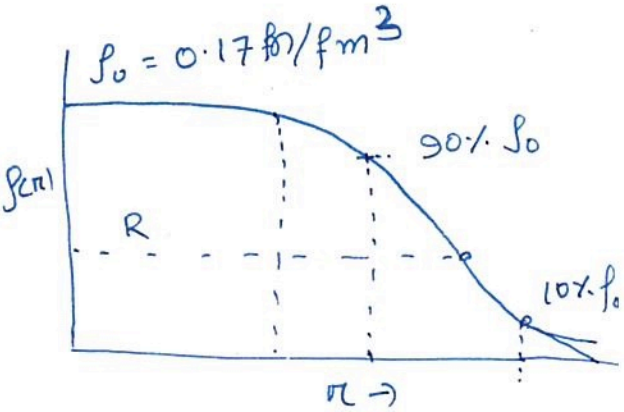
PHY401-Nuclear and Particle Physics			
Academic Year:	2024-25	Instructor:	Dr. Satyajit Jena,
Class	MS21	Lecture	Nuclear Mass and Binding Energy

What we learn so fare ?

By the explanatin of Wood-saxon formulatin we get following



$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

concludes that ~~the~~ various values.

$$\rho_0 = 0.17 \text{ nucleon}/fm^3$$

$$r_0 = 1.18\,fm.$$

$$a = 0.4 - 0.5\,fm.$$

Earlier we discussed that there are several methods to find out the nuclear radious. ~~St~~ The interactin & its spatial variatin between nuclei enables the calculatin of the nuclear radii. If the radius is due to the columbic force then this will be the charaterictic of only proton. However if the radii is charateristic of nuclear force this will be the radii of ~~neutron~~ ~~nucleons~~ the nucleus of both neutron & protons (nucleons). In this context ~~when~~

① when low energy α-particles were used as probe to scatter the nucleus, it reflected purely proton charge distributin.

② when high energy electrons are used it also doesn't reflect upto the nucleonic volume rather it was still a uniform charge distributions.

③ A third method was used to determine the nucleus radius using K-α-X-rays like muonic & mesonic X-rays; here also the energy levels are somewhat calculated using only the columbic interactions.

All these experiments & theoretical models predicts the nuclear volume considering an uniform charge distribution
~~However if to α-particles are managed to be~~

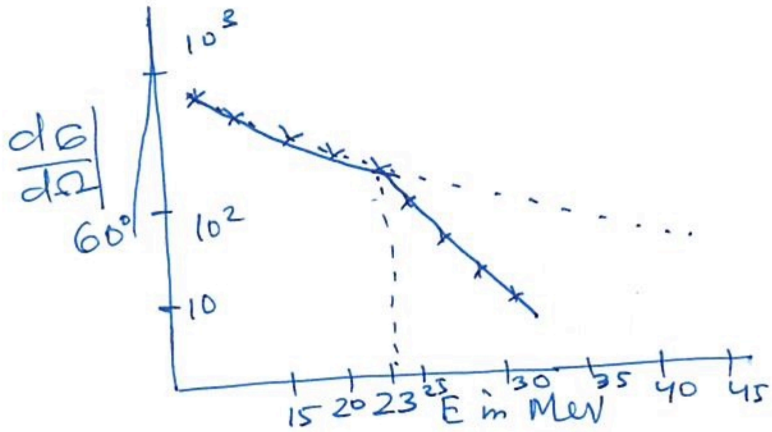
However if we manage to bring the α-particle close to the nucleus, then it must exert the nuclear force. ~~α~~- How we do it? Increase KE of α particle. so that ~~KE~~ ~~be~~ opposing columb force becomes lesser & lesser.

As long as the separation between two nuclei (α & nucleus center test) is greater than the sum of their radii, each of them would always stay beyond the range of their nuclear force; so only columb force acts.

when the KE. of α-particle increases, ~~that~~ to a certain value, the columb repulsion between them will be overcome, and they may approach close enough to allow nuclear force start to act. Thus using this varying incident energy of α-particle to measure the strength of ~~an~~ spatial extent of interaction potential between α-particle & the nucleus.

- Geiger & Marsden were using 4 to 8 MeV α-particle which were very low to overcome columbic repulsion.

During 1950, Gove at MIT cyclotreen experiment measured the 27.5 MeV α-particle scatting with Gold foil. During same decade, Wall, Res, and Ford at Indiana University cyclotreen done series of experiments using variable ~~&~~ energy of α-particle on several metal like Ag, Au, Pb etc. An importomt result was published by R.M. Bisberg & C.E. Porter in 1960, in Rev. Moderen physics v. 33 N. 2 about the ~~fore~~ measurement of scattered α-particles freo Pb foil at 60° as a function of incident energies.



