1.

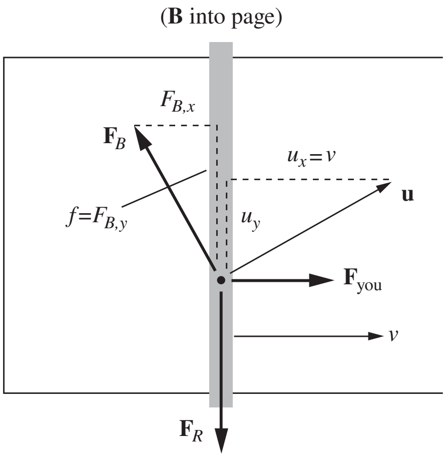


Figure 12.109.

7.2 *What's doing work?*

Figure 12.109 shows the situation. For simplicity, we assume that the mobile charges are positive; this doesn't affect the result. The important point to realize is that there are *two* components to a given charge's velocity **u**, namely the horizontal component $u_x = v$ due to the motion of the rod, and the vertical component u_y due to the current along the rod. This means that the magnetic force $\mathbf{F}_B$ points up and to the left, perpendicular to **u**, as shown. Its magnitude is $F_B = quB$, and its two components have magnitudes $F_{B,x} = qu_yB$ and $F_{B,y} = qu_xB = qvB$. The latter of these is what we called **f** in Eq. (7.5). Assuming that the current is steady and the charge isn't accelerating, the total force on it equals zero. So if you are applying the force to the rod, then your force is given by $F_{\text{you}} = F_{B,x}$, and the resistive force on the charges is given by $F_R = F_{B,y}$. (All of these quantities are magnitudes, so they are defined to be positive.)

Which forces do work? As mentioned in the problem, the magnetic force does no work because $\mathbf{F}_B$ is perpendicular to **u**. But if you wish, you can break this zero work into two equal and opposite pieces. The vertical component of $\mathbf{F}_B$ does work at a rate $F_{B,y}u_y = (qu_xB)u_y$. And the horizontal component does work at a rate $-F_{B,x}u_x = -(qu_yB)u_x$. These two rates are equal and opposite, as they must be. You also do work, because there is a component of **u** in the direction in which you are pulling. The rate at which you do work is $F_{\text{you}}u_x$. And due to the balancing of all the forces, this positive rate is equal and opposite to the negative rate at which $F_{B,x}$ does work. The resistive force also does work, and the rate is $-F_Ru_y$. This negative rate is equal and opposite to the positive rate at which $F_{B,y}$ does work.

We see that the magnetic force does zero net work, while the positive work you do is canceled by the negative work the resistive force does. While it is true that that a *component* of $\mathbf{F}_B$ does positive work (the vertical component, which we called **f** in Section 7.3), the other component of $\mathbf{F}_B$ does an equal and opposite amount of negative work. So it would hardly be accurate to say that *the* magnetic force does work.

This setup is essentially the same as the setup in which you push a block up a frictionless inclined plane at constant speed **u**, by applying a *horizontal* force, as shown in Fig. 12.110. This figure is simply Fig. 12.109 with the forces relabeled. The normal force replaces the magnetic force, and gravity replaces the resistive force. The vertical component of the normal force does positive work, but the horizontal component does an equal and opposite amount of negative work. You are the entity pumping energy into the system (which shows up as gravitational potential energy), just as you were the entity pumping energy into the above circuit (which showed up as heat). Although the vertical component of the normal force is the only force actually lifting the block upward, the *entire* normal force does zero net work. Conversely, you are not lifting the block upward, but you do in fact do positive work.

