Project 3 Report

In this part of the project, the following augmented state vector is used $\mathbf{X} = [x, y, \phi, b]^T$. This gives a new process model with time-varying bias:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \\ b(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \\ b(k) \end{bmatrix} + T \begin{bmatrix} v(k)\cos\big(\phi(k)\big) \\ v(k)\sin\big(\phi(k)\big) \\ \omega_b - b(k) \\ 0 \end{bmatrix}$$

Evaluating the Jacobian matrix of the process model:

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & -Tv(k)\sin\big(\phi(k)\big) & 0 \\ 0 & 1 & Tv(k)\cos\big(\phi(k)\big) & 0 \\ 0 & 0 & 1 & -T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The observation model is:

$$h(\mathbf{X}) = \begin{bmatrix} \sqrt{(x_a - x)^2 + (y_a - y)^2} \\ \tan^{-1}\left(\frac{y_a - y}{x_a - x}\right) - \phi + \frac{\pi}{2} \end{bmatrix}$$

Evaluating the Jacobian matrix of the observation model:

$$\mathbf{H} = \begin{bmatrix} -\frac{x_a - x}{\sqrt{(x_a - x)^2 + (y_a - y)^2}} & -\frac{y_a - y}{\sqrt{(x_a - x)^2 + (y_a - y)^2}} & 0 & 0 \\ \frac{y_a - y}{(x_a - x)^2 + (y_a - y)^2} & -\frac{x_a - x}{(x_a - x)^2 + (y_a - y)^2} & -1 & 0 \end{bmatrix}$$

Initializing the covariance matrix P:

$$\mathbf{P}(0|0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{2\pi}{180}\right)^2 \end{bmatrix}$$

Evaluating the Jacobian matrix at the expected values of the involved variables in the process model:

$$\mathbf{J_u} = \begin{bmatrix} T\cos\left(\hat{\phi}(k|k)\right) & 0 \\ T\sin\left(\hat{\phi}(k|k)\right) & 0 \\ 0 & T \\ 0 & 0 \end{bmatrix}$$

The measurement noise matrix is given by:

$$\mathbf{P_u} = \begin{bmatrix} 0.3^2 & 0 \\ 0 & \left(\frac{1.7\pi}{180}\right)^2 \end{bmatrix}$$

Hence, the covariance measurement noise matrix is given by:

$$\mathbf{Q_u} = \mathbf{J_u} \cdot \mathbf{P_u} \cdot \mathbf{J_u}^T$$

The additional discrepancy instantaneous translation time varying error matrix is given by:

$$\mathbf{Q_2} = \begin{bmatrix} (0.03T)^2 & 0 & 0 & 0 \\ 0 & (0.03T)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{600\pi T}{180}\right)^2 \end{bmatrix}$$

Finally, the covariance matrix is given by:

$$\mathbf{Q} = \mathbf{Q_u} + \mathbf{Q_2}$$

Using the above equations, the estimated bias can be determined and is shown in Figure 1 as a function of time below:

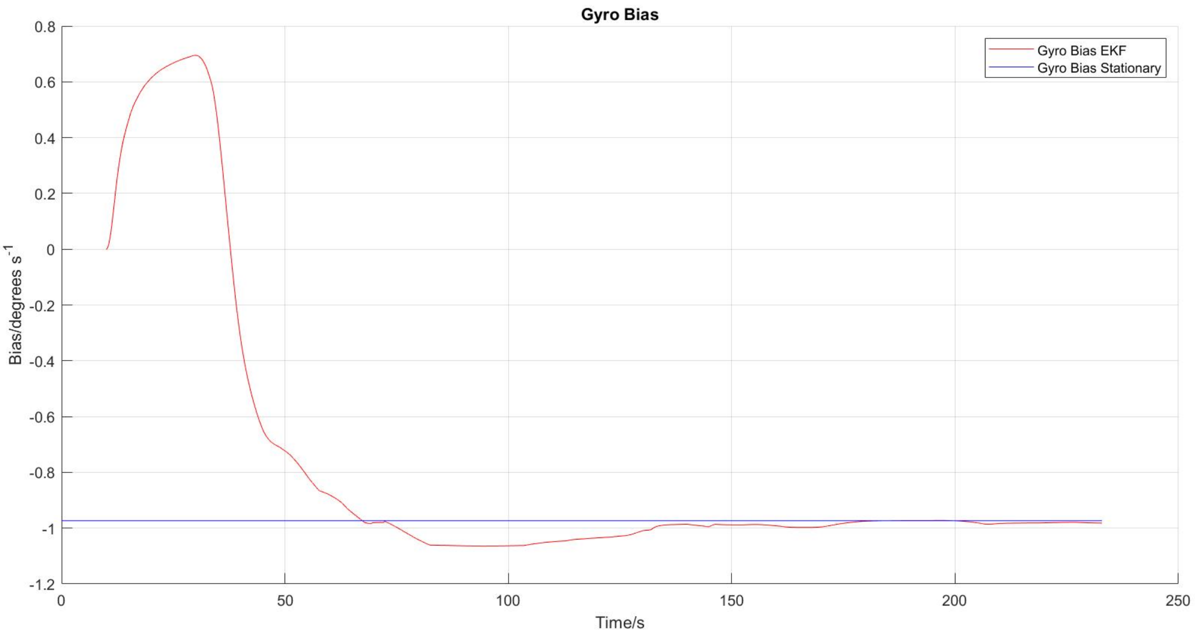

Figure 1: Gyro Bias against Time

As seen from the graph, the bias comes hovers around the real data of -0.97 degrees per second, and hence it matches with the value of the “offline” approach.