PHY310 Assignment – 7

Date – 11/10/2023

No Need to Submit

1. Show that $\int_0^\infty e^{-\alpha x} J_n(x)\,dx = \frac{1}{\sqrt{1+\alpha^2}}\left[\sqrt{1+\alpha^2}-\alpha\right]^n$

2. Show that the solution of the differential equation

$y''(x)+(ae^x-b)y(x)=0$ is given by

$y(x) = A J_\mu(\xi) + B Y_\mu(\xi)$; $\xi = 2\sqrt{a}\, e^{x/2}$; $\mu = 2\sqrt{b}$.

3. Show that $\int_0^a x\, J_n^2(x)\,dx = \frac{1}{2}a^2 J_n^2(a)\left[1-\frac{J_{n-1}(a)J_{n+1}(a)}{J_n^2(a)}\right]$

4. Show that $\int_0^\infty e^{-x} x^3 L_n$

4. Show that $\sum_{i=1}^{\infty} \frac{2J_0(a_i x)}{a_i J_1(a_i)} = 1$, where $a_1, a_2, a_3 \cdots$ are the roots of $J_0(x)$.

5. Show that $x^2 J_n''(x) = (n^2-n-x^2)J_n(x) + x J_{n+1}(x)$.

6. Show that $\int x^2 J_0(x) J_1(x)\,dx = \frac{x^2}{2} J_0'(x) + C$.

7. Prove that $2^r J_n^r = J_{n-r} - r J_{n-r+2} + \frac{r(r-1)}{2!} J_{n-r+4} + \cdots + (-1)^r J_{n+r}$.

r^{th} derivative of $J_n = J_n^n$

8. Show that $\frac{x}{2} J_{n-1} = n J_n - (n+2) J_{n+2} + (n+4) J_{n+4} + \cdots$

9. If $n > -1$, show that $\int_0^x x^n J_{n-1}(x)\,dx = \frac{1}{2^n \Gamma(n+1)} - x^n J_n(x)$

10. Show that $L_n(x) = \frac{e^x}{n!}\int_0^\infty e^{-t} t^n J_0\left[2(xt)^{1/2}\right] dt$.

11. Obtain the 2nd solution of Laguerre eqn for arbitrary n.