Determinant

source: http://physics.stackexchange.com/questions/885/why-does-calculus-of-variations-work

1 Independence

Why is the Lagrangian a function of two variables? Put differently, you can choose position and velocity independently as initial conditions, that's why the Lagrangian function treats them as independent; but the calculus of variation does not vary them independently, a variation in position induces a fitting variation in velocity.

$$S = \int L(q, \dot{q}, t) dt \tag{1}$$

$$\delta S = \int \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} dt = 0 \tag{2}$$

the second step seems suspect; why are q and $\dot{q}$ varied independently?
now

$$\delta \dot{q} = \frac{d}{dt} \delta q \tag{3}$$

the independence of q and $\dot{q}$ is removed by this identity.

$$S = \int L(q, \dot{q}, t) dt \tag{4}$$

$$\delta S = \int \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} dt = 0 \tag{5}$$

$$\delta S = [\frac{\partial L}{\partial \dot{q}} \delta q]_{t_1}^{t_2} + \int_{t_1}^{t_2} (\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}}) \delta q dt = 0 \tag{6}$$

and the bracketed expression is zero because the endpoints are held fixed. And then we can pull out the Euler-Lagrange equation:

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0. \tag{7}$$

Now it makes more sense to me. You start by treating the variables as independent but then remove the independence by imposing a condition *during* the derivation.