Spacecraft Attitude Control using Derivative-free Purely Adaptive Controller

Irene Grace Karot Polson and Dipak Kumar Giri

Abstract— **This paper describes a purely adaptive attitude control system without derivative laws implemented on a spacecraft attitude system. The proposed adaptive control system consists of two components: a proportional adaptation law using the pole-placement method and a derivative-free weight update law for uncertainties. These uncertainties can be caused by sudden changes in the dynamic properties of the system due to docking, structural damage, or measurement errors. The effectiveness of the applied control system is demonstrated through simulations and comparisons are made with a similar controller with derivative weight update laws. They corroborate the robustness of the control system when uncertainties are present.**

I. Introduction

Attitude control plays a vital role in various problems such as spacecraft rendezvous, docking, berthing, and capture for maintenance. Conventional methods of spacecraft attitude control require good knowledge of the system and its dynamics to develop control equations [1-3]. The process of coming up with accurate dynamic laws and system parameters is often time-consuming and tedious. Over the years, methods such as system identification [4-5] have been employed to address this problem.

System identification helps build the mathematical model of dynamical systems using input-output data; system properties are not required. However, it still does not account for sudden changes in dynamics such as structural damage, the release of payload, and uncertainties. This can be tackled by designing a suitable control system that can adapt with time, the system's changing state, and uncertainties while maintaining its robustness.

The use of adaptive controls allow changing the controller in response to changes in the system and environment. This ensures that the system is operating at its best possible state. Various simple adaptive controllers [6-7] have been developed over the decade. In [8], a simple adaptive control has been applied to spacecraft attitude control with applications to docking between spacecraft and free-flying robots. Further, numerous control strategies, from gain scheduling to machine learning [9-12], have been developed for fast and accurate adaptive control laws. This is carried out by computing the control system's parameters that vary with time and state. Multiple derivative-free control systems [13-14] have shown robustness and fast convergence.

However, despite the tremendous improvements in adaptive control systems, their application to real-world problems is limited due to the requirement of high computations, a higher number of variable parameters, and the need for experimental verification and validation of the control algorithms. This paper aims to demonstrate controllers free of derivative components in spacecraft attitude systems represented using modified Rodrigues parameters. The MRP representation of spacecraft attitude gives minimal and non-singular parameterization of attitude. The main contributions of the paper can be summarized as:

- Design a purely adaptive attitude control system consisting of two components, a) pole-placement and b) derivative-free adaptive components. The proportional adaptive gains are assigned using the pole-placement method on a time and state variant system. The adaptive weight for uncertainties is computed using derivative-free weight update laws to ensure improved settling and rise times while maintaining robustness.
- The proposed controller is compared with an adaptive system of similar proprieties having derivative weight update laws. Three cases are simulated to show the efficacy of the applied controller on the spacecraft attitude system.

The rest of the paper is organized as follows, section II shows the derivation of the non-linear state-space equations and covers the general attitude dynamics using modified Rodrigues parameters and Hamiltonian state-space equations. Section III introduces the proposed control law that is used in place of traditional adaptive control laws. Section IV discusses the stability analysis of the proposed control system while section V shows the different simulation results and its interpretations. Section VI concludes the paper while suggesting improvement thoughts that could be considered in future research.

II. Spacecraft Attitude Dynamics

The attitude dynamics of a rigid spacecraft [15] is represented as

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\widehat{\boldsymbol{\omega}}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{\omega}$ is the angular momentum, $\widehat{\boldsymbol{\omega}}$ is the skew-symmetric matrix of $\boldsymbol{\omega}$ given by

$$\widehat{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

Irene Grace Karot Polson and Dipak Kumar Giri are with Dept. of Aerospace Engineering, Indian Institute of Technology Kanpur, India irenegkp@iitk.ac.in, dkgiri@iitk.ac.in

