Rotational dynamics of a diamagnetically levitating rotor

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Abstract

Rotational dynamics of a diamagnetically levitating rotor.

1 Levitating rotor

Figure. 1 shows the schematic of the diamagnetically levitating rotor with the geometric parameters.

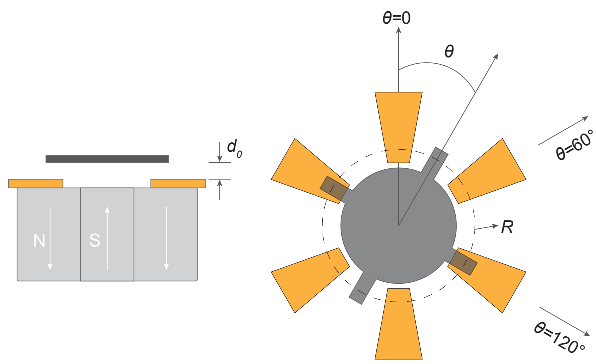

Figure 1: Schematic of diamagnetically levitating rotor.

2 Equation of motion

The rotational dynamics of a levitating rotor driven by 3-phase electrostatic forces can be described by the equation of motion:

$$I\ddot{\theta} + c\dot{\theta} = \tau_{ij}, \tag{1}$$

where:

- I is the moment of inertia of the rotor,
- θ is its angular position,
- τ_{ij} represents the driving torques generated by the 3-phase electrostatic forces,
- $c\dot{\theta}$ accounts for damping proportional to the angular velocity.

Driving Torque

The driving torque, τ_{ij}, is expressed as:

$$\tau_{ij} = \epsilon \frac{AV_j^2}{d_{ij}^2} R \frac{\sin(\theta_{ai} - \theta_{ej})}{|\sin(\theta_{ai} - \theta_{ej})|}, \tag{2}$$

where:

- ϵ is the permittivity of the medium,
- A is the effective overlap area between the electrodes and rotor arms,
- V_j is the voltage of the j-th phase in the 3-phase system,
- d_{ij} is the distance between the i-th rotor arm and the j-th electrode,
- R is the radius of the rotor,
- $\frac{\sin(\theta_{ai}-\theta_{ej})}{|\sin(\theta_{ai}-\theta_{ej})|}$ determines the direction of the torque based on the relative alignment of the rotor arm (θ_{ai}) and electrode (θ_{ej}).

Geometric Configuration

The four rotor arms are oriented as follows:

$$\theta_{a1} = \theta, \quad \theta_{a2} = \theta + \frac{\pi}{2}, \quad \theta_{a3} = \theta + \pi, \quad \theta_{a4} = \theta + \frac{3\pi}{2}, \tag{3}$$

indicating that the arms are evenly spaced 90° apart. The six electrodes are symmetrically distributed around a circle, with angular separations of $\pi/3$ radians, spanning 0 to 2π:

$$\theta_{e1} = 0, \quad \theta_{e2} = \frac{\pi}{3}, \quad \theta_{e3} = \frac{2\pi}{3}, \quad \theta_{e4} = \pi, \quad \theta_{e5} = \frac{4\pi}{3}, \quad \theta_{e6} = \frac{5\pi}{3}. \tag{4}$$

This configuration enables the generation of time-varying electrostatic forces between the rotor arms and electrodes, resulting in torques that drive the rotor's motion. The relative alignment of the arms and electrodes determines both the magnitude and direction of the torque.

Distance Between Rotor Arms and Electrodes

The distance, d_{ij}, between the i-th rotor arm and the j-th electrode is given by:

$$d_{ij}^2 = d_0^2 + \left(2R\sin\frac{\theta_{ai} - \theta_{ej}}{2}\right)^2, \tag{5}$$

where d_0 is the minimum separation between the rotor arms and electrodes.

3-Phase Voltages

The 3-phase voltages, V_j, are triangular waveforms with a 120° phase difference between each phase. These waveforms ensure continuous and balanced excitation to drive the rotor's motion:

$$V_1 = V_4 = V_0\left(2\left|2\left(ft \bmod 1\right) - 1\right| - 1\right), \tag{6}$$

$$V_2 = V_5 = V_0\left(2\left|2\left(\left(ft + \frac{120}{360}\right) \bmod 1\right) - 1\right| - 1\right), \tag{7}$$

$$V_3 = V_6 = V_0\left(2\left|2\left(\left(ft + \frac{240}{360}\right) \bmod 1\right) - 1\right| - 1\right). \tag{8}$$

Here:

- V_0 is the peak voltage amplitude,
- f is the frequency of the triangular waveform,
- t represents time,
- mod ensures periodicity.

The key parameters are listed in Table.

3 Free decay

In the case of free decay

$$I\ddot{\theta} + c\dot{\theta} = 0. \tag{9}$$

The solution is

$$\dot{\theta} = C_1 e^{-\gamma t}, \tag{10}$$

where $\gamma = \frac{c}{I}$.

References