Inti Sets

locked

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Statistics

Difficulty: Hard
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The Inti set of a number N, according to definition, is formed by all positive numbers x, where x and N are coprime. Then the problem asks for the sum of all numbers in range [A, B], that they are coprime with N.

Note that the answer is equivalent to finding the sum of all numbers in range [A, B] and subtract the sum of all numbers in range [A, B] that they are not coprime with N, or as an equation:

SumCoprimeN(A, B) = Sum(A, B) - SumNoCoprimeN(A, B)

One can use a formula for the sum of numbers in range [A, B], but you may end up with integer overflow, and therefore, it would be important to take the result of all operations MOD 1000000007.

Now, the problem is to find the sum of numbers in range[A, B], that are not coprime with N (a number x is not coprime with N if they share a prime factor).

For that, we can factor N in prime factors:

$N = 2^{e_1} 3^{e_2} 5^{e_3} \dots f_n^{e_n}$, where n is the number of prime factors of N.

After that we can define sets S_i, where $1 \le i \le n$. Each set S_i is formed by all numbers in the range [A, B], that share the prime factor f_i with N.

$S_i = \{ \dots, k f_i , (k + 1) f_i , (k + 2) f_i , \dots\}$, where k is a positive integer

We can find the sum of the elements by partial results. In general for a function F, $F(a \le x \le b) = F(b) - F(a-1)$. Using this approach:

$S_i = f_i (B/f_i) (B/f_i + 1) / 2 - f_i ((A - 1)/f_i) ((A - 1)/f_i + 1)/2$

Finally for getting the sum of no coprime numbers, we need to find the sum of the elements in: $S_1 \cup S_2 \cup S_3 \cup \dots \cup S_n$.

We need to note that this sets are not disjoint, so we need to apply [the inclusion-exclusion principle](https://en.wikipedia.org/wiki/Inclusion%E2%80%93exclusion_principle)

For example, when $N = 12 = 2^2 3^1$, so:

S_1: numbers multiples of 2 in [A, B]

S_2: numbers multiples of 3 in [A, B]

$S_1 \cap S_2$: numbers multiples of 6 in range [A, B]

$\text{Sum}(S_1 \cup S_2) = \text{Sum}(S_1) + \text{Sum}(S_2) - \text{Sum}(S_1 \cap S_2)$