

Streamflow Measurement

Overview

1 Introduction

2 Measurement Techniques

- Measurement of Stage
- Measurement of velocity
- Measurement of discharge

Introduction

The water which constitutes the flow in stream is called streamflow. If the streamflow is unaffected by artificial diversions, storage in or on the stream channels, then it is called runoff. It is the only part of the hydrologic cycle that can be measured accurately.

- Streamflow forms the most important data for engineers and hydrologists who use it in the design of water resources projects.
- Streamflow is measured in units of discharge (m^3/s) occurring at a specified time and constitutes historical data.

Measurement Techniques

Streamflow measurement techniques can be broadly classified into two categories:

Direct determination

- Area-velocity methods
- Dilution technique
- Electromagnetic method
- Ultrasonic method

Indirect determination

- Hydraulic structures such as weirs, flumes etc
- Slope-area method

Measurement Techniques

Continuous measurement of stream discharge is very difficult to obtain. Direct measurement is a very time consuming and costly process. Hence a two-step procedure is followed.

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- Then the stage of the stream is observed routinely in a relatively inexpensive manner and the discharge is estimated using the stage-discharge relationship.
- The observation of stage is easy, inexpensive and continuous readings can also be obtained.

Measurement of stage

What is stage?

The stage of a river is defined as its water-surface elevation measured above a datum which can be MSL or any datum connected independently connected to the MSL.

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- The staff is rigidly attached to a permanent structure such as pier, abutment, wall etc.
- The gauge indicates water-surface elevation on a staff that is graduated with clear and accurate markings.

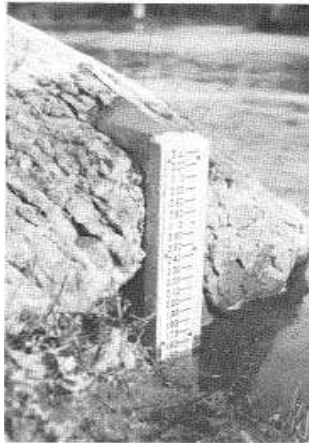
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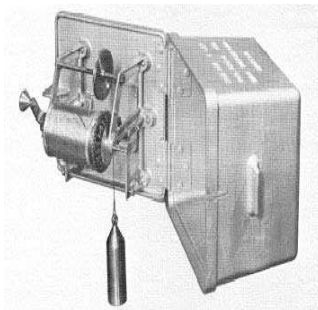
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- The gauge indicates water-surface elevation on a staff that is graduated with clear and accurate markings.
- Sometimes a single gauge is not adequate for all stages and in such cases *sectional gauges* are built in sections at different locations.



Vertical Staff gauge

Measurement of Stage



Wire Gauge

Measurement of stage

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- The operating range of this kind of gauge is about 25 m.

Measurement of stage

Automatic Stage Recorders

- Float gauge recorder
- Bubble gauge

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- In addition to its use in determination of stream discharge, other uses of this data are in flood-insurance studies, design of flood protection works, flood warning, urban development, navigation etc.
- Long term stage data are needed to estimate peak river stages in the design of hydraulic structures such as bridges, culverts, weirs etc.

Measurement of velocity

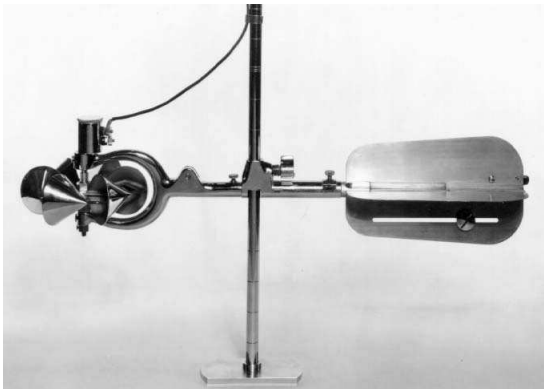
A mechanical device called *current meter*, consisting essentially of a rotating element is probably the most commonly used instrument for accurate determination of the stream-velocity field.

Approximate stream velocities can be determined by *floats*.

Current meter

- Vertical-axis meter
- Horizontal-axis meter

Measurement of velocity



Cup-type Current Meter (Vertical-axis)

Measurement of velocity

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- The Price current meter is typical instrument under this category.

Measurement of velocity

Horizontal axis meter

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Horizontal axis meter

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- Propeller diameters in the range of 6 to 12 cm and can register velocities in the range of 0.15-4.0 m/s.
- Not affected by oblique flows as much as 15° .

Measurement of velocity

Rating Equation

A current meter is so designed that its rotation speed varies linearly with the stream velocity v at the location of the instrument. A typical relationship is

$$v = aN_s + b$$

where v = stream velocity at the instrument location in m/s, N_s = revolutions per second of the meter and a, b = constants of the meter. Typical values of a and b for a standard size 12.5 cm diameter Price

current meter is $a = 0.65$ and $b = 0.03$. The instruments have a provision to count the number of revolutions in a known interval of time which is usually accomplished by making and breaking of an electric circuit either mechanically or electro-magnetically at each revolution of the shaft.

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- In turbulent flow conditions, the variation can be described closely by the logarithmic law or by $1/7$ th power law.
- The velocity at a distance of $0.368d$ from the bottom will be equal to the average velocity in case of logarithmic law and at a distance of $0.393d$ from the bottom in case of $1/7$ th power law.

