INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 : Linear Algebra (2023-24) **Tutorial 03 (September 21, 2023)**

Let us call a matrix to be a **row echelon matrix**[1] if it has the following three properties.

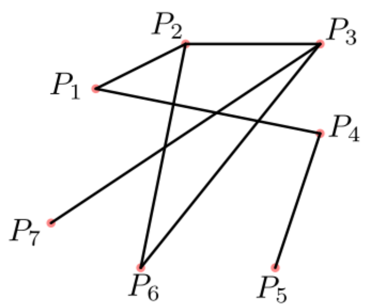
I. First nonzero entry in each row is 1. This entry is to be called the **pivot** of the row.

II. The pivot of a (not entirely 0) row is to the right of the pivot of the preceding row. If a row is entirely 0 then all the subsequent rows are also entirely 0.

III. All entries above pivots are zero. (or equivalently, the pivot element of a row is the only nonzero element of the column it belongs to).

[1] Different books will have a variation in this definition. We stick to the above definition in this course.

1. Consider the following network of seven people.



   (a) Write down the adjacency matrix A for this network.

   (b) Write down A^2 without actually carrying out the matrix multiplication by just looking at the network.

   (c) Can you argue that there is a large enough power r so that none of the entries of A^r is 0.

2. Consider the following system of linear equations:

$$\begin{aligned} x + y - z &= 4 \\ 3x - 2z &= 6 \\ x + 2y - z &= 7 \end{aligned}$$

and express it in the matrix form. Now, compute

$$\begin{pmatrix} -4 & 1 & 2 \\ -1 & 0 & 1 \\ -6 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 3 & 0 & -2 \\ 1 & 2 & -1 \end{pmatrix}.$$

Can you use this computation to obtain values of x, y, z that satisfy the above system of equations?

3. Consider the following chemical reaction:

$$Al_2(SO_4)_3 + Ca(OH)_2 \to Al(OH)_3 + CaSO_4$$

The reaction is unbalanced. Balancing this chemical reaction constitutes finding positive integers x_1, x_2, x_3, x_4 such that:

$$x_1 Al_2(SO_4)_3 + x_2 Ca(OH)_2 \to x_3 Al(OH)_3 + x_4 CaSO_4$$

Express the process of balancing this equation in terms of a system of linear equation. Further, write this system of linear equations in the matrix form.

4. How many 4×4 swapper matrices are there? Of these find distinct matrices A, B such that $AB = BA$. Can you also find distinct swapper matrices A, B such that $AB \neq BA$.

5. Which of the following are row echelon matrices?

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 5 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In each of the cases when matrix is not row echelon, list the condition(s) I, II, III of the definition that it fails to satisfy.

6. Using 0, 1 and 2 write down as many 3×3 row echelon matrices as you can.

7. Convert the following matrices into a row echelon matrix by a suitable sequence of elementary row operations.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 4 & 3 \\ 2 & 1 & 0 & 3 \\ 2 & 1 & 5 & 0 \end{pmatrix}, \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$