SLCO4A - Aula Prática 10

Sumário

Transformada de Fourier

A transformada de Fourier é o caminho para expressar no domínio da frequência um sinal que é dado no domínio do tempo.

A transformada de Fourier é denotado pelo símbolo $\mathfrak{I}\{\ \}$, ou seja, pode-se escrever

$$X(j\omega) = X(\Omega) = \mathfrak{I}\{x(t)\}$$

A expressão matemática da transformada de Fourier é dada por:

$$X(j\omega) = \mathfrak{I}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\mathrm{dt}$$

E para retornar para o domínio do tempo a partir do domínio da frequência aplica-se a transformada inversa de Fourier, dada por

$$x(t) = \mathfrak{I}^{-1}\{X(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\mathrm{dt}$$

Comandos fourier e ifourier

Exercício 1: Compute a transformada de Fourier da função $x(t) = e^{-t^2}$.

```
syms t w
x=exp(-t^2)
```

x = e^{-t^2}

```
fourier(x)
```

ans =

$$\sqrt{\pi}\ \mathrm{e}^{-\frac{w^2}{4}}$$

```
int(x*exp(-j*w*t), t, -inf, inf)
```

ans =

$$\sqrt{\pi}\ \mathrm{e}^{-\frac{w^2}{4}}$$

Exercício 2: Compute a transformada inversa de Fourier da função $X(j\omega) = \dfrac{1}{1+j\omega}$.

```
X=1/(1+j*w);
ifourier(X)
```

ans =

$$\frac{\mathrm{e}^{-x}\ (\mathrm{sign}(x)+1)}{2}$$

```
syms t w n
X=1/(1+j*w);
ifourier(X, t)
```

ans =

$$\frac{\mathrm{e}^{-t}\ (\mathrm{sign}(t)+1)}{2}$$

Exercício 3: Compute a transformada de Fourier da função $x(t) = 1$.

```
sym w
```

ans = w

```
x=1;
fourier(x, w)
```

ans = $2\,\pi\,\delta(w)$

Note que o comando `fourier` deve ser executado como `fourier(x,w).` Usando esta sintaxe é a execução ótima que permite o cômputo de funções constantes e de outras funções.

Exercício 4: Compute a transformada de Fourier (ou espectro) do sinal $x(t) = e^{-t}u(t)$ e compare o resultado com o exercício 2.

```
syms t w
x=exp(-t)*heaviside(t);
```

Note que as funções $x(t) = e^{-\mathrm{at}}u(t) \leftrightarrow X(j\omega) = \dfrac{1}{1+j\omega}$ são um par transformado de Fourier.

Pares transformados

Exercício 5: Considere as seguintes funções obtenha os pares transformados de Fourier.

a) $x(t) = \delta(t)$

```
syms t w
x=dirac(t);
fourier(x, w)
```

ans = 1

b) $x(t) = 1$

```
x=1;
fourier(x, w)
```

ans = $2\pi\,\delta(w)$

c) $X(j\omega) = \dfrac{1}{j\omega} + \pi\,\delta(\omega)$

```
X=1/(j*w)+pi*dirac(w)
```

X =

$$\pi\,\delta(w) - \frac{\mathrm{i}}{w}$$

```
x=ifourier(X,t)
```

x =

$$\frac{\pi + \pi\,\mathrm{sign}(t)}{2\,\pi}$$

```
simplify(x)
```

ans =

$$\frac{\mathrm{sign}(t)}{2} + \frac{1}{2}$$

d) $x(t) = u(t)$

```
x=heaviside(t)
```

x = $\mathrm{heaviside}(t)$

```
fourier(x, w)
```

ans =

$$\pi\,\delta(w) - \frac{\mathrm{i}}{w}$$

e) $x(t) = \delta(t - t_0)$

```
syms t0
x=dirac(t-t0)
```

x = $\delta(t - t_0)$

```
fourier(x, w)
```

ans = $e^{-t_0 w i}$

f) $X(j\omega) = 2\pi\,\delta(\omega - \omega_0)$

```
syms w0
X=2*pi*dirac(w-w0);
ifourier(X, t)
```

ans = $e^{t w_0 i}$

g) $X(j\omega) = \pi\,(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$

```
X=pi*(dirac(w-w0)+dirac(w+w0))
```