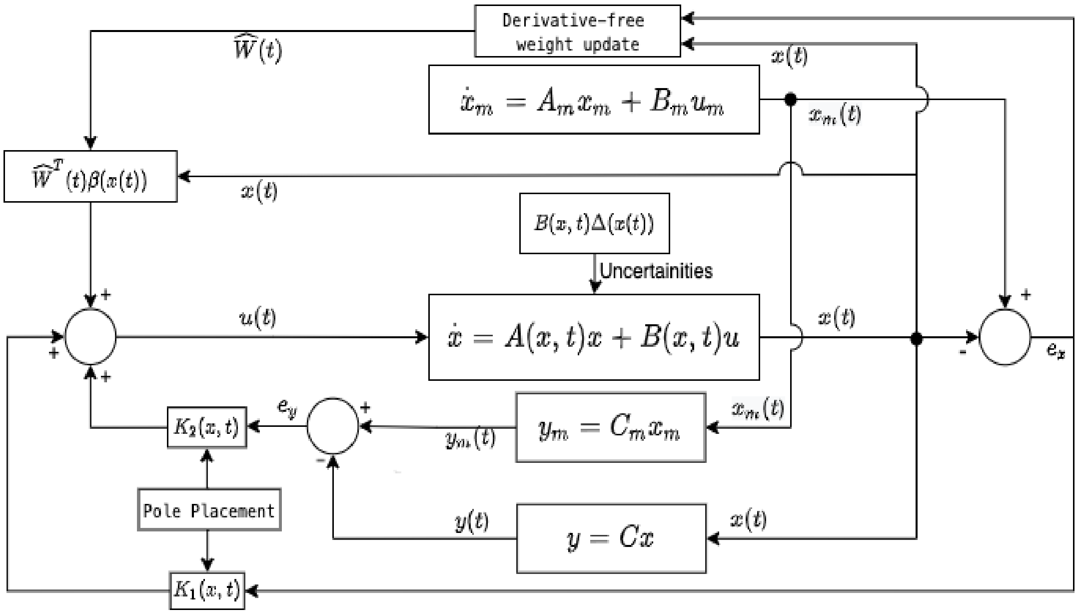

Fig. 1. Derivative-free purely adaptive control system

and $\boldsymbol{J}$ is the moment of inertia matrix given as,

$$\boldsymbol{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{3}$$

The MRP representation is chosen as the suitable representation for satellite attitude. Apart from the extended singularity range, the linearized equations of MRPs [16-17] are more accurate to the non-linear system equations than other angle representations. Let $\hat{\boldsymbol{e}}$ and ϕ represent the unit vector corresponding to the axis of rotation and the angle of rotation, respectively. Then the the MRP parameter is denoted as $\boldsymbol{\sigma} = \tan(\frac{\phi}{4})\hat{\boldsymbol{e}}$ and

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{L}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{4}$$

represents the kinematic equation in terms $\boldsymbol{\sigma}$ where

$$\boldsymbol{L}(\boldsymbol{\sigma}) = \frac{1}{2}\left[\boldsymbol{I}\left(\frac{1-\boldsymbol{\sigma}^T\boldsymbol{\sigma}}{2}\right) + \hat{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \tag{5}$$

and $\hat{\boldsymbol{\sigma}}$ is the skew-symmetric matrix of $\boldsymbol{\sigma}$ similar to that in (2). The spate-space equations of the spacecraft attitude system are written in the Hamiltonian non-linear Euler-Lagrange form [18] as shown below,

$$\boldsymbol{H}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} + \boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = \boldsymbol{u} \tag{6}$$

Using $\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) = 2(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\boldsymbol{I}-\hat{\boldsymbol{\sigma}}]$ and $\hat{\boldsymbol{G}}$ as the skew-symmetric matrix of $\boldsymbol{G} = \boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}}$ yields,

$$\boldsymbol{u} = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{\tau} \tag{7}$$

$$\boldsymbol{H}(\boldsymbol{\sigma}) = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) \tag{8}$$

$$\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = -\boldsymbol{L}^{-T}(\boldsymbol{\sigma})[\boldsymbol{J}\boldsymbol{L}^{-1}\dot{\boldsymbol{L}}\boldsymbol{L}^{-1} + \hat{\boldsymbol{G}}\boldsymbol{L}^{-1}] \tag{9}$$

The control input vector used in the non-linear state-space formulation below is represented as $\boldsymbol{u}$. Let the state vector be defined as $\boldsymbol{x} = [\boldsymbol{\sigma} \quad \dot{\boldsymbol{\sigma}}]^T$, then the state-space equations are formulated as,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{u}, \qquad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{10}$$

and the system's co-efficient matrices are obtained from (6) as,

$$\boldsymbol{A}(\boldsymbol{x},t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I_3} \\ \boldsymbol{0} & -\boldsymbol{H}^{-1}\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}}) \end{bmatrix}, \quad \boldsymbol{B}(\boldsymbol{x},t) = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{H}^{-1} \end{bmatrix} \tag{11}$$

$$\boldsymbol{C} = \begin{bmatrix} \alpha\boldsymbol{I_3} & \boldsymbol{I_3} \end{bmatrix}$$

where $\boldsymbol{A}(\boldsymbol{x},t)$ and $\boldsymbol{B}(\boldsymbol{x},t)$ are varying with time and state and they need to be computed at every instant of time along with the states. The choice of $\boldsymbol{C}$ matrix with α being the scaling factor yields

$$\boldsymbol{y} = \alpha\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}} \tag{12}$$

allowing the spacecraft to be controlled through the position feedback with velocity providing damping.

