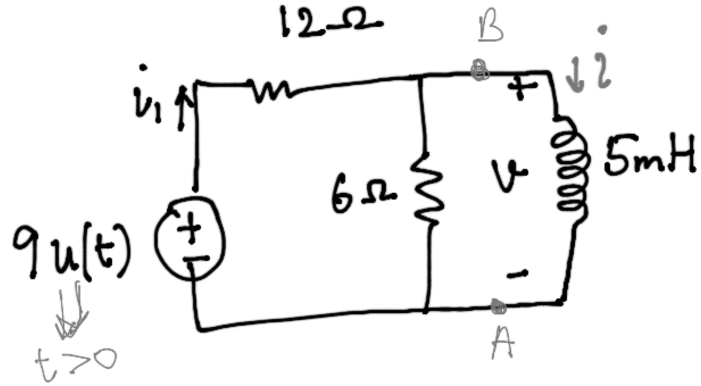
RC & RL circuit examples

Example 1



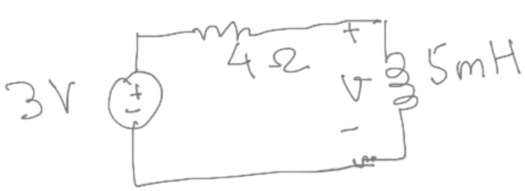
$i = ?$

$i_1 = ?$

$v = ?$

Solution Inductor can be seen as load and Equivalent thevenin values Calculated

||'



From circuit R_{th} is obvious $R_{th} = 6 || 12$

$= \frac{72}{18} = 4\ \Omega$

$V_{th} = \frac{6}{18} * 9 = 3V$

$v = Ae^{-Rt/L}$ where $\tau = \frac{L}{R} = \frac{5m}{4} = 1.25ms$

$t=0$ $v = 3V$ (all drop is across inductor at $i=0$)

$\therefore$ $\boxed{v = 3e^{-\frac{t}{1.25\times10^{-3}}}}$ $\Rightarrow$ $i(t) - i(0) = \frac{1}{5m}\int_0^t v\,dt =$

$$i(t) - i(0) = \frac{1}{5m}\int_0^t 3\, e^{-\frac{t}{1.25m}}\, dt = \frac{1}{\cancel{5m}} * 3\left(-\cancel{1.25m}^{\,0.25}\right)\left(e^{-\frac{t}{1.25m}} - 1\right)$$

$$\| \\ i(t)$$

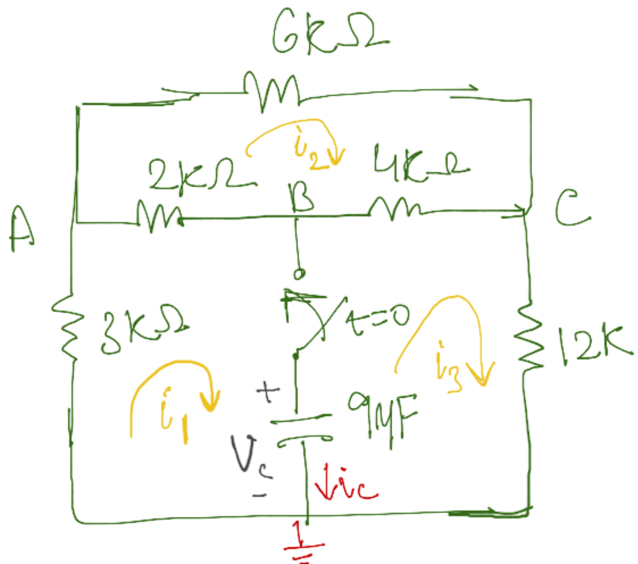
[as $i(0)=0$]

$$\Rightarrow \boxed{i(t) = 0.75 \times \left(1 - e^{-t/1.25m}\right)}$$

$$i_1 = \frac{9V - v(t)}{12\,\Omega} = \frac{9 - 0.75\left(1 - e^{-\frac{t}{1.25m}}\right)}{12}$$

$$= \frac{3}{4} - \frac{3}{4 \times 4} e^{-t/1.25ms} = \boxed{\frac{3}{4} - \frac{1}{16} e^{-\frac{t}{1.25ms}} = i_1(t)}$$

Example 2



$v_C(t=0) = 17V$

Find v_A, v_B, v_C, all currents.

$$R_{eq} = R_{th} = \frac{v_B}{-i_c} = \frac{v_B}{(i_3 - i_1)}$$

Solution: We write KCL or KVL for the circuit for $t>0$

KVL

i_1: $-3K\, i_1 - 2K(i_1 - i_2) - v_C = 0$ ——— (1)

$\Rightarrow \quad -5K\, i_1 + 2K\, i_2 - v_c = 0$

i_2: $-6K\, i_2 - 4K(i_2 - i_3) - 2K(i_2 - i_1) = 0$

$\Rightarrow \quad 2K\, i_1 - 12K i_2 + 4K\, i_3 = 0 \Rightarrow i_1 - 6 i_2 + 2 i_3 = 0$ ——— (2)

i_3: $v_C - 4K(i_3 - i_2) - 12K\, i_3 = 0$ ——— (3)

$\Rightarrow \quad 4K i_2 - 16K\, i_3 + v_c = 0$

check sign

Solve (1), (2) & (3) for i_1, i_2, i_3

$\boxed{i_1 = -\frac{21}{118K} \cdot v_C}$ (4); $\boxed{i_2 = \frac{13}{236K} v_c}$ (5); $\boxed{i_3 = \frac{9}{118K} v_c}$ (6)

Eq. resistance across cap $= \frac{-v_c}{(i_1 - i_3)} = \frac{-v_c}{(-21-9)v_c/118K} = \boxed{+\frac{118K}{30} = R_{th}}$

$$\tau = R_{eq} * C = \frac{118K}{21} * 9\mu F = \frac{118*9^3}{\cancel{21}\,7} \text{ m seconds}$$

$R_{eq} \parallel R_{th}$

$$\tau = \frac{354}{7} \text{ m seconds}$$

General solution

$$\therefore \quad v_c(t) = A \cdot e^{-t/\tau}$$

$$t=0 \quad v_c = 17 \quad \therefore \quad \boxed{v_c = 17\, e^{-t/\tau}}$$

$$\Rightarrow \quad i_c = C\frac{dv_c}{dt} = \boxed{9\mu * \frac{17K}{(354/7)} e^{-t/\tau} = i_c}$$

$$= 3.025 \times 10^{-3} e^{-t/\tau}$$

Using (4): $i_1 = \frac{-21}{118K} * 17 e^{-t/\tau}$ Using (5): $i_2 = \frac{13}{236K} * 17 e^{-t/\tau}$ Value of all currents.

Using (6) $i_3 = \frac{9}{118K} * 17 e^{-t/\tau}$

$$v_A = v_B^{=v_c} + 2K*(i_1 - i_2) = 17 e^{-t/\tau} + 2K*(i_1 - i_2) =$$

$$= 17 e^{-t/\tau} + 4K * (i_1 - i_2) =$$

$$v_C = v_B + 4K(i_2 - i_3) =$$

Extra credits to one who finds calculation mistake & updates value of v_A & v_C.

Example 2+

In the previous problems find the steady state values.

Solution(1) : For steady state values $t \to \infty$ in the obtained solutions.

Solution(2) : Take $L \to$ short (zero resistance wire = zero voltage drop).

and $C \to$ open (zero current flow).

Repeat the calculations.