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(Reaffirmed 1998)

Indian Standard

CRITERIA FOR DESIGN OF REINFORCED CONCRETE SHELL STRUCTURES AND FOLDED PLATES

(First Revision)

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Indian Standard

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0. FOREWORD

0.1 This Indian Standard (First Revision) was adopted by the Bureau of Indian Standards on 15 November 1988, after the draft finalized by the Criteria for Design of Special Structures Sectional Committee had been approved by the Civil Engineering Division Council.

0.2 Shells and folded plates belong to the class of stressed-skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. Wherever shell is referred to in this standard, it refers to thin shell. On account of multiplicity of the types of reinforced concrete shell and folded plate structures used in present day building practice for a variety of applications demanding roofing of large column-free area, it is not practicable to lay down a rigid code of practice to cover all situations. Therefore, this standard lays down certain general recommendations for the guidance of the designers.

0.3 Cylindrical shells have been in use in building construction for the past six decades. Well developed theories exist for their analysis. Labour-saving short cuts, such as, charts and tables for their design, are also available.

0.4 Although shells of double curvature, with the exception of domes, have been introduced on a large scale comparatively recently into building construction, these are likely to be used more and more in future. Being non-developable surfaces, they are more resistant to buckling than cylindrical shells and in general, require less thickness. This saving in materials is, however, often offset by the relatively expensive shuttering required for casting them. Among the doubly-curved shells, the hyperbolic paraboloid and the conoid have, however, the advantage of less expensive shuttering because their ruled surfaces can be formed by straight plank shuttering.

0.5 Folded plates are often competitive with shells for covering large column-free areas. They usually consume relatively more materials compared to shell but this disadvantage is often offset by the simpler framework required for their construction.

0.6 This standard was first published in 1962. The present revision is based on the developments in the design of shell and folded plate structures subsequently and to include more rigorous methods of analysis which have become available to enforce more rational criteria of design.

0.7 For the purpose of deciding whether a particular requirement of this standard is complied with, the final value, observed or calculated, expressing the result of a test or analysis, shall be rounded off in accordance with IS : 2-1960*. The number of significant places retained in the rounded off value should be the same as that of the specified value in this standard.

*Rules for rounding off numerical values (*revised*).

1. SCOPE

1.1 This standard lays down recommendations for the classification, dimensional proportioning, analysis and design of cast *in situ*, reinforced concrete thin shells and folded plates. This standard does not deal with construction practices relating to these structures which have been separately dealt in IS : 2204-1962*.

2. TERMINOLOGY

2.0 For the purpose of this standard, the following definitions shall apply.

2.1 Asymmetrical Cylindrical Shells — Cylindrical shells which are asymmetrical about the crown.

2.2 Barrel Shells — Cylindrical shells which are symmetrical about the crown (*see* Fig. 1).

2.3 Butterfly Shells — Butterfly shells are those which consist of two parts of a cylindrical shell joined together at their lower edges (*see* Fig. 2).

2.4 Chord Width — The chord width is the horizontal projection of the arc of the cylindrical shell.

2.5 Continuous Cylindrical Shells — Cylindrical shells which are longitudinally continuous over the traverses (*see* Fig. 3).

NOTE — Doubly-curved shells continuous in one or both directions may be termed as continuous shells.

2.6 Cylindrical Shells — Shells in which either the directrix or generatrix is a straight line.

*Code of practice for construction of reinforced concrete shell roof.

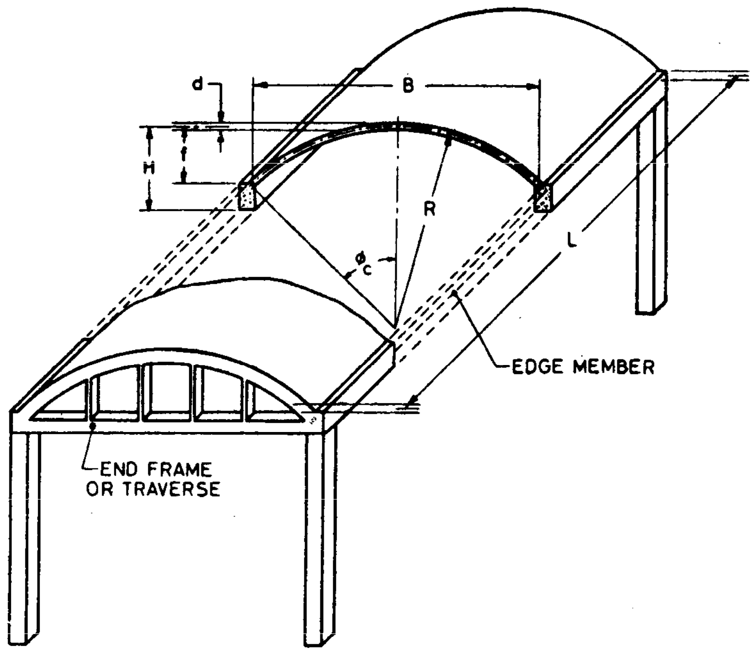

FIG. 1 SINGLE BARREL SHELL

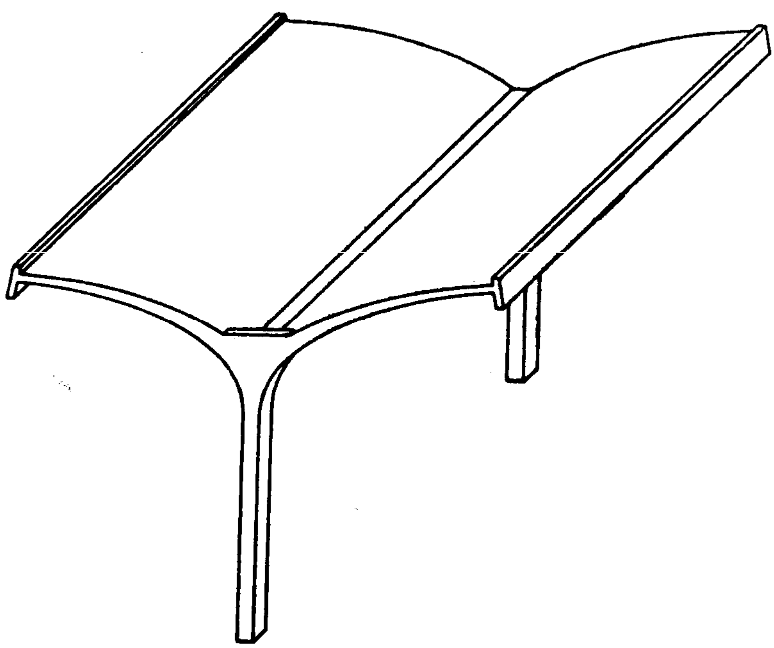

FIG. 2 BUTTERFLY SHELL

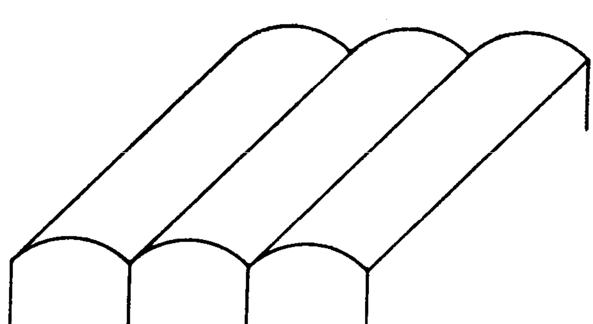

FIG. 3 MULTIPLE BARREL SHELL

2.7 Edge Member — A member provided at the edge of a shell.

NOTE — Edge members increase the rigidity of the shell edge and help in accommodating the reinforcement.

2.8 End Frames or Traverses — End frames or traverses are structures provided to support and preserve the geometry of the shell.

NOTE — They may be solid diaphragms, arch ribs, portal frames or bowstring girders. Where a clear soffit is required, specially for the use of movable formwork, the end frames may consist of upstand ribs.

2.9 Folded Plates — Folded plates consist of a series of thin plates, usually rectangular, joined monolithically along their common edges and supported on diaphragms. They are also known as hipped plates.