III. Derivative-free Adaptive control

This section covers the modification of the state-space formulation with uncertainties and introduction of the derivative-free controller design. Let $\Delta(\boldsymbol{x}(t))$ be the uncertainty introduced, then the system can be redefined as,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{x},t)[\boldsymbol{u}(t) + \Delta(\boldsymbol{x}(t))] \tag{13}$$

The uncertainty in (13) can be linearly parameterized as

$$\Delta(\boldsymbol{x}(t)) = \boldsymbol{W}^T(t)\boldsymbol{\beta}(\boldsymbol{x}) \tag{14}$$

where $\boldsymbol{W}(t) \in \mathbb{R}^{nxm}$ is the weight matrix which is time-varying and needs to be estimated at each time instant and

$\boldsymbol{\beta}(\boldsymbol{x})$ is a vector of basis functions chosen judiciously. The reference model for tracking is defined as,

$$\dot{\boldsymbol{x}}_{\boldsymbol{m}} = \boldsymbol{A}_{\boldsymbol{m}}\boldsymbol{x}_{\boldsymbol{m}} + \boldsymbol{B}_{\boldsymbol{m}}\boldsymbol{u}_{\boldsymbol{m}}, \qquad \boldsymbol{y}_{\boldsymbol{m}} = \boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{x}_{\boldsymbol{m}} \tag{15}$$

where second order dynamics is used to get the desired response by adjusting the damping ratio ζ and natural frequency ω_n given by,

$$\ddot{\boldsymbol{\sigma}}_{\boldsymbol{m}} + 2\zeta\omega_n\dot{\boldsymbol{\sigma}} + \omega_n^2\boldsymbol{\sigma}_{\boldsymbol{m}} = \omega_n^2\boldsymbol{u}_{\boldsymbol{m}} \tag{16}$$

which yields,

$$\boldsymbol{A}_{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I_3} \\ -\omega_n^2\boldsymbol{I_3} & -2\zeta\omega_n \end{bmatrix}, \quad \boldsymbol{B}_{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{0} \\ \omega_n^2\boldsymbol{I_3} \end{bmatrix} \tag{17}$$
$$\boldsymbol{C}_{\boldsymbol{m}} = \begin{bmatrix} \alpha\boldsymbol{I_3} & \boldsymbol{I_3} \end{bmatrix}$$

The derivative-free control law uses derivative-free methods to update the weight that will adjust itself to uncertainties introduced in (13). The control law is designed purely adaptive and is defined as $\boldsymbol{u}(t) = \boldsymbol{u}_{\boldsymbol{a}}(t)$, where $\boldsymbol{u}_{\boldsymbol{a}}(t)$ is the adaptive feedback control represented by,

$$\boldsymbol{u}_{\boldsymbol{a}}(t) = \boldsymbol{K_1}(\boldsymbol{x},t)\boldsymbol{e}_{\boldsymbol{x}}(t) + \boldsymbol{K_2}(\boldsymbol{x},t)\boldsymbol{e}_{\boldsymbol{y}}(t) + \hat{\boldsymbol{W}}^T(t)\boldsymbol{\beta}(\boldsymbol{x}(t)) \tag{18}$$

$\boldsymbol{K_1}(\boldsymbol{x},t)$ and $\boldsymbol{K_2}(\boldsymbol{x},t)$ denoted above are proportional feedback gains obtained using pole-placement methods satisfying the Hurwitz condition at each instant of time as

$$\boldsymbol{A}_{\boldsymbol{m}} = \boldsymbol{A}(\boldsymbol{x},t) + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_1}(\boldsymbol{x},t) \tag{19}$$
$$\boldsymbol{B}_{\boldsymbol{m}} = \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_2}(\boldsymbol{x},t) \tag{20}$$

and $\hat{\boldsymbol{W}}(t)$ is the estimated using the derivative-free weight update law given by

$$\hat{\boldsymbol{W}}(t) = \Omega_1\hat{\boldsymbol{W}}(t-\tau) + \hat{\Omega}_2(t) \tag{21}$$

where $\tau > 0$ and $0 < \Omega_1^T\Omega_1 < 1$. For $k > 0$, $\hat{\Omega}_2(t)$ can be estimated as

