IDC102:Hands-on Electronics
Lecture – 6

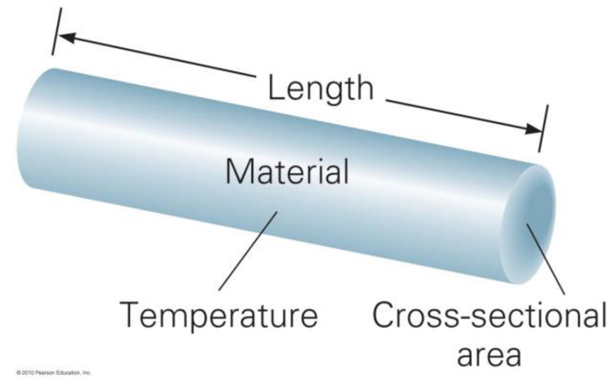
Part-I
Resistor, Capacitor and Inductor Circuits

IISER

Satyajit Jena, 06/06/2022

Review: On what will a wire's resistance depend?

- There are 4 primary factors when determining a wire's resistance:
  - Material composition
  - Length of the wire
  - Cross-sectional Area of the wire
  - Temperature



$$R = \frac{\rho \times length}{section}$$

$$\sigma = \frac{1}{\rho}$$

Electrical Resistivity =

$$\rho = \frac{m_e v_F}{ne^2 \ell} = \frac{m_e}{ne^2\ \tau}$$

n: the number of electrons per unit volume

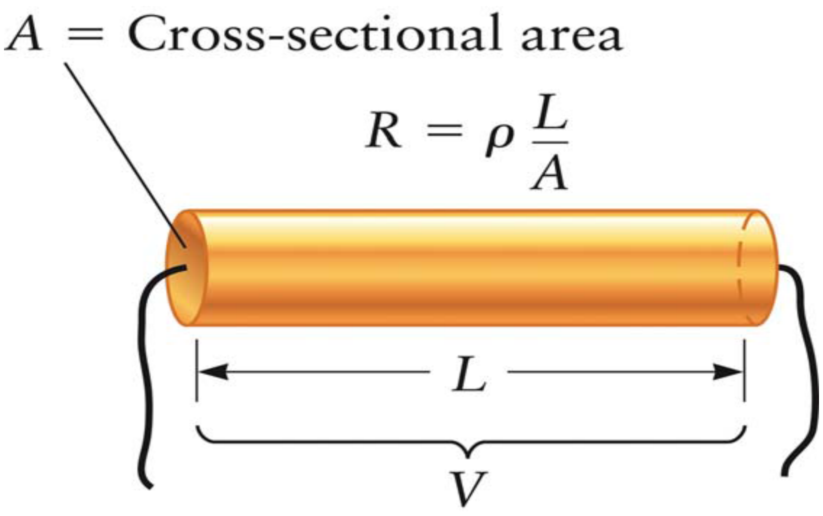
e: the charge carried by an electron

m_e: the mass of an electron

v_F: the average velocity of "conduction electrons"

ℓ: the average distance the electrons travel before being scattered by atomic lattice perturbation (the mean free path)

Review: What determines resistance R for a wire?



Units: L (m); A(m^2); ρ(Ω·m); R(Ω)

Some materials, like metals, offer little resistance to current flow. Other materials, like plastic, offers high resistance to current flow.

Resistivity is used to quantify how much a given material resists the flow of current.

Resistivity is a property of a material.

$$R \propto \frac{L}{A}$$

RESISTIVITIES (Ω · m), at 20° C

Conductors	
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminum	2.7×10^{-8}
Iron	9.6×10^{-8}
Platinum	10.5×10^{-8}
Nichrome	107.5×10^{-8}

Semi-conductors	
Carbon	3.5×10^{-5}
Silicon	2.5×10^{3}

Insulators	
Glass	10^{10} - 10^{14}
Rubber	1.0×10^{13}

Review: Resistor

KVL – Kirchhoff's Voltage Law

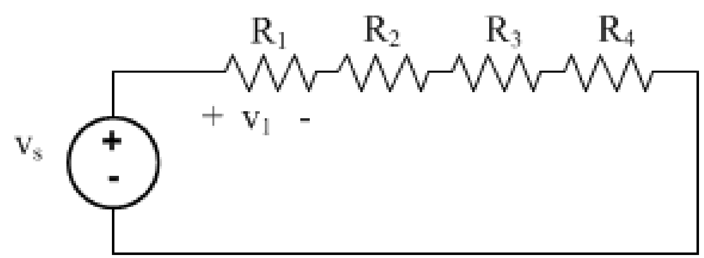
The sum of the voltage drops around a closed path is zero.

KCL – Kirchhoff's Current Law

The sum of the currents leaving a node is zero.

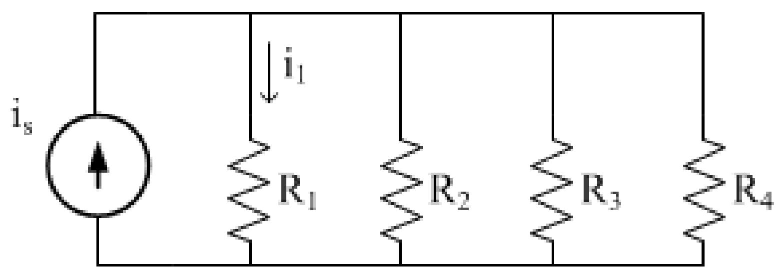
Voltage Divider

$$v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_s$$



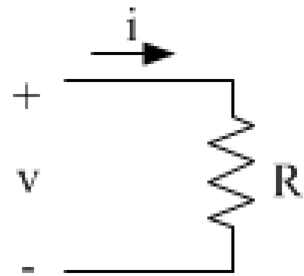
Current Divider

$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} i_s$$



Review: Resistor

Resistor



$$v = Ri$$

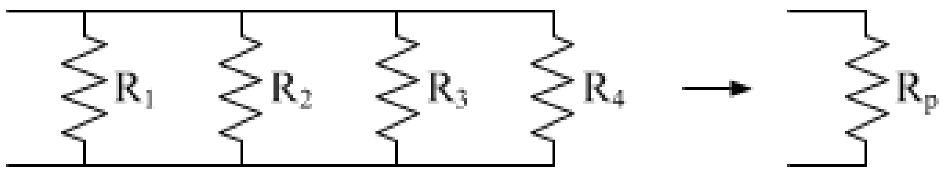
Power absorbed

$$p = vi = Ri^2 = \frac{v^2}{R}$$

Energy dissipated

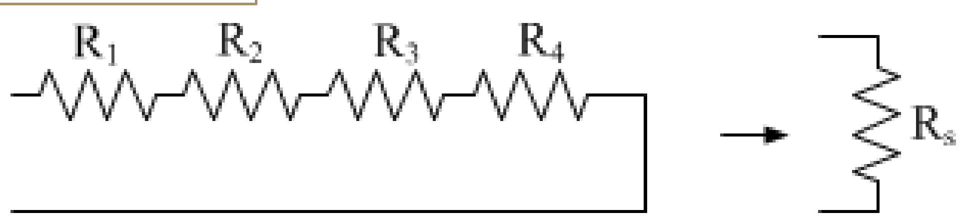
$$w = \int p\,dt = \int_{\text{one period}} p\,dt$$

Parallel Resistors



$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

Series Resistors

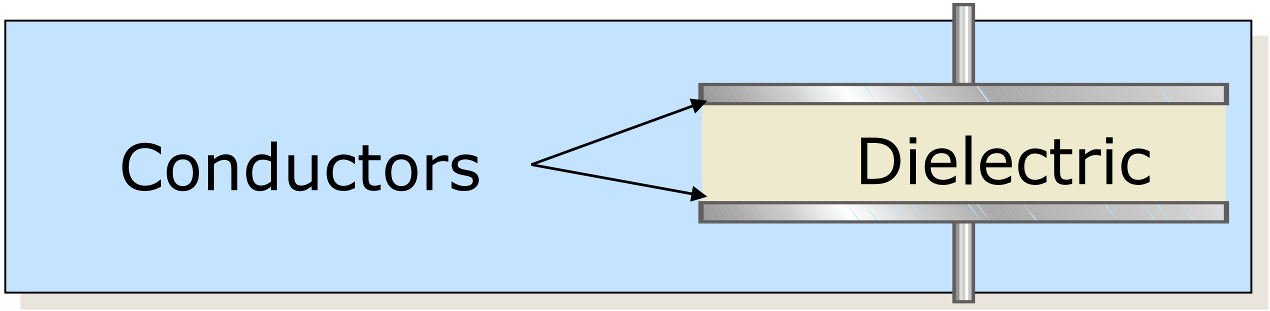


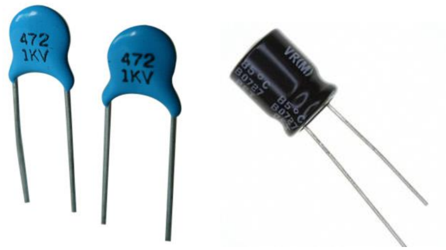
$$R_s = R_1 + R_2 + R_3 + R_4$$

Capacitor:

Electrical device composed of two parallel conductive plates separated by an insulating material (dielectric).

The ability to store charge is the definition of capacitance.





Capacitance is the ratio of charge to voltage

$$C = \frac{Q}{V}$$

C = Capacitance (ability to store charge)

Q = Charge (coulombs)

V = Volts

The amount of charge on a capacitor is determined

$$Q = CV$$

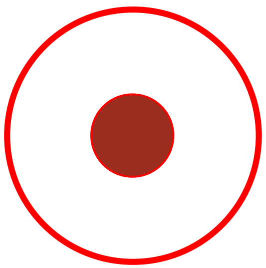
Review: Capacitor Summary

- Any two charged conductors form a capacitor.

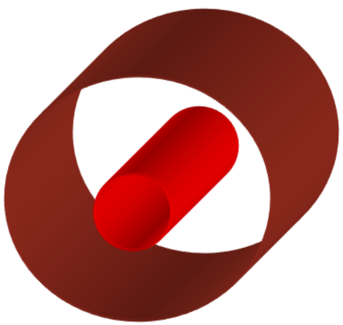
- Capacitance : C= Q/V

- Simple Capacitors:

Parallel plates: $C = \varepsilon_0 A/d$



Spherical : $C = 4\pi\, \varepsilon_0\, ab/(b\text{-}a)$



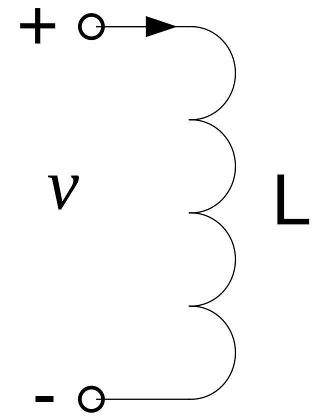
Cylindrical: $C = 2\pi\, \varepsilon_0\, L/\ln(b/a)$

ε_0 is the permittivity of free space, it is an ideal physical constant that represents the absolute dielectric permittivity of a vacuum. => ability of a vacuum to allow electric field lines to flow through => It is approximately 8.854×10^{-12} farads per meter

Review: Inductor

An inductor is a passive element that stores energy in its magnetic field. Generally. An inductor consists of a coil of conducting wire wound around a core. For the inductor

$$v(t) = L\frac{di(t)}{dt}$$

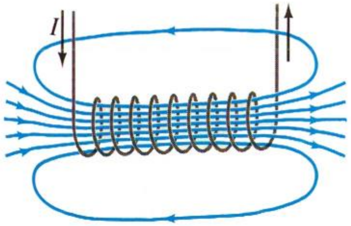


where L is the inductance in henrys (H), and 1 H = 1 volt second/ampere.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.

