Derivative-free Adaptive Satellite Attitude Control

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***Abstract*—This paper describes a fault-tolerant attitude controller with no derivative components for sudden changes in the dynamics of the system. The aim of the paper is to demonstrate the effectiveness of the derivative-free controller when there are matched uncertainties that could be caused by docking, structural damage, or measurement errors. Modified Rodrigues parameters are used in the kinematic equations to increase the mathematical singularity margin in the attitude system representation. Simulations are performed to show the robustness of the applied controller on the satellite attitude system and comparisons are made with the derivative controller.**

***Index Terms*—adaptive control, spacecraft attitude, MRP**

I. Introduction

The control of spacecrafts is a requirement to reach the desired states such as orientation and position in order to perform different tasks. Attitude control plays an important role in spacecraft rendezvous and docking problems. Conventional methods of spacecraft attitude control requires a good knowledge of the system and its dynamics to develop its control equations [1-3]. The task of coming up with accurate dynamic laws as well system parameters is a time consuming and tedious process. Over the years, to address this problem methods such as system identification [4][5] have been employed.

System identification helps in building the mathematical model of dynamical systems using input-output data, hence the knowledge of the system is not a requirement. Though the use of methods such as system identification helps in accurate modelling of the controller, it still does not account for sudden changes in dynamics such as structure damage, release of payload and uncertainties. This leads to the requirement of a control system that has the ability to adapt with time, system's changing state as well as uncertainties while maintaining its robustness.

Use of adaptive controls [6] allows changing the controller according to changes in the system such that the system is operating in its best possible state. The simple adaptive controller [7-9] applied in [10] gives good response to uncertainties in the system. Multiple methods from gain scheduling to machine learning [11] have been explored to develop fast and accurate adaptive control laws which require computing the variable parameters in the control system with time and state.

Systems can be modelled using multiple representations but there exist mathematical singularities associated with angle representations which need to be accounted for when modeling a satellite system. The gimbal lock for Eular Angles is its singularity at $90°$, Classical Rodrigues Parameters (CRPs) have a singularity at $180°$ while Modified Rodrigues Parameters (MRPs) have its singularity at $360°$. Apart from the extended singularity range, the linearized equations of MRPs [13][14] are a lot more accurate and closer to the non-linear system equations than other angle representations.

The intention of this paper is to demonstrate the use of controllers free of derivative components in spacecraft attitude system represented using MRPs. The control law designed in this paper is purely adaptive where the proportional gains have been assigned using pole-placement on a time and state variant system and the weight update for uncertainties coefficient is carried out using the derivative-free weight update law[16]. Extensive work done by T. Yucelen and A.J. Calise on derivative-free control [15][16] has been adopted to show its robustness on our system. Out of the multiple forms and variations of derivative-free control in [15], we have chosen the modification free formula for application to our system and is compared with a constant weight system of similar proprieties.

The rest of the paper is organised as follows, section II shows the derivation of the non-linear state-space equations and covering the general attitude dynamics, use of modified Rodrigues parameters and Hamiltonian state-space equations. Section III introduces the derivative-free control law that is used in place of the traditional Adaptive control. Section IV discusses the stability analysis of the proposed system control approach while section V shows the different simulation results and its interpretations. Section VI concludes the paper while suggesting improvement thoughts that could be considered in future work.

II. System Equations Development

In this section we will look at how to reach the state-space form of spacecraft attitude dynamics in terms of the modified Rodriguez parameters.

A. Spacecraft Attitude Dynamics

The equation of angular momentum is given by (1) where H and ω represents the angular momentum and angular velocity of the spacecraft in the body-fixed frame.

$$\vec{H} = J\vec{\omega} \tag{1}$$

where J is the inertia matrix given as,

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{2}$$

Let M_b be the external moment acting on the spacecraft, then

$$\frac{d\vec{H}}{dt} = \vec{M}_b \tag{3}$$

$$\vec{M}_b = \tau = J\dot{\vec{\omega}} + \widehat{\omega}J\vec{\omega} \tag{4}$$

Equation (4) represents the attitude dynamics of a rigid spacecraft where $\widehat{\omega}$ is the skew-symmetric matrix of ω given by

$$\widehat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{5}$$

In the following sub-section B, the attitude dynamics is represented in terms of MRPs.

