Quantum Mechanics - Griffiths, David J

1 The Wave Function

1.1

For the distribution of ages in section...

1.2

a) Find the standard deviation...

1.3 Gaussian distribution

Consider the Gaussian distribution...

a) $\sqrt{\frac{\lambda}{\pi}}$
b) $\langle x\rangle = a, \langle x^2\rangle = \frac{1}{2\lambda} + a^2, \sigma^2 = \frac{1}{2\lambda}$
c) a smooth gentle hump centered at a

1.4 Triangle wavefunction

At time t=0 a particle is represented by...

a) $A^2 = \frac{3}{b}$
b) a sharp concave up peak
c) at $x = a$
d) $\Pr(x < a) = \frac{a}{b}$
e) ??

1.5 Delta potential

Consider the wave function...

a) $A = \sqrt{\lambda}$
b) $\langle x\rangle = 0, \langle x^2\rangle = \frac{1}{2\lambda^2}$
c) $\sigma = \frac{\sqrt{2}}{2}\frac{1}{\lambda}, \Pr(|x| > \sigma) = e^{-\sqrt{2}}$

1.6

1.7

1.8

1.9 Gaussian wavefunction

A particle of mass m is in the state...

a) $A^2 = \sqrt{\frac{2am}{\pi\hbar}}$
b) $V = 2a^2mx^2$
c)$\langle x\rangle = 0, \langle x^2\rangle = \frac{\hbar}{4am}, \langle p\rangle = 0, \langle p^2\rangle = am\hbar$
d) $\sigma_x^2\sigma_p^2 = \frac{\hbar^2}{4}$

1.10

1.11

1.12

1.13

1.14 Probability current

Let $P_{ab}(t)$ be the probability of finding...

a) ?? b) 0

1.15 Unstable particle

Suppose you wanted to describe an unstable particle...

a) ?? b) $P = P_0 e^{-(2\Gamma/\hbar)t}$

1.16

Done

1.17 Quadratic wavefunction

A particle is represented (at time $t = 0$) by the...

a) $A^2 = \frac{15}{16a^5}$
b) $\langle x \rangle = 0$
c) $\langle p \rangle = 0$
d) $\langle x^2 \rangle = \frac{a^2}{7}$
e) $\langle p^2 \rangle = \frac{5}{2}\frac{\hbar^2}{a^2}$
f,g,h) $\sigma_x^2 \sigma_p^2 = \hbar^2 \frac{5}{14}$

1.18 Quantum mechanical systems

In general, quantum mechanics is relevant...

2 The time-independent Schrödinger equation

2.1

Prove the following three theorems...

2.2

Show that E must exceed the minimum value of $V(x)$...

Done

2.3

Show that there is no acceptable solution to the...

Done

2.4 Uncertainty [ISW]

Calculate $\langle x \rangle, \langle x^2 \rangle, ...$ for the nth stationary state...

$\langle x \rangle = a/2$
$\langle x^2 \rangle = a^2 \left(\frac{1}{3} - \frac{1}{2n^2\pi^2} \right)$
$\langle p \rangle = 0$
$\langle p^2 \rangle = \frac{\hbar^2 n^2 \pi^2}{a^2}$
$\sigma_x^2 = a^2 \left(\frac{1}{12} - \frac{1}{2n^2\pi^2} \right)$
$\sigma_x^2 \sigma_p^2 = \hbar^2 \left(\frac{n^2\pi^2}{12} - \frac{1}{2} \right)$

2.5 Oscillating particle [ISW]

A particle in the infinite square well has as its initial wave function an even mixture of the first two...

a) $A = \frac{\sqrt{2}}{2}$
b) $\psi(x,t) = \frac{\sqrt{a}}{a} \left(\sin(\frac{\pi x}{a}) e^{-i\pi^2 \hbar t/2ma^2} + \sin(\frac{2\pi x}{a}) e^{-4i\pi^2 \hbar t/2ma^2} \right)$
$|\psi|^2 = \frac{1}{a} \left(\sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos \frac{3\pi^2 \hbar}{2ma^2} t \right)$
c) $\langle x \rangle = \frac{a}{2} - \frac{16a}{9\pi^2} \cos 3\omega t$
d) $\frac{8\hbar}{3a} \sin 3\omega t$
e) $\frac{5\pi^2\hbar^2}{4ma^2}$

2.6 Phase constant [ISW]

Although the overall phase constant of the wave function...

