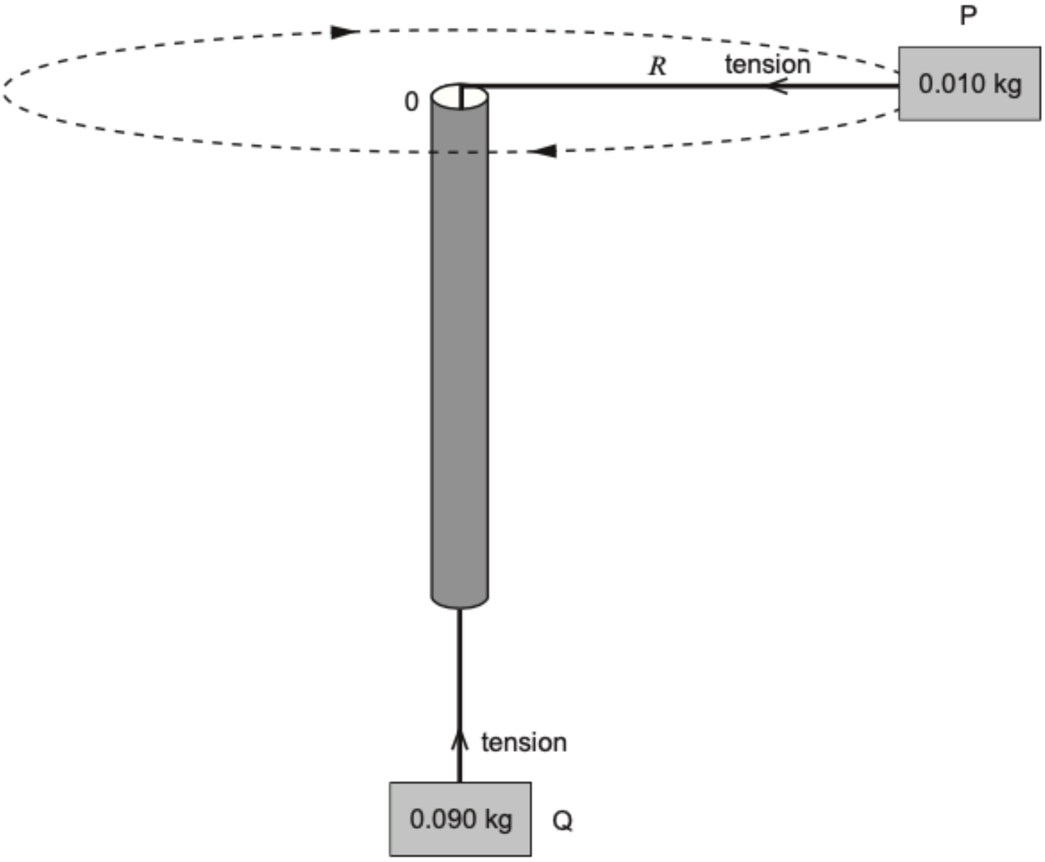
Circular Motion

2017

5. A piece of string is threaded through a hollow narrow cylinder. Two small objects, P and Q, with masses 0.010 kg and 0.090 kg respectively are attached to the ends of the string, as shown.

A student holds the cylinder and sets the 0.010 kg mass rotating in a horizontal circle of radius R, which is kept constant at 0.50 m. The time for 10 rotations is recorded. The tension in the string provides both the centripetal force on P and an upward force to hold Q in equilibrium.

The measurement is repeated for different values of R. All measurements are recorded in the table overleaf.



(a) Show that the speed, v, of mass P for each measurement is given by:

$v = \dfrac{2\pi R}{T}$ where T is the period of rotation. [1]

(b) Complete the table. [4]

R / m	Time for 10 rotations / s	Period T / s	v / m s^{-1}	v^2 / m^2 s^{-2}
0.50	4.7			
0.60	5.2			
0.70	5.6			
0.80	6.0			
0.90	6.3			

(c) (i) Assuming that OP is horizontal, write an equation relating the centripetal force to v and R. [2]

(ii) Hence, by using the equation for the forces acting on mass Q, show that:

$$v^2 = 9g\,R$$

where g is the acceleration due to gravity. [3]

(d) (i) Use the data in the table to plot a graph of v^2 (y-axis) against R (x-axis). [4]

(ii) Determine a value for g. [3]

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(iii) Suggest a way in which the experiment can be improved. [1]

