

# Quantum Mechanics - Griffiths, David J

## 1 The Wave Function

### 1.1

For the distribution of ages in section...

### 1.2

a) Find the standard deviation...

### 1.3 Gaussian distribution

Consider the Gaussian distribution...

- a)  $\sqrt{\frac{\lambda}{\pi}}$
- b)  $\langle x \rangle = a, \langle x^2 \rangle = \frac{1}{2\lambda} + a^2, \sigma^2 = \frac{1}{2\lambda}$
- c) a smooth gentle hump centered at  $a$

### 1.4 Triangle wavefunction

At time  $t=0$  a particle is represented by...

- a)  $A^2 = \frac{3}{b}$
- b) a sharp concave up peak
- c) at  $x = a$
- d)  $\Pr(x < a) = \frac{a}{b}$
- e) ??

### 1.5 Delta potential

Consider the wave function...

- a)  $A = \sqrt{\lambda}$
- b)  $\langle x \rangle = 0, \langle x^2 \rangle = \frac{1}{2\lambda^2}$
- c)  $\sigma = \frac{\sqrt{2}}{2} \frac{1}{\lambda}, \Pr(|x| > \sigma) = e^{-\sqrt{2}}$

### 1.6

### 1.7

### 1.8

### 1.9 Gaussian wavefunction

A particle of mass  $m$  is in the state...

- a)  $A^2 = \sqrt{\frac{2am}{\pi\hbar}}$
- b)  $V = 2a^2mx^2$
- c)  $\langle x \rangle = 0, \langle x^2 \rangle = \frac{\hbar}{4am}, \langle p \rangle = 0, \langle p^2 \rangle = am\hbar$
- d)  $\sigma_x^2 \sigma_p^2 = \frac{\hbar^2}{4}$

**1.10**

**1.11**

**1.12**

**1.13**

**1.14 Probability current**

Let  $P_{ab}(t)$  be the probability of finding...

a) ?? b) 0

**1.15 Unstable particle**

Suppose you wanted to describe an unstable particle...

a) ?? b)  $P = P_0 e^{-(2\Gamma/\hbar)t}$

**1.16**

Done

**1.17 Quadratic wavefunction**

A particle is represented (at time  $t = 0$ ) by the...

a)  $A^2 = \frac{15}{16a^5}$

b)  $\langle x \rangle = 0$

c)  $\langle p \rangle = 0$

d)  $\langle x^2 \rangle = \frac{a^2}{7}$

e)  $\langle p^2 \rangle = \frac{5}{2} \frac{\hbar^2}{a^2}$

f,g,h)  $\sigma_x^2 \sigma_p^2 = \hbar^2 \frac{5}{14}$

**1.18 Quantum mechanical systems**

In general, quantum mechanics is relevant...

## **2 The time-independent Schrödinger equation**

**2.1**

Prove the following three theorems...

**2.2**

Show that  $E$  must exceed the minimum value of  $V(x)$ ...

Done

**2.3**

Show that there is no acceptable solution to the...

Done

## 2.4 Uncertainty [ISW]

Calculate  $\langle x \rangle, \langle x^2 \rangle, \dots$  for the  $n$ th stationary state...

$$\begin{aligned}\langle x \rangle &= a/2 \\ \langle x^2 \rangle &= a^2 \left( \frac{1}{3} - \frac{1}{2n^2\pi^2} \right) \\ \langle p \rangle &= 0 \\ \langle p^2 \rangle &= \frac{\hbar^2 n^2 \pi^2}{a^2} \\ \sigma_x^2 &= a^2 \left( \frac{1}{12} - \frac{1}{2n^2\pi^2} \right) \\ \sigma_x^2 \sigma_p^2 &= \hbar^2 \left( \frac{n^2 \pi^2}{12} - \frac{1}{2} \right)\end{aligned}$$

## 2.5 Oscillating particle [ISW]

A particle in the infinite square well has as its initial wave function an even mixture of the first two...

$$\begin{aligned}\text{a) } A &= \frac{\sqrt{2}}{2} \\ \text{b) } \psi(x, t) &= \frac{\sqrt{a}}{a} \left( \sin\left(\frac{\pi x}{a}\right) e^{-i\pi^2 \hbar t / 2ma^2} + \sin\left(\frac{2\pi x}{a}\right) e^{-4i\pi^2 \hbar t / 2ma^2} \right) \\ |\psi|^2 &= \frac{1}{a} \left( \sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos \frac{3\pi^2 \hbar}{2ma^2} t \right) \\ \text{c) } \langle x \rangle &= \frac{a}{2} - \frac{16a}{9\pi^2} \cos 3\omega t \\ \text{d) } \frac{8\hbar}{3a} \sin 3\omega t \\ \text{e) } \frac{5\pi^2 \hbar^2}{4ma^2}\end{aligned}$$

