Nome: David da Silva Galvão, Ra: 24087440

Lista 4 - Cálculo Vetorial

1) Determinar o Gradiente

a) $f(x,y) = \ln(x+2y)$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j = \frac{1}{x+2y} i + \frac{2}{x+2y} j$$

b) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k = \frac{x}{\sqrt{x^2+y^2+z^2}} i + \frac{y}{\sqrt{x^2+y^2+z^2}} j + \frac{z}{\sqrt{x^2+y^2+z^2}} k$$

c) $f(x,y) = x^2 - y$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j = 2x\, i - j$$

2) a) $f(x,y) = x^2+y^2$ (III)

$\nabla f = 2x i + 2y j$

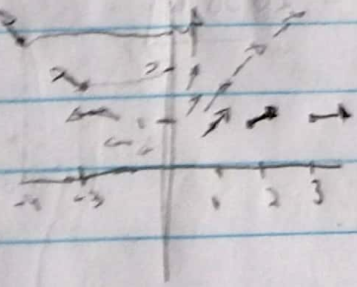
ponto (1,1)

ponto (2,1)

ponto (3,1)

ponto (-3,2)

ponto (-4,4)



b) $f(x,y) = x(x+y)$ (IV)

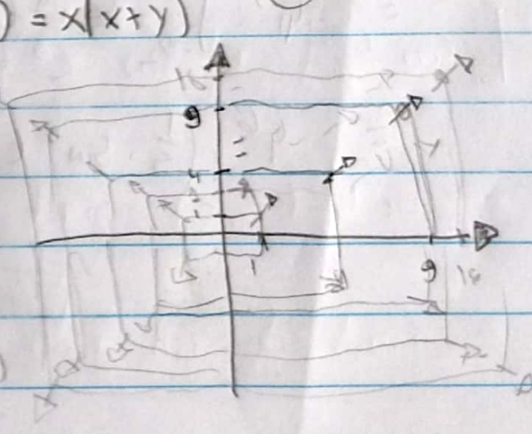
ponto (4,4)

ponto (3,3)

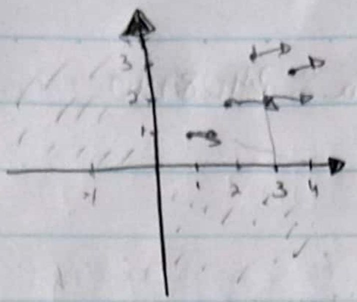
ponto (2,2)

ponto (1,1)

ponto (1,2)

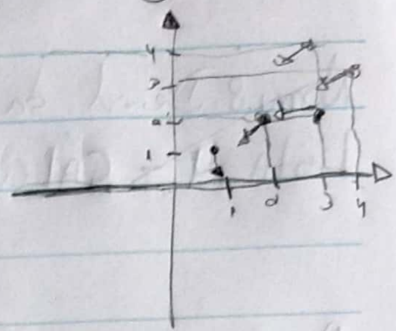


II



S T Q Q S S D

I



c) $f(x,y) = (x+y)^2$

ponto $(1,1) = 4$

ponto $(2,2) = 16$

Ponto $(3,2) = 25$

Ponto $(4,3) = 49$

Ponto $(3,4) = 49$

d) $f(x,y) = \sin\sqrt{x^2+y^2}$

ponto $(1,1) = 0{,}98$

Ponto $(2,2) = 0{,}30$

Ponto $(3,2) = -0{,}44$

Ponto $(4,3) = -0{,}96$

Ponto $(3,4) = -0{,}95$

3- Calcular Integral de linha

a) $\int_C y^3\,ds$, $C: x = t^3, y = t, 0 \leq t \leq 2$

Usando a fórmula temos: $\int_0^2 t^3 \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$$= \int_0^2 t^3 \cdot \sqrt{(3t^2)^2 + (1)^2}\,dt = \int_0^2 t^3 \cdot \sqrt{9t^4+1}\,dt$$

$$= \frac{1}{36} \cdot \frac{2}{3} \cdot (9t^4+1)^{3/2} \Big|_0^2 = \frac{1}{54} \cdot (145^{3/2} - 1)$$

ou $1/54 \cdot (145 \cdot \sqrt{145} - 1)$

b) $\int_C xy^4\,ds$, C é a metade direita do círculo $x^2+y^2=16$

Eq paramétricas

$x = 4\cos t$, $y = 4 \operatorname{sen} t$, para $-\pi/2 \leq t \leq \frac{\pi}{2}$

