Unit Hydrograph

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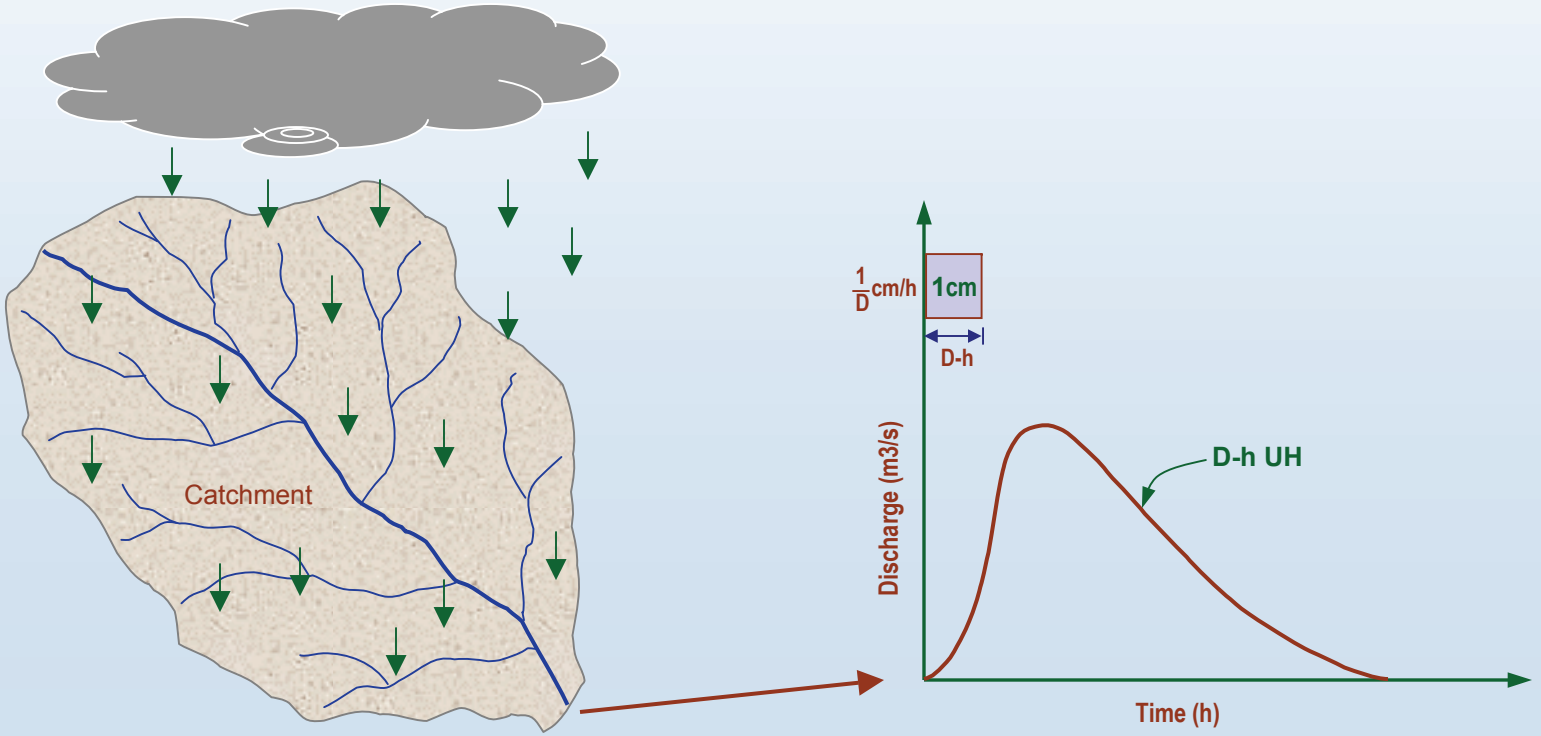
Definition

Sherman (1932) has introduced the concept of unit hydrograph.

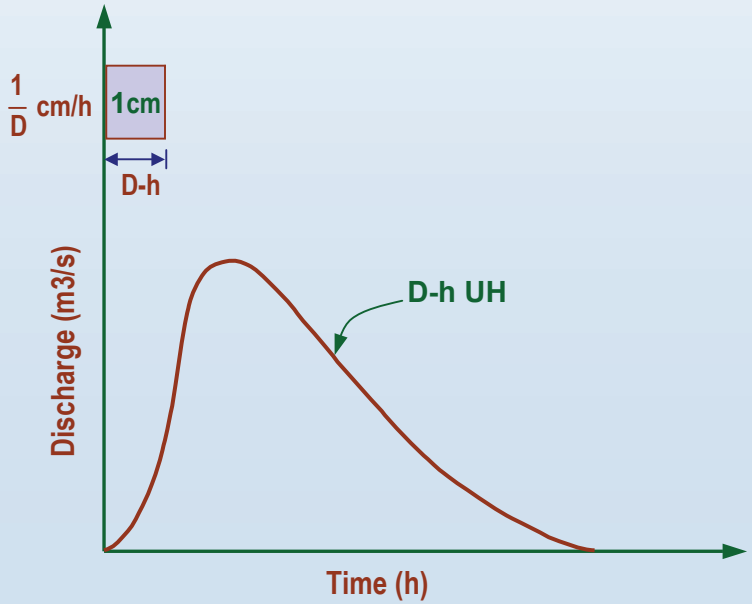
Unit hydrograph is a direct runoff hydrograph resulting from unit (1cm) rainfall excess, occurring uniformly over the entire catchment at a constant rate for a period of D-h.



Characteristics

A D-h Unit Hydrograph has the following characteristics:

- Unit hydrograph is a direct runoff hydrograph. It has no base flow contribution.
- Unit hydrograph is a lumped response of the catchment to a unit rainfall excess of D-h duration.
- Area under the UH curve = 1cm x A km^2
- Intensity of the excess rainfall = 1/D cm/h
- The UH has a steeper rising limb and a flatter falling limb.



Assumptions

Two basic assumptions are involved in the UH theory.

Time Invariance

DRH for a given ER in a catchment is always the same, irrespective of the time of occurrence of the precipitation. It means, shape of the UH will be the same at the beginning, in the middle or at the end of the monsoon.

Linear Response

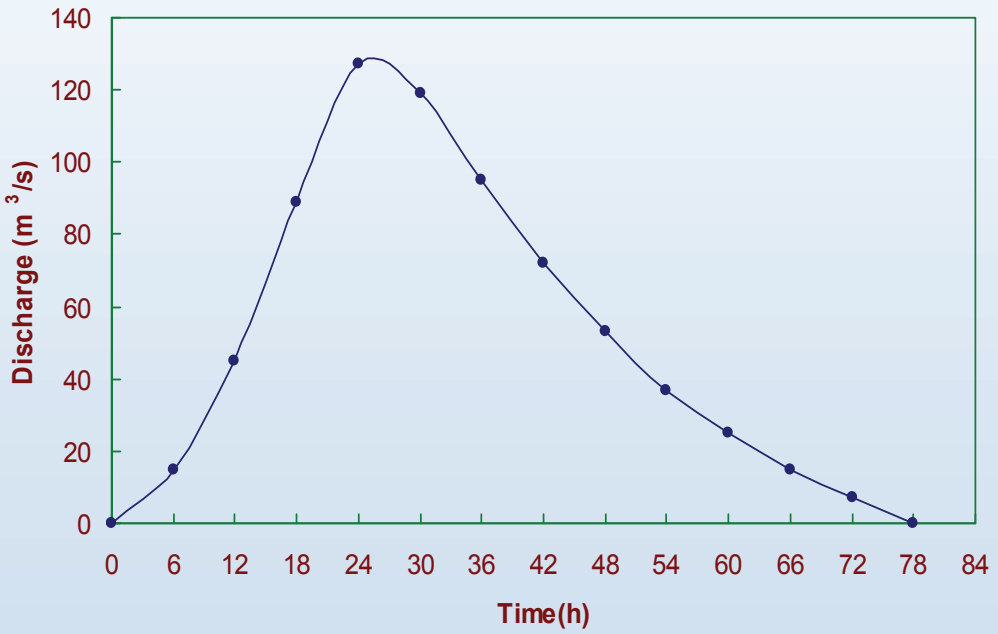
Direct runoff is linearly proportional to the excess rainfall. It implies, if 1cm excess rainfall produces 2.5m^3/s runoff, the 3cm excess rainfall will produce 3 x 2.5 = 7.5m^3/s runoff.

