PHY401:
Nuclear And Particle Physics

Binding Energy

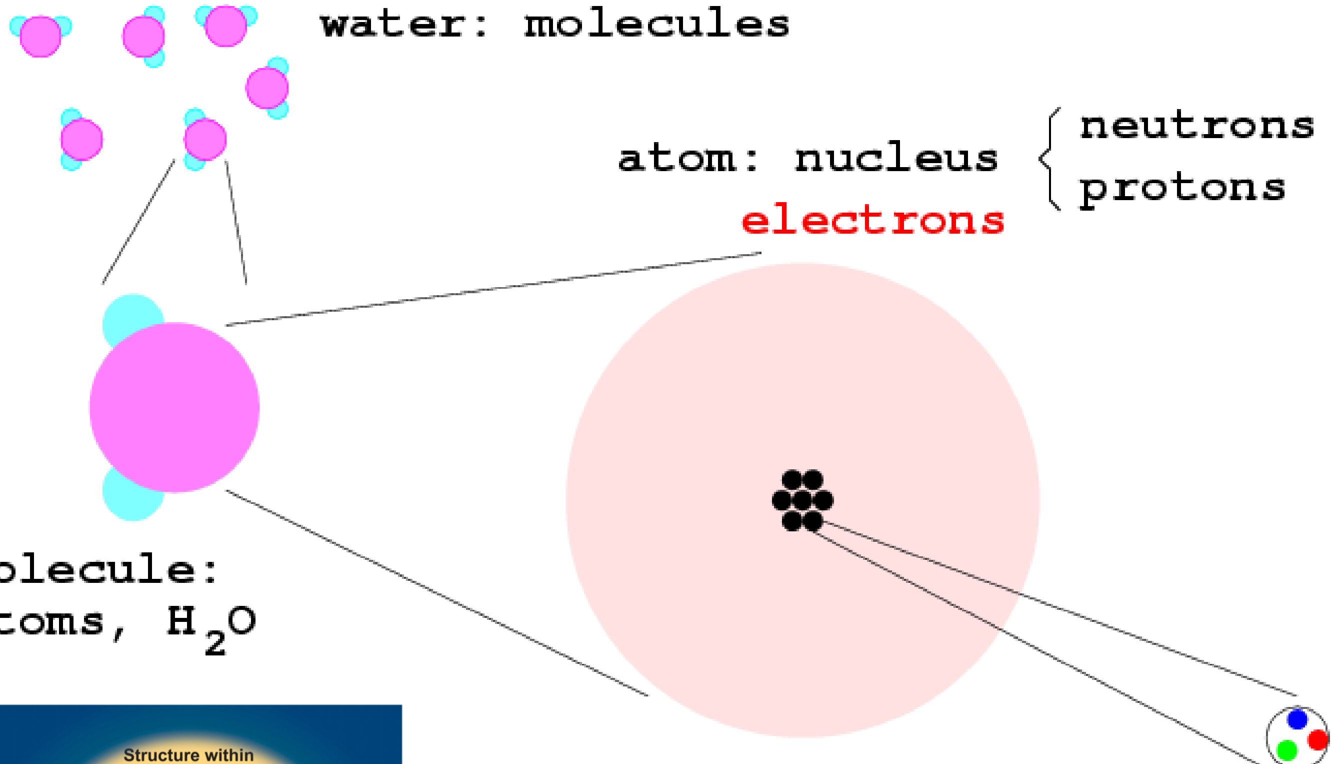
IISER

Satyajit Jena, 02/09/2024

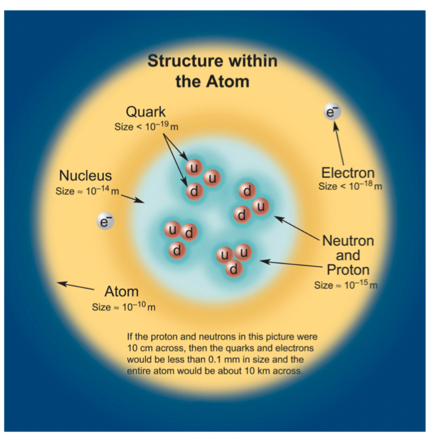
water: molecules

atom: nucleus { neutrons, protons }

electrons



molecule:
atoms, H_2O



neutron/proton:

up quarks

down quarks

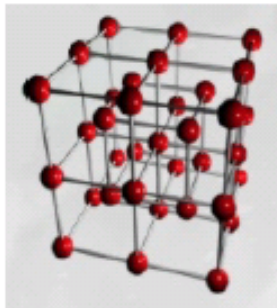
ddu/uud

Nuclear scale

Matter

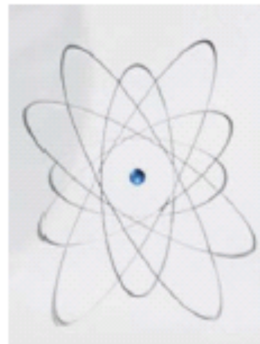
Macroscopic

Crystal



10^{-9} m

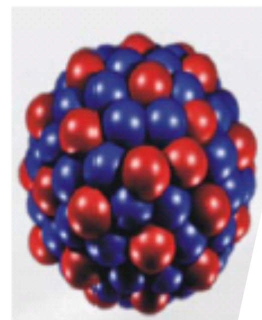
Atom



10^{-10} m

Angstrom

Atomic nucleus



10^{-14} m

Proton

Neutron

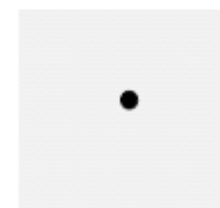
(similar mass)

Nucleon

10^{-15} m

femtometer

Quark



$< 10^{-18}$ m

Nuclear physics:

studies the properties of nuclei and

the interactions inside them (between protons and neutrons) and

between them

every-day” matter

24 Cr 52.00	
24	Atomic number
52.00	Atomic mass

1 1A	2 2A	3 3B	4 4B	5 5B	6 6B	7 7B	8 8B	9 8B	10 8B	11 1B	12 2B	13 3A	14 4A	15 5A	16 6A	17 7A	18 8A
1 **H** 1.008																	2 **He** 4.003
3 **Li** 6.941	4 **Be** 9.012											5 **B** 10.81	6 **C** 12.01	7 **N** 14.01	8 **O** 16.00	9 **F** 19.00	10 **Ne** 20.18
11 **Na** 22.99	12 **Mg** 24.31											13 **Al** 26.98	14 **Si** 28.09	15 **P** 30.97	16 **S** 32.07	17 **Cl** 35.45	18 **Ar** 39.95
19 **K** 39.10	20 **Ca** 40.08	21 **Sc** 44.96	22 **Ti** 47.88	23 **V** 50.94	24 **Cr** 52.00	25 **Mn** 54.94	26 **Fe** 55.85	27 **Co** 58.93	28 **Ni** 58.69	29 **Cu** 63.55	30 **Zn** 65.39	31 **Ga** 69.72	32 **Ge** 72.59	33 **As** 74.92	34 **Se** 78.96	35 **Br** 79.90	36 **Kr** 83.80
37 **Rb** 85.47	38 **Sr** 87.62	39 **Y** 88.91	40 **Zr** 91.22	41 **Nb** 92.91	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.1	45 **Rh** 102.9	46 **Pd** 106.4	47 **Ag** 107.9	48 **Cd** 112.4	49 **In** 114.8	50 **Sn** 118.7	51 **Sb** 121.8	52 **Te** 127.6	53 **I** 126.9	54 **Xe** 131.3
55 **Cs** 132.9	56 **Ba** 137.3	57 **La** 138.9	72 **Hf** 178.5	73 **Ta** 180.9	74 **W** 183.9	75 **Re** 186.2	76 **Os** 190.2	77 **Ir** 192.2	78 **Pt** 195.1	79 **Au** 197.0	80 **Hg** 200.6	81 **Tl** 204.4	82 **Pb** 207.2	83 **Bi** 209.0	84 **Po** (210)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89 **Ac** (227)	104 **Rf** (257)	105 **Ha** (260)	106 **Sg** (263)	107 **Ns** (262)	108 **Hs** (265)	109 **Mt** (266)	110	111	112						

