NAFEMS Benchmarks

Extracted from Abaqus 2023 Documentation

Generated on-demand

Standard benchmarks: linear elastic tests

Standard benchmarks: linear elastic tests

In this section:

LE1: Plane stress elements—elliptic membrane

LE1: Plane stress elements—elliptic membrane

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

CPS3

CPS4

CPS4I

CPS4R

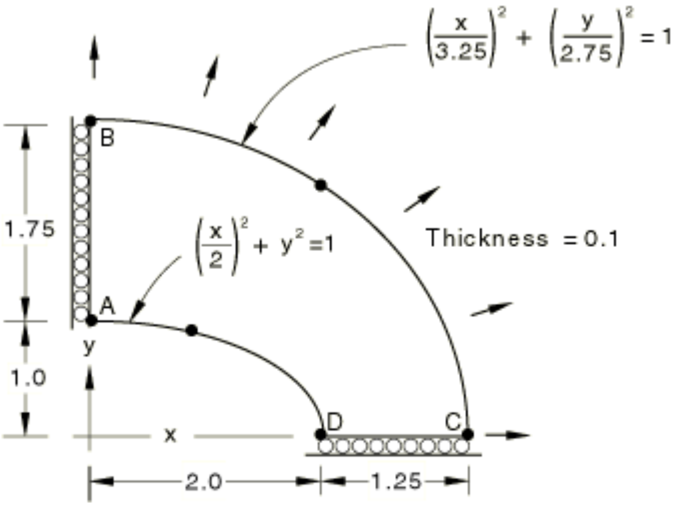
CPS6

CPS6M

CPS8

CPS8R

Problem description



Model:

Plane stress problem with shape defined by ABCD. Functions defining the curves BC and AD are given above.

Mesh:

A coarse and a fine mesh are tested for each element. In addition, a very fine mesh is tested for each element in the explicit dynamic analysis.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

ux=0 along edge AB, uy=0 along edge CD.

Loading:

Uniform outward pressure of 10 MPa at outer edge BC. In the explicit dynamic analysis the loading is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE1 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Tangential edge stress (σyy) at D is 92.7 MPa.

Results and discussion

The results are shown in Table 1 and Table 2. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Table 1. Abaqus/Standard analysis.

Element	Coarse Mesh	Fine Mesh
CPS3	51.04 MPa (−45%)	71.26 MPa (−23%)
CPS4	66.73 MPa (−28%)	84.54 MPa (−9%)
CPS4I	58.82 MPa (−37%)	78.21 MPa (−16%)
CPS4R*	40.48 MPa (−56%)	56.18 MPa (−39%)
CPS6	89.10 MPa (−4%)	94.01 MPa (1%)
CPS6M	85.88 MPa (−7%)	93.71 MPa (1%)
CPS8	84.54 MPa (−9%)	92.81 MPa (0.12%)
CPS8R	85.80 MPa (−7%)	90.07 MPa (−3%)

*A comparison of the results for reduced-integration and full-integration lower-order elements indicates that the full-integration elements perform significantly better for problems with stress concentrations of this type.

Table 2. Abaqus/Explicit analysis.

Element	Coarse Mesh	Fine Mesh	Very Fine Mesh
CPS3	51.2 MPa (−45%)	71.5 MPa (−23%)	85.7 MPa (−8%)
CPS4R	39.6 MPa (−57%)	55.7 MPa (−40%)	87.3 MPa (−6%)
CPS6M	86.12 MPa (−7%)	92.93 MPa (−0.2%)	—

Input files

Abaqus/Standard input files

Coarse mesh tests:

nle1xf3c.inp

CPS3 elements.

nle1xf4c.inp

CPS4 elements.

nle1xi4c.inp

CPS4I elements.

nle1xr4c.inp

CPS4R elements.

nle1xf6c.inp

CPS6 elements.

nle1xm6c.inp

CPS6M elements.

nle1xf8c.inp

CPS8 elements.

nle1xr8c.inp

CPS8R elements.

Fine mesh tests:

nle1xf3f.inp

CPS3 elements.

nle1xf4f.inp

CPS4 elements.

nle1xi4f.inp

CPS4I elements.

nle1xr4f.inp

CPS4R elements.

nle1xf6f.inp

CPS6 elements.

nle1xm6f.inp

CPS6M elements.

nle1xf8f.inp

CPS8 elements.

nle1xr8f.inp

CPS8R elements.

Abaqus/Explicit input files

Coarse mesh tests:

le1_cps3_c.inp

CPS3 elements.

le1_cps4r_c.inp

CPS4R elements.

le1_cps6m_c.inp

CPS6M elements.

Fine mesh tests:

le1_cps3_f.inp

CPS3 elements.

le1_cps4r_f.inp

CPS4R elements.

le1_cps6m_f.inp

CPS6M elements.

Very fine mesh tests:

le1_cps3_vf.inp

CPS3 elements.

le1_cps4r_vf.inp

CPS4R elements.

LE2: Cylindrical shell bending patch test

LE2: Cylindrical shell bending patch test

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

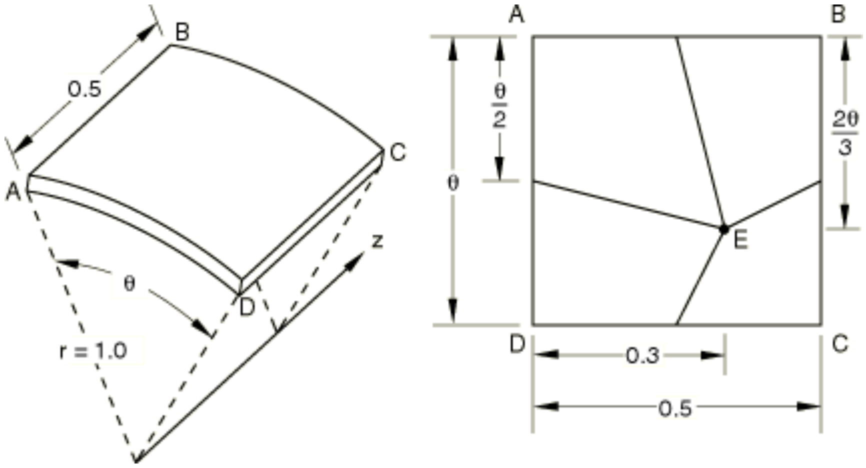
ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

- S3
- S3R
- S3RS
- S4
- S4R
- S4R5
- S4RS
- S4RSW
- S8R
- S8R5
- S9R5

- STRI3
- STRI65
- SC6R
- SC8R

Problem description



Model:

Sector of cylindrical shell with a thickness t= 0.01 m.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

Edge AB is clamped. Axial displacements are constrained along edges AD and BC.

Loading:

Uniform normal edge moment of 1000/unit length along edge DC. In the explicit dynamic analysis the loading is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE2 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Stress: Outer surface tangential stress at point E is 60 MPa.

Results and discussion

The results shown in Table 1 through Table 4 are interpolated from the integration points to the required nodal location. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Table 1. Abaqus/Standard analysis, θ= 30°.

Element	Bottom Surface (MPa)	Top Surface (MPa)
S3/S3R	−44.3 (–26%)	40.6 (–32%)
S4	−63.2 (5%)	54.0 (–10%)
S4R	−58.0 (–3%)	58.0 (–3%)
S4R*	–55.0 (–8%)	55.2 (–8%)
S4R5	−58.6 (–2%)	58.6 (−2%)
S8R	−50.7 (−16%)	50.4 (−16%)
S8R5	−57.8 (−4%)	58.2 (−3%)
S9R5	−57.9 (−4%)	58.3 (−3%)
STRI3	−37.9 (−37%)	36.0 (−40%)
STRI65	−53.6 (−11%)	53.9 (−10%)
SC6R	–43.9 (−27%)	43.9 (−27%)
SC8R	−59.7 (−1%)	59.7 (−1%)
SC8R*	–54.8 (–9%)	–54.8 (–9%)

*Abaqus/Standard results with enhanced hourglass control.

Table 2. Abaqus/Explicit analysis, θ= 30°.

Element	Bottom Surface (MPa)	Top Surface (MPa)
S3R	−43.2 MPa (−28%)	39.7 MPa (−34%)
S3RS	−44.7 MPa (−26%)	42.2 MPa (−30%)
S4R	−58.2 MPa (−3%)	58.3 MPa (−2.8%)
S4RS	−57.0 MPa (−5%)	56.9 MPa (−5.2%)
S4RSW	−57.3 MPa (−4.5%)	57.4 MPa (−4.3%)

These results vary significantly from the target value since the mesh is too coarse to capture a curvature of θ= 30°. The mesh can be refined easily by reducing the arc angle to θ= 10°. The following results show that such mesh refinement greatly improves the accuracy of the results.

Table 3. Abaqus/Standard analysis, θ= 10°.

Element	Bottom Surface (MPa)	Top Surface (MPa)
S3/S3R	−60.1 (0.2%)	59.9 (−0.2%)
S4	−60.5 (0.8%)	59.5 (−0.8%)
S4R	−60.0 (0%)	60.0 (0%)
S4R*	–60.0 (0%)	60.0 (0%)
S4R5	−60.0 (0%)	60.0 (0%)
S8R	−59.6 (−0.7%)	59.7 (−0.5%)
S8R5	−59.9 (−0.2%)	60.0 (0%)

Element	Bottom Surface (MPa)	Top Surface (MPa)
S9R5	−59.7 (–0.5%)	60.0 (0%)
STRI3	−60.8 (1.3%)	60.8 (1.3%)
STRI65	−59.6 (−0.7%)	59.7 (−0.5%)
SC6R	–60.2 (0.3%)	60.2 (0.3%)
SC8R	–60.2 (0.3%)	60.2 (0.3%)
SC8R*	–60.2 (0.3%)	60.2 (0.3%)

*Abaqus/Standard results with enhanced hourglass control.

Table 4. Abaqus/Explicit analysis, θ= 10°.

