Indian Institute of Science Education and Research, Mohali

Integrated MSc, Semester: IV

Probability and Statistics: MTH 202

Tutorial 6 (February, 22, 2023)

Summary

Let X be a real valued random variable on a probability space $(\Omega, \mathcal{A}, P)$. Consider the function $F_X : \mathbb{R} \to [0,1]$, defined by $F_X(x) = P(X^{-1}(-\infty, x]) = P\{w \in \Omega : X(w) \leq x\}$. This function describes the probability law of X completely and is called the **Cumulative Distribution Function** of the random variable X.

The Cumulative Distribution Function F_X is monotonically non-decreasing, right continuous and

$$\lim_{x \to -\infty} F_X(x) = 0, \lim_{x \to \infty} F_X(x) = 1.$$

In particular, for a discrete random variable X with **Probability Mass Function** p_X, the Probability Distribution Function $F_X(x) = \sum_{k \leq x} p_X(k)$ is a step function with set of points of discontinuity is exactly the range of X.

A random variable X is said to be **continuous** if there exists a non negative Riemann integrable function f_X on $\mathbb{R}$ such that $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$.

Which is equivalent to saying $P(\{w \in \Omega : X(w) \in [a, b]\}) = F_X(b) - F_X(a) = \int_a^b f_X(t)dt$. This function f_X is called the **Probability Density Function** of the random variable X.

The **Expectation** of X with Probability Density Function f_X is defined by $E(X) = \int_{-\infty}^{\infty} x f_X(x)dx$ (when it exists !)

Question

1. Suppose a wild life scientiest is exploring a jungle where probability of spoting a tiger in a day is $\frac{1}{100}$. Let X be the random variable which count the number of days he has to explore till spoting a tiger. Write down Probability Mass Function p_X and Cumulative Distribution Function F_X. Compute the expectation $E(X)$.

2. Consider a circle with radious 1 unit in the complex plane. Choose a point Z randomly on the circle. Let the random variable X measures the argument of Z. Write down the cumulative distribution function F_X and density function f_X of the random variables X. Compute the expectation $E(X)$.

3. Let X be a random variable with probability density function $f_X(x) = cx$ for $x \in [0, 1]$ (zero for other $x \in \mathbb{R}$). Find the value of c. Write down the cumulative distribution function F_X and density function f_X of the random variables X. Compute the expectation $E(X)$.

4. Let X be the random variable which describes the life span of a tube light in years. Suppose its probability density function $f_X(x) = ce^{-x}$ for $x \geq 0$ (zero for other $x \in \mathbb{R}$). Determine the value of c. Find the probability that the tube light will atleast last for 9 months. What is the Cumulative distribution function F_X. Find the average life span $E(X)$.