Lecture - 14

Thursday, 17 February 2022 8:36 AM

★ System properties : ① Causality - present output does not depend on future inputs

② Stability - bounded input produces bounded o/p

★ Laplace transform :

Fourier series for periodic signals - representation in terms of sin() & cos() or $e^{j\omega t}$ harmonics

FS coefficients : $a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt$

measure of similarity between the given signal $x(t)$ & basis signal $\cos(k\omega_0 t)$

Laplace transform :

Basis $\rightarrow$ $\{e^{st}\}$ s - complex number

* $s = \sigma + j\omega$ complex number plane

$x(t) \overset{LT}{\longleftrightarrow} X(s)$

s - complex frequency

Analysis * $X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$ .... definition

$e^{-st} = e^{-(\sigma+j\omega)t} = e^{-\sigma t}\, e^{-j\omega t}$ ... plot these

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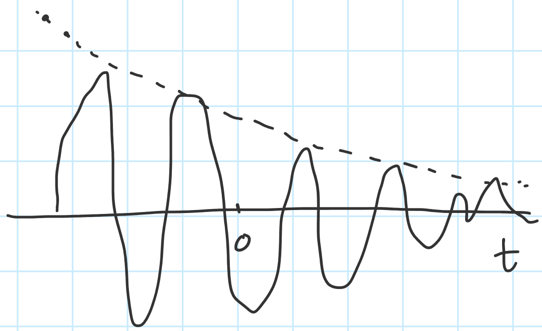
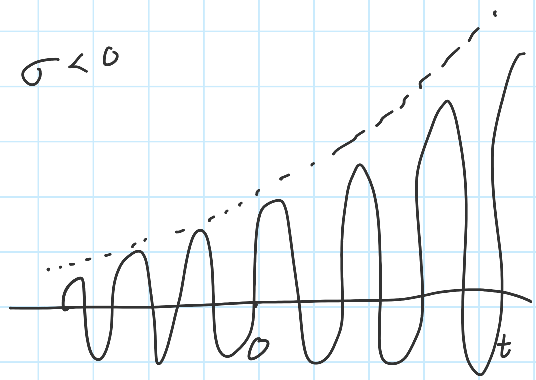
$$\mathrm{Re}\{e^{-st}\} = e^{-\sigma t} \cos(\omega t)$$

$$\mathrm{Im}\{e^{-st}\} = -e^{-\sigma t} \sin(\omega t)$$

$\sigma > 0$



$\sigma < 0$



$$x(t) \xleftrightarrow{LT} X(s)$$

real / cmplx   real   complex

$$x(t) \longrightarrow \underset{h(t)}{\boxed{LTI}} \longrightarrow y(t)$$ any LTI system

Ex. $e^{st} \longrightarrow$
$$y(t) = e^{st} * h(t) \quad \text{... [convolution]}$$
$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$
$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st}\, e^{-s\tau}\, d\tau$$
$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$
$$= H(s)\, e^{st}$$

$$= H(s)\, e^{st}$$

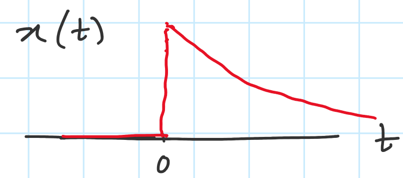
$$e^{st} \longrightarrow H(s)\, e^{st} \quad \cdots \text{[shape is preserved]}$$

→ eigenfunctions for LTI systems

note: all RLC ckts. are LTI systems

* any AC source always gives sinusoidal voltage / current across any part of the ckt.
* amplitude & phase change in general but shape remains the same

Ex. 1 $x(t) = e^{-t}u(t)$. Find $X(s)$.



By definition:

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-t} e^{-st}\, dt = \int_0^{\infty} e^{-(s+1)t}\, dt$$

$$= \left. \frac{e^{-(s+1)t}}{-(s+1)} \right|_0^{\infty}$$

$$= -\left[\cancel{\frac{e^{-(s+1)\infty}}{s+1}} \to 0 - \frac{1}{s+1} \right]$$

$$X(s) = \frac{1}{s+1} \quad \text{IF} \quad Re(s+1) > 0$$

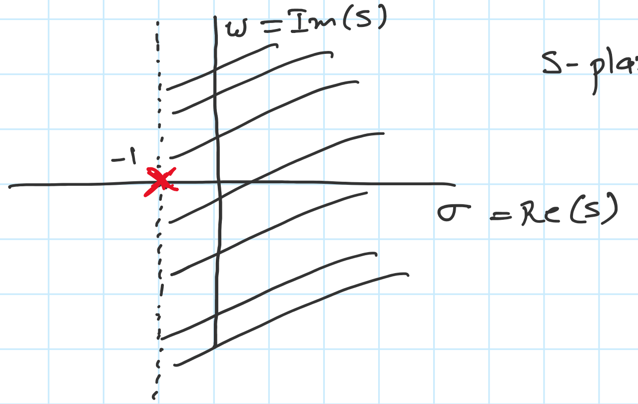
i.e. $Re(s) > -1$

① ②

LT always has two parts ① expression

② region of convergence (ROC)

ROC – region in the s-plane where the expression ① is valid.



s-plane $\boxed{}$ $Re(s) > -1$

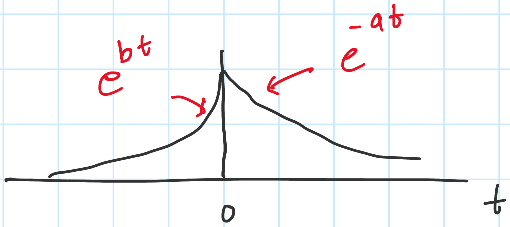
$\times \rightarrow s = -1$ is a point of singularity

There is a pole at $s = -1$.

HW

Ex. ② $x(t) = -e^{-t}\, u(-t)$. Find $X(s)$ & ROC

Ex. ③



. Find $X(s)$ & ROC

for ① $a \neq b$

② $a = b$