INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 (Symmetry)

Tutorial Sheet 08 / March 15, 2022 Spring 2022

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1. Consider the group $\mathrm{GL}_3(\mathbb{R})$ (of 3×3 matrices whose entries are real numbers, under multiplication of matrices) and the set $\mathbb{R}^3$ (of triplets of real numbers). For $A \in \mathrm{GL}_3(\mathbb{R})$ and $(a, b, c) \in \mathbb{R}^3$, define

$$A.(a, b, c) := A \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

   (a) Show that above is an action of $\mathrm{GL}_3(\mathbb{R})$ on $\mathbb{R}^3$.

   (b) What is the orbit of $(1, 2, 3)$? Is it true that the orbit of $(1, 2, 3)$ is same as that of $(-1, -2, -3)$?

   (c) Apart from the identity element of $\mathrm{GL}_3(\mathbb{R})$, find an element in the stabilizer of $(1, 2, 3)$. How many elements are there in this stabilizer?

   (d) Take the 3×3 rotation matrix $R_{x,\theta}$. Find all $P := (a, b, c) \in \mathbb{R}$ such that the stabilizer of P contains $R_{x,\theta}$.

2. Consider the group S_4, consisting of permutations of four elements $1, 2, 3, 4$. Show that the permutation action of S_4 on $S := \{1, 2, 3, 4\}$ has only one orbit. Determine the stabilizer of $3 \in S$. Show that #orbit(3) $\times$ #stab(3) $=$ $\#S_4$. Here the symbol # signifies the number of elements.
