Control System Design for MIMO System using Bond graph Representation - Quadcopter as a Case Study

Irene Grace Karot Polson, R.S. MohanKumar and N. Selvaganesan

***Abstract*— This paper presents the use of power-based graphical representation of system and its controller using bond graph approach. For the detailed case study, the quadcopter system is considered, which exhibits MIMO, non-linear and under-actuated system properties. This system is modelled and PID controller parameters are obtained by adopting manual tuning technique using bond graph approach. The entire closed-loop MIMO system is represented using 20-sim bond graph software and the simulations are performed to meet satisfactory stable responses under tracking and disturbance conditions.**

I. INTRODUCTION

In present days, quadcopters play a major role in various applications like data gathering, steady package deliveries and sight inspections. Similarly, modelling and control of the under-actuated quadcopter system plays a challenging role in the control system domain [1]-[5]. Since the quadcopter requires multi-domian representations, there is a need for a multidisciplinary modelling structure that allows complex systems to be modelled spreading across multiple domains such as mechanical and electrical.

Bond graph is one such methodology that allows the graphical representation of multidisciplinary physical systems. It was developed by Prof. Henry M. Paynter [6]. Engineering systems need to be modelled so that the relationship between design variables and the corresponding designed system's response can be studied, understood and improved in a better way when compared to the conventional approaches of system representations [7]. The bond graph representations gives generalized elemental dynamic information from which detailed insights of the system can be obtained and understood by users specialized in multi-domain systems. By making use of their intuitive knowledge of the physical system to be modelled, required parameters can be included with ease and accurate models can also be made of complex systems that require a large number of factors and produce non-linear responses.

The existing software such as 20-sim and SYMBOLS are excellent tools for bond graph that allows simultaneous modelling, simulation and equations generation [10]. Such accurate models help us to create robust models according to system requirements with objectives such as reliability, cost and performance.

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Literature studies show few contributions making use of bond graph for quadcopter modelling: [8]-[10] have modelled and stabilized quadcopters, while [9] use multi-body systems and linearised Newton-Euler formulations to achieve control using PID. In [10], a reduced bond graph model approach is used to represent a non-linear model with a PID controller. However the existing methods have only considered attitude and altitude control of quadcopters and are not dealing with complete position and attitude control. PID controller and integration models have been developed in bond graph representations are given in [11].

This paper explores more in the application of graphical representations of controller and system modelling using bond graph approach. The controller is chosen as PID since they are extensively used in quadcopters and last literature for comparison with the bond graph model. The contributions are summarized as follows :

- The quadcopter model along with PID controller is represented using bond graph approach.
- Reduced bond graph model is derived and feedback is obtained from inertial elements of the reduced model.
- The PID controller is obtained to achieve stability in both the attitude and position of quadcopter.
- The entire simulation is carried out using 20-sim software under position tracking and disturbance rejection conditions.

Organisation of the paper is as follows, section II shows the basic bond graph components that has been made use of in the quadcopter model. Section III deals with the derivation of mathematical model of quadcopter using Newton-Euler equations. Section IV gives the detailed modelling process to show how the bond graph model and each of its sub-units are modelled. Section V shows how the control system is developed using bond graph. The results and specific simulations are presented in section VI. Section VII concludes the paper and ways to improve the usage of bond graphs for future system engineering.

II. BOND GRAPH METHODOLOGY

Bond graph incorporates physics based representation of system in multi-energy domain i.e. mechanical, electrical, hydraulic etc. by using generalized elements. Since the method of obtaining mathematical model of the system is procedural in this approach, modelling can be automated.
It is more easy to obtain state-space representations of the designed system and gives more insight of the system by converting the flow and effort information generated by this

approach. It is based on the power transfer between the different elements of the considered system.

The exchanged power is :

$$P(t) = e(t)\text{x}f(t) \tag{1}$$

where $e(t)$ is the effort and $f(t)$ is the flow through the bond. There are a set of elements that are used to graphically represent the dynamics of effort and flow [12].

