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Lista 3 - Funções Vetoriais

1- a) $r(t) = \langle t^2, \sqrt{t-1}, \sqrt{5-t} \rangle, t_0 = 1$

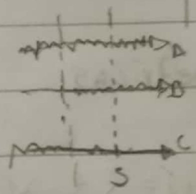
$r(t_0) = \langle 1^2, \sqrt{0}, \sqrt{5-1} \rangle$

$r(t_0) = \langle 1, 0, 2 \rangle \qquad r(1) = i + 2k$

Domínio: 1 - Den $\neq 0$

2 - $\sqrt{\ }\geq 0$

A	B	C
$t \in \mathbb{R}$	$\sqrt{t-1} \geq 0$	$\sqrt{5-t} \geq 0$
	$t \geq 1$	$t \leq 5$



Dom: $1 \leq t \leq 5$

b) $r(t) = \cos(\pi t) i - \ln t j + \sqrt{t-2}k, t_0 = 3$

$r(t_0) = \cos(\pi \cdot 3) i - \ln 3 j + \sqrt{3-2}k$

$r(t_0) = \langle -1, -\ln 3, 1 \rangle$

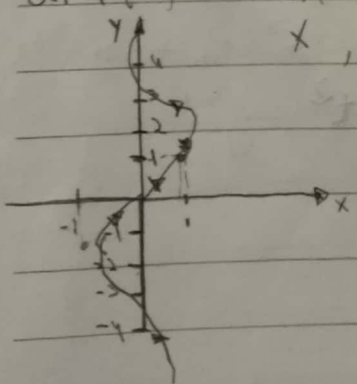
domínio

$\cos(\pi t)$	$\ln(t) \quad t > 0$	$\sqrt{t-2} - t \geq 2 \quad \therefore [2, +\infty)$
	$[2, +\infty]$	$[2, +\infty]$

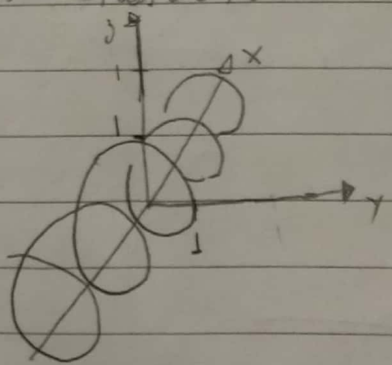
2) Esboce o gráfico da curva

a) $r(t) = \langle \sin(t), t \rangle$

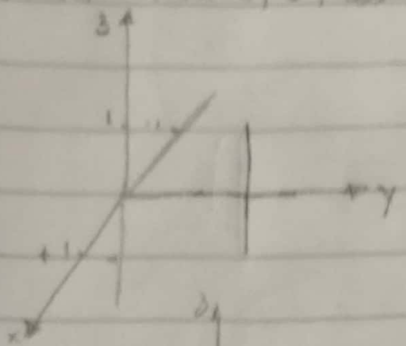
x , y



b) $r(t) = \langle t, \cos 2t, \sin 2t \rangle$



c) $r(t) = \langle \sin t, 3, \cos t \rangle$



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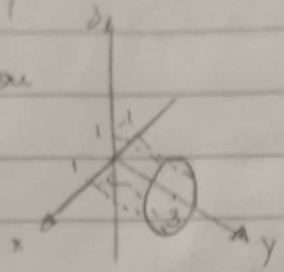
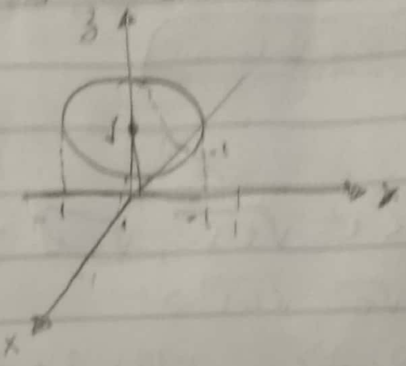