X = $\pi\,(\delta(w - w_0) + \delta(w + w_0))$

```
x=ifourier(X,t)
```

x =

$$\frac{e^{-t w_0 i}}{2} + \frac{e^{t w_0 i}}{2}$$

```
simplify(x)
```

ans = $\cos(t\, w_0)$

h) $X(j\omega) = \frac{\pi}{j}\,(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$

```
X=pi/j*(dirac(w-w0)-dirac(w+w0))
```

X = $-\pi\,(\delta(w - w_0) - \delta(w + w_0))\,i$

```
x=ifourier(X, t)
```

x =

$$\frac{e^{-t w_0 i}\, i}{2} - \frac{e^{t w_0 i}\, i}{2}$$

```
simplify(x)
```

ans = $\sin(t\, w_0)$

j) $x(t) = te^{-at}u(t)$

```
syms a
```

```
x=t*exp(-a*t)*heaviside(t);
X=fourier(x, w)
```

X =

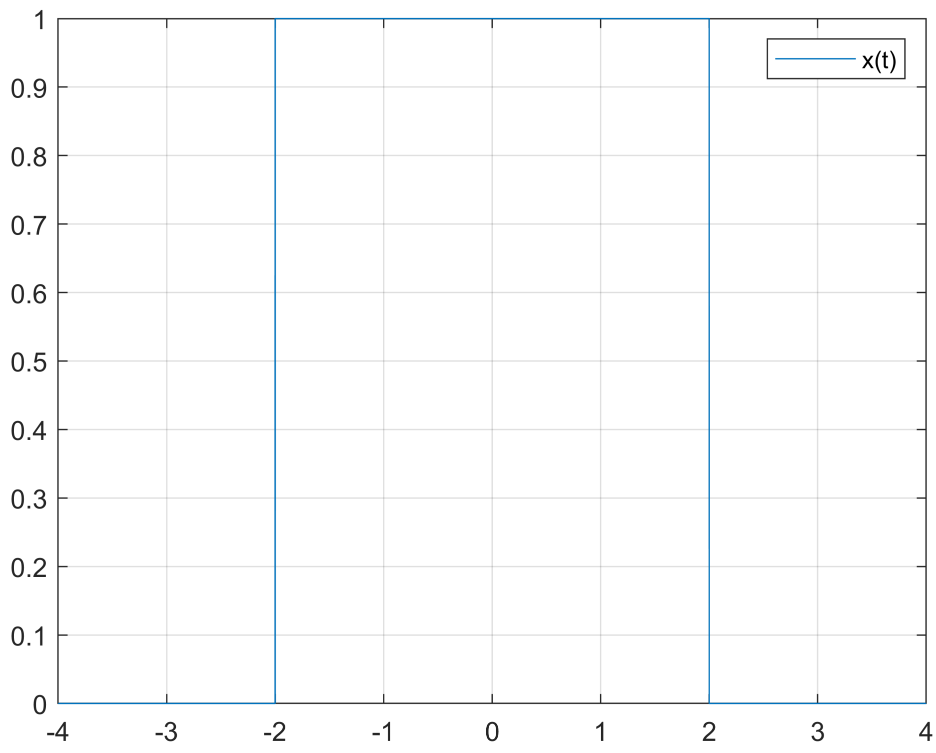
$$-\left(\frac{\mathrm{sign}(\mathrm{real}(a))}{2}-\frac{1}{2}\right)\ \mathrm{fourier}(t\,\mathrm{e}^{-a\,t}, t, w)+\frac{1}{(a+w\,\mathrm{i})^2}$$

```
simplify(x)
```

ans = $t\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)$

k) $\mathrm{pT}(t)=\begin{cases}1 & |t|<\frac{T}{2}\\ 0 & |t|>\frac{T}{2}\end{cases}\leftrightarrow T\frac{\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}\ ,\ T=4$

```
syms t w T
x=heaviside(t+T/2)-heaviside(t-T/2);
xx=subs(x, T, 4);
fplot(xx, [-4 4])
grid on
legend('x(t)')
```




```
X1=fourier(x, w)
```