$$\hat{\Omega}_2(t) = k[\boldsymbol{\beta}(\boldsymbol{x}(t))\boldsymbol{e}_{\boldsymbol{x}}^T\boldsymbol{P}\boldsymbol{B}] \tag{22}$$

where the error $\boldsymbol{e}_{\boldsymbol{x}}$ is defined as

$$\boldsymbol{e}_{\boldsymbol{x}}(t) = \boldsymbol{x}_{\boldsymbol{m}}(t) - \boldsymbol{x}(t) \tag{23}$$

and $\boldsymbol{P}$ is obtained by solving the Riccati equation

$$\boldsymbol{A}_{\boldsymbol{m}}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{\boldsymbol{m}} + \boldsymbol{Q} = 0 \tag{24}$$

choosing $\boldsymbol{Q}$ as any symmetric, positive-definite matrix.

IV. Stability Analysis

In this section, the stability of the proposed controller on the spacecraft attitude system is analysed. The choice of $\boldsymbol{K_1}(\boldsymbol{x},t)$ and $\boldsymbol{K_2}(\boldsymbol{x},t)$ made in (18) is following the Hurwitz stability criterion. This ensures that the pole-placement component of the adaptive control system is stable and follows the reference model.

However, the stability of the entire adaptive system requires error convergence and the bounded nature of the adaptive gains. Let $\boldsymbol{V}$ be a Lyapunov functional such that it is scalar continuous as,

$$\boldsymbol{V}(\boldsymbol{x}(t), \boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t)) = \boldsymbol{V_1}(\boldsymbol{x}(t)) + \boldsymbol{V_2}(\boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t)) \tag{25}$$

where $\boldsymbol{e}_{\boldsymbol{x}}$ is defined in (23) and $\boldsymbol{e}_{\boldsymbol{w}}(t) = \boldsymbol{W}(t) - \hat{\boldsymbol{W}}(t)$, then $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ are defined as

$$\boldsymbol{V_1}(\boldsymbol{x}(t)) = \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} \tag{26}$$
$$\boldsymbol{V_2}(\boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t)) = \boldsymbol{e}_{\boldsymbol{x}}^T\boldsymbol{P}\boldsymbol{e}_{\boldsymbol{x}} + tr\left[\int_{t-\tau}^{t} \boldsymbol{e}_{\boldsymbol{w}}^T(s)\boldsymbol{e}_{\boldsymbol{w}}(s)\,ds\right] \tag{27}$$

$\boldsymbol{V_1}(\boldsymbol{x}(t))$ is the chosen Lyapanov function defined to determine if the states are globally asymptotically stable. $\boldsymbol{V_2}(\boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t))$ is the chosen Lyapunov-Krasovskii [19] function to establish uniformly ultimate bound(UBB) nature of the derivative-free component.

Each of the components ($\boldsymbol{V_1}$ and $\boldsymbol{V_2}$) is independently analyzed in the rest of the section. The derivative of $\boldsymbol{V}_1(\boldsymbol{x}(t))$ is obtained as,

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = \dot{\boldsymbol{x}}^T\boldsymbol{P}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{P}\dot{\boldsymbol{x}} \tag{28}$$

Using the state-space equations and (24), we obtain

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} \tag{29}$$
$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = -\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} \tag{30}$$

As $\boldsymbol{Q}$ is chosen to be a positive definite matrix, the expression in (30) is negative definite, that is $\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) < 0$. As $\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t))$ is negative definite, using the basic theorem of Lyapunov it follows that the states are uniform asymptotically stable with suitable choices of $\boldsymbol{Q}$.

The analysis of $\boldsymbol{V_2}(\boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t))$ is carried using the below preposition which presents the main results adopted from the detailed derivations and corollary in [14], sufficient for the analysis the proposed derivative-free system.

Proposition 1. Let the Lyapunov-Krasovskii functional be defined as in (27). Then if $\epsilon > 0$ and $\eta = 1 + \epsilon \in (1, 1/k)$, its derivative can be reduced into the inequality obtained as,

$$\dot{\boldsymbol{V}}_{\boldsymbol{2}}(\boldsymbol{e}_{\boldsymbol{x}}(t), \boldsymbol{e}_{\boldsymbol{w}}(t)) \leq -d_1||\boldsymbol{e}_{\boldsymbol{x}}(t)||^2 - d_2||\boldsymbol{e}_{\boldsymbol{w}}(t)|| - d_3||\boldsymbol{e}_{\boldsymbol{w}}(t-\tau)|| + d_4 \tag{31}$$

where d_1, d_2, d_3 and d_4 are constants obtained as

$$d_1 = \lambda_{min}(\boldsymbol{Q}) > 0 \tag{32}$$
$$d_2 = \epsilon > 0 \tag{33}$$
$$d_3 = \lambda_{min}(I - k^{-1}\Omega_1^T\Omega_1) > 0 \tag{34}$$
$$d_4 = [1 + \epsilon + (1+\epsilon)^2/2]^2 \geq 0 \tag{35}$$

