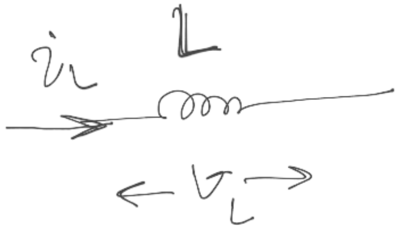
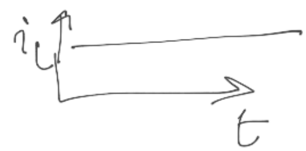
Circuits with Inductors



$$v_L = L \frac{di_L}{dt}$$

$v_L = 0 \Rightarrow$ S.C

i_L = constant $\Rightarrow$ D.C



$\Rightarrow$ Inductor is like a short for DC circuit.

$\Rightarrow$ Even if $V_L = 0$, Energy can be stored.

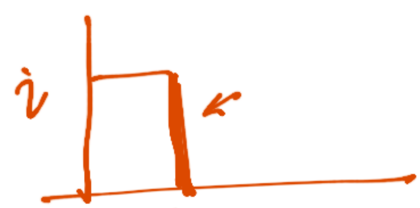
$E = \frac{1}{2} L i^2$

$i_1 \to i_{L2}$ in $\underline{dt \to 0}$ $\Rightarrow$ $V_L = \infty$

Instantaneous

v_L

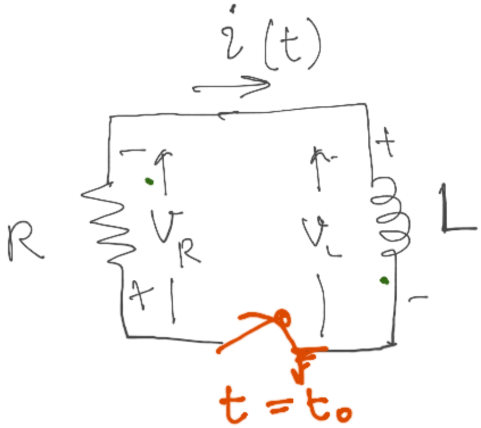
$$v = L \frac{di}{dt} \to \infty$$



$t=0$

$\Delta t \to 0$

L–R circuit



$$v_R + v_L = 0$$

$$v_R = R\, i(t)$$

$$v_L = L\, \frac{di(t)}{dt}$$

$$R\, i(t) + L\, \frac{di(t)}{dt} = 0$$

$$\int_{t_0}^{t} dt = -\frac{L}{R} \int_{i_0}^{i(t)} \frac{di(t)}{i(t)} \qquad \left(i(t) = i \right)$$

Limits :- Initial condition

$$\left.\begin{array}{lll} t : & t_0 \xrightarrow{\text{to}} & t \\ i & i_0 \longrightarrow & i(t) \end{array}\right\}$$

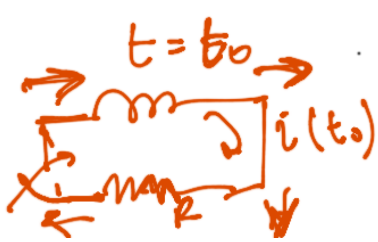
i_0 was current in inductor before $t = t_0$ (flipping to switch)