Temos que

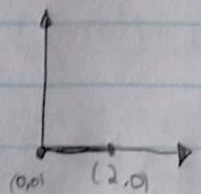
$$\int_C xy^4\,ds = \int_{-\frac{\pi}{2}}^{\pi/2} (4\cos t) \cdot (4 \sin t)^4 \cdot \sqrt{(-4\sin t)^2 + (4\cos t)^2}\,dt$$

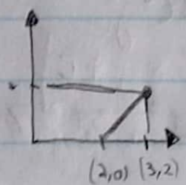
$$= \int_{-\pi/2}^{\pi/2} 4^5 \cdot (\cos t) \cdot (\operatorname{sen}^4 t) \cdot \sqrt{16 \cdot (\sin^2 t + \cos^2 t)}\,dt$$

$$= 4^5 \cdot \int_{-\pi/2}^{\pi/2} (\sin^4 t \cdot \cos t) \cdot 4\,dt$$

$$= 4^6 \cdot \left(\frac{1}{5} \cdot \sin^5 t\right)\Big|_{-\pi/2}^{\pi/2} = \frac{2 \cdot 4^6}{5} = 1638{,}4$$

d) $\int_C xy\,dx + (x-y)\,dy$, C consiste nos segmentos de reta de $(0,0)$ a $(2,0)$ e de $(2,0)$, a $(3,2)$





$v_1 = (2,0)-(0,0) = (2,0)$
$V_2 = (3,2)-(2,0) = (1,2)$

$r(t) = \langle 2t, 0\rangle \Rightarrow \int_0^1 2t.0.2\,dt + \int_0^1 (2t-0).0\,dt = \underline{0}$
$\hookrightarrow 2dt$

$r(t) = \langle 2+t, 2t\rangle \Rightarrow \int_0^1 4t+2t^2\,dt + \int_0^1 (2-t)\,2\,dt$
$\hookrightarrow dt \quad \hookrightarrow 2dt$

$= \int_0^1 4t+2t^2\,dt + \int_0^1 4-2t\,dt$

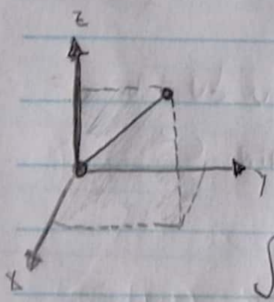
reta 1°: $\begin{cases} x = 0+2t \\ y = 0+0t \end{cases} \Rightarrow 0 \leq t \leq 1$
$x = 2t$; $y=0$

$= \left[2t^2 + \frac{2t^3}{3}\right]_0^1 + \left[4t - t^2\right]_0^1$

$= \frac{8+5}{3} = \frac{17}{3}$

reta 2°: $\begin{cases} x = 2+1t \\ y = 0+2t \end{cases} \Rightarrow \begin{cases} x = 2+t \\ y = 2t \end{cases} \quad 0 \leq t \leq 1$

f) $\int_C xe^{yz}\,ds$, C é o segmento de reta de $(0,0,0)$ a $(1,2,3)$



$v \cdot (1,2,3) - (0,0,0) = (1,2,3)$
$r'(t) = \langle 1,2,3\rangle$
$\|r'(t)\| = \sqrt{1+4+9} = \sqrt{14}$

retas $\begin{cases} x = 0+t \\ y = 0+2t \\ z = 0+3t \end{cases}$

$r(t) = \langle t, 2t, 3t\rangle$

$\int_0^1 t\,e^{6t^2}\sqrt{14}\,dt$
$= \sqrt{14} \cdot \left[\frac{1}{12} e^{6t^2}\right]_0^1$
$= \sqrt{14} \cdot \left[\frac{e^6}{12} - \frac{1}{12}\right] = \sqrt{14} \cdot \frac{1}{12}(e^6 - 1)$