Then it implies that $\boldsymbol{e}_{\boldsymbol{x}}(t)$ and $\boldsymbol{e}_{\boldsymbol{w}}(t)$ are UBB, that is. uniform ultimately bounded if there exists Ψ_1, Ψ_2 and Ψ_3 such that,

$$||\boldsymbol{e}_{\boldsymbol{x}}(t)|| > \Psi_1, \quad ||\boldsymbol{e}_{\boldsymbol{w}}(t)|| > \Psi_2 \tag{36}$$
$$||\boldsymbol{e}_{\boldsymbol{w}}(t-\tau)|| > \Psi_3$$

Since these bounds exist and are obtained as,

$$\Psi_1 = \frac{1}{\sqrt{k\lambda_{min}(\boldsymbol{Q})[1-\eta k]}} \tag{37}$$

$$\Psi_2 = \Psi_1\sqrt{\frac{\lambda_{min}(\boldsymbol{Q})}{\epsilon}} \tag{38}$$

$$\Psi_3 = \Psi_1\sqrt{\frac{\lambda_{min}(\boldsymbol{Q})}{[1+\epsilon+(1+\epsilon)^2/2]^2}} \tag{39}$$

it follows that $\boldsymbol{e_x}(t)$ and $\boldsymbol{e_w}(t)$ are uniformly ultimate bounded leading. This leads to an adaptive system with all state errors, output errors and adaptive control gains bounded providing robustness and stability. The proposed spacecraft attitude control architecture is shown in Fig.1.

V. Simulation and Results

In this section, the efficacy of the proposed spacecraft attitude control system is shown using simulations. The damping ratio and natural frequency of the ideal model reference system is chosen as $\zeta = 0.7$ and $\omega_n = 0.02$ respectively. The initial states are chosen as follows:

$$\boldsymbol{\sigma_o} = \begin{bmatrix} -0.2 & 0.2 & 0.1 \end{bmatrix}^T$$
$$\boldsymbol{\omega_o} = \boldsymbol{\dot{\sigma}_o} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

For the derivative-free controller, $\phi_1 = 0.9$ and $k = 100$ is obtained after tuning and $\boldsymbol{\beta}$ is taken as sigmoidal-type functions that map the states into small ranges defined as,

$$\boldsymbol{\beta}(\boldsymbol{x}(t)) = \Big[\tfrac{1-e^{-\sigma_1}}{1+e^{-\sigma_1}} \quad \tfrac{1-e^{-\sigma_2}}{1+e^{-\sigma_2}} \quad \tfrac{1-e^{-\sigma_3}}{1+e^{-\sigma_3}} \quad \tfrac{1-e^{-\dot{\sigma}_1}}{1+e^{-\dot{\sigma}_1}} \quad \tfrac{1-e^{-\dot{\sigma}_2}}{1+e^{-\dot{\sigma}_2}} \quad \tfrac{1-e^{-\dot{\sigma}_3}}{1+e^{-\dot{\sigma}_3}}\Big] \tag{40}$$

At each iteration, $\boldsymbol{P}$ is calculated as in (24) where $\boldsymbol{Q}$ is taken as an identity matrix.

The computation of the simulation is online using an explicit Runge-Kutta (4,5) formula on MATLAB, the Dormand-Prince pair called ODE45 where calculation of $y(t_n)$ only requires $y(t_{n-1})$.

The performance and robustness of the spacecraft system using the proposed attitude controller has been tested for three cases,

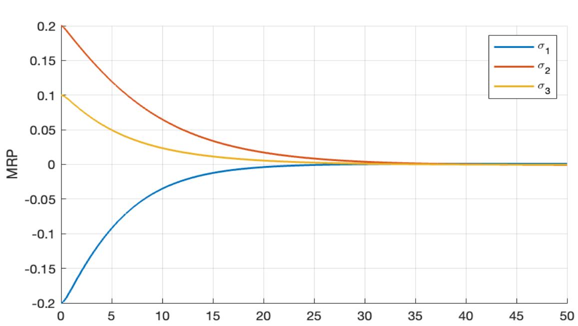

Fig. 2. MRP of derivative-free tuned controller under dynamic properties of case one.

