Loaded Hoop on a conveyer belt

$$v_{m,r} = R\dot{\theta}$$

$$\begin{aligned} v_{m,x} &= \dot{X} + R\dot{\theta}\sin\theta \\ v_{m,y} &= R\dot{\theta}\cos\theta \end{aligned}$$

$$\begin{aligned} K_m &= \frac{1}{2}m(R\dot{\theta}\cos\theta)^2 + \frac{1}{2}m(\dot{X} + R\dot{\theta}\sin\theta)^2 \\ &= ... \end{aligned}$$

$$\begin{aligned} L = &\frac{1}{2}(M+m)R^2\dot{\theta}^2 + \frac{1}{2}(M+m)\dot{X}^2 \\ &+ mR\dot{\theta}\dot{X}\cos\theta + mgR\sin\theta \end{aligned}$$

$$\frac{\partial L}{\partial \theta} = mgR\cos\theta - mR\dot{\theta}\dot{X}\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = (M+m)R^2\dot{\theta} + mR\dot{X}\cos\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = (M+m)R^2\ddot{\theta} + mR\ddot{X}\cos\theta - mR\dot{X}\dot{\theta}\sin\theta$$

$$gR\cos\theta = (M+m)R^2\ddot{\theta} + mR\ddot{X}\cos\theta$$