Lecture - 07

Thursday, 9 December 2021 8:37 AM

* Exponential FS : $x(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$ $\omega_0 = \frac{2\pi}{T}$

$k\omega_0$ frequency of complex sinusoid

* $d_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt$

Q. $x_1(t) = 3\cos(t) - 2\cos(3t) = \frac{3}{2}\left(e^{jt} + e^{-jt}\right) - \frac{2}{2}\left(e^{3jt} + e^{-3jt}\right)$

$T = 2\pi \Rightarrow \omega_0 = 1 \Rightarrow x_1(t) = \sum_{-\infty}^{\infty} d_k e^{jkt}$

$d_0 = 0$, $d_1 = 3/2$, $d_{-1} = 3/2$, $d_2 = d_{-2} = 0$, $d_3 = -1$, $d_{-3} = -1$ } 4 non-zero coeff.

* Fourier spectrum
  - magnitude spectrum $|d_k|$
  - phase spectrum $\measuredangle d_k$

* Time-domain $[x(t)]$ and frequency-domain $[d_k]$

$x(t) \xleftrightarrow{FS} d_k$

* Properties of Fourier series :

given $x(t) \xleftrightarrow{FS} d_k$ *

* $x(t) = \sum_{-\infty}^{\infty} d_k e^{jk\omega_0 t}$

① Amplitude scaling : $\alpha x(t) \xleftrightarrow{FS} \alpha d_k$

② Linearity : $\alpha x_1(t) + \beta x_2(t) \xleftrightarrow{FS} \alpha d_k + \beta f_k$

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$x_1(t) \downarrow d_k \qquad x_2(t) \downarrow f_k$

③ Time shift : $x(t - t_0) \xleftrightarrow{FS} e^{-jk\omega_0 t_0} d_k$

magnitude remains same i.e. $|e^{-jk\omega_0 t_0} d_k| = |d_k|$

only phase spectrum changes

$$\left.\begin{aligned} |e^{j\theta}| &= 1 \\ |z_1 z_2| &= |z_1||z_2| \end{aligned}\right\}$$

④ Time reversal : $x(-t) \xleftrightarrow{FS} d_{-k}$

⑤ Time scaling : $x(at) \xleftrightarrow{FS} d_k$ ..... $\left(\text{period} = \frac{T}{a}\right)$

$a > 0$

⑥ Multiplication : $x_1(t)\, x_2(t) \xleftrightarrow{FS} \sum_{n=-\infty}^{\infty} d_n f_{k-n}$

$x_1(t) \downarrow d_k \qquad x_2(t) \downarrow f_k$

⑦ Parsevals relation : Statement about average power

avg. power : $\frac{1}{T} \int_{\langle T \rangle} |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |d_k|^2$

★ Convergence of Fourier series :

$$x(t) = \sum_{-\infty}^{\infty} d_k e^{jk\omega_0 t} \quad \text{.... (infinite sum)}$$

Q. when does the summation converge ?

Q. do ALL periodic signals have FS representation ?

* Dirichlet conditions : (for convergence of FS)

1. $x(t)$ should be absolutely integrable in a period

$$\int_{\langle T \rangle} |x(t)| \, dt < \infty$$

2. $x(t)$ should have finite number of maxima & minima in a period

3. $x(t)$ should have finite num. of discontinuities

Q. what happens at points of discontinuity ?

let $x(t)$ be discontinuous at $t = t_0$

$$\sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t_0} = \frac{1}{2} \left\{ \lim_{t \to t_0^+} x(t) + \lim_{t \to t_0^-} x(t) \right\}$$

$$x(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

* Gibbs phenomenon : at points of discontinuity, oscillations move towards discontinuity, thier amplitudes don't decrease but energy decreases (of oscillations)