PHY622/Quiz 3

Date: April 20, 2018 **[Total Maximum Marks: 20]**

Enrol. No.: SOLUTION **Name:** —

Instructions:

- Marks: For questions (1-5), **+2** for each correct answer and **−1** for each incorrect answer. For questions (6) and (7), **+5** for each correct answer.
- For multiple choice type questions, mark your answer neatly. Answers with more than one selection will not be taken into account.
- For other questions write **only** the final answer in a space given in the paper.

Questions

1. $SU(n)$ is a group of $n \times n$ unitary matrices with determinant $+1$. The number of independent generators of this group is
   (A) $n+1$ (B) $2n-1$ (C) n^2-1 (D) n^2

2. The dimension of matrices in adjoint representation of $SO(8)$ group is
   (A) 7×7 (B) 8×8 (C) 28×28 (D) 56×56

3. For the generators J_i $(i = 1, 2, 3)$ of $SO(3)$ and $J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$, and for an operator $\mathcal{J} = J_- J_+$ and $J_3|j,m\rangle = m|j,m\rangle$,

   (A) $|j,m\rangle$ is an eigenstate of $\mathcal{J}$ with eigenvalue $\frac{1}{2}(j^2 - m^2)$.
   (B) $|j,m\rangle$ is an eigenstate of $\mathcal{J}$ with eigenvalue $\frac{1}{2}(j-m)(j+m+1)$.
   (C) $|j,m\rangle$ is an eigenstate of $\mathcal{J}$ with eigenvalue $\frac{1}{2}(j+m)(j-m+1)$.
   (D) $|j,m\rangle$ is not an eigenstate of $\mathcal{J}$.

4. Which of the following Lie groups contain an $SU(2)$ as its subgroup,

   (A) $SU(3)$
   (B) $SO(4)$
   (C) $SO(1,3)$
   (D) All of the above.

5. The elements of a general non-abelian Lie group are given as $g(\alpha) = e^{i\alpha_i J_i}$. Consider Hamiltonian of a system is H and $[H, g(\alpha)] = 0$ for all α_i. If each of the J_i corresponds to a physical observable then these observables are

   (A) simultaneously measurable and conserved quantities.
   (B) not simultaneously measurable but conserved quantities.
   (C) simultaneously measurable but not conserved quantities.
   (D) neither simultaneously measurable nor conserved quantities.

6. For the generators J_i of $SO(3)$ and $J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$, write down the 5-dimensional irreducible matrix representation of J_+ in the diagonal basis of J_3.

Answer:

$$J_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

for basis set

$\{|+2\rangle, |+1\rangle, |0\rangle, |-1\rangle, |-2\rangle\}$

<u>OR</u>

$$J_+ = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 \end{pmatrix}$$

for basis set

$\{|-2\rangle, |-1\rangle, |0\rangle, |+1\rangle, |+2\rangle\}$

[ANY ONE OF THE ABOVE IS CORRECT ANSWER]

7. A 5×5 real matrix Λ satisfies $\Lambda^T g \Lambda = g$ where $g = \text{Diag.}(+1, +1, -1, -1, -1)$. How many independent free parameters are required to parametrize the matrix Λ?

Answer:

The total number of free independent Parameters is <u>10</u>.