PHY622/Quiz 2

Date: March 14, 2018 **[Total Maximum Marks: 20]**

Enrol. No.: ....SOLUTION.... **Name:** ....

Instructions:

- Marks: For questions (1-5), **+2** for each correct answer and **−1** for each incorrect answer. For questions (6) and (7), **+5** for each correct answer.
- For multiple choice type questions, mark your answer neatly. Answers with more than one selection will not be taken into account.
- For other questions write **only** the final answer in a space given in the paper.

Questions

1. A finite discrete group of matrices (under the matrix multiplication) is generated by the elements $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The order of this group is

   (A) 2 (B) 4 (C) 8 (D) 12

2. Let $A : V \to V$ is a linear operator and M is invariant subspace of V with respect to A. The operator $AA^\dagger$ is

   (A) always fully reducible.
   (B) fully reducible only if $M^\perp$ is also invariant w.r.t. A.
   (C) fully reducible only if M is also invariant w.r.t. $A^\dagger$.
   (D) fully reducible only if $A = A^\dagger$.

   [Any of (B, C, D) is correct answer.]

3. A group of reflections (with respect to any axis) in 2D Euclidean space is a discrete subgroup of

   (A) U(1)
   (B) O(2)
   (C) SO(2)
   (D) All of the above

4. C_6 is a cyclic group of order 6. The number of its non-trivial subgroups is

   (A) 0 (B) 2 (C) 3 (D) 4

5. G_n is a finite discrete group of order n where n is any prime number. The dimension of irreducible representation of G_n is

   (A) n.
   (B) 1.
   (C) smaller than n but greater than 1.
   (D) always equal to the dimension of irreducible representation of S_n.

6. Write down the matrix $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ in spectral decomposition form.

Answer:

$$R(\theta) = \frac{e^{-i\theta}}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} + \frac{e^{i\theta}}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

7. For elements $g_1, g_2 \in G$, $D(g_1) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$, $D(g_2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $D'(g_1) = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$, $D'(g_2) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ are two different irreducible representations (here $\omega^3 = 1$). Are $D(G)$ and $D'(G)$ *equivalent* representations of G? Justify your answer.

Answer:

$D(G)$ and $D'(G)$ are <u>NOT</u> equivalent representations.

→ Starting from a general 2×2 matrix A, one finds that

$$A\,D(g_i) = D'(g_i)\,A \quad \text{for } i=1,2$$

leads to $\boxed{A=0}$. This implies that $D(G)$ and $D'(G)$ are not equivalent irreducible representations from Schur's Lemma.

→ It would be equivalent to show that there does not exist R such that $R\,D(g_i)R^{-1} = D'(g_i)$ for $i=1,2$ and $\det R \neq 0$.