X1 =

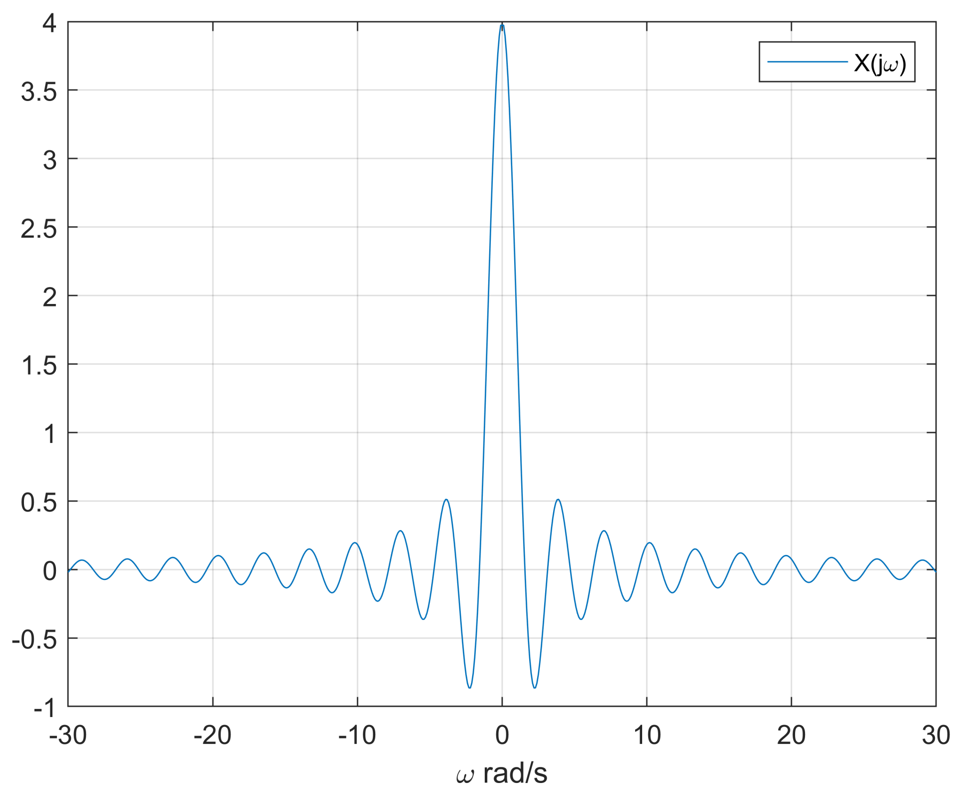
$$\frac{\sin\left(\frac{T\,w}{2}\right)+\cos\left(\frac{T\,w}{2}\right)\,\mathrm{i}}{w}-\frac{-\sin\left(\frac{T\,w}{2}\right)+\cos\left(\frac{T\,w}{2}\right)\,\mathrm{i}}{w}$$

```
simplify(X1)
```

ans =

$$\frac{2\sin\left(\frac{T\,w}{2}\right)}{w}$$

```
figure
X2=T*(sin(w*T/2)/(w*T/2));
ww=[-30:0.1:-0.1 0.1:0.1:30];
X=subs(X1, w, ww);
X=subs(X, T, 4);
plot(ww, X)
grid on
xlabel('\omega rad/s')
legend('X(j\omega)')
```