The dashed curve is the energy dependence of pure columbic interacetion. [This is correceted ~~with the~~ ~~the~~ for the variation of in scattering angle which enters into the experiments]. The result, after nomalization, agrees nicely with Colemb cross-section at law energies; However above certain critical energy, the measured cross-section drop rapidly below the Colemb cross-section

This scattering results ~~sto~~ some what tells that at ~~cer~~ after a certain critical energy, it gives information about the interaction near ~~nucleus neo~~ the nuclear surface. Although, initially ~~the~~ other results are dissapointing about any ~~information~~ ~~of nuel nucl~~

nuclear informatin, ~~kower~~ however the existance of strong absorption of α-particle crossection shows a strong evidence of nuclear force which are totally diffent than the columbic force. And this reesults also shows a marked similarity between nuclees of α-particle and nucleus of heavy ion scattering.

Now by analysing all the above discussion we finally can say or conclude that:

(1) Nuclear density is roughly constent ~~& have~~ throughout its volume & ~~changes~~ almost independent of atomic number. And has a soft skin of 0.4 – 0.5 fm where its density drops from 90% to 10% of maximum value.

(2) Nuclei are positively charged & nuclear charge density is also roughly constant.

(3) The charge and matter radii of nuclei are nearly equal, to within 0.1 fm. Thus the neutron & protons are completely intermixed.

(4) There is a strong force which is attractive & complete ~~with~~ against colombic repulsion to hold all protons & neutron together to form a stable nucleus.

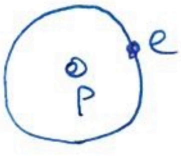
⑤ This force is called nuclear force or strong force which is very short range, falls to zero abruptly with inter-particle separation. It has a hard core which prevents the nucleus from approaching each other to extremely closer (than ~0.4 fm)

⑥ The nuclear force does not depend on any electric charge It is identical for every nucleon like proton & neutron
The nuclear force between p-p, p-n, n-n are all almost identical. p-p has another force repulsive but it's colombic.

Owing to all these properties of nucleus, and nuclear force, the individual nucleus in nucleus is tightly bound. The consequence is, from the attractive/repulsive form of the nuclear force, that the nucleons are in very close proximity. One can almost imagine a nucleus being made up of incompressible nucleonic spheres, sticking to one other, with a "contact" potential, like ping-pong balls smeared with a dense liquid.
Due to the short range, a constant potential is formed by the attraction of its nearby neighbors, i.e. kind of only those are in contact with it.

A nucleon at the surface of a nucleus has fewer neighbors, so, is less tightly bound.
What we find, finally, is that, there an extra energy which is holding the nucleus against colomb repulsion

Due to this attractive force nucleus is not falling apart. Now our task is to findout ~~how~~ what is this energy is? Usually we call is Binding Energy (BE or B) in amu. Before we understand the nuclear binding energy let's understand a bit about atomic binding energy, which is responsible for an electron to stay in orbit of an ~~atom~~ atom. That is, this is the energy required to separate an electron from its nucleus in case of hydrogen atom the binding energy is $-13.6\,eV$.



How electrons are bound to nucleus & their energy levels are beyond the scope of this course. However we will discuss here about an interesting fact.

The mass of bare proton: $M_p = 1.007276466879\,u$

The mass of a bare electron: $M_e = 0.00054857990\,9u$

The mass of ^1_1H atom is : $M(^1_1H_0) = 1.00782503223\,u$

if we ~~so~~ sum the masses of proton & eletron

$$M_p + M_e$$

is pretty close, but not exact. And the missing mass is $\Rightarrow \Delta m = 0.0000000145\,6u$.

Thus the mass of ^1_1H is less than the sum of masses of ~~p~~ its constituent sub particles. & ~~in~~ This mass is stored in terms of energy which we call binding energy around $13.598434005136\,eV$.

Next we will try to find wheather sum of ~~all~~ masses of all nucleons is larger than mass of nucleus.

if lower nuclei fuse together can release some energy or higher nuclei can fragments to lower A nuclei by releasing energy.

At this stage there was no formal theory that can predict or explain properly about preccularity of the binding energy pernucleon curve. The main difficulties arise due to complex structure, nucleus, insufficient knowledge about nucleus, and complexity in dealing with many-body nucleonic system of nuclear force.

In an attempt to understand the structure, we face following challenges

nucleon-nucleon interaction

- A many body problem
  - → easier for $N \leq 10$
  - → very difficult fo $N > 10$ → Diffent forces used depending on the method chosen to solve the problem of many body systen of force.
- not completely known in QCD.
  - ↓ Diffent forces used depending on the method chosen to solve the problem of many body systen of force. → use a model

← use a model

- → simplify the assumptin
- → give resonable accont for the observed properties
- → Make predictin

Liquid drop model
- → uniformly distributed nucleons, within a incompressible fluid

Sheel model
- nucleons are aranged in stable quantn states

Collective model
- neutron & proton show collective behaviour.

Mass of Nucleus:

Earlier we saw that, the actual atomic mass is less than the sum of the individual masses of the constituents of hydrogen atom. considering hydrogen as single proton & electron system, we attributed that missing mass in form of binding energy, -13.6 ev.