Average velocity across a vertical

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Average velocity across a vertical

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$$\bar{v} = (v_{0.2} + v_{0.8})/2$$

- In rivers having flood flows, only the surface velocity v_s is measured within a depth of about 0.5 m below the surface. The average velocity \bar{v} is obtained by using a reduction factor K as

$$\bar{v} = Kv_s$$

Measurement of discharge

The method of discharge measurement consists essentially of measuring the cross-section of the river at a selected section called the gauging site and measuring the velocity of flow through the cross-sectional area. The following criteria are adopted:

- The stream should have a well-defined cross-section which does not change in various seasons
- It should be easily accessible all throughout the year.
- The site should be in a straight, stable reach.
- The gauging site should be free from backwater effects in the channel.

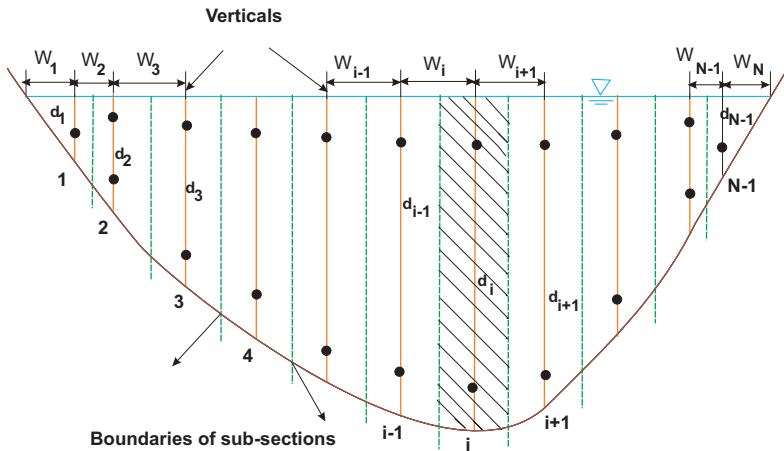
Measurement of discharge

Area-velocity method

The cross-section is considered to be divided into a large number of subsections by verticals. The average velocity in these subsections are measured by current meters or floats. The following are some of the guidelines to select the number of segments.

- The segment width should not be greater than $1/15$ to $1/20$ th of the width of the river.
- The discharge in each segment should be less than 10% of the total discharge.
- The difference in velocities in adjacent segments should not be more than 20%.

Measurement of discharge



Stream-section for area-velocity method

Calculation of discharge

- The streamwidth is divided into N subwidths, $W_i = 1, 2, \dots N$.
- The velocity averaged over the vertical at each section is known.
- The total cross-sectional area can be divided into $N-1$ segments. This means there are $N-1$ verticals.
- The total discharge is calculated by the method of mid-sections.

$$Q = \sum_{i=1}^{N-1} q_i$$

where q_i is the discharge at the i th segment and is obtained as

$$q_i = a_i v_i$$

where a_i = Area of the i th segment

= (depth at the i th vertical) \times (width to the left + width to the right)/2

Calculation of discharge

$$q_i = d_i \times (W_i + W_{i+1})/2 \times v_i \text{ for } i = 2 \text{ to } N-2$$

For the first segment and the last segment, the areas are represented by triangles as

$$a_i = \overline{W_1} d_1$$

$$a_{N-1} = \overline{W_{N-1}} d_{N-1}$$

where

$$\overline{W_1} = \frac{(W_1 + W_2/2)^2}{2W_1}$$

and $\overline{W_{N-1}} = \frac{(W_N + W_{N-1}/2)^2}{2W_N}$

Example

The data pertaining to a stream gauging operation at a gauging site are given below. The rating equation of the current meter is $v = 0.51N_s + 0.03m/s$. Calculate the discharge in the stream.

Distance from left water edge (m)	0	1.0	3.0	5.0	7.0	9.0	11.0	12.0
Depth (m)	0	1.1	2.0	2.5	2.0	1.7	1.0	0
Revolutions of a current meter	0	39	58	112	90	45	30	0
Duration of observation (s)	0	100	100	150	150	100	100	0

Solution

The average width for the first section is

$$\overline{W}_1 = \frac{(1+2/2)^2}{2 \times 1} = 2.0 \text{ m}$$

The average width for the last section is

$$\overline{W}_{N-1} = \frac{(1+2/2)^2}{2 \times 1} = 2.0 \text{ m}$$

The average width for the rest of the sections is $\overline{W}_i = (2 + 2) / 2 = 2.0$ m

Sample Calculation: The cross-sectional flow area of first subsection is :

$$a_1 = d_1 \overline{W}_1 = 1.1 \times 2 = 2.2 \text{ m}^2$$

Average velocity for first vertical $\overline{v}_1 = 0.51 \times 39/100 + 0.03 = 0.229 \text{ m/s}$

Discharge for first subsection $= 2.2 \times 0.229 = 0.504 \text{ m}^3/\text{s}$

Solution

Distance from left water edge (m)	Average width W (m)	Depth d (m) (m)	Area of flow a (m ²)	Velocity v (m/s)	Segmental discharge q (m ³ /s)
0	0	0	-	-	-
1.0	2.0	1.1	2.2	0.229	0.504
3.0	2.0	2.0	4.0	0.326	1.304
5.0	2.0	2.5	5.0	0.411	2.055
7.0	2.0	2.0	4.0	0.336	1.344
9.0	2.0	1.7	3.4	0.260	0.884
11.0	2.0	1.0	2.0	0.183	0.366
12.0	0.0	0.0	0.0	0.0	0.0
					$\Sigma q_i = 6.457 \text{ m}^3/\text{s}$

Total Discharge = $6.457 \text{ m}^3/\text{s}$

Example

The following data were collected during a stream-gauging operation in a river. Compute the discharge.

