Lecture - 17

Thursday, 24 February 2022 8:22 AM

* Properties of Laplace transform

* linear constant coefficient differential equations (LCCDE)

* some standard example signals and their LT

  * $e^{-at}u(t) \xleftrightarrow{LT} \dfrac{1}{s+a}$ & $Re(s) > -a$ $a \in \mathbb{R}$

  * $-e^{-at}u(-t) \xleftrightarrow{LT} \dfrac{1}{s+a}$ & $Re(s) < -a$ $a \in \mathbb{R}$

  * $\delta(t) \xleftrightarrow{LT} 1$ & complete s-plane

  * $\delta(t-t_0) \xleftrightarrow{LT} e^{-st_0}$ & ——〃——〃——

Q. Find signal which has LT given by

① $$X(s) = \frac{s}{s^2 - s - 2}$$ & ROC : $Re(s) > 2$

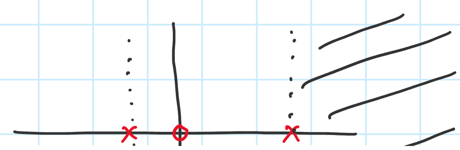
→ using partial fractions :

$$X(s) = \frac{s}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}$$. Find A & B.

$$A = X(s)(s-2)\Big|_{s=2} = 2/3 \quad \& \quad B = 1/3$$

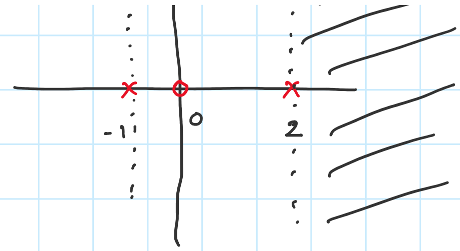
$$X(s) = \frac{2/3}{s-2} + \frac{1/3}{s+1}$$

pole - zero plot



$$x(t) = \frac{2}{\;} e^{2t}u(t) + \frac{1}{\;} e^{-t}u(t)$$

$$x(t) = \frac{2}{3} e^{2t} u(t) + \frac{1}{3} e^{-t} u(t)$$



(2) Same $X(s)$ above but ROC is $Re(s) < -1$. Find $x(t)$.

what are possible ROCs ?

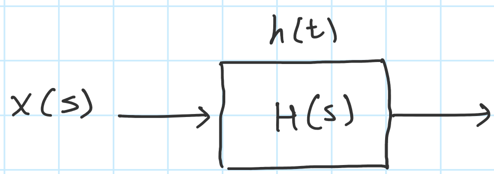
$\longrightarrow$ $$x(t) = -\frac{2}{3} e^{2t} u(-t) - \frac{1}{3} e^{-t} u(-t)$$

(3) Same $X(s)$ but ROC is $-1 < Re(s) < 2$

$$x(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$$

☆ LTI system analysis using Laplace Transform

we know



$Y(s) = X(s)H(s)$

convolution property

we are interested in additional system properties and their effect on impulse response & system function

* Causality of LTI systems

For an LTI system to be causal, we should have

$$h(t) = 0 \quad , \forall\, t < 0$$

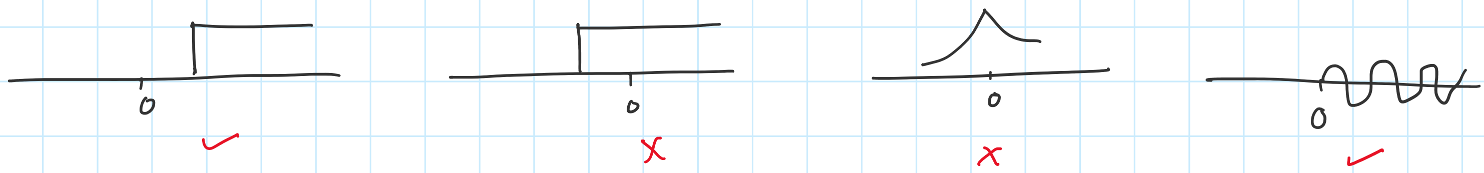
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

For causality $\longrightarrow$ we should only use $x(\tau)$ for $\tau \le t$

$\tau > t \;,\; h(t-\tau) = 0$

$\Rightarrow h(t) = 0 \quad \forall \; t < 0$

poll question:



for a causal system, what is nature of $H(s)$?

$h(t)$ is a right-sided signal.

$\therefore$ ROC of $H(s)$ will be right-sided plane *

converse is not true in general. *

* For a system with rational system function, causality is equivalent to the ROC being right-sided plane to the right of right most pole

Ex. 1 $H(s) = \dfrac{1}{s+1}$ & $\mathrm{Re}(s) > -1 \longleftrightarrow e^{-t}u(t) = h(t)$

causal.

2 $H(s) = \dfrac{e^{s}}{s+1}$ & $\mathrm{Re}(s) > -1 \longleftrightarrow e^{-(t+1)}u(t+1) = h(t)$

non-causal

$e^{s} \times \dfrac{1}{s+1}$

* Stability i.e. BIBO stability.

For an LTI system to be stable, we must have

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

i.e. $h(t)$ is absolutely integrable.

Proof: given $|x(t)| \le B$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \right|$$

$$|y(t)| \le \int_{-\infty}^{\infty} |h(\tau)\, x(t-\tau)|\, d\tau$$

$$\le \int_{-\infty}^{\infty} |h(\tau)|\, B\, d\tau$$

$$|y(t)| \le B \int_{-\infty}^{\infty} |h(\tau)|\, d\tau$$

for BIBO stability, $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$.

Q. If $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau$ is not finite, find a bounded input signal which gives unbounded output.

Ex. $h(t) = \delta(t - t_0) \longrightarrow$ stable & causal $\quad t_0 > 0$

$h(t) = u(t) \longrightarrow$ not BIBO stable. but causal

Shift system $\longrightarrow$ stable & causal

Integrator $\longrightarrow$ Causal but not stable.