Skew sensitivity of shell elements

Skew sensitivity of shell elements

This example illustrates the sensitivity of the shell elements in Abaqus to skew distortion when they are used as thin plates.

An analytical series solution to the boundary value problem is available in Morley (1963), and an identical evaluation of elements in numerous other commercial codes is presented by Robinson (1985).

This page discusses:

ProductsAbaqus/StandardAbaqus/Explicit

Problem description

The geometry of the plate is shown in Figure 1, Figure 2, and Figure 3. The analysis is performed for five different values of the skew angle, δ: 90°, 80°, 60°, 40°, and 30°. Three meshes (4 × 4, 8 × 8, and 14 × 14) are used for each skew angle in the Abaqus/Standard analysis. In the Abaqus/Explicit analysis 4 × 4, 8 × 8, and 14 × 14 meshes are used for each skew angle with the quadrilateral elements and 2 × 2 × 4, 4 × 4 × 4, and 8 × 8 × 4 meshes are used for each skew angle with the triangular elements.

The plate is 10 mm thick. All sides are 1.0 m long. The length/thickness ratio is, thus, 100/1 so that the plate is thin in the sense that transverse shear deformation should not be significant. Young's modulus is 30 MPa, and Poisson's ratio is 0.3. The plate is loaded by a uniform pressure of 1.0×10^{-6} MPa applied over the entire surface. The edges of the plate are all simply supported.

The pressure is applied as a step function in the Abaqus/Explicit analysis. Viscous pressure loading is applied to the structure to damp out dynamic effects. The time period for the step and the viscous pressure are chosen to obtain an optimal static solution.

Results and discussion

Three response quantities are presented: the vertical displacement in the center of the plate, wcenter, and the maximum and minimum bending moments per unit length at the center of the plate, defined as

Mmax=Ma+Mb,Mmin=Ma-Mb,

where

Ma=12(Mx+My),Mb=14(Mx-My)2+Mxy2.

The bending moment values Mx,My, and Mxy are obtained from the average nodal values obtained by requesting element output to the data file in the Abaqus/Standard analysis. These values are calculated by extrapolation from the integration point values in the elements, followed by averaging of these values over all elements attached to the node. They are, therefore, less accurate than the values at the integration points. In the Abaqus/Explicit analysis the bending moment values are obtained from an average of the integration point values for all elements that share the node at the center of the plate.

Abaqus/Standard results

The results for the 3-node triangular shells, S3R and STRI3, are given in Table 1 and Table 2, respectively. These elements give reasonable results for all skew angles with all but the coarsest mesh used (4 × 4 elements).

The results for the 6-node triangular shell STRI65 are given in Table 3. This element gives reasonable results for all the skew angles with the various mesh discretizations, with the exception of the coarsest mesh used.

The results for the 4-node quadrilateral shells are presented in Table 4 (S4R5), Table 5 (S4R), and Table 6 (S4). The performance of these elements in this case is rather similar to that of the triangular elements.

The results for element types S8R5 and S9R5, presented in Table 7, are essentially identical to each other. These second-order elements are more sensitive to the distortion in this problem than the first-order elements. For 80° and 90° angles they give slightly more accurate displacement values than S4R5; but at more severe angles their performance deteriorates noticeably, particularly in the prediction of the minimum moment at the center of the plate. It is possible that this is caused by the extrapolation and averaging technique used to obtain nodal values of bending moments rather than an intrinsic sensitivity of the elements to this type of distortion.

The results for element type S8R are given in Table 8. Except with the finest mesh used, this element generally shows greater loss of accuracy as the plate is skewed than any of the other elements.

The results for the continuum shell elements SC6R and SC8R are presented in Table 9 and Table 10. The performance of these elements is similar to that of the S3R and S4R shell elements.

Abaqus/Explicit results

The explicit dynamic analysis is run until a steady, static solution is obtained. Figure 4 shows an energy balance plot for the 14 × 14 mesh with a skew angle of 40°. It can be seen that inertia effects have died away.

The results for the 3-node triangular shell, S3R, are given in Table 11. These elements exhibit stiff response for the coarsest mesh used (2 × 2 × 4 elements) but converge to the correct answer as the mesh density is increased.

The results for the 4-node quadrilateral shells, S4R and S4RS, are presented in Table 12 and Table 13, respectively. For all but the 40° and 30° skew angles, the S4R elements give reasonable answers for the coarsest mesh used. As the mesh density is increased, the elements converge to the analytical solutions for all skew angles.

The results for the continuum shell element SC8R are presented in Table 14. The performance of this element is similar to that of the S4R shell element.

General remarks

Abaqus gives a warning when quadrilateral elements are defined with skew distortions larger than 45°. The results in this case indicate that, with the possible exception of element type S8R, the elements can provide quite accurate results with reasonable meshes even with large skew distortions. Nevertheless it is also clear that the analyst should attempt to design meshes to avoid distortion of the elements in any region where there are large strain gradients.

