# Streamflow Measurement

# Introduction

### Streamflow

A *stream* is defined as an open channel that carries the surface runoff from a catchment. The discharge it carries is the streamflow. In fact, streamflow is the output or response of the catchment due to a precipitaion over it.

### Streamflow Measurement

In surface water hydrology, the most important task is to estimate the amount of water available from a catchment due to a storm. Runoff from a catchment sue to a storm can be estimated approximately if information about the loss parameters like evaporation, infiltration etc. are available. However, precise measurement of these parameters are difficult and the adopted methods are having their own limitations. Also, it is very difficult to estimate the variation of surface runoff with time.

In contrast with this, direct measurement of the streamflow is relatively easier and accurate.

# Measurement Techniques

Streamflow measurement techniques can be broadly classified as

- Direct determination methods
- Indirect determination methods

**Direct methods**

1. Area-velocity method
2. Dilution technoque
3. Electromagnetic method
4. Ultrasonic method

**Indirect methods**

1. Slope-area method
2. Hydraulic structure

# Stage-Velocity-Discharge



Continuous measurement of stream dischrage is very difficult, time cosuming and costly process. Hence, following procedure is used.

1. First, a series of careful measurements of discharge and corresponding water surface elevation (known as stage) are performed for the stream.
2. Then, a stage-discharge relationship is obtained for the measured set of data.
3. Next time onwards, only the stage is measured routinely. The discharge is obtained from the derived stage-discharge relationship.

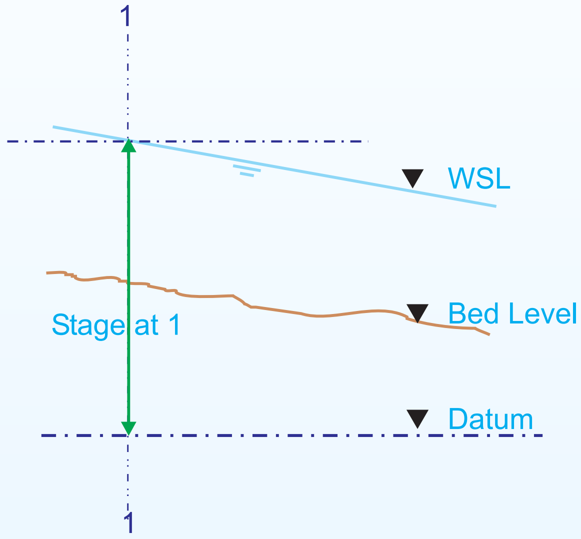
Usually, in step 1 above, discharge is not measured directly. Instead, the stream velocity is measured. Discharge is obtained by multiplying the flow area (obtained from measured stage) by this velocity.

# Measurement of Stage

# Stage Recorder



The stage of a stream is defined as the elevation of the water surface measured above a datum. This datum can be the mean-sea level (MSL) or any other pre-defined level.




**Types**

*Manual Stage Recorder*

- Staff Gauge
- Wire Gauge

*Automatic Stage Recorder*

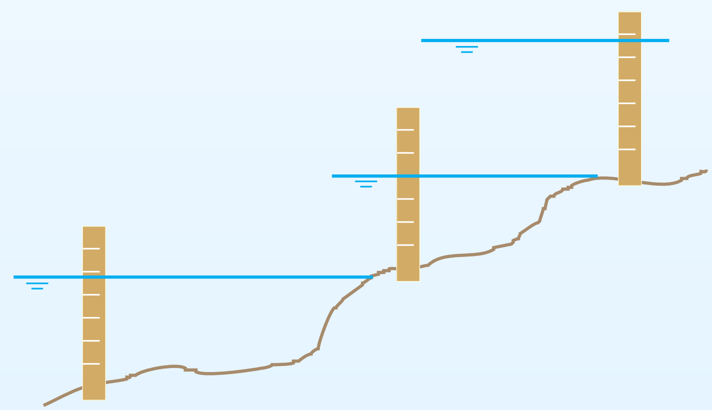
- Float Gauge
- Bubble Gauge

# Staff Gauge

The simplest method of stage measurement is the use of a graduated staff. Is is usually fixed rigidly to a structure like pier, wall etc. It may be

- Vertical
- Inclined
- Sectional

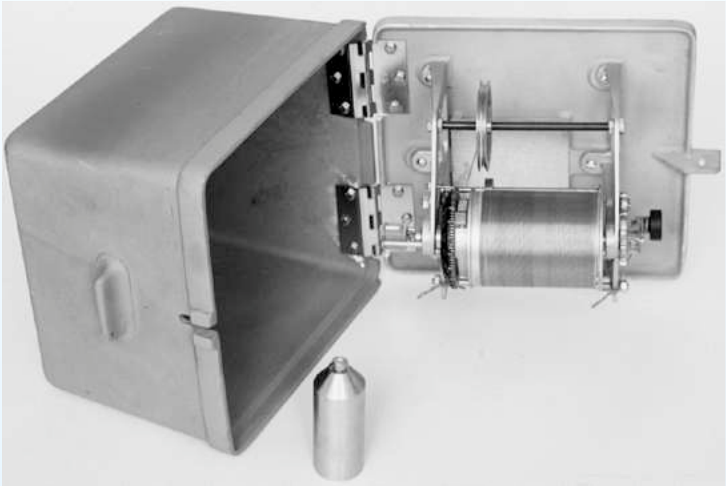
Vertical Staff



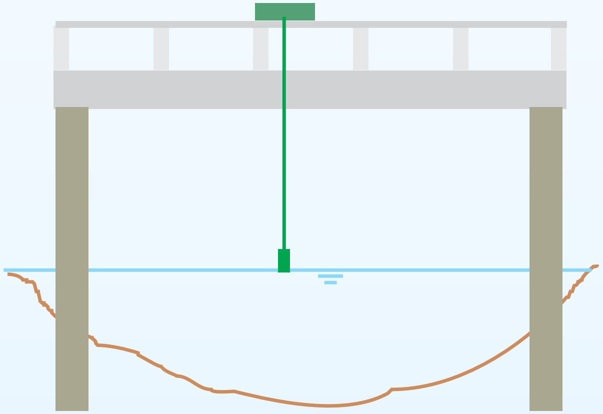
Sectional Gauge

# Wire Gauge

Wire gauge is used to measure the water surface elevation from a structure above water, like a bridge deck. In this a weight is lowered by a reel to touch the water surface. A mechanical counter measures the rotation of the wheel which is proportional to the length of the wire paid out.



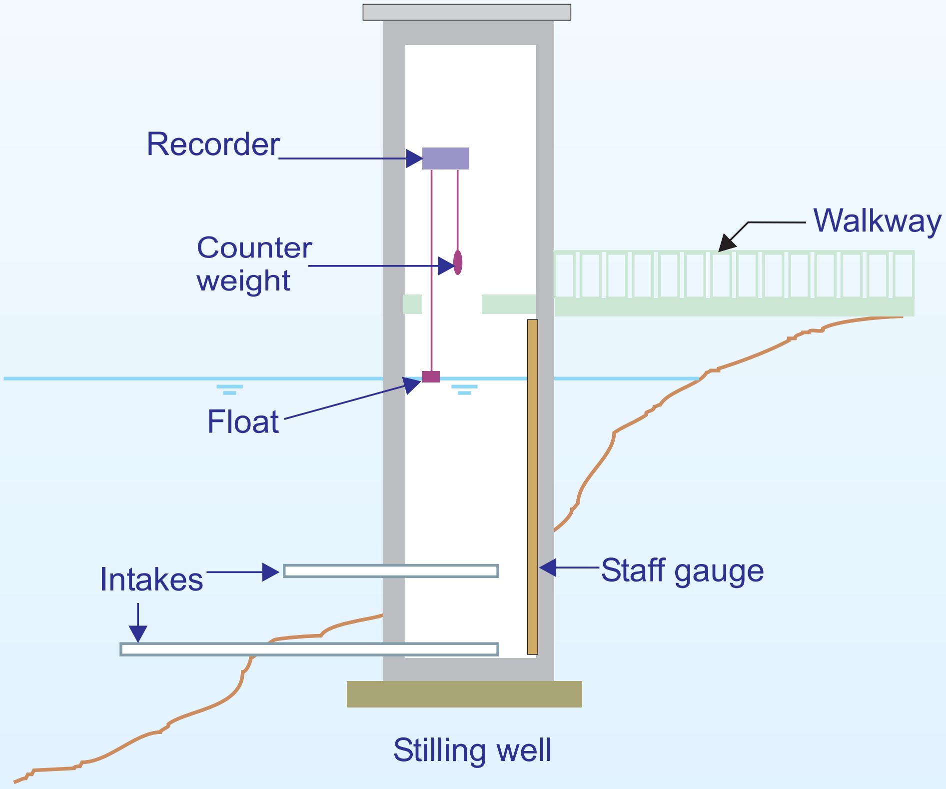
Mechanical counter & weight



Working

# Float Gauge

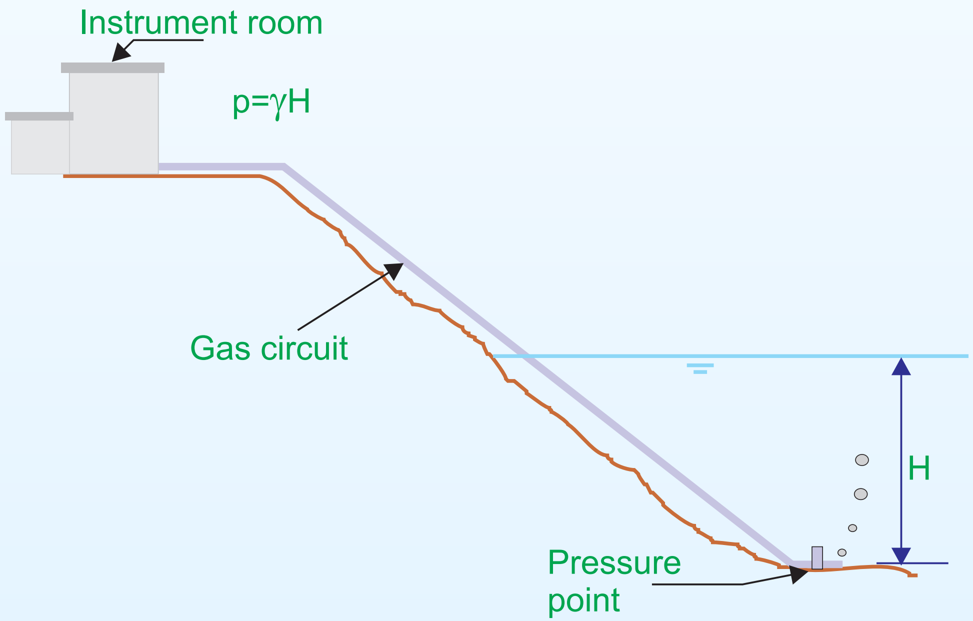
The float gauge is the most common type of automatic stage recorder. In this a float is operating in a stilling well by means of a counterweight over the pulley of a recorder. Displacement of the float due to the rising or lowering of the water surface elevation causes an angular displacement of the pulley. This angular displacement is converted into vertical movement by a mechanical clockwork. A digital converter is also sometimes used. This recorder gives a continuous plot of fluctuation in water surface level.



# Bubble Gauge



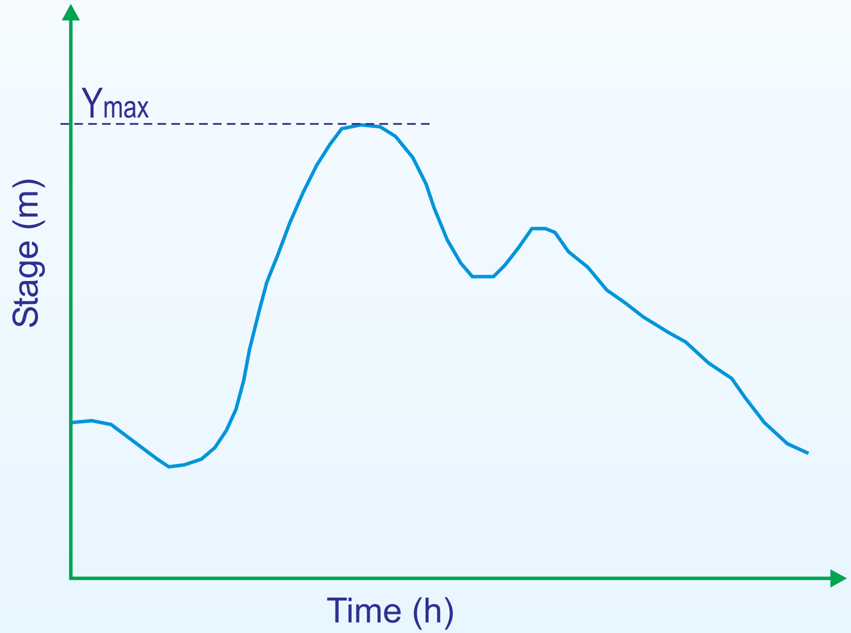
In this gauge, compressed air or gas is made to bleed out at a very small rate through an outlet placed at the bottom of the river. A pressure gauge measures the gas pressure which in turn is equal to the water column above the outlet. A small change in the water surface elevation is felt as a change in pressure from the present value at the pressure gauge and this in turn is adjusted by a servo-mechanism to bring the gas bleed at the original rate under the new head. The pressure gauge reads the new water depth which is transmitted to a recorder.




The bubble gauge setup costs much less than float gauges and it the recorder assembly can be installed far away from the sensing point. Also, there is less chance of blocking or choking at the outlet.

# Stage Hydrograph

The stage data is usually presented in the form of a plot of stage against chronological time. This plot, known as *stage hydrograph*, is very important in flood warning and flood warning works. It is also useful in design of hydraulic structures like bridge, weir, etc.




Stage Hydrograph

# Measurement of Velocity

# Current Meter

The most commonly used instrument to measure the velocity of a stream at a particular cross section is the current meter. It essentially consists of a rotating element which rotates due to the reaction of the stream current with an angular velocity proportional to the stream velocity. Robert Hooke (1663) invented the propeller-type current meter and present-day cup-type current meter was invented by Henry in 1868. There are two main types of current meters

1. Vertical-axis meters

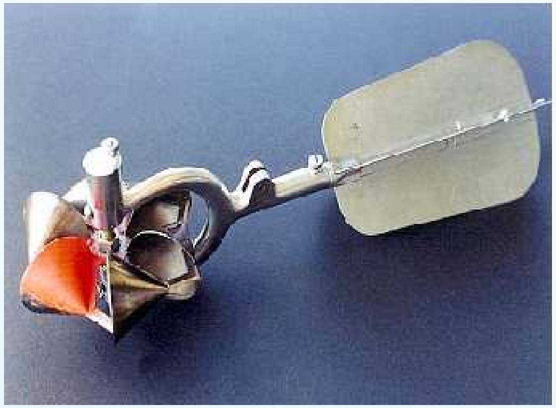
2. Horizontal-axis meters

# Vertical-axis Meters



These instruments consists of a series of conical cups mounted around a vertical axis. The cups rotate in a horizontal plane and a cam attached to the vertical axis spindle records generated signals proportional to the revolutions of the cup assembly.

The *Price current meter* and *Gurley current meters* are two examples of vertical-axis meters.