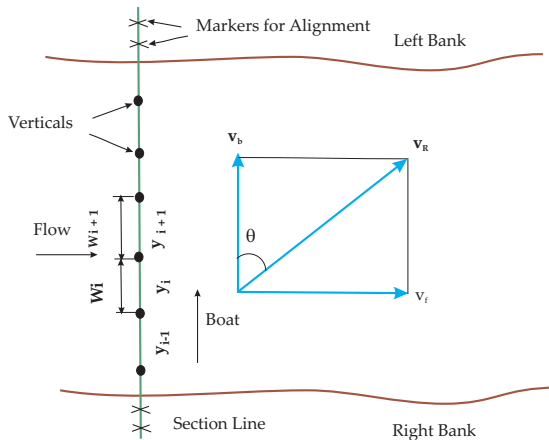
Distance from left water edge (m)	Depth (m)	Velocity at 0.2d (m/s)	Velocity at 0.8d (m/s)
0	0	0	0
1.5	1.3	0.6	0.4
3	2.5	0.9	0.6
4.5	1.7	0.7	0.5
6	1	0.6	0.4
7.5	0.4	0.4	0.3
0	0	0	0

Solution

Distance from left water edge (m)	Depth (m)	Velocity at 0.2d (m/s)	Velocity at 0.8d (m/s)	Average width (m)	Average velocity (m/s)	Discharge (m^3/s)
0	0	0	0	0	0	0
1.5	1.3	0.6	0.4	1.6875	0.5	1.096875
3	2.5	0.9	0.6	1.5	0.75	2.8125
4.5	1.7	0.7	0.5	1.5	0.6	1.53
6	1	0.6	0.4	1.5	0.5	0.75
7.5	0.4	0.4	0.3	1.6875	0.35	0.23625
9	0	0	0	0	0	0
						6.425625

Total Discharge = $6.425 \text{ m}^3/\text{s}$.

Measurement of discharge



Moving - boat method

Moving boat method

- In this method, a special propeller-type current meter which is free to move about a vertical axis is towed in a boat at a velocity v_b at right angles to the stream flow.
- If the flow velocity is v_f , the meter will align itself in the direction of the resultant velocity v_R making an angle θ with the direction of the boat.
- The meter will register the velocity v_R .
- If v_b is normal to v_f ,

$$v_b = v_R \cos \theta \text{ and } v_f = v_R \sin \theta$$

- If the time of transit between two verticals is Δt , then the width between the two verticals is

$$W = v_b \Delta t$$

Moving boat method

The flow in the area between two verticals i and $i+1$ where the depths are y_i and y_{i+1} respectively, by assuming the current meter to measure the average velocity in the vertical is

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2} \right) W_{i+1} v_f = \left(\frac{y_i + y_{i+1}}{2} \right) v_R^2 \sin \theta \cos \theta \Delta t$$

The summation of the partial discharges over the whole width of the stream gives the stream discharge

$$Q = \sum \Delta Q_i$$

Measurement of discharge

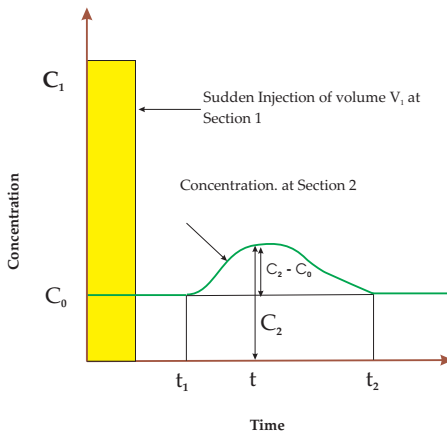
Example

Time (sec)	Angle from indicator θ (deg)	Velocity registered by current meter v_R (m/s)	Depth y (m)	Average θ (deg)	Average Depth (m)	Boat velocity (m/s)	Flow velocity (m/s)	ΔQ_i (m^3/s)
0	0		2.6					
30	21	0.534	3.1	10.5	2.85	0.525	0.097	4.367
60	23	0.8	4.6	22	3.95	0.742	0.3	26.331
90	27	1.134	6.3	25	5.45	1.028	0.479	80.502
120	35	1.284	8.1	31	7.2	1.101	0.661	157.167
150	31	1.167	7.6	33	7.85	0.979	0.635	146.46
180	28	0.717	6.5	29.5	7.05	0.624	0.353	46.585
210	20	0.65	4.3	24	5.4	0.594	0.264	25.423
240	15	0.35	1.8	17.5	3.05	0.334	0.105	3.213

$$\Sigma Q_i = 490.047$$

$$\text{Total Discharge} = 490.047 \text{ m}^3/\text{s}$$

Dilution Technique



Sudden Injection Method

Sudden Injection Method

The dilution method of flow measurement, also known as the chemical method depends upon the continuity principle applied to a tracer which is allowed to mix completely with the flow.

