Evolutionary Optimisation of Adaptive Control Gains: Application to Spacecraft Attitude Control Problem

Irene Grace Karot Polson

Supervisor's Name: Prof. Dipak Kumar Giri

Undergraduate Project Report

Academic Year 2021-22/1

Roll no: 180308

Department of Aerospace Engineering
Indian Institute of Technology Kanpur
November 2021, Kanpur, India

Certificate

It is certified that the work contained this project titled "Evolutionary Optimisation of Spacecraft Attitude Adaptive Control Gains" by Irene Grace Karot Polson has been carried out under my supervision.

Prof. Dipak Kumar Giri
Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Kanpur

Date and Signature

Contents

1 Introduction

Attitude control plays a vital role in various problems such as spacecraft rendezvous, docking, berthing, and capture for maintenance. Conventional methods of spacecraft attitude control require good knowledge of the system and its dynamics to develop control equations [1-3]. The process of coming up with accurate dynamic laws and system parameters is often time-consuming and tedious. Over the years, methods such as system identification [4-5] have been employed to address this problem.

System identification helps build the mathematical model of dynamical systems using input-output data; system properties are not required. However, it still does not account for sudden changes in dynamics such as structural damage, the release of payload, and uncertainties. This can be tackled by designing a suitable control system that can adapt with time, the system's changing state, and uncertainties while maintaining its robustness.

The use of adaptive controls allow changing the controller in response to changes in the system and environment. This ensures that the system is operating at its best possible state. Various simple adaptive controllers [6-7] have been developed over the decade. In [8], a simple adaptive control has been applied to spacecraft attitude control with applications to docking between spacecraft and free-flying robots. Further, numerous control strategies, from gain scheduling to machine learning [9-12], have been developed for fast and accurate adaptive control laws. This is carried out by computing the control system's parameters that vary with time and state. Multiple derivative-free control systems [13-14] have shown robustness and fast convergence.

However, despite the tremendous improvements in adaptive control systems, their application to real-world problems is limited due to the requirement of high computations, a higher number of variable parameters, and the need for experimental verification and validation of the control algorithms. Through the study presented in this paper, the effectiveness of an adaptive control whose gains are optimized using Genetic algorithm. Simulations show the efficacy of the proposed controller and optimization.

The rest of the paper is organised as follows, section II shows the derivation of the non-linear state-space equations and covering the general attitude dynamics, use of modified Rodrigues parameters and Hamiltonian state-space equations. Section III introduces Genetic Algorithm and its terminology. Section IV discusses the control system as well GA optimisation of the adaptive control gains. V shows the different simulation results and its interpretations. Section VI concludes the results of the study and presents inferences made from the simulations. Section VII concludes the paper while suggesting improvement thoughts that could be considered in future work.

2 System Definition

2.1 Modified Rodrigues Parameters

Modified Rodrigues parameters (MRPs) are triplets in $\mathbb{R}^3$ bijectively and rationally mapped to quaternions through stereographic projection. The modified Rodrigues parameters (MRPs) constitute a minimal rotation parameterization with attractive properties. Systems can be modelled using multiple representations but there exist mathematical singularities associated with angle representations which need to be accounted for when modeling spacecraft systems. The gimbal lock for Euler Angles is its singularity at 90°, Classical Rodrigues Parameters (CRPs) have a singularity at 180° while MRPs have its singularity at 360°.
The MRP representation is chosen as the suitable representation for satellite attitude. Apart from the extended singularity range, the linearized equations of MRPs [16-17] are more accurate and closer to the non-linear system equations than other angle representations.

2.2 Spacecraft Attitude Dynamics

The attitude dynamics of a rigid spacecraft [15] is represented as

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\widehat{\boldsymbol{\omega}}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{\omega}$ is the angular momentum, $\widehat{\boldsymbol{\omega}}$ is the skew-symmetric matrix of $\boldsymbol{\omega}$ given by

$$\widehat{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

and $\boldsymbol{J}$ is the moment of inertia matrix given as,

$$\boldsymbol{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{3}$$

Let $\hat{\boldsymbol{e}}$ and ϕ represent the unit vector corresponding to the axis of rotation and the angle of rotation, respectively. Then the the MRP parameter is denoted as $\boldsymbol{\sigma} = \tan\left(\frac{\phi}{4}\right)\hat{\boldsymbol{e}}$ and