B. Modified Rodrigues Parameter Representation

The MRP representation of spacecraft attitude gives minimal and non-singular parameterization of attitude. Let q denote the quaternion corresponding to attitude, then

$$q = q_o + q_1 i + q_2 j + q_3 k \tag{6}$$

$$q = \begin{bmatrix} q_o \\ \vec{q} \end{bmatrix} \tag{7}$$

In principal rotation vector (PRVs) representations, let $\hat{e}$ and ϕ represent the unit vector corresponding to the axis of rotation and the angle of rotation respectively. Then the MRP parameter denoted by σ is given by,

$$\sigma = \frac{\vec{q}}{1 + q_o} = \tan(\frac{\phi}{4})\hat{e} \tag{8}$$

The kinematic equation in terms σ is given by,

$$\dot{\sigma} = L(\sigma)\omega \tag{9}$$

where,

$$L(\sigma) = \frac{1}{2}\left[I\left(\frac{1-\sigma^T\sigma}{2}\right) + \hat{\sigma} + \sigma\sigma^T\right] \tag{10}$$

and $\hat{\sigma}$ is the skew-symmetric matrix of σ similar to that in (5).

C. Non-linear State-Space Equations

In order to write the spate-space equations of the spacecraft attitude system, Hamiltonian non-linear Euler-Lagrange form is adopted [12] as shown below,

$$H(\sigma)\ddot{\sigma} + C(\sigma,\dot{\sigma})\dot{\sigma} = F \tag{11}$$

with

$$\tau = L^T F \tag{12}$$

$$H(\sigma) = L^{-T} J L^{-1} \tag{13}$$

$$C(\sigma,\dot{\sigma})\dot{\sigma} = -L^{-T} J L^{-1} \dot{L} L^{-1} - L^{-T}\hat{P}L^{-1} \tag{14}$$

where, $\hat{P}$ is the skew-symmetric matrix of momentum P that is expressed as,

$$P = R(\sigma)P^i \tag{15}$$

where,

$$R(\sigma) = 2(1+\sigma^T\sigma)^1[I + \sigma\sigma^T - \hat{\sigma}] - I \tag{16}$$

$$P^i = R(-\sigma(0))J^a[\omega(0) + \Omega(0)] + R(-\sigma(0))J\omega(0) \tag{17}$$

Here, J^a and $\Omega(0)$ are the axial wheel inertia and initial angular velocity respectively. Equation (14) can be simplified further as in [10] to get,

$$C(\sigma,\dot{\sigma})\dot{\sigma} = -L^{-T}[JL^{-1}\dot{L}L^{-1} + \hat{G}L^{-1}] \tag{18}$$

where,

$$G = JL^{-1}\dot{\sigma} \tag{19}$$

$$L^{-1}(\sigma) = 2(1+\sigma^T\sigma)^{-1}[I - \hat{\sigma}] \tag{20}$$

In (11), F is the control input vector which can be denoted by u, that is used to write the the non-linear state-space formulation.

State and output equations are given by,

$$\dot{x} = A(x,t)x + B(x,t)u \qquad \text{and} \qquad y = Cx \tag{21}$$

State vector is given by,

$$x = \begin{bmatrix} \sigma & \dot{\sigma} \end{bmatrix}^T \tag{22}$$

and system co-efficient matrices are given by,

$$A(x,t) = \begin{bmatrix} 0 & I_3 \\ 0 & -H^{-1}C(\sigma,\dot{\sigma}) \end{bmatrix}, \quad B(x,t) = \begin{bmatrix} 0 \\ H^{-1} \end{bmatrix} \tag{23}$$
$$C = \begin{bmatrix} I_3 & I_3 \end{bmatrix}$$

Here, $A(x,t)$ and $B(x,t)$ are varying with time and state. Hence they need to be computed at every instant of time along with the states.

III. Derivative-free Adaptive control

In this section, we modify our system with uncertainties and introduce the derivative-free controller design.

A. State-Space Equations with Uncertainty

To our system in (21), Δ (x(t)) is added as uncertainty as shown below,

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)[u(t) + \Delta(x(t))] \tag{24}$$

where the matched uncertainty is linearly parameterised as

$$\Delta(x(t)) = W^T(t)\beta(x) \tag{25}$$

where $W(t)$ is the weight matrix which is time-varying and needs to be estimated at each time instant and where $\beta(x)$ is a vector of basis functions.

