Derivative-free Adaptive Control Application on Satellite Attitude System

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***Abstract*—This paper describes an attitude controller with no derivative components for sudden changes in the system's dynamics. The paper aims to demonstrate the effectiveness of the derivative-free controller when there are matched uncertainties that could be caused by docking, structural damage, or measurement errors. Modified Rodrigues parameters are used in the kinematic equations to increase the mathematical singularity margin in the attitude system representation. Simulations are performed to show the robustness of the applied controller on the satellite attitude system, and comparisons are made with the derivative controller.**

***Index Terms*—adaptive control, spacecraft attitude, MRP**

I. Introduction

Spacecraft control is a requirement to reach the desired states such as orientation and position to perform different tasks. Attitude control plays a vital role in spacecraft rendezvous and docking problems. Conventional methods of spacecraft attitude control require a good knowledge of the system and its dynamics to develop its control equations [1-3]. The task of coming up with accurate dynamic laws and system parameters is a time-consuming and tedious process. Over the years, methods such as system identification [4-5] have been employed to address this problem.

System identification helps build the mathematical model of dynamical systems using input-output data; hence, the system's knowledge is not a requirement. Though methods such as system identification help inaccurate controller modeling, it still does not account for sudden changes in dynamics such as structure damage, the release of payload, and uncertainties. This leads to the requirement of a control system that can adapt with time, the system's changing state, and uncertainties while maintaining its robustness.

The use of adaptive controls [6] allows changing the controller according to changes in the system such that the system is operating in its best possible state. The simple adaptive controller [7-9] applied in [10] responds to uncertainties in the system. Multiple methods from gain scheduling to machine learning [11] have been explored to develop fast and accurate adaptive control laws that require computing the control system's variable parameters with time and state.

Systems can be modeled using multiple representations, but mathematical singularities are associated with angle representations that need to be accounted for when modeling a satellite system. Euler angles have their singularity at $\pi/2$ radians while the classical Rodrigues parameters have a singularity at π radians and modified Rodrigues parameters have their singularity at 2π radians. Apart from the extended singularity range, the linearized equations of MRPs [13-14] are a lot more accurate and closer to the non-linear system equations than other angle representations.

This paper intends to demonstrate the use of controllers free of derivative components in spacecraft attitude systems represented using MRPs. Extensive work done by T. Yucelen and A.J. Calise on derivative-free control [15-16] has been adopted to show its robustness on our system. Out of the multiple forms and variations of derivative-free control in [15], we have chosen the modification-free formula for application to our system, and it is compared with a constant weight system of similar proprieties. The control law designed in this paper is purely adaptive. The proportional gains have been assigned using pole-placement on a time and state variant system. The weight update for uncertainties coefficient is carried out using the derivative-free weight update law.

The rest of the paper is organized as follows, section II shows the derivation of the non-linear state-space equations and covering the general attitude dynamics, use of modified Rodrigues parameters, and Hamiltonian state-space equations. Section III introduces the derivative-free control law that is used in place of the traditional adaptive control. Section IV discusses the stability analysis of the proposed system control approach while section V shows the different simulation results and its interpretations. Section VI concludes the paper while suggesting improvement thoughts that could be considered in future work.

II. System Equations Development

This section will look at how to reach the state-space form of spacecraft attitude dynamics in terms of the modified Rodriguez parameters.

A. Spacecraft Attitude Dynamics

The equation of angular momentum is given by (1) where H and ω represents the angular momentum and angular velocity of the spacecraft in the body-fixed frame.

$$\vec{H} = J\vec{\omega} \tag{1}$$

where J is the inertia matrix given as,

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \quad (2)$$

Let M_b be the external moment acting on the spacecraft, then

$$\frac{d\vec{H}}{dt} = \vec{M}_b = \tau = J\dot{\vec{\omega}} + \widehat{\omega}J\vec{\omega} \quad (3)$$

Equation (3) represents the attitude dynamics of a rigid spacecraft where $\widehat{\omega}$ is the skew-symmetric matrix of ω given by

$$\widehat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (4)$$

In the following sub-section B, the attitude dynamics are represented in terms of MRPs.

