Assignment-1

Date-14-Aug-2023
No Need to Submit.

1. In each of the following differential equation, state whether it is linear or nonlinear, homogeneous or inhomogeneous and what is the order of each eqn?

a) $y'''(x) - 5y'(x) - 3x = 0$

b) $y''(x) - yy'(x) + 7y(x) = 3$

c) $x^2 y''(x) + e^x y'(x) + (x^2 - 1) y(x) = 0$

d) $x y''(x) + 7y'(x) + 2y(x) = \sin x$

e) $y'^2(x) + xy^2(x) = 0$

2. Find the solution of each of the following equations:

a) $(D^2 - D - 2) y(x) = 0$

b) $(D^2 + a^2) y(x) = 0$

c) $(D^4 + 1) y(x) = 0$

d) $D(D^2 - D - 2) y(x) = 0$

e) $(D^2 - a^2) y(x) = 0$

3. Find the general solution (C.F + P.I.) of each of the following differential equations - (all 1st order)

a) $x y'(x) + 2y(x) = -x^5$

b) $y'(x) + (\cot x) y(x) = 3$

c) $y'(x) - (\tan x) y(x) = 2\cos x$.

d) $y'(x) + \frac{6x}{1+x^2} y(x) = \frac{e^x}{(1+x^2)^2}$.

e) $y'(x) - \frac{3x^2}{1-x^3} y(x) = \frac{\tan x}{1-x^3}$.

4) Determine the general solution of each of the following differential equations - (2nd order).

a) $(D^2 - 5D + 4) y(x) = 4x^2$

b) $(D^2 - a^2)\, y(x) = x + \sin ax + e^{ax}$

c) $(D^2 - 1)\, y(x) = -\frac{2}{1+e^x}$

d) $(D^2 - 2D + 5)\, y(x) = x^2 + e^{2x}\cos x$. ⊛

e) $(D^2 - 5D + 6)\, y(x) = x^2 e^{3x} \sin x$. ⊛

5. Find the general solution of each of the following equations- (Euler - Cauchy Equation)

a) $x^2 y''(x) - 3xy'(x) + 4y(x) = x^2 + 1$ [Hint → $y = x^m$]

b) $x^3 y'''(x) - 5xy'(x) + 9y(x) = \frac{4}{x}$ [Hint → $y = x^m$]

c) $x^3 y'''(x) - x^2 y''(x) + xy'(x) - 16y(x) = 1 - \ln x$ [Hint $z = \ln x$]

d) $x^2 y''(x) - 2xy'(x) + 2y(x) = \frac{6}{x} + 1 + 2\ln x$. [Hint $z = \ln x$]