d) $r(t) = \langle 2\cos t\, \hat{i} - 3\sin t\, \hat{j} + \hat{k} \rangle$

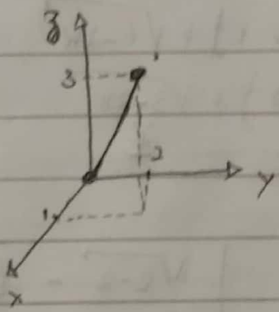


3a) Encontrar Eq vetorial e paramétricas

$P(0,0,0)$, $Q(1,2,3)$

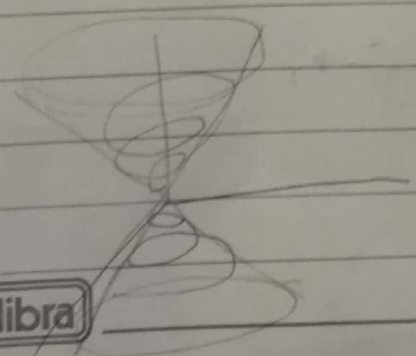
$$r: \begin{cases} x = t \rightarrow 0 \leq t \leq 1 \\ y = 2t \rightarrow 0 \leq t \leq 1 \\ z = 3t \rightarrow 0 \leq t \leq 1 \end{cases}$$

$G(t) = \langle t, 2t, 3t \rangle,\ 0 \leq t \leq 1$



4) Mostre que a curva com equações Paramétricas $x = t\cos t$, $y = t\sin t$, $z = t$ está no cone $z^2 = x^2 + y^2$ e esboce a curva.

$$x^2 + y^2 = (t\cos(t))^2 + (t\,\mathrm{sen}(t))^2 = t^2\cos^2(t) + t^2\,\mathrm{sen}^2(t)$$
$$= t^2(\cos^2(t) + \mathrm{sen}^2(t))$$
$$= t^2 = z^2$$



5) Em Quais pontos a curva $r(t) = t\,\hat{i} + (2t - t^2)\,k$ intercepta o parabolóide $z = x^2 + y^2$?

$x = t$

$z = 2t - t^2$

$y = 0$

$z = x^2 + y^2$

$2t - t^2 = t^2 + 0^2$

$2t - t^2 - t^2 = 0$

$2t - 2t^2 = 0 \to t = 1$

$\to t = 0$

$r(t) = t\,i + (2t - t^2)\,k$

$r(1) = 1\,i + (2 - 1^2)\,k = \boxed{\langle 1, 0, 1 \rangle}$

$r(0) = 0\,i + (2 \cdot 0 - 0^2)\,k = \boxed{\langle 0, 0, 0 \rangle}$

8) Mostre que a curva com Eqs paramétricas $x = t^2$, $y = 1 - 3t$, $z = 1 + t^3$ passa pelos pontos $(1, 4, 0)$ e $(9, -8, 28)$, mas não passa pelo ponto $(4, 7, -6)$

$P(1, 4, 0)$

$x = t^2$ $\qquad t^2 = 1 \Rightarrow t = \pm 1$

$y = 1 - 3t$ $\qquad 1 - 3t = 4$

$z = 1 + t^3$ $\qquad 3t = -3$

$\boxed{t = -1}$

$0 = 1 + t^3$

$t^3 = -1$

$\boxed{t = -1}$

$P\ t = -1 \Rightarrow P(1, 4, 0)$

$P(9, -8, 28)$	$P(4, 7, -6)$
$x = t^2 = 9 \quad t = \pm 3$	$x = t^2 = 4 \Rightarrow \pm 2$
$y = 1 - 3t = -8 \quad t = +3$	$y = 1 - 3t = 7 \Rightarrow -2$
$z = 1 + t^3 = 28 \quad t = \sqrt[3]{27} = 3$	$z = 1 + t^3 = -6 \Rightarrow \sqrt[3]{-7} \neq -2$
como $t = 3$, ela passa pelos pontos	logo a curva não passa pelos pontos $(4, 7, -6)$

9) Determine a função vetorial que representa a curva obtida pela interseção do cone $z = \sqrt{x^2+y^2}$ com o plano $z = 1+y$

