PHY 101 : Problem Sheet 5

For a point on earth which is at latitude λ north of equator, find out :

1. The magnitude of $|\mathbf{\Omega}_H|$, the projection of angular velocity vector of the earth's rotation $\mathbf{\Omega}$ along the surface. Find out the magnitude of the projection of $\mathbf{\Omega}$ normal to the earth's surface.

2. If a particle moves with speed v_0 as seen by a person standing on the surface of earth, along the direction of $\mathbf{\Omega}_H$ find out the Coriolis force experienced by the particle according to the person standing.

3. If a particle moves with speed v_0 as seen by a person standing on the surface of earth, perpendicular to the direction of $\mathbf{\Omega}_H$ and along the surface, find out the Coriolis force experienced by the particle according to the person standing.

4. If a particle moves with speed v_0 as seen by a person standing on the surface of earth, normal to the surface of earth, find out the Coriolis force experienced by the particle according to the person standing.

We have learnt in the class that a circularly rotating non-inertial observer calls the vector $\mathbf{v}_{ni} = \dot{x}'\hat{\mathbf{i}}' + \dot{y}'\hat{\mathbf{j}}' + \dot{z}'\hat{\mathbf{k}}'$ as the velocity vector without worrying about rate of change of $\hat{\mathbf{i}}', \hat{\mathbf{j}}', \hat{\mathbf{k}}'$ themselves. The velocity of the particle as seen by an inertial observer $\mathbf{v}_{in}$ is related as $\mathbf{v}_{in} = \mathbf{v}_{ni} + \mathbf{\Omega} \times \mathbf{r}$.

1. If $\mathbf{a}_{ni} = \ddot{x}'\hat{\mathbf{i}}' + \ddot{y}'\hat{\mathbf{j}}' + \ddot{z}'\hat{\mathbf{k}}'$ verify that $\mathbf{a}_{ni} \neq d\mathbf{v}_{ni}/dt$ in general.

2. If there is any non-trivial condition for which $\mathbf{a}_{ni} = d\mathbf{v}_{ni}/dt$?

3. Obtain the expression connecting $\mathbf{v}_{in}$ to $\mathbf{v}_{ni}$ if $\mathbf{\Omega}$ also changes in time.

4. Obtain the expression connecting $\mathbf{a}_{in}$ to $\mathbf{a}_{ni}$ if $\mathbf{\Omega}$ also changes in time.

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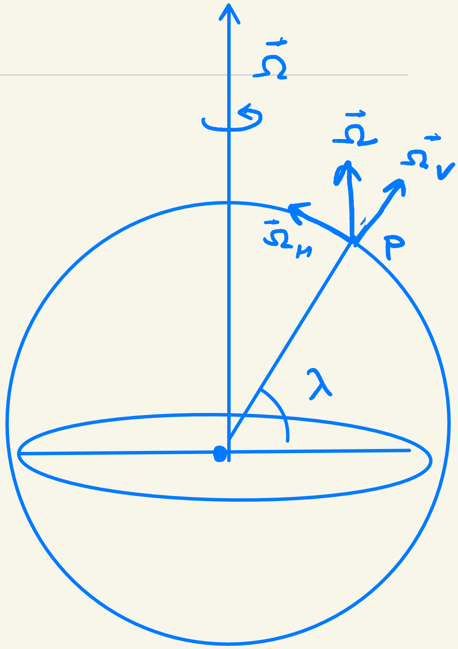
1. If $\vec{\Omega}$ is the angular velocity, at point P, its projection along the surface is $\vec{\Omega}_H$ and normal to the surface is $\vec{\Omega}_V$.



$$\vec{\Omega}_V = \vec{\Omega} \cos\left(\frac{\pi}{2} - \lambda\right) = \vec{\Omega} \sin\lambda$$

$$\vec{\Omega}_H = \vec{\Omega} \sin\left(\frac{\pi}{2} - \lambda\right) = \vec{\Omega} \cos\lambda$$

$$|\vec{\Omega}_V| = \Omega \sin\lambda, \quad |\vec{\Omega}_H| = \Omega \cos\lambda$$