4- Calcular integral de linha

a) $F(x,y) = xy\, i + 3y^2 j$, $r(t) = 11t^4 i + t^3 j$, $0 \le t \le 1$

$F(r(t)) = 11t^7 + 3t^6$ $\quad r'(t) = 44t^3 i + 3t^2 j$

$$\int_c \vec{F}\, dr = \int_0^1 \langle 11t^7, 3t^6 \rangle \cdot \langle 44t^3, 3t^2 \rangle dt$$

$$= \int_0^1 484t^{10} + 9t^8\, dt = \left[\frac{484}{11}t^{11} + t^9 \right]_0^1$$

$$= \frac{484}{11} + 1 = \frac{495}{11} = 45$$

b) $F(x,y,z) = \sin x\, i + \cos y\, j + xz\, K$, $r(t) = t^3 i - t^2 j + t K$, $0 \le t \le 1$

$F(r(t)) = \sin t^3 i + \cos(-t^2) j + t^4 K$ $\quad r'(t) = 3t^2 i - 2t + 1K$

$$\int_c \langle \sin t^3 + \cos(-t^2) + t^4 \rangle \cdot \langle 3t^2, -2t, +1 \rangle dt$$

$$= \int_0^1 3t^2 \cdot \sin t^3 + 2t \cdot \cos(-t^2) + t^4\, dt$$

$t^2 = v$

$dv = -2t\,dt$

$$2\int_0^1 t \cos(-t^2)\, dt$$

$$3\int_0^1 t^2 \cdot \sin t^3 \rightarrow t^3 = u$$

$du = 3t^2\, dt$

$$= 2\int \frac{\cos v}{-2} dv \quad \frac{dv}{-2} = t\,dt$$

$$3\int \frac{\sin u}{3} du = \int \sin u\, du \quad \frac{du}{3} = t^2\, dt$$

$$= -\int \cos v\, dv = \sin v =$$

$-\cos u = -\cos t^3$

$\sin -t^2$

$$\Rightarrow \left[-\cos t^3\right]_0^1 + \left[\sin -t^2\right]_0^1 + \left[\frac{t^5}{5}\right]_0^1$$

$$= -\cos 1 - (\cos 0) - \sin 1 - (-\sin 0) + 1/5$$

$$= -\cos 1 + 1 - \sin 1 + 1/5 = 6/5 - \cos 1 - \sin 1$$

5) Integral de linha $\int_C F \cdot dr$, onde $F(x,y) = e^{x-1}\,i + xy\,j$ e C é $r(t) = t^2 i + t^3 j$, $0 \le t \le 1$

$F(r(t)) = e^{t^2-1}\,i + t^5\,j$ $\qquad$ $r'(t) = 2t\,i + 3t^2\,j$

$\int_0^1 \langle e^{t^2-1}, t^5 \rangle \cdot \langle 2t + 3t^2 \rangle \, dt$

$\int_0^1 2t \cdot e^{t^2-1} + 3t^7 \, dt$ $\qquad$ $t^2-1 = u$

$du = 2t\,dt$

$\int_0^1 2t\, e^{t^2-1}\, dt + \int_0^1 3t^7\, dt = \int_0^1 e^u\, du + \int_0^1 3t^7\, dt$

$= [e^u]_0^1 + [3t^8/8]_0^1 = [e^{t^2-1}]_0^1 + [3t^8/8]_0^1$

$= -e^{-1} + 3/8 = -e^0 + \frac{[illegible]}{8} = \frac{11}{8} - 1/e$

6) Determinar se F é um campo vetorial conservativo, se for determine uma função f tal que $F = \nabla f$.

a) $F(x,y) = \underbrace{(2x - 3y)}_{M}\,i + \underbrace{(-3x + 4y - 8)}_{N}\,j$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow 0 - 3 = -3 + 0 + 0$ ✓ conservativo

$\frac{\partial f}{\partial x} = (2x - 3y)$

$(M, N) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

Integrando $\left(\frac{\partial f}{\partial x}\right)$ em relação a X temos

$\int 2x - 3y\, dx + g(y)$ (2) $\qquad$ $F(x,y) = x^2 - 3xy + g(y)$ (3)

Derivando (3) em relação a Y

$\partial f/\partial y = -3x + g'(y)$ $\qquad$ $\partial f/\partial y = -3x + 4y - 8$ (5)

Igualando (4) (5): $\cancel{-3x} + g'(y) = \cancel{-3x} + 4y - 8 \Rightarrow g'(y) = 4y - 8$