$(\sqrt{x^2+y^2})^2 = (1+y)^2$

$x^2 + \cancel{y^2} = 1^2 + 2y + \cancel{y^2}$

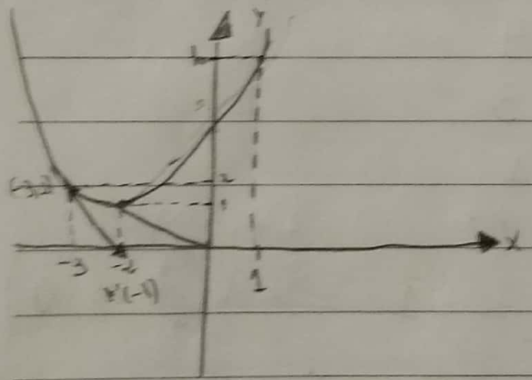
$2y = x^2 - 1$

$y = \frac{x^2-1}{2} \Rightarrow t = x, \; y = \frac{t^2-1}{2} \quad r(t) = \langle t, \frac{t^2-1}{2} \rangle$

11) Esboçar gráfico da curva, determinar $r'(t)$, esboçar vetor tangente e posição para o t dado

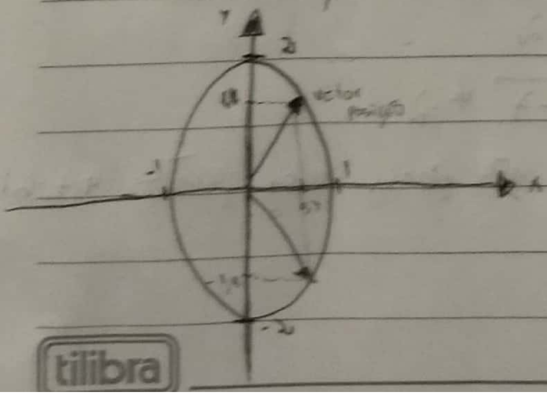
a) $r(t) = \langle t-2, t^2+1 \rangle \quad t = -1$ → Vetor Posição: $r(-1) = \langle -3, 2 \rangle$

$r'(t) = \langle 1, 2t \rangle$ → Vetor tangente: $r'(-1) = \langle 1, -2 \rangle$



b) $r(t) = \sin t \, i + 2 \cos t \, j, \quad t = \pi/4$ → Vetor Posição: $r(\frac{\pi}{4}) = \langle 0{,}7071, 1{,}4142 \rangle$ ($\frac{\sqrt{2}}{2}$, $\sqrt{2}$)

$r'(t) = \langle \cos(t), -2\sin(t) \rangle$ → Vetor tangente: $r'(\frac{\pi}{4}) = \langle 0{,}7071, -1{,}4142 \rangle$ ($\frac{\sqrt{2}}{2}$, $-\sqrt{2}$)



12) Derivada da função vetorial

a) $r(t) = \langle t \sin t, t^2, t \cos 2t \rangle$ $\quad gh' + g'h$

$r'(t) = \langle t \cos t + \sin t, 2t, t \cdot -\sin 2(2) + \cos 2t \rangle$ $\quad t \cos t + \sin(t)$

$t \cdot -\sin 2t + \cos 2t$

b) $r(t) = i - j + e^{4t} K$

$r(t) = \langle 1, -1, e^{4t} \rangle$

$r'(t) = \langle 0, 0, 4e^{4t} \rangle$

c) $r(t) = e^{t^2} i - j + \ln(1+3t) K$

$r(t) = \langle e^{t^2}, -1, \ln(1+3t) \rangle \rightarrow \ln(u)' \cdot u'$

$r'(t) = \langle 2te^{t^2}, 0, \frac{3}{1+3t} \rangle$ $\quad \frac{1}{1+3t} \cdot 0+3$