Element	Bottom Surface (MPa)	Top Surface (MPa)
S3R	−60.2 MPa (−0.3%)	59.9 MPa (−0.1%)
S3RS	−60.0 MPa (0%)	59.8 MPa (−0.3%)
S4R	−60.0 MPa (0%)	60.0 MPa (0%)
S4RS	−60.1 MPa (−0.1%)	60.1 MPa (−0.1%)
S4RSW	−60.0 MPa (0%)	59.9 MPa (−0.2%)

Input files

Abaqus/Standard input files

θ = 30°:

nle2xf3c.inp

S3/S3R elements.

nle2xe4c.inp

S4 elements.

nle2xf4c.inp

S4R elements.

nle2xf4c_eh.inp

S4R elements with enhanced hourglass control.

nle2x54c.inp

S4R5 elements.

nle2x68c.inp

S8R elements.

nle2x58c.inp

S8R5 elements.

nle2x59c.inp

S9R5 elements.

nle2x63c.inp

STRI3 elements.

nle2x56c.inp

STRI65 elements.

nle2_std_sc6r_30.inp

SC6R elements.

nle2_std_sc8r_30.inp

SC8R elements.

nle2_std_sc8r_30_eh.inp

SC8R elements with enhanced hourglass control.

$\theta = 10°$:

nle2xf3f.inp

S3/S3R elements.

nle2xe4f.inp

S4 elements.

nle2xf4f.inp

S4R elements.

nle2xf4f_eh.inp

S4R elements with enhanced hourglass control.

nle2x54f.inp

S4R5 elements.

nle2x68f.inp

S8R elements.

nle2x58f.inp

S8R5 elements.

nle2x59f.inp

S9R5 elements.

nle2x63f.inp

STRI3 elements.

nle2x56f.inp

STRI65 elements.

nle2_std_sc6r_10.inp

SC6R elements.

nle2_std_sc8r_10.inp

SC8R elements.

nle2_std_sc8r_10_eh.inp

SC8R elements with enhanced hourglass control.

Abaqus/Explicit input files

θ = 30°:

le2_s3r_c.inp

S3R elements.

le2_s3rs_c.inp

S3RS elements.

le2_s4r_c.inp

S4R elements.

le2_s4rs_c.inp

S4RS elements.

le2_s4rsw_c.inp

S4RSW elements.

θ = 10°:

le2_s3r_f.inp

S3R elements.

le2_s3rs_f.inp

S3RS elements.

le2_s4r_f.inp

S4R elements.

le2_s4rs_f.inp

S4RS elements.

le2_s4rsw_f.inp

S4RSW elements.

LE3: Hemispherical shell with point loads

LE3: Hemispherical shell with point loads

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

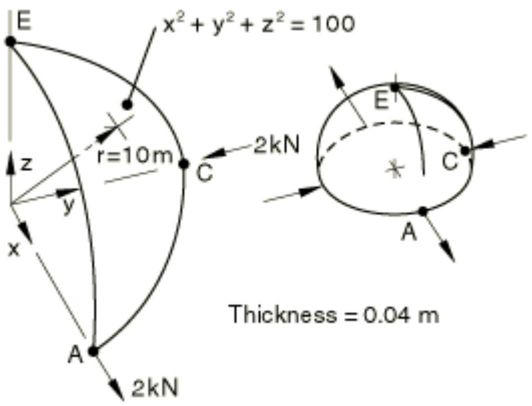
ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

- S3
- S3R
- S4
- S4R
- S4R5
- S8R
- S8R5
- S9R5
- STRI3
- STRI65
- SC6R
- SC8R

- SAXA12
- SAXA22

Problem description



Model:

The model is illustrated in the figure above. In addition, two input files are provided for the continuum shell element model to illustrate the use of the STACK DIRECTION=ORIENTATION parameter to define the element thickness (stacking) direction independent of the nodal connectivity using a spherical system.

Material:

Linear elastic, Young's modulus = 68.25 GPa, Poisson's ratio = 0.3.

Boundary conditions:

ux=uy=uz= 0 at point E. Along edge AE, symmetry about the z–x plane. Along edge CE, symmetry about the y–z plane.

Loading:

Concentrated radial loads of 2 kN outward at A, inward at C.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE3 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: ux= 185 mm at point A.

Results and discussion

The values enclosed in parentheses are percentage differences with respect to the reference solution.

Element	ux at A (Coarse)	ux at A (Fine)
S3/S3R	0.080 (−57%)	0.161 (−13%)
S4	0.083 (−55%)	0.175 (−5%)
S4R	0.180 (−2.7%)	0.180 (−2.7%)
S4R*	0.072 (−61%)	0.170 (−8.1%)
S4R**	0.058 (–68%)	0.168 (–9.1%)
S4R5	0.190 (2.7%)	0.183 (−1.1%)
S8R	0.101 (−45%)	0.178 (−3.8%)
S8R5	0.179 (−3.2%)	0.185 (0.0%)
S9R5	0.179 (−3.2%)	0.185 (0.0%)
STRI3	0.173 (−1.2%)	0.185 (0.0%)
STRI65	0.169 (−8.6%)	0.182 (−1.6%)
SC6R	0.088 (−52.4%)	0.167 (−9.7%)
SC8R	0.210 (13.5%)	0.188 (1.6%)
SC8R***	0.194(4.9%)	0.185(0.0%)
SAXA12****	0.179 (−3.2%)	
SAXA22****	0.178 (−3.8%)	

* Abaqus/Explicit finite-strain element with enhanced hourglass control.
**Abaqus/Standard finite-strain element with enhanced hourglass control.
*** Abaqus/Explicit continuum shell element with the default “relax stiffness” hourglass control. **** Due to the loading position, only the Mode 2 and Mode 4 elements can be used. Furthermore, due to the symmetries of the problem, only the Fourier interpolator cos2θ contributes to the solution. Thus, the Mode 4 elements produce identical results. Since Mode 4 is the highest-order Fourier term provided, no further circumferential mesh refinement is possible, and only coarse mesh results can be obtained.

The continuum shell element meshes using the STACK DIRECTION=ORIENTATION parameter yield identical results to the continuum shell element meshes in which the thickness direction is defined by the element nodal connectivity.

Input files

Abaqus/Standard input files

nle3xf3x.inp

S3/S3R elements.

nle3xe4x.inp

S4 elements.

nle3xf4x.inp

S4R elements.

nle3xf4x_eh.inp

S4R elements with enhanced hourglass control.

nle3x54x.inp

S4R5 elements.

nle3x68x.inp

S8R elements.

nle3x58x.inp

S8R5 elements.

nle3x59x.inp

S9R5 elements.

nle3x63x.inp

STRI3 elements.

nle3x56x.inp

STRI65 elements.

nle3xntx.inp

SAXA12 elements.

nle3xnxx.inp

SAXA22 elements.

nle3_std_sc6r.inp

SC6R elements.

nle3_std_sc8r.inp

SC8R elements.

nle3_std_sc6r_stackdir_sphori.inp

SC6R elements using the STACK DIRECTION=ORIENTATION parameter with a spherical orientation system to define the element thickness direction.

nle3_std_sc8r_stackdir_sphori.inp

SC8R elements using the STACK DIRECTION=ORIENTATION parameter with a spherical orientation system to define the element thickness direction.

nle3_std_sc8r_sgs.inp

SC8R elements using SHELL GENERAL SECTION to define section properties.

Abaqus/Explicit input files

le3_s4r.inp

S4R elements with enhanced hourglass control.

le3_sc8r.inp

SC8R elements with the default “relax stiffness” hourglass control.

LE4: Axisymmetric hyperbolic shell under uniform internal pressure

LE4: Axisymmetric hyperbolic shell under uniform internal pressure

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This page discusses:

ProductsAbaqus/Standard

Elements tested

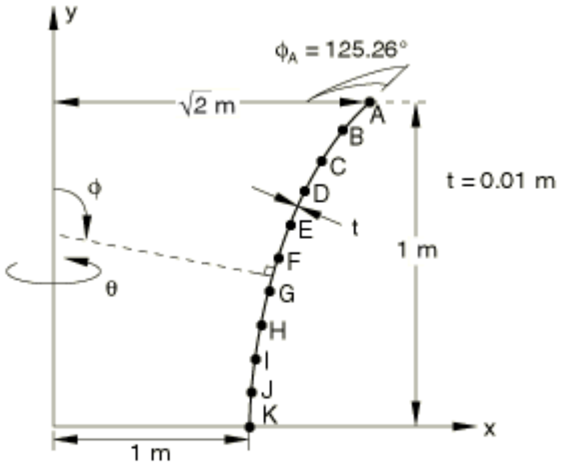
SAX1

SAX2

SAXA11

SAXA21

Problem description



Mesh:

A coarse and a fine mesh are tested for each element.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3.

Boundary conditions:

At point B, uz=0.

Loading:

Uniform internal pressure of 1 MPa.

General:

Gauss integration is used for the shell cross-section for the SAXA11 elements.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE4 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: On the midsurface at point K the meridional stress, $\sigma_{\phi\phi}$, is −50.0 MPa and the hoop stress, $\sigma_{\theta\theta}$, is 50.0 MPa.

Results and discussion

The results are shown in the following table. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Element Type	$\sigma_{\phi\phi}$	$\sigma_{\theta\theta}$
SAX1 (Coarse)	−49.69 (−0.62%)	49.99 (−0.02%)
SAX1 (Fine)	−49.99 (−0.02%)	49.92 (−0.16%)
SAX2 (Coarse)	−50.09 (0.18%)	48.33 (−3.3%)
SAX2 (Fine)	−50.02 (0.04%)	48.34 (−3.3%)
SAXA11 (Coarse)	−49.69 (−0.62%)	49.92 (−0.16%)
SAXA11 (Fine)	−49.99 (−0.02%)	49.20 (−1.6%)
SAXA21 (Coarse)	−50.09 (0.18%)	48.33 (−3.3%)
SAXA21 (Fine)	−50.02 (0.04%)	48.34 (−3.3%)

Input files

Coarse mesh tests:

esa2smsf.inp

SAX1 elements.

esa3smsf.inp

SAX2 elements.

esnssmsf.inp

SAXA11 elements.

esnwsmsf.inp

SAXA21 elements.

Fine mesh tests:

esa2sfsf.inp

SAX1 elements.

esa3sfsf.inp

SAX2 elements.

esnssfsf.inp

SAXA11 elements.

esnwsfsf.inp

SAXA21 elements.

LE5: Z-section cantilever

LE5: Z-section cantilever

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

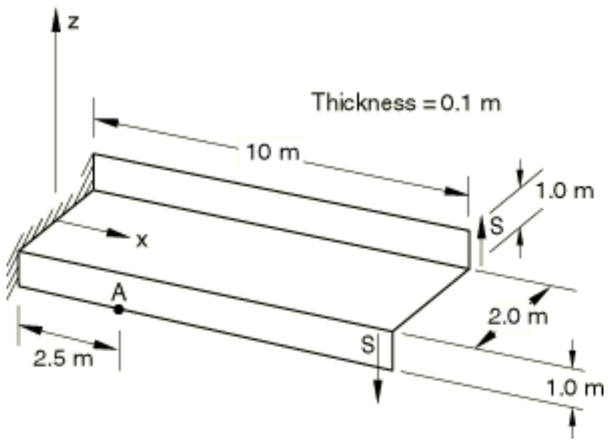
ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

- S3
- S3R
- S4R
- S4R5
- S4RS
- S4RSW
- S8R
- S8R5
- S9R5
- STRI3
- STRI65

- B31OS
- B32OS

Problem description



Model:

Z-section cantilever under torsional loading.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

All displacements are zero along the edge at x= 0.