Note:

- The normal range of velocities that can be measured is from 0.15 m/s to 4.0 m/s.
- The accuracy of these instruments is about 0.3% at speeds higher than 1.0 m/s.
- These cannot be used where there is appreciable vertical component of velocities.

# Horizontal-axis Meters

These meters consist of a propeller mounted at the end of a horizontal shaft.

*Ott current meter* and *Neyrtec current meter* are two examples of horizontal-axis current meters.

Note:

- These instruments can register velocities in the range of 0.15 m/s to 4.0 m/s.
- The accuracy of these instruments is about 1% at the threshold value and about 0.25% at velocities higher than 3 m/s.
- These meters are not affected by oblique flows of as much as $15^\circ$.

# Relationship



A current meter is so designed that its rotation speed varies linearly with the stream velocity $v$ at the location of the instrument. A typical relationship is

$$v = a_s + b$$

where

$v$ = stream velocity at the instrument location in m/s,

$N_s$ = revolutions per second of the meter,

$a, b$ = constants of the meter. Typical values of a and be are: $a$ = 0.65 and $b$ = 0.033.

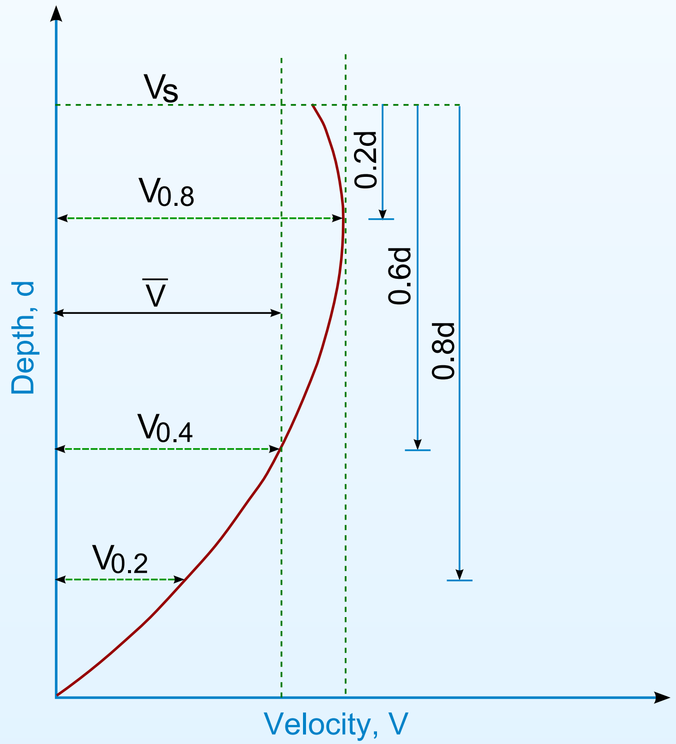
Note: Each instrument has a threshold velocity below which the above equation is not applicable.

**Calibration**

The relationship between the stream velocity and revolutions per second of the meter, as shown above, is called the calibration equation. The calibration equation is unique to each instrument and is determined by towing the instrument in a special tank known as *Towing tank*.

# Average Velocity

The velocity in an open channel at any vertical varies from zero at the bottom to a maximum value little below the water surface. It can be shown that the average velocity $\bar{V}$ for the vertical is approximately equal to the velocity at $0.4d$ from bottom or ($0.6d$ from top). It is also observed that the arithmetic average of the velocities at $0.2d$ and $0.8d$ is approximately equal to the average velocity, $\bar{V}$. This $\bar{V}$ can also be approximated as $0.85\,V_s$.



## Field Use

As seen earlier, velocity distribution in a stream across a vertical section is logarithmic in nature. To accurately determine the average velocity in a vertical cross section, one has to measure the velocity at a large number of points on the vertical. This is time consuming and costly. Hence, in practice, following procedures are used.

1. In shallow streams of depth $\leq$ 3.0 m, the velocity is measured at a depth 0.6d (*d = depth of flow*) below the water surface is taken as the average velocity in the vertical.

$$\bar{v} = v_{0.6}$$

2. In moderately deep streams, velocity is observed at two points: i) at 0.2d and ii) at 0.8d, below the water surface. The average velocity in the vertical is calculated as

$$\bar{v} = \frac{v_{0.2} + v_{0.8}}{2}$$

3. In rivers having flood flows, only the surface velocity, $v_s$ is measured within a depth of about 0.5 m below the surface. The average velocity $\bar{v}$ is obtained by muliplying it with a factor $K$ $(0.85 \leq K \leq 0.95)$ as

$$\bar{v} = K\, v_s$$

# Estimation of Discharge

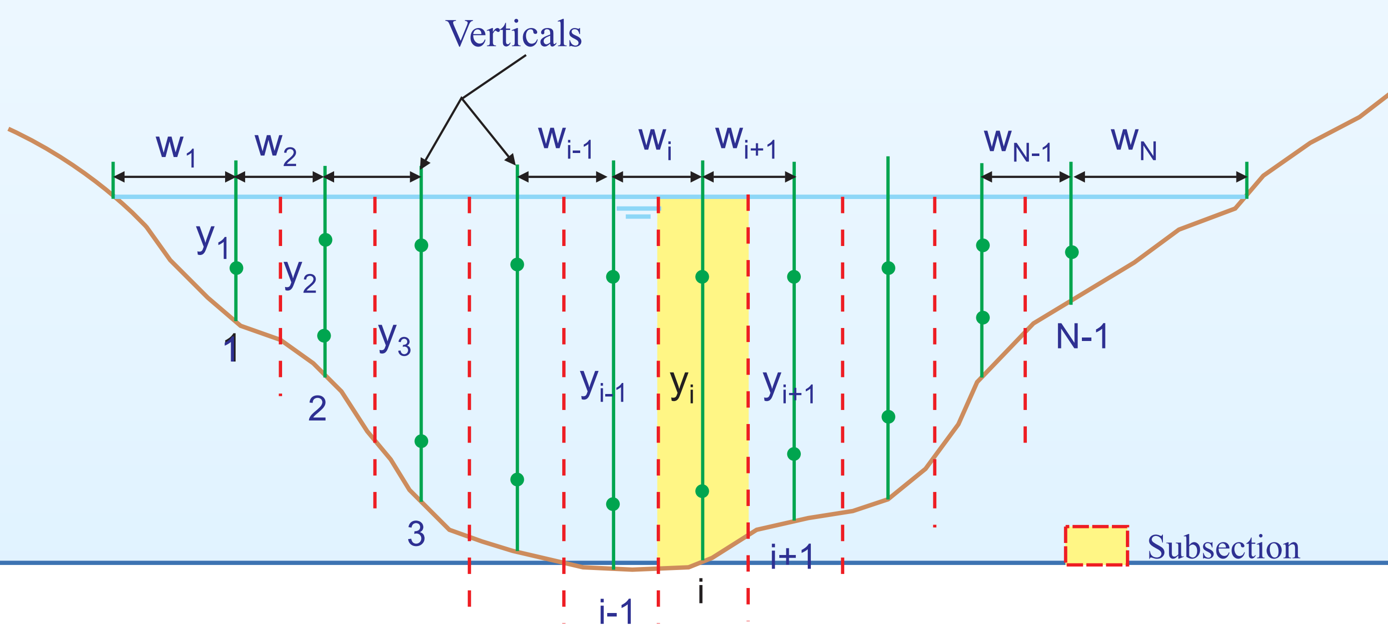
# Direct Methods

# Area-Velocity Method



This method essentially consists of measuring the depth of flows and velocities at a number of verticals within the cross section.

- The section is divided into a number of verticals ($i = 1, \ldots, N-1$).
- Depth of flow at various verticals are measured ($y_i, i = 1, \ldots, N-1$).
- Velocities are measured at 0.6d or 0.2d and 0.8d, depending on d. ($V_i, i = 1, \ldots, N-1$).
- Distance between two verticals are measued ($W_i, i = 1, \ldots, N$).
- Subsections are formed taking half-widths from both sides of a vertical.



# Calculation of Discharge

There are $1,\ 2, \ldots, (N-2), (N-1)$ sub-sections. First consider the itermediate sub-sections, $i = 2, \ldots, (N-2)$. For each sub-section $i$,

$\Delta Q_i = \Delta A_i \times V_i$ and $\Delta A_i = y_i \times \left( \frac{W_i}{2} + \frac{W_{i+1}}{2} \right)$

For the first sub-section ($i = 1$), consider it as a triangle.

Its width = $[W_1 + \frac{W_2}{2}]$ and height = $\frac{y_1}{W_1}[W_1 + \frac{W_2}{2}]$. So,

$$\Delta A_1 = \frac{1}{2} \times \frac{y_1}{W_1} \left( W_1 + \frac{W_2}{2} \right)^2 = y_1 \times \frac{1}{2W_1} \left( W_1 + \frac{W_2}{2} \right)^2 = y_1 \times \overline{W}_1$$

$$\Delta Q_1 = \Delta A_1 \times V_1$$

Similarly assuming the last sub-section ($i = N - 1$) as a triangle,

$$\Delta A_{N-1} = y_{N-1} \times \frac{1}{2W_N} \left( W_N + \frac{W_{N-1}}{2} \right)^2 = y_{N-1} \times \overline{W}_{N-1}$$

$$\Delta Q_{N-1} = \Delta A_{N-1} \times V_{N-1}$$

Total discharge through the section = $\sum \Delta Q_i \qquad i = 1, \ldots, N-1$

# Example

The data pertaining to a stream-gauging operation at a gauging site are given below. The rating equation of the current meter is $v = 0.51\,N_s + 0.03$ m/s. Calculate the discharge in the stream using area-velocity method.

| Distance from Left water edge (m) | Depth (m) | Current meter revolutions at 0.6d | Duration of observations (s) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1.0 | 1.1 | 39 | 100 |
| 3.0 | 2.0 | 58 | 100 |
| 5.0 | 2.5 | 112 | 150 |
| 7.0 | 2.0 | 90 | 150 |
| 9.0 | 1.7 | 45 | 100 |
| 11 | 1.0 | 30 | 100 |
| 12.0 | 0 | 0 | 0 |

# Solution

1. Calculate widths: $W_1 = 1 - 0 = 1; W_2 = 3 - 1 = 2; W_3 = 5 - 3 = 2; W_4 = 2$ ; $W_5 = 2; W_6 = 1$

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2. Calculate average widths: $\overline{W}_1 = \frac{1+(W_2/2)]^2}{2W_1} = \frac{[1+2/2]^2}{2\times 1} = 2m$ ; $\overline{W}_6 = \frac{6+(W_5/2)]^2}{2W_6} = \frac{[1+2/2]^2}{2\times 1} = 2m$ ; $\overline{W}_2 = (2+2)/2 = 2m$

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3. Calculate velocities: $v = 0.51\ N_s + 0.03$; For 1st vertical, $N_s = 39/100 = 0.39$ rps; $v_1 = 0.51 \times 0.39 + 0.03 = 0.2289$ m/s

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4. Calculate discharges:
$\Delta Q_1 = \Delta A_1 \times v_1 = y_1 \times \overline{W}_1 \times v_1 = 1.1 \times 2 \times 0.2289 = 0.5036$ m$^3$/s.

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| Distance from Left water edge (m) | Depth (m) | Average Width (m) | Revolutions per sec | Velocity, $v$ (m/s) | $\Delta Q$ (m$^3$/s) |
|---|---|---|---|---|---|
| 0 | 0 | | | | |
| 1 | 1.1 | 2 | 0.390 | 0.2289 | 0.5036 |
| 3 | 2.0 | 2 | 0.580 | 0.3258 | 1.3032 |
| 5 | 2.5 | 2 | 0.747 | 0.4110 | 2.0550 |
| 7 | 2.0 | 2 | 0.600 | 0.3360 | 1.3440 |
| 9 | 1.7 | 2 | 0.450 | 0.2595 | 0.8823 |
| 11 | 1.0 | 2 | 0.300 | 0.1830 | 0.3660 |

# Solution



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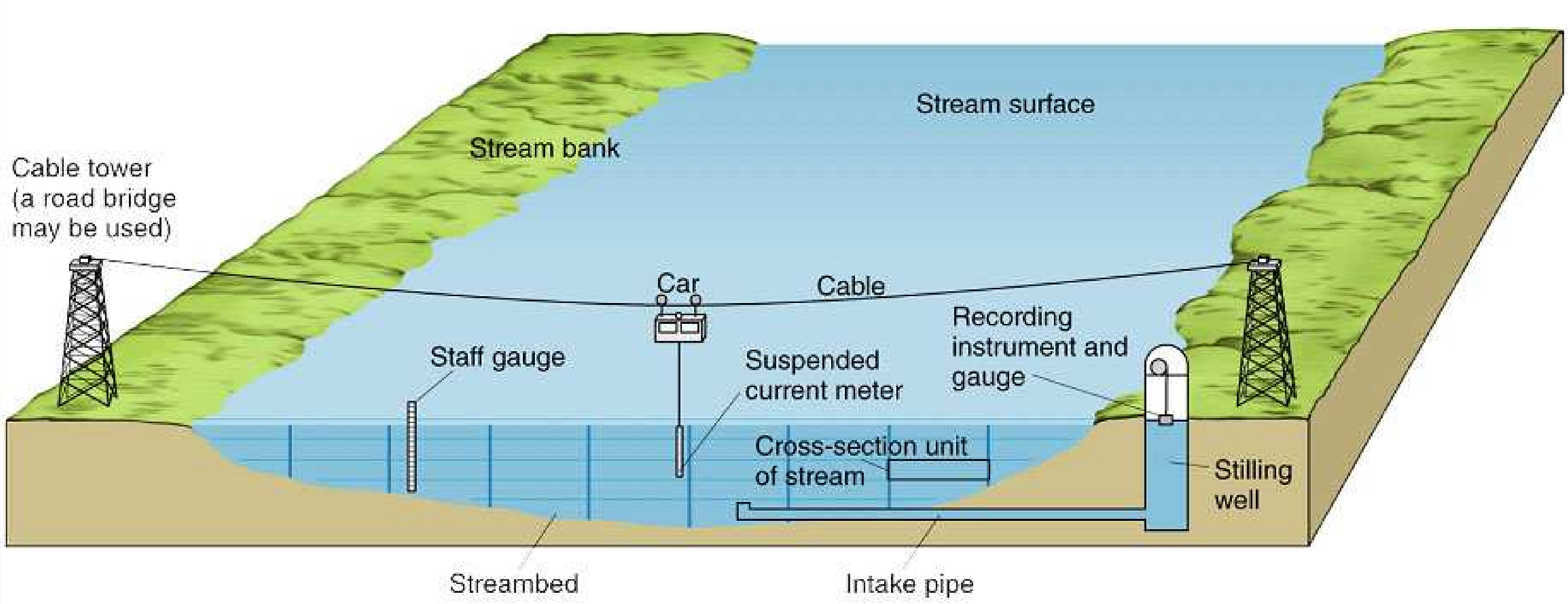
# Measurement at Site



