Ayudantía 11

Reinforcement Learning

Sarah Everke - Daniel Florea

Reinforcement Learning

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El problema se modela como un MDP

Markov decision process

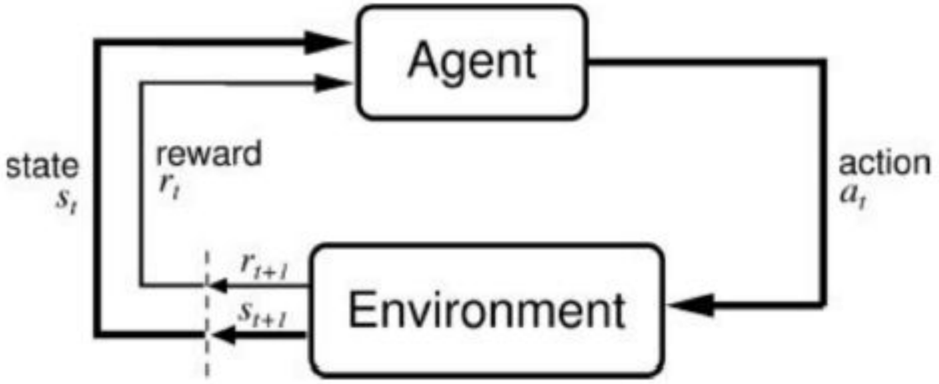
MDP se compone:

- Set finito de estados: s_1, …., s_n
- Set de recompensas: r_1, …., r_n
- Set de acciones: a_1, …., a_n
- Set de probabilidades de transición entre estados:

$$P_{ij}^{k} = P(s_j | s_i, a_k)$$

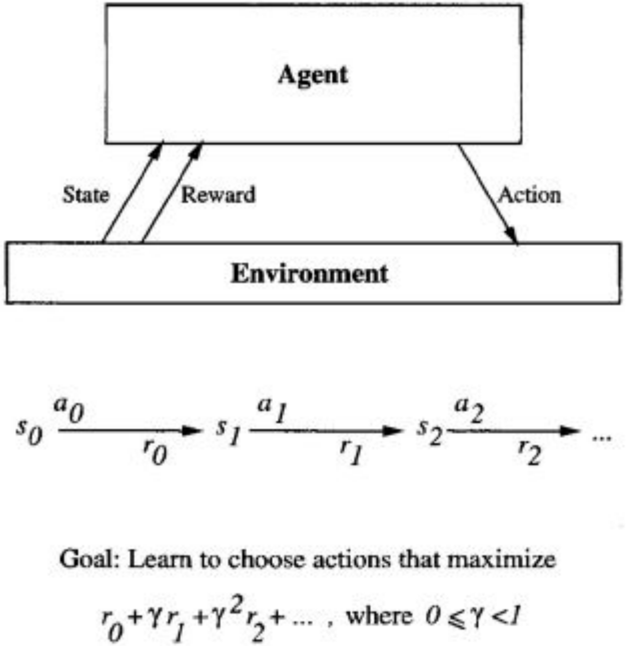
Reinforcement Learning

Se busca **aprender a comportarse en el entorno** (aprender una política).



Factor de descuento

Determina la importancia de las **recompensas futuras**



Value function

$$V(s) = E\left\{\sum_{t=0}^{\infty} \gamma^t r_t^\pi\right\}$$

Recompensa total que el agente recibe al iniciar en el estado *s* y seguir la política π

Política de recompensa π

Value function

- Se busca encontrar la **política π que maximice la *value function***:

$$V^*(s) = \max_{\pi} E\left\{\sum_{t=0}^{\infty} \gamma^t r_t^{\pi}\right\}$$

Value function

- Se utilizan las ecuaciones de Bellman:

$$V^*(s) = \max_a \left\{ R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^*(s') \right\}, \quad \forall s \in S$$

Recompensa del estado actual

Recompensa futura esperada

Value function

- Interesan las acciones que maximizan la función

$$V^*(s) = \arg\max_a \left\{ R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^*(s') \right\}, \quad \forall s \in S$$

Value iteration

Value iteration

```
Initialize V(s) arbitrarily
loop until policy good enough
     loop for s ∈ S
          loop for a ∈ A
               Q(s,a) := R(s,a) + γ Σ_{s'∈S} T(s,a,s') V̂(s')
          end loop
          V̂(s) := max_a Q(s,a)
     end loop
end loop
return{V̂(s)}
```

Where:

$$Q(s,a) := R(s,a) + \gamma \sum_{s' \in S} T(s,a,s')\hat{V}(s')$$

$$\hat{V}(s) := \max_a Q(s,a)$$

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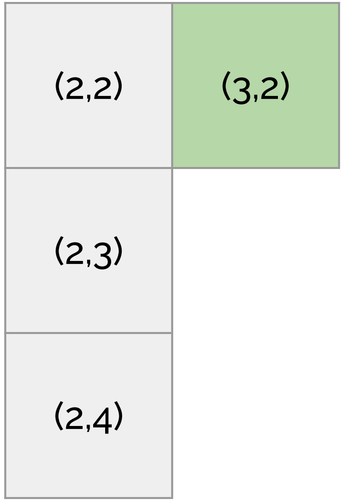

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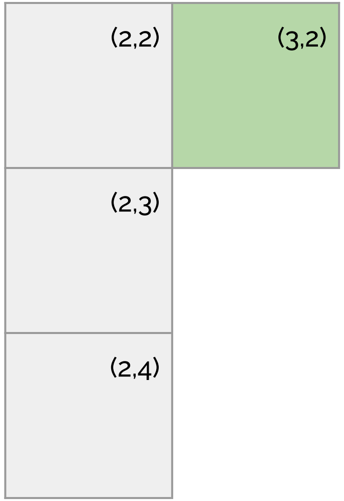

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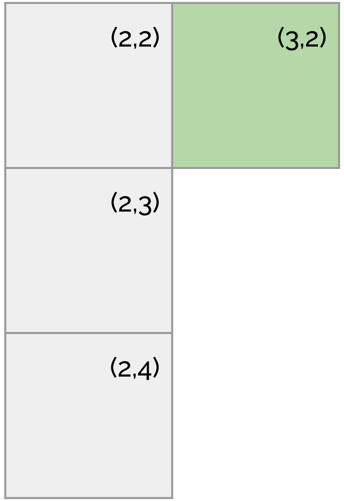

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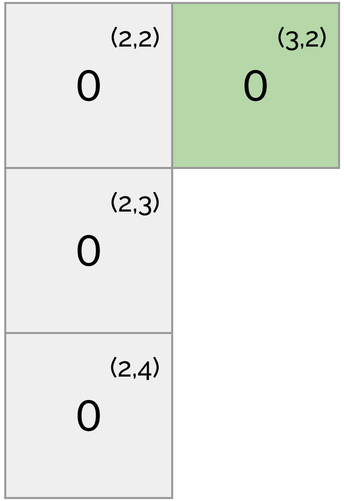

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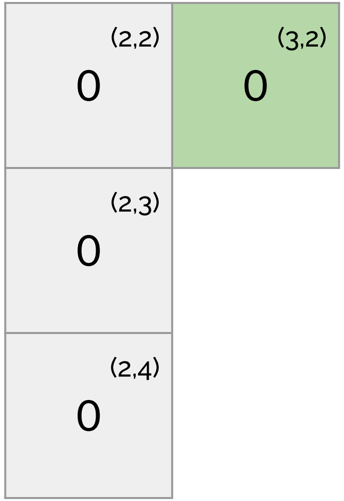

