# When too much wind is a bad thing for wind power:

# Short-term forecasting of extreme and non-extreme wind speeds — an INLA-based approach

Talk based on:

Castro-Camilo, D., Huser, R., & Rue, H. (2019). A spliced Gamma-Generalized Pareto model for short-term extreme wind speed probabilistic forecasting. **JABES**, **24**(3), 517-534.

SCAN ME

D. Castro-Camilo

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- When wind farms are located in windy areas, they can provide relatively reliable power.
- However, wind turbines can be damaged when wind speeds exceed their engineered limit, causing them to shut down.
- Predicting the frequency and intensity of high wind speeds is more challenging than the estimation of averages.
- By splicing together two models describing normal and rare extreme conditions, we develop a method to predict the frequency and intensity of winds strong enough to shut down wind turbines–even if such winds haven't yet been observed.

**Figure 1:** Towers located along the Columbia River, on the border between Oregon and Washington, US.

# Data & Challenges & Goals

# Data& Challenges

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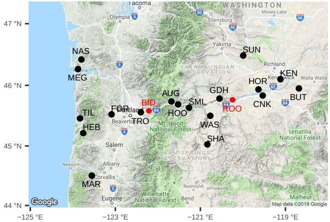

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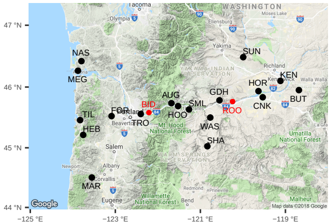


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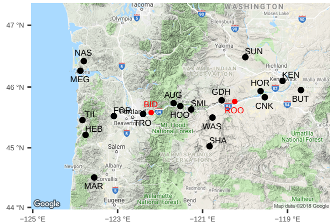


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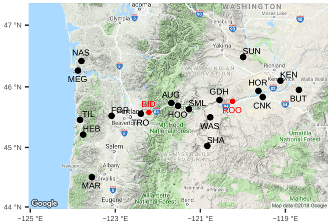


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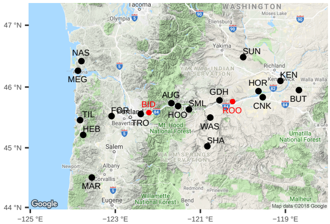


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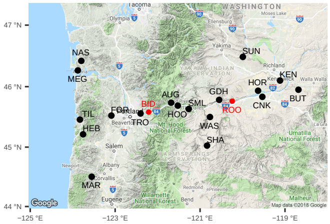


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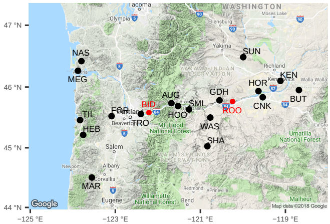


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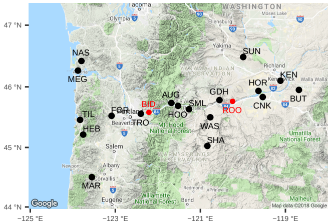


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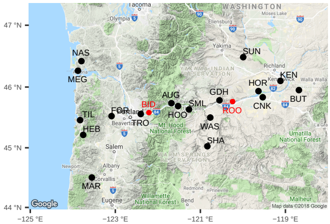


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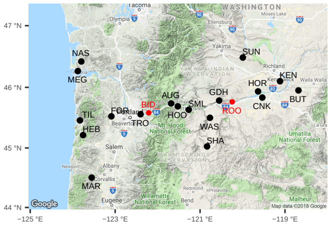


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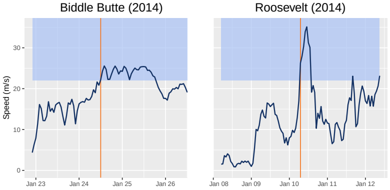


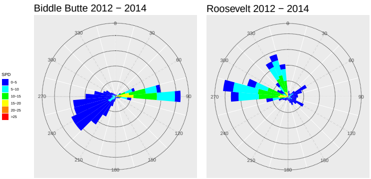

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**Extreme wind speeds:** We define extreme wind speeds (WS) as those values that exceed a certain high threshold, since a succession of large values over a period of time may pose great risk to the wind turbines.