Let's check with next nearest atom ${}_2^4He_2$ in this case we can go further to divide all constituents of ${}_2^4He_2$. Thus Helium is a system of 2 protons, 2 neutrons & 2 electrons. Thus in principle the atomic mass should be equal to

$$M^{atm}({}_2^4He_2) = 2m_p + 2m_n + 2m_e - [b.e]^{atm}/c^2 \quad -①$$

$[be]^{atm} \rightarrow$ Energy required to remove both electrons

$\downarrow$

{atomic binding energy}

we know the value of LHS from various experiments. i.e. $M^{atm}({}_2^4He_2) = 4.002603\ u.$

Now it is better to convert every mass into form of energy via $E = mc^2$, this gives $1u = 931.5$ Mev.

$$M^{atm}({}_2^4He_2)c^2 = 4.002603 \times 931.5 \text{ Mev}$$

$$= 3728.42507 \text{ Mev}.$$

Let's look into R.H.S of right side of the equation

$$2m_p + 2m_n + 2m_e - \frac{[be]^{atm}}{c^2}$$

$$= 2 \times 1.007276 + 2 \times 1.008665 + 2 \times 0.000549 - \frac{be}{c^2}$$

$$RHS = [2.014552u + 2.017330u + 0.0010984u] - \frac{[be]^{atm}}{c^2}$$

$$= 4.03298u - be/c^2$$

$$(RHS)c^2 = 4.03298 \times 931.5 \text{ Mev} - [be]^{atm}$$

$$= 3756.72087 \text{ Mev} - [be]^{atm}$$

what is the value of $(be)^{atm.}$ for hellium atom.. It is the binding energy by which the electrons are bound to hellium nucleus.

$$[be]^{atm} = 24.5878 \text{ ev} + 54.4177 \text{ ev}$$

$$\underset{\text{First electron}}{\downarrow} \qquad \underset{\text{second electron}}{\downarrow}$$

$$= 79.0055 \text{ ev}.$$

$$\Rightarrow (RHS)c^2 = 3756720870 \text{ ev} - 79.0055 \text{ ev}$$

$$= 3756720790.9945 \text{ ev}$$

$$= 3756.72079 \text{ Mev}$$

what we observe is that LHS $\neq$ RHS of equation (1) even after ~~incor~~ removing the atomic binding energy. And the difference is

$$\Delta m c^2 = (3756.72079 - 3728.42507) \text{ Mev}$$

$$= 28.29572 \text{ Mev}.$$

Note the diffence, this is much larger than electron binding energy. ~~Thus~~ Since we already incorporated the electron binding energy into the calculation this this missing mass must be something to do with ~~a~~ nucleus → ~~a~~ And we call it nuclear binding energy. which is much larger than atomic binding energy.

Thus we can write that the mass of nucleus is always less than the sum of masses of all constituent nucleons.

$$M^{nuc}({}^{A}_{Z}X_N) < Zm_p + Nm_n \quad - ②$$

A = mass number = Z + N
Z = Atomic number = number of proton
N = number of neutron
m_p = mass of proton
m_n = mass of neutron.

Since RHS is more massive than the LHS, we can subtract the excess from RHS to equate this equation

$$M^{nu}({}^{A}_{Z}X_N) = Zm_p + Nm_n - \Delta m \quad - ③$$

Δm = axcess mass called mass defect

It is easier to measure or find the atomic mass that the mass of nucleus. Ton's of experiments have done to find the atomic mass & its being done with great accuracy. Thu rearrangen the equatin such a way that we could use some of the known quanties to find out Δm.

First let's add Zm_e both sides.

$$M^{nu}({}^{A}_{Z}X_N) + Zm_e = Zm_p + Zm_e + Nm_n - \Delta m$$

we know that the atomic mass is equal to the sum of mass of nucleus, mass of electrons, and subtracted by the atomic binding energy. i.e. $M^{atm}({}^{A}_{Z}X_N) = M^{nu}({}^{A}_{Z}X_N) + Zm_e - \frac{(be)^{atm}}{c^2}$

$$\Rightarrow M^{atm}({}^{A}_{Z}X_N) + \frac{[be({}^{A}_{Z}X_N)]^{atm}}{c^2} = Z(m_p + m_e) + Nm_n - \Delta m \quad - ④$$

similarly the mass of hydrogen atom is

$$M^{atm}\left(m_p + m_e - \frac{13.6\,eV}{c^2}\right)$$

Let's also convert everything into energy using mc^2 i.e. by multiplying c^2 in all sides.

$$\Rightarrow \quad M^{atm}\left({}^{A}_{Z}X_N\right)c^2 + \left[be\left({}^{A}_{Z}X_N\right)\right]^{atm} = Z \cdot M^{atm}\left({}^{1}_{1}H_0\right)c^2 + Z\,13.6\,eV + N m_n c^2 - \Delta m c^2 \quad -④$$

$$\Rightarrow \Delta m c^2 = Z M^{atm}\left({}^{1}_{1}H_0\right)c^2 + N m_n c^2 - M^{atm}\left({}^{A}_{Z}X_N\right)c^2 + \left\{Z\,13.6\,eV - \left[be\left({}^{A}_{Z}X_N\right)\right]^{atm}\right\} \quad -⑤$$

Let's calculate Δmc^2 for ${}^{16}_{8}O_8$

we know each term of RHC of above equation in great detail. like m_n, Z, N, atomic mass of ${}^{1}_{1}H$, atomic mass of ${}^{A}_{Z}X_N$, c^2, binding energy of ${}^{A}_{Z}X_N$ atom, Thus the calculation become easy.