## 2.6 Phase constant [ISW]

Although the overall phase constant of the wave function...

$$\begin{aligned}\psi(x, t) &= \frac{\sqrt{a}}{a} \left( \sin\left(\frac{\pi x}{a}\right) e^{-i\omega t} + \sin\left(\frac{2\pi x}{a}\right) e^{-4i\omega t + \phi} \right), \\ |\psi|^2 &= \frac{1}{a} \left( \sin^2 \frac{\pi x}{a} + \sin^2 \frac{2\pi x}{a} + 2 \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} \cos(3\omega t - \phi) \right) \\ \langle x \rangle &= \frac{a}{2} - \frac{16a}{9\pi^2} \cos(3\omega t - \phi)\end{aligned}$$

## 2.7 Triangular wave function [ISW]

A particle in the infinite square well has the initial wave function...

$$\begin{aligned}\langle x \rangle &= \frac{a}{2}, \quad \langle x^2 \rangle = \frac{2}{7}a^2, \quad \sigma_x^2 = \frac{5}{14}a^2 \\ \langle p \rangle &= 0, \quad \langle p^2 \rangle = \frac{10\hbar^2}{a^2} \\ \text{a) } A^2 &= \frac{12}{a^3} \\ \text{b) } c_n &= \frac{4\sqrt{6}}{(n\pi)^2} (-1)^{\frac{n-1}{2}} \\ \psi(x, t) &= \frac{4}{\pi^2} \sqrt{\frac{12}{a}} \sum_{n \text{ odd}} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \sin\left(\frac{n\pi}{a} x\right) e^{-iE_n t / \hbar} \\ \text{c) } c_1 &= \frac{16\sqrt{6}}{\pi^4} = 0.9855 \\ \text{d) } &\end{aligned}$$

## 2.8 Half-flat potential [ISW]

$$c_1 = -2/\pi \quad \text{XXX} \quad 2/\pi$$

## 2.9 Explicit calculation of energy [ISW]

$$\langle H \rangle = \frac{\pi \hbar^2}{ma^2} \quad \text{XXX} \quad 5\hbar \dots$$

## 2.10 First three states [QHO]

Construct explicitly...

$$\begin{aligned}\psi_0 &= \alpha e^{-\xi^2/2} \\ \psi_1 &= \alpha \sqrt{2} \xi e^{-\xi^2/2} \\ \psi_2 &= \alpha (2\xi^2 - 1) e^{-\xi^2/2}\end{aligned}$$

### 2.11 Uncertainty [QHO]

$$\begin{aligned}\psi_0 : \langle x^2 \rangle &= \hbar/2m\omega, \langle p^2 \rangle = \hbar m\omega/2 \\ \psi_1 : \langle x^2 \rangle &= 3\hbar/2m\omega, \langle p^2 \rangle = 3\hbar m\omega/2\end{aligned}$$

### 2.12 Uncertainty by operator method [QHO]

$$\begin{aligned}\langle x^2 \rangle &= \frac{\hbar}{m\omega} (n + \frac{1}{2}) \\ \langle p^2 \rangle &= \hbar m\omega (n + \frac{1}{2})\end{aligned}$$

### 2.13 Linear combination of states [QHO]

$$\begin{aligned}\text{a) } A &= \frac{1}{5} \\ \text{b) } \psi &= \frac{3}{5} \alpha e^{-\xi^2/2} e^{-i\omega t/2} + \frac{4}{5} \alpha \sqrt{2} \xi e^{-\xi^2/2} e^{-3i\omega t/2} \\ |\psi|^2 &= \frac{9}{25} \alpha^2 e^{-\xi^2} + \frac{32}{25} \alpha^2 \xi^2 e^{-\xi^2} + \frac{24\sqrt{2}}{25} \alpha^2 \xi e^{-\xi^2} \cos \omega t \\ \langle x \rangle &= \frac{24\sqrt{2}}{50} \sqrt{\frac{\hbar}{m\omega}} \cos \omega t \\ \langle p \rangle &= -\frac{24\sqrt{2}}{50} \sqrt{m\omega\hbar} \sin \omega t\end{aligned}$$

