Derivative-free Adaptive Control Application on Satellite Attitude System

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***Abstract*—This paper describes an attitude controller with no derivative components for sudden changes in the system's dynamics. The aim of the study is to demonstrate the effectiveness of derivative-free adaptive controller when there are matched uncertainties that could be caused by docking, structural damage, or measurement errors. Modified Rodrigues parameters are used in the kinematic equations to increase the mathematical singularity margin in the attitude system representation. Simulations are performed to show the robustness of the applied controller on the satellite attitude system, and comparisons are made with constant weight update controller.**

***Index Terms*—adaptive control, spacecraft attitude, MRP**

I. Introduction

Spacecraft control is crucial to reach the desired states such as orientation and position to perform different tasks. Attitude control plays a vital role in spacecraft rendezvous and docking problems. Conventional methods of spacecraft attitude control require good knowledge of the system and its dynamics to develop control equations [1-3]. The process of coming up with accurate dynamic laws and system parameters is often time-consuming and tedious. Over the years, methods such as system identification [4-5] have been employed to address this problem.

System identification helps build the mathematical model of dynamical systems using input-output data; system properties are not required. Though methods such as system identification help design accurate controllers, it still does not account for sudden changes in dynamics such as structure damage, the release of payload, and uncertainties. This is achieved through a control system that can adapt with time, the system's changing state, and uncertainties while maintaining its robustness.

The use of adaptive controls [6] allows changing the controller according to changes in the system such that the system is operating in its best possible state. The simple adaptive controller [7-9] applied in [10] responds to uncertainties in the system. Multiple methods from gain scheduling to machine learning [11] have been explored to develop fast and accurate adaptive control laws that require computing the control system's variable parameters with time and state.

Mathematical singularities are associated with angle representations that need to be accounted for when modeling a satellite system and modified Rodrigues parameters have a few advantages associated with the representation. For instance, Euler angles and classical Rodrigues parameters are singular at $\pi/2$ and π radians respectively. On the other hand, MRPs have their singularity at 2π radians. Apart from the extended singularity range, the linearized equations of MRPs [13-14] are a lot more accurate and closer to the non-linear system equations than other angle representations.

Extensive work done by T. Yucelen and A.J. Calise on derivative-free control [15-16] has been adopted to show its robustness on our system. Out of the multiple forms and variations of derivative-free control in [15]. The main contributions of the paper can be summarised as:

- Demonstration of the use of controllers free of derivative components in spacecraft attitude systems represented using MRPs
- Design of a purely adaptive controller where the proportional gains are assigned using pole-placement on a time and state variant system.
- Comparison of the proposed derivative-free controller with a constant weight update adaptive system of similar proprieties.

The rest of the paper is organized as follows, section II shows the derivation of the non-linear state-space equations and covering the general attitude dynamics, use of modified Rodrigues parameters, and Hamiltonian state-space equations. Section III introduces the derivative-free control law that is used in place of the traditional adaptive control. Section IV discusses the stability analysis of the proposed system control approach while section V shows the different simulation results and its interpretations. Section VI concludes the paper while suggesting improvement thoughts that could be considered in future work.

II. System Equations Development

This section will define the state-space form of spacecraft attitude dynamics in terms of modified Rodriguez parameters.

A. Spacecraft Attitude Dynamics

The equation of angular momentum is given by (1) where H and ω represents the angular momentum and angular velocity of the spacecraft in the body-fixed frame.

$$\boldsymbol{H} = \boldsymbol{J}\boldsymbol{\omega} \tag{1}$$

and $\boldsymbol{J}$ is the inertia matrix given as,

$$\boldsymbol{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{2}$$

Let $\boldsymbol{M_b}$ be the external moment acting on the spacecraft, then

$$\frac{d\boldsymbol{H}}{dt} = \boldsymbol{M}_b = \boldsymbol{\tau} = \boldsymbol{J}\dot{\boldsymbol{\omega}} + \widehat{\boldsymbol{\omega}}\boldsymbol{J}\boldsymbol{\omega} \tag{3}$$

represents the attitude dynamics of a rigid spacecraft where $\widehat{\omega}$ is the skew-symmetric matrix of ω given by

$$\widehat{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{4}$$

In the following sub-section B, the attitude dynamics are represented in terms of MRPs.