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<table>
<tr><th colspan="3" rowspan="2">Question</th><th rowspan="2">Marking details</th><th colspan="4">Marks available</th><th></th><th></th></tr>
<tr><th>AO1</th><th>AO2</th><th>AO3</th><th>Total</th><th>Maths</th><th>Prac</th></tr>
<tr><td>5</td><td>(a)</td><td></td><td>Distance moved by mass P in 1 period, T (i.e. in one rotation) =
$2\pi R$ and speed = distance / time = $\frac{2\pi R}{T}$
Alternative:
$\omega = \frac{2\pi}{T}$ and $v = \omega R$</td><td></td><td>1</td><td></td><td>1</td><td>1</td><td>1</td></tr>
<tr><td></td><td>(b)</td><td></td><td><table>
<tr><th>R /m</th><th>Time of 10 rot /s</th><th>T /s</th><th>v / m s^{-1}</th><th>v^2 / m^2 s^{-2}</th></tr>
<tr><td>0.50</td><td>4.7</td><td>0.47</td><td>6.68</td><td>44.6</td></tr>
<tr><td>0.60</td><td>5.2</td><td>0.52</td><td>7.25</td><td>52.6</td></tr>
<tr><td>0.70</td><td>5.6</td><td>0.56</td><td>7.85</td><td>61.6</td></tr>
<tr><td>0.80</td><td>6.0</td><td>0.60</td><td>8.38</td><td>70.2</td></tr>
<tr><td>0.90</td><td>6.3</td><td>0.63</td><td>8.98</td><td>80.6</td></tr>
</table>
T column: All values correct (1)
For column v all values correct (1)
For column v^2 all values correct ecf (1)
Consistent use of sig figs in each column and 2 or 3 sig figs (1)</td><td></td><td>4</td><td></td><td>4</td><td>4</td><td>4</td></tr>
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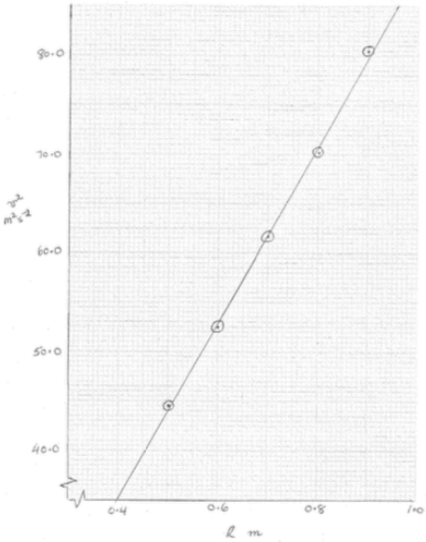
<tr><td></td><td>(c)</td><td>(i)</td><td>Centripetal force = 0.010 $\frac{v^2}{R}$
(1 for $\frac{mv^2}{R}$; 1 if value inserted for m)</td><td>1</td><td>1</td><td></td><td>2</td><td>1</td><td>2</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>Forces acting on mass Q: $0.090\,g - \tau = 0$ τ: tension (1)
So $\tau = 0.090g$. Substitution for τ into (c)(i) (1)
$0.090g = 0.010\frac{v^2}{R}$
$v^2 = \frac{0.090g}{0.010}R$
$v^2 = 9g\ R$ clear and convincing working (1)</td><td></td><td>3</td><td></td><td>3</td><td>2</td><td>3</td></tr>
</table>

Question			Marking details	Marks available				Maths	Prac
				AO1	AO2	AO3	Total		
	(d)	(i)	 Axes, suitable choice scales (no multiples of 3) and labels on both axes (1) – scales to occupy more than half of paper All points plotted correctly to ±½ small square division (2) 4 points plotted correctly to ±½ small square division award 1 mark 1-3 points plotted correctly to ±½ small square division award 0 marks Line of best fit (1)		4		4	4	4

Question			Marking details	Marks available				Maths	Prac
				AO1	AO2	AO3	Total		
		(ii)	gradient = $\frac{50.0}{0.56} = 89.286$ m s^{-2} ; find gradient from best fit line (1) also gradient = $9g$ general method (1) $9g = 89.286$ $g = \frac{1}{9}\ 89.286 = 9.92$ m s^{-2} **unit mark** (1) (Accept g = 8.8 to 10.8 m s^{-2} i.e. uncertainty of ~10%.) Use of single data point award a maximum of 2 marks			3	3	3	3
		(iii)	Take measurements for each value of R several times **or** measure time of more rotations **or** use of video capture or increase radius and period Accept repeat readings Don't accept have an assistant			1	1		1
			Question 5 total	**1**	**13**	**4**	**18**	**15**	**18**

2018

1. *(a)* A car travels at a constant speed of 45.0 km h^{-1} around a curve in the road with a radius of 80 m.

