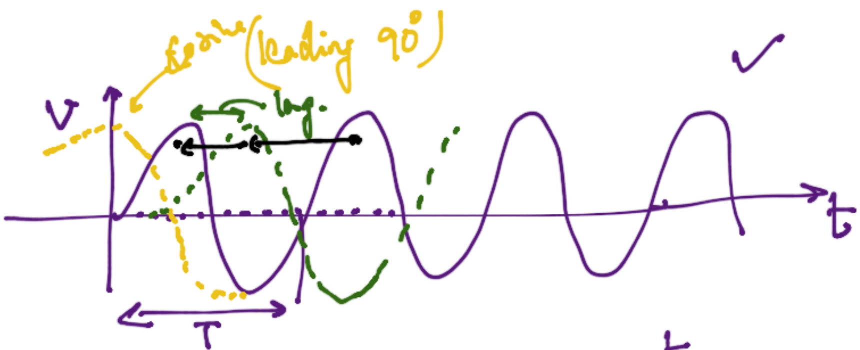
25 Jan

V constant ✓ Battery, ✓

V source $v(t) = V e^{-kt}$ → Input. ← first principle.



✓

$V = V_m \sin \omega t$

$\omega \to 2\pi f$

$f \to 50\,Hz$ (India)

$f = 60\,Hz$ (U.S.)

$f = \frac{1}{T}$

$\omega = 2\pi f = \frac{2\pi}{T}$

rad. or degree

$v_1 = V_{m_1} \cos (5t + 10^\circ)$

$= V_{m_1} \sin (5t + 90^\circ + 10^\circ)$

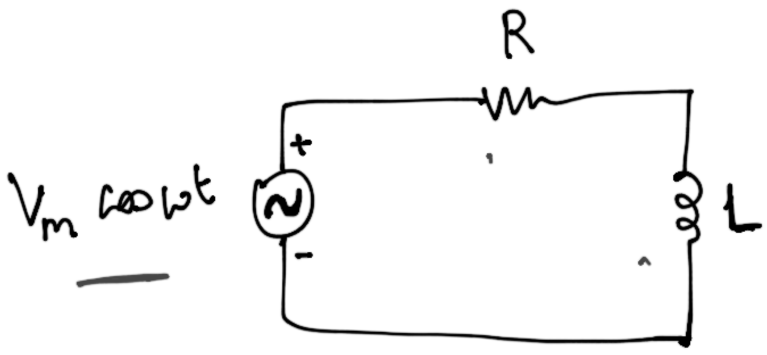
$V_2 = V_{m_2} \sin (5t - 30^\circ)$.

→ V_1 is leading V_2 by $(90 + 10 + 30^\circ) = 130^\circ$

$360^\circ - 130^\circ = 230^\circ$

→ V_1 is lagging V_2 by 230°

..



$$V_R + V_L = V_s$$

$$iR + L\frac{di}{dt} = V_m \cos\omega t \rightarrow \boxed{1}$$

$\Downarrow$

solution

$$i = I_1 \cos\omega t + I_2 \sin\omega t \rightarrow \boxed{2}$$

Substitute $\boxed{2}$ in $\boxed{1}$

$$(I_1 \cos\omega t + I_2 \sin\omega t)R + L\frac{d}{dt}(I_1 \cos\omega t + I_2 \sin\omega t) = V_m \cos\omega t$$

$$\Rightarrow \cos\omega t\left[I_1 R + \omega L I_2 - V_m\right] + \left[I_2 R - I_1 L\omega\right]\sin\omega t = 0$$

Coeff. of sine & cosine have to $= 0$

$$\left.\begin{aligned} I_1 R + \omega L I_2 - V_m &= 0 \rightarrow \boxed{3} \\ I_2 R - I_1 L\omega &= 0 \rightarrow \boxed{4} \end{aligned}\right] \quad I_1 \;\&\; I_2 \rightarrow \text{unknown}$$

Solve $\boxed{3}$ & $\boxed{4}$ simultaneously

$$I_1 = \frac{RV_m}{R^2 + w^2 L^2}$$

$$I_2 = \frac{\omega L V_m}{R^2 + w^2 L^2}$$

$$\boxed{i = \overbrace{\frac{RV_m}{R^2 + w^2 C^2}}^{\cos\theta} \cos\omega t + \overbrace{\frac{\omega L V_m}{R^2 + w^2 L^2}}^{\sin\theta} \sin\omega t} \rightarrow \boxed{5} = A\cos(\omega t + \theta)$$

