Module
13

Working Stress Method

Lesson
35

Numerical Problems

Instructional Objectives:

At the end of this lesson, the student should be able to:

- employ the equations established for the analysis and design of singly and doubly-reinforced rectangular beams,

- use tables of SP-16 for the analysis and design of singly and doubly-reinforced rectangular beams,

- understand the economy in the design by limit state of collapse.

13.35.1 Introduction

Lesson 34 explains the equations for the analysis and design of singly and doubly-reinforced rectangular beams. The applications of the equations are illustrated in this lesson through the solutions of several numerical problems. Direct computation method and use of tables of SP-16 are the two approaches for the analysis and design of singly and doubly-reinforced rectangular beams. In direct computation method, the derived equations are employed directly, while the use of tables of SP-16 gives the results quickly with several alternatives avoiding tedious calculations. Some of the numerical problems are solved using both methods. Problems having the same width and effective depth but of different grades of steel are solved to compare the results. Moreover, problems of singly and doubly-reinforced rectangular beams, solved earlier by limit state of collapse method are also taken up here to compare the results. It is shown that the beams designed by limit state of collapse method are economical. In addition to illustrative examples, practice and test problems are also given in this lesson. Understanding the solved numerical examples and solving the practice and test problems will help in following the applications of the equations and the use of tables of SP-16 in analysing and designing singly and doubly-reinforced rectangular beams.
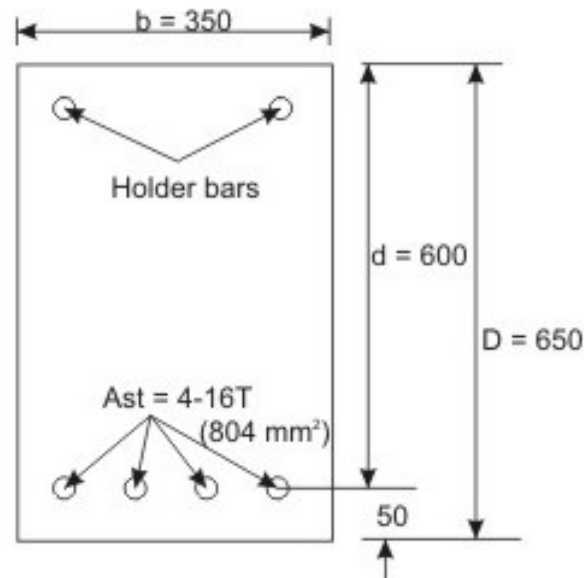
13.35.2 Numerical Problems

Fig. 13.35.1: Problems 1, 2, 3, 4, 7 and 8

Problem 1. (a) Determine the moment of resistance of the rectangular beam of Fig. 13.35.1 having b = 350 mm, d = 600 mm, D = 650 mm, A_{st} = 804 mm^2 (4-16T), σ_{cbc} = 7 N/mm^2 and σ_{st} = 230 N/mm^2. (b) Determine the balanced moment of resistance of the beam and the balanced area of tension steel. (c) Determine the actual compressive stress of concrete f_{cbc} and tensile stress of steel f_{st} when 60 kNm is applied on the beam. Use direct computation method for all three parts.

Solution 1.

1 (a): Given data are: b = 350 mm, d = 600 mm, A_{st} = 804 mm^2, σ_{cbc} = 7 N/mm^2 and σ_{st} = 230 N/mm^2.

We have: $p_t = A_{st}$ (100) / bd = 80400 / (350) (600) = 0.383 per cent and m = 93.33 / σ_{cbc} = 93.33 /7 = 13.33. We determine the value of k from Eq. 13.16.

$$k = - (p_t\, m\, /100) + \{(p_t\, m/100)^2 + (p_t\, m/50)\}^{1/2}$$
(13.16)
$$= - 0.051 + 0.323 = 0.272$$

$$j = 1 - k/3 = 0.909$$

Equation 13.17 gives the moment of resistance of the beam as,

$$M = (p_t/100)\ \sigma_{st}\ (1 - k/3)\ bd^2$$
(13.17)

$$= (0.383/100) \ (230) \ (0.909) \ (350) \ (600) \ (600) = 100.89 \text{ kNm.}$$

1 (b): The balanced moment of resistances and $p_{t, \, bal}$ of the beam is obtained from Eqs. 13.10, 13.11 and 13.13.

