Lecture -16

Tuesday, 22 February 2022 8:12 AM

* Laplace transform :

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

* Inverse Laplace transform using a combination of :

- standard examples
- LT properties
- partial fractions

* we are interested in signals with rational form of $X(s)$

i.e. $X(s) = \frac{A(s)}{B(s)}$ i.e ratio of polynomials

poles & zeros in the s-plane

poles decide the possible region of convergence (ROC)

ROC cannot contain any poles

* Properties of Laplace transform :

given $x(t) \xleftrightarrow{LT} X(s)$ with ROC: R_x

1. Linearity i.e. $\alpha x_1(t) + \beta x_2(t)$

2. Time - shift i.e. $x(t-t_0)$

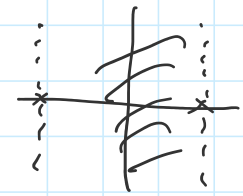
3. Frequency -shift i.e. $X(s-s_0)$

(4) Time-reversal   i.e. $x(-t)$

$$x(-t) \xleftrightarrow{LT} X(-s) \qquad \text{ROC: } R_{-x}$$

Ex. ROC: $Re(s) < 6$ i.e. R_x $s = \sigma + j\omega$

$\sigma < 6$



R_{-x} : $s = -\sigma - j\omega$

$\Rightarrow -\sigma > -6$

R_{-x} : $Re(s) > -6$

(5) Time-scaling   i.e. $x(at)$ $a \in \mathbb{R}$

$a > 0 \Rightarrow x(at) \xleftrightarrow{LT} \frac{1}{a} X(s/a)$

$a < 0 \Rightarrow x(at) \xleftrightarrow{LT} -\frac{1}{a} X\left(\frac{s}{a}\right)$

$\Rightarrow x(at) \xleftrightarrow{LT} \frac{1}{|a|} X\left(\frac{s}{a}\right)$ $a \in \mathbb{R}$

ROC: $a R_x$

(6) Derivative in time   i.e. $\frac{dx}{dt}$

$$\frac{dx}{dt} \xleftrightarrow{LT} s X(s) \qquad \text{ROC: at least } R_x$$

(7) Derivative in frequency domain

$$-t\, x(t) \xleftrightarrow{LT} \frac{dX}{ds} \qquad \text{ROC: } R_x$$

(8) Integration   i.e. $\int_{-\infty}^{t} x(\tau)\, d\tau = y(t)$

$$x(t) \longrightarrow \boxed{\mathcal{A}} \longrightarrow y(t)$$

$$x(t) \downarrow X(s) \qquad y(t) \downarrow Y(s) = \frac{1}{s} X(s)$$

impulse response of Integrator system is unit step signal.
i.e. $h(t) = u(t)$

ROC: $R_x \cap [Re(s) > 0]$

⑨ Convolution

LTI

Time : $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$

LT $\downarrow$

Frequency : $X(s) \qquad H(s) \qquad Y(s) = X(s)\,H(s)$

ROC: at least $R_x \cap R_h$

$$Y(s) = \int_{-\infty}^{\infty} y(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right] e^{-st}\, dt$$

$$= X(s)\, H(s)$$

* Differential equations :

linear constant coefficient differential equation * (LCCDE)

Ex.

$$\frac{dy}{dt} + a\,y(t) = x(t) \quad \cdots \text{(system)}$$

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$$X(s) \qquad\qquad Y(s)$$

$\downarrow$ LT

$$s\,Y(s) + a\,Y(s) = X(s)$$

$$Y(s)\,[a+s] = X(s)$$

$$Y(s) = \frac{X(s)}{s+a}$$

convolution property

$$\frac{Y(s)}{X(s)} = \frac{1}{s+a} = H(s)$$

system function
transfer function

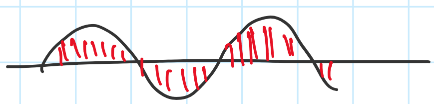
any LCCDE in Laplace domain can be expressed as ratio of polynomials.

☆ Discrete-time signals & systems (review for mini-project)

time is sampled $\qquad x(t) \longrightarrow x(nT_s) = x[n]$

T_s - sampling interval

$F_s = \frac{1}{T_s}$ - sampling frequency.



$x(t) = \sin(\omega_0 t)$

$x[n] = \sin(\omega_0 n T_s)$

N - period of DT signal

NT_s - period of CT

* periodic DT signals.

$x[n] = x[n+N] \quad \forall n \qquad$ period $= N$

Fourier series for DT periodic signals:

$\sum^{\infty} \quad e^{jk\omega_0 t}$

Fourier series of DT periodic signals

recall CT : $x(t) = \sum_{-\infty}^{\infty} d_k e^{jk\omega_0 t}$ ... synthesis

$\omega_0 = \frac{2\pi}{T}$

$d_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$ .... analysis

For DT: only finite basis signals with period N

Basis : $\{ e^{jk\omega_0 n} \}$, $\omega_0 = \frac{2\pi}{N}$ N is integer

$k = 0, 1, 2, \dots N-1$

DT FS : synthesis $\rightarrow$ $x[n] = \sum_{k=0}^{N-1} d_k e^{jk\omega_0 n}$

DT FS : analysis $\rightarrow$ $d_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\omega_0 n}$

discrete - time system (LTI)

$x[n] \longrightarrow \boxed{h[n]}^{\text{LTI}} \longrightarrow y[n] = x[n] * h[n]$

Convolution sum : $y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$