We are choosing a neural network sigmoidal-type functions for our basis given by,

$$\beta(x(t)) = \begin{bmatrix} \frac{1-e^{-\sigma_1}}{1+e^{-\sigma_1}} & \frac{1-e^{-\sigma_2}}{1+e^{-\sigma_2}} & \frac{1-e^{-\sigma_3}}{1+e^{-\sigma_3}} & \frac{1-e^{-\dot{\sigma}_1}}{1+e^{-\dot{\sigma}_1}} & \frac{1-e^{-\dot{\sigma}_2}}{1+e^{-\dot{\sigma}_2}} & \frac{1-e^{-\dot{\sigma}_3}}{1+e^{-\dot{\sigma}_3}} \end{bmatrix}^T \tag{26}$$

B. Reference Model

The controller tries to track or force the closed loop dynamics of the system to follow the reference model as defined below,

$$\dot{x}_m = A_m x_m + B_m u_m \qquad \text{and} \qquad y_m = C_m x_m \tag{27}$$

We use 2nd order dynamics to get our desired response by adjusting the damping ratio ζ and natural frequency ω_n to get the 2nd order equation given by,

$$\ddot{\sigma}_m + 2\zeta\omega_n\dot{\sigma} + \omega_n^2\sigma_m = \omega_n^2 u_m \tag{28}$$

which gives,

$$A = \begin{bmatrix} 0 & I_3 \\ -\omega_n^2 I_3 & -2\zeta\omega_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \omega_n^2 I_3 \end{bmatrix}, \qquad c = \begin{bmatrix} I_3 & I_3 \end{bmatrix} \tag{29}$$

C. Control Law and Design

The derivative-free control law uses derivative-free methods to update the weight that will adjust itself corresponding to the uncertainties at any instant.

The control law is given by

$$u(t) = u_a(t) \tag{30}$$

Here, $u_a(t)$ is the adaptive feedback controller is given by,

$$u_a(t) = K_1(x,t)x(t) + K_2(x,t)u_m(t) + \hat{W}^T(t)\beta(x(t)) \tag{31}$$

where K_1 and K_2 are control feedback gains that follow the assumptions in [15] :

$$A_m = A(x,t) + B(x,t)K_1(x,t) \quad B_m = B(x,t)K_2(x,t) \tag{32}$$

This is pole-placement method such that the Hurwitz condition is satisfied where $K_1(x,t)$ and $K_2(x,t)$ are proportional feedback gains obtained using pole-placement methods at each instant of time such that the above conditions are satisfied as our $A(x,t)$ and $B(x,t)$ are a function of time as well as state.

In (31), $\hat{W}(t)$ is the estimated using the derivative-free weight update law given by

$$\hat{W}(t) = \Omega_1\hat{W}(t-\tau) + \hat{\Omega}_2(t) \tag{33}$$

where $\tau > 0$ and $0 < \Omega_1^T\Omega_1 < 1$

$$\hat{\Omega}_2(t) = k[\beta(x(t))e^T PB] \tag{34}$$

where $k > 0$ and e is the error given by

$$e_x(t) = x_m(t) - x(t) \tag{35}$$

and P is obtained by solving the Riccati equation

$$A_m^T P + PA_m + Q = 0 \tag{36}$$

where Q is any symmetric, positive-definite matrix

IV. Stability Analysis

In this section we look at the stability of the proposed controller on the spacecraft attitude system using Lyapunov stability criterion by choosing a Lyapunov–Krasovskii Functional [17] for uniformly ultimate bound and globally asymptotically stable conditions.

A. Lyapunov Stability Criterion

Lyapunov Stability theory allows is to determine if our system is globally asymptotically stable [17]. Consider the Lyaponov function V such that it is scalar continuous as,

$$V(x(t), e_x(t), e_w(t)) = V_1(x(t)) + V_2(e_x(t)) + V_3(e_w(t)) \tag{37}$$

$$V_1(x(t)) = x^T Px \tag{38}$$

$$V_2(e_x(t)) = e_x^T Pe_x \tag{39}$$

$$V_3(e_w(t)) = \int_{t-\tau}^{t} e_w^T(s)e_w(s)\,ds \tag{40}$$

Equation (37)'s derivative is given as,

$$\dot{V}(x(t), e_x(t), e_w(t)) = \dot{V}_1(x(t)) + \dot{V}_2(e_x(t)) + \dot{V}_3(e_w(t)) \tag{41}$$

Each of these derivatives are independently calculated in the rest of the section.