(i) Explain why the car is accelerating. [2]

(ii) Calculate the angular velocity of the car (in rad s^{-1}) as it travels around the curve. [2]

(iii) Calculate the acceleration of the car and state its direction. [3]

(b) Discuss how the application of science enables cars to travel safely around curves. [2]

Question			Marking details	Marks available					
				AO1	AO2	AO3	Total	Maths	Prac
1	*(a)*	(i)	Velocity [of car] is changing [with time] (1) because its direction is changing [so the car is accelerating] (1) **or** There is a resultant force (1) [towards the centre] due to friction / grip (1)	2			2		
		(ii)	$v=\frac{45\times10^3}{60\times60}$ = 12.5 m s^{-1} (conversion) (1) $\omega=\frac{v}{r}=\frac{12.5}{80}$ = 0.156 rad s^{-1} substitution and calculation (1)		2		2	2	
		(iii)	$a=\frac{v^2}{r}=\frac{(12.5)^2}{80}$ substitution (**ecf**) (1) [Alt: use $a=\omega^2 r$] = 1.95 m s^{-2} (1) Direction: towards centre (of 'circular' motion) (1)	1 1	1		3	2	
	(b)		Either: Any two × (1) of these points - Appropriate tyre design for friction - Banking of road [for contribution from normal contact force] - Appropriate surface - Suspension set-up - Anti-roll bars. (or any sensible answers, one referring to road and the other to the car) **Or** any one sensible point (1) + explanation of the role of physics (1) [			2	2		
			Question 1 total	**4**	**3**	**2**	**9**	**4**	**0**

2022

4. A satellite orbiting the Earth completes one revolution in 105 minutes.

Mass of the Earth = 6.0×10^{24} kg, radius of the Earth = 6400 km

(a) Explain what is meant by the term *radian*. [2]

(b) Calculate the satellite's angular velocity, ω, in rad s^{-1}. [3]

(c) The gravitational force on the satellite is given by $\frac{GM_E m}{R^2}$ where m is the mass of the satellite, M_E is the mass of the Earth and R is the radius of the orbit. Show that:

$$R = \sqrt[3]{\frac{GM_E}{\omega^2}}$$ [2]

(d) Determine the height of the satellite above the surface of the Earth. [3]

(e) The mass of the Moon is 7.3×10^{22} kg and its radius is 1 740 km. Discuss whether a satellite would be able to orbit the Moon with a period of 105 minutes. [3]

Question			Marking details	Marks available					
				AO1	AO2	AO3	Total	Maths	Prac
4	(a)		<u>Angle</u> where (1) accept diagram showing angle arc length equals the radius **or** approximately 57.3° or $2\pi = 360°$ (1)	2			2		
	(b)		Period, $T = 105 \times 60 = 6300$ [s] or 105 [min] (1) Substitution (regardless of unit): $\omega = \frac{2\pi}{T} = \frac{2\pi}{6300}$ (1) $= 9.97 \times 10^{-4}$ [rad s^{-1}](1)	1	1 1		3	3	
	(c)		$m\omega^2 R = \frac{GM_{\mathrm{E}}m}{R^2}$ (1) Accept $\frac{mv^2}{r} = \frac{GM_{\mathrm{E}}m}{R^2}$ $R^3 = \frac{GM_{\mathrm{E}}}{\omega^2}$ [so: $R = \sqrt[3]{\frac{GM_{\mathrm{E}}}{\omega^2}}$] (1)		2		2	1	
	(d)		Substitution: $R = \sqrt[3]{\frac{(6.67\times10^{-11})\,(6.0\times10^{24})}{(9.97\times10^{-4})^2}}$ **ecf** (1) R = 7 380 k[m] (1) Altitude above the surface of the Earth = 7 380 – 6 400 = 980 k[m] (1)	1	1 1		3	3	
	(e)		$R = \sqrt[3]{\frac{(6.67\times10^{-11})\,(7.3\times10^{22})}{(9.97\times10^{-4})^2}}$ **ecf (1)** R = 1 698 k[m] (1) Correct conclusion based on candidate's answer e.g. not possible since inside Moon (1) **Alternative:** Substituting into equation to calculate period or to calculate mass (1) Correct answer e.g. to period (1) Correct conclusion based on period / mass e.g. period longer so not possible, mass smaller so not possible (1)			3	3	1	

Question			Marking details	Marks available					
				AO1	AO2	AO3	Total	Maths	Prac
			Alternative: Can also work centripetal acceleration (1.7) (1) Calculate gravitational acceleration (1.6) (1) Not possible (since gravity too weak) (1)						
			Question 4 total	**4**	**6**	**3**	**13**	**8**	**0**

2023

2. (a) Explain what is meant by centripetal force and state its direction. [2]

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(b) A small sphere of mass 30 g at the end of a light string rotates in a horizontal circle of radius 0.80 m and completes 10 revolutions in 15 s.