B. Modified Rodrigues Parameter Representation

The MRP representation of spacecraft attitude gives minimal and non-singular parameterization of attitude. In principal rotation vector representations, let $\hat{e}$ and ϕ represent the unit vector corresponding to the axis of rotation and the angle of rotation, respectively. Then the MRP parameter denoted by σ is given by,

$$\sigma = \tan(\frac{\phi}{4})\hat{e} \quad (5)$$

The kinematic equation in terms σ is given by,

$$\dot{\sigma} = L(\sigma)\omega \quad (6)$$

where,

$$L(\sigma) = \frac{1}{2}\left[I\left(\frac{1-\sigma^T\sigma}{2}\right) + \hat{\sigma} + \sigma\sigma^T\right] \quad (7)$$

and $\hat{\sigma}$ is the skew-symmetric matrix of σ similar to that in (4).

C. Non-linear State-Space Equations

In order to write the spate-space equations of the spacecraft attitude system, Hamiltonian non-linear Euler-Lagrange form is adopted [12] as shown below,

$$H(\sigma)\ddot{\sigma} + C(\sigma,\dot{\sigma})\dot{\sigma} = F \quad (8)$$

with

$$\tau = L^T(\sigma)F \quad (9)$$

$$H(\sigma) = L^{-T}(\sigma)JL^{-1}(\sigma) \quad (10)$$

$$C(\sigma,\dot{\sigma})\dot{\sigma} = -L^{-T}(\sigma)JL^{-1}(\sigma)\dot{L}(\sigma)L^{-1}(\sigma) - L^{-T}(\sigma)\hat{P}L^{-1}(\sigma) \quad (11)$$

where,

$$G = JL^{-1}(\sigma)\dot{\sigma} \quad (12)$$

$$L^{-1}(\sigma) = 2(1+\sigma^T\sigma)^{-1}[I-\hat{\sigma}] \quad (13)$$

In (9), F is the control input vector which can be denoted by u, that is used to write the the non-linear state-space formulation.

State and output equations are given by,

$$\dot{x} = A(x,t)x + B(x,t)u \qquad \text{and} \qquad y = Cx \quad (14)$$

State vector is given by,

$$x = \begin{bmatrix}\sigma & \dot{\sigma}\end{bmatrix}^T \quad (15)$$

and system co-efficient matrices are given by,

$$A(x,t) = \begin{bmatrix} 0 & I_3 \\ 0 & -H^{-1}C(\sigma,\dot{\sigma}) \end{bmatrix}, \quad B(x,t) = \begin{bmatrix} 0 \\ H^{-1} \end{bmatrix} \quad (16)$$
$$C = \begin{bmatrix}\alpha I_3 & I_3\end{bmatrix}$$

Here, $A(x,t)$ and $B(x,t)$ are varying with time and state. Hence they need to be computed at every instant of time along with the states.

The choice of C matrix, with $alpha$ being the scaling factor yields

$$y = \alpha\sigma + \dot{\sigma} \quad (17)$$

which allows for the spacecraft to be controlled through the position feedback with the velocity providing damping.

III. Derivative-free Adaptive control

In this section, we modify our system with uncertainties and introduce the derivative-free controller design.

A. State-Space Equations with Uncertainty

To our system in (15), Δ (x(t)) is added as uncertainty as shown below,

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)[u(t) + \Delta(x(t))] \quad (18)$$

where the matched uncertainty is linearly parameterised as

$$\Delta(x(t)) = W^T(t)\beta(x) \quad (19)$$

where $W(t) \in \mathbb{R}^{nxm}$ is the weight matrix which is time-varying and needs to be estimated at each time instant and where $\beta(x)$ is a vector of basis functions.

We are choosing a neural network sigmoidal-type functions for our basis given by,

$$\beta(x(t)) = \begin{bmatrix} \frac{1-e^{-\sigma_1}}{1+e^{-\sigma_1}} & \frac{1-e^{-\sigma_2}}{1+e^{-\sigma_2}} & \frac{1-e^{-\sigma_3}}{1+e^{-\sigma_3}} & \frac{1-e^{-\dot{\sigma}_1}}{1+e^{-\dot{\sigma}_1}} & \frac{1-e^{-\dot{\sigma}_2}}{1+e^{-\dot{\sigma}_2}} & \frac{1-e^{-\dot{\sigma}_3}}{1+e^{-\dot{\sigma}_3}} \end{bmatrix}^T \quad (20)$$