# Criteria



### Selection of gauging site

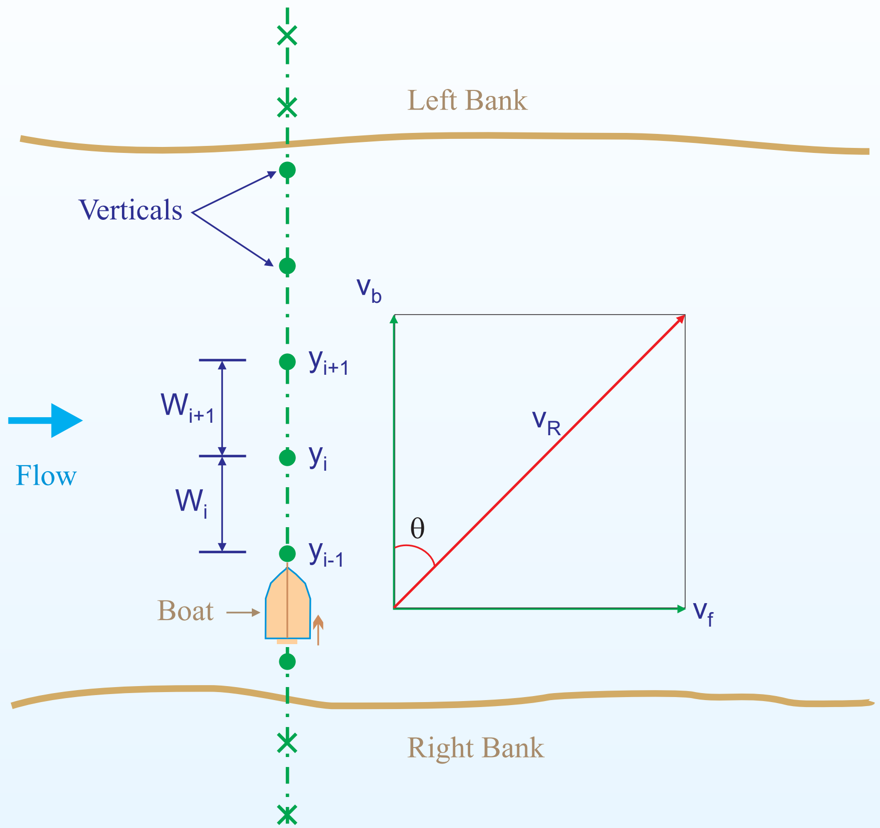
- The site should have a well-defined cross section that does not change with seasons.
- It should be easily accesible throught the year.
- The site should be in a straight, stable reach
- The gauging site should be free from backwater effects in the channel.

### Selection of number of sub-section segments

It is obvious that the accuracy of discharge estimation increases with the number of subsections used. However, time and cost involvement limit the number of segments to be used. As a guideline,

- The segment width $\ngtr$ 1/15 or 1/20 of the width of the river.
- The discharge in each segment $<$ 10% of the total discharge.
- Difference of velocities in adjacent segments $\ngtr$ 20%

# Moving Boat Method




Here,

$$v_b = v_R \cos\theta$$

$$v_f = v_R \sin\theta$$

$$W = v_b\,\Delta t = v_R \cos\theta \times \Delta t$$

Flow in the sub-area between two verticals $i$ and $i+1$ where depths are $y_i$ and $y_{i+1}$ is,

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2}\right) \times W_{i+1} \times v_f = \left(\frac{y_i + y_{i+1}}{2}\right) v_R^2 \sin\theta \cos\theta\,\Delta t$$

Total discharge through the section, $Q = \sum Q_i$

# Dilution Technique



This method is based on the continuity principle.

- A tracer is added to water at a section.
- Changes in concentration are then measured at two (u/s and d/s) sections.
- Discharge is calculated using principle of conservation of mass.

Tracer can be added to water in two ways:-

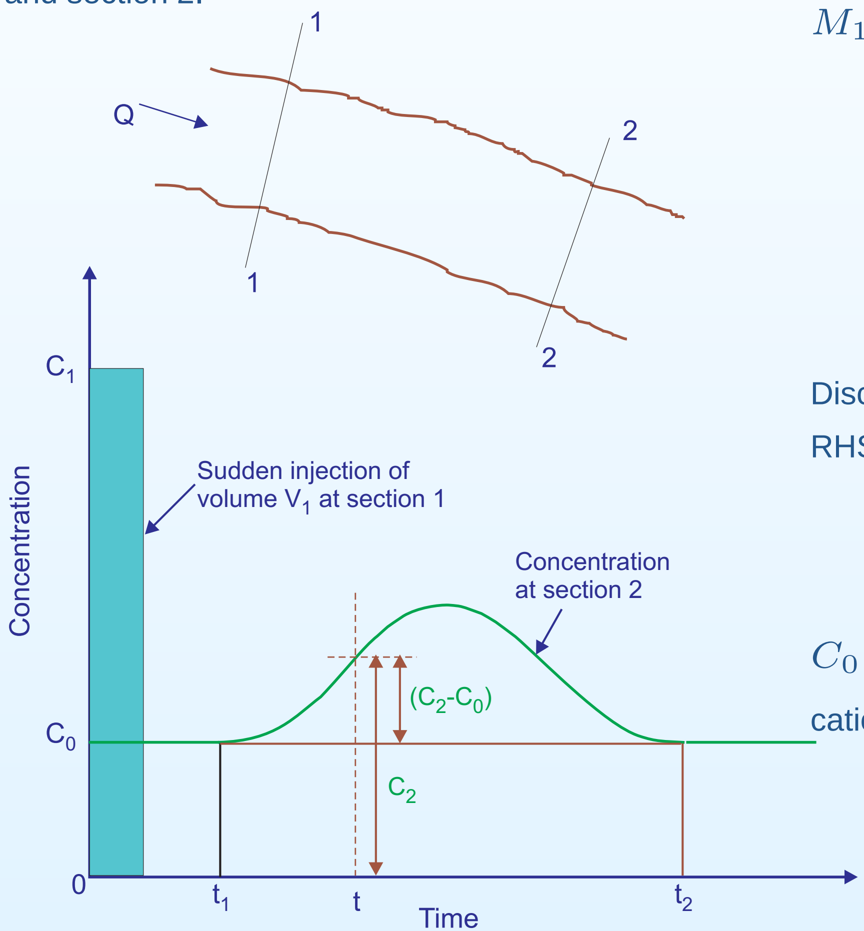
- Sudden Injection
- Constant Rate Injection

**Type of Tracer**

- Sodium Chloride (common salt)
- Sodium Dichromate
- Radioactive Tracer

# Sudden Injection

In this method, a small quantity of tracer of volume $V_1$ with high concentration $C_1$ is added at section 1. At section 2, sufficiently far away from section 1, water samples are collected at regular time intervals and corresponding concentrations are measured. A plot of concentration vs time is developed for both section 1 and section 2.




Sudden Injection Technique

$M_1$ = mass of tracer at section 1 = $V_1 \times C_1$

$$= \int_{t_1}^{t_2} Q\,(C_2 - C_0)\,dt$$

$$+ \frac{V_1}{(t_2 - t_1)} \int_{t_1}^{t_2} (C_2 - C_0)\,dt$$

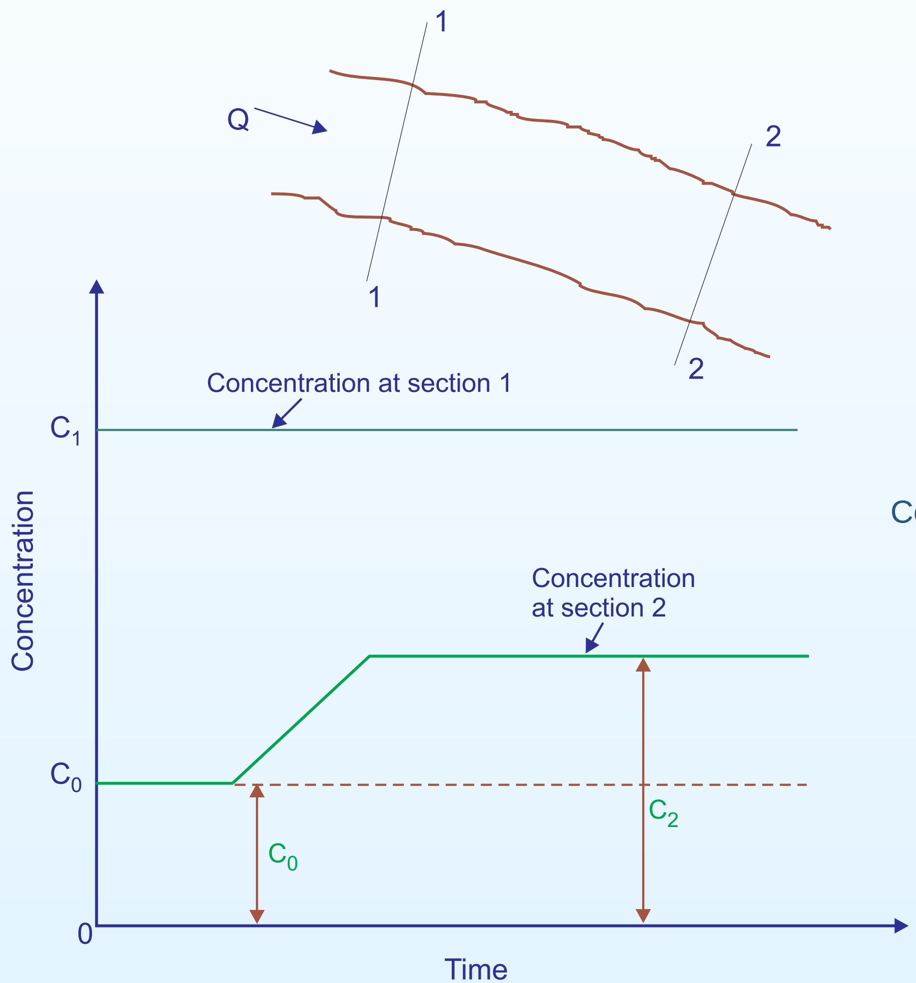
Discarding the negligibly small second term on RHS and re-arranging,

$$Q = \frac{V_1 C_1}{\int_{t_1}^{t_2} (C_2 - C_0)\,dt}$$

$C_0$ = Background concentration (prior to the application of the tracer).

# Constant Rate Injection

In this method a tracer of concentration $C_1$ is added at section 1 at a constant rate $Q_t$. Concentration at section 2 are measured at regular time intervals. Stream discharge $Q$ is obtained from this plot.



Constant Rate Injection Technique

Continuity equation for the tracer

$$Q_t C_1 + Q C_0 = (Q + Q_t) C_2$$

$$Q = \frac{Q_t (C_1 - C_2)}{(C_2 - C_0)}$$

# Examples



### Example 1

A 500 g/l solution of sodium dichromate was used as chemical tracer. It was dosed at a constant rate of 4 l/s and at a downstream section the equilibrium concentration was measured as 4 parts per million (ppm). Estimate the discharge in the stream.

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**Solution:**

Here, $Q_t$ = 4 l/s = 0.004 m$^3$/s, $C_1$ = 500 g/l = 0.50, $C_2$ = 4 ppm = 4$\times$10$^{-6}$ ,

## Examples



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A 500 g/l solution of sodium dichromate was used as chemical tracer. It was dosed at a constant rate of 4 l/s and at a downstream section the equilibrium concentration was measured as 4 parts per million (ppm). Estimate the discharge in the stream.

**Solution:**

Here, $Q_t$ = 4 l/s = 0.004 m$^3$/s, $C_1$ = 500 g/l = 0.50, $C_2$ = 4 ppm = 4$\times$10$^{-6}$ , $C_0$= 0

So, stream discharge

# Examples

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So, stream discharge

$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - C_0)} = \frac{0.004 \times (0.50 - 4 \times 10^{-6})}{(4 \times 10^{-6} - 0)} = 500\, m^3/s$$

# Examples



### Example 2

A 200 g/l solution of common salt was discharged into a stream at a constant rate of 25 l/s. The background concentration of the salt in the stream was found to be 10 ppm. At a downstream section the equilibrium concentration was found to be 45 ppm. Estimate the discharge in the stream.

# Examples



**Example 2**

A 200 g/l solution of common salt was discharged into a stream at a constant rate of 25 l/s. The background concentration of the salt in the stream was found to be 10 ppm. At a downstream section the equilibrium concentration was found to be 45 ppm. Estimate the discharge in the stream.

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# Examples



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**Solution:**

Here, $Q_t$ =

# Examples



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**Solution:**

Here, $Q_t$ = 25 l/s = 0.025 m$^3$/s,

## Examples



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**Solution:**

Here, $Q_t$ = 25 l/s = 0.025 m$^3$/s, $C_1$ = 200 g/l = 0.20,

## Examples



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**Solution:**

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# Examples



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Here, $Q_t$ = 25 l/s = 0.025 m$^3$/s, $C_1$ = 200 g/l = 0.20, $C_2$ = 45 ppm = 45$\times$10$^{-6}$, $C_0$ = 10 ppm = 10$\times$10$^{-6}$

# Examples



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Here, $Q_t$ = 25 l/s = 0.025 m$^3$/s, $C_1$ = 200 g/l = 0.20, $C_2$ = 45 ppm = 45$\times$10$^{-6}$,
$C_0$ = 10 ppm = 10$\times$10$^{-6}$
So, stream discharge

$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - C_0)} = \frac{0.025 \times (0.20 - 45 \times 10^{-6})}{(45 \times 10^{-6} - 10 \times 10^{-6})} = 142.83\, m^3/s \simeq 143\, m^3/s$$

# Tracer Properties

The tracer should have the following properties:

1. It should not be absorbed by the sediment, channel boundary and vegetation.
2. It should not react with channel boundary, sediment or vegetation.
3. It should not be lost by evaporation.
4. It should be non-toxic.
5. It should be capable of being detected in a distinctive manner in small concentrations.
6. It should not be very expensive.

# Length of Reach



The length of the reach between the two sections should be adequate to have complete mixing of the tracer with the flow. An approximate length suggested by Rimmar (1960) is as follows

$$L = \frac{0.13 B^2 \, C \, (0.7 \, C + 2\sqrt{g})}{g \, d}$$

where

$L$ = mixing length (m); $B$ = average width of the stream (m); $d$ = average depth of the stream (m); $C$ = Chezy roughness coefficient and $g$ = acceleration due to gravity.

### Example

It is proposed to adopt the dilution method of stream gauging for a river whose hydraulic properties at average flow are as follows: Width = 45m, depth = 2.0m, discharge = 85m$^3$/s, Chezy coefficient = 30. Determine the safe mixing length for this stream.