For ${}^{16}O$

$\Rightarrow \quad Z = 8 \qquad m_n = 1.00893\,u$

$\quad N = 8 \qquad M^{atm}({}^{1}H) = 1.00784\,u$

$$M^{atm}\left({}^{16}_{8}O_8\right) = 15.9949\,u.$$

$[be({}^{16}O)]^{atm}$ = sum of all energies that holds all electrons in their orbital in ${}^{16}O$

$$= 13.6\,eV + 35.12\,eV + 54.94\,eV + 77.42\,eV + 113.89\,eV + 138.12\,eV + 739.29\,eV + 871.41\,eV$$

$$= 2043.77\,eV.$$

$2\times 13.6 = 8\times 13.6\,eV = 108.8\,eV$

$Z\,M(H) = 8\times 1.00784$

$= 8.06272u$

$N m_n = 8\times 1.00893u = 8.07144u.$

Equation (5) will

$$\Delta m c^2 = (8.06272u)c^2 + (8.071472u)^2 - (15.9949u)^2 - 1934.97eV$$

convert everything in MeV by using $1u = 931.5$ MeV.

$$\Delta m c^2 = (7510.42368 + 7518.576168 - 14899.24935)\text{MeV} - 1934.97eV$$

$$= 129.75049\,8\text{ MeV} - 1934.97\text{ eV}$$

$$= 129750498\text{ eV} - 1934.97\text{ eV}$$

$$= 129755\,2433\text{ eV}$$

$$= 129.755\text{ MeV}$$

one of the important observation is that the the atomic binding energy is in eV & the mass defect coming from nucleus is in order of MeV, thus ~~their~~ considering the significance of nuclear mass defect we can safely neglect the atomic binding energy, it will not make any change upto 5th decimal place of ~~nucl~~ the value of nuclear mass defect.

Thus for oxygen ^{16}O

$$\boxed{\Delta m c^2 = 129.755\text{ MeV}}$$

we can safely write LHS ~~BE~~ as a form of energy some how is required to separate all nucleons of ^{16}O

Thus we call it nuclear Binding Energy

$$\boxed{B.E = \Delta m c^2 = 129.755\text{ MeV}}$$

After neglectin atomic binding energy the equation(5) can be rewritten as:

$$\boxed{B.E = \left[Z\, M^{atm}({}^{1}_{1}H_0) + N m_n - M^{atm}({}^{A}_{Z}X_N)\right] c^2} \quad -(6)$$

B.E : Nuclear bind energy

$M^{at}(H)$: atomic mass of hydrogen

m_n : mass of neutron

$M^{atm}({}^{A}_{Z}X_N)$: Atomic mass of ${}^{A}_{Z}X_N$

Z : atomic number

N : number of neutron.

Binding energy of a nucleus is the energy required to break the nucleus into all of its constituent nucleons (proton & neutrons) or the energy by which the nucleons ~~are~~ bind up to form nucleus.

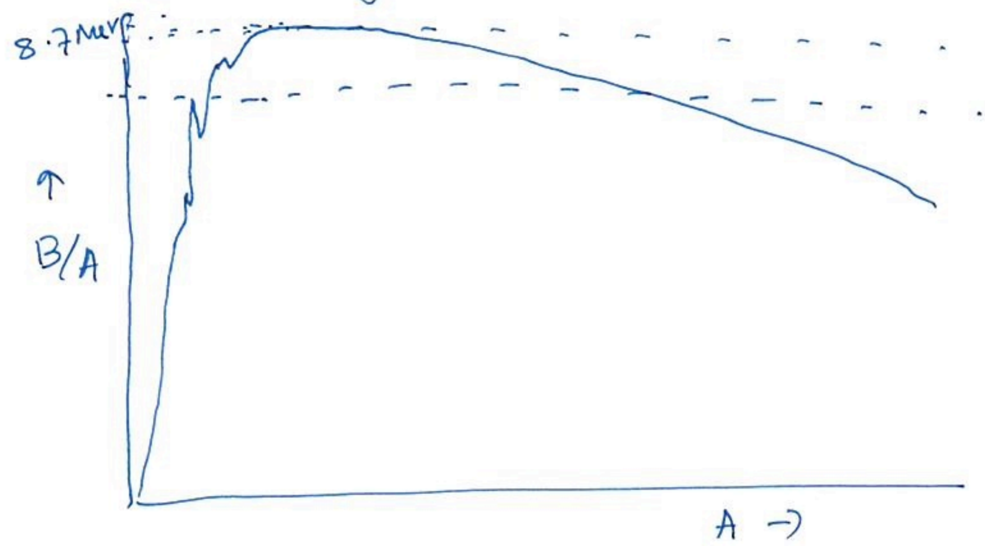
Binding Energy per nucleons:

