PHY 310 - Mathematical Methods for Physicists I

Odd Term 2019, IISER Mohali

Instructor: Dr. Anosh Joseph

Homework 8

---

1. Consider the set of functions $\{u_1, u_2, u_3\} = \{1, x, \sin x\}$. Also consider the inner product

$$\langle u_m | u_n \rangle = \int_{-\pi}^{\pi} dx u_m(x) u_n(x). \tag{1}$$

   (i.) Are these functions orthogonal with respect to the inner product?

   (ii.) If not, find the corresponding orthogonal functions using the Gram-Schmidt orthogonalization process.

2. Use the Gram-Schmidt orthogonalization process to convert the set of polynomials $\{1, x, x^2\}$ to a set of orthogonal polynomials with respect to the inner product

$$\langle u_m | u_n \rangle = \int_{0}^{\infty} dx u_m(x) w(x) u_n(x), \tag{2}$$

   where $w(x) = \exp(-ax)$ and $a > 0$.

   Hint:

$$\int_0^\infty dx x^n e^{-ax} = \frac{n!}{a^{n+1}}. \tag{3}$$

3. Consider the boundary value problem

$$y'' + 4y = x^2, \tag{4}$$

   where $0 \leq x \leq 1$ and $y(0) = y(1) = 0$.

   (i.) Construct the Greens function for this problem using the method of eigenfunction expansion.

   (ii.) Find the solution $y(x)$ using the Green's function computed above.

4. Show that

$$\int_0^\infty dy e^{-ay} y^{n-1} = a^{-n} \Gamma(n). \tag{5}$$

5. Show that
$$B(m,n) = B(n,m). \tag{6}$$
Hint: Use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

6. Show that
$$\int_0^{\frac{\pi}{2}} d\theta \sin^p\theta \cos^q\theta = \frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{2\Gamma(\frac{p+q+2}{2})}. \tag{7}$$
Hint: Use $x = \sin^2\theta$ in the standard definition of the beta function.