$$M_b = R_b b d^2$$
(13.10)
$$R_b = (1/2) \ \sigma_{cbc} \ k_b \ j_b = (p_{t, \, bal} / 100) \ \sigma_{st} \ j_b$$
(13.11)
$$p_{t, \, bal} = 50 \ k_b \ (\sigma_{cbc} / \sigma_{st})$$
(13.13)

where $k_b = 93.33 / (\sigma_{st} + 93.33)$ (13.3)
            $j_b = 1 - k_b / 3$
(13.12)

The values of R_b and $p_{t, \, bal}$ may also be taken from Tables 13.3 and 13.4, respectively of Lesson 34.

Here, $k_b = 93.33 / (230 + 93.33) = 0.288$ and $j_b = 1 - 0.288/3 = 0.904$.

$$p_{t, \, bal} = 50 \ k_b \ (\sigma_{cbc} / \sigma_{st}) = 50 \ (0.288) \ (7/230) = 0.438$$
$p_{t, \, bal}$ is 0.44 from Table 13.4.

Therefore, $M_b = (1/2) \sigma_{cbc} \ k_b \ j_b \ (b d^2) \ = \ 114.81$ kNm,
and $M_b = (p_{t, \, bal} / 100) \ \sigma_{st} \ j_b \ (b d^2) \ = \ 114.75$ kNm

Taking R_b from Table 13.3, we get:
$$M_b = R_b \ (b d^2) = 0.91 \ (350) \ (600)^2 = 114.66 \text{ kNm}$$

The balanced area of steel = $p_{t, bal} \ (bd \ /100) = 0.438 \ (350) \ (600) \ /100 = 919.8$ mm^2. The beam has 4-16 mm diameter bars having 804 mm^2. So, additional amount of steel = 919.8 − 804 = 115.8 mm^2 is needed to make the beam balanced.

1`(c): The actual stresses in steel and concrete f_{st} and f_{cbc} are obtained from Eqs. 13.24 and 13.25, respectively.

$$f_{st} = M / \{A_{st} \ d \ (1 - k/3)\}$$
(13.24)
$$f_{cbc} = (2 \ A_{st} \ f_{st}) \ / \ (b \ k \ d)$$
(13.25)

where $A_{st} = 804$ mm^2 and $k = 0.272$ (obtained in part a of the solution of this problem). Thus, we have:

$f_{st} = 60000000 / 804 (600) (0.909) = 136.83 \text{ N/mm}^2 \langle 230 \text{ N/mm}^2$

$f_{cbc} = 2(804) (136.83) / (350) (0.272) (600) = 3.85 \text{ N/mm}^2 \langle 7 \text{ N/mm}^2$

Problem 2. Solve part a of Problem 1 (Fig. 13.35.1) by employing table of SP-16.

Solution 2.

Given data are: $p_t = 0.383$ per cent, $\sigma_{cbc} = 7 \text{ N/mm}^2$ and $\sigma_{st} = 230 \text{ N/mm}^2$. Table 69 of SP-16 gives $R = 0.7956$ (by linear interpolation). Accordingly, the moment of resistance of the beam is $M = R \, bd^2 = (0.7956) (350) (600) (600) = 100.246$ kNm.

The moment of resistance of the beam is 100.89 kNm by direct computation, as obtained in part a of Problem 1.

Problem 3. Solve Problem 1 (Fig. 13.35.1) when $\sigma_{cbc} = 7 \text{ N/mm}^2$ and $\sigma_{st} = 140$ N/mm^2 (mild steel) for all three parts. For part c, the applied moment is 40 kNm. Other values/data remain unchanged.

Solution 3.

3 (a): Given data are: $b = 350$ mm, $d = 600$ mm, $A_{st} = 804 \text{ mm}^2$, σ_{cbc} 7 N/mm^2 and $\sigma_{st} = 140 \text{ N/mm}^2$. We have from Problem 1: $p_t = 0.383$ per cent, $m = 13.33$, $k = 0.272$, and $j = 0.909$.

