Aniket Aggarwal
MS20040

IISER Mohali End-Sem Examination
2021-2022 Spring Semester
Course Code: PHY202

Time: 150 minutes **Max. Marks: 50**

1. Write if the following statements are *True* or *False*.

   [*Warning: Negative marking for this question. 1/2 mark for each correct answer. -1/2 mark for each wrong answer. No credit for cutting and overwriting an answer.*]

   (i) Free expansion of a gas is an irreversible process.
   (ii) Heat capacity of a thermodynamic system is an extensive quantity.
   (iii) For $S = AU^n V^m N^r$ with $A > 0$, to be a valid fundamental relation for a simple thermodynamic system, we must have $n > 0$ and $m < 0$.
   (iv) An equation of state of a system contains the same thermodynamic information as the fundamental relation.
   (v) T, P and μ are the intensive parameters in entropy representation.
   (vi) Work supplied to the system in an irreversible process is always more than for a reversible process.
   (vii) An exchange of heat with the surroundings may or may not lead to a change in the entropy of the system.
   (viii) A quasi-static process must be reversible, but a reversible process may not be quasi-static. [8 × 1/2]

2. A system is described by the fundamental relation:

$$S = \tfrac{RUV}{N} - \tfrac{RN^3}{UV}, \quad R > 0.$$

   (i) Show that the temperature of the system is intrinsically positive.
   (ii) Find the form of pressure: $P = P(T, v)$, where $v = V/N$. [2+2]

3. (a) From the fundamental relation $S(U, V, N)$, derive the expression for $d\hat{s}$ in terms of $d\hat{u}$ and $d\hat{n}$, where $\hat{s} = S/V, \hat{u} = U/V, \hat{n} = N/V$.

   (b) Calculate the work done in isobaric expansion of two moles of monoatomic ideal gas from temperature 200K to 350K. [4+2]

4. Electromagnetic radiation inside a cavity at temperature T obeys: $u = bT^4$ and $P = u/3$, where u is energy per unit volume (U/V) and b is some positive constant. Find an expression for entropy per unit volume in terms of u. [4]

5. Enthalpy is given by $H = U + PV$. Also H is given to be a function only of the variables S, P, N. Show that

$$\frac{\partial H}{\partial P} = V, \quad \frac{\partial H}{\partial N} = \mu,$$

   where the symbols have their usual thermodynamic meanings. [4]

6. Using the following equation of state for the rubber band of length L and at temperature T:

$$\Pi = \frac{bT(L-L_0)}{L_1-L_0},$$

where Π is the tension, and $L_0 < L < L_1$, and $b > 0$. Evaluate the following product by an explicit calculation of each factor:

$$\left(\frac{\partial \Pi}{\partial T}\right)_L \left(\frac{\partial T}{\partial L}\right)_\Pi \left(\frac{\partial L}{\partial \Pi}\right)_T.$$

[4]

7. Calculate and show the total entropy change is positive definite in the process in which a monoatomic ideal gas at temperature T_A is put in thermal contact with a heat reservoir at temperature T_B. Take $T_A > T_B$. [4]

8. An irreversible heat engine runs between two reservoirs at temperatures T_h and T_c (where $T_c < T_h$). Let Q_h be the heat absorbed in a cycle from the hot reservoir. Show that the work performed in an irreversible heat cycle is given by $W = W_{\max} - T_c \Delta S$, where ΔS is the net increase in the entropy of the universe and $W_{\max}$ is the maximum work that may be extracted in a cycle for a given amount Q_h. [4]

9. A monoatomic ideal gas is at volume V_0 and pressure P_0 inside an insulated cyclinder with a piston. The piston is suddenly moved out so that the gas undergoes a free expansion and the final volume V_f.
(i) Calculate the change in entropy of the gas.
(ii) How will you depict the process on a $P-V$ plane?
(iii) What is the work done by the gas in the process ? [2+1+1]

10. A two-level atom with energies $(-\epsilon, +\epsilon)$ is in contact with a reservoir at temperature T. Both the energy levels are then shifted by an energy Δ. Show the effect on (i) probabilities p_j to occupy a level j, and (ii) the mean energy of the atom. [4]

11. A four-level atom has energy levels with energies as: $(-1, 1, 3)$, where the highest energy level is doubly degenerate. Find the mean entropy and mean energy of the atom at temperature T. [4]

12. Variance of a random quantity x, is defined as: $\mathcal{V}(x) = \sum_i (x_i)^2 p_i - \left(\sum_i x_i p_i\right)^2$. Express the variance of the energy of a two level system with energy levels $(0, a)$ and in equilibrium with a heat reservoir at temperature T, in terms of the heat capacity of this system. [4]