58 **Ce** 140.1	59 **Pr** 140.9	60 **Nd** 144.2	61 **Pm** (147)	62 **Sm** 150.4	63 **Eu** 152.0	64 **Gd** 157.3	65 **Tb** 158.9	66 **Dy** 162.5	67 **Ho** 164.9	68 **Er** 167.3	69 **Tm** 168.9	70 **Yb** 173.0	71 **Lu** 175.0
90 **Th** 232.0	91 **Pa** (231)	92 **U** 238.0	93 **Np** (237)	94 **Pu** (242)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (249)	99 **Es** (254)	100 **Fm** (253)	101 **Md** (256)	102 **No** (254)	103 **Lr** (257)

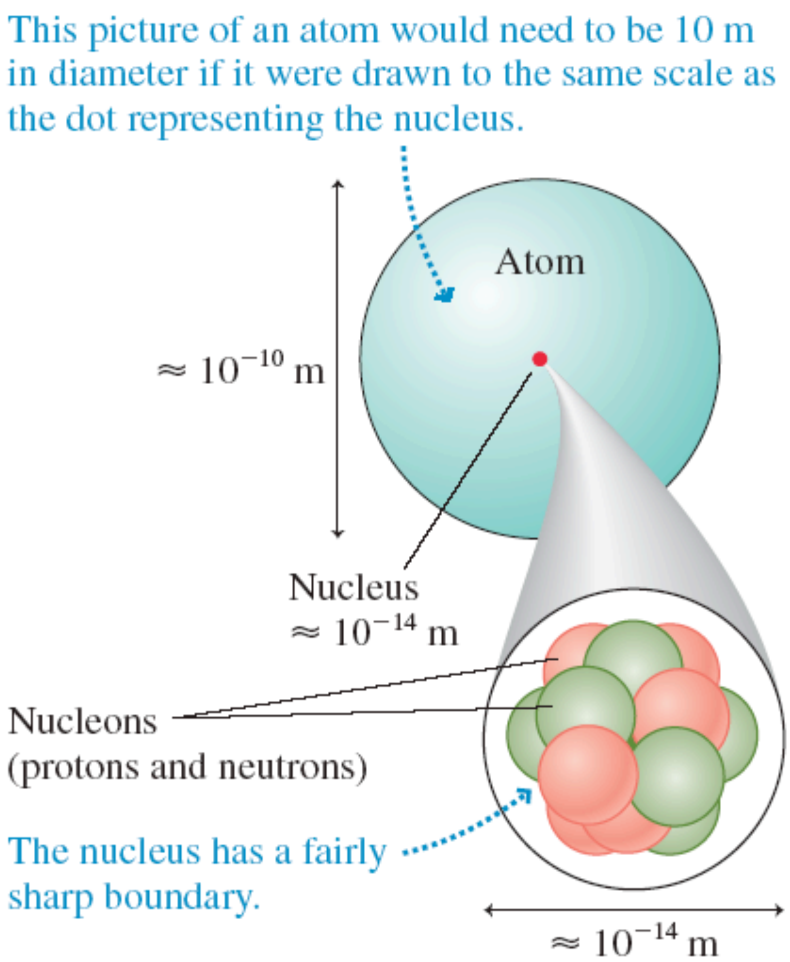
Metals

Metalloids

Nonmetals

Nucleus

FIGURE 43.1 The nucleus is a tiny speck within an atom.



Atomic number – in a neutral atom of any element, the number of protons or electrons (since p=e). (small blue number in upper right corner of each element box on periodic table): The atomic number is an element's most basic property since this determines how many electrons its atoms have and how they are arranged, which in turn determines the physical and chemical behavior of the element.

Mass number – Also called nucleon number is the number of **protons and neutrons** in an atom.

What is the difference between atomic mass and mass number?

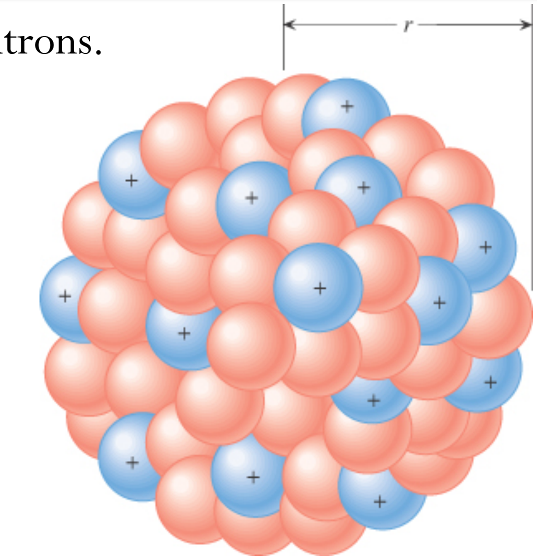
- **Mass number** and **atomic mass** are easily confused. Let's clear up the difference.
- Concentrate on the 2^{nd} word of each: *number* and *mass*
- **Mass number** is the **count of the number** of nucleons in an isotope and requires no units because it is simply a count.
- **Atomic mass** is a measure of the **total mass** of an atom whose units are the amu (atomic mass units).

Nuclear Structure

The atomic nucleus consists of positively charged protons and neutral neutrons.

Table 31.1 Properties of Select Particles

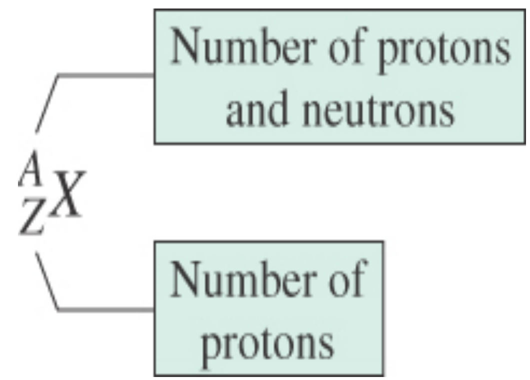
		Mass	
Particle	Electric Charge (C)	Kilograms (kg)	Atomic Mass Units (u)
Electron	-1.60×10^{-19}	$9.109\,382 \times 10^{-31}$	$5.485\,799 \times 10^{-4}$
Proton	$+1.60 \times 10^{-19}$	$1.672\,622 \times 10^{-27}$	1.007 276
Neutron	0	$1.674\,927 \times 10^{-27}$	1.008 665
Hydrogen atom	0	$1.673\,534 \times 10^{-27}$	1.007 825



atomic mass number

atomic number

$$\underbrace{A}_{\text{Number of protons and neutrons}} = \underbrace{Z}_{\text{Number of protons}} + \underbrace{N}_{\text{Number of neutrons}}$$



Nuclei that contain the same number of protons but a different number of neutrons are known as ***isotopes***.

Structure and Properties of the Nucleus

A nucleus is made of protons and neutrons. A proton is positively charged. A neutron is electrically neutral.

