① $f(x)$ can be regarded as a p.d.f if

(i) $f(x) \geq 0, \forall x \in \mathbb{R}$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

(i) $f(x) \geq 0, \forall x \in R$

$\Rightarrow x-2 \geq 0$ or $\boxed{k \geq 2}$.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$

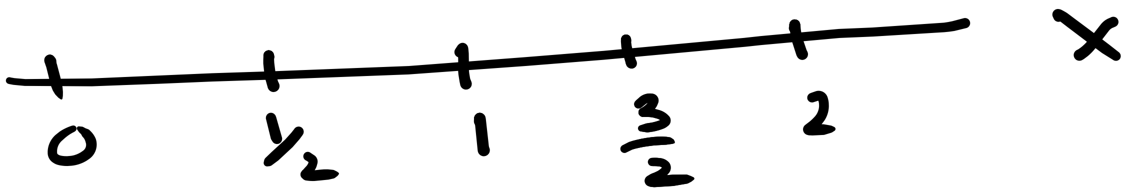
$\Rightarrow \int_{0}^{1} x\,dx + \int_{1}^{2} (k-x)\,dx = 1$

$\Rightarrow \frac{1}{2} + \left[kx - \frac{x^2}{2}\right]_1^2 = 1$

$\Rightarrow k-1 = 1 \Rightarrow k=2$

Thus,

$$f(x) = \begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & \text{elsewhere.} \end{cases}$$



Required probability

$= P\left[\frac{1}{2} < X < \frac{3}{2}\right]$

$$= \int_{1/2}^{3/2} f(x)\,dx$$

$$= \int_{1/2}^{1} f(x)\,dx + \int_{1}^{3/2} f(x)\,dx$$

$$= \int_{1/2}^{1} x\,dx + \int_{1}^{3/2} (2-x)\,dx$$

$$= \frac{3}{4}.$$

(i) R.T.P. $P(a<X<b) = F(b-0) - F(a)$.

The events $(a<X<b)$ and $(X=b)$ are mutually exclusive, and

$$(a<X<b) \cup (X=b) = (a<X\le b)$$

$$\Rightarrow P(a<X<b) + P(X=b) = P(a<X\le b)$$

$$\Rightarrow P(a<X<b) = P(a<X\le b) - P(X=b)$$

$$= [F(b) - F(a)] - [F(b) - F(b-0)]$$

$$= F(b-0) - F(a).$$

(ii) R.T.P. $P(a\le X\le b) = F(b) - F(a-0)$.

The events $(X=a)$ and $(a<X\le b)$ are mutually exclusive, and

$$(X=a) \cup (a<X\le b) = (a\le X\le b)$$

$$P(X=a) + P(a<X\le b) = P(a\le X\le b)$$

$$\Rightarrow P(a\le X\le b) = [F(a) - F(a-0)] + [F(b) - F(a)]$$

$$= F(b) - F(a-0)$$

$X \sim \text{Binomial}(n, p)$

p.m.f of X

$$f_i = P(X=x_i) = P(X=i)$$

$$= {}^nC_i\, p^i q^{n-i}, \quad i = 0, 1, 2, \ldots, n$$

(*) The number of lines in operation, X, has a binomial distribution with parameters $n = 20$ and $p = 0.6$

Then, $q = 1 - p = 0.4$

The p.m.f. of X is then given by:

$$f_i = P(X = x_i) = P(X = i)$$

$$= {}^{n}C_i \, p^i q^{n-i}$$

$$= {}^{20}C_i (0.6)^i (0.4)^{20-i}, \quad i = 0, 1, 2, \dots, 20$$

Required probability

$$= P[X \geq 10]$$

$$= \sum_{k=10}^{20} {}^{20}C_i (0.6)^i (0.4)^{20-i}$$

$$= 0.872478$$

$$\approx 87\%$$

Let X be the number of inquiries that arrive per second at the central computer system.