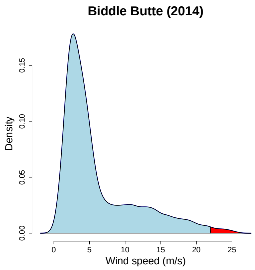


**Figure 2:** Illustration showing possible extreme and non-extreme WS. In practice, the threshold is not fixed and varies with space and time.

# Methodology

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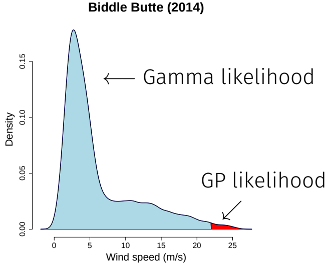

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- We compare our models in terms of forecasting ability with their natural competitors (without the tail correction).

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Stage 3. Threshold exceedances defined as $x_{\mathbf{s}}(t) = \{y_{\mathbf{s}}(t) - \psi_{\mathbf{s},\alpha}(t)\} \mid y_{\mathbf{s}}(t) > \psi_{\mathbf{s},\alpha}(t)$, are characterized by a **GP distribution** parametrised in terms of a shape parameter $\xi_{\mathbf{s}} \geq 0$ (constant in time) and the time-varying median $\phi_{\mathbf{s},0.5}(t) > 0$.

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- How can we choose the neighbours? $\longrightarrow$ based on **wind direction.**

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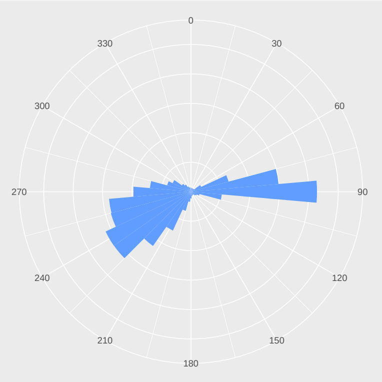
Intuition: Exploiting wind direction information for improving wind speed forecasting.

Biddle Butte ( BID )



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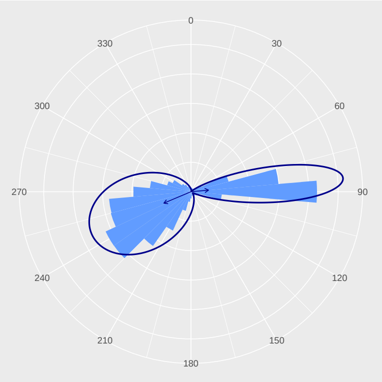


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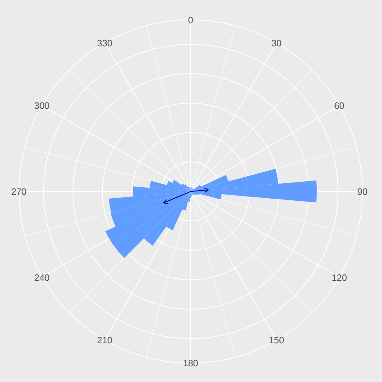


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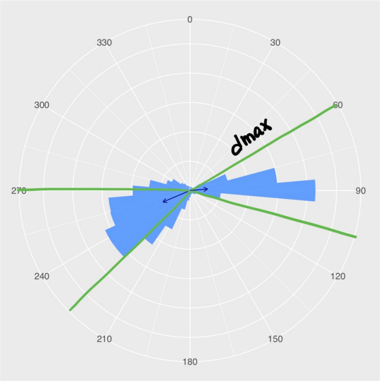