To be able to compare binding energy ies among all nuclei we need to normalized to each of its number of nucleus Recalling all our previous discussin on nuclear shape & radius, it seems that when we go up in atomic number but the experimental finds are very different. let's calculate ~~previously~~ same for previously calculated B.E for ^4He & ^{16}O

$$B.E|_{He} = 28.29572\,MeV \qquad (B.E/n)^{He} = 7.0739\,MeV$$

$$B.E.|_{O} = 129.755\,MeV \qquad (BE/n)^{O} = 8.1096\,MeV$$

Grossly the binding energy per nucleon for a nucleus is defined as the energy required for a single nucleon to bind up with a particular nucleus (BE/A). if we draw (BE/A) as a funtion of A we get.



B/A is neither going up to remaining same. what we observe that the binding energy per nucleon curve is rapidly increasing as a function of A for the lighter nuclei, it attains a maximum height at mass number 58. Then gradually it decreases. This was a big issue during early time to explain exactly why the nature of curve is like this.

- Observation & possible explanations.

(1) with the exception of ^4He, ^8Be, ^{12}C, ^{16}O, the values fall in close proximity to a single curve.

(2) The binding per ~~nucleons~~ nucliedes are very low, however, over a very considrable range of mass number, the binding energy per nucleon are close to 7.5-8.0 MeV

(3) The possitive value of BE/A of all nucleus shows that, this is the attreactive force responsible. for nuclear stability. A nucleus doesnot collapse, that mean the force becomes abreuptly repulsive for a very close distance of approach of the nucleons.

(4) The values of BE/A vary in an erratic maner from lighter nucleus to hevier nucleus. There is an rapid increase of the values for lighter nuclei with a notable peaks at $A = 4n$ [^{4}He, ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg] This kind of reflects the peculiarity of the stability of the α-particle structure.

(5) A greater than 20, BE/A is almost remain same (change is few MeV). This must reflect an important property of nuclear force. If A number of nucleons interact with all the remaining (A-1) nucleons with same strength, it would expect BE to increase as the function of (A^2-A) or approximately $\sim A^2$ for the heavy nuclei. However, the observation ~~not~~ matches only if it is postulated that the neucleons interact with only a limited number of their neighbours who are in close contact with.

(6) There are several discontinuity in the BE/A curve. which are related to some very specific nuclei & their incredible stability, called magic number nuclei.

(7) Curve indicates that the value of BE/A increases to a maximum of about 8.7MeV around the mass number 56-58 then decreases slowly to about 7.6 MeV per nucleon at A = 238. This means if

if lower nuclei fuse together can release some energy or higher nuclei can fragments to lower A nuclei by releasing energy.

At this stage there was no formal theory that can predict or explain properly about preccularity of the binding energy pernucleon curve. The main difficulties arise due to complex structure, nucleus, insufficient knowledge about nucleus, and complexity in dealing with many-body nucleonic system of nuclear force.

In an attempt to understand the structure, we face following challenges

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A many body problem
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Liquid drop model
→ uniformly distributed nucleons, within a incompressible fluid

Sheel model
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Collective model
neutron & proton show collective behaviour.

In an attempt to overcome the problems in explaining the BE/A curve, Bethe & Weizsacker developed a formula taking the experimental results & theoretical views in every stages of development. Thus this type of formula is called semiempirical formula. And this model assumes the nuclear system to be a liquid drop.

Liquid Drop Model:

The liquid drop model in nuclear physics treats the nucleus as a drop of incompressible nuclear fluid of very high density. This was first proposed by Geoge Gamow, and Weizsacher in 1935 & Later developed by Niels Bohr & John wheeler. Assumpts are:

- → The volume of nucleus is proportinal to the A
- → The matter density is constant, and decreases rapidly at boundary.
- → Nuclear force is identical betwem nn, np, pp.
- → Like the molecules in liquid, the nucleus are Imagined interact with each other.
- → Just like liquid molecules can collide with eachother due to the thermal agitatin, but remain well inside the drop, ~~so~~ a given nucleon can collide frequently with other nucleons within the nucleare volume, thus the free mean path of motin is much less than ~~the~~ the nuclear diameter.

→ The liquid drop is assumed to be incompressible meaning its density can't be changed, so the density of nucleus remain constant throughout.

→ Liquid drop is spherical due to surface tension, the nucleus is also spherical due to surface tension

→ cohesive force saturates in case of liquid drop, ~~similar~~ as it also required to happen in nuclear force.

→ The heat of vapourization represents the amount of energy required to take out molecule from liquid drop, binding energy also required to pull nucleons apart from nucleus.

→ due to the limited number of nucleons in nucleus (< 270), there are large number of nucleon in surface. of the ~~dr~~ sphere.

→ Nucleus is a two component system compose of neutron & protons.

The main assumption is that liquid drop model treats nucleus as a drop of incompressible ~~nucleus~~ fluid made of nucleons & held together by a nuclear strong force. The nucleons interact with each other just like ~~a~~ the molecules of a liquid drop.