1) Dynamic properties to which the controller is explicitly tuned: the control system has been tuned using trial and error method to obtain desired response.
2) Dynamic properties are altered from $t = 0$ without additional tuning: the performance of the system in this case determines how well it can perform with inherent uncertainties in the system.
3) Dynamic properties are suddenly varied during the flight at $t > 0$: this is carried out to determines how well and quickly the control system can adapt when uncertainties arise mid-flight.

In all cases, the proposed system is compared with a similar control system consisting of derivative laws for weight adaptation. The controller with derivative weight update laws [10] is constructed keeping all other factors constant for comparison.

Case 1. Dynamic properties to which the controller is explicitly tuned

In case 1, the value of moment of inertia $\boldsymbol{J}$ for an asymmetrical spacecraft is given by [20] :

$$\boldsymbol{J} = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

The performance of the proposed derivative-free control system, that is, the asymptotic stability of $\boldsymbol{\sigma}$ with time, can be seen in Fig.2. In Fig. 3., angular velocities obtained from the derivative and derivative-free weight update controllers are super imposed for comparison. It is observed that the derivative-free controller can track the reference state with less overshoot, rise and settling time.

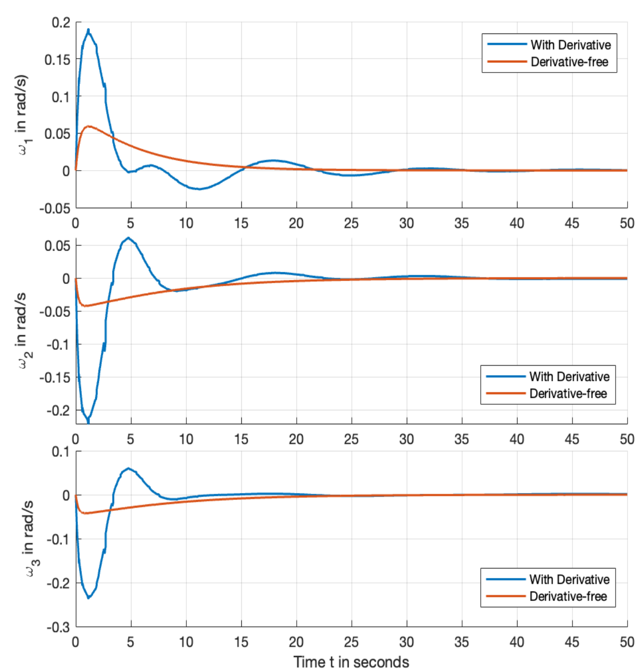

Fig. 3. Comparison between the derivative-free controller and controller with derivative weight update laws for Case one.

Manuscript 633 submitted to 2022 American Control Conference.
Received October 12, 2021.

Case 2. Dynamic properties are drastically altered without additional tuning

In this case, while the controllers are tuned to the dynamic properties in case 1, the robustness of the proposed controller to changes in dynamics is observed. The moment of inertia $\boldsymbol{J}$ is modified to

$$\boldsymbol{J} = \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

where the value of J_x is drastically changed and both the controllers have not been tuned after the modification.

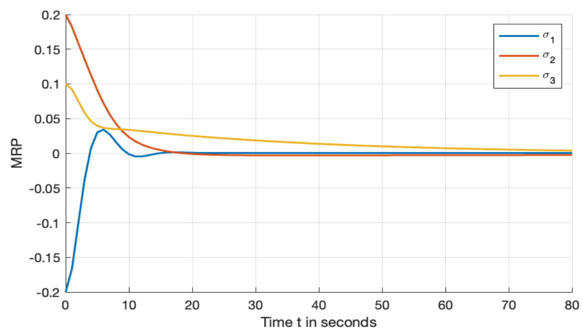

Fig. 4. MRP of derivative-free tuned controller under dynamic properties of Case two.

For the simulation results shown in Fig. 5, it can be seen that the derivative-free weight update law seems to adjust faster to the changes in dynamics and also has lesser oscillations in comparison to the derivative weight update controller.