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{L}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{4}$$

represents the kinematic equation in terms $\boldsymbol{\sigma}$ where

$$\boldsymbol{L}(\boldsymbol{\sigma}) = \frac{1}{2}\left[\boldsymbol{I}\left(\frac{1-\boldsymbol{\sigma}^T\boldsymbol{\sigma}}{2}\right) + \hat{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \tag{5}$$

and $\hat{\boldsymbol{\sigma}}$ is the skew-symmetric matrix of $\boldsymbol{\sigma}$ similar to that in (2). The spate-space equations of the spacecraft attitude system are written in the Hamiltonian non-linear Euler-Lagrange form [18] as shown below,

$$\boldsymbol{H}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} + \boldsymbol{C}(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = \boldsymbol{u} \tag{6}$$

Using $\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) = 2(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}[\boldsymbol{I} - \hat{\boldsymbol{\sigma}}]$ and $\hat{\boldsymbol{G}}$ as the skew-symmetric matrix of $\boldsymbol{G} = \boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}}$ yields,

$$\boldsymbol{u} = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{\tau} \tag{7}$$

$$\boldsymbol{H}(\boldsymbol{\sigma}) = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma}) \tag{8}$$

$$\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = -\boldsymbol{L}^{-T}(\boldsymbol{\sigma})[\boldsymbol{J}\boldsymbol{L}^{-1}\dot{\boldsymbol{L}}\boldsymbol{L}^{-1} + \hat{\boldsymbol{G}}\boldsymbol{L}^{-1}] \tag{9}$$

The control input vector used in the non-linear state-space formulation below is represented as $\boldsymbol{u}$.

2.3 State-Space Formulation

Let the state vector be defined as $\boldsymbol{x} = \begin{bmatrix}\boldsymbol{\sigma} & \dot{\boldsymbol{\sigma}}\end{bmatrix}^T$, then the state-space equations are formulated as,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{u}, \qquad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{10}$$

and the system's co-efficient matrices are obtained from (6) as,

$$\boldsymbol{A}(\boldsymbol{x},t) = \begin{bmatrix}\mathbf{0} & \boldsymbol{I_3}\\ \mathbf{0} & -\boldsymbol{H}^{-1}\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\end{bmatrix}, \quad \boldsymbol{B}(\boldsymbol{x},t) = \begin{bmatrix}\mathbf{0}\\ \boldsymbol{H}^{-1}\end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix}\alpha\boldsymbol{I_3} & \boldsymbol{I_3}\end{bmatrix} \tag{11}$$

where $\boldsymbol{A}(\boldsymbol{x},t)$ and $\boldsymbol{B}(\boldsymbol{x},t)$ are varying with time and state and they need to be computed at every instant of time along with the states. The choice of $\boldsymbol{C}$ matrix with α being the scaling factor yields

$$\boldsymbol{y} = \alpha\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}} \tag{12}$$

allowing the spacecraft to be controlled through the position feedback with velocity providing damping.

3 Control System Design

In this section, we first look at the control system development and further move on to fine tuning the adaptive gains using genetic algorithm introduced in the previous section.

3.1 Reference Model Definition

The reference model for tracking is defined as,

$$\dot{\boldsymbol{x}}_{\boldsymbol{m}} = \boldsymbol{A_m}\boldsymbol{x_m} + \boldsymbol{B_m}\boldsymbol{u_m}, \qquad \boldsymbol{y_m} = \boldsymbol{C_m}\boldsymbol{x_m} \tag{13}$$

where second order dynamics is used to get the desired response by adjusting the damping ratio ζ and natural frequency ω_n given by,

$$\ddot{\boldsymbol{\sigma}}_{\boldsymbol{m}} + 2\zeta\omega_n\dot{\boldsymbol{\sigma}} + \omega_n^2\boldsymbol{\sigma_m} = \omega_n^2\boldsymbol{u_m} \tag{14}$$

which yields,

$$\boldsymbol{A_m} = \begin{bmatrix}\mathbf{0} & \boldsymbol{I_3}\\ -\omega_n^2\boldsymbol{I_3} & -2\zeta\omega_n\end{bmatrix}, \quad \boldsymbol{B_m} = \begin{bmatrix}\mathbf{0}\\ \omega_n^2\boldsymbol{I_3}\end{bmatrix}, \quad \boldsymbol{C_m} = \begin{bmatrix}\alpha\boldsymbol{I_3} & \boldsymbol{I_3}\end{bmatrix} \tag{15}$$

