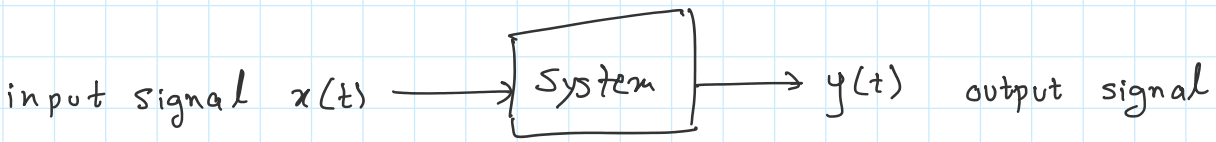
Lecture - 08

Tuesday, 1 February 2022 8:44 AM

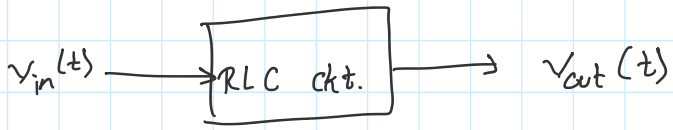
* Part 1 - Signals, Fourier Series for periodic signals
* Part 2 - Network analysis, RLC circuits
* Part 3 - Systems & signals

Electrical network is an example of a system

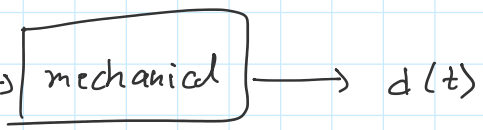
* Black box representation of systems



Ex. Electrical system



Ex. mechanical system

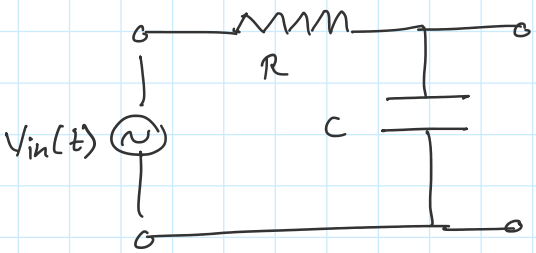


* Various questions of interest *

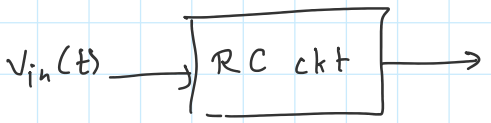
1. How to describe a system ? Representation
2. Given a system & input to it, find the output ? Analysis
3. Given input - output description, how to design a system ? Design

1. System description or representation

Ex. RC circuit example



series RC circuit



Black box representation

(a) verbal / oral description

(b) mathematical : differential equation

$$i = C\frac{dv_c}{dt} = \frac{v_{in} - v_c}{R}$$

$$\frac{dv_c}{dt} = \frac{1}{RC}v_{in} - \frac{1}{RC}v_c \qquad \frac{1}{RC} = \alpha$$

$$v_{in}(t) = v_c + RC\frac{dv_c}{dt}$$

ODE

ordinary diff. eqn.

$$v_c(t) = \alpha\int v_{in}(\tau)\,d\tau - \alpha\int v_c(\tau)\,d\tau$$

(c) Operator : $\mathcal{A}$ — integration operator

$$\mathcal{A}x = \int x(\tau)\,d\tau$$

$$V_c = \alpha A V_{in} - \alpha A V_c$$

$$V_c = \alpha A (V_{in} - V_c) \quad \text{.... compact} \quad (1)$$

operator based representation

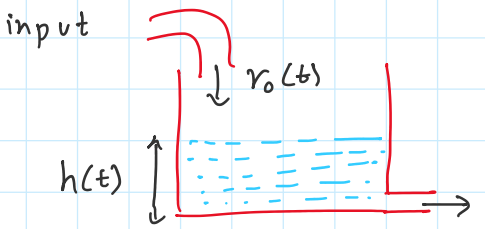
Ex. $x \rightarrow$ [Sys.] $\rightarrow y$

given : $y = A(1-A)x = (A - A^2)x$

$$= Ax - A^2 x = \int x(\tau)\,d\tau - \int \left[\int x(\tau)\,d\tau \right] dx$$

double integration

* Ex Leaky tank example



fluid mechanical system

assume o/p flow rate $\propto h(t) \Rightarrow r_1(t) \propto h(t)$

$$\frac{dh}{dt} \propto r_0(t) - r_1(t)$$

$$\frac{dh}{dt} \propto \frac{dr_1}{dt}$$

$$\Rightarrow \frac{dr_1}{dt} \propto r_0(t) - r_1(t)$$

ODE representation $\rightarrow$ $\frac{dr_1}{dt} = \beta \left[r_0(t) - r_1(t) \right]$

