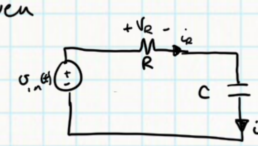
Given



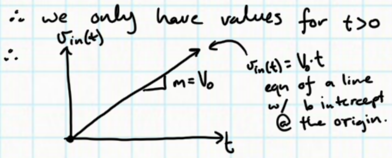
&

$v_{in}(t) = V_0 \cdot t \cdot u(t)$

all the u(t) does it set all values of v_{in}: for $t<0$ then $v_{in} = 0$.

∴ we only have values for $t>0$

∴



Therefore to solve for the current in this problem we must 1st solve the O.D.E. WRT the $v_C(t)$.

Let's make our equations:

<u>KVL</u> $v_{in} = v_R + v_C$

<u>KCL</u> $i_{in} = i_R = i_C$

<u>time equations</u> for $t>0$

$v_R = R \cdot i_R$ given: $v_{in} = V_0 \cdot t$

$i_C = C v_C'$

starting w/ the KVL and plugging in time domain eqn's as needed:

$$V_0 \cdot t = R \cdot \cancel{i_R}^{\,i_C} + v_C$$

↳ $V_0 \cdot t = RC v_C' + v_C$

∴ the O.D.E. becomes

$$v_C' + \frac{1}{RC} v_C = \frac{V_0 t}{RC}$$

Solving for homogeneous soln:

we set driving function to 0:

$$v_C' + \frac{1}{RC} v_C = 0$$

the general soln form is

$$v_{C,H}(t) = K_1 e^{s_1 t}$$

we solve the characteristic eqn to find, s:

$$s + \frac{1}{RC} = 0 \rightarrow s = -\frac{1}{RC}$$

∴

$$v_{C,H}(t) = K_1 e^{-\frac{t}{RC}}$$

Solving for the particular soln:

we find a general form that is of the same form as the driving function. In this case, the driving function is a polynomial to the 1st power:

∴ $v_{C,P}(t) = A \cdot t + B$ & $v_{C,P}'(t) = A$

plugging this back into the O.D.E. we get:

$$\underbrace{(A)}_{v_C'} + \underbrace{\left(\frac{A \cdot t + B}{RC}\right)}_{\frac{1}{RC} v_C} = \frac{V_0 \cdot t}{RC}$$

Rearranging we get:

$$\left(\frac{A}{RC}\right)t + \left(\frac{B}{RC} + A\right) = \left(\frac{V_0}{RC}\right)t + 0$$

we see for these 2 eqn to be equal, then

$$\frac{A}{RC} = \frac{V_0}{RC} \quad \& \quad \frac{B}{RC} + A = 0$$

∴ $A = V_0$ & $B = -ARC = -V_0 RC$

∴ the particular soln becomes

$$v_{C,P}(t) = A \cdot t + B = V_0 \cdot t - V_0 RC$$

and the general solution becomes for $t>0$

$$v_C(t) = v_{C,H}(t) + v_{C,P}(t)$$

$$v_C(t) = K_1 e^{-\frac{t}{RC}} + V_0 t - V_0 RC$$

if we make $\tau = RC$ we get

$$v_C(t) = K_1 e^{-\frac{t}{\tau}} + (t - \tau) V_0$$

using our i.c. we get

$$v_C(0) = 0^V = K_1 \cancel{e}^{\,1} + (0 - \tau) V_0$$

∴ $K_1 = V_0 \cdot \tau$

and the solution becomes:

$$\boxed{v_C(t) = V_0 \tau e^{-\frac{t}{\tau}} + V_0 t - V_0 \tau}$$

to find i we can find i_C This is $i_C = C v_C'$

∴ for $t>0$:

$$i(t) = C\left[-V_0 e^{-\frac{t}{\tau}} + V_0\right] = V_0 C\left[1 - e^{-\frac{t}{\tau}}\right]$$

if we integrate the input because this is a LTI system the answer should be integrated.

∴ if the input is

$$v_p \quad \int V_0 \cdot t \, dt = \frac{V_0 t^2}{2} + \cancel{C_1}^{\,0}$$

$$\therefore v_p = \int V_0 \tau e^{-\frac{t}{\tau}} + V_0 t - V_0 \tau \, dt$$

$$= -V_0 \tau^2 e^{-\frac{t}{\tau}} + \frac{V_0 t^2}{2} - V_0 \tau \cdot t + C_2$$

if $v_C(0) = 0$ then $C_2 = V_0 \tau^2$

$$\therefore \boxed{v_C^{new}(t) = -V_0 \tau^2 e^{-\frac{t}{\tau}} + \frac{V_0}{2} t^2 - V_0 \tau \cdot t + V_0 \tau^2}$$

Check this: $V_{in} = \frac{V_0 t^2}{2}$

Homogeneous solution is still the same

$$v_{C,H}(t) = K_1 e^{-\frac{t}{\tau}}$$

$v_{C,P} = At^2 + Bt + C$ ∴ $v_{C,P}' = 2At + B$

$$\therefore (2At + B) + \frac{(At^2 + Bt + C)}{\tau} = \frac{V_0 t^2}{2\tau}$$

$$\left(\frac{A}{\tau}\right)t^2 + \left(2A + \frac{B}{\tau}\right)t + \left(B + \frac{C}{\tau}\right) = \left(\frac{V_0}{2\tau}\right)t^2 + 0t + 0$$

$\frac{A}{\tau} = \frac{V_0}{2\tau}$ & $2A + \frac{B}{\tau} = 0$ & $B + \frac{C}{\tau} = 0$

$A = \frac{V_0}{2}$

$B = -2\tau A$

$B = -2\tau \frac{V_0}{2}$

$B = -\tau V_0$

$C = -\tau B$

$C = -\tau(-\tau V_0)$

$C = \tau^2 V_0$

∴ The general solution becomes:

$$v_C(t) = K_1 e^{-\frac{t}{\tau}} + \frac{V_0}{2} t^2 + (-\tau V_0)t + \tau^2 V_0$$

using the i.c. $v_C(0) = 0$ we get:

$$0 = K_1 \cancel{e^{-\frac{t}{\tau}}}^{\,1} + \frac{V_0}{2}\cancel{(0)}^{2\,0} - \tau V_0 \cancel{(0)}^{\,0} + \tau^2 V_0$$

↳ $K_1 = -\tau^2 V_0$

and the solution becomes:

$$v_C(t) = \tau^2 V_0 e^{-\frac{t}{\tau}} + \frac{V_0}{2} t^2 - \tau V_0 t + \tau^2 V_0$$

So we see w/ LTI systems the Linear part allows us to take an already known solution and if we do something to the input we do the same to the O/P and find a new solution. something linear is like multiplication/division/integration/derivatives.

No need to go through the whole process twice once you prove this to yourself when this works.