NOTE — *Shapes of Folded Plates and Their Applications* — A few of the commonly used shapes of folded plates are shown in Fig. 4. The simplest is V-shaped unit (Fig. 4A) but this may not provide enough area of concrete at the top and bottom to resist the compressive forces due to bending and to accommodate the reinforcement. The trough-shaped or the trapezoidal unit (Fig. 4B and 4E) eliminates these disadvantages. Asymmetrical sections of the 'Z' shape (Fig. 4C) provided with window glazing between two adjacent units serve as north-light roofs for factory buildings. The shape shown in Fig. 4D is obtained by replacing the curved cross-section of a cylindrical shell by a series of straight plates. This has the advantage of greater structural depth compared to other shapes. Butterfly type of folded plates shown in Fig. 4F are also employed to cover factory roofs as there are provisions for window glazing. Tapering plates are also used as roofs mainly for aesthetic reasons. Hipped plate structures of the pyramidal types are used for tent-shaped roofs, cooling towers, etc.

2.10 Gauss Curvature — The product of the two principal curvatures, $1/R_1$ and $1/R_2$ at any point on the surface of the shell.

2.11 Generatrix, Directrix — A curve which moves parallel to itself over a stationary curve generates a surface. The moving curve is called the generatrix and the stationary curve the directrix. One of them may be a straight line.

NOTE — The common curves used for cylindrical shells are, arc of a circle, semi-ellipse, parabola, catenary and cycloid.

2.12 Junction Member — The common edge member at the junction of two adjacent shells.

2.13 Multiple Cylindrical Shells — A series of parallel cylindrical shells which are transversely continuous.

2.14 North-Light Shells — Cylindrical shells with two springings at different levels and having provisions for north-light glazing (*see* Fig. 5).

2.15 Radius — Radius at any point of the shell in one of the two principal directions.

NOTE — If cylindrical shell of a circular arc is used, the radius of the arc is the radius of the shell. In other cases, the radius R at any point is related to the radius R_0 at the crown by $R = R_0 \cos n\phi$, where ϕ is the angle of inclination of the tangent to the curve at that point. The parabola, value of n is 1, −2 and −3 for cycloid, catenary and parabola, respectively. For an ellipse:

$$R = \frac{a^2 b^2}{(a^2 \sin^2\phi + b^2 \cos^2\phi)^{3/2}}$$

where a and b are the semi-major and semi-minor axes, respectively, and ϕ is the slope of the tangent at the point.

2.16 Rise — The vertical distance between the apex of the curve representing the centre line of the shell and the lower most springing.

2.17 Ruled Surfaces — Surfaces which can be generated entirely by straight lines. The surface is said to be 'singly ruled' if at every point, a single straight line only can be ruled and 'doubly ruled' if at every point, two straight lines can be ruled. Cylindrical shells, conical shells and conoids are examples of singly ruled surfaces; hyperbolic paraboloids and hyperboloids of revolution of one sheet are examples of doubly ruled surface (*see* Fig. 6).

2.18 Semi-Central Angle — Half the angle subtended by the arc of a symmetrical circular shell at the centre.

2.19 Shells — Thin shells are those in which the radius to thickness ratio should not be more than 20.

2.20 Shells of Revolution — Shells which are obtained when a plane curve is rotated about the axis of symmetry. Examples are segmental domes, cones, paraboloids of revolution, hyperboloids of revolution, etc (*see* Fig. 7).

2.21 Shells of Translation — Shells which are obtained when the plane of the generatrix and the directrix are at right angles. Examples are cylindrical shells, elliptic paraboloids, hyperbolic paraboloids, etc (*see* Fig. 8).

2.22 Span — The span of a cylindrical shell is the distance between the centre lines of two adjacent end frames of traverses (*see* Fig. 1).

3. NOTATIONS

3.1 For the purpose of this standard, unless otherwise defined in the text, the following notations shall have the meaning indicated against each:

a = semi-major axis of an elliptical shell;
B = chord width;
b = semi-minor axis of an elliptical shell;
D = flexural rigidity;
d = thickness of shell;
E_c = Modulus of elasticity of concrete (long term);
E_s = Modulus of elasticity of steel;
F = stress function which gives the in-plane stress in doubly-curved shells when bending is also considered;
f_{ck} = characteristic strength of concrete;
f_{cr} = critical buckling stress;
h = rise of shell;

f_{ac} = permissible compressive stress from buckling consideration;

H = total depth of shell, measured from the crown of the shell to the bottom of the edge member;

L = span;

M_x = bending moment in the shell in the x-direction;

M_y = bending moment in the shell in the y-direction;

M_{xy} = twisting moment in the shell;

N_x, N_y and N_{xy} = real membrane stresses in the shell;

N_{xp}, N_{yp} and N_{xyp} = projected membrane forces;

P = permissible buckling load per unit area of the surface of doubly-curved shells;

$n = \frac{\delta z}{\delta x}$;

$q = \frac{\delta z}{\delta y}$;

$r = \frac{\delta^2 z}{\delta x^2}$;

$s = \frac{\delta^2 z}{\delta x . \delta y}$;

$t = \frac{\delta^2 z}{\delta y^2}$;

R = radius;

R_c = radius at crown;

R_1 and R_2 = principal radii of curvature at any point on the surface of shell;

S = shear stress;

T_x = normal stress in the x-direction;

T_y = normal stress in the y-direction;

W_x, W_y and W_z = real forces on unit area of the shell in the x, y and z-direction;

w = deflection in the direction of z-axis;

X, Y and Z = fictitious forces on unit projected area of the shell in the x, y and z-directions;

x, y and z = axes of co-ordinates;

Φ = stress function used in the membrane analysis of doubly-curved shell;

ϕ = angle of inclination of tangent to the curve at any point;

ϕ_c = semi-central angle of a symmetrical circular cylindrical shell;

ρ and κ = Aas Jakobsen's parameters for cylindrical shells;

ν = Poisson's ratio; and

$\nabla^2 = \left[\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2}\right]$

4. CLASSIFICATION OF SHELLS

4.1 General — Shells may be broadly classified as 'singly-curved' and 'doubly-curved'. This is based on Gauss curvature. The gauss curvature of singly-curved shells is zero because one of their principal curvatures is zero. They are, therefore, developable. Doubly-curved shells are non-developable and are classified as synclastic or anticlastic according as their Gauss curvature is positive or negative.

4.1.1 The governing equations of membrane theory of singly curved shells are parabolic. It is elliptic for synclastic shells and hyperbolic for anticlastic shells. If $z = f(x, y)$ is the equation to the surface of a shell, the surface will be synclastic, developable or anticlastic according as $s^2 - rt \lesseqgtr 0$ where r, s and t are as defined in **3.1**.

4.1.2 There are other special types of doubly curved shells, such as, funicular shells, which are synclastic and anticlastic in parts and corrugated shells which are alternately synclastic and anticlastic. The gauss curvature for such shells is positive where they are synclastic and negative where they are anticlastic.

4.2 The detailed classification of shell structures is given in Appendix A.

5. MATERIALS

5.1 Concrete — Controlled concrete shall be used for all shell and folded plate structures. The concrete is of minimum grade M20. The quality of materials used in concrete, the methods of proportioning and mixing the concrete shall be done in accordance with the relevant provisions of IS : 456-1978*.

NOTE — High cement content mixes are generally undesirable as they shrink excessively giving rise to cracks.

5.2 Steel — The steel for the reinforcement shall be:

a) mild steel and medium tensile steel bars and hard-drawn steel wire conforming to IS : 432 (Part 1)-1982 and IS : 432 (Part 2)-1982†;

b) hard-drawn steel wire fabric for concrete reinforcement conforming to IS : 1566-1982‡; and

c) high strength deformed bars conforming to IS : 1786-1985§.

5.2.1 Welding may be used in reinforcement in accordance with IS : 456-1978*.

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*Code of practice for plain and reinforced concrete (*third revision*).

