Quiz - 6

- A damped oscillator is at equilibrium location at t=0 moving with speed 3π cm/s. If its natural oscillations had amplitude and time period 3 cm and 1s respectively, find out the damping coefficient.

- A unit mass forced oscillator with very small damping is oscillating in its steady state. It is observed that its speed becomes maximum 1.5 seconds after its acceleration magnitude becomes maximum. If its natural oscillation frequency was (π/5) rad/s and external force applied is a sine periodic function with an amplitude of 0.1 N, what is the amplitude of oscillation?

Q-6

1. $x(t) = Ce^{-\beta t/2}\sin(\tilde{\omega}t + \phi)$

$x(0) = C\sin\phi = 0 \Rightarrow \phi = 0$

$\dot{x}(t) = Ce^{-\frac{\beta t}{2}}\left[-\frac{\beta}{2}\sin(\tilde{\omega}t+\phi) + \tilde{\omega}\cos(\tilde{\omega}t+\phi)\right]$

$\dot{x}(0) = C\left[\tilde{\omega}\right]$

$\dot{x}(0) = 3\pi \times 10^{-2}\ m\,s^{-1}$, $C = 3\times10^{-2}\ m$

$T = 1\ s$

$\tilde{\omega} = \pi\ rad\,s^{-1}$ $\quad \omega = \frac{2\pi}{T}\ rad\,s^{-1} = 2\pi\ rad\,s^{-1}$

$\tilde{\omega}^2 = \frac{1}{4}(4\omega^2 - \beta^2) \Rightarrow \beta^2 = 4(\omega^2 - \tilde{\omega}^2)$

$\beta = 2\pi\sqrt{4-1}\ rad\,s^{-1} = 2\sqrt{3}\pi\ rad\,s^{-1}$

2. For steady state

$x_p(t) = C\sin(\omega_0 t + \phi)$

$\dot{x}_p(t) = C\omega_0\cos(\omega_0 t + \phi)$

$|\dot{x}_p| = |-C\omega_0^2\sin(\omega_0 t + \phi)|$

If $|\dot{x}_p(t)|$ becomes maximum at t_*, $|\ddot{x}_p(t)|$ will become maximum at $t_* + t$ s.t.

$\omega_0 t = \pi/2 \Rightarrow \omega_0 = \frac{\pi}{2\times1.5} = \frac{\pi}{3}\ rad\,s^{-1}$

$\omega = \pi/5\ rad\,s^{-1}$, $f_0 = 0.1$

$$C = \frac{f_0/m}{\left[(\omega_0^2-\omega^2)^2 + \beta^2\omega_0^2\right]^{1/2}} \simeq \frac{0.1}{\pi^2\left(\frac{16}{9\times25}\right)} = \frac{22.5}{16\pi^2}$$