1 Arithmetic and Geometric Progressions

2 Summation of Series

2.1 Sigma Notation

$$\sum_{T=1}^{n} T_r = T_1 + T_2 + T_3 + \ldots + T_n$$

Number of terms $= m - n + 1$
Express the following in sigma notation

1. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$
2. $3 + 5 + 7 + 9 + ... + 41$
3. $1 \times 3 - 2 \times 5 + 3 \times 7 - 4 \times 9 + 5 \times 11$

Write down the first 3 terms of the following sums

1. $\sum_{k=1}^{100} (3k - 1)$
2. $\sum_{k=1}^{40} (2r^2)$
3. $\sum_{k=3}^{10} (r - 1)(2r + 1)$

2.2 Basic Properties of Sigma

$$\sum_{r=1}^{n} kT_r = k \sum_{r=1}^{n} T_r$$
$$k \sum_{r=1}^{n} (T_r + G_r) = \sum_{r=1}^{n} T_r + \sum_{r=1}^{n} G_r$$
$$\sum_{r=m}^{n} T_r = \sum_{r=1}^{n} T_r - \sum_{r=1}^{m-1} T_r$$

True or False?

1. $\sum_{r=1}^{100} (2r + 1) = \sum_{r=1}^{100} (2r) + 1$
2. $\sum_{n=1}^{100} (2n + 1) = \sum_{m=1}^{100} (2n + 1)$
3. $\sum_{n=1}^{100} a_n = \sum_{n=0}^{99} T_{n+1}$
4. $\sum_{m=1}^{100} m^2 = \sum_{m=0}^{100} m^2$

5. $\sum_{r=1}^{100} a = 100a$
6. $\sum_{n=1}^{100}(2n+1) = \sum_{m=2}^{101}(2m-1)$
7. $\sum_{m=1}^{k}(m+2) = \sum_{m=0}^{k} m + \sum_{m=0}^{k} 2$

2.3 Basic Formulas

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(2n+1)$$
$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Evaluate $\sum_{r=1}^{n}(r+2)(2r-1)$ in terms of n.
Find $\sum_{k=0}^{n}(2n+1-2k)$ in terms of n.
Find an expression, in simplified form, for $\sum_{r=n+1}^{2n}(2r-1)^2$.

2.4 Method of difference

If a general term T_r can be expressed as $G_{r+1} - G_r$, then

$$\begin{aligned}\sum_{r=1}^{n} T_r &= \sum_{r=1}^{n}(G_{r+1} - G_r)\\ &= G_n - G_0\end{aligned}$$

Evaluate $\sum_{r=1}^{100} \dfrac{1}{r(r+1)}$

Find the partial fractions of $\dfrac{1}{(2x-1)(2x+1))}$. Hence, find the sum of $\dfrac{1}{1\times 3} + \dfrac{1}{3\times 5} + \ldots + \dfrac{1}{(2n-1)(2n+1)}$

Express $\dfrac{2}{y(y+1)(y+2)}$ in partial fractions. Using this result, show that

$$\sum_{r=1}^{N} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(N+1)(N+2)}$$

By considering $\dfrac{1}{1+a^{n-1}} - \dfrac{1}{1+a^n}$ or otherwise, show that $\sum_{n=1}^{N} \dfrac{a^{n-1}}{(1+a^{n-1})(1+a^n)} = \dfrac{a^N - 1}{2(a-1)(a^N+1)}$, where a is positive and $a \neq 1$. Deduce that $\sum_{n=1}^{N} \dfrac{2^n}{(1+2^{n-1})(1+2^n)} < 1$.

Express $\dfrac{2}{n(n+1)(n+2)}$ in partial fractions. Hence, evaluate $\sum_{k=1}^{99} \dfrac{1}{k(k+1)(k+2)}$

Expand $n^2-(n-1)^2e$. Hence or otherwise, prove that $\sum_{n=1}^{N} e^n \left[(1-e)n^2+2ne-e\right] = N^2e^N$.

Find $\sum_{r=2}^{n} \left[r-2-2n\left(\dfrac{1}{r-1}-\dfrac{1}{r}\right)\right]$ in terms of n.

Show that $\dfrac{1}{n^2-n+2}-\dfrac{1}{n^2+n+2}=\dfrac{2n}{n^4+3n^2+4}$. Find an expression in terms of N for the sum S_N where $S_N=\sum_{n=1}^{N} \dfrac{n}{n^4+3n^2+4}$. Deduce that $S_N<\dfrac{1}{4}$.