
prpy: Probabilistic Robot Localization Python Library

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Pere Ridao

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Probabilistic Robot Localization is Python Library containing the main algorithms explained in the **Probabilistic Robot Localization** Book used in the **Probabilistic Robotics** and the **Hands-on Localization** Courses of the **Intelligent Field Robotic Systems (IFRoS)** European Erasmus Mundus Master.

Note: This documentation is still under construction.

1.1 Pose Representation

1.1.1 Pose 3DOF

class Pose3D.Pose3D(input_array)

Bases: ndarray

Definition of a robot pose in 3 DOF (x, y, yaw). The class inherits from a ndarray. This class extends the ndarray with the \$plus\$ and \$minus\$ operators and the corresponding Jacobians.

oplus(BxC)

Given a Pose3D object AxB (the self object) and a Pose3D object BxC , it returns the Pose3D object AxC .

$$\begin{aligned} \mathbf{A}_{\mathbf{x}_B} &= [{}^A x_B \quad {}^A y_B \quad {}^A \psi_B]^T \\ \mathbf{B}_{\mathbf{x}_C} &= [{}^B x_C \quad {}^B y_C \quad {}^B \psi_C]^T \end{aligned}$$

The operation is defined as:

$$\mathbf{A}_{\mathbf{x}_C} = \mathbf{A}_{\mathbf{x}_B} \oplus \mathbf{B}_{\mathbf{x}_C} = \begin{bmatrix} {}^A x_B + {}^B x_C \cos({}^A \psi_B) - {}^B y_C \sin({}^A \psi_B) \\ {}^A y_B + {}^B x_C \sin({}^A \psi_B) + {}^B y_C \cos({}^A \psi_B) \\ {}^A \psi_B + {}^B \psi_C \end{bmatrix} \quad (1.1)$$

Parameters

BxC – C-Frame pose expressed in B-Frame coordinates

Returns

C-Frame pose expressed in A-Frame coordinates

J_1oplus(BxC)

Jacobian of the pose compounding operation (eq. (1.1)) with respect to the first pose:

$$J_{1\oplus} = \frac{\partial \mathbf{A}_{\mathbf{x}_B} \oplus \mathbf{B}_{\mathbf{x}_C}}{\partial \mathbf{A}_{\mathbf{x}_B}} = \begin{bmatrix} 1 & 0 & -{}^B x_C \sin({}^A \psi_B) - {}^B y_C \cos({}^A \psi_B) \\ 0 & 1 & {}^B x_C \cos({}^A \psi_B) - {}^B y_C \sin({}^A \psi_B) \\ 0 & 0 & 1 \end{bmatrix} \quad (1.2)$$

The method returns a numerical matrix containing the evaluation of the Jacobian for the pose AxB (the self object) and the ${}^2\text{nd}$ pose BxC .

Parameters

BxC – 2nd pose

Returns

Evaluation of the $J_{1\oplus}$ Jacobian of the pose compounding operation with respect to the first pose (eq. (1.2))

J_2oplus()

Jacobian of the pose compounding operation ((1.1)) with respect to the second pose:

$$J_{2\oplus} = \frac{\partial^A x_B \oplus^B x_C}{\partial^B x_C} = \begin{bmatrix} \cos(^A\psi_B) & -\sin(^A\psi_B) & 0 \\ \sin(^A\psi_B) & \cos(^A\psi_B) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1.3)$$

The method returns a numerical matrix containing the evaluation of the Jacobian for the 1^{st} posepose AxB (the self object).

Returns

Evaluation of the $J_{2\oplus}$ Jacobian of the pose compounding operation with respect to the second pose (eq. (1.3))

ominus()

Inverse pose compounding of the AxB pose (the self objetc):

$${}^B x_A = \ominus^A x_B = \begin{bmatrix} -{}^A x_B \cos(^A\psi_B) - {}^A y_B \sin(^A\psi_B) \\ {}^A x_B \sin(^A\psi_B) - {}^A y_B \cos(^A\psi_B) \\ -{}^A\psi_B \end{bmatrix} \quad (1.4)$$

Returns

A-Frame pose expressed in B-Frame coordinates (eq. (1.4))

J_ominus()

Jacobian of the inverse pose compounding operation ((1.1)) with respect the pose AxB (the self object):

$$J_{\ominus} = \frac{\partial \ominus^A x_B}{\partial^A x_B} = \begin{bmatrix} -\cos(^A\psi_B) & -\sin(^A\psi_B) & {}^A x_B \sin(^A\psi_B) - {}^A y_B \cos(^A\psi_B) \\ \sin(^A\psi_B) & -\cos(^A\psi_B) & {}^A x_B \cos(^A\psi_B) + {}^A y_B \sin(^A\psi_B) \\ 0 & 0 & -1 \end{bmatrix} \quad (1.5)$$

Returns the numerical matrix containing the evaluation of the Jacobian for the pose AxB (the self object).