This assumption also enables the principle of superposition to be used. If two or more excess rainfall occur successively, their effects (i.e., resulting DRH) can be estimated independently. These DRH can then be added together maintaining proper time lag, to get the combined DRH.

Example 1

Ordinates of a 6-h UH for a catchment are given below. Calculate the ordinates of a DRH due a rainfall excess of 4.5cm occurring in 6-h on this catchment.

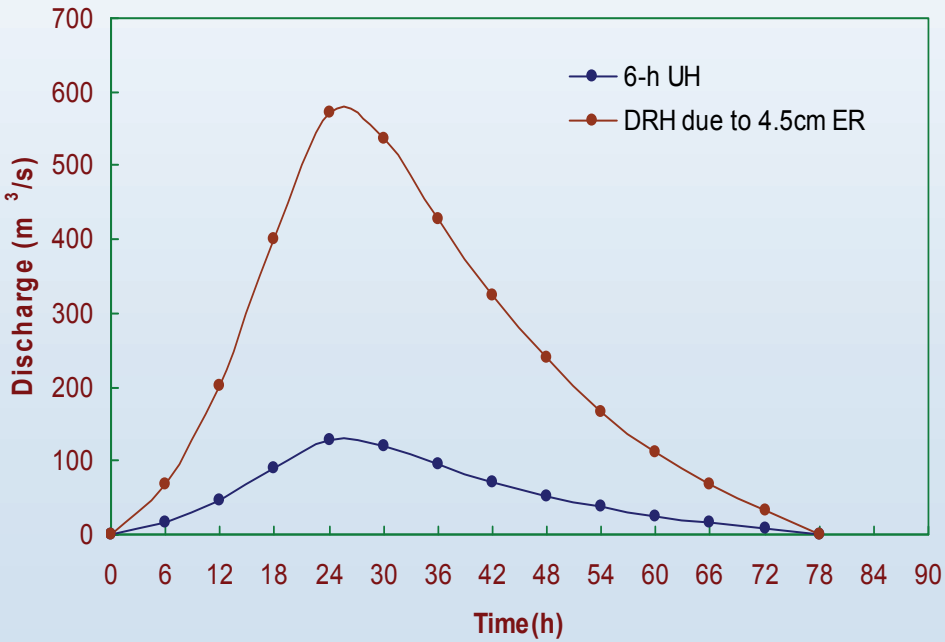
Time (h)	6-h UH Ordinates (m^3/s)
0	0
6	15
12	45
18	89
24	127
30	119
36	95
42	72
48	53
54	37
60	25
66	15
72	7
78	0



Solution

Multiply all the ordinates of the 6-h UH by 4.5, to get the ordinates of the resulting DRH due to 4.5cm ER.

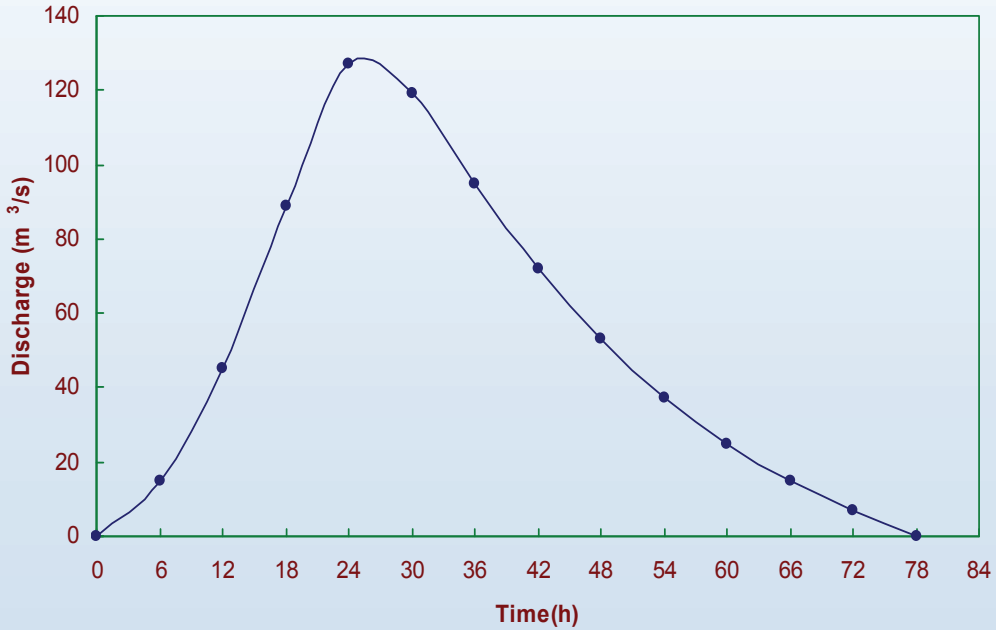
Time (h)	6-h UH Ordinates (m^3/s)	DRH due to 4.5cm ER (m^3/s)
0	0	0
6	15	67.5
12	45	202.5
18	89	400.5
24	127	571.5
30	119	535.5
36	95	427.5
42	72	324
48	53	238.5
54	37	166.5
60	25	112.5
66	15	67.5
72	7	31.5
78	0	0



Example 2

Ordinates of a 6-h UH for a catchment are given below. Calculate the ordinates of a DRH due a rainfall excess of 2cm occurring during first 6-h and a rainfall excess of 3.5cm occurring in the next 6-h over this catchment.

Time (h)	6-h UH Ordinates (m^3/s)
0	0
6	15
12	45
18	89
24	127
30	119
36	95
42	72
48	53
54	37
60	25
66	15
72	7
78	0

