1 Binary Logistic Regression

Model

1. Binary classification: $y \in \{0, 1\}$
2. Want to predict probability of being in a particular class: $P(y = 1|\mathbf{x}; \mathbf{w})$
3. Could fit a linear model: $f(\mathbf{x}; \mathbf{w}) = \mathbf{w}^T\mathbf{x}$
4. But this could give predictions outside $[0, 1]$ for some test inputs (invalid probabilities)
5. Use the sigmoid function to force the output to lie in the $[0, 1]$ range:

$$f(\mathbf{x}; \mathbf{w}) = \frac{1}{1 + e^{-\mathbf{w}^T\mathbf{x}}}$$

6. Interpret $f(\mathbf{x}; \mathbf{w}) = P(y = 1|\mathbf{x}; \mathbf{w})$, implying $P(y = 0|\mathbf{x}; \mathbf{w}) = 1 - f(\mathbf{x}; \mathbf{w})$

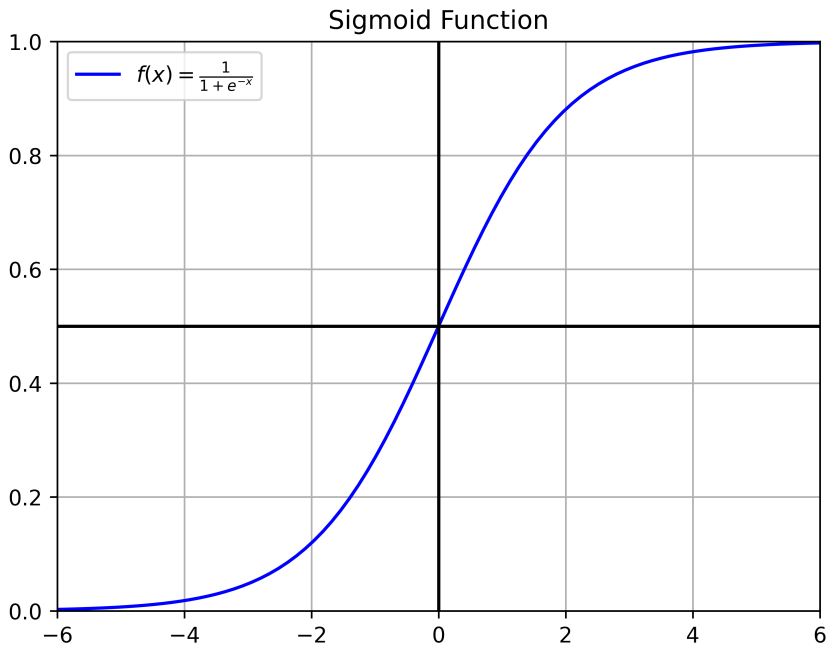

Figura 1: Function used to force the output to lie in the $[0, 1]$ range

Loss Funciton

We observe data $\{(x^{(n)}, y^{(n)})\}_{n=1}^{N}$, with $y \in \{0, 1\}$, Using maximum likehood:

$$L(w)$$