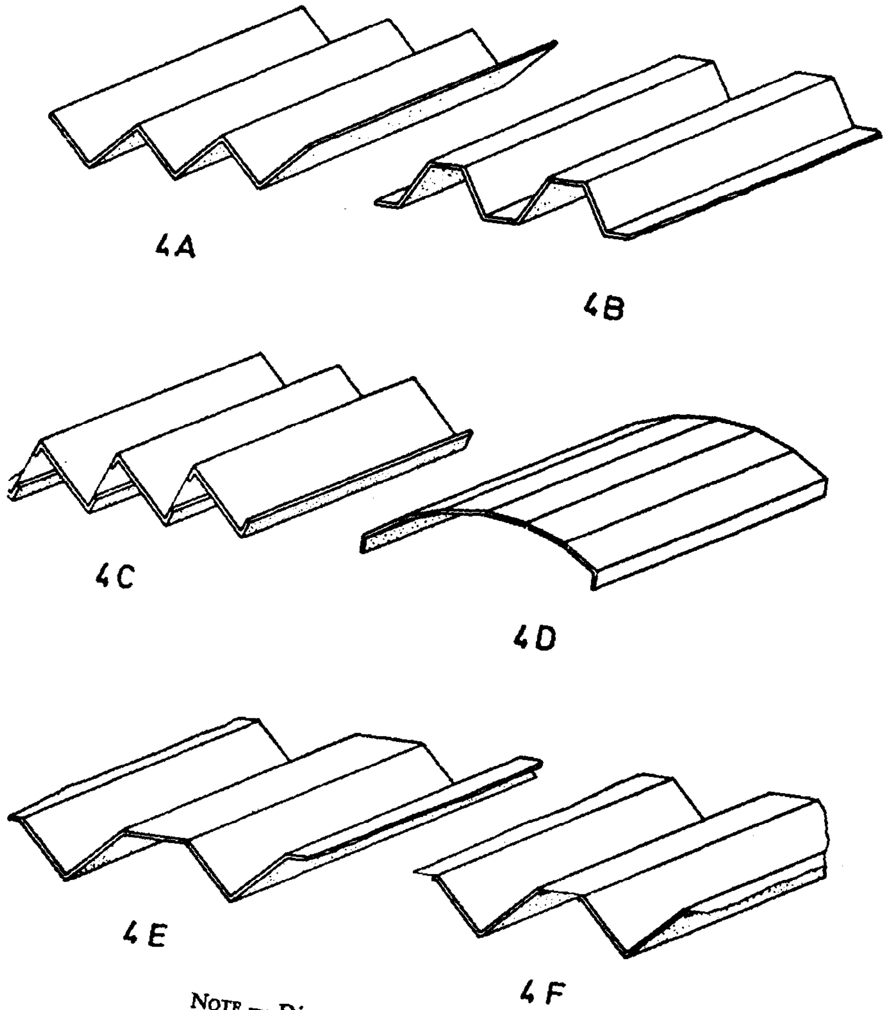
†Specification for mild steel and medium tensile steel bars and hard-drawn steel wire for concrete reinforcement:

Part 1 Mild steel and medium tensile steel bars (*third revision*).

Part 2 Hard-drawn steel wire (*third revision*).

‡Specification for hard-drawn steel wire fabric for concrete reinforcement (*second revision*).

§Specification for high strength deformed steel bars and wires for concrete reinforcement (*third revision*).



NOTE — Diaphragms are not shown.

FIG. 4 FOLDED PLATES

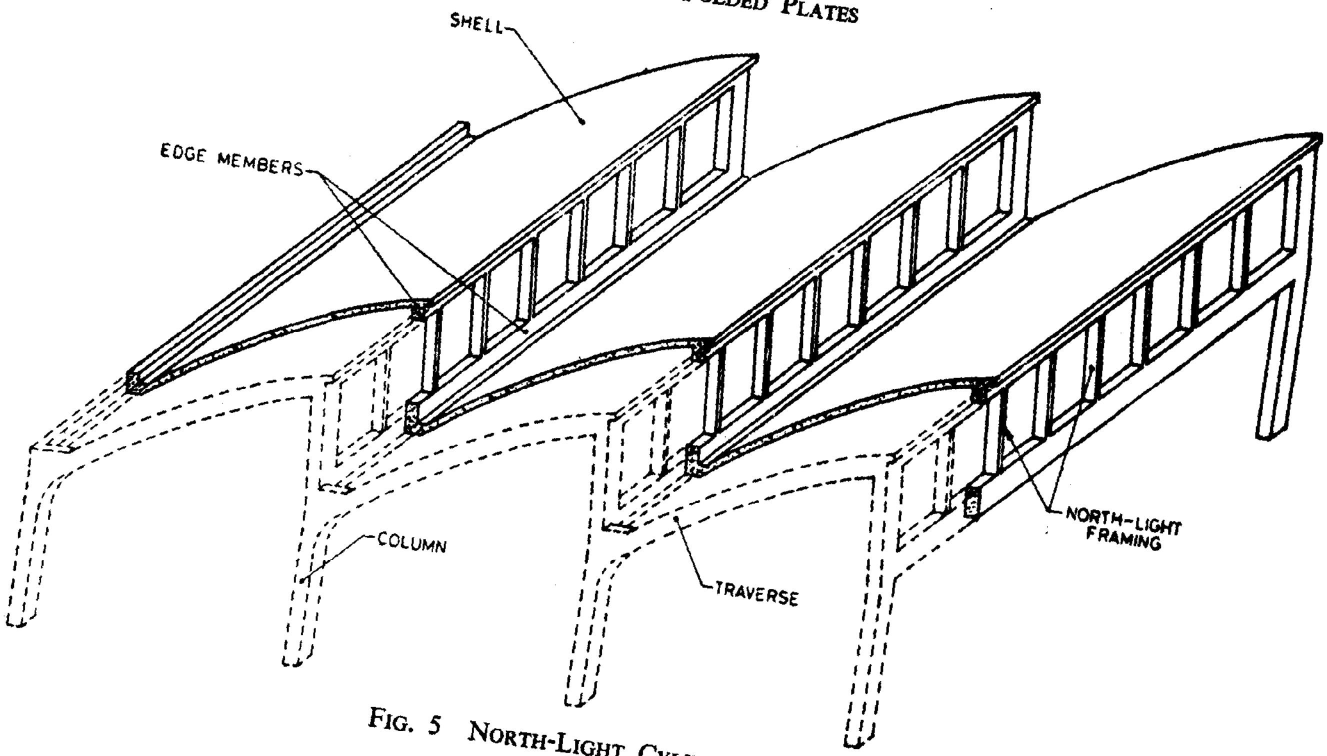
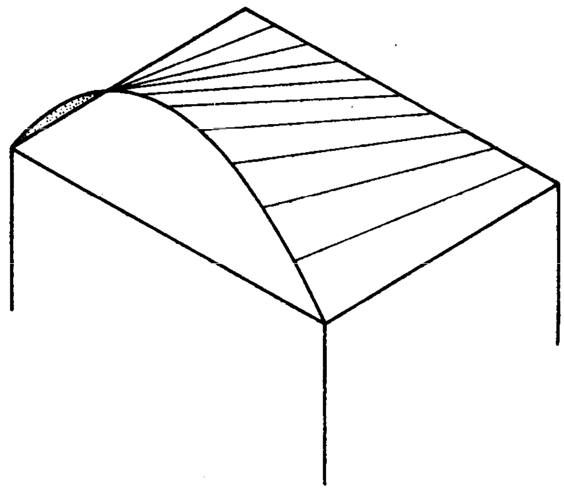
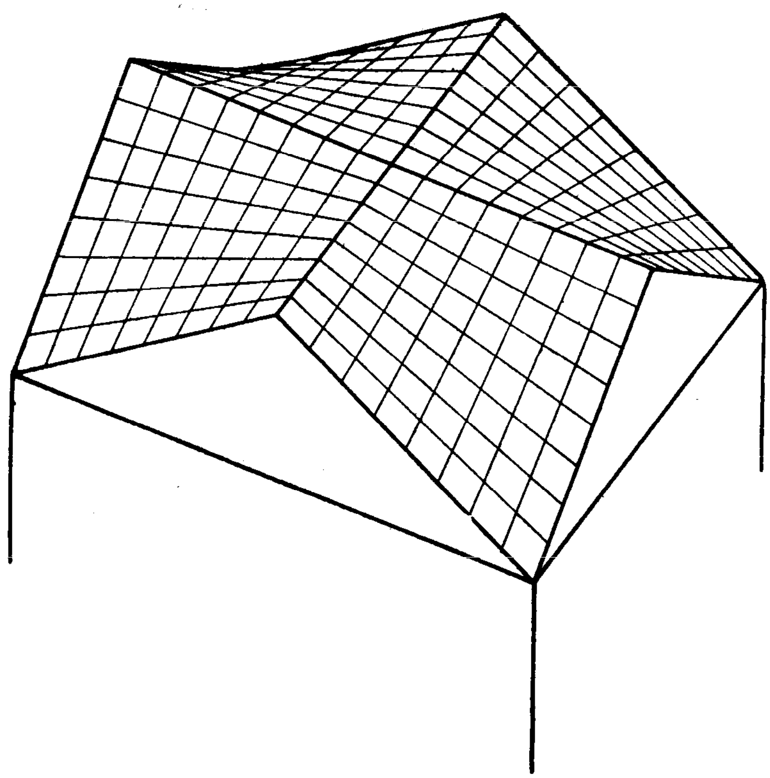