___/___/___

$g(y) = \int 4y - 8\, dy$

$g(y) = 2y^2 - 8y + C$ ∴

Substituindo em (3): $\boxed{f(x,y) = x^2 - 3xy + 2y^2 - 8y + C}$ ✓

6) c) $F(x,y) = (\overset{m}{ye^x + \sin y})\, i + (\overset{n}{e^x + x\cos y})\, j$

$\frac{\partial m}{\partial y} = \frac{\partial n}{\partial x} \Rightarrow e^x + \cos y = e^x + \cos y$ conservativo ✓

$(M,N) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ $\frac{\partial f}{\partial x} = M = y e^x + \sin y$ (1)

Integrando em relação a x:

$\int y e^x + \sin y\, dx = ye^x + x \sin y + g(y)$ (3)

Derivando (3) em relação a y = $\frac{\partial f}{\partial y} = e^x + x\cos y + g'(y)$

$\frac{\partial f}{\partial y} = N = e^x + x\cos y$

Igualando (4)

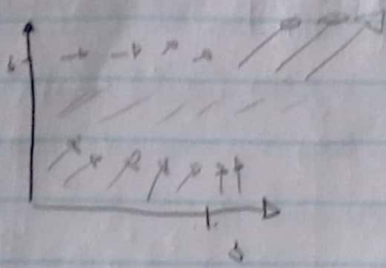
$e^x + x\cos y + g'(y) = e^x + x\cos y$

$g'(y) = 0$ (5)

$g(y) = \int 0\, dy \Rightarrow g(y) = 0$

Substituindo em (3) = $f(x,y) = \underline{y e^x + x \sin y}$ //

7)



$\Rightarrow f(x,y) = \langle 2xy, x^2 \rangle$ e 3 curvas começam em (1,2) e terminam em (3,2)

$\vec{F} = \vec{F}_x\, i + \vec{F}_y\, j \rightarrow \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \Rightarrow \frac{\partial F_x}{\partial y} = \frac{\partial(2xy)}{\partial y} = 2x$

$\int \frac{\partial F_x}{\partial x} = \int (2xy)\, dx = \int x^2 y + g(y) \Rightarrow \frac{\partial f}{\partial y} = x^2 + g(y)$

$F_y = x^2 \quad , \quad x^2 = x^2 + g'(y) \rightarrow g'(y) = 0$

$\int g'(y)\, dy = \int 0\, dy = C \quad , \quad g(y) = C \quad , \quad f = x^2 y + C$

$f(3,2) = 3^2 \cdot 2 + C = 18 + C \; / \; f(1,2) = 1^2 \cdot 2 + C = 2 + C$

$f(3,2) = 18 - 2$

$f(3,2) = 16 \,//$

8) Determine a função f tal que $F = \nabla f$ e calcule $\int_C F \cdot dr$ sobre a curva C dada.

$$b)\; F(x,y,z) = \overset{M}{yz}\, i + \overset{N}{xz}\, j + \overset{D}{(xy + 2z)}\, k$$

C é o segmento de reta de (1,0,-2) a (4,6,3)

$\frac{\partial D}{\partial y} = \frac{\partial N}{\partial z} \Rightarrow x = x$

$\frac{\partial M}{\partial z} = \frac{\partial D}{\partial x} \Rightarrow y = y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow z = z$

Conservativo ✓

$(M, N, D) \; / \; \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$\frac{\partial f}{\partial x} = M \Rightarrow \frac{\partial f}{\partial x} = yz$ ①

$\frac{\partial f}{\partial y} = xz$ ② $\qquad \frac{\partial f}{\partial y} = xy + 2z$ ③

Integrando ① em relação a x:

$f(x,y,z) = xyz + g(y,z)$ ④

Derivando em relação a y

$xz + g(y,z) = xz \quad g'(y,z)$

Integrando em relação a y

$g(y,z) = h(z)$

Substituindo em ④

$f(x,y,z) = xyz + h(z)$

$\frac{\partial f}{\partial z} = xy + h'(z)$

Comparando com ③

$\cancel{xy} + h'(z) = \cancel{xy} + 2z \therefore h'(z) = 2z$

$h(z) = \int 2z\, dz = z^2 \therefore f(x,y,z) = xyz + z^2$

$\int f\, dr = f(\text{final}) - f(\text{Inicial}) = (72 + 9) - (4)$

$= 81 - 4 = \boxed{77}$

c) $f(x,y,z) = y^2 \cos z\, i + 2xy \cos z\, j - xy^2 \sin z\, K$

$C: r(t) = t^2 i + \sin t\, j + t\, K,\ 0 \leq t \leq \pi$

$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z} \rightarrow -2xy \sin z = -2xy \sin z$