(i) Show that the speed of the sphere is 3.35 m s^{-1}. [2]

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(ii) Calculate the centripetal force on the sphere. [2]

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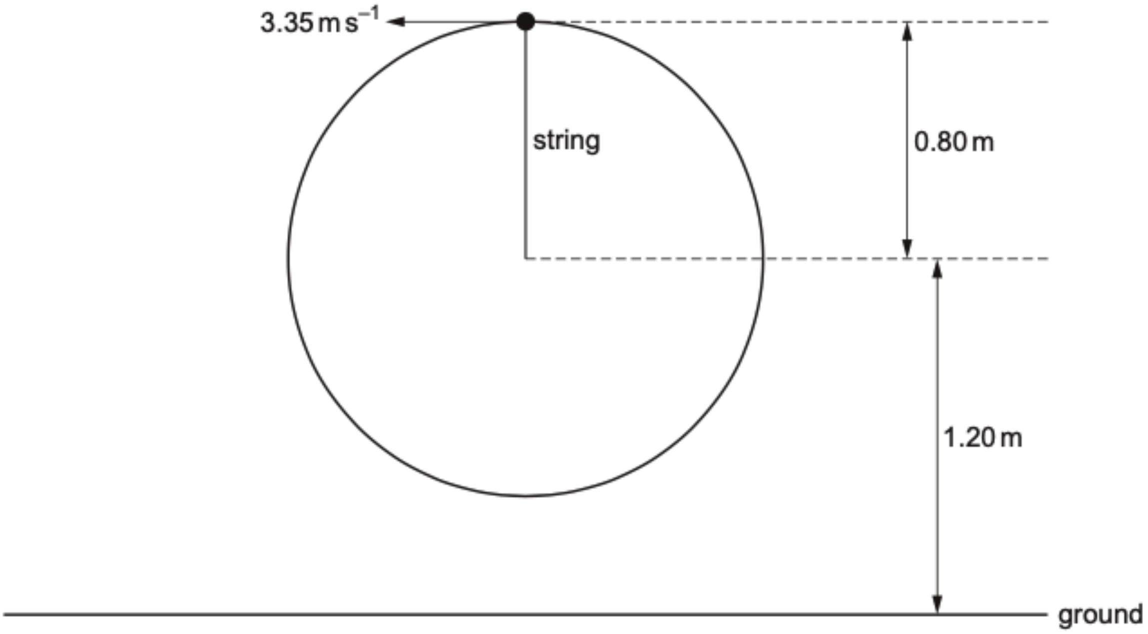
(c) The sphere now rotates in a **vertical** circle of the same radius, from a point that is 1.20 m above the ground. The speed at the top of the circle is 3.35 m s^{-1}.



Calculate the tension in the string at the **top of the circle**. [2]

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(d) (i) Calculate the speed of the sphere when it reaches the **bottom of the circle**. [3]