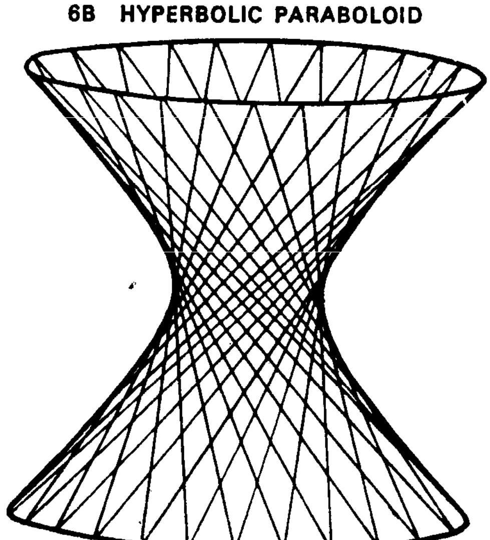
FIG. 5 NORTH-LIGHT CYLINDRICAL SHELLS



6A CONOID

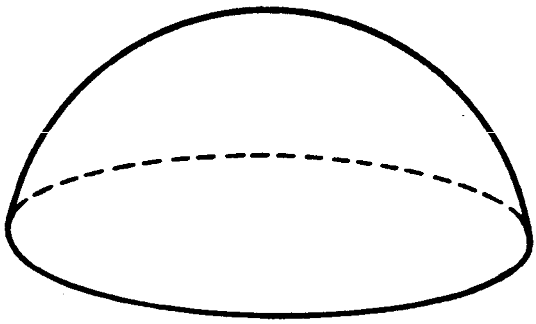


6B HYPERBOLIC PARABOLOID

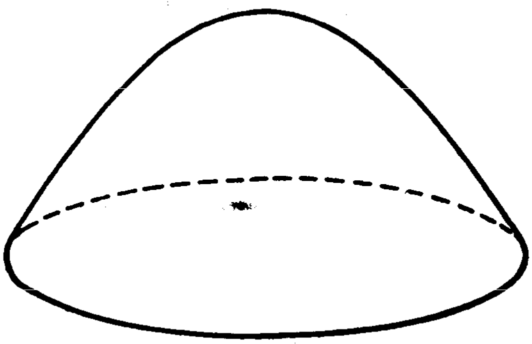


6C HYPERBOLOID OF REVOLUTION OF ONE SHEET

FIG. 6 RULED SURFACES

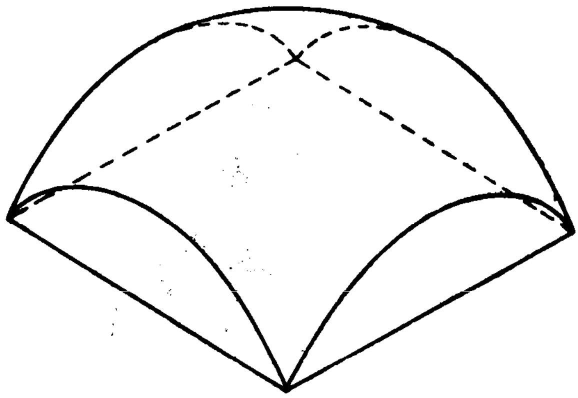


7A SEGMENTAL DOME

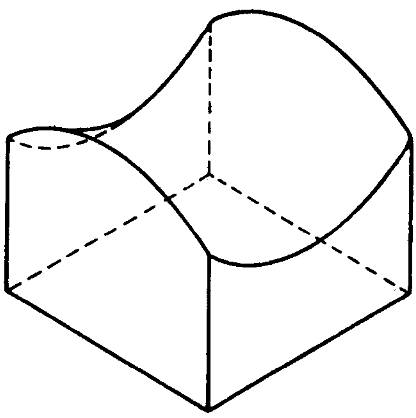


7B PARABOLOID OF REVOLUTION

FIG. 7 SHELLS OF REVOLUTION



8A ELLIPTIC PARABOLOID



8B HYPERBOLIC PARABOLOID

FIG. 8 SHELLS OF TRANSLATION

6. LOADS

6.1 Unless otherwise specified, shells and folded plates shall be designed to resist the following load combinations:

a) Dead load,
b) Dead load + appropriate live load or snow load,
c) Dead load + appropriate live load + wind load, and
d) Dead load + appropriate live load + seismic load.

6.2 Dead loads shall be calculated on the basis of the unit weights taken in accordance with IS : 875 (Part 1)-1987*.

6.3 Live loads, wind loads and snow loads shall be taken as specified in IS : 875 (Parts 2 to 4)-1987*.

6.4 Seismic loads shall be taken in accordance with IS : 1893-1984†.

6.5 Where concentrated loads occur, special considerations should be given in analysis and design.

7. SELECTION OF DIMENSIONS

7.1 Thickness

7.1.1 *Thickness of Shells* — Thickness of shells shall not normally be less than 50 mm if singly-curved and 40 mm if doubly-curved. This requirement does not, however, apply to small precast concrete shell units in which the thickness may be less than that specified above but it shall in no case be less than 25 mm (*see* IS : 6332-1984‡).

7.1.1.1 The reinforcement shall have a minimum clear cover of 15 mm or its nominal size whichever is greater.

7.1.2 Shells are usually thickened for some distance from their junction with edge members and traverses. The thickening is usually of the order of 30 percent of the shell thickness. It is, however, important to note that undue thickening is undesirable. In the case of singly-curved shells, the distance over which the thickening at the junction of the shell and traverse is made should be between $0{\cdot}38\ \sqrt{R.d}$ and $0{\cdot}76\ \sqrt{R.d}$, where R and d are the radius and the thickness, respectively. The thickening of shell at straight edges shall depend on the transverse bending moment. For doubly-curved shells, this distance will depend upon the geometry of the shell and the boundary conditions as the extent of bending penetration is governed by these factors.

7.1.3 *Thickness of Folded Plates* — The thickness of folded plates shall not normally be less than 75 mm.

7.2 Other Dimensions

7.2.1 *Cylindrical Shells*

7.2.1.1 The span should preferably be less than 30 m. Shells longer than 30 m will involve special design considerations, such as the application of prestressing techniques.

7.2.1.2 The width of the edge member shall generally be limited to three times the thickness of the shell.

7.2.1.3 The radius of shell structures shall be selected keeping acoustic requirements in view. Coincidence of the centre of curvature with the working level should be avoided unless suitable acoustic correction is made. It is, however, important to note that even where coincidence of centre of curvature with the working level is avoided, acoustic treatment may be necessary in certain cases.

7.2.1.4 A single cylindrical shell whose span is larger than three times the chord width shall have a total depth, H, between 1/6 and 1/12 of its span (the former value being applicable to smaller spans). The rise in the case of a shell without edge members shall not be less than 1/10 of its span.

7.2.1.5 For a shell with chord width larger than three times the span, the rise of the shell shall not be less than 1/8 of its chord width.

7.2.1.6 The chord width of shells shall preferably be restricted to six times the span as otherwise arch action is likely to predominate.

7.2.1.7 The semi-central angle shall preferably be between 30 and 40°.

NOTE — Keeping the semi-central angle between these limits is advisable for the following reasons:

a) If the angle is below 40°, the effect of wind load on the shell produces only suction; and
b) With slopes steeper than 40°, backforms may become necessary.

Within these limits the semi-central angle shall be as high as possible consistent with the functional requirements.

7.2.2 *Folded Plates* — For folded plates of type shown in Fig. 4D, the selection of depth may be based on the rules applicable to cylindrical shells. With other shapes, such as, the 'V' or the trough, the depth may be taken as about 1/15 of the span for preliminary designs.

*Code of practice for design loads (other than earthquake) for building and structures:

Part 1 Dead loads (*second revision*).
Part 2 Imposed loads (*second revision*).
Part 3 Wind loads (*second revision*).
Part 4 Snow loads (*second revision*).

†Criteria for earthquake resistant design of structures (*fourth revision*).

‡Code of practice for construction of floors and roofs using precast doubly-curved shell units (*first revision*).