2. $$\vec{\Omega} = \vec{\Omega}_V + \vec{\Omega}_H$$
$$= \Omega \sin\lambda \, \hat{k} + \Omega \cos\lambda \, \hat{j}$$

$$\vec{v} = v_0 \hat{\Omega}_H = v_0 \hat{j}$$

$$\vec{F}_{cor} = -2m(\vec{\Omega} \times \vec{v})$$
$$= -2m\left[(\Omega \cos\lambda \, \hat{j} + \Omega \sin\lambda \, \hat{k}) \times v_0 \hat{j}\right]$$
$$= 2m\,\Omega \sin\lambda \, v_0 \, \hat{i}$$

3. If the particle moves $\perp$ to

$$\vec{v}_0 = v_0 \hat{i}$$

$$\vec{F}_{cor} = -2m\left[(\Omega \cos\lambda \hat{j} + \Omega \sin\lambda \hat{k}) \times v_0 \hat{i}\right]$$

$$= 2m v_0 \Omega \cos\lambda \hat{k} - 2m v_0 \Omega \sin\lambda \hat{j}$$

4. If the particle moves normal to the surface

$$\vec{v} = v_0 \hat{k}$$

$$\vec{F}_{cor} = -2m\left[(\Omega \cos\lambda \hat{j} + \Omega \sin\lambda \hat{k}) \times v_0 \hat{k}\right]$$

$$= -2m\Omega \cos\lambda \, v_0 \hat{i}$$

5- $$\vec{V}_{in} = \vec{V}_{ni} + \vec{\Omega} \times \vec{r}$$

$$\vec{V}_{ni} = \dot{x}' \hat{i}' + \dot{y}' \hat{j}' + \dot{z}' \hat{k}'$$

$$\frac{d\vec{V}_{ni}}{dt} = \dot{x}'' \hat{i}' + \dot{y}'' \hat{j}' + \dot{z}'' \hat{k}' + \dot{x}' \dot{\hat{i}}' + \dot{y}' \dot{\hat{j}}' + \dot{z}' \dot{\hat{k}}'$$

Since $\frac{d\hat{i}'}{dt} = \vec{\Omega} \times \hat{i}'$, $\frac{d\hat{j}'}{dt} = \vec{\Omega} \times \hat{j}'$

$\frac{d\hat{k}'}{dt} = \vec{\Omega} \times \hat{k}'$ (done in the class)

$$\therefore \frac{d\vec{v}_{ni}}{dt} = \vec{a}_{ni} + \dot{x}'(\vec{\Omega}\times\hat{i}') + \dot{y}'(\vec{\Omega}\times\hat{j}') + \dot{z}'(\vec{\Omega}\times\hat{k}')$$

$$= \vec{a}_{ni} + \vec{\Omega}\times(\dot{x}'\hat{i}' + \dot{y}'\hat{j}' + \dot{z}'\hat{k}')$$

$$= \vec{a}_{ni} + \vec{\Omega}\times\vec{v}_{ni}$$

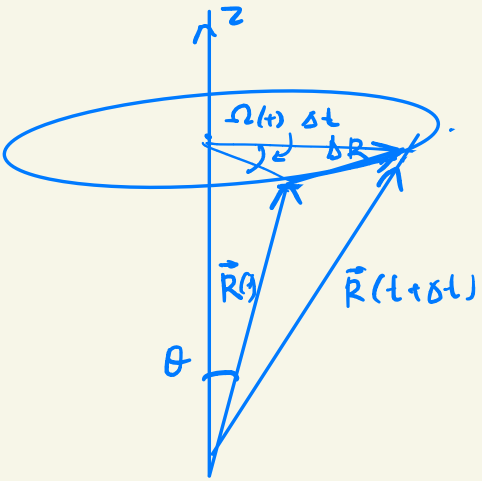
$\therefore \frac{d\vec{v}_{ni}}{dt} \neq \vec{a}_{ni}$ in general.

