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SPACECRAFT ATTITUDE CONTROL USING DERIVATIVE-FREE PURELY ADAPTIVE CONTROLLER

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MOTIVATION AND OBJECTIVE

Attitude control plays a vital role in various spacecraft maneuvers. But uncertainties in system and environment is inevitable. These uncertainties can be caused by sudden changes in the dynamic properties of the system due to docking, structural damage, or measurement errors.

The main objective behind our work can be summarized as:

- Design a purely adaptive attitude control system consisting of two components, a) pole-placement and b) derivative-free adaptive components.
  - The proportional adaptive gains are assigned using the pole-placement method on a time and state variant system.
  - The adaptive weight for uncertainties is computed using derivative-free weight update laws to ensure improved settling and rise times while maintaining robustness.
- The proposed controller is compared with an adaptive system of similar proprieties having derivative weight update laws. Three cases are simulated to show the efficacy of the applied controller on the spacecraft attitude system.

ATTITUDE DYNAMICS AND KINEMATICS

The attitude dynamics of a rigid spacecraft is represented as : $J\dot{\omega} = -\widehat{\omega} J\omega + \tau$

It is converted to MRP representations to give the Hamiltonian nonlinear Euler-Lagrange equation :

$$H(\sigma)\ddot{\sigma} + C(\sigma, \dot{\sigma})\dot{\sigma} = u$$

The Modified Rodrigues parameter representation is chosen as the suitable representation for spacecraft attitude.

MRPs are triplets rationally mapped to quaternions through stereographic projection.

Advantages:

- Minimal rotation parameterization
- MRPs have its singularity at 360 degrees. (Euler Angles: 90 degrees, Classical Rodrigues Parameters: 180 degrees)
- Apart from the extended singularity range, the linearized equations of MRPs are more accurate to the non-linear system equations than other angle representations.

ATTITUDE DYNAMICS AND KINEMATICS

Using the Hamiltonian nonlinear Euler-Lagrange equation, we formulate the State-space equations

$$H(\sigma)\ddot{\sigma} + C(\sigma, \dot{\sigma})\dot{\sigma} = u$$

The state space formulated as below:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x}, t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x}, t)\boldsymbol{u}, \qquad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$$

$$\boldsymbol{A}(\boldsymbol{x}, t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I_3} \\ \boldsymbol{0} & -\boldsymbol{H}^{-1}\boldsymbol{C}(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}}) \end{bmatrix}, \quad \boldsymbol{B}(\boldsymbol{x}, t) = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{H}^{-1} \end{bmatrix} \quad \boldsymbol{C} = [\alpha \boldsymbol{I_3} \quad \boldsymbol{I_3}]$$

where,

$$\boldsymbol{C}(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = -\boldsymbol{L}^{-T}(\boldsymbol{\sigma})[\boldsymbol{J}\boldsymbol{L}^{-1}\dot{\boldsymbol{L}}\boldsymbol{L}^{-1} + \hat{\boldsymbol{G}}\boldsymbol{L}^{-1}]$$

$$\boldsymbol{L}(\boldsymbol{\sigma}) = \frac{1}{2}\left[\boldsymbol{I}\left(\frac{1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma}}{2}\right) + \hat{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\sigma}^T\right] \qquad \boldsymbol{H}(\boldsymbol{\sigma}) = \boldsymbol{L}^{-T}(\boldsymbol{\sigma})\boldsymbol{J}\boldsymbol{L}^{-1}(\boldsymbol{\sigma})$$

CONTROLLER DESIGN

The system is introduced with uncertainties as shown below:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x}, t)\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{x}, t)[\boldsymbol{u}(t) + \Delta(\boldsymbol{x}(t))]$$

The uncertainties are linearized as,

$$\Delta(\boldsymbol{x}(t)) = \boldsymbol{W}^T(t)\boldsymbol{\beta}(\boldsymbol{x})$$

Reference Model:

$$\dot{\boldsymbol{x}}_{\boldsymbol{m}} = \boldsymbol{A}_{\boldsymbol{m}}\boldsymbol{x}_{\boldsymbol{m}} + \boldsymbol{B}_{\boldsymbol{m}}\boldsymbol{u}_{\boldsymbol{m}}, \qquad \boldsymbol{y}_{\boldsymbol{m}} = \boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{x}_{\boldsymbol{m}}$$

