

Basics

write Newton's law for the i th particle.

$$\mathbf{F}_i = \dot{\mathbf{p}}_i \quad (1)$$

expanding out F_i as $\sum \mathbf{F}_{ji} + \mathbf{F}_i^e$, where \mathbf{F}_{ji} is the constraint force of particle j on i ,

$$\sum_j \mathbf{F}_{ji} + \mathbf{F}_i^e = \dot{\mathbf{p}}_i \quad (2)$$

sum this over i ,

$$\sum_{i,j} \mathbf{F}_{ji} + \sum_i \mathbf{F}_i^e = \sum_i \dot{\mathbf{p}}_i \quad (3)$$

due to the law of action and reaction, $\mathbf{F}_{ji} = -\mathbf{F}_{ij}$, the sum over pairs of reaction forces cancel out, so the first term is 0. The second term we write as \mathbf{F}^e , or total external force. To interpret the third term we define the center-of-mass vector \mathbf{R} as the mass-weighted sum of the \mathbf{r}_i ,

$$\mathbf{R} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} \quad (4)$$

to obtain

$$\mathbf{F}^e = M \ddot{\mathbf{R}} \quad (5)$$