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# **prpy: Probabilistic Robot Localization Python Library**

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**Probabilistic Robot Localization** is Python Library containing the main algorithms explained in the **Probabilistic Robot Localization** Book used in the **Probabilistic Robotics** and the **Hands-on Localization** Courses of the **Intelligent Field Robotic Systems (IFRoS)** European Erasmus Mundus Master.

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**Note:** This documentation is still under construction.

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## 1.1 Pose Representation

### 1.1.1 Pose 3DOF

**class** Pose3D.Pose3D(input\_array)

Bases: ndarray

Definition of a robot pose in 3 DOF (x, y, yaw). The class inherits from a ndarray. This class extends the ndarray with the \$plus\$ and \$minus\$ operators and the corresponding Jacobians.

**oplus**(BxC)

Given a Pose3D object  $AxB$  (the self object) and a Pose3D object  $BxC$ , it returns the Pose3D object  $AxC$ .

$$\begin{aligned} \mathbf{A}_{\mathbf{x}_B} &= [{}^A x_B \quad {}^A y_B \quad {}^A \psi_B]^T \\ \mathbf{B}_{\mathbf{x}_C} &= [{}^B x_C \quad {}^B y_C \quad {}^B \psi_C]^T \end{aligned}$$

The operation is defined as:

$$\mathbf{A}_{\mathbf{x}_C} = \mathbf{A}_{\mathbf{x}_B} \oplus \mathbf{B}_{\mathbf{x}_C} = \begin{bmatrix} {}^A x_B + {}^B x_C \cos({}^A \psi_B) - {}^B y_C \sin({}^A \psi_B) \\ {}^A y_B + {}^B x_C \sin({}^A \psi_B) + {}^B y_C \cos({}^A \psi_B) \\ {}^A \psi_B + {}^B \psi_C \end{bmatrix} \quad (1.1)$$

**Parameters**

**BxC** – C-Frame pose expressed in B-Frame coordinates

**Returns**

C-Frame pose expressed in A-Frame coordinates

**J\_1oplus**(BxC)

Jacobian of the pose compounding operation (eq. (1.1)) with respect to the first pose:

$$J_{1\oplus} = \frac{\partial {}^A x_B \oplus {}^B x_C}{\partial {}^A x_B} = \begin{bmatrix} 1 & 0 & -{}^B x_C \sin({}^A \psi_B) - {}^B y_C \cos({}^A \psi_B) \\ 0 & 1 & {}^B x_C \cos({}^A \psi_B) - {}^B y_C \sin({}^A \psi_B) \\ 0 & 0 & 1 \end{bmatrix} \quad (1.2)$$

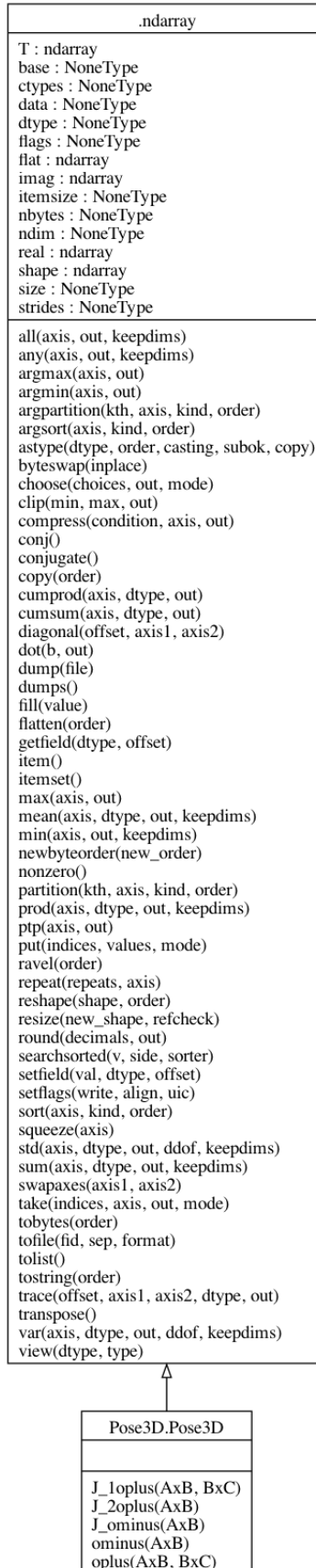
The method returns a numerical matrix containing the evaluation of the Jacobian for the pose  $AxB$  (the self object) and the  ${}^2\text{nd}$  pose  $BxC$ .