B. Modified Rodrigues Parameter Representation

The MRP representation of spacecraft attitude gives minimal and non-singular parameterization of attitude. In principal rotation vector representations, let $\hat{e}$ and ϕ represent the unit vector corresponding to the axis of rotation and the angle of rotation, respectively. Then the MRP parameter denoted by $\boldsymbol{\sigma}$ is defined as,

$$\boldsymbol{\sigma} = \tan(\frac{\phi}{4})\hat{\boldsymbol{e}} \tag{5}$$

The kinematic equation in terms $\boldsymbol{\sigma}$ is obtained as,

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{L}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{6}$$

where,

$$\boldsymbol{L}(\boldsymbol{\sigma}) = \frac{1}{2}\left[\boldsymbol{I}\left(\frac{1-\boldsymbol{\sigma}^T\boldsymbol{\sigma}}{2}\right) + \hat{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \tag{7}$$

and $\hat{\boldsymbol{\sigma}}$ is the skew-symmetric matrix of $\boldsymbol{\sigma}$ similar to that in (4).

C. Non-linear State-Space Equations

In order to write the spate-space equations of the spacecraft attitude system, Hamiltonian non-linear Euler-Lagrange form is adopted [12] as shown below,

$$\boldsymbol{H}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} + \boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = \boldsymbol{F} \tag{8}$$

with

$$\boldsymbol{\tau} = \boldsymbol{L}^T(\boldsymbol{\sigma})\boldsymbol{F} \tag{9}$$

$$\boldsymbol{H}(\boldsymbol{\sigma}) = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) \tag{10}$$

$$\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = -\boldsymbol{L}^{-T}[\boldsymbol{J}\boldsymbol{L}^{-1}\dot{\boldsymbol{L}}\boldsymbol{L}^{-1} + \hat{\boldsymbol{G}}\boldsymbol{L}^{-1}] \tag{11}$$

where,

$$\boldsymbol{G} = \boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}} \tag{12}$$

$$\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) = 2(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\boldsymbol{I} - \hat{\boldsymbol{\sigma}}] \tag{13}$$

In (8), $\boldsymbol{F}$ denotes the control input vector which corresponds to $\boldsymbol{u}$, which is used in the non-linear state-space formulation. State and output equations are defined as,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{u} \qquad \text{and} \qquad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{14}$$

State vector is defined as,

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\sigma} & \dot{\boldsymbol{\sigma}} \end{bmatrix}^T \tag{15}$$

and system co-efficient matrices are defined as,

$$\boldsymbol{A}(\boldsymbol{x},t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I_3} \\ \boldsymbol{0} & -\boldsymbol{H}^{-1}\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}}) \end{bmatrix}, \quad \boldsymbol{B}(\boldsymbol{x},t) = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{H}^{-1} \end{bmatrix} \tag{16}$$

$$\boldsymbol{C} = \begin{bmatrix} \alpha\boldsymbol{I_3} & \boldsymbol{I_3} \end{bmatrix}$$

Here, $\boldsymbol{A}(\boldsymbol{x},t)$ and $\boldsymbol{B}(\boldsymbol{x},t)$ are varying with time and state. Hence they need to be computed at every instant of time along with the states.
The choice of $\boldsymbol{C}$ matrix, with α being the scaling factor yields

$$\boldsymbol{y} = \alpha\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}} \tag{17}$$

which allows for the spacecraft to be controlled through the position feedback with velocity providing damping.

III. Derivative-free Adaptive control

In this section, we modify our system with uncertainties and introduce the derivative-free controller design.

