Norton & Thevenin Equivalent circuits

Supplemental Problem

Thevenin

1. Remove resistance (R_L)
2. Find open circuit voltage across the two nodes ($V_{oc} = V_{Th}$)
3. Short voltage sources & open current sources and calculate equivalent resistance across the two nodes
   $R_{eq} = R_{th}$
4. Replace the circuit with Thevenin equivalent.

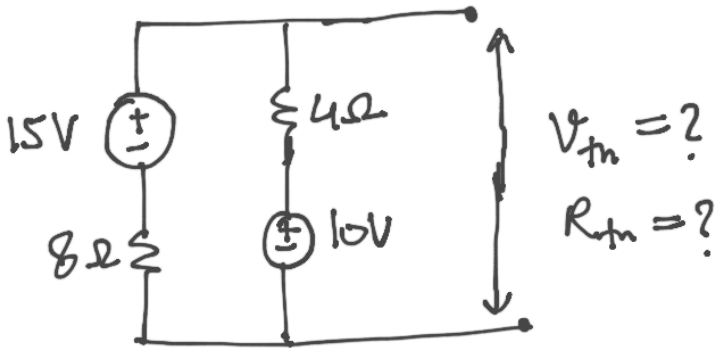
Note (1) V_{oc} has influence of all the sources.

(2) When dependent & independent sources are present, i_{sc} is found between the two nodes.

$$R_{th} = \frac{V_{oc}}{i_{sc}}$$

(3) When dependent source is present, its best to connect test current source of 1A across the nodes & find voltage across the terminals. $R_{th} = \frac{v}{1A}$ ← node voltage ← test current

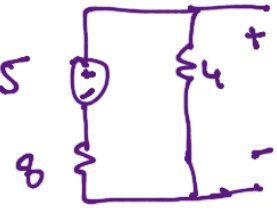
Example 1



Step 1 find V_{oc} due to the sources

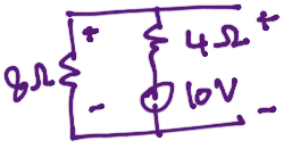
Superposition:

(A)



$$v_{oc_1} = \frac{4}{12} * 15 = 5\ V$$

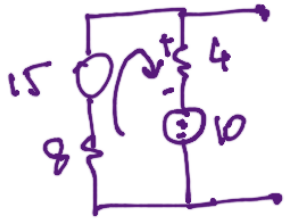
(B)



$$V_{oc_2} = \frac{2\cancel{8}}{3+\cancel{8}} * 10 = \frac{20}{3}\ V$$

$$V_{oc} = 5 + \frac{20}{3} = \boxed{\frac{35}{3}\ V = V_{th}.}$$

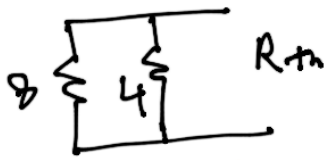
KVL

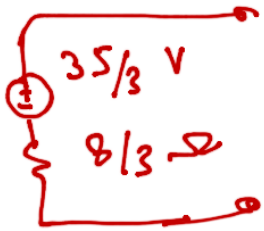


$$i = \frac{-10+15}{12} =$$

$$\text{Drop} = \cancel{4} * \frac{5}{\cancel{12}_3} + 10 = \frac{35}{3}\ V = V_{Th}$$

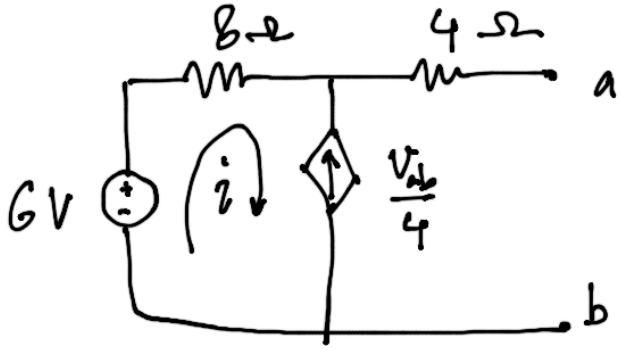
Step 2 find R_{th}.





