RLC Circuits

22 Jan

(From charc eqn)

$$s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

$$\alpha = \text{Damping coefficient} = \frac{1}{2RC}$$

$$\omega_0^2 = \frac{1}{LC} = \text{Resonant frequency}$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 = -\alpha + \sqrt{\alpha^2-\omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2-\omega_0^2}$$

	$\alpha^2-\omega_0^2$	s_1	s_2	Name
$\alpha > \omega_0$	+	−	−	Over damping
$\alpha = \omega_0$	0	$-\alpha$	$-\alpha$	Critical Damping
$\alpha < \omega_0$	−	$-\alpha + j\omega_d$ $\omega_d = \sqrt{\omega_0^2-\alpha^2}$	$-\alpha - j\omega_d$	Under damped

Overdamping

$$\alpha^2 > \omega_0^2$$

$$\Rightarrow \frac{1}{(2RC)^2} > \frac{1}{LC}$$

$$\Rightarrow \frac{1}{4R^2C^2} > \frac{1}{L\cancel{C}}$$

$$\Rightarrow L > 4R^2C$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Real — Real

$$v = Ae^{st} = Ae^{s_1 t} + Be^{s_2 t}$$

[Natural Response]

$$v_n = Ae^{-t/\tau_1} + Be^{-t/\tau_2}$$

Time constants

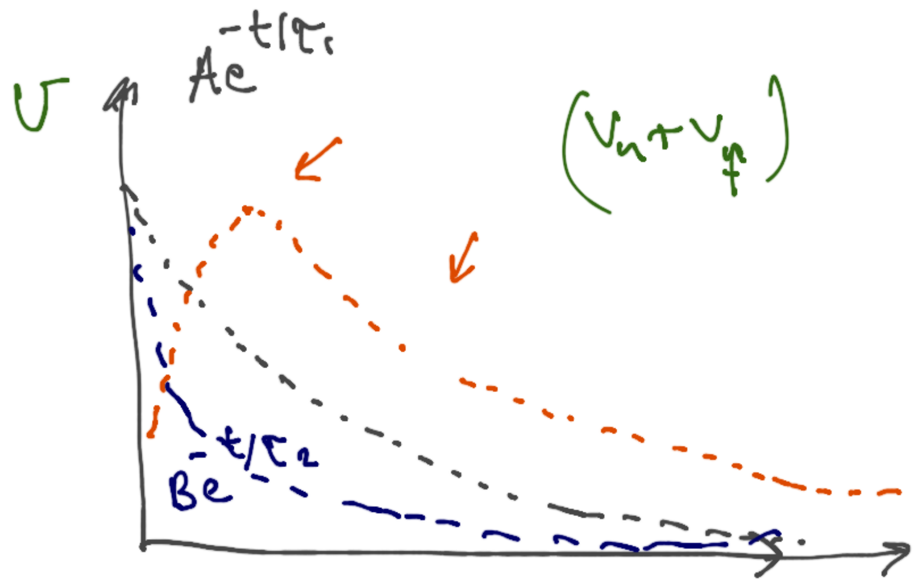
$$\tau_1 = \frac{1}{s_1} \quad \& \quad \tau_2 = \frac{1}{s_2}$$

$$|s_1| < |s_2| \Rightarrow \tau_2 < \tau_1$$

$v = v_f + v_n$

$\downarrow$

v at $t \to \infty$



Critical Damping.