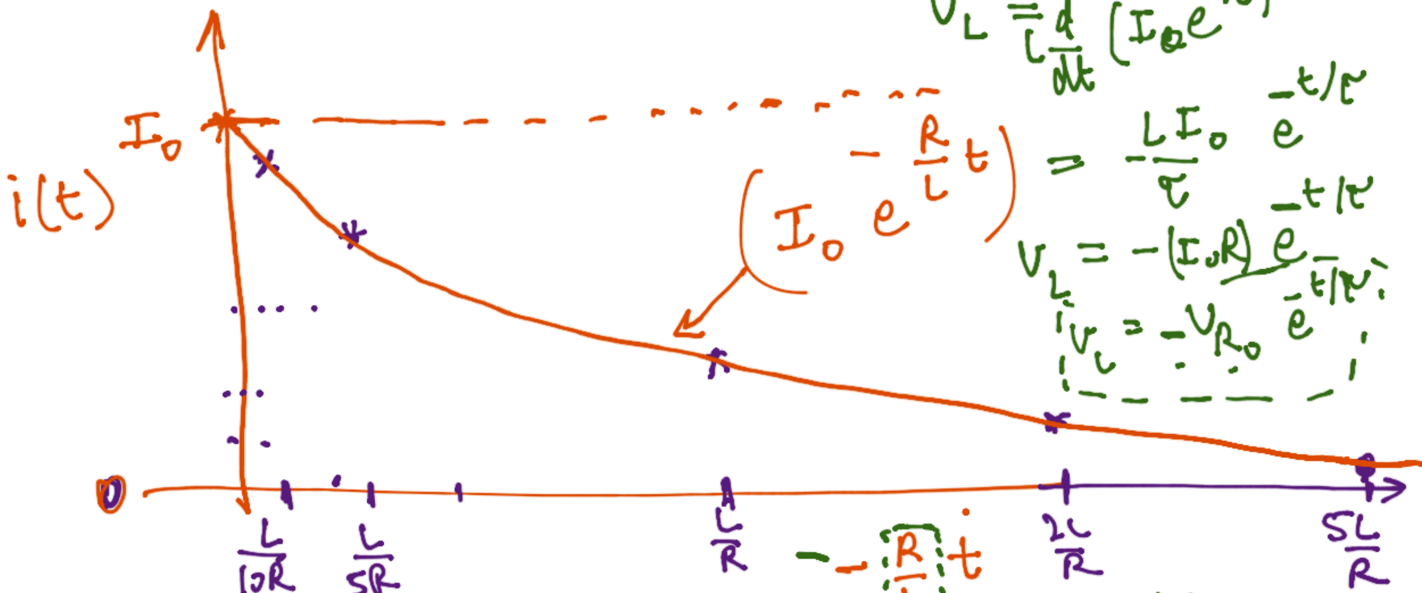
natural log

$$t = -\frac{L}{R}\left[\ln i(t) - \ln i_0\right]$$

$$-\frac{R}{L}t = \ln \frac{i(t)}{i_0}$$

$$\boxed{i(t) = i_0\, e^{-\frac{R}{L}t}}$$





$V_L = L\frac{d}{dt}\left(I_0 e^{-t/\tau}\right) = -\frac{L I_0}{\tau} e^{-t/\tau}$

$V_L = -(I_0 R)\, e^{-t/\tau}$

$V_L = -V_{R_0}\, e^{-t/\tau}$

$$i(t) = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau}$$

$t=0 \qquad i(t=0) = I_0$

$t_1 = \frac{L}{R}\times\frac{1}{10} = \frac{\tau}{10} \qquad i(t_1) = I_0 e^{-0.1} = 0.9048\, I_0$

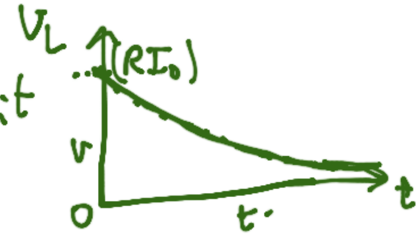
$t_2 = \frac{L}{R}\times\frac{1}{5} = \frac{\tau}{5} \qquad i(t_2) = I_0 e^{-.2} = 0.8187\, I_0$