Further converting $\boxed{5}$ into 'cos'

$\boxed{6}$ — $\underline{A\cos(\omega t+\theta)} = \underbrace{A\cos\omega t\,\cos\theta}\ - \underbrace{A\sin\omega t\,\sin\theta}$

From $\boxed{5}$ $\cos\theta$ & $\sin\theta$

Comparing $\boxed{5}$ with $\boxed{6}$

$$\Rightarrow \tan\theta = \frac{\omega L}{R}$$

$$\Rightarrow \theta = \tan^{-1}\frac{\omega L}{R}$$

Using $(A\cos\theta)^2 + (A\sin\theta)^2 = A^2 \Rightarrow \boxed{6}$

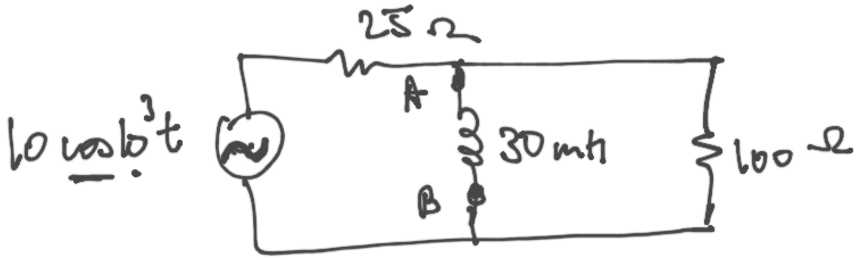
$$= \left[\frac{(RV_m)^2 + (\omega L V_m)^2}{(R^2+\omega^2L^2)^2}\right] \Rightarrow \text{Using } \boxed{5}$$

$$= \frac{V_m^2}{R^2+\omega^2L^2}\cdot 1 = A^2 \Rightarrow A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\Rightarrow \boxed{i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\theta)}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

27 Jan



$i_L(t) = ?$

- AC
- Phasor

AC power.

2 port
Op-amp
Dual circuit

Method 1 $I_s \to V_L \, , \, V_R$

$V_s \longrightarrow I_s$

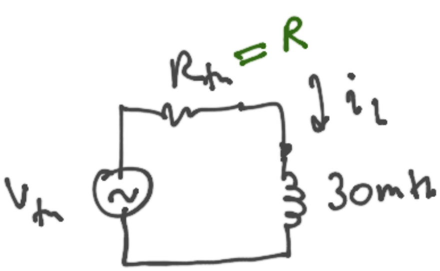
Method 2 Find Thevenin voltage and resistance across L

($30\,mH \to Z_L$ Load)

$$R_{th} = \frac{\cancel{25} \times \cancel{100}^{20}}{\cancel{125}_{5}} = 20$$

$10\cos 10^3 t$
$\downarrow$
ω

$$V_{th} = \underbrace{\frac{100}{125} \times 10}_{V_m} \cos \underset{10^3}{\omega} t$$



$$i_L(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi)$$

V_m

$$\phi = \tan^{-1} \frac{\omega L}{R}$$

$$i_L(t) = \frac{\frac{100}{125} \times 10}{\sqrt{20^2 + (10^3)^2 (30 \times 10^{-3})^2}} \cos\left(10^3 t + \tan^{-1} \frac{2}{3}\right)$$

Complex forcing function

$V_m \cos(\omega t + 0^\circ) = \underline{V_m \angle 0^\circ}$

sinusoidal function 'ωt'

$[\theta \rightarrow$ ang phase$]$

$e^{j\omega t} = \cos \omega t + j \sin \omega t$

$V_m \cos(\omega t + \theta) = Re\left[V_m e^{j\omega t + \theta}\right]$

$= Re\left[\mathbf{V_m} \angle \theta\right]$

→ 2 part stimulus : Re , Im.

