Lecture – 15

Saturday, 19 February 2022 8:18 AM

* Laplace transform (LT)

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$ .... Analysis equation

$\{e^{st}\}$ – basis signals

s – complex frequency variable $(s = \sigma + j\omega)$

$X(s)$ – complex valued function of s

* LTI systems & LT

eigenfunction

$$e^{st} \longrightarrow \underset{\text{LTI}}{\boxed{h(t)}} \longrightarrow H(s)\, e^{st}$$

$$h(t) \overset{LT}{\longleftrightarrow} H(s)$$

$h(t)$ – impulse response of LTI system

$H(s)$ – system transfer function

system descriptors *

* Examples :

1. $x(t) = e^{-t}\, u(t)$ right-sided signal

$$X(s) = \frac{1}{s+1} \quad \& \quad Re(s) > -1$$

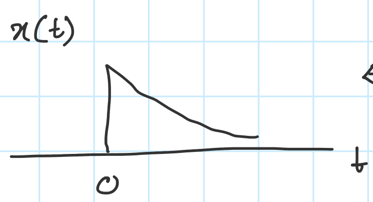
ROC

* Laplace transform always has two parts :

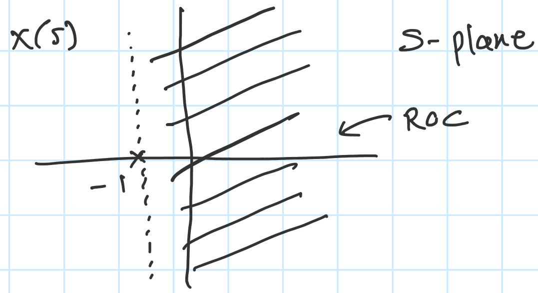
(a) expression (b) ROC } both must be always specified.

ⓐ expression ⓑ ROC } both must be always specified.

* note: $x(t)$ is right-sided signal & ROC is also right-sided in the s-plane.