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**Solution:**

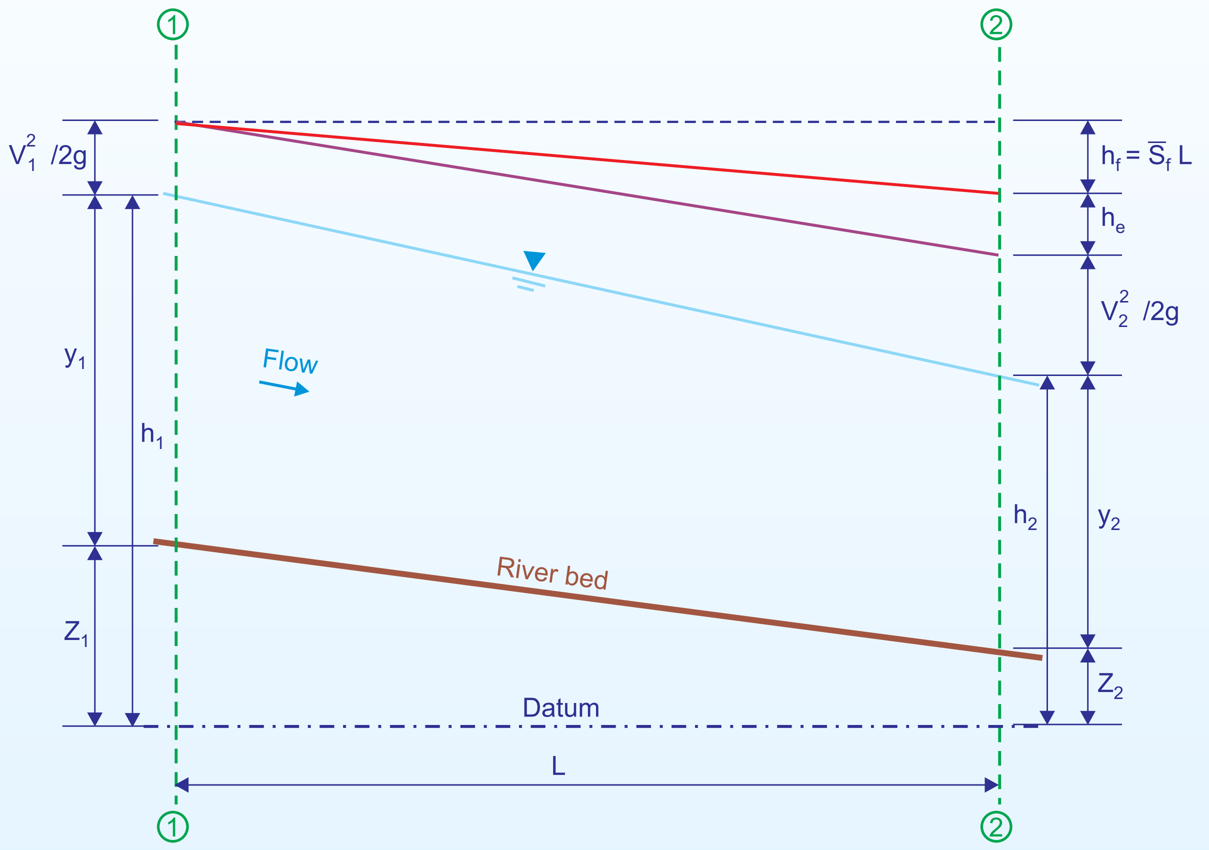
Here, $B$ = 45m, $d$ = 2.0m, $C$ = 30. So, desired safe mixing length,

$$L = \frac{0.13 \times 45^2 \times 30 \times (0.7 \times 30 + 2\sqrt{9.807})}{9.807 \times 2}\,m = 10977.43\,m \simeq 11\,km$$

# Indirect Methods

# Slope Area Method

This method is particularly used to determine the discharge during a flood. From the known water surface elevations at two sections, the flood discharge is estimated using Manning's formula.




Applying energy equation to sections 1 and 2,

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + h_L$$

## Slope Area Method



$h_L$ is the head loss in the reach = friction loss $h_f$ + eddy loss $h_e$.
Now, let $Z_1 + y_1 = h_1$ and $Z_2 + y_2 = h_2$. Then the energy equation becomes

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_e + h_f$$

$$h_f = (h_1 - h_2) + \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_e \tag{1}$$

If $\overline{S}_f$ is the average slope of the energy line in reach length $L$, then $h_f = \overline{S}_f \times L$.

Now, from Manning's equation, $Q = (1/n)\, A\, R^{2/3}\, S_f^{1/2} = K\sqrt{S_f}$. Or, $S_f = Q^2/K^2$, where $K$ = conveyance = $(1/n)\, A\, R^{2/3}$
For section 1, $K_1 = (1/n_1)\, A_1\, R_1^{2/3}$ and for section 2, $K_2 = (1/n_2)\, A_2\, R_2^{2/3}$.
So, average conveyance, $K = \sqrt{K_1 K_2}$ and average energy slope, $\overline{S}_f = Q^2/K^2$. Hence,

$$Q = K\sqrt{h_f/L} \tag{2}$$

From continuity equation, $Q = A_1\, V_1 = A_2\, V_2$. So,

$$V_1 = Q/A_1 \qquad V_2 = Q/A_2 \tag{3}$$

Eddy loss may be estimated as

$$h_e = \frac{K_e}{2g}\left(V_1^2 - V_2^2\right) \tag{4}$$

# Slope Area Method



**Steps:**

Known: Section 1: $h_1$, $A_1$, $R_1$, $n_1$; Section 2: $h_2$, $A_2$, $R_2$, $n_2$; Eddy loss coefficient: $K_e$

Step 0: Calculate $K_1 = (1/n_1)\,A_1 R_1^{2/3}$ for section 1 and $K_2 = (1/n_2)\,A_2 R_2^{2/3}$ for section 2. Then calculate $K = \sqrt{K_1 K_2}$. Now assume $V_1 = V_2$.

Step 1: Calculate $h_f$ from equation 1.

Step 2: Calculate $Q$ from equation 2.

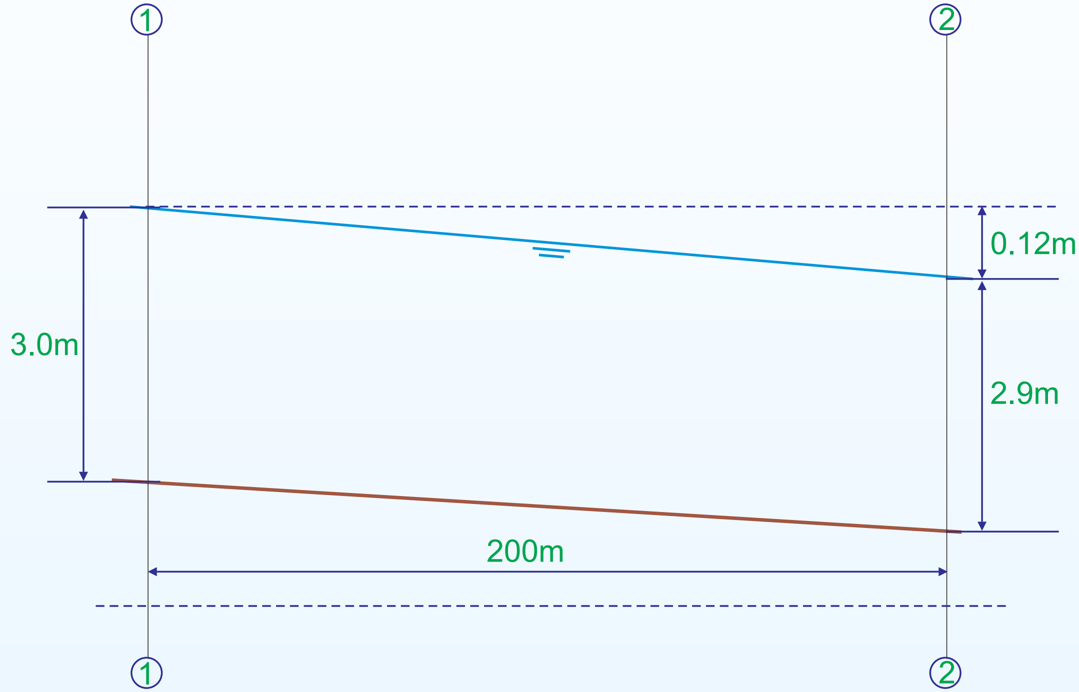
Step 3: Calculate $V_1$ and $V_2$ from equation 3.

Step 4: Calculate $h_e$ from equation 4.

Step 5: Repeat steps 1 to 4, until two successive values of $Q$ or $h_f$ becomes almost equal.

# Example



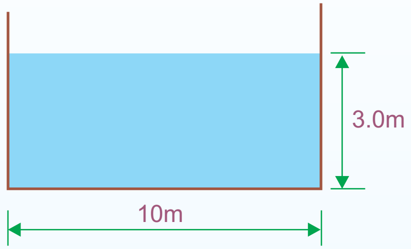



During a flood flow, the depth of water in a 10m wide rectangular channel was found to be 3.0m and 2.9m at two sections 200m apart. The drop in the water surface elevation was found to be 0.12m. Assuming Manning's coefficient to be 0.025, estimate the flood discharge through the channel.

# Solution



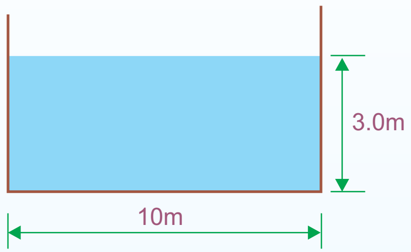
*Section 1*




$y_1 = 3.0m,\ B_1 = 10m,\ A_1 = 30m^2,\ P_1 = 16m,$
$R_1 = 1.875m,\ K_1 = (1/n_1)\ A_1 R_1^{2/3} = 1824.661$

# Solution

*Section 1*

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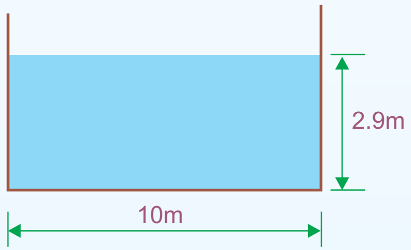
*Section 2*




$$y_2 = 2.9m,\ B_2 = 10m,\ A_2 = 29m^2,\ P_2 = 15.8m,$$
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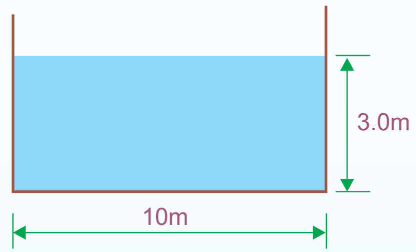
# Solution

*Section 1*

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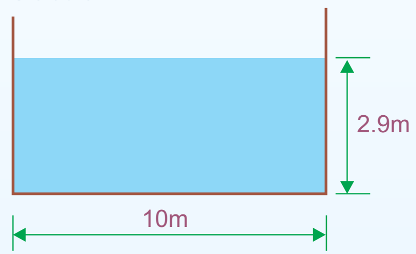


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$K = \sqrt{K_1 K_2} = 1781.143$; $L = 200m$; assume $K_e = 0$; so, $h_e = 0$

# Solution

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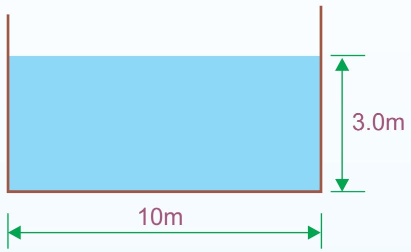


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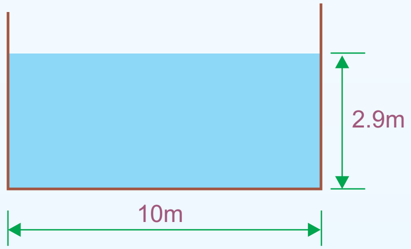


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# Solution

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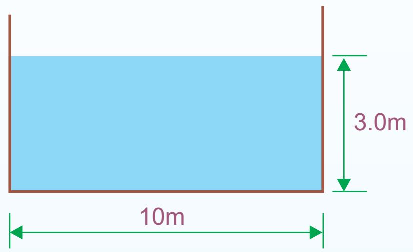


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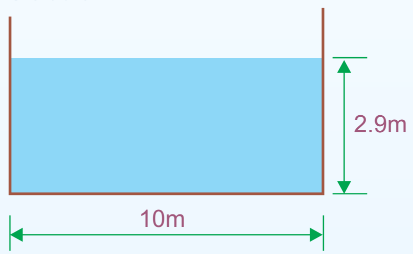


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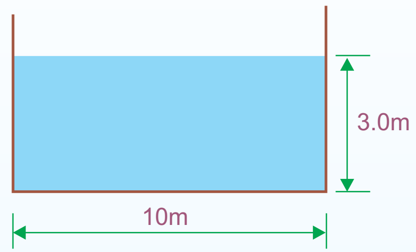
Step 1: From equation 1

# Solution



*Section 1*

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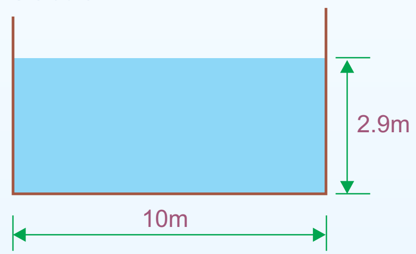


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Step 1: From equation 1 $h_f = (h_1 - h_2) + 0 = 0.12m$ [Note: $h_1 - h_2 =$ drop in water level].

# Solution

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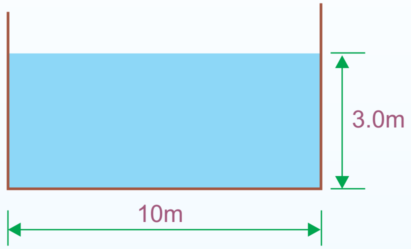


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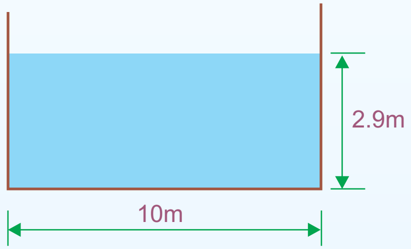


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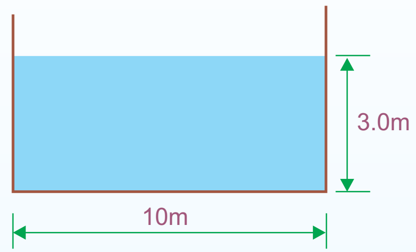


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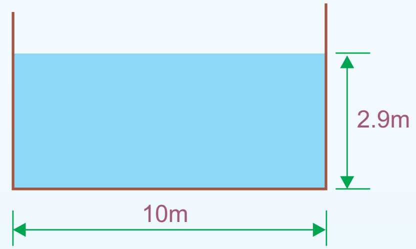


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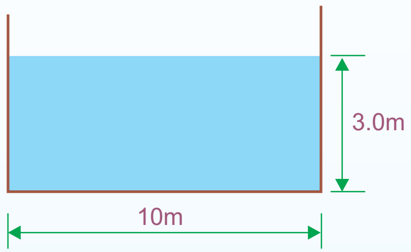
# Solution

*Section 1*

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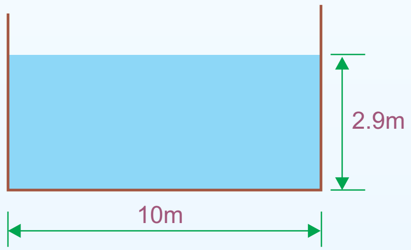


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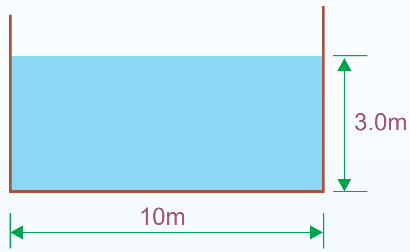
# Solution



*Section 1*

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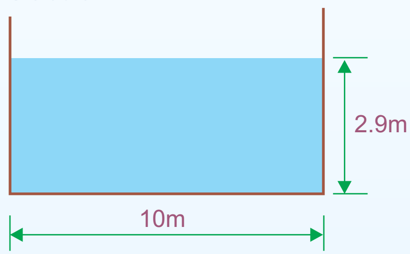
*Section 2*




$y_2 = 2.9m,\ B_2 = 10m,\ A_2 = 29m^2,\ P_2 = 15.8m,$
$R_2 = 1.835m,\ K_2 = (1/n_2)\ A_2 R_2^{2/3} = 1738.663$

$K = \sqrt{K_1 K_2} = 1781.143$; $L = 200m$; assume $K_e = 0$; so, $h_e = 0$

First trial: Assume $V_1 = V_2$. Then

Step 1: From equation 1 $h_f = (h_1 - h_2) + 0 = 0.12m$ [Note: $h_1 - h_2 =$ drop in water level].