3.2 Simple Adaptive Control

The control law is designed purely adaptive and is defined as $\boldsymbol{u}(t) = \boldsymbol{u_a}(t)$, where $\boldsymbol{u_a}(t)$ is the adaptive feedback control represented by,

$$\boldsymbol{u_a}(t) = \boldsymbol{K}_y(\boldsymbol{x},t)\boldsymbol{e_y}(t) + \boldsymbol{K}_x(\boldsymbol{x},t)\boldsymbol{e_x}(t) + \boldsymbol{K}_u(\boldsymbol{x},t)\boldsymbol{u_m}(t) \tag{16}$$

In (16), $\boldsymbol{K_y}(\boldsymbol{x},t)$ and $\boldsymbol{K_x}(\boldsymbol{x},t)$ are stabilizing gains while $\boldsymbol{K_u}(\boldsymbol{x},t)$ is provided for tracking. These are defined as,

$$\begin{aligned} \boldsymbol{K}_y(\boldsymbol{x},t) &= \boldsymbol{K}_{Iy}(\boldsymbol{x},t) + \boldsymbol{K}_{Py}(\boldsymbol{x},t) \\ \boldsymbol{K}_x(\boldsymbol{x},t) &= \boldsymbol{K}_{Ix}(\boldsymbol{x},t) + \boldsymbol{K}_{Px}(\boldsymbol{x},t) \\ \boldsymbol{K}_u(\boldsymbol{x},t) &= \boldsymbol{K}_{Iu}(\boldsymbol{x},t) + \boldsymbol{K}_{Pu}(\boldsymbol{x},t) \end{aligned} \tag{17}$$

If c_1, c_2 and c_3 are defined as constants that determine the rate of adaptation of the law, then the integral controller gains are obtained as

$$\begin{gathered} \dot{\boldsymbol{K}}_{Iy}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{e_y}^T(t)c_1\boldsymbol{I}, \quad \dot{\boldsymbol{K}}_{Ix}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{e_x}^T(t)c_2\boldsymbol{I}, \\ \dot{\boldsymbol{K}}_{Iu}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{u_m}^T(t)c_3\boldsymbol{I} \end{gathered} \tag{18}$$

Though the integral gains are sufficient for convergence, the inclusion of proportional gains will increase the rate of conference. Let c_4, c_5 and c_6 be the constants that determine the rate of adaptation of the proportional component of adaptive gains, then

$$\begin{gathered} \boldsymbol{K}_{Py}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{e_y}^T(t)c_4\boldsymbol{I}, \quad \boldsymbol{K}_{Px}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{e_x}^T(t)c_5\boldsymbol{I}, \\ \boldsymbol{K}_{Pu}(\boldsymbol{x},t) = \boldsymbol{e_y}(t)\boldsymbol{u_m}^T(t)c_6\boldsymbol{I} \end{gathered} \tag{19}$$

3.3 Stability Analysis

In this section we look at the stability of the proposed controller on the spacecraft attitude system using Lyapunov stability criterion. Lyapunov Stability theory allows is to determine if our system is globally asymptotically stable [17].
Consider the Lyaponov function V such that it is scalar continuous as,

$$V(\boldsymbol{x}(t),\boldsymbol{e_x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{e_x}(t)) \tag{20}$$

$$V_1(\boldsymbol{x}(t)) = \boldsymbol{e_x}^T P(x,t)\boldsymbol{e_x} \tag{21}$$

$$V_2(\boldsymbol{e_x}(t)) = tr\left[(\boldsymbol{K_I}(t))\boldsymbol{\chi_I}^{-1}(\boldsymbol{K_I}(t))^T\right] \tag{22}$$

where $\boldsymbol{\chi_I}$ is the resulting adaptation matrix obtained from c_1, c_2 and c_3. The derivative of (20) is obtained as,

$$\dot{V}(\boldsymbol{x}(t),\boldsymbol{e_x}(t)) = -\boldsymbol{e_x}^T Q(x,t)\boldsymbol{e_x} - 2\boldsymbol{e_x}^T C^T C\boldsymbol{e_x}\boldsymbol{r}^T\boldsymbol{\chi_P}\boldsymbol{r} \tag{23}$$

where C is obtained from the detailed proof that could be referred in []. As we can control the value of χ such that $\dot{\boldsymbol{V}}(\boldsymbol{x}(t),\boldsymbol{e_x}(t))$ is negative definitive. Therefore, the stability of the adaptive system is guaranteed from Lyapunov stability theory. In the following section we look at how the $\boldsymbol{\chi}$ values can be obtained to guarantee the stability of system as well allow the system to perform at its optimal state.