Since it is easier to deal with the liquid drop to develop a formula to explain its properties, we will explain the same for the nucleus as liquid. And the data we will take for the B.E. formula as masses are well known.

Thus the B.E can be defined ~~to~~ interms of some propeties ~~we have~~ as per liquid drop.

Semi Empirical Mass Formula.

The binding energy of the nucleus & the average binding energy per nucleons can be approximated through a contribution from various effect & characteristics of nucleous shape and size. we will discuss each of these contribution & how they are arising.

= Contribution from volume term.

For a drop of liquid to evaporate a certain amount of heat must be supplied. This heat is the energy required to overcome all the interactions ~~of~~ between molecules bound within a liquid drop, which therefore must be proportional to the volume of the liquid drop. Analogues to this nucleons inside the nucleus also have binding interaction which depends on the volume of the nucleus. Thus the contribution due to the volume term is

$$BE_V \propto V \qquad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(r_0 A^{1/3}\right)^3$$

$$\Rightarrow BE_V = a_V A \quad - (1) \qquad = \left(\frac{4}{3}\pi r_0^3\right) A$$

↳ constant

a_V = constant term arising due to the volume of the nucleus

Contribution from the surface term.

Analogues to liquid drop, the proportionality between BE & A depends on the assumption that the size of the drop is so large that nearly every molecule is surrounded by its full of neighbours with which it may interacts. But molecules are not surrounded on all sides at the surface region. Similarly the nucleons on the surface of the nucleus have fewer neighbours, thus fewer interactions than those in the interior of the nucleus. The effect is analogous to the surface tension of a ~~big~~ liquid drop. Thus this will be a correction to the interactions calculated considering volume contribution. There, ~~each~~ the interactions of each nucleons are calculate considering each nucleons are surrounded by others. But at surface this is not the case, and a correction is required, because the binding energy will be less as some nucleus will not have interaction pairs all around it at surface. The surface energy term takes this into account, hence it ~~be to~~ become negative. That is more is the surface area of a nucleus unstable the nucleus is going to be.

$$BE_s \propto -S \qquad S = 4\pi r^2 = \underbrace{(4\pi r_0^2)}_{\text{const.}} A^{2/3}$$

$$\Rightarrow BE_s = -a_s A^{2/3} \quad -(2)$$

a_s is the constant from surface term.

Contribution from columb ~~term~~ repulsion:

This is the contribution arising due to the electrostatic repulsion of protons inside nucleus. There are some energy required to bring each protons to the nucleare volume & hold there inside.

Thus, a work must be done against the repulsive columb forces in order to assemble neucleons within nucleus. This will reduce the binding energy further ~~with~~, a negative sign to it. $BE_c \propto -C$ C columb Energy.

Earlier we calculated that the nuclear charge distributions are almost uniform through out its volume. i.e. $\rho_c = \frac{Ze}{\frac{4}{3}\pi R^3}$ — ⓐ

The work dw required to bring a thin spherical shell of width dr of this charge upto its radius r is

$$dw = \frac{4}{3}\pi r^3 \rho_c \times 4\pi r^2 dr \frac{\rho_c}{4\pi\epsilon_0 r} \quad \text{— ⓑ}$$

& by integrating r from 0 to R, the energy can be written $BE_c = \frac{\left(\frac{3}{5}\right) Z^2 e^2}{4\pi\epsilon_0 R}$ — ⓒ

If the system is having only one proton then the

$$-BE_c = \frac{(3/5)\, e^2}{4\pi\epsilon_0 R} \quad \text{— ⓓ}$$

But there is no work required for one proton system. as it has nothing to get repulsion from. Thus the total must be less by a ~~term~~ factor of ⓓ for each of the proton.

$$-BE_c = (c) - Z(d)$$

$$= \frac{\left(\frac{3}{5}\right) Z^2 e^2}{4\pi\epsilon_0 R} - Z\left(\frac{\left(\frac{3}{5}\right) e^2}{4\pi\epsilon_0 R}\right)$$

$$= \left(\frac{3}{5}\right)\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{1}{R}\right) e^2 Z(Z-1)$$

$R = r_0 A^{1/3}$

$$= \left(\frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\right)\frac{1}{r_0}\left[\frac{Z(Z-1)}{A^{1/3}}\right]$$

$$\Rightarrow BE_C = -a_c \frac{Z(Z-1)}{A^{1/3}} \qquad a_c = \frac{\left(\frac{3}{5}\right) e^2}{4\pi\epsilon_0 r_0}.$$

The spacing between energy levels is inversely proportional to the volume of nucleus, this can be seen by treating nucleus as a three dimentional potential well & therefore inversely proportional to volume $\left(\frac{4}{3}\pi r^3\right) \sim \left(\frac{4}{3}\pi r_0^3 A\right)$