Loading:

Torque of 1.2 MN-m applied at x= 10. The torque is applied by two uniformly distributed edge shears of 0.6 MN at each flange when shell elements are used. In the explicit dynamic analysis the loading rate is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE5 from NAFEMS publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Axial stress, σxx= −108 MPa at midsurface, point A.

Results and discussion

The results are shown in Table 1 and Table 2. The values enclosed in parentheses are percentage differences with respect to the reference solution. Slow convergence toward the target solution is seen as the mesh is refined.

Table 1. Abaqus/Standard analysis.

Element	σxx, Coarse Mesh	σxx, Refined Mesh
S3/S3R	−24.266 MPa (−78%)	−92.166 MPa (−15%)
S4	−110.36 MPa (2.2%)	−110.38 MPa (2.2%)
S4R	−50.480 MPa (−53%)	−96.732 MPa (−10%)
S4R5	−50.116 MPa (−54%)	−96.378 MPa (−11%)
S8R	−109.85 MPa (1.7%)	—
S8R5	−109.72 MPa (1.6%)	—
S9R5	−109.72 MPa (1.6%)	—
STRI3	−30.389 MPa (−72%)	−94.532 MPa (−12%)
STRI65	−107.32 MPa (−0.63%)	—
B31OS	−108.09 MPa (0.08%)	—
B32OS	−107.34 MPa (−0.61%)	—

Table 2. Abaqus/Explicit analysis.

Element	σxx, Coarse Mesh	σxx, Refined Mesh
S4R	−49.5 MPa (−54%)	−100.3 MPa (−7.1%)
S4RS	−87.5 MPa (−19%)	−100.3 MPa (−7.1%)
S4RSW	−87.7 MPa (−19%)	−100.3 MPa (−7.1%)

Input files

Abaqus/Standard input files

Coarse mesh tests:

nle5xf3c.inp

S3/S3R elements.

nle5xe4c.inp

S4 elements.

nle5xf4c.inp

S4R elements.

nle5x54c.inp

S4R5 elements.

nle5x68c.inp

S8R elements.

nle5x58c.inp

S8R5 elements.

nle5x59c.inp

S9R5 elements.

nle5x63c.inp

STRI3 elements.

nle5x56c.inp

STRI65 elements.

nle5xb2c.inp

B31OS elements.

nle5xb3c.inp

B32OS elements.

Fine mesh tests:

nle5xf3f.inp

S3/S3R elements.

nle5xe4f.inp

S4 elements.

nle5xf4f.inp

S4R elements.

nle5x54f.inp

S4R5 elements.

nle5x63f.inp

STRI3 elements.

Abaqus/Explicit input files

Coarse mesh tests:

le5_c.inp

S4R elements.

le5_c_s4rs.inp

S4RS elements.

le5_c_s4rsw.inp

S4RSW elements.

Fine mesh tests:

le5_f.inp

S4R elements.

le5_f_s4rs.inp

S4RS elements.

le5_f_s4rs_subcyc.inp

S4RS elements and subcycling.

le5_f_s4rsw.inp

S4RSW elements.

LE6: Skew plate under normal pressure

LE6: Skew plate under normal pressure

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

S4R

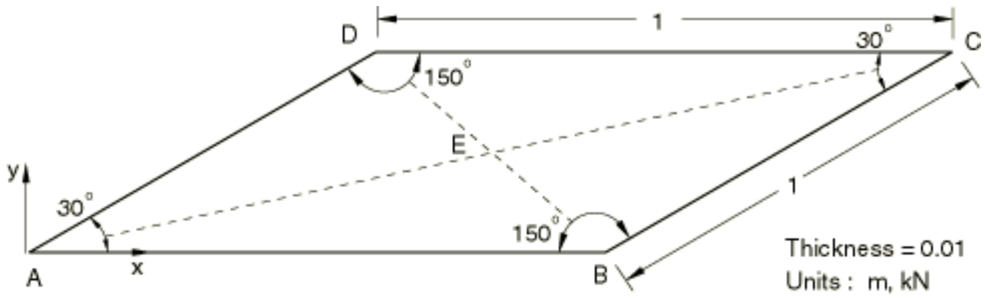
S4RS

S4RSW

S8R5

S9R5

Problem description



Model:

Skew plate under normal pressure.

Mesh:

A coarse (2 × 2) and a fine (4 × 4) are tested for each element. In addition, a very fine (8 × 8) mesh is tested for each element in the explicit dynamic analysis.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

uz=0 along edges AB, BC, CD, and AD. ux=uy= 0 at point A and uy=0 at point B to prevent rigid body motion.

Loading:

Uniform pressure of −7.0 kPa in the vertical z-direction. In the explicit dynamic analysis the loading is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE6 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Maximum principal stress = 0.802 MPa on the lower surface at point E.

Results and discussion

The results are shown in Table 1 and Table 2. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Table 1. Abaqus/Standard analysis.

Element	Coarse Mesh	Fine Mesh
S8R5	1.156 MPa (+44.1%)	0.862 MPa (+7.5%)
S9R5	1.156 MPa (+44.1%)	0.862 MPa (+7.5%)

Table 2. Abaqus/Explicit analysis.

Element	Coarse Mesh	Fine Mesh	Very Fine Mesh
S4R	0.338 MPa (−58%)	0.703 MPa (−12.3%)	0.765 MPa (−4.61%)
S4RS	0.343 MPa (−57%)	0.745 MPa (−7.11%)	0.8021 MPa (+0.01%)
S4RSW	0.341 MPa (−57%)	0.674 MPa (−16.0%)	0.8034 MPa (+0.17%)

Remarks

The skew sensitivity of shell elements is discussed in Skew sensitivity of shell elements.

Input files

Abaqus/Standard input files

Coarse mesh tests:

nle6x58c.inp

S8R5 elements.

nle6x59c.inp

S9R5 elements.

Fine mesh tests:

nle6x58f.inp

S8R5 elements.

nle6x59f.inp

S9R5 elements.

Abaqus/Explicit input files

Coarse mesh tests:

le6_c.inp

S4R elements.

le6_c_s4rs.inp

S4RS elements.

le6_c_s4rsw.inp

S4RSW elements.

Refined mesh tests:

le6_f.inp

S4R elements.

le6_f_s4rs.inp

S4RS elements.

le6_f_s4rsw.inp

S4RSW elements.

Very refined mesh tests:

le6_vf.inp

S4R elements.

le6_vf_s4rs.inp

S4RS elements.

le6_vf_s4rsw.inp

S4RSW elements.

Skew sensitivity of shell elements

Skew sensitivity of shell elements

This example illustrates the sensitivity of the shell elements in Abaqus to skew distortion when they are used as thin plates.

An analytical series solution to the boundary value problem is available in Morley (1963), and an identical evaluation of elements in numerous other commercial codes is presented by Robinson (1985).

This page discusses:

ProductsAbaqus/StandardAbaqus/Explicit

Problem description

The geometry of the plate is shown in Figure 1, Figure 2, and Figure 3. The analysis is performed for five different values of the skew angle, δ: 90°, 80°, 60°, 40°, and 30°. Three meshes (4 × 4, 8 × 8, and 14 × 14) are used for each skew angle in the Abaqus/Standard analysis. In the Abaqus/Explicit analysis 4 × 4, 8 × 8, and 14 × 14 meshes are used for each skew angle with the quadrilateral elements and 2 × 2 × 4, 4 × 4 × 4, and 8 × 8 × 4 meshes are used for each skew angle with the triangular elements.

The plate is 10 mm thick. All sides are 1.0 m long. The length/thickness ratio is, thus, 100/1 so that the plate is thin in the sense that transverse shear deformation should not be significant. Young's modulus is 30 MPa, and Poisson's ratio is 0.3. The plate is loaded by a uniform pressure of 1.0×10^{-6} MPa applied over the entire surface. The edges of the plate are all simply supported.

The pressure is applied as a step function in the Abaqus/Explicit analysis. Viscous pressure loading is applied to the structure to damp out dynamic effects. The time period for the step and the viscous pressure are chosen to obtain an optimal static solution.

Results and discussion

Three response quantities are presented: the vertical displacement in the center of the plate, wcenter, and the maximum and minimum bending moments per unit length at the center of the plate, defined as

Mmax=Ma+Mb,Mmin=Ma-Mb,

where

Ma=12(Mx+My),Mb=14(Mx-My)2+Mxy2.

The bending moment values Mx,My, and Mxy are obtained from the average nodal values obtained by requesting element output to the data file in the Abaqus/Standard analysis. These values are calculated by extrapolation from the integration point values in the elements, followed by averaging of these values over all elements attached to the node. They are, therefore, less accurate than the values at the integration points. In the Abaqus/Explicit analysis the bending moment values are obtained from an average of the integration point values for all elements that share the node at the center of the plate.

Abaqus/Standard results

The results for the 3-node triangular shells, S3R and STRI3, are given in Table 1 and Table 2, respectively. These elements give reasonable results for all skew angles with all but the coarsest mesh used (4 × 4 elements).

The results for the 6-node triangular shell STRI65 are given in Table 3. This element gives reasonable results for all the skew angles with the various mesh discretizations, with the exception of the coarsest mesh used.

The results for the 4-node quadrilateral shells are presented in Table 4 (S4R5), Table 5 (S4R), and Table 6 (S4). The performance of these elements in this case is rather similar to that of the triangular elements.

The results for element types S8R5 and S9R5, presented in Table 7, are essentially identical to each other. These second-order elements are more sensitive to the distortion in this problem than the first-order elements. For 80° and 90° angles they give slightly more accurate displacement values than S4R5; but at more severe angles their performance deteriorates noticeably, particularly in the prediction of the minimum moment at the center of the plate. It is possible that this is caused by the extrapolation and averaging technique used to obtain nodal values of bending moments rather than an intrinsic sensitivity of the elements to this type of distortion.

