1. If A and B be two events, $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$.

Proof. By De Morgan's law,

$$\bar{A} \cup \bar{B} = \overline{A \cap B}$$

$$\therefore P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B).$$

$[P(\bar{A}) = 1 - P(A)]$

Theorem: For events A and B in a sample space,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

For three events A, B and C in a sample space,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Generalizing for n events, say $A_1, A_2, \cdots A_n$, in a sample space,

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{\substack{i,j=1 \\ i \neq j}}^{n} P(A_i \cap A_j) + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n-1} P(A_1 \cap A_2 \cap \cdots \cap A_n).$$

[Proof by Mathematical Induction]

[Boole's Inequality]

for any n events $A_1, A_2, \ldots, A_n$

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$$

Proof [by Mathematical Induction]

[Basis] $n = 2$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$[P(A_i) \geq 0]$

$$\leq P(A_1) + P(A_2)$$

The theorem holds for $n = 2$.

[Hypothesis] Assume that the theorem holds for $n = k$, $n > 2$.

$\therefore$ By assumption,

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) \leq P(A_1) + P(A_2) + \cdots + P(A_k)$$

[Induction] R.T.P. theorem holds for $n = k+1$.

Now, $P(A_1 \cup A_2 \cup \cdots \cup A_k \cup A_{k+1})$

$$= P[(A_1 \cup A_2 \cup \cdots \cup A_k) \cup A_{k+1}]$$

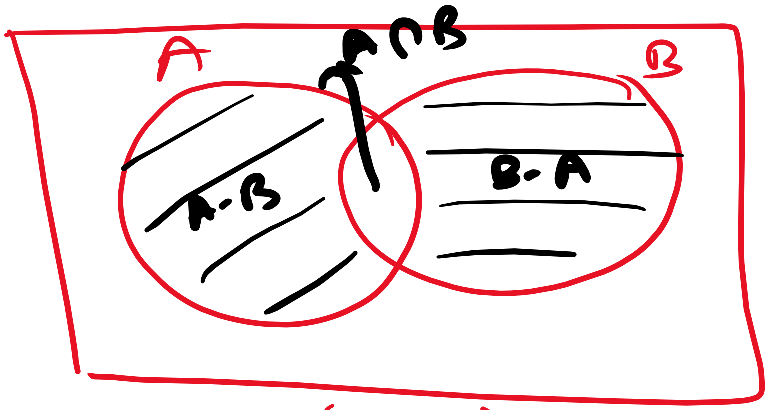
$$= P(A_1 \cup A_2 \cup \cdots \cup A_k) + P(A_{k+1}) - P[(A_1 \cup A_2 \cup \cdots \cup A_k) \cap A_{k+1}]$$

$$\leq \underline{P(A_1 \cup A_2 \cup \cdots \cup A_k)} + P(A_{k+1})$$

$$\leq P(A_1) + P(A_2) + \cdots + P(A_k) + P(A_{k+1})$$

by hypothesis.

Problem: Find the probability of occurrence of only one of the events A and B.



$A \cup B = (A-B) \cup (A \cap B) \cup (B-A)$

and $(A-B)$, $(A \cap B)$ and $(B-A)$ are pairwise mutually exclusive events.

Required probability $= P[(A-B) \cup (B-A)]$

Now, $P(A \cup B) = P(A-B) + P(A \cap B) + P(B-A)$

$\Rightarrow \underline{P(A-B) + P(B-A)} = P(A \cup B) - P(A \cap B)$

$\Rightarrow P[(A-B) \cup (B-A)]$

$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

We have:

$$P(A_1 \cap A_2 \cap \cdots \cap A_{r-1} \cap A_r)$$
$$= P[(A_1 \cap A_2 \cap \cdots \cap A_{r-1}) \cap A_r]$$
$$= P(A_1 \cap A_2 \cap \cdots \cap A_{r-1})\, P(A_r \mid A_1 \cap A_2 \cap \cdots \cap A_{r-1})$$
$$\Rightarrow \frac{P(A_1 \cap A_2 \cap \cdots \cap A_{r-1} \cap A_r)}{P(A_1 \cap A_2 \cap \cdots \cap A_{r-1})} = P(A_r \mid A_1 \cap A_2 \cdots \cap A_{r-1})$$

Substituting $r = 2, 3, \ldots, n$ in succession, we obtain:

$$\frac{P(A_1 \cap A_2)}{P(A_1)} = P(A_2 \mid A_1)$$
$$\frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)} = P(A_3 \mid A_1 \cap A_2)$$
$$\vdots$$
$$\frac{P(A_1 \cap A_2 \cap \cdots \cap A_n)}{P(A_1 \cap A_2 \cap \cdots \cap A_{n-1})} = P(A_n \mid A_1 \cap A_2 \cap \cdots \cap A_{n-1})$$

Multiplying both sides:

$$\frac{P(A_1 \cap A_2 \cap \cdots \cap A_n)}{P(A_1)} = P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2) \cdots P(A_n \mid A_1 \cap A_2 \cdots \cap A_{n-1})$$

$$\therefore P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)\, P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2) \cdots P(A_n \mid A_1 \cap A_2 \cap \cdots \cap A_{n-1}).$$

$x = \langle 1 \quad 1 \quad 1 \quad 0 \rangle$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

$x_3 \; x_5 \quad x_6 \quad x_7$

$y = \langle y_1, y_2, y_3, y_4, y_5, y_6, y_7 \rangle$

$$\left.\begin{aligned} y_3 &= x_3 = 1 \\ y_5 &= x_5 = 1 \\ y_6 &= x_6 = 1 \\ y_7 &= x_7 = 0 \end{aligned}\right\}$$

$y_1 \oplus y_3 \oplus y_5 \oplus y_7 = 0$

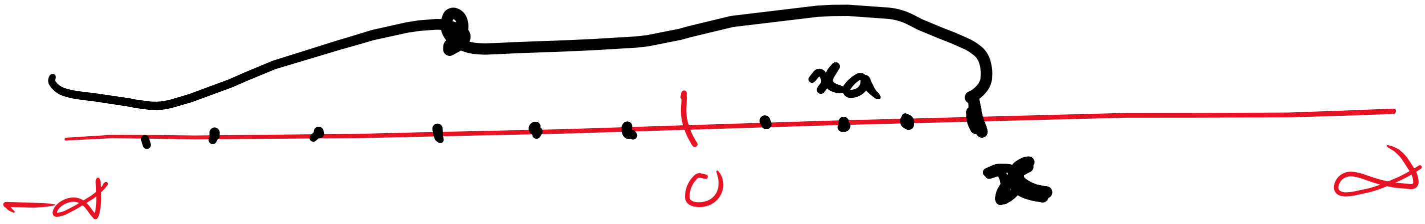
$$\Rightarrow \boxed{\begin{aligned} y_1 &= y_3 \oplus y_5 \oplus y_7 \\ &= 1 \oplus 1 \oplus 0 = 0 \end{aligned}}.$$

$$x = x' \oplus \epsilon$$

error tuple

$$\epsilon = \boxed{x \oplus x'}$$

$$x = i = x_i$$



$$P(-\infty < X \leq x) = F(x)$$

$$\therefore F(x) = P\left[\sum_{a=-\infty}^{i} \underbrace{(X = x_a)}\right]$$

$$= \sum_{a=-\infty}^{i} P(X = x_a)$$