**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**


Due Thursday, March 9 at 4pm

1. **Time delay in photoelectric effect.** (15 points)
   A beam of ultraviolet light ($\lambda$=121 nm) of intensity 10nW/cm$^2$ and area A=1cm$^2$ is turned on suddenly and falls on a metal surface, ejecting electrons through the photoelectric effect. The work function of the metal is 5 eV. How soon after the beam is turned on might one expect photoelectric emission to occur?
   (a) (5 points) Classically, one can estimate this as the time needed for the work-function energy (5 eV) to be accumulated over the area of one atom (radius 1Å). Calculate how long this would be, assuming the energy of the light beam to be uniformly distributed over its cross-section.
   (b) (5 points) Actually the estimate from part (a) is too pessimistic. An atom can represent a cross section $\sigma_{\text{eff}} = (3/2\pi)\lambda^2$ for the absorption of resonant light. Calculate a classical delay time on this basis.
   (c) (5 points) In the quantum picture, it is possible for photoelectric emission to begin as soon as the first photon strikes the emitting surface. However, in order to obtain a time that may be compared to the classical estimates, calculate the average time interval between the arrivals of successive photons. This would also be the average time delay between switching on the beam and observing the first photoelectron. Express the ratio of the classical to the quantum mechanical delay time in terms of quantities give above.

2. **Photons interacting with electrons. (**20 points)
   a) (10 points) Ultraviolet light of wavelength 350nm falls on a potassium surface. The maximum energy of the photoelectrons is 1.6eV. What is the work function of potassium? Above what wavelength will no photoemission be observed?
   b) (10 points) A beam of X-rays is scattered by electrons at rest. What is the energy of the X-rays if the wavelength of the X-rays scattered at 60º to the beam axis is 0.035Å?

3. **Momentum shift and spatial wavefunction. (**10 points)
   When discussing the double-slit experiment with photon scattering, we have postulated how the wave function of the electron changes due to momentum conservation as it scatters a photon. Prove the relationship that we have used, i.e. show the following:
   If the momentum of a particle is changed by a given amount $p_0=\hbar k_0$,
   $\phi(p) \to \tilde{\phi}(p) = \phi(p - \hbar k_0)$, then the spatial wavefunction acquires a position-dependent phase factor, $\psi(x) \to \tilde{\psi}(x) = \exp(ik_0 x)\psi(x)$.

4. **Double-slit interference of electrons. (**15 points)
French&Taylor 2.10.
   a) (5 points) Electrons of momentum $p$ fall normally on a pair of slits separated by a distance $d$. What is the distance between adjacent maxima of the fringe pattern formed on a screen a distance $D$ beyond the slits?
   b) (5 points) In the actual experiment performed by Joensson, the electrons were accelerated through 50kV, the slit separation $d$ was about 2 $\mu$m, and $D$ = 35cm. Calculate $\lambda$ and the fringe spacing. You will then appreciate why subsequent magnification using an electron microscope was required.
   c) (5 points) What would be the corresponding values of $d$, $D$, and the fringe spacing if Joenssons's apparatus were simply scaled up for use with visible light (all dimensions simply multiplied up by the ratio of the wavelengths)?

5. **Hydrogen atom ground state as structure of minimum energy allowed by Heisenberg uncertainty. (**20 points)

Modified from French & Taylor, problem 2-16.
A simple but sophisticated argument holds that the hydrogen atom has its observed size because this size minimizes the total energy of the system. The argument rests on the assumption that the lowest-energy state corresponds to a physical size comparable to a deBroglie wavelength of the electron. Larger size means larger deBroglie wavelength, hence smaller momentum and kinetic energy. In contrast smaller size means lower potential energy, since the potential well is deepest near the proton. The observed size is a compromise between kinetic and potential energies that minimizes the total energy of the system. Develop the argument explicitly, for example as follows:

*a)* (5 points) Write down the classical expression for the total energy of the hydrogen atom with an electron of momentum $p$ in a circular orbit of radius $r$. Keep kinetic and potential energies separate.

*b)* (5 points) *Failure of classical energy minimization.* Use the force law to obtain the total energy as a function of radius. What radius corresponds to the lowest possible energy?

*c)* (5 points) For the lowest-energy state, demand that the orbit *circumference* be one deBroglie wavelength. Obtain an expression for the total energy as a function of radius. Note how a larger radius decreases the kinetic and increases the potential energy, whereas a smaller radius increases the kinetic and decreases the potential energy.

*d)* (5 points)Take the derivative of the energy versus radius function and find the radius that minimizes the total energy. How large is that radius for a hydrogen atom? For a $He^+$ ion with one electron?

6. **Rutherford scattering.** (20 points)

a) (10 points) A beam of $\alpha$-particles, of kinetic energy E = 5.30 MeV and intensity $10^3$ particles/s, is
incident normally on a gold foil of density $19.3 \times 10^4$ kg m$^{-3}$, atomic weight 197, and thickness 150nm. An $\alpha$-particle detector of area 1.0 cm$^2$ is placed at a distance 10 cm from the foil. Let $\theta$ be the scattering angle. Use the differential cross-section for Rutherford scattering,

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 q^4}{64\pi^2 \varepsilon_0^2 E^2} \frac{1}{\sin^4(\theta/2)},$$

to find the number of counts per minute for $\theta = \pi/10$ and $\theta = \pi/4$. The charge of the gold nucleus is 79q (that is, Z = 79), where $q$ is the elementary charge. Note that by measuring the count rate we can determine the value of the nuclear charge Z.

(b) (10 points) Although the Rutherford cross-section correctly predicts the scattering rate of 5.3 MeV $\alpha$-particles from gold foil, serious discrepancies occur when 32 MeV $\alpha$-particles are scattering through large angles ($\theta \sim \pi$). In order for the Rutherford formula to be correct, the $\alpha$-particle must not penetrate the nucleus. As a crude indication of the size of a nucleus, calculate the closest approach of a 32 MeV $\alpha$-particle in a head-on collision with a gold nucleus (Z = 79). Neglect the recoil of the gold nucleus. Briefly explain why we may ignore in this calculation the electrons surrounding the gold nucleus.