Comparison of the results reported here with the evaluations given by Robinson (1985) indicate that the elements in Abaqus are among the most accurate and least sensitive to skew angle.

Parametric study using a parametric study script

The skew sensitivity investigation discussed in this example can be performed conveniently as a parametric study using the Python scripting capabilities offered in Abaqus. As an example we perform a parametric study in Abaqus/Standard in which 15 analyses are automatically executed; these analyses correspond to combinations of five different values of the skew angle (δ: 90°, 80°, 60°, 40°, and 30°) for three different element types (S8R, S4R, and S4). We also perform a parametric study in Abaqus/Explicit in which 12 analyses are executed automatically; these analyses correspond to combinations of three different values of the skew angle (δ: 90°, 60°, and 30°), two different element types (S4R and S4RS), and two mesh discretizations (4 × 4 and 8 × 8 elements).

skewshell_parametric.inp shows the parametrized template input data used to generate the parametric variations of the Abaqus/Standard parametric study. The parametric study script file (skewshell_parametric.psf) is used to perform the parametric study. The vertical displacement in the center of the plate is reported in the following table for each of the analyses of the parametric study:

```
Parametric study: skewshell_parametric

     elemType,        delta,      N405_U.3,

          s8r,           90,    -0.00150858,
          s4r,           90,    -0.00149891,
           s4,           90,    -0.00144697,
          s8r,           80,    -0.00141673,
          s4r,           80,    -0.00143168,
           s4,           80,    -0.00137446,
          s8r,           60,   -0.000845317,
          s4r,           60,   -0.000969093,
           s4,           60,   -0.000885679,
          s8r,           40,   -0.000258699,
          s4r,           40,   -0.000371343,
           s4,           40,   -0.000315966,
          s8r,           30,    -9.5434e-05,
          s4r,           30,   -0.000153366,
           s4,           30,   -0.000130785,
```

These results match the corresponding results found in Table 5 to Table 8.

skew_discr.inp shows the parametrized template input data used to generate the parametric variations for the Abaqus/Explicit parametric study. The parametric study script file (skew_discr.psf) is used to perform the parametric study. The vertical displacement at the center of the plate is reported in the following table for each analysis of the parametric study:

```
                          Parametric study: skewXpl

        level,     elemType,        delta,      N405_U.3,

           1,          s4r,           90,    -0.00144092,
           2,          s4r,           90,    -0.00144511,
```

```
1,          s4rs,              90,   -0.00155302,
2,          s4rs,              90,   -0.00147813,
1,           s4r,              60,  -0.000954238,
2,           s4r,              60,  -0.000925741,
1,          s4rs,              60,   -0.00102325,
2,          s4rs,              60,  -0.000963277,
1,           s4r,              30,  -0.000151794,
2,           s4r,              30,  -0.000148982,
1,          s4rs,              30,  -0.000161744,
2,          s4rs,              30,  -0.000162303,
```

The results match the corresponding results found in Table 11 to Table 13.

Input files

Abaqus/Standard input files

skewshell_typ_tri.inp

Typical input data for a triangular element.

skewshell_typ_quad.inp

Typical input data for a quadrilateral element.

skewshell_parametric.inp

Parametrized template input data used to generate the parametric variations of the parametric study.

S3R elements:

skewshell_s3r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s3r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s3r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s3r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s3r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s3r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s3r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s3r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s3r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s3r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s3r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s3r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s3r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s3r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s3r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S4 elements:

skewshell_s4_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s4_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s4_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s4_4x4_ang80.inp

4×4 mesh, skew angle = 80°.

skewshell_s4_4x4_ang90.inp

4×4 mesh, skew angle = 90°.

skewshell_s4_8x8_ang30.inp

8×8 mesh, skew angle = 30°.

skewshell_s4_8x8_ang40.inp

8×8 mesh, skew angle = 40°.

skewshell_s4_8x8_ang60.inp

8×8 mesh, skew angle = 60°.

skewshell_s4_8x8_ang80.inp

8×8 mesh, skew angle = 80°.

skewshell_s4_8x8_ang90.inp

8×8 mesh, skew angle = 90°.

skewshell_s4_14x14_ang30.inp

14×14 mesh, skew angle = 30°.

skewshell_s4_14x14_ang40.inp

14×14 mesh, skew angle = 40°.

skewshell_s4_14x14_ang60.inp

14×14 mesh, skew angle = 60°.

skewshell_s4_14x14_ang80.inp

14×14 mesh, skew angle = 80°.

skewshell_s4_14x14_ang90.inp

14×14 mesh, skew angle = 90°.

