PHY 303 : Assignment 5

Submit by 09 November 2024 midnight

1. Obtain the wave equations for $\mathbf{A}, \mathbf{B}$ and $\mathbf{E}$ fields in electrodynamics and identify their sources. Does the Poynting vector also satisfy a wave equation ?

2. From the matching conditions of electric and magnetic fields of an EM wave across an interface with incident, reflection and transmission angles α, β and γ respectively, find out the ratio of magnetic fields of the transmitted to the reflected components for the case when the magnetic field is (i) in the same plane as that of incident and reflected wave (ii) perpendicular to the plane of incident and reflected wave.

3. An inertial observer finds a charge density ρ_0 at rest in her frame. How will another observer moving with respect to the first observer with a relative velocity $\mathbf{v}$ find the charge and current densities to be ? Using the Lorentz transformation rules of $dx^0 = cdt$ and dx^i find out the transformation relation for $\partial/\partial x^0$ and $\partial/\partial x^i$. Further show that the spacetime D'Alembertian $\Box \equiv -(\partial/\partial x^0)^2 + \sum_i (\partial/\partial x^i)^2$ is an invariant operator under Lorentz transformation.

4. Show that in the Lorentz gauge both the scalar and vector potentials satisfy the wave equations with the charge density and the current density as the sources. From here then argue that ϕ and $\mathbf{A}$ transform like dt and $\mathbf{dx}$ under Lorentz transformation. Therefore we can define a Lorentz 4-vector $A_\mu = (\phi, \mathbf{A})$.

5. If we define a tensorial quantity $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, show that these quantities do not change under gauge transformations as well as space and time translations by constant amounts. Identify the electric field and magnetic fields as the components of $F_{\mu\nu}$ and find out how do they transform if we do a boost in $y-$ direction.