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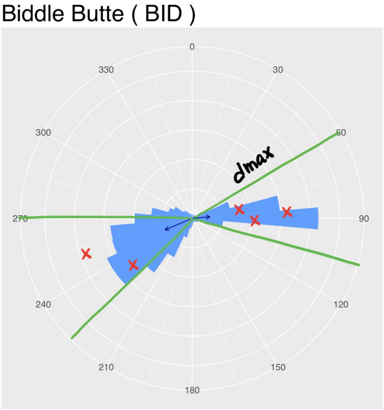


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For comparison, let's take another look at the off-site model:

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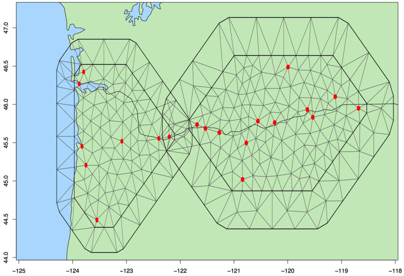
- We split the study region into two parts (West and East side of the Cascade Mountains) and use two discretisation meshes. Red dots indicate the towers' locations.



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- This is a principled method (Occam's razor is one of the principles), where priors are develop in such a way that overfitting is prevented.

# Wind speed probabilistic forecasting results

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- We test our models against a baseline Gamma model (with off-site and SPDE latent models) that forecasts wind speeds using only the first stage, i.e., without the tail correction.

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- Average performance measures for one-hour ahead forecast using the off-site model, the off-site baseline model, the SPDE model, and the SPDE baseline model.

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- Both the off-site and the SPDE latent models outperform their baseline counterparts when focusing on the upper tail of the distribution, showing that the GP correction is useful to improve the forecasting of strong wind speeds.

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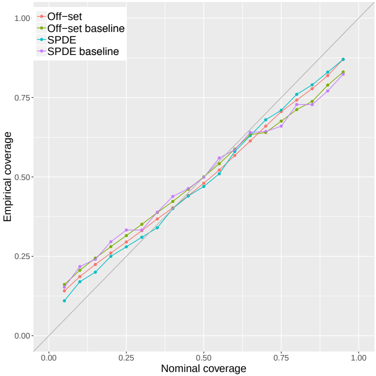
- **Construction:** for every station, we compute the proportion of times that the predictive CDF is below a certain threshold. Our model is well calibrated if this proportion is close to the observed frequencies.



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  - **Modelling assumptions:** data are conditionally independent given the latent process. **Is it reasonable?**

# References


- Rue, H., Martino, S. and Chopin, N. (2009). Approximate Bayesian inference for latent Gaussian models by using integrated nested Laplace approximations. Journal of the Royal statistical society: Series B (Statistical Methodology) 71(2), 319–392.

- Simpson, D., Rue, H., Riebler, A., Martins, T. G., & Sørbye, S. H. (2017). Penalising model component complexity: A principled, practical approach to constructing priors. Statistical science, 32(1), 1-28.

Gracias!

# More on prior specification

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- [Likelihood parameters] We assume a slightly informative prior over the **Gamma shape** $\kappa_{\mathbf{s}}$, by considering a Gamma distribution with shape 10 and rate 1, which gives a high probability to values between 5 and 15. A strong PC prior is assumed for the **shape parameter of the GP distribution** $\xi_{\mathbf{s}}$; since large values of the shape parameter are usually unrealistic for wind speeds, we here assume that $\Pr(\xi_{\mathbf{s}} > 0.4) \approx 0.01$.

- [Hyperparameters off-site model] We assume fairly informative PC priors for the **correlation hyperparameter of the AR(1) process**, and the **precision hyperparameter of the random walk of order 2**. Specifically, $\Pr(\rho_{\mathbf{s},1} > 0.9) = 0.95$ and $\Pr(1/\sqrt{\tau_{\mathbf{s},2}} > \mathrm{sd}_{\mathrm{wind}}) = 0.01$, where $\mathrm{sd}_{\mathrm{wind}}$ denotes the empirical standard deviation of the temporally aggregated wind speeds.