Inductors: solenoids

Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They "pack a lot of field in a small region". Also, the higher the current, the higher the magnetic field they produce.



Capacitance → how much **potential** for a given charge: **Q=CV**

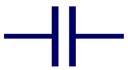
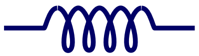
Inductance → how much **magnetic flux** for a given current: **Φ=L*i***

Using Faraday's law:

$$EMF = -L\frac{di}{dt}$$

$$\text{Units}: \quad [L] = \frac{\text{Tesla}\cdot\text{m}^2}{\text{Ampere}} \equiv \text{H} \;\; (\text{Henry})$$

Joseph Henry (1799-1878)

CAPACITOR	INDUCTOR
Has a capacity to "hold" charge, called capacitance… $C = \dfrac{Q}{\Delta V}$ [$^{C}/_{V}$ = farad, F] …determined by its geometry: $C = \varepsilon_0 \dfrac{A(N-1)}{d}$	Has an ability to "hold" flux, called inductance… $L = \dfrac{\Phi_m}{I}$ [$^{Wb}/_{A} = {}^{T\,m^2}/_{A}$ = henry, H] …determined by its geometry: $L = \mu_0 \dfrac{AN^2}{\ell}$
Circuit diagram symbol: 	Circuit diagram symbol: 
Stores energy in its electric field: $U_C = ½\, C(\Delta V)^2$ $u_E = \dfrac{\varepsilon_0}{2} E^2$	Stores energy in its magnetic field: $U_L = ½\, LI^2$ $u_B = \dfrac{1}{2\mu_0} B^2$
Regulates current through a series resistor… …at switch on: $I = \dfrac{\mathcal{E}}{R} e^{-t/\tau}$ …and switch off: $I = I_0 e^{-t/\tau}$	…at switch on: $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$ …and switch off: $I = I_0 e^{-t/\tau_L}$

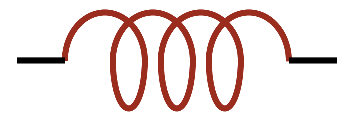
Review: Resistor (R), Capacitor(C), Inductor (L)

Relation	Resistor (R)	Capacitor(C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$

Resistor (R), Capacitor(C), Inductor (L) => Now onward we will refer to as R, C, L

R, C, R in circuits in the next slide

Review: C, R, L in circuits

	Capacitor 	Resistor 	Inductor 
Series	$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$	$R=R_1+R_2$	$L=L_1+L_2$
Parallel	$C=C_1+C_2$	$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$	$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}$
Fundamental Formula	$\Delta V=\frac{Q}{C}$	$\Delta V=IR$	$\mathrm{E}_L=-L\frac{dI}{dt}$

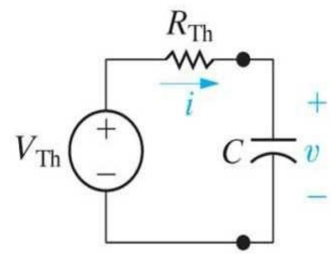
Review: units for L, R and C

Quantity	Symbol	SI Unit	Symbol for the Unit
Current	I or i	ampere	A
Voltage	V or v	volt	V
Resistance	R	ohm	Ω
Charge	Q or q	coulomb	C
Time	t	second	s
Energy	W or w	joule	J
Power	P or p	watt	W
Conductance	G	siemens	S
Capacitance	C	farad	F
Inductance	L	henry	H

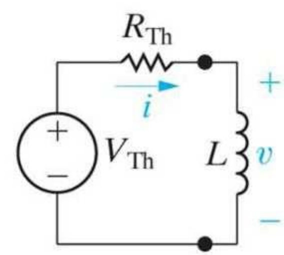
Combined Circuit of L, R, C

Inductors and capacitors can be charged or discharged through a resistive element

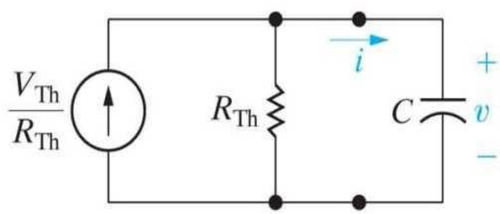
With a Source



(c)



(a)



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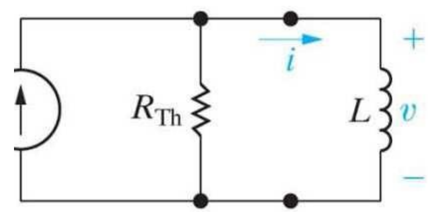
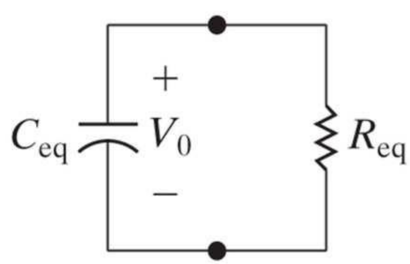
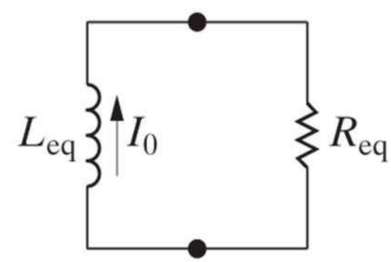


(b)

Charge

Without Source, but energy is stored

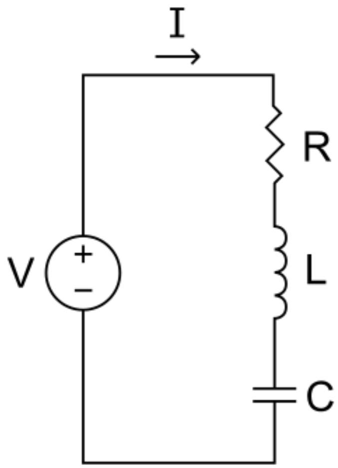


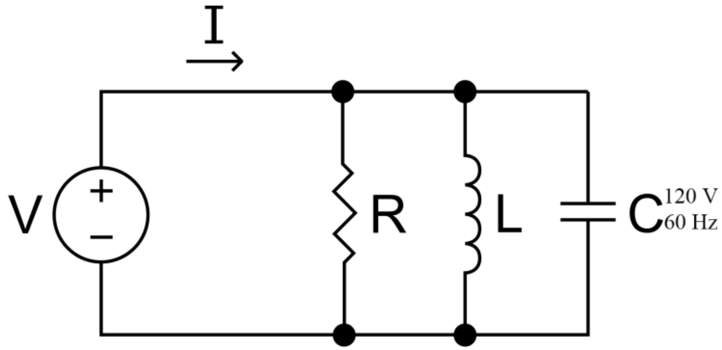


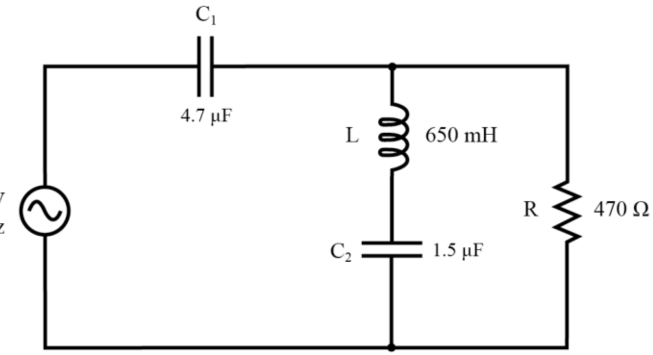
Discharge

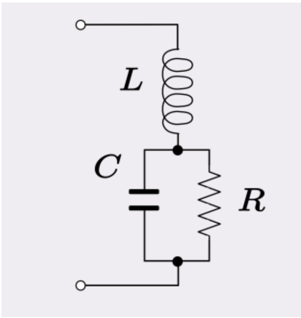
We can also have L, R, and C together

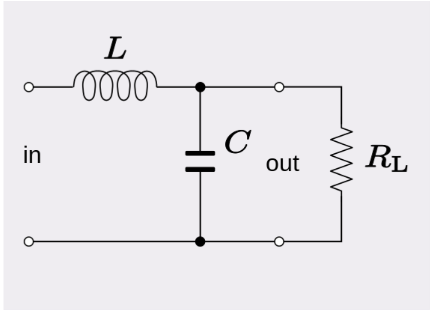
Combined Circuit of L, R, C

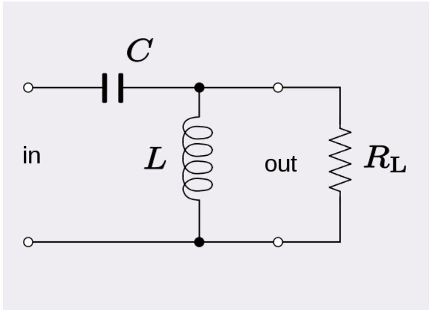


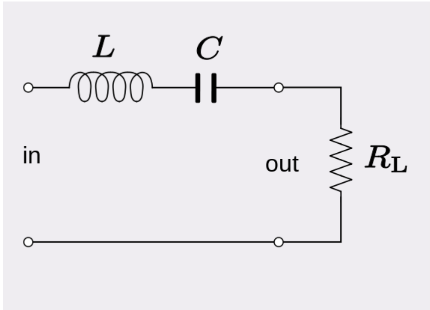


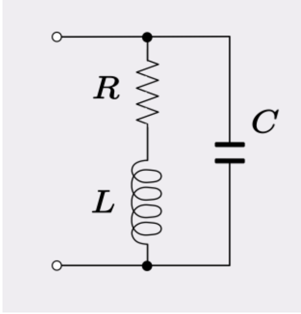


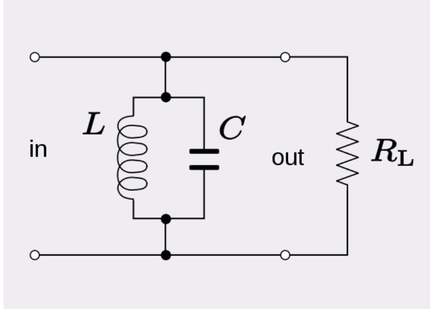


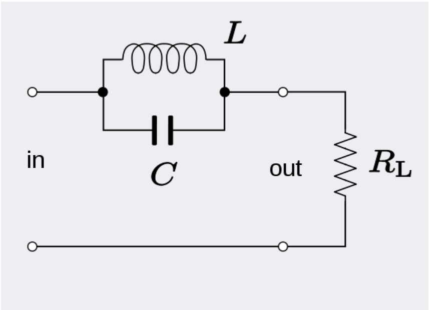


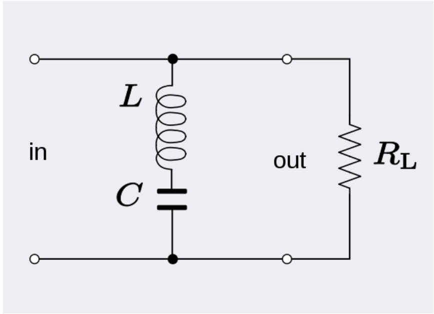




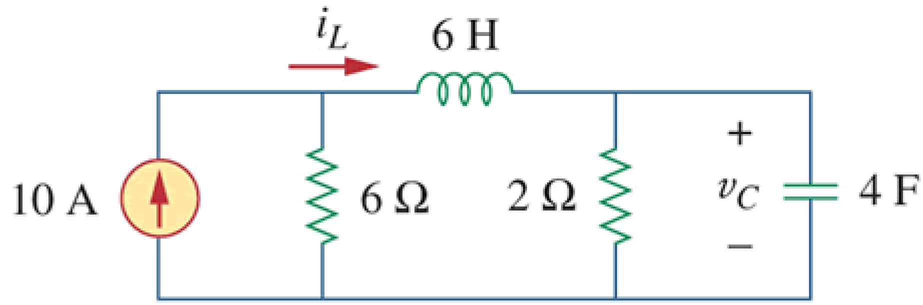








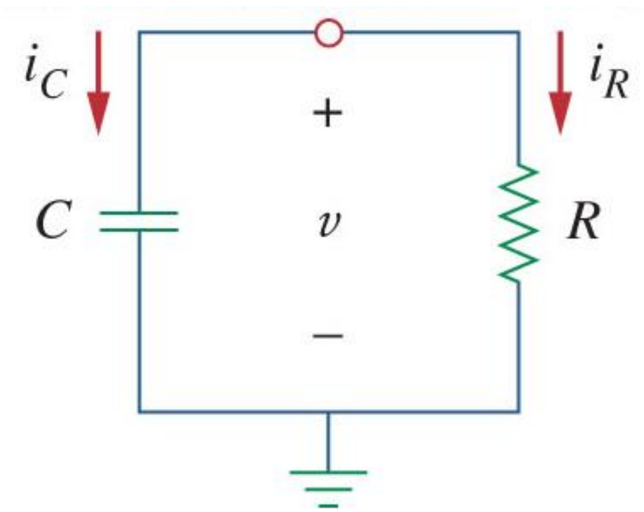
DC Conditions in a Circuit with Inductors or Capacitors