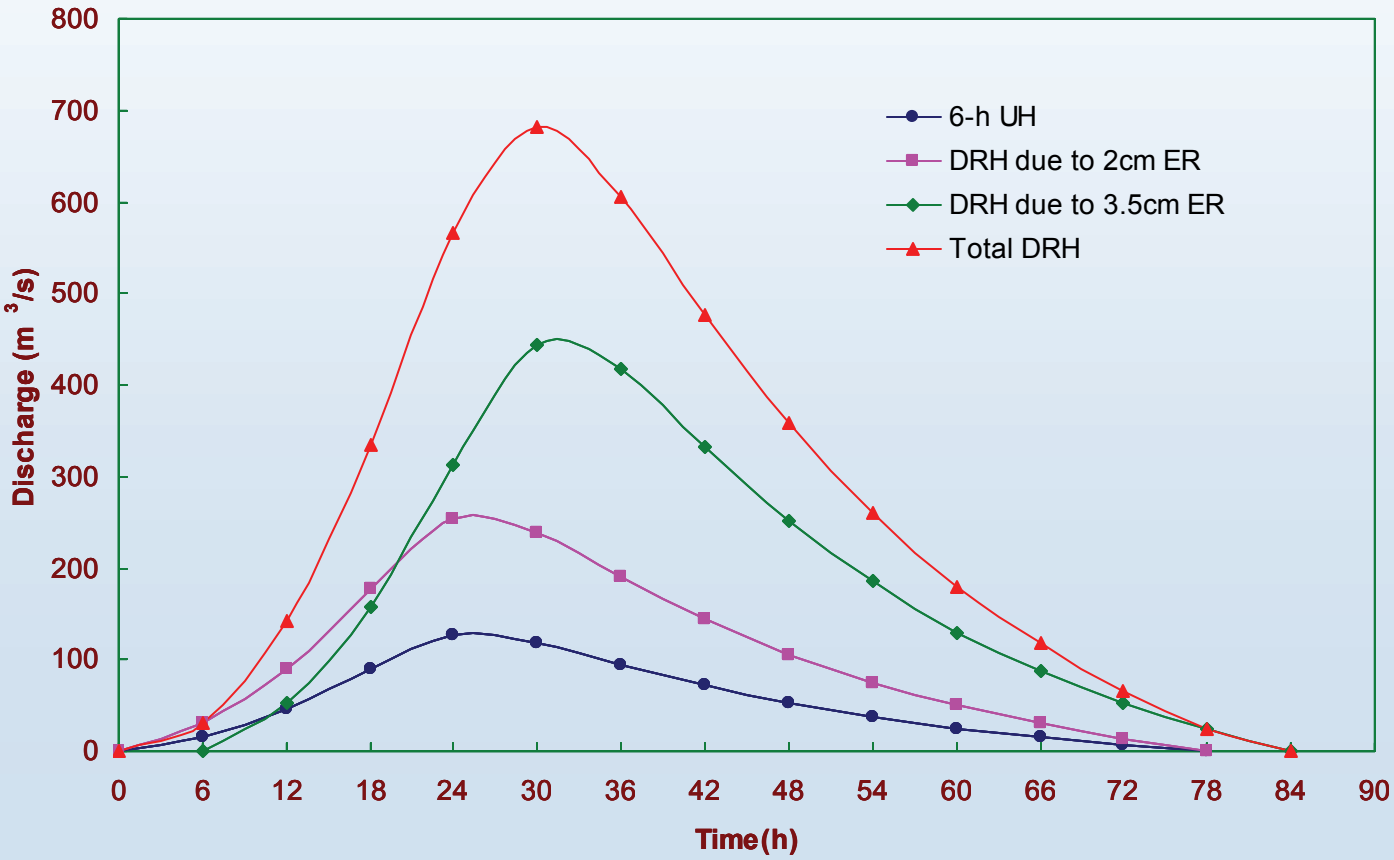


Solution

Time (h)	6-h UH Ordinates (m^3/s)	DRH due to		Total DRH (m^3/s)
		2 cm ER (m^3/s)	3.5 cm ER (m^3/s) lag = 6h	
(1)	(2)	(3) = (2) x 2	(4) = (2) x 3.5	(3) + (4)
0	0	0		0.0
6	15	30	0.0	30.0
12	45	90	52.5	142.5
18	89	178	157.5	335.5
24	127	254	311.5	565.5
30	119	238	444.5	682.5
36	95	190	416.5	606.5
42	72	144	332.5	476.5
48	53	106	252.0	358.0
54	37	74	185.5	259.5
60	25	50	129.5	179.5
66	15	30	87.5	117.5
72	7	14	52.5	66.5
78	0	0	24.5	24.5
84	0		0.0	0.0

Solution

- Draw 6-h UH
- Multiply all ordinates of 6-h UH by 2. Draw the DRH due to 2cm ER.
- Multiply all ordinates of 6-h UH by 3.5. Draw the DRH due to 3.5cm ER starting from 6-h (i.e., lag = 6h), because 3.5cm ER starts at 6-h.
- Add ordinates of the two DRH to get the combined DRH.

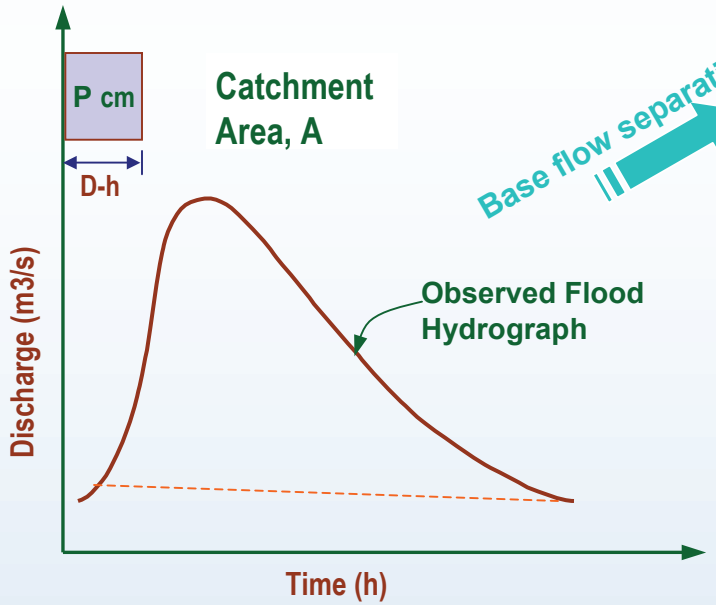


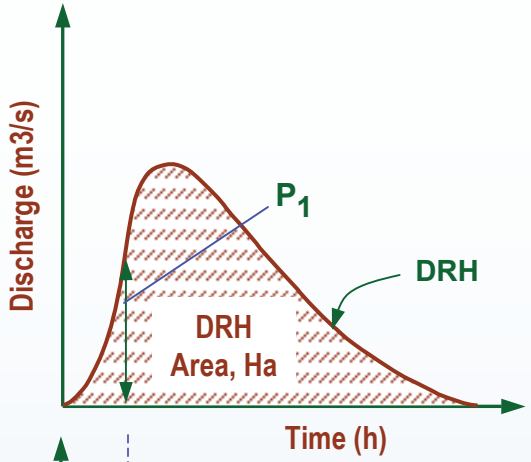
Derivation of UH

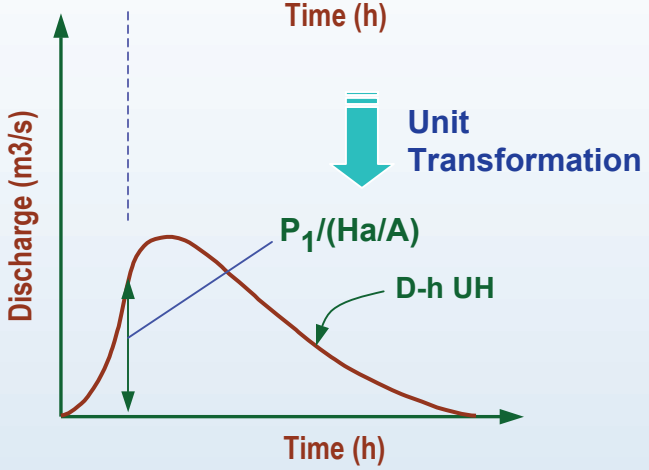
A D-h UH for a catchment is to developed from observed rainfall and corresponding flood hydrograph at the catchment outlet.

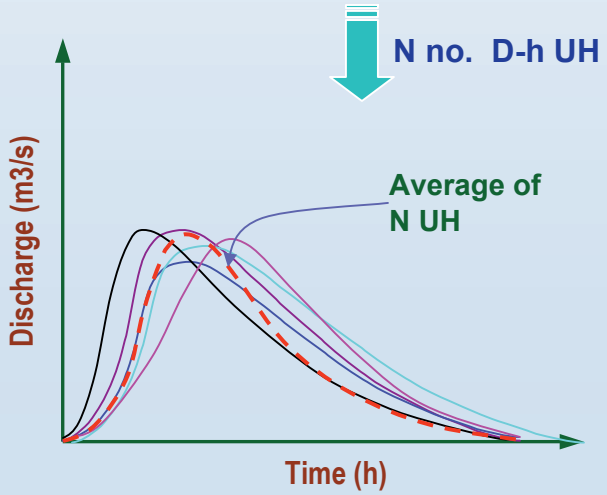
- Select D
- Select a number of isolated storms of duration 10% ± of D.
- Select corresponding flood hydrographs at the catchment outlet.
- For each of these flood hydrographs, separate base flow and get DRHs.
- For each of these DRH,
  - calculate area under the DRH
  - divide the area by the catchment area to get ER
  - divide the ordinates by ER to make it unit hydrograph
- Draw all UH thus derived on a graph paper.
- Calculate average T_B, T_P and Q_P. Draw the average DRH graph.
- Calculate the area under this average graph.
- Divide the area by the catchment area to get ER.
- Divide the ordinates of the average graph by ER to get D-h UH.