S4R elements:

skewshell_s4r_4x4_ang30.inp

4×4 mesh, skew angle = 30°.

skewshell_s4r_4x4_ang30_eh.inp

4×4 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s4r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s4r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s4r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s4r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s4r_8x8_ang30_eh.inp

8 × 8 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s4r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s4r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s4r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s4r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s4r_14x14_ang30_eh.inp

14 × 14 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s4r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s4r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s4r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S4R5 elements:

skewshell_s4r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s4r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s4r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s4r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s4r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s4r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s4r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s4r5_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s4r5_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s4r5_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s4r5_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s4r5_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s4r5_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s4r5_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s4r5_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S8R elements:

skewshell_s8r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s8r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s8r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s8r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s8r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s8r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s8r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s8r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s8r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s8r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s8r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s8r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s8r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s8r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s8r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S8R5 elements:

skewshell_s8r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s8r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s8r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s8r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s8r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s8r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s8r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s8r5_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s8r5_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s8r5_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s8r5_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s8r5_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s8r5_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s8r5_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s8r5_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S9R5 elements:

skewshell_s9r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s9r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s9r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s9r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s9r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s9r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s9r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s9r5_8x8_ang60.inp

8×8 mesh, skew angle = 60°.

skewshell_s9r5_8x8_ang80.inp

8×8 mesh, skew angle = 80°.

skewshell_s9r5_8x8_ang90.inp

8×8 mesh, skew angle = 90°.

skewshell_s9r5_14x14_ang30.inp

14×14 mesh, skew angle = 30°.

skewshell_s9r5_14x14_ang40.inp

14×14 mesh, skew angle = 40°.

skewshell_s9r5_14x14_ang60.inp

14×14 mesh, skew angle = 60°.

skewshell_s9r5_14x14_ang80.inp

14×14 mesh, skew angle = 80°.

skewshell_s9r5_14x14_ang90.inp

14×14 mesh, skew angle = 90°.

STRI3 elements:

skewshell_stri3_4x4_ang30.inp

4×4 mesh, skew angle = 30°.

skewshell_stri3_4x4_ang40.inp

4×4 mesh, skew angle = 40°.

skewshell_stri3_4x4_ang60.inp

4×4 mesh, skew angle = 60°.

skewshell_stri3_4x4_ang80.inp

4×4 mesh, skew angle = 80°.

skewshell_stri3_4x4_ang90.inp

4×4 mesh, skew angle = 90°.

skewshell_stri3_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_stri3_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_stri3_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_stri3_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_stri3_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_stri3_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_stri3_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_stri3_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_stri3_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_stri3_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

STRI65 elements:

skewshell_stri65_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_stri65_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_stri65_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_stri65_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_stri65_4x4_ang90.inp

4×4 mesh, skew angle = 90°.

skewshell_stri65_8x8_ang30.inp

8×8 mesh, skew angle = 30°.

skewshell_stri65_8x8_ang40.inp

8×8 mesh, skew angle = 40°.

skewshell_stri65_8x8_ang60.inp

8×8 mesh, skew angle = 60°.

skewshell_stri65_8x8_ang80.inp

8×8 mesh, skew angle = 80°.

skewshell_stri65_8x8_ang90.inp

8×8 mesh, skew angle = 90°.

skewshell_stri65_14x14_ang30.inp

14×14 mesh, skew angle = 30°.

skewshell_stri65_14x14_ang40.inp

14×14 mesh, skew angle = 40°.

skewshell_stri65_14x14_ang60.inp

14×14 mesh, skew angle = 60°.

skewshell_stri65_14x14_ang80.inp

14×14 mesh, skew angle = 80°.

skewshell_stri65_14x14_ang90.inp

14×14 mesh, skew angle = 90°.

SC6R elements:

skewshell_sc6r_4x4_ang30.inp

4×4 mesh, skew angle = 30°.

skewshell_sc6r_4x4_ang40.inp

4×4 mesh, skew angle = 40°.

skewshell_sc6r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_sc6r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_sc6r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_sc6r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_sc6r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_sc6r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_sc6r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_sc6r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_sc6r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_sc6r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_sc6r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_sc6r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_sc6r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

SC8R elements:

skewshell_sc8r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_sc8r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_sc8r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_sc8r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_sc8r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_sc8r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_sc8r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_sc8r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_sc8r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_sc8r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_sc8r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_sc8r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_sc8r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_sc8r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_sc8r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

Abaqus/Explicit input files

skew_discr.inp

Parametrized template input data used to generate the parametric variations of the parametric study.

S3R element tests:

skew_coarse_30_s3r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s3r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s3r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s3r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s3r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s3r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s3r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s3r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s3r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s3r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s3r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s3r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s3r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s3r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s3r.inp

Medium mesh, skew angle = 90°.

S4R element tests:

skew_coarse_30_s4r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s4r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s4r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s4r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s4r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s4r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s4r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s4r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s4r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s4r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s4r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s4r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s4r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s4r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s4r.inp

Medium mesh, skew angle = 90°.