6. For $\vec{a}_{ni} = \frac{d\vec{v}_{ni}}{dt}$ we need

$\vec{\Omega}\times\vec{v}_{ni} = 0$, i.e. $\vec{v}_{ni} \parallel \vec{\Omega}$

Other conditions: $\vec{\Omega} = 0$, $\vec{v}_{ni} = 0$; which will lead to inertial motion ($\vec{\Omega} = 0$) or no acceleration ($\vec{a}_{ni} = 0$)

7. Ω changing with time:



If $\vec{\Omega}$ changes in time, at any time t let it be $\vec{\Omega}(t)$

- At the same time the position vector be $\vec{R}(t)$

→ At time $t+\Delta t$ the vector is $\vec{R}(t+\Delta t)$

$$\Delta\vec{R} = \vec{R}(t+\Delta t) - \vec{R}(t)$$

$$\Delta\vec{R} = R\sin\theta\,\Delta\phi\,\hat{\phi}$$

For small Δt,

the angle made at the centre $\Delta\phi = \Omega(t)\,\Delta t$

$$\therefore \frac{\Delta\vec{R}}{\Delta t} = |\vec{R}\sin\theta|\,\Omega(t)\,\hat{\phi}$$

$$\lim_{\Delta t\to 0}\frac{\Delta R}{\Delta t} = \lim_{\Delta t\to 0}\frac{\vec{R}(t+\Delta t) - \vec{R}(t)}{\Delta t} = \lim_{\Delta t\to 0} R\sin\theta\cdot\Omega(t)\,\hat{\phi}$$

$$\frac{d\vec{R}}{dt} = R\sin\theta\,\Omega(t)\cdot\hat{\phi} = \vec{\Omega}(t)\times\vec{R}$$

So, the relation remains the same, leading to

$$\vec{V}_{in} = \vec{V}_{ni} + \vec{\Omega}(t)\times\vec{r}$$

8. $\left(\frac{d}{dt}-\right)_{in} = \left(\frac{d}{dt}-\right)_{ni} + \vec{\Omega}(t) \times - \quad (1)$

$$\left(\frac{d\vec{v}_{in}}{dt}\right)_{in} = \left(\frac{d\vec{v}_{in}}{dt}\right)_{ni} + \vec{\Omega}(t) \times \vec{v}_{in}$$

$$\vec{a}_{in} = \left[\frac{d}{dt}\left(\vec{v}_{ni} + \vec{\Omega}(t)\times\vec{r}\right)\right]_{ni} + \vec{\Omega}(t) \times \left(\vec{v}_{ni} + \vec{\Omega}(t)\times\vec{r}\right)$$

$$= \left(\frac{d\vec{v}_{ni}}{dt}\right)_{ni} + \left(\frac{d\vec{\Omega}}{dt}\right)_{ni} \times \vec{r} + \vec{\Omega}(t) \times \left(\frac{d\vec{r}}{dt}\right)_{ni}$$
$$+ \vec{\Omega}(t) \times \vec{v}_{ni} + \vec{\Omega}(t) \times \vec{\Omega}(t) \times \vec{r}$$

$$\vec{a}_{in} = \vec{a}_{ni} + 2\left(\vec{\Omega}(t)\times\vec{v}_{ni}\right) + \vec{\Omega}(t)\times\vec{\Omega}(t)\times\vec{r}$$
$$+ \left(\frac{d\vec{\Omega}}{dt}\right)_{ni} \times \vec{r}$$

From (1)

$$\left(\frac{d\vec{\Omega}}{dt}\right)_{in} = \left(\frac{d\vec{\Omega}}{dt}\right)_{ni} + \vec{\Omega} \times \vec{\Omega}$$

$$\Rightarrow \quad \vec{\alpha}_{in} = \left(\frac{d\vec{\Omega}}{dt}\right)_{ni}$$

angular acceleration as seen by inertial observer

$$\boxed{\therefore \quad \vec{a}_{in} = \vec{a}_{ni} + 2\left(\vec{\Omega}(t)\times\vec{v}_{ni}\right) + \vec{\Omega}\times\vec{\Omega}\times\vec{r} + \vec{\alpha}_{in}\times\vec{r}}$$