Lecture - 12

Saturday, 12 February 2022 8:27 AM

* Linear & Time - invariant (LTI) systems

here H is an LTI system

① $\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$

② $\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow h(t-\tau)$

③ $x(\tau)\,\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow x(\tau)\,h(t-\tau)$

④ $x(t) \longrightarrow \boxed{H} \longrightarrow x(t) * h(t) = y(t)$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

* LTI systems are fully desbribed by their impulse response

H

$$\delta(t) \xrightarrow{H} h(t)$$

$$\Rightarrow \quad x(t) \xrightarrow{H} x(t) * h(t)$$

* Convolution operator & convolution integral

* Properties of convolution operator

ⓐ Commutative : $x(t) * h(t) = h(t) * x(t)$

ⓑ Associative : $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$

ⓒ Distributive : $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

* Impulse response of some systems

Ex. ① $y(t) = x(t+1) + x(t-1)$ ⋯ LTI

$\delta(t) \xrightarrow{H} \delta(t+1) + \delta(t-1) = h(t)$

② $y(t) = t\, x(t)$ ⋯ not LTI

③ $y(t) = x^2(t)$ ⋯ not LTI

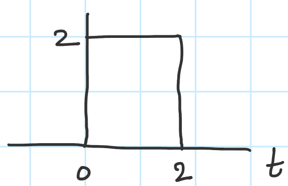
④ $y(t) = \mathcal{A}\{x(t)\}$

$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ ⋯ LTI

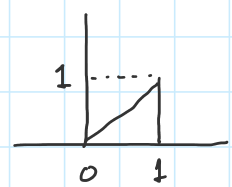
$\delta(t) \xrightarrow{H} u(t) = h(t)$

* Convolution integral example

① consider $x(t) =$



& $h(t) =$



Find $y(t) = x(t) * h(t)$

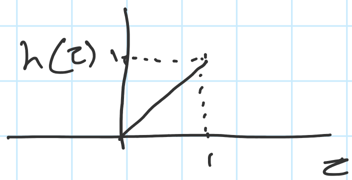
→ $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Steps of convolution : ① flip i.e. $h(-\tau)$

② shift i.e. $h(t-\tau)$

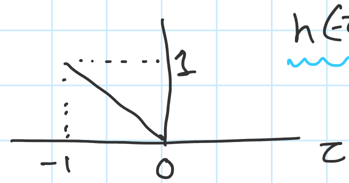
③ product i.e. $x(\tau)h(t-\tau)$

④ integrate i.e. $\int_{-\infty}^{\infty}$



$$h(\tau) = \begin{cases} \tau & 0 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$h(-\tau)$

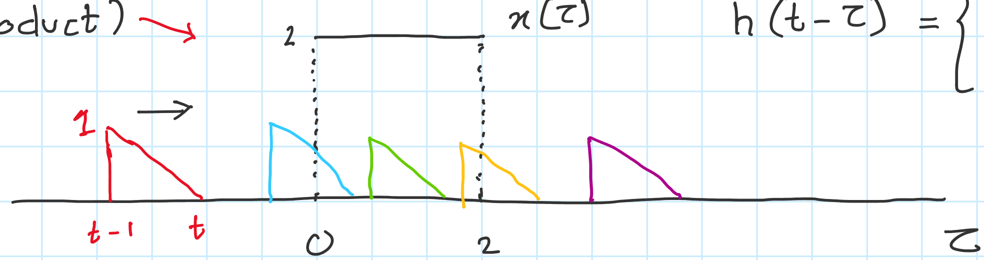


$h(-\tau)$ ...(flip)

...(shift)

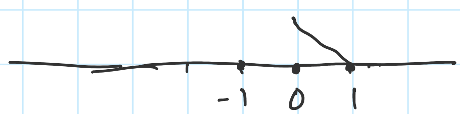
$$h(t-\tau) = \begin{cases} t-\tau, & t-1 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

(product)



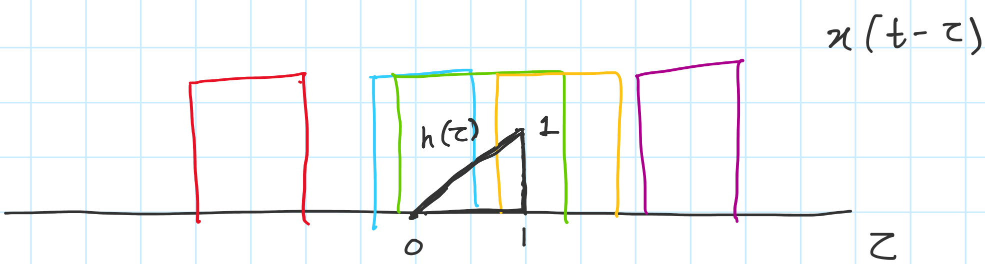
$h(t-\tau)$ at $t = +1$

i.e. $h(1-\tau)$



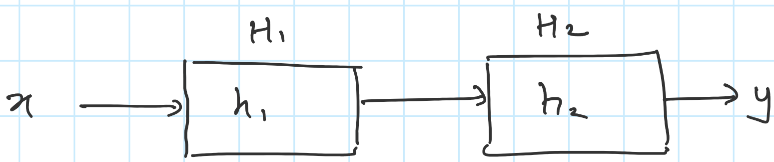
$$x(t) * h(t) = \text{(integrate)} \rightarrow \begin{cases} ① = 0 & t < 0 \\ ② = \int_0^t 2(t-\tau)\,d\tau & 0 \le t \le 1 \\ ③ = \int_{t-1}^t 2(t-\tau)\,d\tau & 1 \le t \le 2 \\ ④ = \int_{t-1}^2 2(t-\tau)\,d\tau & 2 \le t \le 3 \\ ⑤ = 0 & t > \end{cases}$$

* Alternately $\quad y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$



* For an LTI system: impulse response is another way to describe the system

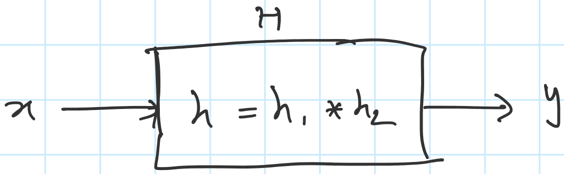
* combination of systems



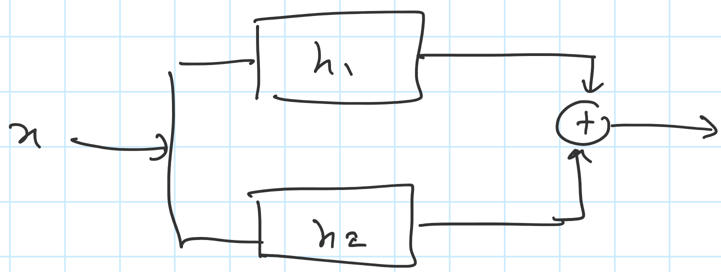
1) Cascade connection

$$y = [x * h_1] * h_2 = x * [h_1 * h_2] = x * h$$

using associativity



2) Parallel connection



$$y = x * h_1 + x * h_2$$

$$= x * [h_1 + h_2]$$

using distributivity