From Eq. 13.17, we get:

$$M = (p_t / 100) \, \sigma_{st} \, (1 - k/3) \, bd^2$$
(13.17)
$$= (0.383 / 100) (140) (0.909) (350) (600) (600) = 61.41 \text{ kNm}$$

3 (b): Here $k_b = 93.33 / (140 + 93.33) = 0.4$; $j_b = 0.87$,

$p_{t,bal} = 50 \, k_b \, (\sigma_{cbc} / \sigma_{st}) = 50 (0.4) (7/140) = 1.0$ (Table 13.4 also gives $p_{t,bal}$ = 1).

Therefore, $M_b = (1/2) \, \sigma_{cbc} \, k_b \, j_b \, (bd^2)$
(13.10)
$$= (1/2) (7) (0.4) (0.87) (350) (600) (600) = 153.47 \text{ kNm}$$

$$M_b = (p_{t, bal}/100) \, \sigma_{st} \, (j_b) \, (bd^2)$$
(13.11)
$$= (1/100) (140) (0.87) (350) (600) (600) = 153.47 \text{ kNm}$$

and, taking R_b from Table 13.3, we have from Eq. 13.10,
$$M_b = R\,bd^2 = (1.21)\,(350)\,(600)\,(600) = 152.46\text{ kNm}.$$

The balanced area of steel is obtained from taking $p_{t,\,bal}$ from Table 13.4. Accordingly, we get balanced area of steel $= 1\,(350)\,(600)/100 = 2100\text{ mm}^2$. The beam is having 4-16 mm diameter bars of area 804 mm². The additional area of steel $= 1296\text{ mm}^2$ is needed to make the beam balanced.

3 (c): $f_{st} = M/\{A_{st}\,d\,(1-k/3)\}$
 (13.24)
This gives $f_{st} = 40000000 / (804)\,(600)\,(0.909) = 91.22\text{ N/mm}^2 \langle 140\text{ N/mm}^2$

$f_{cbc} = (2\,A_{st}\,f_{st}) / b\,kd$
(13.25)

So, $f_{cbc} = 2\,(804)\,(91.22) / (350)\,(0.272)\,(600) = 2.57\text{ N/mm}^2 \langle 7\text{ N/mm}^2$

Problem 4. Solve part a of Problem 3 (Fig. 13.35.1) by using table of SP-16.

Solution 4.

Given data are: $p_t = 0.383$ per cent, $\sigma_{cbc} = 7\text{ N/mm}^2$ and $\sigma_{st} = 140\text{ N/mm}^2$. Table 69 of SP-16 gives $R = 0.48436$ (by linear interpolation). Accordingly, the moment of resistance of the beam is
$$M = R\,b\,d^2 = (0.48436)\,(350)\,(600)\,(600) = 61.03\text{ kNm}.$$
The moment of resistance of this beam is 61.41 kNm by direct computation, as obtained in part a of Problem 3.
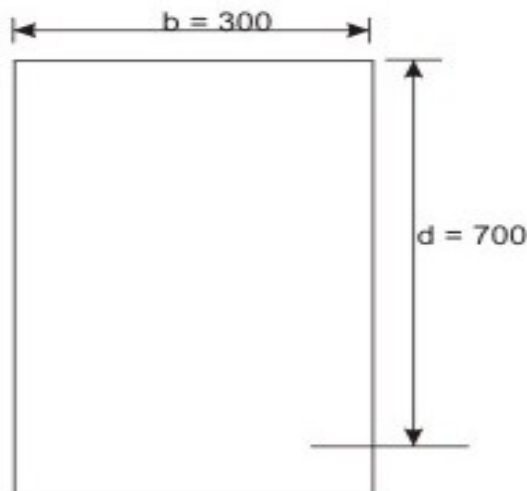

Fig. 13.35.2: Problems 5, 6, Q.1 and Q.2

Problem 5. Design a singly-reinforced rectangular beam to carry the design moment = 100 kNm using M 25 concrete and Fe 415 grade of steel. Use b = 300 mm and d = 700 mm (Fig. 13.35.2) as preliminary dimensions.

Solution 5.

Steps 1 and 2 are not needed as the preliminary dimension of b = 300 mm and d = 700 mm are given.