The atomic mass of the proton and the neutron is approximately:
Proton (mp) = 1.6726 x 10^{-24} grams = 1.0073 amu
Neutron (mn)= 1.6749 x 10^{-24} grams = 1.0087 amu

Thus, the neutron is just a little heavier than the proton.

Because of wave-particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx \left(1.2 \times 10^{-15}\,\mathrm{m}\right)\left(A^{\frac{1}{3}}\right)$$

Structure and Properties of the Nucleus

A nucleus is made of protons and neutrons.

A proton is positively charged. Its mass is:

$$m_{\mathrm{p}} = 1.67262 \times 10^{-27}\,\mathrm{kg}$$

A neutron is electrically neutral:

$$m_{\mathrm{n}} = 1.67493 \times 10^{-27}\,\mathrm{kg}$$

Because of wave-particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx \left(1.2 \times 10^{-15}\,\mathrm{m}\right)\left(A^{\frac{1}{3}}\right)$$

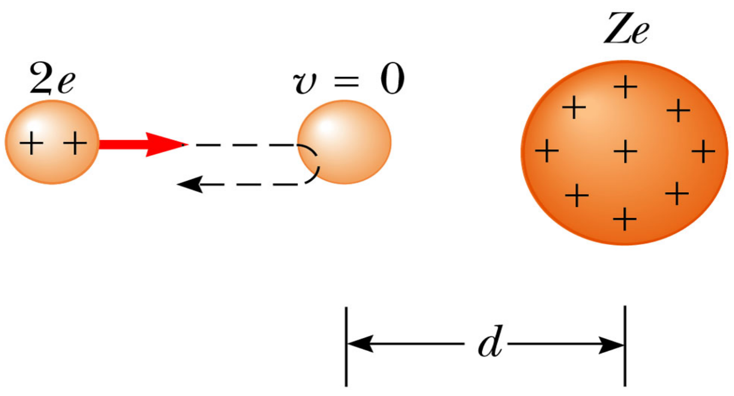
Size & density of Nucleus

- From scattering experiments, Rutherford found an expression for how close an alpha particle moving toward the nucleus can come before being turned around by the Coulomb force

- The KE of the particle must be completely converted to PE

$$\frac{mv^2}{2} = k_e \frac{q_1 q_2}{r} = k_e \frac{(2e)(Ze)}{d} \qquad d = \frac{4k_e Ze^2}{mv^2}$$

- d gives an upper limit for the size of the nucleus (e.g., for gold, d = 3.2 x 10^{-14} m and for silver, d = 2 x 10^{-14} m)



Size & density of Nucleus

- Such small lengths are often expressed in femtometers where 1 fm = 10^{-15} m Also called a fermi
- Since the time of Rutherford, many other experiments have concluded that most nuclei are approximately spherical with the average radius of $r = r_o A^{1/3}$
- $r_o = 1.2 \times 10^{-15}$ m
- The volume of the nucleus (assumed to be spherical) is directly proportional to the total number of nucleons
- This suggests that all nuclei have nearly the same density
- Nucleons combine to form a nucleus as though they were tightly packed spheres

Structure and Properties of the Nucleus

Masses of atoms are measured with reference to the carbon-12 atom, which is assigned a mass of exactly 12u. A u is a *unified atomic mass unit*.

$$1\ \mathrm{u} = 1.6605 \times 10^{-27}\ \mathrm{kg} = 931.5\ \mathrm{MeV}/c^2.$$

Mass-energy relation: $E = mc^2$

From the following table, you can see that the electron is considerably less massive than a nucleon.

Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

	Mass		
Object	**kg**	**u**	**MeV/c^2**
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Binding Energy and Nuclear Forces

The total mass of a stable nucleus is always less than the sum of the masses of its separate protons and neutrons.

Where has the mass gone?

It has become energy, such as radiation or kinetic energy, released during the formation of the nucleus. Again, $E = mc^2$.

This difference between the total mass of the constituents and the mass of the nucleus is called the total binding energy of the nucleus. (The binding energy is the energy needed to break a nucleus into individual nucleons.)

$$E_{Binding} = \left[\left(total\ mass\ of\ nucleons\right) - \left(mass\ of\ nucleus\right)\right]c^2$$

Mass Defect (Δm)

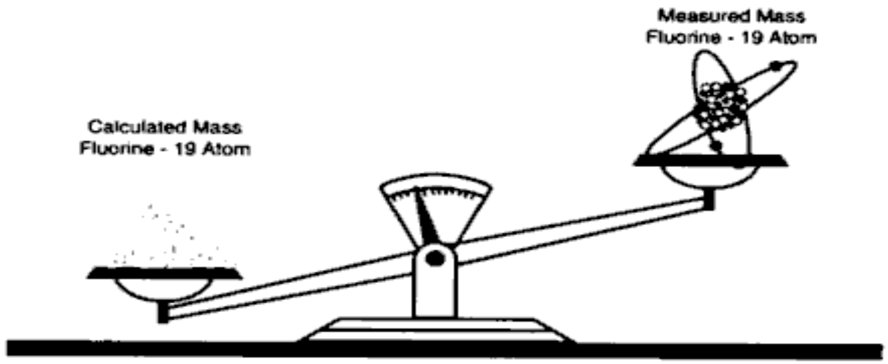


Figure 2-V. Illustration of a Mass Defect

- **Sum individual nucleons weigh more than the nucleus.**
- **Mass/ E difference = mass defect/deficit.**
- **The mass defect equals the BE.**
- **Find dif btw atomic mass and mass of constituents.**

Calculate mass defect & BE/nucleon

- 26 p^+ x 1.00782 u = 26.20332
- 28 n^o x 1.00866 u = 28.24248
- 26 e- x 0.0005490.014274
- Mass constituents <u>54.4458 u</u>

