习题1.

3. 解 (1) $\because \int_{-\infty}^{+\infty} f(x) = \int_{-\infty}^{0} Ae^{x}dx + \int_{0}^{2}\frac{1}{4}dx + \int_{2}^{+\infty} 0\,dx$

$= A + \frac{1}{2} + 0 = 1$

$\therefore A = \frac{1}{2}$

(2) ①当 $x<0$ 时.

$F(x) = \int_{-\infty}^{x} \frac{1}{2}e^{x}dx = \frac{1}{2}e^{x}$

②当 $0\le x<2$ 时

$F(x) = \int_{-\infty}^{0}\frac{1}{2}e^{x}dx + \int_{0}^{x}\frac{1}{4}dx = \frac{1}{2}+\frac{1}{4}x$

③当 $x\ge 2$ 时.

$F(x) = 1$

得:

$$F(x) = \begin{cases} \frac{1}{2}e^{x}, & x<0 \\ \frac{1}{2}+\frac{1}{4}x, & 0\le x<2 \\ 1, & x\ge 2 \end{cases}$$

(3) $P(1\le X<3) = F(3)-F(1) = 1-\frac{3}{4} = \frac{1}{4}$

5. (1) $X\sim B(1,p)$ 两点分布

$\prod_{i=1}^{5} p^{x_i}q^{1-x_i}, \quad x_i = 0,1$

(2) $X\sim P(\lambda)$ Poisson分布

$\prod_{i=1}^{5} \frac{\lambda^{x_i}}{x_i!}\cdot e^{-\lambda}, \quad x_i = 0,1,2\cdots$

(3) $X\sim U[a,b]$ 均匀分布

$$\begin{cases} \frac{1}{(b-a)^5}, & a\le x_i\le b, \quad i=1,2,3,4,5 \\ 0, & \text{else} \end{cases}$$

(4) $X\sim N(\mu,1)$ 正态分布

$$\prod_{i=1}^{5}\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{(x_i-\mu)^2}{2}} \quad -\infty<x_i<+\infty,\ i=1,2,3,4,5$$

6. (1) ① X的边缘密度函数

$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy$$

$$=\begin{cases}\int_{-x}^{x} 1\,dy, & 0<x<1\\ 0, & \text{else}\end{cases}$$

$$=\begin{cases}2x, & 0<x<1\\ 0, & \text{else}\end{cases}$$

② Y的边缘密度函数

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx$$

$$=\begin{cases}\int_{y}^{1} 1\,dx, & 0<y\le 1\\ \int_{-y}^{1} 1\,dx, & -1<y<0\\ 0, & \text{else}\end{cases}$$

$$=\begin{cases}1-|y|, & |y|<1\\ 0, & \text{else}\end{cases}$$

(2) $f(x,y)\neq f_X(x)\cdot f_Y(y)$，故 X 与 Y 不独立.

(3). $$r=\frac{cov(X,Y)}{\sqrt{DX}\ \sqrt{DY}}$$

$$cov(X,Y)=E(X-EX)(Y-EY)=E(XY)-EXEY$$

$$E(X)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x f(x,y)\,dxdy=\int_{-x}^{x}\int_{0}^{1} x\,dxdy=x$$

$$E(X^2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2 f(x,y)\,dxdy=\int_{-x}^{x}\int_{0}^{1} x^2\,dxdy=\frac{2}{3}x$$

$$E(XY)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy f(x,y)\,dxdy=\int_{-x}^{x}\int_{0}^{1} xy\,dxdy=0$$

同理 $E(Y)=0$， $E(Y^2)=\frac{2}{3}x^3$.