$\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \rightarrow -y^2 \sin z = -y^2 \sin z$

$\frac{\partial M}{\partial z} = \frac{\partial N}{\partial x} \rightarrow 2y \cos z = 2y \cos z$

F é conservativo

$\frac{\partial f}{\partial x} = M \rightarrow \frac{\partial f}{\partial x} = y^2 \cos z$ ①

$\frac{\partial f}{\partial y} = N \rightarrow \frac{\partial f}{\partial y} = 2xy \cos t$ ②

$\frac{\partial f}{\partial z} = P \rightarrow \frac{\partial f}{\partial z} = -xy^2 \sin z$ ③

Integrando ① em relação a x 3

$f(x,y,z) = xy^2 \cos z + g(y,z)$ ④

Derivando em relação a y e igualando a ②

$2xy \cos z + g'(y,z) = 2xy \cos z \quad g'(y,z) = 0$ ⑤

Integrando em relação a y e substituindo em ④

$g(y,z) = h(z) \quad , \quad f(x,y,z) = xy^2 \cos z + h(z)$

$\frac{\partial f}{\partial z} = -xy^2 \sin z + h'(z)$

comparando com ③ → $-xy^2 \sin z + h'(z) = -xy^2 \sin z$

$f(x,y,z) = xy^2 \cos z \qquad h(z) = 0$

$F(r(t)) = \langle \sin^2 t \cos t, 2t^2 \sin t \cos t, -t^2 \sin^2 t \rangle$

$r'(t) = \langle 2t, \cos t, 1 \rangle$

$\int_C F \cdot dr = \int_0^{\pi} \langle \sin^2 t \cos t, 2t^2 \sin t \cos t, -t^2 \sin^2 t \sin t \rangle \langle 2t, \cos t, 1 \rangle$

$= \int_0^{\pi} 2t \sin^2 t \cos t + 2t^2 \sin t \cos^2 t - t^2 \sin^2 t \sin t \, dt$

$-[t(\sin t)^0 - \frac{1}{2} \sin t - \frac{1}{3} \sin t \cos^2 t) - (-\cos t) + \frac{1}{4}(\cos 3t)]$

$[\pi(0) - (1 - \frac{1}{4})] - [t(0) - (-1 + \frac{1}{4})]$

$= 0 - \frac{3}{4} - 0 + \frac{3}{4} = 0$

2) Mostre que a integral de linha $\int_C 2x \sin y \, dx + (x^2 \cos y - 3y^2) dy$ é independente do caminho e calcule a integral onde C é qualquer caminho de $(-1,0)$ a $(5,1)$.

$\int_{C_1} 2x \sin y \, dx + (x^2 \cos y - 3y^2) dy$

$= \int_{C_2} 2x \sin y \, dx + (x^2 \cos y - 3y^2) dy$

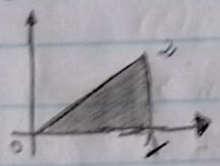
$C_1 : \iint_R \left(\frac{\partial}{\partial x}(x^2 \cos y - 3y^2) - \frac{\partial}{\partial y}(2x \sin y) \right) dx \, dy$

$$= \iint_R 2x \cos y - 2x \cos y \, dx\, dy = \iint dx\, dy$$

Portanto, $\int_C 2x \sin y \, dx + x^2 \cos y - 3y^2 \, dy$ é independente do caminho.

10) Calcular Integral de linha Teorema de Green

10a) $\oint_C xy\, dx + x^2y^3\, dy$, C é o Triângulo com vértices $(0,0), (1,0)$ e $(1,2)$



$P(x,y) = xy \quad \frac{\partial P}{\partial y} = x$

$G(x,y) = x^2y^3 \quad \frac{\partial G}{\partial x} = 2xy^3$

$\Rightarrow \iint_D 2xy^3 - x \, dA$

$$\int_1^2 \int_0^1 2xy^3 - x\,dx\,dy = \int_1^2 \left[x^2y^3 - \frac{x^2}{2}\right]_0^1 dy$$

$$= \int_1^2 y^3 - \frac{1}{2}\, dy = \left[\frac{y^4}{4} - \frac{y}{2}\right] = \left[\left(\frac{2^4}{4} - \frac{2}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= (4-1) - \frac{1-2}{4} = 3 + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4}$$

11) Teorema de Green

b) $\int_C (y + e^{\sqrt{x}})\, dx + (2x + \cos y^2)\, dy$

C é a fronteira da região enfolada pelas parábolas $y = x^2$ e $x = y^2$.

