Lecture - 13

Tuesday, 15 February 2022 8:48 AM

* LTI systems & impulse response

* Convolution operator & its properties

* computing the convolution integral using the steps of (flip , shift , multiply , integrate)

* interconnection of LTI systems
  1. cascade
  2. parallel

* System properties

  1. Linear systems

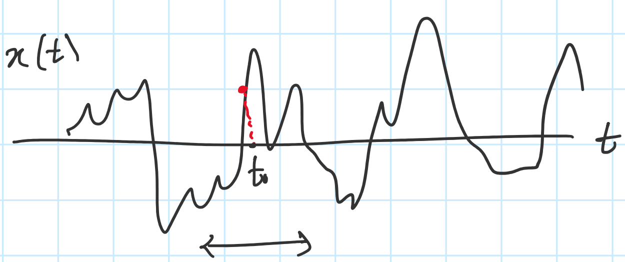
  2. Time-invariant systems

  3. Causal systems

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

* A system is said to causal if the current output depends only on current & past inputs
  (i.e. it does not depend on future inputs)

$$y(t) = f\left(\{x(\tau) \mid \tau \leq t\}\right)$$



$$y(t_0) = \left\{ \begin{matrix} t - t_0 \\ t + t_0 \end{matrix} \right\} \text{ of } x(t)$$

Ex. 1 $y(t) = x(-t)$ non-causal system $\left\{ \begin{array}{l} t = -1 \\ y(-1) = x(1) \end{array} \right.$

2 $h(t) = \delta(t-1)$

* $y(t) = x(t) * \delta(t-1) = x(t-1)$ causal system

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau-1)\,d\tau = x(t-1)$$

In general: $x(t) * \delta(t-t_0) = x(t-t_0)$

3 $y(t) = x(t-1)$ causal system

4 $h(t) = \delta(t+1)$ $x(t) \longrightarrow x(t+1) = y(t)$

non-causal system

note: $$x(t) * \underbrace{\delta(t-t_0)}_{h(t)} = \int_{-\infty}^{\infty} x(\tau)\,\underbrace{\delta(t-t_0-\tau)}_{h(t-\tau)}\,d\tau = x(t-t_0)$$

* non-causal systems can also be useful.

4 Stable systems

* A system is said to be stable if for bounded inputs the output is also bounded.

i.e. Bounded input bounded output (BIBO) stability

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

i.e. if $|x(t)| \le B < \infty \quad \forall t$, then for BIBO

Stability $\Rightarrow \quad |y(t)| \le M < \infty \quad \forall\, t$

Ex.

1. $y(t) = e^{x(t)} \qquad |x(t)| \le B$

$\Rightarrow |y(t)| \le e^{B} = M \qquad \therefore$ Stable

2. $y(t) = t\, x(t) \qquad |x(t)| \le B$

$y(t)$ cannot be bounded $\therefore$ unstable

3. $x(t) \longrightarrow \boxed{A} \longrightarrow y(t)$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{unstable}$$

★ Laplace transform

recall FS : basis signals — Sinusoids
complx. Sinusoids.

$$x(t) \xleftrightarrow{FS} \{a_k, b_k\} \text{ or } \{c_k, \theta_k\} \text{ or } \{d_k\}$$

coefficients $\longrightarrow$ find by comparing $x(t)$ with basis signals
i.e. analysis equation

*Laplace transform is for any general signals (not restricted to periodic signals)

Ex. LTI or not ? $\quad y(t) = \mathcal{A}\{x(t)\} = \int_{-\infty}^{t} x(\tau)\,d\tau$

linear ✓ $\because$ integration is linear

time-invariance ✓ $\because$ ⓐ $x(t-t_0) \longrightarrow y_1(t) = \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$

ⓑ $y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\,d\tau$ same integral

$\therefore$ system is LTI.