# T-Fold Sequential Validation Technique for Out-Of-Distribution Generalization with Financial Time Series Data

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**Hipothesis:** There exists a set of conditions under which a cross-validation process can be defined and conducted in order to achieve Out-Of-Sample and Out-Of-Distribution Generalization when performing a Predictive Modeling Process using Financial Time Series Data.

**Dataset:** Continuous futures prices of the UsdMxn (U.S. Dollar Vs Mexican Peso), extracted from CME group MP Future Contract. Prices are Open, High, Low, Close in intervals of 8 Hours, **OHLC** data. GMT timezone-based and a total of 66,500 from 2010-01-03 18:00:00 to 2021-06-14 16:00:00.

**Experiment:** A classification problem is formulated as to predict the target variable, $CO_{t+1}$, which is defined as the sign$(Close_{t+1} - Open_{t+1})$. For the explanatory variables, the base definition is to use only those of endogenous nature, that is, to create them using only **OHLC** values.

## A discrete representation

Let $V_t$ be the value of a financial asset at any given time $t$, and $S_t$ as a discrete representation of $V_t$ if there is an observable transaction $Ts_t$. Similarly, if there is a set of discrete $Ts_t$ observed during an interval of time $T$ of $n = 1, 2, ..., n$ units of time, $\{S_T\}_{T=1}^{n}$, can be represented by $OHLC_T$ : $\{Open_t, High_t, Low_t, Close_t\}$. The frequency of sampling $T$, can be arbitrarly defined.

## OHLC data

*Timestamp*: The date and time for each interval.
**O**pen: The first price of the interval.
**H**igh: The highest price during the interval.
**L**ow: The lowest price during the interval.
**C**lose: The last price of the interval

Intra-day micro-information:
**volatility:** $HL_t$ , **price-change:** $CO_t$
**uptrend:** $HO_t$ , **downtrend:** $OL_t$

## Candlestick Visual Representation *(Figure 1)*

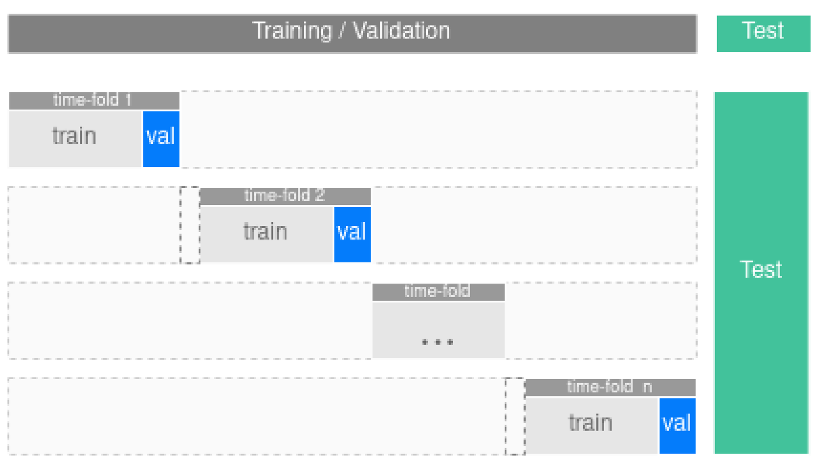
The base calculations are:

$$HL_t = High_t - Low_t$$
$$OL_t = Open_t - Low_t$$
$$CO_t = Close_t - Open_t$$
$$HO_t = High_t - Open_t$$


## T-Fold-SV *(Steps)*

**1.- Folds Formation**
Depends on labeling, can be calendar based.
**2.- Target and Feature Engineering**
In-Fold exclusive or Global and then divide.
**3.- Information matrix**
To asses information sparsity among Folds.
**4.- Predictive Modeling**
Hyperparameter optimization Train-Val sets.
**5.- Generalization Assesment**
Out-Of-Sample and/or Out-Of-Distribution.

## 1: Folds Formation *(Figure 2)*




## 2: Target Variable (labeling)

A continuous variable prediction (regression problem), into a discrete variable prediction (classification problem), a time-based labeling can be stated as:

$$\hat{y}_t = sign\{CO_t\}$$

Interesting enough, this target variable never had an imbalace of classes more than 5.5%

## 2: Feature Engineering

with $\{OL\}_{t-k}$, $\{HO\}_{t-k}$, $\{HL\}_{t-k}$, $\{CO\}_{t-k}$ for values of $k = 1, 2, ...K$, with $K$ as a proposed *memory* parameter. Then perform some fundamental operations: Simple Moving Average $SMA_t$, lag: $LAG_t$, Standard Deviation: $SD_t$ and Cumulative Sumation: $CUMSUM_t$.

Then symbolic variables where generated using Genetic Programming.

