Transforming I

1 Introduction

I is the moment of inertia tensor. A tensor is a matrix that obeys certain transformation rules; here we derive the rules for I.

We know that the angular momentum L can be written as $I\omega$. The only things we need to know to derive the transformation laws are that:

1. L and ω are vectors and
2. In any frame there exists a matrix (tensor) I such that $L = I\omega$

So if we have

$$L = I\omega \tag{1}$$

then in another frame

$$L' = I'\omega' \tag{2}$$

where, since L and ω are vectors, L' =AL and likewise $\omega' =$ Aω for some transformation matrix A. Then I must transform as such

$$I' = AIA^{-1} \tag{3}$$

and we note that this is a similarity transformation

2 Example

[diagram]

First we write down I in a basis where it is diagonal; then we only need to calculate moment of principle moments of inertia.

$$I = \begin{bmatrix} \frac{1}{4} & & \\ & \frac{1}{2} & \\ & & \frac{1}{4} \end{bmatrix} MR^2 \tag{4}$$

then we write the transformation matrix we use

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ & & 1 \end{bmatrix} \tag{5}$$

We then apply the transformation

$$I' = RIR^{-1} = [calculate] \begin{bmatrix} \frac{1}{4} & & \\ & \frac{1}{2} & \\ & & \frac{1}{4} \end{bmatrix} \tag{6}$$

and as desired, $I'_x = \frac{1}{4}\sin^2\theta + \frac{1}{2}\cos^2\theta$.