

1. In triangle $\triangle ABC$, point D lies on AC and point E lies on BC . Angles ABC and EDC are right angles. Given that $CD = 6\text{cm}$, $AD = 7\text{cm}$ and $AB = 5\text{cm}$, what is the area of $ABED$?

2. Four congruent circles, each of which is tangent externally to two of the other three circles, are circumscribed by a square of area 144cm^2 . If a small circle is then placed in the centre so that it is tangent to each of the circles, what is the diameter of the small circle?

3. In quadrilateral $PQRS$, side PQ is parallel to and 3 times as long as side SR . Diagonals PR and QS intersect at O , $PO : OR = 1 : 3$ and $QO : OS = 1 : 4$. If $\triangle PQO$ has area 2cm^2 , find the area of $PQRS$.

4. $ABCD$ is an isosceles trapezium with AB parallel to DC , $AC = DC$ and $AD = BC$. If the height of the trapezium is equal to AB , find the ratio of $AB : DC$.

5. In triangle ABC , M and N are points on AB and AC respectively such that $AM : MB = 1 : 3$ and $AN : NC = 3 : 5$. What is the ratio of $[MNC] : [ABC]$?

6. Six right-angled triangles are assembled together. Given $PQ = a$ and $QR = 8a$, find $(b - a)(b + a)$ in terms of a .

7. On side BC of $\triangle ABC$ point A_1 is taken so that $BA_1 : A_1C = 2 : 1$. What is the ratio in which median CC_1 divides segment AA_1 ?

8. Square $PQRS$ is inscribed into $\triangle ABC$ so that vertices P and Q lie on sides AB and AC and vertices R and S lie on BC . Express the length of the squares side through a and h_a .
- * 9. Four isosceles right triangles are removed from the four corners of a square piece of paper so that a rectangle remains. What is the length of the diagonal of the rectangle if the sum of the areas of the cut-off pieces is 200cm^2 ?
- ** 10. Congruent radii PS and QR intersect tangent SR . If the two disjoint shaded regions have equal area and if $PS = 10\text{cm}$, what is the area of rectangle $PRQS$?
- ** 11. An equilateral triangle ABC has area $\sqrt{3}$ and side length 2. Point P is an arbitrary point in the interior of the triangle. What is the sum of the distances from P to AB , AC and BC ?

Proofs

12. Consider heights AA_1 and BB_1 in acute triangle $\triangle ABC$. Prove that $A_1C \cdot BC = B_1C \cdot AC$.
13. Consider height CH in right triangle ABC with right angle C . Prove that $AC_2 = AB \cdot AH$ and $CH_2 = AH \cdot BH$.

- ** 14. Prove that the medians of a triangle meet at one point and this point divides each median in the ratio of $2 : 1$ counting from the vertex. (A median is a line from a vertex to the opposite side that divides the side equally)