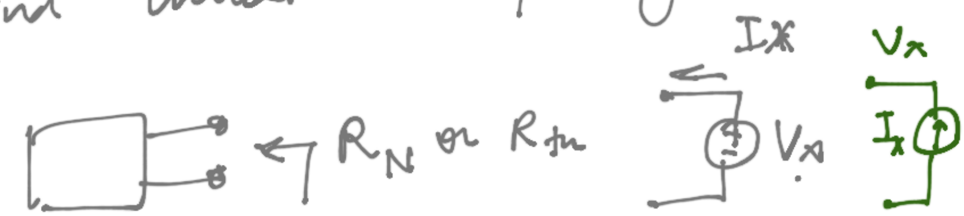
13-Jan

# Solving circuits with dependent current or voltage source

$R_{th}$ $V_{Th}$

$R_N$ $I_N$

- $R_{th}$, $R_N$ . Apply test voltage $V_x$ and find current $I_x$ flowing through it.




$$R_N,\ R_{Th} = \frac{V_x}{I_x}$$

- $V_{th}$, $I_N$


① ⓐ $V_{oc}$ →
ⓑ $I_{sc}$ →

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$


② Apply test voltage (1V) or current (1A) source.

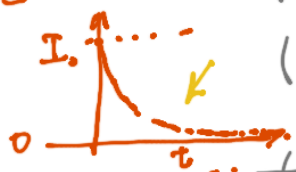
$$R_{th} = \frac{1}{I_x}$$ [when 1V test voltage applied]

# RL & RC

**Recap**

R-L (no source)

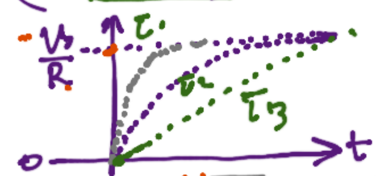
$$\tau = \frac{L}{R} \quad \Big\{ \quad I_L = I_0\, e^{-\frac{R}{L}t}$$




R-L (Voltage Source)

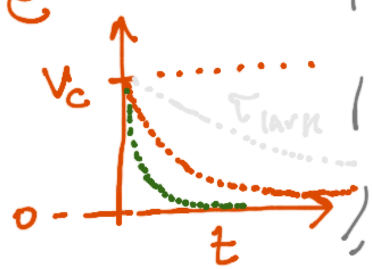
$$I_L = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

source voltage




R-C (no source)

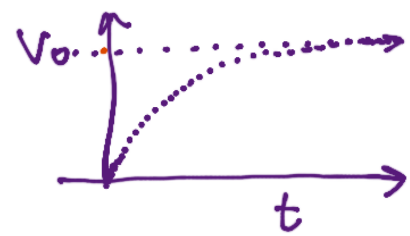
$$V_c = V_{c_0}\, e^{-\frac{t}{RC}}$$




R-C (Voltage Source)

$$V_c = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

source volt.




$\tau$ (time constant)

$\tau = \frac{L}{R}$ 'time' (R-L)

fast decay : $\tau$ small

Slow response $\tau$ large.

$\tau = RC$ 'time' (R-C)

$\tau_{small} \rightarrow$ fast

$\tau_{large} \rightarrow$ slow.

Eg. $(\tau_1 < \tau_2 < \tau_3)$

Characteristic equation :- Defines the circuit.

For any circuit let the following be it Charc. equation

$$\frac{di}{dt} + Pi = Q$$

$\boxed{P, Q \rightarrow \text{constants.}}$

* R-L or R-C type circuit will be $1^{st}$ O.D.E

$$di + Pi\,dt = Q\,dt \quad : \text{—} (1)$$

* $e^{-t/\tau}$

$e^{Pt} \rightarrow$ integration

$$(1) \Rightarrow \int e^{Pt}\,di + \int Pi\,e^{Pt}\,dt = \int Q\,e^{Pt}\,dt.$$

$$\Rightarrow \int_0^t d(ie^{Pt}) = Q\int e^{Pt}\,dt$$

$$\Rightarrow i\,e^{Pt} = \int Q\,e^{Pt}\,dt + A$$

$A$ ↳ integration constant

$$\Rightarrow \boxed{i = \underbrace{e^{-Pt}\int Q\,e^{Pt}\,dt}_{\text{Forced Response } 'i_f'} + \underbrace{Ae^{-Pt}}_{\text{Natural / Transient } 'i_n'}}$$

'Transient' = change with time

From prev. example: $\underbrace{K_1 e^{-t/\tau}}$ : Transient response (Even without $V_s$, $I_s$)

Natural Response of the circuit.