The results for element type S8R are given in Table 8. Except with the finest mesh used, this element generally shows greater loss of accuracy as the plate is skewed than any of the other elements.

The results for the continuum shell elements SC6R and SC8R are presented in Table 9 and Table 10. The performance of these elements is similar to that of the S3R and S4R shell elements.

Abaqus/Explicit results

The explicit dynamic analysis is run until a steady, static solution is obtained. Figure 4 shows an energy balance plot for the 14 × 14 mesh with a skew angle of 40°. It can be seen that inertia effects have died away.

The results for the 3-node triangular shell, S3R, are given in Table 11. These elements exhibit stiff response for the coarsest mesh used (2 × 2 × 4 elements) but converge to the correct answer as the mesh density is increased.

The results for the 4-node quadrilateral shells, S4R and S4RS, are presented in Table 12 and Table 13, respectively. For all but the 40° and 30° skew angles, the S4R elements give reasonable answers for the coarsest mesh used. As the mesh density is increased, the elements converge to the analytical solutions for all skew angles.

The results for the continuum shell element SC8R are presented in Table 14. The performance of this element is similar to that of the S4R shell element.

General remarks

Abaqus gives a warning when quadrilateral elements are defined with skew distortions larger than 45°. The results in this case indicate that, with the possible exception of element type S8R, the elements can provide quite accurate results with reasonable meshes even with large skew distortions. Nevertheless it is also clear that the analyst should attempt to design meshes to avoid distortion of the elements in any region where there are large strain gradients.

Comparison of the results reported here with the evaluations given by Robinson (1985) indicate that the elements in Abaqus are among the most accurate and least sensitive to skew angle.

Parametric study using a parametric study script

The skew sensitivity investigation discussed in this example can be performed conveniently as a parametric study using the Python scripting capabilities offered in Abaqus. As an example we perform a parametric study in Abaqus/Standard in which 15 analyses are automatically executed; these analyses correspond to combinations of five different values of the skew angle (δ: 90°, 80°, 60°, 40°, and 30°) for three different element types (S8R, S4R, and S4). We also perform a parametric study in Abaqus/Explicit in which 12 analyses are executed automatically; these analyses correspond to combinations of three different values of the skew angle (δ: 90°, 60°, and 30°), two different element types (S4R and S4RS), and two mesh discretizations (4 × 4 and 8 × 8 elements).

skewshell_parametric.inp shows the parametrized template input data used to generate the parametric variations of the Abaqus/Standard parametric study. The parametric study script file (skewshell_parametric.psf) is used to perform the parametric study. The vertical displacement in the center of the plate is reported in the following table for each of the analyses of the parametric study:

Parametric study: skewshell_parametric

elemType,	delta,	N405_U.3,
s8r,	90,	-0.00150858,
s4r,	90,	-0.00149891,
s4,	90,	-0.00144697,
s8r,	80,	-0.00141673,
s4r,	80,	-0.00143168,
s4,	80,	-0.00137446,
s8r,	60,	-0.000845317,
s4r,	60,	-0.000969093,
s4,	60,	-0.000885679,
s8r,	40,	-0.000258699,
s4r,	40,	-0.000371343,
s4,	40,	-0.000315966,
s8r,	30,	-9.5434e-05,
s4r,	30,	-0.000153366,
s4,	30,	-0.000130785,

These results match the corresponding results found in Table 5 to Table 8.

skew_discr.inp shows the parametrized template input data used to generate the parametric variations for the Abaqus/Explicit parametric study. The parametric study script file (skew_discr.psf) is used to perform the parametric study. The vertical displacement at the center of the plate is reported in the following table for each analysis of the parametric study:

Parametric study: skewXpl

level,	elemType,	delta,	N405_U.3,
1,	s4r,	90,	-0.00144092,
2,	s4r,	90,	-0.00144511,

```
1,          s4rs,             90,   -0.00155302,
2,          s4rs,             90,   -0.00147813,
1,           s4r,             60,  -0.000954238,
2,           s4r,             60,  -0.000925741,
1,          s4rs,             60,   -0.00102325,
2,          s4rs,             60,  -0.000963277,
1,           s4r,             30,  -0.000151794,
2,           s4r,             30,  -0.000148982,
1,          s4rs,             30,  -0.000161744,
2,          s4rs,             30,  -0.000162303,
```

The results match the corresponding results found in Table 11 to Table 13.

Input files

Abaqus/Standard input files

skewshell_typ_tri.inp

Typical input data for a triangular element.

skewshell_typ_quad.inp

Typical input data for a quadrilateral element.

skewshell_parametric.inp

Parametrized template input data used to generate the parametric variations of the parametric study.

S3R elements:

skewshell_s3r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s3r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s3r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s3r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s3r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s3r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s3r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s3r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s3r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s3r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s3r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s3r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s3r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s3r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s3r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S4 elements:

skewshell_s4_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s4_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s4_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s4_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s4_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s4_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s4_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s4_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s4_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s4_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s4_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s4_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s4_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s4_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s4_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S4R elements:

skewshell_s4r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s4r_4x4_ang30_eh.inp

4 × 4 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_4x4_ang40.inp

4×4 mesh, skew angle = 40°.

skewshell_s4r_4x4_ang60.inp

4×4 mesh, skew angle = 60°.

skewshell_s4r_4x4_ang80.inp

4×4 mesh, skew angle = 80°.

skewshell_s4r_4x4_ang90.inp

4×4 mesh, skew angle = 90°.

skewshell_s4r_8x8_ang30.inp

8×8 mesh, skew angle = 30°.

skewshell_s4r_8x8_ang30_eh.inp

8×8 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_8x8_ang40.inp

8×8 mesh, skew angle = 40°.

skewshell_s4r_8x8_ang60.inp

8×8 mesh, skew angle = 60°.

skewshell_s4r_8x8_ang80.inp

8×8 mesh, skew angle = 80°.

skewshell_s4r_8x8_ang90.inp

8×8 mesh, skew angle = 90°.

skewshell_s4r_14x14_ang30.inp

14×14 mesh, skew angle = 30°.

skewshell_s4r_14x14_ang30_eh.inp

14×14 mesh, skew angle = 30° with ENHANCED hourglass control.

skewshell_s4r_14x14_ang40.inp

14×14 mesh, skew angle = 40°.

skewshell_s4r_14x14_ang60.inp

14×14 mesh, skew angle = 60°.

skewshell_s4r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s4r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S4R5 elements:

skewshell_s4r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s4r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s4r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s4r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s4r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s4r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s4r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s4r5_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s4r5_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s4r5_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s4r5_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s4r5_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s4r5_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s4r5_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s4r5_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S8R elements:

skewshell_s8r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s8r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s8r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s8r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s8r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s8r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s8r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s8r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s8r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s8r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s8r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s8r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s8r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s8r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s8r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S8R5 elements:

skewshell_s8r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s8r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s8r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s8r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s8r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s8r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s8r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s8r5_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s8r5_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s8r5_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s8r5_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s8r5_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s8r5_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s8r5_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s8r5_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

S9R5 elements:

skewshell_s9r5_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_s9r5_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_s9r5_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_s9r5_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_s9r5_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_s9r5_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_s9r5_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_s9r5_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_s9r5_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_s9r5_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_s9r5_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_s9r5_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_s9r5_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_s9r5_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_s9r5_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

STRI3 elements:

skewshell_stri3_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_stri3_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_stri3_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_stri3_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_stri3_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_stri3_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_stri3_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_stri3_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_stri3_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_stri3_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_stri3_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_stri3_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_stri3_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_stri3_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_stri3_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

STRI65 elements:

skewshell_stri65_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_stri65_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_stri65_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_stri65_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_stri65_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_stri65_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_stri65_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_stri65_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_stri65_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_stri65_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_stri65_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_stri65_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_stri65_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_stri65_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_stri65_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

SC6R elements:

skewshell_sc6r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_sc6r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_sc6r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_sc6r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_sc6r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_sc6r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_sc6r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_sc6r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_sc6r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_sc6r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_sc6r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_sc6r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_sc6r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_sc6r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_sc6r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

SC8R elements:

skewshell_sc8r_4x4_ang30.inp

4 × 4 mesh, skew angle = 30°.

skewshell_sc8r_4x4_ang40.inp

4 × 4 mesh, skew angle = 40°.

skewshell_sc8r_4x4_ang60.inp

4 × 4 mesh, skew angle = 60°.

skewshell_sc8r_4x4_ang80.inp

4 × 4 mesh, skew angle = 80°.

skewshell_sc8r_4x4_ang90.inp

4 × 4 mesh, skew angle = 90°.

skewshell_sc8r_8x8_ang30.inp

8 × 8 mesh, skew angle = 30°.

skewshell_sc8r_8x8_ang40.inp

8 × 8 mesh, skew angle = 40°.

skewshell_sc8r_8x8_ang60.inp

8 × 8 mesh, skew angle = 60°.

skewshell_sc8r_8x8_ang80.inp

8 × 8 mesh, skew angle = 80°.

skewshell_sc8r_8x8_ang90.inp

8 × 8 mesh, skew angle = 90°.

skewshell_sc8r_14x14_ang30.inp

14 × 14 mesh, skew angle = 30°.

skewshell_sc8r_14x14_ang40.inp

14 × 14 mesh, skew angle = 40°.

skewshell_sc8r_14x14_ang60.inp

14 × 14 mesh, skew angle = 60°.

skewshell_sc8r_14x14_ang80.inp

14 × 14 mesh, skew angle = 80°.

skewshell_sc8r_14x14_ang90.inp

14 × 14 mesh, skew angle = 90°.

Abaqus/Explicit input files

skew_discr.inp

Parametrized template input data used to generate the parametric variations of the parametric study.

S3R element tests:

skew_coarse_30_s3r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s3r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s3r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s3r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s3r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s3r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s3r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s3r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s3r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s3r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s3r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s3r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s3r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s3r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s3r.inp

Medium mesh, skew angle = 90°.

S4R element tests:

skew_coarse_30_s4r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s4r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s4r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s4r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s4r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s4r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s4r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s4r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s4r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s4r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s4r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s4r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s4r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s4r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s4r.inp

Medium mesh, skew angle = 90°.