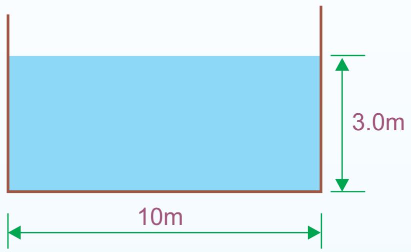
Step 2: From equation 2 $Q = K\sqrt{h_f/L} = 1781.143\sqrt{0.12/200} = 43.628\, m^3/s$

Step 3: From equation 3 $V_1 = Q/A_1 = 1.4543m/s$; $V_2 = Q/A_2 = 1.5045m/s$

# Solution

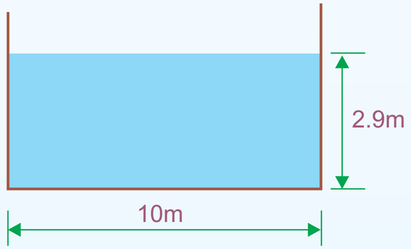


*Section 1*




$y_1 = 3.0m,\ B_1 = 10m,\ A_1 = 30m^2,\ P_1 = 16m,$
$R_1 = 1.875m,\ K_1 = (1/n_1)\ A_1 R_1^{2/3} = 1824.661$

*Section 2*




$y_2 = 2.9m,\ B_2 = 10m,\ A_2 = 29m^2,\ P_2 = 15.8m,$
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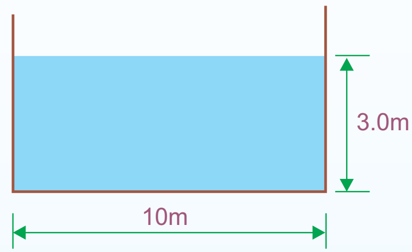
Step 4: Again from equation 1

# Solution

*Section 1*

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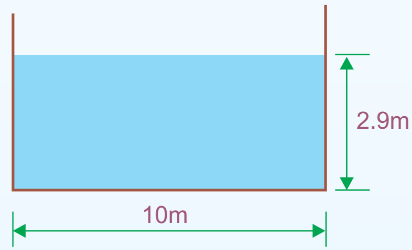


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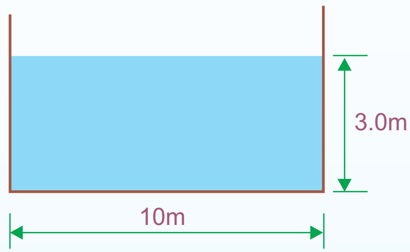
Step 4: Again from equation 1 $h_f = (h_1 - h_2) + (1/2g)(V_1^2 - V_2^2) = 0.1124m$

# Solution

*Section 1*

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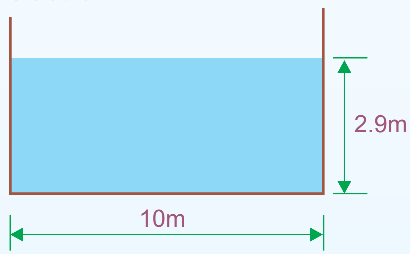
*Section 2*




$y_2 = 2.9m,\ B_2 = 10m,\ A_2 = 29m^2,\ P_2 = 15.8m,$
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First trial: Assume $V_1 = V_2$. Then

Step 1: From equation 1 $h_f = (h_1 - h_2) + 0 = 0.12m$ [Note: $h_1 - h_2 =$ drop in water level].

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Step 4: Again from equation 1 $h_f = (h_1 - h_2) + (1/2g)(V_1^2 - V_2^2) = 0.1124m$

The calculations are shown in a tabular format.

# Solution

| Trial | $h_f^{old}$ (m) | $Q$ (m$^3$/s) | $V_1$ (m/s) | $V_2$ (m/s) | $h_f^{new}$ (m) | Remarks |
|---|---|---|---|---|---|---|
| 1 | 0.12 | 43.628 | 1.4543 | 1.5045 | 0.1124 | $h_f^{old} > h_f^{new}$ |
| 2 | 0.1124 | 42.2314 | 1.4077 | 1.4563 | 0.1129 | $h_f^{old} < h_f^{new}$ |
| 3 | 0.1129 | 42.3208 | 1.4107 | 1.4593 | 0.1129 | $h_f^{old} \simeq h_f^{new}$ |

(Note: $h_f^{new}$ in one trial will be $h_f^{old}$ for the next trial)

# Solution

| Trial | $h_f^{old}$ (m) | $Q$ (m$^3$/s) | $V_1$ (m/s) | $V_2$ (m/s) | $h_f^{new}$ (m) | Remarks |
|---|---|---|---|---|---|---|
| 1 | 0.12 | 43.628 | 1.4543 | 1.5045 | 0.1124 | $h_f^{old} > h_f^{new}$ |
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(Note: $h_f^{new}$ in one trial will be $h_f^{old}$ for the next trial)

So, the discharge in the channel = 42.321 m$^3$/s.

# Questions



### Question 1

During a high flow, water surface elevations of a small stream were noted at two sections A and B, 10km apart. Section A is upstream of B. Elevations and other salient hydraulic properties at these sections are given below. The eddy loss coefficients of 0.3 for gradual expansion and 0.1 for gradual contraction are appropriate. Assuming Manning's $n = 0.020$, estimate the discharge in the stream.

| Section | Water surface elevation (m) | Area of cross-section ($m^2$) | Hydraulic radius (m) |
|---|---|---|---|
| A | 104.771 | 73.293 | 2.733 |
| B | 104.550 | 93.375 | 3.089 |

# Questions



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During a high flow, water surface elevations of a small stream were noted at two sections A and B, 10km apart. Section A is upstream of B. Elevations and other salient hydraulic properties at these sections are given below. The eddy loss coefficients of 0.3 for gradual expansion and 0.1 for gradual contraction are appropriate. Assuming Manning's $n = 0.020$, estimate the discharge in the stream.

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**Answer:** 44.25 $m^3$/s.

# Questions



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**Answer:** 44.25 $m^3$/s.

### Question 2

A small stream has a trapezoidal cross section with base width of 12 m and side slope 2 horizontal:1 vertical in a reach of 8 km. During a flood the high water levels record at the ends of the reach are as below. Assuming Manning's $n = 0.030$, estimate the discharge in the stream.

| Section | Elevation of bed (m) | Water surface elevation (m) |
|---|---|---|
| Upstream | 100.20 | 102.70 |
| Downstream | 98.60 | 101.30 |

# Questions



### Question 1

During a high flow, water surface elevations of a small stream were noted at two sections A and B, 10km apart. Section A is upstream of B. Elevations and other salient hydraulic properties at these sections are given below. The eddy loss coefficients of 0.3 for gradual expansion and 0.1 for gradual contraction are appropriate. Assuming Manning's $n = 0.020$, estimate the discharge in the stream.

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| Section | Elevation of bed (m) | Water surface elevation (m) |
|---|---|---|
| Upstream | 100.20 | 102.70 |
| Downstream | 98.60 | 101.30 |

**Answer:** 30.18 $m^3$/s.

# Slope-Area Method (contd..)

The discharge can also be calculated from

$$h_f = (h_1 - h_2) + (1 - k_e)(\frac{V_1^2}{2g} - \frac{V_2^2}{2g})$$

$$Q = K\sqrt{\bar{S_f}} = K\sqrt{h_f/L}$$

$$h_f = Q^2\frac{L}{K^2}$$

$$V_1 = \frac{Q}{A_1} \qquad V_2 = \frac{Q}{A_2}$$

$$h_f = F + \frac{(1 - K_e)}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right)Q^2 = F + X\,Q^2$$

$$Q^2\frac{L}{K^2} = F + X\,Q^2$$

$$Q = \sqrt{\frac{F}{\left[\frac{L}{K^2} - X\right]}}$$

# Hydraulic Structures



## Weirs

- Sharp crested weir
- Broad crested weir
- Rectangular weir
- Trapezoidal weir
- V-notch weir

## Flumes

- Large-crested flume
- Long-throated flume
- Short-throated flume
- Pashall flume
- H flume

For all these structures, $Q = f(H)$
where $Q$ is the discharge and $H$ is the water surface elevation measured from a datum. Usually, for weirs,

# Stage-Discharge Relation

# Rating Curve



Direct method of discharge measurement involves

- Measurement of stage (G) and discharge (Q) values
- Establishing relationship between G and Q

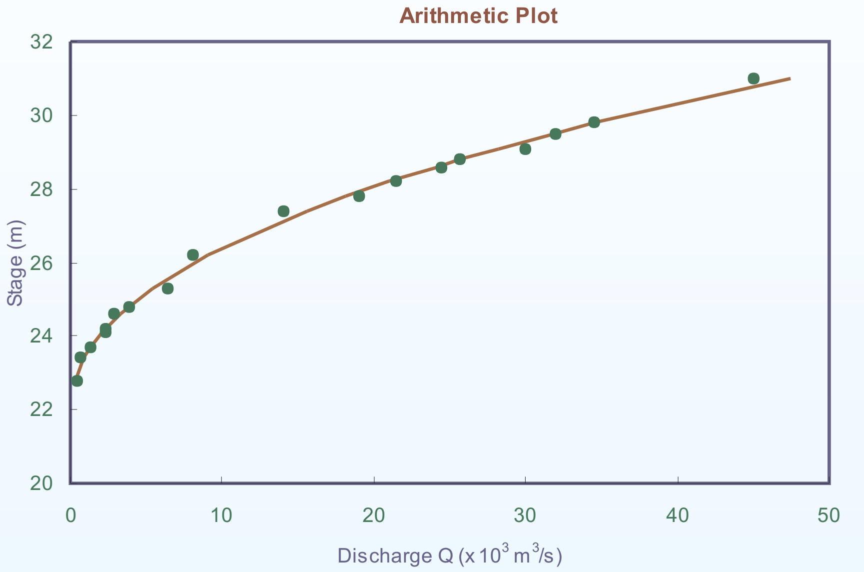
The G-Q relationship is used subsequently to estimate the discharge for a measured stage. This stage-discharge relationship is known as *Rating Curve*.

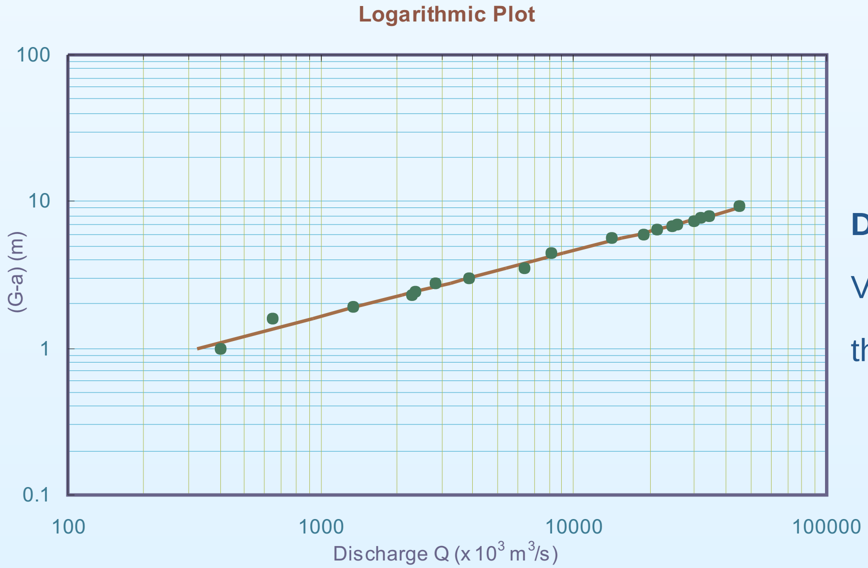
**Control**

This relationship reflects the combined effect of all hydraulic flow parameters on discharge. This combined effect is known as control. If the G-Q relationship for a section does not change with time, then the control is said to be *permanent control*. If it changes with time, then it is called a *shifting control*.

# Permanent Control

# Graphical Plot




**Determination of $C_r$ and $\beta$**

Values of $C_r$ and $\beta$ can be obtained from the intercept and slope of the line.

# Least-square Method

The stage-discharge relationship is

$$Q = C_r (G - a)^{\beta}$$

Taking logarithm of both sides

$$\ln Q = \beta \ln(G - a) + \ln C_r$$

Or,

$$Y = \beta X + b$$

where $Y = \ln Q$, $X = \ln(G - a)$, $b = \ln C_r$
Using least-square method

$$\beta = \frac{N\left(\sum XY\right) - \left(\sum X\right)\left(\sum Y\right)}{N\left(\sum X^2\right) - \left(\sum X\right)^2} \tag{5}$$

$$b = \frac{\sum Y - \beta\left(\sum X\right)}{N}$$

$$C_r = e^b \tag{6}$$

From $N$ observed dataset of $Q$ and $G$, $N$ sets of $X$, $Y$ data are to be calculated. Then the values of $\beta$, and $C_r$ can be obtained from equations 5 and 6.

# Correlation



The coefficient of correlation is given by

$$r = \frac{N\left(\sum XY\right) - \left(\sum X\right)\left(\sum Y\right)}{\left(\sqrt{N\left(\sum X^2\right) - \left(\sum X\right)^2}\right)\left(\sqrt{N\left(\sum Y^2\right) - \left(\sum Y\right)^2}\right)}$$

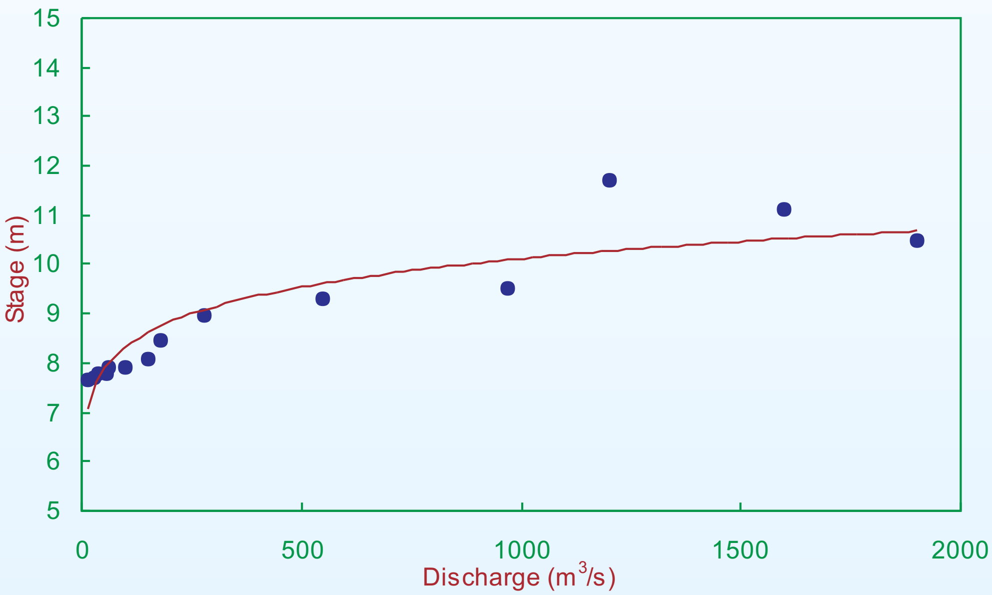
If $r = 1$, then it is a perfect correlation. Usually a value of $0.6 < r < 1.0$ is considered as an indication of good fit.