4 Genetic Algorithm

The GA optimization technique is based on Darwin's Theory Of Evolution and natural genetics (which follows the sequence of Natural Selection → Fittest Individuals → Crossover → Mutation). This algorithm reflects the process of natural selection where the fittest individuals are selected for reproduction in order to produce offspring of the next generation. It usually provides approximate solutions to various problems. Genetic algorithms can be used in a wide variety of fields. It is mainly used to solve optimization problems.
GA can be applied in various fields like bio-informatics, computational science, electrical engineering etc. Real-coded GA does not require binary encoding and decoding hence it is faster than binary GA. GA can manipulate both continuous and discrete variables. It can deal with a large number of variables and does not require derivative information.

4.1 Parameter Definitions

The evaluation scheme of chromosomes is called fitness functions, which is defined by the designer. In Fig. 1, the genes represent each parameter to be optimized, chromosomes are the set of parameters that are optimized as a group and population is the different sets of parameters evaluated to find the best set.

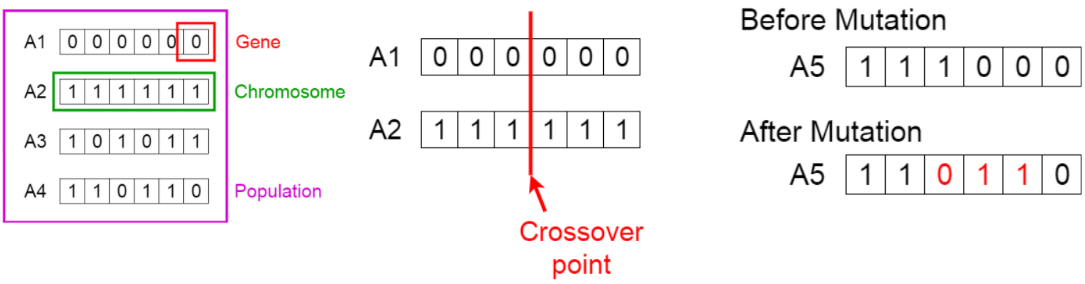

Figure 1: Example of GA with six parameters

During each successive generation, a portion of the existing population is selected to breed a new generation. The next step is to generate a second generation population of solutions from those selected through a combination of genetic operators: crossover, and mutation. A new population is created every generation which typically shares many of the characteristics of its "parents". Crossover represents what portion of parent genes are carried forward while mutations introduce new genes. The termination of the program can be achieved using multiple stopping criteria. These include setting the maximum number of generation, tolerance for fitness function evaluation that satisfies a minimum criteria and allocating time or memory.

4.2 Adaptive Gains Tuning

The constant from c_1 to c_6 represents are χ in (23) needs to chosen such that the system is stable. This requires (23) to be negative definitive and the most basic bound on χ is for it to be non-negative. Using this bound, we make use of Genetic algorithm to search for the optimal values of these constants.

Genetic algorithm, as we had seen in section 3, has multiple parameters that need proper tuning to ensure fast convergence and solution. When we increase the population size and number of generations, the time taken for optimisation increases. So we need to arrive at a trade-off between the two such that at the end of optimization, we have satisfying results. The GA parameters obtained for our problem is presented in the table below.

Parameter	Value
Population Size	15
Maximum Generations	10
Initial value	10^4
Elite Ratio	5
Crossover ratio	0.6
Mutation rate	0.01

It can be observed that the mutation rate is quite low while the crossover ratio is high. This is kept so as we have found viable initial values and the requirement to spread rapidly is minimized. We have prioritized finding accurate solutions in the region of the initial values provided after rounds of trail for this spacecraft attitude system.

5 Simulation

In this section, the efficacy of the proposed spacecraft attitude control system is shown using simulations. The damping ratio and natural frequency of the ideal model reference system is chosen as $\zeta = 0.7$ and $\omega_n = 0.02$ respectively. The initial states are chosen as follows:

$$\boldsymbol{\sigma_o} = \begin{bmatrix} -0.2 & 0.2 & 0.1 \end{bmatrix}^T$$

$$\boldsymbol{\omega_o} = \boldsymbol{\dot{\sigma}_o} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

The value of moment of inertia $\boldsymbol{J}$ for an asymmetrical spacecraft is given by [20] :

$$\boldsymbol{J} = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

The computation of the simulation is online using an explicit Runge-Kutta (4,5) formula on MATLAB, the Dormand-Prince pair called ODE45 where calculation of $y(t_n)$ only requires $y(t_{n-1})$.
We tested the performance of multiple fitness functions and the results have been presented. The Mean Square Error(MSE) has been chosen as the base fitness function after literature survey on different such fitness functions. The error used in the MSE calculation is varied as combinations of $\boldsymbol{e_x}$ and $\boldsymbol{e_y}$.