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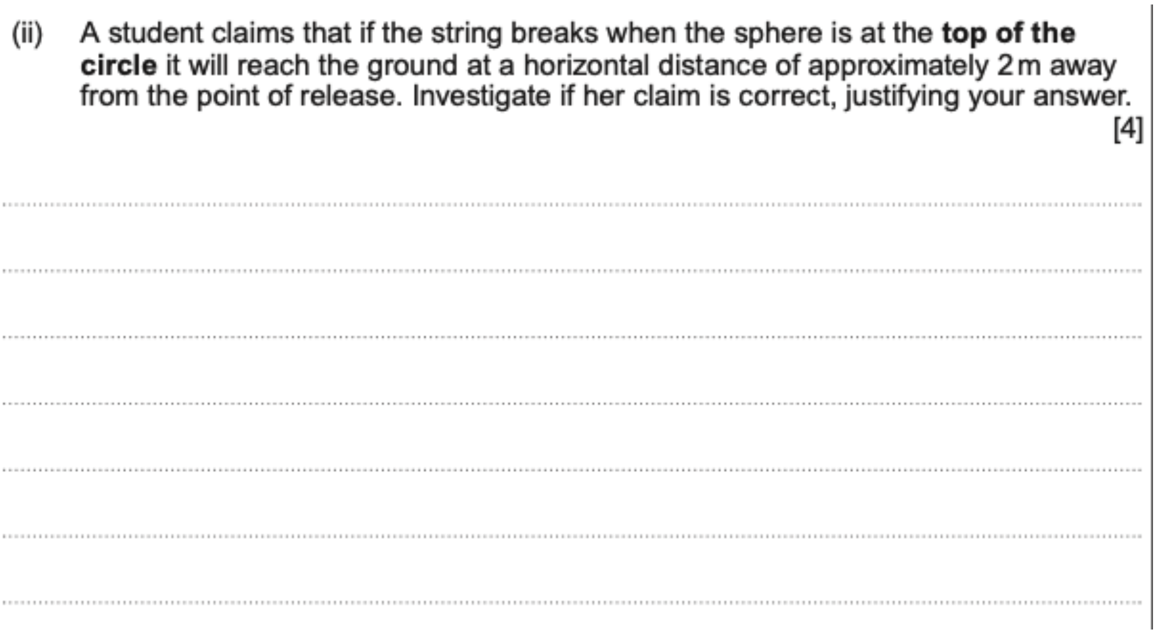

(ii) A student claims that if the string breaks when the sphere is at the **top of the circle** it will reach the ground at a horizontal distance of approximately 2 m away from the point of release. Investigate if her claim is correct, justifying your answer. [4]

Question				Marking details	Marks available					
					AO1	AO2	AO3	Total	Maths	Prac
2	(a)			Is the [resultant] force for circular motion (1) It is directed towards the centre [of the circle] (1)	2			2		
	(b)	(i)		$T = \frac{15}{10} = 1.5$ s **or** $f = \frac{10}{15} = 0.67$ [Hz] (1) $v = \frac{2\pi r}{T} = \frac{2\pi(0.8)}{1.5}$ (1)[= 3.35 m s^{-1}] **Alternative:** Total distance = 1.6π × 10 (1) $v = \frac{1.6\pi \text{x} 10}{15}$ (1) [= 3.35 m s^{-1}]	1	1		2	2	
		(ii)		Substitution: $F = \frac{mv^2}{r} = \frac{(30 \times 10^{-3})\text{x}\, 3.35^2}{0.8}$ (1) = 0.42 [N] (1) **Alternative:** $F = mr\omega^2 = (30 \times 10^{-3}) \times 0.8 \times \left(\frac{2\pi}{1.5}\right)^2$ (1) = 0.42 [N](1)	1	1		2	2	
	(c)			Force = Tension + mg (1) Tension = Force − mg = 0.42 **ecf** − $(30 \times 10^{-3})(9.81)$ = 0.13 [N] (1) Award 1 mark for 0.7 [N]		2		2	1	

Question				Marking details	Marks available					
					AO1	AO2	AO3	Total	Maths	Prac
	(d)	(i)		Conservation of energy or implied (1) Full substitution or good algebra e.g. $\frac{1}{2}mv_b{}^2 = mgh + \frac{1}{2}mv^2$ **OR** Initial KE = 0.17 [J] and PE loss = 0.47 [J] (implied by final KE = 0.64 J) **OR** $v_b = \sqrt{2gh + v^2}$ (1) Final velocity $= 6.53$ [m s^{-1}] (1) **Alternative:** Use of $v^2 = u^2 + 2ax$ with u = 3.35 m s^{-1} and a = 9.81 m s^{-2} (1) Correct answer = 6.53 [m s^{-1}] (1) Some statement of why this equation works e.g. due to conservation of energy, we can consider a particle dropping with initial downward speed of 3.35 (1) Accept – this equation shouldn't work but it does! **Alternative:** Use of $v^2 = u^2 + 2ax$ with u = 0 gives v = 5.6 [m s^{-1}] (1) Pythagoras applied i.e. $\sqrt{3.35^2 + 5.6^2}$ (1) Correct answer = 6.53 [m s^{-1}] (1)		3		3	2	
		(ii)		Use of $x = ut + \frac{1}{2}at^2$ for the vertical motion **or** 2 other usable equations (1) $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(1.2+0.8)}{9.81}} = 0.64$ [s] (1) For the horizontal motion: horizontal distance from point of released = vt = 3.35 × 0.64 **ecf** (1) = 2.14 [m] so claim is correct (1) **ecf** Accept 2.14 ≈ 2			4	4	3	
				Question 2 total	**4**	**7**	**4**	**15**	**10**	**0**