$$R_{th} = \frac{8*4}{\cancel{12}_3} = \boxed{\frac{8}{3}\ \Omega}$$

Example 2



Step 1 $v_{ab} = v_{oc}$

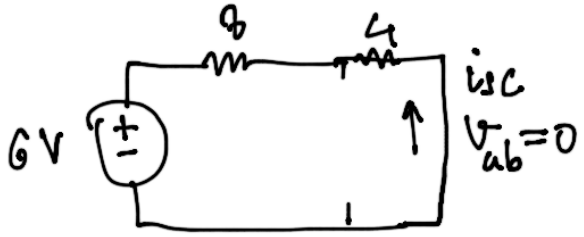
By KVL $6 - 8i - v_{ab} = 0$

$$i = -\frac{v_{ab}}{4}$$

$$\therefore \quad 6 - 8\left(-\frac{v_{ab}}{4}\right) - v_{ab} = 0$$

$$6 + v_{ab} = 0 \Rightarrow V_{ab} = -6\,V = v_{oc}$$

Step 2 i_{sc}

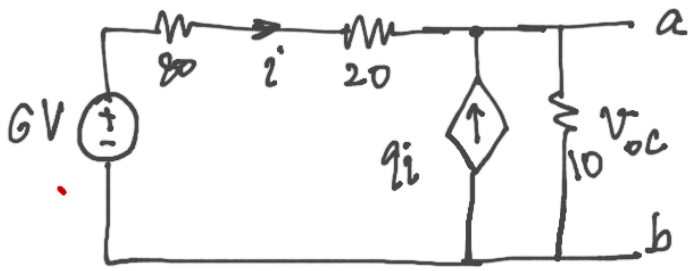


$$V_{ab} = 0 \Rightarrow \frac{v_{ab}}{4} = 0 \Rightarrow i_{sc} = -\frac{6}{12} = 0.5A$$

Step 3 $$R_{th} = \frac{-6}{-0.5} = \boxed{12\,\Omega}$$

Thevenin Eq.

Example 3



$V_{th} = ?$

$R_{th} = ?$

Step 1 Find v_{oc}

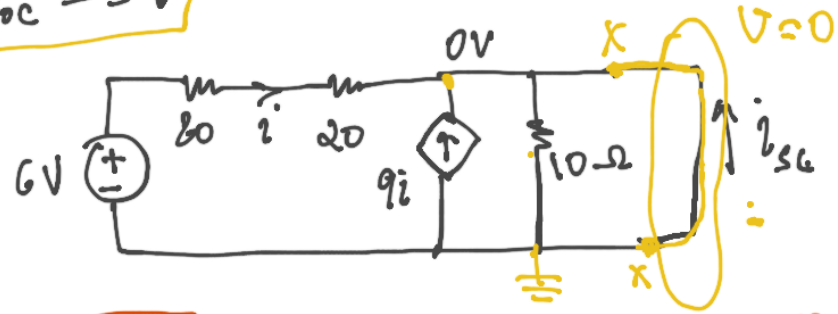
KCL (node a) $\quad \dfrac{(6 - v_{oc})}{100} + 9i - \dfrac{v_{oc}}{10} = 0$

$$i = \frac{6 - v_{oc}}{100}$$

$$\therefore \quad 10\,\frac{(6 - v_{oc})}{100} - \frac{v_{oc}}{10} = 0 \Rightarrow 6 - 2v_{oc} = 0$$

$$\Rightarrow \boxed{v_{oc} = 3\ V}$$

Step 2 I_{sc} ? $(v_{oc} = 0)$



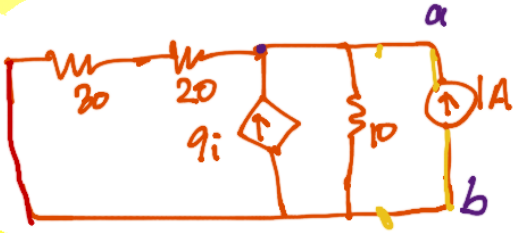
Method 1

$$i = \frac{6 - 0}{100} = -9i \quad : \text{Not valid}$$

$-i_{sc}$

$i_{sc} = -10$

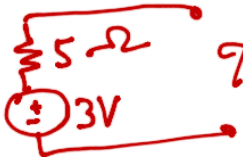
Method 2 Connect 1A current (test) and Remove independent Sources



KCL at node a $\quad -\dfrac{v_{ab}}{100} - \dfrac{9\,v_{ab}}{100} - \dfrac{v_{ab}}{10} + 1 = 0$