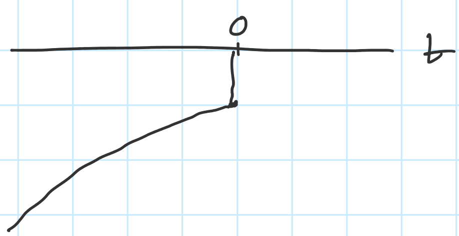


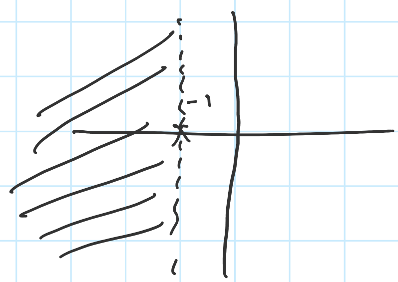
$\longleftrightarrow$



② $x(t) = -e^{-t}\,u(-t)$ left-sided signal

$$X(s) = \frac{1}{s+1} \quad \& \quad Re(s) < -1$$





③

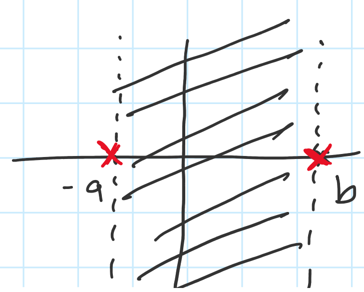
two-sided signal

$a, b \in \mathbb{R}^+$

$$X(s) = \frac{1}{b-s} + \frac{1}{s+a} \qquad -a < Re(s) < b$$

ⓐ expression

$$\frac{(a+b)}{(s+a)(b-s)}$$



ⓑ ROC

$$\hookrightarrow \frac{(a+b)}{(s+a)(b-s)}$$

* Rational form of Laplace transform

$\hookrightarrow$ $X(s) = \dfrac{A(s)}{B(s)}$ polynomials in s variable

roots of the polynomials $A(s)$ & $B(s)$ are important

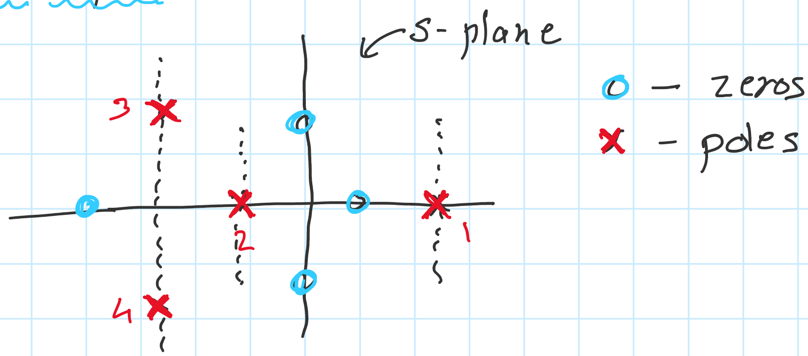
roots of $A(s)$ i.e. $A(s) = 0$ $\rightarrow$ zeros

roots of $B(s)$ i.e. $B(s) = 0$ $\rightarrow$ poles

poles are important to decide ROC

* pole-zero plot

Ex.



ROC cannot have poles within them *

Q. what are possible ROC in above example?

Q. given pole-zero plot, can we find $x(t)$? we also have to specify the ROC.

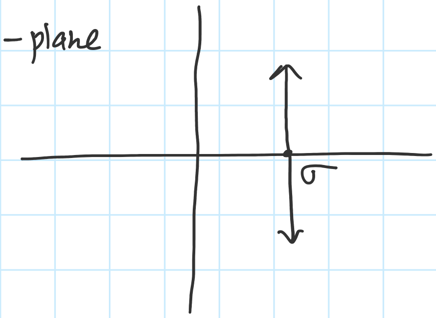
* Inverse Laplace transform :

i.e. given $X(s)$ & ROC, find the signal $x(t)$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s)\, e^{st}\, ds \qquad \cdots \text{synthesis}$$

contour integration

s-plane



σ should be in ROC

* In practice to compute $x(t)$ given $X(s)$ & ROC, we will be using partial fractions & properties of LT & standard example signals

* Properties of LT

① Linearity :

$$x_1(t) \overset{LT}{\longleftrightarrow} X_1(s) \qquad R_1$$

$$x_2(t) \overset{LT}{\longleftrightarrow} X_2(s) \qquad R_2$$

$$\alpha x_1(t) + \beta x_2(t) \overset{LT}{\longleftrightarrow} \alpha X_1(s) + \beta X_2(s) \quad \& \text{ ROC: } R_1 \cap R_2$$

② Time - shift :

$$x(t) \longleftrightarrow X(s) \quad \& \quad R_x$$

$$x(t - t_0) \longleftrightarrow e^{-st_0} X(s) \quad \& \text{ ROC: } R_x$$

③ Freq - shift : $\quad x(t) \longleftrightarrow X(s) \quad R_x$

$$e^{s_0 t} x(t) \longleftrightarrow X(s-s_0) \quad \& \quad \text{ROC: } R_x + Re(s_0)$$

Ex. ① $x(t) = \delta(t)$

$$X(s) = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt = 1 \quad \forall s$$

$$X(s) = 1 \quad \& \quad \text{ROC: complete s-plane}$$

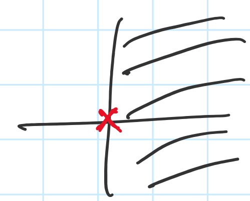
② $x(t) = \delta(t - t_0)$

$$X(s) = e^{-st_0} \quad \& \quad \text{ROC: complete s-plane}$$

③ $x(t) = u(t)$

$$X(s) = \int_0^{\infty} e^{-st} dt = \left. \frac{e^{-st}}{-s} \right|_0^{\infty}$$

$$X(s) = \frac{1}{s} \quad \& \quad Re(s) > 0$$



④ $X(s) = \dfrac{1}{s-a} \quad \& \quad Re(s) > a \quad , \quad a \in \mathbb{R}$

$$x(t) = e^{at} u(t)$$

Standard LT pairs:

$$e^{-at}\,u(t) \longleftrightarrow \frac{1}{s+a} \quad \& \quad Re(s) > -a$$

$$-\,e^{-at}\,u(-t) \longleftrightarrow \frac{1}{s+a} \quad \& \quad Re(s) < -a$$