SC8R element tests:

skew_coarse_30_sc8r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_sc8r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_sc8r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_sc8r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_sc8r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_sc8r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_sc8r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_sc8r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_sc8r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_sc8r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_sc8r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_sc8r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_sc8r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_sc8r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_sc8r.inp

Medium mesh, skew angle = 90°.

S4RS element tests:

skew_coarse_30_s4rs.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s4rs.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s4rs.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s4rs.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s4rs.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s4rs.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s4rs.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s4rs.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s4rs.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s4rs.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s4rs.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s4rs.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s4rs.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s4rs.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s4rs.inp

Medium mesh, skew angle = 90°.

References

1. Morley, L. S. D., Skew Plates and Structures, Pergamon Press, London, 1963.
2. Robinson, J., "An Evaluation of Skew Sensitivity of Thirty-Three Plate Bending Elements in Nineteen FEM Systems," paper presented at the Finite Element Standards Forum at the AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics, and Materials Conference, April 1985.

Tables

Table 1. Skewed plate results: S3R, Abaqus/Standard analysis.

Skew angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
90°	Series solution	1.478		4.79		4.79	

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
	4 × 4	1.214	−17.9%	4.03	−15.9%	3.97	−17.1%
	8 × 8	1.425	−3.6%	4.86	1.5%	4.84	1.0%
	14 × 14	1.462	−1.1%	4.81	0.4%	4.80	0.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.148	−18.5%	4.09	−15.8%	3.60	−19.6%
	8 × 8	1.343	−4.7%	4.91	1.0%	4.44	−0.9%
	14 × 14	1.391	−1.3%	4.87	0.2%	4.51	0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.615	−34.0%	2.98	−29.9%	1.93	−42.0%
	8 × 8	0.812	−12.9%	3.82	−10.1%	2.82	−15.3%
	14 × 14	0.913	−2.0%	4.19	−1.4%	3.31	−0.6%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.213	−39.0%	1.86	−33.8%	0.88	−51.1%
	8 × 8	0.292	−16.3%	2.42	−13.9%	1.39	−22.8%
	14 × 14	0.346	−0.8%	2.81	0.0%	1.82	1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.080	−45.9%	1.14	−40.3%	0.46	−57.4%
	8 × 8	0.125	−15.5%	1.60	−16.2%	0.80	−25.9%
	14 × 14	0.148	0.0%	1.89	−1.0%	1.08	0.0%

Table 2. Skewed plate results: STRI3, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.488	0.7%	5.22	8.9%	5.22	8.9%
	8 × 8	1.481	0.2%	4.89	2.0%	4.89	2.0%
	14 × 14	1.480	0.1%	4.82	0.6%	4.82	0.6%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.419	0.7%	5.37	10%	4.83	7.8%
	8 × 8	1.410	0.1%	4.98	2.4%	4.57	2.0%
	14 × 14	1.409	0.0%	4.89	0.7%	4.51	0.7%
60°	Series solution	0.932		4.25		3.33	

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.965	3.5%	4.86	14%	3.62	8.8%
	8 × 8	0.940	0.8%	4.43	4.2%	3.41	2.4%
	14 × 14	0.935	0.3%	4.31	1.4%	3.36	0.9%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.390	12%	3.40	21%	2.15	19%
	8 × 8	0.363	4.2%	3.05	8.5%	1.93	7.4%
	14 × 14	0.357	2.4%	2.91	3.4%	1.87	4.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.173	16%	2.35	23%	1.35	25%
	8 × 8	0.158	6.6%	2.12	11%	1.22	13%
	14 × 14	0.154	3.8%	2.01	5.3%	1.16	7.5%

Table 3. Skewed plate results: STRI65, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.481	−0.2%	5.11	6.7%	4.99	4.2%
	8 × 8	1.486	0.5%	4.91	2.5%	4.87	1.7%
	14 × 14	1.484	0.4%	4.83	0.8%	4.81	0.4%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.377	−2.3%	4.89	0.6%	4.65	3.8%
	8 × 8	1.413	0.3%	4.93	1.4%	4.61	2.9%
	14 × 14	1.414	0.3%	4.89	0.6%	4.54	1.3%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.825	−3.5%	3.95	−7%	3.06	−8.2%
	8 × 8	0.919	−0.8%	4.27	0.5%	3.36	0.9%
	14 × 14	0.934	0.3%	4.27	0.5%	3.36	0.9%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.273	−22%	2.45	−13%	1.41	−21%
	8 × 8	0.333	−4.8%	2.76	−1.8%	1.73	−3.8%
	14 × 14	0.350	0.6%	2.81	0.0%	1.82	1.1%
30°	Series solution	0.148		1.91		1.08	