- Recall that when power is first applied to a dc circuit with inductors or capacitors, voltages and currents change briefly as the inductors and capacitors become energized.
- But once they are fully energized (i.e., "under dc conditions"), all voltages and currents in the circuit have constant values.
- To analyze a circuit under dc conditions, replace all capacitors with open circuits and replace all inductors with short circuits.

Circuit Analysis: R and C

- Consider the circuit shown. Assume that at time $t=0$, the capacitor is charged and has an initial voltage, V_0.
- As time passes, the initial charge on the capacitor will flow through the resistor, gradually discharging the capacitor.



- This results in changing voltage $v(t)$ and currents $i_C(t)$ and $i_R(t)$, which we wish to calculate.

Circuit Analysis: R and C

- Applying KCL,

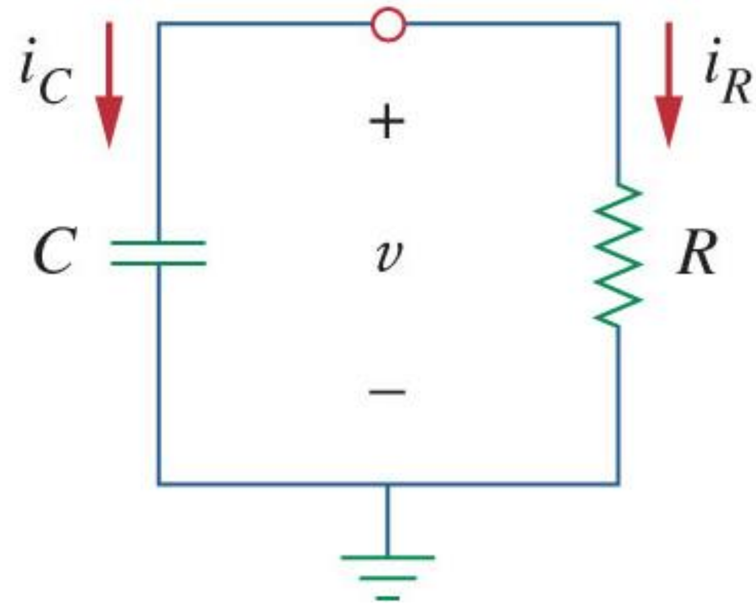
$$i_C + i_R = 0$$

- Therefore

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

- Therefore

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$



- This equation is an example of a **first-order differential equation**. How do we solve it for *v(t)*?

Circuit Analysis: R and C

- Note that our equation, $\frac{dv}{dt} + \frac{v}{RC} = 0$, contains two constants, R and C.
- It also contains two variables, v and t.
- Also, t is the independent variable, while v is the dependent variable. We sometimes indicate this by writing $v(t)$ instead of just v.
- Our goal is to write down an equation that expresses $v(t)$ in terms of t

- To solve our equation, $\frac{dv}{dt} + \frac{v}{RC} = 0$, use a technique called **separation of variables**.
- First, separate the variables v and t:

$$\frac{dv}{v} = -\frac{dt}{RC}$$

- Then integrate both sides:

$$\ln v = -\frac{t}{RC} + \ln A$$

- Then raise *e* to both sides:

$$v(t) = Ae^{-\frac{t}{RC}}$$

Circuit Analysis: R and C

- At this point we have:

$$v(t) = Ae^{-\frac{t}{RC}}$$

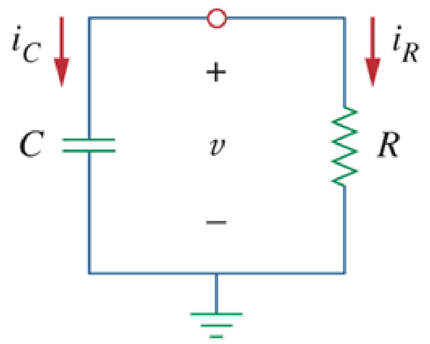
- The last step is to note that if we set t equal to 0, we get:

$$v(0) = A$$

- But we assumed earlier that the initial voltage is some value that we called V_0. So A must be equal to V_0, and therefore:

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

- I won't expect you to be able to reproduce the derivation on the previous slides.
- The important point is to realize that whenever we have a circuit like this



the solution for $v(t)$ is:

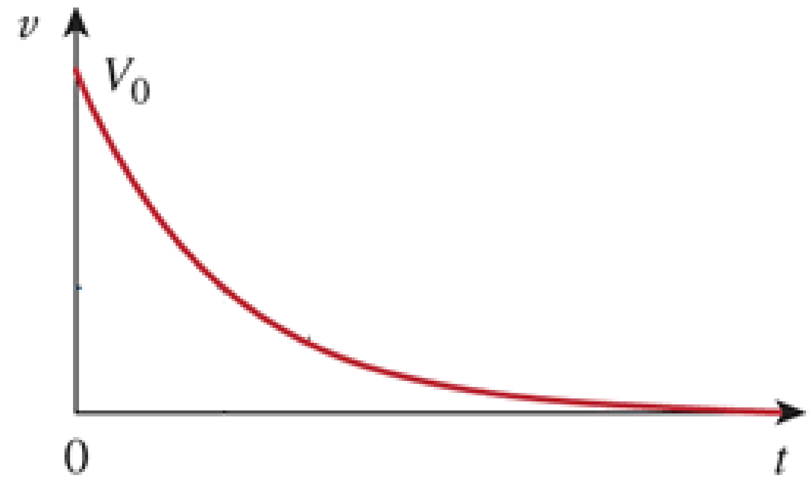
$$v(t) = V_0 e^{-\frac{t}{RC}}$$

Circuit Analysis: R and C: Graph

- Here's a graph of

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

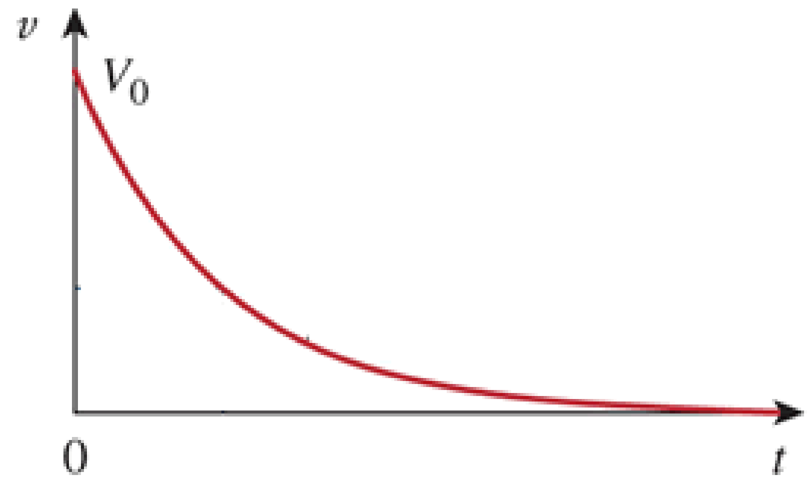
- This curve is called a **decaying exponential** curve.



- Note that at first the voltage falls steeply from its initial value (V_0). But as time passes, the descent becomes less steep.

Circuit Analysis: R and C : Time constant

- The values of R and C determine how rapidly the voltage descends.

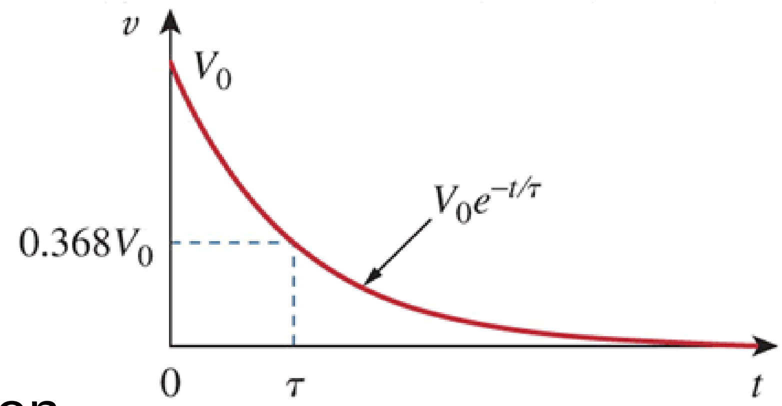


- The product RC is given a special name (the **time constant**) and symbol (τ) is used:

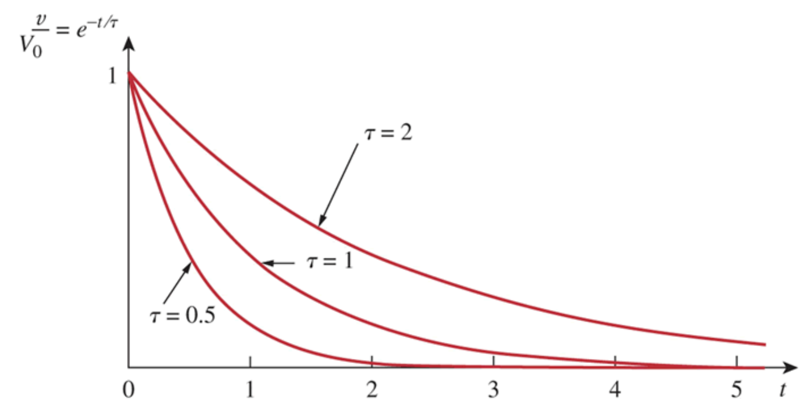
$$\tau = RC$$

- The greater τ is, the more slowly the voltage descends.

- After **one time constant** (i.e., when $t = \tau$), the voltage has fallen to about 36.8% of its initial value.
- After **five time constants** (i.e., when $t = 5\tau$), the voltage has fallen to about 0.7% of its initial value. For most practical purposes we say that the capacitor is completely discharged and $v = 0$ after five time constants.

