18.014 pset 2

18 For each of the following functions f

19 Show that a finite set is Jordan-measurable, and that its Jordan measure is 0

Let E be the finite set. Then for a fixed-width partition $\int = |E|/N \to 0$ as $N \to \infty$. No, it is not true; example is $\mathbb{Q}$

20 Show that

the union of two Jordan-measurable sets is Jordan-measurable

$\chi_{E_1 \cup E_2} = \chi_{E_1} + \chi_{E_2} - \chi_{E_1}\chi_{E_2}$
To prove that products RI functions are RI: $s_1 s_2 < f_1 f_2 < t_1 t_2$

the intersection of two Jordan-measurable sets is Jordan-measurable

$\chi_{E_1 \cap E_2} = \chi_{E_1}\chi_{E_2}$

the complement of a Jordan-measurable sets is Jordan-measurable

$\chi_{E'} = 1 - \chi_E$

21 Show that if E is a Jordan...

$\mu_{f(E)} = \int \chi_{f(E)}(x)dx = \int \chi_E(f^{-1}(x))dx = a \int \chi_E(x)dx$
because
$\chi_{f(E)}(x) = 1 \implies x \in \chi_{f(E)} \implies x = f(u) \implies f^{-1}(x) = u \implies f^{-1}(x) \in \chi_E \implies \chi_E(f^{-1}(x)) = 1$

22 Define the floor function...