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.114	−23%	1.64	23%	0.80	−25%
	8 × 8	0.143	−3.4%	1.87	11%	1.03	−5%
	14 × 14	0.152	2.7%	1.92	5.3%	1.11	2.7%

Table 4. Skewed plate results: S4R5, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.502	1.6%	4.23	−12%	4.23	−12%
	8 × 8	1.485	0.5%	4.65	−2.8%	4.65	−2.8%
	14 × 14	1.482	0.3%	4.75	−0.9%	4.75	−0.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.436	1.9%	4.29	−11%	3.96	−12%
	8 × 8	1.415	0.5%	4.71	−3.0%	4.36	−2.6%
	14 × 14	1.412	0.2%	4.81	−1.0%	4.45	−0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.981	5.3%	3.78	−11%	2.88	−14%
	8 × 8	0.943	1.2%	4.12	−3.1%	3.25	−2.3%
	14 × 14	0.937	0.5%	4.21	−0.9%	3.31	−0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.384	10%	2.58	−8.1%	1.45	−19%
	8 × 8	0.365	4.8%	2.74	−2.6%	1.80	−0.0%
	14 × 14	0.357	2.3%	2.79	−0.8%	1.83	−1.7%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.160	7.7%	1.74	−9.0%	0.80	−26%
	8 × 8	0.160	7.9%	1.89	−1.3%	1.08	0.0%
	14 × 14	0.155	4.6%	1.91	−0.2%	1.13	4.3%

Table 5. Skewed plate results: S4R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.498	1.4%	4.22	−12%	4.22	−12%

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	8 × 8	1.485	0.5%	4.65	−2.9%	4.65	−2.9%
	14 × 14	1.483	0.3%	4.75	−0.9%	4.75	−0.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.431	1.6%	4.28	−12%	3.94	−12%
	8 × 8	1.415	0.4%	4.71	−3.0%	4.36	−2.7%
	14 × 14	1.414	0.4%	4.81	−0.9%	4.45	−0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.969	4.0%	3.76	−12%	2.84	−15%
	8 × 8	0.937	0.5%	4.11	−3.4%	3.23	−3.1%
	14 × 14	0.936	0.4%	4.21	−0.9%	3.30	−1.0%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.371	6.3%	2.52	−10%	1.41	−22%
	8 × 8	0.353	1.1%	2.68	−4.5%	1.72	−4.4%
	14 × 14	0.351	0.4%	2.76	−1.9%	1.77	−1.4%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.153	3.4%	1.70	−11%	0.78	−27%
	8 × 8	0.151	2.0%	1.82	−4.8%	1.00	−6.9%
	14 × 14	0.149	0.7%	1.86	−2.7%	1.05	−2.5%
	4 × 4*	0.156	5.4%	1.72	11%	0.79	–27%
	8 × 8*	0.155	4.7%	1.86	2.6%	1.05	–2.7%
	14 × 14*	0.150	1.3%	1.88	–1.5%	1.10	–1.8%

*Abaqus/Standard finite-strain element with enhanced hourglass control.

Table 6. Skewed plate results: S4, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.447	−2.1%	4.78	−0.2%	4.78	−0.2%
	8 × 8	1.474	−0.3%	4.80	0.2%	4.80	0.2%
	14 × 14	1.481	0.2%	4.80	0.2%	4.80	0.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.375	−2.4%	4.84	−0.4%	4.52	0.9%
	8 × 8	1.402	−0.5%	4.86	0.0%	4.50	0.4%

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	14 × 14	1.410	0.1%	4.86	0.0%	4.50	0.4%
60°	Series solution	0.932		4.86		3.33	
	4 × 4	0.886	−2.4%	4.84	−0.4%	3.38	1.5%
	8 × 8	0.910	−0.5%	4.86	0.0%	3.31	−0.6%
	14 × 14	0.925	0.1%	4.86	0.0%	3.32	−0.3%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.316	−4.9%	2.63	−6.4%	1.70	−5.6%
	8 × 8	0.323	−2.4%	2.71	−3.6%	1.74	−3.3%
	14 × 14	0.327	−0.8%	2.76	−1.9%	1.77	−1.4%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.131	−11.0%	1.67	−12.6%	0.92	−14.8%
	8 × 8	0.133	−10.1%	1.80	−5.8%	1.02	−5.6%
	14 × 14	0.141	−4.7%	1.85	−3.1%	1.06	−1.9%

Table 7. Skewed plate results: S8R5, S9R5; Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.483	0.3%	5.16	7.8%	5.16	7.8%
	8 × 8	1.481	0.2%	4.88	1.9%	4.88	1.9%
	14 × 14	1.483	0.3%	4.82	0.7%	4.82	0.7%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.413	0.3%	5.18	6.5%	4.91	9.6%
	8 × 8	1.411	0.2%	4.94	1.6%	4.59	2.4%
	14 × 14	1.413	0.3%	4.89	0.6%	4.53	1.0%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.945	1.4%	4.43	4.3%	3.84	15%
	8 × 8	0.937	0.6%	4.31	1.4%	3.45	3.5%
	14 × 14	0.938	0.7%	4.28	0.8%	3.38	1.5%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.370	6.0%	2.92	4.0%	2.35	31%
	8 × 8	0.357	2.5%	2.85	1.3%	1.97	9.3%
	14 × 14						