2nd Order Dynamics for desired responses:

$$\ddot{\boldsymbol{\sigma}}_{\boldsymbol{m}} + 2\zeta\omega_n\dot{\boldsymbol{\sigma}} + \omega_n^2\boldsymbol{\sigma}_{\boldsymbol{m}} = \omega_n^2\boldsymbol{u}_{\boldsymbol{m}}$$

Resulting ideal state-space coefficients:

$$\boldsymbol{A}_{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{\boldsymbol{3}} \\ -\omega_n^2\boldsymbol{I}_{\boldsymbol{3}} & -2\zeta\omega_n \end{bmatrix}, \quad \boldsymbol{B}_{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{0} \\ \omega_n^2\boldsymbol{I}_{\boldsymbol{3}} \end{bmatrix}, \quad \boldsymbol{C}_{\boldsymbol{m}} = \begin{bmatrix} \alpha\boldsymbol{I}_{\boldsymbol{3}} & \boldsymbol{I}_{\boldsymbol{3}} \end{bmatrix}$$

CONTROL LAW DESIGN

Derivative free control law:

$$\boldsymbol{u_a}(t) = \boldsymbol{K_1}(\boldsymbol{x},t)\boldsymbol{e_x}(t) + \boldsymbol{K_2}(\boldsymbol{x},t)\boldsymbol{e_y}(t) + \hat{\boldsymbol{W}}^T(t)\boldsymbol{\beta}(\boldsymbol{x}(t))$$

Here, K_1 and K_2 are feedback gains obtained using pole-placement methods such that,

$$\boldsymbol{A_m} = \boldsymbol{A}(\boldsymbol{x},t) + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_1}(\boldsymbol{x},t)$$
$$\boldsymbol{B_m} = \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{K_2}(\boldsymbol{x},t)$$

The derivative free component is defined as below,

$$\hat{\boldsymbol{W}}(t) = \Omega_1\hat{\boldsymbol{W}}(t-\tau) + \hat{\Omega}_2(t)$$

where,

$$\hat{\Omega}_2(t) = k[\boldsymbol{\beta}(\boldsymbol{x}(t))\boldsymbol{e}_{\boldsymbol{x}}^T\boldsymbol{PB}] \qquad \boldsymbol{e_x}(t) = \boldsymbol{x_m}(t) - \boldsymbol{x}(t)$$

and P is obtained by the solving the Riccati equation. $\boldsymbol{A}_m^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A_m} + \boldsymbol{Q} = 0$

STABILITY ANALYSIS

- The system's stability is shown using the Lyapunov–Krasovskii function.

$$\boldsymbol{V}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) = \boldsymbol{e_x}^T \boldsymbol{P} \boldsymbol{e_x} + tr\left[\int_{t-\tau}^{t} \boldsymbol{e_w}^T(s)\boldsymbol{e_w}(s)\, ds\right]$$

$$\dot{\boldsymbol{V}}(\boldsymbol{e_x}(t), \boldsymbol{e_w}(t)) \leq -d_1||\boldsymbol{e_x}(t)||^2 - d_2||\boldsymbol{e_w}(t)|| - d_3||\boldsymbol{e_w}(t-\tau)|| + d_4$$

such that

$$d_1 = \lambda_{min}(\boldsymbol{Q}) > 0$$
$$d_2 = \epsilon > 0$$
$$d_3 = \lambda_{min}(I - k^{-1}\Omega_1^T\Omega_1) > 0$$
$$d_4 = [1 + \epsilon + (1+\epsilon)^2/2]^2 \geq 0$$