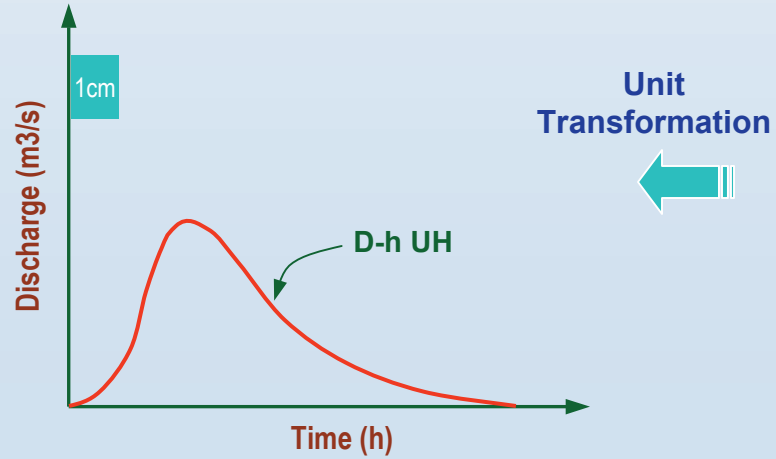
Derivation of UH



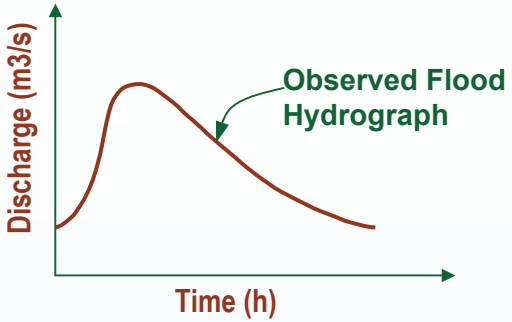








UH from Compound Storm



Time (h)	UH ordinate (m^3/s)	DRH Due to ER rainfall				Total DRH (m^3/s)		Observed DRH (m^3/s)
		x1 cm	x2 cm	x3 cm	x4 cm			
0	0	0	--	--	--	0	=	0
D	y1	x1y1	0	--	--	x1y1	=	Q1
2D	y2	x1y2	x2y1	0	--	x1y2+x2y1	=	Q2
3D	y3	x1y3	x2y2	x3y1	0	x1y3+x2y2+x3y1	=	Q3
4D	y4	x1y4	x2y3	x3y2	x4y1	x1y4+x2y3+x3y2+x4y1	=	Q4
5D	y5	x1y5	x2y4	x3y3	x4y2	x1y5+x2y4+x3y3+x4y2	=	Q5
6D	y6	x1y6	x2y5	x3y4	x4y3	x1y6+x2y5+x3y4+x4y3	=	Q6
7D	0	0	x2y6	x3y5	x4y4	x2y6+x3y5+x4y4	=	Q7
8D			0	x3y6	x4y5	x3y6+x4y5	=	Q8
9D				0	x4y6	x4y6	=	Q9
10D					0	0	=	0

UH from Compound Storm

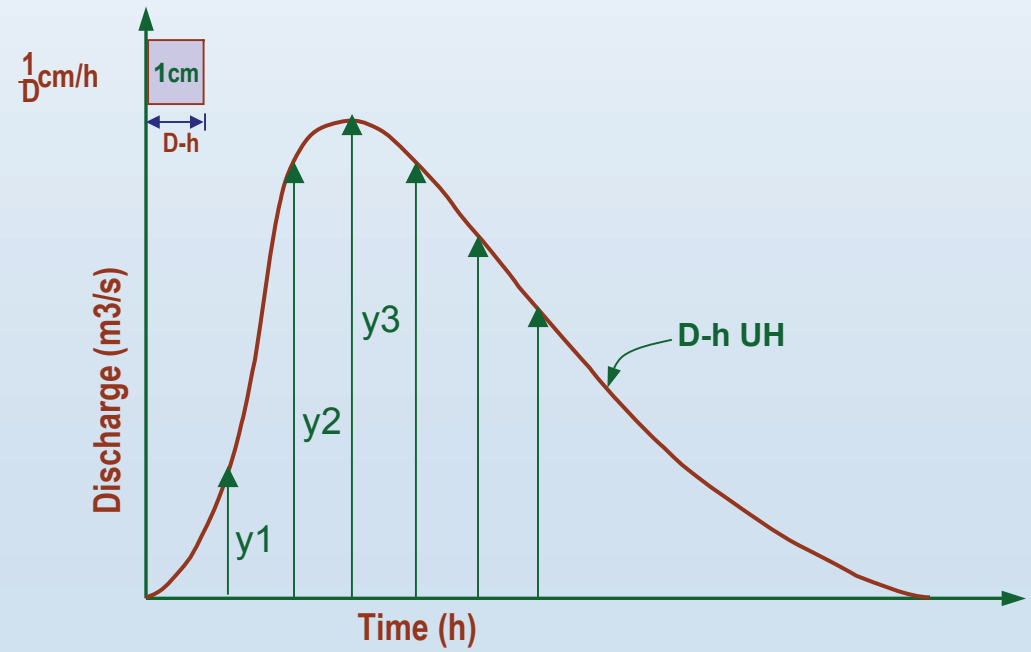
$x1y1 = Q1 \rightarrow y1 = Q1/x1$

$x1y2+x2y1 = Q2 \rightarrow y2 = (Q2 – x2y1)/x1$

$x1y3 + x2y2 + x3y1 = Q3 \rightarrow y3 = (Q3 – x2y2 – x3y1)/x1$

……..

$x1y6 + x2y5 + x3y4 + x4y3 = Q6 \rightarrow y6 = (Q6 – x2y5 – x3y4 – x2y5)/x1$



UH of Different Durations

For determining flood hydrograph using UH, the type of UH to be used depends on the time interval of the hyetograph. If the hyetograph is having 2-h time interval, then a 2-h UH is to be used. If the time interval is 4-h, then a 4-h UH is to be used. Any other UH cannot be used.

As the development of the UH from observed data is an involved task, usually only one UH is made available for a catchment. Any other UH is to be derived from this UH. That is, if a 2-h UH is available and a 3-h UH or 4-h UH is needed, then these are to be derived from the 2-h UH.

Two different types of tasks are involved while developing a T-h UH from a D-h UH

- T is an integer multiple of D (i.e., T = 1D, 2D, 3D etc.)
- T is a fractional multiple of D (i.e., T = 0.5D, 1.25D, 1.5D etc.)