mass Fe nucl 53.9396 u

subtract

Mass defect 0.5062 u

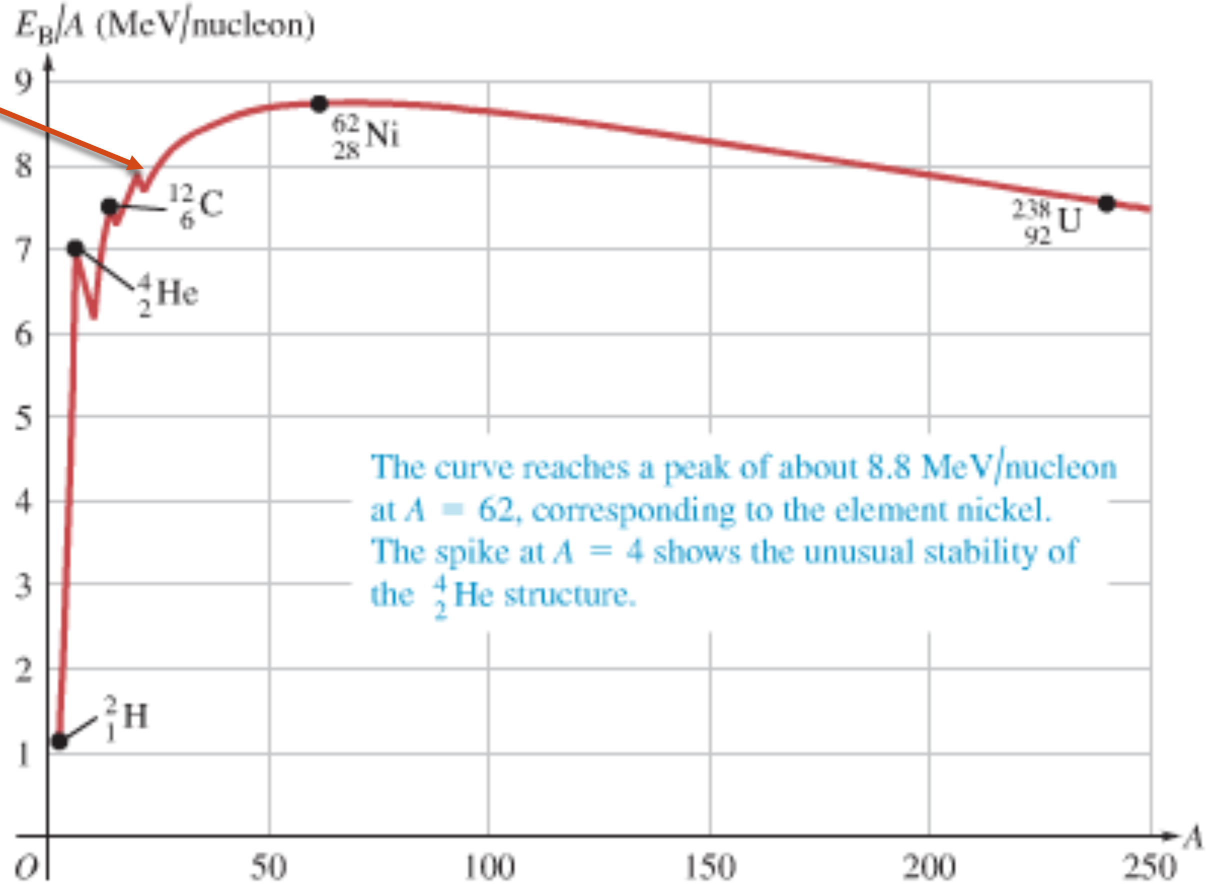
energy x 931.5 MeV/u

- BE = 471.5 MeV ÷ 54 nucl = 8.7 MeV/nucl

Nuclide	Z	N	A	Stability[a]	Mass[b] (u)	Spin[c]	Binding Energy (MeV/nucleon)
^{1}H	1	0	1	99.985%	1.007 825	$\frac{1}{2}$	—
^{7}Li	3	4	7	92.5%	7.016 004	$\frac{3}{2}$	5.60
^{31}P	15	16	31	100%	30.973 762	$\frac{1}{2}$	8.48
^{84}Kr	36	48	84	57.0%	83.911 507	0	8.72
^{120}Sn	50	70	120	32.4%	119.902 197	0	8.51
^{157}Gd	64	93	157	15.7%	156.923 957	$\frac{3}{2}$	8.21
^{197}Au	79	118	197	100%	196.966 552	$\frac{3}{2}$	7.91
^{227}Ac	89	138	227	21.8 y	227.027 747	$\frac{3}{2}$	7.65
^{239}Pu	94	145	239	24 100 y	239.052 157	$\frac{1}{2}$	7.56

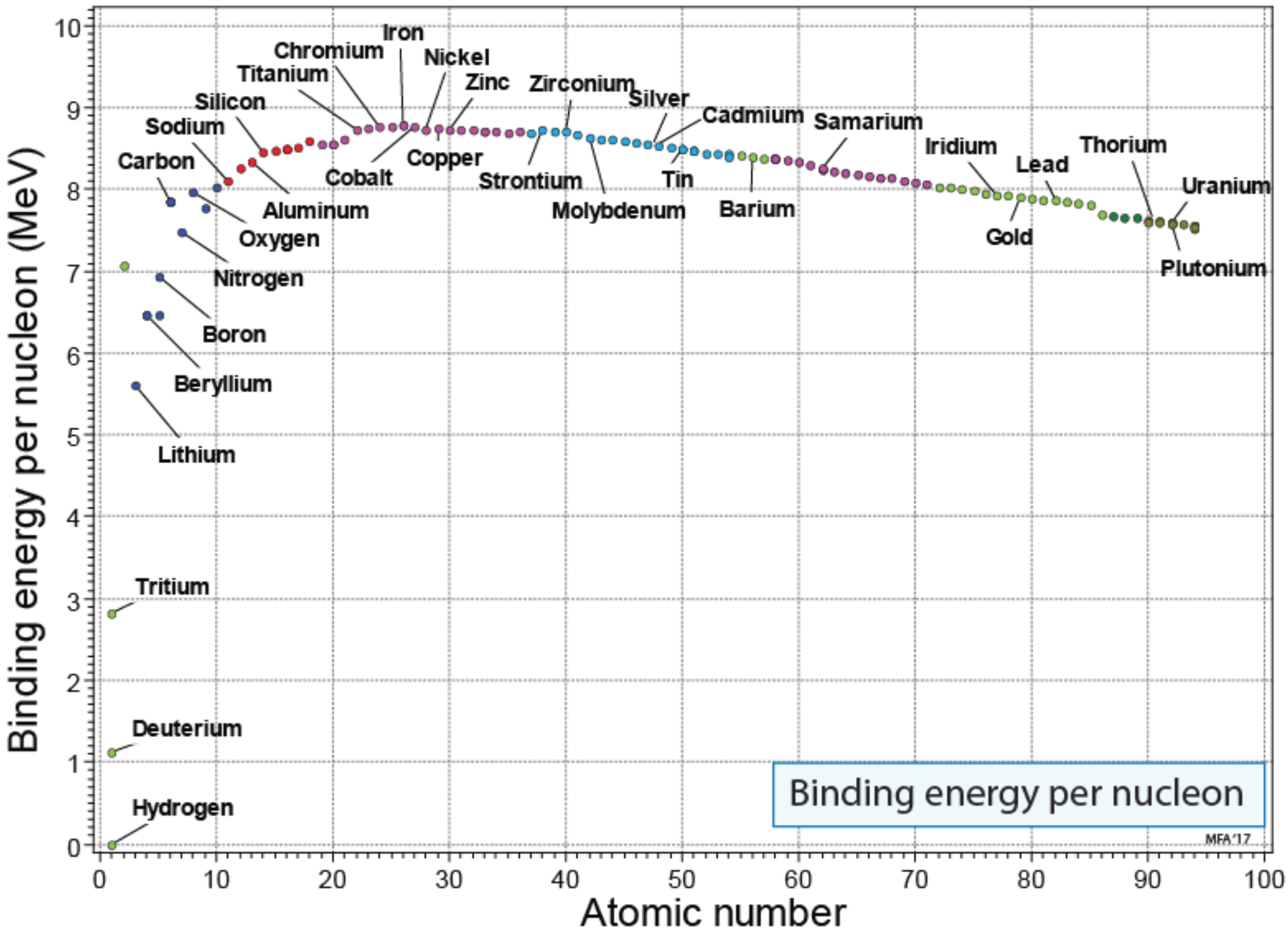
Binding Energy per Nucleon Curve

Oxygen-16 is here.



▲ **FIGURE 30.1** Approximate binding energy per nucleon as a function of mass number A (the total number of nucleons) for stable nuclides.

