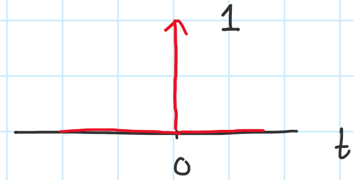
Lecture - 10

Saturday, 5 February 2022 8:48 AM

* Unit impulse signal $\delta(t)$: defined as follows

① $\delta(t) = 0 \quad \forall\, t \neq 0$

② $\int_{-\infty}^{\infty} \delta(t) = 1$



* $\delta(t)$ sifting property

① $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

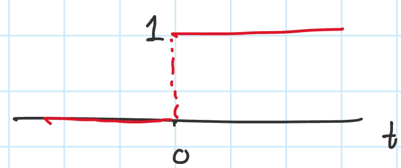
② $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$ .... "sifts" out $x(t_0)$

* Unit step signal $u(t)$

integrator

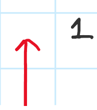
$$\delta(t) \longrightarrow \boxed{A} \longrightarrow u(t)$$

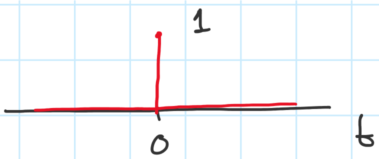
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

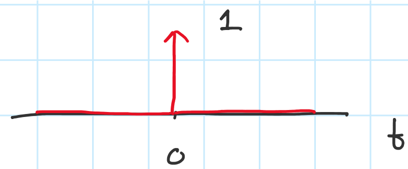


* Note : In signals & systems, we are always interested in signals with non-zero area / energy

$f(t)$ vs $\delta(t)$





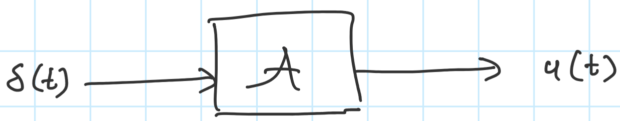


$$\int_{-\infty}^{\infty} f(t)\,dt = 0 \qquad\qquad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

($\therefore$ not of practical interest in S&S)

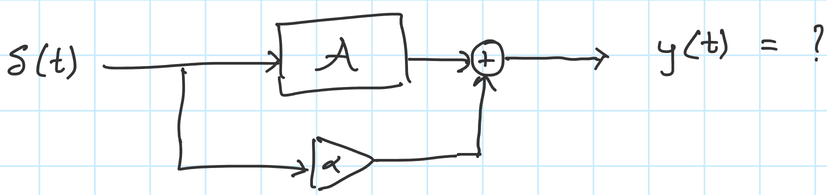
* Response of systems to unit impulse :

①



$\delta(t) \longrightarrow u(t)$

②

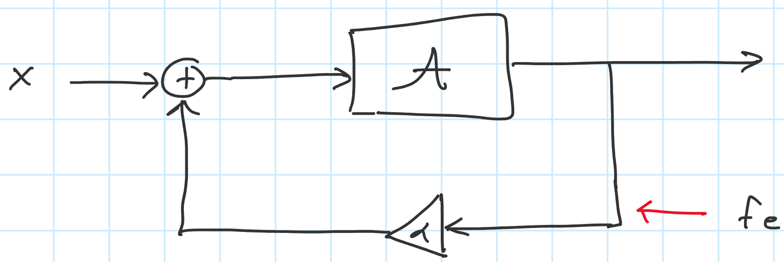


operator : $Y = Ax + \alpha x = (A+\alpha)x$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau + \alpha x(t) \quad *$$

$$y(t) = u(t) + \alpha\,\delta(t)$$

③



← feedback arm

operator : $y = \mathcal{A}[x + \alpha y] = \mathcal{A}x + \alpha \mathcal{A} y$

$$y(1 - \alpha\mathcal{A}) = \mathcal{A}x$$

assume this is valid!

$$* \quad y = \frac{\mathcal{A}}{1 - \alpha\mathcal{A}} x$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$

$$\frac{1}{1-\alpha\mathcal{A}} = 1 + \alpha\mathcal{A} + \alpha^2\mathcal{A}^2 + \cdots$$

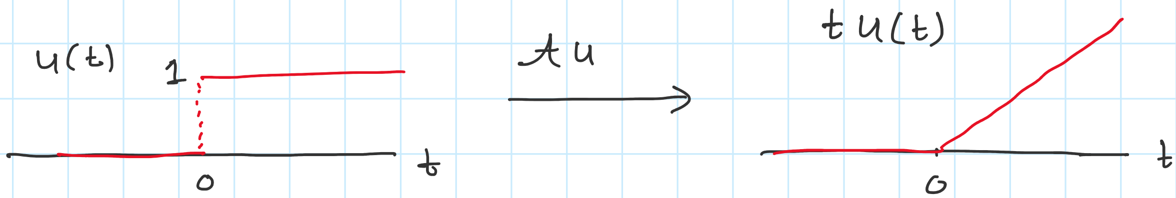
$$\Rightarrow \quad y = \mathcal{A}\left(1 + \alpha\mathcal{A} + \alpha^2\mathcal{A}^2 + \alpha^3\mathcal{A}^3 + \cdots\right) x$$

$$= \left(\mathcal{A} + \alpha\mathcal{A}^2 + \alpha^2\mathcal{A}^3 + \cdots\right) x$$

$$x(t) = \delta(t)$$

$$\mathcal{A}\delta = u(t)$$

$$\mathcal{A}^2\delta = \mathcal{A}(\mathcal{A}\delta) = \mathcal{A}(u(t)) = t\,u(t)$$



$$\mathcal{A}^3\delta = \mathcal{A}(\mathcal{A}^2\delta) = \mathcal{A}(t\,u(t)) = \frac{t^2}{2} u(t)$$

$$\vdots$$

$$\mathcal{A}^k\delta = \frac{t^{k-1}}{\quad} u(t)$$ .... (verify . HW)

$(k-1)!$

$$y(t) = u(t) + \alpha t\, u(t) + \alpha^2 \frac{t^2}{2} u(t) + \cdots \alpha^k \frac{t^k}{k!} u(t) + \cdots$$

$$= u(t) \left[1 + \alpha t + \frac{\alpha^2 t^2}{2!} + \frac{\alpha^3 t^3}{3!} + \cdots \right]$$

$$y(t) = e^{\alpha t} u(t)$$

$$\underset{\text{(input)}}{\delta(t)} \xrightarrow{\text{(system)}} \underset{\text{(output)}}{e^{\alpha t} u(t)}$$

* α - comes from feedback part

* Linear and time-invariant systems *

(a) Linearity (L)

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

If system is linear

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t) = \alpha y_1(t) + \beta y_2(t)$$

In general: $\sum_i \alpha_i x_i(t) \longrightarrow \sum_i \alpha_i y_i(t)$

Ex. ① $y(t) = c\,x(t)$ ② $y(t) = x(t - t_0)$

③ $y(t) = a\,x^2(t)$

ⓑ Time - Invariance (TI)

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

consider the delay operator

$$x(t) \longrightarrow \boxed{D_\Delta} \longrightarrow x(t-\Delta)$$

* A system is TI if the following systems have same output

① $x(t) \longrightarrow \boxed{D_\Delta} \longrightarrow \boxed{H} \longrightarrow y_1(t)$

② $x(t) \longrightarrow \boxed{H} \longrightarrow \boxed{D_\Delta} \longrightarrow y_2(t)$

* If $\boxed{H}$ and $\boxed{D_\Delta}$ blocks commute,

then the system $\boxed{H}$ is said to be time-invariant *