7.2.2.1 The angle of inclination of the plates to horizontal shall be limited to about 40 for *in-situ* construction in order to facilitate placing of concrete without the use of the backforms.

8. ANALYSIS

8.0 General — Shells may be analyzed either by linear elastic analysis based on theory of elasticity or yield line theory. Methods based on yield line theory for shells are still the subject of research and experimentation and, therefore, for the present, it is recommended that they may be used along with model tests to check the load carrying capacity.

The finite element method has become a practical and popular method of analysis for all types of structures. Many common and important features of shell and folded plate structures that cannot be considered by classical methods can now be analyzed satisfactorily by the finite element method. For example:

a) Complex support or boundary conditions;

b) Openings large enough to disturb global stress distribution;

c) Irregular surface geometry;

d) Highly variable or localized loads;

e) Tapering folded plates or boxes or silo and bunker bottoms;

f) Branching shells;

g) Large deformations;

h) Heavy and eccentric stiffners;

j) Thermoelastic strains;

k) Elastoplastic, viscoelastic or any inelastic behaviour;

m) Material non-homogeneity;

n) Irregular surface geometry;

p) Sudden changes in curvature;

q) Shells under dynamic wind action; and

r) Possible effect of settlement.

When one or more of these complexities occur in shell structures, it is advisable to use finite element method, at least for a final acceptance of the design. Even normal shell structures of spans larger than 30 m should be analyzed by finite element method if it is expected that there would be serious and significant structural participation in the shell behaviour by the supporting units, such as, edge and intermediate beams, stiffners, and or intermediate traverses (specially flexible traverses) cable supports, columns, etc.

For many shells, finite strip method (FSM) (a particular form of finite element method) of analysis is easier to apply and also more economical to use than the finite element method. All types of prismatic folded plates and cylindrical shells can be handled by FSM since these shells can be discretized into long strip elements. Shells of revolution can also be efficiently analyzed by this method, after discretization of such shells into finite ring elements. Since FSM can be used even when the loads are not uniform, it may be advisable to use the method even for simple shells that are amenable to analysis by common classical methods.

The common classical methods of analysis of shells are mentioned in the following clauses for use for the analysis of common types of shells that are without any of the complexities.

8.1 Cylindrical Shells

8.1.1 *Analytical Methods* — The analytical methods consist of two parts, membrane analysis and edge disturbance analysis.

8.1.1.1 *Membrane analysis* — In the membrane analysis, the shell is regarded as a perfectly flexible membrane which is infinite in extent and is assumed to carry loads by means of forces in its plane only. This analysis gives the two normal stress resultants N_x and N_y in the longitudinal and the transverse directions and the shear stress resultant N_{xy}.

8.1.1.2 *Edge disturbance analysis* — Shells, in practice, are always limited by finite boundaries where the boundary conditions demanded by the membrane theory are not obtained with the result that a pure membrane state would seldom exist. Edge disturbances emanate from the boundaries, altering the membrane state and causing bending stresses in the shell. These are accounted for by carrying out the edge disturbance analysis. Usually edge disturbance analysis is confined to disturbance emanating from straight edge as any disturbance emanating from curved edges is damped quite fast. Even in the case of disturbance from straight edges, the bending stresses would get damped out more rapidly in shells having chord width larger than the span and would seldom travel beyond the crown, with the result that the effect of the further edge may be ignored without appreciable error.

The superposition of the membrane and the edge disturbance stresses gives the final stress pattern in the shell.

8.1.1.3 *Tables for the analysis of circular cylindrical shells* — Simplifications in the analysis of shells are possible by systematizing the calculations making use of tables compiled for this purpose (*see* Appendix B).

8.1.2 *Applicability of the Methods of Analysis*

8.1.2.1 Cylindrical shells with $\frac{L}{R}$ ratio less than π shall be analyzed using any of the accepted analytical methods (*see* Appendix B).

In such shells, if ρ exceeds 10 and κ exceeds 0·15, the effect at any point on the shell of the disturbances emanating from the farther edge may be ignored, where

$$\rho = 8\sqrt{\frac{12\,\pi^4 R^6}{L^4 d^2}} \text{ and } \kappa = \frac{\pi^2 R^2}{L^2 \rho^2}$$

For shells with ρ less than 7 and κ less than 0·12, the effect of the disturbances from both the edges shall be considered. Shells with ρ values between 7 and 10 and κ between 0·12 and 0·15 are relatively infrequent. However, should such cases arise, the effects of both the edges shall be considered.

8.1.2.2 Cylindrical shells with L/R greater than or equal to π may be treated as beams of curved cross section spanning between the traverses and the analysis carried out using an approximate method known as the beam method (*see* Appendix B) which consists of the following two parts:

a) The beam calculation which gives the longitudinal stress resultant N_x and the shear stress resultant N_{xy}, and

b) The arch calculation which gives the transverse stress resultant N_y and the transverse moment M_y.

8.1.3 *Continuous Cylindrical Shells*

8.1.3.1 *Analytical methods* — In the analytical methods, the problem of continuous shells is solved in two stages. In the first, the shell is assumed to be simply supported over one span and all the stress resultants worked out. In the second stage, corrections for continuity are worked out and are superimposed on the values corresponding to the simply supported span. Long cylindrical shells can be analyzed by approximating the cylindrical profile by a folded plate shape and applying well known analysis methods for continuous folded plates (*see* Appendix B).

8.1.3.2 *Beam method* — Solution of continuous shell with $\frac{L}{R} \geqslant \pi$ is simpler by the beam method. The bending moment factors are obtained by solving the corresponding continuous beam. Thereafter, analysis can be continued as in **8.1.2.2**.

8.2 Doubly-Curved Shells

8.2.1 *Membrane Analysis* — In the membrane analysis, it is assumed that the shell carry loads by in-plane stress resultants and usually only deep doubly-curved shells behave like membranes. The governing equations for membrane analysis of doubly-curved are usually solved by using stress functions (*see* Appendix C).

8.2.2 *Shallow Shells* — Shells may be considered shallow if the rise to span ratio is less than or equal to 1/5 and p^2 and q^2 may be ignored in all the expressions. The shorter side shall be considered as the span for this purpose for shells of rectangular ground plan. A shell with a circular ground plan may be considered shallow if the rise does not exceed 1/5 of the diameter.

NOTE — Based on the same consideration of ignoring p^2, q^2 and $P\,q$ being very small, it is sometimes suggested that the shells may be treated as shallow if the surface is such that the values of p and q do not exceed 1/8 at any point on it. However, in normal cases, for practical purposes, higher values of p and q up to half may be considered as shallow provided the span to rise ratio does not exeeed 5.

8.2.3 *Boundary Conditions for Doubly-Curved Shells* — In general, in the membrane analysis of synclastic shells, only one boundary condition is admissible on each boundary. For an anticlastic shell, the boundary conditions have to be specified in a special manner as the characteristic lines of such surfaces play a significant role in the membrane theory. The type of boundary conditions that can be specified depend on whether or not the boundaries of the shell are characteristic lines.

Further, a membrane state of stress can be maintained in a shell only if the boundaries are such that the reactions exerted by the boundary members on the shells correspond to stresses in the shell at the boundaries given by the membrane theory. It is seldom possible to provide boundary conditions which would lead to a pure membrane state of stress in the shell. In most practical cases, a resort to bending theory becomes necessary.

8.2.3.1 Only deep doubly-curved shells behave like membranes and it is only for such shells that a membrane analysis is generally adequate for design (*see also* **8.0**). Bending analysis is necessary for all singly-curved shells and shallow doubly-curved shells. The governing equation for bending analysis of shallow shells are given in Appendix C.

8.2.4 *Bending Theory of Doubly-Curved Shells* — The governing equations for bending analysis of shallow doubly-curved shells are given in Appendix C.

8.2.5 *Bending Theory of Shells of Revolution, Symmetrically Loaded* — In synclastic shells of revolution, such as, domes which are symmetrically loaded, a more or less membrane state of stress exists. The bending stresses are confined to a very narrow strip close to edge members and get rapidly damped out. The bending stresses in domes can be calculated with sufficient accuracy by approximate methods like Geckler's approximation.

8.2.6 *Funicular Shells* — The shapes of these shells are so choosen that, under uniformly distributed vertical loads, in a membrane state of stress, they develop only pure compression unaccompanied by shear stresses. Thus theoretically no reinforcement will be necessary except in the edge members. Small precast funicular shells without any reinforcement except in edge beams are suitable for roofs and floors of residential, industrial, and institutional buildings (*see* IS : 6332-1984*). For roofs of larger size, *in situ* construction may be resorted to; in such shells, provision of reinforcement is necessary to take care of the effects of shrinkage, temperature and bending.