The elements used in the bond graph model in this paper are as follows:

Table 1: Bond graph elements and symbols

Element Name	Symbol
I-type elements or Inductor	I
C-type elements or Capacitor	C
R-type elements or R	R
Source of Effort	Se
Modulated Gyrator Element	MGY
Modulated Source of Effort	MSe
Modulated Tranformer Element	MTF
Power Mux and Demux	$\|\|$
Zero Junction	0
One Junction	1

The causal stroke determines the direction of effort and flow while the direction of the half arrow is the conventional direction of positive power flow. With the use of bond graph modelling we will be able to access the effort and flow of each bond which simplifies the process of parameter identifications and fault detection. The rules for the causality assignment and steps to developing the model can be found in [13].

III. Quadcopter Dynamics

The equations of motions of the quadcopter are derived and modelled in the body-fixed reference of the quadcopter [15]-[17]. The absolute position and angle references are provided and controlled in the inertial frame of reference. Let $F_{external}$ and $M_{external}$ be the external force and moment acting on the quadcopter about its center of gravity, in its body-fixed frame of reference.

Translation Kinematics is given by,

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = R_b^I \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{2}$$

Then using the Newton-Euler formulation, the translation dynamics of the quadcopter is,

$$\vec{F}_{external} = m\dot{\vec{v}} + \widehat{\omega}J\vec{v} \tag{3}$$

where, m is the mass and $\widehat{\omega}$ is the skew-symmetric matrix of ω given by

$$\widehat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{4}$$

The rotational kinematics of the body is given by,

$$\dot{R} = R_b^I \vec{\omega} \tag{5}$$

where R_b^I is the rotation matrix given by,

$$R_b^I = \begin{bmatrix} 1 & 0 & -sin(\theta) \\ 0 & cos(\phi) & sin(\phi)cos(\theta) \\ 0 & -sin(\phi) & cos(\phi)cos(\theta) \end{bmatrix} \tag{6}$$

and ϕ, θ and ψ correspond to the Euler angles roll, pitch and yaw in the inertial frame.

The rotational dynamics of the quadcopter is given by,

$$\vec{M}_{external} = I\dot{\vec{\omega}} + \widehat{\omega}I\vec{\omega} \tag{7}$$

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{8}$$

Hence, the non-linear equations represented using bond graph are given by,

$$\begin{aligned} F_{x,external} &= m\ddot{x} + m(\dot{z}\omega_y - \dot{y}\omega_z) \\ F_{y,external} &= m\ddot{y} + m(\dot{x}\omega_z - \dot{z}\omega_x) \\ F_{z,external} &= m\ddot{z} + m(\dot{y}\omega_x - \dot{x}\omega_y) \\ M_{x,external} &= I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z \\ M_{y,external} &= I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x \\ M_{z,external} &= I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y \end{aligned} \tag{9}$$

The external forces acting on the quadcopter are mainly the thrust from the rotors, drag force and gravity. The force due to gravity and thrust are given as:

$$F_{gravity} = R_I^b \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \tag{10}$$

$$F_{Thrust} = \begin{bmatrix} 0 \\ 0 \\ \sum T_{rotor} \end{bmatrix} \tag{11}$$

The drag for each body-fixed axis can be approximated as proportional to the square of velocity in that directions, as given below:

$$F_{drag} = \begin{bmatrix} -k_d\dot{x}^2 \\ -k_d\dot{y}^2 \\ -k_d\dot{z}^2 \end{bmatrix} \tag{12}$$

The moments acting on the quadcopter are the torques created by the differential spinning of the rotors,

$$\vec{M}_b = \vec{\tau} \tag{13}$$

IV. Bond Graph Representations

In this section, the detailed process of modelling the dynamics in bond graph representation is discussed. The quadcopter mass m and its moment of inertia I are both I-type elements. As all the bonds connected to through a 1-Junction have equal flow; 1-Junctions are used to represent $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\omega_x}$, $\dot{\omega_y}$ and $\dot{\omega_z}$. As the translation of a quadcopter is coupled with the angular velocities ω, we pass this information to the MGY element as shown in Fig. 1.