$\alpha^2 = \omega_0^2$

$$\frac{1}{(2RC)^2} = \frac{1}{LC}$$

$$\boxed{L = 4R^2C}$$ Critical Damp-

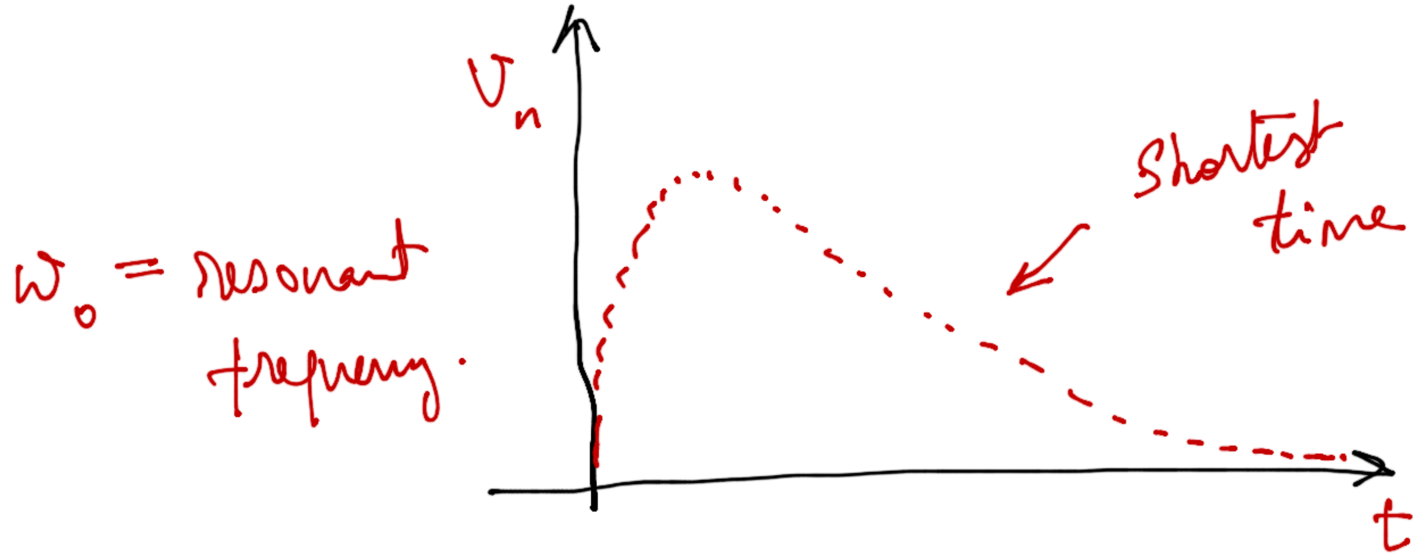
$s_1, s_2 = -\alpha$

Solution

$\hookrightarrow v_n = Ce^{-\alpha t} + Dte^{-\alpha t}$

[Math-sol 2nd ODE with same roots]

$\rightarrow \quad v_n = (C + Dt)e^{-\alpha t}$



$$v = v_f + v_n$$

Under damping.

$$\alpha < \omega_0 \Rightarrow \frac{1}{(2RC)^2} < \frac{1}{LC}$$

$$\Rightarrow \boxed{L < 4R^2C}$$

$S_1 = -\alpha + j\omega_d$ $\qquad$ $S_2 = -\alpha - j\omega_d$

$$\omega_d^2 = \omega_0^2 - \alpha^2$$

$S_1, S_2 \rightarrow$ Complex numbers

$j \rightarrow$ Imaginary $\sqrt{-1}$

$$V_n = \underline{A_1 e^{-\alpha t}}\, \underbrace{e^{j\omega_d t}}_{(\cos\omega_d t + j\sin\omega_d t)} + A_2 e^{-\alpha t} \underbrace{e^{-j\omega_d t}}_{(\cos\omega_d t - j\sin\omega_d t)}$$

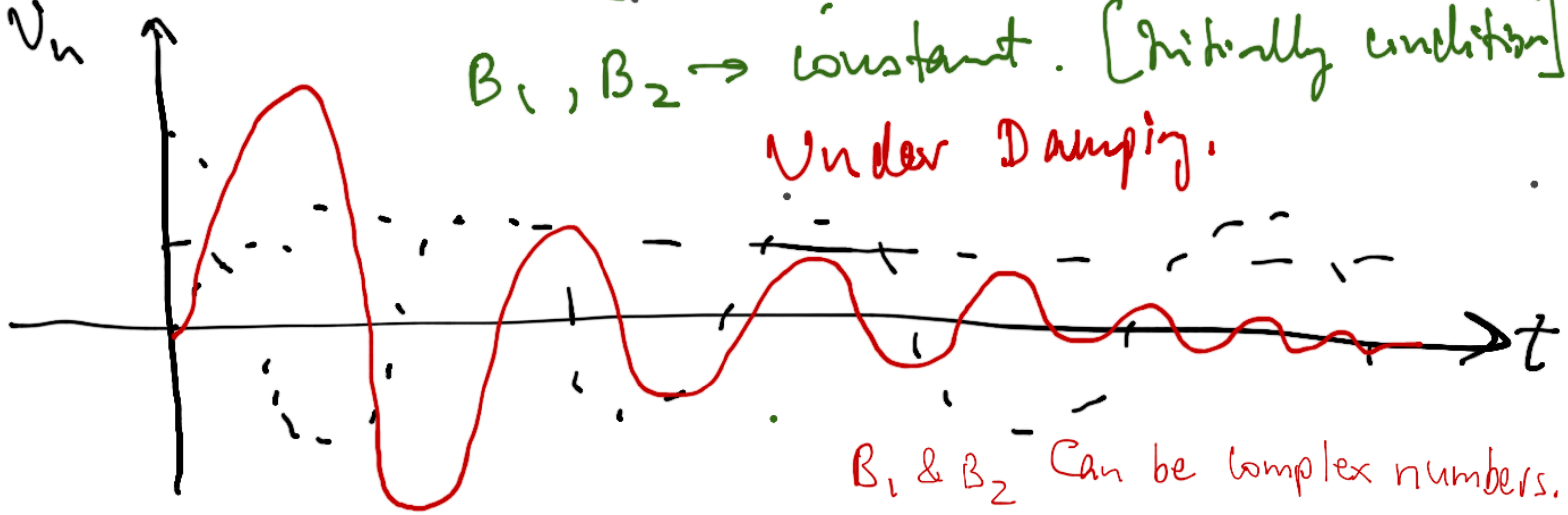
Natural
Complete Solution.