# Example



Develop a stage-discharge relationship for the following set of data. Assume a value of $a = 7.50m$. What is the discharge for a gauge reading of $10.05m$?

| Gauge reading ($m$) | Discharge ($m^3/s$) |
|---|---|
| 7.65 | 15 |
| 7.70 | 30 |
| 7.77 | 57 |
| 7.80 | 39 |
| 7.90 | 60 |
| 7.91 | 100 |
| 8.08 | 150 |
| 8.48 | 180 |
| 8.98 | 280 |
| 9.30 | 550 |
| 9.50 | 970 |
| 10.50 | 1900 |
| 11.10 | 1600 |
| 11.70 | 1200 |



# Solution



| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.65 | 15 | 0.15 | -1.8971 | 2.7081 | -5.1375 | 3.5991 | 7.3335 |
| 7.70 | 30 | 0.20 | -1.6094 | 3.4012 | -5.4740 | 2.5903 | 11.5681 |
| 7.77 | 57 | 0.27 | -1.3093 | 4.0431 | -5.2937 | 1.7144 | 16.3463 |
| 7.80 | 39 | 0.30 | -1.2040 | 3.6636 | -4.4108 | 1.4496 | 13.4217 |
| 7.90 | 60 | 0.40 | -0.9163 | 4.0943 | -3.7516 | 0.8396 | 16.7637 |
| 7.91 | 100 | 0.41 | -0.8916 | 4.6052 | -4.1060 | 0.7949 | 21.2076 |
| 8.08 | 150 | 0.58 | -0.5447 | 5.0106 | -2.7294 | 0.2967 | 25.1065 |
| 8.48 | 180 | 0.98 | -0.0202 | 5.1930 | -0.1049 | 0.0004 | 26.9668 |
| 8.98 | 280 | 1.48 | 0.3920 | 5.6348 | 2.2091 | 0.1537 | 31.7509 |
| 9.30 | 550 | 1.80 | 0.5878 | 6.3099 | 3.7089 | 0.3455 | 39.8151 |
| 9.50 | 970 | 2.00 | 0.6931 | 6.8773 | 4.7670 | 0.4805 | 47.2972 |
| 10.50 | 1900 | 3.00 | 1.0986 | 7.5496 | 8.2941 | 1.2069 | 56.9966 |
| 11.10 | 1600 | 3.60 | 1.2809 | 7.3778 | 9.4504 | 1.6408 | 54.4313 |
| 11.70 | 1200 | 4.20 | 1.4351 | 7.0901 | 10.1749 | 2.0595 | 50.2692 |
| $\sum$ | | | -2.9051 | 73.5584 | 7.5964 | 17.1718 | 419.2744 |

## Solution

| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.65 | 15 | 0.15 | -1.8971 | 2.7081 | -5.1375 | 3.5991 | 7.3335 |
| 7.70 | 30 | 0.20 | -1.6094 | 3.4012 | -5.4740 | 2.5903 | 11.5681 |
| 7.77 | 57 | 0.27 | -1.3093 | 4.0431 | -5.2937 | 1.7144 | 16.3463 |
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| 9.30 | 550 | 1.80 | 0.5878 | 6.3099 | 3.7089 | 0.3455 | 39.8151 |
| 9.50 | 970 | 2.00 | 0.6931 | 6.8773 | 4.7670 | 0.4805 | 47.2972 |
| 10.50 | 1900 | 3.00 | 1.0986 | 7.5496 | 8.2941 | 1.2069 | 56.9966 |
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$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N\left(\sum X^2\right) - (\sum X)^2} = 1.379696$$

# Solution

| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
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| 7.65 | 15 | 0.15 | -1.8971 | 2.7081 | -5.1375 | 3.5991 | 7.3335 |
| 7.70 | 30 | 0.20 | -1.6094 | 3.4012 | -5.4740 | 2.5903 | 11.5681 |
| 7.77 | 57 | 0.27 | -1.3093 | 4.0431 | -5.2937 | 1.7144 | 16.3463 |
| 7.80 | 39 | 0.30 | -1.2040 | 3.6636 | -4.4108 | 1.4496 | 13.4217 |
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| 8.08 | 150 | 0.58 | -0.5447 | 5.0106 | -2.7294 | 0.2967 | 25.1065 |
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| 9.30 | 550 | 1.80 | 0.5878 | 6.3099 | 3.7089 | 0.3455 | 39.8151 |
| 9.50 | 970 | 2.00 | 0.6931 | 6.8773 | 4.7670 | 0.4805 | 47.2972 |
| 10.50 | 1900 | 3.00 | 1.0986 | 7.5496 | 8.2941 | 1.2069 | 56.9966 |
| 11.10 | 1600 | 3.60 | 1.2809 | 7.3778 | 9.4504 | 1.6408 | 54.4313 |
| 11.70 | 1200 | 4.20 | 1.4351 | 7.0901 | 10.1749 | 2.0595 | 50.2692 |
| $\sum$ | | | -2.9051 | 73.5584 | 7.5964 | 17.1718 | 419.2744 |

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N\left(\sum X^2\right) - (\sum X)^2} = 1.379696$$

$$b = \frac{\sum Y - \beta(\sum X)}{N} = 5.540467$$

# Solution

| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
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| 7.70 | 30 | 0.20 | -1.6094 | 3.4012 | -5.4740 | 2.5903 | 11.5681 |
| 7.77 | 57 | 0.27 | -1.3093 | 4.0431 | -5.2937 | 1.7144 | 16.3463 |
| 7.80 | 39 | 0.30 | -1.2040 | 3.6636 | -4.4108 | 1.4496 | 13.4217 |
| 7.90 | 60 | 0.40 | -0.9163 | 4.0943 | -3.7516 | 0.8396 | 16.7637 |
| 7.91 | 100 | 0.41 | -0.8916 | 4.6052 | -4.1060 | 0.7949 | 21.2076 |
| 8.08 | 150 | 0.58 | -0.5447 | 5.0106 | -2.7294 | 0.2967 | 25.1065 |
| 8.48 | 180 | 0.98 | -0.0202 | 5.1930 | -0.1049 | 0.0004 | 26.9668 |
| 8.98 | 280 | 1.48 | 0.3920 | 5.6348 | 2.2091 | 0.1537 | 31.7509 |
| 9.30 | 550 | 1.80 | 0.5878 | 6.3099 | 3.7089 | 0.3455 | 39.8151 |
| 9.50 | 970 | 2.00 | 0.6931 | 6.8773 | 4.7670 | 0.4805 | 47.2972 |
| 10.50 | 1900 | 3.00 | 1.0986 | 7.5496 | 8.2941 | 1.2069 | 56.9966 |
| 11.10 | 1600 | 3.60 | 1.2809 | 7.3778 | 9.4504 | 1.6408 | 54.4313 |
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| $\sum$ | | | -2.9051 | 73.5584 | 7.5964 | 17.1718 | 419.2744 |

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N\left(\sum X^2\right) - (\sum X)^2} = 1.379696$$

$$b = \frac{\sum Y - \beta(\sum X)}{N} = 5.540467 \qquad C_r = e^b = 254.797$$

# Solution



| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.65 | 15 | 0.15 | -1.8971 | 2.7081 | -5.1375 | 3.5991 | 7.3335 |
| 7.70 | 30 | 0.20 | -1.6094 | 3.4012 | -5.4740 | 2.5903 | 11.5681 |
| 7.77 | 57 | 0.27 | -1.3093 | 4.0431 | -5.2937 | 1.7144 | 16.3463 |
| 7.80 | 39 | 0.30 | -1.2040 | 3.6636 | -4.4108 | 1.4496 | 13.4217 |
| 7.90 | 60 | 0.40 | -0.9163 | 4.0943 | -3.7516 | 0.8396 | 16.7637 |
| 7.91 | 100 | 0.41 | -0.8916 | 4.6052 | -4.1060 | 0.7949 | 21.2076 |
| 8.08 | 150 | 0.58 | -0.5447 | 5.0106 | -2.7294 | 0.2967 | 25.1065 |
| 8.48 | 180 | 0.98 | -0.0202 | 5.1930 | -0.1049 | 0.0004 | 26.9668 |
| 8.98 | 280 | 1.48 | 0.3920 | 5.6348 | 2.2091 | 0.1537 | 31.7509 |
| 9.30 | 550 | 1.80 | 0.5878 | 6.3099 | 3.7089 | 0.3455 | 39.8151 |
| 9.50 | 970 | 2.00 | 0.6931 | 6.8773 | 4.7670 | 0.4805 | 47.2972 |
| 10.50 | 1900 | 3.00 | 1.0986 | 7.5496 | 8.2941 | 1.2069 | 56.9966 |
| 11.10 | 1600 | 3.60 | 1.2809 | 7.3778 | 9.4504 | 1.6408 | 54.4313 |
| 11.70 | 1200 | 4.20 | 1.4351 | 7.0901 | 10.1749 | 2.0595 | 50.2692 |
| $\sum$ | | | -2.9051 | 73.5584 | 7.5964 | 17.1718 | 419.2744 |

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N\left(\sum X^2\right) - (\sum X)^2} = 1.379696$$

$$b = \frac{\sum Y - \beta(\sum X)}{N} = 5.540467 \qquad C_r = e^b = 254.797$$

So, the stage-discharge relationship (rating curve ) is given by

## Solution



| G | Q | (G-a) | X=ln(G-a) | Y=ln(Q) | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.65 | 15 | 0.15 | -1.8971 | 2.7081 | -5.1375 | 3.5991 | 7.3335 |
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So, the stage-discharge relationship (rating curve ) is given by

$$Q = 254.797\,(G - a)^{1.3797}$$

# Solution

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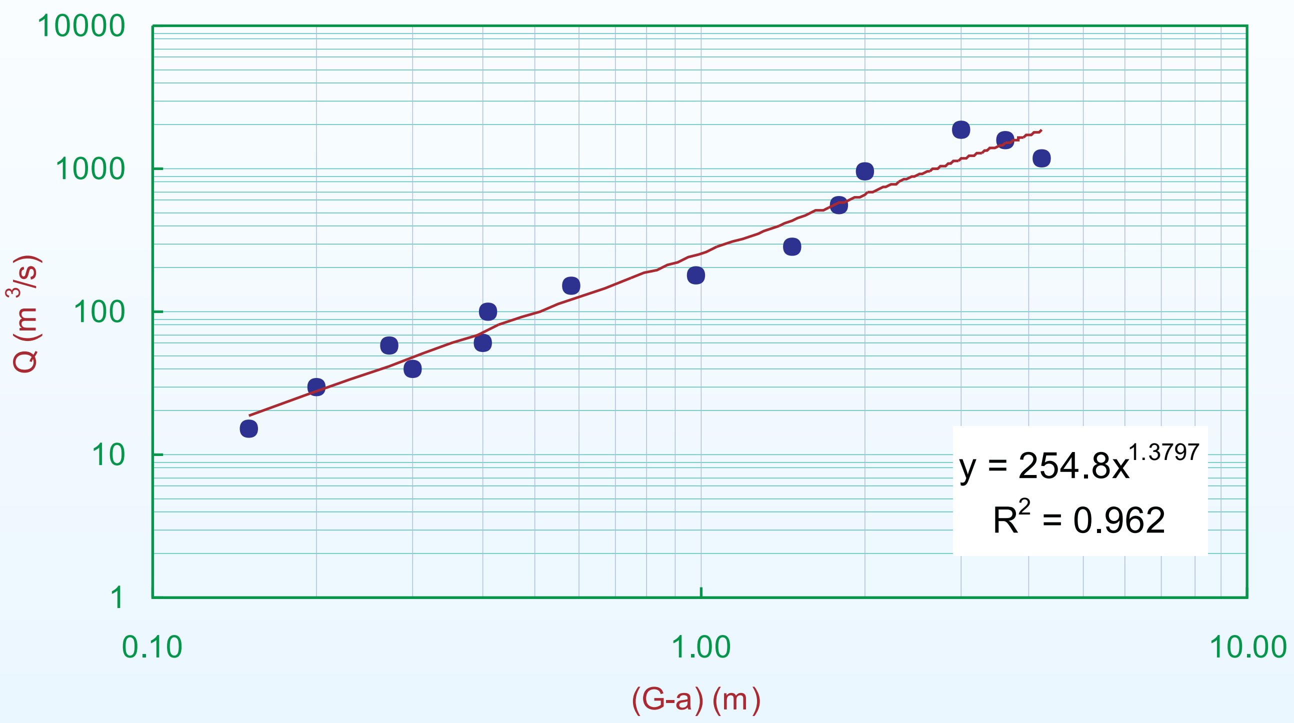
So, the stage-discharge relationship (rating curve ) is given by

$$Q = 254.797\,(G-a)^{1.3797}$$

For a gauge height of $10.05m$, $Q = 254.797\,(10.05-7.5)^{1.3797} m^3/s = 927^3/s$

# Solution

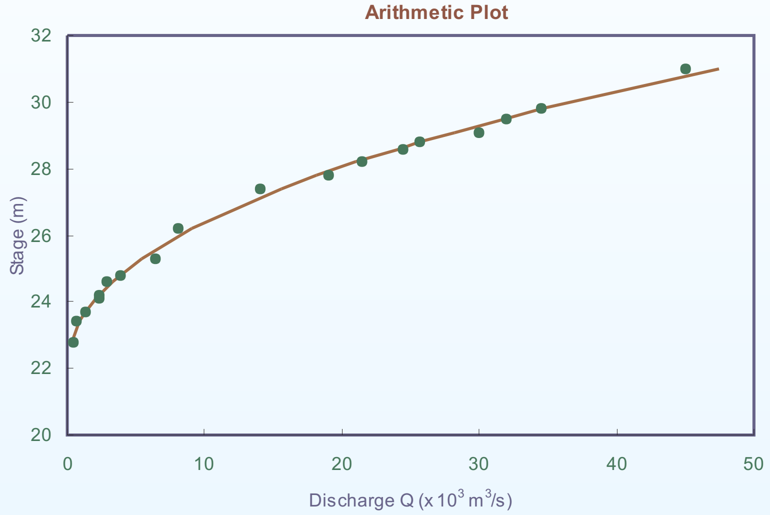


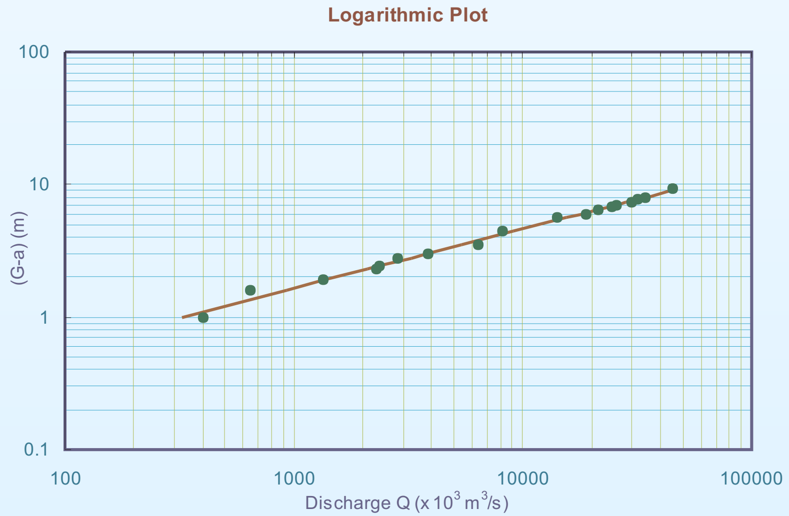


# Stage for Zero Discharge



The constant $a$ in the rating curve is the stage for zero discharge. This is a hypothetical quantity and cannot be measured in the field. It is to be estimated empirically.



