1 Time evolution of wavefunction in box potential

a) $|\psi|^2 = C^2(2^2 + 3^2 + 1)\frac{a}{2} = 7a = 1, C = \frac{1}{\sqrt{7a}}$
b) $c_1 = 2C\sqrt{\frac{2}{a}}\frac{a}{2} = 2C\sqrt{\frac{a}{2}}, c_2 = 3C\sqrt{\frac{a}{2}}, c_3 = C\sqrt{\frac{a}{2}}$
$S = 14C^2a/2 = 14/7aa/2 = 1$
c)$\psi(t) = C(2\sin(kx)e^{-iE_2t/\hbar} + 3\sin(2kx)e^{-iE_2t/\hbar} + \sin(3kx)e^{-iE_3t/\hbar})$
d) Yes
e) 4:9:1 (/14)
f) No
g) Particle's energy not known prior to measurement.

2 Diabatic (sudden) expansion of infinite box

Griffiths

3 Hermitian operators

a)

I will use the definition that Hermitian operators are H such that $H^\dagger = H$

Sum of Hermitian operators is Hermitian because $(H_1 + H_2)^\dagger = H_1^\dagger + H_2^\dagger = H_1 + H_2$

V is Hermitian becauese it is multiplication by a real function; in fact it only has δ eigenvectors if it is not constant

p^2 is Hermitian; if we write $\psi_{xx} = -k^2\psi$ we find that k must be real, otherwise we cannot normalize the state

b) $\int \psi^* H\psi dx = \int \psi^* E\psi dx = E$
c) $\hbar i\psi_p = k\psi, i\hbar \ln\psi = kp, \psi = Ce^{i\hbar p/k}$, k must be real for this to be normalizable

4 Square well centered at origin, parity

`http://web.mit.edu/jlee08/Public/7.91/8.04/sol6.pdf`

a)

$$f(x) = Ae^{ikx} + Be^{-ikx}$$
$$f(-a/2) = Ae^{-ika/2} + Be^{ika/2} = 0$$
$$Ae^{-ika} + B = 0$$

substituting,

$$\begin{aligned} f(x) &= Ae^{ikx} - Ae^{-ika}e^{-ikx} \\ &= Ae^{-ika/2}(e^{ikx}e^{ika/2} - e^{-ikx}e^{-ika/2}) \\ &= C\sin k(x + a/2) \end{aligned}$$

Now the rest is the same, $k = \frac{n\pi}{a}$ and $C = \sqrt{\frac{2}{a}}$

b)

$$
\begin{aligned}
P \sin \frac{n\pi}{a}(x + a/2) &= \sin \frac{n\pi}{a}(-x + a/2) \\
&= \sin \frac{n\pi}{a}(-x - a/2 + a) \\
&= \sin \frac{n\pi}{a}(-x - a/2) + n\pi \\
&= -\sin \frac{n\pi}{a}(x + a/2) - n\pi \\
&= -\psi(x)(-1)^n
\end{aligned}
$$

c)

$$
\begin{aligned}
H\psi &= E\psi \\
PH\psi &= EP\psi \\
HP\psi &= EP\psi - [P, H]\psi
\end{aligned}
$$

so $[P, H]\psi = \lambda P\psi$ but obviously $[P, D] = 0$ so

$$
\begin{aligned}
[P, V]\psi &= \lambda P\psi \\
PV\psi - VP\psi &= \lambda P\psi \\
V(-x)\psi(-x) - V(x)\psi(-x) &= \lambda\psi(-x) \\
V(-x) &= V(x) + \lambda \\
&= V(-x) + 2\lambda \\
\lambda &= 0
\end{aligned}
$$

V must be an even function