AMMM - Course project

Master in Research and Innovation in Informatics

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11 de diciembre de 2022

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Problem Statement

Main requirements

1. Each contestant will play exactly once against each of the other contestants.
2. Each round will consist of $\frac{n-1}{2}$ matches.
3. Players will play 50 % of their games as white, 50 % will be played as black.

Subtle requirements

- A player can only play up to 1 game per round
- A player can't play against himself

Problem Statement: Inputs & Outputs

Inputs

- Number of contestants, n. Has to be odd
- Matrix of points per day per player, $p_{n \times n}$

Outputs

- Schedule with the set of pairings $\{\{r_1, p_i, p_j\}, \ldots, \{r_n, p_k, p_h\}\}$ that maximizes total score. Ensured to be optimal if it's obtained through the ILP.

Problem Statement: Definitions

In order to specify the constraints, we need to specify the sets and variables we're going to work with:

- $M(x, y)$ matches played among x and y (1)
- $R(r)$ matches played at round r (2)
- $W(p)$ matches played by player p as white (3)
- $B(p)$ matches played by player p as black (3)
- $G(p, r)$ games played by p at round r (4)
- $F(r)$ free players at round r (5)

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4. Players can play up to 1 match per round
5. Objective function

Integer Linear Programming Model: Variables

Every set will be constructed from a boolean multidimensional array. $m[w][b][r]$ will be 1 whenever player w plays player b in round r, and 0 otherwise.

Set constructions

$$
\begin{aligned}
M(x,y) &= \{\{x,y,r\}, & r \in [1, Rounds] \mid \mathsf{m}[x][y][r] = 1 \vee \mathsf{m}[y][x][r] = 1 & \} \\
F(r) &= \{p, & p \in [1,n] \mid \mathsf{m}[p][o][r] = 0 \wedge \mathsf{m}[o][p][r] = 0 \quad \forall o \in [1,n] & \} \\
W(p) &= \{\{p,b,r\}, & r \in [1, Rounds], b \in [1,n] \mid \mathsf{m}[p][b][r] = 1 & \} \\
B(p) &= \{\{w,p,r\}, & r \in [1, Rounds], w \in [1,n] \mid \mathsf{m}[w][p][r] = 1 & \} \\
R(r) &= \{\{w,b,r\}, & w,b \in [1,n] \mid \mathsf{m}[w][b][r] = 1 & \} \\
G(p,r) &= \{\{o,p,r\}, & o \in [1,n] \mid \mathsf{m}[p][o][r] = 1 \vee \mathsf{m}[o][p][r] = 1 & \}
\end{aligned}
$$

Integer Linear Programming Model: Constraints

$$|M(x,y)| = 1 \quad \forall x, y \in P \mid x \neq y \tag{1}$$

$$|R(r)| = \frac{n-1}{2} \quad \forall r \in [1,\ \text{Rounds}] \tag{2}$$

$$|W(p)| = \frac{n-1}{2} \quad \forall p \in P \tag{3}$$

$$|G(p,r)| \leq 1 \quad \forall p \in P, r \in [1,\ \text{Rounds}] \tag{4}$$

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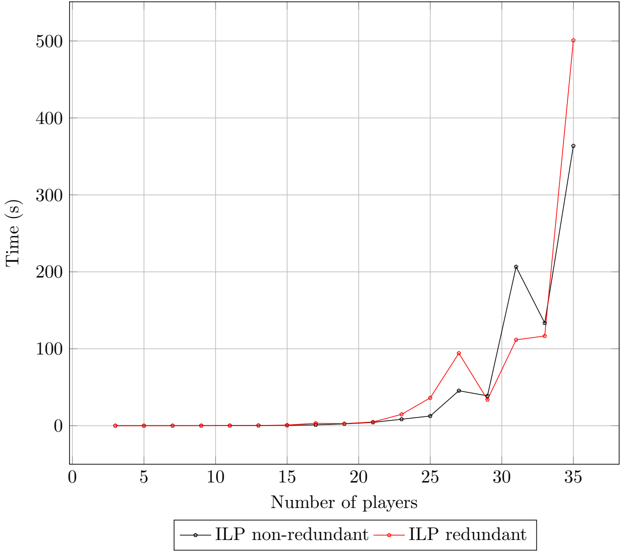
Integer Linear Programming Model: Redundant constraints