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
		0.357	2.5%	2.85	1.4%	1.88	4.7%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.164	10%	2.05	7.4%	1.51	40%
	8 × 8	0.156	4.9%	1.94	1.7%	1.26	16%
	14 × 14	0.155	4.5%	1.95	2.2%	1.17	8.2%

Table 8. Skewed plate results: S8R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.509	2.1%	5.22	9.1%	5.22	9.1%
	8 × 8	1.494	1.1%	4.91	2.5%	4.91	2.5%
	14 × 14	1.492	1.0%	4.85	1.2%	4.85	1.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.417	0.6%	5.25	8.0%	4.89	9.1%
	8 × 8	1.421	0.9%	4.97	2.3%	4.60	2.8%
	14 × 14	1.421	0.9%	4.91	1.1%	4.55	1.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.845	−9.3%	4.46	4.8%	3.39	1.8%
	8 × 8	0.915	−1.8%	4.30	1.2%	3.32	−0.1%
	14 × 14	0.933	0.2%	4.29	0.8%	3.35	0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.259	−26%	2.73	−3.0%	1.50	−17%
	8 × 8	0.308	−12%	2.68	−4.7%	1.57	−13%
	14 × 14	0.332	−4.7%	2.75	−2.2%	1.70	−5.5%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.095	−36%	1.72	−10%	0.78	−28%
	8 × 8	0.119	−20%	1.70	−11%	0.82	−24%
	14 × 14	0.149	0.5%	1.91	0.2%	1.09	0.8%

Table 9. Skewed plate results: SC6R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.214	–17.8%	4.00	–6.5%	4.00	–16.5%
	8 × 8	1.425	–3.6%	4.85	1.3%	4.85	1.3%
	14 × 14	1.462	–1.1%	4.80	0.3%	4.80	0.3%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.147	–18.5%	4.07	–16.3%	3.54	–20.9%
	8 × 8	1.343	–4.7%	4.92	1.1%	4.43	–1.1%
	14 × 14	1.391	–1.3%	4.87	0.2%	4.50	0.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.615	−34%	3.02	−28.9%	1.94	–41.8%
	8 × 8	0.812	–12.9%	3.83	–9.8%	2.81	–15.5%
	14 × 14	0.913	−2.1%	4.19	–1.5%	3.31	-0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.213	−38.9%	1.89	−32.9%	0.89	−50.7%
	8 × 8	0.292	−16.3%	2.43	−13.6%	1.39	–22.8%
	14 × 14	0.346	−0.8%	2.81	0%	1.82	−1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.080	−46.2%	1.16	−39.4%	0.46	−56.9%
	8 × 8	0.125	−15.8%	1.61	−15.9%	0.80	−26.1%
	14 × 14	0.148	0.1%	1.91	1.9%	1.07	0.5%

Table 10. Skewed plate results: SC8R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.503	1.7%	4.23	–11.6%	4.23	–11.6%
	8 × 8	1.487	0.6%	4.66	–2.8%	4.66	–2.8%
	14 × 14	1.485	–0.5%	4.75	–0.8%	4.75	–0.8%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.436	1.9%	4.29	–11.8%	3.96	—11.6%
	8 × 8	1.417	0.6%	4.72	–2.9%	4.43	–2.5%

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	14 × 14	1.414	–0.4%	4.82	–0.8%	4.46	–0.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.981	−5.2%	3.78	–11.0%	2.87	–13.7%
	8 × 8	0.943	1.2%	4.12	–3.1%	3.25	–2.3%
	14 × 14	0.938	0.6%	4.22	–0.7%	3.31	–0.5%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.383	9.6%	2.58	−8.2%	1.45	−19.5%
	8 × 8	0.363	4.1%	2.73	−2.8%	1.79	–0.5%
	14 × 14	0.355	1.7%	2.78	–0.9%	1.82	−1.3%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.158	6.8%	1.74	–9.1%	0.80	−25.9%
	8 × 8	0.158	6.8%	1.88	−1.6%	1.07	−0.8%
	14 × 14	0.153	3.4%	1.90	–0.7%	1.11	3.1%