Step 3. The balanced moment of resistance of the beam = $M_b = R_b\, d^2$ = (1.11) (300) (700) (700) = 163.17 kNm (taking R_b = 1.11 from Table 13.3 for σ_{cbc} = 8.5 N/mm^2 and σ_{st} = 230 N/mm^2). The balanced area of steel = $p_{t,\,bal}$ (bd) / 100 = (0.53) (300) (700) / 100 = 1113 mm^2 (taking $p_{t,\,bal}$ = 0.53 per cent from Table 13.4 for σ_{cbc} = 8.5 N/mm^2 and σ_{st} = 230 N/mm^2). We provide 4-16 mm diameter bars of A_{st} = 804 mm^2 $\langle$ 1113 mm^2 to have p_t = 804 (100) / (300) (700) = 0.383 per cent. This is more than the minimum tensile steel = 0.85 bd / f_y = 430.12 mm^2, as stipulated in cl. 26.5.1.1 of IS 456.

Step 4. Now, the section has to be checked for the stresses in concrete and steel, f_{cbc} and f_{st}, respectively. For this problem m = 93.33 /σ_{cbc} = 10.98 and p_t = 0.383 per cent, obtained in Step 3.

Equation 13.16 gives: k = - $(p_t\, m\, /100)$ + $\{(p_t\, m/100)^2 + (p_t\, m\, /50)\}^{1/2}$ = 0.251. So, j = 1 – k/3 = 1 – 0.251/3 = 0.916.

Equation 13.24 gives: f_{st} = M / $\{A_{st}\, d\, (1\text{-}k/3)\}$= 100 (10^6) / (804) (700) (0.916) = 193.98 N/mm^2 $\langle$ 230 N/mm^2.

Equation 13.25 gives: f_{cbc} = 2 $A_{st}\, f_{st}$ / b kd = 2(804) (193.98) / (300) (0.251) (700) = 5.92 N/mm^2 $\langle$ 8.5 N/mm^2

Therefore, the beam having b = 300 mm, d = 700 mm and A_{st} = 4-16 mm diameter bars is safe.

Problem 6. Design the singly-reinforced rectangular beam of Problem 5 (Fig. 13.35.2) by using tables of SP-16.

Solution 6.

Given data are: M = 100 kNm, b = 300 mm and d = 700 mm. M/bd^2 = 100 (10^6) / (300) (700) (700) = 0.6803. Using Table 70 of SP-16, we get p_t = 0.321, which gives A_{st} = 0.321 (300) (700)/100 = 674 mm^2. This area of steel is the minimum

amount of steel required to resist the design moment of 100 kNm. In fact, we have several alternatives of selecting area of steel from 674 mm^2 to the balanced area of steel = 1113 mm^2, as obtained in Step 3 of Problem 5. While solving Problem 5 by direct computation method, steel area of 804 mm^2 is selected (4-16 mm diameter bars), as the amount is less than the balanced area of steel = 1113 mm^2. However, selecting area of 674 mm^2 is difficult as 3-16 mm diameter bars have the area of steel 603 mm^2. Only one 12 mm diameter is to be added to give additional area of 113 mm^2, which satisfies the requirement. However, 12 mm bars are normally not preferred in beams. So, for the practical considerations, we prefer 4-16 mm diameter bars, as assumed in the direct computation method.

The other alternatives are changing the depth of the beam. For studying all these options, use of tables of SP-16 is very much time-saving.

Moreover, in the direct computation method, the selected area of steel has to be checked so that it is more than the minimum area of steel, as stipulated in cl. 26.5.1.1 of IS 456. On the other hand, the areas obtained from the table of SP-16 already take care so that they are more than the minimum area of steel.

Problem 7. Determine the area of steel of the beam of Problem 3 to carry a design moment of 200 kNm. The relevant data of Problem 3 are: b = 350 mm, d = 600 mm, D = 650 mm (Fig. 13.35.1) σ_{cbc} = 7 N/mm^2 and σ_{st} = 140 N/mm^2.

Solution 7.

The moment of resistance of the balanced beam = 152.46 kNm obtained by using R_b = 1.21 in the equation $M_b = R_b\, bd^2$, in part b of the solution of Problem 3. The balanced area of the steel = 2100 mm^2, which is taken from the solution of part b of Problem 3. The values of m = 13.33, k_b = 0.4, j_b = 0.87 and $p_{t,\,bal}$ = 1 are also taken from the solution of part b of Problem 3.