$$D(X)=E(X^2)-[E(X)]^2=\frac{2}{3}x-x^2$$

$$D(Y)=E(Y^2)-[E(Y)]^2=\frac{2}{3}x^3.$$

$cov(X,Y)=0$

得: $$r=\frac{cov(X,Y)}{\sqrt{DX}\cdot\sqrt{DY}}=0.$$

8. 解: $X-Y\sim N(0,1)$， 设 $Z=X-Y$

$$E(|X-Y|)=E(|Z|)$$

$$=\int_{-\infty}^{+\infty}|z|\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{|z|^2}{2}}dz$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}|z|\cdot e^{-\frac{z^2}{2}}dz$$

$$=\frac{2}{\sqrt{2\pi}}\int_{0}^{+\infty}z\cdot e^{-\frac{z^2}{2}}dz$$

$$=\frac{2}{\sqrt{2\pi}}=\sqrt{\frac{2}{\pi}}$$

9. 解 (1) $EZ = E(\frac{1}{3}X+\frac{1}{2}Y) = E(\frac{1}{3}X)+E(\frac{1}{2}Y) = \frac{1}{3}E(X)+\frac{1}{2}E(Y) = \frac{1}{3}$

$\cancel{DZ = D(\frac{1}{3}X+\frac{1}{2}Y) = DX+DY+2cov(X,Y)}$

$\cancel{DX = \frac{1}{9}DX}$, $\cancel{DY = \frac{1}{4}DY}$

~~Cov~~

$DZ = D(\frac{1}{3}X+\frac{1}{2}Y) = D(\frac{1}{3}X)+D(\frac{1}{2}Y)+2cov(\frac{1}{3}X,\frac{1}{2}Y)$

$= \frac{1}{9}DX+\frac{1}{4}DY+\frac{1}{3}cov(X,Y)$

$= \frac{1}{9}\cdot 9+\frac{1}{4}\cdot 16+\frac{1}{3}\cdot r\cdot\sqrt{DX}\cdot\sqrt{DY}$

$= 1+\cancel{8}4-2 = 3.$

(2). $r = \dfrac{cov(X,Z)}{\sqrt{DX}\cdot\sqrt{DZ}}$

$= \cancel{\dfrac{E(XZ)-EXEZ}{\sqrt{DX}\sqrt{DZ}}}\quad \dfrac{cov(X,\frac{1}{3}X+\frac{1}{2}Y)}{\sqrt{DX}\cdot\sqrt{DY}}$

$= \dfrac{cov(X,\frac{1}{3}X)+cov(X,\frac{1}{2}Y)}{12}$

$= \frac{1}{12}\cdot(\frac{1}{3}cov(X,X)+\frac{1}{2}cov(X,Y))$

$= \frac{1}{12}\cdot(\frac{1}{3}\cdot DX+\frac{1}{2}\cdot r\cdot\sqrt{DX}\cdot\sqrt{DY})$

$= \frac{1}{12}\cdot(\frac{1}{3}\cdot 9+\frac{1}{2}\cdot(-\frac{1}{2})\cdot 3\cdot 4) = 0.$

习题2.

3. 解 $\sigma^2 = 4$, $n = 100$.

类出~~中心极限~~定理

$P\left(\left|\dfrac{\bar{X}-a}{\sigma/\sqrt{n}}\right| \le x\right) = P\left(\left|\dfrac{\bar{X}-a}{2/\sqrt{100}}\right| \le 5k\right) = P(5|\bar{X}-a| \le 5k)$

$\because \dfrac{\bar{X}-a}{\sigma\cdot\sqrt{n}} \overset{近似}{\sim} N(0,1)$

$\therefore P(5|\bar{X}-a| \le 5k) = P(-5k \le 5|\bar{X}-a| < 5k)$

$= \Phi(5k)-\Phi(-5k)$

$= \Phi(5k)-(1-\Phi(5k)) = 2\Phi(5k)-1 = 0.9$

$\therefore \Phi(5k) = 0.95 \Rightarrow 5k = 1.65 \Rightarrow k = 0.33$

5. 解 $\bar{X} \sim N(20,\frac{3}{10})$, $\bar{Y} \sim N(20,\frac{3}{15})$. $\bar{X}-\bar{Y} \sim N(0, 0.5)$.

($\bar{X},\bar{Y}$ 相互独立).

$\frac{3}{10}+\frac{3}{15} = \frac{45}{150}+\frac{30}{150}$ $\quad \frac{75}{150}$ $\quad 0.5$

$(\sqrt{2})\cdot(\bar{X}-\bar{Y}) \sim N(0,1)$.

