MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Spring 2006 Physics 8.04 Vuletic
Assignment #8
Due Thursday, April 20 at 4pm

1. **Qualitative bound state solutions I** (20 points).
French & Taylor, Problem 3-15.
(a) Draw the wave functions associated with E_1, E_2, and E_3.
(b)-(e) Draw the ground state wave function only.
(f) Draw a wave function for small barrier height and one for very large barrier height.
(g) Draw one wave function for a narrow central barrier.
State in words what happens when the barrier grows to the full width.

2. **Qualitative bound state solutions II** (20 points).
French & Taylor, Problem 3-16.

3. **Characteristics of wave functions for stationary states** (20 points).
French & Taylor, Problem 3-18.

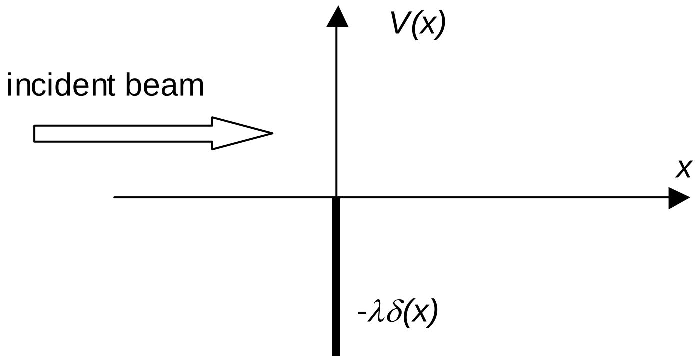
4. **Relation between wavefunction phase and probability current** (20 points).
Suppose that for a particle of mass m we write the complex wavefunction in the amplitude-phase form
$\psi(x) = A(x)\exp(i\varphi(x))$, where $A(x)$ and $\phi(x)$ are real quantities.

a) (5 points) Show that the probability current is given by $j(x) = |A(x)|^2 \frac{\hbar}{m}\frac{\partial\varphi}{\partial x}(x)$, i.e. the probability current is proportional to the gradient of the complex phase.
b) (5 points) Show that there can be no current in a region where the wavefunction is real.
c) (5 points) How large is the current for a plane wave of wavevector k?
d) (5 points) Calculate the current in a region where the wavefunction is given by $Be^{-\kappa x} + Ce^{+\kappa x}$, where κ is a real constant, and B, C are complex numbers, i.e. B, C are not position-dependent. (This is the solution inside a potential barrier if the particle energy is insufficient to overcome the barrier.) Is it correct to say that since $e^{\pm\kappa x}$ are real functions, the current inside the barrier must be zero? Find a condition on B and C such that the current vanishes.

5. **Scattering by an attractive δ-potential** (20 points)

A beam of particles, each of mass m and energy E>0, is incident from the left on a delta function well located at the origin,

$$V(x) = -\lambda\delta(x), \text{ with } \lambda > 0.$$



a) (5 points) Show that the first derivative of the wavefunction is not continuous at x=0, and obeys $\psi'(+\varepsilon)-\psi'(-\varepsilon)=-\frac{2m\lambda}{\hbar^2}\psi(0)$.

b) (15 points) Calculate the fraction of particles in the incident beam that are reflected by this potential, i.e. find the reflection coefficient R. Write your answer in terms of E, m, λ, and fundamental constants. Check if your result is reasonable in the limits $\lambda\to 0$ for constant E, and $E\to 0$ for constant λ.