5.1 Case 1: MSE with e_x

From (18) and (19), gains c_1 and c_4 represents the adaptation rate of $\boldsymbol{e_x}$ and it dominates for this choice of fitness function. It can be seen that the gains arrange themselves such that the e_x portion of the controller is prioritized. If $\boldsymbol{e_x} = \boldsymbol{x_m}(t) - \boldsymbol{x}(t)$, then the fitness function is chosen as,

$$J_1 = \frac{1}{n} \sum_{n=1}^{\infty} \boldsymbol{e_x}^2 \tag{24}$$

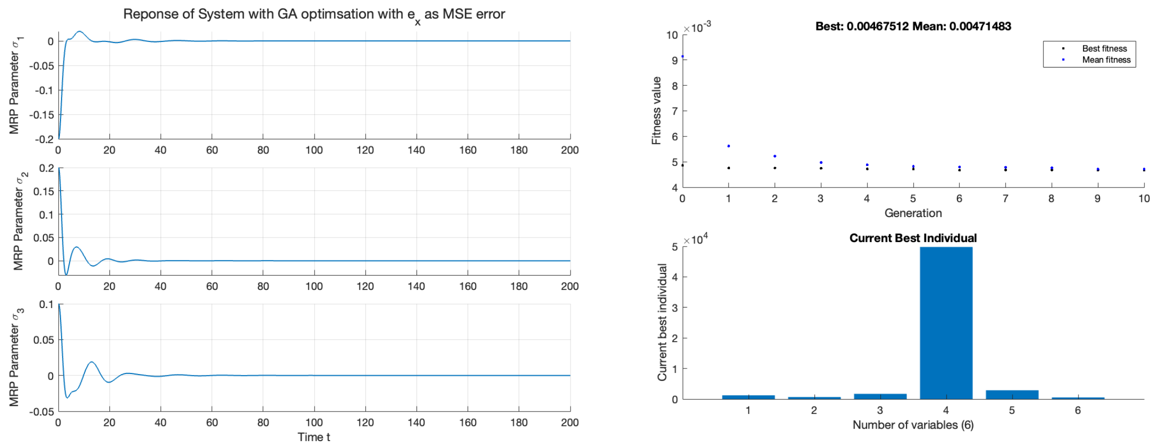

Figure 2: Response of System with the optimized GA parameters and distribution of GA parameters for case 1 where fitness function uses $\boldsymbol{e_x}$.

The value of each adaptation obtained for this case is presented below. c_4 is obtained significantly larger than other gains in case 1.

c_1	c_2	c_3	c_4	c_5	c_6
0.1217	0.0657	0.1718	4.9798	0.2883	0.0511

5.2 Case 2: MSE with e_y

From (18) and (19), gains c_2 and c_5 represents the adaptation rate of $\boldsymbol{e_y}$ and it dominates for this choice of fitness function. It can be seen that the gains arrange themselves such that the $\boldsymbol{e_y}$ portion of the controller is prioritized.
If $\boldsymbol{e_y} = \boldsymbol{y_m}(t) - \boldsymbol{y}(t)$, then the fitness function is chosen as,