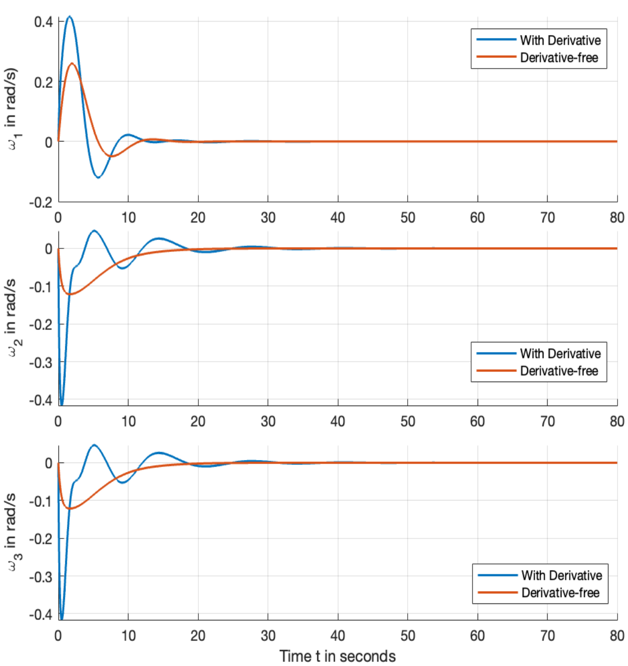

Fig. 5. Comparison between the derivative-free controller and controller with derivative weight update laws for Case two.

Case 3. Dynamic properties are suddenly varied mid-flight

In the third case, a sudden change in dynamic property is made at $t = 20$s. The moment of inertia J_x is suddenly changed such that, $\boldsymbol{J}$ changes from

$$\boldsymbol{J} = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} kg.m^2$$

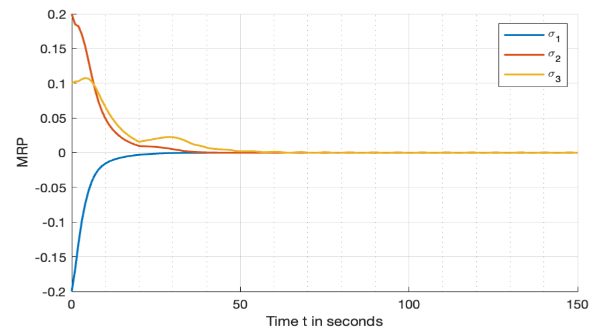

Fig. 6. MRP of derivative-free tuned controller under dynamic properties of Case three.

For the simulation result shown in Fig. 7 it can be seen that, in the case of derivative-free controller, the performance with and without sudden changes is steady. On the other hand, the derivative weight update controller seems to have more difficulties in adapting when there is an unexpected change in the structural property. It can also be seen that, the angular velocity $\boldsymbol{\omega}$ about the x-axis, ω_1, is oscillatory

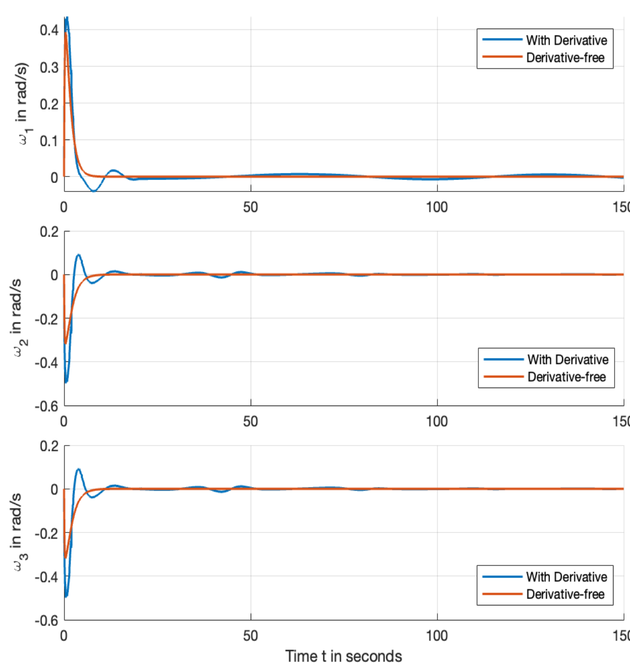

Fig. 7. Comparison between the derivative-free controller and controller with derivative weight update laws for Case three.

Manuscript 633 submitted to 2022 American Control Conference.
Received October 12, 2021.

for the derivative weight update controller once the change in J_x is made. The other components of $\boldsymbol{\omega}$ settle down, but later than the corresponding derivative-free controller settling time.

VI. Conclusions

This paper shows the efficacy of a purely adaptive controller on a spacecraft attitude system. The developed control system uses pole-placement to assign the proportional gains of the adaptive controller while it uses derivative-free weight update laws for dealing with uncertainties. The system is defined using modified Rodrigues parameters. The state-space equations are developed into a non-linear Hamiltonian-based system to have more accurate representations. The system's stability is shown using the Lyapunov–Krasovskii function. As the system is adaptive, error convergence and bounded nature of the adaptive gains are also shown. Simulations show the robustness of the controller and its ability to adapt when major system dynamic properties are significantly and suddenly changed. The above-presented purely adaptive control system can be used in cases that require fast convergence and minimal overshoot conditions. In the future, the effectiveness and advantages of other modified derivative-free laws can be explored to expand their application in spacecraft control.