- [Hyperparameters SPDE model] PC priors on the parameters of the Gaussian field in the SPDE latent model, namely the **marginal variance** $\sigma^2 > 0$ and the **range of dependence** $r > 0$, are chosen such that the variance is shrunk towards zero, whereas the range is shrunk towards infinity. Specifically, we set $\Pr(\sigma > 2 \times \mathrm{sd}_{\mathrm{wind}}) = 0.01$ and $\Pr(r < r_{\mathrm{median}}) = 0.5$, where $r_{\mathrm{median}}$ is the median of the distances between stations. For stations to the East of the Cascade Mountains, $r_{\mathrm{median}} = 94.6$ km, and for stations to the West, $r_{\mathrm{median}} = 113.3$ km. A PC prior is also chosen for the correlation coefficient of the autoregressive term in $u(\mathbf{s}, t)$, specifically $\Pr(\rho_2 > 0.9) = 0.95$. The PC prior for $\tau_2$ is the same as for $\tau_{\mathbf{s},2}$ in the off-site latent model.

# Posterior predictive distribution

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How to obtain posterior predictive distributions for the 1-hour, 2-hour, and 3-hour ahead probabilistic forecasts of hourly wind speeds, produced by our three-stage hierarchical Bayesian model, using the two linear predictors.

- For the SPDE latent model, we use a rolling training period of length 5 days, whereas for the off-site latent model, we multiply this period by the number of stations in each side of the Cascade Mountains, as a way to balance the effective sample sizes of the SPDE and the off-site latent models.

- We generate $10,000$ samples from the posterior predictive distribution, for each station, each forecasting time horizon, and each latent model, as follows: we extract the posterior means of the linear predictor and hyperparameters for each stage, and use the link between the linear predictor and the likelihood parameters to obtain $10,000$ samples for the Gamma, Bernoulli, and GP predictive distributions.

- We replace Gamma samples by threshold exceedances (GP samples) whenever the threshold is exceeded, i.e., whenever the associated Bernoulli sample is equal to 1. In other words, the tail of the Gamma distribution is *corrected* by the GP distribution in the presence of exceedances.

# Inference based on INLA

# Inference based on INLA: technical details

- Here we describe the form of the joint posterior distribution for each stage of our spliced Gamma-GP model.

- Let $\mathbf{y}$ denote the vector of observations for any of the three stages (note that we remove the spatial component), with associated hyperparameters $\boldsymbol{\theta}_1 = \boldsymbol{\kappa}$ (Gamma likelihood) or $\boldsymbol{\theta}_1 = \xi$ (GP likelihood).

- Let $\mathbf{x}$ be the latent Gaussian random field, $\boldsymbol{\theta}_2$ be the vector of hyperparameters of any of the latent models, and $\boldsymbol{\theta} = (\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T)^T$.
- The joint posterior distribution of parameters and hyperparameters for any of the three stages, can be written as

$$
\begin{aligned}
\mathrm{p}(\mathbf{x}, \boldsymbol{\theta} \mid \mathbf{y}) &\propto \mathrm{p}(\boldsymbol{\theta})\mathrm{p}(\mathbf{x} \mid \boldsymbol{\theta}_2) \prod_{t \in \mathscr{T}} \mathrm{p}(y(t) \mid x(t), \boldsymbol{\theta}_1) \\
&\propto \mathrm{p}(\boldsymbol{\theta}) |\mathbf{Q}_{\boldsymbol{\theta}_2}|^{1/2} \exp\left( -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_{\boldsymbol{\theta}_2})^T \mathbf{Q}_{\boldsymbol{\theta}_2} (\mathbf{x} - \boldsymbol{\mu}_{\boldsymbol{\theta}_2}) + \sum_{t \in \mathscr{T}} \log \mathrm{p}(y(t) \mid x(t), \boldsymbol{\theta}_1) \right).
\end{aligned}
$$