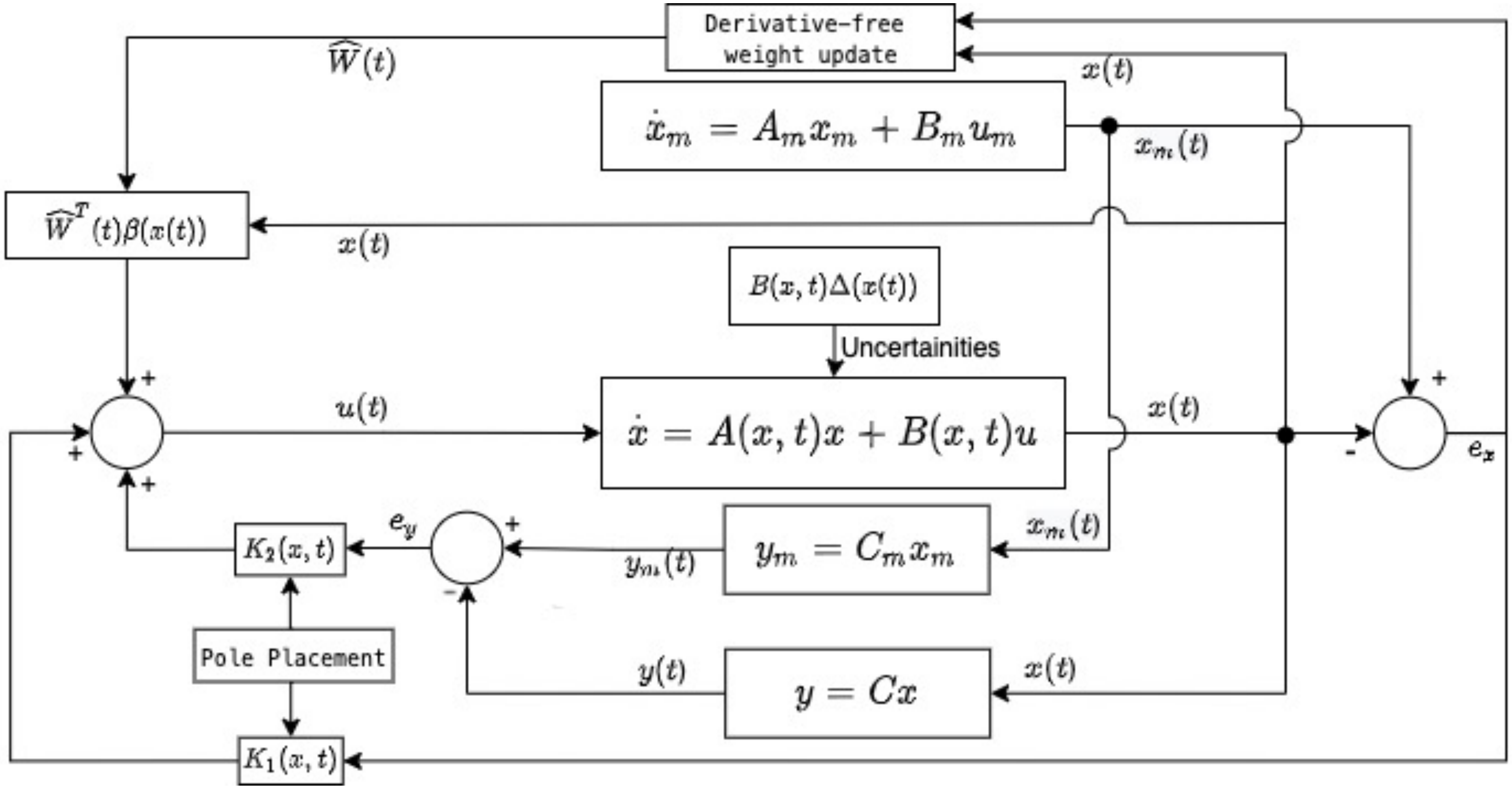
As the system is adaptive, error convergence and bounded nature of the adaptive gains are also shown.

$$||\boldsymbol{e_x}(t)|| > \Psi_1, \quad ||\boldsymbol{e_w}(t)|| > \Psi_2$$
$$||\boldsymbol{e_w}(t-\tau)|| > \Psi_3$$

such that

$$\Psi_1 = \frac{1}{\sqrt{k\lambda_{min}(\boldsymbol{Q})[1-\eta k]}}$$
$$\Psi_2 = \Psi_1\sqrt{\frac{\lambda_{min}(\boldsymbol{Q})}{\epsilon}}$$
$$\Psi_3 = \Psi_1\sqrt{\frac{\lambda_{min}(\boldsymbol{Q})}{[1+\epsilon+(1+\epsilon)^2/2]^2}}$$

SYSTEM BLOCK DIAGRAM



SIMULATION

The performance and robustness of the spacecraft system using the proposed attitude controller has been tested for three cases,

- Dynamic properties to which the controller is explicitly tuned: the control system has been tuned using trial and error method to obtain desired response.
- Dynamic properties are altered from t = 0 without additional tuning: the performance of the system in this case determines how well it can perform with inherent uncertainties in the system.
- Dynamic properties are suddenly varied during the flight at t > 0: this is carried out to determines how well and quickly the control system can adapt when uncertainties arise mid-flight.