B. Reference Model

The controller tries to track or force the closed loop dynamics of the system to follow the reference model as defined below,

$$\dot{x}_m = A_m x_m + B_m u_m \qquad \text{and} \qquad y_m = C_m x_m \quad (21)$$

We use 2nd order dynamics to get our desired response by adjusting the damping ratio ζ and natural frequency ω_n to get the 2nd order equation given by,

$$\ddot{\sigma}_m + 2\zeta\omega_n\dot{\sigma} + \omega_n^2\sigma_m = \omega_n^2 u_m \quad (22)$$

which gives,

$$A_m = \begin{bmatrix} 0 & I_3 \\ -\omega_n^2 I_3 & -2\zeta\omega_n \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ \omega_n^2 I_3 \end{bmatrix} \quad (23)$$
$$C_m = \begin{bmatrix}\alpha I_3 & I_3\end{bmatrix}$$

C. Control Law and Design

The derivative-free control law uses derivative-free methods to update the weight that will adjust itself to the uncertainties at any instant.

The control law is given by

$$u(t) = u_a(t) \tag{24}$$

Here, $u_a(t)$ is the adaptive feedback controller is given by,

$$u_a(t) = K_1(x,t)x_m(t) + K_2(x,t)e_y(t) + \hat{W}^T(t)\beta(x(t)) \tag{25}$$

where K_1 and K_2 are control feedback gains that follow the assumptions in [15] :

$$A_m = A(x,t) + B(x,t)K_1(x,t) \quad B_m = B(x,t)K_2(x,t) \tag{26}$$

This is a pole-placement method such that the Hurwitz condition is satisfied where $K_1(x,t)$ and $K_2(x,t)$ are proportional feedback gains obtained using pole-placement methods at each instant of time such that the above conditions are satisfied as our $A(x,t)$ and $B(x,t)$ are a function of time as well as the state.

In (26), $\hat{W}(t)$ is the estimated using the derivative-free weight update law given by

$$\hat{W}(t) = \Omega_1 \hat{W}(t-\tau) + \hat{\Omega}_2(t) \tag{27}$$

where $\tau > 0$ and $0 < \Omega_1^T \Omega_1 < 1$

$$\hat{\Omega}_2(t) = k[\beta(x(t))e^T PB] \tag{28}$$

where $k > 0$ and e is the error given by

$$e_x(t) = x_m(t) - x(t) \tag{29}$$

and P is obtained by solving the Riccati equation

$$A_m^T P + PA_m + Q = 0 \tag{30}$$

where Q is any symmetric, positive-definite matrix

IV. Stability Analysis

In this section, we look at the stability of the proposed controller on the spacecraft attitude system using the Lyapunov stability criterion by choosing a Lyapunov–Krasovskii Functional [17] for uniformly ultimate bound and globally asymptotically stable conditions.

Lyapunov Stability theory allows is to determine if our system is globally asymptotically stable [17]. Consider the Lyaponov function V such that it is scalar continuous as,

$$V(x(t), e_x(t), e_w(t)) = V_1(x(t)) + V_2(e_x(t), e_w(t)) \tag{31}$$

$$V_1(x(t)) = x^T Px \tag{32}$$

Each of these derivatives is independently calculated in the rest of the section.

Let $\dot{V}_1(x(t))$ is given by,

$$\dot{V}_1(x(t)) = \dot{x}^T Px + x^T P\dot{x} \tag{33}$$

From (31),

$$A^T P + PA = -Q \tag{34}$$

Equation (42) is simplified as,

$$\dot{V}_1(x(t)) = x^T(A^T P + PA)x \tag{35}$$

$$\dot{V}_1(x(t)) = -x^T Qx \tag{36}$$

Q is chosen to be a positive definite matrix, hence the expression in (34) is negative definite given by,

$$\dot{V}_1(x(t)) < 0 \tag{37}$$

For the stability of an adaptive system, to show the asymptotic stability, a Lyapunov function that considers the error convergence and the bounded nature of the adaptive gains is required. Krasovskii's theorem helps find suitable Lyapunov function with the above requirements as demonstrated in [17].