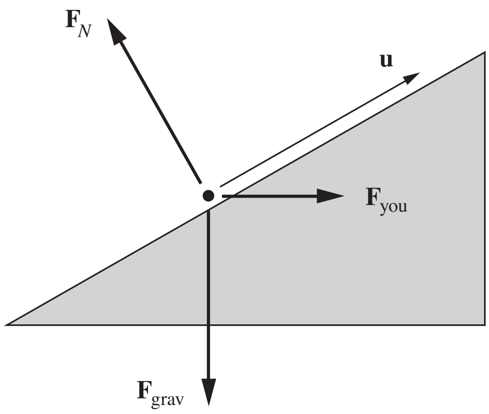
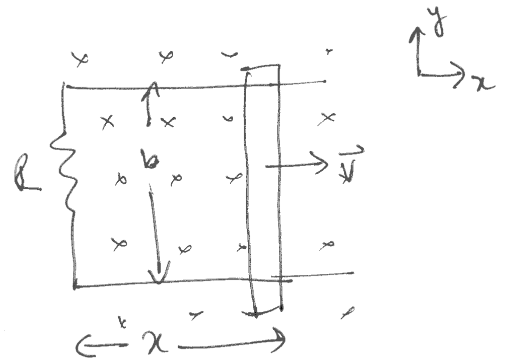


Figure 12.110.

2.



$\times \Rightarrow$ denotes magnetic field direction

$\therefore \vec{B} = -B\hat{z}$ pointing into the page.

$\vec{V} = V\hat{x}$.

The rod moves ~~in~~ to the right in the $\hat{x}$ direction due to some external force.

Let at a time, t, the crossbar of mass m is at a distance x as shown.

Then it forms closed loop and during its motion, area of loop increases with time.

With the magnetic field acting downwards, flux increases with time.

flux, $\phi = BA = Bbx$

$$\therefore \varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(Bbx) = -Bb\frac{dx}{dt} = -Bbv.$$

This induced emf will cause current in the loop in the counterclockwise direction so that it opposes the increase in flux by giving rise to a magnetic field to oppose the external magnetic field $\vec{B}$.

Now, induced current, $I = \frac{|\varepsilon|}{R} = \frac{Bbv}{R}$

∴ Magnetic force experienced by bar,

$$\vec{F}_B = I\,\vec{l}\times\vec{B} = I\,b\hat{y}\times(-B\hat{z})$$

$$= -IbB(\hat{y}\times\hat{z}) = -IbB\,\hat{x}$$

$$= -\frac{B^2b^2v}{R}\,\hat{x}$$

→ opposite to $\vec{v}$.

∴ For the bar to keep moving at a constant speed, an external force needs to be present so that,

$$\vec{F}_{ext} = -\vec{F}_B = \frac{B^2b^2v}{R}\,\hat{x}.$$

Suppose at $t=0$, speed of rod $= v_0$ & the external agent stops pushing. Then,

$$F_B = -\frac{B^2b^2v}{R} = ma = m\frac{dv}{dt}.$$

$$\Rightarrow \quad \frac{dv}{v} = -\frac{B^2b^2}{mR}\,dt = -\frac{1}{\tau}\,dt, \quad \tau = \frac{mR}{B^2b^2}$$

Integrating,

$$v(t) = v_0 e^{-t/\tau}.$$

∴ speed decreases exponentially in the absence of external force doing work.

~~Power delivered~~

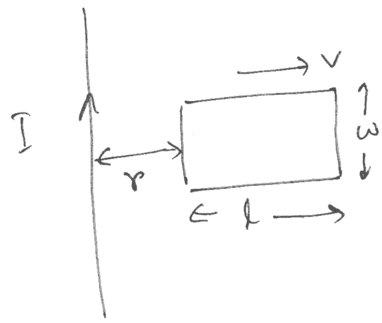
$$x = \int_0^\infty v(t)\,dt = \int_0^\infty v_0 e^{-t/\tau}\,dt = v_0\tau.$$

$$= \frac{mR}{B^2b^2}v_0.$$

Power delivered by $\vec{F}_{ext}$ = power dissipated in resist

$$\therefore P = \vec{F}_{ext}\cdot\vec{v} = \frac{B^2b^2v}{R}\cdot v = \frac{(Bbv)^2}{R} = \frac{\mathcal{E}^2}{R} = I^2R.$$

3.