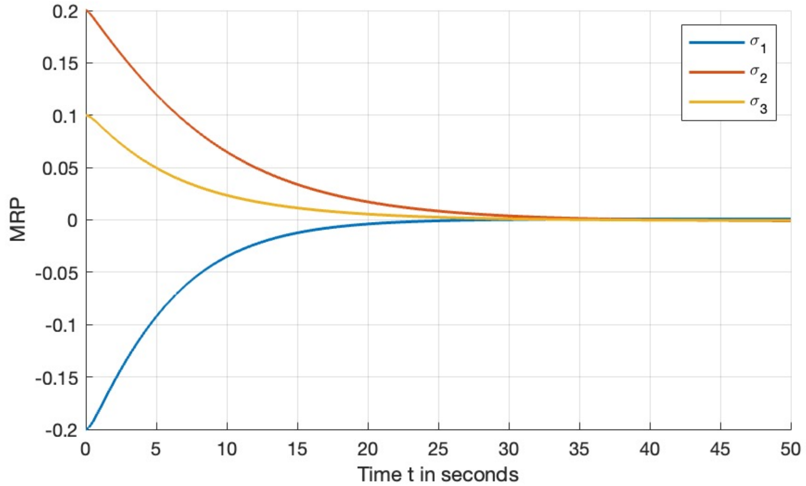
In all cases, the proposed system is compared with a similar control system consisting of derivative laws for weight adaptation.

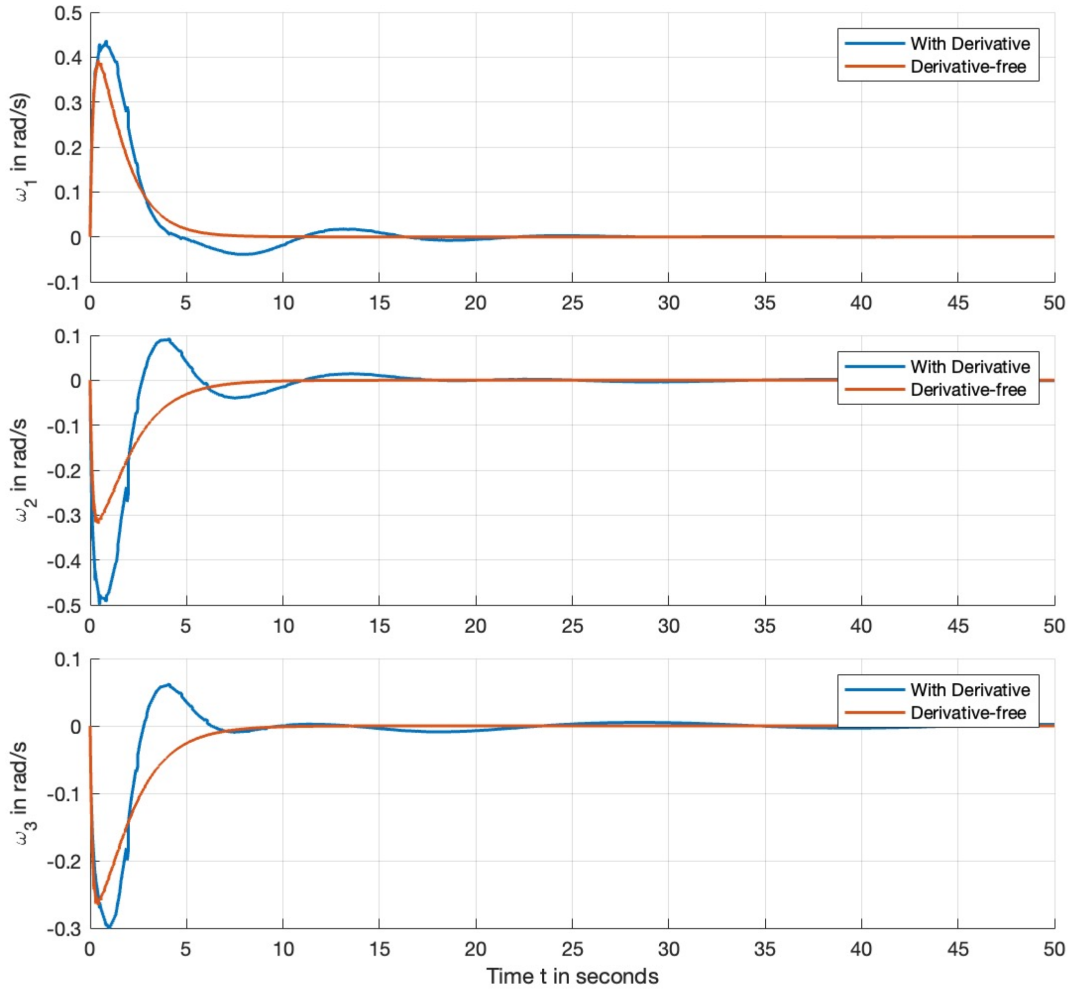
The controller with derivative weight update laws is constructed keeping all other factors constant for comparison.