$\downarrow \cos \omega t \qquad \downarrow j \sin \omega t$

By principle of superposition $[L, R, C, V_s, I_s \rightarrow$ linear$]$

the output will also be a sum of Re & Im

Input → $V_m (\cos \omega t + j \sin \omega t) \longrightarrow e^{j\omega t}$

Output → $I_m \left(\cos(\omega t + \phi) + j \sin(\omega t + \phi)\right] \leftarrow$

[Imaginary sources cause Im response
Real sources cause Re response]

$V_m \cos(\omega t + \theta)$

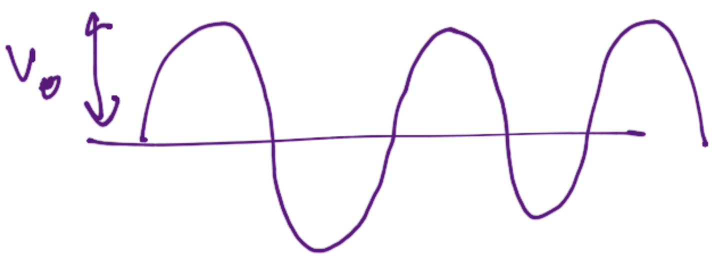
Step 1 $\quad V_m e^{j(\omega t + \theta)}$

Step 2 $\quad \underline{I_m e^{j(\omega t + \phi)}}$, $V_o e^{j(\omega t + \phi')}$

$I_m \rightarrow$ Amplitude

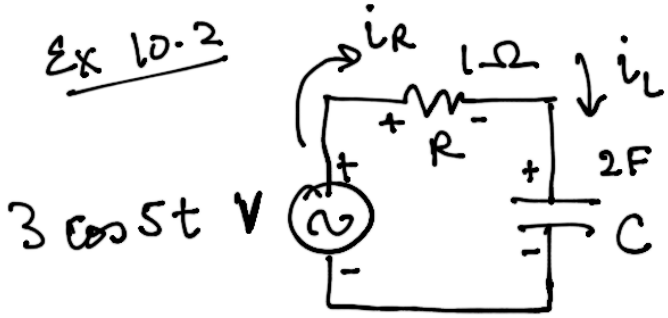
$\downarrow$

final output : $Re\left[I_m e^{j(\omega t + \phi)}\right]$



$V_o \sin \omega t$

Ex 10.2



Solve 'steady-state $\underbrace{\text{cap volt.}}_{V_c}$'

$\rightarrow$ $3\cos 5t \Rightarrow \underbrace{Re\left[3e^{j5t}\right]}_{\text{forcing function.}}$

KVL $\quad V = V_R + V_c$

$$3e^{j5t} = i_R R + V_c$$

$$V_c \qquad i_c = C\frac{dv_c}{dt} = \underline{i_R}$$

$$3e^{j5t} = RC\frac{dv_c}{dt} + V_c \quad \text{———} \boxed{A}$$

Solution V_c will be of the form $V_c = \underline{V_m e^{j(5t+\phi)}}$

$\boxed{A} \Rightarrow$
$$3e^{j5t} = RC\,V_m(j5)\,e^{j(5t+\phi)} + V_m e^{j(5t+\phi)}$$

$$3 = RC\,V_m(j5)\,e^{j\phi} + V_m e^{j\phi}$$

$$3 = (jRCV_m5 + V_m)\,e^{j\phi}$$

$$V_m = \left(\frac{3}{1+j5RC}\right)\cdot e^{-j\phi} = \left(\frac{3(1-j5RC)}{1+(5RC)^2}\right)e^{-j\phi}$$

$$V_m = \frac{3(1 - j\,5RC)}{1+(5RC)^2}\, e^{-j\phi}$$

$$\text{phase} = \tan^{-1}\left(\frac{-\overset{\leftarrow \omega RC}{5RC}}{1}\right) = \tan^{-1}(-10) \overset{\leftarrow(\text{check})}{=} -84^\circ$$

$$\text{Amplitude} = \frac{3}{\sqrt{1+10^2}} = 0.3 \quad (\text{approx})$$

Solution →

$$V_c = 0.3 \tan^{-1}(-10) = 0.3 \text{ phase } (-84^\circ)$$

$$V_c = 0.3 \underbrace{e^{j5t}}_{\text{coming from forcing function}} e^{-j84^\circ}$$

Voltage across the capacitor is $Re\left[0.3\, e^{j(5t-84^\circ)}\right]$

* No differential equation in time was solved.
* Solution was done in freq. domain 'ωt'.