## 3.1: Information Representation and Sparsity

A gamma distribution to fit the PDF of two set of variables, and the Kullback-Leibler Divergence to measure the similarity between the two:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad \text{for} \quad x > 0 \quad \alpha, \beta > 0 \qquad (1)$$

$\Gamma(\alpha)$: The gamma function $\forall\ \alpha \in \mathbb{Z}^+$ and the $D_{KL}(P||Q)$: Kullback-Liebler Divergence, which for unknown continuous random variables, $P, Q$, or for $p, q$ as empirically adjusted Probability Density Functions (PDF) is denoted by:

$$D_{KL}(P||Q) = \textstyle\int_{-\infty}^{\infty} p(x) log\Big(\frac{p(x)}{q(x)}\Big) dx \qquad (2)$$

## 3.2: Information matrix

An *Information Matrix* (IM) represents the similarity in information, for the target varible, among every Fold.

$$IM = \begin{vmatrix} D_{KL(1,1)} & D_{KL(1,2)} & \dots & D_{KL(1,n)} \\ D_{KL(2,1)} & & & \vdots \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & D_{KL(n-1,n)} \\ D_{KL(n,1)} & \dots & D_{KL(n,n-1)} & D_{KL(n,n)} \end{vmatrix}$$

$D_{KL}$ is a non-conumtative operation, hence $D_{KL}(P||Q) \neq D_{KL}(Q||P)$. That means the *Information Matrix* (IM), is not symmetric, but has 0's in its diagonal.

## 3.3: Matrix Characterization

If an *Information Threshold* is defined, and then applied to every value in IM, then the latter can be characterized according to a counting of following:

- **Sparse**:
All the elements of IM are sufficient disimilar among each other.
- **Weakly Sparse**:
There exists one or more very similar pairs of elements.
- **Non-sparse**:
ALll elements are highly similar to each other.

The ideal in theory is to have a *Sparse Information Matrix* to train any model, so to use non-repeated data.

## 4.1: Cost Function and Regularization

One common component of the predictive modeling process is binary-logloss cost function with *elasticnet* regularization:

$$J(w) = J(w) + C\frac{\lambda}{m}\sum_{j=1}^{n} \|w_j\|_1 + (1 - C)\frac{\lambda}{2m}\sum_{j=1}^{n} \|w_j\|_2^2$$

Where $\Sigma_{j=1}^{n} \|w_j\|_1 = L_1$ and $\Sigma_{j=1}^{n} \|w_j\|_2^2 = L_2$ are also known as *Lasso* and *Ridge* respectively, with $C$ as the coefficient to regulate the effect between the two.

## 4.2: Model's Params

**Logistic Regression**
- L1_L2_Ratio = 1.0 (Lasso) - Inverse of regularization (C): 1.5 - Parameter repetitions (Stability): Yes

**ANN-MLP**
- Hidden Layers: 2, 80 neurons each
- Activations: ReLU
- Dropout: 10% all layers

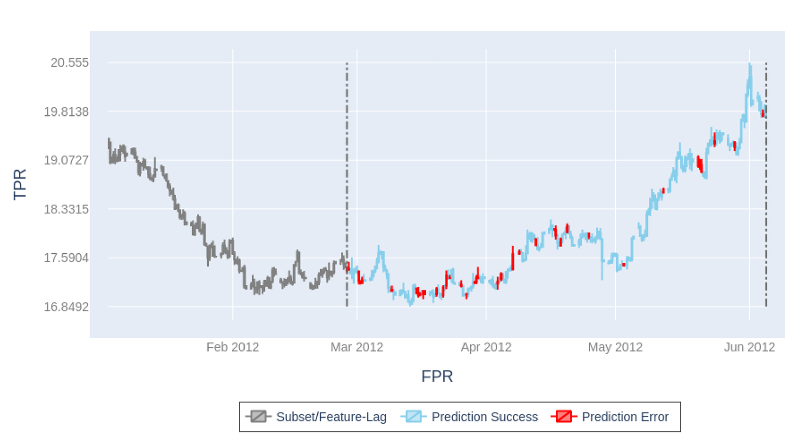
## 4.3: Results

Two models were defined, Logistic-Regression and Multi-layer Feedforward Perceptron.

| Metric | ann-mlp | logistic |
|---|---|---|
| acc-train | 0.9155 | 0.8311 |
| acc-val | 0.8245 | 0.7368 |
| acc-weighted | 0.4486 | 0.4061 |
| acc-inv-weighted | 0.4213 | 0.3778 |
| auc-train | 0.9924 | 0.9300 |
| auc-val | 0.8401 | 0.8017 |

| Metric | ann-mlp | logistic |
|---|---|---|
| auc-weighted | 0.4810 | 0.4521 |
| auc-inv-weighted | 0.4353 | 0.4137 |
| logloss-train | 0.2290 | 5.8333 |
| logloss-val | 6.0595 | 9.0892 |
| logloss-weighted | 0.6975 | 3.2422 |
| logloss-inv-weighted | 2.4467 | 4.2190 |