$t_3 = \frac{L}{R} = \tau \qquad i(t_3) = I_0 e^{-1} = \boxed{0.3678\, I_0}$

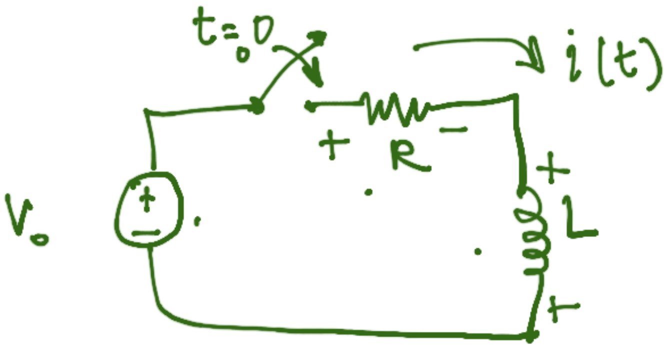
$t_4 = 2\frac{L}{R} = 2\tau \qquad i(t_4) = I_0 e^{-2} = .1353\, I_0$

$t_5 = 5\frac{L}{R} = 5\tau \qquad i(t_5) = I_0 e^{-5} = 6.7\times10^{-3}\, I_0$

$\frac{L}{R}$ → time constant 'τ' for L-R ckt (s, ms, μs) : Unit



R-L circuit with Source



$$V_0 = R\,i(t) + L\frac{di}{dt}$$

$$dt\left[V_0 - R\,i(t)\right] = L\,di$$

$$dt = \frac{L\,di}{\left[V_0 - R\,i(t)\right]} \Rightarrow \frac{R}{L}\int_{t=0}^{t} dt = \int_{i_0}^{i(t)} \frac{di}{\left[\frac{V_0}{R} - i(t)\right]}$$

$i_0 \rightarrow$ initial through the inductor

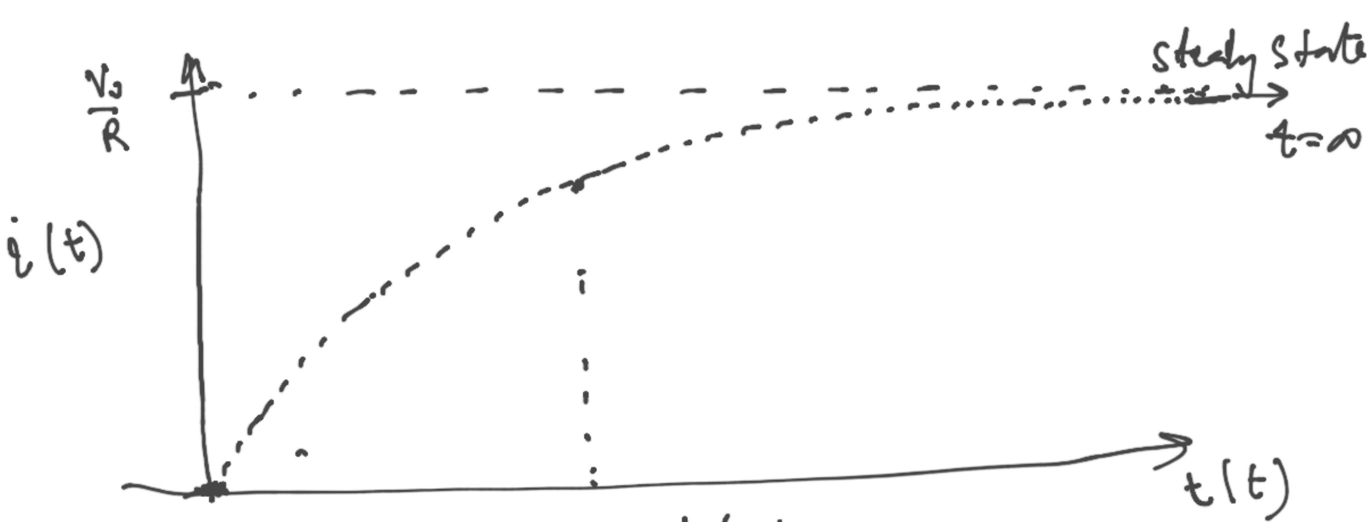
$$\Rightarrow \quad \frac{R}{L}\,t = -\left[\ln\left(\frac{V_0}{R} - i(t)\right)\right]_{i_0}^{i(t)}$$

$$-\frac{R}{L}t = \ln\left[\frac{\frac{V_0}{R} - i(t)}{\frac{V_0}{R} - i_0}\right]$$

$$\frac{V_0}{R} - i(t) = \left[\frac{V_0}{R} - i_0\right] e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = \frac{V_0}{R} - \left(\frac{V_0}{R} - i_0\right) e^{-\frac{R}{L}t}}$$

