Vectors

1 Geometry of Linear Equations

Given a system of linear equations, what is the solution set?

Let us work in equations of three variables x, y, z. We will consider a solution to be a vector in $\mathbb{R}^3$. We assume that each individual linear equation is consistent.

With 0 equations our solution set is R^3.

With 1 equation our solution set is a plane.

With 2 equations our solution set is either

1. a line
2. the empty set
3. a plane

The first case is the most common, while the second occurs if the planes are parallel; this corresponds to an inconsistent set of equations. The third occurs if the two equations are "the same".

With 3 equations our solution set is

1. a point
2. a line
3. the empty set
4. a plane

notice that there is now a greater variety in how it can be inconsistent. Also, it appears that the only way to have a point as a solution is to have the number of equations greater or equal to the number of unknowns.

2 Gauss Algorithm

Now we develop an algebraic way to solve this.

Gauss's Algorithm. If a linear system is changed to another by one of these operations

1. an equation is swapped with another ($\rho_i \leftrightarrow \rho_j$)
2. an equation has both sides multiplied by a nonzero constant ($n\rho_i$)
3. an equation is replaced by the sum of itself and a multiple of another ($n\rho_i + \rho_j$)

then the two systems have the same set of solutions.

There is an algorithm (gaussian elimination) that uses these operations to write the equation in reduced row-echelon form

3 Algebraic proofs

solution sets are $\mathbb{R}^n$

Theorem. For any linear system there are vectors $\vec{p}, \vec{\beta}_1 ... \vec{\beta}_k$ such that the solution set can be described as a $\vec{p}$ and a linear combination of β.

$m \geq n$ for unique solution

4 General = Particular + Homogenous

Such a split occurs in ordinary differential equations, linear diophantine equations, and many more (vector space!)