SC8R element tests:

skew_coarse_30_sc8r.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_sc8r.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_sc8r.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_sc8r.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_sc8r.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_sc8r.inp

Fine mesh, skew angle = 30°.

skew_fine_40_sc8r.inp

Fine mesh, skew angle = 40°.

skew_fine_60_sc8r.inp

Fine mesh, skew angle = 60°.

skew_fine_80_sc8r.inp

Fine mesh, skew angle = 80°.

skew_fine_90_sc8r.inp

Fine mesh, skew angle = 90°.

skew_medium_30_sc8r.inp

Medium mesh, skew angle = 30°.

skew_medium_40_sc8r.inp

Medium mesh, skew angle = 40°.

skew_medium_60_sc8r.inp

Medium mesh, skew angle = 60°.

skew_medium_80_sc8r.inp

Medium mesh, skew angle = 80°.

skew_medium_90_sc8r.inp

Medium mesh, skew angle = 90°.

S4RS element tests:

skew_coarse_30_s4rs.inp

Coarse mesh, skew angle = 30°.

skew_coarse_40_s4rs.inp

Coarse mesh, skew angle = 40°.

skew_coarse_60_s4rs.inp

Coarse mesh, skew angle = 60°.

skew_coarse_80_s4rs.inp

Coarse mesh, skew angle = 80°.

skew_coarse_90_s4rs.inp

Coarse mesh, skew angle = 90°.

skew_fine_30_s4rs.inp

Fine mesh, skew angle = 30°.

skew_fine_40_s4rs.inp

Fine mesh, skew angle = 40°.

skew_fine_60_s4rs.inp

Fine mesh, skew angle = 60°.

skew_fine_80_s4rs.inp

Fine mesh, skew angle = 80°.

skew_fine_90_s4rs.inp

Fine mesh, skew angle = 90°.

skew_medium_30_s4rs.inp

Medium mesh, skew angle = 30°.

skew_medium_40_s4rs.inp

Medium mesh, skew angle = 40°.

skew_medium_60_s4rs.inp

Medium mesh, skew angle = 60°.

skew_medium_80_s4rs.inp

Medium mesh, skew angle = 80°.

skew_medium_90_s4rs.inp

Medium mesh, skew angle = 90°.

References

1. Morley, L. S. D., Skew Plates and Structures, Pergamon Press, London, 1963.
2. Robinson, J., “An Evaluation of Skew Sensitivity of Thirty-Three Plate Bending Elements in Nineteen FEM Systems,” paper presented at the Finite Element Standards Forum at the AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics, and Materials Conference, April 1985.

Tables

Table 1. Skewed plate results: S3R, Abaqus/Standard analysis.

Skew angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
90°	Series solution	1.478		4.79		4.79	

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
	4 × 4	1.214	−17.9%	4.03	−15.9%	3.97	−17.1%
	8 × 8	1.425	−3.6%	4.86	1.5%	4.84	1.0%
	14 × 14	1.462	−1.1%	4.81	0.4%	4.80	0.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.148	−18.5%	4.09	−15.8%	3.60	−19.6%
	8 × 8	1.343	−4.7%	4.91	1.0%	4.44	−0.9%
	14 × 14	1.391	−1.3%	4.87	0.2%	4.51	0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.615	−34.0%	2.98	−29.9%	1.93	−42.0%
	8 × 8	0.812	−12.9%	3.82	−10.1%	2.82	−15.3%
	14 × 14	0.913	−2.0%	4.19	−1.4%	3.31	−0.6%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.213	−39.0%	1.86	−33.8%	0.88	−51.1%
	8 × 8	0.292	−16.3%	2.42	−13.9%	1.39	−22.8%
	14 × 14	0.346	−0.8%	2.81	0.0%	1.82	1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.080	−45.9%	1.14	−40.3%	0.46	−57.4%
	8 × 8	0.125	−15.5%	1.60	−16.2%	0.80	−25.9%
	14 × 14	0.148	0.0%	1.89	−1.0%	1.08	0.0%

Table 2. Skewed plate results: STRI3, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.488	0.7%	5.22	8.9%	5.22	8.9%
	8 × 8	1.481	0.2%	4.89	2.0%	4.89	2.0%
	14 × 14	1.480	0.1%	4.82	0.6%	4.82	0.6%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.419	0.7%	5.37	10%	4.83	7.8%
	8 × 8	1.410	0.1%	4.98	2.4%	4.57	2.0%
	14 × 14	1.409	0.0%	4.89	0.7%	4.51	0.7%
60°	Series solution	0.932		4.25		3.33	

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.965	3.5%	4.86	14%	3.62	8.8%
	8 × 8	0.940	0.8%	4.43	4.2%	3.41	2.4%
	14 × 14	0.935	0.3%	4.31	1.4%	3.36	0.9%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.390	12%	3.40	21%	2.15	19%
	8 × 8	0.363	4.2%	3.05	8.5%	1.93	7.4%
	14 × 14	0.357	2.4%	2.91	3.4%	1.87	4.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.173	16%	2.35	23%	1.35	25%
	8 × 8	0.158	6.6%	2.12	11%	1.22	13%
	14 × 14	0.154	3.8%	2.01	5.3%	1.16	7.5%

Table 3. Skewed plate results: STRI65, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.481	−0.2%	5.11	6.7%	4.99	4.2%
	8 × 8	1.486	0.5%	4.91	2.5%	4.87	1.7%
	14 × 14	1.484	0.4%	4.83	0.8%	4.81	0.4%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.377	−2.3%	4.89	0.6%	4.65	3.8%
	8 × 8	1.413	0.3%	4.93	1.4%	4.61	2.9%
	14 × 14	1.414	0.3%	4.89	0.6%	4.54	1.3%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.825	−3.5%	3.95	−7%	3.06	−8.2%
	8 × 8	0.919	−0.8%	4.27	0.5%	3.36	0.9%
	14 × 14	0.934	0.3%	4.27	0.5%	3.36	0.9%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.273	−22%	2.45	−13%	1.41	−21%
	8 × 8	0.333	−4.8%	2.76	−1.8%	1.73	−3.8%
	14 × 14	0.350	0.6%	2.81	0.0%	1.82	1.1%
30°	Series solution	0.148		1.91		1.08	

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.114	−23%	1.64	23%	0.80	−25%
	8 × 8	0.143	−3.4%	1.87	11%	1.03	−5%
	14 × 14	0.152	2.7%	1.92	5.3%	1.11	2.7%

Table 4. Skewed plate results: S4R5, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.502	1.6%	4.23	−12%	4.23	−12%
	8 × 8	1.485	0.5%	4.65	−2.8%	4.65	−2.8%
	14 × 14	1.482	0.3%	4.75	−0.9%	4.75	−0.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.436	1.9%	4.29	−11%	3.96	−12%
	8 × 8	1.415	0.5%	4.71	−3.0%	4.36	−2.6%
	14 × 14	1.412	0.2%	4.81	−1.0%	4.45	−0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.981	5.3%	3.78	−11%	2.88	−14%
	8 × 8	0.943	1.2%	4.12	−3.1%	3.25	−2.3%
	14 × 14	0.937	0.5%	4.21	−0.9%	3.31	−0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.384	10%	2.58	−8.1%	1.45	−19%
	8 × 8	0.365	4.8%	2.74	−2.6%	1.80	−0.0%
	14 × 14	0.357	2.3%	2.79	−0.8%	1.83	−1.7%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.160	7.7%	1.74	−9.0%	0.80	−26%
	8 × 8	0.160	7.9%	1.89	−1.3%	1.08	0.0%
	14 × 14	0.155	4.6%	1.91	−0.2%	1.13	4.3%

Table 5. Skewed plate results: S4R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.498	1.4%	4.22	−12%	4.22	−12%

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
	8 × 8	1.485	0.5%	4.65	−2.9%	4.65	−2.9%
	14 × 14	1.483	0.3%	4.75	−0.9%	4.75	−0.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.431	1.6%	4.28	−12%	3.94	−12%
	8 × 8	1.415	0.4%	4.71	−3.0%	4.36	−2.7%
	14 × 14	1.414	0.4%	4.81	−0.9%	4.45	−0.7%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.969	4.0%	3.76	−12%	2.84	−15%
	8 × 8	0.937	0.5%	4.11	−3.4%	3.23	−3.1%
	14 × 14	0.936	0.4%	4.21	−0.9%	3.30	−1.0%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.371	6.3%	2.52	−10%	1.41	−22%
	8 × 8	0.353	1.1%	2.68	−4.5%	1.72	−4.4%
	14 × 14	0.351	0.4%	2.76	−1.9%	1.77	−1.4%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.153	3.4%	1.70	−11%	0.78	−27%
	8 × 8	0.151	2.0%	1.82	−4.8%	1.00	−6.9%
	14 × 14	0.149	0.7%	1.86	−2.7%	1.05	−2.5%
	4 × 4*	0.156	5.4%	1.72	11%	0.79	–27%
	8 × 8*	0.155	4.7%	1.86	2.6%	1.05	–2.7%
	14 × 14*	0.150	1.3%	1.88	–1.5%	1.10	–1.8%

*Abaqus/Standard finite-strain element with enhanced hourglass control.

Table 6. Skewed plate results: S4, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.447	−2.1%	4.78	−0.2%	4.78	−0.2%
	8 × 8	1.474	−0.3%	4.80	0.2%	4.80	0.2%
	14 × 14	1.481	0.2%	4.80	0.2%	4.80	0.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.375	−2.4%	4.84	−0.4%	4.52	0.9%
	8 × 8	1.402	−0.5%	4.86	0.0%	4.50	0.4%

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	14 × 14	1.410	0.1%	4.86	0.0%	4.50	0.4%
60°	Series solution	0.932		4.86		3.33	
	4 × 4	0.886	−2.4%	4.84	−0.4%	3.38	1.5%
	8 × 8	0.910	−0.5%	4.86	0.0%	3.31	−0.6%
	14 × 14	0.925	0.1%	4.86	0.0%	3.32	−0.3%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.316	−4.9%	2.63	−6.4%	1.70	−5.6%
	8 × 8	0.323	−2.4%	2.71	−3.6%	1.74	−3.3%
	14 × 14	0.327	−0.8%	2.76	−1.9%	1.77	−1.4%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.131	−11.0%	1.67	−12.6%	0.92	−14.8%
	8 × 8	0.133	−10.1%	1.80	−5.8%	1.02	−5.6%
	14 × 14	0.141	−4.7%	1.85	−3.1%	1.06	−1.9%

Table 7. Skewed plate results: S8R5, S9R5; Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.483	0.3%	5.16	7.8%	5.16	7.8%
	8 × 8	1.481	0.2%	4.88	1.9%	4.88	1.9%
	14 × 14	1.483	0.3%	4.82	0.7%	4.82	0.7%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.413	0.3%	5.18	6.5%	4.91	9.6%
	8 × 8	1.411	0.2%	4.94	1.6%	4.59	2.4%
	14 × 14	1.413	0.3%	4.89	0.6%	4.53	1.0%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.945	1.4%	4.43	4.3%	3.84	15%
	8 × 8	0.937	0.6%	4.31	1.4%	3.45	3.5%
	14 × 14	0.938	0.7%	4.28	0.8%	3.38	1.5%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.370	6.0%	2.92	4.0%	2.35	31%
	8 × 8	0.357	2.5%	2.85	1.3%	1.97	9.3%
	14 × 14						

Skew angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
		0.357	2.5%	2.85	1.4%	1.88	4.7%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.164	10%	2.05	7.4%	1.51	40%
	8 × 8	0.156	4.9%	1.94	1.7%	1.26	16%
	14 × 14	0.155	4.5%	1.95	2.2%	1.17	8.2%

Table 8. Skewed plate results: S8R, Abaqus/Standard analysis.

