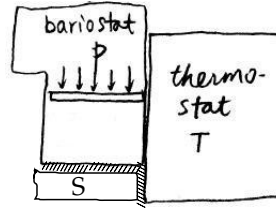


1. Consider a hydrostatic system \mathcal{S} which is connected to a bath, with which it can exchange energy and volume.



- a) Recall, how we calculated the canonical and the grand canonical distribution functions. Using the same line of argument, calculate the probability $\rho(E, V)$ of finding the system with energy E and volume V .
- b) Using the relation $S = \langle S \rangle$ and the Boltzmann relation for the entropy S , relate the partition function Q , that defines the normalization in $\rho(E, V)$, to one of the thermodynamic potentials.
- c) Using the expression for $\rho(E, V)$ calculate the relative fluctuation $\sqrt{\langle \Delta V^2 \rangle} / \langle V \rangle$.
- d) Now, the partition function Q can be written down as

$$Q = \int_0^\infty dV \int \prod_{i=1}^N d^3 p_i d^3 q_i \rho(E, V).$$

Using this expression, relate Q to the canonical partition function $Z(N, V, T)$. Hence, derive an expression for Q of an ideal gas, and the chemical potential μ for the same system.

2. The energy of a system of N localized magnetic spins, at temperature T and in the presence of magnetic field H is given by

$$\mathcal{H} = D \sum_{i=1}^N S_i^2 - \mu_0 H \sum_{i=1}^N S_i,$$

where the parameters D, μ_0, H are positive and spin variables S_j may assume values ± 1 or 0, for $i = 1, 2, 3, \dots$

- a) Obtain an expression for the internal energy, the entropy and the magnetization per site.
 - b) In zero field limit ($H = 0$), sketch the graphs of the internal energy, entropy and the specific heat as a function of temperature.
 - c) Indicate the behavior of these quantities in the limits of $T \rightarrow 0$ and $T \rightarrow \infty$.
 - d) Calculate the average of the quadrupole moment defined as $\mathcal{Q} = \frac{1}{N} \sum_{i=1}^N S_i^2$.
3. Consider a one-dimensional magnetic system of N localized spins, at temperature T , with the Hamiltonian given by

$$\mathcal{H} = -J \sum_{i=1,3,5,\dots,N-1} \sigma_i \sigma_{i+1} - \mu_0 H \sum_{i=1}^N \sigma_i$$

where the parameters J, μ_0, H are positive numbers, and $\sigma_i = \pm 1$ for all sites i . Assume that N is even and note that the first sum is over the odd spins.

- a) Obtain an expression for the canonical partition function and calculate the internal energy per spin $u = u(T, H)$. Sketch a graph of $u(T, H = 0)$ versus temperature.
- b) Obtain an expression for the entropy per spin $s = s(T, H)$. Sketch a graph of $s(T, H = 0)$ versus temperature.
- c) Calculate the magnetization per spin defined as

$$m = m(T, H) = \frac{1}{N} \left\langle \mu_0 \sum_{i=1}^N \sigma_i \right\rangle$$

and the magnetic susceptibility $\chi(T, H)$.

- 4. Consider a system of N classical and non-interacting particles in contact with a thermal reservoir at temperature T . Each particle may have energies $0, \epsilon >$, or 3ϵ . Obtain the expression for the canonical partition function, and calculate the internal energy and the entropy per particle for the system. Sketch the results, indicating the values for $T \rightarrow 0$ and $T \rightarrow \infty$.
- 5. Consider N non-interacting spins in a magnetic field $\mathbf{B} = B\hat{z}$, and at a temperature T . The work done by the field is given by BM_z , with a magnetization $M_z = \sum_{i=1}^N m_i$. For each spin, m_i can take only the $(2s + 1)$ values $-s, -s + 1, \dots, s - 1, s$.
 - a) Calculate the canonical partition function and the free energy of the system.
 - b) Expand the free energy for small magnitude of the magnetic field and show that the magnetic susceptibility obeys Curie's law $\chi = c/T$.
 - c) Show that $C_B - C_M = cB^2/T^2$, where C_B and C_M are the heat capacities at constant field and magnetization, respectively.
- 6. An ideal gas of N spin-less particles of mass m is inserted in between two parallel surfaces. To make sure that the particles won't escape a harmonic two dimensional potential is created in such a way that:

$$U(x, y, z) = \begin{cases} \frac{1}{2}m\omega^2(x^2 + y^2), & z_1 \leq z \leq z_2 \\ \infty, & \text{otherwise} \end{cases}$$

Denote $L = z_2 - z_1$.

- a) Calculate the N -particle classical partition function and the specific heat of the system.
- b) Assuming that L is large enough, calculate the N -particle quantum partition function and the specific heat of the system. What is the value of the specific heat as $T \rightarrow \infty$? (*Hint: Large enough L would mean that the sum can be replaced by an integral.*)
- c) In the previous question, can you estimate how large L should be?