Returns

Evaluation of the J_{\ominus} Jacobian of the inverse pose compounding operation with respect to the pose (eq. (1.5))

1.2 Robot Simulation

class SimulatedRobot.**SimulatedRobot**(xs0, map=[], *args)

Bases: object

This is the base class to simulate a robot. There are two operative frames: the world N-Frame (North East Down oriented) and the robot body frame body B-Frame. Each robot has a motion model and a measurement model. The motion model is used to simulate the robot motion and the measurement model is used to simulate the robot measurements.

All Robot simulation classes must derive from this class .

dt = 0.1

class attribute containing sample time of the simulation

__init__(xs0, map=[], *args)

Parameters

- **xs0** – initial simulated robot state x_{s_0} used to initialize the the motion model

- **map** – feature map of the environment $M = [{}^N x_{F_1}^T, \dots, {}^N x_{F_{nf}}^T]^T$

Constructor. First, it initializes the robot simulation defining the following attributes:

- **k** : time step
- **Qsk** : **To be defined in the derived classes.** Object attribute containing Covariance of the simulation motion model noise
- **usk** : **To be defined in the derived classes.** Object attribute containing the simulated input to the motion model
- **xsk** : **To be defined in the derived classes.** Object attribute containing the current simulated robot state
- **zsk** : **To be defined in the derived classes.** Object attribute containing the current simulated robot measurement
- **Rsk** : **To be defined in the derived classes.** Object attribute containing the observation noise covariance matrix
- **xsk** : current pose is the initial state
- **xsk_1** : previous state is the initial robot state
- **M** : position of the features in the N-Frame
- **nf** : number of features

Then, the robot animation is initialized defining the following attributes:

- **vehicleIcon** : Path file of the image of the robot to be used in the animation
- **vehicleFig** : Figure of the robot to be used in the animation
- **vehicleAxes** : Axes of the robot to be used in the animation
- **xTraj** : list containing the x coordinates of the robot trajectory
- **yTraj** : list containing the y coordinates of the robot trajectory
- **visualizationInterval** : time-steps interval between two consecutive frames of the animation

PlotRobot()

Updates the plot of the robot at the current pose

fs(xsk_I, uk)

Motion model used to simulate the robot motion. Computes the current robot state x_k given the previous robot state x_{k-1} and the input u_k . It also updates the object attributes xsk , xsk_1 and usk to be made them available for plotting purposes. *To be overridden in child class.*

Parameters

- **xsk_1** – previous robot state x_{k-1}
- **uk** – model input u_k

Returns

current robot state x_k

SetMap(map)

Initializes the map of the environment.

`_PlotSample(x, P, n)`

Plots n samples of a multivariate gaussian distribution. This function is used only for testing, to plot the uncertainty through samples. :param x: mean pose of the distribution :param P: covariance of the distribution :param n: number of samples to plot

1.2.1 3 DOF Diferential Drive Robot Simulation

class DifferentialDriveSimulatedRobot.DifferentialDriveSimulatedRobot(xs0, map=[], *args)

Bases: *SimulatedRobot*

This class implements a simulated differential drive robot. It inherits from the *SimulatedRobot* class and overrides some of its methods to define the differential drive robot motion model.