Skew angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.509	2.1%	5.22	9.1%	5.22	9.1%
	8 × 8	1.494	1.1%	4.91	2.5%	4.91	2.5%
	14 × 14	1.492	1.0%	4.85	1.2%	4.85	1.2%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.417	0.6%	5.25	8.0%	4.89	9.1%
	8 × 8	1.421	0.9%	4.97	2.3%	4.60	2.8%
	14 × 14	1.421	0.9%	4.91	1.1%	4.55	1.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.845	−9.3%	4.46	4.8%	3.39	1.8%
	8 × 8	0.915	−1.8%	4.30	1.2%	3.32	−0.1%
	14 × 14	0.933	0.2%	4.29	0.8%	3.35	0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.259	−26%	2.73	−3.0%	1.50	−17%
	8 × 8	0.308	−12%	2.68	−4.7%	1.57	−13%
	14 × 14	0.332	−4.7%	2.75	−2.2%	1.70	−5.5%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.095	−36%	1.72	−10%	0.78	−28%
	8 × 8	0.119	−20%	1.70	−11%	0.82	−24%
	14 × 14	0.149	0.5%	1.91	0.2%	1.09	0.8%

Table 9. Skewed plate results: SC6R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.214	–17.8%	4.00	–6.5%	4.00	–16.5%
	8 × 8	1.425	–3.6%	4.85	1.3%	4.85	1.3%
	14 × 14	1.462	–1.1%	4.80	0.3%	4.80	0.3%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.147	–18.5%	4.07	–16.3%	3.54	–20.9%
	8 × 8	1.343	–4.7%	4.92	1.1%	4.43	–1.1%
	14 × 14	1.391	–1.3%	4.87	0.2%	4.50	0.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.615	−34%	3.02	−28.9%	1.94	–41.8%
	8 × 8	0.812	–12.9%	3.83	–9.8%	2.81	–15.5%
	14 × 14	0.913	−2.1%	4.19	–1.5%	3.31	-0.7%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.213	−38.9%	1.89	−32.9%	0.89	−50.7%
	8 × 8	0.292	−16.3%	2.43	−13.6%	1.39	–22.8%
	14 × 14	0.346	−0.8%	2.81	0%	1.82	−1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.080	−46.2%	1.16	−39.4%	0.46	−56.9%
	8 × 8	0.125	−15.8%	1.61	−15.9%	0.80	−26.1%
	14 × 14	0.148	0.1%	1.91	1.9%	1.07	0.5%

Table 10. Skewed plate results: SC8R, Abaqus/Standard analysis.

Skew angle	Mesh	wcenter (mm)	Error	Mmax (× 10^{-2} N-m/m)	Error	Mmin (× 10^{-2} N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.503	1.7%	4.23	–11.6%	4.23	–11.6%
	8 × 8	1.487	0.6%	4.66	–2.8%	4.66	–2.8%
	14 × 14	1.485	–0.5%	4.75	–0.8%	4.75	–0.8%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.436	1.9%	4.29	–11.8%	3.96	—11.6%
	8 × 8	1.417	0.6%	4.72	–2.9%	4.43	–2.5%

Skew angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
	14 × 14	1.414	–0.4%	4.82	–0.8%	4.46	–0.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.981	−5.2%	3.78	–11.0%	2.87	–13.7%
	8 × 8	0.943	1.2%	4.12	–3.1%	3.25	–2.3%
	14 × 14	0.938	0.6%	4.22	–0.7%	3.31	–0.5%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.383	9.6%	2.58	−8.2%	1.45	−19.5%
	8 × 8	0.363	4.1%	2.73	−2.8%	1.79	–0.5%
	14 × 14	0.355	1.7%	2.78	–0.9%	1.82	−1.3%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.158	6.8%	1.74	–9.1%	0.80	−25.9%
	8 × 8	0.158	6.8%	1.88	−1.6%	1.07	−0.8%
	14 × 14	0.153	3.4%	1.90	–0.7%	1.11	3.1%

Table 11. Skewed plate results: S3R, Abaqus/Explicit analysis.

Skew Angle	**Mesh**	**wcenter (mm)**	**Error**	**Mmax ($\times 10^{-2}$ N-m/m)**	**Error**	**Mmin ($\times 10^{-2}$ N-m/m)**	**Error**
90°	Series solution	1.478		4.79		4.79	
	2 × 2 × 4	0.949	−36.2%	2.69	−44.0%	2.69	−44.0%
	4 × 4 × 4	1.325	−10.4%	4.30	−10.2%	4.30	−10.2%
	8 × 8 × 4	1.413	−4.4%	4.56	−4.8%	4.56	−4.8%
80°	Series solution	1.409		4.86		4.48	
	2 × 2 × 4	0.941	−33.0%	2.71	−44.1%	2.65	−40.8%
	4 × 4 × 4	1.257	−10.7%	4.26	−12.3%	4.18	−6.6%
	8 × 8 × 4	1.347	−4.4%	4.65	4.0%	4.26	−4.9%
60°	Series solution	0.932		4.25		3.33	
	2 × 2 × 4	0.783	−15.9%	2.60	−38.1%	2.19	−34.3%

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4 × 4	0.822	−11.8%	3.99	−6.1%	2.98	−10.5%
	8 × 8 × 4	0.897	−3.7%	4.14	−2.5%	3.21	−3.6%
40°	Series solution	0.349		2.81		1.80	
	2 × 2 × 4	0.332	−4.9%	1.78	−36.6%	1.10	−38.8%
	4 × 4 × 4	0.326	−6.6%	2.63	−6.4%	1.65	−8.3%
	8 × 8 × 4	0.348	0.2%	2.84	1.0%	1.80	0.0%
30°	Series solution	0.148		1.91		1.08	
	2 × 2 × 4	0.145	−2.0%	1.16	−39.2%	0.60	−44.4%
	4 × 4 × 4	0.148	−0.0%	1.78	−6.8%	0.98	−9.3%
	8 × 8 × 4	0.152	2.7%	1.86	2.6%	1.10	1.9%

Table 12. Skewed plate results: S4R, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.444	−2.3%	3.95	−17%	3.95	−17%
	8 × 8	1.450	−1.9%	4.49	−6.0%	4.49	−6.0%
	14 × 14	1.445	−2.2%	4.60	−3.9%	4.60	−3.9%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.356	−4.0%	4.08	−16%	3.71	−17%
	8 × 8	1.372	−2.6%	4.57	−6.0%	4.20	−6.2%
	14 × 14	1.378	−2.2%	4.68	−2.5%	4.32	−3.5%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.957	2.7%	3.53	−16%	2.54	−23%
	8 × 8	0.930	−0.2%	4.02	−5.4%	3.09	−7.2%
	14 × 14	0.922	−1.0%	4.15	−2.3%	3.23	−3.0%
40°	Series solution	0.349		2.81		1.80	

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	4 × 4	0.305	−12%	2.21	−21%	1.17	−35%
	8 × 8	0.328	−6.0%	2.59	−7.9%	1.57	−13%
	14 × 14	0.344	−1.5%	2.74	−2.5%	1.73	−4.0%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.152	2.7%	1.45	−24%	0.63	−42%
	8 × 8	0.151	2.0%	1.71	−10%	0.88	−26%
	14 × 14	0.144	−2.7%	1.84	−3.7%	1.01	−6.4%

Table 13. Skewed plate results: S4RS, Abaqus/Explicit analysis.

Skew Angle	Mesh	wcenter (mm)	Error	Mmax ($\times 10^{-2}$ N-m/m)	Error	Mmin ($\times 10^{-2}$ N-m/m)	Error
	Series solution	1.478		4.79		4.79	
90°	4 × 4	1.553	+5.1%	4.29	−10%	4.29	−10%
	8 × 8	1.477	−0.1%	4.60	−4.0%	4.60	−4.0%
	14 × 14	1.458	−1.4%	4.63	−3.3%	4.63	−3.3%
	Series solution	1.409		4.86		4.48	
80°	4 × 4	1.516	+7.6%	4.46	−8.2%	4.08	−8.9%
	8 × 8	1.409	0.0%	4.72	−2.9%	4.26	−4.9%
	14 × 14	1.391	−1.3%	4.73	−2.7%	4.32	−3.6%
	Series solution	0.932		4.25		3.33	
60°	4 × 4	1.026	+10%	3.91	−8.0%	2.96	−11%
	8 × 8	0.954	+2.1%	4.20	−1.2%	3.21	−3.6%
	14 × 14	0.942	+1.0%	4.19	−1.4%	3.33	0.0%
	Series solution	0.349		2.81		1.80	
40°	4 × 4	0.397	+14%	2.65	−5.7%	1.49	−17%
	8 × 8	0.371	+6.3%	2.78	−1.1%	1.82	+1.1%
	14 × 14	0.361	+3.4%	2.80	−0.5%	1.87	+4.0%
	Series solution	0.148		1.91		1.08	
30°	4 × 4	0.162	+9.5%	1.76	−7.9%	0.82	−24%
	8 × 8	0.162	+9.5%	1.91	0.0%	1.09	+1.0%
	14 × 14	0.156	+5.4%	1.92	+0.5%	1.14	+5.6%

Table 14. Skewed plate results: SC8R, Abaqus/Explicit analysis.