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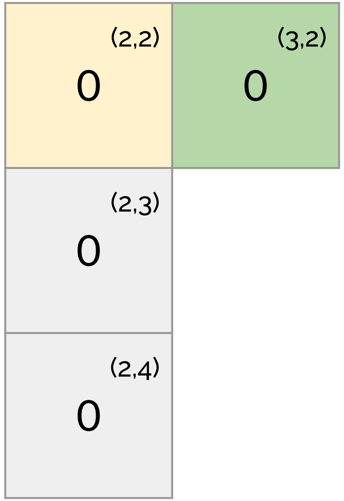

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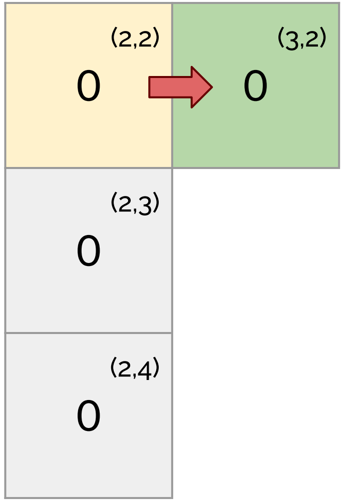

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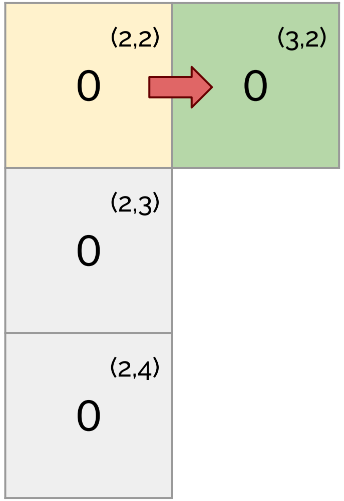

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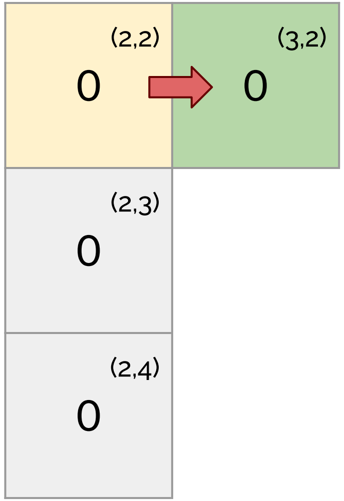
$$Q(s,a) := R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') \hat{V}(s')$$

$$Q([2,2],r) = R([2,2],r) + \gamma \cdot T([2,2],r,[3,2]) \cdot V_0([3,2])$$



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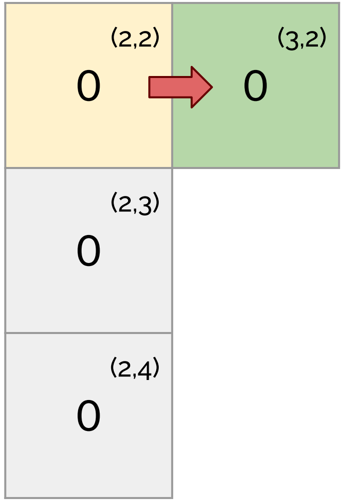
$$\begin{aligned} Q([2,2],r) &= R([2,2],r) + \gamma \cdot T([2,2],r,[3,2]) \cdot V_0([3,2]) \\ &= 1 + 0{,}9 \cdot 0{,}25 \cdot 0 \end{aligned}$$



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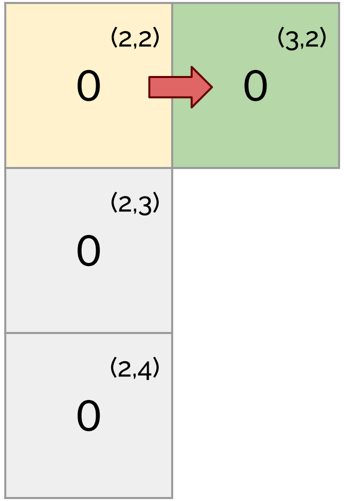

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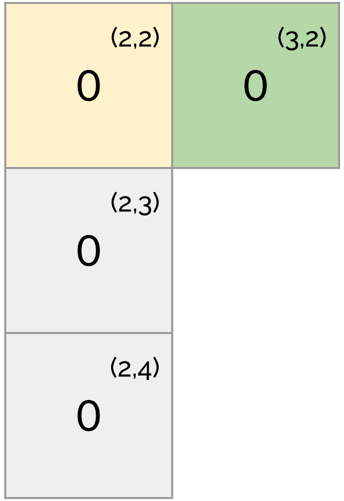

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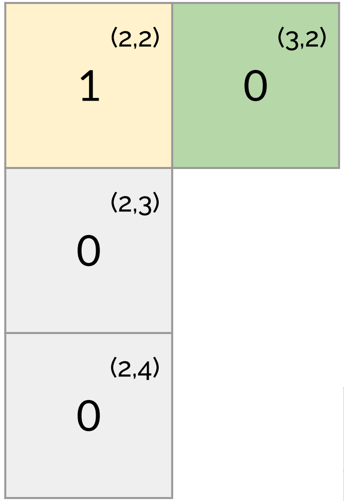
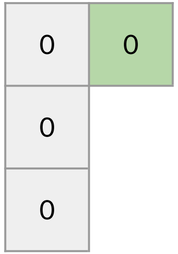


t = 0



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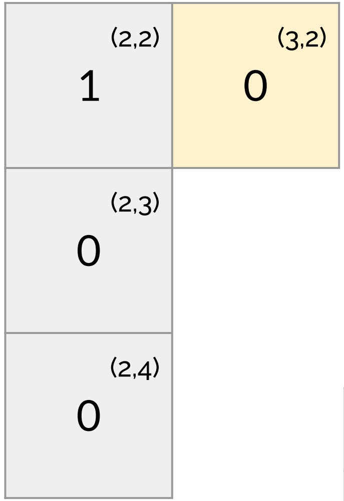

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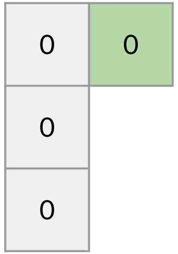

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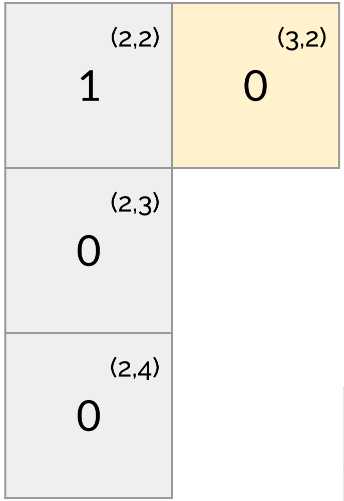

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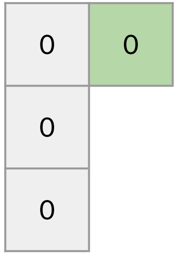

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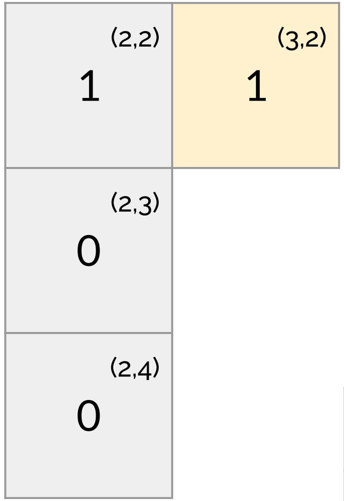

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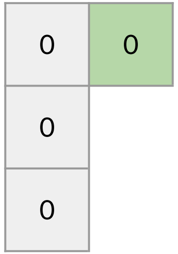