References

[1] E. Crouch, Peter, "Spacecraft attitude control and stabilization: Applications of geometric control theory to rigid body models," *IEEE Transactions on Automatic Control*, Vol. 29, No. 4, pp. 321 - 331, 1984.

[2] Nicosia S., Tomei P. , "Non-linear observer and output feedback attitude control of spacecraft," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 28, No. 4, pp. 970–977, 1992.

[3] Oda M., "Coordinated control of spacecraft attitude and its manipulator," *IEEE International Conference on Robotics and Automation*, pp. 732-738, 2002.

[4] Khemaissia S., Morris A. S., "Fast-robot system identification based on neural network models," *IEEE International Symposium on Intelligent Control*, pp. 13-16, 1992.

[5] Schoukens J., Marconato A., Pintelon R., Rolain Y., Schoukens M., Tiels K., … Van Mulders A., "System Identification in a Real World," *IEEE 13th International Workshop on Advanced Motion Control (AMC)*, pp. 1-9, 2014.

[6] Nishiyama T., Suzuki S., Sato M., Masui K., "Simple Adaptive Control with PID for MIMO Fault Tolerant Flight Control Design. ," *AIAA Infotech @ Aerospace*, 2016.

[7] Ulrich, S., Hayhurst, D. L., Saenz Otero, A., Miller, D., Barkana, I., "Simple Adaptive Control for Spacecraft Proximity Operations," *AIAA Guidance, Navigation, and Control Conference*, 2014.

[8] Shi, J.-F., Ulrich, S., Allen, A. , "Spacecraft Adaptive Attitude Control with Application to Space Station Free-Flyer Robotic Capture," *AIAA Guidance, Navigation, and Control Conference*, 2015.

[9] Wang, Q., Zhou, B., Duan, G.-R. (2015). Robust gain scheduled control of spacecraft rendezvous system subject to input saturation. *Aerospace Science and Technology*, 42, 442–450, 2014.

[10] Nishiyama, T., Suzuki, S., Sato, M., Masui, K., "Adaptive Control Strategy using Lyapunov Stability Theory ,", *International Journal of Engineering Research Technology (IJERT)*, Vol. 3, No. 9, pp. 1557-1562, 2014.

[11] Mooij, E., "Simple Adaptive Control System Design Trades," *AIAA Guidance, Navigation, and Control Conference*, 2017

[12] J. E. Gaudio, T. E. Gibson, A. M. Annaswamy, M. A. Bolender, E. Lavretsky, "Connections Between Adaptive Control and Optimization in Machine Learning," *IEEE 58th Conference on Decision and Control (CDC)*, pp 4563 - 4568, 2019.

[13] Yucelen, T. and Calise, A.J., "Robustness of a Derivative-Free Adaptive Control Law," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 5, pp. 1583-1594, 2014.

[14] Yucelen, T. and Calise, A.J., "Derivative-Free Model Reference Adaptive Control," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 4, pp. 933-950, 2011.

[15] Tewari, Ashish, "Atmospheric and Space Flight Dynamics," Birkhäuser Boston, *Birkhäuser Basel*, 2007

[16] Schaub, H., and Junkins, J. L., "Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters," *The Journal of the Astronautical Sciences*, Vol. 44, No. 1, pp. 1–19, 1996.

[17] Terzakis, G. and Lourakis, M. and Ait-Boudaoud, D., "Modified Rodrigues Parameters: An Efficient Representation of Orientation in 3D Vision and Graphics," *Journal of Mathematical Imaging and Vision*, Vol. 60, No. 2, pp. 422–442, 2018.

[18] Slotine, J.-J. E. and Benedetto, M. D. D., "Hamiltonian Adaptive Control of Spacecraft," *IEEE Transactions on Automatic Control*, Vol. 35, No. 7, pp. 848–852, 1990.

[19] Ariba, Y., and Gouaisbaut, F., "Construction of Lyapunov-Krasovskii functional for time-varying delay systems," *47th IEEE Conference on Decision and Control*, 2008.

[20] T. A. W. D. and Sira-Ramirez, H., "Variable Structure Control of Spacecraft Reorientation Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 3, pp. 262–270, 1988.

Manuscript 633 submitted to 2022 American Control Conference.
Received October 12, 2021.