2+36s+36}{s(s+2)^2}$

$I_L = \frac{9s^2+38s+48}{s(s+2)^2}$ ✓

we see from earlier that

$\mathcal{L}\{-3e^{-2t} - 4te^{-2t} + 12\} = \frac{9s^2+38s+48}{s(s+2)^2}$

This example is a demonstration of applying the transform technique and using regular resistance techniques on Impedance blocks and finding the equivalence of the solution in lieu of solving O.D.E.

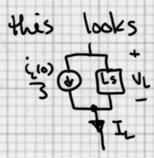
Things to watch out for:

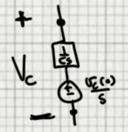
- simple algebra mistakes.
- incorrect transform assumptions

remember $I_L = \frac{V_L}{Ls} + \frac{i_L(0)}{s}$

$V_C = \frac{I_C}{Ls} + \frac{v_C(0)}{s}$

this looks like





- Also be careful when taking partial fraction decomposition values.
- After each transform & invers transform see if your circuit behaves correctly.
  - Does it satisify initial conditions (i.c.)?
  - Does the system respond as it should? (underdamped, overdamped, etc.)
  - Does it come to the correct steady state? (does it settle or doe it oscillate?)

Remember,

Laplace yields

- transient & steady state info
- can handle initial conditions (i.c.)
- Can hande sinusoids [$\sin(\omega t)$ $\cos(\omega t)$], DC [$u(t)$], exponential [$e^{\alpha t}$] & pulse [$\delta(t)$] signals for inputs
- You are using a Unilateral Laplace Transform, which means all functions of time are $t>0$. When inverse Laplace transforms are taken (even if the table isn't showing it) every function has a $u(t)$ attached to it.

Phasor Technique (Fourier Transform) is the way simplified version of the Laplace.

- $s \to j\omega$
- no initial conditions can be present
- only sinusoids for the I/P of one frequency
- only yields steady state info. (no transient info)