Propriedades da transformada de Fourier

Linearidade

$\mathfrak{F}\,\{a_1x_1(t)+a_2x_2(t)\}=a_1X_1(j\omega)+a_2X_2(j\omega)$

Exercício 6: Considere os seguintes sinais $x_1(t) = e^{-3t}u(t)$, $x_2(t) = \text{te}^{-t}u(t)$, $a_1 = 3$ e $a_2 = 4$.

```
syms w t
a1=3; a2=4;
x1=exp(-3*t)*heaviside(t);
x2=t*exp(-t)*heaviside(t);
Le=a1*x1+a2*x2;
Lef=fourier(Le, w)
```

Lef =

$$\frac{4}{(1+w\,\mathrm{i})^2}+\frac{3}{3+w\,\mathrm{i}}$$

```
X1=fourier(x1, w);
X2=fourier(x2, w);
Right=a1*X1+a2*X2
```

Right =

$$\frac{4}{(1+w\,\mathrm{i})^2}+\frac{3}{3+w\,\mathrm{i}}$$

Deslocamento temporal

$\mathfrak{F}\{x(t-t_0)\} = e^{-j\omega t_0}X(j\omega)$

Exercício 7: Considere o sinal $x(t) = \cos(t)$, $t_0 = 2$.

```
syms t w
x=cos(t);
t0=2;
xt0=cos(t-t0);
Left=fourier(xt0);
simplify(Left)
```

ans = $\pi\,\delta(w-1)\,\mathrm{e}^{-2\,\mathrm{i}} + \pi\,\delta(w+1)\,\mathrm{e}^{2\,\mathrm{i}}$

```
X=fourier(x, w)
```

X = $\pi\,(\delta(w-1)+\delta(w+1))$

```
Right=exp(-j*w*t0)*X
```

Right = $\pi\,\mathrm{e}^{-2\,w\,\mathrm{i}}\,(\delta(w-1)+\delta(w+1))$

```
simplify(Right)
```

ans = $\pi\,\delta(w-1)\,\mathrm{e}^{-2\,\mathrm{i}} + \pi\,\delta(w+1)\,\mathrm{e}^{2\,\mathrm{i}}$

Deslocamento em frequência

$\mathfrak{I}\left\{e^{j\omega_0 t}x(t)\right\} = X(j(\omega - \omega_0))$

Exercício 8: Considere o sinal $x(t) = t^3$, $\omega_0 = 3$.