It is evident that the design moment (= 200 kNm) being greater than balanced moment (= 152.46 kNm), a doubly-reinforced beam has to be designed. The additional moment M' is obtained from Eq.13.26, as

$M' = M - M_b = 200 - 152.46 = 47.54$ kNm.

The additional tensile steel, obtained from Eq.13.35, is: $A_{st2} = M'/\sigma_{st}\,(d-d') = 47.54\,(10^6)/140\,(600-50) = 617.4$ mm^2 (assuming d' = 50 mm).

The amount of compression steel, obtained from Equation 13.36 is: $A_{sc} = (A_{st2})\,/\,(\sigma_{st})/\{\sigma_{cbc}(1.5m-1)(1-d'/kd)\} = (617.4)(140)/7(18.995)(0.792) = 820.79$ mm^2.

Total area of steel, obtained from Eq.13.33 is $A_{st} = A_{st1} + A_{st2} = 2100 + 617.4 = 2717.4$ mm^2.

Problem 8. Solve Problem 7 (Fig. 13.35.1) using tables of SP-16.

Solution 8.

The data of Problem 7 are: b = 350 mm, d = 600 mm, D = 650 mm, σ_{cbc} = 7 N/mm^2 and σ_{st} = 140 N/mm^2. We assume d' = 50 mm, as in Problem 7.

The design of doubly-reinforced beam is vary simple with the help of tables of SP-16. There are two ways. One is the direct solution using the appropriate table from Tables 72 to 79. The other option is to determine A_{st1} and A_{st2} from the equations and then to obtain A_{sc} from Table M of SP-16. Both the methods are explained below.

Method 1: Here, the appropriate table is Table 73. From the values of M/bd^2 and d'/d, we get the percentages of tensile (total) and compressive reinforcement A_{st} and A_{sc}. Thereafter, the total tensile area of steel and compression area of steel are determined.

$M/bd^2 = 200(10^6) / (350) (600) (600) = 1.59$ and $d'/d = 50/600 = 0.083$.

All tables from 72 to 79 of SP-16 have step values of M/bd^2 and d'/d. So, the linear interpolations are to be done three times. The adjacent M/bd^2 and d'/d are 1.55 and 1.60, and 0.05 and 0.1. First, we carry out linear interpolation for $d'/d = 0.5$. For $M/bd^2 = 1.59$ (when $d'/d = 0.5$)

$p_t = 1.253 + (0.038 / 0.05) 0.04 = 1.2834$
and $p_c = 0.305 + (0.045 /0.05) 0.04 = 0.341$

Again for $M/bd^2 = 1.59$, when $d'/d = 0.1$
$p_t = 1.267 + (0.04/0.05) (0.04) = 1.299$
and $p_c = 0.375 + (0.056 / 0.05) (0.04) = 0.4198$.

Now, the linear interpolation is done for $M/bd^2 = 1.59$ and $d'/d = 0.083$. The results are given below:
$p_t = 1.2834 + (0.033) (1.299 − 1.2834) / 0.05 = 1.2936$
$p_c = 0.341 + (0.033) (0.4198 − 0.341) / 0.05 = 0.404$

Thus, we have area of total tensile steel, A_{st} = (1.2936) (350) (600)/100 = 2716.56 mm^2 and the area of compression steel, A_{sc} = 0.404 (350) (600)/100 = 848.4 mm^2. In the above calculations, more number of digits are taken to minimise the truncation error.

We get almost the same values of A_{st} and A_{sc} for Problem 7 by direct computation method (same as those of Problem 8). They are as follows: A_{st} = 2717.4 mm^2 and A_{sc} = 820.79 mm^2. The values are reasonably comparable.

Method 2: Here, in this method the values of A_{st1} and A_{st2} are obtained by direct computation as in Problem 7 giving A_{st1} = 2100 mm^2 and A_{st2} = 617.4 mm^2. Table M of SP-16 gives the ratio of A_{sc} to A_{st2} for different d''/d ratios. For d''/d = 0.083, by linear interpolation, we get

A_{sc} / A_{st2} = 1.2 + (0.2) (0.333)/(0.05) = 1.332

So, A_{sc} = (1.332) (617.4) = 822.37 mm^2. This value is close to that of direct computation method as the ratio is depending on A_{st2} obtained by direct computation method.

