MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Spring 2006 Physics 8.04 Vuletic
Optional assignment #11

Suggested reading for assignment #11: G8-1 to 8-3, F&T 12-1 to 12-6

1. Hydrogen atom

Assume that Ψ_{nlm} denotes an eigenfunction of the hydrogen atom with principal quantum number *n*, and angular momentum quantum numbers *l* and *m*. Let the hydrogen atom be in a state described by the wave function

$$\psi(\mathbf{r}) = C\left[4\psi_{100}(\mathbf{r}) + 3\psi_{211}(\mathbf{r}) - 4\psi_{210}(\mathbf{r}) + \sqrt{10}\psi_{21-1}(\mathbf{r})\right]$$

a) Find a normalization constant C.
b) What is the expectation value of the energy?
c) What is the expectation value of $\boldsymbol{L}^2$?
d) What is the expectation value of L_z?
e) Write down $\psi(\boldsymbol{r},t)$ at some later time t.

2. Expansion in terms of angular momentum eigenstates

At a given instant in time the angular wavefunction of a system is given by

$$Y(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\phi\ .$$

a) What possible values of L_z will be found in a measurement, and with what probabilities will they occur?
b) What is $\langle L_x \rangle$ for this state?
c) What is $\langle L^2 \rangle$ for this state?

3. Orthonormality of spherical harmonics

The spherical harmonics are orthonormal in the sense

$$\int d\Omega Y^*_{l'm'}(\theta,\phi)\, Y_{lm}(\theta,\phi) = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, Y^*_{l'm'}(\theta,\phi)\, Y_{lm}(\theta,\phi) = \delta_{l'l}\delta_{m'm}$$

Verify the orthonormality of the spherical harmonics for the following three cases:

a) l=0, m=0; l'=1, m'=0
b) l=0, m=0; l'=1, m'=1
c) l=1, m=0; l'=1, m'=-1
d) l=1, m=-1; l'=1, m'=-1

It is helpful to use $\cos\theta$ as a variable of integration, $\sin\theta\, d\theta = -d(\cos\theta)$.

4. Commutation relations for $\vec{r}$ and $\vec{p}$.

Prove the following commutation relations for the Cartesian components $x = r_1, y = r_2, z = r_3$

a) $\left[\hat{L}_j, \hat{r}_k\right] = i\hbar\varepsilon_{jkl}\hat{r}_l$
b) $\left[\hat{L}_j, \hat{p}_k\right] = i\hbar\varepsilon_{jkl}\hat{p}_l$
c) $\left[\vec{L}, \hat{p}^2\right] = \left[\vec{L}, \hat{r}^2\right] = 0$