Method of Superposition can be used for the integer case, whereas

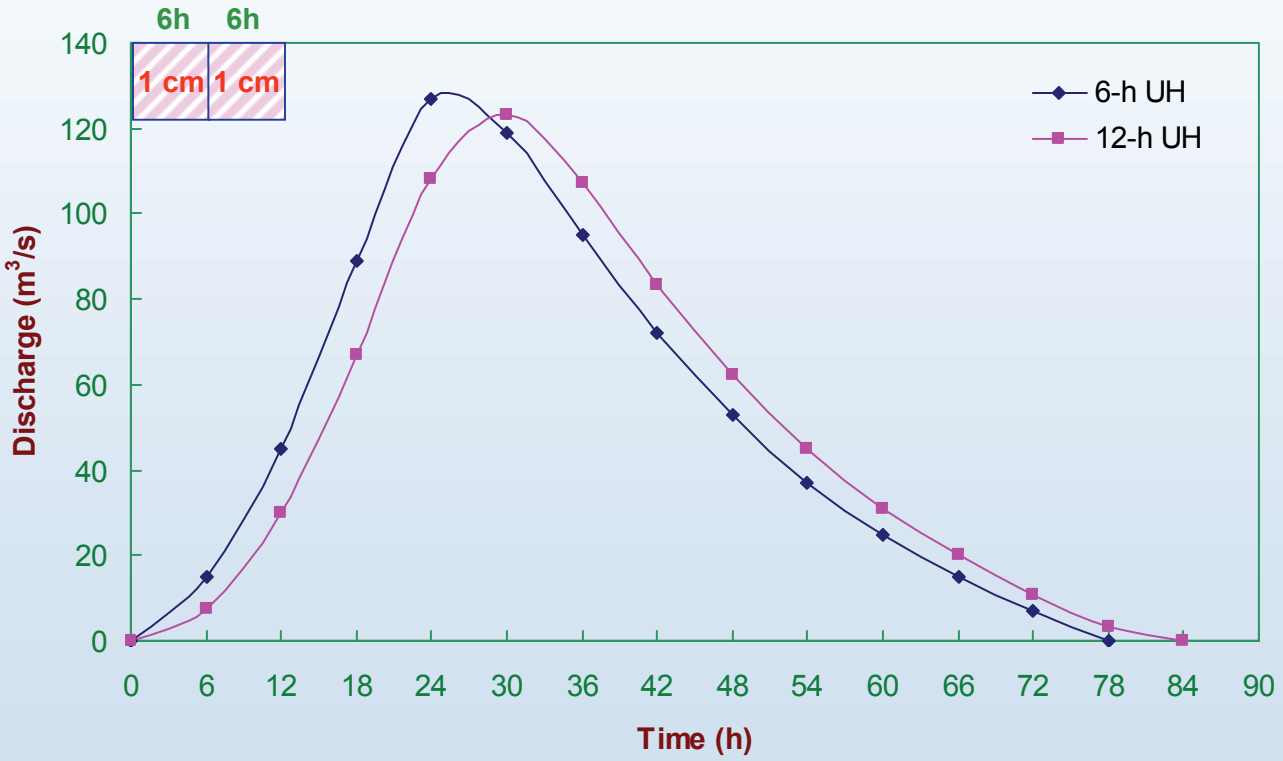
S-curve Technique can be used for both the cases.

Method of Superposition

A 6-h UH for a catchment is given below. Derive a 12-h UH.

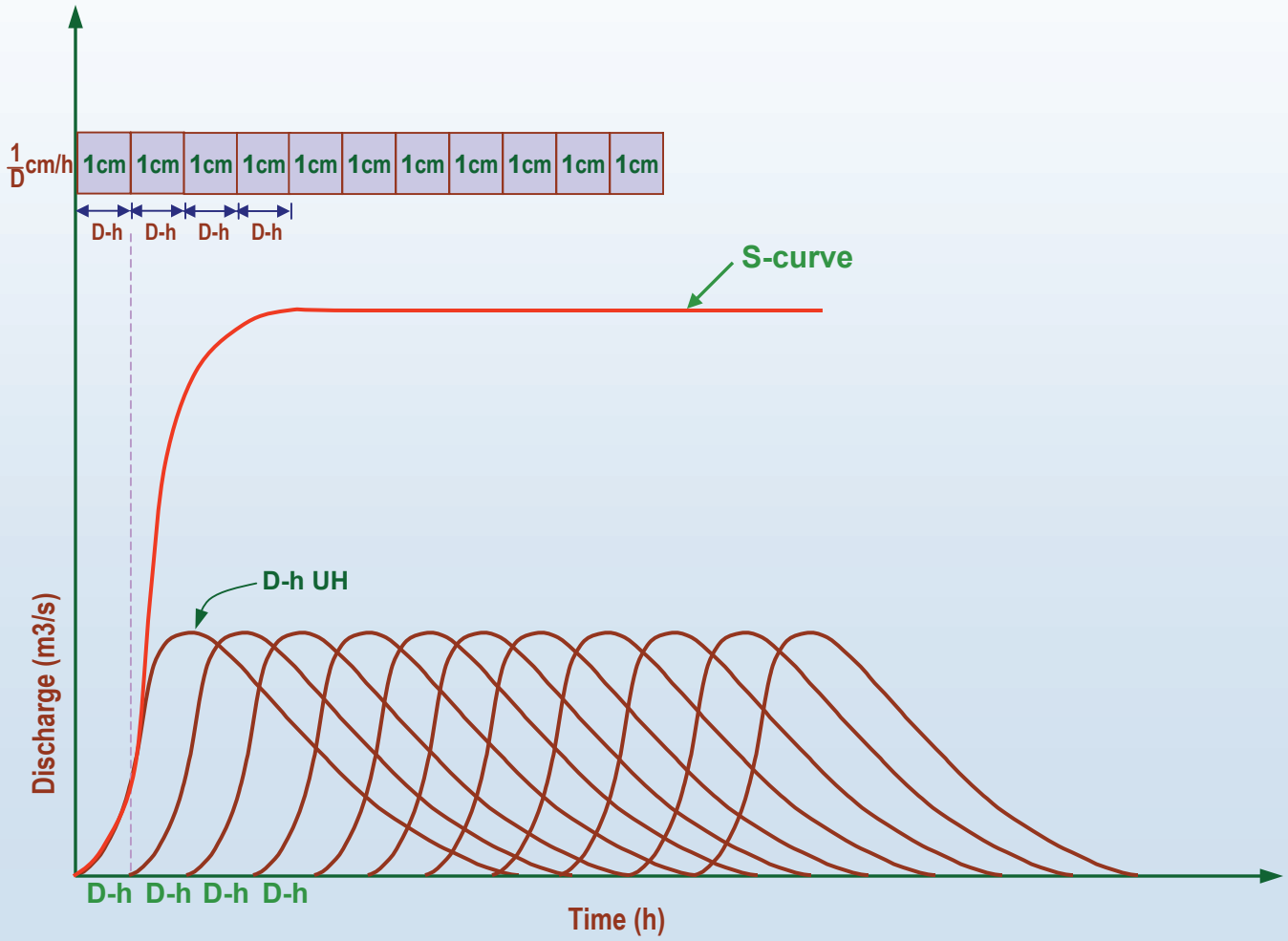
Time (h)	6-h UH Ordinates (m3/s)	6-h UH Lagged by 6-h (m3/s)	12-h DRH due to 1+1 cm ER (m3/s)	12-h UH (m3/s)
col 1	col 2	col 3	col 4	col 4/(1+1)
0	0		0	0.0
6	15	0	15	7.5
12	45	15	60	30.0
18	89	45	134	67.0
24	127	89	216	108.0
30	119	127	246	123.0
36	95	119	214	107.0
42	72	95	167	83.5
48	53	72	125	62.5
54	37	53	90	45.0
60	25	37	62	31.0
66	15	25	40	20.0
72	7	15	22	11.0
78	0	7	7	3.5
84		0	0	0.0

Method of Superposition



S-curve

If a rainfall of intensity 1/D cm/h occurs continuously over a catchment the resulting DRH at the catchment outlet is known as S-curve or S-hydrograph.

