

Determinant

1 Row operations

2 Volume

We can interpret $\det(M)$ to measure the volume of the volume element with edges defined by the row vectors of M .

3 Axioms

Shear axiom: Volume is invariant under shearing

Scale axiom: Scaling is multiplicative

Identity axiom: $\det(I) = 1$

4 Construction

Gaussian elimination

5 Explicit form

$$\det(X) = \sum_{\pi \in S_n} \text{sgn}(\pi) \prod x_{i, \pi(i)} \quad (1)$$