- A tracer is considered which does not react with the fluid or boundary.
- Let C_0 be the initial concentration of the tracer.
- At section 1, a small quantity (volume V_1) of high concentration C_1 of this tracer is added.
- Let section 2 be far away on the downstream of section 1 so that the tracer mixes thoroughly with the fluid while passing through the reach.
- The concentration will have a base value of C_0 , increases from time t_1 to a peak value and gradually reaches the base value at time t_2 .

Sudden Injection Method

By continuity of the tracer material,

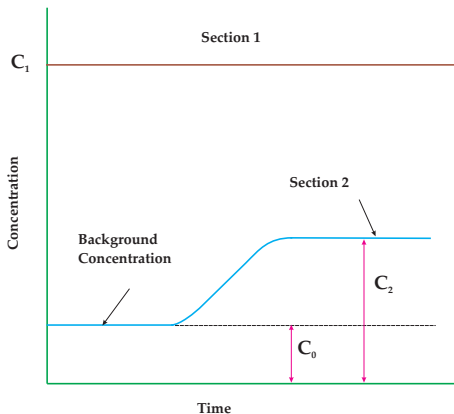
$$M_1 = \text{mass of tracer added at section 1} = V_1 C_1$$
$$= \int_{t_1}^{t_2} Q(C_2 - C_0) dt + \frac{V_1}{t_2 - t_1} \int_{t_1}^{t_2} (C_2 - C_0) dt$$

Neglecting the second term on the R.H.S. as insignificantly small,

$$Q = \frac{V_1 C_1}{\int_{t_1}^{t_2} (C_2 - C_0) dt}$$

Thus the discharge Q in the stream can be estimated if for a known M_1 , the variation of C_2 with time at section 2 and C_0 are determined.

Dilution Technique



Constant rate injection method

Constant rate injection method

- The tracer of concentration C_1 is injected at a constant rate Q_t at section 1.
- At section 2, the concentration gradually rises from the background value of C_0 to a constant value C_2 .
- At the steady state, the continuity equation for the tracer is

$$Q_t C_1 + Q C_0 = (Q + Q_t) C_2$$
$$Q = \frac{Q_t (C_1 - C_2)}{(C_2 - C_0)}$$

Tracers

Tracers are of three main types:

- Chemicals (common salt and sodium dichromate)
- Fluorescent dyes
- Radioactive materials

The tracer should have the following properties:

- It should not be absorbed by the sediment, channel boundary and vegetation. It should not chemically react with any one of the above surfaces and should not be lost by evaporation.
- It should be non-toxic.
- It should be capable of being detected in a distinctive manner in small concentrations.
- It should not be very expensive.

Example

A 25g/l solution of a fluorescent tracer was discharged into a stream at a constant rate of $10 \text{ cm}^3/\text{s}$. The background concentration of the dye in the stream water was found to be zero. At a downstream section sufficiently far away, the dye was found to reach an equilibrium concentration of 5 parts per billion. Estimate the stream discharge.

Solution

For the constant rate injection method,

$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - C_0)}$$

$$Q_t = 10 \text{ cm}^3/\text{s} = 10 \times 10^{-6} \text{ m}^3/\text{s}$$

$$C_1 = 0.025, C_2 = 5 \times 10^{-9}, C_0 = 0$$

$$Q = \frac{10 \times 10^{-6}}{5 \times 10^{-9}} (0.025 - 5 \times 10^{-9}) = 50 \text{ m}^3/\text{s}$$

Example

A 200 g/l solution of common salt was discharged into a stream of constant rate of 25 l/s. The background concentration of the salt in the stream water was found to be 10 ppm. At a downstream section where the solution was believed to be completely mixed, the salt concentration was found to reach an equilibrium value of 45 ppm. Estimate the discharge in the stream.

Solution

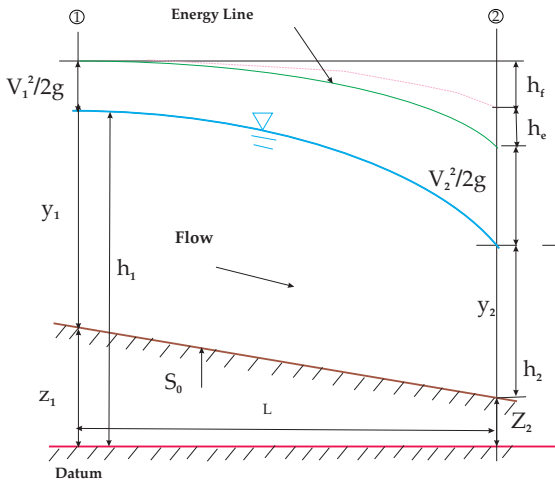
$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - C_0)}$$

$$Q_t = 25 \text{ l/s} = 25 \times 10^{-3} \text{ m}^3/\text{s}$$

$$C_1 = 0.2, C_2 = 45 \times 10^{-9}, C_0 = 10 \times 10^{-9}$$

$$Q = \frac{25 \times 10^{-3}}{(45 - 10) \times 10^{-6}} (0.2 - 45 \times 10^{-6}) = 143.73 \text{ m}^3/\text{s}.$$

Measurement of discharge



Slope Area Method

Slope Area Method

- The method is based on the principle of energy conservation.
- From Bernoulli's equation applied to the ends of a stream reach (sections 1 and 2),

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_L$$

where z is the elevation above the chosen datum, y is the depth of flow, V is the velocity of flow and h_L is the head loss in the reach.