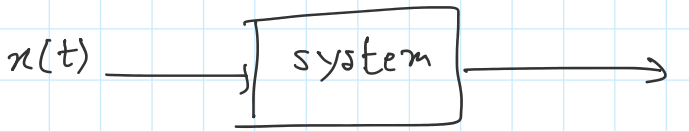
$R_0 \equiv r_0(t)\ \forall t$

$R_1 \equiv r_1(t)\ \forall t$

$$R_1 = \beta A R_0 - \beta A R_1$$

operator representation → $R_1 = \beta A (R_0 - R_1)$ → (2)

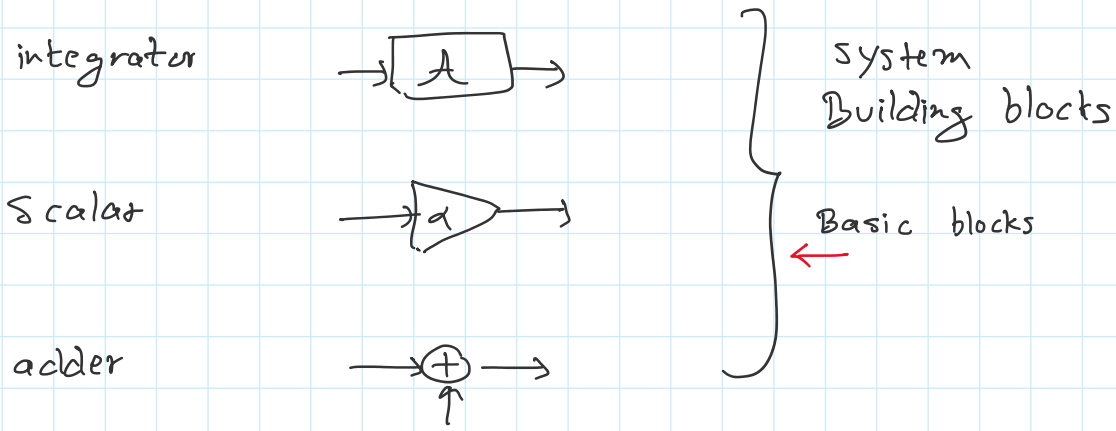
* Both systems [electrical in (1) and mechanical in (2)] can be represented by same / similar ODE rep. and operator representation (given below)



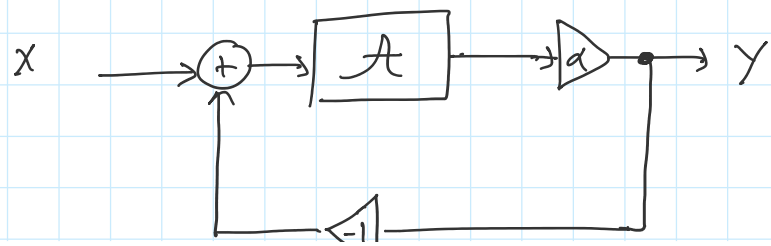
$$y = \alpha A (x - y) \quad \rightarrow (3)$$

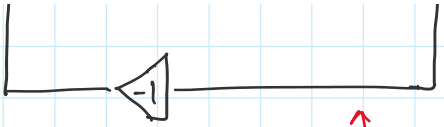
Eq. (3) includes / describes both (1) & (2)

(d) Block diagram rep.



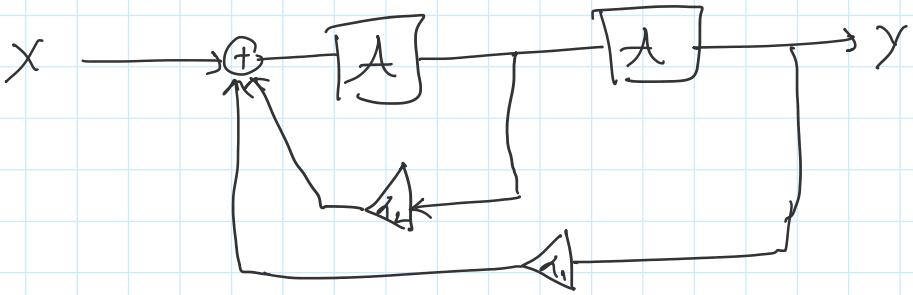
Ex. $y = \alpha A (x - y)$





feedback

HW. Ex.



find the operator & ODE rep. of this system