$$\boxed{i_n = A e^{-Pt}}$$

Natural

- 'P' is +ve R, L or C
- Natural response becomes '0' zero at $t \to \infty$

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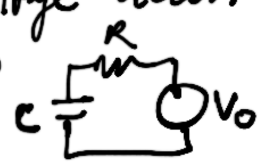
Forced Response (F.R)

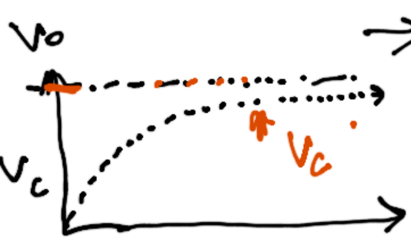
- Due to $V_s$ or $I_s$
- Final value after natural response die

F.R = Steady State Response. (s.s) (stays even $t \to \infty$)

$$V_c = V_0 \left(1 - e^{-t/RC}\right)$$

Voltage across cap




$$V_c = \underset{\uparrow}{V_0} - \underbrace{V_0 e^{-t/RC}}$$

Forced supply.

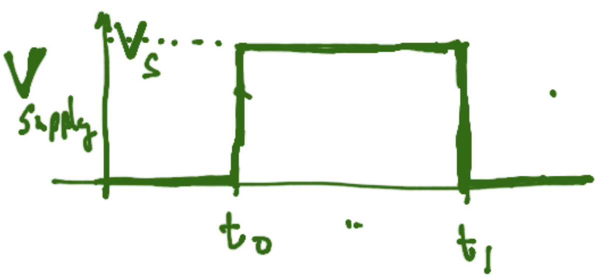

$V_0 e^{-t/RC} \to 0$ $(t \to \infty)$ Natural.

# Response of a circuit

$$i = i_f + i_n$$

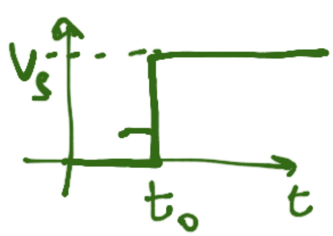
## Step Function

$$u(t) = \begin{cases} 0 & t < t_0 \\ V_s & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$