- The head loss h_L can be considered to be made up of two parts, frictional loss h_f and eddy loss h_e .
- If the water surface elevation is h , then $h = z + y$

Slope Area Method

The equation can be expressed as

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_e + h_f$$

or

$$h_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) - h_e \quad (1)$$

If L = Length of the reach, by Manning's formula for uniform flow,

$$\frac{h_f}{L} = S_f = \text{Energy slope} = \frac{Q^2}{K^2}$$

where K = conveyance of the channel = $\frac{1}{n}AR^{2/3}$,

R = hydraulic radius and n = Manning's roughness factor.

In nonuniform flow, an average conveyance is used to estimate the average energy slope.

$$\frac{h_f}{L} = \overline{S_f} = \frac{Q^2}{K^2} \quad (2)$$

where $K = \sqrt{K_1 K_2}$, $K_1 = \frac{1}{n}A_1R_1^{2/3}$ and $K_2 = \frac{1}{n}A_2R_2^{2/3}$

Slope Area Method

The eddy loss is estimated as

$$h_e = K_e \left| \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) \right| \quad (3)$$

where K_e is the eddy loss coefficient.

Equation (1), (2) and (3) together with the continuity equation

$$Q = A_1 V_1 = A_2 V_2 \quad (4)$$

enable the discharge Q to be estimated for known values of h , cross-sectional properties and n .

Slope Area Method

The discharge is calculated by a trial and error procedure using the following steps:

- Assume $V_1 = V_2$. This leads to $\frac{V_1^2}{2g} = \frac{V_2^2}{2g}$
- Then from equation (1), $h_f = h_1 - h_2 = F =$ fall in water surface between sections 1 and 2.
- Use equation (2) to calculate discharge Q .
- Compute $V_1 = Q/A_1$ and $V_2 = Q/A_2$. Calculate velocity heads and eddy loss.
- Calculate a refined value of h_f by eqn. (1) and go to step 3.
- Repeat the calculations till two successive calculations give values of h_f differing by a negligible margin.

Example

During a flood flow, the depth of water in a 10 m wide rectangular channel was found to be 3.0 m and 2.9 m at two sections 200 m apart. The drop in the water surface elevation was found to be 0.12 m. Assume Manning's coefficient to be 0.025, estimate the flood discharge through the channel.

Solution

The cross- sectional properties at two sections are calculated as follows:

Section 1

$$y_1 = 3.0m, A_1 = 3 \times 10 = 30m^2$$

$$P_1 = 10 + 2 \times 3 = 16m, R_1 = A_1/P_1 = 1.875m$$

$$K_1 = \frac{1}{0.025} \times 30 \times (1.875)^{2/3} = 1824.7$$

Section 2

$$y_2 = 2.9m, A_2 = 2.9 \times 10 = 29m^2$$

$$P_2 = 10 + 2 \times 2.9 = 15.8m, R_2 = A_2/P_2 = 1.835m$$

$$K_2 = \frac{1}{0.025} \times 29 \times (1.835)^{2/3} = 1738.9$$

$$\text{Aveage K for the reach} = \sqrt{K_1 K_2} = 1781.3$$

Solution

To start with, $h_f = \text{fall} = 0.12 \text{ m}$ is assumed.

Eddy loss $h_e = 0$

$$\bar{S}_f = h_f/L = h_f/200$$

$$Q = K\sqrt{S_f} = 1781.3\sqrt{S_f}$$

$$V_1^2/2g = (Q/30)^2/19.62, V_2^2/2g = (Q/29)^2/19.62$$

$$h_f = h_1 - h_2 + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_f = 0.12 + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

Trial	h_f (trial)	S_f (10^{-4})	Q (m^3/s)	$V_1^2/2g$ (m)	$V_2^2/2g$ (m)	h_f (m)
1	0.1200	6.000	43.63	0.1078	0.1154	0.1124
2	0.1124	5.615	42.21	0.1009	0.1080	0.1129
3	0.1129	5.645	42.32	0.1014	0.1085	0.1129

The discharge in the channel is $42.32 \text{ m}^3/\text{s}$.

Example

A small stream has a trapezoidal cross-section with base width of 12 m and side slope 2 horizontal : 1 vertical in a reach of 8 km.

During a high flood the high water levels record at the ends of the reach are given below.

Section	Elevation of bed (m)	Water surface elevation (m)	Remarks
Upstream	100.20	102.70	Manning's $n = 0.030$
Downstream	98.60	101.30	

Estimate the discharge in the stream.

Solution

The cross- sectional properties at two sections are calculated as follows:

Section 1

$$y_1 = 102.70 - 100.20 = 2.5m$$

$$A_1 = (b_1 + zy_1)y_1 = (12 + 2 \times 2.5)2.5 = 42.5m^2$$

$$P_1 = (b_1 + zy_1)\sqrt{(1 + z^2)} = (12 + 2 \times 2.5)\sqrt{5} = 23.1803m$$

$$K_1 = \frac{1}{0.030} \times 42.5 \times (42.5/23.1803)^{2/3} = 2126.464$$

Section 2

$$y_2 = 101.3 - 98.6 = 2.7m, A_2 = (12 + 2 \times 2.7)2.7 = 46.98m^2$$

$$P_2 = (12 + 2 \times 2.7)\sqrt{5} = 24.0747m$$

$$K_2 = \frac{1}{0.030} \times 46.98 \times (46.98/24.0747)^{2/3} = 2450.907$$

$$\text{Aveage K for the reach} = \sqrt{K_1 K_2} = 2282.929$$

Solution

To start with, $h_f = \text{fall} = 102.7 - 101.3 = 1.4 \text{ m}$ is assumed.