Let $\dot{V}_1(x(t))$ is given by,

$$\dot{V}_1(x(t)) = \dot{x}^T Px + x^T P\dot{x} \tag{42}$$

From (36),

$$A^T P + PA = -Q \tag{43}$$

Equation (42) is simplified as,

$$\dot{V}_1(x(t)) = x^T(A^T P + PA)x \tag{44}$$

$$\dot{V}_1(x(t)) = -x^T Qx \tag{45}$$

Q is chosen to be a positive definite matrix, hence the expression in (42) is negative definite given by,

$$\dot{V}_1(x(t)) < 0 \tag{46}$$

For the stability of an adaptive system, in order to show the asymptotic stability, a Lyaponov function that takes into consideration the error convergence as well as the bounded nature of the adaptive gains is required. Krasovskii's theorem helps find suitable Lyapanov function with the above requirements as demonstrated in [17].

Let $\dot{V}_2(x(t))$ is given by,

$$\dot{V}_2(e_x(t)) = \dot{e}_x(t)^T Pe_x(t) + e_x^T(t)P\dot{e}_x(t) \tag{47}$$

From (24) and (27) using the conditions in (32) we obtain,

$$\dot{x}(t) = (A_m - BK_1)x(t) + B[u(t) + \Delta(x(t))] \tag{48}$$

Substituting the value of u(t) and $\Delta(x(t))$ from (31) and (25),

$$\begin{aligned}\dot{x}(t) = A_m x(t) - BK_1x(t) + B[(K_1x(t) + K_2u_m(t) \\ + \hat{W}^T(t)\beta(x(t))) + W^T(t)\beta(x)]\end{aligned} \tag{49}$$

Grouping similar terms and cancelling gives,

$$\dot{x}(t) = A_m x(t) + BK_2 u_m(t) + B\hat{W}^T(t)\beta(x(t)) + BW^T(t)\beta(x) \tag{50}$$

From (35) its derivative is given by,

$$\dot{e}_x(t) = \dot{x}_m(t) - \dot{x}(t) \tag{51}$$

From (27) and (49),

$$\begin{aligned}\dot{e}_x(t) = A_m x_m + B_m u_m - A_m x(t) + BK_2 u_m(t) \\ + B\hat{W}^T(t)\beta(x(t)) + BW^T(t)\beta(x)\end{aligned} \tag{52}$$

The error in the weight update is the difference between the actual weight and estimated weight denoted by,

$$e_w(t) = W(t) - \hat{W}(t) \tag{53}$$

Substituting (33) into (53) yields,

$$e_w(t) = W(t) - \Omega_1 \hat{W}(t-\tau) + \hat{\Omega}_2(t) \tag{54}$$

and

$$W(t) = \Omega_1 W(t-\tau) + \Omega_2(t) \tag{55}$$

which yields,

$$e_w(t) = \Omega_1 W(t-\tau) + \Omega_2(t) - \Omega_1 \hat{W}(t-\tau) - \hat{\Omega}_2(t) \tag{56}$$

$$e_w(t) = \Omega_1 e_w(t-\tau) + \Omega_2(t) - \hat{\Omega}_2(t) \tag{57}$$

Substituting (57) into (51), yields

$$\begin{aligned}\dot{e}_x(t) = A_m e_x(t) + B[\Omega_1 e_w(t-\tau) \\ + \Omega_2(t) - \hat{\Omega}_2(t)]^T \beta(x(t))\end{aligned} \tag{58}$$

For simplicity, let us define

$$\Phi = \Omega_1 e_w(t-\tau) + \Omega_2(t) - \hat{\Omega}_2(t) \tag{59}$$

which yields,

$$\begin{aligned}\dot{V}_2(e_x(t)) = [A_m e_x(t) + B\Phi^T \beta(x(t))]^T P e_x(t) \\ + e_x^T(t) P [A_m e_x(t) + B\Phi^T \beta(x(t))]\end{aligned} \tag{60}$$