13.35.3 Practice Questions and Problems with Answers

Q.1: Design the beam of Problem 5 of sec. 13.35.2 (Fig. 13.35.2) using σ_{cbc} = 8.5 N/mm^2 and σ_{st} = 140 N/mm^2. The given data of Problem 5 are: b = 300 mm, d = 700 mm and M = 100 kNm.

A.1: Steps 1 and 2 are not needed as preliminary dimensions of b and d are given.

Step 3.

M_b = R_b bd^2 = (1.47) (300) (700) (700) = 216.09 kNm (Using R_b = 1.47 from Table 13.3). $A_{st, bal}$ = $p_{t, bal}$ $(bd/100)$ = (1.21) (300) (700) / 100 = 2541 mm^2 (using $p_{t, bal}$ = 1.21 from Table 13.4). We provide 4-20 + 2-16 (=1256 + 402 = 1658 mm^2) as A_{st}, giving p_t = 1658 (100) / (300) (700) = 0.79 per cent. The minimum area of steel = 0.85 bd / f_y = (0.85) (300) (700) / 250 = 714 mm^2 ⟨1658 mm^2. Hence, ok.

Step 4. Check for stresses

With the modular ratio m = 93.33 /8.5 = 10.98 and p_t = 0.79; Eq. 13.16 gives:
k = - (p_t $m/100$) + {(p_t $m/100$)^2 + (p_t $m/50$)}$^{1/2}$ = 0.339 ; and j = 1- $k/3$ = 0.887.

Equation 13.24 gives: $f_{st} = M /\{A_{st}\ d\ (1 - k/3)\} = 100(10^6) / (1658)\ (700)\ (0.887) =$
    97.14 N/mm^2 ⟨ 140 N/mm^2

Equation 13.25 gives: $f_{cbc} = 2\ A_{st}\ f_{st}\ /\ bkd = 2(1658)\ (97.14)\ /\ (300)\ (0.339)\ (700)$
    $= 4.53$ N/mm^2 ⟨ 8.5 N/mm^2.

Hence, this design is ok.

In fact, there are several possible alternatives as explained in the solution of Problem 5. These alternatives are obtained quickly using tables of SP-16.

Q.2: Design the beam of Q.1 (Fig. 13.35.2) using tables of SP-16.

A.2: Steps 1 and 2 are not needed as the given data are: $b = 300$ mm, $d = 700$ mm and $M = 100$ kNm.

Step 3.

$M/bd^2 = 100(10^6) / (300)\ (700)\ (700) = 0.6803$. Table 70 of SP-16 gives $p_t = 0.53775$ by linear interpolation. Accordingly, $A_{st} = (0.53775)\ (300)\ (700)/100 = 1129.275$ mm^2. Provide 4-20 mm diameter bars of area 1256 mm^2, which has $p_t = 1256\ (100) / (300)\ (700) = 0.598$

Step 4. Check for stresses

The modular ratio $m = 10.98$, $k = 0.339$ and $j = 0.887$ from A.1 of Q.1. Equation 13.24 gives: $f_{st} = M/A_{st}\ d(j) = 100\ (10^6) / (1256)\ (700)\ (0.887) = 128.23$ N/mm^2 ⟨ 140 N/mm^2. Equation 13.25 gives: $f_{cbc} = 2A_{st}\ f_{st}\ /\ b\ kd = (2)\ (1256)\ (128.23) / (300)\ (0.339)\ (700) = 4.52$ N/mm^2 ⟨ 8.5 N/mm^2. Hence, the design is ok.

Q.3: The problem of Q.4 of sec. 3.6.11 of Lesson 6 (Fig. 3.6.4) has the following data: $b = 300$ mm, $d = 550$ mm, $D = 600$ mm, $M_{u,\ lim} = 220.45$ kNm, $A_{st,\ lim} = 1107.14$ mm^2, M 20 and Fe 500.

(a) Determine the balanced moment of resistance and $A_{st,b}$ in working stress method.
(b) Determine the areas of steel required to resist the design moment $= M_{u,lim}$ /1.5 = 146.97 kNm.

A.3: (a) To determine M_b and $A_{st,\ b}$

$M_b = R_b \, bd^2 = (0.99) (300) (550) (550) = 89.84$ kNm (using $R_b = 0.99$ from Table 13.3).