$\therefore P(|\bar{X}-\bar{Y}| > 0.3) = 1-P(|\bar{X}-\bar{Y}| \le 0.3) = 1-P(\sqrt{2}|\bar{X}-\bar{Y}| \le 0.3\sqrt{2})$

$\cancel{= P(\sqrt{2}|\bar{X}-\bar{Y}| > 0.3\sqrt{2})}$

$\cancel{= \Phi(0.3\sqrt{2})-\Phi(-0.3\sqrt{2})}$

$\cancel{= 2\Phi(0.3\sqrt{2})+1 = 1-0.38 = 0.62}$

$= 1-[\Phi(0.3\sqrt{2})-\Phi(-0.3\sqrt{2})]$

$= 2-2\Phi(0.3\sqrt{2}).$

$= 2-2\cdot 0.6628$

$= 0.6744$

6. 解: $X \sim N(0,4) \Rightarrow \frac{1}{2}X \sim N(0,1)$

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{X_i}{2}\right)^2 \sim \chi^2(10)$$

$$P\left(\sum_{i=1}^{10} X_i^2 > c\right) = P\left(\sum_{i=1}^{10}\left(\frac{X_i}{2}\right)^2 > \frac{c}{4}\right)$$

$$= P\left(\frac{1}{4}\sum_{i=1}^{10} X_i^2 > \frac{c}{4}\right)$$

$$= 1 - P\left(\frac{1}{4}\sum_{i=1}^{10} X_i^2 \le \frac{c}{4}\right)$$

$$= 0.05$$

$$\therefore P\left(\frac{1}{4}\sum_{i=1}^{10} X_i^2 \le \frac{c}{4}\right) = 0.95$$

查表得 $\frac{c}{4} = 18.31 \Rightarrow c = 73.24$

9. (1) $X_1 + X_2 \sim (0,2)$

$X_3 + X_4 + X_5 \sim (0,3)$.

$\frac{1}{\sqrt{2}}(X_1+X_2) \sim (0,1)$; $\frac{1}{\sqrt{3}}(X_3+X_4+X_5) \sim (0,1)$.

$$\left[\frac{1}{\sqrt{2}}(X_1+X_2)\right]^2 + \left[\frac{1}{\sqrt{3}}(X_3+X_4+X_5)\right]^2$$

$$= \frac{1}{2}(X_1+X_2)^2 + \frac{1}{3}(X_3+X_4+X_5)^2$$

$$= \chi^2(2) \Rightarrow c_1 = \frac{1}{2},\ d_1 = \frac{1}{3},\ n = 2.$$

(2). 由(1)得 $\frac{1}{3}(X_3+X_4+X_5)^2 \sim \chi^2(1)$.

且 $X_1^2 + X_2^2 \sim \chi^2(2)$.

$$\frac{(X_1^2+X_2^2)/2}{\left[\frac{1}{3}(X_3+X_4+X_5)^2\right]/1} \sim F(2,1)$$

得 $c_2 = \frac{3}{2}$. $m = 2$. $n = 1$.

11. 解: 由~~…~~抽样分布定理得:

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\therefore P(|\bar{X}-\mu| \le 0.25\sigma) = P\left(\left|\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right| \le \frac{0.25\sigma}{\sigma/\sqrt{n}}\right)$$

$$= P\left(-0.25\sqrt{n} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le 0.25\sqrt{n}\right)$$

$$= 2\Phi(0.25\sqrt{n}) - 1 \ge 0.95.$$

得: $\Phi(0.25\sqrt{n}) \ge 0.975$

$\Phi(0.25\sqrt{n}) \ge \Phi(1.96)$

$\sqrt{n} \ge 7.84$

$n \ge 61.4656$

$n \ge 62$.

11.6

Chapter 3. 2.8.9.12.14.

习题 3.