**Parameters**

**BxC** – 2nd pose

**Returns**

Evaluation of the  $J_{1\oplus}$  Jacobian of the pose compounding operation with respect to the first pose (eq. (1.2))





### J\_2oplus()

Jacobian of the pose compounding operation ((1.1)) with respect to the second pose:

$$J_{2\oplus} = \frac{\partial^A x_B \oplus^B x_C}{\partial^B x_C} = \begin{bmatrix} \cos({}^A\psi_B) & -\sin({}^A\psi_B) & 0 \\ \sin({}^A\psi_B) & \cos({}^A\psi_B) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1.3)$$

The method returns a numerical matrix containing the evaluation of the Jacobian for the  ${}^A\psi_B$  posepose  $AxB$  (the self object).

#### Returns

Evaluation of the  $J_{2\oplus}$  Jacobian of the pose compounding operation with respect to the second pose (eq. (1.3))

### ominus()

Inverse pose compounding of the  $AxB$  pose (the self objetc):

$${}^B x_A = \ominus^A x_B = \begin{bmatrix} -{}^A x_B \cos({}^A\psi_B) - {}^A y_B \sin({}^A\psi_B) \\ {}^A x_B \sin({}^A\psi_B) - {}^A y_B \cos({}^A\psi_B) \\ -{}^A\psi_B \end{bmatrix} \quad (1.4)$$

#### Returns

A-Frame pose expressed in B-Frame coordinates (eq. (1.4))

### J\_ominus()

Jacobian of the inverse pose compounding operation ((1.1)) with respect the pose  $AxB$  (the self object):

$$J_{\ominus} = \frac{\partial \ominus^A x_B}{\partial^A x_B} = \begin{bmatrix} -\cos({}^A\psi_B) & -\sin({}^A\psi_B) & {}^A x_B \sin({}^A\psi_B) - {}^A y_B \cos({}^A\psi_B) \\ \sin({}^A\psi_B) & -\cos({}^A\psi_B) & {}^A x_B \cos({}^A\psi_B) + {}^A y_B \sin({}^A\psi_B) \\ 0 & 0 & -1 \end{bmatrix} \quad (1.5)$$

Returns the numerical matrix containing the evaluation of the Jacobian for the pose  $AxB$  (the self object).

#### Returns

Evaluation of the  $J_{\ominus}$  Jacobian of the inverse pose compounding operation with respect to the pose (eq. (1.5))

## 1.2 Robot Simulation

**class** SimulatedRobot.**SimulatedRobot**(xs0, map=[], \*args)

Bases: object

This is the base class to simulate a robot. There are two operative frames: the world N-Frame (North East Down oriented) and the robot body frame body B-Frame. Each robot has a motion model and a measurement model. The motion model is used to simulate the robot motion and the measurement model is used to simulate the robot measurements.

**All Robot simulation classes must derive from this class .**

**dt = 0.1**

class attribute containing sample time of the simulation

**\_\_init\_\_**(xs0, map=[], \*args)

#### Parameters

- **xs0** – initial simulated robot state  $x_{s_0}$  used to initialize the the motion model

SimulatedRobot.SimulatedRobot
M : list Qsk : NoneType Rsk : NoneType dt : float k : int nf plt_samples : list trajectory usk : NoneType vehicleAxes vehicleFig : NoneType vehicleIcon : VehicleIcon visualizationInterval : int xTraj : list xsk : NoneType xsk_1 yTraj : list
PlotRobot() SetMap(map) fs(xsk_1, uk)

Fig. 1: SimulatedRobot Class Diagram.

- **map** – feature map of the environment  $M = [^N x_{F_1}^T, \dots, ^N x_{F_{n_f}}^T]^T$

Constructor. First, it initializes the robot simulation defining the following attributes:

- **k** : time step
- **Qsk** : **To be defined in the derived classes.** Object attribute containing Covariance of the simulation motion model noise
- **usk** : **To be defined in the derived classes.** Object attribute contining the simulated input to the motion model
- **xsk** : **To be defined in the derived classes.** Object attribute contining the current simulated robot state
- **zsk** : **To be defined in the derived classes.** Object attribute contining the current simulated robot measurement
- **Rsk** : **To be defined in the derived classes.** Object attribute contining the observation noise covariance matrix
- **xsk** : current pose is the initial state
- **xsk\_1** : previous state is the initial robot state
- **M** : position of the features in the N-Frame
- **nf** : number of features

Then, the robot animation is initialized defining the following attributes:

- **vehicleIcon** : Path file of the image of the robot to be used in the animation
- **vehicleFig** : Figure of the robot to be used in the animation
- **vehicleAxes** : Axes of the robot to be used in the animation

- **xTraj** : list containing the x coordinates of the robot trajectory
- **yTraj** : list containing the y coordinates of the robot trajectory
- **visualizationInterval** : time-steps interval between two consecutive frames of the animation

#### PlotRobot()

Updates the plot of the robot at the current pose

#### fs(xsk\_1, usk)

Motion model used to simulate the robot motion. Computes the current robot state  $x_k$  given the previous robot state  $x_{k-1}$  and the input  $u_k$ . It also updates the object attributes  $xsk$ ,  $xsk_1$  and  $usk$  to be made them available for plotting purposes. *To be overridden in child class.*

##### Parameters

- **xsk\_1** – previous robot state  $x_{k-1}$
- **usk** – model input  $u_{sk}$

##### Returns

current robot state  $x_k$

#### SetMap(map)

Initializes the map of the environment.

#### \_PlotSample(x, P, n)

Plots n samples of a multivariate gaussian distribution. This function is used only for testing, to plot the uncertainty through samples. :param x: mean pose of the distribution :param P: covariance of the distribution :param n: number of samples to plot

## 1.2.1 3 DOF Diferential Drive Robot Simulation

**class** DifferentialDriveSimulatedRobot.DifferentialDriveSimulatedRobot(xs0, map=[], \*args)

Bases: *SimulatedRobot*

This class implements a simulated differential drive robot. It inherits from the *SimulatedRobot* class and overrides some of its methods to define the differential drive robot motion model.

**\_\_init\_\_**(xs0, map=[], \*args)

##### Parameters

- **xs0** – initial simulated robot state  $\mathbf{x}_{s0} = [^N x_{s0} \ ^N y_{s0} \ ^N \psi_{s0}]^T$  used to initialize the motion model
- **map** – feature map of the environment  $M = [^N x_{F_1}, \dots, ^N x_{F_{nf}}]$

Initializes the simulated differential drive robot. Overrides some of the object attributes of the parent class *SimulatedRobot* to define the differential drive robot motion model:

- **Qsk** : Object attribute containing Covariance of the simulation motion model noise.

$$Q_k = \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{bmatrix} \quad (1.6)$$

- **usk** : Object attribute containing the simulated input to the motion model containing the forward velocity  $u_k$  and the angular velocity  $r_k$

