CLASSMATE
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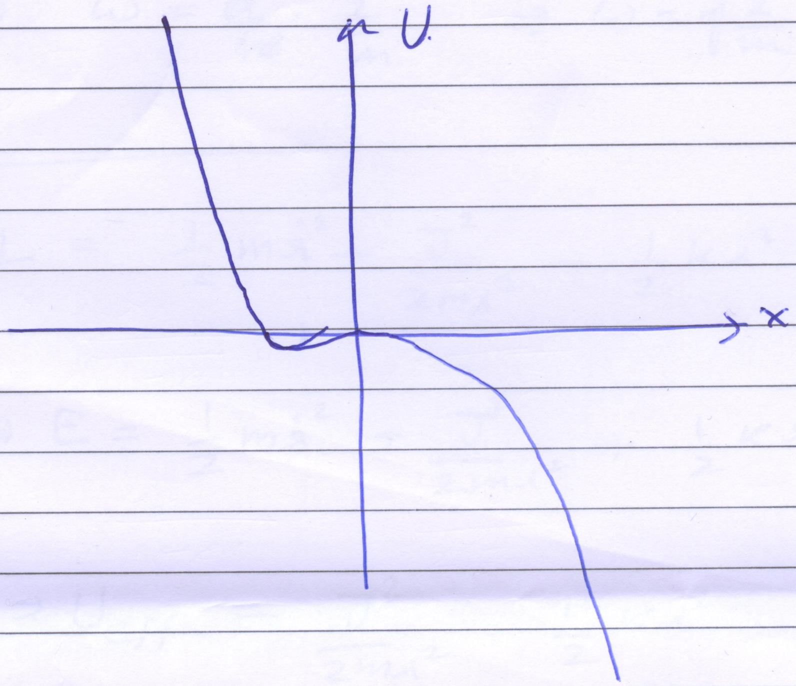
(1) $U(x) = -ax^3 - bx^2$

(a) $a=1, b=1 \Rightarrow U = -x^3 - x^2$.




(b) $\frac{\partial U}{\partial x} = 0 \Rightarrow -3x^2 - 2x = 0$

$\Rightarrow -x(3x+2) = 0.$

$\Rightarrow$ extrema at $x=0, x = -\frac{2}{3}$

$\frac{\partial^2 U}{\partial x^2} = -6x - 2$

$U'' < 0$ @ $x = 0$. (maxima)

$U'' = +6 \times \frac{2}{3} - 2 = 2 > 0$ (minima)

$x = -2/3$ is a minima for $U$.

(c) $\omega^2 = \frac{k}{m}, \quad k = U''\big|_{minima.}$

$$\Rightarrow \quad \omega^2 = \cdot \frac{2}{m} \quad \Rightarrow \omega = \sqrt{\frac{2}{m}}$$

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(2) $$L = \frac{1}{2} m\dot{r}^2 + \frac{J^2}{2mr^2} - \frac{1}{2} k r^2$$

(a) $$\Rightarrow E = \frac{1}{2} m\dot{r}^2 + \frac{J^2}{2mr^2} + \frac{1}{2} k r^2$$

$$\Rightarrow U_{eff} = \frac{J^2}{2mr^2} + \frac{1}{2} k r^2$$

$$\frac{\partial U_{eff}}{\partial r} = -\frac{J^2}{mr^3} + kr = 0$$

$$\Rightarrow r^4 = \frac{J^2}{km} \quad \Rightarrow r = \left[\frac{J^2}{km}\right]^{1/4}$$

$$\frac{\partial^2 U_{eff}}{\partial r^2} = +\frac{3J^2}{mr^4} + k > 0 \quad \Rightarrow \text{minima.}$$

(b) $$\omega^2 = \frac{1}{m} \cdot U'' = \frac{1}{m}\left[k + \frac{3J^2 k\cancel{m}}{\cancel{m}J^2}\right] = \frac{4k}{m}.$$

$$\omega = 2\sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{\omega} = \pi\sqrt{\frac{m}{k}}$$

(c) Orbital frequency : $J = m r^2 \omega_{orb}$

$$\Rightarrow \omega_{orb} = \frac{J}{mr^2} = \frac{J}{m} \cdot \frac{\sqrt{km}}{J} = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T_{orb} = 2\pi\sqrt{\frac{m}{k}} = 2T$$

$\uparrow$ Time period for radial oscillations.

(d) as $\frac{T_{orb}}{T} = 2$, orbit closes after each azimuthal orbit. Radial oscillations complete two time periods in this time.