2. 解(1) $EX = \int_0^1 x(\alpha+1)x^{\alpha}dx$

$= \frac{\alpha+1}{\alpha+2}\left[(x^{\alpha+2})\Big|_0^1\right] = \frac{\alpha+1}{\alpha+2}$

则 $\frac{\hat{\alpha}_1+1}{\hat{\alpha}_1+2} = \bar{x} \Rightarrow \hat{\alpha}_1 = \frac{2\bar{x}-1}{1-\bar{x}}$

(2). $L\alpha = \prod_{i=1}^{n}(\alpha+1)x_i^{\alpha}$

$\ln(L\alpha) = n\ln(\alpha+1) + \alpha\sum_{i=1}^{n}\ln x_i$

对 α 求导得:

$\frac{n}{\alpha+1} + \sum_{i=1}^{n}\ln x_i$

则 $\frac{n}{\hat{\alpha}_2+1} + \sum_{i=1}^{n}\ln x_i = 0 \Rightarrow \hat{\alpha}_2 = -\left(1+\frac{n}{\sum_{i=1}^{n}\ln x_i}\right)$

(3) 将样本观测值代入得:

$\bar{x} = 0.566667$

$\hat{\alpha}_1 = 0.3079$ $\hat{\alpha}_2 = 0.21117$

8. 解: $E(\bar{X}^2 - CS^2) = E\bar{X}^2 - CES^2 = D\bar{X} + (E\bar{X})^2 - CES^2$

$= \frac{\sigma^2}{n} + \mu^2 - C\sigma^2 = \mu^2$

得 $C = \frac{1}{n}$

Chapter 4: 3.6.9.10.12

9. 解: $E(\hat{\theta}_1) = \theta$, $E(\hat{\theta}_2) = \theta$ 且 $D(\hat{\theta}_1) = 2D(\hat{\theta}_2) = 2\sigma^2$.

$E(c_1\hat{\theta}_1 + c_2\hat{\theta}_2) = c_1E(\hat{\theta}_1) + c_2E(\hat{\theta}_2) = (c_1+c_2)E(\theta) = \theta$.

得: $c_1 + c_2 = 1$.

$D(c_1\hat{\theta}_1 + c_2\hat{\theta}_2) = c_1^2D(\hat{\theta}_1) + c_2^2D(\hat{\theta}_2) = (2c_1^2 + c_2^2)\sigma^2 = (3c_1^2 - 2c_1 + 1)\sigma^2$

对 $3c_1^2 - 2c_1 + 1$ 求导等于0得. $6c_1 + 2 = 0$

即 $c_1 = \frac{1}{3}$ 时, 为最小方差无偏估计量. 此时 $c_2 = \frac{2}{3}$.

12. (1) 方差 σ^2 已知, μ 的置信度为 $1-\alpha$ 的置信区间为

$\left(\bar{x} - \frac{\sigma}{\sqrt{n}}u_{1-\frac{\alpha}{2}},\ \bar{x} + \frac{\sigma}{\sqrt{n}}u_{1-\frac{\alpha}{2}}\right)$

观测值的统计量如下:

$\bar{x} = 2.125$ $\alpha = 0.1$ $u_{1-\frac{\alpha}{2}} = u_{0.95} = 1.65$

$n = 16$ $1-\frac{\alpha}{2} = 0.95$

$\sigma = 0.01$, 代入上式得:

μ 的置信度为 90% 的置信区间为 (2.1209, 2.129).

(2) 方差 σ^2 未知. μ 的置信度为 $1-\alpha$ 的置信区间为

$\left(\bar{x} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}},\ \bar{x} + t_{1-\frac{\alpha}{2}}(n-1)\frac{S}{\sqrt{n}}\right)$

统计 $t_{0.95}(15) = 1.753$, $S = 0.0171$

μ 的置信度为 90% 的置信区间为 (2.115, 2.135).

14. 解：σ_1^2, σ_2^2 未知，$\mu_1-\mu_2$ 的置信度为 $1-\alpha$ 的置信区间为

$$\cancel{(\bar{X}-\bar{Y}) \pm u_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

$$(\bar{X}-\bar{Y}) \pm t_{1-\frac{\alpha}{2}}(n_1+n_2-2) \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \cdot \sqrt{\frac{(n_1-1)S_X^2+(n_2-1)S_Y^2}{n_1+n_2-2}}$$

计算得：$\bar{x}=0.14125$，$\bar{y}=0.1392$。

$t_{0.975}(7)=2.365$.

