

$f(\sigma_A, \sigma_B, \rho_{AB})$

Activos	A	y	B
rend. (A.)	$r_A$	y	$r_B$
rend. Esp.	$E[r_A]$		$E[r_B]$
vol.	$\sigma_A$		$\sigma_B$
ponder.	$w_A$	+	$w_B = 1$
	$\downarrow$		$\downarrow$
	0		0

$$w := w_A \geq 0$$

$$1 - w = w_B \geq 0$$

$$0 \leq w \leq 1$$

$$E[r_P] = w E[r_A] + (1-w) E[r_B]$$

$$= w (E[r_A] - E[r_B]) + E[r_B]$$

$$\sigma_P^2 = w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w) \sigma_A \sigma_B \rho_{AB}$$

Queremos analizar el efecto de  $\rho_{AB}$ .

$$\rho_{AB} = 1$$

$$\sigma_P^2 = w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w) \sigma_A \sigma_B$$

$$= [w \sigma_A + (1-w) \sigma_B]^2$$

$$\sigma_P = w(\sigma_A - \sigma_B) + \sigma_B$$

$$\begin{bmatrix} E[r_P] \\ \sigma_P \end{bmatrix} = \begin{bmatrix} E[r_A] - E[r_B] \\ \sigma_A - \sigma_B \end{bmatrix} w + \begin{bmatrix} E[r_B] \\ \sigma_B \end{bmatrix}$$

$$0 \leq w \leq 1$$

$$\boxed{\rho_{AB} = -1}$$

$$\begin{aligned}\sigma_P^2 &= w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 - 2w(1-w)\sigma_A \sigma_B \\ &= [w\sigma_A - (1-w)\sigma_B]^2\end{aligned}$$

$$\begin{aligned}\sigma_P &= |w\sigma_A - (1-w)\sigma_B| \\ &= |w(\sigma_A + \sigma_B) - \sigma_B| \\ &= \begin{cases} w(\sigma_A + \sigma_B) - \sigma_B & \text{si } w \geq \frac{\sigma_B}{\sigma_A + \sigma_B} \\ \sigma_B - w(\sigma_A + \sigma_B) & \text{si } w < \frac{\sigma_B}{\sigma_A + \sigma_B} \end{cases}\end{aligned}$$

$$\text{si } w = \frac{\sigma_B}{\sigma_A + \sigma_B} \Rightarrow \sigma_P = 0$$

$$\boxed{-1 < \rho_{AB} < 1}$$

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multiplicamos por  $2w(1-w)\sigma_A\sigma_B \geq 0$

$$-2w(1-w)\sigma_A\sigma_B < 2w(1-w)\sigma_A\sigma_B \rho_{AB}$$

$$[w\sigma_A - (1-w)\sigma_B]^2$$

$$< 2w(1-w)\sigma_A\sigma_B \text{ suma } w^2\sigma_A^2 + (1-w)^2\sigma_B^2$$

$$w^2\sigma_A^2 + (1-w)^2\sigma_B^2 - 2w(1-w)\sigma_A\sigma_B <$$

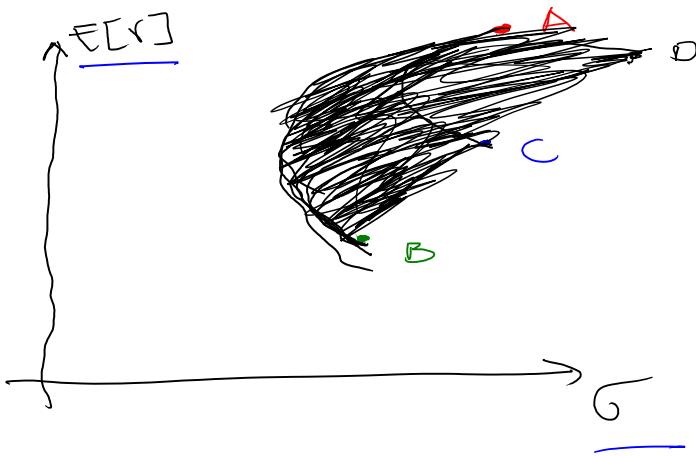
$$\sigma_P^2 \leadsto w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B \rho_{AB} <$$

$$w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B$$

$$[w\sigma_A + (1-w)\sigma_B]^2$$

$$[w\sigma_A - (1-w)\sigma_B]^2 < \sigma_P^2 < [w\sigma_A + (1-w)\sigma_B]^2$$

$$\underbrace{w\sigma_A - (1-w)\sigma_B}_{\rho = -1} < \sigma_P < \underbrace{w\sigma_A + (1-w)\sigma_B}_{\rho = 1}$$



$$\begin{aligned}
 \sigma_P^2 &= \sum_{i=1}^n \sum_{k=1}^n w_i w_k \sigma_{ik} \\
 &= \sum_{i=1}^n \left[ \sum_{\substack{k=1 \\ k \neq i}}^n w_i w_k \sigma_{ik} + \sum_{k=1}^n w_i w_k \sigma_{ii} \right] \\
 &= \underbrace{\sum_{i=1}^n \underbrace{w_i w_i}_{w_i^2} \underbrace{\sigma_{ii}}_{\sigma_i^2}}_{n \text{ términos}} + \underbrace{\sum_{i=1}^n \sum_{\substack{k=1 \\ k \neq i}}^n w_i w_k \sigma_{ik}}_{n(n-1) \text{ términos}}
 \end{aligned}$$

$n=2$

$$\sigma_P^2 = \underbrace{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2}_{i=1} + \underbrace{w_1 w_2 \sigma_{12}}_{i=2} + \underbrace{w_2 w_1 \sigma_{21}}_{i=2}$$

$n=3$

$$\begin{aligned}
 \sigma_P^2 &= \underbrace{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2}_{k=2, 3} + \\
 &\quad \underbrace{w_1 w_2 \sigma_{12} + w_1 w_3 \sigma_{13}}_{k=3} + \\
 &\quad \underbrace{w_2 w_1 \sigma_{21} + w_2 w_3 \sigma_{23}}_{k=1} + \\
 &\quad \underbrace{w_3 w_1 \sigma_{31} + w_3 w_2 \sigma_{32}}_{k=2}
 \end{aligned}$$

$$\sum_{i=1}^n a_i : n \text{ términos}$$

$$\sum_{i=1}^n \sum_{\substack{k=1 \\ k \neq i}}^n a_{ik} : n(n-1) \text{ términos}$$

$$n \qquad n(n-1)$$

$$2 \qquad 2$$

$$10 \qquad 90$$

$$100 \quad \ll \quad 9900$$

$$1000 \quad \ll \quad 999000$$