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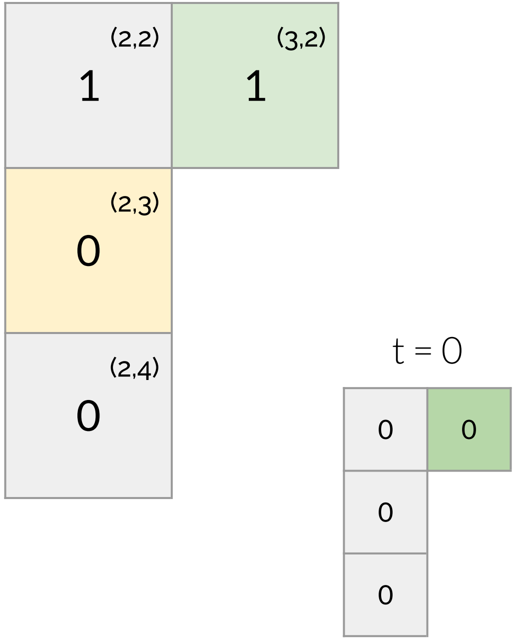

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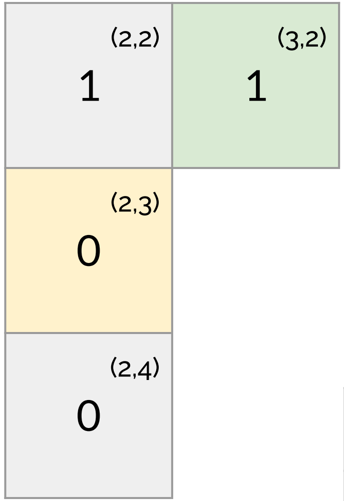

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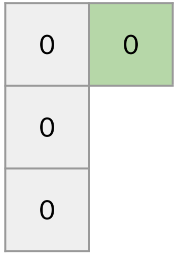

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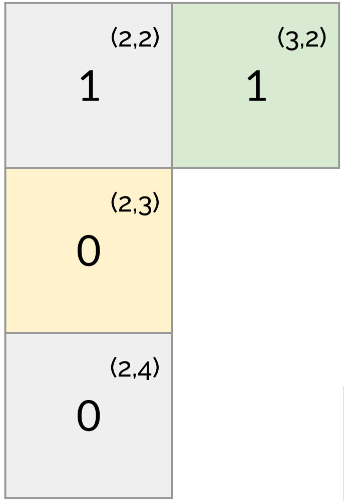

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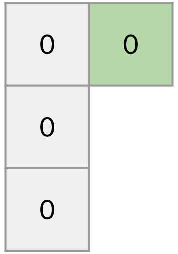

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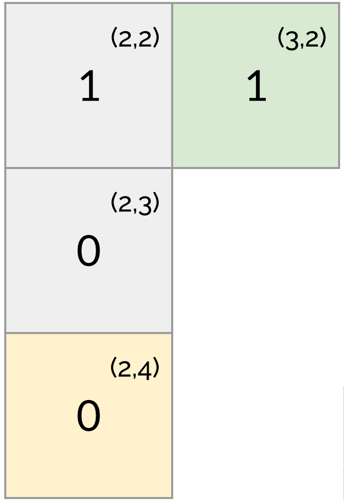

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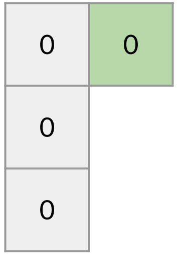

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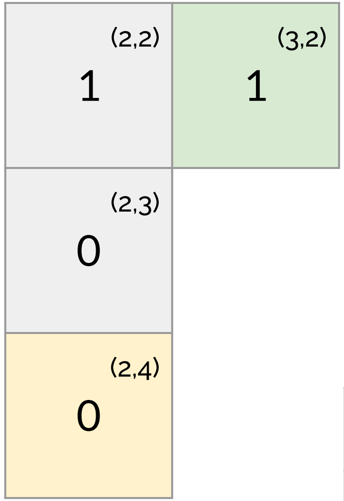

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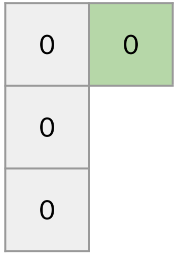

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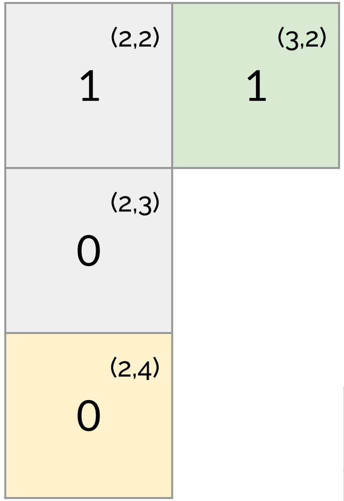

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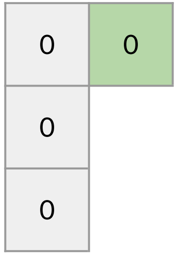

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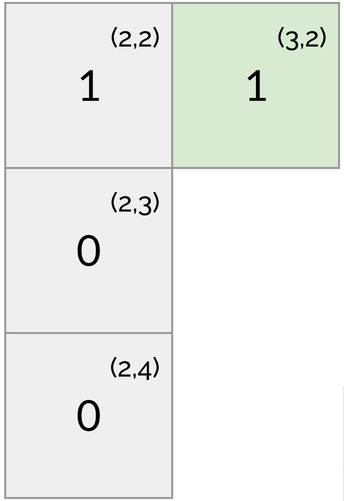