$$V_n = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$

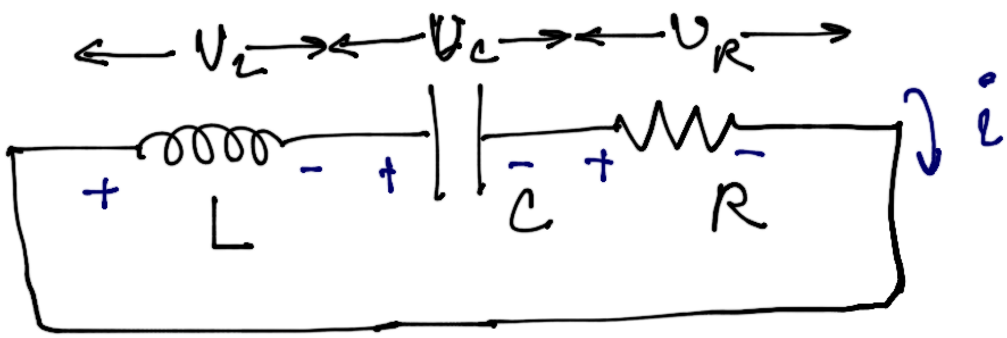
$B_1 = A_1 + A_2$ $\qquad$ $B_2 = (A_1 - A_2)j$

$B_1, B_2 \rightarrow$ constant. [Initially condition]

Under Damping.



B_1 & B_2 Can be Complex numbers.



Natural response of SERIES R-L-C

KVL $\quad v_L + v_C + v_R = 0$

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt + iR = 0$$

$$\underbrace{v_L = L\frac{di}{dt}} \qquad \underbrace{i_C = C\frac{dv_C}{dt}}$$

Taking derivative wrt 't'

$$\Rightarrow \quad L\frac{d^2 i}{dt} + \frac{1}{C}i + R\frac{di}{dt} = 0$$

2^{nd} ODE in 'i'

Characteristic Eqn

$$\frac{d^2 i}{dt} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0$$

As we know sol^n $i = Ae^{st}$

Substitute back in Charac. Eq^n.

$$\underbrace{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}_{\text{quad Eq}^n}Ae^{st} = 0$$

<u>Sol</u>: $s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$

α = Damping coefficient = $\boxed{\frac{R}{2L}}$

w_0 = Charac. frequency = $\frac{1}{\sqrt{LC}}$

$$s = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

3 Solution

	$\alpha^2 - \omega_0^2$	S_1	S_2	
$\alpha > \omega_0$	+	Real	Real	Overdamping
$\alpha = \omega_0$	0	$-\alpha$	$-\alpha$	Critically damped.
$\alpha < \omega_0$	Imaginary	$-\alpha + j\omega_d$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$-\alpha - j\omega_d$	Under damped.