$S_x^2=8.25\times10^{-6}$，$S_y^2=5.2\times10^{-6}$.

$n_1=4$，$n_2=5$.

得：90% 的置信区间为 $(-0.0022, 0.0063)$.

习题 4.

3. 解：此题是要检验假设：$H_0: \sigma=12$，$H_1: \sigma\neq12$.

故 H_0 的拒绝域为：$\begin{cases}\chi^2 \le \chi^2_{\frac{\alpha}{2}}(n-1) \\ \chi^2 \ge \chi^2_{1-\frac{\alpha}{2}}(n-1)\end{cases}$

$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}$，$k=0,1,2,3.$

又因 $n=10$，$S^2=70.889$，$\sigma_0=12$

故 $\chi^2=\frac{(n-1)S^2}{\sigma_0^2}=\frac{9\times70.889}{12^2}=4.4368$.

又因 $\chi^2_{\frac{\alpha}{2}}(n-1)=\chi^2_{0.025}(9)=2.7$

$\chi^2_{1-\frac{\alpha}{2}}(n-1)=\chi^2_{0.975}(9)=19.02$

则 H_0 的拒绝域为 $\{\chi^2\le2.7, \chi^2\ge19.02\}$.

当 $\sigma_0=12$ 时，$\chi^2=4.4368$ 不在 H_0 的拒绝域.

则说明可以认为总体标准差 $\sigma=12$.

6. 解：此题是要检验：$H_0: \mu_1=\mu_2$. $H_1: \mu_1\neq\mu_2$，σ_1^2, σ_2^2 未知，但 $\sigma_1^2=\sigma_2^2$。

因 $\bar{x}=21.5$，$\bar{y}=18$，$n_1=5$，$n_2=4$，$S_1^2=7.505$，$S_2^2=2.5933$

$$S_w^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}=5.4$$

$$T=\frac{(21.5-18)}{\sqrt{\frac{1}{5}+\frac{1}{4}}+S_w^2}=2.2453$$

H_0 的拒绝域

$|T|\ge t_{1-\frac{\alpha}{2}}(n_1+n_2-2)=t_{0.975}(5+4-2)=2.365$.

$T=2.2453$ 不在 H_0 的拒绝域内，则 $\mu_1=\mu_2$.

即甲、乙两煤矿的含灰量无显著差异.

9. 解：因 $n=200$，$m=10$，考虑到部分区间 $[0,1), [1,2), \dots, [9,10)$，本题为检验假设

$H_0: F(x)=F_0(x)$. $H_1: F(x)\neq F_0(x)$.

其中 $F_0(x)$ 为均匀分布，由题意得 $p_i=0.1$，$i=0,1\dots9$，可得

$$\chi^2=\sum_{i=1}^{m}\frac{(\nu_i-np_i)^2}{np_i}=\sum_{i=1}^{10}\frac{(\nu_i-20)^2}{20}=24.9$$

H_0 的拒绝域为

$\{\chi^2>\chi^2_{1-\alpha}(m-1)=\chi^2_{0.95}(9)=16.916\}$.

$\chi^2=24.9$ 在 H_0 的拒绝域内，得

这些数字不服从均匀分布.

10. 解：本题是检验假设

$H_0: F(x) = F_0(x)$ ，$H_1: F(x) \neq F_0(x)$.

其中 $F_0(x)$ 为二项分布． $P\{\xi = k\} = C_N^k p^k (1-p)^{N-k}$.

$$L(p) = \prod_{i=1}^{n} C_N^{\xi_i} p^{\xi_i} (1-p)^{n-\xi_i}$$

故 $\ln L(p) = \ln\left(\prod_{i=1}^{n} C_N^{\xi_i}\right) + n\bar{\xi}\ln p + n(N-\bar{\xi})\ln(1-p)$.