__init__(xs0, map=[], *args)

Parameters

- **xs0** – initial simulated robot state $\mathbf{x}_{s0} = [{}^N x_{s0} \ {}^N y_{s0} \ {}^N \psi_{s0}]^T$ used to initialize the motion model
- **map** – feature map of the environment $M = [{}^N x_{F_1}, \dots, {}^N x_{F_{nf}}]$

Initializes the simulated differential drive robot. Overrides some of the object attributes of the parent class *SimulatedRobot* to define the differential drive robot motion model:

- **Qsk** : Object attribute containing Covariance of the simulation motion model noise.

$$Q_k = \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{bmatrix} \quad (1.6)$$

- **usk** : Object attribute containing the simulated input to the motion model containing the forward velocity u_k and the angular velocity r_k

$$\mathbf{u}_k = [u_k \ r_k]^T \quad (1.7)$$

- **xsk** : Object attribute containing the current simulated robot state

$$\mathbf{x}_k = [{}^N x_k \ {}^N y_k \ {}^N \theta_k \ {}^B u_k \ {}^B v_k \ {}^B r_k]^T \quad (1.8)$$

where ${}^N x_k$, ${}^N y_k$ and ${}^N \theta_k$ are the robot position and orientation in the world N-Frame, and ${}^B u_k$, ${}^B v_k$ and ${}^B r_k$ are the robot linear and angular velocities in the robot B-Frame.

- **zsk** : Object attribute containing $\mathbf{z}_{s_k} = [n_L \ n_R]^T$ observation vector containing number of pulses read from the left and right wheel encoders.
- **Rsk** : Object attribute containing $R_{s_k} = \text{diag}(\sigma_L^2, \sigma_R^2)$ covariance matrix of the noise of the read pulses`.
- **wheelBase** : Object attribute containing the distance between the wheels of the robot ($w = 0.5$ m)
- **wheelRadius** : Object attribute containing the radius of the wheels of the robot ($R = 0.1$ m)
- **pulses_x_wheelTurn** : Object attribute containing the number of pulses per wheel turn ($\text{pulseXwheelTurn} = 1024$ pulses)
- **Polar2D_max_range** : Object attribute containing the maximum Polar2D range ($\text{Polar2D}_{max_range} = 50$ m) at which the robot can detect features.
- **Polar2D_feature_reading_frequency** : Object attribute containing the frequency of Polar2D feature readings (50 tics -sample times-)

- **Rfp** : Object attribute containing the covariance of the simulated Polar2D feature noise ($R_{fp} = \text{diag}(\sigma_\rho^2, \sigma_\phi^2)$)

Check the parent class `prpy.SimulatedRobot` to know the rest of the object attributes.

fs(*xsk_1*, *usk*)

Motion model used to simulate the robot motion. Computes the current robot state x_k given the previous robot state x_{k-1} and the input u_k :

$$\begin{aligned}
 \eta_{s_{k-1}} &= [x_{s_{k-1}} \quad y_{s_{k-1}} \quad \theta_{s_{k-1}}]^T \\
 \nu_{s_{k-1}} &= [u_{s_{k-1}} \quad v_{s_{k-1}} \quad r_{s_{k-1}}]^T \\
 x_{s_{k-1}} &= [\eta_{s_{k-1}}^T \quad \nu_{s_{k-1}}^T]^T \\
 u_{s_k} = \nu_d &= [u_d \quad r_d]^T \\
 w_{s_k} &= \dot{\nu}_{s_k} \\
 x_{s_k} &= f_s(x_{s_{k-1}}, u_{s_k}, w_{s_k}) \\
 &= \begin{bmatrix} \eta_{s_{k-1}} \oplus (\nu_{s_{k-1}} \Delta t + \frac{1}{2} w_{s_k} \Delta t^2) \\ \nu_{s_{k-1}} + K(\nu_d - \nu_{s_{k-1}}) + w_{s_k} \Delta t \end{bmatrix} \quad ; \quad K = \text{diag}(k_1, k_2, k_3) \quad k_i > 0
 \end{aligned} \tag{1.9}$$

Where $\eta_{s_{k-1}}$ is the previous 3 DOF robot pose (x,y,yaw) and $\nu_{s_{k-1}}$ is the previous robot velocity (velocity in the direction of x and y B-Frame axis of the robot and the angular velocity). u_{s_k} is the input to the motion model containing the desired robot velocity in the x direction (u_d) and the desired angular velocity around the z axis (r_d). w_{s_k} is the motion model noise representing an acceleration perturbation in the robot axis. The w_{s_k} acceleration is the responsible for the slight velocity variation in the simulated robot motion. K is a diagonal matrix containing the gains used to drive the simulated velocity towards the desired input velocity.

Finally, the class updates the object attributes *xsk*, *xsk_1* and *usk* to made them available for plotting purposes.

To be completed by the student.

