1 Parseval's Theorem

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_{-\infty}^{\infty} |\phi(k)|^2 dk$$

$$\begin{aligned}
\int_{-\infty}^{\infty} |\psi(x)|^2 dx &= \int \left(\int \frac{1}{\sqrt{2\pi}} \phi(k) e^{ikx} dk \right) \left(\int \frac{1}{\sqrt{2\pi}} \phi^*(k') e^{-ik'x} dk' \right) dx \\
&= \frac{1}{2\pi} \int \int \int \phi(k) \phi^*(k') e^{i(k-k')x} dk' dk dx \\
&= \frac{1}{2\pi} \int \int \phi(k) \phi^*(k') 2\pi \delta(k-k') dk dk' \\
&= \int \phi(k) \phi^*(k) dk
\end{aligned}$$

$$\int_{-L}^{L} e^{ikx} dx = \frac{2\sin(kL)}{L}$$

The central peak between the two roots closest to $x = 0$ is roughly triangular, height $2L$, width $\frac{2\pi}{L}$, so the area is

$$\frac{1}{2} 2L \frac{2\pi}{L} = 2\pi$$

letting $L \to \infty$,

$$\int_{-\infty}^{\infty} e^{ikx} dx = 2\pi\delta(k)$$

2 Fourier transform of a square wave packet

$$\begin{aligned}
\phi(k) &= \frac{1}{\sqrt{2\pi}} \int_{-d/2}^{d/2} \frac{1}{\sqrt{d}} e^{-ikx} dx \\
&= \frac{1}{\sqrt{2\pi d}} \int_{-d/2}^{d/2} e^{-ikx} dx \\
&= \frac{1}{\sqrt{2\pi d}} \frac{2\sin(\frac{dk}{2})}{k}
\end{aligned}$$

Width of $\psi(x) : d$
Width of $\phi(k) : 4\pi/d$

3 Momentum distribution due to slit and their diffraction patterns

$$
\begin{aligned}
\psi(x) &\sim [-d/2 < x < d/2] \\
\phi(k_x) &\sim \frac{2\sin(\frac{dk_x}{2})}{k_x}
\end{aligned}
$$

For the particle to land on x', the ratio momenta (= ratio of wavenumber) must be $x' : L$. Hence $k_x = k_0 x'/L$. Also we have $k_0 = 2\pi/\lambda$. The argument to the sin function is then

$$
\begin{aligned}
\frac{dk_x}{2} &= \frac{d(2\pi/\lambda)(x'/L)}{2} \\
&= \frac{d\pi x'}{\lambda L} \\
&= Ax'
\end{aligned}
$$

Which is the answer up to normalization.

4 de Broglie wavelength of macroscopic objects

5 Gaussian wavepacket in free space

$\int e^{-ikx - x^2/4w^2} = 2\sqrt{\pi} w e^{-k^2 w^2}$, $w_0 = 1/2k_0$, $w_0 k_0 = 1/2$