13) Vetor unitário $T(t)$ no ponto t dado

a) $r(t) = \langle 6t^5, 4t^3, 2t \rangle$, $t = 1$

$r'(t) = \langle 30t^4, 12t^2, 2 \rangle$

para $t = 1$ $r'(1) = \langle 30, 12, 2 \rangle$ ou $30i + 12j + 2K$

$\|r'(t)\| = \sqrt{30^2 + 12^2 + 2^2} = \sqrt{1048}$

Vetor unitário $\Rightarrow \frac{r'(1)}{\|r'(1)\|} = \frac{\langle 30, 12, 2 \rangle}{\sqrt{1048}} = \left\langle \frac{30}{\sqrt{1048}}, \frac{12}{\sqrt{1048}}, \frac{2}{\sqrt{1048}} \right\rangle$

b) $r(t) = \cos t\, i + 3t j + 2 \sin 2t K$, $t = 0$ $\quad \cos(u) \cdot u'$

$r'(t) = \langle -\sin t, 3, 4 \cos 2t \rangle$ $\quad 2\cos(2t) \cdot 2$

para $t = 0$ $r'(0) = \langle 0, 3, 4 \rangle$

$\|r'(t)\| = \sqrt{0^2 + 3^2 + 4^2} = \sqrt{25}$

vetor unitário $\rightarrow \frac{r'(0)}{\|r'(0)\|} = \frac{\langle 0, 3, 2 \rangle}{5} = \left\langle \frac{0}{5}, \frac{3}{5}, \frac{2}{5} \right\rangle$

14) $r(t) = \langle t, t^2, t^3 \rangle$, encontre $r'(t)$, $T(1)$, $r''(t)$ e $r'(t) \times r''(t)$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$T(1) = \dfrac{r'(1)}{\|r'(1)\|} = \dfrac{\langle 1, 2t, 3t^2 \rangle}{\sqrt{1^2+2^2+3^2}} = \left\langle \dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right\rangle$

$r'(t) \cdot r''(t)$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 2t & 3t^2 & 1 & 2t \\ 0 & 2 & 6t & 0 & 2 \end{array}$$

$0 + 6t^2 i + 6t j - (12t^2 i + 0 + 2k)$

$\langle -6t^2 i + 6t j - 2k \rangle$

$r'(t) \cdot r''(t) = \langle -6t^2 i + 6t j - 2k \rangle$

15) Determine as eqs paramétricas para a reta tangente à curva dada pelas eqs paramétricas, no ponto especificado.

a) $x = t^5$, $y = t^4$, $z = t^3$, $(1,1,1)$

Vetor posição $r(t) = \langle t^5, t^4, t^3 \rangle$

$\begin{cases} x = t^5 = 1 \\ y = t^4 = 1 \\ z = t^3 = 1 \end{cases} \Rightarrow t = 1$

- Vetor tangente: $r'(t) = \langle 5t^4, 4t^3, 3t^2 \rangle$

$r'(1) = \langle 5, 4, 3 \rangle$

Soma $r(t)$ com $r'(t)$

$\begin{cases} x = 1 + 5t \\ y = 1 + 4t \\ z = 1 + 3t \end{cases}$

b) $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$, $(1,0,1)$

Vetor posição: $r(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t}\rangle$

$$\begin{cases} x = e^{-t}\cos t = 1 \\ y = e^{-t}\sin t = 0 \Rightarrow t = 0 \\ z = e^{-t} = 1 \end{cases}$$

- Vetor tangente: $r'(t) = \langle -e^{-t}\cdot\sin t - e^{-t}\cos t, \; te^{-t}\cos t + e^{-t}\sin t, -e^{-t}\rangle$

$r'(0) = \langle -1, 1, -1\rangle$

$$\begin{cases} x = 1 - 1t \\ y = 0 + 1t \\ z = 1 - 1t \end{cases}$$

16) Encontrar as eqs paramétricas para a reta tangente à curva dada pelas eqs paramétricas, no ponto especificado

a) $x = t$, $y = e^{-t}$, $z = 2t - t^2$; $(0,1,0)$

∘ Vetor posição: $r(t) = \langle t, e^{-t}, 2t - t^2\rangle$

$$\begin{cases} x = t = 0 \\ y = e^{-t} = 1 \Rightarrow t = 0 \\ z = 2t - t^2 = 0 \end{cases}$$