Table 11. Skewed plate results: S3R, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	2 × 2 × 4	0.949	−36.2%	2.69	−44.0%	2.69	−44.0%
	4 × 4 × 4	1.325	−10.4%	4.30	−10.2%	4.30	−10.2%
	8 × 8 × 4	1.413	−4.4%	4.56	−4.8%	4.56	−4.8%
80°	Series solution	1.409		4.86		4.48	
	2 × 2 × 4	0.941	−33.0%	2.71	−44.1%	2.65	−40.8%
	4 × 4 × 4	1.257	−10.7%	4.26	−12.3%	4.18	−6.6%
	8 × 8 × 4	1.347	−4.4%	4.65	4.0%	4.26	−4.9%
60°	Series solution	0.932		4.25		3.33	
	2 × 2 × 4	0.783	−15.9%	2.60	−38.1%	2.19	−34.3%

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4 × 4	0.822	−11.8%	3.99	−6.1%	2.98	−10.5%
	8 × 8 × 4	0.897	−3.7%	4.14	−2.5%	3.21	−3.6%
40°	Series solution	0.349		2.81		1.80	
	2 × 2 × 4	0.332	−4.9%	1.78	−36.6%	1.10	−38.8%
	4 × 4 × 4	0.326	−6.6%	2.63	−6.4%	1.65	−8.3%
	8 × 8 × 4	0.348	0.2%	2.84	1.0%	1.80	0.0%
30°	Series solution	0.148		1.91		1.08	
	2 × 2 × 4	0.145	−2.0%	1.16	−39.2%	0.60	−44.4%
	4 × 4 × 4	0.148	−0.0%	1.78	−6.8%	0.98	−9.3%
	8 × 8 × 4	0.152	2.7%	1.86	2.6%	1.10	1.9%

Table 12. Skewed plate results: S4R, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.444	−2.3%	3.95	−17%	3.95	−17%
	8 × 8	1.450	−1.9%	4.49	−6.0%	4.49	−6.0%
	14 × 14	1.445	−2.2%	4.60	−3.9%	4.60	−3.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.356	−4.0%	4.08	−16%	3.71	−17%
	8 × 8	1.372	−2.6%	4.57	−6.0%	4.20	−6.2%
	14 × 14	1.378	−2.2%	4.68	−2.5%	4.32	−3.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.957	2.7%	3.53	−16%	2.54	−23%
	8 × 8	0.930	−0.2%	4.02	−5.4%	3.09	−7.2%
	14 × 14	0.922	−1.0%	4.15	−2.3%	3.23	−3.0%
40°	Series solution	0.349		2.81		1.80	

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.305	−12%	2.21	−21%	1.17	−35%
	8 × 8	0.328	−6.0%	2.59	−7.9%	1.57	−13%
	14 × 14	0.344	−1.5%	2.74	−2.5%	1.73	−4.0%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.152	2.7%	1.45	−24%	0.63	−42%
	8 × 8	0.151	2.0%	1.71	−10%	0.88	−26%
	14 × 14	0.144	−2.7%	1.84	−3.7%	1.01	−6.4%

Table 13. Skewed plate results: S4RS, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	Series solution	1.478		4.79		4.79	
90°	4 × 4	1.553	+5.1%	4.29	−10%	4.29	−10%
	8 × 8	1.477	−0.1%	4.60	−4.0%	4.60	−4.0%
	14 × 14	1.458	−1.4%	4.63	−3.3%	4.63	−3.3%
	Series solution	1.409		4.86		4.48	
80°	4 × 4	1.516	+7.6%	4.46	−8.2%	4.08	−8.9%
	8 × 8	1.409	0.0%	4.72	−2.9%	4.26	−4.9%
	14 × 14	1.391	−1.3%	4.73	−2.7%	4.32	−3.6%
	Series solution	0.932		4.25		3.33	
60°	4 × 4	1.026	+10%	3.91	−8.0%	2.96	−11%
	8 × 8	0.954	+2.1%	4.20	−1.2%	3.21	−3.6%
	14 × 14	0.942	+1.0%	4.19	−1.4%	3.33	0.0%
	Series solution	0.349		2.81		1.80	
40°	4 × 4	0.397	+14%	2.65	−5.7%	1.49	−17%
	8 × 8	0.371	+6.3%	2.78	−1.1%	1.82	+1.1%
	14 × 14	0.361	+3.4%	2.80	−0.5%	1.87	+4.0%
	Series solution	0.148		1.91		1.08	
30°	4 × 4	0.162	+9.5%	1.76	−7.9%	0.82	−24%
	8 × 8	0.162	+9.5%	1.91	0.0%	1.09	+1.0%
	14 × 14	0.156	+5.4%	1.92	+0.5%	1.14	+5.6%

