Schrodinger

A mix of experimental and mathematical whacking

1 Starting assumptions

Mostly experimental

$$p = \hbar k \tag{1}$$

2 Schrodinger's equation is linear

Wave packet

3 Photons

For a monochromatic wave in vacuum, with no currents or charges present, Maxwells wave equation,

$$\nabla^2 E - \frac{1}{c^2} \partial_t E = 0 \tag{2}$$

admits the plane wave solution,

$$\mathbf{E} = \mathbf{E_0} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \tag{3}$$

$$= \mathbf{E_0} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} \tag{4}$$

where the second equation, expressed in terms of p and E, allow us to connect more readily with particles.

4 TDSE

Treating a particle as a wave, we write the "phase"

$$\psi = A e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar} \tag{5}$$

$$\partial_t \psi = iE/\hbar \tag{6}$$

$$\partial_{\mathbf{r}}^2 \psi = -p^2/\hbar^2 \tag{7}$$

Using the classical relation that $E = p^2/2m + V$, we get

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \partial_{\mathbf{r}}^2 \psi + V\psi \tag{8}$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial \mathbf{r}} + V\psi \tag{9}$$

If we instead use $E^2 = (cp)^2 + m^2c^4$ and do some other stuff we end up with the Klein-Gordon equation

5 TISE

If we separate the time dependance to write (can we always do this? it turns out yes) $\psi(x,t) = e^{iEt}\psi(x)$ and V s independent of time then

$$-\frac{\hbar^2}{2m}\partial_x^2\psi + V\psi = E\psi \tag{10}$$

for some constants E. Various ways to derive this, including from the heuristic, a formal definition to prove that any state can be written as a sum of stationary states, etc.