$$\mathbf{u}_k = [u_k \ r_k]^T \quad (1.7)$$

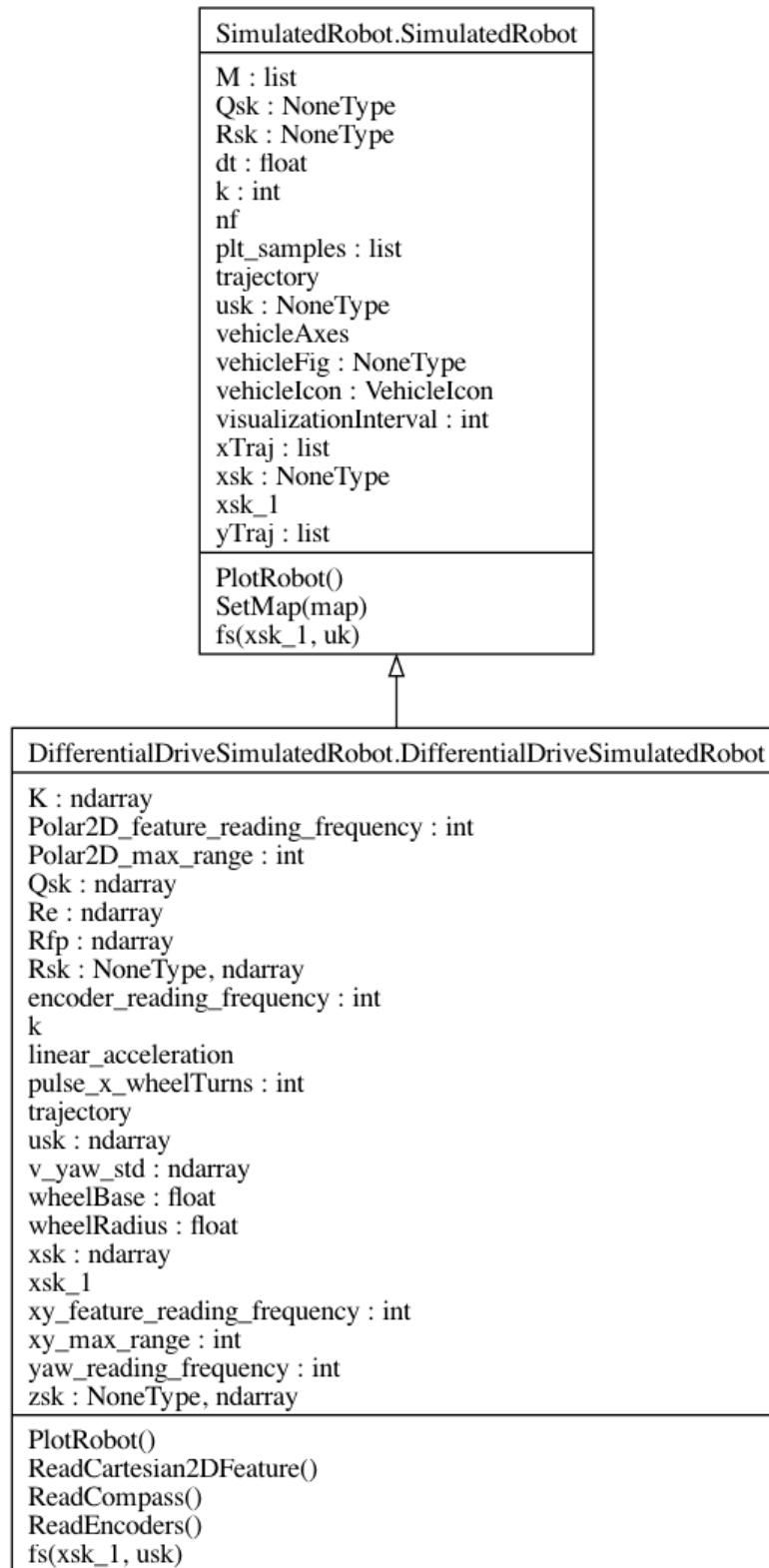


Fig. 2: DifferentialDriveSimulatedRobot Class Diagram.

- **xsk** : Object attribute containing the current simulated robot state

$$x_k = [{}^N x_k \quad {}^N y_k \quad {}^N \theta_k \quad {}^B u_k \quad {}^B v_k \quad {}^B r_k]^T \quad (1.8)$$

where  ${}^N x_k$ ,  ${}^N y_k$  and  ${}^N \theta_k$  are the robot position and orientation in the world N-Frame, and  ${}^B u_k$ ,  ${}^B v_k$  and  ${}^B r_k$  are the robot linear and angular velocities in the robot B-Frame.

- **zsk** : Object attribute containing  $z_{s_k} = [n_L \ n_R]^T$  observation vector containing number of pulses read from the left and right wheel encoders.
- **Rsk** : Object attribute containing  $R_{s_k} = \text{diag}(\sigma_L^2, \sigma_R^2)$  covariance matrix of the noise of the read pulses`.
- **wheelBase** : Object attribute containing the distance between the wheels of the robot ( $w = 0.5$  m)
- **wheelRadius** : Object attribute containing the radius of the wheels of the robot ( $R = 0.1$  m)
- **pulses\_x\_wheelTurn** : Object attribute containing the number of pulses per wheel turn ( $pulseXwheelTurn = 1024$  pulses)
- **Polar2D\_max\_range** : Object attribute containing the maximum Polar2D range ( $Polar2Dmaxrange = 50$  m) at which the robot can detect features.
- **Polar2D\_feature\_reading\_frequency** : Object attribute containing the frequency of Polar2D feature readings (50 tics -sample times-)
- **Rfp** : Object attribute containing the covariance of the simulated Polar2D feature noise ( $R_{fp} = \text{diag}(\sigma_\rho^2, \sigma_\phi^2)$ )

Check the parent class `prpy.SimulatedRobot` to know the rest of the object attributes.

