Homework - 02

CSE-121
Discrete Mathematics
Monsoon 2014

Total : 30 points
Due: Friday, 29/08/2014
10:00 am (in lecture)

1. (3 points) Express the following statements into logic formula:
   (a) The weak can never forgive.
   (b) You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time.
   (c) [*Goldbach Conjecture*] Every even integer greater than 2 can be written as the sum of any two primes.

2. (2 points) Define a function that maps the natural numbers from 0 to $2^n - 1$ onto the 2's complement numbers -2^{n-1} to $2^{n-1} - 1$.

3. (5 points) Let $B = \{0, 1\}$ be the set of binary numbers, and B^n be the set of all strings of length n. How many functions can be constructed from B^n to B using AND ($\wedge$), OR ($\vee$), and NOT ($\neg$) operations ?

4. (5 points) If n is a perfect square [1] then $n + 2$ is not a perfect square.

5. (5 points) Prove the statement: "The integer $3n + 2$ is odd if and only if $9n + 5$ is even, where n is an integer."

6. (5 points) Suppose you begin with a pile of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute rs. Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n - 1)/2$.

7. (5 points) Consider a $2^n \times 2^n$ chessboard with one (arbitrarily chosen) square removed. Prove that any such chessboard can be tiled without gaps or overlaps by L-shaped pieces, each composed of 3-squares.

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[1] A perfect square is a number of the form n^2 where n is any integer. For example, $0, 1, 4, 9, 16, 25, \ldots$