PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 9

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1. Consider scattering from the potential $V(r) = V_0 e^{-r^2/a^2}$. Find the differential cross section in the first Born approximation and the total cross section.

2. Consider scattering of a particle by an attractive square well potential $V(r) = -V_0, r < a$ and $V(r) = 0, r > a$. Find the differential cross section in the first Born approximation.

3. Consider elastic scattering from the potential $V(r) = V_0 e^{-r/a}$, V_0, a being constants. Find the differential cross section in the first Born approximation.

4. Find the differential cross section in the first Born approximation for the elastic scattering of a particle of mass m, which is intially traveling along the z axis, from a nonspherical double-delta potential $V(r) = V_0\delta(r - ak) - V_0\delta(r + ak)$, where k is the unit vector along the z-axis.

5. Consider the Coulomb potential and write the Klein-Gordon equation for the hydrogen atom. Write $\psi(r,t) = R(r)Y_l^m(\theta,\phi)e^{-iEt/\hbar}$ and write the radial part of the Klein-Gordon equation in terms of the defined quantities
$$\gamma = \frac{Ze^2}{\hbar c};\ \alpha^2 = \frac{4(m^2c^4 - E^2)}{\hbar^2 c^2};\ \lambda = \frac{2E\gamma}{\hbar c\alpha}$$
and $\rho = \alpha r$. Assume the solution to be $R(\rho) = \rho^s e^{-\rho 2} v(\rho)$. Expand $v(\rho)$ as a power series $\sum_{k=0}^{\infty} a_k \rho^k$. Put this into the radial Klein-Gordon equation and derive a recursion relation for a_k. Put in that for $k \geq N, a_k = 0$ and derive that
$$E_n = mc^2(1 + \frac{\gamma^2}{n^2})^{-1/2}$$
where $n = N + s + 1$.

6. Consider a system described by the Dirac Hamiltonian $H = -i\alpha.\nabla + \beta m$, and let $\hbar = c = 1$. Show that the Heisenberg equation
$$-i\frac{\partial Q(t)}{\partial t} = (H, Q)$$

applied to the position operator $x(t)$ gives

$$\frac{\partial x(t)}{\partial t} = \alpha_x$$

Hence α_x can be viewed as a velocity (in units of c) in the x direction. Show that by itself $\alpha_x(t)$ satisfies the equation of motion

$$\frac{\partial \alpha_x(t)}{\partial t} = 2i[p_x - \alpha_x H]$$

Solve the two equations of motion for α_x and x to get $\alpha_x(t)$ in terms of $\alpha_x(0)$, H, p_x. Hence find $x(t)$ in terms of $x(0), \alpha_x(0)$, H, p_x.

7. Consider a Dirac particle in a radial potential $V(r)$. Define the operator $K = \beta(\frac{\sigma'.\mathbf{L}}{\hbar} + 1)$ and the radial momentum operator $p_r = -i\hbar\frac{1}{r}\frac{\partial}{\partial r}r = \frac{1}{r}(\mathbf{r}.\mathbf{p} - i\hbar)$ and $\alpha_r = \frac{1}{r}(\alpha.\mathbf{r})$. Show that

$$(\alpha.\mathbf{r})(\alpha.\hat{\mathbf{p}}) = rp_r + i(\sigma.\mathbf{L} + \hbar)$$

. Also show that

$$\alpha.\mathbf{p} = \alpha_r[p_r + \frac{i}{r}\beta K]$$

Hence write the Dirac Hamiltonian H_D in terms of $\alpha_r, p_r, \beta, K, V(r)$.

8. Consider the Diract Hamiltonian H_D from the above problem. Show that $[K, H] = 0$. The matrix K is block diagonal and operates separately on two spinor wave functions ϕ_L, ϕ_S. Consider the basis wave functions $Y_l^m\chi_\pm$ where Y_m^l are usual spherical harmonics, $\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the spin-up and spin-down spinors. Show that these basis functions are eigentates of $\hbar K$ and find the eigenvalues.