Using (43) to simplify gives,

$$\begin{aligned}\dot{V}_2(e_x(t)) = e_x^T(t) A_m^T P e_x(t) + e_x^T(t) P A_m e_x(t) \\ + \beta^T(x(t))^{TT} P e_x(t) + e_x^T(t) P B \Phi^T \beta(x(t)) \\ = -e_x^T(t) Q e_x(t) + \beta^T(x(t))^{TT} P e_x(t) \\ + e_x^T(t) P B \Phi^T \beta(x(t))\end{aligned} \tag{61}$$

Expanding for Φ yields,

$$\begin{aligned}\dot{V}_2(e_x(t)) = -e_x^T(t) Q e_x(t) + 2e_x^T(t) P B \Phi^T \beta(x(t)) \\ = -e_x^T(t) Q e_x(t) + 2e_x^T(t) P B [\Omega_1 e_w(t-\tau) \\ + \Omega_2(t) - \hat{\Omega}_2(t)]^T \beta(x(t))\end{aligned} \tag{62}$$

Using the fundamental theorem of Calculus, let $\dot{V}_2(x(t))$ be given by,

$$\dot{V}_3(W(t)) = e_w^T(t) e_w(t) - e_w^T(t-\tau) e_w(t-\tau) \tag{63}$$

We can substitute for e_w using (57) to obtain,

$$\begin{aligned}\dot{V}_3(W(t)) = [\Omega_1 e_w(t-\tau) + \Omega_2(t) - \hat{\Omega}_2(t)]^T [\Omega_1 e_w(t-\tau) \\ + \Omega_2(t) - \hat{\Omega}_2(t)] - e_w^T(t-\tau) e_w(t-\tau)\end{aligned} \tag{64}$$

$$\begin{aligned}\dot{V}_3(W(t)) = e_w^T(t-\tau)\Omega_1^T \Omega_1 e_w(t-\tau) + e_w^T(t-\tau)\Omega_1^T \Omega_2(t) \\ - e_w^T(t-\tau)\Omega_1^T \hat{\Omega}_2(t) - \Omega_2^T(t)\Omega_1 e_w(t-\tau) + \Omega_2^T(t)\Omega_2(t) \\ - \Omega_2^T \hat{\Omega}_2(t) - \hat{\Omega}_2^T(t)\Omega_1 e_w(t-\tau) - \hat{\Omega}_2^T(t)\Omega_2(t) \\ + \hat{\Omega}_2^T(t)\hat{\Omega}_2(t) - e_w^T(t-\tau) e_w(t-\tau)\end{aligned} \tag{65}$$

Using young's inequality of products [18],

$$ab = \frac{p-1}{p} a^{\frac{p}{p-1}} + \frac{1}{p} b^p \tag{66}$$

where p = 2, on $a = [\Omega_1 e_w(t-\tau)]^T$ b = $\Omega_2(t)$ we obtain,

$$\begin{aligned}[\Omega_1 e_w(t-\tau)]^T \Omega_2(t) = \frac{1}{2} e_w^T(t-\tau)\Omega_1^T \Omega_1 e_w(t-\tau) \\ + \frac{1}{2}\Omega_2^T(t)\Omega_2(t)\end{aligned} \tag{67}$$

Substituting (67) into (64) simplifies as,

$$\begin{aligned}\dot{V}_2(W(t)) + \dot{V}_3(W(t)) = -e_x^T(t) Q e_x(t) \\ + 2e_x^T(t) P B [\Omega_1 e_w(t-\tau) + \Omega_2(t) - \hat{\Omega}_2(t)]^T \beta(x(t)) \\ + \Omega_2^T(t)\Omega_2(t) - \Omega_2^T \hat{\Omega}_2(t) - e_w^T(t-\tau) e_w(t-\tau)\end{aligned} \tag{68}$$

With Eq.(34) in place of $\hat{\Omega}_2(t)$ and using $\epsilon \geq 0$ as in [15] for simplification and norm is applied on the different variables with $c_i \geq 0$ where $i = 1, 2, 3, 4$ as shown below,

$$\begin{aligned}\dot{V}_2(W(t)) + \dot{V}_3(W(t)) = -c_1 |e_x(t)|^2 - c_2 ||e_w(t)|| \\ - c_3 ||e_w(t-\tau)|| + c_4\end{aligned} \tag{69}$$