Using $p_{t, \, bal} = 0.39$ from Table 13.4, $A_{st, \, b} = (0.39) \, bd/100 = 643.5$ mm^2.

(b) Design of doubly-reinforced beam

From Eq.13.26: $M^{'} = 146.97 - 89.84 = 57.13$ kNm. From Eq.13.34: $A_{st1} = 643.5$ mm^2 for

the balanced moment of 89.84 kNm (see sol.a of this question when $p_{t, \, bal} = 0.39$).

From Eq.13.35: $A_{st2} = M^{'}/\sigma_{st} \, (d - d^{'}) = (57.13) \, (10^6)/ (275) (500) = 415.49$ mm^2 (assuming $d^{'} = 50$ mm).

Total $A_{st} = A_{st1} + A_{st2} = 643.5 + 415.49 = 1058.99$ mm^2.

From Eq.13.36, $A_{sc} = (A_{st2}) \, (\sigma_{st}) \, / \, \{(\sigma_{cbc}) \, (1.5 \, m - 1) \, (1 - d^{'} /kd)\} = (415.49)(275) / \{(7) (18.995) (0.636)\} = 1350.36$ mm^2.

The beam requires $A_{st} = 1058.99$ mm^2 and $A_{sc} = 1350.36$ mm^2. In the limit state method the values are: $A_{st} = 1107.14$ mm^2 and $A_{sc} = 0$, i.e., only singly-reinforced beam with almost the same amount of A_{st} is sufficient. This shows the economy in the design when done by employing limit state method.

This problem, however, cannot be solved by using tables of SP-16 as the tables of SP-16 do not have the values for Fe 500 grade of steel.

13.35.4 Limitations of Working Stress Method

The basis of the analysis by the working stress method is very simple. This method was used for the design of steel and timber structures also. However, due to the limitations of the method, now the limit state methods are being used. The limitations of working stress method are the following:

i)   The assumptions of linear elastic behaviour and control of stresses within specially defined permissible stresses are unrealistic due to several reasons viz., creep, shrinkage and other long term effects, stress concentration and other secondary effects.

ii) The actual factor of safety is not known in this method of design. The partial safety factors in the limit state method is more realistic than the

concept of permissible stresses in the working stress method to have factor of safety in the design.

iii) Different types of load acting simultaneously have different degrees of uncertainties. This cannot be taken into account in the working stress method.

Accordingly, the working stress method is gradually replaced by the limit state method. The Indian code IS 456 has given working stress method in Annex B to give greater emphasis to limit state design. Moreover, cl. 18.2.1 of IS 456 specifically mentions of using limit state method normally for structures and structural elements. However, cl.18.2.2 recommends the use of working stress method where the limit state method cannot be conveniently adopted. Due to its simplicity in the concept and applications, better structural performance in service state and conservative design, working stress method is still being used for the design of reinforced concrete bridges, water tanks and chimneys. In fact, design of tension structures and liquid retaining structures are not included in IS 456 for the design guidelines in the limit state method of design.

Accordingly, Lessons 34 and 35 include the basic concept of this method. The design of T-beam in flexure, rectangular and T-beams under shear, torsion and other topics of the limit state method, covered in different lessons, are not included adopting working stress method. However, the designs of tension structures and liquid retaining structures are taken up in the next lesson, as these are not included by the limit state method.

13.35.5 References

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13.35.6 Test 35 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: Problem 4.1 of sec. 4.9.2 of Lesson 4 has the following data: (Fig. 4.9.1) b = 300 mm, d = 630 mm, d' = 70 mm, D = 700 mm, M 20, Fe 415, $M_{u,\ lim}$ = 492.96 kNm, A_{st1} =1809.14 mm^2, A_{st2} = 783.621 mm^2 (A_{st} = 2572.834 mm^2), A_{sc} = 806.517 mm^2 (by direct computation method). Tension steel reinforcement = 4-25 mm diameter bars + 2-20 mm diameter bars (= 2591 mm^2) and compression steel reinforcement = 2-20 mm diameter bars + 2-12 mm diameter bars (= 854 mm^2).

Determine the areas of steel by working stress method when the design moment = $M_{u,\ lim}$ / 1.5 = 328.64 kNm. Use the same dimensions of the beam as mentioned above. Use direct computation method and also tables of SP-16.