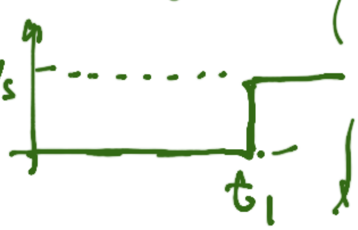



$\Downarrow$

$u(t-t_0)$

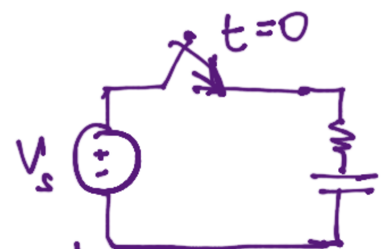



$u(t-t_1)$



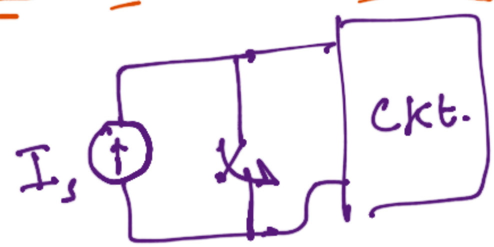

$\Downarrow$

$$V_{supply} = u(t-t_0) - u(t-t_1)$$

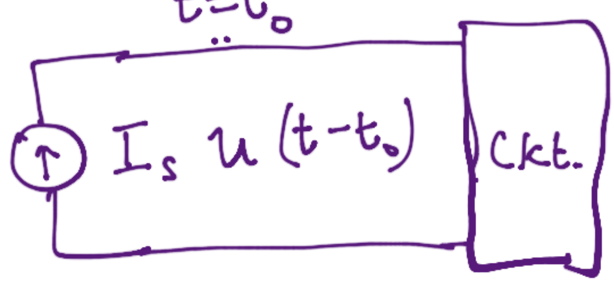



$\Downarrow$

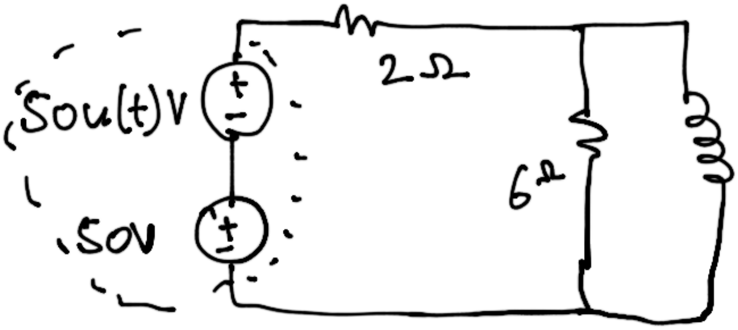

$\boxed{V_s\, u(t)}$



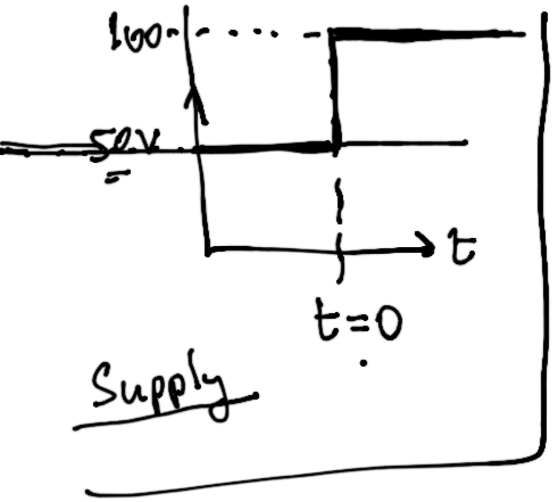

$t = t_0$



## Ex 8.8

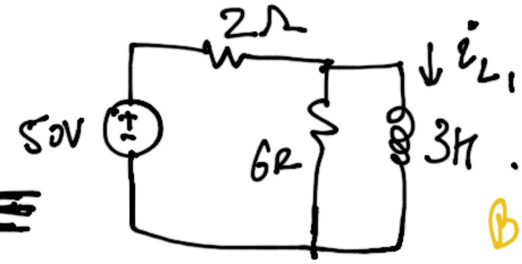



$i(t)$



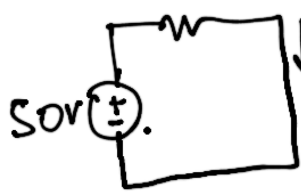

$\underbrace{t<0}$

[for extremely long time] $\equiv$



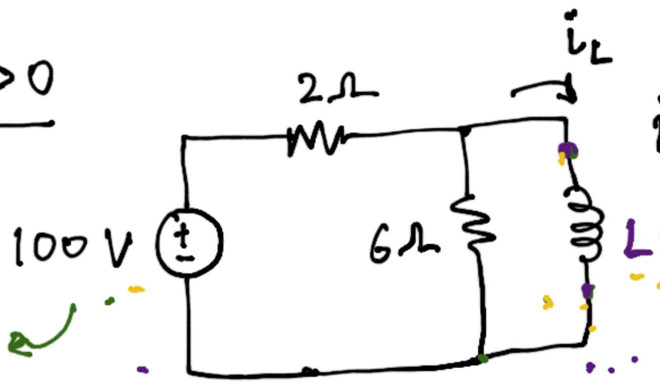

Before $t=0$

Inductor is like a S.C (short ckt)




$i_{L1} = \frac{50}{2} = \underline{\underline{25}}\,A$ : Initial condition

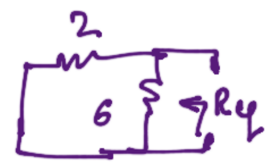
After $t>0$




$\underline{i_L(0) = 25A}$ → natural response ←

forced response

Let the Natural response be $i_n = A\, e^{-t/\tau}$ ←




'$\tau$' : $L=3H$, $R_{eq} = \frac{3}{2}\Omega \Rightarrow \tau = \frac{L}{R} = \frac{3}{3/2} = 2\,sec$

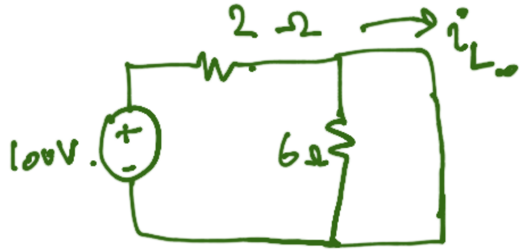
$R_{eq} = \frac{2\times6}{6+2} = \frac{12}{8} = \frac{3}{2}$

$i_n = A\, e^{-t/2}$

$i_f$ : Natural part will die & constant or S.S current will remain.

The S.S

$t \to \infty$




For forced response $(t \to \infty)$ $L \to$ S.C

$$i_{L\infty} = \frac{100}{2} = 50 \text{ Amp.} = i_f$$

: forced response current in 'L'

General Total response

$$i_L = i_f + i_N$$

$$i_L = 50 + A e^{-t/2} \longrightarrow \boxed{A}$$

$t=0$ $\quad i_L(0) = 25A$

$$i_L \Rightarrow 25 = 50 + A \Rightarrow A = -25$$

Finally putting 'A' back in $\boxed{A}$

$$\boxed{i_L = 50 - 25e^{-t/2}}$$