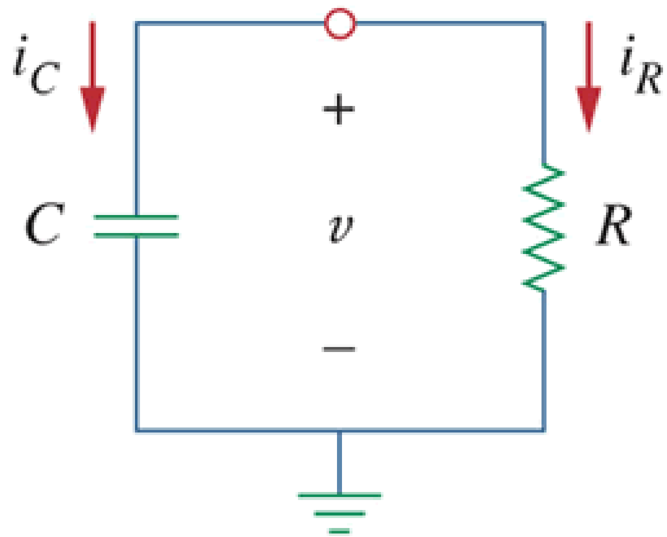


- The greater τ is, the more slowly the voltage descends, as shown below for a few values of τ.



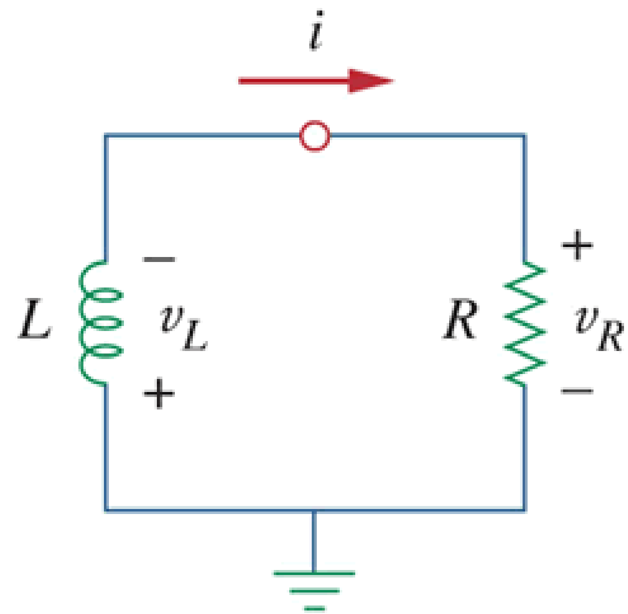
- We've seen that $v(t) = V_0 e^{-\frac{t}{\tau}}$
- From this equation we can use our prior knowledge to find equations for other quantities, such as current, power, and energy. For example, using Ohm's law we find that

$$i_R(t) = \frac{V_0}{R} e^{-\frac{t}{\tau}}$$



Circuit Analysis: R and L

- Consider the circuit shown. Assume that at time $t=0$, the inductor is energized and has an initial current, I_0.
- As time passes, the inductor's energy will gradually dissipate as current flows through the resistor.



- This results in changing current $i(t)$ and voltages $v_L(t)$ and $v_R(t)$, which we wish to calculate.

Circuit Analysis: R and L

- Applying KVL,

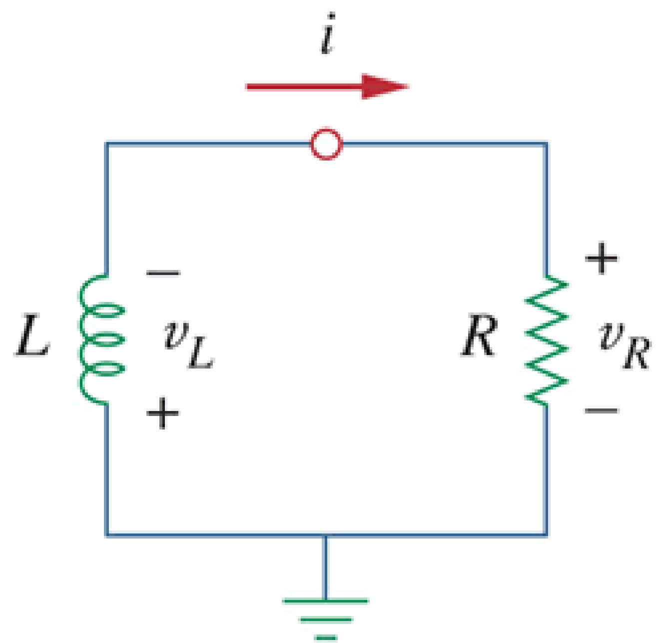
$$v_L + v_R = 0$$

- Therefore

$$L\frac{di}{dt} + iR = 0$$

- Therefore

$$\frac{di}{dt} + \frac{R}{L}i = 0$$



- This first-order differential equation is similar to the equation we had for source-free *RC* circuits.

Circuit Analysis: R and L

- To solve our equation, $\frac{di}{dt} + \frac{R}{L} i = 0$, first separate the variables *i* and *t*:

$$\frac{di}{i} = -\frac{R}{L} dt$$

- Then integrate both sides:

$$\ln i = -\frac{Rt}{L} + \ln A$$

- Then raise *e* to both sides:

$$i(t) = Ae^{-\frac{Rt}{L}}$$

Circuit Analysis: R and L

- At this point we have:

$$i(t) = Ae^{-\frac{Rt}{L}}$$

- The last step is to note that if we set *t* equal to 0, we get:

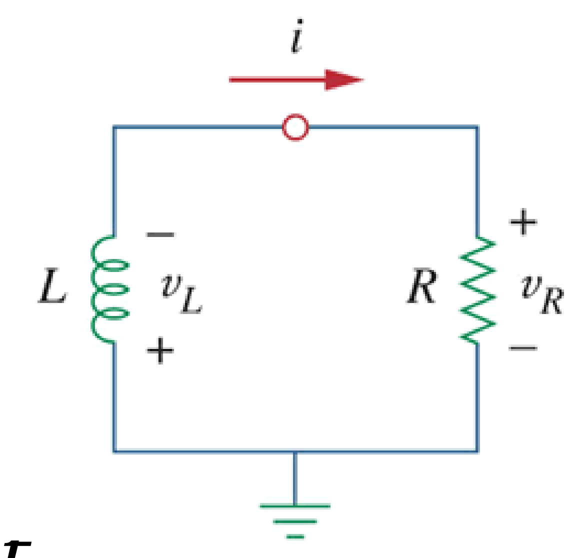
$$i(0) = A$$

- But we assumed earlier that the initial current is some value that we called I_0. So A must be equal to I_0, and therefore:

$$i(t) = I_0 e^{-\frac{Rt}{L}}$$

Circuit Analysis: R and L

- I won't expect you to be able to reproduce the derivation on the previous slides.
- The important point is to realize that whenever we have a circuit like this



the solution for $i(t)$ is:

$$i(t) = I_0 e^{-\frac{R\tau}{L}}$$

- I won't expect you to be able to reproduce the derivation on the previous slides.
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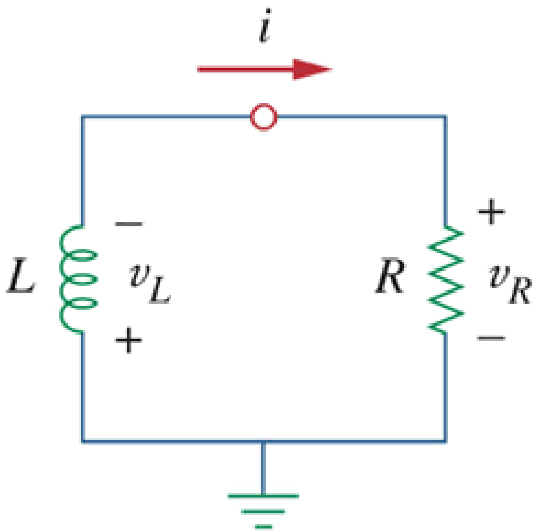


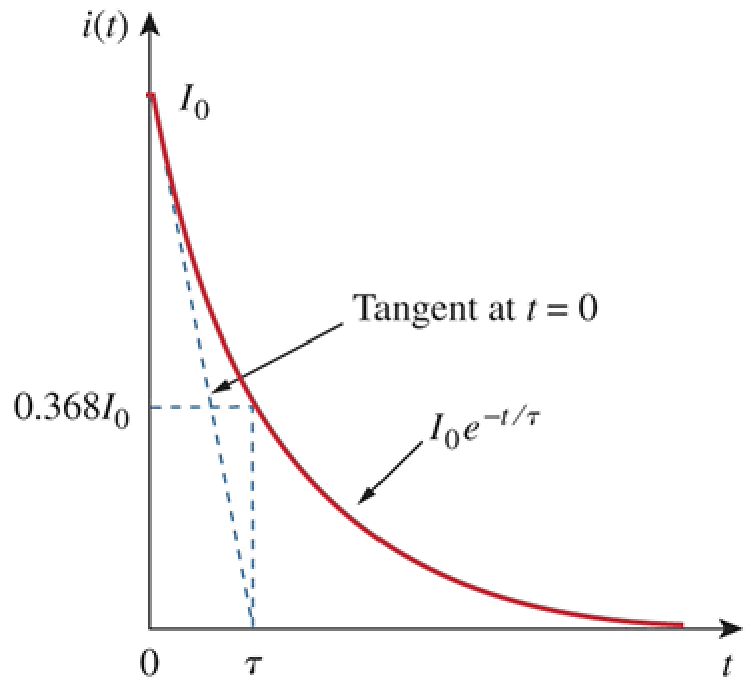
the solution for $i(t)$ is:

$$i(t) = I_0 e^{-\frac{R\tau}{L}}$$

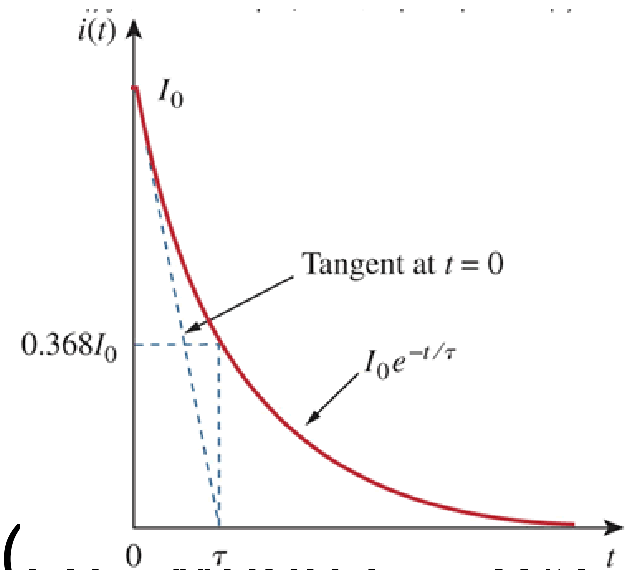
Circuit Analysis: R and L

- Here's a graph of

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$



  - It's a decaying exponential curve, with the current falling steeply from its initial value (I_0). But as time passes, the descent becomes less steep.

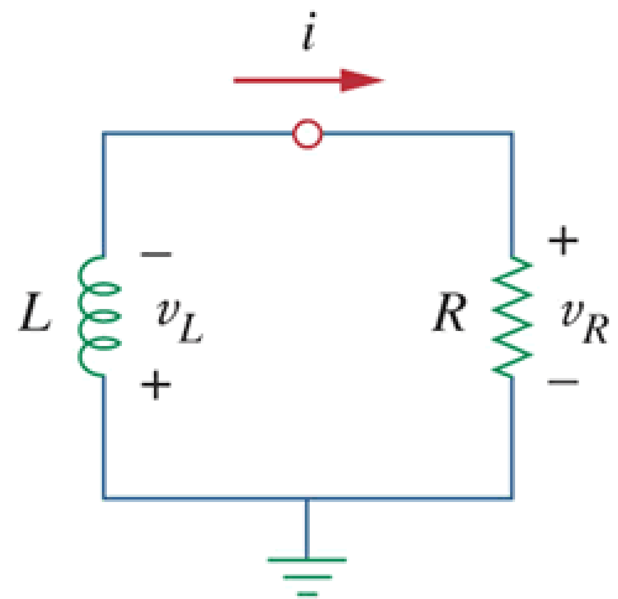


- After **one time constant** (i.e., when $t = \tau$), the current has fallen to about 36.8% of its initial value.
- After **five time constants** (i.e., when $t = 5\tau$), the current has fallen to about 0.7% of its initial value. For most practical purposes we say that the inductor is completely de-energized and $i = 0$ after five time constants.