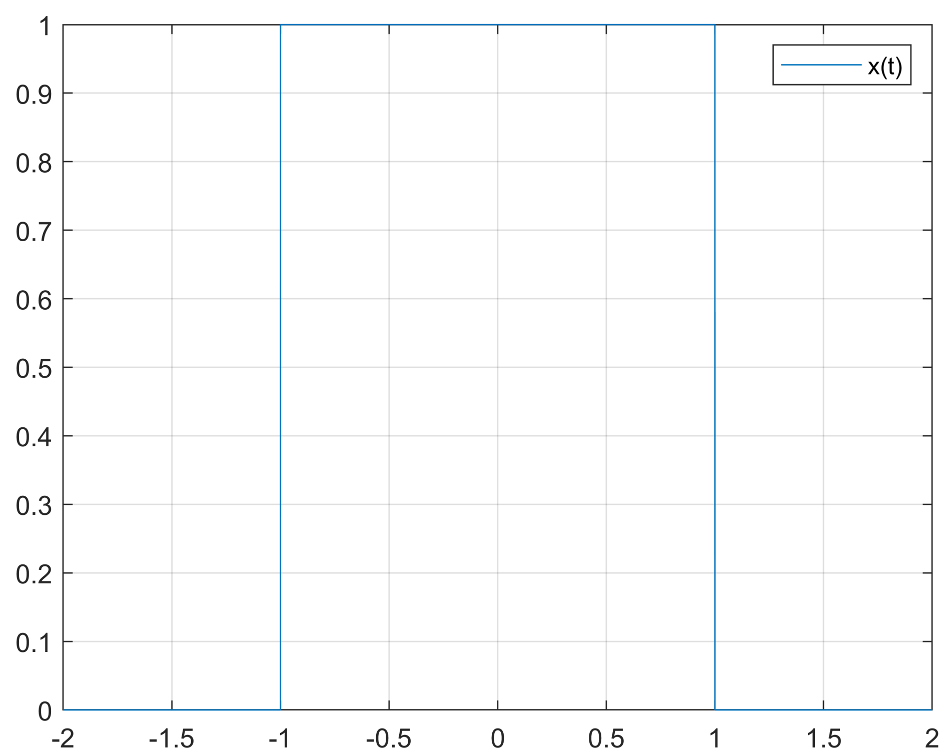
Escalonamento temporal e na frequência

$\mathfrak{I}\left\{x(\text{bt})\right\} = \frac{1}{|b|}X\left(\frac{j\omega}{b}\right)$

$\mathfrak{I}\left\{\frac{1}{|b|}x\left(\frac{t}{b}\right)\right\} = X(\text{bj}\omega)$

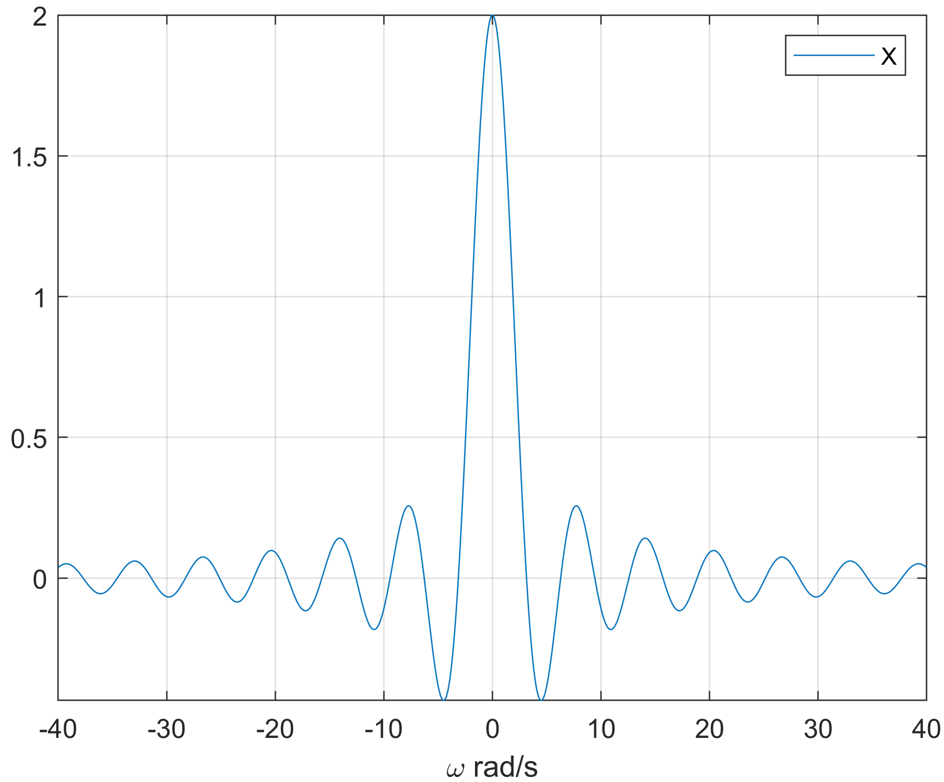
Exercício 9: Considere o sinal $x(t) = u(t+1) - u(t-1)$, $b = 3$.

```
syms t w
b=3;
x=heaviside(t+1)-heaviside(t-1);
fplot(x, [-2 2])
grid on
legend('x(t)')
```