(25+25 = 50 marks)

A.TQ.1: (1) Direct computation method

Given data are: b = 300 mm, d = 630 mm, σ_{cbc} = 7 N/mm^2 and σ_{st} = 230 N/mm^2.

$M_b = R_b\ bd^2$ = 0.91 (300) (630) (630) = 108.35 kNm (using R_b = 0.91 from Table 13.3). Therefore, $M' = M - M_b$ = 328.64 – 108.35 = 220.29 kNm.

$A_{st\,b} = p_{t,\,bal}\,(bd/100) = 0.44\,(300)\,(630)\,/\,100 = 831.6$ mm^2 (using $p_{t,\,bal} = 0.44$ from Table 13.4) and $A_{st2} = M'\,/\,\{\sigma_{st}\,(d\text{-}d')\} = 220.29\,(10^6)\,/\,\{(230)\,(630-70)\} = 1710.32$ mm^2. Therefore, $A_{st} = 831.6 + 1710.32 = 2541.92$ mm^2.

The modular ratio $m = 93.33\,/\,\sigma_{cbc} = 13.33$ and $k_b = 93.33/(\sigma_{st}+93.33) = 0.289$.

$A_{sc} = (A_{st2})\,(\sigma_{st})\,/\,\{\sigma_{cbc}\,(1.5\,m-1)\,(1\text{-}d'/kd) = 4806.37$ mm^2, giving $p_c = A_{sc}\,(100)/\,bd = (4806.37)\,(100)\,/\,(300)\,(630) = 2.54$ per cent $\langle$ maximum percentage allowed (4 per cent) as per cl.26.5.1.2 of IS 456.

So, $A_{st} = 2541.92$ mm^2 = (4-25 + 2-20 giving 2591 mm^2)
and $A_{sc} = 4806.37$ mm^2 = (8-25 + 3-20 giving 4869 mm^2)

(2) Using tables of SP-16

$$M/bd^2 = 328.64\,(10^6)\,/\,(300)\,(630)\,(630) = 2.76$$
$$d'/d = 70/630 = 0.11$$

Table 77 has the values of p_t and p_c for different values of M/bd^2 and d'/d. Therefore, the final values of p_t and p_c are to be determined by carrying out the linear interpolation three times, as explained in the solution of Problem 8. The results are given below:

(a) For $M_u/bd^2 = 2.75$, $d'/d = 0.11$

   $p_t = 1.327 + 0.052\,(0.01)\,/\,0.05 = 1.3374$
   $p_c = 2.348 + 1.03\,(0.01)\,/\,0.005 = 2.555$

(b) For $M_u/bd^2 = 2.8$ and $d'/d = 0.11$

   $p_t = 1.351 + (0.053)\,(0.01)\,/\,0.05 = 1.3616$
   $p_c = 2.412 + (1.063)\,(0.01)\,/\,0.05 = 2.6246$

(c) Therefore, for $M_u/bd^2 = 2.76$ and $d'/d = 0.11$

   $p_t = 1.3374 + (0.0342)\,(0.01)\,/\,(0.05) = 1.34424$
   $p_c = 2.555 + (0.0696)\,(0.01)\,/\,(0.05) = 2.5689$

So, $A_{st} = (1.34424)\,(300)\,(630)\,/\,100 = 2540.6$ mm^2

and A_{sc} = (2.5689) (300) (630) / 100 = 4855.2 mm^2.

The values of A_{st} = 2541.92 mm^2 and A_{sc} = 4806.37 mm^2, obtained by direct computation method in part (1) of this problem, are reasonably comparable with the values obtained by using tables of SP-16.

13.35.7 Summary of this Lesson

This lesson illustrates the applications of the equations developed in Lesson 34 for the analysis and design of singly and doubly-reinforced rectangular beams by working stress method. For this purpose, several numerical problems are solved. Some of the problems have the same dimensions of width and effective depth but with different grades of steel so that the results are compared. Moreover, problems solved earlier on singly and doubly-reinforced beams are also taken up to compare the results. The method of direct computation i.e., using the appropriate equations directly and the use of tables of SP-16 are employed for some of the problems to illustrate the simplicity in using the tables of SP-16. Several practice problems, test problem and illustrative examples will help in understanding the applications for the analysis and design of singly and doubly-reinforced rectangular beams.