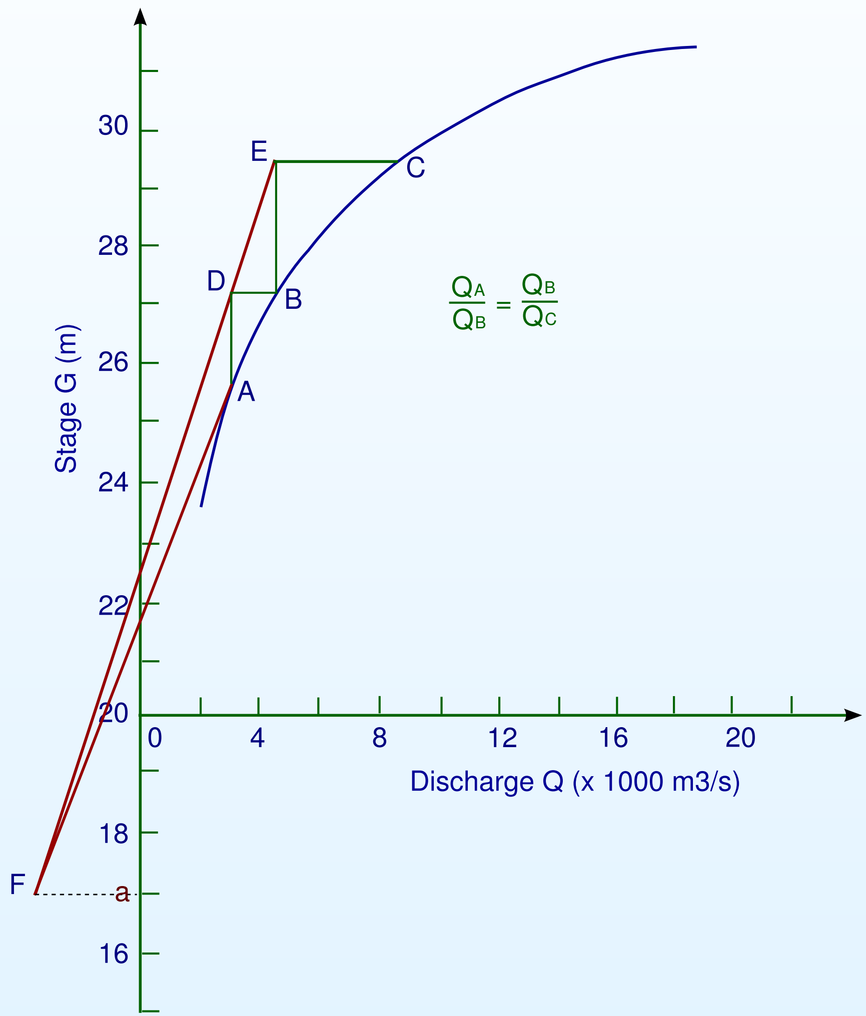

## Method 1

- Plot $Q$ vs $G$ on arithmetic graph paper as a best-fit curve.
- Estimate $a$ from the curve for $Q = 0$ by eye estimation.
- Plot $\ln Q$ vs $\ln(G - a)$ and verify whether it fits as a straight line.
- If not, select another value of $a$ and redo the previous steps.

# Stage for Zero Discharge



## Method 2(Running's Method)




- Draw Q vs G data on arithmetic graph paper
- Select points A, B, C on the curve such that $(Q_A/Q_B) = (Q_B/Q_C)$
- Draw vertical lines at A and B, and horizontal lines at B and C. These will meet at D and E.
- Join AB and extend. Also join DE and extend. These two will meet at F.
- Ordinate of F is a.

# Stage for Zero Discharge



## Method 3

- Plot Q vs G data on an arithmetic graph paper and draw a best-fit curve.
- Select three discharges $Q_A$, $Q_B$ and $Q_C$ such that $(Q_A/Q_B) = (Q_B/Q_C)$.
- Now $Q = C_r(G-a)^\beta$, so

$$\frac{C_r(G_A-a)^\beta}{C_r(G_B-a)^\beta} = \frac{C_r(G_B-a)^\beta}{C_r(G_C-a)^\beta}$$

# Stage for Zero Discharge



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Or

$$\frac{G_A-a}{G_B-a} = \frac{G_B-a}{G_C-a}$$

# Stage for Zero Discharge



## Method 3

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Or

$$\frac{G_A-a}{G_B-a} = \frac{G_B-a}{G_C-a}$$

So,

$$a = \frac{G_A G_C - G_B^2}{(G_A+G_C) - 2G_B}$$

# Example



The following are the coordinates of a smooth curve drawn to best represent the stage-discharge data of a river. Determine the stage corresponding to zero discharge.

| Stage (m) | 20.80 | 21.42 | 21.95 | 23.37 | 23.00 | 23.52 | 23.00 |
|---|---|---|---|---|---|---|---|
| Discharge ($m^3$/s) | 100 | 200 | 300 | 400 | 600 | 800 | 1000 |

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| Stage (m) | 20.80 | 21.42 | 21.95 | 23.37 | 23.00 | 23.52 | 23.00 |
|---|---|---|---|---|---|---|---|
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**Solution:**

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**Solution:**

Let $Q_A = 100\, m^3/s;\;\; Q_B = 200\, m^3/s;\;\; Q_C = 400\, m^3/s$. Then

# Example



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So,

$$a = \frac{G_A G_C - G_B^2}{(G_A + G_C) - 2G_B} = \frac{20.80 \times 23.37 - 21.42^2}{(20.80 + 23.37) - 2 \times 21.42} = 20.511m$$

# Example

The following are the coordinates of a smooth curve drawn to best represent the stage-discharge data of a river. Determine the stage corresponding to zero discharge.

| Stage (m) | 20.80 | 21.42 | 21.95 | 23.37 | 23.00 | 23.52 | 23.00 |
|---|---|---|---|---|---|---|---|
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**Solution:**

Let $Q_A = 100\,m^3/s;\;\; Q_B = 200\,m^3/s;\;\; Q_C = 400\,m^3/s$. Then

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So,

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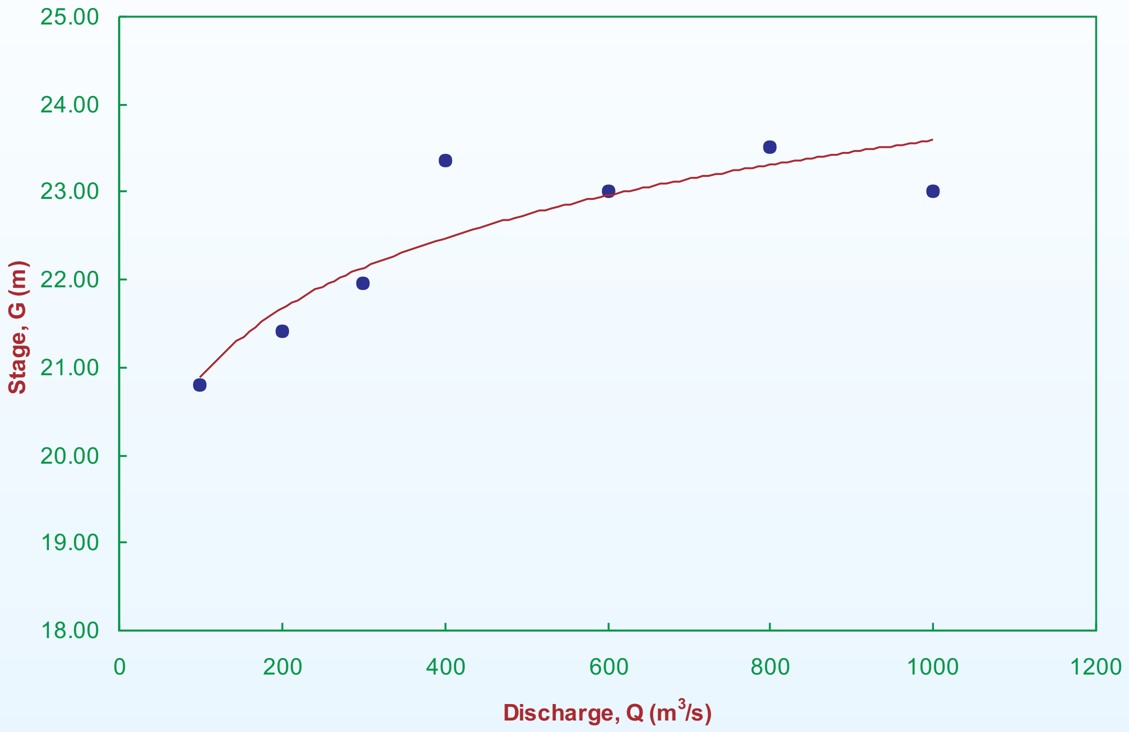
Again, if we take $Q_A = 200\,m^3/s;\;\; Q_B = 400\,m^3/s;\;\; Q_C = 800\,m^3/s$. Then
$a = 23.533m\,(> G_{max})$

# Solution


Streamflow Measurement
Measurement of Stage
Measurement of Velocity
Estimation of Discharge
Direct Methods
Indirect Methods
Stage-Discharge Relation
Permanent Control
- Graphical Plot
- Least-square Method
- Correlation
- Example
- Solution
- Solution
- Stage for Zero Discharge
- Stage for Zero Discharge
- Stage for Zero Discharge
- Example
- Solution
- Solution

Shifting Control

## Solution



Let us vary the value of $a$ and calculate $r$ :

| G | Q | (G-20.5) | (G-19.5) | (G-18.5) |
|---|---|---|---|---|
| 20.80 | 100 | 0.30 | 1.30 | 2.30 |
| 21.42 | 200 | 0.92 | 1.92 | 2.92 |
| 21.95 | 300 | 1.45 | 2.45 | 3.45 |
| 23.37 | 400 | 2.87 | 3.87 | 4.87 |
| 23.00 | 600 | 2.50 | 3.50 | 4.50 |
| 23.52 | 800 | 3.02 | 4.02 | 5.02 |
| 23.00 | 1000 | 2.50 | 3.50 | 4.50 |
| r value | | 0.9150 | 0.9202 | 0.9167 |

# Solution

Let us vary the value of $a$ and calculate $r$ :

| G | Q | (G-20.5) | (G-19.5) | (G-18.5) |
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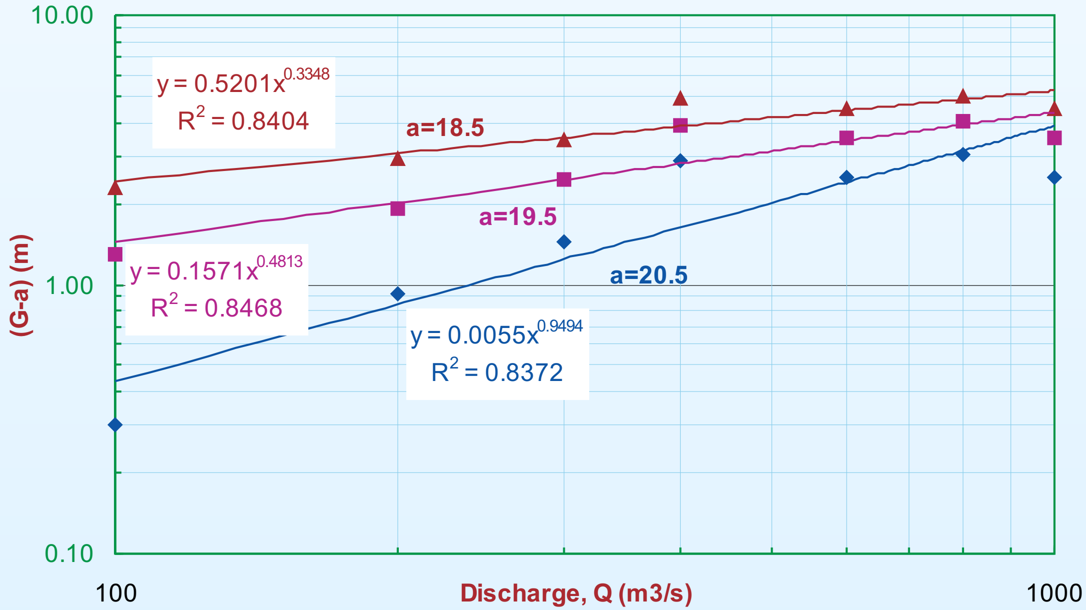
| 20.80 | 100 | 0.30 | 1.30 | 2.30 |
| 21.42 | 200 | 0.92 | 1.92 | 2.92 |
| 21.95 | 300 | 1.45 | 2.45 | 3.45 |
| 23.37 | 400 | 2.87 | 3.87 | 4.87 |
| 23.00 | 600 | 2.50 | 3.50 | 4.50 |
| 23.52 | 800 | 3.02 | 4.02 | 5.02 |
| 23.00 | 1000 | 2.50 | 3.50 | 4.50 |
| r value | | 0.9150 | 0.9202 | 0.9167 |



# Shifting Control

# Causes



The stage-discharge relationship at a gauging station may change due to

- Change in channel geometry due to
  - weed growth
  - dredging
  - channel encraoachment
- aggradation or degradation phenomenon in an alluvial channel
- variable backwater effects
- unsteady flow effects

There are no permanent corrective measure for the first two above, except for regular updating of the rating curve. The other two effects can be corrected.

# Backwater Effect

At the gauging station, the same stage will indicate different discharges due to backwater effect. To counter this effect, another gauge, known as *secondary gauge* or *auxiliary* gauge is installed at a section further downstream. For any discharge $Q$, both the gauge readings are taken. The difference between the readings of the main gauge and secondary gauge is known as *Fall* $(F)$. Now for a given main gauge reading $G$, discharge $Q$ is a function of $F$ also.

$$Q = f(G, F)$$

Instead of a single 3-D plot, two parametric plots are developed from which $Q$ can be estimated for a known $G$ and $F$.

