Rigged Hilbert Space

What interesting subsets of function space can we identify?

Let's start with a basis for this space $\{\phi_i\}$. We can define V, the linear combination of a finite number of ϕ_i's:

$$\psi = \sum_{i=1}^{n} c_i \phi_i \tag{1}$$

1 Conjugate space

Let $V^\times$ = set of all f such that $\langle f, \psi \rangle < \infty$. $V^\times$ is any linear combination of ϕ_i's, even very funny ones which don't converge, because $\langle f, \psi \rangle < \infty$ always consists of a finite number of terms.

2 Competeness

One problem with V is that this space isn't complete, that a series $\{\psi_i\}$ of could converge to a function that is not in V. If we use mean convergence and complete V (find the smallest space that is complete and contains V) we get Hilbert space. Some other definitions of Hilbert space:

$\sum |c_n|^2 < \infty$

h is square-integrable

$H \sim H^\times$

$c_n \sim n^{-1/2}$