In General form

$V_m \cos\omega t = Ri + L\frac{di}{dt}$.

Step 1 Complex exp. source $V_m e^{j\omega t} = V_m \angle 0° \ (\omega)$

Step 2 Response will be $i = I_m e^{j(\omega t + \phi)}$

Step 3 Write KVL (or KCL) $\Rightarrow$ differential equation 'C' or 'L'

Step 4 Substitute solution $i(t) = I_m e^{j(\omega t+\phi)}$

Step 5 Solve the Diff. eqn with $I_m e^{j(\omega t+\phi)}$

$$Ri + L\frac{di}{dt} = V_m e^{j\omega t}$$

$$\text{LHS} = R\, I_m e^{j(\omega t+\phi)} + L(j\omega) I_m e^{j(\omega t+\phi)}$$

$$V_m = R I_m e^{j\phi} + j L\omega I_m e^{j\phi}$$

$$= (R + j\omega L)\, \underline{\underline{I_m}}\, e^{j\phi}$$

$$I_m e^{j\phi} = \frac{V_m}{R + j\omega L} \rightarrow \text{Mag \& phase}$$

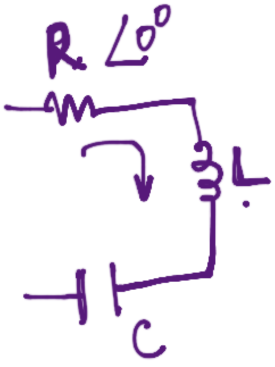
Mag $\Downarrow$ $I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$

phase $\Downarrow$ $\tan^{-1} -\frac{\omega L}{R}$

Step 6 Express $I_m e^{j\phi}$ in terms of V_m, R, ω, L

Solution: $I_m e^{j\phi} = I_m e^{j(\omega t + \phi)} \Rightarrow I_m \angle\phi \ (\omega) \Rightarrow \boxed{\frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \angle -\tan^{-1} \omega L/R}$

$I_m \angle \phi$: Phasor representation. Phase $\Downarrow$ phasor



$R \& L : \phi = \tan^{-1}\left[-\frac{\omega L}{R}\right] \leftarrow$

$R \& C : \phi = \tan^{-1}(\omega RC) \leftarrow$

$\boxed{\text{Inductor}}$ $V_L = L\frac{di}{dt}$ Let $i = I_m e^{j(\omega t+\phi)}$

$$V_L = I_m (j\omega) L e^{j(\omega t+\phi)}$$

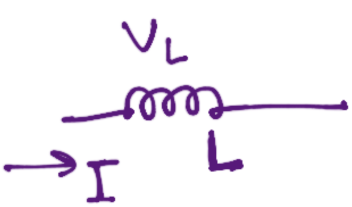
$$V_L = V_m e^{j(\omega t+\theta)} = j\omega L I_m e^{j(\omega t+\phi)}$$

$$\Rightarrow \underbrace{V_m e^{j\theta}}_{V} = j\omega L \overbrace{I_m e^{j\phi}}^{I}$$

Amp of voltage across Ind $= j\omega L$ for a current $I_m e^{j\phi}$

$$\boxed{V = j\omega L I}$$

$\frac{V}{I} = j\omega L =$ Impedance



Capacitor

$$i_c = C \frac{dv_c}{dt}$$

$$v_c = V_m e^{j(\omega t + \phi)}$$

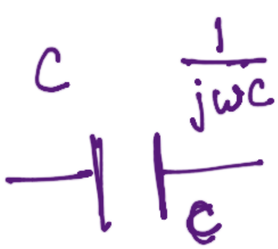
$$i_c = C V_m j\omega \, e^{j(\omega t + \phi)}$$

$$\overbrace{I_m e^{j(\omega t + \theta)}}^{i_c} = j\omega C \, V_m e^{j\omega t + \phi}$$

