Laplacian

1 ∇

We assume you are familiar with the humble ∇ (del) vector and its physcal interpretation, including in the operator form ∇ (scalar $\to$ vector), $\nabla\cdot$ (vector $\to$ scalar) and $\nabla\times$ (vector $\to$ vector).

There are 9 ($3 \cdot 3$) combinations of two del operators. Three of them have the correct type signature:

1. $\nabla \times \nabla\times$
2. $\nabla \times \nabla$
3. $\nabla \cdot \nabla\times$
4. $\nabla \cdot \nabla$
5. $\nabla\nabla\cdot$

Items 2 and 3 are always zero.

2 ∇^2

The ∇^2operator, otherwise as the laplacian operator, is very important operator in physics. In cartesian coordinates it is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Note that this value is necessarily a scalar, ie, independent of coordinates chosen. So what is its physical interpretation?

3 Concavity

In single-variable calculus the second derivative represents concavity, or the rate of change of the rate of change.

A slightly more useful way to think about this is in terms of average values; the concavity of a function f at a point x_0 measures much the average value of $f(x)$ about x_0 exceeds $f(x_0)$. [pics]

This argument generalizes to higher dimensions too.

4 Poisonn's equation

$$\nabla^2 = 0$$