

Quantum Mechanics - Griffiths, David J

1 The Wave Function

1.1

For the distribution of ages in section...

1.2

a) Find the standard deviation...

1.3 Gaussian distribution

Consider the Gaussian distribution...

- a) $\sqrt{\frac{\lambda}{\pi}}$
- b) $\langle x \rangle = a, \langle x^2 \rangle = \frac{1}{2\lambda} + a^2, \sigma^2 = \frac{1}{2\lambda}$
- c) a smooth gentle hump centered at a

1.4 Triangle wavefunction

At time $t=0$ a particle is represented by...

- a) $A^2 = \frac{3}{b}$
- b) a sharp concave up peak
- c) at $x = a$
- d) $\Pr(x < a) = \frac{a}{b}$
- e) ??

1.5 Delta potential

Consider the wave function...

- a) $A = \sqrt{\lambda}$
- b) $\langle x \rangle = 0, \langle x^2 \rangle = \frac{1}{2\lambda^2}$
- c) $\sigma = \frac{\sqrt{2}}{2} \frac{1}{\lambda}, \Pr(|x| > \sigma) = e^{-\sqrt{2}}$

1.6

1.7

1.8

1.9 Gaussian wavefunction

A particle of mass m is in the state...

- a) $A^2 = \sqrt{\frac{2am}{\pi\hbar}}$
- b) $V = 2a^2mx^2$
- c) $\langle x \rangle = 0, \langle x^2 \rangle = \frac{\hbar}{4am}, \langle p \rangle = 0, \langle p^2 \rangle = am\hbar$
- d) $\sigma_x^2 \sigma_p^2 = \frac{\hbar^2}{4}$

1.10

1.11

1.12

1.13

1.14 Probability current

Let $P_{ab}(t)$ be the probability of finding...

a) ?? b) 0

1.15 Unstable particle

Suppose you wanted to describe an unstable particle...

a) ?? b) $P = P_0 e^{-(2\Gamma/\hbar)t}$

1.16

Done

1.17 Quadratic wavefunction

A particle is represented (at time $t = 0$) by the...

a) $A^2 = \frac{15}{16a^5}$

b) $\langle x \rangle = 0$

c) $\langle p \rangle = 0$

d) $\langle x^2 \rangle = \frac{a^2}{7}$

e) $\langle p^2 \rangle = \frac{5}{2} \frac{\hbar^2}{a^2}$

f,g,h) $\sigma_x^2 \sigma_p^2 = \hbar^2 \frac{5}{14}$

1.18 Quantum mechanical systems

In general, quantum mechanics is relevant...

2 The time-independent Schrödinger equation

2.1

Prove the following three theorems...

2.2

Show that E must exceed the minimum value of $V(x)$...

Done

2.3

Show that there is no acceptable solution to the...

Done

2.4 Uncertainty [ISW]

Calculate $\langle x \rangle, \langle x^2 \rangle, \dots$ for the n th stationary state...

$$\begin{aligned}\langle x \rangle &= a/2 \\ \langle x^2 \rangle &= a^2 \left(\frac{1}{3} - \frac{1}{2n^2\pi^2} \right) \\ \langle p \rangle &= 0 \\ \langle p^2 \rangle &= \frac{\hbar^2 n^2 \pi^2}{a^2} \\ \sigma_x^2 &= a^2 \left(\frac{1}{12} - \frac{1}{2n^2\pi^2} \right) \\ \sigma_x^2 \sigma_p^2 &= \hbar^2 \left(\frac{n^2\pi^2}{12} - \frac{1}{2} \right)\end{aligned}$$

2.5 Oscillating particle [ISW]

A particle in the infinite square well has as its initial wave function an even mixture of the first two...

$$\begin{aligned}\text{a) } A &= \frac{\sqrt{2}}{2} \\ \text{b) } \psi(x, t) &= \frac{\sqrt{a}}{a} \left(\sin\left(\frac{\pi x}{a}\right) e^{-i\pi^2 \hbar t / 2ma^2} + \sin\left(\frac{2\pi x}{a}\right) e^{-4i\pi^2 \hbar t / 2ma^2} \right), |\psi|^2 = \frac{1}{a} \left(\sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos \frac{3\pi^2 \hbar}{2ma^2} t \right) \\ \text{c) } \langle x \rangle &= \frac{a}{2} - \frac{16a}{9\pi^2} \cos 3\omega t \\ \text{d) } \frac{8\hbar}{3a} \sin 3\omega t \\ \text{e) } \frac{5\pi^2 \hbar^2}{4ma^2}\end{aligned}$$

2.6 Phase constant [ISW]

Although the overall phase constant of the wave function...

$$\begin{aligned}\psi(x, t) &= \frac{\sqrt{a}}{a} \left(\sin\left(\frac{\pi x}{a}\right) e^{-i\omega t} + \sin\left(\frac{2\pi x}{a}\right) e^{-4i\omega t + \phi} \right), \\ |\psi|^2 &= \frac{1}{a} \left(\sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos(3\omega t - \phi) \right) \\ \langle x \rangle &= \frac{a}{2} - \frac{16a}{9\pi^2} \cos(3\omega t - \phi)\end{aligned}$$

2.7 Triangular wave function [ISW]

A particle in the infinite square well has the initial wave function...

$$\begin{aligned}\langle x \rangle &= \frac{a}{2}, \langle x^2 \rangle = \frac{2}{7}a^2, \sigma_x^2 = \frac{5}{14}a^2 \\ \langle p \rangle &= 0, \langle p^2 \rangle = \frac{10\hbar^2}{a^2} \\ \text{a) } A^2 &= \frac{12}{a^3} \\ \text{b) } c_n &= \frac{4\sqrt{6}}{(n\pi)^2} (-1)^{\frac{n-1}{2}} \\ \psi(x, t) &= \frac{4}{\pi^2} \sqrt{\frac{12}{a}} \sum_{n \text{ odd}} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \sin\left(\frac{n\pi}{a} x\right) e^{-iE_n t / \hbar} \quad \text{c) } c_1 = \frac{16 \cdot 6}{\pi^4} = 0.9855\end{aligned}$$