Overdamped case

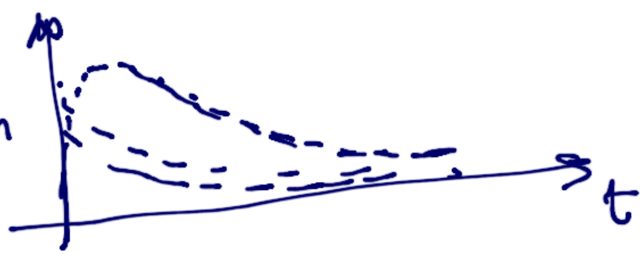
Solution:

$$i_n = A_1 e^{s_1 t} + B e^{s_2 t}$$

$$\alpha^2 > \omega_0^2$$

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

$$\boxed{L < \frac{R^2 C}{4}}$$



Critically Case

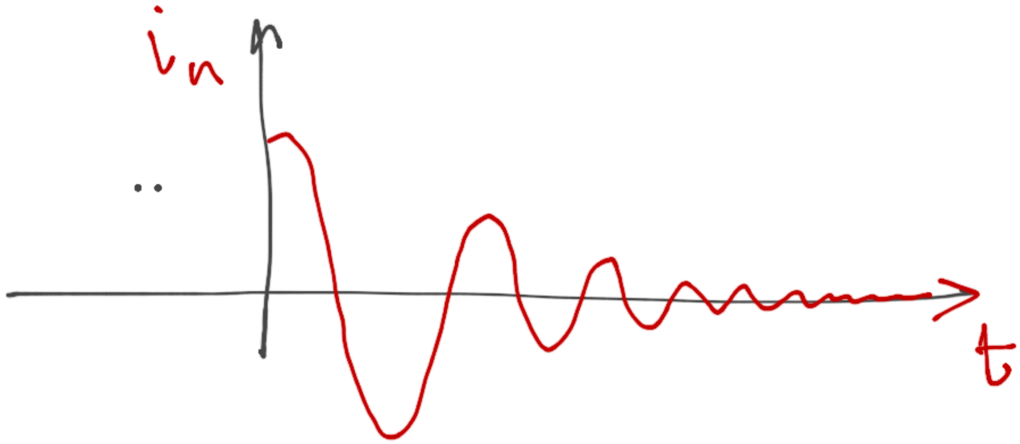
$$i_n = (A + Bt)\, e^{-\alpha t}$$

fastest way to steady state

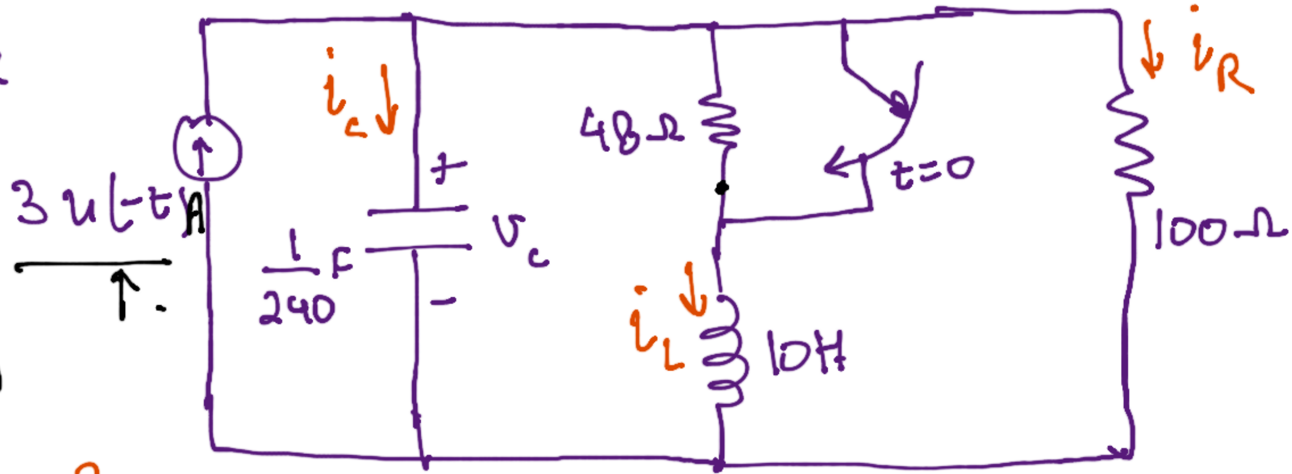
Under damped

$\alpha < \omega_0$

$$i_n = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$



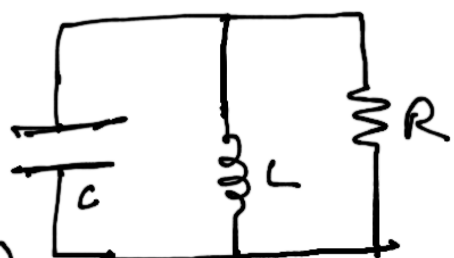
Ex 9.6



3A	t<0
0	t>0

$i_L(t) = ?$

t>0



CKt A

$C = \frac{1}{240}$ $R = 100$ $L = 10$

✓ $\alpha = \frac{1}{2RC} = 1.2$

✓ $\omega_0 = \frac{1}{\sqrt{LC}} \approx 5$

$\alpha < \omega_0$

$\Downarrow$

Under damped.

$$i_L(t) = e^{-\alpha t}\left(A_1 \cos \omega_d t + A_2 \sin \omega_d t\right)$$

↳ form of solution remains same for the complete circuit.