$P(x,y) = y + e^{\sqrt{x}} \quad \partial P/\partial y = 1$

$G(x,y) = 2x + \cos y^2 \quad \frac{\partial G}{\partial x} = 2$

$$\iint_R 2 - 1 \, dA$$

$$\int_0^1 \int_0^{x^2} 1 \, dy \, dx = \int_0^1 y \Big|_0^{x^2} dx = \int_0^1 x^2 \, dx$$

$$= \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

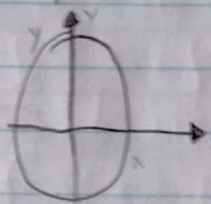
d) $\int_C y^3 \, dx - x^3 \, dy$, C é o círculo $x^2 + y^2 = 4$

$P(x,y) = y^3 \quad \frac{\partial P}{\partial y} = 3y^2$

$Q(x,y) = -x^3 \quad \frac{\partial Q}{\partial x} = -3x^2$

$$\iint_R -3x^2 - 3y^2 \, dA$$

Coord. Esféricas



$$\int_0^2 \int_0^{2\pi} -3(r\cos\theta)^2 - 3(r\sin\theta)^2 \, r \, dr \, d\theta$$

$$= \int_0^2 \int_0^{2\pi} -3r^3 (\underbrace{\cos^2\theta + \sin^2\theta}_{1}) \, dr \, d\theta$$

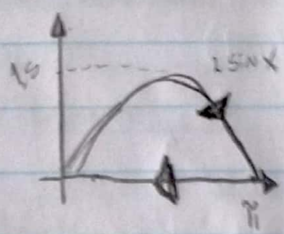
$$= -3 \int_0^2 r^3 \, dr \cdot \int_0^{2\pi} d\theta$$

$$= -3 \left[\frac{r^4}{4}\right]_0^2 \cdot [\theta]_0^{2\pi}$$

$$= -3[4 \cdot 2\pi] = -3 \cdot 8\pi = -24\pi$$

12) teorema de Green (verifique a orientação da curva)

a) $F(x,y) = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$, C consiste no arco da curva $y = \sin x$ de $(0,0)$ a $(\pi, 0)$ e no segmento de reta $(\pi, 0)$ a $(0,0)$.



$P(x,y) = \sqrt{x} + y^3 \quad \frac{\partial P}{\partial y} = 0 + 3y^2$

$Q(x,y) = x^2 + \sqrt{y} \quad \frac{\partial Q}{\partial x} = 2x + 0$

$$\iint_R 2x-3y^2\,dA = \int_0^{\pi}\int_0^{\sin x} 2x-3y^2\,dy\,dx = \int_0^{\pi}\left[2xy-y^3\right]_0^{\sin x} dx$$

$$= \int_0^{\pi}\left[2x\sin x-\sin^3 x\right]dx = \left[-2x\cos x+2\sin x+\frac{\cos x^3}{3}-\cos x\right]_0^{\pi}$$

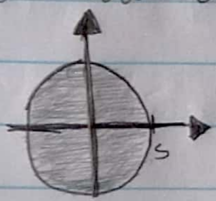
$$= -2\pi\cos\pi+2\sin\pi+\cos\pi-\frac{\cos 3\pi}{3}-\left(2-\frac{2}{3}\right)$$

$$= 2\pi-1+\sqrt{3}-2/3 = 2\pi-4/3$$

$$\int_C F\,dr = -\int_C F\,dr \Rightarrow \int_C F\,dr = -(2\pi-4/3)$$

$$\Rightarrow \int_C F\,dr = 4/3-2\pi$$

12)b) $F(x,y) = \langle e^x+x^2, e^y-xy^2\rangle$, C é a circunferência $x^2+y^2=25$ orientada no sentido anti-horário.