A. State-Space Equations with Uncertainty

To our system in (15), $\Delta(\boldsymbol{x}(t))$ is introduced as uncertainty,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{x},t)[\boldsymbol{u}(t) + \Delta(\boldsymbol{x}(t))] \tag{18}$$

where the matched uncertainty is linearly parameterised as

$$\Delta(\boldsymbol{x}(t)) = \boldsymbol{W}^T(t)\boldsymbol{\beta}(\boldsymbol{x}) \tag{19}$$

Here, $\boldsymbol{W}(t) \in \mathbb{R}^{nxm}$ is the weight matrix which is time-varying and needs to be estimated at each time instant and $\boldsymbol{\beta}(\boldsymbol{x})$ is a vector of basis functions.

neural network sigmoidal-type functions are chosen for our basis vector defined as,

$$\boldsymbol{\beta}(\boldsymbol{x}(t)) = \Big[\tfrac{1-e^{-\boldsymbol{\sigma_1}}}{1+e^{-\boldsymbol{\sigma_1}}} \quad \tfrac{1-e^{-\boldsymbol{\sigma_2}}}{1+e^{-\boldsymbol{\sigma_2}}} \quad \tfrac{1-e^{-\boldsymbol{\sigma_3}}}{1+e^{-\boldsymbol{\sigma_3}}} \quad \tfrac{1-e^{-\dot{\boldsymbol{\sigma}}_1}}{1+e^{-\dot{\boldsymbol{\sigma}}_1}} \quad \tfrac{1-e^{-\dot{\boldsymbol{\sigma}}_2}}{1+e^{-\dot{\boldsymbol{\sigma}}_2}} \quad \tfrac{1-e^{-\dot{\boldsymbol{\sigma}}_3}}{1+e^{-\dot{\boldsymbol{\sigma}}_3}}\Big] \tag{20}$$

B. Reference Model

The controller tries to track or force the closed-loop dynamics of the system to follow the reference model as defined as,

$$\dot{\boldsymbol{x}}_{\boldsymbol{m}} = \boldsymbol{A_m}\boldsymbol{x_m} + \boldsymbol{B_m}\boldsymbol{u_m} \qquad \text{and} \qquad \boldsymbol{y_m} = \boldsymbol{C_m}\boldsymbol{x_m} \tag{21}$$

We use 2nd order dynamics to get our desired response by adjusting the damping ratio ζ and natural frequency ω_n given by,

$$\ddot{\boldsymbol{\sigma}}_{\boldsymbol{m}} + 2\zeta\omega_n\dot{\boldsymbol{\sigma}} + \omega_n^2\boldsymbol{\sigma_m} = \omega_n^2\boldsymbol{u_m} \tag{22}$$

which yields,

$$\boldsymbol{A_m} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I_3} \\ -\omega_n^2\boldsymbol{I_3} & -2\zeta\omega_n \end{bmatrix}, \quad \boldsymbol{B_m} = \begin{bmatrix} \boldsymbol{0} \\ \omega_n^2\boldsymbol{I_3} \end{bmatrix} \tag{23}$$

$$\boldsymbol{C_m} = \begin{bmatrix} \alpha\boldsymbol{I_3} & \boldsymbol{I_3} \end{bmatrix}$$

