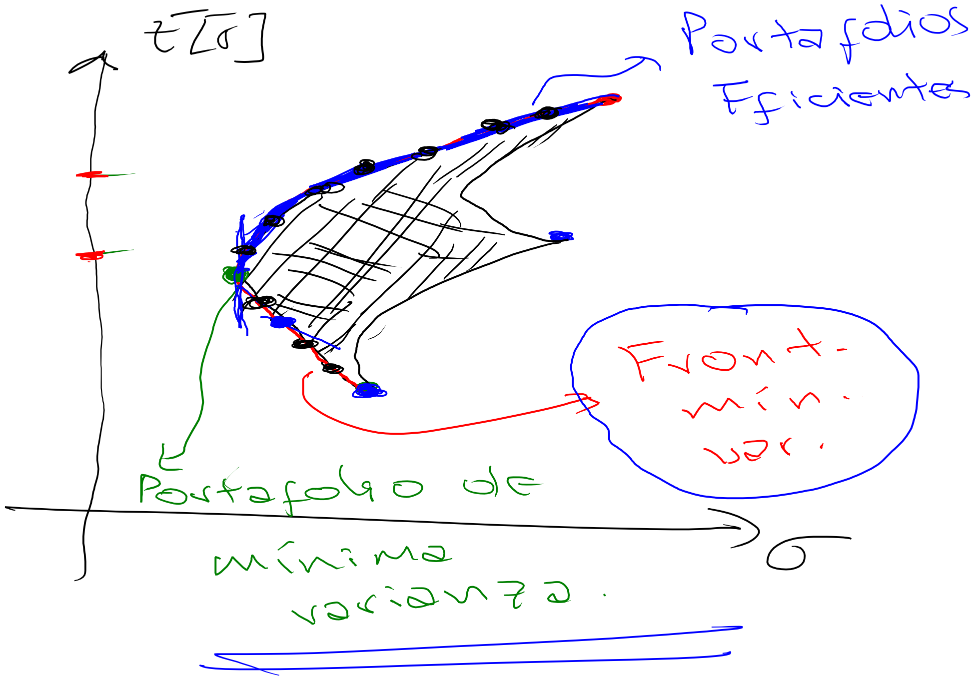

Portafolio de mín. var. (con 2 activos)

$$\min_{0 \leq w \leq 1} \quad f(w) := w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2\underbrace{w(1-w)}\sigma_{12}$$

$$\frac{d}{dw}(w - w^2)$$

$$1 - 2w$$

$$f'(w) = 0$$

$$\iff \cancel{2}w\sigma_1^2 - \cancel{2}(1-w)\sigma_2^2 + \cancel{2}(1-2w)\sigma_{12} = 0$$

$$2w(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) - \sigma_2^2 + \sigma_{12} = 0$$

$$\boxed{w^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}}$$

$$f''(w) = 2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) \geq 0$$

Queremos ver que

$$\sigma_1^2 + \sigma_2^2 - 2\underbrace{\sigma_{12}}_{\sigma_1\sigma_2\rho_{12}} \geq 0$$

$\underline{\underline{\rho_{12} \leq 1}}$; multiplicar por $-2\sigma_1\sigma_2$

$$\underbrace{-2\sigma_1\sigma_2\rho_{12}}_{\sigma_{12}} \geq -2\sigma_1\sigma_2 \text{ ; sumar } \sigma_1^2 + \sigma_2^2$$

$$\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \geq \underline{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2}$$

$$= (\sigma_1 - \sigma_2)^2 \geq 0$$

Por el criterio de la II derivada

$$w^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

es el punto de mínimo.