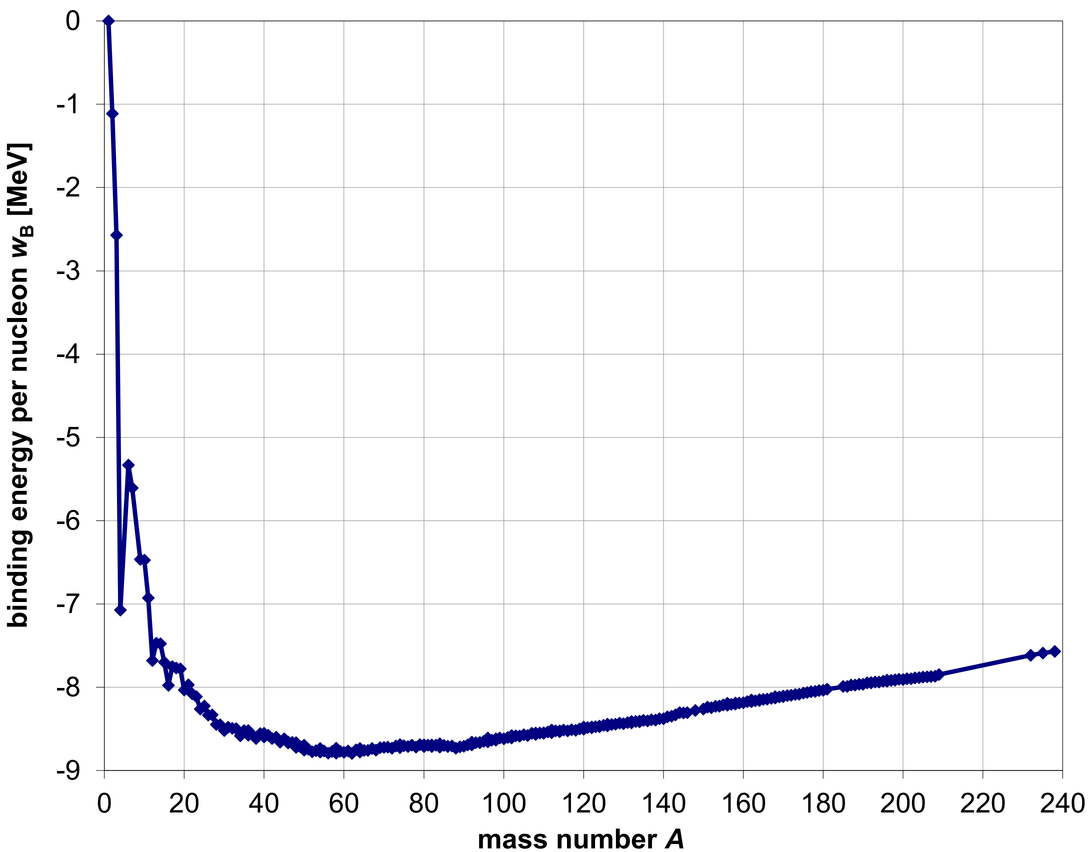
Our calculation for oxygen-16 gives 7.98 MeV/nucleon. The most stable nuclei are metals like iron, cobalt and nickel, with values just below 9 MeV/nucleon.



Binding energy per nucleon (MeV)
Atomic number
Binding energy per nucleon
Hydrogen
Deuterium
Tritium
Lithium
Beryllium
Boron
Nitrogen
Oxygen
Carbon
Sodium
Silicon
Aluminum
Titanium
Chromium
Iron
Nickel
Zinc
Cobalt
Copper
Zirconium
Strontium
Molybdenum
Silver
Cadmium
Tin
Barium
Samarium
Iridium
Gold
Lead
Thorium
Uranium
Plutonium
MFA'17

Binding energy of a nucleon

- Average energy needed to release one nucleon from the nucleus

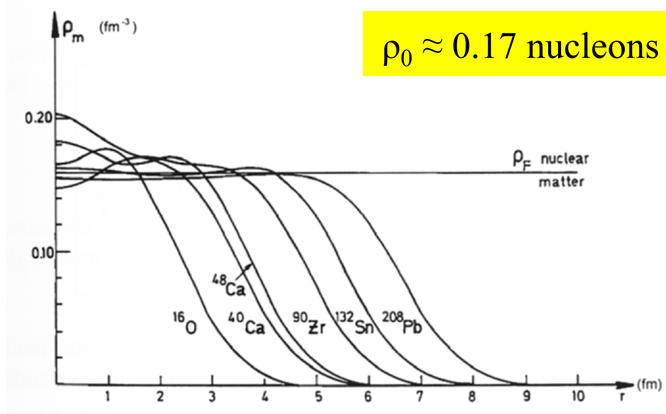


Task: Find the Nuclides and elements find out these curves

A charged drop of incompressible liquid

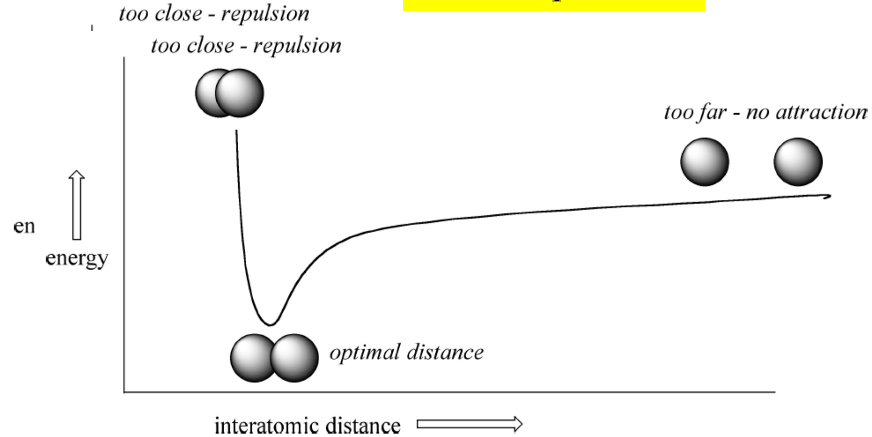
The scattering experiments we saw previously suggested that nuclei have approximately constant density. We were then able to calculate the nuclear radius assuming a uniform sphere. A drop of uniform liquid has the same property.