令 $\frac{\partial}{\partial p}\ln L(p) = \frac{n\bar{\xi}}{p} - \frac{n(N-\bar{\xi})}{1-p} = 0$

解得：$\hat{p} = \frac{\bar{\xi}}{N}$

在本题中 $n=100$，$N=10$，故 $\hat{p}=0.1$

当 H_0 成立时，$\xi \sim b(10, 0.1)$

$p_i = P_{0.1}\{\xi = i-1\} = F_0(i) - F_0(i-1)$ ，$i=1,2,\cdots 11$.

由皮尔逊定理得．

$$\chi^2(11-2) = \sum_{i=0}^{10}\frac{(n_i - np)^2}{n_i p} = 5.08$$

查表得：$\chi^2_{0.05}(9) = 16.919 > \chi^2(9)$，故不否认 H_0

即认为次品服从二项分布．

12. 解：两车间各抽取的12产品混合秩序表为

秩：	1	2	3	4	5	6	7	8	9.5	11	12
数据：	0.62	0.86	0.99	1.12	1.13	1.16	1.18	1.20	1.21	1.22	1.24
秩：	13.5	15	16	17	18.5	20	21	22.5	24		
数据：	1.31	1.34	1.38	1.39	1.41	1.48	1.59	1.60	1.84		

$T = 112$ （T denotes the sum of the ranks）

two sample hypothesis：

$H_0: F_X(x) = F_Y(y)$ ； $H_1: F_X(x) \neq F_Y(y)$.

$\because m = n > 10$

$\therefore$ test statistic is

$$U^* = \frac{T - n(n+m+1)/2}{\sqrt{nm(n+m+1)/12}} = -2.19391$$

H_0 should rejected if

$$|U^*| > U_{1-\frac{\alpha}{2}} = U_{0.975} = 1.96$$

$\because |-2.19391| > 1.96$ $\therefore$ 拒绝 H_0．即两项指标的分布显著不同．

习题 5.

1. 解：From the table，we find that

$$\sum_{i=1}^{k}(x_i - \bar{x})^2 = l_{xx}$$

$\sum_{i=1}^{6} x_i = 3300$ ， $\sum_{i=1}^{6} x_i^2 = 1990000$

$\sum_{i=1}^{6} x_i y_i = 198400$

$\sum_{i=1}^{6} y_i = 342$ ， $\sum_{i=1}^{6} y_i^2 = 20114$

$\bar{x} = 550$ $\bar{y} = 57$

$L_{xx} = \sum_{i=1}^{6} x_i^2 - \frac{1}{6}\left(\sum_{i=1}^{6} x_i\right)^2 = 175000$ $L_{xy} = \sum_{i=1}^{6} x_i y_i - \frac{1}{6}\sum_{i=1}^{6} x_i \sum_{i=1}^{6} y_i = 10300$

$\hat{b} = \frac{L_{xy}}{L_{xx}} = 0.05886$ ； $\hat{a} = \bar{y} - \hat{b}\bar{x} = 24.62857$

making the least squares line $y = 24.6286 + 0.0589x$

2.解: From the table find that

$\sum_{i=1}^{18} x_i = 2791251$ $\sum_{i=1}^{18} x_i^2 = 640857446919$

$\sum_{i=1}^{18} y_i = 117697$ $\sum_{i=1}^{18} y_i^2 = 1390554019$

$\bar{x} = 155069.5$ $\bar{y} = 6538.7222$

$L_{xx} = 208019549974.5$ $L_{xy} = 11197130334.5$

$L_{yy} = 620966029.6111$

$\hat{b} = 0.0538$ $\hat{a} = -1808.25$

making the least squares line

$y = -1808.25 + 0.0538x$

Hypothesis:

$H_0: b=0$ $H_1: b\neq 0$

① T-test:

the statistic $T = \frac{\hat{b}\sqrt{L_{xx}}}{\hat{\sigma}}$

H_0 will be rejected if $|T| \geq t_{1-\frac{\alpha}{2}}(n-2)$

In this case:

$T = \frac{\hat{b}\sqrt{L_{xx}}}{\hat{\sigma}} = 22.9841$

while $Q = L_{yy} - \hat{b}L_{xy} = 18254756.8334$

$\sigma^2 = Q/18-2 = 1140922.3021$

$t_{1-\frac{\alpha}{2}}(n-2) = t_{0.975}(16) = 2.120 \leq |22.9841| = T$

our conclusion is reject H_0. 即X与Y显著线性相关.