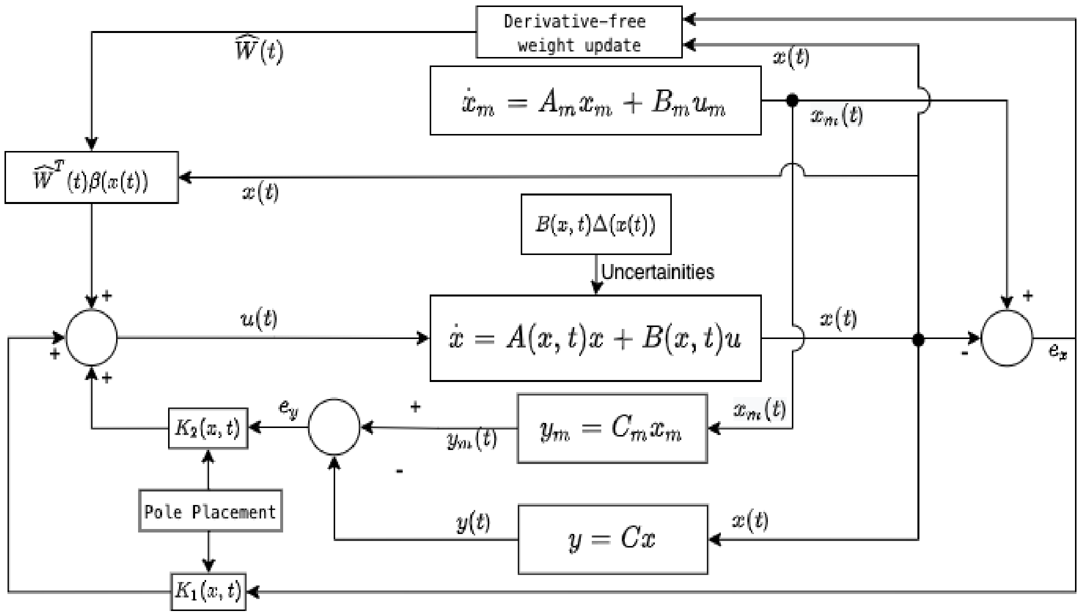

Fig. 1: a. Derivative-free purely adaptive control system

C. Control Law and Design

The derivative-free control law uses derivative-free methods to update the weight that will adjust itself to the uncertainties at any instant.

The control law is purely adaptive and is defined as

$$\boldsymbol{u}(t) = \boldsymbol{u_a}(t) \tag{24}$$

Here, $\boldsymbol{u_a}(t)$ is the adaptive feedback controller is given by,

$$\begin{aligned}\boldsymbol{u_a}(t) = \boldsymbol{K_1}(\boldsymbol{x},t)\boldsymbol{e_x}(t) + \boldsymbol{K_2}(\boldsymbol{x},t)\boldsymbol{e_y}(t) \\ + \hat{\boldsymbol{W}}^T(t)\boldsymbol{\beta}(\boldsymbol{x}(t))\end{aligned} \tag{25}$$

where $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are control feedback gains that follow the assumptions in [15] :

$$\boldsymbol{A_m} = \boldsymbol{A}(\boldsymbol{x},t) + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_1}(\boldsymbol{x},t) \tag{26}$$

$$\boldsymbol{B_m} = \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_2}(\boldsymbol{x},t) \tag{27}$$

Pole-placement method is employed such that the Hurwitz condition is satisfied. $\boldsymbol{K_1}(\boldsymbol{x},t)$ and $\boldsymbol{K_2}(\boldsymbol{x},t)$ are proportional feedback gains obtained using pole-placement methods at each instant of time ,as $\boldsymbol{A}(\boldsymbol{x},t)$ and $\boldsymbol{B}(\boldsymbol{x},t)$ are a function of time as well as the state, and such that the (26-27) is satisfied.

In (25), $\hat{\boldsymbol{W}}(t)$ is the estimated using the derivative-free weight update law given by

$$\hat{\boldsymbol{W}}(t) = \Omega_1\hat{\boldsymbol{W}}(t-\tau) + \hat{\Omega}_2(t) \tag{28}$$

where $\tau > 0$ and $0 < \Omega_1^T\Omega_1 < 1$

$$\hat{\Omega}_2(t) = k[\boldsymbol{\beta}(\boldsymbol{x}(t))\boldsymbol{e_x}^T\boldsymbol{P}\boldsymbol{B}] \tag{29}$$

Here, $\boldsymbol{P}$ is obtained by solving the Riccati equation

$$\boldsymbol{A_m}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A_m} + \boldsymbol{Q} = 0 \tag{30}$$

where $\boldsymbol{Q}$ is any symmetric, positive-definite matrix and $k > 0$. The error, $\boldsymbol{e_x}$ is defined as

$$\boldsymbol{e_x}(t) = \boldsymbol{x_m}(t) - \boldsymbol{x}(t) \tag{31}$$

IV. Stability Analysis

In this section, we look at the stability of the proposed controller on the spacecraft attitude system using the Lyapunov stability criterion by choosing a Lyapunov–Krasovskii Functional [17] for uniformly ultimate bound and globally asymptotic stable conditions.

