CTCS - graphs

Sets

1

Produce a graph that expresses the concept of graph with nodes labeled from a set L.

$A \overset{s}{\underset{t}{\rightrightarrows}} N \longrightarrow L$

2

Let G be a graph with a set of nodes N and a set of arrows A. Show that G is simple iif the function $\langle source, target \rangle : A \to N \times N$ is injective.

$\Longrightarrow$: We are given that G is simple.

$$\begin{aligned} \langle s, t \rangle(a_1) &= \langle s, t \rangle(a_2) \\ \langle s_1, t_1 \rangle &= \langle s_2, t_2 \rangle \\ s_1 &= s_2 \\ t_1 &= t_2 \end{aligned}$$

Since a_1 and a_2 have the same source and target, they are the same arrow. Hence $a_1 = a_2$.

$\Longleftarrow$: We are given that $\langle source, target \rangle : A \to N \times N$ is injective. Suppose a_1 and a_2 have the same source and target. Then $\langle s, t \rangle(a_1) = \langle s, t \rangle(a_2)$. Since $\langle s, t \rangle$ is injective, $a_1 = a_2$. Hence G is simple.