1 Circular Polarization

$|L\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle), |R\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle)$
a) Show that these two states are orthonormal

$$\langle L|L\rangle = \langle R|R\rangle = \frac{1}{2}(\langle x|x\rangle + \langle y|y\rangle)$$
$$\langle L|R\rangle = \langle R|L\rangle = \frac{1}{2}(\langle x|x\rangle - \langle y|y\rangle)$$

b) Given

$$\psi = a|x\rangle + b|y\rangle$$

find a', b' such that

$$\psi = a'|L\rangle + b'|R\rangle$$

Write the linear polarization states in terms of the circular ones:

$$|x\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$$
$$|y\rangle = \frac{i}{\sqrt{2}}(|L\rangle - |R\rangle)$$

So

$$\begin{aligned}\psi &= a|x\rangle + b|y\rangle \\ &= \frac{a}{\sqrt{2}}(|L\rangle + |R\rangle) + \frac{bi}{\sqrt{2}}(|L\rangle - |R\rangle) \\ &= (\frac{a+bi}{\sqrt{2}})|L\rangle + (\frac{a-bi}{\sqrt{2}})|R\rangle\end{aligned}$$

c) Write down the operators that correspond to left- and right-circular polarization filters. Determine the probability that an x-polarized photon will pass through a left circular polarization filter.

$$\begin{aligned}|L\rangle\langle L| &= \frac{1}{2}(|x\rangle - i|y\rangle)(\langle x| + i\langle y|) \\ &= \frac{1}{2}(|x\rangle\langle x| + i|x\rangle\langle y| - i)|y\rangle\langle x| + |y\rangle\langle y| \\ |R\rangle\langle R| &= \frac{1}{2}(|x\rangle + i|y\rangle)(\langle x| - i\langle y|) \\ &= \frac{1}{2}(|x\rangle\langle x| - i|x\rangle\langle y| + i)|y\rangle\langle x| + |y\rangle\langle y|\end{aligned}$$

and

$$\langle L|x\rangle = \frac{1}{\sqrt{2}}$$

it is $\frac{1}{2}$.