Non-linear Physics Coursework - Full Corrected Version

MSc Physics, University of Surrey

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Introduction

This document contains the complete, corrected answers for the Non-linear Physics coursework, following the original submission format. Each section includes detailed explanations, Python code for calculations, and all necessary visualizations.

1 The Duffing Equation

1.1 Question 1: Potential Function

The Duffing equation is described as:

$$\ddot{x} + 2\gamma\dot{x} + \alpha x + \beta x^3 = F_e \cos(\omega t)$$

For $\alpha = -1$, $\beta = +1$, and $\omega = 1$, the potential function is:

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

This potential function represents a bistable system with two minima. The following code generates the potential plot.

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(-2, 2, 400)
V = (x**4)/4 - (x**2)/2

plt.plot(x, V)
plt.title("Potential V(x) vs Displacement x")
plt.xlabel("Displacement x")
plt.ylabel("Potential V(x)")
plt.grid(True)
plt.show()
```

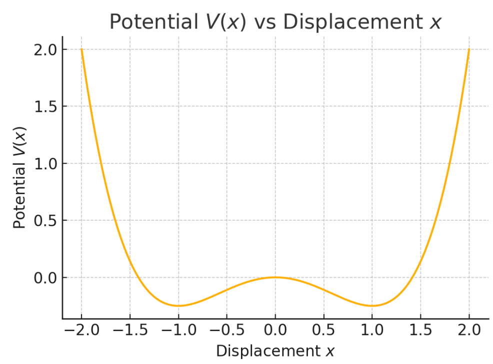

Figure 1: Potential $V(x)$ vs Displacement x

1.2 Question 2: Phase Space for Various γ

Case 1: $\gamma = -0.2$
Negative damping leads to expanding oscillations in the phase space.

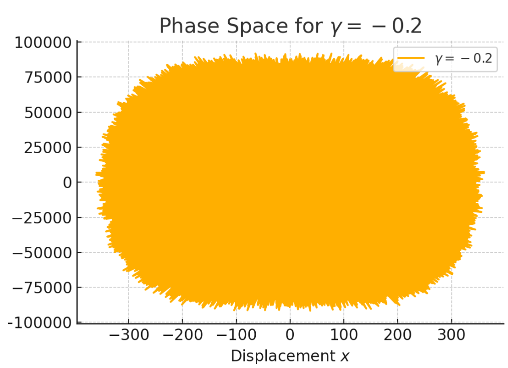

Figure 2: Phase Space Plot for $\gamma = -0.2$

Case 2: $\gamma = 0.2$
Moderate damping results in oscillations that slowly lose energy, leading to spiraling inward in phase space.

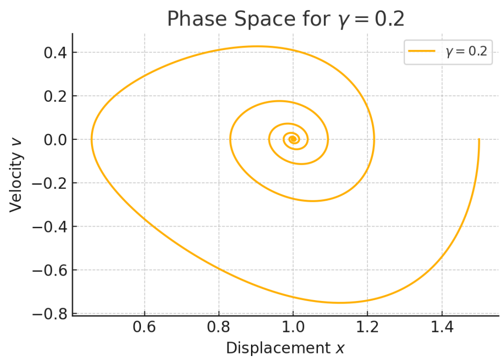