Theorem 1. Let the Lyapunov-Krasovskii functional be defined as,

$$V_2(e_x(t), e_w(t)) = e_x^T Pe_x + \int_{t-\tau}^{t} e_w^T(s)e_w(s)\,ds \tag{38}$$

then its derivative can be reduced into the inequality obtained as,

$$\dot{V}_2(e_x(t), e_w(t)) \leq -c_1||e_x(t)||^2 - c_2||e_w(t)|| - c_3||e_w(t-\tau)|| + d \tag{39}$$

If (28) is satisfied, then the $\dot{V}_2(e_x(t), e_w(t)) < 0$, i.e. uniform ultimately bounded if,

$$||e_x(t)|| > \Psi_1, \quad ||e_w(t)|| > \Psi_2 \quad and \quad ||e_w(t-\tau)|| > \Psi_3 \tag{40}$$

Proof: The corollaries in [16] estimates the ultimate bound as well as proves that the error trajectory reaches the bound exponentially. Moreover, from the detailed derivations in [16] when $\eta \in (1, 1/k)$, we obtain

$$\Psi_1 = \frac{1}{\sqrt{k\lambda_{min}(Q)[1-\eta k]}} \tag{41}$$

$$\Psi_2 = \Psi_1 \sqrt{\frac{c_1}{c_2}} \tag{42}$$

$$\Psi_3 = \Psi_1 \sqrt{\frac{c_1}{c_3}} \tag{43}$$

Hence, it can be followed that $e_x(t)$ and $e_w(t)$ are uniformly ultimate bounded.

The spacecraft attitude system with the proposed controller is asymptotic Lyapunov stable. There is a convergence of derivative-free weight update error, which is a feature of adaptive control robustness.

V. Simulation and Results

The validation and robustness of the above controller for the spacecraft attitude dynamics has been tested on a reference following maneuver for three cases,

1) Realistic dynamic properties to which controller is tuned.
2) Unrealistic dynamic properties without further tuning.
3) Sudden change in the dynamic property mid-flight.

Case 1. The realistic value of J for an asymmetrical spacecraft is given by [18] :

$$J = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

The initial states are taken as follows:

$$\sigma_o = \begin{bmatrix} -0.1 & 0.1 & 0.1 \end{bmatrix}^T$$
$$\omega_o = \dot{\sigma}_o = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

The damping ratio and natural frequency of the ideal model reference system is chosen as $\zeta = 0.7$ and $\omega_n = 0.02$ respectively.

For the derivative-free controller, β is taken as in (21) while $\phi_1 = 0.9$ and $k = 100$ and P is calculated as in (31) where Q is taken as identity matrix.

The computation of the simulation is online using an explicit Runge-Kutta (4,5) formula on MATLAB, the Dormand-Prince pair called ODE45 where calculation of $y(t_n)$ only requires $y(t_{n-1})$.

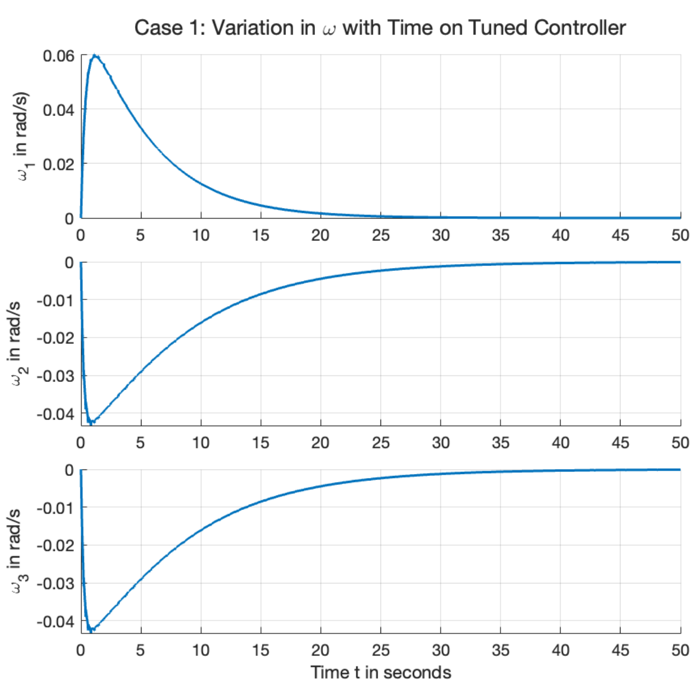

Fig. 1. Omega of derivative-free tuned controller under dynamic properties of case one.

The performance of the proposed derivative-free controller, i.e., the asymptotic stability of ω with time, can be seen in Fig.1.