CASE 1. DYNAMIC PROPERTIES TO WHICH THE CONTROLLER IS EXPLICITLY TUNED

Both the control systems have been tuned to the Moment of inertia shown below of the asymmetrical spacecraft:

$$J = \begin{bmatrix} 114.562 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

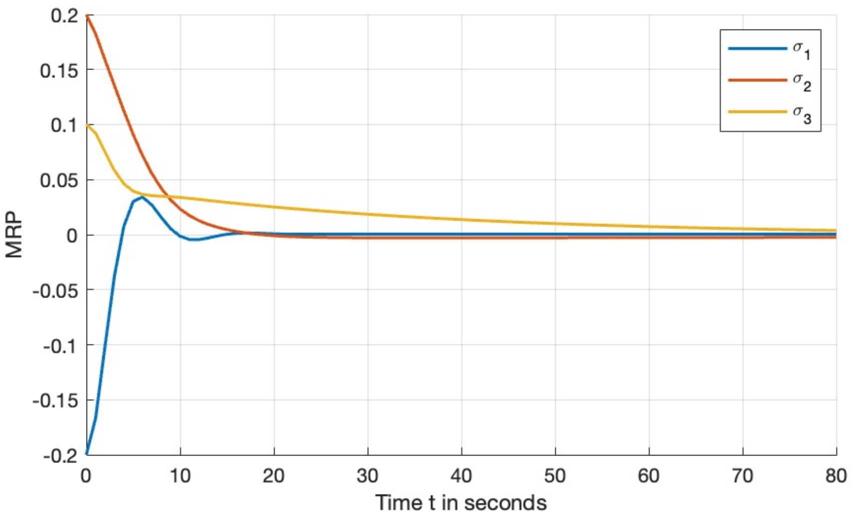


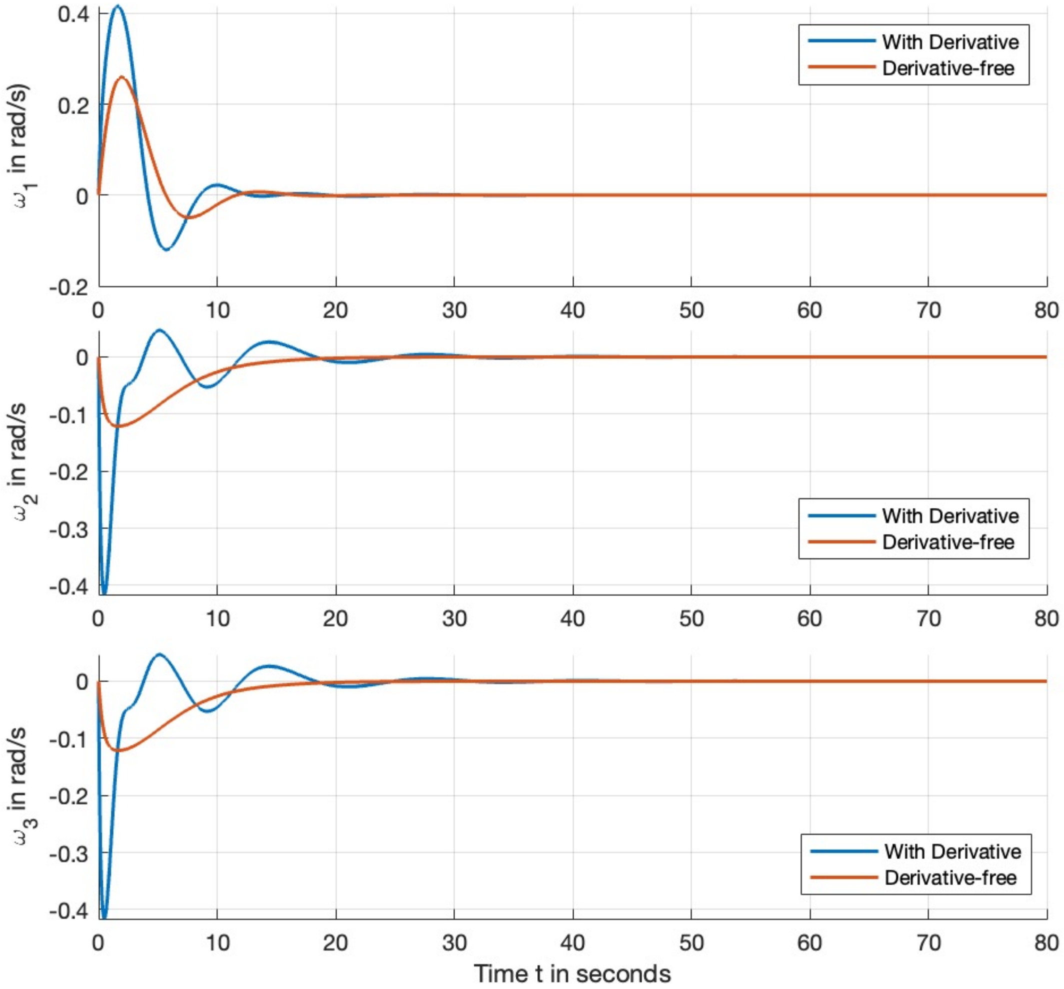


CASE 2. DYNAMIC PROPERTIES ARE DRASTICALLY ALTERED WITHOUT ADDITIONAL TUNING

The Moment of inertia is changed without any further tuning as shown:

$$\boldsymbol{J} = \begin{bmatrix} 171 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 87.212 \end{bmatrix} kg.m^2$$