$$J_2 = \frac{1}{n} \sum_{n=1}^{\infty} \boldsymbol{e_y}^2 \tag{25}$$

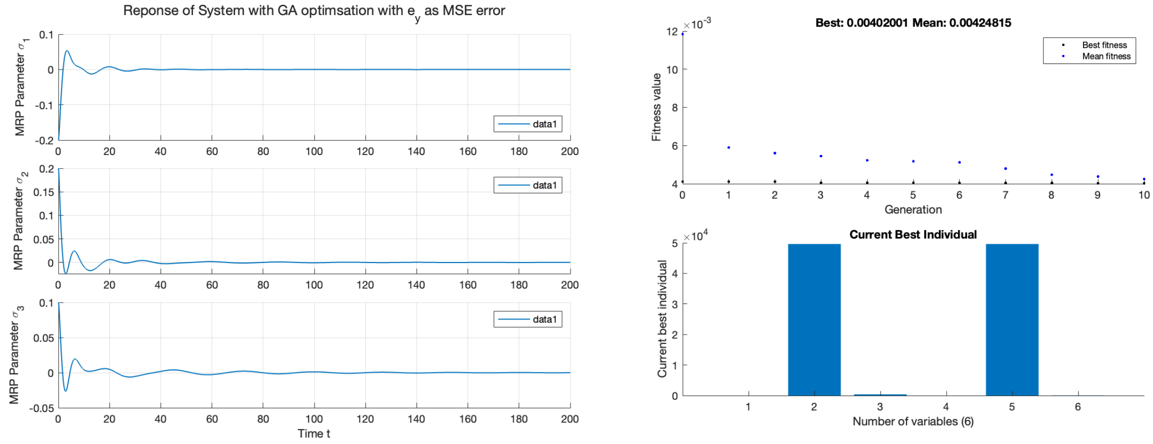

Figure 3: Response of System with the optimized GA parameters and distribution of GA parameters in case 2 where fitness function uses $\boldsymbol{e_y}$.

The value of each adaptation obtained for this case is presented below. c_2 and c_5 are obtained significantly larger than other gains in case 2.

c_1	c_2	c_3	c_4	c_5	c_6
0.0005	4.9614	0.0379	0.0006	4.9617	0.0056

5.3 Case 3: MSE with e_x and e_y

From (18) and (19), gains c_1 and c_4 represents the adaptation rate of $\boldsymbol{e_x}$, while c_2 and c_5 represents the adaptation rate of $\boldsymbol{e_y}$. It can be seen that the gains arrange themselves more uniformly when both $\boldsymbol{e_x}$ and $\boldsymbol{e_y}$ components are included in the fitness function.
If $\boldsymbol{e_y} = \boldsymbol{y_m}(t) - \boldsymbol{y}(t)$ and $\boldsymbol{e_x} = \boldsymbol{x_m}(t) - \boldsymbol{x}(t)$, then the fitness function is chosen as,

$$J_3 = \frac{1}{n}\left[\sum_{n=1}^{\infty} \boldsymbol{e}_{\boldsymbol{y}}^2 + \sum_{n=1}^{\infty} \boldsymbol{e}_{\boldsymbol{x}}^2\right] \tag{26}$$

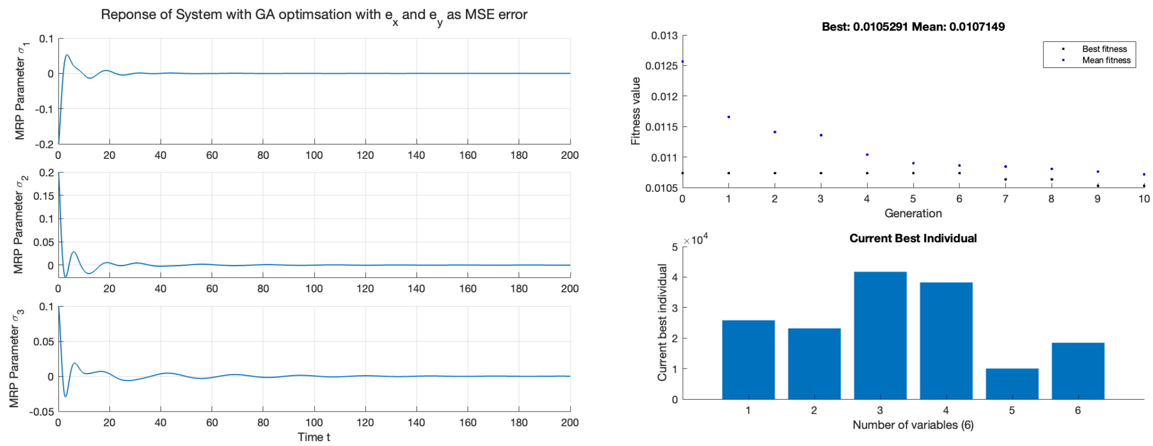

Figure 4: Response of System with the optimized GA parameters and distribution of GA parameters in case 3 where fitness function uses $\boldsymbol{e_x}$ and $\boldsymbol{e_y}$.

The value of each adaptation obtained for this case is presented below. It can bee seen that the GA optimization happens uniformly distributed to all gains in case 3.

c_1	c_2	c_3	c_4	c_5	c_6
0.0533	0.4940	0.2890	0.4222	1.3047	0.0819

5.4 Comparison with regular simple adaptive controller

The proposed controller with GA optimization has been compared with the simple adaptive controller without optimization. The adaptation rates are taken as in [],

c_1	c_2	c_3	c_4	c_5	c_6
1e5	1e3	1e3	1e5	1e3	1e3

The responses obtained in with and without GA optimization has been super imposed as shown in Fig.().

