PHY306 Advanced Quantum Mechanics Jan-Apr 2025: Assignment 7

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1. Find an upper bound on the ground-state energy of the one-dimensional infinite square well using the triangular trial wave function

$$\begin{aligned} \psi(x) &= Ax, \;\; 0 \leq x \leq a/2, \\ &= A(a-x), \;\; a/2 < x < a, \\ &= 0, \;\; \text{otherwise} \end{aligned}$$

   Find A from the normalization.

2. Find the best bound on E_g for the delta-function potential $-\alpha\delta(x-a/2)$ using the triangular trial wave function given above, where a is an adjustable parameter.

3. Assume a Yukawa potential of the form

$$V(r) = -\frac{e^2 e^{-\mu r}}{4\pi\epsilon_0 r}$$

   where $\mu = m_\gamma c/\hbar$. Choose any trial wave function you want to and estimate the binding energy of a "hydrogen" atom with this potential. Assume $\mu a << 1$ and give your answer correct to order $(\mu a)^2$.

4. Consider a particle of mass m moving under a central force potential $V(r) = -\alpha e^{-2\mu r}$, where $\alpha, \mu > 0$. Use a variational trial wavefunction $\psi_\lambda(r) = Ce^{-\lambda r}$. Find C using the normalization of the wavefunction. Using the variational method find the lowest energy. A useful integral is $\int_0^\infty dx x^n e^{-x} = n!$.

5. Prove the corollary to the variational principle that if $\langle\psi|\psi_g\rangle = 0$, then $\langle H\rangle \geq E_f$ where E_f is the energy of the first excited state. What kinds of trial functions should one use in order to get an upper bound on the first excited state?

6. Find the best bound on the first excited state of the one-dimensional harmonic oscillator using the trial function $\psi(x) = Axe^{-bx^2}$

7. Use the variational principle to prove that first-order nondegenerate perturbation theory always overestimates (or never underestimates) the ground state energy. Use this to confirm that second-order correction to the ground state is always negative.

8. Consider a two-level system with unperturbed Hamiltonian H_0 and eigenstates ψ_a, ψ_b and $E_a < E_b$. Turn on a perturbation H' with diagonal elements 0 and off-diagonal elements h. Estimate the ground-state energy of the perturbed system using the variational principle with a trial wavefunction $\psi = \cos\phi\psi_a + \sin\phi\psi_b$, where ϕ is an adjustable parameter. Now consider an electron at rest in a uniform magnetic field $B = B_z\hat{k}$ with the Hamiltonian $H_0 = \frac{eB_z}{m}S_z$ with eigen spin wave functions χ_a, χ_b (which are eigenstates of S_z) and energies E_a, E_b. Turn on a perturbation which is a uniform field in the x direction with the Hamiltonian $H' = \frac{eB_x}{m}S_x$. Find the matrix elements of H' and what is h in this case? Use the above more general results to find the variational bound on the ground-state energy.