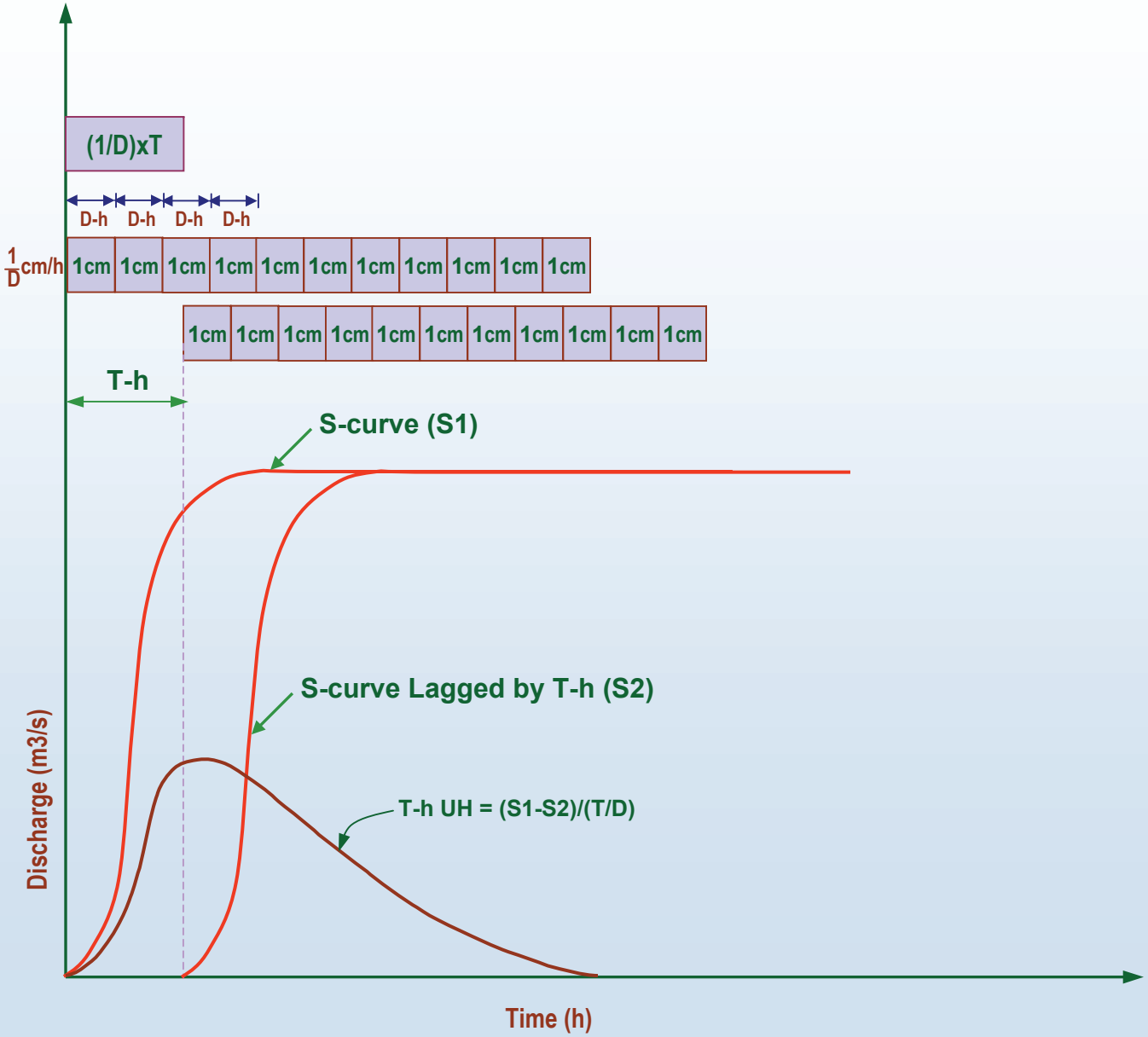


S-curve

If a rainfall of intensity 1/D cm/h occurs continuously over a catchment the resulting DRH at the catchment outlet is known as S-curve or S-hydrograph.

Time (h)	D-h UH (m^3/s)	D-h UH Lagged by D-h	D-h UH Lagged by 2D-h	D-h UH Lagged by 3D-h	D-h UH Lagged by 4D-h	D-h UH Lagged by 4D-h		S-curve ordinates (m^3/s)
0	0						c	0
D	y1	0					o	y1
2D	y2	y1	0				n	y1+y2
3D	y3	y2	y1	0			t	y1+y2+y3
4D	y4	y3	y2	y1	0		i	y1+y2+y3+y4
5D	y5	y4	y3	y2	y1	0	n	y1+y2+y3+y4+y5
6D	0	y5	y4	y3	y2	y1	u	y1+y2+y3+y4+y5
7D		0	y5	y4	y3	y2	e	y1+y2+y3+y4+y5
8D			0	y5	y4	y3		y1+y2+y3+y4+y5
				0	y5	y4		y1+y2+y3+y4+y5

S-curve



S-curve

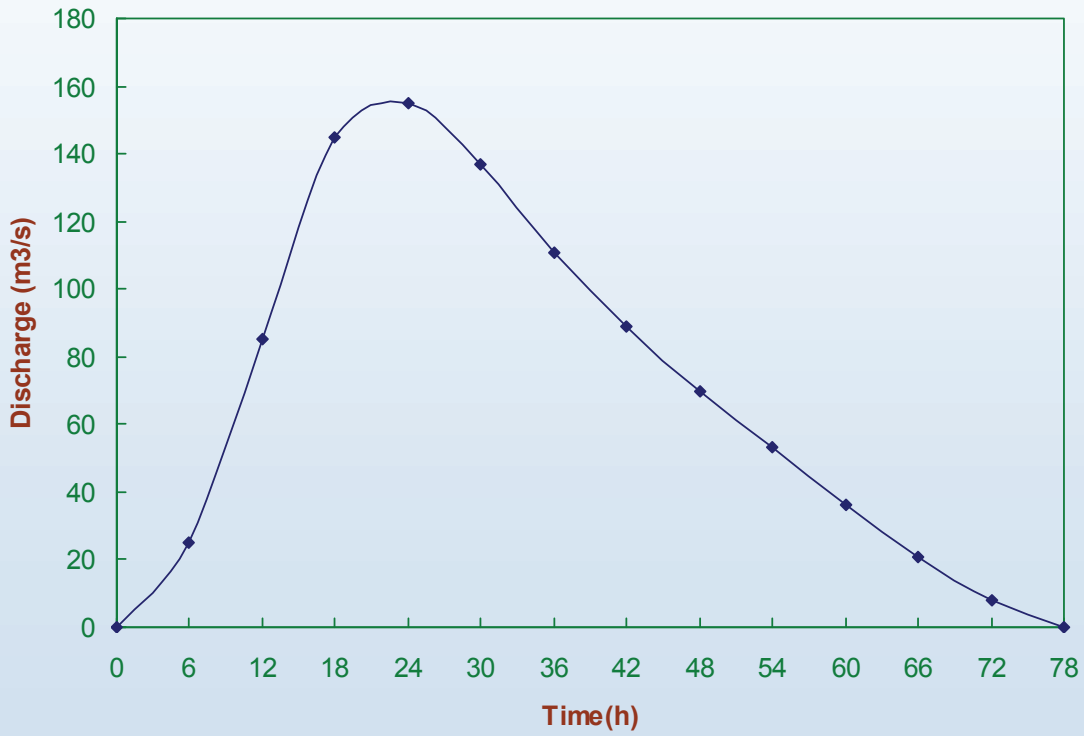
A 6-h UH for a catchment is given below. Derive a 12-h UH using S-curve.