*Code of practice for construction of floors and roofs using precast doubly-curved shell units (*first revision*).

8.4 Folded Plates — The structural action of folded plates may be thought of as consisting of two parts. the 'slab action' and the 'plate action'. By the slab action, the loads are transmitted to the joints by the transverse bending of the slabs. The slabs, because of their large depth and relatively small thickness, offer considerable resistance to bending in their own planes and are flexible out of their planes. The loads are, therefore, carried to the end diaphragms by the longitudinal bending of the slabs in their own planes. This is known as 'plate action'. The analysis of folded slabs is carried out in two stages.

8.3.1 *Transverse Slabs Action Analysis* — The transverse section of the slab, of unit length, is analyzed as a continuous beam on rigid supports. The joint loads obtained from this analysis are replaced by their components in the planes of the slabs and these are known as plate loads.

8.3.2 *In Plane Plate Action Analysis* — Under the action of 'plate loads' obtained above, each slab is assumed to bend independently between the diaphragms, and the longitudinal stresses at the edges are calculated. Continuity demands that the longitudinal stresses at the common edges of the adjacent slabs be equal. The corrected stresses are obtained by introducing edge shear forces.

8.4 Expansion Joints — The expansion joint shall conform to provisions laid down in IS : 456-1978*. In the case of folded plates, it is recommended that the joint may be located in the ridge slab. In the cases of large spans where it is not feasible to provide expansion joints, effects of shrinkage shall be taken care of in the design.

8.5 Openings in Shells — Openings in shells shall preferably be avoided in zones of critical stresses. Small openings of size not exceeding five times the thickness in shells may be treated in the same way as in the case of reinforced concrete structures. For larger openings, detailed analysis should be carried out to arrive at stresses due to the openings.

9. ELASTIC STABILITY

9.1 Permissible Stresses

9.1.1 Permissible stresses in steel reinforcement, and concrete for shells and folded plates shall be in accordance with the provisions given in IS : 456-1978*.

9.2 Causes of Instability — Instability in a cylindrical shell may be caused by:

a) local buckling in zones submitted to compressive stresses;

b) flattening of shells, known as the 'Brazier Effect', which occurs particularly in shells without edge members; and

c) instability caused by the combined effect of bending and torsion in the shell as a whole. This occurs particularly in asymmetrical shells.

9.3 Buckling in Cylindrical Shells

9.3.1 The permissible buckling stress f_{ac} in cylindrical shells shall be calculated as follows:

$$f_{ac} = \frac{0.25 f_{ck}}{1 + \frac{f_{ck}}{f_{cr}}}$$

where

f_{ck} = characteristic strength of concrete at 28 days; and

f_{cr} = critical buckling stress determined in accordance with (a), (b) and (c) below:

a) *Shells with* $\rho < 7$ *and* $\kappa < 0.12$ — In such shells, buckling is caused by excessive longitudinal compression near the crown of the shell and the critical buckling stress f_{cr} shall be calculated as follows:

$$f_{cr} = 0.20 \frac{E_c . d}{R}$$

b) *Shells with* $\rho > 10$ *and* $\kappa > 0.15$ — In such shells, the transverse stresses tend to be critical from the point of view of buckling and the critical buckling stress f_{cr} shall be determined as follows:

1) For shells with $L < 2.3\sqrt{dR}$

$$f_{cr} = E_c \left[3.4 \left(\frac{d}{L} \right)^2 + 0.025 \left(\frac{L}{R} \right) \right]$$

2) For shells with $L > 2.3\sqrt{dR}$

$$f_{cr} = E_c \left[\frac{0.89 \frac{d}{L} \sqrt{\frac{d}{R}}}{1 - 1.18 \sqrt{\frac{d.R}{L}}} \right]$$

where

E_c = modulus of elasticity of concrete, which may be taken as $= \frac{E_s}{280} \times 3 f_{ck}$;

E_s = modulus of elasticity of steel;

d = thickness of the shell; and

R = radius of curvature.

c) Shells with ρ values between 7 and 10 and κ between 0.12 and 0.15 are relatively infrequent. For such shells, formulae given in (a) or (b) shall apply depending upon whether longitudinal or transverse stresses are critical from considerations of elastic stability.

The value of modulus of elasticity of concrete to be used in the above formulae for calculating the buckling stresses should be the value for long term modulus including the effect of creep also.

*Code of practice for plain and reinforced concrete (*third revision*).

9.4 Buckling in Doubly-Curved Shells — For spherical shells, the permissible buckling load per unit area of surface, P_{perm} from considerations of elastic stability, is given by:

$$P_{perm} = 0{\cdot}1 \frac{E_c d^2}{R^2}$$

where

E_c d and R are as defined in **3.1**.

For other types of doubly-curved shells, the permissible buckling load per unit area of surface, P_{perm} shall be calculated from the formula:

$$P_{perm} = 0{\cdot}1 \frac{E_c d^2}{R_1 R_2}$$

where

R_1 and R_2 are principal radii of curvature at any point, and E_c and d are as defined in **3.1**.

9.5 Buckling in Folded Plates — The folded plate may be replaced by the corresponding cylindrical shell where possible and the appropriate formula used to check for elastic stability.

10. DESIGN OF TRAVERSES

10.1 Types of Traverses — Traverses may be solid diaphragms, arches, portal frames, trusses or bowstring girders. For shells with large chord widths, it is advantageous to have trusses in the form of arches, trusses or bowsting girder.

10.1.1 Traverses may be placed below or above the shells. Where a clear soffit is required, specially to facilitate the use of movable formwork, they may be in the form of upstand ribs.

10.1.2 The simplest diaphragms for folded plates are rectangular beams with depth equal to the height of the plate. The diaphragms are subject to the action of the plate loads on one-half of the span of the folded plate.

10.2 Load on Traverses — Traverses shall be designed to carry, in addition to their own weight, reactions transferred from the shell in the form of shear forces, and the loads directly acting on them. For preliminary trial designs, however, the total load on half the span of the structure may be considered as a uniformly distributed vertical load on the diaphragm.

10.3 Design — The shear forces transferred on to the end frames from the shell shall be resolved into vertical and horizontal components and the analysis made by the usual methods. Owing to the monolithic connection between the traverse and the shell, the latter participates in the bending action of the traverse. The effective width of a cylindrical shell that acts along with the traverse may be assumed as $0{\cdot}38\sqrt{Rd}$ to $0{\cdot}76\sqrt{Rd}$, on either side in the case of intermediate traverses and on one side in the case of end ones; the higher value being applicable to an infinitely rigid rib that can prevent the shell from rotating and the lower value to a flexible rib. Where solid diaphragm traverses are used, adequate reinforcement to distribute shrinkage cracking shall be provided throughout the area of the traverses.

10.3.1 In the design of tied arches, it may be necessary to determine the elastic extension of the tie member due to tension and the consequent effect on the horizontal thrust on the arch.

10.3.2 The bottom member of a bowstring girder, or the tie in the case of a tied arch, is usually subjected to heavy tension. Welding or the provision of threaded sleeve couplings (*see* **25.2.5.2** of IS : 456-1978*) or laps may be used for joints in the reinforcement rods. Where lapping is done, the length of the overlap shall be as specified in the relevant clause of IS : 456-1978* and the composite tension shall be restricted to $0{\cdot}1\ f_{ck}$, where f_{ck} is the characteristics cube strength of concrete at 28 days. Where the composite tension exceeds $0{\cdot}1\ f_{ck}$, the entire length of the lap shall be bound by a helical binder of 6 mm diameter at a pitch not exceeding 75 mm. The joints in the bars shall always be staggered. Prestressing the tension member offers a simple and satisfactory solution. The detailing of inclined or vertical members of trusses or bowstring girders and suspenders of tied arches should be done with great care. The reinforcements in the tie of tied-arches shall be securely anchored at their ends.

10.4 The traverses may be hinged to the columns, except where the traverses and columns are designed as one unit, such as in a portal frame. Provision shall be made in the design of columns to allow for the expansion or contraction of traverses due to temperature changes.