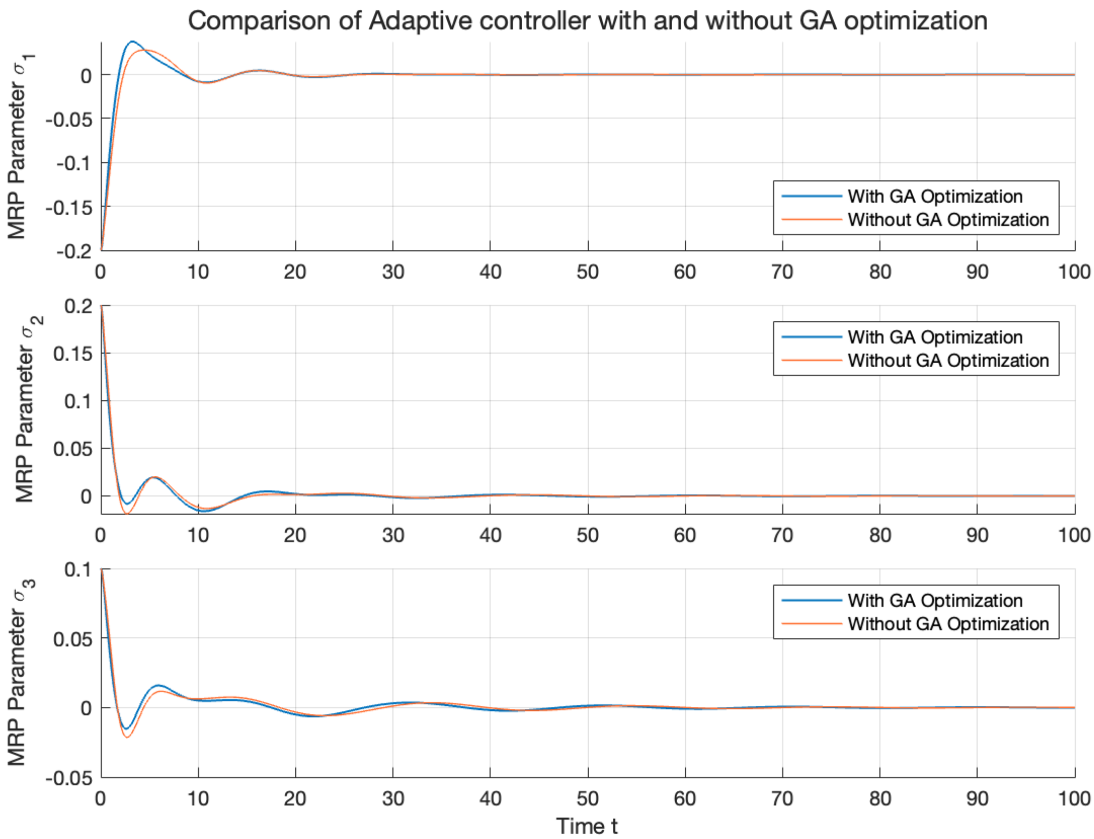

Figure 5: Comparison plot with and without GA optimization

6 Results and Discussion

From the simulation carried out in section 6, we can make many inferences that could help better formulate genetic algorithm. The results obtained by studying the graphs are given below.

- Choosing the fitness function plays a significant role in obtaining good optimization using Genetic Algorithm.
- From simulations for the first three cases, it is evident that the fitness function should incorporate all components of the inherent controller whose gains we are tuning.
- In our case, the simple adaptive controller had $\boldsymbol{e_x}$ and $\boldsymbol{e_y}$ both of which needed to be controlled. Hence, the fitness chosen in case 3 has both components for better results.
- From the comparison of GA optimized controller with one that is not optimised, it was seen that the optimized controller has better performance as it has lower overshoot, and lower settling time.

7 Conclusion

This paper shows the efficacy of a simple adaptive controller on a spacecraft attitude system whose adaptation rates are tuned using Genetic algorithm. The developed control system has both proportional and adaptive control gains and the control input makes use of state and output error as well as reference control input from the reference model. The system is defined using modified Rodrigues parameters. The state-space equations are developed into a non-linear Hamiltonian-based system to have more accurate representations. The system's stability is shown using the Lyapunov stability criterion. As the system is adaptive, error convergence and bounded nature of the adaptive gains are also shown. Simulations show the robustness of the controller and its ability to adapt when major system dynamic properties are significantly and suddenly changed. The above-presented GA optimized adaptive control system can be used in cases that require fast convergence and minimal overshoot conditions. In the future, conditions that lead to the requirement of optimization of gains can be explored to expand their application in spacecraft control.