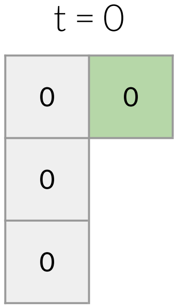

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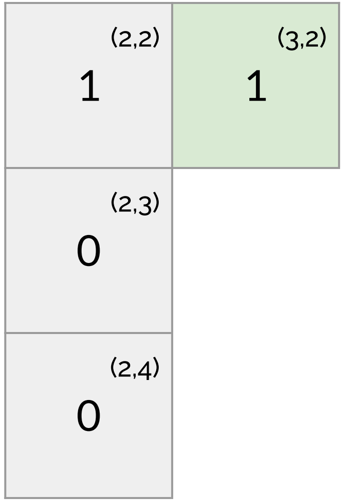
    end loop
end loop
return{V̂(s)}
```

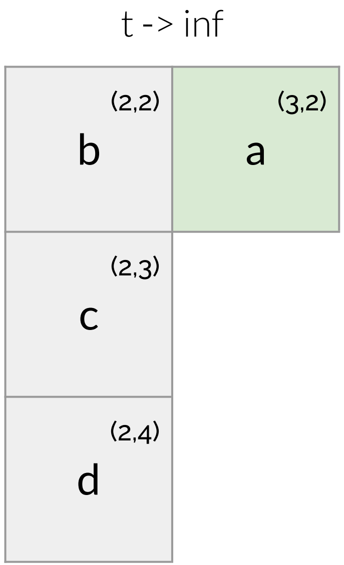
t = 1



Value iteration

(...)

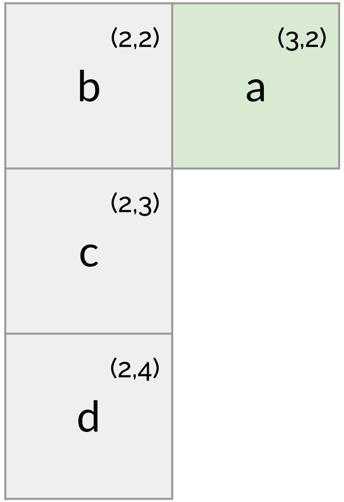
Value iteration



¿ Qué relación existirá entre a, b, c y d?

Value iteration

t -> inf

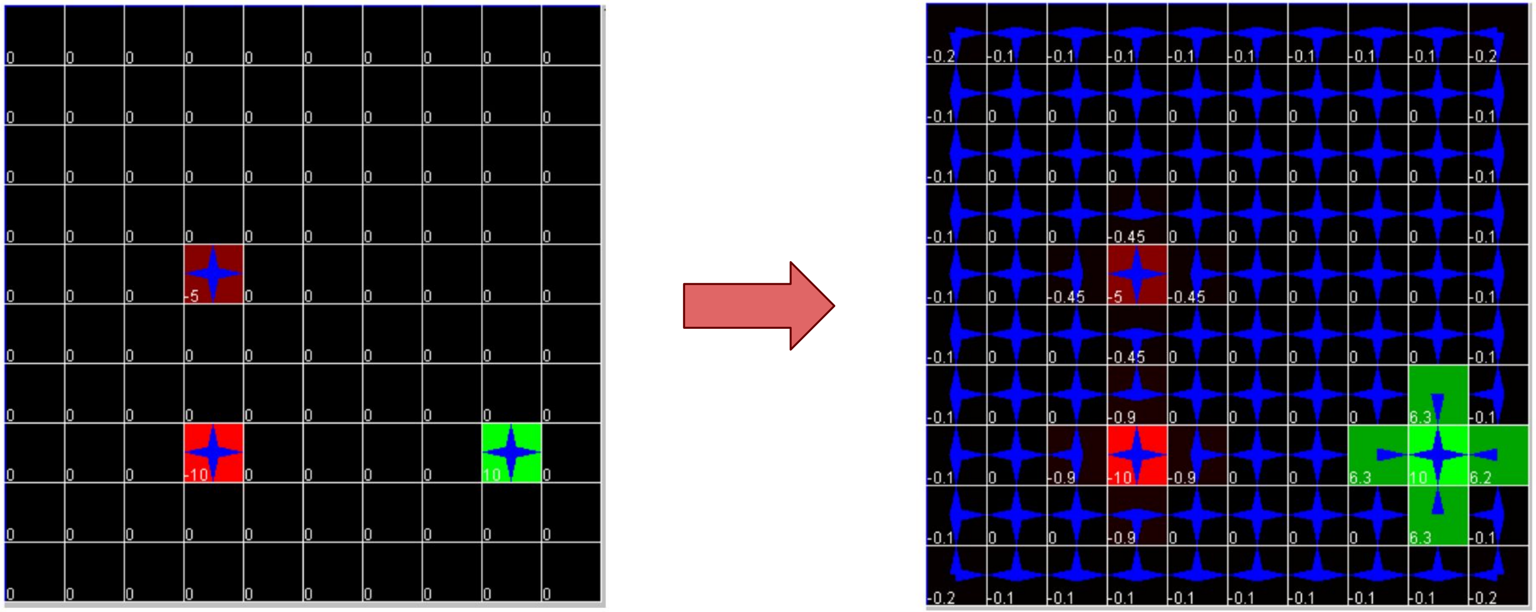


Para este caso en particular

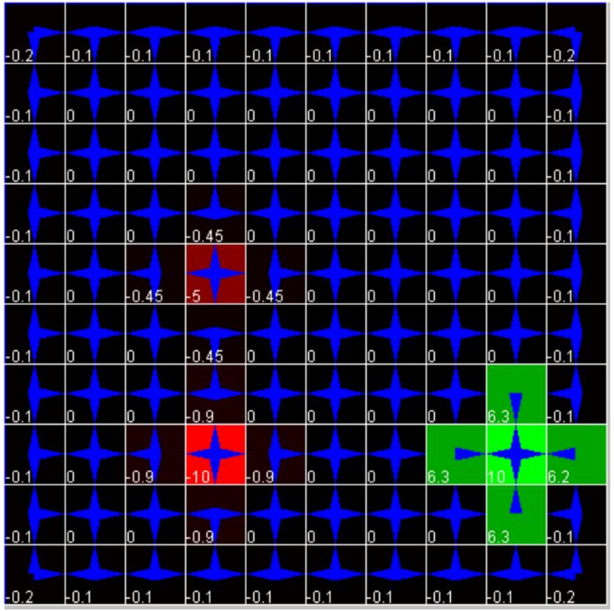
$$a > b > c > d$$

Value iteration

Iteration 1

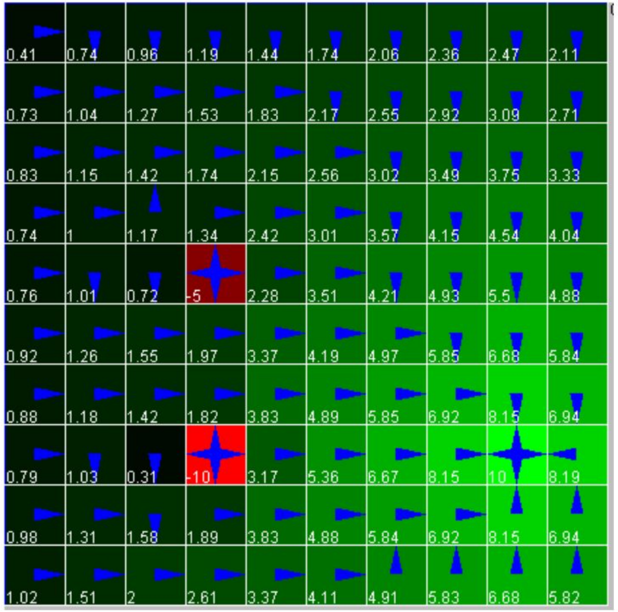


Value iteration



(...)

Iteration 10



Policy iteration

Policy iteration

```
Choose an arbitray policy π′
Loop
        π := π′
        Compute value function of policy π:
              #solve linear equations
              V_π(s) := R(s, π(s)) + γ ∑_{s′∈S} T(s, π(s), s′) V_π(s′)
        Improve the policy at each state
              π′(s) := argmax_a (R(s, a) + γ ∑_{s′∈S} T(s, a, s′) V_π(s′))
until π = π′
```

Choose an arbitray policy π'

Loop

  $\pi := \pi'$

  Compute value function of policy π:

    #solve linear equations

    $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

  Improve the policy at each state

    $\pi'(s) := \arg\max_a \left(R(s, a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s')\right)$

until $\pi = \pi'$

Policy iteration

Choose an arbitray policy π'

Loop

    $\pi := \pi'$

    Compute value function of policy π:

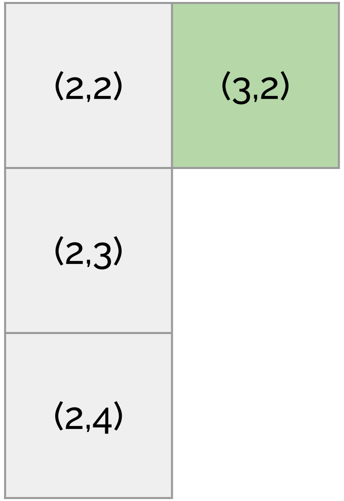
        #solve linear equations

        $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

    Improve the policy at each state

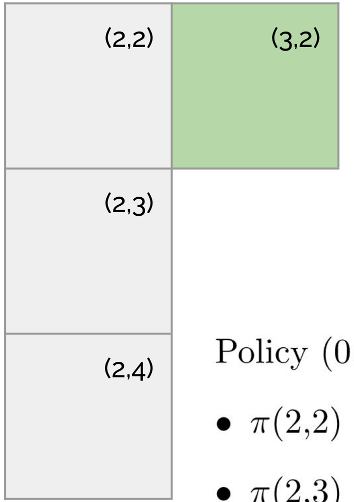
        $\pi'(s) := \arg\max_a \left(R(s, a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s')\right)$

until $\pi = \pi'$



Policy iteration