Eddy loss $h_e = 0$

Trial	h_f	Q	h_f
1	1.4	30.20031	1.404674
2	1.404674	30.25069	1.404690

m^3/s .

The discharge in the channel is 30.25

Selection of Reach

- The quality of high water marks should be good.
- The reach should be straight and uniform as far as possible.
- The recorded fall in the water surface elevation should be larger than the velocity head. It is preferable if the fall is greater than 0.15 m.
- The longer the reach, the greater is the accuracy in the estimated discharge. A length greater than 75 times the mean depth provides an estimate of the reach length required.

Stage-Discharge Relationship

Once the stage-discharge (G-Q) relationship is established, the subsequent procedure consists of measuring the stage (G) and reading the discharge (Q) from the (G-Q) relationship. The relationship is also known as the *rating curve*.

- The measured value of discharges when plotted against the corresponding stages given relationship that represents the integrated effect of a wide range of channel and flow parameters.
- The combined effect of these parameters is termed *control*.
- If the (G-Q) relationship for a gauging section is constant and does not change with time, the control is said to be *permanent*.
- If it changes with time, it is called *shifting control*.

Stage-Discharge Relationship

Permanent Control

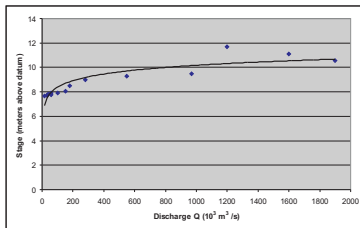
A majority of streams and rivers, exhibit permanent control. For such a case, the relationship between the stage and the discharge is a single-valued relation which is expressed as

$$Q = C_r(G - a)^\beta \quad (5)$$

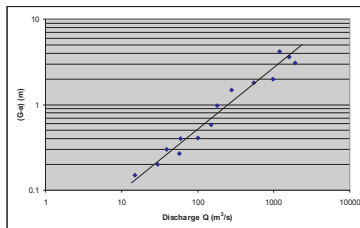
in which Q = stream discharge, G = gauge height(stage), a = a constant which represent the gauge reading corresponding to zero discharge, C_r and β are rating curve constants.

The relationship can be expressed graphically by plotting the observed stage against the corresponding discharge values in an arithmetic or logarithmic plot. Logarithmic plotting is advantageous as it plots as a straight line in logarithmic coordinates.

Measurement of discharge



Stage - discharge curve : arithmetic plot



Stage - discharge curve : logarithmic plot

Stage-Discharge Relationship

Permanent Control

The best values of C_r and β in equation (5) for a given range of stage are obtained by least square method. Thus by taking logarithms,

$$\log Q = \beta \log(G - a) + \log C_r \quad (6)$$

$$\text{or } Y = \beta X + b \quad (7)$$

in which dependent variable $Y = \log Q$, independent variable $X = \log(G - a)$ and $b = \log C_r$.

For the best-fit straight line of N observations of X and Y , by regressing $X = \log(G - a)$ on $Y = \log Q$

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2} \quad (8)$$

$$b = \frac{\sum Y - \beta(\sum X)}{N} \quad (9)$$

Stage-Discharge Relationship

The coefficient of correlation is given by

$$r = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{(N(\sum X^2) - (\sum X)^2)(N(\sum Y^2) - (\sum Y)^2)}} \quad (10)$$

For a perfect correlation, $r = 1.0$. If r is between 0.6 and 1.0 it is generally taken as a good correlation.

Stage-Discharge Relationship

Stage for zero discharge, a

In the equation, constant a represents the stage for zero discharge in the stream is a hypothetical parameter and can not be measured in the field. The following alternative methods are available for its determination :

Method 1

- Plot Q vs G on arithmetic graph paper and draw a best fit curve.
- By extrapolating the curve by eye judgement, find a as the value of G corresponding to $Q = 0$.
- Using the value of a , plot $\log Q$ vs $\log (G-a)$ and verify whether the data plots as a straight line.
- If not, select another value in the neighbourhood of previously assumed value and by trial and error, find the acceptable value of a which gives a straight line plot of $\log Q$ vs $\log (G-a)$.

Stage-Discharge Relationship

Stage for zero discharge, a

Method 2

A graphical method due to Running is as follows:

- The Q vs G data are plotted to an arithmetic scale.
- A smooth curve through the plotted points are drawn.
- Three points A, B and C on the curve are selected such that their discharges are in geometric progression, i.e,

$$\frac{Q_A}{Q_B} = \frac{Q_B}{Q_C}$$

- At A and B, vertical lines are drawn and then horizontal lines are drawn at B and C to get D and E as intersection points with the verticals.
- Two straight lines ED and BA are drawn to intersect at F.
- The ordinate at F is the required value of a.