Lyapunov stability theory allows is to determine if our system is globally asymptotically stable [17]. Consider the Lyaponov function V such that it is scalar continuous as,

$$\boldsymbol{V}(\boldsymbol{x}(t),\boldsymbol{e_x}(t),\boldsymbol{e_w}(t)) = \boldsymbol{V_1}(\boldsymbol{x}(t)) + \boldsymbol{V_2}(\boldsymbol{e_x}(t),\boldsymbol{e_w}(t)) \tag{32}$$

where

$$\boldsymbol{V_1}(\boldsymbol{x}(t)) = \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} \tag{33}$$

$$\boldsymbol{V_2}(\boldsymbol{e_x}(t),\boldsymbol{e_w}(t)) = \boldsymbol{e_x}^T\boldsymbol{P}\boldsymbol{e_x} + \int_{t-\tau}^{t}\boldsymbol{e_w}^T(s)\boldsymbol{e_w}(s)\,ds \tag{34}$$

Each of the components is independently analysed in the rest of the section.

Let $\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t))$ is given by,

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = \dot{\boldsymbol{x}}^T\boldsymbol{P}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{P}\dot{\boldsymbol{x}} \tag{35}$$

Using (30), we obtain

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = \boldsymbol{x}^T(\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{x} \tag{36}$$

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) = -\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} \tag{37}$$

Here, $\boldsymbol{Q}$ is chosen to be a positive definite matrix, hence the expression in (35) is negative definite.

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}}(\boldsymbol{x}(t)) < 0 \tag{38}$$

For the stability of an adaptive system, to show the asymptotic stability, a Lyapunov function that considers the error convergence and the bounded nature of the adaptive gains is necessary. Krasovskii's theorem helps find suitable Lyapunov function with the above requirements as demonstrated in [17].

Theorem 1. Let the Lyapunov-Krasovskii functional be defined as,

$$\boldsymbol{V_2}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) = \boldsymbol{e_x}^T \boldsymbol{P} \boldsymbol{e_x} + \int_{t-\tau}^{t} \boldsymbol{e_w}^T(s) \boldsymbol{e_w}(s)\, ds \tag{39}$$

then its derivative can be reduced into the inequality obtained as,

$$\begin{aligned}\dot{\boldsymbol{V}}_{\boldsymbol{2}}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) \leq -c_1||\boldsymbol{e_x}(t)||^2 - c_2||\boldsymbol{e_w}(t)|| \\ -c_3||\boldsymbol{e_w}(t-\tau)|| + d\end{aligned} \tag{40}$$

If (28) is satisfied, then the $\dot{\boldsymbol{V}}_{\boldsymbol{2}}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) < 0$, i.e. uniform ultimately bounded if,

$$\begin{aligned}||\boldsymbol{e_x}(t)|| > \Psi_1, \quad ||\boldsymbol{e_w}(t)|| > \Psi_2 \\ \boldsymbol{e_w}(t-\tau)|| > \Psi_3\end{aligned} \tag{41}$$

Proof: The corollaries in [16] estimates the ultimate bound as well as proves that the error trajectory reaches the bound exponentially. The main results adopted are from the detailed derivations in [16], sufficient for our analysis, is stated below.

When $\epsilon > 0$ and $\eta \in (1, 1/k)$,

$$c_1 = \lambda_{min}(\boldsymbol{Q}) > 0 \tag{42}$$

$$c_2 = \epsilon > 0 \tag{43}$$

$$c_3 = \lambda_{min}(I - k^{-1}\Omega_1^T\Omega_1) > 0 \tag{44}$$

$$c_4 = [1 + \epsilon + (1+\epsilon)^2/2]^2 \geq 0 \tag{45}$$

and we obtain the bounds in (41) as,

$$\Psi_1 = \frac{1}{\sqrt{k\lambda_{min}(\boldsymbol{Q})[1-\eta k]}} \tag{46}$$

$$\Psi_2 = \Psi_1\sqrt{\frac{c_1}{c_2}} \tag{47}$$

$$\Psi_3 = \Psi_1\sqrt{\frac{c_1}{c_3}} \tag{48}$$

It can be concurred that $\boldsymbol{e_x}(t)$ and $\boldsymbol{e_w}(t)$ are uniformly ultimate bounded.