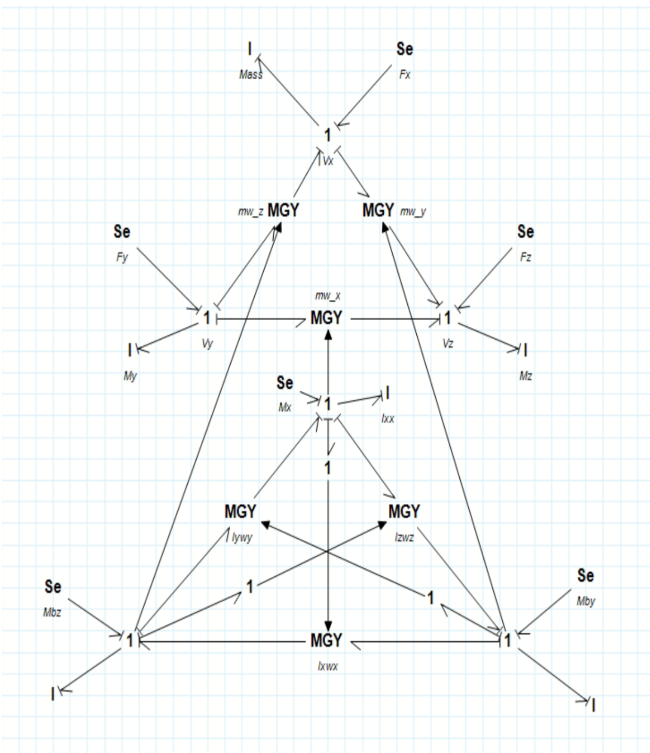

Fig. 1. Quadcopter model

The top triangle represents the translation motion of the quadcopter with each 1-junction having $\dot{x}$, $\dot{y}$ and $\dot{z}$ as flows. The bottom triangle represents the rotational dynamics of the quadcopter with each of the 1-junction having $\dot{\omega_x}$, $\dot{\omega_y}$ and $\dot{\omega_z}$ as flows.

As the Newton Euler equations are in the body-fixed frame and the references are in the inertial frame, frame transformation [13] is required before estimating state errors. For this purpose, the angular rates and velocities are coupled into a vector to transform it using the rotation matrix in (6) using MTF. As the R_b^I rotation matrix requires angles as inputs, we need to compute the angles as shown in Fig. 2

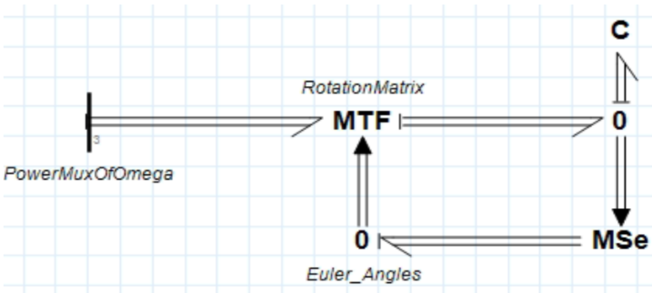

Fig. 2. Integration of omega to get Euler angles

V. Position and Attitude Control

In this section, PID controller is designed to track the position trajectory and simultaneously stabilizing the Euler angles. A PID controller is chosen to demonstrate the use of bond graph representation for the entire system as opposed to only the quadcopter dynamics. Any controller can be used in place of the PID to achieve stability of the quadcopter system based on requirements.

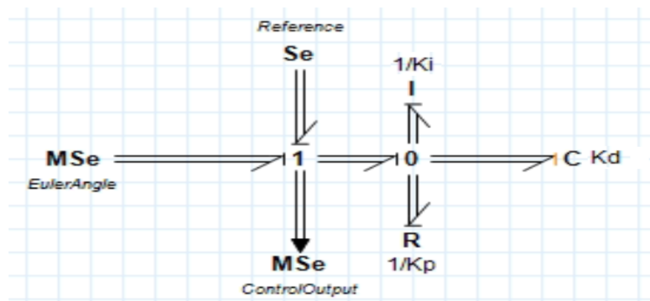

Fig. 3. PID controller structure for attitude

The design of PID controller using bond graph [11] relies on the angles and position which are represented as an effort in our case as shown in Fig. 3. The control outputs that we get are transformed back into the body-frame and passed to the reduced bond graph model system and we obtain external forces($F_{external}$) and moments($M_{external}$) as the effort from the inertial elements. The use of such a reduced model eliminates the need for finding individual rotor forces and torques as shown in Fig. 4.