Thus any excess nucleon will occupy the energy level to a higher value

$$(BE)_{ass} \propto (N-Z)^2$$
$$\propto \left(\frac{1}{\frac{4}{3}\pi r_0^3}\right)\frac{1}{A}$$

$$\Rightarrow \quad BE_{ass} = -a_{ass}\frac{(N-Z)^2}{A} = -a_{ass}\frac{(A-2Z)^2}{A} \quad -(4)$$

Contribution from Pairing:

An extra correction appears from the tendency of proton pairs and neutron pairs to occur which actually comes because of the different overlap of the wavefunctions for pairs of nucleons in various states. That the interactions are more favorable when the pairs are either proton-proton or ~~neutron and neutron~~ neutron-neutron. In order to account for the binding energy, if number of proton & number of neutrons are both even the pairing energy is +ve, we substract the same term if these are both odd, and do nothing ~~on~~ if one is odd other is even. Experimentally it has been found that the pairing energy goes inversely as

$$(BE)_p \propto \frac{1}{A^{3/4}} \quad -(5)$$

$$(BE)_p = a_p \frac{1}{A^{3/4}} \quad \text{even-even}$$
$$= 0 \quad \text{even-odd odd-even}$$
$$= -a_p \frac{1}{A^{3/4}} \quad \text{odd-odd}$$

Adding up all the contribution of the Binding Energy terms as discussed above we can write the final semi-empirical formula as:

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{ass} \frac{(A-2Z)^2}{A} + a_p \frac{\delta}{A^{3/4}}$$

where a_v, as a_c, a_{ass}, and a_p are the constants for volume, surface columb, assymetric and pair terms. And δ is

$$\delta = \begin{cases} +1 & \text{even-even nuclei} \\ 0 & \text{odd-even, even-odd nuceli} \\ -1 & \text{odd-odd nuclei} \end{cases}$$

After fitting to various data it is established ~~the~~ each term in the semiempirical formula. The value of constants are

$a_v = 15.5$ MeV

$a_s = 16.8$ MeV

$a_c = 0.72$ MeV

$a_{ass} = 23$ MeV

$a_p = 34$ MeV

~~A better~~

Extra note:

The better way the pairing term can be written as

$$(BE)_p = a_p \left\{ \frac{(-1)^Z + (-1)^N}{2} \right\} \frac{1}{A^{3/4}}$$

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{ass} \frac{(A-2Z)^2}{A} + a_p \frac{(-1)^Z + (-1)^N}{2} \frac{1}{A^{3/4}}$$

The values of constants ~~vary~~ differ from book to book & references to references but we will follow the values which are menmed in Keneth Krane.

This can also be derived using simple pair formation of each protons within the proton pair potential. (eventually it is same though).

$$V = \frac{e^2}{4\pi\epsilon_0 R}$$ → Potential of pair of protons.

if there are Z number of protons in the nucleus then ${}^{Z}C_2$ number of pairs of protons interaction can happen. which is $\frac{Z(Z-1)}{2}$ pairs of repulsive force

Thus

$$BE_c = -\frac{Z(Z-1)}{2} V = -\frac{Z(Z-1)}{2}\frac{e^2}{4\pi\epsilon R}$$

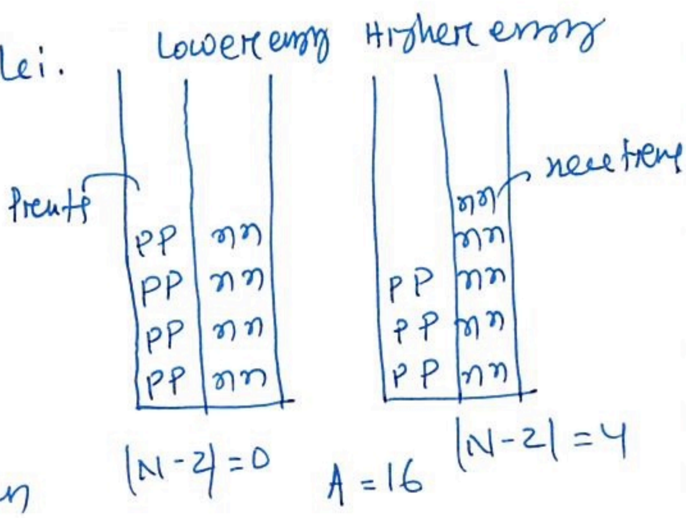
$$= -\frac{Z(Z-1)}{8\pi\epsilon_0 r_0 A^{1/3}}$$

$$\Rightarrow BE_c = -a_c \frac{Z(Z-1)}{A^{1/3}} \quad — (3)$$

Please note the a_c calculate earlier & here are notabley different by a factor of 1.2. This is purely due to the fact of work done calculation.

Contribution From Symmetry Energy Term:

This is a quantum effects arising from Pauli's exclusion principle which only allows two protons or two neutrons (with opposite spin direction) in each energy state. If a nucleus contains the same number of protons and neutrons then all the protons and neutrons will be filled up to the same maximum energy level. If, on the other hand, we exchange one of the neutrons by a proton then that proton would be guided by the exclusion princi to occupy a higher energy state, since all the below level stat are already. ~~Well~~ For nuclei with equal numbers of ~~the~~ protons and neutrons, the nucleus is symmetric and it tends to be very stable. But what if the number of neutrons is greate than the number of protons. This energy associated is a as a correction in types of nuclei.