$$\dot{\boldsymbol{V}}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) < 0 \tag{49}$$

Hence, the spacecraft attitude system with the proposed controller is asymptotic Lyapunov stable and there exists convergence of derivative-free weight update error, which provides adaptive control robustness.

V. Simulation and Results

The performance and robustness of system of the proposed controller for has been tested for three cases,

1) Dynamic properties to which the controller is tuned.
2) Unrealistically altered dynamic properties without additional tuning.
3) Dynamic properties are suddenly varied mid-flight.

Case 1. The value of moment of inertia $\boldsymbol{J}$ for an asymmetrical spacecraft is given by [18] :

$$\boldsymbol{J} = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

The initial states are chosen as follows:

$$\boldsymbol{\sigma_o} = \begin{bmatrix} -0.1 & 0.1 & 0.1 \end{bmatrix}^T$$
$$\boldsymbol{\omega_o} = \dot{\boldsymbol{\sigma}}_{\boldsymbol{o}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

The damping ratio and natural frequency of the ideal model reference system is chosen as $\zeta = 0.7$ and $\omega_n = 0.02$ respectively.

For the derivative-free controller, $\boldsymbol{\beta}$ is taken as in (21) while $\phi_1 = 0.9$ and $k = 100$ after tuning. $\boldsymbol{P}$ is calculated as in (30) where $\boldsymbol{Q}$ is taken as identity matrix.

The computation of the simulation is online using an explicit Runge-Kutta (4,5) formula on MATLAB, the Dormand-Prince pair called ODE45 where calculation of $y(t_n)$ only requires $y(t_{n-1})$.

The performance of the proposed derivative-free controller, i.e., the asymptotic stability of ω with time, can be seen in Fig.2.

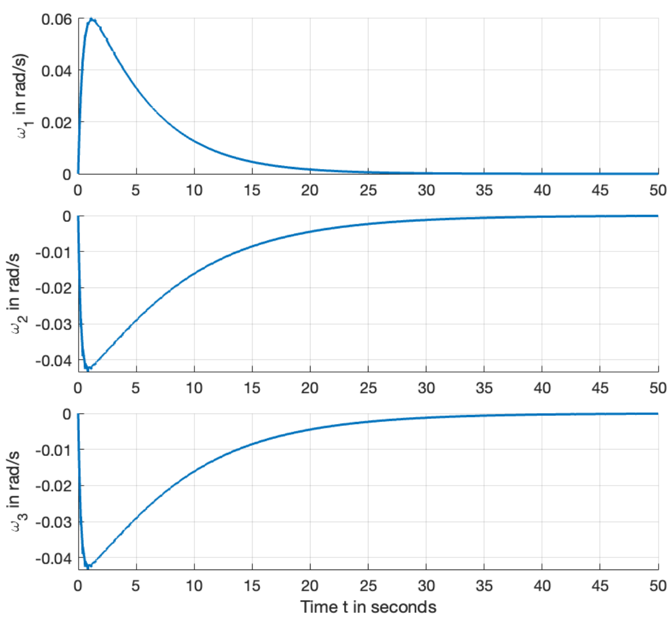

Fig. 2: Angular velocity of derivative-free tuned controller under dynamic properties of case one.

Comparison with constant weight update controller: A controller with constant weight update law is considered keeping all other factors same. The angular velocities obtained

from the constant and derivative-free weight update controllers are super imposed to observe the differences in Fig. 3.
The angular velocity in the derivative-free controller can be seen to track the reference state with less overshoot and settling time. The derivative-free weight update technique also has lower rise-time in comparison to the traditional constant weight update law.

