IIISER Mohali

[August 2024 Session]

PHY 403 (Atomic and molecular physics)

Midsem-I

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Max. Marks : 20

1. At time $t = 0$, the hydrogen atom is found in a state given by,
$$|\phi\rangle = N[a\,|100\rangle + b\,|200\rangle + c\,|322\rangle] \tag{1}$$
where, N is the normalization factor.
   (a) Is this an eigenstate of parity? Explain your answer.
   (b) What is the energy of this state?
   (c) Calculate the expectation values of L^2 and L_z operators.

2. Argue that hyperfine splitting for muonic atom (bound state of muon with a proton) should be larger than for the hydrogen atom. Provide an estimation of the splitting in eV. Take muon mass as 200 times the mass of electron.

3. Consider the spin states of two electrons denoted by,
$$\chi_1 = |\uparrow\uparrow\rangle,\ \chi_2 = |\uparrow\downarrow\rangle,\ \chi_3 = |\downarrow\uparrow\rangle,\ \chi_4 = |\downarrow\downarrow\rangle. \tag{2}$$
Which of the above are eigenstates of the $\mathbf{S}^2$ operator ($\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$)? Explain your answer. Construct the states which are eigenstates of $\mathbf{S}^2$.

4. What is the degeneracy of the ground state of the Helium atom? Consider a hypothetical Helium atom in which the two electrons are replaced by two identical spin-one particles of negative charge. If we ignore spin-dependent forces, what is the degeneracy of the ground state of the hypothetical atom? Explain you answer.

5. Analyze the linear Stark effect in the Hydrogen atom for $n = 1$ and $n = 2$ states when an electric field $\mathcal{E}_{\text{ext}} = \mathcal{E}_0\hat{x}$ is applied. Will there be a first order correction to the $n = 1$ state? Explain the splitting of $n = 2$ energy levels using degenerate perturbation theory. Explicit computation of matrix elements is not required. Identify the *good* states.

Useful results

1. Hyperfine shift in hydrogen atom

$$\Delta E = \frac{4}{3} g \frac{m_e}{m_p} \alpha^4 m_e c^2 \; (g \simeq 5.56) \tag{3}$$

2. Spin operators

$$S_x \left|\uparrow\right\rangle = \frac{\hbar}{2} \left|\downarrow\right\rangle; \; S_x \left|\downarrow\right\rangle = \frac{\hbar}{2} \left|\uparrow\right\rangle; \; S_y \left|\uparrow\right\rangle = -i\frac{\hbar}{2} \left|\downarrow\right\rangle; \; S_y \left|\downarrow\right\rangle = -i\frac{\hbar}{2} \left|\uparrow\right\rangle \tag{4}$$

3. Spherical harmonics

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}}; \; Y_{1,0} = \sqrt{\frac{3}{4\pi}} \cos\theta; \; Y_{1,\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\phi} \tag{5}$$

4. Commutation relations

$$[L_z, x] = i\hbar y; \; [L_z, y] = -i\hbar x \tag{6}$$