11. DESIGN OF EDGE BEAMS

11.1 Edge beams stiffen the shell edges and act together with the shell in carrying the load of the supporting system. They can either be vertical or horizontal. Vertical beams are usually employed in long cylindrical shells wherein the cylindrical action is predominant. Horizontal beams are employed in short cylindrical shells where transverse arch action is predominant. It is preferable to completely isolate the structural system of the shell structure without adding any other structure to it.

In most of the shell forms, edge beams form part of the shell structure itself. An analysis of the shell structure is carried out along with the edge beam. Analysis and design of edge beams should ensure compatibility of boundary conditions at the shell edge. Analysis should take into account the eccentricities, if any, between the central line of the shell and the edge beam. Analysis should also take into account the type of edge beam, interior or exterior, as well as supporting arrangement of the edge beam.

*Code of practice for plain and reinforced concrete (*third revision*).

11.1.1 *Thickness* — A width of two to three times the thickness of the shell subject to a minimum of 15 cm is usually necessary for the edge beams.

11.1.2 *Reinforcements* — Edge beams carry most of the longitudinal tensile forces due to N_x in the shell and hence main reinforcements have to be provided for carrying these forces. It may be necessary to provide many layers of reinforcement in the edge beam. Design of reinforcements should ensure that the stresses in the farthermost layer does not exceed the permissible stresses. Edge beams should also be designed for carrying its self-weight, live load on the part of the shell, wind load and horizontal forces due to earthquakes.

12. DESIGN OF REINFORCEMENT

12.1 Shells — The ideal arrangement would be to lay the reinforcement in the shell to follow the isostatics, that is, directions of the principal stresses. However, for practical purposes, one of the following methods may be used:

One is the diagonal grid at 45° to the axis of the shell, and the other the rectangular grid in which the reinforcing bars run parallel to the edges of the shell. The rectangular grid needs additional reinforcement at 45° near the supports to take up the tension due to shear.

12.1.1 In the design of the rectangular grid for cylindrical shells, the reinforcement shall usually be divided into the following three groups:

a) Longitudinal reinforcement to take up the longitudinal stress N_x or N_y as the case may be,

b) Shear reinforcement to take up the principal tension caused by shear N_{xy}, and

c) Transverse reinforcement to resist N_y and M_y.

12.1.2 Longitudinal reinforcement shall be provided at the junction of the shell and the traverse to resist the longitudinal moment M_x. Where M_x is ignored in the analysis, nominal reinforcement shall be provided.

12.1.3 To ensure monolithic connection between the shell and the edge members, the shell reinforcement shall be adequately anchored into the edge members and traverses or *vice versa* by providing suitable bond bars from the edge members and traverses to lap with the shell reinforcement.

12.2 Folded Plates

12.2.1 *Transverse Reinforcement* — Transverse reinforcement shall follow the cross section of the folded plate and shall be designed to resist the transverse moment.

12.2.2 *Longitudinal Reinforcement* — Longitudinal reinforcement, in general, may be provided to take up the longitudinal tensile stresses, in individual slabs.

12.2.3 Diagonal reinforcement may be provided for shear.

12.2.4 The section of the plate at its junction with the traverse shall be checked for shear stress caused by edge shear forces.

12.2.5 Reinforcement bars shall preferably be placed, as closely as possible, so that the steel is well distributed in the body of the plate. Nominal reinforcement consisting of minimum 8 mm diameter bars may be provided in the compression zones at about 200 mm centre-to-centre.

12.3 General — The minimum reinforcement shall conform to the requirements of IS : 456-1978*.

12.3.1 *Diameters of Reinforcement Bars* — The following diameters of bars may be provided in the body of the shell. Larger diameters may be provided in the thickened portions, transverse and beams:

a) Minimum diameter : 8 mm, and

b) Maximum diameter : $\frac{1}{4}$ of shell thickness or 16 mm whichever is smaller.

12.3.2 *Spacing of Reinforcement* — The maximum spacing of reinforcement in any direction in the body of the shell shall be limited to five times the thickness of the shell and the area of unreinforced panel shall in no case exceed 15 times the square of thickness.

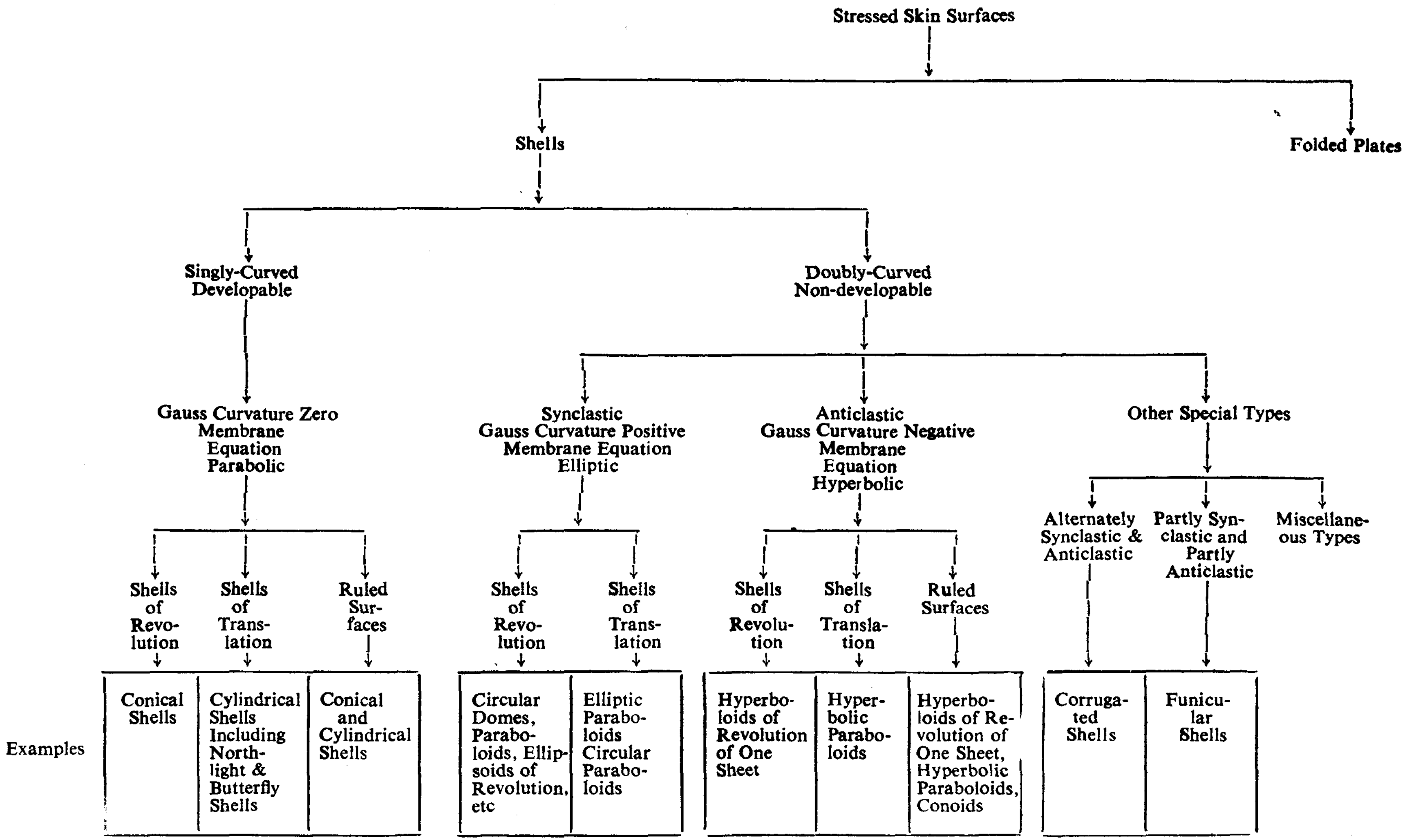
NOTE — These limitations do not apply to edge members which are governed by IS : 456-1978*.

*Code of practice for plain and reinforced concrete (*third revision*).

APPENDIX A

(*Clause* 4.2)

DETAILED CLASSIFICATION OF STRESSED SKIN SURFACES



APPENDIX B

(*Clauses* 8.1.1.3, 8.1.2.1, 8.1.2.2 *and* 8.1.3.1)

TABLES AND THE METHODS OF ANALYSIS OF CIRCULAR CYLINDRICAL SHELLS AND FOLDED PLATES

B-1. TABLES FOR THE ANALYSIS OF CIRCULAR CYLINDRICAL SHELLS

B-1.1 Tables given in the ASCE Manual of Engineering Practice No. 31 entitled 'Design of Cylindrical Concrete Shell Roofs' are based on a rigorous analytical method and are accurate enough for design of shells of all proportions. However, as values corresponding only to two ratios of R/d are given, interpolation involved for ratios does not give accurate results. A subsequent publication by the Portland Cement Association, Chicago, entitled 'Coefficients for Design of Cylindrical Concrete Shell Roofs', gives the coefficients for 4 values of R/d making interpolation easier.