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

5

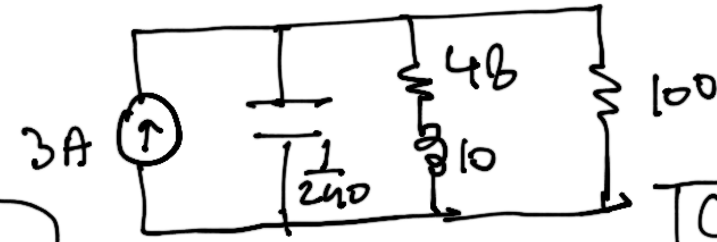
A_1 ? A_2 ?

$i_L(t=0) =$

$\frac{di_L}{dt}(t=0) =$

Initial conditions

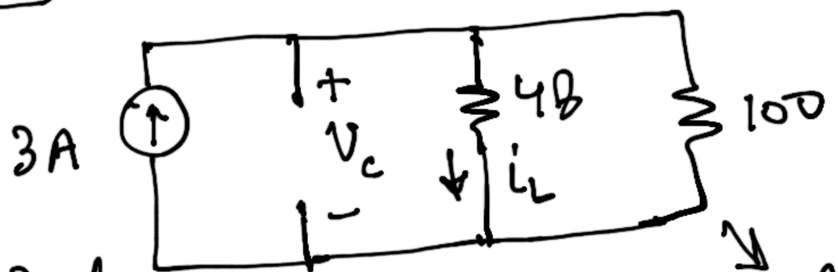
$\boxed{t < 0}$



$\boxed{\text{Ckt Z}}$

$\boxed{\begin{array}{l}\text{ckt Z at } t \to \infty \\ = \text{ckt A at } t = 0\end{array}}$

$t = -\infty$

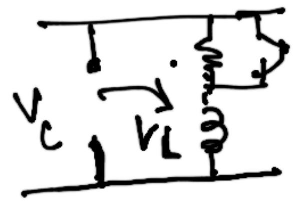


S.S circuit

A

$$i_L\begin{pmatrix}t=\infty\\ Z\end{pmatrix} = \frac{100}{148} * 3A = 2\ A$$

$$V_c\begin{pmatrix}t=\infty\\ Z\end{pmatrix} = 48 * 2 = 97\ V$$



For $\boxed{\text{ckt A}} \rightarrow$

$$i_L\left(t \underset{A}{=} 0\right) = i_L\begin{pmatrix}t=\infty\\ Z\end{pmatrix} = 2A$$

$$V_c\begin{pmatrix}t=0\\ A\end{pmatrix} = V_c\begin{pmatrix}t=\infty\\ Z\end{pmatrix} = \underline{97V}$$

$$i_L = e^{-\alpha t}\left(A_1 \cos w_d t + A_2 \sin w_d t\right) \quad \text{— (1)}$$

$t=0$ $$2 = e^{-\alpha 0}\left(A_1 + 0\right) \Rightarrow A_1 = 2 \quad \text{— (2)}$$

$$\frac{di_L}{dt} = e^{-\alpha t}\left(-A w_d \sin w_d t + A w_d \cos w_d t\right) - \alpha e^{-\alpha t}\left(A_1 \cos w_d t + A_2 \sin w_d t\right) \quad \text{— (3)}$$

$V_C(t=0_A) = V_L(t=0) \quad \because$ 4Ω Ω was shorted

$\therefore$ ③ $\quad L\dfrac{di}{dt} = L\left[e^{-\alpha t}(\,\cdot + \cdot\,) - \alpha e^{-\alpha t}(\,\cdot + \cdot\,)\right]$

$L\dfrac{di}{dt}(t=0) = L(A_2 w_d - \alpha A_1) = 97$ —— ③'

Solve ③' to find A_2

$$i_L = e^{-1.2t}\left(2.027\cos(4.7t) + 2.56\sin(4.75t)\right)$$