Tutorial-7 Questions

1. Show that {1, 2, 3} under multiplication modulo 4 is not a group but that {1, 2, 3, 4} under multiplication modulo 5 is a group.
2. In Z^{*}_{9} , find the inverse of 2, 7, and 8.
3. An abstract algebra teacher intended to give a typist a list of nine integers that form a group under multiplication modulo 91. Instead, one of the nine integers was inadvertently left out, so that the list appeared as 1, 9, 16, 22, 53, 74, 79, 81. Which integer was left out?
4. Let G be a group with the property that for any x, y, z in the group, xy = zx implies y = z. Prove that G is Abelian.
5. Suppose n is an even positive integer and H is a subgroup of Zn. Prove that either every member of H is even or exactly half of the members of H are even
6. If H and K are subgroups of G, show that H ∩ K is a subgroup of G.
7. Let G be an Abelian group and H = {x ∈ G | |x| is odd}. Prove that H is a subgroup of G.
8. Let H = {0, ±1, ±3, ±6, . . . }. Find all the left cosets of H in Z.
9. Suppose that a has order 15. Find all the left cosets of ha 5 i in hai.
10. Show that Z ∗ 8 is isomorphic to Z ∗ 12.