Lecture - 18

Saturday, 26 February 2022 8:43 AM

* using partial fractions to find inverse LT

* causal system
  - nature of $h(t)$
  - nature of $H(s)$ ROC

* stable system

BIBO stability $\Longrightarrow \int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

LT : $H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$

$$H(s)\Big|_{s=0} = \int_{-\infty}^{\infty} h(t)\, dt < \infty$$

$\Rightarrow$ Laplace trasform converges at $s = 0$

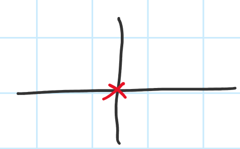
In fact, for any $s = j\omega = 0 + j\omega$

$$H(s)\Big|_{s=j\omega} < \infty \quad \Rightarrow \text{system is stable}$$

$\therefore$ $j\omega$ - axis i.e. $Re(s) = 0$ is part of the ROC.

* Examples: 1) Shift : $h(t) = \delta(t - t_0) \xleftrightarrow{LT} e^{st_0}$, full s-plane

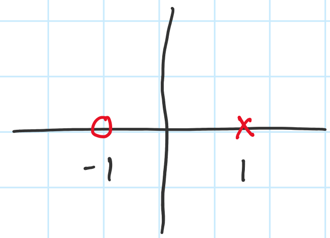
2) Integrator : $h(t) = u(t) \xleftrightarrow{LT} \frac{1}{s}$, $Re(s) > 0$



* Causal & Stable systems:

A causal system with rational system function is stable if and only if: all the poles have negative real part.*

Ex. $H(s) = \dfrac{s+1}{s-1}$

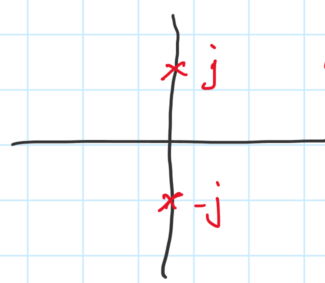


can be stable or causal, not both

a) causal ✓

b) stable ✓

Ex. $H(s) = \dfrac{1}{s^2+1}$



can never be stable

$h(t) = ?$

→ find using partial fractions when the system is causal → $Re(s) > 0$

$$H(s) = \frac{1}{(s+j)(s-j)} = \frac{A}{s+j} + \frac{B}{s-j}$$

$$A = -\frac{1}{2j}, \quad B = \frac{1}{2j}$$

$$A = H(s)(s+j)\Big|_{s=-j}$$

$$B = H(s)(s-j)\Big|_{s=j}$$

$$\Rightarrow \quad h(t) = A \cdot e^{-jt} u(t) + B e^{jt} u(t)$$

$$= \frac{1}{2j}\left[e^{jt} - e^{-jt} \right] u(t)$$

$$h(t) = \sin(t)\, u(t)$$

NOTE: For a real valued signal $x(t)$, the zeros [or poles] appear in complex conjugate pairs.

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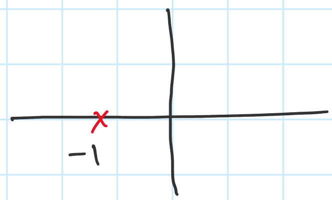
$$X(s) = \frac{A(s)}{B(s)}$$

$A(s)$, $B(s)$ are polynomials with real valued coefficients.
$\Rightarrow$ roots in complex conjugate pairs

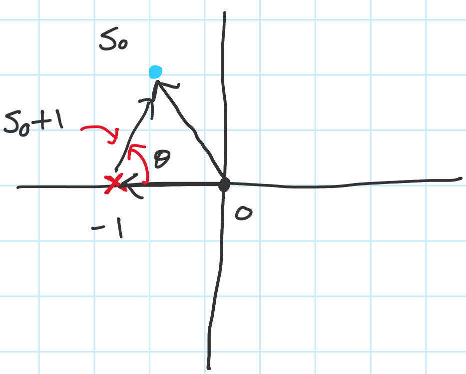
* **Geometric** Interpretation: Frequency Analysis

Ex. $H(s) = \frac{1}{s+1}$



evaluate $H(s)$ at any point $s = s_0$ in the s-plane

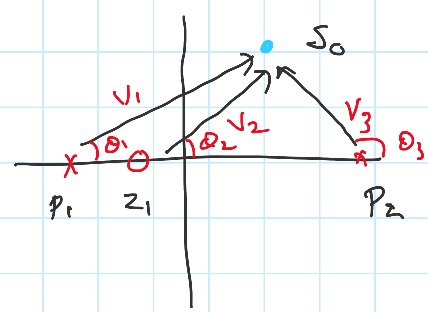
$$H(s)\Big|_{s=s_0} = \frac{1}{s_0+1}$$



$$H(s)\Big|_{s=s_0} = |H(s_0)|\, e^{j \measuredangle H(s_0)}$$

$$|H(s_0)| = \frac{1}{|s_0+1|} \quad \& \quad \measuredangle H(s_0) = \theta$$

Ex. $H(s) = \frac{s-z_1}{(s-p_1)(s-p_2)}$



$$|H(s_0)| = \frac{|s-z_1|}{|s-p_1||s-p_2|} = \frac{|v_2|}{|v_1||v_3|}$$

$\measuredangle\, H(s_0) = \theta_2 - (\theta_1 + \theta_3)$