From [15] it can be seen that,

$$c_1 = Q > 0 \tag{70}$$

$$c_2 = \epsilon \geq 0 \tag{71}$$

$$c_3 = (I - k^{-1}\Omega_1^T \Omega_1) \tag{72}$$

$$c_4 = [1 + \epsilon + (1+\epsilon)^2/2]^2 \geq 0 \tag{73}$$

Equation (45) and (69) yields,

$$\dot{V}_1(W(t)) + \dot{V}_2(W(t)) + \dot{V}_3(W(t)) \leq 0 \tag{74}$$

Hence it can be concluded that,

$$\dot{V}(x(t), e_x(t), e_w(t)) \leq 0 \tag{75}$$

The spacecraft attitude system with the proposed controller is asymptotic Lyapunov stable as well as there exists a convergence of derivative free weight update error which is a feature of adaptive control robustness.

V. Simulation and Results

The validation and robustness of the above controller for the spacecraft attitude dynamics has been tested on a reference following maneuver for two cases, with and without sudden system property changes.

The value of J for an asymmetrical spacecraft is given by [19] and is controlled by external jet thrusters :

$$J = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

The initial states are taken as follows:

$$\sigma_o = \begin{bmatrix} 0.1 & 0.5 & 1.5 \end{bmatrix}^T$$
$$\omega_o = \dot{\sigma}_o = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

The reference given in the form of x_m is taken as follows:

$$x_m = \begin{cases} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T & 0s \leq t < 200s \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0 & 0 & 0 \end{bmatrix}^T & 200s \leq t < 600s \\ \begin{bmatrix} -0.2 & -0.2 & -0.2 & 0 & 0 & 0 \end{bmatrix}^T & 600s \leq t \end{cases}$$

The damping ratio and natural frequency of the ideal model reference system is chosen as $\zeta = 0.7$ and $\omega_n = 0.02$ respectively. For the derivative-free controller, β is taken as in (26) while $k = 1.5$ and P is calculated as in (36) where Q is taken as identity matrix.

For the purpose of comparison, a controller with derivative weight update law is considered keeping all other factors same. The computation of the simulation is online using an explicit Runge-Kutta (4,5) formula on MATLAB, the Dormand-Prince pair called ODE45 where calculation of $y(t_n)$ only requires $y(t_{n-1})$.

The interpretations of the results can be summurised as the following for the two cases shown by Fig. 1 and Fig. 2 respectively.

For the simulation result shown in Fig. 1 without any sudden change in the system parameters:

- Each of the state vectors in the derivative-free controller can be seen to track the reference state with less overshoot and settling time.
- The derivative-free weight update technique also has lower rise-time in comparison to the traditional derivative weight update law.

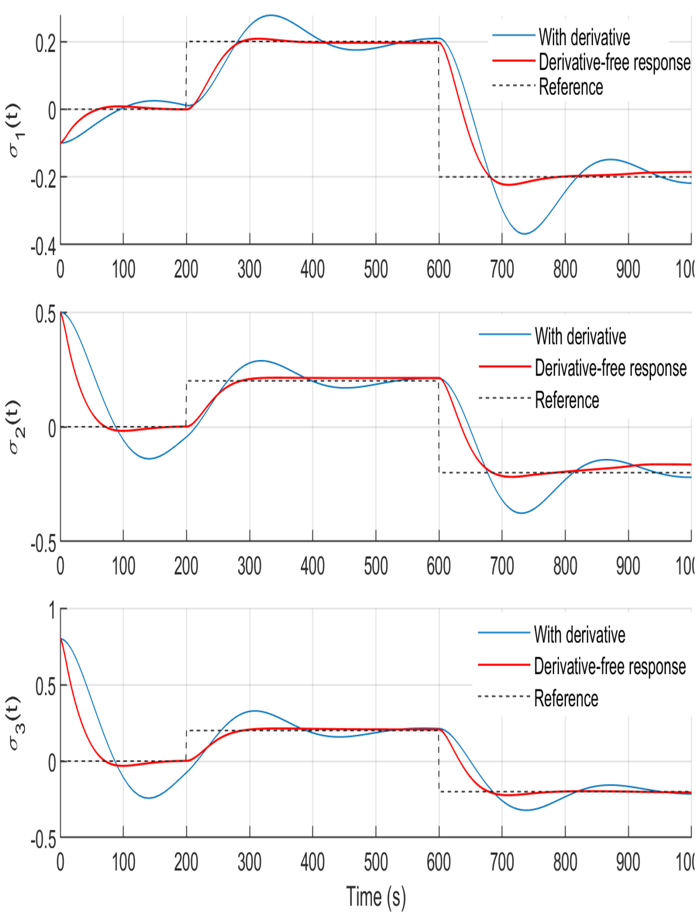

Fig. 1. MRP parameters Response to x_m without system property variation such as J_x [Case one].