Parameters

- **xsk_1** – previous robot state $x_{s_{k-1}} = [\eta_{s_{k-1}}^T \quad \nu_{s_{k-1}}^T]^T$
- **usk** – model input $u_{s_k} = \nu_d = [u_d \quad r_d]^T$

Returns

current robot state x_{s_k}

ReadEncoders()

Simulates the robot measurements of the left and right wheel encoders.

To be completed by the student.

Return zsk,Rsk

$z_k = [n_L \ n_R]^T$ observation vector containing number of pulses read from the left and right wheel encoders. $R_{s_k} = \text{diag}(\sigma_L^2, \sigma_R^2)$ covariance matrix of the read pulses.

ReadCompass()

Simulates the compass reading of the robot.

Returns

yaw and the covariance of its noise R_{yaw}

ReadCartesian2DFeature()

Simulates the reading of 2D cartesian features. The features are placed in the map in cartesian coordinates.

Returns

zsk: $[[x_1 \ y_1], \dots, [x_n \ y_n]]$

Cartesian position of the feature observations.

Rsk: `block_diag(R_1, ..., R_n)`, where $R_i = \begin{bmatrix} r_{xx} & r_{xy} \\ r_{xy} & r_{yy} \end{bmatrix}$ is the

2x2 i-th feature observation covariance. Covariance of the Cartesian feature observations.

Note the features are uncorrelated among them. They are independent. However, the x and y coordinates of each feature are correlated.

`PlotRobot()`

Updates the plot of the robot at the current pose

1.3 Robot Localization

1.3.1 Robot Localization

class `Localization.Localization(index, kSteps, robot, x0, *args)`

Bases: `object`

Localization base class. Implements the localization algorithm.

__init__(*index, kSteps, robot, x0, *args*)

Constructor of the DRLocalization class.

Parameters

- **index** – Logging index structure (`prpy.Index`)
- **kSteps** – Number of time steps to simulate
- **robot** – Simulation robot object (`prpy.Robot`)
- **args** – Rest of arguments to be passed to the parent constructor
- **x0** – Initial Robot pose in the N-Frame

`GetInput()`

Gets the input from the robot. To be overridden by the child class.

Return **uk**

input variable

`Localize(xk_1, uk)`

Single Localization iteration invoked from `prpy.DRLocalization.Localization()`. Given the previous robot pose, the function reads the inout and computes the current pose.

Parameters

xk_1 – previous robot pose

Return **xk**

current robot pose

`LocalizationLoop(x0, usk)`

Given an initial robot pose x_0 and the input to the `prpy.SimulatedRobot` this method calls iteratively `prpy.DRLocalization.Localize()` for k steps, solving the robot localization problem.

Parameters

x0 – initial robot pose

Log(x_{sk}, x_k)

Logs the results for later plotting.

Parameters

- **xsk** – ground truth robot pose from the simulation
- **xk** – estimated robot pose

PlotXY()

Plots, in a new figure, the ground truth (orange) and estimated (blue) trajectory of the robot at the end of the Localization Loop.

PlotTrajectory()

Plots the estimated trajectory (blue) of the robot during the localization process.

1.3.2 Dead Reckoning

3 DOF Differential Drive Mobile Robot Example

class DR_3DOFDifferentialDrive.DR_3DOFDifferentialDrive(*index, kSteps, robot, x0, *args*)

Bases: *Localization*

Dead Reckoning Localization for a Differential Drive Mobile Robot.

__init__(*index, kSteps, robot, x0, *args*)

Constructor of the prlab.DR_3DOFDifferentialDrive class.

Parameters

args – Rest of arguments to be passed to the parent constructor

Localize(*xk_1, uk*)

Motion model for the 3DOF ($x_k = [x_k \ y_k \ \psi_k]^T$) Differential Drive Mobile robot using as input the readings of the wheel encoders ($u_k = [n_L \ n_R]^T$).

Parameters

- **xk_1** – previous robot pose estimate ($x_{k-1} = [x_{k-1} \ y_{k-1} \ \psi_{k-1}]^T$)
- **uk** – input vector ($u_k = [u_k \ v_k \ r_k]^T$)

Return xk

current robot pose estimate ($x_k = [x_k \ y_k \ \psi_k]^T$)

GetInput()

Get the input for the motion model. In this case, the input is the readings from both wheel encoders.

Returns

uk: input vector ($u_k = [n_L \ n_R]^T$)

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