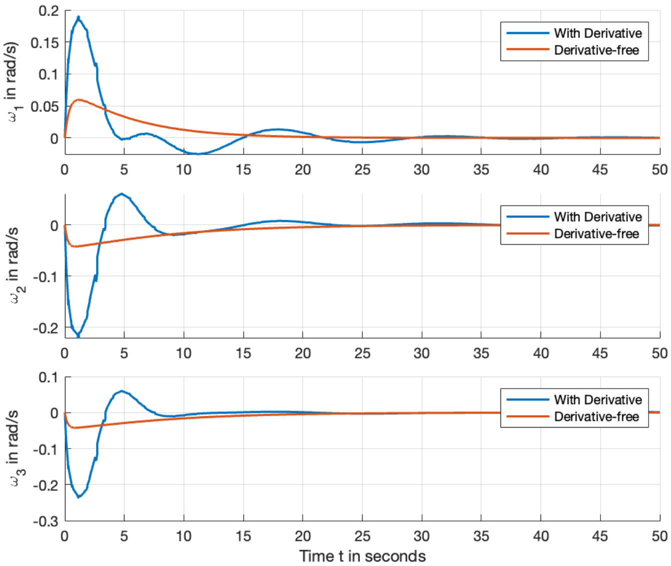

Fig. 3: Comparison between the derivative-free controller and controller with constant weight update laws.

Case 2. For the simulation results shown in Fig. 4, While the controllers are tuned to the dynamic properties in case 1, we are looking at the robustness of the proposed controller to changes in dynamics.

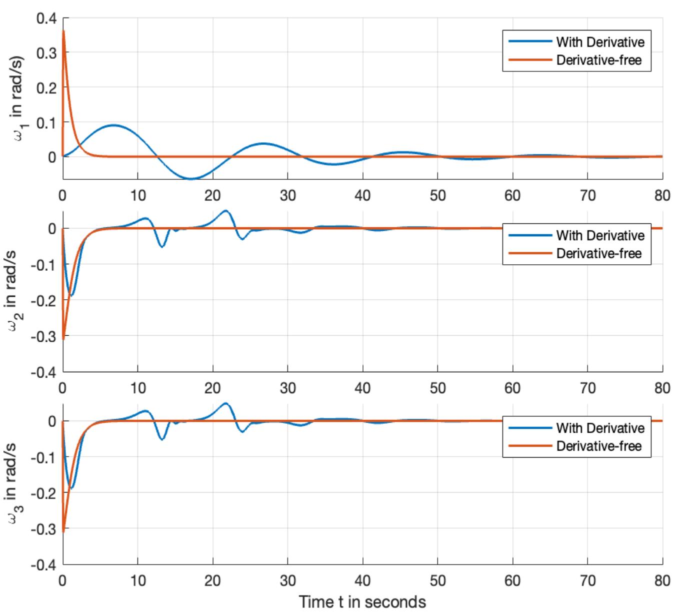

Fig. 4: Controller performances for modified dynamic properties of case 2.

The moment of inertia $\boldsymbol{J}$ is modified.

$$\boldsymbol{J} = \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

where the value of $\boldsymbol{J_x}$ is unrealistically changed.

It can be seen that the derivative-free weight update law seems to adjust faster to the changes in dynamics. The derivative-free controller also has lower steady-state error and lesser oscillations in comparison to the constant weight update controller.

Case 3. For the simulation result shown in Fig. 5, a sudden change in dynamic property is made at t = 20s. The moment of inertia J_x is suddenly changed such that, $\boldsymbol{J}$ changes from

$$\boldsymbol{J} = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} 350 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} kg.m^2$$

It can be seen that the constant weight update controller seems to have more difficulties in adapting when the unexpected change in the structural property of J_x was provided. It can also be seen that its performance with and without sudden changes is similar in the case of derivative-free controller.