Time (h)	6-h UH Ordinates (m^3/s)	S-Curve Ordinates S1 (m^3/s)	S-curve lagged by 12-h S2 (m^3/s)	S1 - S2 (m^3/s)	12-h UH Ordinates (S1-S2)/2 (m^3/s)
col 1	col 2	col 3	col 4	col 5	col 6
0	0	0		0	0.0
6	15	15		15	7.5
12	45	60	0	60	30.0
18	89	149	15	134	67.0
24	127	276	60	216	108.0
30	119	395	149	246	123.0
36	95	490	276	214	107.0
42	72	562	395	167	83.5
48	53	615	490	125	62.5
54	37	652	562	90	45.0
60	25	677	615	62	31.0
66	15	692	652	40	20.0
72	7	699	677	22	11.0
78	0	699	692	7	3.5
84		699	699	0	0.0

S-curve

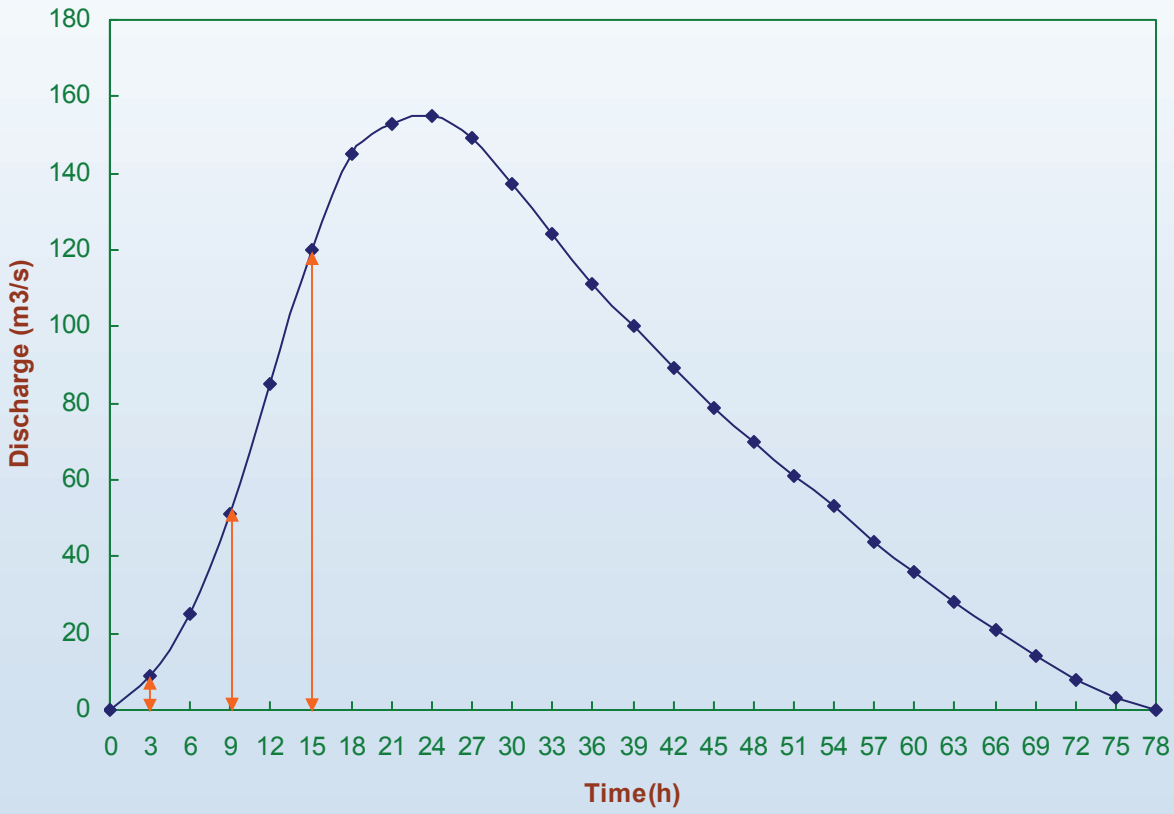
A 6-h UH for a catchment is given below. Derive a 3-h UH using S-curve.

Time (h)	6-h UH (m3/s)
0	0
6	25
12	85
18	145
24	155
30	137
36	111
42	89
48	70
54	53
60	36
66	21
72	8
78	0



S-curve

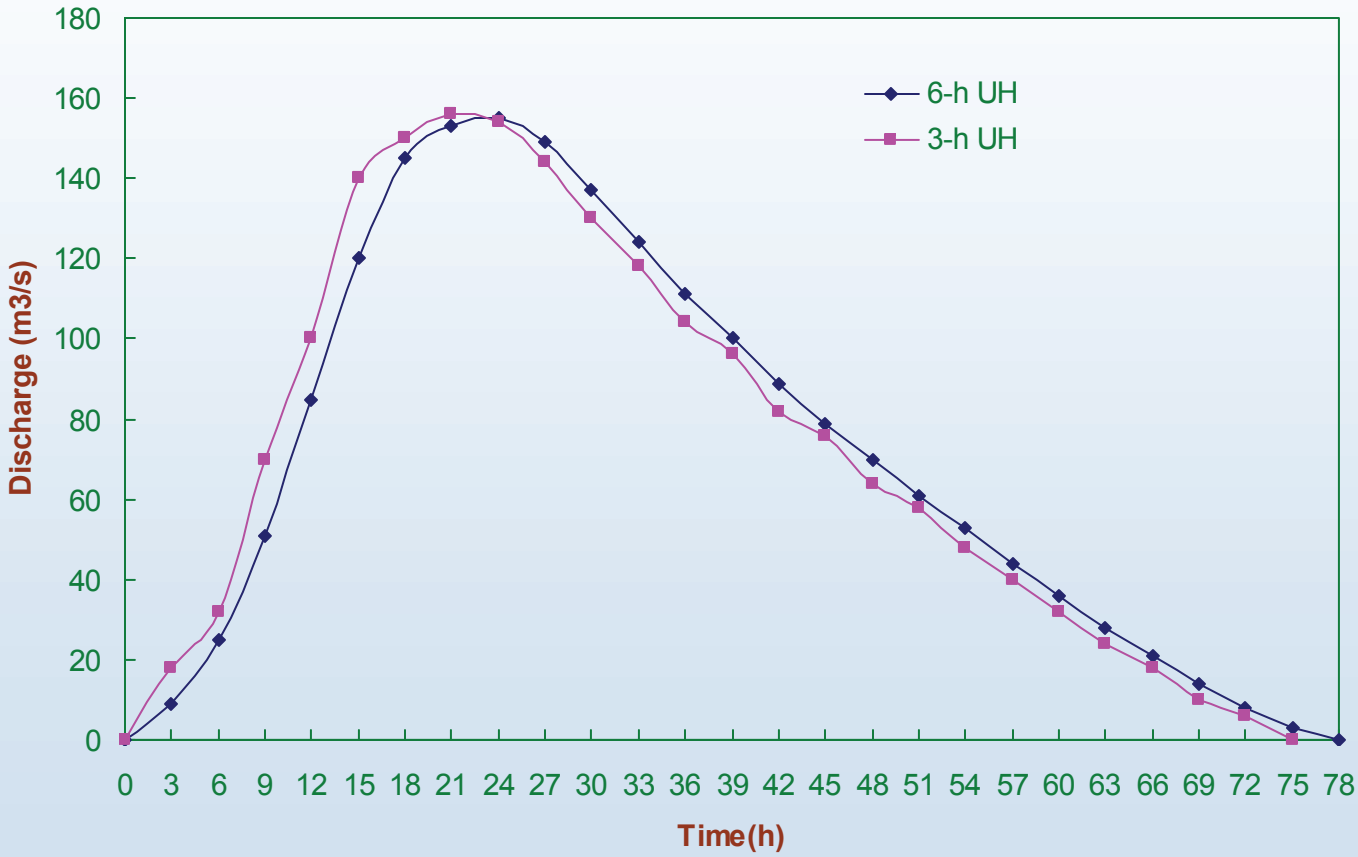
Get ordinates at 3-h interval from the graph (or interpolate).