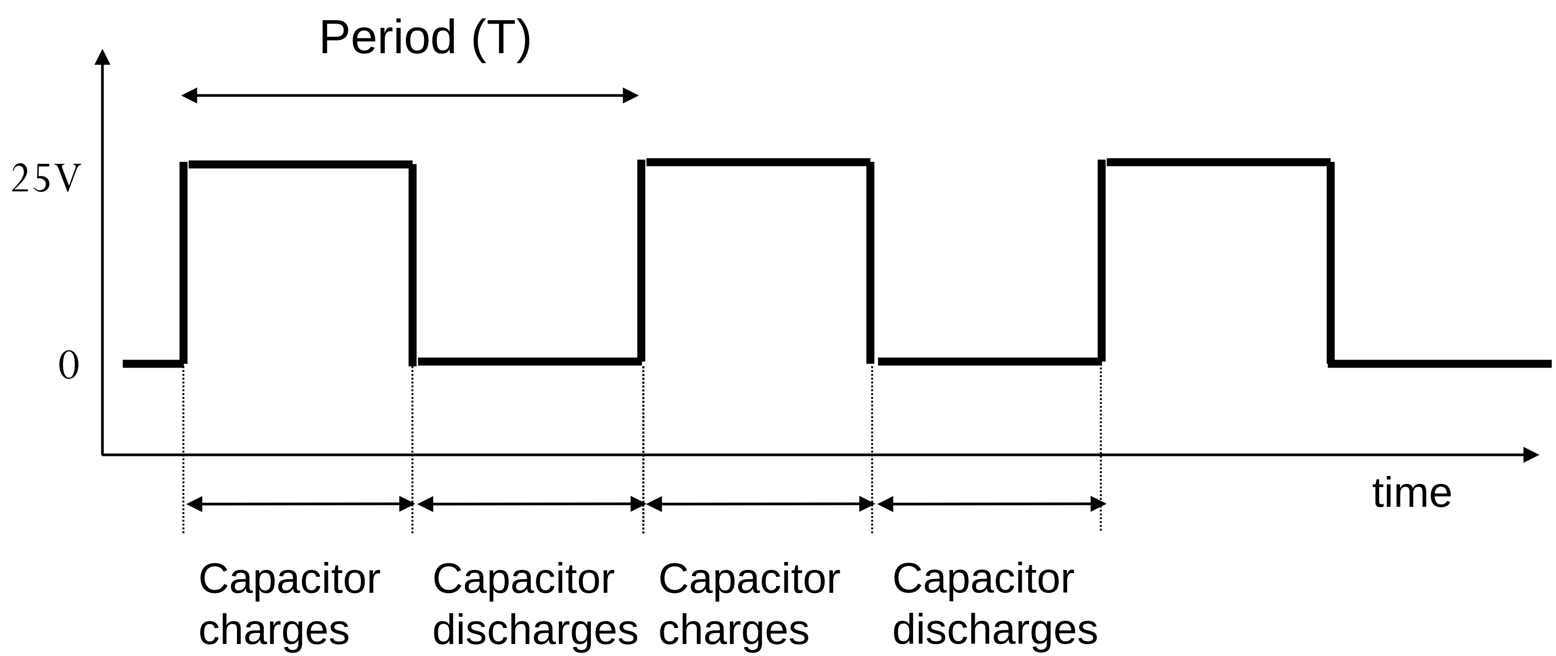
Circuit Analysis: R and L

- We've seen that $i(t) = I_0 e^{-\frac{t}{\tau}}$
- From this equation we can use our prior knowledge to find equations for other quantities, such as voltage, power, and energy. For example, using Ohm's law we find that