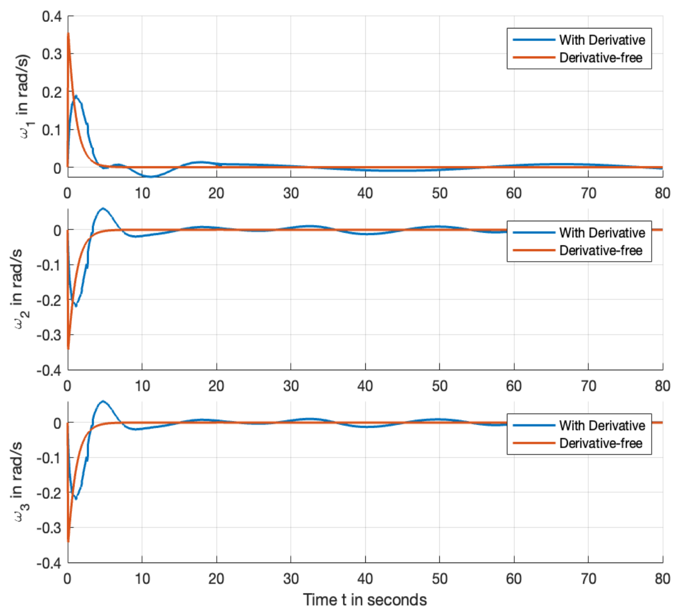

Fig. 5: Controllers' performance when J is suddenly changed mid-flight.

Variation of MRP state variables: The results in Fig. 6 shows the variation of MRP variables, i.e σ with time for case 3 for both the derivatie and derivative-free controllers.

It can be observed that the $\boldsymbol{\sigma}$ about the x-axis, σ_x, is highly oscillatory for the constant weight update controller once the change in $\boldsymbol{J}$ is made in the x-component. The other components of $\boldsymbol{\sigma}$ settle down, but later than the corresponding derivative-free controller.

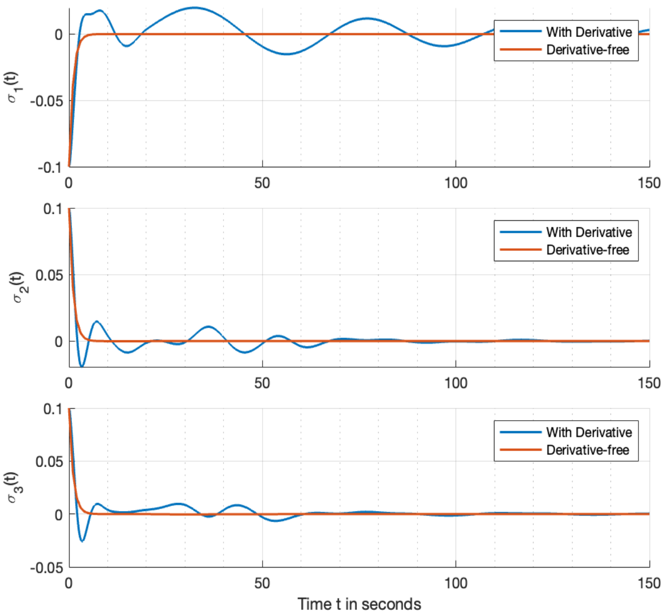

Fig. 6: Variation of state variable σ for Case 3.

VI. Conclusions

This paper has shown the robustness of employing derivative-free weight update laws, which do not require complete information from the system's past to generate control inputs. The developed controller used pole-placement to assign proportional gains of the purely adaptive controller while it used a derivative-free weight update for the weight gains dealing with matched uncertainties. The system was defined using modified Rodrigues parameters, and the state-space equations were developed into a non-linear Hamiltonian-based system to have more accurate representations. The system's stability was shown using the Lyapunov–Krasovskii functional and as the system being adaptive, the error convergence and the bounded nature of the adaptive gains were also shown. The different simulations carried out show the robustness of the controller and its ability to adapt when major system dynamic properties are significantly and suddenly changed. The derivative-free weight update law used in this paper is not modified to fit the system perfectly. Many modified derivative-free controllers have been proposed, and their effectiveness and advantages need to be explored and tested so that the use of such a computationally less expensive control system can be extensively used in spacecraft applications.

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