```
Choose an arbitray policy π′
Loop
        π := π′
        Compute value function of policy π:
              #solve linear equations
              V_π(s) := R(s, π(s)) + γ Σ_{s′∈S} T(s, π(s), s′) V_π(s′)
        Improve the policy at each state
              π′(s) := argmax_a (R(s, a) + γ Σ_{s′∈S} T(s, a, s′) V_π(s′))
until π = π′
```



Policy (0.3 noise):

- $\pi(2{,}2) = \text{Right}$
- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

Choose an arbitray policy π'

Loop

$\pi := \pi'$

Compute value function of policy π:

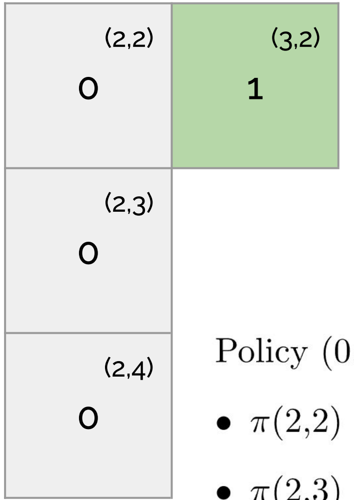
#solve linear equations

$V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

Improve the policy at each state

$\pi'(s) := \arg\max_a \left(R(s, a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s')\right)$

until $\pi = \pi'$



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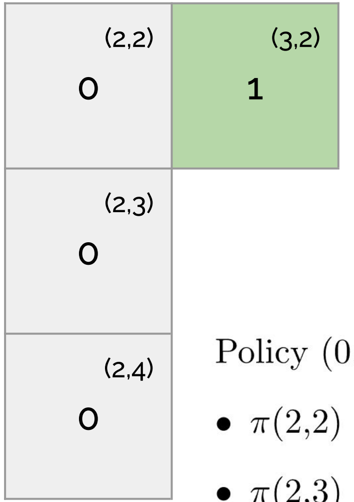
- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

Choose an arbitray policy π'
Loop
    $\pi := \pi'$
    Compute value function of policy π:
        #solve linear equations
        $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$
    Improve the policy at each state
        $\pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s')\right)$
until $\pi = \pi'$



Policy (0.3 noise):

- $\pi(2{,}2)$ = Right
- $\pi(2{,}3)$ = Right
- $\pi(2{,}4)$ = Right
- $\pi(3{,}2)$ = Stay

Policy iteration

Choose an arbitray policy π'

Loop

  $\pi := \pi'$

  Compute value function of policy π:

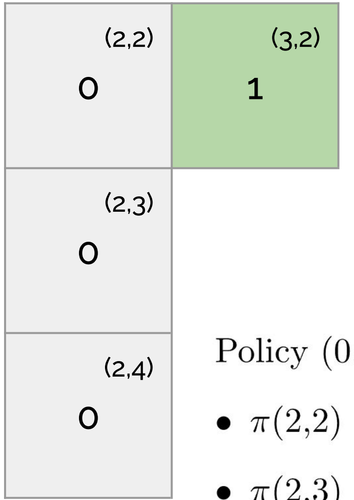
    #solve linear equations

    $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

  Improve the policy at each state

    $\pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s') \right)$

until $\pi = \pi'$



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- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

Choose an arbitray policy π'

Loop

  $\pi := \pi'$

  Compute value function of policy π:

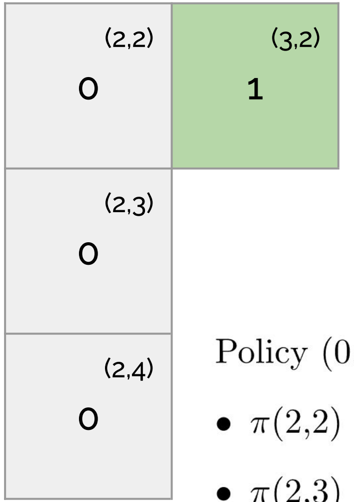
    #solve linear equations

    $V_\pi(s) := R(s,\pi(s)) + \gamma \sum_{s' \in S} T(s,\pi(s),s') V_\pi(s')$

  Improve the policy at each state

    $\pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s') \right)$

until $\pi = \pi'$



Policy (0.3 noise):

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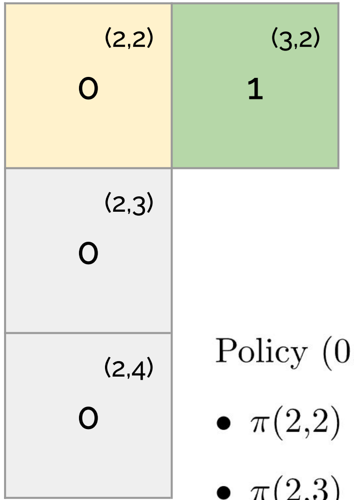
- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

$$\boxed{\gamma = 0.9}$$

$$V_\pi(s) := R(s,\pi(s)) + \gamma \sum_{s' \in S} T(s,\pi(s),s')V_\pi(s')$$

$$\begin{aligned}
V([2,2]) = R([2,2],\pi) + \gamma(& \\
& T([2,2],\pi,[3,2]) \cdot V([3,2]) + \\
& T([2,2],\pi,[2,2]) \cdot V([2,2]) + \\
& T([2,2],\pi,[2,3]) \cdot V([2,3]) \\
&)
\end{aligned}$$



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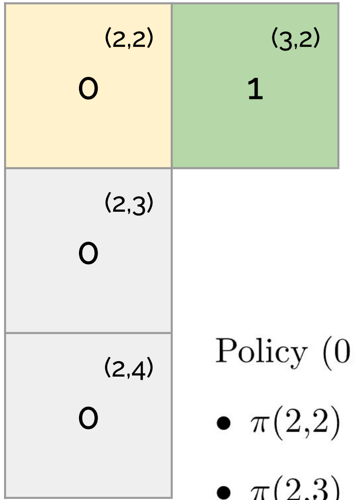
Policy iteration

$$\boxed{\gamma = 0.9}$$

$$
\begin{aligned}
V([2,2]) = R([2,2],\pi) + \gamma(&\\
&T([2,2],\pi,[3,2]) \cdot V([3,2]) + \\
&T([2,2],\pi,[2,2]) \cdot V([2,2]) + \\
&T([2,2],\pi,[2,3]) \cdot V([2,3]) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
V([2,2]) = 1 + \gamma(&\\
&0{,}7 \cdot V([3,2]) + \\
&0{,}2 \cdot V([2,2]) + \\
&0{,}1 \cdot V([2,3]) \\