Table 14. Skewed plate results: SC8R, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.466	−0.8%	4.15	−13%	4.15	−13%
	8 × 8	1.446	−2.1%	4.53	−5.4%	4.53	−5.4%
	14 × 14	1.443	−2.4%	4.61	−3.8%	4.61	−3.8%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.403	−0.4%	4.20	−14%	3.87	−14%
	8 × 8	1.381	−2.0%	4.59	−5.6%	4.25	−5.1%
	14 × 14	1.378	−2.2%	4.68	−3.7%	4.33	−3.3%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.957	1.6%	3.72	−13%	2.80	−16%
	8 × 8	0.926	−0.6%	4.06	−4.5%	3.18	−4.5%
	14 × 14	0.924	−0.9%	4.16	−2.1%	3.26	−2.1%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.367	5.2%	2.50	−11%	1.39	−23%
	8 × 8	0.350	0.3%	2.67	−5.0%	1.71	−5%
	14 × 14	0.349	0.0%	2.76	−1.8%	1.78	−1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.152	2.0%	1.68	−12%	0.77	−29%
	8 × 8	0.149	0.7%	1.81	−5.2%	0.99	−8.3%
	14 × 14	0.149	0.7%	1.87	−2.1%	1.06	−1.9%

Figures

Figure 1. Simply supported skew plate with uniform distributed load. A 4 × 4 mesh for the complete plate of quadrilateral elements is shown. The corresponding mesh of triangular elements is shown by

the dotted line.

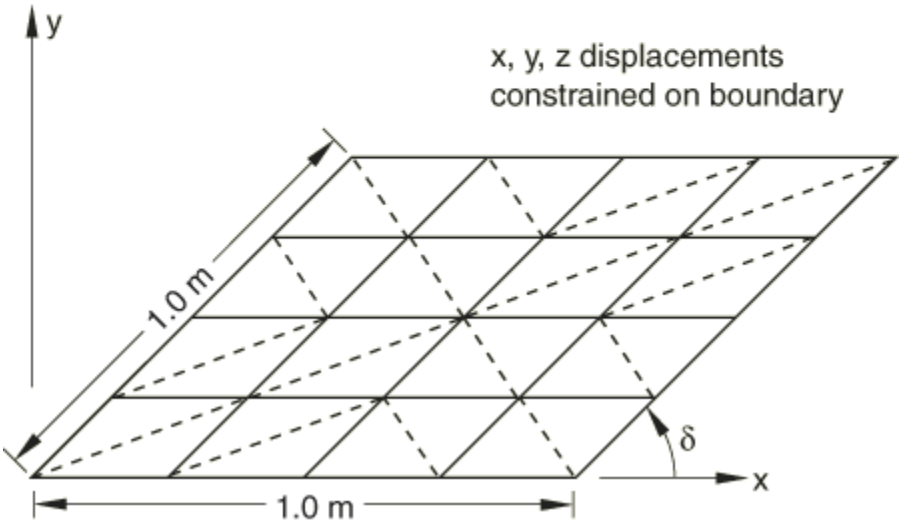

Figure 2. 4 × 4 mesh for the complete plate of quadrilateral elements.

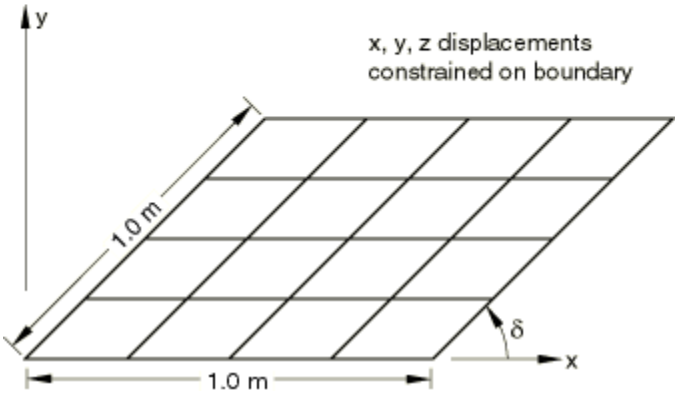

Figure 3. 2 × 2 × 4 mesh for the complete plate of triangular elements.

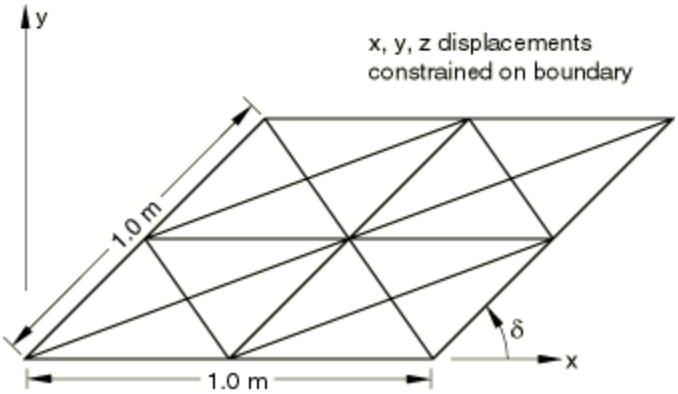

Figure 4. Energy balance for 14 × 14 mesh at 40° (Abaqus/Explicit analysis).