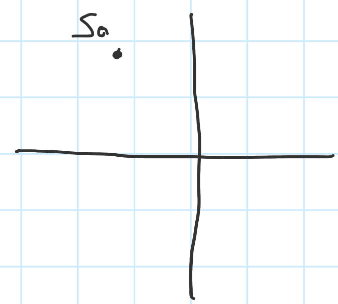
Lecture - 19

Thursday, 3 March 2022 8:34 AM

* Causal & Stable systems

* Geometric interpretation

given a rational system function :



$$H(s) = \frac{\prod_{i=1}^{M} (s - z_i)}{\prod_{i=1}^{N} (s - p_i)} \quad \& \text{ RoC}$$

$N > M$

$$|H(s)|\Big|_{s=s_0} = \frac{\prod_{i=1}^{M} |s_0 - z_i|}{\prod_{i=1}^{N} |s_0 - p_i|}$$

$$\measuredangle H(s)\Big|_{s=s_0} = \sum_{i=1}^{M} \measuredangle (s_0 - z_i) - \sum_{i=1}^{N} \measuredangle (s_0 - p_i)$$

* Eigenfunctions of LTI system

$$X(s) \longrightarrow \underset{h(t)}{\boxed{H(s)}} \longrightarrow Y(s) = X(s)H(s)$$

$$e^{s_0 t} \longrightarrow H(s_0)\, e^{s_0 t}$$

Ex. $H(s) = \dfrac{1}{s+1}$, $Re(s) > -1$

Q. Find system output when input signal is :

(a) $x(t) = e^{j5t}$ (b) $x(t) = e^{(2+j5)t}$

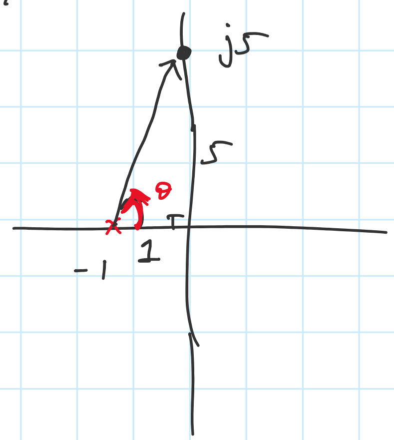
(c) $x(t) = e^{-j5t}$ (d) $x(t) = \cos(5t)$

use geometric interpretation method

(a) $e^{j5t} \longrightarrow\ ?\ H(j5)\, e^{j5t}.$

$H(j5)$ evaluate geometrically.

$$H(s) = \frac{1}{s+1}$$

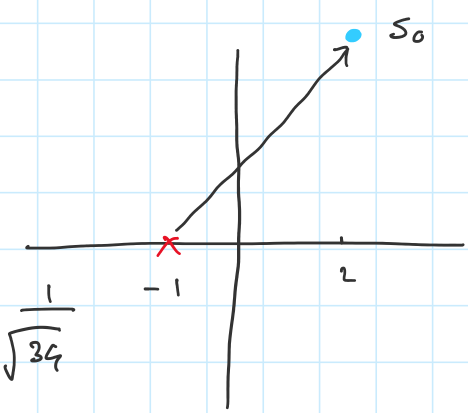


$$\left| H(s)\Big|_{s=j5} \right| = \frac{1}{\sqrt{1+25}} = \frac{1}{\sqrt{26}}$$

$$\measuredangle H(s)\Big|_{s=j5} = -\tan^{-1}(5) = -\theta$$

$$\text{output } y(t) = \frac{1}{\sqrt{26}}\, e^{-j\tan^{-1}(5)}\, e^{j5t}$$

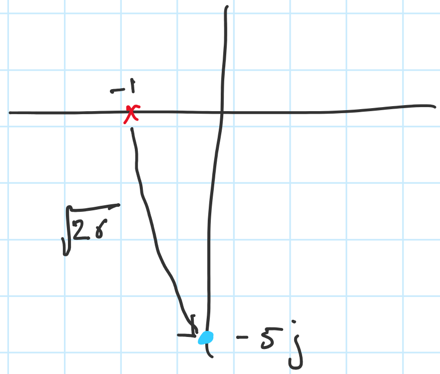
(b) $x(t) = e^{(2+j5)t}$



$$\left| H(s)\Big|_{s=2+j5} \right| = \frac{1}{\sqrt{9+25}} = \frac{1}{\sqrt{34}}$$

$$\measuredangle H(s)\Big|_{s=2+j5} = -\tan^{-1}\left(5/3\right)$$

(c) $s_0 = -j5$



$$|H(s)|\Big|_{s=-5j} = \frac{1}{\sqrt{26}}$$

$$\measuredangle H(s)\Big|_{s=-5j} = -\tan^{-1}(-5) = \tan^{-1}(5)$$

(d) $x(t) = \cos(5t) = \frac{1}{2}\left[e^{j5t} + e^{-j5t}\right].$

$$y(t) = \frac{1}{2}\left[\frac{1}{\sqrt{26}} e^{-j\theta} e^{j5t} + \frac{1}{\sqrt{26}} e^{j\theta} e^{-j5t}\right]$$

$$y(t) = \frac{1}{2\sqrt{26}}\left[e^{j(5t-\theta)} + e^{-j(5t-\theta)}\right]$$

$$y(t) = \frac{1}{\sqrt{26}}\cos(5t-\theta) \quad , \quad \theta = \tan^{-1}(5)$$