B-1.2 Tables given in 'Circular Cylindrical Shells' by Rudiger and Urban (Published by E.G. Teubner Verlagsgesellschaft, Leipzig, 1955) are based on the Donnel-Karman-Jenkins Method. These are particularly accurate for shells with $\frac{L}{R} < \pi$.

B-2. BEAM METHOD

B-2.1 This method, due to Lundgren, consists of two parts. In the first, known as the 'beam calculation', the shell is treated as a beam of curved cross section spanning between the traverses and the longitudinal stress T_x and the shear stress S are determined. In the second, known as the 'arch calculation', a unit length of the shell is treated as an arch subject to the action of applied loads and specific shear which is defined as the rate of change of shear or $\frac{dS}{d_x}$. This calculation yields the transverse stress T and the transverse moment M_y. For a detailed treatment, reference may be made to 'Cylindrical Shells' Vol I, by H. Lundgren published by the Danish Technical Press, the Institution of Danish Civil Engineers, Copenhagen, 1949.

APPENDIX C

(*Clauses* 8.2.1, 8.2.3.1 *and* 8.2.4)

GOVERNING EQUATIONS FOR ANALYSIS OF DOUBLY-CURVED SHELLS

C-1. MEMBRANE ANALYSIS

C-1.1 Real and Projected Forces and Stress Resultant — For membrane analysis of doubly-curved shells, it is usual to deal with projected stress resultants N_{xp}, N_{yp} and N_{xyp} instead of real stress resultants N_x, N_y and N_{xy}. The real and projected stress resultants are related as follows:

$$N_x = N_{xp}\sqrt{\frac{1+p^2}{1+q^2}}$$

$$N_y = N_{yp}\sqrt{\frac{1+q^2}{1+p^2}}$$

$$\text{and } N_{xy} = N_{xyp}$$

Similarly, the real forces W_x, W_y and W_z on shell per unit area on its surface with x, y and z-directions are replaced by the fictitious forces x, y and z and the relationships between them are as follows:

$$X = W_x\sqrt{1+p^2+q^2}$$

$$Y = W_y\sqrt{1+p^2+q^2}$$

$$\text{and } Z = W_z\sqrt{1+p^2+q^2}$$

Equilibrium Equations

The three equations of equilibrium can now be written using the projected stress resultants and fictitious forces as follows:

$$\frac{\partial N_{xp}}{\partial_x} + \frac{\partial N_{xyp}}{\partial_y} + X = 0 \qquad (1)$$

$$\frac{\partial N_{yp}}{\partial_y} + \frac{\partial N_{xyp}}{\partial_x} + Y = 0 \qquad (2)$$

$$\text{and } r\,N_{xp} + 2s\,N_{xyp} + tN_{yp} = p_x + q_y - Z \qquad (3)$$

Analysis Using Stress Function

Analysis of the equations of equilibrium is simplified by using a stress function ϕ which reduces the three equations to one. The stress function Φ is related to the projected stress resultants as follows:

$$N_{xp} = \frac{\partial^2\Phi}{\partial y^2} - \int X\,dx$$

$$N_{yp} = \frac{\partial^2\Phi}{\partial x^2} - \int Y\,dy$$

$$\text{and } N_{xyp} = \frac{-\partial^2\Phi}{\partial x.\partial y} \qquad (4)$$

On introducing the stress function, the third equilibrium equation reduce to the following:

$$r\left[\frac{\partial^2\Phi}{\partial y^2}\right]-2\,s\left[\frac{\partial^2\Phi}{\partial x\partial y}\right]+t\left[\frac{\partial^2\Phi}{\partial x^2}\right]=$$

$$pX+qY-Z+r\int X\,dx+t\int Y\,dy \qquad (5)$$

The homogeneous part of the partial differential equation given above will be of the elliptic, parabola or hyperbolic type depending upon whether $s^2-rt=0$ which is also the test for classifying shells as synclastic, developable or anticlastic.

C-2. BENDING ANALYSIS

C-2.1 Bending analysis is necessary for shallow doubly-curved shells. For shallow shells under vertical loading, this would involve the solution, of two partial simultaneous equations given below:

a) for shells of variable curvature:

$$\left.\begin{aligned}&\frac{1}{E_{cd}}\nabla^4F+\left(\frac{\partial}{\partial x}t\frac{\partial}{\partial x}-2s\frac{\partial^2}{\partial x\partial y}+\frac{\partial r}{\partial y}\frac{\partial}{\partial y}\right)w=0,\\&\qquad\qquad\text{and}\\&D\nabla^4w-\left(\frac{\partial}{\partial x}t\frac{\partial}{\partial x}-2s\frac{\partial^2}{\partial x\partial y}+\frac{\partial}{\partial y}r\frac{\partial}{\partial y}\right)F-Z=0\end{aligned}\right\}(6)$$

b) For shells of constant curvature, that is, for shells for which r, s and t are constant, above equations simplify to:

$$\left.\begin{aligned}&\frac{1}{E_{cd}}\nabla^4F+\left[t\frac{\partial^2w}{\partial x^2}-2s\frac{\partial^2w}{\partial x\partial y}+r\frac{\partial^2w}{\partial y^2}\right]=0\\&D\nabla^4w-\left[t\frac{\partial^2F}{\partial x^2}-2s\frac{\partial^2F}{\partial x\partial y}+r\frac{\partial^2F}{\partial y^2}\right]-Z=0\end{aligned}\right\}(7)$$

where

F = stress function which gives in-plane stresses, when bending is also considered;

w = deflection along z-axis;

D = flexural rigidity $= E_c d^3/12\,(1-\nu^2)$;

Z = vertical load per unit area of shell surface, assumed positive in the positive direction of Z-axis;

ν = Poisson ratio; and

r,s,t = curvature as defined in **3.1**.

The equations given above are based on the co-ordinate system in Fig. 9.

Bending Stress Resultants

From the values of F and w satisfying equations (6) or (7), the stress resultants may be obtained from the following relations:

$$T_x=\frac{\partial^2F}{\partial y^2}$$

$$T_y=\frac{\partial^2F}{\partial x^2}$$

$$S=-\frac{\partial^2F}{\partial x\partial y}$$

$$M_x=D\left[\frac{\partial^2w}{\partial x^2}+\nu\frac{\partial^2w}{\partial y^2}\right]$$

$$M_y=D\left[\frac{\partial^2w}{\partial y^2}+\nu\frac{\partial^2w}{\partial x^2}\right]$$

$$M_{xy}=-D\,(1-\nu)\,\frac{\partial^2w}{\partial x\partial y}$$

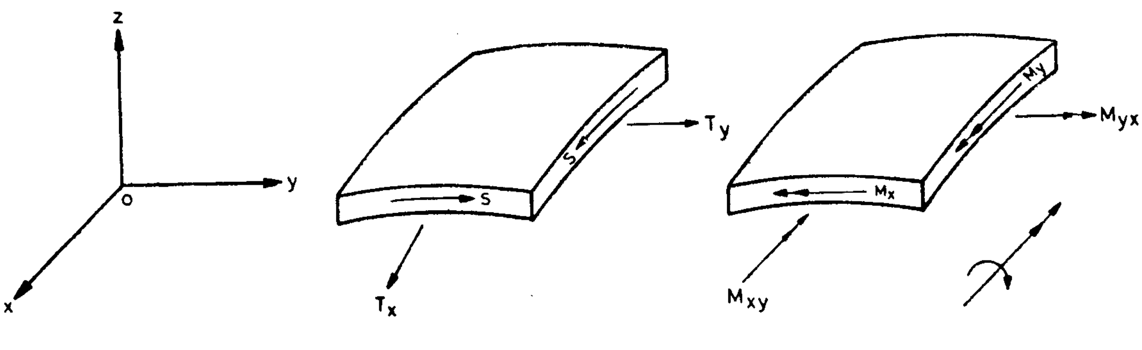


FIG. 9 SIGN CONVENTION FOR STRESSES AND MOMENTS IN A SHELL ELEMENT

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