**fs**(*xsk\_I*, *usk*)

Motion model used to simulate the robot motion. Computes the current robot state  $x_k$  given the previous robot state  $x_{k-1}$  and the input  $u_k$ :

$$\begin{aligned} \eta_{s_{k-1}} &= [x_{s_{k-1}} \quad y_{s_{k-1}} \quad \theta_{s_{k-1}}]^T \\ \nu_{s_{k-1}} &= [u_{s_{k-1}} \quad v_{s_{k-1}} \quad r_{s_{k-1}}]^T \\ x_{s_{k-1}} &= [\eta_{s_{k-1}}^T \quad \nu_{s_{k-1}}^T]^T \\ u_{s_k} &= \nu_d = [u_d \quad r_d]^T \\ w_{s_k} &= \dot{\nu}_{s_k} \\ x_{s_k} &= f_s(x_{s_{k-1}}, u_{s_k}, w_{s_k}) \\ &= \begin{bmatrix} \eta_{s_{k-1}} \oplus (\nu_{s_{k-1}} \Delta t + \frac{1}{2} w_{s_k} \Delta t^2) \\ \nu_{s_{k-1}} + K(\nu_d - \nu_{s_{k-1}}) + w_{s_k} \Delta t \end{bmatrix} \quad ; \quad K = \text{diag}(k_1, k_2, k_3) \quad k_i > 0 \end{aligned} \quad (1.9)$$

Where  $\eta_{s_{k-1}}$  is the previous 3 DOF robot pose (x,y,yaw) and  $\nu_{s_{k-1}}$  is the previous robot velocity (velocity in the direction of x and y B-Frame axis of the robot and the angular velocity).  $u_{s_k}$  is the input to the motion model containing the desired robot velocity in the x direction ( $u_d$ ) and the desired angular velocity around the z axis ( $r_d$ ).  $w_{s_k}$  is the motion model noise representing an acceleration perturbation in the robot axis. The  $w_{s_k}$  acceleration is the responsible for the slight velocity variation in the simulated robot motion.  $K$  is a diagonal matrix containing the gains used to drive the simulated velocity towards the desired input velocity.

Finally, the class updates the object attributes *xsk*, *xsk\_1* and *usk* to made them available for plotting purposes.

**To be completed by the student.**

**Parameters**

- **xsk\_1** – previous robot state  $x_{s_{k-1}} = [\eta_{s_{k-1}}^T \quad \nu_{s_{k-1}}^T]^T$
- **usk** – model input  $u_{s_k} = \nu_d = [u_d \quad r_d]^T$

**Returns**

current robot state  $x_{s_k}$

**ReadEncoders()**

Simulates the robot measurements of the left and right wheel encoders.

**To be completed by the student.**

**Return zsk,Rsk**

$z_k = [\Delta n_L \quad \Delta n_R]^T$  observation vector containing number of pulses read from the left and right wheel encoders during the last differential motion.  $R_{s_k} = \text{diag}(\sigma_L^2, \sigma_R^2)$  covariance matrix of the read pulses.

**ReadCompass()**

Simulates the compass reading of the robot.

**Returns**

yaw and the covariance of its noise  $R_{\text{yaw}}$

**ReadCartesian2DFeature()**

Simulates the reading of 2D cartesian features. The features are placed in the map in cartesian coordinates.

**Returns**

**zsk:**  $[[x_1 \ y_1], \dots, [x_n \ y_n]]$

Cartesian position of the feature observations.

**Rsk:**  $\text{block\_diag}(R_1, \dots, R_n)$ , where  $R_i = [[r_{xx} \ r_{xy}], [r_{xy} \ r_{yy}]]$  is the

2x2 i-th feature observation covariance. Covariance of the Cartesian feature observations.

Note the features are uncorrelated among them. They are independent. However, the x and y coordinates of each feature are correlated.

**PlotRobot()**

Updates the plot of the robot at the current pose

## 1.3 Robot Localization

### 1.3.1 Robot Localization

**class** Localization.**Localization**(index, kSteps, robot, x0, \*args)

Bases: object

Localization base class. Implements the localization algorithm.

**\_\_init\_\_**(index, kSteps, robot, x0, \*args)

Constructor of the DRLocalization class.

**Parameters**

- **index** – Logging index structure (prpy.Index)
- **kSteps** – Number of time steps to simulate
- **robot** – Simulation robot object (prpy.Robot)

Localization.Localization
index k : int kSteps log_x : ndarray log_xs : ndarray plot_xy_estimation : bool robot trajectory xTraj : list xk xk_1 yTraj : list
GetInput() LocalizationLoop(x0, usk) Localize(xk_1, uk) Log(xsk, xk) PlotTrajectory() PlotXY()

- **args** – Rest of arguments to be passed to the parent constructor
- **x0** – Initial Robot pose in the N-Frame

#### GetInput()

Gets the input from the robot. To be overridden by the child class.