Comparison with derivative controller: A controller with derivative weight update law is considered keeping all other factors same. The angular velocities obtained from the derivative and derivative-free controllers are super imposed to observe the differences in Fig. 2.

- The angular velocity ω in the derivative-free controller can be seen to track the reference state with less overshoot and settling time.
- The derivative-free weight update technique also has lower rise-time in comparison to the traditional derivative weight update law.

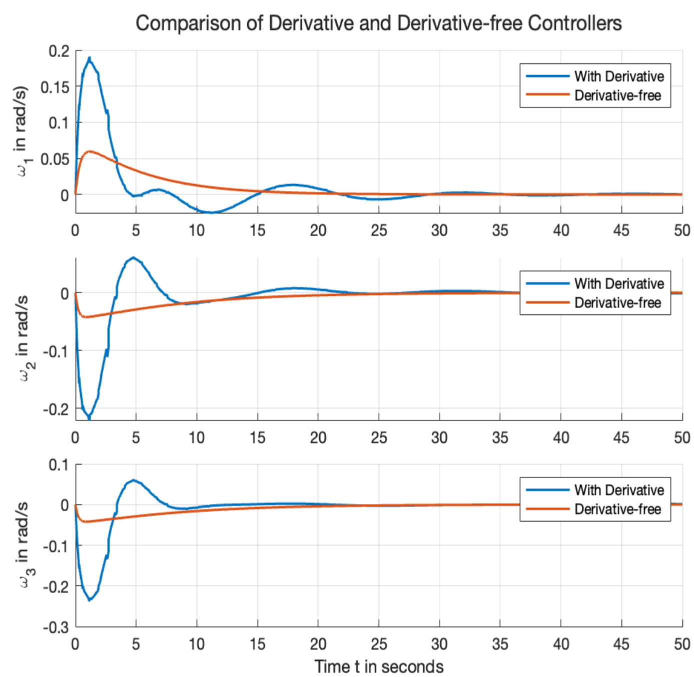

Fig. 2. Comparison between the derivative-free controller and controller with derivative weight update laws.

Case 2. For the simulation result shown in Fig. 3, the moment of inertia J is taken as where the value of J_X is unrealistically changed.

$$J = \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

While the controllers are tuned to the dynamic properties in Case 1, we are looking at the robustness of the proposed controller to changes in dynamics.

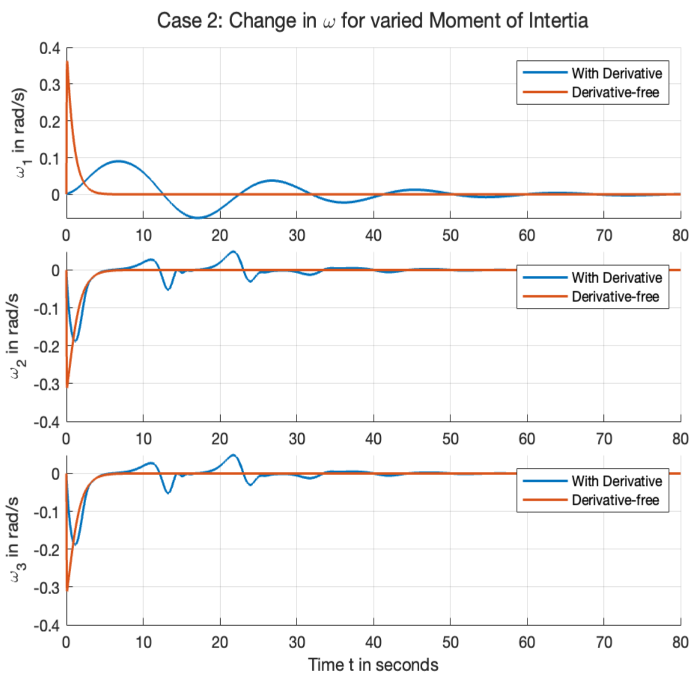

Fig. 3. Controllers' performance with unrealistic dynamic properties.