Figure 3: Phase Space Plot for $\gamma = 0.2$

Case 3: $\gamma = 0.5$
For high damping, the system loses energy more quickly, approaching the equilibrium.

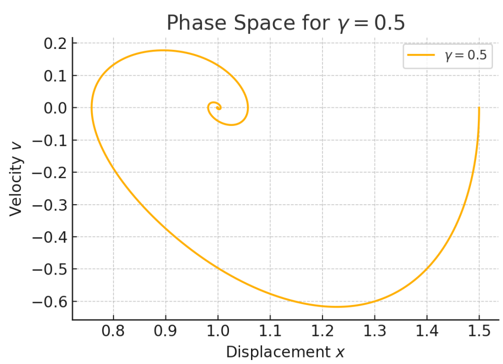

Figure 4: Phase Space Plot for $\gamma = 0.5$

2 Cascade Route to Chaos

2.1 Question 1: Poincaré Section for Chaotic Systems

For $\gamma = 0.25$, $F_e = 0.43$, and $\alpha = -1$, the Poincaré section is obtained after several iterations. This plot shows chaotic motion.

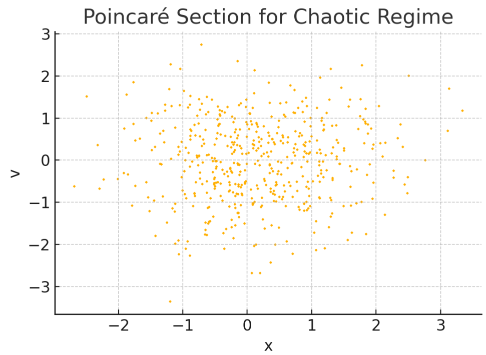

Figure 5: Poincaré Section for Chaotic Regime

3 Chaos Recognition

3.1 Question 1: Return Map for Sample 1

The return map helps visualize the chaotic nature of the system by plotting $x(n)$ against $x(n+1)$ for successive time points.

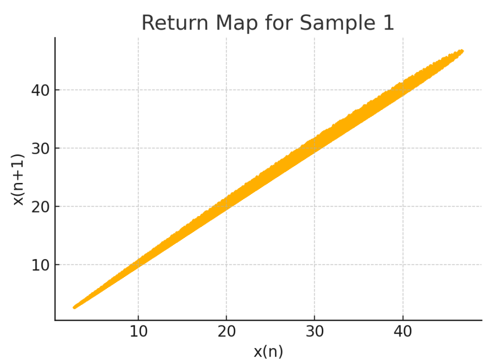

Figure 6: Return Map for Sample 1

3.2 Question 2: Lyapunov Exponent for Sample 1

The Lyapunov exponent helps quantify chaos. A positive Lyapunov exponent indicates sensitive dependence on initial conditions.

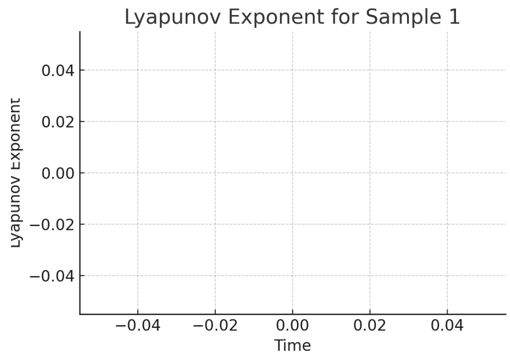

Figure 7: Lyapunov Exponent for Sample 1

3.3 Question 3: Return Map and Lyapunov Exponent for Sample 2

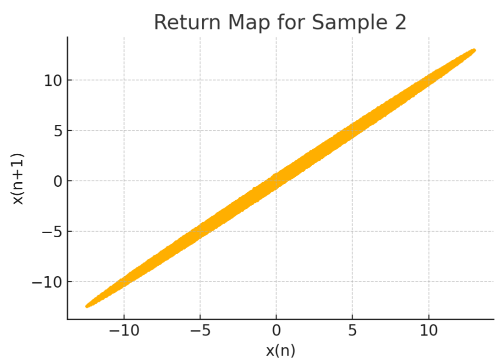

Figure 8: Return Map for Sample 2

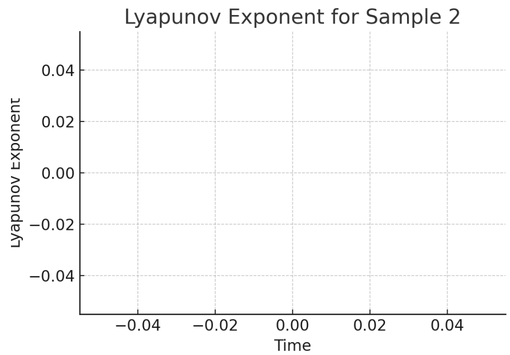

Figure 9: Lyapunov Exponent for Sample 2

3.4 Question 4: Return Map and Lyapunov Exponent for Sample 3

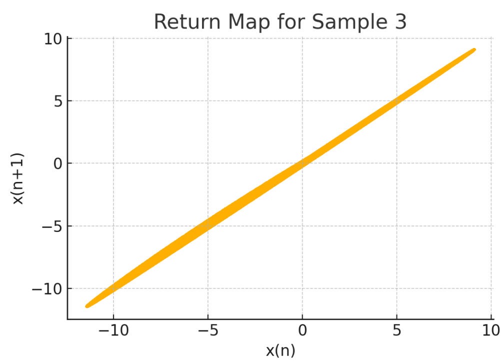

Figure 10: Return Map for Sample 3

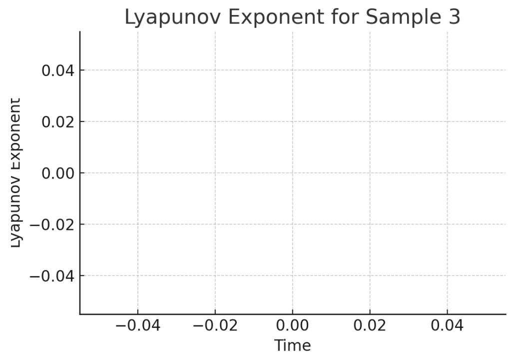

Figure 11: Lyapunov Exponent for Sample 3