Lecture - 09

Thursday, 3 February 2022 8:36 AM

* Representation of systems

(a) verbal / ord

(b) mathematical i.e ODE

(c) operators

(d) block diagram

* Operator properties

commutative : $A(1-A)x = (1-A)Ax$

distributive : $A(1-A)x = (A - A^2)x$

associative : $[A(1-A)](\alpha - A)x = A[(1-A)(\alpha-A)]x$

* Analysis of systems - introduction to elementary signals

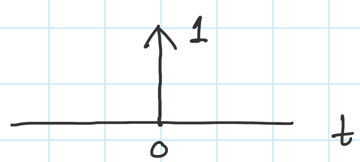
* Elementary signals

* Unit impulse signal : $\delta(t)$ (Dirac delta function)

$\delta(t)$ is defined as :

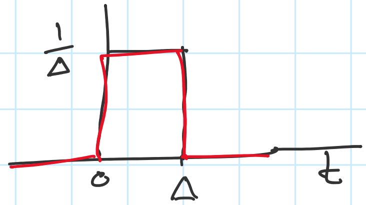
Diagram of $\delta(t)$

* (1) $\delta(t) = 0 , \quad t \neq 0$
* (2) $\displaystyle\int_{-\infty}^{\infty} \delta(t)\, dt = 1$



Intuition for $\delta(t)$

pulse $\quad p_\Delta(t) \equiv$



$\Delta > 0$

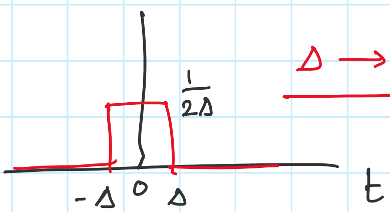
consider : $\Delta \longrightarrow 0$ *

area under pulse $= \Delta \times \frac{1}{\Delta} = 1$

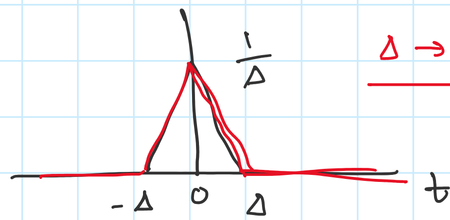
$$\lim_{\Delta \to 0} p_\Delta(t) = \delta(t)$$

$$\int_{-\infty}^{\infty} p_\Delta(t)\,dt = 1$$

Above intuition works for other pulses as well. For Ex.



$\xrightarrow{\Delta \to 0} \delta(t)$



$\xrightarrow{\Delta \to 0} \delta(t)$

* $\delta(t)$ as input to systems :

$$\delta(t) \longrightarrow \boxed{\mathcal{A}} \longrightarrow \; ? \quad y(t) = ?$$

definition : $\quad \mathcal{A}x = \int_{-\infty}^{t} x(\tau)\,d\tau$

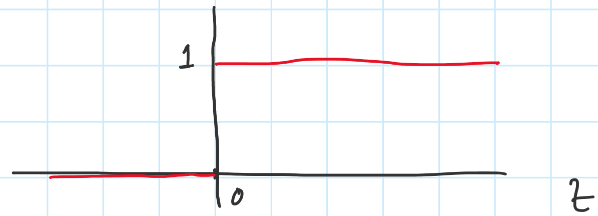
running integral : $\quad y(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

discontinuity at $t = 0$

Note: $\displaystyle\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \longrightarrow \int_{-\epsilon}^{\epsilon} \delta(t)\,dt = 1 \quad \forall\, \epsilon > 0$

Sketch $u(t)$:



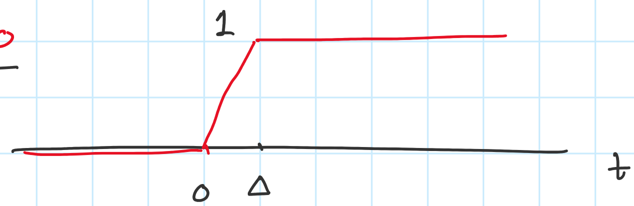
* unit step signal

"Switch" signal

$$\delta(t) \longrightarrow \boxed{A} \longrightarrow u(t)$$

alternately, $\dfrac{d\,u(t)}{dt} = \delta(t)$ ---- (justification)

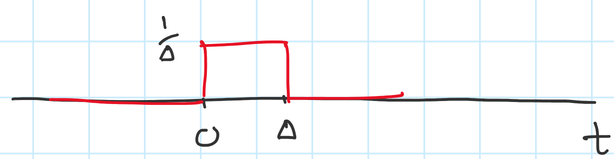
$$u(t) \xleftarrow{\Delta \to 0}$$



in a limiting sense:

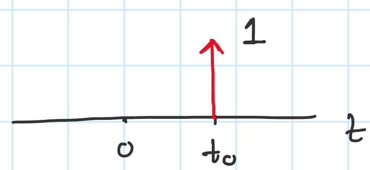
derivative

$$\delta(t) \xleftarrow{\Delta \to 0}$$



* we also have:

shifted impulse : $\delta(t - t_0)$



scaled impulse : $k\,\delta(t)$

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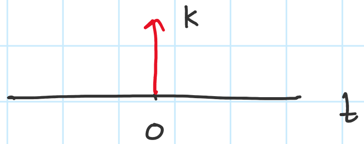
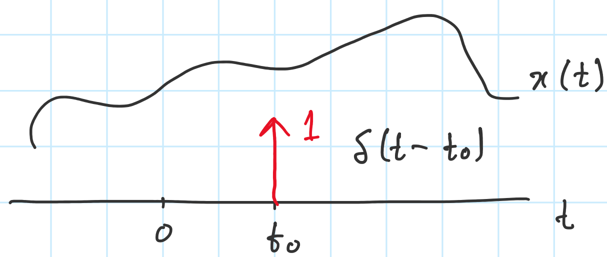


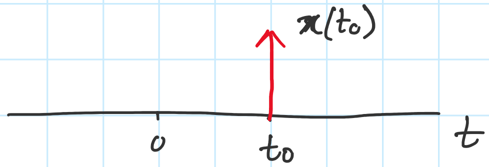
$k \in \mathbb{R}$

* Some proterties of $\delta(t)$:

consider: (1) $\delta(t - t_0)\, x(t)$

$x(t)$ is any signal which is continuous at $t = t_0$

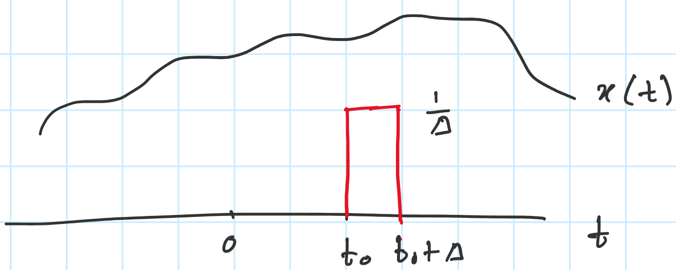




* $x(t)\,\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$

$\Delta \to 0$ $\qquad$ $\Delta \to 0$

$x(t)\, p_\Delta(t - t_0)$



(2) $$\int_{-\infty}^{\infty} x(t)\,\delta(t - t_0)\,dt = \int_{-\infty}^{\infty} x(t_0)\,\delta(t - t_0)\,dt$$

$$= x(t_0) \int_{-\infty}^{\infty} \delta(t - t_0)\,dt = x(t_0)$$

"Sifting" property

"Sifting" property of unit impulse