Skew Angle	Mesh	**wcenter** (mm)	**Error**	**Mmax** ($\times 10^{-2}$ N-m/m)	**Error**	**Mmin** ($\times 10^{-2}$ N-m/m)	**Error**
90°	Series solution	1.478		4.79		4.79	
	4 × 4	1.466	−0.8%	4.15	−13%	4.15	−13%
	8 × 8	1.446	−2.1%	4.53	−5.4%	4.53	−5.4%
	14 × 14	1.443	−2.4%	4.61	−3.8%	4.61	−3.8%
80°	Series solution	1.409		4.86		4.48	
	4 × 4	1.403	−0.4%	4.20	−14%	3.87	−14%
	8 × 8	1.381	−2.0%	4.59	−5.6%	4.25	−5.1%
	14 × 14	1.378	−2.2%	4.68	−3.7%	4.33	−3.3%
60°	Series solution	0.932		4.25		3.33	
	4 × 4	0.957	1.6%	3.72	−13%	2.80	−16%
	8 × 8	0.926	−0.6%	4.06	−4.5%	3.18	−4.5%
	14 × 14	0.924	−0.9%	4.16	−2.1%	3.26	−2.1%
40°	Series solution	0.349		2.81		1.80	
	4 × 4	0.367	5.2%	2.50	−11%	1.39	−23%
	8 × 8	0.350	0.3%	2.67	−5.0%	1.71	−5%
	14 × 14	0.349	0.0%	2.76	−1.8%	1.78	−1.1%
30°	Series solution	0.148		1.91		1.08	
	4 × 4	0.152	2.0%	1.68	−12%	0.77	−29%
	8 × 8	0.149	0.7%	1.81	−5.2%	0.99	−8.3%
	14 × 14	0.149	0.7%	1.87	−2.1%	1.06	−1.9%

Figures

Figure 1. Simply supported skew plate with uniform distributed load. A 4 × 4 mesh for the complete plate of quadrilateral elements is shown. The corresponding mesh of triangular elements is shown by

the dotted line.

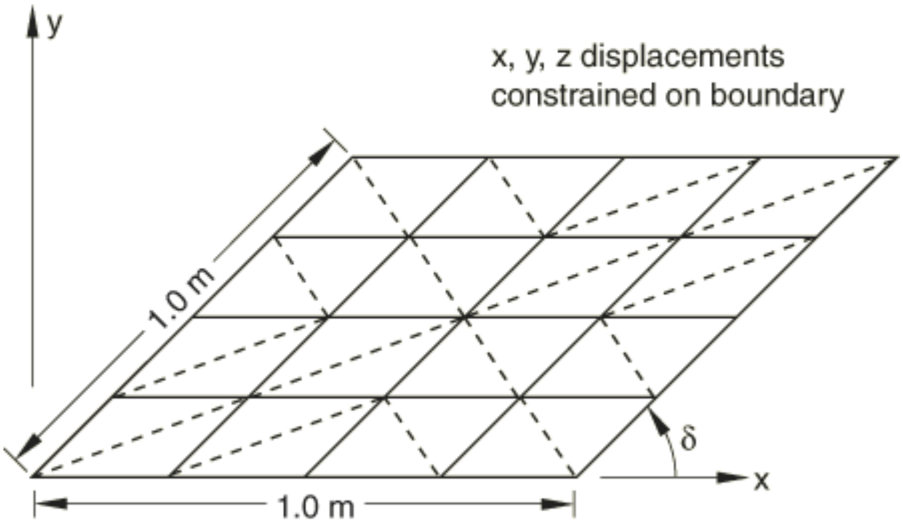


Figure 2. 4 × 4 mesh for the complete plate of quadrilateral elements.

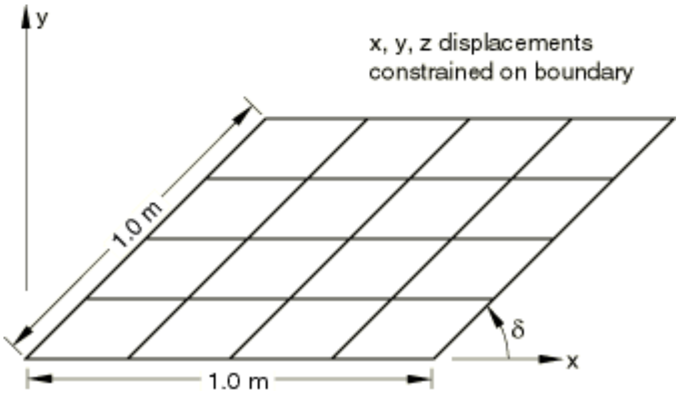


Figure 3. 2 × 2 × 4 mesh for the complete plate of triangular elements.

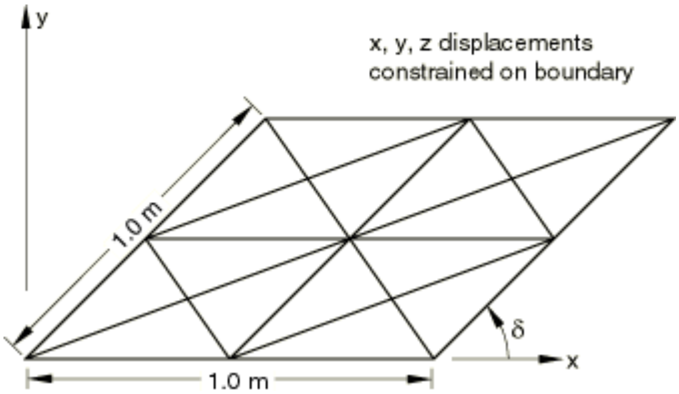
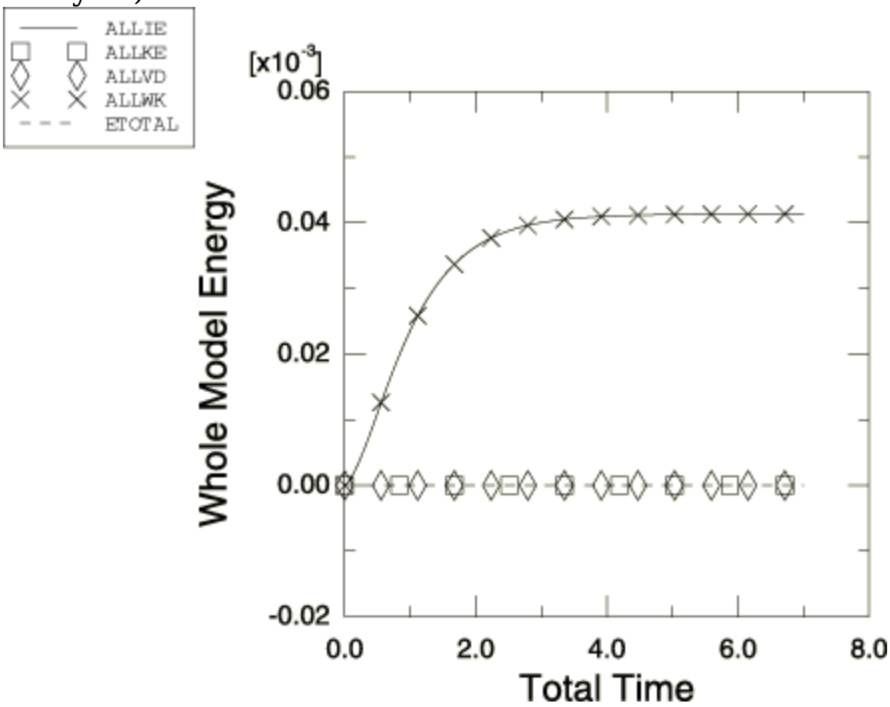


Figure 4. Energy balance for 14 × 14 mesh at 40° (Abaqus/Explicit analysis).



LE7: Axisymmetric cylinder/sphere under pressure

LE7: Axisymmetric cylinder/sphere under pressure

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

- Elements tested
- Problem description
- Reference solution
- Results and discussion
- Input files

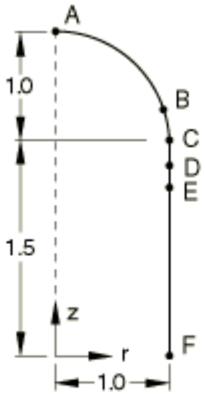
ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

SAX1

SAX2

Problem description



Units: m, kN

Thickness = 0.025

Point	r	z
B	0.9814	1.6920
D	1.0	1.4034
E	1.0	1.1136

Model:

Thin-walled pressure vessel.

Mesh:

A coarse and a fine mesh are tested.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

ur=ϕ= 0 at point A. uz= 0 at point F.

Loading:

Uniform internal pressure of 1.0 MPa. In the explicit dynamic analysis the loading is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE7 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Axial stress, σ_{zz}= 25.9 MPa on the outer surface at point D.

Results and discussion

The results are shown in the following table. The values enclosed in parentheses are percentage differences from the target solution.

Element	σ_{zz}, Coarse Mesh	σ_{zz}, Fine Mesh
SAX1 (Abaqus/Explicit)	25.6 MPa (−1%)	25.7 MPa (−0.5%)
SAX2 (Abaqus/Standard)	26.034 MPa (+0.67%)	25.878 MPa (+0.07%)

Input files

Coarse mesh tests:

le7_c.inp

SAX1 elements.

nle7xa3c.inp

SAX2 elements.

Fine mesh tests:

le7_f.inp

SAX1 elements.

nle7xa3f.inp

SAX2 elements.

LE8: Axisymmetric shell under pressure

LE8: Axisymmetric shell under pressure

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

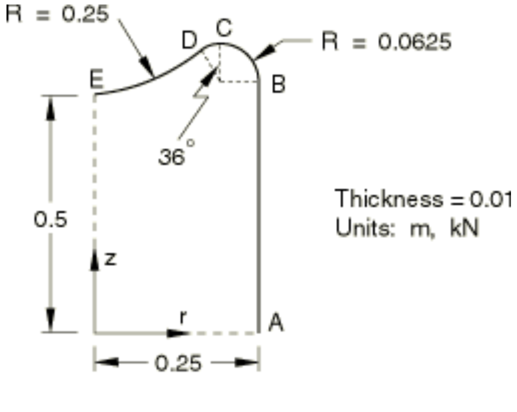
ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

SAX1

SAX2

Problem description



Model:

Axisymmetric shell under pressure.

Mesh:

A coarse and a fine mesh are tested.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

uz= 0 at point A. ur=ϕ= 0 at point F.

Loading:

Uniform internal pressure of 1.0 MPa. In the explicit dynamic analysis the loading is applied such that a quasi-static solution is obtained.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE8 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Hoop stress, σθθ= 94.5 MPa on the outer surface at point D.

Results and discussion

The results are shown in the following table. The values enclosed in parentheses are percentage differences from the target solution.