## Train-Logistic

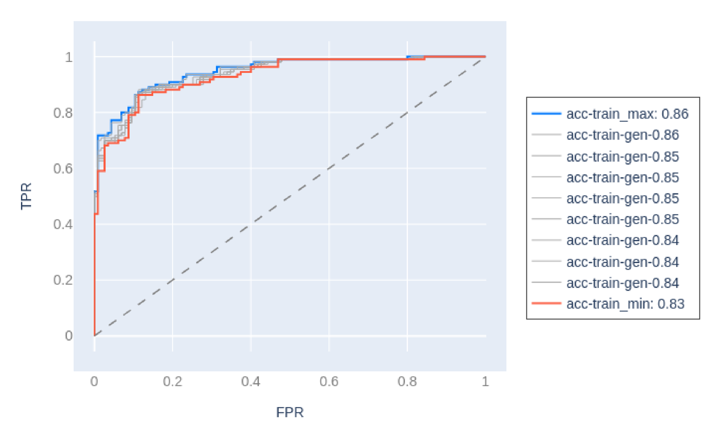



## Validation-Logistic

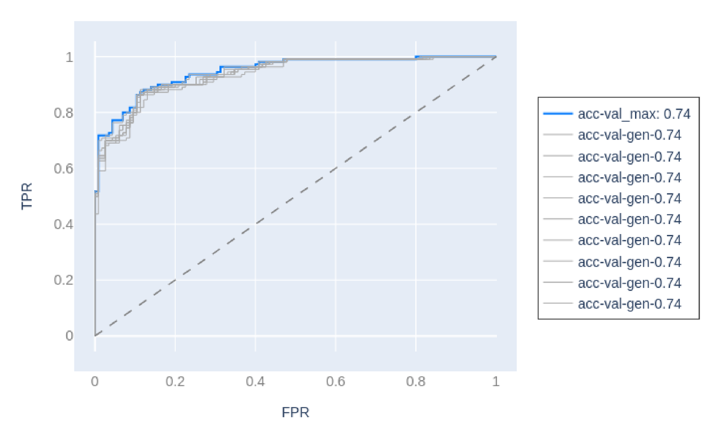

## Train-NeuralNet


## Validation-NeuralNet

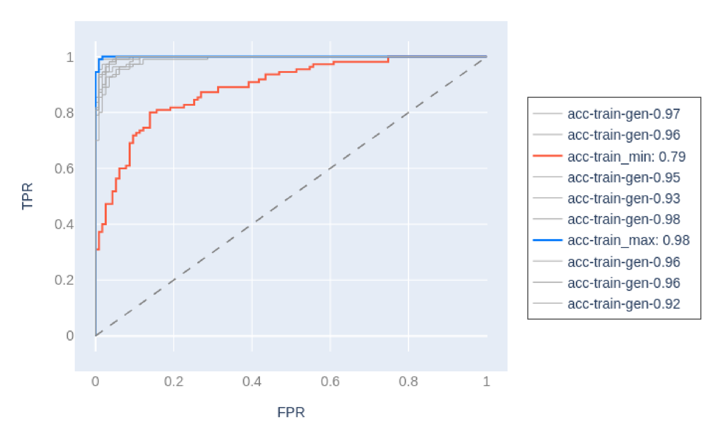

## ROCs-Train-Logistic



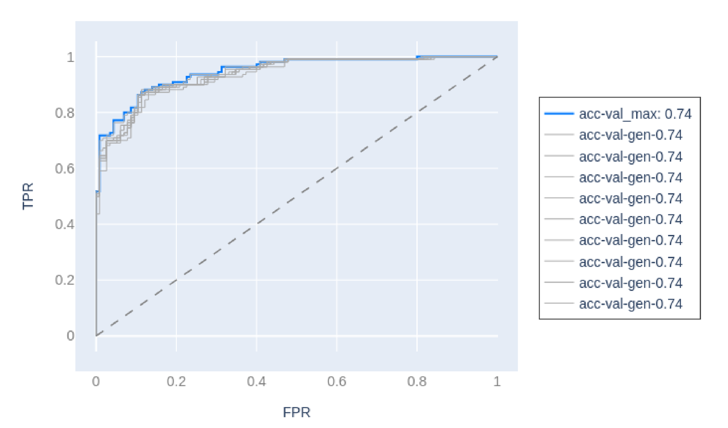

## ROCs-Validation-Logistic




## ROCs-Train-NeuralNet




## ROCs-Validation-NeuralNet




## References

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- Pezeshki et al (2020). *Gradient Starvation:A Learning Proclivity in Neural Networks*, Mohammad Pezeshki, Sekou-Oumar Kaba, Yoshua Bengio, Aaron Courville, Doina Precup, Guillaume Lajoie, arXiv:2011.09468.
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