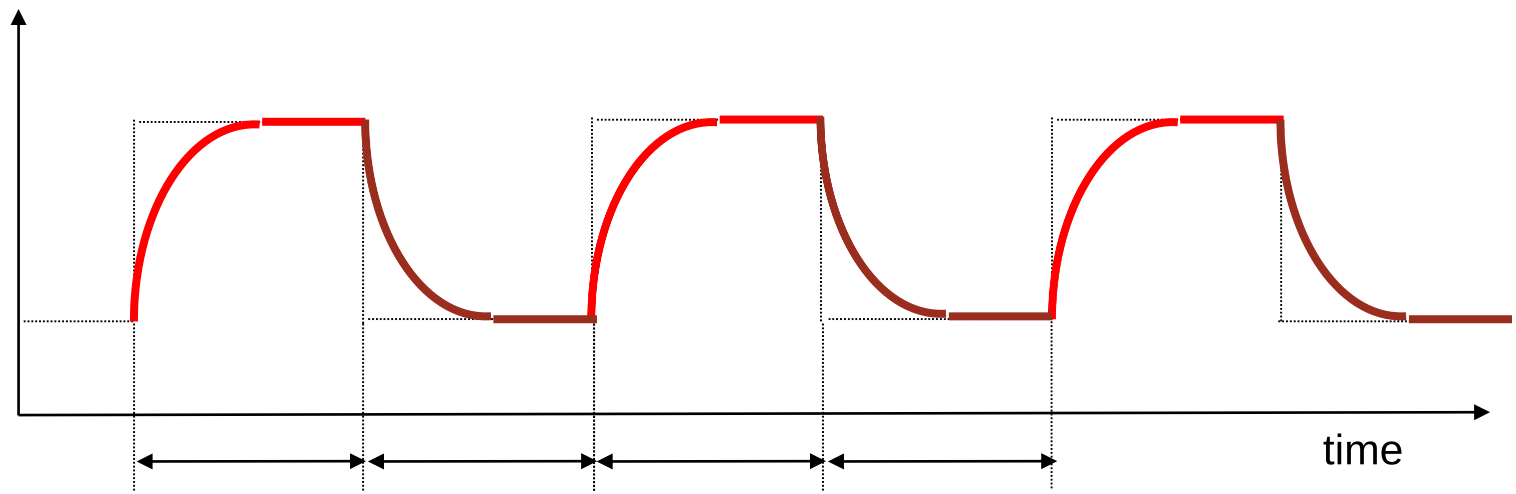
$$v_R(t) = I_0 R e^{-\frac{t}{\tau}}$$



A capacitor Charging and discharging



Charging and Discharging



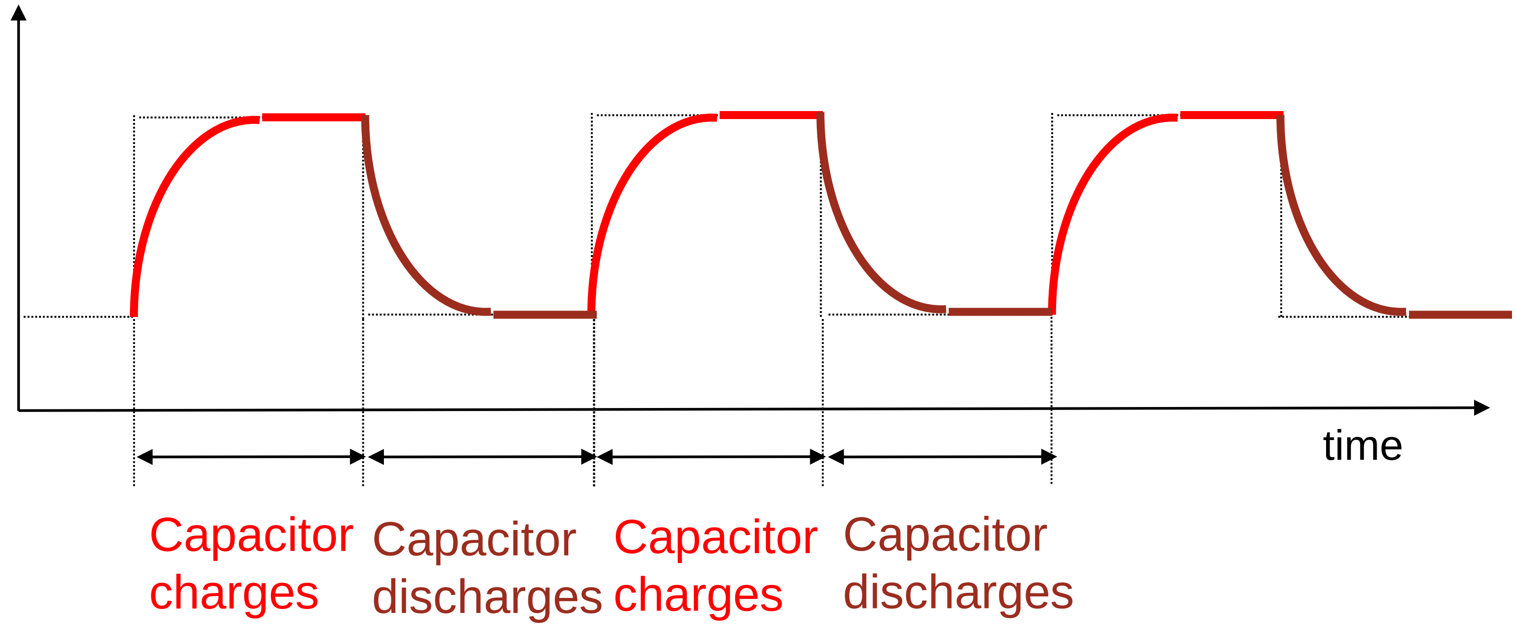
Capacitor charges

Capacitor discharges

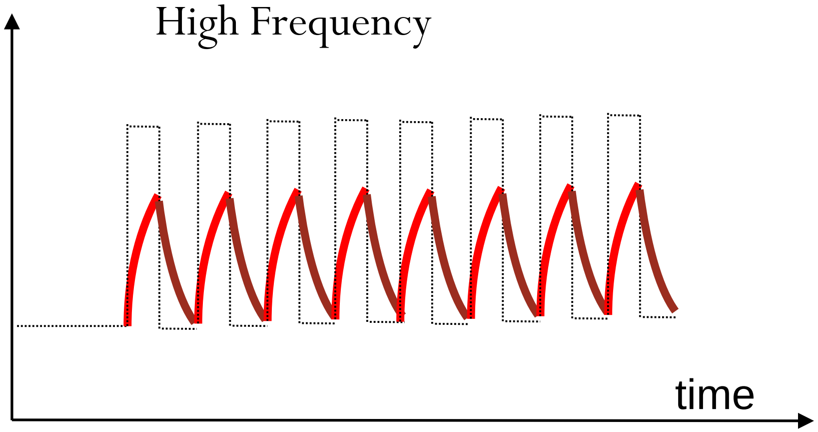
Capacitor charges

Capacitor discharges

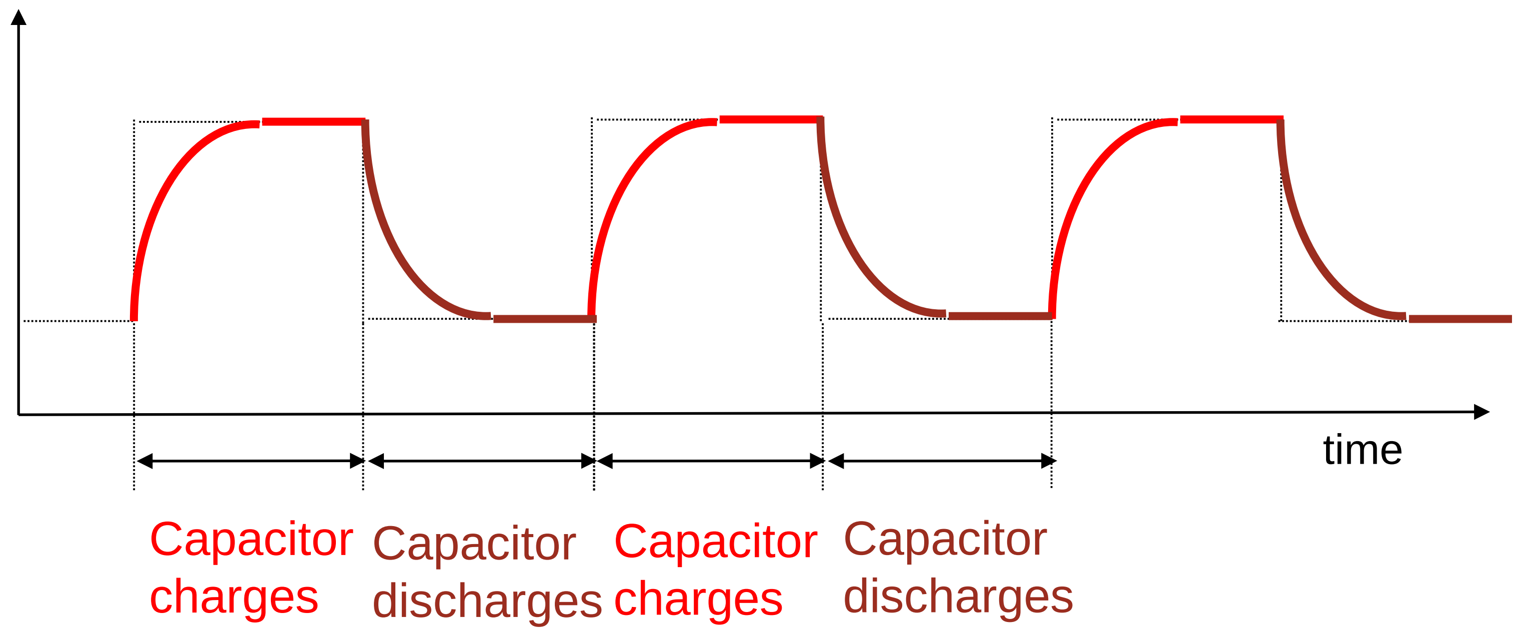
Charging and Discharging

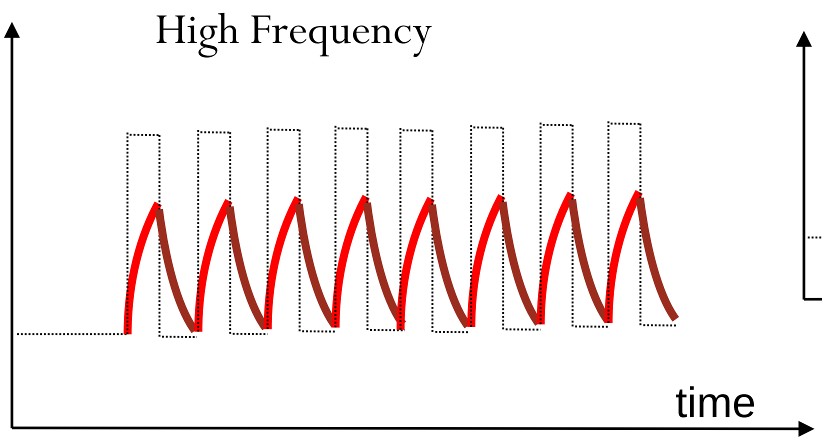


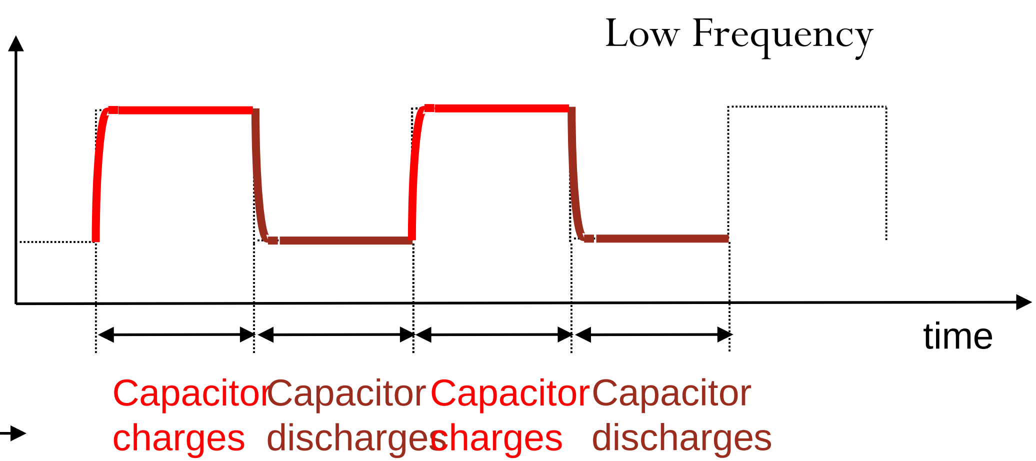
High Frequency



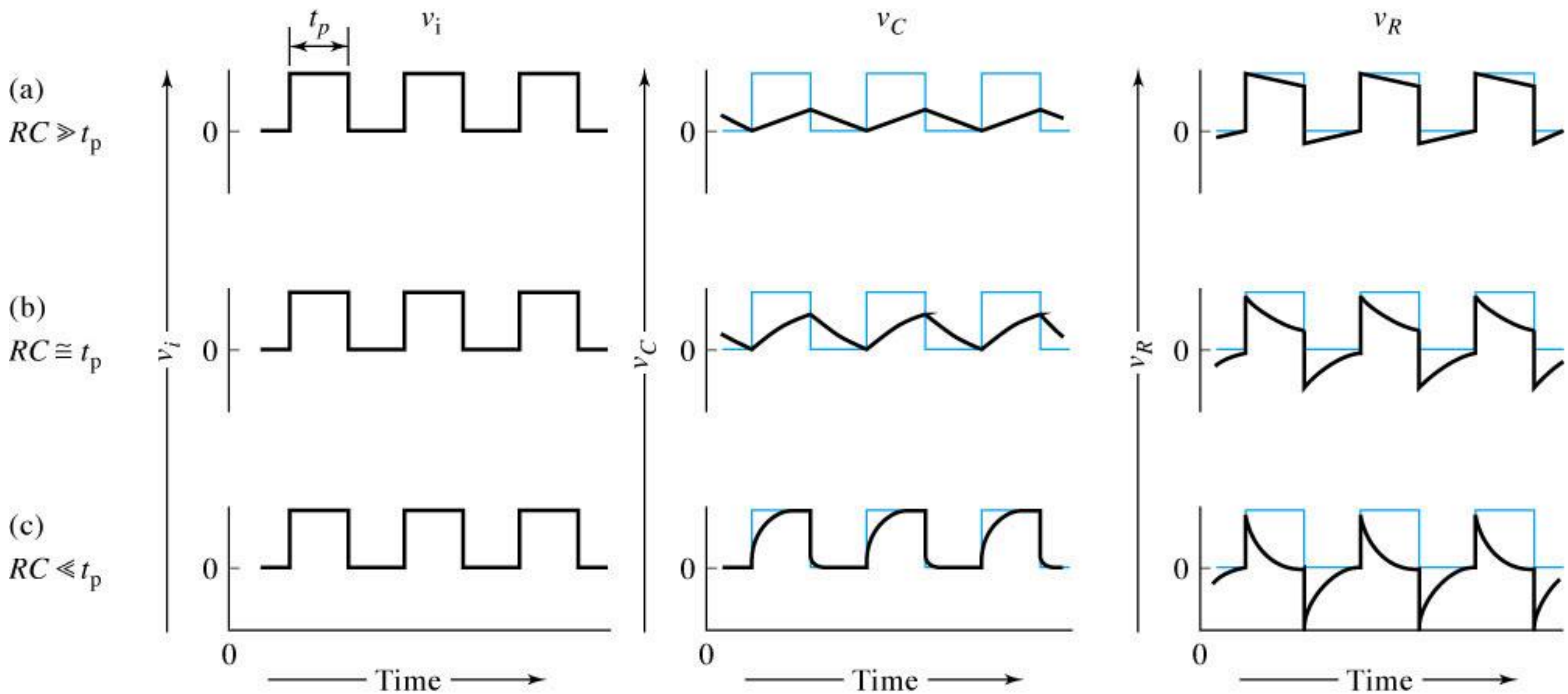
Charging and Discharging





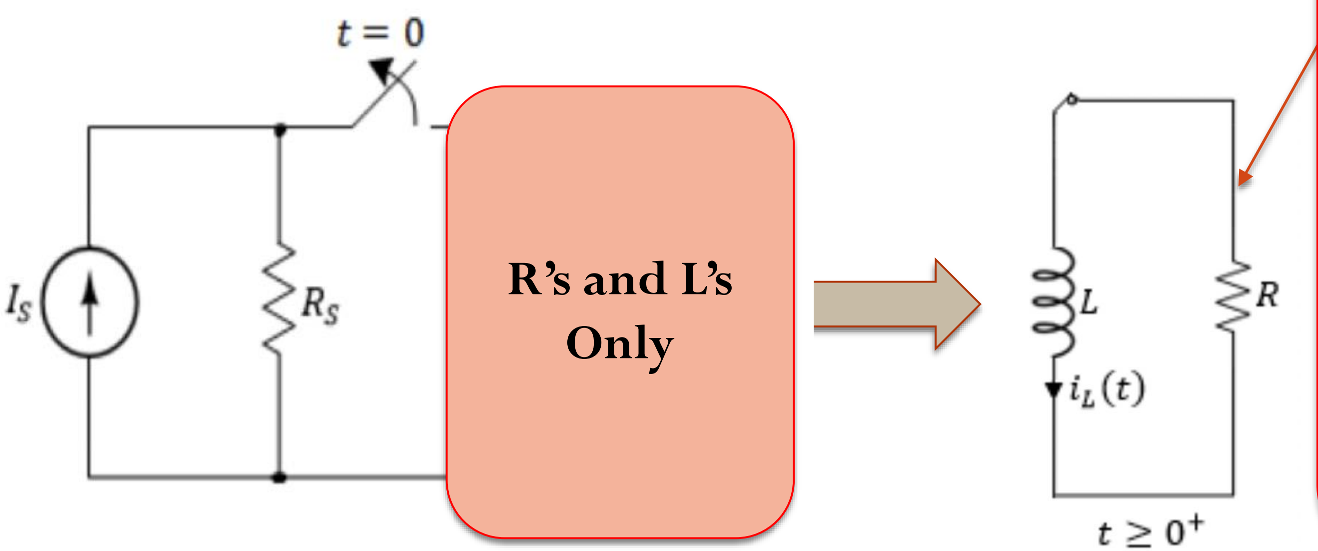


Charging and Discharging



Natural Response RL circuits

- The natural response for an RL circuits are obtained when the external independent source is set to zero or removed as shown below.
- For the natural responses to exist it is required that the inductor have energy stored in it at the moment of switching and the equivalent RL circuits are obtained.
- After certain action (switching):