∘ Vetor tangente $= r'(t) = \langle 1, -e^{-t}, 2 - 2t\rangle$

$r'(0) = \langle 1, -1, 2\rangle$

$$\begin{cases} x = 0 + 1t \\ y = 1 - 1t \\ z = 0 + 2t \end{cases}$$

a) $x = t\cos t$, $y = t$, $z = t\sin t$; $(-\pi, \pi, 0)$

∘ Vetor posição: $r(t) = \langle t\cos t, t, t\sin t\rangle$

$$\begin{cases} x = t\cos t = -\pi \\ y = t = \pi \Rightarrow t = \pi \\ z = t\sin t = 0 \end{cases}$$

∘ Vetor tangente $= r'(t) = \langle t\cdot\sin t + \cos t, 1, t\cos t + \sin t\rangle$

$r'(\pi) = \langle -1, 1, -\pi\rangle$

$$\begin{cases} x = -\pi - 1t \\ y = \pi + 1t \\ z = 0 - \pi t \end{cases}$$

17) Comprimento da curva dada

a) $r(t) = \langle 2\sin t, 5t, 2\cos t \rangle$, $-10 \leq t \leq 10$

$\|r'(t)\| = \langle 2\cos t, 5, -2\sen t \rangle$

$\|r'(t)\| = \sqrt{(2\cos t)^2 + (5)^2 - (2\sin t)^2}$

$\|r'(t)\| = \sqrt{4.(\cos t^2 + \sin t^2) + 25}$

$\|r'(t)\| = \sqrt{4.\underset{1}{1} + 25} = \sqrt{29}$

$L = \int_{-10}^{10} \sqrt{29}\, dt = \sqrt{29} \int_{-10}^{10} dt = \sqrt{29}\, t \Big|_{-10}^{10}$

$= \sqrt{29}\ 10 + 10 = 20\sqrt{29}$ //

b) $r(t) = \sqrt{2}t\, i + e^t j + e^{-t} K$, $0 \leq t \leq 1$

$\|r'(t)\| = \sqrt{(\sqrt{2})^2 + (e^t)^2 + e^{t^2}}$

$\|r'(t)\| = \sqrt{2 + e^{2t} - e^{-2t}}$

$L = \int_0^1 \sqrt{2 + e^{2t} - e^{-2t}}\, dt \rightarrow 2\cos 2t$

$L = e - e^{-1}$ //

c) $r(t) = i + t^2 j + t^3 K$, $0 \leq t \leq 1$

$\|r'(t)\| = \sqrt{0^2 + 2t^2 + (3t^2)^2}$

$\|r'(t)\| = \sqrt{4t^2 + 9t^4}$

$L = \int_0^1 \sqrt{4t^2 + 9t^4}\, dt = \int_0^1 t\sqrt{4 + 9t^2}\, dt = \frac{1}{18} \cdot \int u^{1/2}\, du$

$= \frac{1}{18} \cdot \frac{2u\sqrt{u}}{3} = \frac{1}{18} \cdot \frac{2(4 + 9t^2)\sqrt{4 + 9x^2}}{3} = \frac{(4 + 9x^2)\sqrt{4 + 9x^2}}{27} \Bigg|_0^1$

$\frac{4 + 9\sqrt{4 + 9}}{27} \quad \frac{13\sqrt{13}}{27} - \frac{8}{27}$

18) Seja C a curva de intersecção do cilindro parabólico $x^2 = 2y$ e da superfície $3z = xy$. Encontre o comprimento de C da origem até o ponto $(6, 18, 36)$

$t - 0 \to 6$

$y = \frac{x^2}{2}$, $z = \frac{x \cdot x^2}{6}$

$r(t) = \langle t, t^2/2, t^3/6 \rangle$

$\|r'(t)\| = \langle 1, t, t^2/2 \rangle$

$L = \int_0^6 \sqrt{1 + t^2 + \left(\frac{t^2}{2}\right)^2}\, dt \to \int_0^6 1 + \frac{t^2}{2}\, dt$

$\left[t + \frac{t^3}{6}\right]_0^6 = 6 + \frac{6^3}{6} - 0 = \underline{42}$ //