By hypothesis, $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$

$\therefore f_i =$ p.m.f. of $X = P(X = x_i) = P(X = i)$

$$= e^{-\lambda} \cdot \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \ldots$$

Here $\lambda =$ average rate of messages per second

$$= E(X)$$

$$= 10$$

$$\therefore f_i = P(X = i) = e^{-10} \cdot \frac{10^i}{i!}, \quad i = 0, 1, 2, \ldots$$

The required probability that 15 or fewer inquires arrive in a one-second period $= P(X \leq 15)$

$$= \sum_{i=0}^{15} P[X = i]$$

$$= \sum_{i=0}^{15} e^{-10} \cdot \frac{10^i}{i!}$$

$$= e^{-10} \cdot \sum_{i=0}^{15} \frac{10^i}{i!} = 0.95126$$

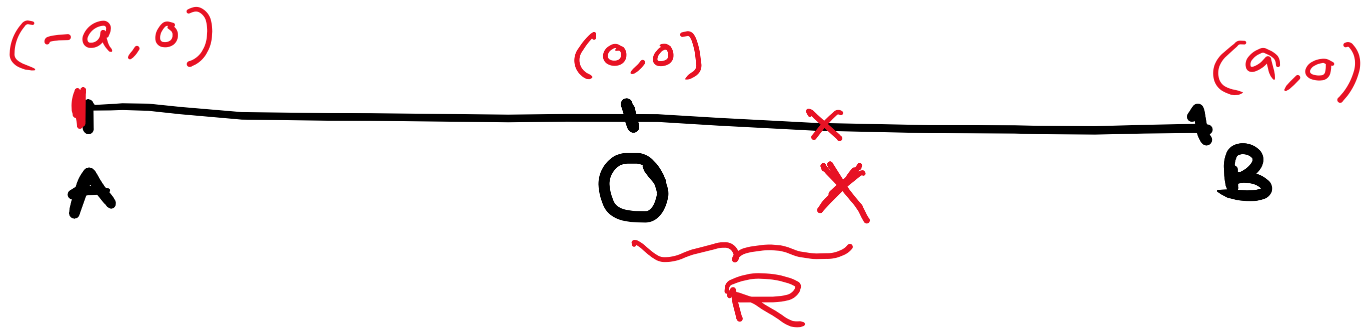
$$\approx \underline{95}\%.$$

$$\Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1}\, dx$$

$$\begin{aligned}
\Gamma(n+1) &= n \cdot \Gamma(n) \\
&= n \cdot (n-1)\,\Gamma(n-1) \\
&\;\;\vdots \\
&= n \cdot (n-1) \cdot (n-2) \cdots 1 \cdot \Gamma(1) \\
&= n \cdot (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1
\end{aligned}$$

$$\boxed{\Gamma(n+1) = n!}$$

$\Gamma(1) = 1$, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.



Let $AB = 2a$ and $OX = R$.

Since X can be taken anywhere on AB, R has a uniform distribution in $(-a, a)$.

$\therefore f(r) =$ p.d.f. of R

$$= \begin{cases} \dfrac{1}{a-(-a)} = \dfrac{1}{2a}, & \text{if } -a < r < a \\ 0, & \text{elsewhere.} \end{cases}$$

Now, $AX = AO + OX$

$= a + R$

$BX = OB - OX$

$= a - R$

$AO = a$

AX, BX and AO can form the sides of a triangle, if

(i) $AX + BX > AO$

$\Rightarrow 2a > a$ ✓

(ii) $AX + AO > BX$

$\Rightarrow (a+R) + a > a - R$

$\Rightarrow 2a + R > a - R$

$\Rightarrow 2R > -a \Rightarrow \boxed{R > -\frac{a}{2}}$

(iii) $BX + AO > AX$

$\Rightarrow (a-R) + a > a+R$

$\Rightarrow 2a - R > a+R$

$\Rightarrow a > 2R$

$\Rightarrow \boxed{R < \frac{a}{2}}$

Thus, AX, BX and AO can form the sides of a triangle if (i), (ii) & (iii) hold simultaneously

ie, $R > -\frac{a}{2}$ and $R < \frac{a}{2}$

ie, $\boxed{-\frac{a}{2} < R < \frac{a}{2}}$

$\therefore$ Required probability

$$= P\left[-\frac{a}{2} < R < \frac{a}{2}\right]$$

$$= \int_{-\frac{a}{2}}^{a/2} f(r)\, dr$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{2a}\, dr$$

$$= \frac{1}{2a} \int_{-\frac{a}{2}}^{a/2} dr = \frac{1}{2a} \left[r\right]_{-\frac{a}{2}}^{a/2}$$

$$= \frac{1}{2a} \times a = \frac{1}{2}$$