Let $i_0 = 0$ $\quad i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$



$$i(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$

$$\left[\, i_0 \neq 0 \qquad i(t) = I_0 \,\right]$$

$t \to \infty$ $\quad i(t) \to \frac{V_0}{R}$ $\Rightarrow$ Inductor current becomes constant ($i_L = \text{const}$)

$\Downarrow$

Steady state current

$\Downarrow$

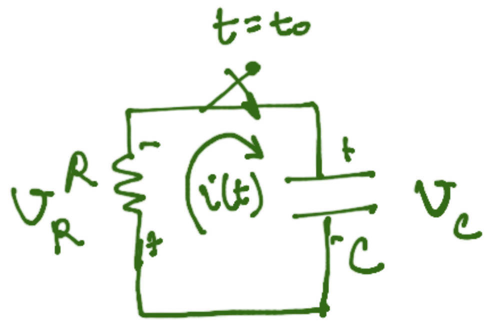
$v_L = 0$

$$v_L = L\frac{di(t)}{dt} = L\,\frac{V_0}{R\tau}\,e^{-t/\tau}$$

$$\boxed{v_L = V_0\, e^{-t/\tau}}$$

$t \to \infty$ $\quad v_L \to 0$

Capacitor with Resistor



Let $t \le t_0$ $\quad v_c = V_{c0}$

$$v_R + v_c(t) = 0$$

$$R\,i(t) + \frac{1}{C}\int i\,dt = 0$$

$$i(t) = C\frac{dv_c}{dt} = \frac{v_R}{R} \leftarrow$$

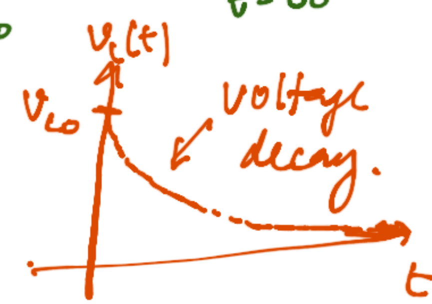
$$C\frac{dv_c}{dt} - \frac{v_R}{R} = 0$$

$$C\frac{dv_c}{dt} + \frac{v_c}{R} = 0$$

$$C\frac{dv_c}{dt} = -\frac{v_c}{R} \Rightarrow \int_{V_{c0}}^{v_c(t)} \frac{dv_c}{v_c} = \frac{-1}{RC}\int_{t=t_0}^{t=t} dt$$

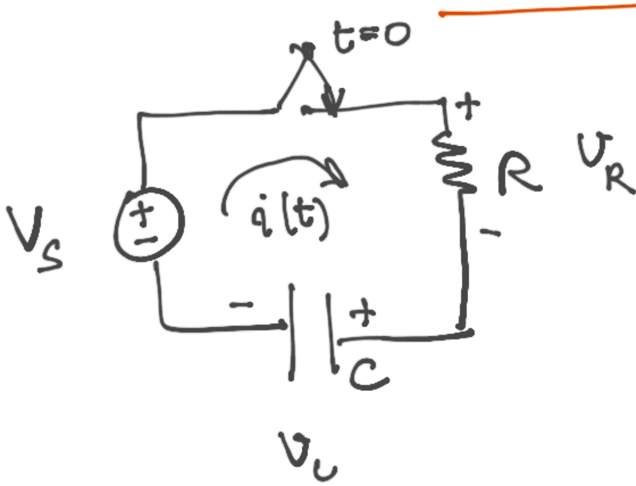
$$\Rightarrow \quad -\frac{1}{RC}t = \ln\left(\frac{v_c(t)}{V_{c0}}\right)$$

$$\Rightarrow \quad \boxed{v_c(t) = V_{c0}\, e^{-t/RC}}$$



$\tau = RC$ (unit: time) is time constant for RC circuit.