&)
\end{aligned}
$$



Policy (0.3 noise):

- $\pi(2{,}2)$ = Right
- $\pi(2{,}3)$ = Right
- $\pi(2{,}4)$ = Right
- $\pi(3{,}2)$ = Stay

Policy iteration

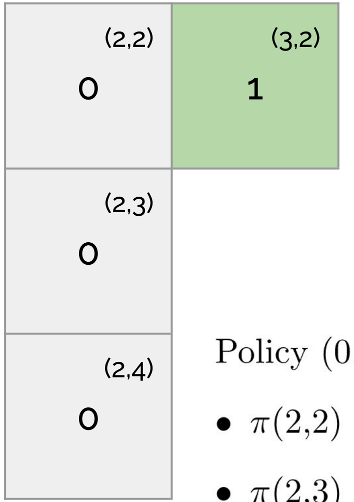
$$\gamma = 0.9$$

$$V([2,2]) = 1 + \gamma(\; 0{,}7 \cdot V([3,2]) + 0{,}2 \cdot V([2,2]) + 0{,}1 \cdot V([2,3]) \;)$$

$$V([3,2]) = 1 + \gamma(\; 1 \cdot ([3,2]) \;)$$

$$V([2,3]) = 0 + \gamma(\; 0{,}8 \cdot V([2,3]) + 0{,}1 \cdot V([2,2]) + 0{,}1 \cdot V([2,4]) \;)$$

$$V([2,4]) = 0 + \gamma(\; 0{,}9 \cdot V([2,4]) + 0{,}1 \cdot V([2,3]) \;)$$



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- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

$$\gamma = 0.9$$

$$
\begin{aligned}
V([2,2]) = 1 + \gamma(& \\
&0{,}7 \cdot V([3,2]) + \\
&0{,}2 \cdot V([2,2]) + \\
&0{,}1 \cdot V([2,3]) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
V([2,3]) = 0 + \gamma(& \\
&0{,}8 \cdot V([2,3]) + \\
&0{,}1 \cdot V([2,2]) + \\
&0{,}1 \cdot V([2,4]) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
V([2,4]) = 0 + \gamma(& \\
&0{,}9 \cdot V([2,4]) + \\
&0{,}1 \cdot V([2,3]) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
V([3,2]) = 1 + \gamma(& \\
&1 \cdot ([3,2]) \\
&)
\end{aligned}
$$

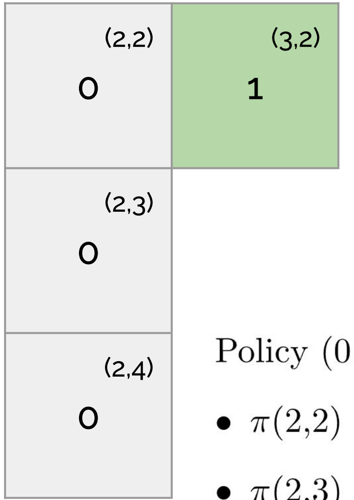
solve

$$a = 1 + 0.9\,(0.2\,a + 0.1\,b + 0.7\,d)$$

$$b = 0.9\,(0.1\,a + 0.8\,b + 0.1\,c)$$

$$c = 0.9\,(0.1\,b + 0.9\,c)$$

$$d = 1 + 0.9\,d$$



Policy (0.3 noise):

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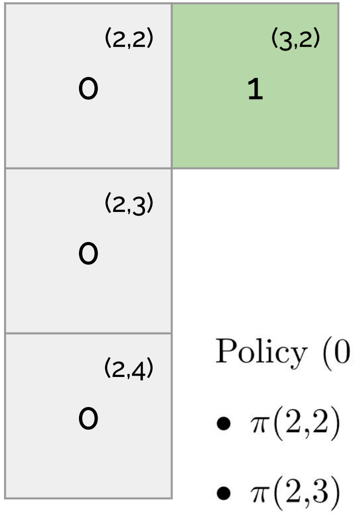
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

$$\boxed{\gamma = 0.9}$$

$$V([2,2]) = 9{,}289$$
$$V([2,3]) = 3{,}522$$
$$V([2,4]) = 1{,}668$$
$$V([3,2]) = 10$$



Policy (0.3 noise):

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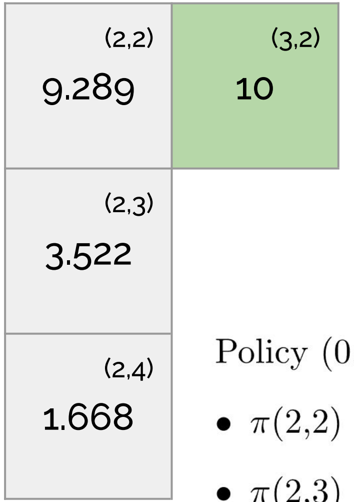
Policy iteration

$$\boxed{\gamma = 0.9}$$



$$V([2,2]) = 9{,}289$$
$$V([2,3]) = 3{,}522$$
$$V([2,4]) = 1{,}668$$
$$V([3,2]) = 10$$

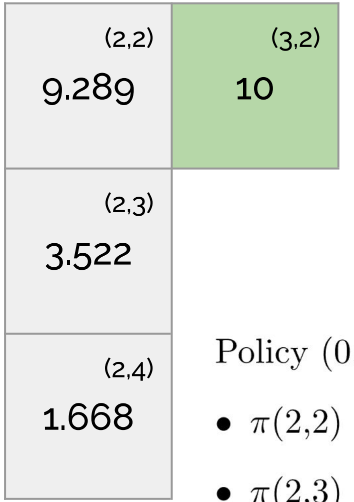
Policy (0.3 noise):

- $\pi(2,2)$ = Right
- $\pi(2,3)$ = Right
- $\pi(2,4)$ = Right
- $\pi(3,2)$ = Stay

Policy iteration

$\boxed{\gamma = 0.9}$

Choose an arbitray policy π'
Loop
    $\pi := \pi'$
    Compute value function of policy π:
        #solve linear equations
        $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$
    Improve the policy at each state
        $\pi'(s) := \arg\max_a \left(R(s, a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s') \right)$
until $\pi = \pi'$



Policy (0.3 noise):

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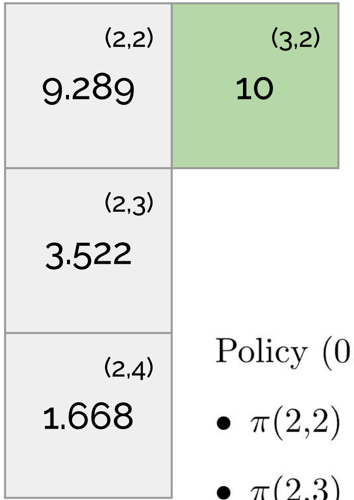
- $\pi(2{,}3) =$ Right
- $\pi(2{,}4) =$ Right
- $\pi(3{,}2) =$ Stay

Policy iteration

$$\gamma = 0.9$$



Choose an arbitray policy π'

Loop

$\quad \pi := \pi'$

$\quad$ Compute value function of policy π:

$\quad\quad$ #solve linear equations

$\quad\quad V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

$\quad$ Improve the policy at each state

$\quad\quad \pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s') \right)$

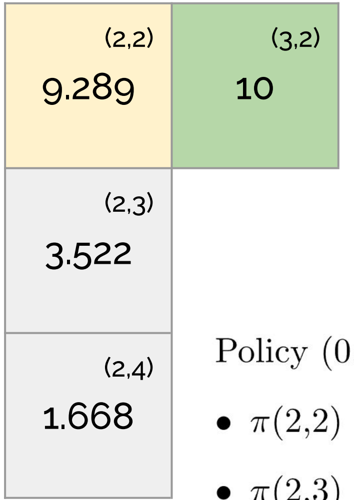
until $\pi = \pi'$

Policy (0.3 noise):

- $\pi(2,2)$ = Right
- $\pi(2,3)$ = Right
- $\pi(2,4)$ = Right
- $\pi(3,2)$ = Stay