$\psi(x,t) = \frac{\sqrt{a}}{a} \left(\sin(\frac{\pi x}{a}) e^{-i\omega t} + \sin(\frac{2\pi x}{a}) e^{-4i\omega t + \phi} \right),$
$|\psi|^2 = \frac{1}{a} \left(\sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos(3\omega t - \phi) \right)$
$\langle x \rangle = \frac{a}{2} - \frac{16a}{9\pi^2} \cos(3\omega t - \phi)$

2.7 Triangular wave function [ISW]

A particle in the infinite square well has the initial wave function...

$\langle x \rangle = \frac{a}{2}$, $\langle x^2 \rangle = \frac{2}{7} a^2$, $\sigma_x^2 = \frac{5}{14} a^2$
$\langle p \rangle = 0$, $\langle p^2 \rangle = \frac{10\hbar^2}{a^2}$

a) $A^2 = \frac{12}{a^3}$
b) $c_n = \frac{4\sqrt{6}}{(n\pi)^2} (-1)^{\frac{n-1}{2}}$
$\psi(x,t) = \frac{4}{\pi^2} \sqrt{\frac{12}{a}} \sum_{\text{n odd}} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \sin(\frac{n\pi}{a} x) e^{-iE_n t/\hbar}$
c) $c_1 = \frac{16 \cdot 6}{\pi^4} = 0.9855$
d)
check! —v

2.8 Half-flat potential [ISW]

$c_1 = -2/\pi$

2.9 Explicit calculation of energy [ISW]

$\langle H \rangle = \frac{\pi\hbar^2}{ma^2}$

2.10 First three states [QHO]

Construct explicitly...

$\psi_0 = \alpha e^{-\xi^2/2}$
$\psi_1 = \alpha\sqrt{2}\xi e^{-\xi^2/2}$
$\psi_2 = \alpha(2\xi^2 - 1)e^{-\xi^2/2}$

2.11 Uncertainty [QHO]

$\psi_0 : \langle x^2\rangle = \hbar/2m\omega, \langle p^2\rangle = \hbar m\omega/2$
$\psi_1 : \langle x^2\rangle = 3\hbar/2m\omega, \langle p^2\rangle = 3\hbar m\omega/2$

2.12 Uncertainty by operator method [QHO]

$\langle x^2\rangle = \frac{\hbar}{m\omega}(n + \frac{1}{2})$
$\langle p^2\rangle = \hbar m\omega(n + \frac{1}{2})$

2.13 Linear combination of states [QHO]

a) $A = \frac{1}{5}$
b) $\psi = \frac{3}{5}\alpha e^{-\xi^2/2}e^{-iwt/2} + \frac{4}{5}\alpha\sqrt{2}\xi e^{-\xi^2/2}e^{-3iwt/2}$
$|\psi|^2 = \frac{9}{25}\alpha^2 e^{-\xi^2} + \frac{32}{25}\alpha^2\xi^2 e^{-\xi^2} + \frac{24\sqrt{2}}{25}\alpha^2\xi e^{-xi^2}\cos\omega t$
$\langle x\rangle = \frac{24\sqrt{2}}{50}\sqrt{\frac{\hbar}{m\omega}}\cos\omega t$
$\langle p\rangle = -\frac{24\sqrt{2}}{50}\sqrt{m\omega t}\sin\omega t$