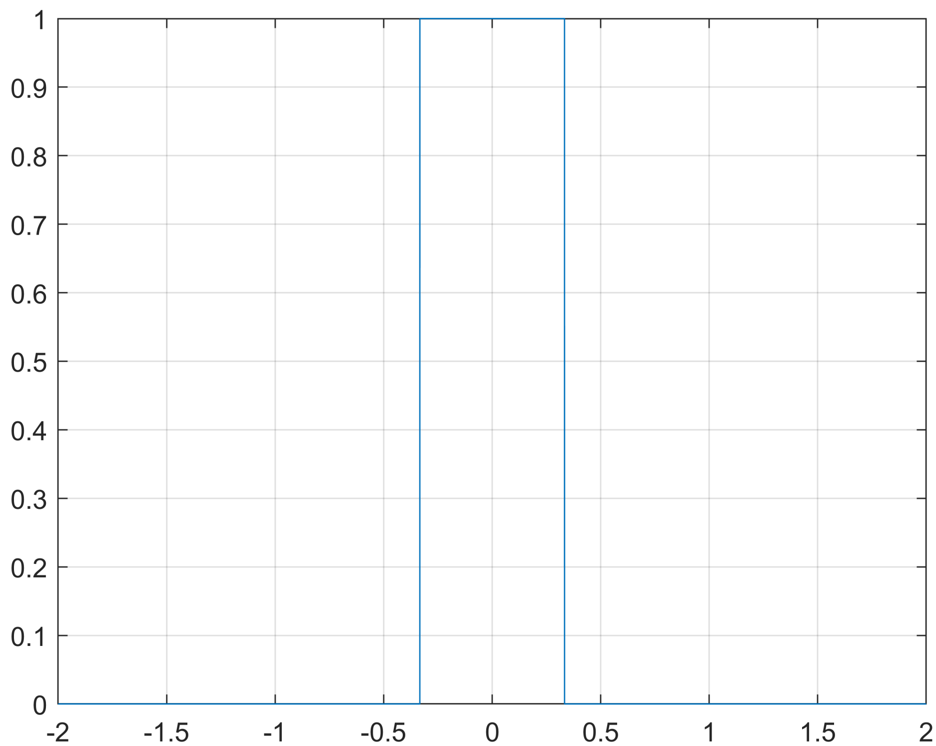



```
%F{x(t)}
X=fourier(x, w);
```

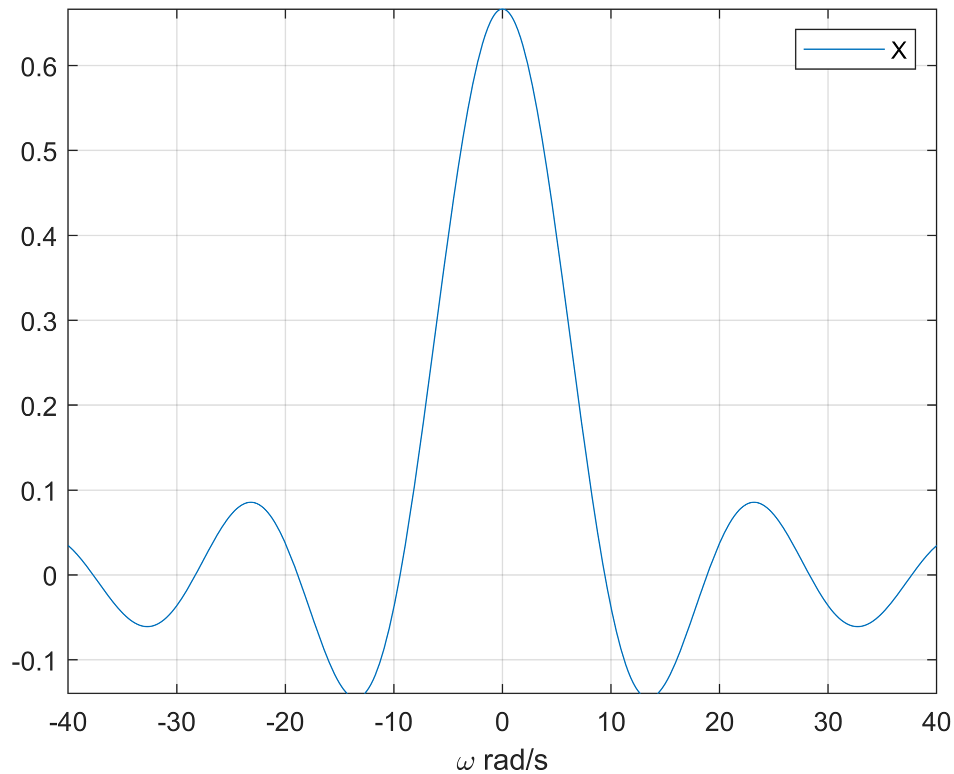
```
fplot(X, [-40 40])
grid on
legend('X')
xlabel('\omega rad/s')
```




```
%F{x(bt)}
xb=subs(x, t, b*t);
figure
fplot(xb, [-2 2])
grid on
```




```
Xb=fourier(xb, w);
fplot(Xb, [-40 40])
grid on
legend('X')
xlabel('\omega rad/s')
```