$\rho_0 \approx 0.17$ nucleons / fm^3



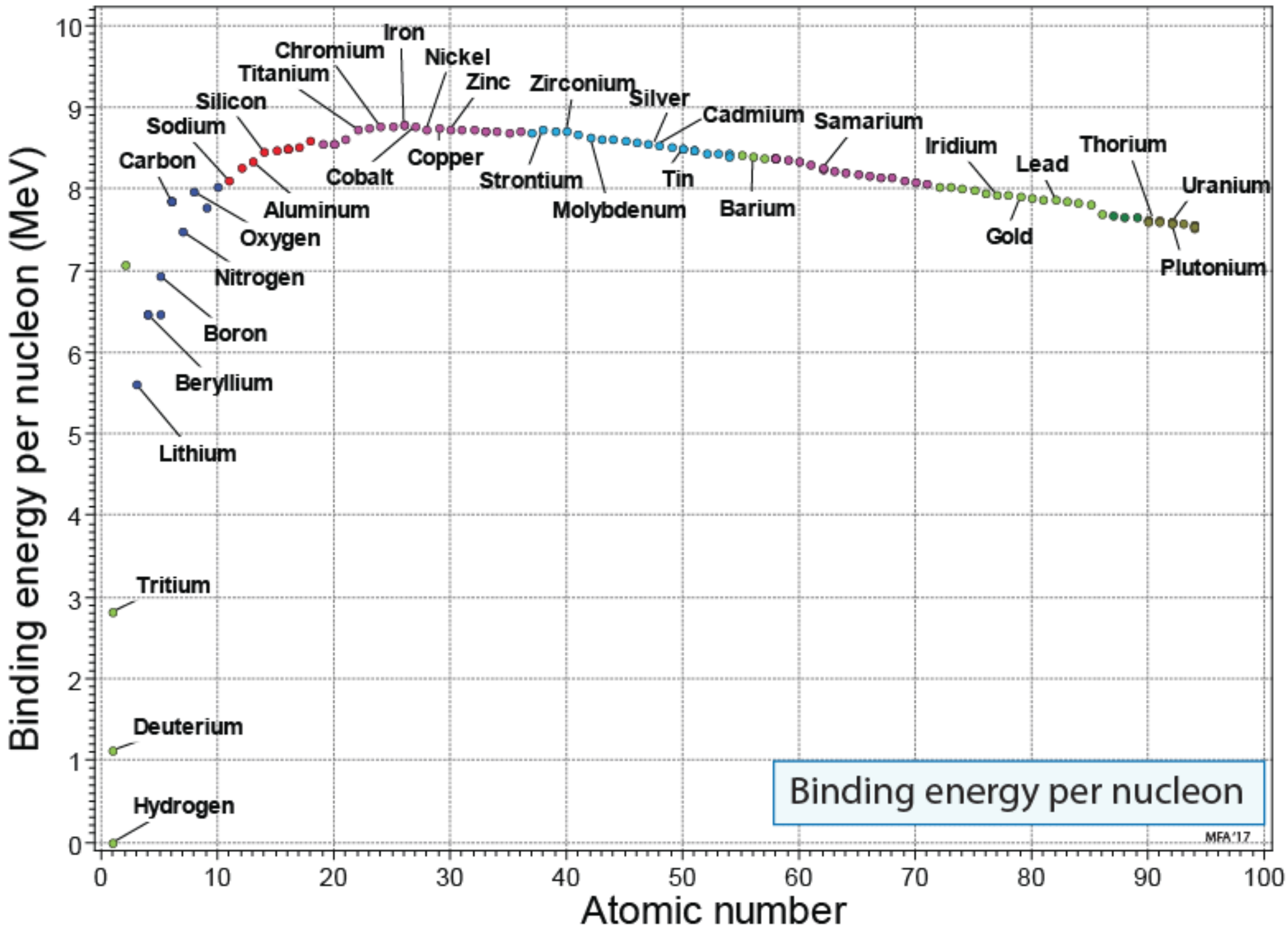
The nuclear force is short-range, but does not allow for compression of nuclear matter. Molecules in a liquid drop have the same basic properties.

Yukawa potential

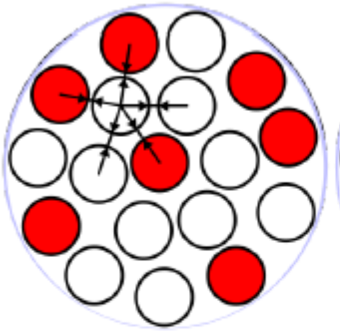


A charged drop of incompressible liquid

For the nucleus we assume a liquid drop with a uniform positive charge.



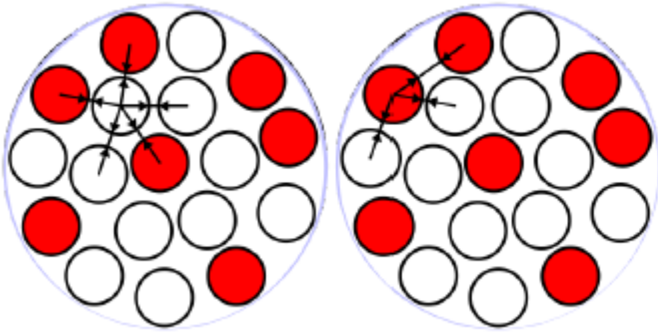
A charged drop of incompressible liquid



Interactions coming due to nucleons within the volume and sounded by it

$$\boxed{a_V \cdot A}$$

A charged drop of incompressible liquid

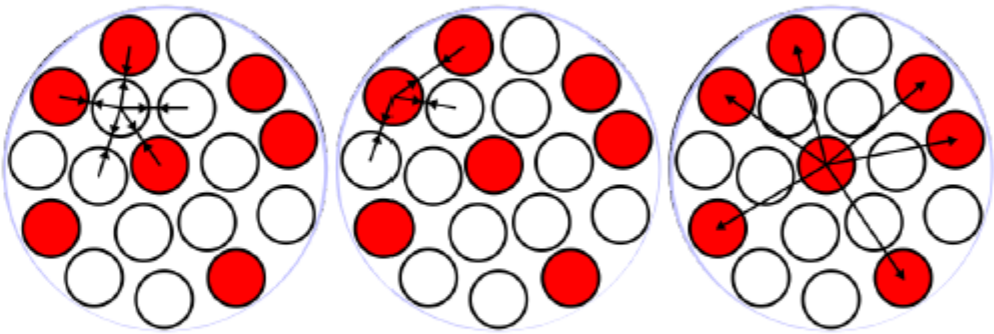


$$\boxed{a_V \cdot A}$$

First, we need to account for the fact that the nucleons on the surface have less neighbours, and do not exhibit the same binding as those in the interior (volume)....

$$\boxed{-a_S \cdot A^{2/3}}$$

A charged drop of incompressible liquid

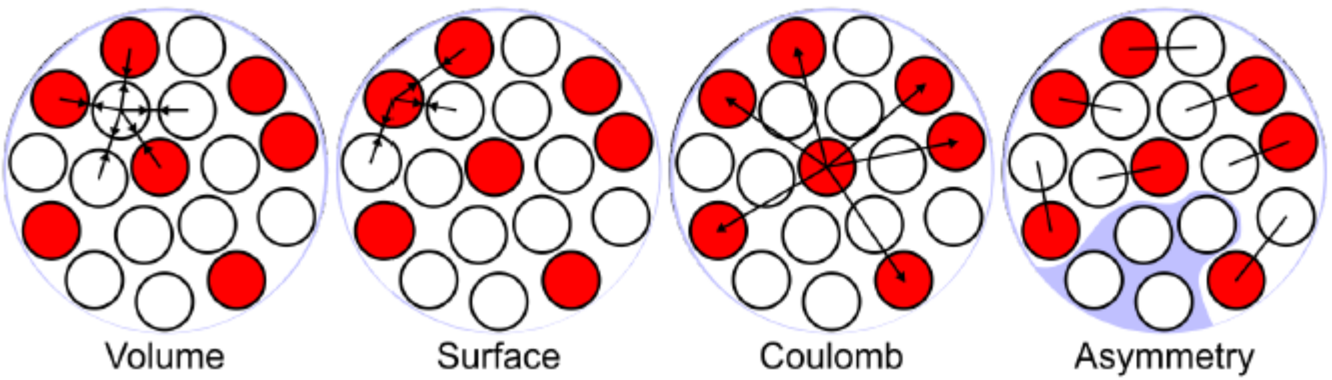


$\boxed{a_V \cdot A}$ $\boxed{-a_S \cdot A^{2/3}}$

Protons in the nucleus repel each other due to their mutual positive charge, this reduces the binding energy further....

