PHY660: Non-linear Optics
Final Examination, May, 2025

Date: May 01, 2025

Total marks: 50

1. Consider the following Jones vectors.

$$J_1 = \begin{pmatrix} \cos\theta_1 \\ i\sin\theta_1 \end{pmatrix}, \qquad J_2 = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}.$$

Find the coordinates on the Poincaré sphere corresponding to the Jones vector $J = J_1 + J_2$.
5 marks

2. Consider the permittivity tensor

$$\epsilon = \begin{pmatrix} 2 & 0.3 & 0 \\ 0.3 & 2 & 0 \\ 0 & 0 & 1.5 \end{pmatrix}.$$

Calculate the principle axes and principle values of susceptibilities. **5 marks**

3.

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = -\frac{e}{m}E(t),$$

where $x \equiv x(t)$ is the displacement of the electron from its equilibrium position, γ is the damping factor, ω_0 is the natural frequency of the oscillation and $E(t) = E_0 e^{i\Omega t} + cc$ is the driving field. Calculate the first order susceptibility from this equation.
5 marks

4. **Electromagnetically Induced Transparency:** Consider a three-level atom with energy levels $|1\rangle, |2\rangle, |3\rangle$ such that $E_1 < E_2 < E_3$, where E_i is the energy of level $|i\rangle$. A weak probe field of frequency ω_p couples the $|1\rangle \leftrightarrow |3\rangle$ transition, while a strong control field of frequency ω_s couples the $|2\rangle \leftrightarrow |3\rangle$ transition, with Rabi frequency Ω_s. The linear susceptibility for the probe field is given by:

$$\chi^{(1)} = \frac{N}{\hbar}\frac{|\mu_{31}|^2(\delta - \Delta + i\gamma_2)}{|\Omega_s|^2 - (\delta + i\gamma_3)(\delta - \Delta + i\gamma_2)},$$

where $\delta = \omega_p - \omega_{31}$, $\Delta = \omega_s - \omega_{32}$, and γ_i is the decay rate of level $|i\rangle$.
Calculate the expression for the group velocity v_g of the *probe field* in this medium at two-photon resonance ($\delta = \Delta$).
Hint: $v_g = c/n_g$; $n_g(\omega) = n(\omega) + \omega\frac{d}{d\omega}n(\omega)$.
5 marks

5. Consider a two-level atom with:

- Ground state $|g\rangle$ and excited state $|e\rangle$,
- Transition frequency ω_0,
- Dipole matrix element $\mu_{ge} = \langle g|\hat{\mu}|e\rangle$ (real and nonzero),

The atom is driven by a classical electric field $\vec{E}(t)$ which may contain a discrete set of frequencies.

Assume:

- The electric field is weak (perturbative regime),
- The rotating wave approximation (RWA) applies,
- The atom is initialized in the ground state $|g\rangle$.

Calculate the following

(a) Find the formal recursive perturbative solution of the time-dependent state of the atom, for all orders.
5 marks

(b) Find the explicit first order perturbative solution for the time-dependent state of the atom.
5 marks

(c) Find the explicit second order perturbative solution for the time-dependent state of the atom.
7 marks

(d) Find the expression for the linear susceptibility $\chi^{(1)}(\omega)$ (assuming Γ as the decay rate).
5 marks

(e) Find the expression for the second order susceptibility for second harmonic generation process $\chi^{(2)}(2\omega;\omega,\omega)$ (assuming Γ as the decay rate).
8 marks