Conjugação

$\Im\left\{x^*(t)\right\} = X^*(-j\omega)$ e $\Im\left\{x^*(-t)\right\} = X^*(j\omega)$

Exercício 10: Considere o sinal $x(t) = e^{-2t}u(t)$.

```

```

Diferenciação no tempo e na frequência

$\Im\left\{\frac{\mathrm{d}}{\mathrm{d}t}x(t)\right\} = j\omega X(j\omega)$ e $\Im\left\{\mathrm{tx}(t)\right\} = j\frac{\mathrm{d}}{\mathrm{d}\omega}X(j\omega)$

Exercício 11: Considere o sinal $x(t) = e^{-3t}u(t)$, avalie a diferenciação no tempo e na frequência.

```
x=exp(-3*t)*heaviside(t);
%F{x'(t)}
xd=diff(x, t);
Left=fourier(xd, w)
```

Left =

$$1 - \frac{3}{3 + w\,\mathrm{i}}$$

```
simplify(Left)
```

```
ans =
```

$$\frac{w}{w-3\,\mathrm{i}}$$

```
X=fourier(x, w);
Right=j*w*X
```

```
Right =
```

$$\frac{w\,\mathrm{i}}{3+w\,\mathrm{i}}$$

```
simplify(Right)
```

```
ans =
```

$$\frac{w\,\mathrm{i}}{3+w\,\mathrm{i}}$$

```
%F{t x(t)}
Left=fourier(t*x, w)
```

```
Left =
```

$$\frac{1}{(3+w\,\mathrm{i})^2}$$

```
der=diff(X, w);
Right=j*der
```

```
Right =
```

$$\frac{1}{(3+w\,\mathrm{i})^2}$$

Integração

$$\Im\left\{\int_{-\infty}^{t} x(\tau)d\tau\right\} = \frac{1}{j\omega}X(j\omega) + \pi\, X(0)\delta\,(\omega)$$

Exercício 12: Considere o sinal $x(t) = \delta(t)$.

```
x=heaviside(t);
X=fourier(x, w)
```

```
X =
```

$$\pi\,\delta(w) - \frac{\mathrm{i}}{w}$$

```
syms t tau w
x=dirac(tau)
```

`x =` $\delta(\tau)$

```
%F{int(x(tau))
```

```
integrau=int(x, tau, -inf, t);
```

integrau =

$$\frac{\mathrm{sign}(t)}{2} + \frac{1}{2}$$

```
Left=fourier(integrau)
```

Left =

$$\pi\ \delta(w) - \frac{\mathrm{i}}{w}$$