PHY304: Statistical Mechanics

2nd Mid Semester Examination 2025

March 07, 2025

Instructor: Rajeev Kapri **Max. Marks 20**

1. Consider a system of N distinguishable non-interacting objects, each of which can be in one of two possible states, "up" and "down", with energies ϵ and 0. Assume that N is large.

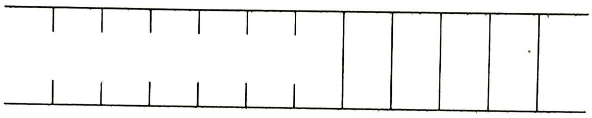
   (a) Working in the microcanonical ensemble, find the entropy of the system $S(E, N)$ as a function of fixed total energy E and number N. [2]

   (b) Using your result from (a) find the temperature T as a function of energy E and number N. [2]

   (c) Can this system shows negative temperature? If your answer is yes find the relation among E, N and ϵ for which T is negative. [2]

   (d) Obtain the number of objects in the "up" state as a function of T at equilibrium. [2]

2. The unwinding of a double-stranded DNA molecule is like unzipping a zipper. The DNA has N bonds, each of which can be in one of two states: a closed state with energy $-\epsilon$ ($\epsilon > 0$), and open state with energy 0. A bond can be open only if all the bonds to its left are already open, as illustrated in the sketch.



   (a) Obtain the partition function of the DNA chain. [3]

   (b) Find the average number of open bonds in the low-temperature ($\epsilon \gg kT$) and the high-temperature ($\epsilon \ll kT$) limits. [3]

3. Consider a box containing an ideal classical gas at pressure P and temperature T. The walls of the box have N_0 absorbing sites, each of which can absorb one molecule of the gas. Let $-\epsilon$ be the energy of an absorbed molecule.

   (a) Find the fugacity $z = e^{\beta\mu}$ of the gas in terms of temperature and pressure. [3]

   (b) Find the mean number of absorbed molecules $\langle N \rangle$ and investigate its low and high-pressure limits. [3]

Some important mathematical identities:

$$\ln N! \approx N \ln N - N; \qquad e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$$

The partial sum of the first $n+1$ terms of a geometric series

$$S_n = \sum_{k=0}^{n} ar^k = a\left(\frac{1 - r^{n+1}}{1 - r}\right) \qquad r \neq 1$$