The main argument for the symmetry effect comes from the exchange character of the nuclear force. Exchange force of nuclei are attractive between two particle if the wave function is symmetric with respect to the space exchange of these particles; they are repulsive if the wave function is antisymmetric. Thus the exchange forces are attractive between particles in same level, but less attractive or even repulsive between particle in diffent levels. Hence we obtain largest ammount of attraction if the number of particles in equal levels ~~are~~ is highest.

Thus the assumption of exchange forces provides a natural explanation for the increased stability of nuclei with equal numbers of neutrons and protons, for these are the nuclei which can have the largest number of symmetric pairs. Nuclei with an excess of either type of nucleon must have more anti-symmetric pairs, according to pauli ~~excl~~ principle. The anti-symmetric pairs donot contribute to the potential energy so the stablest nuclei are the ones with the fewest anti-symmetric pairs. This is the effect ~~wh~~ which we call the symmetry effect.

The upshot of this is that nucleides with $Z = N = (A-Z)$ have a higher binding energy, where as for nuclei with diffent numbers of protons & neutrons (for fix A) the binding energy decrease as the square of the numbers differences.

Let's take an example & calculate binding energy using the semiempirical formula & using the mass of each of the components in the nucleus. & let's compare the both. Let's take $^{64}_{30}Zn$ for this calculation, Zn has 30 proton & 34 neutrons. Therefore using atomic mass & constituent nucleon's mass we can write the equation.

$$BE = \left(Nm_n + Zm_p - M_{Zn}\right) \times 931.5 \text{ MeV}.$$

$$= \left(34 \times 1.0086 + 30 \times 1.0072 - 63.929\right) 931.5 \text{ MeV}$$

$$= 559.1 \text{ MeV}.$$ [we have used working formula)

By using semi-emperical formula, we will put $A = 64$ & $Z = 30$ & insert all the constants.

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + a_p \frac{\delta}{A^{3/4}}$$

$$= 15.5 \times 64 - 16.8\,(64)^{2/3} - 0.72 \times \frac{30(30-1)}{(64)^{1/3}} - 23\,\frac{(64-2\times30)^2}{64} + \frac{34}{(64)^{3/4}}$$

a_p is possive (even-even)

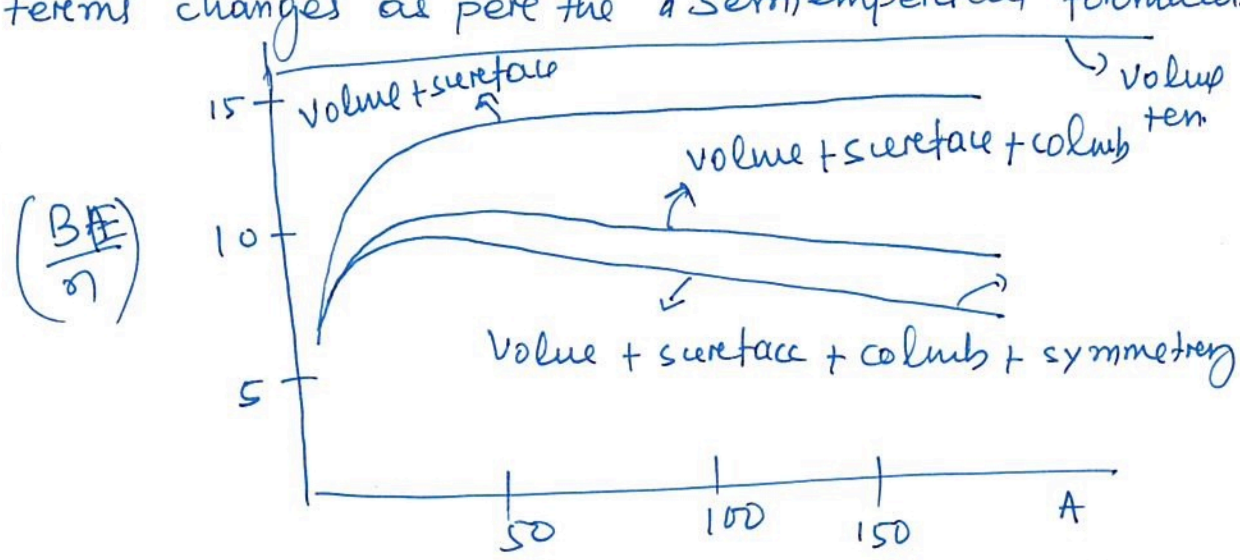
$$= 992 - 16.8 \times 16 - 0.72 \times \frac{870}{4} - 23 \times \frac{16}{64} + \frac{34}{22.63}$$

$$= 992 - 268.8 - 43.5 - 5.75 + 1.502$$

$$= 675.452.$$

The values are 559.1 & 675.452

How each terems changes as pere the a semiempereical foremula.



This picture shows the contribution of each term is the semi emperical mass foremula.