Policy iteration

$\boxed{\gamma = 0.9}$



Choose an arbitray policy π'

Loop

  $\pi := \pi'$

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    $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

  Improve the policy at each state

    $\pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s') \right)$

until $\pi = \pi'$

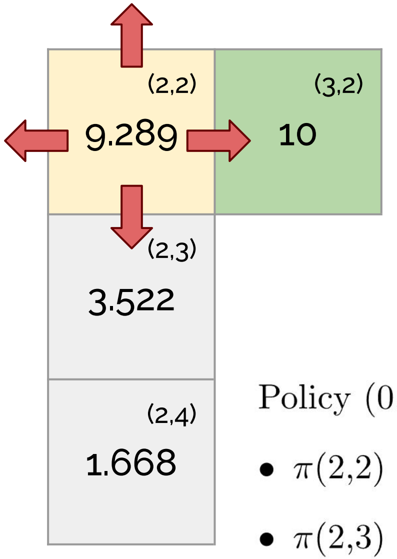
Policy (0.3 noise):

- $\pi(2,2)$ = Right
- $\pi(2,3)$ = Right
- $\pi(2,4)$ = Right
- $\pi(3,2)$ = Stay

Policy iteration

$\boxed{\gamma = 0.9}$

$R(s, a) + \gamma \sum_{s' \in S} T(s, a, s') V_\pi(s')$



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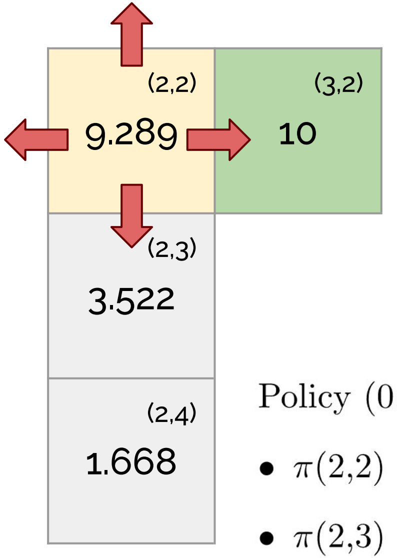
Policy iteration

$$\boxed{\gamma = 0.9}$$



$$R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s')$$

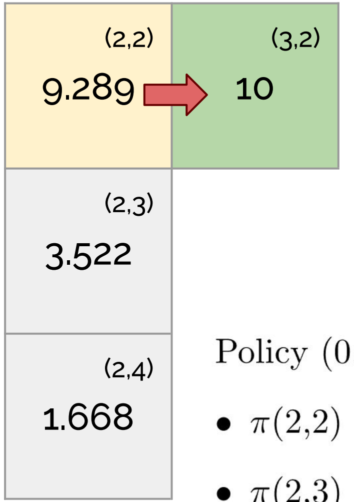
$$right \rightarrow 1 + 0{,}9 \cdot (0{,}7 \cdot 10 + 0{,}2 \cdot 9{,}289 + 0{,}1 \cdot 3{,}522) = 9{,}289$$
$$left \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 9{,}289 + 0{,}1 \cdot 10 + 0{,}1 \cdot 3{,}522) = 7{,}90506$$
$$up \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 9{,}289 + 0{,}1 \cdot 10 + 0{,}1 \cdot 3{,}522) = 7{,}90506$$
$$down \rightarrow 0 + 0{,}9 \cdot (0{,}7 \cdot 3{,}522 + 0{,}2 \cdot 9{,}289 + 0{,}1 \cdot 10) = 4{,}79088$$

Policy (0.3 noise):

- $\pi(2{,}2) = \text{Right}$
- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

$$\boxed{\gamma = 0.9}$$



$$\arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s')\right)$$

$$right \rightarrow 1 + 0{,}9 \cdot (0{,}7 \cdot 10 + 0{,}2 \cdot 9{,}289 + 0{,}1 \cdot 3{,}522) = 9{,}289$$

$$left \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 9{,}289 + 0{,}1 \cdot 10 + 0{,}1 \cdot 3{,}522) = 7{,}90506$$

$$up \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 9{,}289 + 0{,}1 \cdot 10 + 0{,}1 \cdot 3{,}522) = 7{,}90506$$

$$down \rightarrow 0 + 0{,}9 \cdot (0{,}7 \cdot 3{,}522 + 0{,}2 \cdot 9{,}289 + 0{,}1 \cdot 10) = 4{,}79088$$

Policy (0.3 noise):

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- $\pi(2{,}3) = \text{Right}$
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

Policy iteration

$$\boxed{\gamma = 0.9}$$

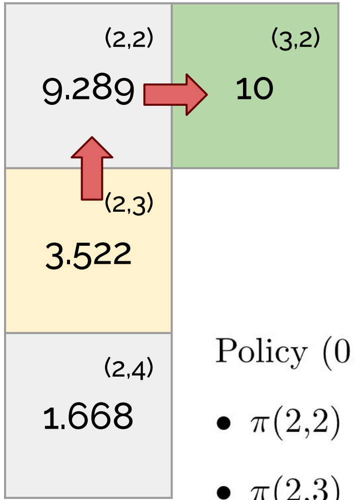
$$\arg\max_a \left(R(s,a) + \gamma \textstyle\sum_{s' \in S} T(s,a,s') V_\pi(s')\right)$$

$$right \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 3{,}522 + 0{,}1 \cdot 9{,}289 + 0{,}1 \cdot 1{,}668) = 3{,}52197$$

$$left \rightarrow 0 + 0{,}9 \cdot (0{,}8 \cdot 3{,}522 + 0{,}1 \cdot 9{,}289 + 0{,}1 \cdot 1{,}668) = 3{,}52197$$

$$up \rightarrow 1 + 0{,}9 \cdot (0{,}7 \cdot 9{,}289 + 0{,}2 \cdot 3{,}522 + 0{,}1 \cdot 1{,}668) = 7{,}63615$$

$$down \rightarrow 0 + 0{,}9 \cdot (0{,}7 \cdot 1{,}668 + 0{,}2 \cdot 3{,}522 + 0{,}1 \cdot 9{,}289) = 2{,}52081$$



Policy (0.3 noise):

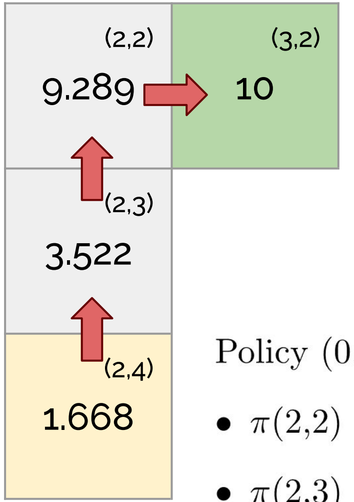
- $\pi(2{,}2)$ = Right
- $\pi(2{,}3)$ = Right
- $\pi(2{,}4)$ = Right
- $\pi(3{,}2)$ = Stay

Policy iteration

$$\boxed{\gamma = 0.9}$$

$$\arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s')\right)$$

$$right \rightarrow 0 + 0{,}9 \cdot (0{,}9 \cdot 1{,}668 + 0{,}1 \cdot 3{,}522) = 1{,}66806$$
$$left \rightarrow 0 + 0{,}9 \cdot (0{,}9 \cdot 1{,}668 + 0{,}1 \cdot 3{,}522) = 1{,}66806$$