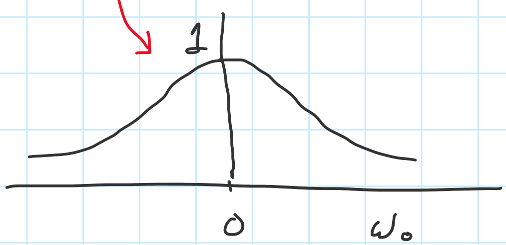
$$\cos(5t) \longrightarrow \frac{1}{\sqrt{26}}\cos(5t-\theta)$$

$$\cos(\omega_0 t) \longrightarrow \frac{1}{\sqrt{1+\omega_0^2}}\cos(\omega_0 t-\theta)$$

$\theta = \tan^{-1}(\omega_0)$

Q. as ω_0 is changed. what happens to output signal magnitude?

$$|H(j\omega_0)| = \frac{1}{\sqrt{1+\omega_0^2}}$$



low-pass filter

$$e^{j\omega_0 t} \longrightarrow H(j\omega_0)\, e^{j\omega_0 t}$$

$$\underbrace{e^{-j\omega_0 t}}_{\cos(\omega_0 t)} \longrightarrow H(-j\omega_0)\, e^{-j\omega_0 t}$$

$$H(j\omega_0) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega_0 t}\, dt$$

$$H(-j\omega_0) = \int_{-\infty}^{\infty} h(t)\, e^{+j\omega_0 t}\, dt$$

$h(t)$ is a real impulse response

$$\therefore H(-j\omega_0)^* = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega_0 t}\, dt = H(j\omega_0)$$

$$|H(j\omega_0)| = |H(-j\omega_0)| \quad \& \quad \measuredangle H(j\omega_0) = -\measuredangle H(-j\omega_0)$$

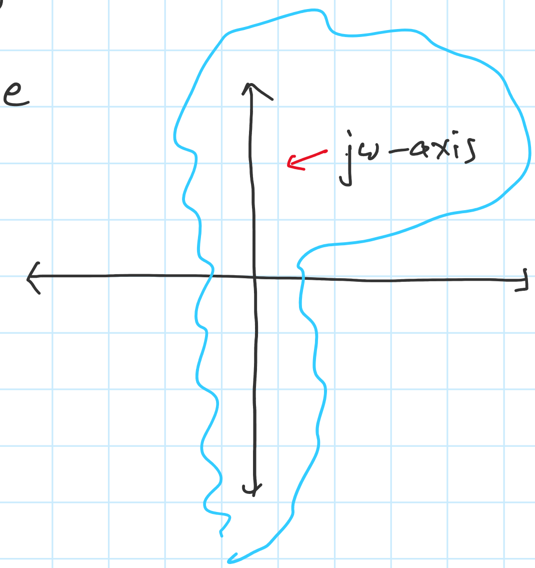
For any LTI system,

$$\cos(\omega_0 t) \longrightarrow |H(j\omega_0)| \cos(\omega_0 t - \theta)$$

$$\theta = \measuredangle H(j\omega_0)$$

$|H(j\omega_0)|$ — magnitude response

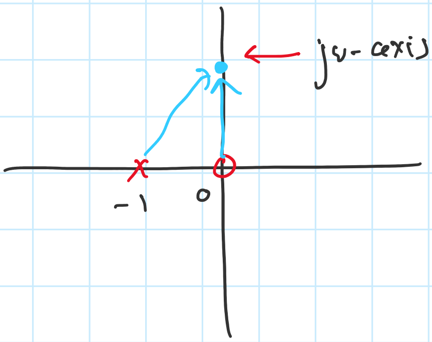
$\measuredangle H(j\omega_0)$ — phase response

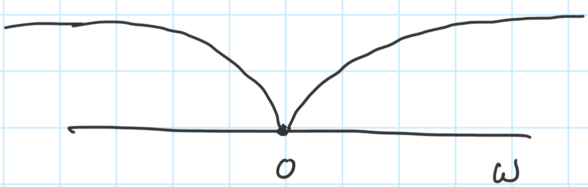


$e^{j\omega_0 t}$ & $e^{-j\omega_0 t}$

$s = j\omega_0$ & $s = -j\omega_0$

$$\frac{s}{s+1}$$





high-pass filter