Stage-Discharge Relationship

Stage for zero discharge, a

Method 3

- Plot Q vs G on arithmetic scale and draw a smooth good fitting curve by eye judgement.
- Select three discharges Q_1, Q_2, Q_3 such that $Q_1/Q_2 = Q_2/Q_3$
- Note from the curve the corresponding value of gauge readings G_1, G_2, G_3 .

From equation (5) , $(G_1 - a)/(G_2 - a) = (G_2 - a)/(G_3 - a)$

$$a = \frac{G_1 G_3 - G_2^2}{(G_1 + G_3) - 2G_2} \quad (11)$$

Example

Following are the data of gauge and discharge collected at a particular section of the river by stream gauging operation. (a) Develop a gauge - discharge relationship for this stream at this section for use in estimating the discharge for a known gauge reading. What is the coefficient of correlation of the derived relationship? Use a value of $a = 7.5$ m for the gauge reading corresponding to zero discharge. (b) estimate the discharge corresponding to gauge reading of 10.5 m at this section.

Gauge reading (m)	Discharge (m^3/s)
7.65	15
7.70	30
7.77	57
7.80	39
7.90	60
7.91	100
8.08	150
8.48	180
8.98	280
9.30	550
9.50	970
10.50	1900
11.10	1600
11.70	1200

Measurement of discharge

Solution

Gauge G (m)	(G-a)	Q (m^3/s)	X = log (G-a)	Y = log Q	XY
7.65	0.15	15	-0.824	1.177	-0.97
7.7	0.2	30	-0.699	1.478	-1.034
7.77	0.27	57	-0.569	1.756	-1
7.8	0.3	39	-0.523	1.592	-0.833
7.9	0.4	60	-0.398	1.779	-0.709
7.91	0.41	100	-0.388	2	-0.776
8.08	0.58	150	-0.237	2.177	-0.516
8.48	0.98	180	-0.009	2.256	-0.021
8.98	1.48	280	0.171	2.448	0.419
9.3	1.8	550	0.256	2.741	0.702
9.5	2	970	0.302	2.987	0.903
10.55	3.05	1900	0.485	3.279	1.591
11.1	3.6	1600	0.557	3.205	1.786
11.7	4.2	1200	0.624	3.08	1.922

Solution

$$\sum X = -1.2545, \sum Y = 31.9460, \sum XY = 1.456$$

$$\sum X^2 = 3.2457, \sum Y^2 = 79.08$$

$$\sum X^2 = 1.5378, \sum Y^2 = 1020.5$$

$$\text{By using eqn (8), } \beta = 1.378$$

$$\text{From eqn. (9), } b = 2.4054 = \log(C_r)$$

$$C_r = 254.3$$

$$\text{The required relationship is therefore } Q = 254.3(G - a)^{1.378}$$

$$\text{From equn. (10), } r = 0.981$$

Shifting Control

When the control at a gauging station changes, its rating curve changes. The change may result from

- Scour or deposition
- variable backwater effects
- unsteady flow effects of a rapidly changing stage
- changes in flow caused by dredging, channel encroachment and weed growth etc.

For the shifting control due to (1) and (4), frequent current meter gaugings and updating the rating curve are required. The effect of backwater and unsteady flow is amenable to analytical treatment.

Shifting Control

Backwater effect

- The backwater may develop as a result of an obstruction downstream or high stages in an intersection stream and may be variable in time.
- Under the conditions of shifting control due to backwater effects, a given stage will indicate different discharge values which results from difference in water surface slope at the control.
- The discharge is estimated by establishing another gauge called auxillary gauge some distance downstream of the main gauging station.

Shifting Control

Backwater effect

- The difference between the main gauge and the auxillary gauge gives the fall of water surface in the reach.
- For a given main stage reading, the discharge under variable backwater condition is a function of the stage (at main gauge) and fall F i.e, $Q = f(G,F)$.
- Instead of having a three paramter plot, the observed data is normalized with respect to a constant fall value.

Shifting Control

Backwater effect

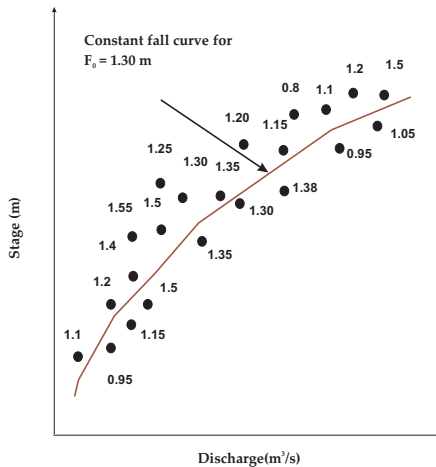
Let F_0 be a normalizing value of the fall taken to be constant at all stages and F is the actual fall at a given stage when the actual discharge is Q .

$$\frac{Q}{Q_0} = \left(\frac{F}{F_0} \right)^m \quad (12)$$

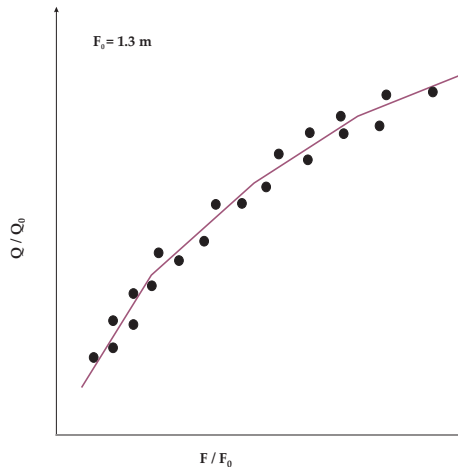
Q_0 = Normalized discharge at the given stage when the fall is F_0 , m is an exponent value close to 0.5.