VI. Simulation and Results

This section presents the results of various simulation conducted using 20-sim to check the stability and robustness of the controller. The simulation shows a trajectory tracker while stabilizing the angles to the required reference at each point of time. The trajectory followed is,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

In each case, a step disturbance of 2N is given at t = 200s which needs to be stabilized by the system for robustness. The variation in the X, Y and Z positions in the inertial frame for 2 reference cases as well as the variation of ω when disturbance is applied has been shown for when reference is

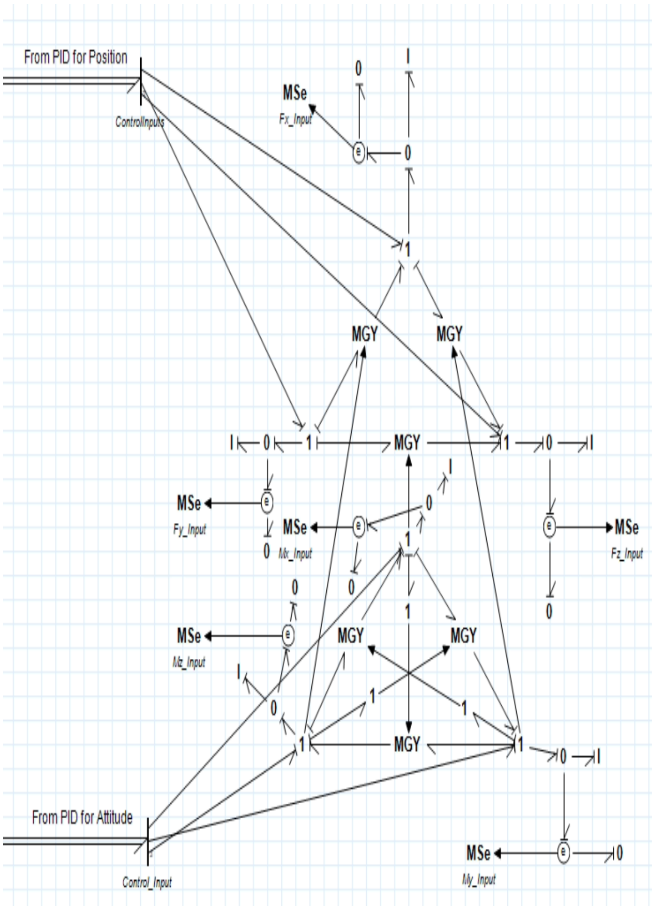

Fig. 4. Reduced bond graph model

set to [0,0,1]. The values of the parameters considered in the simulation are shown in Table 2.

Table 2: System and controller parameters

Parameters	Value
Mass of quadcopter m	1 kg
Moment of inertia I_x	0.1 kg m^2
Moment of inertia I_y	0.1 kg m^2
Moment of inertia I_z	0.1 kg m^2
Position control K_p, K_i, K_d	10, 1, 1
Attitude control K_p, K_i, K_d	1, 0.1, 1

From Fig.5 where the reference is set as [0,0,1] and 2N disturbance is applied after 200s, it can be observed that, (i) the quadcopter initially tries to stabilize itself in the Z-direction as response to gravity force, (ii) the quadcopter is able to return back to its command position after the disturbance and (iii) the responses for the disturbance are similar in the X and Y direction due to symmetric in moment of inertia i.e. $I_x = I_y$.

The variation in angular velocities are shown in Fig. 6 and it is noted that (i) when the disturbance is applied to the system, the angles adjust itself to facilitate translation of position back to its reference and further stabilizes itself and (ii) the angular stabilization along the Z-axis takes longer as the moment of inertia I_z is smaller in magnitude than I_x and I_y.