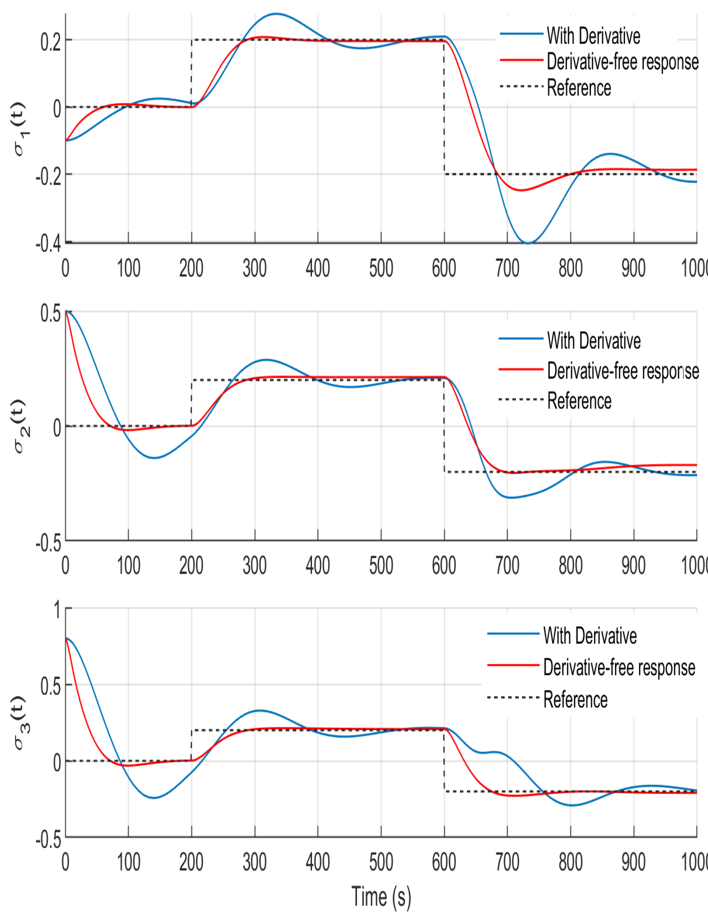

Fig. 2. MRP parameters Response to x_m when there is unexpected change to moment of inertia of spacecraft J_x [Case two].

For the simulation result shown in Fig. 2 where at t = 500s the moment of inertia J_x is suddenly changed from

$$J = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} kg.m^2$$

- It can be seen that the derivative weight update law seems to have more difficulties in adapting when the unexpected change in the structural property of J_x was provided.
- The sudden change in J_x is being handled more efficiently by the derivative-free adaptive controller and it can be seen that its performance with and without sudden changes is similar.

VI. Conclusions

This paper has shown the robustness of using derivative-free weight update laws which do not require complete information from the system's past to generate control inputs. The developed controller used pole-placement to assign proportional gains of the adaptive controller while it used a derivative-free weight update for the weight gain dealing with uncertainties. The system was defined using modified Rodrigues parameters and the state-space formed is non-linear by the use of Hamiltonian based system to have more accurate representations. The stability of the system was shown using the Lyapunov–Krasovskii functional as the system being adaptive the error convergence as well as the bounded nature of the adaptive gains also had to be additionally satisfied at all times. The different simulations carried out show the robustness of the controller as well as ability to adapt when major system properties are faulty, uncertain, or suddenly changed. The derivative-free weight update law used in this paper is not modified to perfectly fit the system. Many modified derivative-free controllers have been proposed and their effectiveness and advantages need to be explored and tested so that the use of such a computationally less expensive control system can be extensively used in spacecraft applications.

References

[1] E. Crouch, Peter, "Spacecraft attitude control and stabilization: Applications of geometric control theory to rigid body models," IEEE Transactions on Automatic Control, Vol. 29, No. 4, Apr 1984, pp. 321 - 331.