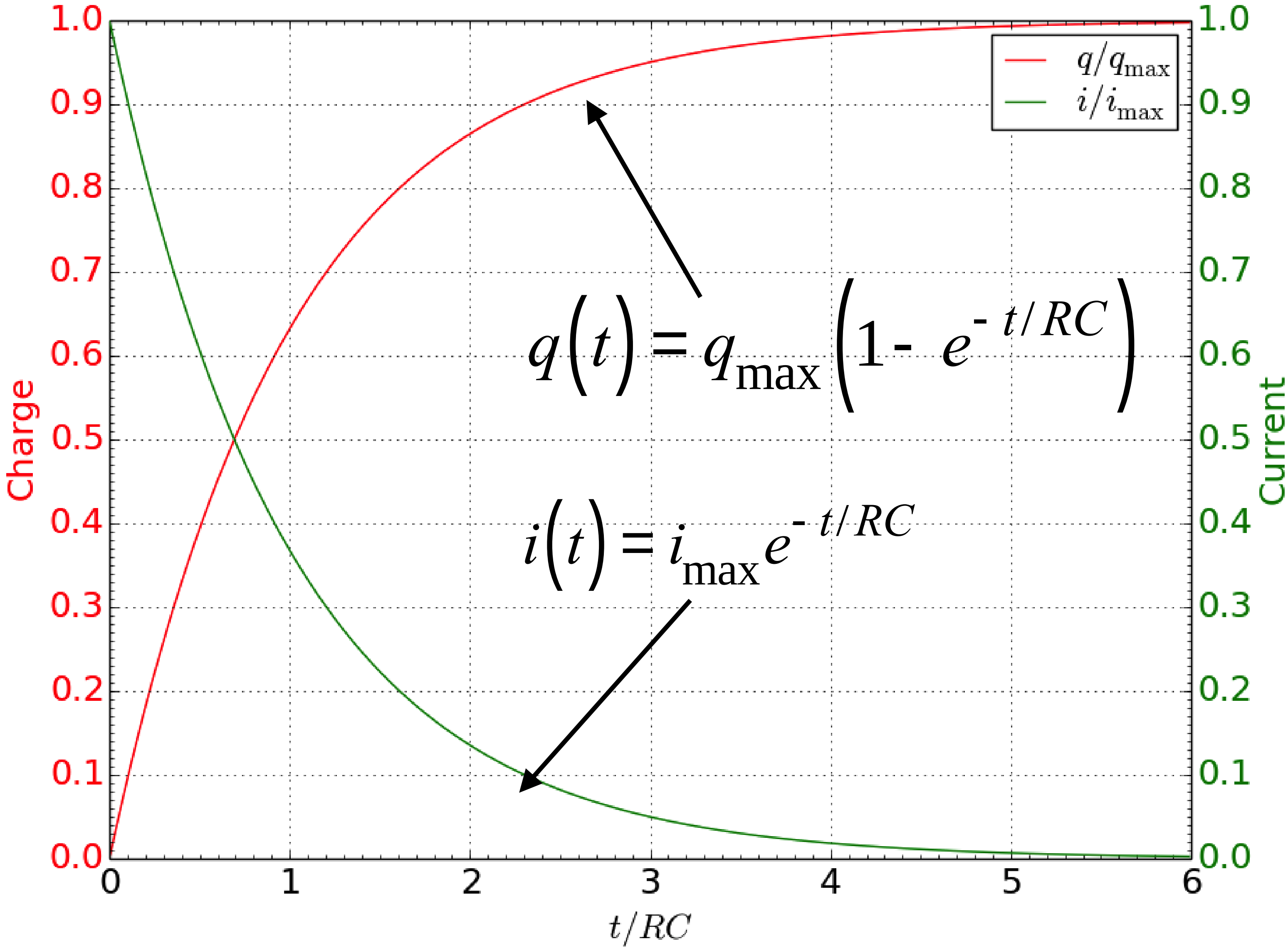
This circuit will responds naturally due to its L &R equivalent values.

We call it natural response or transient response.

No External source

Our goal to find the voltages, currents and power after t > 0.

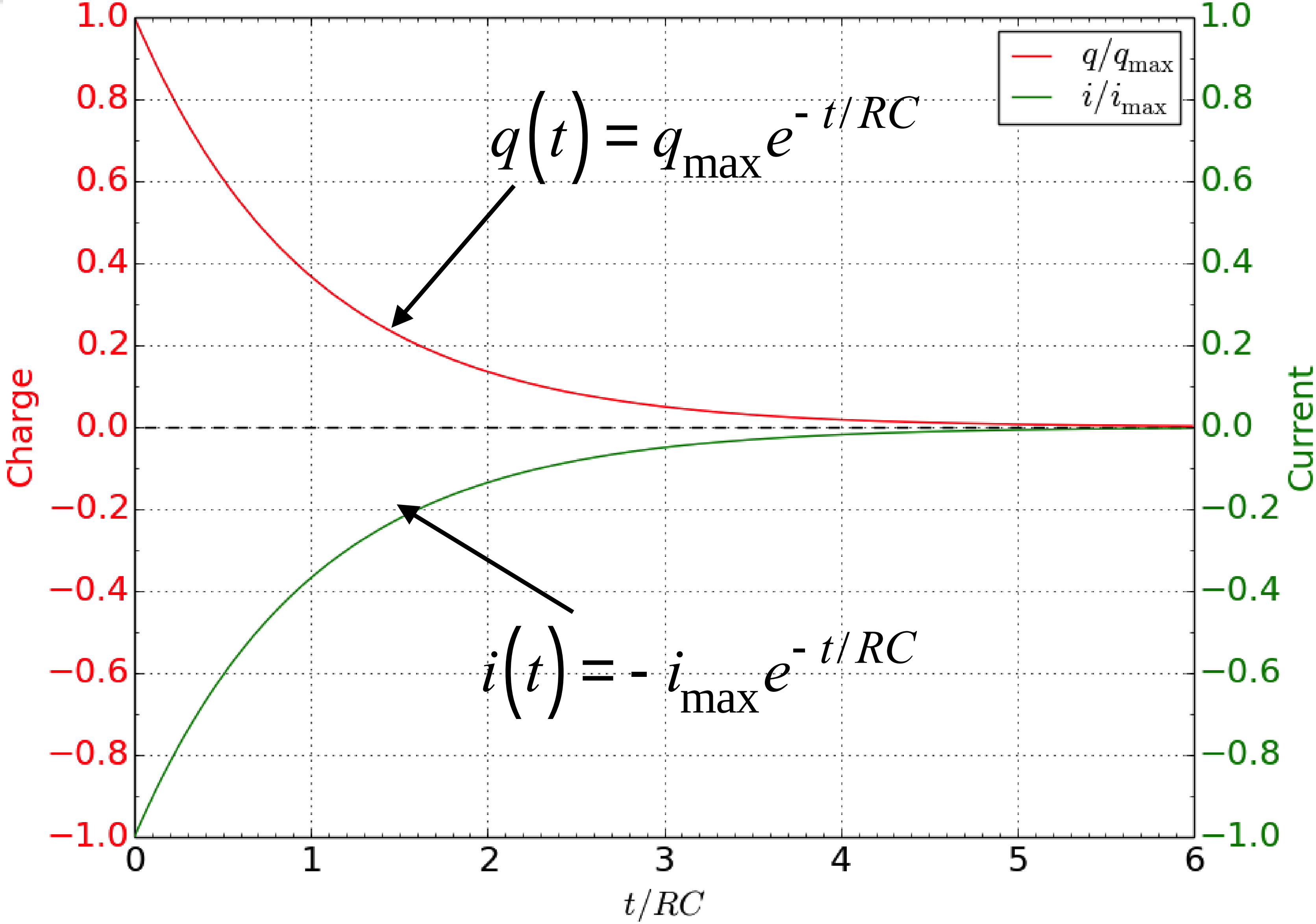
Charging Capacitor (Capacitor Initially Uncharged)



RC Example

- Let $\mathcal{E} = 12$ volts, $C = 6\mu\text{F}$, $R = 100\Omega$
  - $\tau = RC = 0.6$ msec
- Charge vs time $\quad q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = 72\left(1 - e^{-t/0.6}\right)$
  - Using units of μC and msec: $q_{\max} = 72$ μC
- Current vs time $\quad i = \dfrac{\mathcal{E}}{R} e^{-t/RC} = 0.12 e^{-t/0.6}$
  - (Using units of mA and msec: $i_{\max} = 0.12$ mA)
- This circuit will fully charge in a few msec
  - Initial values: $i = 0.12$ mA $\quad q = 0$ μC
  - Final values: $i = 0 \quad\quad q = 72$ μC

Discharging Capacitor (Capacitor Initially Charged)



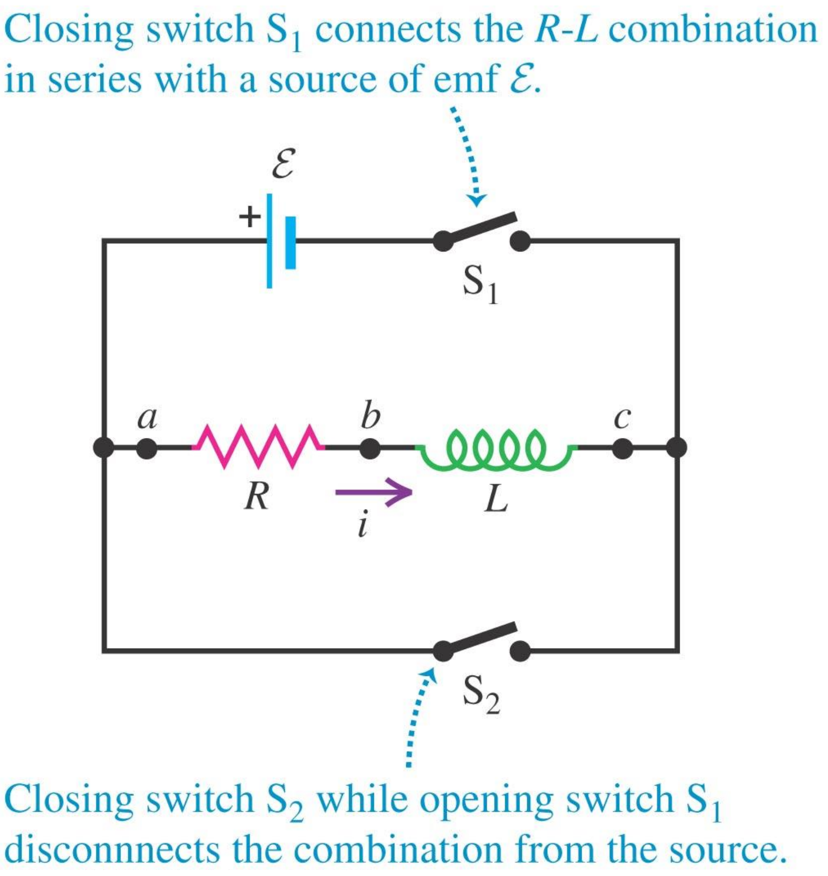
Same RC Example

- $C = 6\mu F$, $R = 100\Omega$
  - $\tau = RC = 0.6$ msec
- Charge capacitor to 12 volts, then pull out battery and let the *RC* circuit discharge
- Charge vs time $q = CVe^{-t/RC} = 72e^{-t/0.6}$
  - Using units of μC and msec: $q_{\max} = 72\ \mu C$
- Current vs time $i = \dfrac{V}{R}e^{-t/RC} = 0.12e^{-t/0.6}$
  - Using units of mA and msec: $i_{\max} = 0.12$ mA
- This circuit will fully discharge in a few msec

R-L Circuit: Discussion

When connected in a circuit with a resistor, an inductor will have the effect of slowing down changes in the current through the resistor.

When the current is steady (the switch has been closed for a long time), the inductor has no effect, but there is potential energy stored in the inductor.