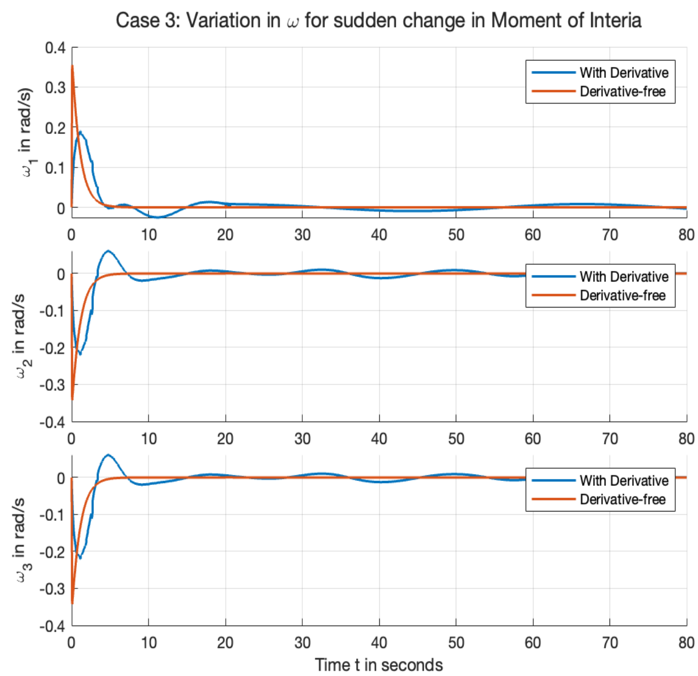

Fig. 4. Controllers' performance when J is suddenly changed mid-flight.

Case 3. For the simulation result shown in Fig. 4, at t = 20s the moment of inertia J_x is suddenly changed from

$$J = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} kg.m^2$$

- It can be seen that the derivative weight update law seems to have more difficulties in adapting when the unexpected change in the structural property of J_x was provided.
- The sudden change in J_x is being handled more efficiently by the derivative-free adaptive controller and it can be seen that its performance with and without sudden changes is similar.

VI. Conclusions

This paper has shown the robustness of using derivative-free weight update laws, which do not require complete information from the system's past to generate control inputs. The developed controller used pole-placement to assign proportional gains of the adaptive controller while it used a derivative-free weight update for the weight gain dealing with uncertainties. The system was defined using modified Rodrigues parameters, and the state-space formed is non-linear by using a Hamiltonian-based system to have more accurate representations. The system's stability was shown using the Lyapunov–Krasovskii functional as the system being adaptive the error convergence and the bounded nature of the adaptive gains also had to be additionally satisfied at all times. The different simulations carried out show the robustness of the controller and the ability to adapt when major system dynamic properties are significantly and suddenly changed. The derivative-free weight update law used in this paper is not modified to fit the system perfectly. Many modified derivative-free controllers have been proposed, and their effectiveness and advantages need to be explored and tested so that the use of such a computationally less expensive control system can be extensively used in spacecraft applications.

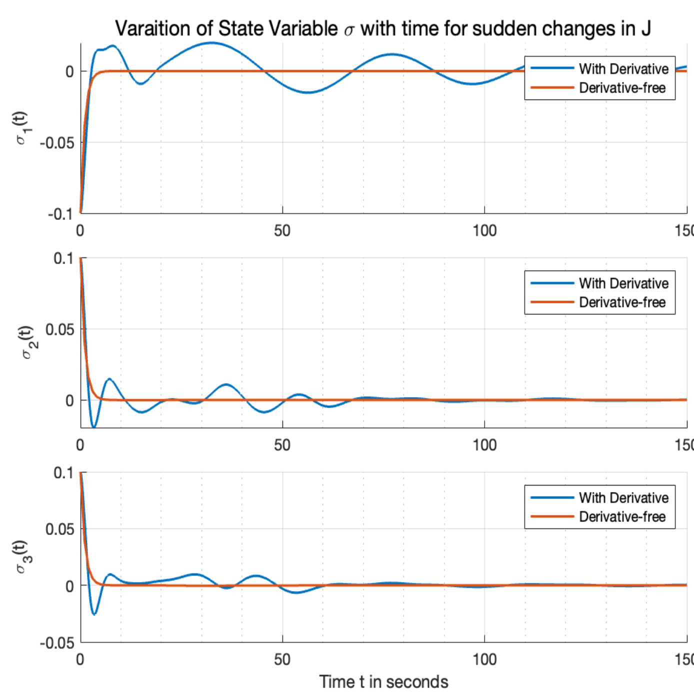

Fig. 5. Variation of state variable σ for Case 3.

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