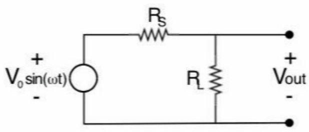
1. [40 points total] For the circuit shown below the voltage source is sinusoidal with an amplitude V_0 and frequency w. R_S is **time varying** such that:

$R_s = R_0 + R_1\sin(w_1 t)$ where $R_0 > R_1$ (so that R_s is always positive)



a. [15 points] Write down a relationship for V_{out} in terms of V_{in}, R_L, R_0, and R_1.

b. [15 points] Further assume that $R_0 - R_1 >> R_L$, and $R_0 >> R_1$. Using these relationships, simplify the result from part a. and obtain a solution to first order in R_1. [Hint: this means you can't ignore R_1 altogether; you may find the Taylor series for $1/(1 + x)$ for small x useful: $1/(1+x) = (1-x)$].

c. [10 points] The result from part b. should have a term of the form sin(wt) sin(w_1t). Expand this to obtain the different frequency components of V_{out} [hint: see appendix F attached]

time varying series resistor

$R_S = R_0 + R_1 \sin(\omega_1 t)$

where $R_0 > R_1$ due to resistance cannot be negative.

Using ohm's Law, KVL, & KCL we get:

KVL

$V_{in} = V_{R_S} + V_{R_L}$ where $V_{R_x} = R_x \cdot I_{R_x}$ Ohm's Law

$V_{out} = V_{R_L}$

KCL

$I_{in} = I_{R_S} = I_{R_L}$

R, I_{R_x}, + V_{R_x} -

$\therefore$

$V_{in} = R_S \cdot \cancel{I_{R_S}} + V_{out}$ I_{in} also $I_{in} = \frac{V_{in}}{R_S + R_L}$

$\therefore V_{in} = R_S\left(\frac{V_{in}}{R_S+R_L}\right) + V_{out}$

$\therefore V_{out} = \left(1 - \frac{R_S}{R_S+R_L}\right)V_{in} = \frac{R_L}{R_S+R_L} \cdot V_{in}$

Just do Voltage Divider eqn if current is the same going through all the components.

a)

$V_{out} = \frac{R_L}{R_S+R_L} \cdot V_{in}$ or $\frac{R_L \cdot V_{in}}{R_0 + R_1\sin(\omega_1 t) + R_L}$

using the assumptions given

$R_0 >> R_1$ & $R_{S,min} >> R_L$

we can generalize our eqn:

$V_{out} \approx \frac{R_L \cdot V_{in}}{R_S} \rightarrow V_o \approx \frac{R_L V_{in}}{R_0 + R_1\sin(\omega_1 t)}$

$\therefore V_{out} \approx R_L V_{in} \cdot \left(\frac{1}{R_0 + R_1 \sin(\omega_1 t)}\right)$

pulling out the R_0 on the denominator we get

$V_{out} \approx \frac{R_L}{R_0} \cdot V_{in}\left(\frac{1}{1 + \frac{R_1}{R_0}\cdot\sin(\omega_1 t)}\right)$

using Taylor series expansion you would find for the function $\frac{1}{1+x}$ would be approximately equal to $1-x$ for values of x being very small $x << 1$

since $R_0 >> R_1$ this condition is satisfied

$\therefore \frac{1}{1+\frac{R_1}{R_0}\sin(\omega_1 t)} \approx 1 - \frac{R_1}{R_0}\sin(\omega_1 t)$

$\hookrightarrow V_{out} \approx \frac{R_L}{R_0} \cdot V_{in} \cdot \left(1 - \frac{R_1}{R_0}\sin(\omega_1 t)\right)$

b)

$V_{out} \approx \left(\frac{R_L}{R_0} - \frac{R_L R_1}{R_0^2}\sin(\omega_1 t)\right)V_{in}$

plugging in $V_{in} = V_0 \sin(\omega t)$

$V_{out} \approx \left(\frac{R_L}{R_0}\sin(\omega t) - \frac{R_L R_1}{R_0^2}\sin(\omega t)\sin(\omega_1 t)\right)V_0$

Need to see Appendix F, but I'm assuming it has

$\sin(a)\cdot\sin(b) = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

$V_{out} \approx \left[\frac{R_L}{R_0}\cdot\sin(\omega t) - \frac{R_L R_1}{2R_0^2}\left[\cos(\omega_1 t - \omega t) - \cos(\omega_1 t + \omega t)\right]\right]V_0$

$V_{out} \approx \left[\frac{R_L}{R_0}\sin(\omega t) - \frac{R_L R_1}{2R_0^2}\left[\cos((\omega_1 - \omega)t) - \cos((\omega_1+\omega)t)\right]\right]V_0$

$\therefore$ we look @ the terms multiplied by t in sinusoids to find freq. components

$\cos(\omega t)$ or $\sin(\omega t)$ where $\omega = 2\pi f$

radial frequency, frequency

$\therefore$

c)

freq components are

$\text{Freq}[V_0] = \omega + (\omega_1 - \omega) + (\omega_1 + \omega)$