1 Continuity equation for probability density and probability current

a)

$$\begin{aligned}
\psi_t &= i\frac{\hbar}{2m}\psi_x - i\frac{V}{\hbar}\psi \\
\frac{\partial}{\partial t}\psi\psi^* &= \psi\psi_t^* + \psi^*\psi_t \\
&= \psi(-i\frac{\hbar}{2m}\psi_{xx}^* + i\frac{V}{\hbar}\psi^*) + \psi^*(i\frac{\hbar}{2m}\psi_x - i\frac{V}{\hbar}\psi) \\
&= \psi(-i\frac{\hbar}{2m}\psi_{xx}^*) + \psi^*(i\frac{\hbar}{2m}\psi_{xx}) \\
&= i\frac{\hbar}{2m}(\psi^*\psi_{xx} - \psi\psi_{xx}^*)
\end{aligned}$$

$$\begin{aligned}
J_x &= \frac{\hbar}{2im}(\psi_x^*\psi_x + \psi^*\psi_{xx} - \psi_x^*\psi_x - \psi_{xx}^*\psi_x) \\
&= -i\frac{\hbar}{2m}(\psi^*\psi_{xx} - \psi_{xx}^*\psi_x)
\end{aligned}$$

b) $P_t(a,b) = J(a) - J(b)$

2 Fictitious Bohr Atom

$$\begin{aligned}
V &= -C_6 r^{-6} \\
F_r &= 6C_6 r^{-7} \\
&= \frac{m_e v^2}{r} \\
v^2 &= \frac{6C_6}{m_e r^6} \\
v &= \sqrt{\frac{6C_6}{m_e}} r^{-3} \\
L &= \sqrt{6m_e C_6} r^{-2} \\
&= n\hbar \\
r^2 &= \frac{\sqrt{6m_e C_6}}{n\hbar} \\
v^2 &= \frac{6C_6}{m_e}(\frac{n\hbar}{\sqrt{6m_e C_6}})^3 \\
\frac{1}{2}m_e v^2 &= 3C_6(\frac{n\hbar}{\sqrt{6m_e C_6}})^3 \\
E &\sim n^3
\end{aligned}$$

3 Sommerfeld-Wilson quantization for linear potential in one dimension