2.14 Quadrapoled spring constant [QHO]

$|c_0|^2 = \sqrt{\frac{8}{9}}$

2.15 Probabiliity of particle outside classical region [QHO]

2.16 Ok

2.17 Ok

2.18 Exponentials, Sines and Cosines [FP]

$C = (A + B)$
$D = (A - B)i$

2.19 Probability current [FP]

$J = \frac{A^2\hbar k}{m}$

2.20 Proof of Plancheral's theorem [FP]

a) $a_n = i(c_n - c_{-n})$, $b_n = c_n + c_{-n}$
b) $\frac{1}{2a}\int_{-a}^{+a} c_n e^{in\pi x/a}e^{-in\pi x/a}dx = c_n$
c) $\Delta k = \frac{pi}{a}$

2.21 Exponential decay function [FP]

a) $A = \sqrt{a}$
b) $\phi(k) = \frac{2a}{\pi}\sqrt{\frac{a}{a^2+k^2}}$
c) $\psi(x,t) = \frac{a^{\frac{3}{2}}}{\pi}\int \frac{1}{a^2+k^2}e^{i(kx-\omega t)}dk$

d) $\phi(0) = a^{-\frac{1}{2}}$ and $\phi(k)$ goes as $\frac{a}{k^2}$
$\psi(0,0) = a^{\frac{1}{2}}$ and $\psi(x)$ goes as e^{-ax}

2.22 The gaussian wave packet [FP]

a) $A^2 = \sqrt{2a}\pi$
b) $\psi(x,t) = A\frac{e^{-ax^2/(1+i\omega t)}}{\sqrt{(1+i\omega t)}}[\omega = 2\hbar a/m]$
c) $|\psi(x,t)|^2 = \sqrt{\frac{2}{a\pi}}\omega^2 e^{-2\omega^2 x^2}[\omega = \sqrt{\frac{a}{1+(\omega' t)^2}}]$
d) $\langle x \rangle = 0, \langle p \rangle = 0, \langle x^2 \rangle = (1+\omega t)^2/4a, \langle p^2 \rangle = a\hbar^2$
We must evaluate $\int \psi^* \frac{d}{dx^2}\psi dx = \int |\frac{d\psi}{dx}|^2 dx$; for $\psi = e^{-ax^2}$ this is $-\frac{2|a|^2}{\lambda}\sqrt{\frac{\pi}{\lambda}}$ where $\lambda = a^* + a$
$\langle x^2 \rangle \langle p^2 \rangle = \frac{\hbar^2}{4}(1 + (\frac{2\hbar a t}{m})^2)$

2.23

2.24 Delta and step functions [δFP]

$\int f\frac{d\theta}{dx}dx = f\theta - \int \frac{df}{dx}\theta dx = f(\infty) - [f(\infty) - f(0)] = f(0)$

2.25 Uncertainty [δFP]

$\langle x^2 \rangle = \frac{1}{4k^2}, \langle p^2 \rangle = \hbar^2 k^2$

2.26 Fourer transform [δFP]

$\phi(k) = \frac{1}{\sqrt{2\pi}}$

2.27 Double well [δFP]

2.28 Long LA problem [δFP]

$T = \frac{1}{(\frac{\lambda^4\beta}{4k})^2 + \frac{1}{(2k)^4}(\lambda^4\beta - \lambda^4\beta^2 - \beta^2 - 4k^2)^2}$

2.29 Even states [FSW]

$\tan(z) = -\sqrt{\frac{z_0}{z})^2 - 1}$
same formula for E, but with n even

2.30 Normalization [FSW]

$D = \lambda F, \lambda = \frac{e^{-p}\tan(p)}{\cos(p)}, p = \frac{a\sqrt{2m(E+V_0)}}{\hbar}$
$F = \frac{k^2}{\sqrt{e^{-2ka} + \lambda^2 ka}}, k = \frac{\sqrt{-2mE}}{\hbar}$

2.31 Infinite square well [FSW]

$z_0 = \frac{\sqrt{2mV_0 aa}}{\hbar}, a \to 0$
as $V_0 a \to$ a constant