References

[1] E. Crouch, Peter, "Spacecraft attitude control and stabilization: Applications of geometric control theory to rigid body models," *IEEE Transactions on Automatic Control*, Vol. 29, No. 4, pp. 321 - 331, 1984.

[2] Nicosia S., Tomei P. , "Non-linear observer and output feedback attitude control of spacecraft," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 28, No. 4, pp. 970–977, 1992.

[3] Oda M., "Coordinated control of spacecraft attitude and its manipulator," *IEEE International Conference on Robotics and Automation*, pp. 732-738, 2002.

[4] Khemaissia S., Morris A. S., "Fast-robot system identification based on neural network models," *IEEE International Symposium on Intelligent Control*, pp. 13-16, 1992.

[5] Schoukens J., Marconato A., Pintelon R., Rolain Y., Schoukens M., Tiels K., ... Van Mulders A., "System Identification in a Real World," *IEEE 13th International Workshop on Advanced Motion Control (AMC)*, pp. 1-9, 2014.

[6] Nishiyama T., Suzuki S., Sato M., Masui K., "Simple Adaptive Control with PID for MIMO Fault Tolerant Flight Control Design. ," *AIAA Infotech @ Aerospace*, 2016.

[7] Ulrich, S., Hayhurst, D. L., Saenz Otero, A., Miller, D., Barkana, I., "Simple Adaptive Control for Spacecraft Proximity Operations," *AIAA Guidance, Navigation, and Control Conference*, 2014.

[8] Shi, J.-F., Ulrich, S., Allen, A. , "Spacecraft Adaptive Attitude Control with Application to Space Station Free-Flyer Robotic Capture," *AIAA Guidance, Navigation, and Control Conference*, 2015.

[9] Wang, Q., Zhou, B., Duan, G.-R. (2015). Robust gain scheduled control of spacecraft rendezvous system subject to input saturation. *Aerospace Science and Technology*, 42, 442–450, 2014.

[10] Nishiyama, T., Suzuki, S., Sato, M., Masui, K., "Adaptive Control Strategy using Lyapunov Stability Theory ,", *International Journal of Engineering Research Technology (IJERT)*, Vol. 3, No. 9, pp. 1557-1562, 2014.

[11] Mooij, E., "Simple Adaptive Control System Design Trades," *AIAA Guidance, Navigation, and Control Conference*, 2017

[12] J. E. Gaudio, T. E. Gibson, A. M. Annaswamy, M. A. Bolender, E. Lavretsky, "Connections Between Adaptive Control and Optimization in Machine Learning," *IEEE 58th Conference on Decision and Control (CDC)*, pp 4563 - 4568, 2019.

[13] Yucelen, T. and Calise, A.J., "Robustness of a Derivative-Free Adaptive Control Law," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 5, pp. 1583-1594, 2014.

[14] Yucelen, T. and Calise, A.J., "Derivative-Free Model Reference Adaptive Control," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 4, pp. 933-950, 2011.

[15] Tewari, Ashish, "Atmospheric and Space Flight Dynamics," Birkhäuser Boston, *Birkhäuser Basel*, 2007

[16] Schaub, H., and Junkins, J. L., "Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters," *The Journal of the Astronautical Sciences*, Vol. 44, No. 1, pp. 1–19, 1996.

[17] Terzakis, G. and Lourakis, M. and Ait-Boudaoud, D., "Modified Rodrigues Parameters: An Efficient Representation of Orientation in 3D Vision and Graphics," *Journal of Mathematical Imaging and Vision*, Vol. 60, No. 2, pp. 422–442, 2018.

[18] Slotine, J.-J. E. and Benedetto, M. D. D., "Hamiltonian Adaptive Control of Spacecraft," *IEEE Transactions on Automatic Control*, Vol. 35, No. 7, pp. 848–852, 1990.

[19] Ariba, Y., and Gouaisbaut, F., "Construction of Lyapunov-Krasovskii functional for time-varying delay systems," *47th IEEE Conference on Decision and Control*, 2008.

[20] T. A. W. D. and Sira-Ramirez, H., "Variable Structure Control of Spacecraft Reorientation Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 3, pp. 262–270, 1988.