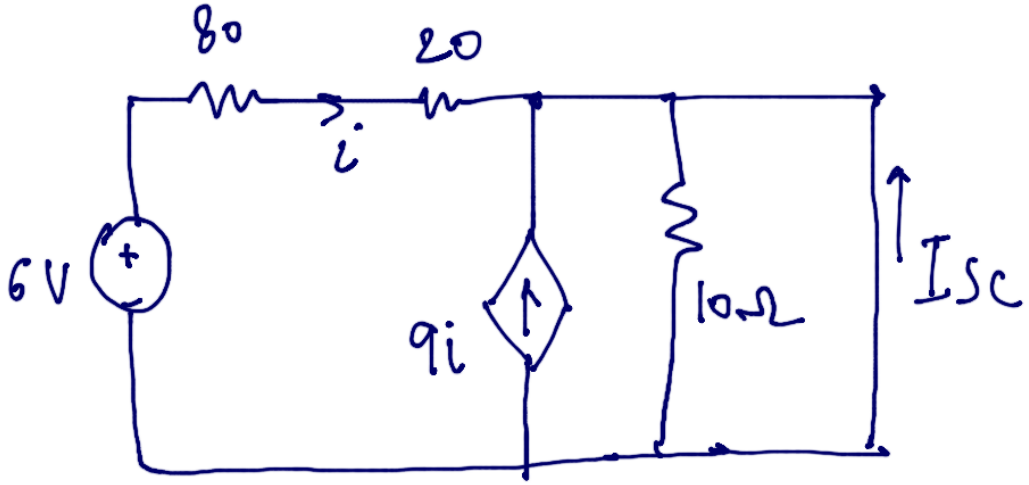
$v_{ab} = 5\ V \Rightarrow R_{th} = \dfrac{5V}{1A} = \boxed{5\Omega}$

Thevenin Eq



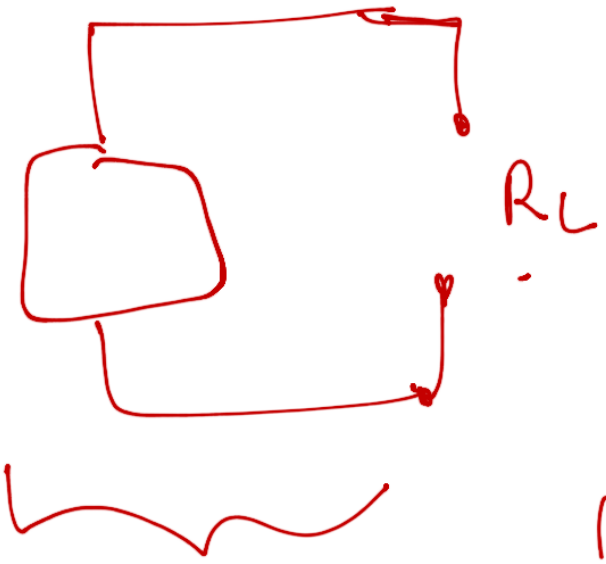
Example 3

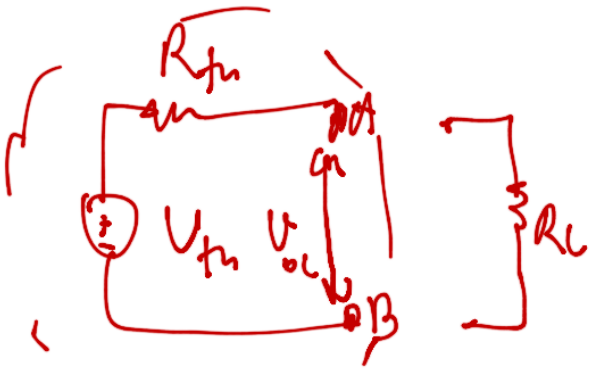
i_{sc}



$$\frac{0-6}{100} - 9\left(\frac{0-6}{100}\right) - I_{sc} = 0$$

$$I_{sc} = 10$$





V_{oc} (open circuit voltage) ; i_{sc} (short circuit)

Superposition

V_1 V_{oc1}

V_2 V_{oc_2}

$V_{oc} = V_{oc1} + V_{oc2}$: Superposition

$R_{th} = R_N$; $R_{th} = \frac{V_{oc}}{i_{sc}}$;