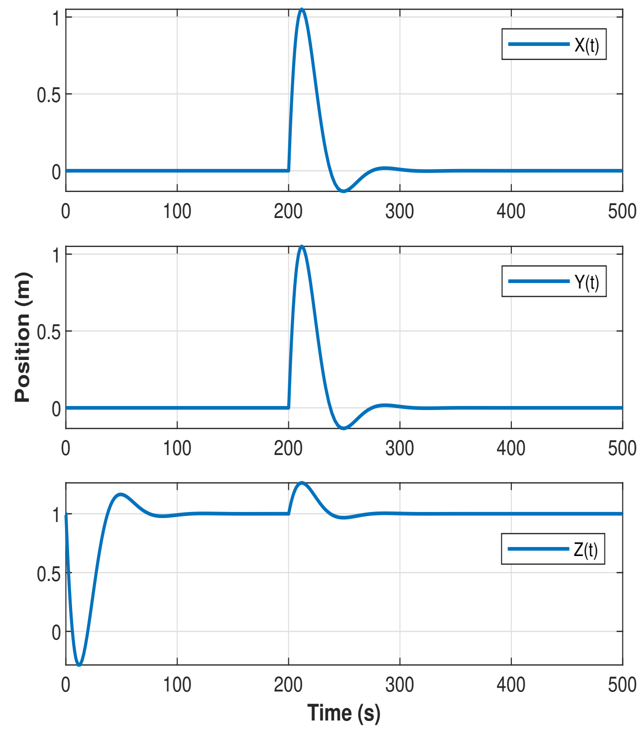

Fig. 5. Response in Position with time

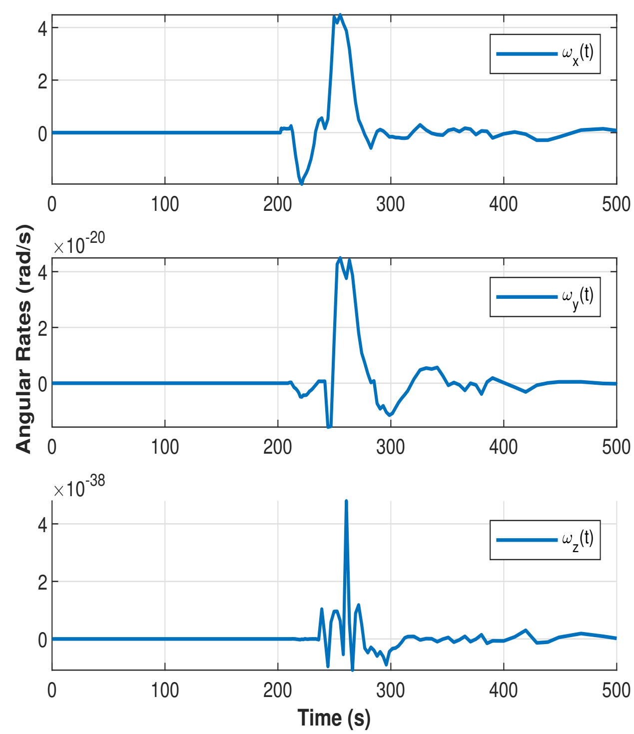

Fig. 6. Response in omega i.e. angular rates with time

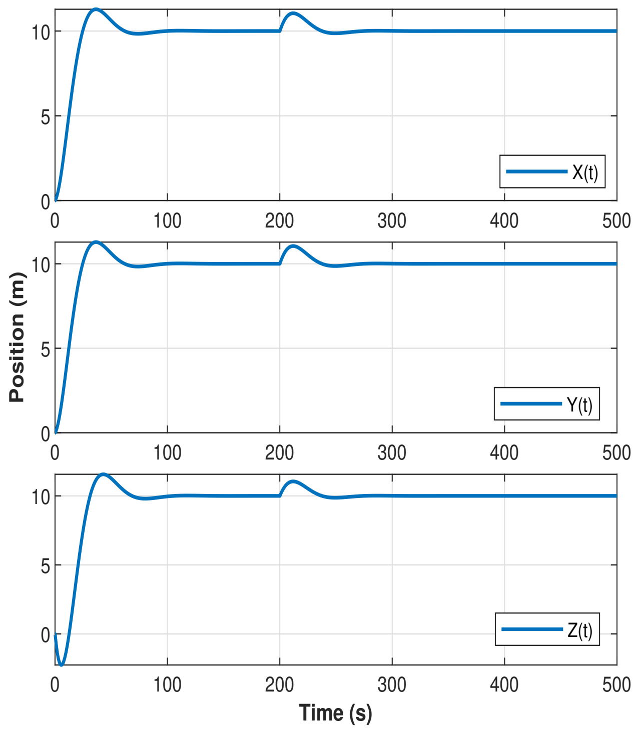

Fig. 7. Response in position with time with non-zero reference

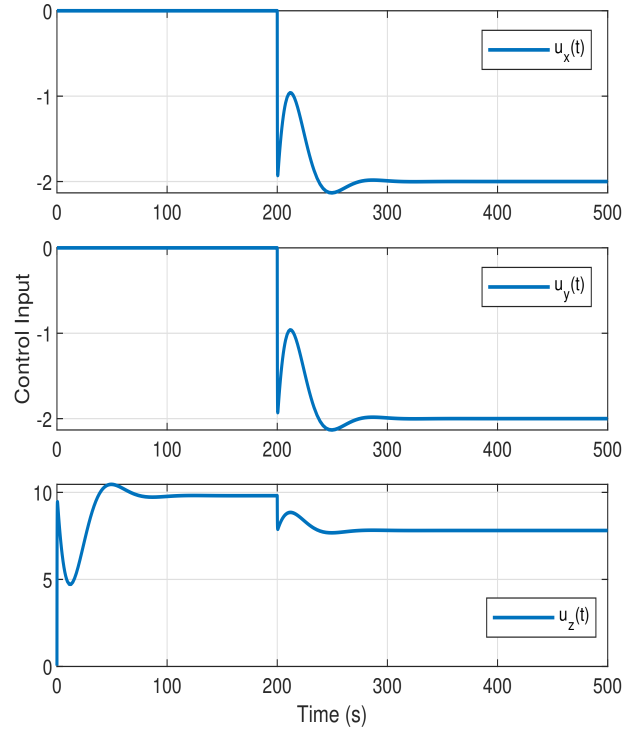

Fig. 8. Postion control inputs from PID controller

From Fig. 7 where the reference is set as [10,10,10]m and 2N disturbance is applied after 200s, it can be observed that the quadcopter is able to track and stabilize at the reference position.

The corresponding control efforts for position and attitude are shown in Fig. 8 and Fig. 9. From the above analysis, it is concluded that the quadcopter is able to track the desired trajectories and is robust towards disturbance. It can be observed that the angular velocities in the inertial frame adjust itself to retain stability and to facilitate translational motion required for position control.

VII. Conclusions