$$up \rightarrow 1 + 0{,}9 \cdot (0{,}7 \cdot 3{,}522 + 0{,}3 \cdot 1{,}668) = 3{,}66922$$
$$down \rightarrow 0 + 0{,}9 \cdot (0{,}9 \cdot 1{,}668 + 0{,}1 \cdot 3{,}522) = 1{,}66806$$



Policy (0.3 noise):

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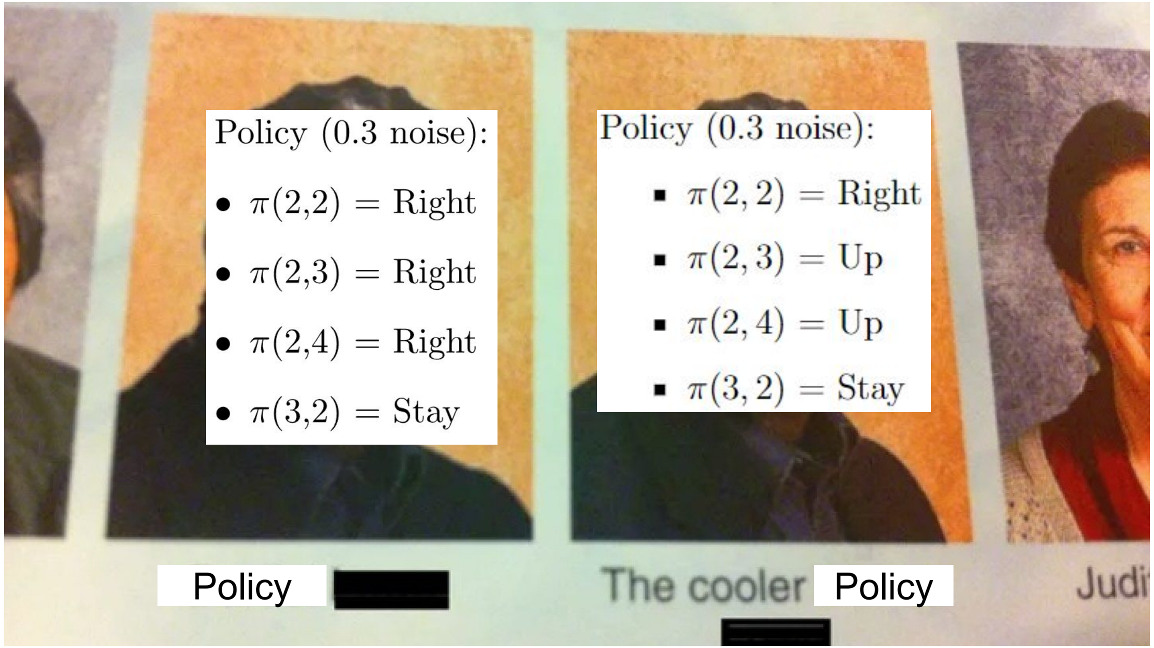
- $\pi(2{,}4) = \text{Right}$
- $\pi(3{,}2) = \text{Stay}$

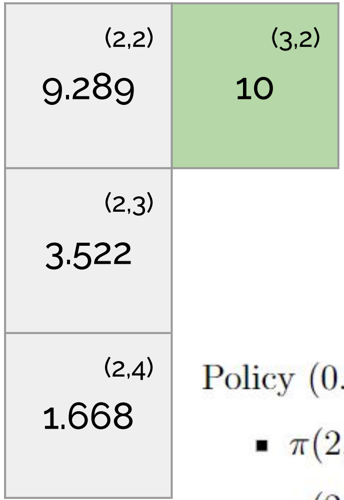
Policy iteration



Policy iteration

$\gamma = 0.9$

Choose an arbitray policy π'
Loop
    $\pi := \pi'$
    Compute value function of policy π:
        #solve linear equations
        $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$
    Improve the policy at each state
        $\pi'(s) := \arg\max_a \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_\pi(s') \right)$
until $\pi = \pi'$



Policy (0.3 noise):

- $\pi(2,2) = \text{Right}$
- $\pi(2,3) = \text{Up}$
- $\pi(2,4) = \text{Up}$
- $\pi(3,2) = \text{Stay}$

Policy iteration

$\boxed{\gamma = 0.9}$

Choose an arbitray policy π'

Loop

    $\pi := \pi'$

    Compute value function of policy π:

        #solve linear equations

        $V_\pi(s) := R(s, \pi(s)) + \gamma \sum_{s' \in S} T(s, \pi(s), s') V_\pi(s')$

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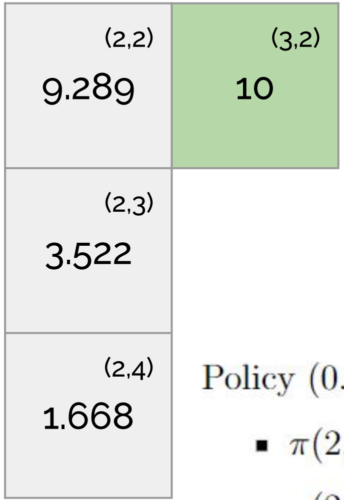


Policy (0.3 noise):

- $\pi(2,2) = \text{Right}$
- $\pi(2,3) = \text{Up}$
- $\pi(2,4) = \text{Up}$
- $\pi(3,2) = \text{Stay}$

Policy iteration

(...)

Q-Learning

Q-Learning

Q learning algorithm

For each s, a initialize the table entry $\hat{Q}(s, a)$ to zero.
Observe the current state s
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s, a)$ as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

- $s \leftarrow s'$

Q-Learning

$$Q(s,a) = r(s,a) + \gamma \arg\max_{a'} Q(s',a')$$

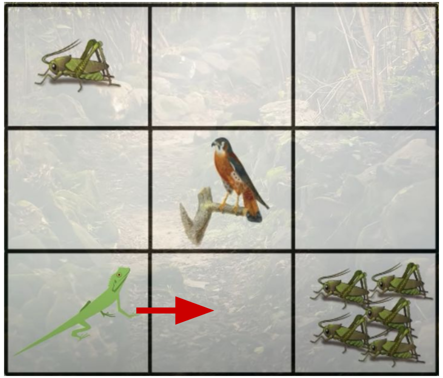
Recompensa adquirida tras realizar la acción *a* en *s*

Valor que entrega la mejor acción en el próximo estado

Estado	Recompensa
Un grillo	+1
Vacío	-1
Cinco grillos	+10
Pájaro	-10

Estado	Recompensa
Un grillo	+1
Vacío	-1
Cinco grillos	+10
Pájaro	-10

	Arriba	Abajo	Izq.	Der.
1 grillo	0	0	0	0
Vacío 1	0	0	0	0
Vacío 2	0	0	0	0
Vacío 3	0	0	0	0
Pájaro	0	0	0	0
Vacío 4	0	0	0	0
Vacío 5	0	0	0	0
Vacío 6	0	0	0	0
5 grillos	0	0	0	0



Estado	Recompensa
Un grillo	+1
Vacío	-1
Cinco grillos	+10
Pájaro	-10

$$Q(s, a) = \cancelto{-1}{r(s, a)} + \gamma \arg \cancelto{0}{\max_{a'} Q(s', a')}$$

Estado	Recompensa
Un grillo	+1
Vacío	-1
Cinco grillos	+10
Pájaro	-10

	Arriba	Abajo	Izq.	Der.
1 grillo	0	0	0	0
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Vacío 3	0	0	0	0
Pájaro	0	0	0	0
Vacío 4	0	0	0	0
Vacío 5	0	0	0	-1
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Q-Learning

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ε- greedy

- ϵ-greedy exploration policy:
  - With probability $1 - \epsilon$:
    - Choose the current optimal action: $\arg\max_a \hat{Q}(s, a)$.
  - With probability ϵ:
    - Select a random action.

Q-Learning

$$Q(s,a) = (1-\alpha)Q(s,a)+\alpha(r(s,a)+\gamma \arg\max_{a'} Q(s',a'))$$

Cuánta importancia le doy a la experiencia nueva

Ayudantía 11

Reinforcement Learning

Sarah Everke - Daniel Florea