Element	σθθ, Coarse Mesh	σθθ, Fine Mesh
SAX1 (Abaqus/Explicit)	99.1 MPa (+5%)	89.3 MPa (−6%)
SAX2 (Abaqus/Standard)	90.12 MPa (−4.7%)	90.41 MPa (−4.4%)

Input files

Coarse mesh tests:

le8_c.inp

SAX1 elements.

nle8xa3c.inp

SAX2 elements.

Fine mesh tests:

le8_f.inp

SAX1 elements.

nle8xa3f.inp

SAX2 elements.

LE9: Axisymmetric branched shell under pressure

LE9: Axisymmetric branched shell under pressure

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

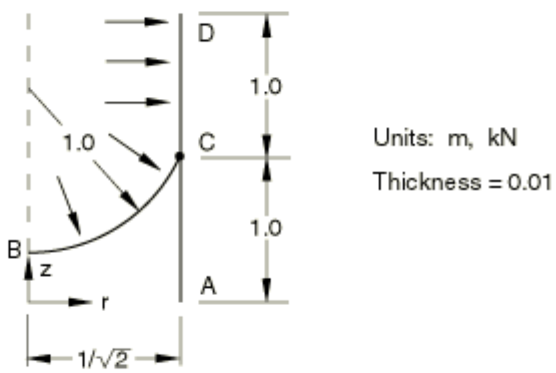
This page discusses:

ProductsAbaqus/Standard

Elements tested

SAX2

Problem description



Mesh:

A coarse and a fine mesh are tested.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3.

Boundary conditions:

$u_r = u_z = \phi = 0$ at point A.

Loading:

Uniform internal pressure of 1.0 MPa along edge BCD.

General:

Gauss integration is used for the shell cross-section in input file nle9xa3f.inp.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE9 from NAFEMS Publication TNSB, Rev. 3, "The Standard NAFEMS Benchmarks," October 1990.

Target solution: Axial stress, $\sigma_{zz} = -319.9$ MPa on the outer surface of the upper cylinder at point C.

Results and discussion

The results are shown in the following table. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Element	σ_{zz}, Coarse Mesh	σ_{zz}, Fine Mesh
SAX2	−307.24 MPa (−4.0%)	−314.81 MPa (−1.6%)

Input files

nle9xa3c.inp

Coarse mesh analysis.

nle9xa3f.inp

Fine mesh analysis.

LE10: Thick plate under pressure

LE10: Thick plate under pressure

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

- Elements tested
- Problem description
- Reference solution
- Results and discussion
- Input files

ProductsAbaqus/StandardAbaqus/Explicit

Elements tested

C3D20

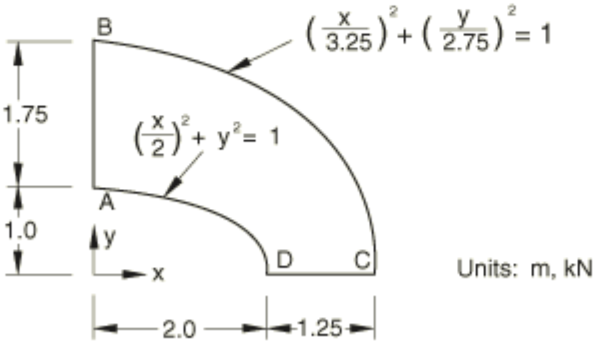
C3D20R

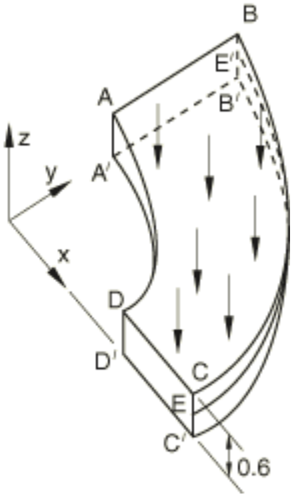
C3D10

C3D10HS

C3D10M

Problem description





Model:

Thick plate under uniform pressure.

Mesh:

A coarse and a fine mesh are tested.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, density = 7800 kg/m^3.

Boundary conditions:

uy= 0 on face DCD′C′. ux= 0 on face ABA′B′. ux=uy= 0 on face BCB′C′. uz= 0 on line EE′ (E is the midpoint of edge CC′; E′ is the midpoint of edge BB′).

Loading:

Uniform normal pressure of 1.0 MPa on the upper surface of the plate.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE10 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Direct stress, σyy= 5.38 MPa at point D.

Results and discussion

The Abaqus/Standard results are shown in Table 1. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Table 1. Abaqus/Standard analysis.

Element	σyy, Coarse Mesh	σyy, Fine Mesh
C3D20	−6.72 MPa (+25.00%)	−5.64 MPa (+4.83%)
C3D20R	−7.93 MPa (+47.39%)	−5.53 MPa (+2.78%)
C3D10	−5.44 MPa (+1.15%)	−5.77 MPa (+7.24%)
C3D10HS	–5.08 MPa (−3.72%)	–5.51 MPa (+2.42%)
C3D10M	−5.57 MPa (+3.53%)	−5.89 MPa (+9.48%)

The C3D10 and C3D10M elements are more accurate with the coarse mesh than with the fine mesh: in the coarse meshes four elements come together at the point of interest, giving a more accurate result after averaging to the nodes. In the more refined mesh, only one element contains the point of interest; therefore, the extrapolation to the nodes is less accurate.

Unlike Abaqus/Standard, Abaqus/Explicit does not have the option for extrapolating integration point outputs (such as stresses) to the nodes. Consequently, the desired stress component at point D cannot be extracted except by rough interpretation of color contour plots. As an alternative, the value of σyy at an integration point near point D is compared between an Abaqus/Standard simulation and an Abaqus/Explicit simulation.

In the Abaqus/Explicit analyses the pressure is ramped up smoothly from zero to its final value of 1.0 MPa over a time period of 0.4 seconds, which is slow enough to be considered quasi-static (inertial effects play a minimal role).

Analysis Type	σyy, Coarse Mesh	σyy, Fine Mesh
Abaqus/Standard	−3.70 MPa	−4.61 MPa
Abaqus/Explicit	−3.79 MPa	−4.55 MPa

For the coarse mesh the point of comparison is at element 18, integration point 3. For the fine mesh the point of comparison is at element 199, integration point 1. Both are close neighbors of the physical corner point D.

Input files

Abaqus/Standard input files

Coarse mesh tests:

nle10fkc.inp

C3D20 elements.

nle10rkc.inp

C3D20R elements.

nle10c_c3d10.inp

C3D10 elements.

nle10c_c3d10hs.inp

C3D10HS elements.

nle10c_c3d10m.inp

C3D10M elements.

Fine mesh tests:

nle10fkf.inp

C3D20 elements.

nle10rkf.inp

C3D20R elements.

nle10f_c3d10.inp

C3D10 elements.

nle10f_c3d10hs.inp

C3D10HS elements.

nle10f_c3d10m.inp

C3D10M elements.

Abaqus/Explicit input files

exxle10_c.inp

C3D10M elements, coarse mesh.

exxle10_f.inp

C3D10M elements, fine mesh.

LE11: Solid cylinder/taper/sphere—temperature loading

LE11: Solid cylinder/taper/sphere—temperature loading

This problem provides evidence that Abaqus can reproduce the result from the benchmark defined by NAFEMS and cited as the reference solution.

This page discusses:

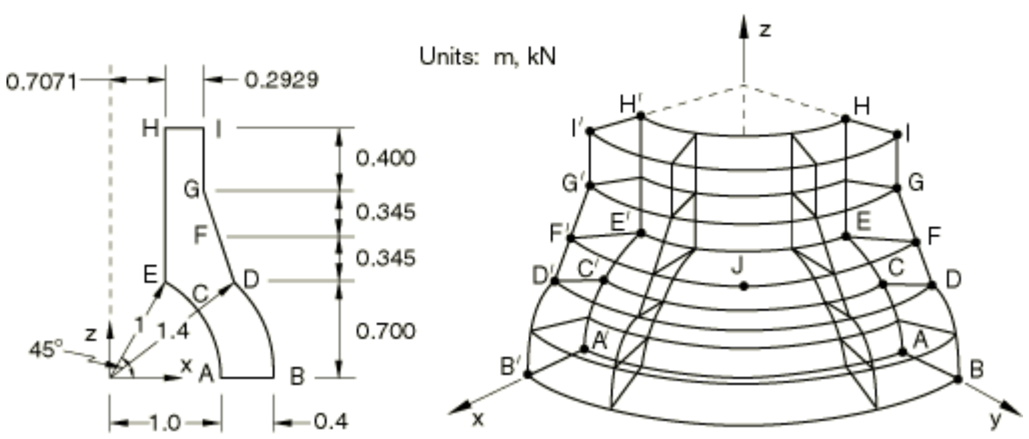
ProductsAbaqus/Standard

Elements tested

C3D20

C3D20R

Problem description



Mesh:

A coarse and a fine mesh are tested.

Material:

Linear elastic, Young's modulus = 210 GPa, Poisson's ratio = 0.3, coefficient of thermal expansion = 2.3E−4/°C.

Boundary conditions:

uy= 0 on the plane y= 0. ux= 0 on the plane x= 0. uz= 0 on the plane z= 0 and the face HIH′I′.

Loading:

Linear temperature gradient in the radial and axial directions is given by

Δθ=(x2+y2)+z.

This is applied using user subroutine UTEMP.

Reference solution

This is a test recommended by the National Agency for Finite Element Methods and Standards (U.K.): Test LE11 from NAFEMS Publication TNSB, Rev. 3, “The Standard NAFEMS Benchmarks,” October 1990.

Target solution: Direct stress, σzz = −105 MPa at point A.

Results and discussion

The results are shown in the following table. The values enclosed in parentheses are percentage differences with respect to the reference solution.

Element	σzz, Coarse Mesh	σzz, Fine Mesh
C3D20	−96.71 MPa (−7.9%)	−103.26 MPa (−1.7%)
C3D20R	−93.04 MPa (−11.4%)	−99.60 MPa (−5.1%)

Input files

Coarse mesh tests:

nle11fkc.inp

C3D20 elements.

nle11fkc.f

User subroutine used in nle11fkc.inp.

nle11rkc.inp

C3D20R elements.

nle11rkc.f

User subroutine used in nle11rkc.inp.

Fine mesh tests:

nle11fkf.inp

C3D20 elements.

nle11fkf.f

User subroutine used in nle11fkf.inp.

nle11rkf.inp

C3D20R elements.

nle11rkf.f

User subroutine used in nle11rkf.inp.