Lecture - 11

Thursday, 10 February 2022 8:13 AM

* response of some systems to unit impulse signal

* Linear systems : H is a linear system

If $x_1(t) \xrightarrow{H} y_1(t)$ & $x_2(t) \xrightarrow{H} y_2(t)$

Then, $\alpha x_1(t) + \beta x_2(t) \xrightarrow{H} \alpha y_1(t) + \beta y_2(t)$

* Time invariant systems :

delay operator : $x(t) \longrightarrow \boxed{D_\Delta} \longrightarrow x(t-\Delta)$

H is a time invariant system if :

(a) $x(t) \longrightarrow \boxed{D_\Delta} \longrightarrow \boxed{H} \longrightarrow y_1(t)$

&

(b) $x(t) \longrightarrow \boxed{H} \longrightarrow \boxed{D_\Delta} \longrightarrow y_2(t)$

(a) & (b) are the same systems (i.e. $y_1(t) = y_2(t)$)

Ex. (1) $y(t) = x(t-1)$ i.e. $x(t) \xrightarrow{H} x(t-1)$

(a) $x(t) \xrightarrow{D_\Delta} x(t-\Delta) \xrightarrow{H} y_1(t) = x(t-\Delta-1)$

(b) $x(t) \xrightarrow{H} x(t-1) \xrightarrow{D_\Delta} y_2(t) = x(t-1-\Delta)$

$\Big\}$ $y_1(t) = y_2(t)$

$\Rightarrow$ System is TI

Ex. 2 $y(t) = t\,x(t)$ *

a) $x(t) \xrightarrow{D_\Delta} x(t-\Delta) \xrightarrow{H} y_1(t) = t\,x(t-\Delta)$

b) $x(t) \xrightarrow{H} t\,x(t) \xrightarrow{D_\Delta} y_2(t) = (t-\Delta)\,x(t-\Delta)$

$y_1(t) \neq y_2(t) \Rightarrow$ system is time variant

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = t\,x(t)$$

$$x(t-\Delta) \longrightarrow \boxed{H} \longrightarrow t\,x(t-\Delta) = y(t)$$

$$t\,x(t) = g(t) \longrightarrow \boxed{D_\Delta} \longrightarrow g(t-\Delta) = (t-\Delta)\,x(t-\Delta) \quad *$$

Ex. $y(t) = g(t)\,x(t)$

$$x(t) \longrightarrow \boxed{H} \longrightarrow g(t)\,x(t)$$

$$x(t-\Delta) = z(t) \longrightarrow \boxed{H} \longrightarrow g(t)\,z(t) = g(t)\,x(t-\Delta) \quad *$$

★ Impulse input & LTI systems

H is an LTI system ↓ Linear & time-invariant

$$\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$$

$$\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow h(t-\tau) \quad \dots (TI)$$

$$x(\tau)\,\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow x(\tau)\,h(t-\tau) \qquad \ldots (L)$$

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \longrightarrow \boxed{H} \longrightarrow \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

using "sifting" prop. $\quad x(t)$ $\qquad$ ... using (linearity & TI)

For LTI : $\quad \delta(t) \xrightarrow{H} h(t)$ → impulse response

for any arbitrary input $\longrightarrow x(t) \xrightarrow{H} \displaystyle\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = y(t)$

τ is dummy variable, can use any other notation *

NOTE

$\delta(t-1)$ & $\delta(1-t)$ are same signals.*

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(r)\,h(t-r)\,dr$$

* Convolution Integral : $\qquad y(t) = x(t) * h(t)$

Convolution operator

* properties of convolution operator :

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

prove it by change of variables (HW)

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$\therefore$ convolution operator is commutative

Interpretation

role of signal & system can be interchanged

$$h(t) \longrightarrow \underset{\text{LTI}}{\boxed{x(t)}} \longrightarrow y(t) = x(t) * h(t)$$

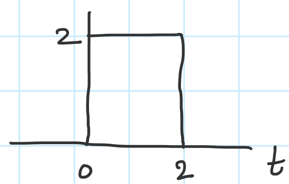
impulse response

$$\longrightarrow \boxed{H} \longrightarrow \quad \text{or} \quad \longrightarrow \underset{\text{LTI}}{\boxed{h(t)}} \longrightarrow$$

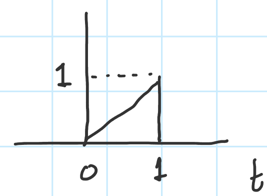
we will use either notation

* Convolution integral example

1. consider $x(t) \equiv$



& $h(t) \equiv$



Find $y(t) = x(t) * h(t)$