# Inference based on INLA: technical details

$$\begin{aligned}
\mathrm{p}(\mathsf{x},\boldsymbol{\theta} \mid \mathsf{y}) &\propto \mathrm{p}(\boldsymbol{\theta})\mathrm{p}(\mathsf{x} \mid \boldsymbol{\theta}_2) \prod_{t\in\mathcal{T}} \mathrm{p}(y(t) \mid x(t),\boldsymbol{\theta}_1) \\
&\propto \mathrm{p}(\boldsymbol{\theta})|\mathsf{Q}_{\boldsymbol{\theta}_2}|^{1/2} \exp\left(-\frac{1}{2}(\mathsf{x}-\boldsymbol{\mu}_{\boldsymbol{\theta}_2})^T \mathsf{Q}_{\boldsymbol{\theta}_2}(\mathsf{x}-\boldsymbol{\mu}_{\boldsymbol{\theta}_2}) + \sum_{t\in\mathcal{T}} \log \mathrm{p}(y(t) \mid x(t),\boldsymbol{\theta}_1)\right).
\end{aligned}$$

- The main objectives of the statistical inference are to extract from $\mathrm{p}(\mathsf{x},\boldsymbol{\theta} \mid \mathsf{y})$ the marginal posterior distributions for each of the elements of the linear predictor vector, and for each element of the hyperparameter vector, i.e.,

$$\mathrm{p}(x(t) \mid \boldsymbol{y}) = \int \mathrm{p}(\boldsymbol{\theta} \mid \boldsymbol{y})\mathrm{p}(x(t) \mid \boldsymbol{\theta},\boldsymbol{y})\mathrm{d}\boldsymbol{\theta}, \qquad \mathrm{p}(\theta_k \mid \boldsymbol{y}) = \int \mathrm{p}(\boldsymbol{\theta} \mid \boldsymbol{y})\mathrm{d}\boldsymbol{\theta}_{-k},$$

from which predictive distributions may be derived.

- We use INLA, where these posterior distributions are numerically approximated using the Laplace approximation.

# GP parametrization

# GP parametrization

To avoid confounding problems due to the correlation between estimated GP parameters, we parametrize the GP in terms of the shape parameter $\xi$ and a $\beta$-quantile $\phi_{\mathsf{s},\beta} > 0$. The choice of the optimal quantile (i.e., the optimal $\beta$) is an open question, but the simulation results presented below shows that the GP parametrized in terms of $\xi$ and $\phi_{\mathsf{s},\beta}$ produces less correlated estimations than the GP parametrized in terms of $\xi$ and $\sigma$, where $\sigma > 0$ is the GP scale parameter.

Simulation study. We generate a sample of length $n = 1000$ from a generalized Pareto distribution with fixed scale $\sigma = 1$ and shape $\xi \in \{0, 0.1, \ldots, 1\}$. we compute the maximum likelihood estimators $\hat{\sigma}$ and $\hat{\xi}$ as well as the plug-in maximum likelihood estimator $\hat{\phi}_{\mathsf{s},0.5}$. We replicate the experiment $R = 1000$ times. Using the $R$ replicates, we compute the sample correlation between $\hat{\xi}$ and $\hat{\sigma}$ and between $\hat{\xi}$ and $\hat{\phi}_{\mathsf{s},0.5}$. The table shows these sample correlations for every true value of the $\xi$ parameter. As we can see, the correlation between $\xi$ and $\phi_{\mathsf{s},\beta}$ is smaller than the correlation between $\xi$ and $\sigma$ in all the cases.

| $\xi$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\sigma}$ | -0.70 | -0.73 | -0.71 | -0.68 | -0.66 | -0.64 | -0.62 | -0.61 | -0.59 | -0.58 | -0.56 |
| $\hat{\phi}_{\mathsf{s},0.5}$ | -0.53 | -0.57 | -0.52 | -0.48 | -0.43 | -0.40 | -0.36 | -0.32 | -0.29 | -0.26 | -0.23 |