**Return uk**  
input variable

#### Localize(xk\_1, uk)

Single Localization iteration invoked from prpy.DRLocalization.Localization(). Given the previous robot pose, the function reads the inout and computes the current pose.

**Parameters**  
**xk\_1** – previous robot pose

**Return xk**  
current robot pose

#### LocalizationLoop(x0, usk)

Given an initial robot pose  $x_0$  and the input to the prpy.SimulatedRobot this method calls iteratively prpy.DRLocalization.Localize() for k steps, solving the robot localization problem.

**Parameters**  
**x0** – initial robot pose

#### Log(xsk, xk)

Logs the results for later plotting.

**Parameters**

- **xsk** – ground truth robot pose from the simulation
- **xk** – estimated robot pose

#### PlotXY()

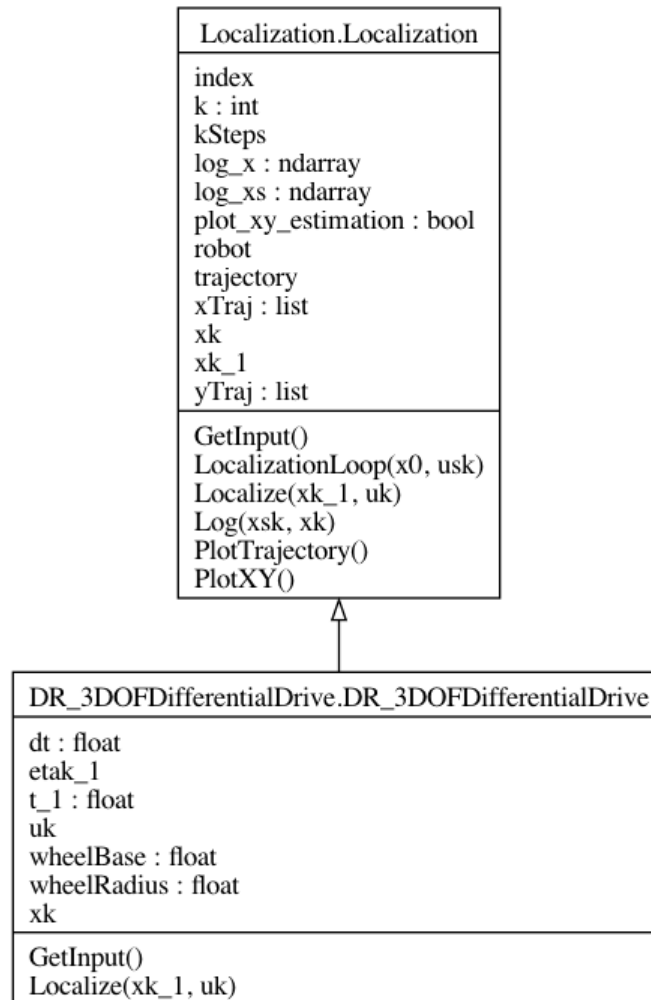
Plots, in a new figure, the ground truth (orange) and estimated (blue) trajectory of the robot at the end of the Localization Loop.

**PlotTrajectory()**

Plots the estimated trajectory (blue) of the robot during the localization process.

### 1.3.2 Dead Reckoning

#### 3 DOF Differential Drive Mobile Robot Example



```
class DR_3DOFDifferentialDrive.DR_3DOFDifferentialDrive(index, kSteps, robot, x0, *args)
```

Bases: `Localization`

Dead Reckoning Localization for a Differential Drive Mobile Robot.

```
__init__(index, kSteps, robot, x0, *args)
```

Constructor of the `prlab.DR_3DOFDifferentialDrive` class.

**Parameters**

**args** – Rest of arguments to be passed to the parent constructor

```
Localize(xk_1, uk)
```

Motion model for the 3DOF ( $x_k = [x_k \ y_k \ \psi_k]^T$ ) Differential Drive Mobile robot using as input the readings of the wheel encoders ( $u_k = [n_L \ n_R]^T$ ).



**Parameters**

- **xk\_1** – previous robot pose estimate ( $x_{k-1} = [x_{k-1} \ y_{k-1} \ \psi_{k-1}]^T$ )
- **uk** – input vector ( $u_k = [u_k \ v_k \ r_k]^T$ )

**Return xk**

current robot pose estimate ( $x_k = [x_k \ y_k \ \psi_k]^T$ )

**GetInput()**

Get the input for the motion model. In this case, the input is the readings from both wheel encoders.

**Returns**

uk: input vector ( $u_k = [n_L \ n_R]^T$ )



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