Positive-definite matrices

1 Tests

Eigenvalues > 0

Leading submatrices test

Pivots positive

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} \tag{1}$$

$\lambda_1 > 0, \lambda_2 > 0$ kk work out what this means

$a > 0, ac - b^2 > 0$

$a > 0, \frac{ac-b^2}{a} > 0$

But probably the most important test / definition is

$\langle x|Ax\rangle > 0$

$$\begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix} \tag{2}$$

positive semi-definite

$\lambda = 0, 20$

pivots $= 2$

Now let's look at $x^T Ax = 2x_1^2 + 12x_1x_2 + 18x_2^2 = 2(x + 3y)^2$

the coefficients are $a, 2b, c$

quadratic form

The form will be positive-definite if we can factor it into a sum of squares. This shows the connection between positive-definiteness and pivots. The things that go outside the squares are the pivots. Then we have a "bowl". Otherwise we could have a saddle point, minimum in one direction and maximum in another. The important directions to look turns out to be the eigenvector directions.

For a function f, the matrix of second derivatives at a point is positive-definite implies it is a true minimum. The matrix of second derivatives is obviously symmetric.

If I cut the bowl at a constant, it's obviously an ellipse.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{3}$$

dets 2,3,4 pivots 2,3/2,4/3 eigenvalues $2-\sqrt{2}, 2, 2+\sqrt{2}$

ellipsoid, axes in the directions of the eigenvectors