Mid-semester examination

MTH201 - Curves and Surfaces

Total marks: 25

22 October, 2021

*All parametrizations are assumed to be smooth and regular. You **must** give complete justifications for everything. Answers with no justifications will **not** be given credit.*

Here are the Frenet-Serret equations, just in case you need them for any of the questions below. Here, κ denotes the curvature and τ the torsion.

$$\begin{aligned}
\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t) \\
\dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t) \\
\dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)
\end{aligned}$$

1. Consider the curve parametrized by $\gamma(t) = (3\cos(-7t) + 3, -3\sin(-7t) + 1)$, and let $t_0 = -1$.
   (a) Compute its arc length from $t_1 = -2$ to $t_2 = 3$ (*3 marks*)
   (b) Find a unit speed reparametrization (*3 marks*)
   (c) Compute the unit tangent vector at t_0 (*3 marks*)
   (d) Compute the curvature at t_0 (*2 marks*)
   (e) Compute the signed unit normal vector at t_0 (*2 marks*)
   (f) Compute the signed curvature at t_0 (*2 marks*)
2. Given a curve parametrized by $\gamma : (\alpha, \beta) \to \mathbb{R}^3$, assume that the curvature at some point t_0 is 2, while its derivative at t_0 is 2; the torsion at t_0 is 3, while its derivative at t_0 is 3. Compute the dot product, $\ddot{\mathbf{N}}(t_0).\mathbf{B}(t_0)$ (note the double dot, i.e. second derivative, over the first term). (*4 marks*)
3. Let $\mathbf{v} := -(2, 4, 4)$, Given a curve parametrized by a unit speed parametrization $\gamma : (\alpha, \beta) \to \mathbb{R}^3$, let $\delta : (\alpha, \beta) \to \mathbb{R}^3$ be defined by $\delta(t) = \gamma(-4t+2)+\mathbf{v}$ and p be a point on the curve traced by γ. Derive a relationship between the curvature of γ at p and the curvature of δ at the point $p+\mathbf{v}$. (*3 marks*)
4. Consider a space curve parametrized by $\delta : (\alpha, \beta) \to \mathbb{R}^3$. If there is a q, a point, so that the vector $q - \delta(t)$ is perpendicular to $\mathbf{B}(t)$ and $\mathbf{T}(t)$, then show that the distance of $\delta(t)$ from q is always constant. (*3 marks*)