

Translational Alignment

The simplest way to establish an alignment between two images or image patches is to shift one image relative to the other

$$E_{\text{SSD}}(\mathbf{u}) = \sum_i [I_1(\mathbf{x}_i + \mathbf{u}) - I_0(\mathbf{x}_i)]^2 = \sum_i e_i^2,$$

Hierarchical Motion Estimation

an image pyramid is constructed and a search over a smaller number of discrete pixel is first performed at coarser level. The motion estimate from one level of the pyramid is then used to initialize a smaller local search at the next finer level. Alternatively, several good solutions from the coarse level can be used to initialize the fine-level search. While this is not guaranteed to produce the same result as a full search, it usually works very well and is much faster.

Fourier Based Alignment

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$$\mathcal{F}\{I_1(\mathbf{x} + \mathbf{u})\} = \mathcal{F}\{I_1(\mathbf{x})\} e^{-j\mathbf{u} \cdot \boldsymbol{\omega}} = \mathcal{I}_1(\boldsymbol{\omega}) e^{-j\mathbf{u} \cdot \boldsymbol{\omega}},$$

vector-valued angular frequency of the Fourier transform
we denote the Fourier transform of $I_1(\mathbf{x})$ as $\mathcal{I}_1(\boldsymbol{\omega}) = \mathcal{F}\{I_1(\mathbf{x})\}$ to denote the Fourier transform

Incremental Refinement

To obtain better sub-pixel estimates, This Technique is used. In general, image stabilization and stitching applications require much higher accuracies to obtain acceptable result. This is based on a Taylor series expansion of the image function.

Spline Based Motion

> Most image motion is too complicated to be captured by parametric motion model (low dimensional model) Represent the motion field as a two-dimensional spline controlled by a smaller number of control vertices.

$$u_i = \sum_j \hat{u}_j B_j(x_i) = \sum_j \hat{u}_j w_{i,j}$$

where B is basis functions and only non-zero over finite support interval and

w_{ij} are the weights to emphasize the motion field region

Spline Based Motion \Rightarrow Advantage

>> Coarse-to-fine Strategy reduces computational time.

>> Excel at representing smooth elastic deformation fields.

>> That is why it is used in Medical Imaging.