This paper had implemented the use of bond graph methodology for control system designs on a quadcopter. The entire model as well as the PID control had been represented using bond graphs and hence intricate details of each element of the system can be obtained and used for further improvements of the system and its life-cycle. The reduced bond graph model served the purpose of obtaining the dynamic inversion of the non-linear system. PID controllers were used for both position tracking and attitude stabilization. Simulations were conducted to show the position tracking, angle stabilizing capabilities of the system as well as the robustness towards disturbances. Moreover, advanced controllers can be used in place of the modelled PID controller to obtain different stability characteristics. The controller gains in this paper have been manually tuned using the simulation of the bond graph model. This process can be improved and optimal controller gains can be obtained by innovative use of bond graph modelling methods.

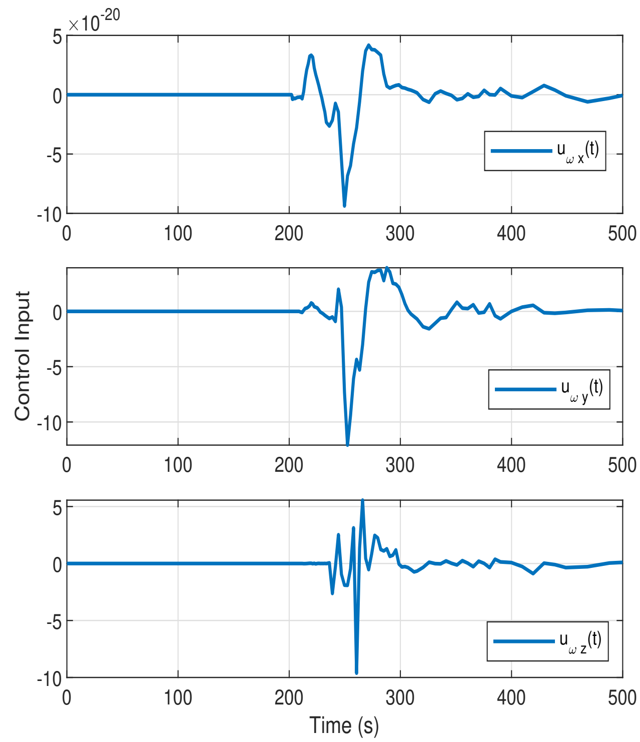

Fig. 9. Attitude control inputs from PID controller

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