Capacitor with Voltage Source



$$V_s = V_R + V_C$$

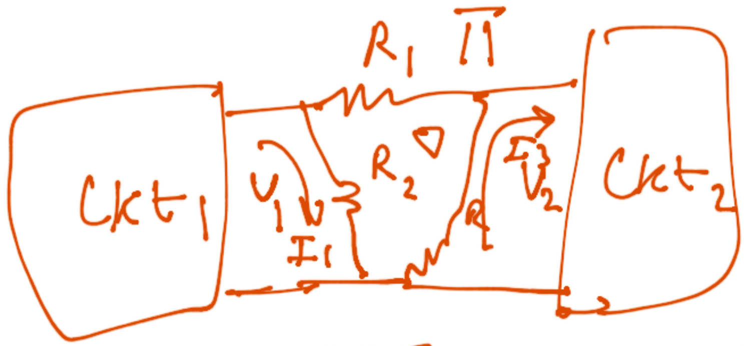
$$V_s = i(t) + \frac{1}{C}\int i\,dt$$

$$\boxed{V_C = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)}$$

$(V_{C0} = 0)$

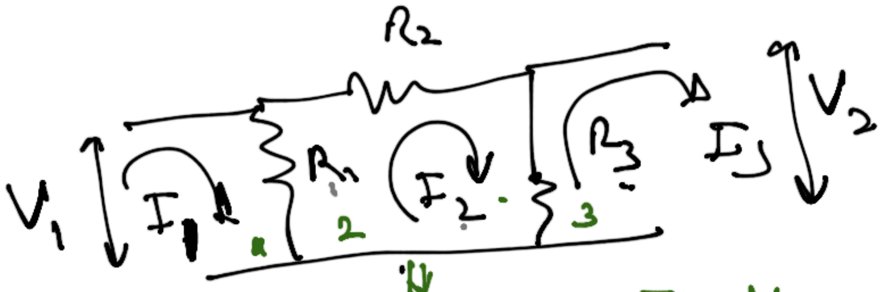
$Y - \Delta$

$\pi - Y$



Δ or π

Equivalent circuit



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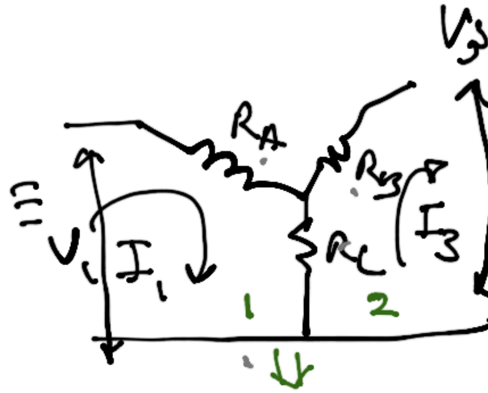

$$\text{KVL}\begin{cases} - I_1 - I_2 - I_3 = V_1 \\ = 0 \\ = V_2 \end{cases}$$

$$\text{KVL}\begin{cases} A\, I_1 + B\, I_3 = V_1 \\ C\, I_1 + D\, I_3 = V_2 \end{cases}$$

Eliminate I_2

$$\Rightarrow \quad \begin{aligned} X\, I_1 + Y\, I_3 &= V_1 \\ Z\, I_1 + V\, I_3 &= V_2 \end{aligned} \quad \Leftrightarrow \quad R_1, R_2, R_3 \;\&\; R_A, R_B, R_C$$

R_A, R_B, R_C