$$P = e^x+x^2 \Rightarrow \frac{\partial P}{\partial y} = 0$$

$$Q = e^y-xy^2 \Rightarrow \frac{\partial Q}{\partial x} = -y^2$$

$$\iint_D y^2\,dA$$

$0\leq r\leq 5$
$0\leq\theta\leq 2\pi$ $\Rightarrow \int_0^5\left[\int_0^{2\pi} -r^2\sin^2\theta\, r\,dr\right]d\theta$

$$= -\int_0^5 r^3\,dr\int_0^{2\pi}\sin^2\theta\,d\theta$$

$$= -\left[\frac{r^4}{4}\right]_0^5\cdot\left[\frac{1}{2}\left(x-\frac{1}{2}\sin(2x)\right)\right]_0^{2\pi}$$

$$= -\frac{625}{2}\pi$$

S T Q Q S S D

__/__/__

13) Use as fórmulas: $A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - y\,dx$ para achar a área sob um arco da ciclóide $x = t - \sin t$, $y = 1 - \cos t$

$x = t - \sin t \rightarrow (1-\cos t)\,dt = dx$, $y = 1 - \cos t \rightarrow dy = \sin t\,dt$

$$\frac{1}{2}\oint_C x\,dy - y\,dx \rightarrow \frac{1}{2}\int_0^{2\pi} (1-\cos t)^2\,dt$$

$$= \int_0^{2\pi} (1 - 2\cos t + \cos^2 t)\,dt$$

$$= \int_0^{2\pi} \left(1 - 2\cos t + \frac{1+\cos(2t)}{2}\right) dt$$

$$= \left[t - 2\sin t + \frac{t}{2} + \frac{\sin(2t)}{4}\right]_0^{2\pi}$$

$$= 2\pi + \frac{2\pi}{2} = \underline{3\pi}$$

14) Determine o rotacional e o divergente do campo vetorial.

c) $F(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}(x\,i + y\,j + z\,k)$

$$\text{rotacional } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix} \begin{matrix} i & j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} \end{matrix}$$

Esse determinante é $= 0$ ∴ Rotacional F é 0

$$\text{divergente } \vec{F} = \frac{1}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}} \underbrace{(y^2+z^2+x^2+z^2+x^2+y^2)}_{2x^2+2y^2+2z^2}$$

$$\frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}} = \boxed{\frac{2}{\sqrt{x^2+y^2+z^2}}} = \text{div } \vec{F}$$

14) d) $f(x,y,z) = \langle \ln x, \ln(x,y), \ln(xyz) \rangle$

$$\text{Rotacional } f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \ln x & \ln(xy) & \ln(xyz) \end{vmatrix} \begin{matrix} i & j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \ln x & \ln(xy) \end{matrix}$$

$$\left(\frac{\partial}{\partial y}(\ln(xyz)) - \frac{\partial}{\partial z}(\ln(xy))\right); -\left(\frac{\partial}{\partial x}(\ln(xyz)) - \frac{\partial}{\partial z}(\ln x)\right)j + \left(\frac{\partial}{\partial x}(\ln(xy)) - \frac{\partial}{\partial y}(\ln x)\right)$$

$$= \left(\frac{1}{xyz}\, xz\right) i - \left(\frac{1}{xyz}\, yz\right) j + \left(\frac{1}{xy}\, y\right) k$$

$\rightarrow$ Rotacional $f = \left(\frac{1}{y}, \frac{1}{x}, \frac{1}{x}\right)$

$$\text{Divergente } f = \frac{\partial}{\partial x}\ln(x) + \frac{\partial}{\partial y}\ln(xy) + \frac{\partial}{\partial z}\ln(xyz)$$

$$= 1/x + 1/xy\; x + 1/xyz\; xy \rightarrow \text{divergente } f = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

15) Determine se é conservativo e caso sim calcule f tal que $F = \nabla f$

a) $F(x,y,z) = y^2 z^3\, i + 2xyz^3\, j + 3xy^2 z^2\, k$

$$\text{Rotacional } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 z^3 & 2xyz^3 & 3xy^2 z \end{vmatrix} \begin{matrix} i & j \\ \partial/\partial x & \partial/\partial y \\ y^2 z^3 & 2xyz^3 \end{matrix}$$

