$\underline{r} = (x, y, z)$, $\underline{x}_i = (x_i, y_i, z_i)$, $i = 1, \dots, N$

$\underline{v} = (v_x, v_y, v_z)$, $\underline{u}_i = (u_{xi}, u_{yi}, u_{zi})$

$\underline{r} + t_i \underline{v} = \underline{x}_i + t_i \underline{u}_i$

$\underline{r} - \underline{x}_i = t_i(\underline{u}_i - \underline{v}_i) \implies \underline{r} - \underline{x}_i$ and $\underline{u}_i - \underline{v}_i$ are collinear

$(\underline{r} - \underline{x}_i) \times (\underline{v}_i - \underline{u}_i) = \underline{0}$

$3N$ eqns

$$(x - x_i)(v_y - u_{yi}) - (y - y_i)(v_x - u_{xi}) = 0$$
$$(y - y_i)(v_z - u_{zi}) - (z - z_i)(v_y - u_{yi}) = 0$$
$$(z - z_i)(v_x - u_{xi}) - (x - x_i)(v_z - u_{zi}) = 0$$

Nonlinear terms xv_y, yv_x, ..., etc are independent of i

So: choose $i = 1, 2$ then subtract

$$\begin{aligned} &- x_1 v_y - x u_{y_1} + x_1 u_{y_1} + y_1 v_x + y u_{x_1} - y_1 u_{x_1} \\ &+ x_2 v_y + x u_{y_2} - x_2 u_{y_2} - y_2 v_x - y u_{x_2} + y_2 u_{x_2} \end{aligned} = 0$$

$$(x_2 - x_1) v_y + (u_{y_2} - u_{y_1}) x + (y_1 - y_2) v_x + (u_{x_1} - u_{x_2}) y = y_1 u_{x_1} - x_1 u_{y_1} + x_2 u_{y_2} - y_2 u_{x_2}$$

similarly get eqns with $x \to y$, $y \to z$ and $x \to z$, $y \to x$

Get another 3 eqns from $2 \to 3$.

We write this system of linear eqns as a matrix,

$$\begin{pmatrix} (u_{y_2} - u_{y_1}) & (u_{x_1} - u_{x_2}) & 0 & (y_1 - y_2) & (x_2 - x_1) & 0 \\ 0 & (u_{z_2} - u_{z_1}) & (u_{y_1} - u_{y_2}) & 0 & (z_1 - z_2) & (y_2 - y_1) \\ (u_{z_1} - u_{z_2}) & 0 & (u_{x_2} - u_{x_1}) & (z_2 - z_1) & 0 & (x_1 - x_2) \\ & & \vdots & & & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} y_1 u_{x_1} - x_1 u_{y_1} + x_2 u_{y_2} - y_2 u_{x_2} \\ z_1 u_{y_1} - y_1 u_{z_1} + y_2 u_{z_2} - z_2 u_{y_2} \\ x_1 u_{z_1} - z_1 u_{x_1} + z_2 u_{x_2} - x_2 u_{z_2} \\ \vdots \end{pmatrix}$$

$\cdots$ means same but $2 \to 3$