# 1 Compton Scattering

Photon energy $= hc/\lambda$ Photon momentum $= h/\lambda$ $v = 2c$

we have

$$\begin{aligned} mc^2 + \frac{hc}{\lambda_0} &= \frac{hc}{\lambda_1} + \sqrt{m^2c^4 + p^2c^2} \\ \frac{h}{\lambda_0} &= \frac{h}{\lambda_1}\cos\theta + p\cos\phi \\ \frac{h}{\lambda_1}\sin\theta &= p\sin\phi \end{aligned}$$

From the last 2,

$$p^2 = \left(\frac{h}{\lambda_0} - \frac{h}{\lambda_1}\right)^2 + \frac{2h^2}{\lambda_0\lambda_1}(1-\cos\theta)$$

From the first,

$$p^2 = \left(\frac{h}{\lambda_0} - \frac{h}{\lambda_1}\right)^2 + 2mc\left(\frac{h}{\lambda_0} - \frac{h}{\lambda_1}\right)$$

$$\lambda_c = \frac{h}{mc}$$

c,d,e

# 2 Practice with delta functions

44
-2.5

# 3 Gaussian wave packets and Heisenberg uncertainty relation

a)

$$\begin{aligned} |\psi^2| &= \frac{1}{\sqrt{2\pi}w_0} e^{-x^2/2w_0} \\ I &= \frac{1}{\sqrt{2\pi}w_0}\int e^{-t^2}\sqrt{2}w_0 dt \\ &= 1 \end{aligned}$$

b) 0, 0
c)

$$\begin{aligned} I &= \frac{1}{\sqrt{2\pi}} w_0 \int x^2 e^{-x^2/2w_0^2} dx \\ &= \frac{1}{\sqrt{2\pi}} w_0 \int 2w_0^2 t^2 e^{-t^2}\sqrt{2}w_0 dt \\ &= w_0^2 \end{aligned}$$

d)

$$\begin{aligned}\langle x^2\rangle\langle p^2\rangle &= \hbar^2 k_0^2 w_0^2 \\ &= \frac{\hbar^2}{4}\end{aligned}$$

e)

$$\begin{aligned}\bar{\phi}(k) &= A\int e^{-x'^2/4w_0^2} e^{-ikx'} e^{-ikx_0}\,dx' \\ &= e^{-ikx_0}\phi(k)\end{aligned}$$

same momentum distribution

# 4 Childish precision experiment

$$\begin{aligned}\Delta p &= \frac{\hbar}{\Delta x} \\ \Delta v &= \frac{\hbar}{m\Delta x} \\ t &= \sqrt{\frac{2H}{g}} \\ \Delta x' &= \Delta x + t\Delta v \\ &= \Delta x + \sqrt{\frac{2H}{g}}\frac{\hbar}{m\Delta x}\end{aligned}$$

Differentiating wrt $\Delta x$ and setting to 0,

$$(\Delta x)^2 = \sqrt{\frac{2H}{g}}\frac{\hbar}{m}$$

$9.08 \times 10^{-17} m$
$1.02 \times 10^{-5} m$