1. Two points of mass m are joined by a rigid weightless rod of length ℓ , the center of which is constrained to move on a circle of radius a. Set up the kinetic energy in the generalised coordinate.

2. Show that the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \left(\frac{\partial T}{\partial q_j}\right) = Q_j$$

can be written as

$$\left(\frac{\partial \dot{T}}{\partial \dot{q}_j}\right) - 2\left(\frac{\partial T}{\partial q_j}\right) = 0$$

3. If L is a Lagrangian for a system of n degrees of freedom satisfying the Lagrange equations, show by direct substitution that

$$L' = L + \frac{dF(\{q_i\}, t)}{dt}$$

also satisfies Lagrange's equations where F is any arbitrary, but differentiable, function of its arguments.

4. The electromagnetic field is invariant under a gauge transformation of the scalar and vector potential given by

$$\mathbf{A} \to \mathbf{A} + \nabla\psi(\mathbf{r}, t)$$

and

$$\phi \to \phi - \frac{\partial \psi}{\partial t}$$

. where ψ is arbitrary (but differentiable). What effect does this gauge transformation have on the Lagrangian of a moving particle in the electromagnetic field? Is the equation of motion affected?

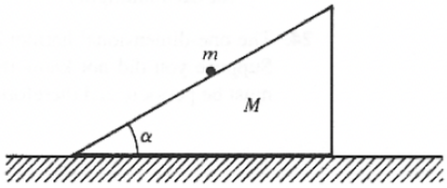
5. Show that the geodesics of a spherical surface are great circles, i.e., circles whose centers lie at the center of the sphere.

6. A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder.

7. A particle of mass m slides without friction on a wedge of angle α and mass M that can move without friction on a smooth horizontal surface, as shown in the figure. Treating the constraint of the particle on the wedge by the method of Lagrange multipliers, find the equation of motion for particle and wedge. Also obtain an expression for the forces of constraint. Calculate the work done in time t by the forces of constraint acting on the particle and on the wedge. What are the constants of motion for the system?

8. Show that the motion of a particle in the potential field

$$V(r) = -\frac{k}{r} + \frac{h}{r^2}$$



is the same as that of the motion under the Kepler potential alone when expressed in terms of a coordinate system rotating or precessing around the center of force. For negative total energy, show that if the additional potential term is very small compared to the Kepler potential, then the angular speed of precession of the elliptical orbit is

$$\dot{\Omega} = \frac{2\pi m h}{l^2 \tau}$$

9. Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$.

10. Suppose a satellite is moving around a planet in a circular orbit of radius r_0. Due to a collision with another object, satellite's orbit gets perturbed. Show that the radial position of the satellite will execute simple harmonic motion with $\omega = l/mr_0^2$, where l is the initial angular momentum of the satellite.

11. A particle of mass m is moving under the central force $\mathbf{F(r)} = -\frac{C}{r^3}\hat{\mathbf{r}}$ with $C > 0$. Find the non-zero values of angular momentum l for which the particle will move in a circular orbit.

12. A particle of mass m is constrained to move under gravity without friction on the inside of a paraboloid of revolution whose axis is vertical. Find the one-dimensional problem equivalent to its motion. What is the condition on the particle's initial velocity to produce circular motion? Find the period of small oscillations about this circular motion.

13.
    a) Show that if a particle describes a circular orbit under the influence of an attractive central force directed at a point on the circle, then the force varies as the inverse fifth power of the distance.
    b) Show that for the orbit described the total energy of the particle is zero.
    c) Find the period of the motion.

14. At perigee of an elliptic gravitational orbit a particle experiences an impulse S in the radial direction, sending the particle into another elliptic orbit. Determine the new semimajor axis, eccentricity, and orientation of major axis in terms of the old.

15. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Determine the differential cross section.