② F-test:

the statistic $F = \frac{\hat{b}^2 L_{xy}}{\hat{\sigma}^2} = \frac{U}{Q/(n-2)}$

H_0 will be rejected if $F \geq F_{1-\alpha}(1, n-2)$

In this case:

$F = \frac{\hat{b}^2 L_{xy}}{\hat{\sigma}^2} = 528.$

while: $\sigma^2 = 1140922.3021$

$F_{1-\alpha}(1, n-2) = F_{0.95}(1, 16) = 4.49$

$F \geq F_{1-\alpha}(1, n-2)$. therefore reject H_0. 即X.Y显著线性相关.

3.解: From the table can find that

$\sum_{i=1}^{8} x_i = 19.2$ $\sum_{i=1}^{8} x_i^2 = 60.58$ $\sum_{i=1}^{8} x_i y_i = 3.18$

$\sum_{i=1}^{8} y_i = 1.61$ $\sum_{i=1}^{8} y_i^2 = 0.3647$

$\bar{x} = 2.4$ $\bar{y} = 0.2013$

$L_{xx} = 14.5$ $L_{xy} = -0.684$ $L_{yy} = 0.0407$

$\Rightarrow y = 0.3145 - 0.0472x$

$= 0.3145 - 0.0472t$

0.972·*：①

Hypothesis: $H_0: b=0$. $H_1: b\neq 0$.

① T-test.

$$T=\frac{\hat{b}\sqrt{L_{xx}}}{\hat{\sigma}^2}=-4.7945$$

while $Q=0.0842$, $\hat{\sigma}^2=0.0140$

H_0 will be reject if,

$$|T|\geq t_{1-\frac{\alpha}{2}}(n-2)=t_{0.975}(6)=\cancel{2.\ldots}\,2.447$$

$\because |T|=4.7945\geq t_{1-\frac{\alpha}{2}}(n-2)=\cancel{2.\ldots}\,2.447$

$\therefore$ reject H_0. 即流动时间t与氧浓度显著相关.

② F-test

$$F=\frac{\hat{b}^2L_{xx}}{\hat{\sigma}^2}=22.9880$$

H_0 is rejected while $F\geq F_{1-\alpha}(1,n-2)=F_{0.95}(1,6)=5.99$

即流动时间与氧浓度显著相关.

10. 解: 记. $\beta=(\beta_0,\beta_1,\beta_2)^T$. $\cancel{Y}$,

$$X=\begin{pmatrix}1&1000&5\\1&600&7\\1&1200&6\\1&500&6\\1&300&8\\1&400&7\\1&1300&5\\1&1100&4\\1&1300&3\\1&300&9\end{pmatrix}\quad Y=\begin{pmatrix}100\\75\\80\\70\\50\\65\\90\\100\\110\\60\end{pmatrix}$$

$$\hat{\beta}=(X^TX)^{-1}X^T\cdot Y=\begin{pmatrix}1.116918\times10^{2}\\1.4291\times10^{-2}\\-7.1882\times10^{0}\end{pmatrix}$$

即 $\hat{y}=111.6918+0.0143x_1-7.1882x_2$

11. 用Excel得:

$y=-3.7623x^2+34.827x-8.3595$. $R^2=0.9937$

$F=476.518>F_{0.99}(2,7)=9.55$ 说明回归效果显著.

$$\text{记}X=\begin{array}{c}\begin{array}{ccc}&x&x^2\end{array}\\\begin{pmatrix}1&1&1\\1&2&4\\1&3&9\\1&4&16\\1&5&25\\1&6&36\\1&7&49\\1&8&64\\1&9&81\end{pmatrix}\end{array}\quad Y=\begin{pmatrix}21.9\\42.1\\61.9\\70.8\\72.8\\66.4\\50.3\\25.3\\3.2\end{pmatrix}$$

$$\hat{\beta}=(X^TX)^{-1}X^TY=\begin{pmatrix}-8.3595\\34.8267\\-3.7623\end{pmatrix}\begin{array}{l}\to(c)\\\to(x)\\\to(x^2)\end{array}$$

即 $y=-8.3595+34.8267x-3.7623x^2$.