S-curve

Time (h)	6-h UH (m3/s)	6-h S-Curve (m3/s) S1	6-h S-curve 3-h lagged S2	S1-S2	3-h UH m3/s col 5 x 2
col 1	col 2	col 3	col 4	col 5	col 6
0	0	0		0	0
3	9	9	0	9	18
6	25	25	9	16	32
9	51	60	25	35	70
12	85	110	60	50	100
15	120	180	110	70	140
18	145	255	180	75	150
21	153	333	255	78	156
24	155	410	333	77	154
27	149	482	410	72	144
30	137	547	482	65	130
33	124	606	547	59	118
36	111	658	606	52	104
39	100	706	658	48	96
42	89	747	706	41	82
45	79	785	747	38	76
48	70	817	785	32	64
51	61	846	817	29	58
54	53	870	846	24	48
57	44	890	870	20	40
60	36	906	890	16	32
63	28	918	906	12	24
66	21	927	918	9	18
69	14	932	927	5	10
72	8	935	932	3	6
75	3	935	935	0	0
78	0	935	935	0	0

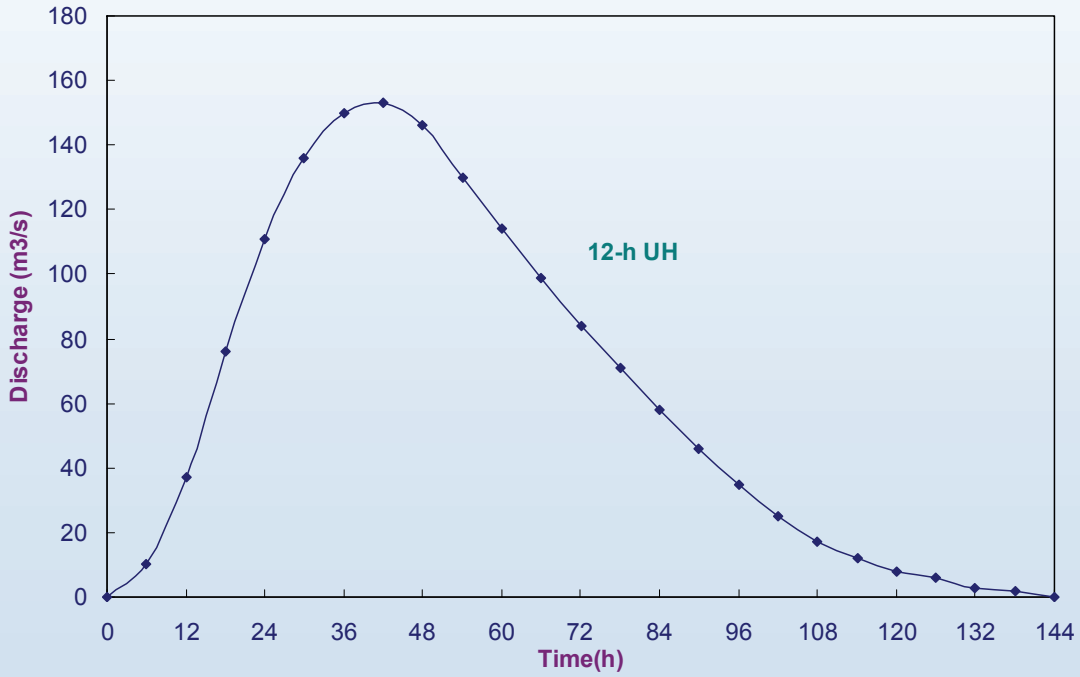
S-curve



S-curve

The ordinates of a 12-h UH are given below, compute the ordinates of a 6-h UH.

Time (h)	UH Ordinate (m3/s)
0	0
6	9
12	37
18	75
24	111
30	136
36	150
42	153
48	146
54	130
60	114
66	99
72	84
78	71
84	58
90	46
96	35
102	25
108	17
114	12
120	8
126	6
132	3
138	1
144	0

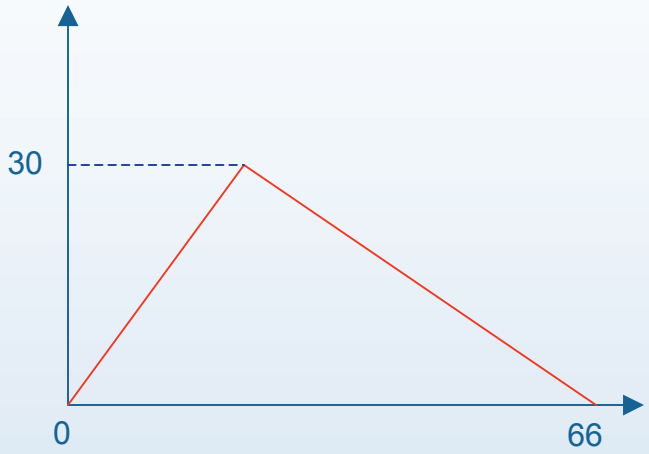


S-curve

Time (h)	UH Ordinate (m^3/s)	12-h S-Curve (m^3/s) (S1)	12-h S-Curve 6-h lagged (S2)	S1-S2 (m^3/s)	6-h UH (m^3/s) Col. 5 x 2
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6
0	0	0		0	0
6	9	9	0	9	18
12	37	37	9	28	56
18	75	84	37	47	94
24	111	148	84	64	128
30	136	220	148	72	144
36	150	298	220	78	156
42	153	373	298	75	150
48	146	444	373	71	142
54	130	503	444	59	118
60	114	558	503	55	110
66	99	602	558	44	88
72	84	642	602	40	80
78	71	673	642	31	62
84	58	700	673	27	54
90	46	719	700	19	38
96	35	735	719	16	32
102	25	744	735	9	18
108	17	752	744	8	16
114	12	756	752	4	8
120	8	760	756	4	8
126	6	762	760	2	4
132	3	763	762	1	2
138	1	763	763	0	0
144	0	763	763		

S-curve

A 6-h UH of a catchment is triangular in shape with a base width of 66h and a peak ordinate of 30 m3/s. Calculate the equilibrium discharge of an S-curve derived from this 6-h UH.



S-curve

By definition, S-curve is the hydrograph resulted from a continuous rainfall excess of intensity (1/D) cm/h, occurring uniformly over the catchment area, A km^2.

So,

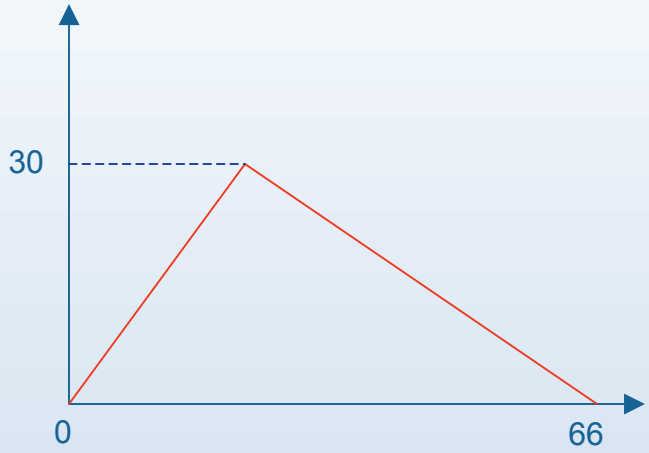
Equilibrium discharge for S-curve = Qs = (1/D) x A x (1/100)x(1/3600)x10^6 m^3/s

= 2.778 A/D m^3/s

Now,

Catchment area,

A = area under the UH / 1 cm



Area under UH = 0.5 x 66 x 30 m^3/s-h

= 3564000 m^2

Then, A = 3564000 / 0.01 = 356.4 km^2

So, equilibrium discharge, Qs = 2.778 x 356.4 / 6 = 165 m^3/s