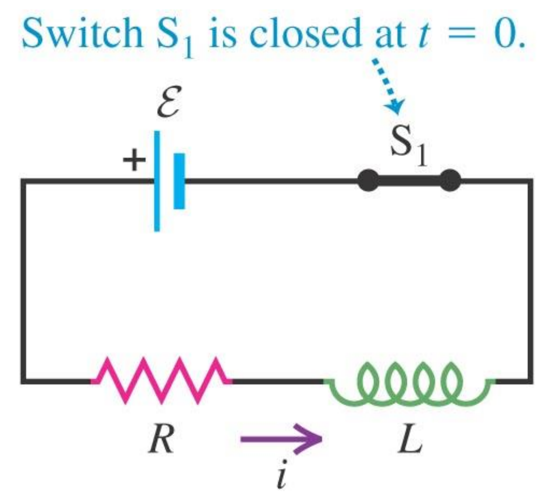
R-L Circuit: Discussion

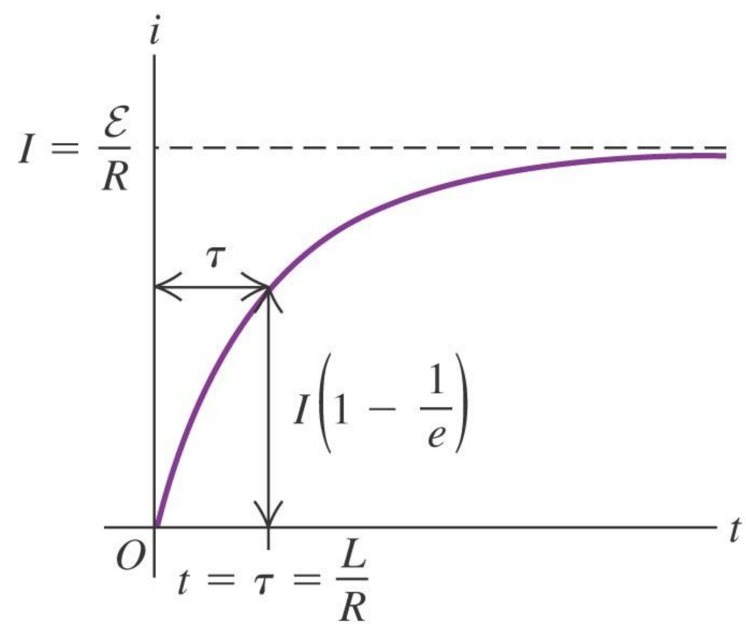
If switch S_1 is closed in the circuit, current will begin to flow through the resistor and inductor as shown. This increasing current will induce current to flow the opposite direction, slowing the growth of the current.

We can write down a formula for the current as a function of time:

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-(R/L)t}\right)$$

The quantity $\tau = \frac{L}{R}$ is called the "time constant" for this exponential decay.



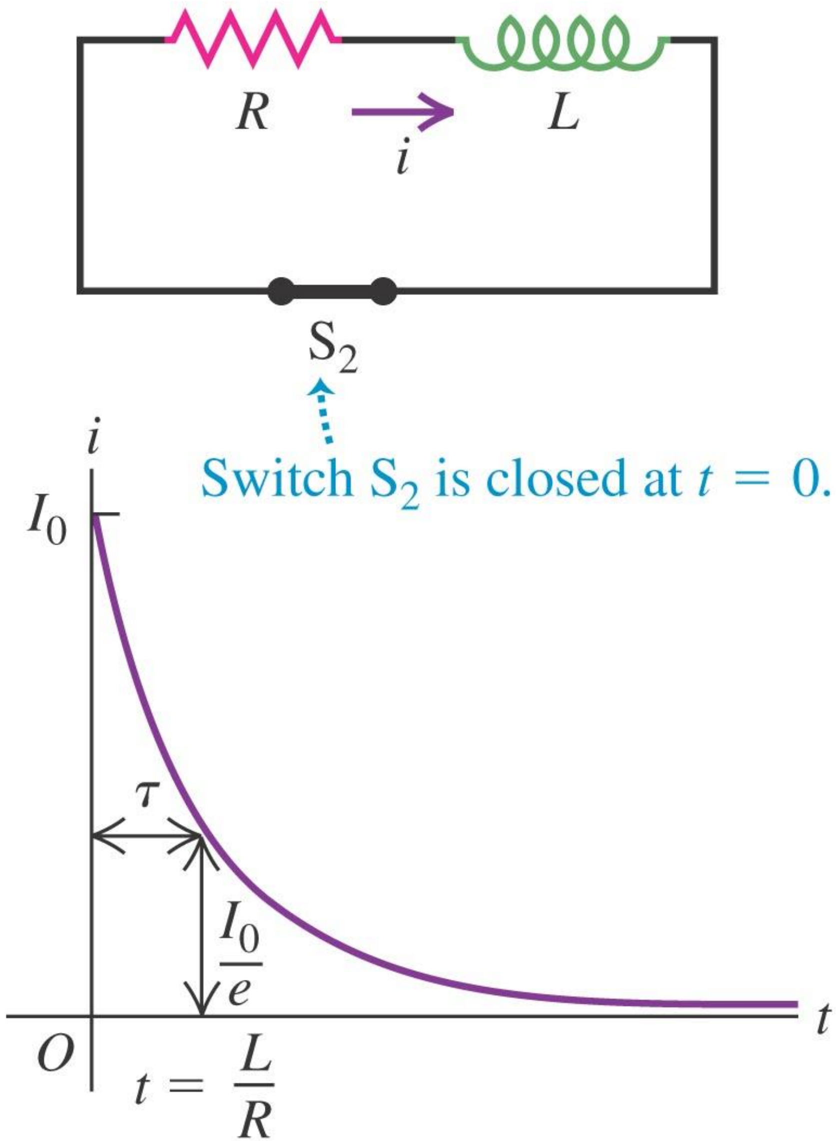


R-L Circuit: Discussion

Once the current reaches a steady value we can flip the switches, opening S_1 and closing S_2. Then current will keep flowing for while as the inductor opposes this decreasing current.

A similar formula describes this decaying current as a function of time:

$$i(t) = I_0\left(e^{-(R/L)t}\right)$$



R-L Circuit: Discussion

Example: A 35.0V battery with negligible internal resistance, a 50.0Ω resistor and a 1.25mH inductor are connected in series with an open switch. The switch is suddenly closed.

(a) How long after closing the switch will the current through the inductor reach half of its maximum value?

(b) How long after closing the switch will the energy stored in the inductor reach half its maximum value?

As soon as the switch is closed, current begins to flow around the circuit, increasing toward a maximum value given by Ohm's Law. Here is the formula:

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\left(R/L\right)t}\right)$$

We want to find the time when the current is half of the maximum.

$$i(t) = \tfrac{1}{2}\left(\frac{\varepsilon}{R}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\left(R/L\right)t}\right) \rightarrow \tfrac{1}{2} = \left(1 - e^{-\left(R/L\right)t}\right) \rightarrow e^{-\left(R/L\right)t} = \frac{1}{2}$$

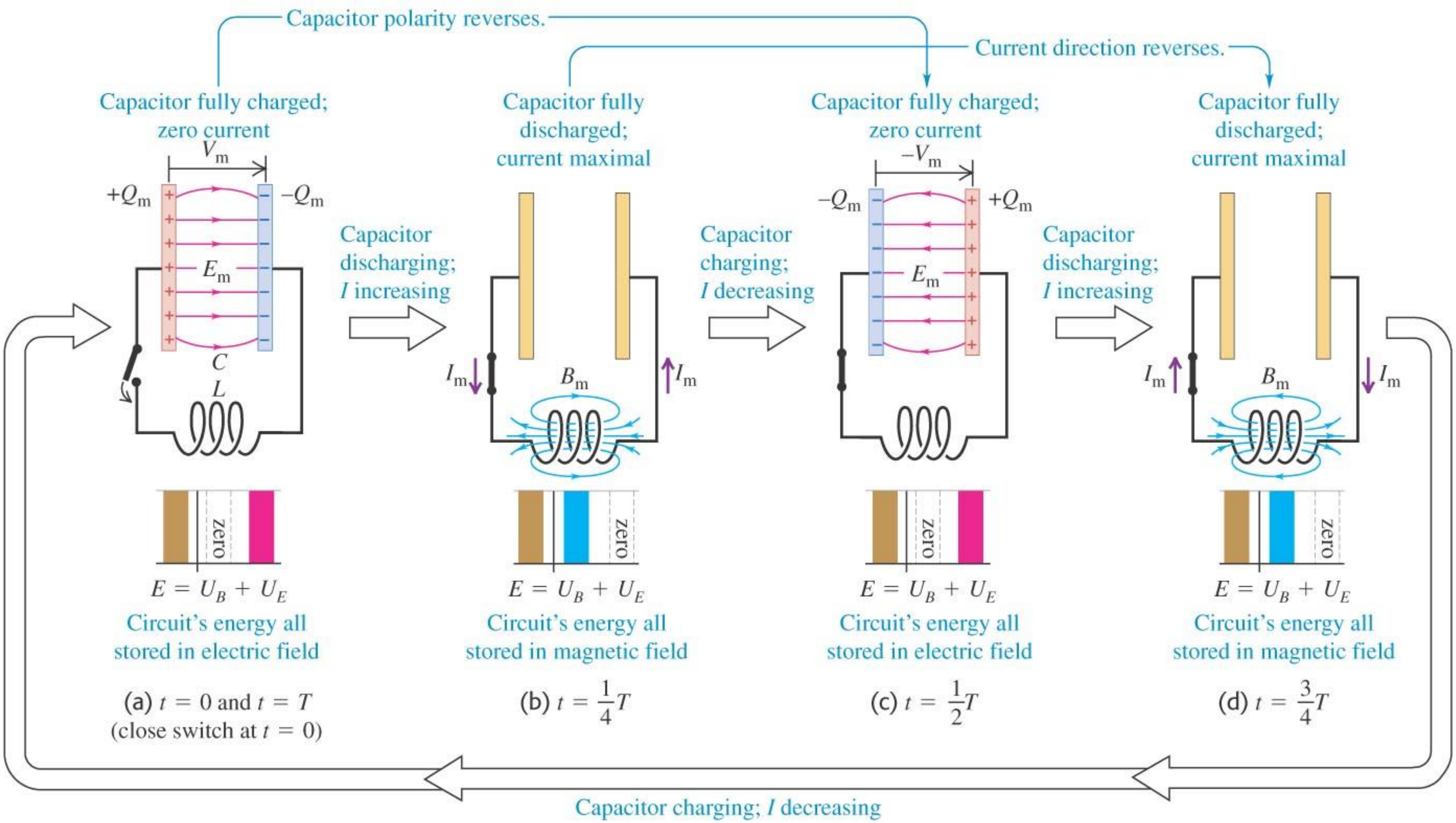
$$-\left(R/L\right)t = \ln\left(\tfrac{1}{2}\right) \rightarrow t = -\left(\frac{L}{R}\right)\cdot\ln\left(\tfrac{1}{2}\right) = -\left(\frac{1.25\cdot 10^{-3}H}{50\Omega}\right)\cdot\ln\left(\tfrac{1}{2}\right) = 1.73\cdot 10^{-5}s = 17.3\mu s$$

For part b) we want the energy to be half of its maximum, so use the energy formula:

$$U = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left(\tfrac{1}{2}LI^2\right) \rightarrow i(t) = \frac{I}{\sqrt{2}}$$

Using the formula for current again: $\frac{1}{\sqrt{2}} = \left(1 - e^{-\left(R/L\right)t}\right) \rightarrow t = 30.7\mu s$

What about LC circuit??



L-C Circuit: Next class

A circuit containing a capacitor and an inductor will exhibit an oscillating current, with potential energy transferring back and forth.