$$\boxed{-a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}}}$$

A charged drop of incompressible liquid



$$\boxed{a_V \cdot A}$$

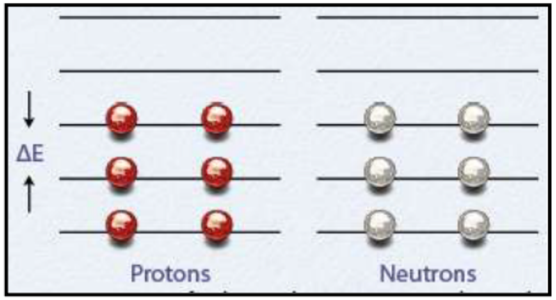
$$\boxed{-a_S \cdot A^{2/3}}$$

$$\boxed{-a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}}}$$

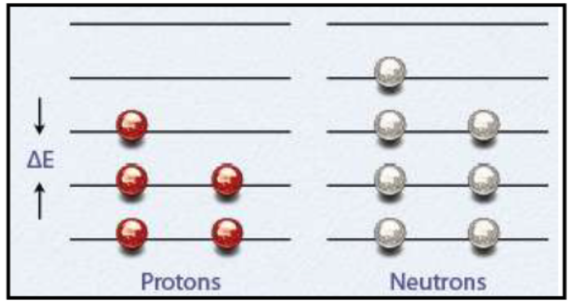
For light nuclei, N ~ Z (for heavy nuclei N is only slightly larger than Z). Where the Coulomb term would always favour Z = 0 for any A, we must account for the fact that nuclei are quantum objects (specifically that nucleons are fermions), and must obey the Pauli exclusion principle....

$$\boxed{-a_{asym} \cdot \frac{(N-Z)^2}{A}}$$

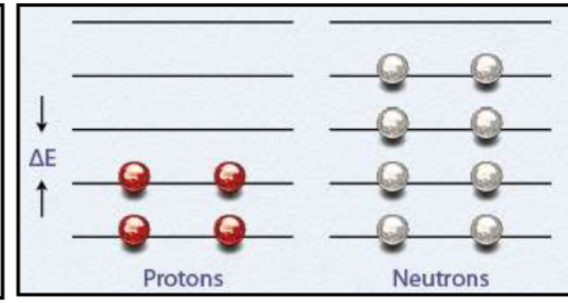
A charged drop of incompressible liquid



Most stable configuration



N–Z = 2: Reduced p-p repulsion
But: moving proton has cost **ΔE**



N–Z = 4: has cost **2ΔE**

For more states:

N - Z	Step energy (ΔE)	Cumulative energy (ΔE)
2	1	1
4	1	2
6	3	5
8	3	8
10	5	13
12	5	18
14	7	25

Cumulative energy change from lowest energy $N = Z$ given by $\sim (N - Z)^2 / 8 \times \Delta E$

This can be rewritten as $\sim \mathbf{(A - 2Z)^2 / 8 \times \Delta E}$

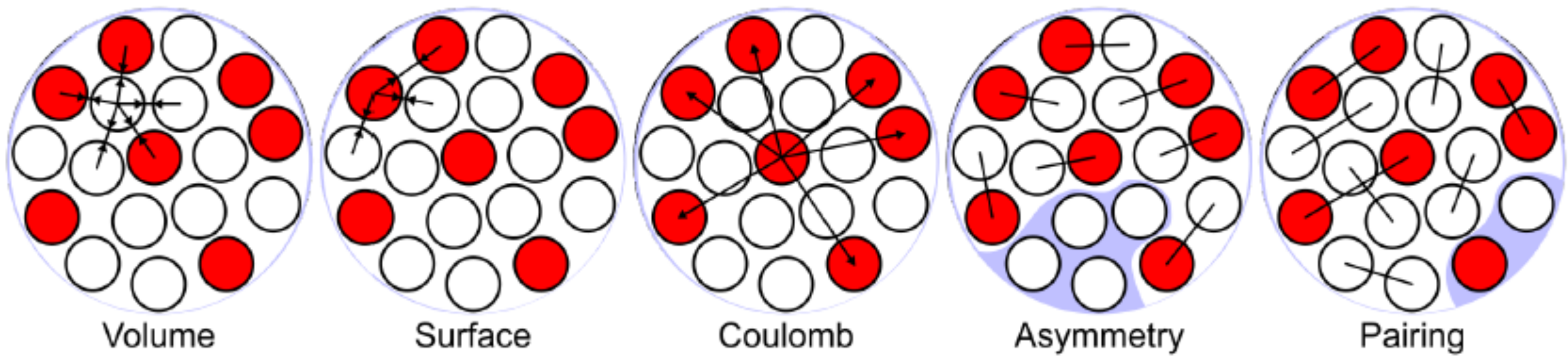
However, ΔE is not constant.
From QM: energy levels for particle in 3D finite well follow $\Delta E \sim 1 / R^3$. Again using $R \sim n^{1/3} = A^{1/3}$

$$\rightarrow \mathbf{\Delta E \sim 1 / A}$$

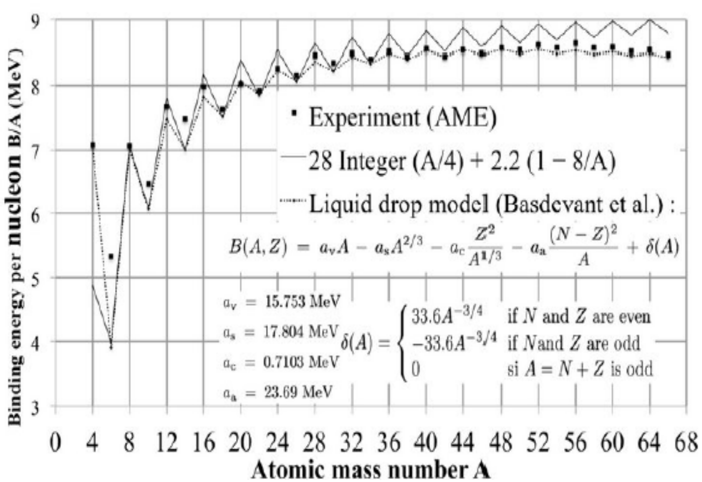
Collecting constants together, change in binding energy given by: $\boxed{- a_a (A - 2Z)^2 A^{-1}}$

i.e. asymmetry between protons and neutrons again **reduces** the binding energy, hence -ve

A charged drop of incompressible liquid



$\boxed{a_V \cdot A}$ $\boxed{-a_S \cdot A^{2/3}}$ $\boxed{-a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}}}$ $\boxed{-a_{asym} \cdot \frac{(N-Z)^2}{A}}$



There is still one observation that can tell us something about the binding energy, and how nucleons interact with one another. How many nuclei with an even or odd number of protons and neutrons are stable?

$$\text{Pairing} = \begin{cases} +\delta & for\ even - even\ nuclei \\ 0 & for\ odd - even\ nuclei \\ -\delta & for\ odd - odd\ nuclei \end{cases}$$

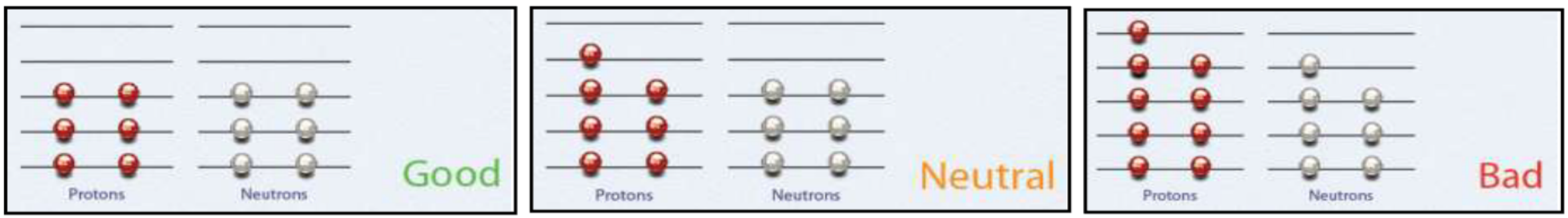
$\boxed{\text{nuclear pairing}}$

A charged drop of incompressible liquid

<u>Spin pairing in the liquid drop model</u>:

Spin pairing favours pairs of fermionic nucleons (similar to electrons in atoms)
i.e. a pair with opposite spin have lower energy than pair with same spin

Best case: even numbers of both protons and neutrons
Worst case: odd numbers of both protons and neutrons
Intermediate cases: odd number of protons, even number of neutrons or vice versa



→ Subtract small energy δ required to decouple nucleons from binding energy:

$$\begin{aligned} \delta &= +a_p A^{-1/2} && \text{for both N \& Z odd} \\ &= 0 && \text{for N even, Z odd / Z even, N odd} \\ &= -a_p A^{-1/2} && \text{for both N \& Z even} \end{aligned}$$

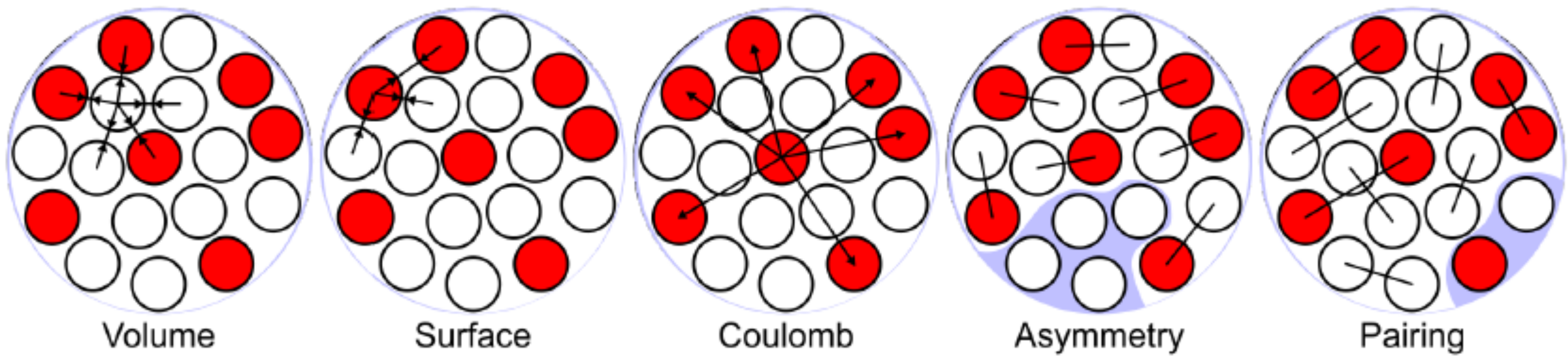
→ a_p collects constants, $A^{-1/2}$ dependence provides best empirical fit to data

→ subtracting δ **<u>reduces</u>** BE for N and Z both odd

→ subtracting δ **<u>adds</u>** small amount to BE for N and Z both even

1

A charged drop of incompressible liquid



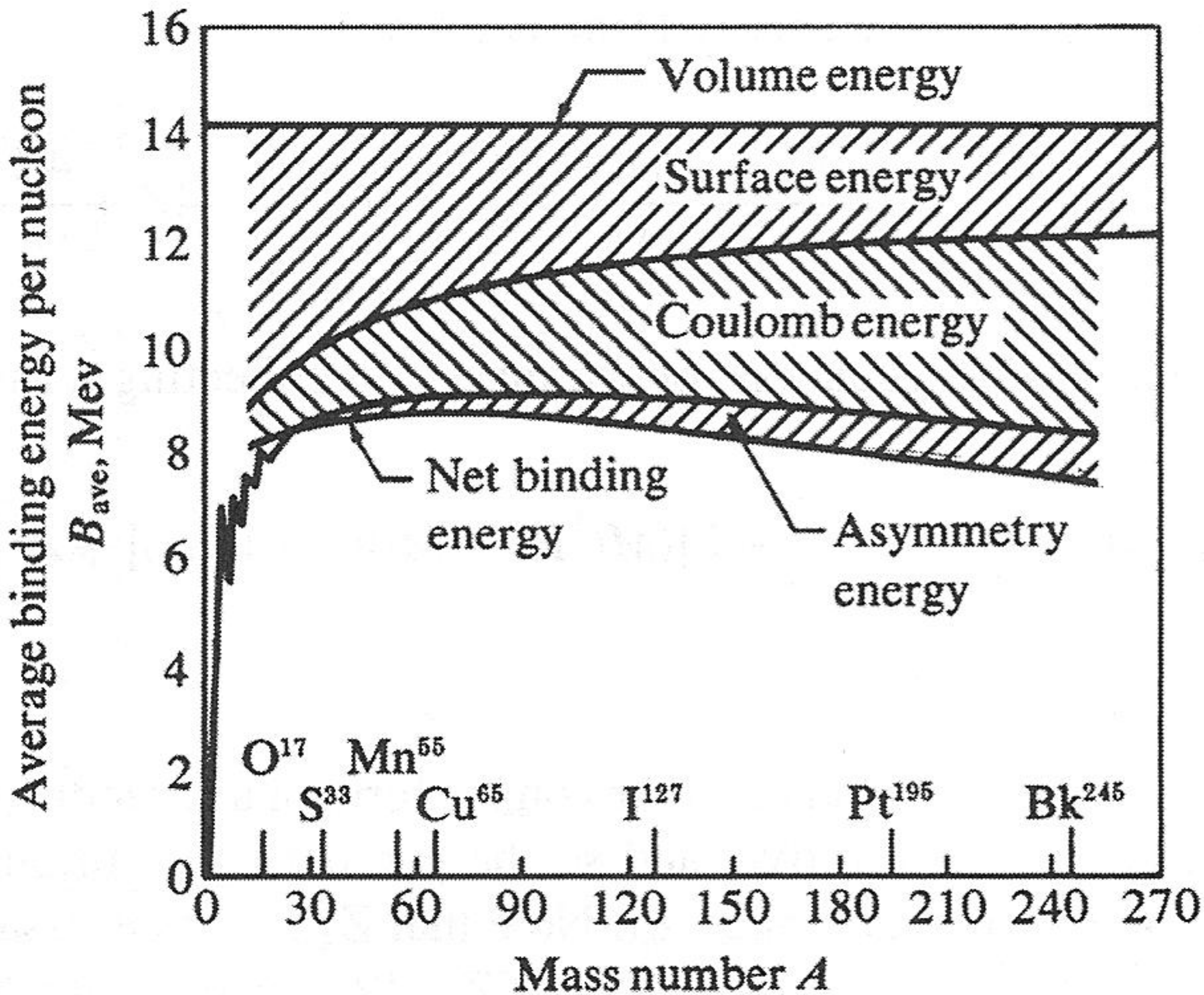
The Semi-Empirical Mass Formula

$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z\cdot(Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$

a_V	15.85 MeV
a_S	18.34 MeV
a_C	0.71 MeV
a_{asym}	23.21 MeV
a_{pair}	12 MeV

$$\delta = \begin{cases} +1 & for\ even - even\ nuclei \\ 0 & for\ odd - even\ nuclei \\ -1 & for\ odd - odd\ nuclei \end{cases}$$

A charged drop of incompressible liquid



A charged drop of incompressible liquid