### 2.14 Quadrapoled spring constant [QHO]

$$|c_0|^2 = \sqrt{\frac{8}{9}}$$

### 2.15 Probabiliity of particle outside classical region [QHO]

2.16 Ok

2.17 Ok

### 2.18 Exponentials, Sines and Cosines [FP]

$$\begin{aligned}C &= (A + B) \\ D &= (A - B)i\end{aligned}$$

### 2.19 Probability current [FP]

$$J = \frac{A^2 \hbar k}{m}$$

### 2.20 Proof of Plancheral's theorem [FP]

$$\begin{aligned}\text{a) } a_n &= i(c_n - c_{-n}), b_n = c_n + c_{-n} \\ \text{b) } \frac{1}{2a} \int_{-a}^{+a} c_n e^{in\pi x/a} e^{-in\pi x/a} dx &= c_n \\ \text{c) } \Delta k &= \frac{p_i}{a}\end{aligned}$$

### 2.21 Exponential decay function [FP]

$$\begin{aligned}\text{a) } A &= \sqrt{a} \\ \text{b) } \phi(k) &= \frac{2a}{\pi} \sqrt{\frac{a}{a^2 + k^2}} \text{ XXX} \\ \text{c) } \psi(x, t) &= \frac{a^{\frac{3}{2}}}{\pi} \int \frac{1}{a^2 + k^2} e^{i(kx - \omega t)} dk \\ \text{d) } \phi(0) &= a^{-\frac{1}{2}} \text{ and } \phi(k) \text{ goes as } \frac{a}{k^2} \\ \psi(0, 0) &= a^{\frac{1}{2}} \text{ and } \psi(x) \text{ goes as } e^{-ax}\end{aligned}$$

## 2.22 The gaussian wave packet [FP]

a)  $A^2 = \sqrt{\frac{2a}{\pi}}$

b)  $\psi(x, t) = A \frac{e^{-ax^2/(1+i\omega t)}}{\sqrt{(1+i\omega t)}} [\omega = 2\hbar a/m]$

c)  $|\psi(x, t)|^2 = \sqrt{\frac{2}{a\pi}} \omega^2 e^{-2\omega^2 x^2} [\omega = \sqrt{\frac{a}{1+(\omega' t)^2}}]$

d)  $\langle x \rangle = 0, \langle p \rangle = 0, \langle x^2 \rangle = (1 + \omega t)^2/4a, \langle p^2 \rangle = a\hbar^2$

We must evaluate  $\int \psi^* \frac{d}{dx^2} \psi dx = \int |\frac{d\psi}{dx}|^2 dx$ ; for  $\psi = e^{-ax^2}$  this is  $-\frac{2|a|^2}{\lambda} \sqrt{\frac{\pi}{\lambda}}$  where  $\lambda = a^* + a$   
 $\langle x^2 \rangle \langle p^2 \rangle = \frac{\hbar^2}{4} (1 + (\frac{2\hbar a t}{m})^2)$

## 2.23

## 2.24 Delta and step functions [ $\delta$ FP]

$$\int f \frac{d\theta}{dx} dx = f\theta - \int \frac{df}{dx} \theta dx = f(\infty) - [f(\infty) - f(0)] = f(0)$$

## 2.25 Uncertainty [ $\delta$ FP]

$$\langle x^2 \rangle = \frac{1}{4k^2}, \langle p^2 \rangle = \hbar^2 k^2 \text{ XXX}$$

## 2.26 Fourer transform [ $\delta$ FP]

$$\phi(k) = \frac{1}{\sqrt{2\pi}}$$

## 2.27 Double well [ $\delta$ FP]

??

## 2.28 Long LA problem [ $\delta$ FP]

??

$$T = \frac{1}{(\frac{\lambda^4 \beta}{4k})^2 + \frac{1}{(2k)^4} (\lambda^4 \beta - \lambda^4 \beta^2 - \beta^2 - 4k^2)^2}$$

## 2.29 Even states [FSW]

$$\tan(z) = -\sqrt{\frac{z_0}{z}}^2 - 1 \text{ XXX}$$

same formula for  $E$ , but with  $n$  even

## 2.30 Normalization [FSW]

XXX

$$D = \lambda F, \lambda = \frac{e^{-p} \tan(p)}{\cos(p)}, p = \frac{a\sqrt{2m(E+V_0)}}{\hbar}$$

$$F = \frac{k^2}{\sqrt{e^{-2ka} + \lambda^2 ka}}, k = \frac{\sqrt{-2mE}}{\hbar}$$

## 2.31 Infinite square well [FSW]

???

$$z_0 = \frac{\sqrt{2mV_0}aa}{\hbar}, a \rightarrow 0$$

as  $V_0 a \rightarrow$  a constant