[2] Nicosia, S., Tomei, P. , "Nonlinear observer and output feedback attitude control of spacecraft," IEEE Transactions on Aerospace and Electronic Systems, Vol. 28, No. 4, 1992, pp. 970–977

[3] Oda, M. , "Coordinated control of spacecraft attitude and its manipulator,"IEEE International Conference on Robotics and Automation., 6 August 2002, pp. 732-738.

[4] Khemaissia, S., Morris, A. S., "Fast-robot system identification based on neural network models," IEEE International Symposium on Intelligent Control, 11-13 Aug. 1992, pp. 13-16..

[5] Schoukens, J., Marconato, A., Pintelon, R., Rolain, Y., Schoukens, M., Tiels, K., … Van Mulders, A., "System Identification in a Real World," IEEE 13th International Workshop on Advanced Motion Control (AMC),02 June 2014, pp. 1-9. Adaptive Control Strategy using Lyapunov Stability Theory

[6] Nishiyama, T., Suzuki, S., Sato, M., Masui, K., "Adaptive Control Strategy using Lyapunov Stability Theory ," ,International Journal of Engineering Research Technology (IJERT), Vol. 3, No. 9, September -2014, pp. 1557-1562.

[7] Nishiyama, T., Suzuki, S., Sato, M., Masui, K., "Simple Adaptive Control with PID for MIMO Fault Tolerant Flight Control Design. ," AIAA Infotech @ Aerospace, 1 Jan 2016

[8] Mooij, E. , "Simple Adaptive Control System Design Trades," AIAA Guidance, Navigation, and Control Conference., Jan 9, 2017

[9] Ulrich, S., Hayhurst, D. L., Saenz Otero, A., Miller, D., Barkana, I., "Simple Adaptive Control for Spacecraft Proximity Operations," AIAA Guidance, Navigation, and Control Conference, 10 Jan 2014.

[10] Shi, J.-F., Ulrich, S., Allen, A. , "Spacecraft Adaptive Attitude Control with Application to Space Station Free-Flyer Robotic Capture," AIAA Guidance, Navigation, and Control Conference, 2 Jan 2015.

[11] J. E. Gaudio, T. E. Gibson, A. M. Annaswamy, M. A. Bolender, E. Lavretsky, "Connections Between Adaptive Control and Optimization in Machine Learning," IEEE 58th Conference on Decision and Control (CDC),11-13 Dec. 2019, pp 4563 - 4568.

[12] Slotine, J.-J. E. and Benedetto, M. D. D., "Hamiltonian Adaptive Control of Spacecraft," IEEE Transactions on Automatic Control, Vol. 35, No. 7, 1990, pp. 848–852..

[13] Schaub, H. and Junkins, J. L., "Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters," The Journal of the Astronautical Sciences, Vol. 44, No. 1, 1996, pp. 1–19.

[14] Terzakis, G. and Lourakis, M. and Ait-Boudaoud, D.,"Modified Rodrigues Parameters: An Efficient Representation of Orientation in 3D Vision and Graphics," Journal of Mathematical Imaging and Vision, Vol. 60, No. 2, March 2018, pp. 422–442.

[15] Yucelen, T. and Calise , A.J., "Robustness of a Derivative-Free Adaptive Control Law," Journal of Guidance, Control, and Dynamics, Vol. 37, No. 5, September–October 2014, pp. 1583-1594.

[16] Yucelen, T. and Calise , A.J., "Derivative-Free Model Reference Adaptive Control," Journal of Guidance, Control, and Dynamics, Vol. 34, No. 4, July–August 2011, pp. 933-950.

[17] Ariba, Y., Gouaisbaut, F. , "Construction of Lyapunov-Krasovskii functional for time-varying delay systems,", 47th IEEE Conference on Decision and Control, Dec. 9-11, 2008

[18] Young's, Minkowski's, and H¨older's inequalities, September 13, 2011 , ¡https://www2.math.upenn.edu/ brweber/Courses/2011/Math361/Notes/YMandH.pdf¿.

[19] T. A. W. D. and Sira-Ramirez, H., "Variable Structure Control of Spacecraft Reorientation Maneuvers," Journal of Guidance, Control, and Dynamics, Vol. 11, No. 3, 1988, pp. 262–270