Redundant constraints might appear to make the model faster but they seem make it slower in the long run

$$|M(x,x)| = 0 \quad \forall x \in P$$

$$|B(p)| = \frac{n-1}{2} \quad \forall p \in P$$



Greedy cost function

$$q(c, day) = c.points\_per\_day[day]$$

Algorithm Greedy algorithm

```
Players ← Set of Players
rests ← {}
for day in 0..days do
    playersToRest ← filter Players(p) | p.hasNotRested
    sortedPlayers ← sort playersToRest(p) by q(p,day) (DESC)
    rests[day] ← sortedPlayers.first()
```

Algorithm Local Search

```
for i in 0..days do
    best_swap_points ← 0
    best_swap ← i
    for j in 0..days do
        change = EvaluateRestSwap(i,j)
        if change > best_swap_points then
            best_swap_points ← change
            best_swap ← j
    rests[i] ↔ rests[best_swap]
```

Algorithm constructRCL(day)

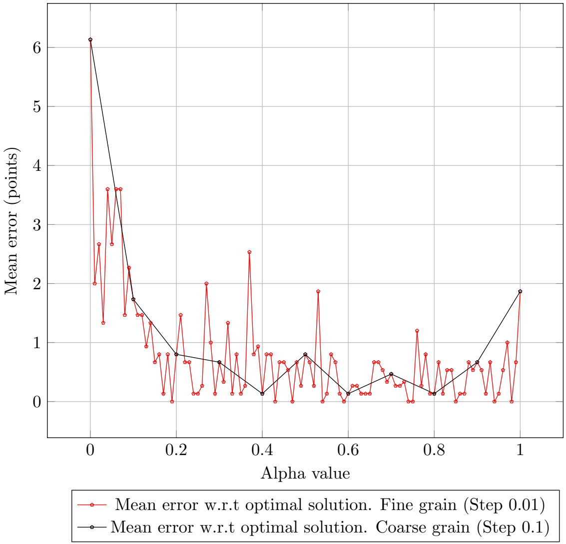
$q_{max} \leftarrow$ sortedPlayers.first().points[d]
$q_{min} \leftarrow$ sortedPlayers.last().points[d]
$RCL_{max} \leftarrow \{p \in sortedPlayers \mid p.points[d] >= q_{max} - \alpha \cdot (q_{max} - q_{min})\}$

Algorithm GRASP

rests $\leftarrow$ {}
for day in 0..days **do**
    playersToRest $\leftarrow$ filter Players(p) | p.hasNotRested
    sortedPlayers $\leftarrow$ sort playersToRest(p) by q(p,day) (DESC)
    RCL $\leftarrow$ constructRCL(day)
    select p $\in$ RCL randomly
    rests[day] $\leftarrow$ p

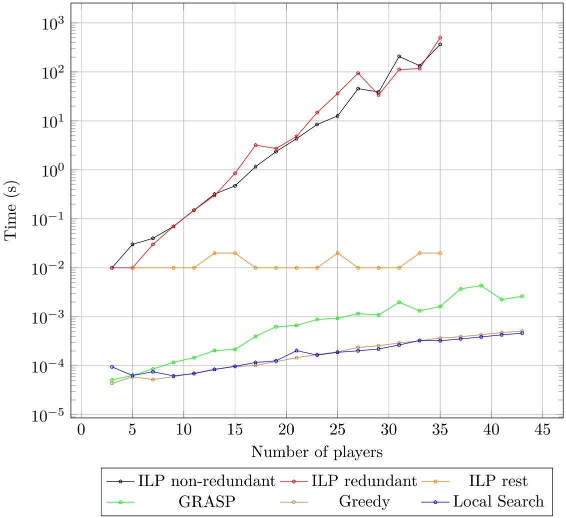
GRASP: Parameter tuning

- Figure shows the arithmetic mean error with respect to the optimal solution for each of the instances.
- We keep the smallest alpha that gives the minimum mean error.



Results: Time

- Greedy and Local Search need approximately same time to reach the solution. They're fastest but their quality of the solution is too low.
- GRASP sacrifices some runtime performance to improve the quality of the solution.
- ILPs obtain the optimal solution at cost of being several orders of magnitude slower due to the complexity of creating valid pairings.
- ILP rest obtains the optimal solution just computing the rest day for each player. It is not much slower than GRASP and for large instances it can be even faster.



Results: Quality of solutions

- Greedy offers the worst solution for every instance
- Greedy + Local Search improves the quality of the solution taking practically the same time as Greedy.
- GRASP commonly reaches the optimal solution, having a good improvement over Local Search.