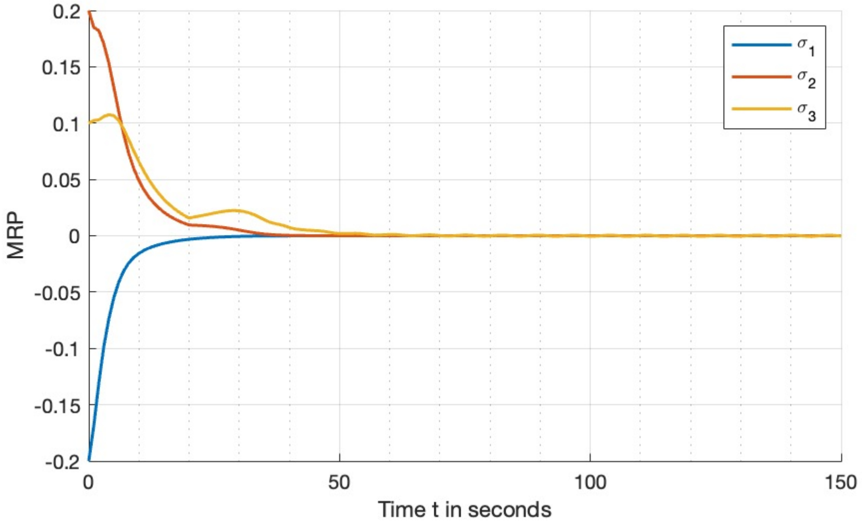


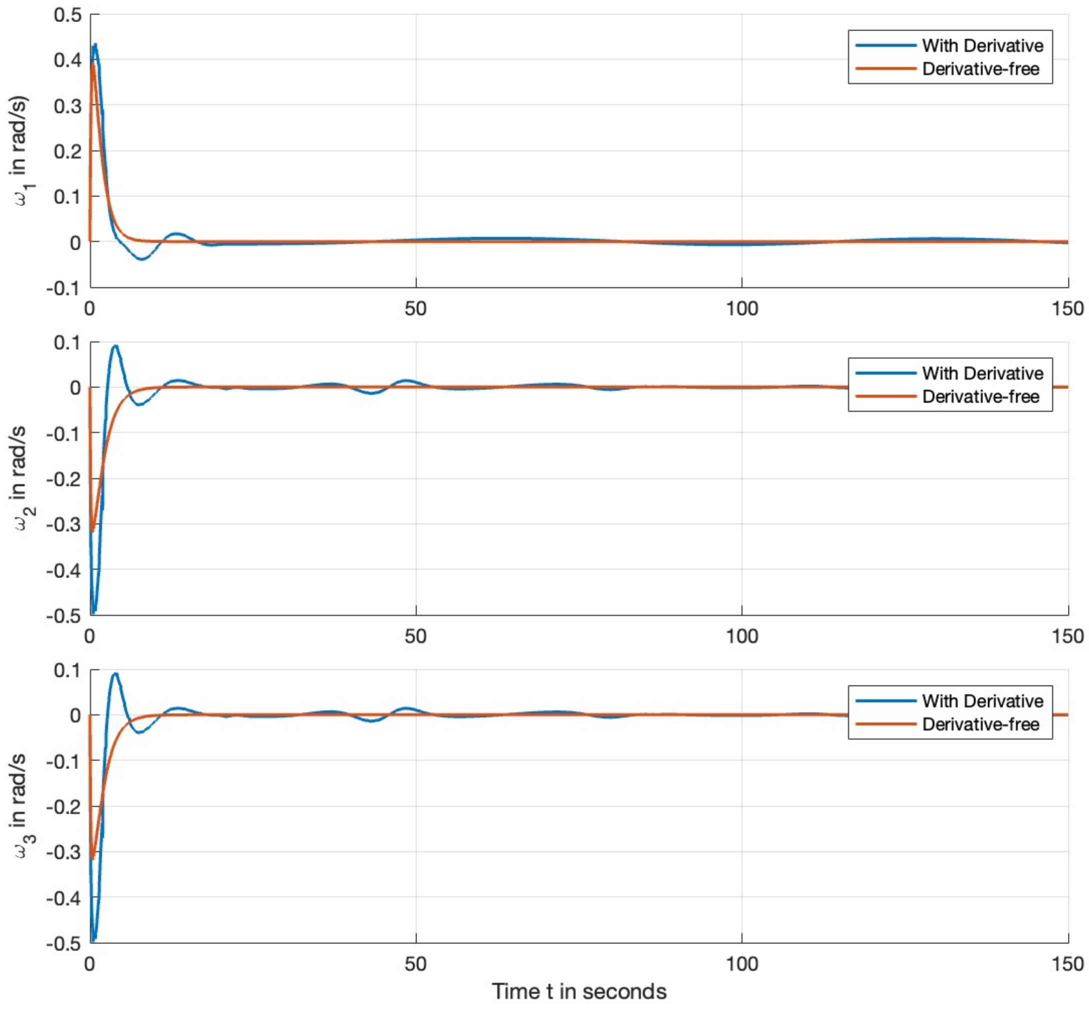


CASE 3. DYNAMIC PROPERTIES ARE SUDDENLY VARIED MID-FLIGHT

A sudden change in dynamic property is made at t = 20s. The moment of inertia is suddenly changed such that,

$$\boldsymbol{J} = \begin{bmatrix} 114 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} \text{ to } \begin{bmatrix} 171 & 0 & 0 \\ 0 & 86 & 0 \\ 0 & 0 & 87 \end{bmatrix} kg.m^2$$





OBSERVATIONS FROM SIMULATION

- From the simulation results, it is observed that the derivative-free controller can track the reference state with less overshoot, rise and settling time.
- In the case of derivative-free controller, the performance with and without sudden changes is steady. On the other hand, the derivative weight update controller seems to have more difficulties in adapting when there is an unexpected change in the structural property.
- It can also be seen that, the angular velocity about the x-axis is oscillatory for the derivative weight update controller once the change in moment of inertia is made or other components of omega settle down, but later than the corresponding derivative-free controller settling time.
- The presented purely adaptive control system can be used in cases that require fast convergence and minimal overshoot conditions.

CONCLUSION

- The efficacy of a purely adaptive controller on a spacecraft attitude system is shown.
- The control system uses pole-placement to assign the proportional gains of the adaptive controller while it uses derivative-free weight update laws to deal with uncertainties.
- The system is defined using modified Rodrigues parameters.
- A non-linear Hamiltonian-based system is used to develop the state-space to have more accurate representations.
- Simulations were carried out to show the robustness of the controller and its ability to adapt when major system dynamic properties are significantly and suddenly changed.
- In the future, the effectiveness and advantages of other modified derivative-free laws can be explored to expand their application in spacecraft control.

THANK YOU