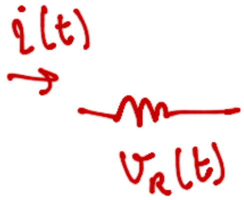
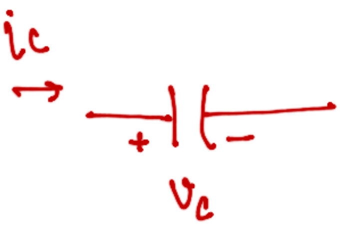
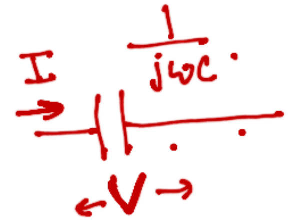
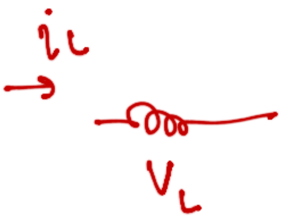
$$\underbrace{I_m e^{j\theta}}_{I} = j\omega C \underbrace{V_m e^{j\phi}}_{V}$$

$$I = j\omega C \, V$$

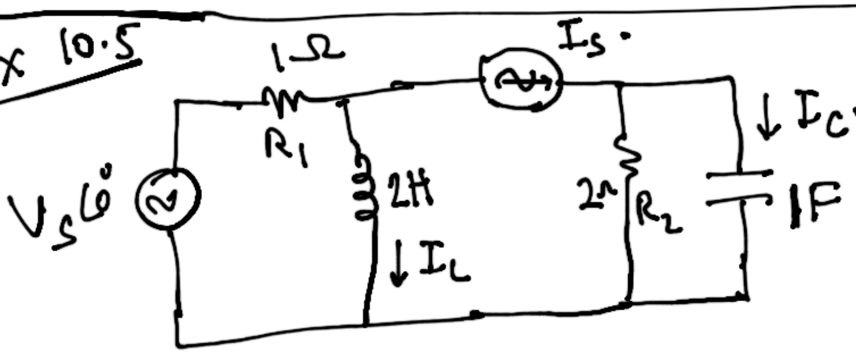
$$\Rightarrow \frac{V}{I} = \text{Impedance of Capacitor} = \frac{1}{j\omega C}$$



Resistor: no V_R & I_R → no lag ∴ 'R'

	Time Domain	Frequency Domain	
	$v_R(t) = R\, i_R(t)$	$V = R I$	
	$v_c = \frac{1}{C}\int i_c\, dt$	$V = \frac{I}{j\omega C}$	
	$V_L = L \frac{di_L}{dt}$	$V = j\omega L I$	

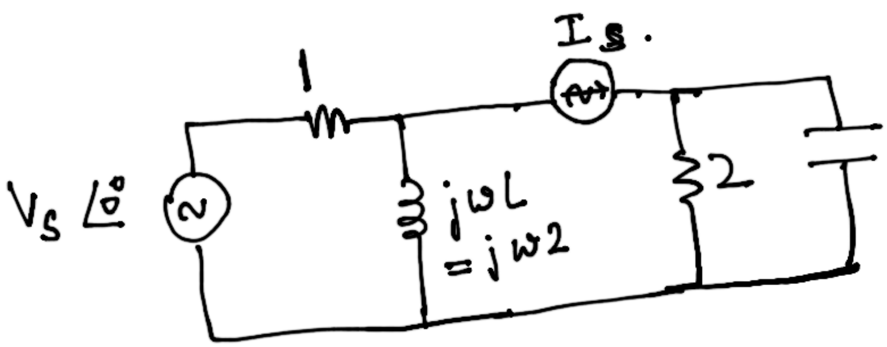
Ex 10.5



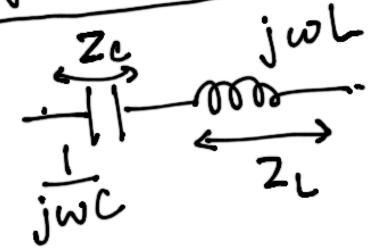
Q: I_s & $i_s(t)$ = ?

$\omega = 2$ rad/s

$I_c = 2\angle 28^\circ$ A



$\frac{1}{j\omega C} = \frac{1}{j\omega 1}$



Apply :

Z is series $Z_1 + Z_2$

Z in $\|$ $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$Z = Z_c + Z_L$$
$$= \frac{1}{j\omega C} + j\omega L = \frac{-j}{\omega C} + j\omega L$$
$$= j\left(\omega L - \frac{1}{\omega C}\right)$$