- From the observed set of $G$, $F$ and $Q$, a constant fall value, $F_0$ is selected.
- The observed set is plotted on an arithmetic graph paper and a best fit curve is drawn for the points having $F_0$ as fall (constant fall curve).
- All fall values are normalized as $F/F_0$ and all discharge values are normalized as $Q/Q_0$. $Q_0$ is the discharge due to $F_0$ for the corresponding $G$, for which $Q$ is obtained.
- A plot of $Q/Q_0$ vs $F/F_0$ is developed (adjustment curve). $(Q/Q_0) = (F/F_0)^m$
- For the new measured $F$, $F/F_0$ is calculated and from the adjustment curve $Q/Q_0$ is obtained.
- From the constant fall curve, $Q_0$ is obtained for new measured $G$.
- $Q$ is estimated as $Q = (Q/Q_0) \times Q_0$

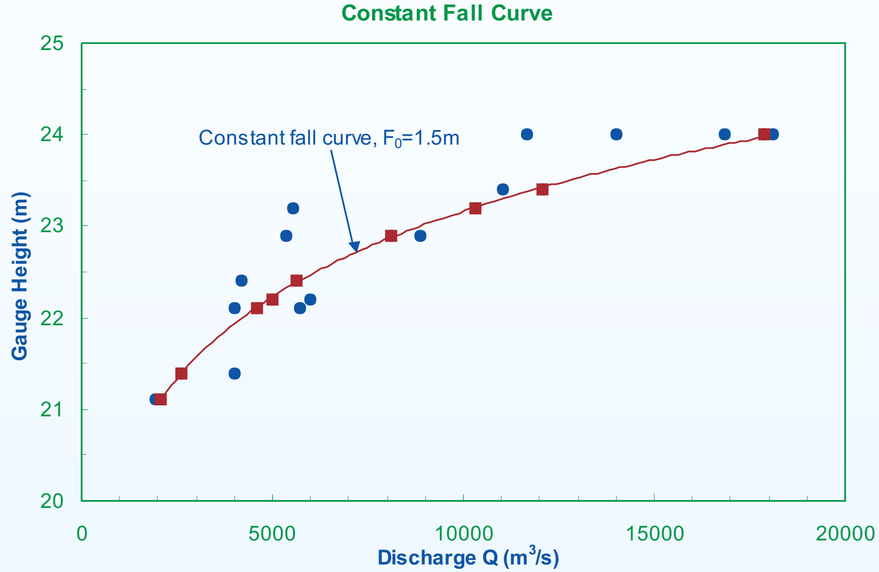
# Example

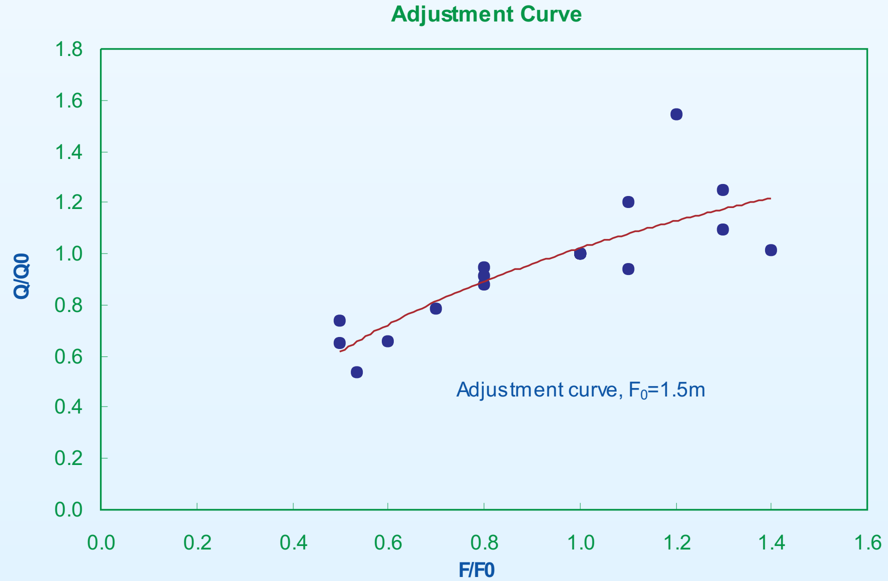


| Primary Gauge Raeding G (m) | Secondary Gauge Reading G1(m) | Fall = (G-G1) (m) | Discharge, Q (m3/s) | Q/Q0 | F/F0 |
|---|---|---|---|---|---|
| 21.1 | 19.9 | 1.20 | 1950 | 0.947 | 0.800 |
| 21.1 | 19.6 | **1.50** | **2060** | 1.000 | 1.000 |
| 21.4 | 19.9 | **1.50** | **2595** | 1.000 | 1.000 |
| 21.4 | 19.6 | 1.80 | 4010 | 1.545 | 1.200 |
| 22.1 | 20.9 | 1.20 | 4030 | 0.881 | 0.800 |
| 22.1 | 20.6 | **1.50** | **4575** | 1.000 | 1.000 |
| 22.1 | 20.2 | 1.95 | 5715 | 1.249 | 1.300 |
| 22.2 | 20.7 | **1.50** | **4985** | 1.000 | 1.000 |
| 22.2 | 20.6 | 1.65 | 5980 | 1.200 | 1.100 |
| 22.4 | 21.7 | 0.75 | 4175 | 0.739 | 0.500 |
| 22.4 | 20.9 | **1.50** | **5650** | 1.000 | 1.000 |
| 22.9 | 22.0 | 0.90 | 5340 | 0.658 | 0.600 |
| 22.9 | 21.4 | **1.50** | **8120** | 1.000 | 1.000 |
| 22.9 | 21.0 | 1.95 | 8870 | 1.092 | 1.300 |
| 23.2 | 22.4 | 0.80 | 5540 | 0.537 | 0.533 |
| 23.2 | 21.7 | **1.50** | **10325** | 1.000 | 1.000 |
| 23.4 | 22.2 | 1.20 | 11025 | 0.912 | 0.800 |
| 23.4 | 21.9 | **1.50** | **12085** | 1.000 | 1.000 |
| 24.0 | 23.3 | 0.75 | 11650 | 0.652 | 0.500 |
| 24.0 | 23.0 | 1.05 | 14015 | 0.784 | 0.700 |
| 24.0 | 22.5 | **1.50** | **17870** | 1.000 | 1.000 |
| 24.0 | 22.4 | 1.65 | 16850 | 0.943 | 1.100 |
| 24.0 | 21.9 | 2.10 | 18110 | 1.013 | 1.400 |

# Parametric Plot





## Example



An auxiliary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effect. The following data were noted at a certain main gauge reading.

| Main gauge | Auxiliary gauge | Discharge |
|---|---|---|
| (m above datum) | (m above datum) | ($m^3$/s) |
| 86.00 | 85.50 | 275 |
| 86.00 | 84.80 | 600 |

If the gauge reading is still 86.00m and the auxiliary gauge reads 85.30m, estimate the discharge in the river.

**Solution:**

Fall $F_1 = 86.00 - 85.50 = 0.50m; \quad F_2 = 86.00 - 84.80 = 1.20m$

# Example

An auxiliary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effect. The following data were noted at a certain main gauge reading.

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|---|---|---|
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If the gauge reading is still 86.00m and the auxiliary gauge reads 85.30m, estimate the discharge in the river.

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Fall $F_1 = 86.00 - 85.50 = 0.50m;\quad F_2 = 86.00 - 84.80 = 1.20m$

Discharge $Q_1 = 275\, m^3/s;\quad Q_2 = 600\, m^3/s$

# Example

An auxiliary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effect. The following data were noted at a certain main gauge reading.

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|---|---|---|
| (m above datum) | (m above datum) | ($m^3/s$) |
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If the gauge reading is still 86.00m and the auxiliary gauge reads 85.30m, estimate the discharge in the river.

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Fall $F_1 = 86.00 - 85.50 = 0.50m$; $F_2 = 86.00 - 84.80 = 1.20m$

Discharge $Q_1 = 275\, m^3/s$; $Q_2 = 600\, m^3/s$

Now, $Q_1/Q_2 = (F_1/F_2)^m$ or

# Example

An auxiliary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effect. The following data were noted at a certain main gauge reading.

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Discharge $Q_1 = 275\, m^3/s; \quad Q_2 = 600\, m^3/s$

Now, $Q_1/Q_2 = (F_1/F_2)^m$ or $m$ = $[\ln(Q_1/Q_2)/\ln(F_1/F_2)]$ =0.8911

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| 86.00 | 85.50 | 275 |
| 86.00 | 84.80 | 600 |

If the gauge reading is still 86.00m and the auxiliary gauge reads 85.30m, estimate the discharge in the river.

**Solution:**

Fall $F_1 = 86.00 - 85.50 = 0.50m$; $\quad F_2 = 86.00 - 84.80 = 1.20m$

Discharge $Q_1 = 275\, m^3/s$; $\quad Q_2 = 600\, m^3/s$

Now, $Q_1/Q_2 = (F_1/F_2)^m$ or $m$ = $[\ln(Q_1/Q_2)/\ln(F_1/F_2)]$ $=$0.8911

So, for $G = 86.00m$, $F = 86.00 - 85.30 = 0.70\, m$

# Example



An auxiliary gauge was used downstream of a main gauge in a river to provide corrections to the gauge-discharge relationship due to backwater effect. The following data were noted at a certain main gauge reading.

| Main gauge | Auxiliary gauge | Discharge |
|---|---|---|
| (m above datum) | (m above datum) | ($m^3/s$) |
| 86.00 | 85.50 | 275 |
| 86.00 | 84.80 | 600 |

If the gauge reading is still 86.00m and the auxiliary gauge reads 85.30m, estimate the discharge in the river.

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So, for $G = 86.00m$, $F = 86.00 - 85.30 = 0.70\, m$

$Q = Q_2 \times (F_1/F_2)^m = 600 \times (0.70/1.20)^{0.8911}$= $371.16\, m^3/s$

# Example



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|---|---|---|
| (m above datum) | (m above datum) | ($m^3$/s) |
| 86.00 | 85.50 | 275 |
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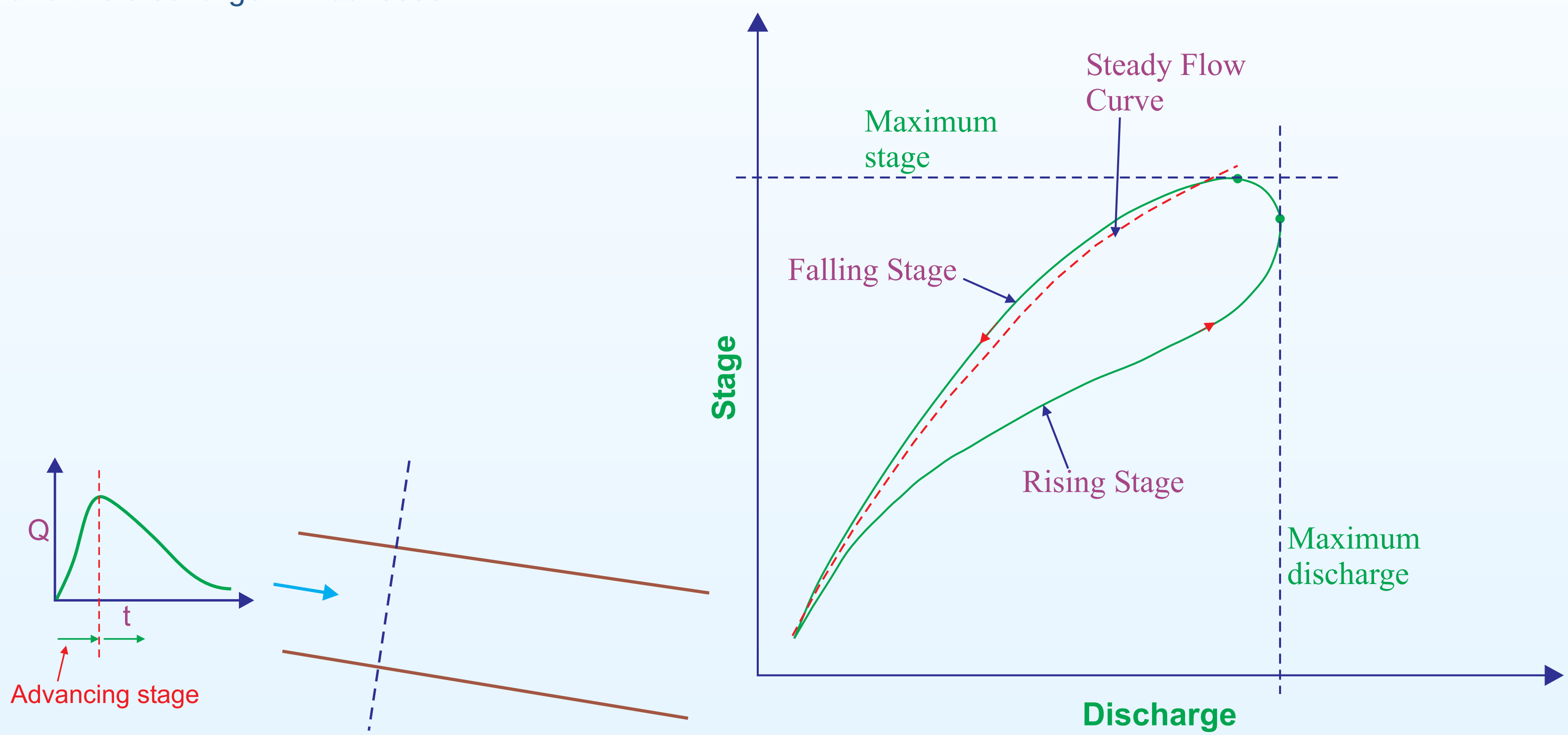
So, for $G = 86.00m$, $F = 86.00 - 85.30 = 0.70\, m$

$Q = Q_2 \times (F_1/F_2)^m = 600 \times (0.70/1.20)^{0.8911}$= $371.16\, m^3/s$

Hence, the new discharge, $Q$= 371 $m^3$/s.

# Unsteady Flow Effect

When a flood wave passes a gauging station in the advancing portion of the wave, the approach velocities are larger than in the steady flow at the corresponding stage. So, for the same stage there will be more discharge. Similarly, in the retreating stage of the flood wave, velocity will be less than the steady velocity and the discharge will be lesser.



# Unsteady Flow Effect



If $Q_n$ is the normal discharge at a given stage under steady uniform flow and $Q_M$ is the measured unsteady flow, then

$$\frac{Q_M}{Q_n} = \sqrt{1 + \frac{1}{V_w S_0}\frac{dh}{dt}} \tag{7}$$

where $S_0$ = channel slope = water surface slope at uniform flow, $dh/dt$ = rate of change of stage and $V_w$ = velocity of flood wave. For natural channels, $V_w \simeq 1.4V$, where $V$ = average velocity for a given stage as estimated by Manning's formula with $S_f$. In that case $S_0$ in equation 7 is to be replaced by $S_f$.

**Example**

During a flood the water surface at a section in a river was found to increase at a rate of 11.2 cm/h. The slope of the river is 1/3600 and the normal discharge for the river stage read from a steady-flow rating curve was 160 m$^3$/s. If the velocity of the flood wave can be assumed as 2.0 m/s, determine the actual discharge.

Answer: 164.4 m$^3$/s