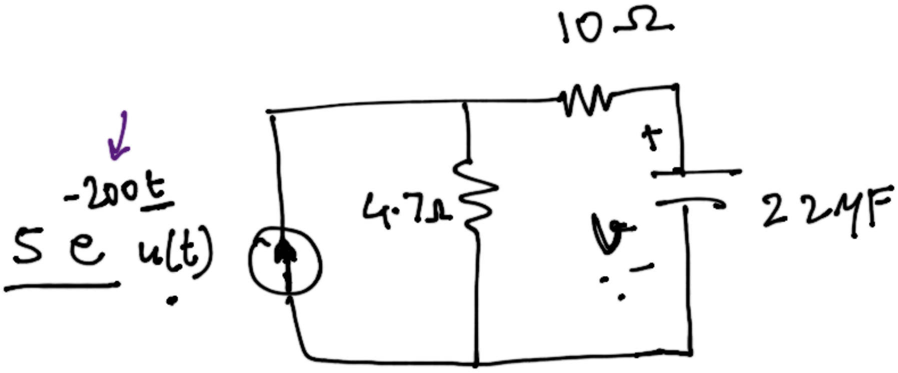
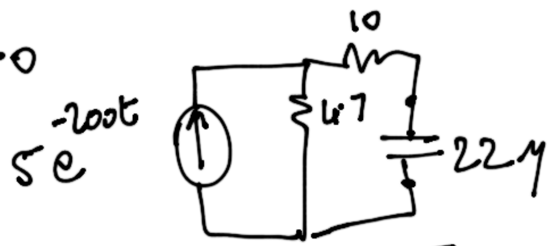
20 Jan

Ex

Find $v(t)$



$t>0$



$v = v_f + v_n$

$v = v_f + Ae^{-t/\tau}$

'τ'

$C = 22\,\mu F$

R across C =?

$R_{th} = 14.7$

$\tau = RC$

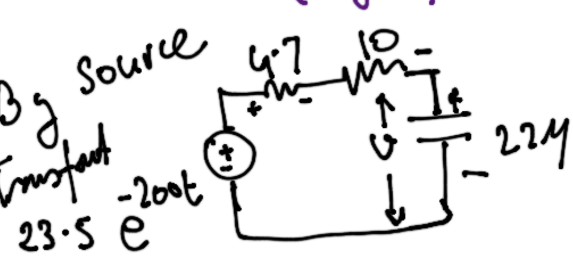
$= 14.7\Omega * 22\,\mu F = \_\_\_$

* V_s or I_s is constant → Forced is also constant (s.s response)

V_s or I_s is changing, the Forced is also a function of time.

Method 1 write KVL or KCL & solve differential equation. (by first principle)

By source Transform



$$23.5\,e^{-200t} - \left(22\mu \frac{dv}{dt}\right)(4.7+10) - v = 0$$

$i = C\frac{dv}{dt}$ ⇒ Solve this Diff eq.

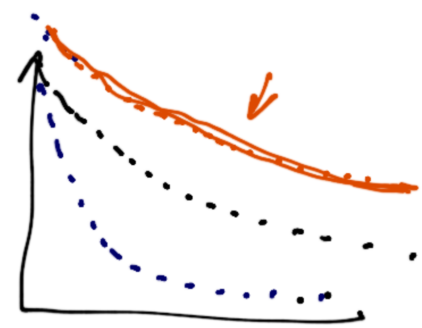
Solve the 1st ODE

$$v = \underline{\quad} \int \underline{\quad} (t)\, dt + A e^{-t/\tau}$$

$$= K \underline{e}^{-2000t} + A e^{-t/\tau}$$

$$v = \underbrace{K e^{-2000t}}_{\text{Forced}} + \underbrace{A e^{-3092t}}_{\text{Natural}}$$

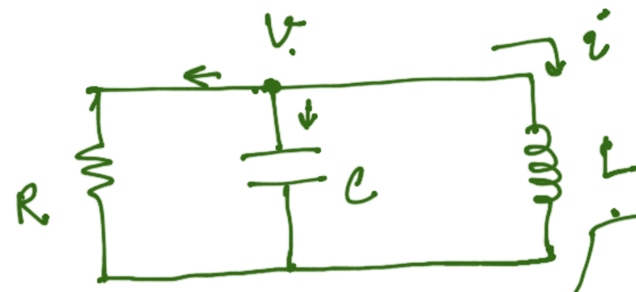
$$\boxed{v = \underline{\quad}\, e^{-2000t}}$$



C R

L R

RLC



$$v_L = L\frac{di}{dt}$$

$$\Rightarrow i = \frac{1}{L}\int v_L \, dt$$

KCL

$$-\frac{v}{R} - C\frac{dv}{dt} - \frac{1}{L}\int v\, dt = 0$$

Differentiate it wrt 't'

$$\rightarrow \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt} + \frac{1}{L}v = 0 \quad \text{—} \ \boxed{A}$$

2^{nd} ODE

Borrowing solution of 2^{nd} ODE from Mathematics

$\equiv$ Solution of ODE is also $v = \underline{Ae^{st}}$'s' → constant.

Substituting Ae^{st} into $\boxed{A}$

$$\frac{1}{R}As\,e^{st} + CAs^2e^{st} + \frac{1}{L}Ae^{st} = 0$$

$$\left(Cs^2 + \frac{s}{R} + \frac{1}{L}\right)Ae^{st} = 0$$

Characteristic Equation

Solve $Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$

$$Cs^2 + \frac{s}{R} + \frac{1}{L} = 0$$

$$s = \frac{-\frac{1}{R} \pm \sqrt{\frac{1}{R^2} - 4 \cdot \frac{C}{L}}}{2C}$$

$$s = -\frac{1}{2RC} \pm \frac{1}{2}\sqrt{\frac{1}{R^2C^2} - \frac{4}{LC}}$$

Solutions

$$s_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

$\to$ decay

The solution Ae^{st}

$$v = Ae^{s_1 t} + Be^{s_2 t}$$

} Solution to R-L-C circuit.

Analyze s_1 & s_2

Let $\frac{1}{2RC} = \alpha$ Damping factor. $\frac{1}{LC} = \omega_0^2$ Resonant frequency.

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

α, ω_0

	s_1	s_2	
$\alpha > \omega_0$			Overdn
$\alpha = \omega_0$	$-\alpha$	$-\alpha$	Critically damped
$\alpha < \omega_0$	Im	Im	Under damped.