Chapter 6. 3.4.5.6

3. 解: Two - Factor Without Replication.

SUMMARY	Count	Sum	Average	Variance
0.2%	4	26	6.5	9.2433
0.4%	4	35.7	8.925	7.6225
0.8%	4	48	12	6.36
20°C	3	36.7	12.13333	4.1033
0°C	3	31.3	10.4333	10.1633
-20°C	3	22.5	7.5	13.69
-40°C	3	19.2	6.4	5.07

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
含铜量	60.78	2	30.39	34.59	0.000508	5.143253
试验温度	64.32	3	21.44	24.48	0.000924	4.75063
Error	5.27	6	0.88			
Total	130.3692	11				

$F_{0.95}(3,6) = 4.76$ 34.59 > 4.76

$F_{0.95}(2,6) = 5.14$ 24.4

$F(\text{含铜量}) = 34.59 > F_{0.95}(2,6) = 5.14$

$F(\text{试验温度}) = 24.4 > F_{0.95}(3,6) = 4.76$ ，可得:

两者皆会对冲击值产生影响.

4. 解: ANOVA

Source of Var.	SS	df	MS	F	P-value	F crit.
Rows	1243.8	3	414.6	10.558	0.001103	3.49
Columns	778.8	4	194.7	4.958	0.013595	3.26.
Error	471.2	12	39.2667			
Total	2493.8	19				

$F_{0.95}(3,12) = 3.49$

$F_{0.95}(4,12) = 3.26$

由分差分析表中的F值与P-value值判断

两者皆会对抗断强度与显著影响.

5. 解: ANOVA

Source of Var.	SS	df	MS	F	P-value	F crit
Sample	2.75	3	0.9167	0.532	0.664528	3.01
Columns	27.167	2	13.58	7.888	0.00233	3.403
Interaction	73.5	6	12.25	7.113	0.000192	2.508.
Within	41.333	24	1.72			
Total	144.75	35				

$F_{0.95}(2,24) = 3.4 < 7.888$ $F_{0.95}(3,24) = 3.01 > 0.532$, $F_{0.95}(6,24) = 2.51 < 7.113$.

操作工和交互作用对日产量有显著影响，机器无显著影响.

6.解：ANOVA

Source of Var.	SS	df	MS	F	P-value	F crit.
Sample	12.46	3	4.153	2.431	0.187	3.49
Columns	14.33	2	7.167	4.195	0.042	3.89
Interaction	43.67	6	7.278	4.260	0.457	2.996
Within	20.5	12	1.708			
Total	90.96	23				

$F_{0.95}(3,12) = 3.49 > 2.431$ $F_{0.95}(6,12) = 3 < 4.260$

$F_{0.95}(2,12) = 3.89 < 4.195$

得：浓度和交互作用对产量有显著影响。

12.1

习题 8

1.

变量	因子载荷估计 $\hat{a}_{ij} = \sqrt{\lambda_j}\eta_{ij}$ F_1	F_2	变量共同度估计 $\hat{h}_i^2$	特殊因子方差 $\hat{\sigma}_i^2 = 1 - \hat{h}_i^2$
1	0.8757	-0.1802	0.7994	0.2006
2	0.8312	-0.4048	0.8547	0.1453
3	0.7111	0.6951	0.98878	0.01122
方差贡献	1.9634	0.6795		

受公因子影响最大的变量的 X_3；最有影响的公因子是 F_1.

2. 因子旋转前的两个因子载荷表：

	因子 1	因子 2
创汇	0.841	0.451
创润	0.880	0.320
均汇	0.199	0.968
出口合同履约率	0.929	0.045

因子与原变量的相关系数表

	因子 1	因子 2
创汇	0.297	0.149
创润	0.373	-0.12
均汇	-0.286	0.983
出口合同履约率	0.515	-0.335