- The value of F is computed for each discharge measurement.
- If the variation in observed F values is not high, an average value of F is computed and this average value is taken as F_0 .
- All observed values of stage (G) and discharge values (Q) for values of $F = F_0$ are plotted as a constant fall rating curve.

Measurement of discharge



Measurement of discharge



Adjustment curve

Shifting Control

Backwater effect

- For each observed data, $\frac{Q}{Q_0}$ and $\frac{F}{F_0}$ values are calculated and plotted as $\frac{Q}{Q_0}$ vs $\frac{F}{F_0}$. This is called adjustment curve.
- Both the constant fall curve and adjustment curve are refined, by trial and error to get the best fit curves.
- If the observed stage is G_1 and fall F_1 , first by using the adjustment curve, the value of $\frac{Q_1}{Q_0}$ is read for a known value of $\frac{F_1}{F_0}$.
- Using the constant fall rating curve, for the given stage G_1 , Q_0 is read and the actual discharge is calculated as $\frac{Q_1}{Q_0} \times Q_0$.

Example

An auxillary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effects. The following data were noted.

Main Gauge (m above datum)	Auxillary Gauge (m above datum)	Discharge (m^3/s)
86.00	85.50	275
86.00	84.80	600

If the main gauge reading is still 86.0 m and the auxillary gauge reads 85.30 m, estimate the discharge in the river.

Solution

Fall = Main gauge reading - Auxillary gauge reading

When $F_1 = (86.0 - 85.50) = 0.50$ m , $Q_1 = 275$ m³/s.

$F_2 = (86.0 - 84.80) = 1.20$ m , $Q_2 = 600$ m³/s.

$$\begin{aligned}\frac{Q_1}{Q_2} &= \left(\frac{F_1}{F_2}\right)^m \\ (275/600) &= (0.50/1.20)^m \\ m &= 0.891\end{aligned}$$

When the auxillary gauge reads 85.30 m, at a main gauge reading of 86.00 m,

Fall $F = (86.0 - 85.30) = 0.70$ m and

$$Q = Q_2 \left(\frac{F}{F_2}\right)^m = 600(0.70/1.20)^{0.891} = 371 \text{ m}^3/\text{s}.$$

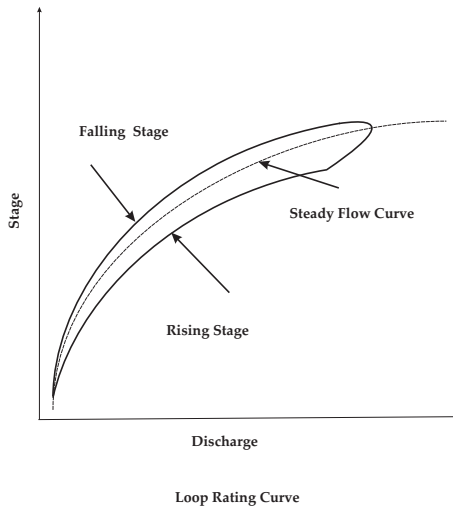
Shifting Control

Unsteady Flow Effect

When a flood wave passes a gauging station in the advancing portion of the wave, the approach velocities are larger than in steady flow at corresponding stage.

- For the same stage, more discharge than in a steady uniform flow occurs.
- In the retreating phase of the flood wave the converse situation occurs with reduced approach velocities giving lower discharges than in a equivalent steady flow case.
- So the stage-discharge relationship for an unsteady flow will not be a single-valued relationship as in steady flow but it will be a looped curve.
- At the same stage, more discharge passes through the river during rising stages than in falling ones.
- Different flood will give different loops.

Measurement of discharge



Shifting Control

Unsteady Flow Effect

If Q_n is the normal discharge at a given stage under steady uniform flow and Q_M is the measured (actual) unsteady flow, it can be expressed as

$$\frac{Q_M}{Q_n} = \sqrt{1 + \frac{1}{V_w S_0} \frac{dh}{dt}} \quad (13)$$

where S_0 = Channel Slope = Water surface slope at uniform flow

$\frac{dh}{dt}$ = Rate of change of stage, V_w = Velocity of flood wave.

For natural channels, V_w is assumed equal to $1.4V$, where V = average velocity for a given stage is estimated by applying Manning's formula and the energy slope S_f .

- The energy slope is used in place of S_0 in the equation.
- If enough field data about the flood magnitude and $\frac{dh}{dt}$ are available, the term $\frac{1}{V_w S_0}$ can be calculated and plotted against the stage for use in the equation.
- For estimating the actual discharge at an observed stage, $\frac{Q_M}{Q_n}$ is calculated by using the observed data of $\frac{dh}{dt}$.

Shifting Control

Unsteady Flow Effect

Another method is to construct the rating curve for those observations for which the flow is uniform. Assume V_w to be a constant, a plot of $\frac{Q_M}{Q_n}$ vs $\frac{dh}{dt}$ is prepared which serves as a correction curve. Thus for a given stage, the discharge is first obtained from the rating curve and $\frac{dh}{dt}$ is computed for which the actual discharge is obtained from the correction curve.