$$6 + yz^2 i + 3y^2 z^2 j + 2xz^2 k - (2yz^3 k + 3y^2 z^2 j + 6xyz^2 i)$$

$$= 0i + 0j + 0k \Rightarrow \text{Rotacional } f = 0$$

$$f_x = y^2 z^3,\; f_y = 2xyz^3,\; f_z = 3xy^2 z^2$$

$$f(xyz) = xy^2 z^3 + c,\; f(xyz) = xy^2 z^3 + c,\; f(xyz) = xy^2 z^3 + c$$

$$f(x,y,z) = x\, y^2 + k$$

spiral

c) $F(x,y,z) = ye^{-x} i + e^{-x} j + 2z K$

Rotacional $F = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ye^{-x} & e^{-x} & 2z \end{vmatrix} \begin{matrix} i & j \\ \partial/\partial x & \partial/\partial y \\ ye^{-x} & e^{-x} \end{matrix}$

$0i + 0j - e^{-x} K - (e^{-x} K + 0j + 0i)$

Rotacional $= -2e^{-x}$ ∴ Não Conservativo

16) Existe um campo vetorial G em $\mathbb{R}^3$ tal que rot $G = \langle x \sin y, \cos y, z - xy \rangle$?

$\operatorname{div}(\operatorname{rot} F) = \frac{\partial P}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial Q}{\partial z}$

$= \frac{\partial x \sin y}{\partial x} + \frac{\partial \cos y}{\partial y} + \frac{\partial z - xy}{\partial z}$

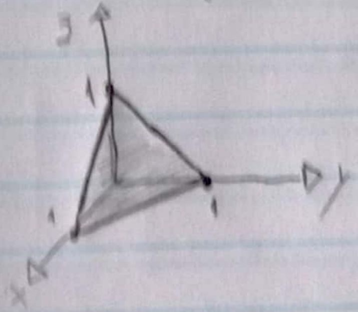
$= \sin y - \sin y + 0 = 0$

∴ Não existe um campo vetorial em $\mathbb{R}^3$ para seu rotacional.

18) Teorema de Stokes (C orientada no sentido anti-horário quando vista de cima.

a) $F(x,y,z) = (x+y^2) i + (y+z^2) j + (z+x^2) K$.

C é um triângulo com vértices $(1,0,0)$, $(0,1,0)$ e $(0,0,1)$



$\operatorname{Rot} F = -2y i - 2x j - 2z K$

$\vec{P}_{21} = (0,1,0) - (1,0,0) = (-1,1,0)$

$\vec{P}_{31} = (0,0,1) - (1,0,0) = (-1,0,1)$

$\vec{P}_{21} \times \vec{P}_{31} = (1,1,1)$

$(x-x_0, y-y_0, z-z_0) \cdot \vec{n} = 0$

$x-1, y-0, z-0) \cdot (1,1,1) = 0$

$x-1+y+z=0 \quad \sqrt{z^2+1^2+1^2}$

$g(x,y) = 1-x-y$

$\iint \langle -2x, -2y, -2z \rangle \langle 1,1,1 \rangle \, dA$

$\iint -2(1-x-y) - 2x - 2y \, dA = \iint -2 \, dA$

$\begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 1-x \end{cases} \quad \int_0^1 \int_0^{1-y} -2 \, dy \, dx = \int_0^1 [-2y]_0^{1-x} = \int_0^1 -2+2x \, dx$

$= [-2x + x^2]_0^1 = -2+1 = \underline{-1}$ //

b) $f(x,y,z) = yz\,i + 2xz\,j + e^{xy}\,k$

C é a circunferência $x^2+y^2=16$, $z=5$

$\vec{N} = \vec{k} = (0,0,1) \quad \begin{cases} 0 \leq r \leq 4 \\ 0 \leq \theta \leq 2\pi \end{cases}$

$\text{Rot } \vec{F} = \langle xe^{xy} - 2x, y - ye^{xy}, z \rangle$

$\langle xe^{xy} - 2x, y - ye^{xy}, z \rangle \langle 0,0,1 \rangle = z = 5$

$= \int_0^{2\pi} \int_0^4 5 r \, dr \, d\theta$

$= 5 \int_0^{2\pi} \left[\frac{r^2}{2} \right]_0^4 d\theta$

$= 5 \cdot \frac{4^2}{2} \int_0^{2\pi} d\theta$

$= 5 \cdot 8 \, [\theta]_0^{2\pi} = 40 \cdot 2\pi = \underline{80\pi}$ //