Method 1:

$$|\mathcal{E}| = vw(B_1 - B_2)$$

$$B_1 = \frac{\mu_0 I}{2\pi r}, \quad B_2 = \frac{\mu_0 I}{2\pi (r+l)}..$$

$r = 15\text{ cm} = 0.15\text{ m}$; $l = 10\text{ cm} = 0.1\text{ m}$.

$v = 5\text{ m/sec}$; $w = 8\text{ cm} = 0.08\text{ m}$, $I = 100\text{ A}$

$$\therefore |\mathcal{E}| = 5 \times 0.08 \cdot \frac{\mu_0}{2\pi}\left(\frac{100}{0.15} - \frac{100}{0.25}\right)$$

$$= 0.4 \times 2 \cdot 10^{-7} \times 10^{2} \; \frac{0.25 - 0.15}{0.25 \times 0.15}$$

$$= 2.13 \times 10^{-5} \text{ volts.}$$

Method 2:

$$\phi = \int \vec{B} \cdot d\vec{a} = \int_{r}^{r+l} \frac{\mu_0 I}{2\pi x} w\, dx$$

Note. $x \equiv x(t)$, $r \equiv r(t)$

$$= \frac{\mu_0 I w}{2\pi} \ln x \Big|_{r}^{r+l} = \frac{\mu_0 I w}{2\pi} \ln\left(\frac{r+l}{r}\right)$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{\mu_0 I w}{2\pi} \cdot \frac{r}{r+l}\left(\frac{1}{r}\cdot\frac{dr}{dt} - \frac{1}{r^2}\cdot\frac{dr}{dt}(r+l)\right)$$

$$= -\frac{\mu_0 I w}{2\pi} \cdot \frac{r}{r+l}\left(\frac{v}{r} - \frac{v(r+l)}{r^2}\right).$$

$$= \frac{\mu_0 I w}{2\pi} \frac{l v}{r(r+l)} = vw\left(\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi(r+l)}\right)$$

$$= vw(B_1 - B_2)$$ (Same as above)

Direction of $\mathcal{E}$ will be in a direction to drive current such that flux decreases. Since flux is downward & decreases, induced current is clockwise.

The current in the loop at a time t is $I(t) = \frac{\mathcal{E}(t)}{R}$ where R is a non-negligible resistance in the loop. This current creates a field of its own and therefore a flux linked with it. Since $\mathcal{E}$ is changing with time, so does this flux. This creates an additional induced emf $\mathcal{E}'$ which we have ignored. We need to therefore find R so that, $\mathcal{E} \gg \mathcal{E}'$ & we can safely ignore $\mathcal{E}'$.

The field created by the current, $B' \sim \frac{\mu_0 I(t)}{2\pi . L}$

where L is some length ~ 10 cm.

$$\therefore \phi' \sim B' . \text{area of loop} = \frac{\mu_0 \times I(t) \times 0.08 \times 0.1}{2\pi \times 0.1}$$

$$= 2 \times 10^{-7} \times 0.08 \times I(t)$$

$$\therefore \phi' \sim 0.16 \times 10^{-5} \times I(t)$$

A typical time scale in the problem would be,

$$\tau' \sim \frac{l}{v} = \frac{0.1}{5} \text{ sec}$$

$$\therefore \mathcal{E}' \sim \frac{\phi'}{\tau'} = \frac{0.16 \times 10^{-5}}{0.1} \times 5 \times I(t) = 8 \times 10^{-5} I(t)$$

$$= 8 \times 10^{-5} \frac{\mathcal{E}(t)}{R}$$

$$\therefore R \sim \left(8 \times 10^{-5} \frac{\mathcal{E}}{\mathcal{E}'}\right) \Omega$$

$\therefore$ Even for $R \gg 10^{-5}\,\Omega$ we will have $\mathcal{E} \gg \mathcal{E}'$.

4. (a) $B(t) = \mu_0 n I(t) = \mu_0 n I_0 \cos\omega t.$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(B.\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

$$= \pi r^2 \mu_0 n I_0 \omega \sin\omega t$$

With the +ve current direction shown in the figure, right hand rule gives direction of B as upward. This also gives the +ve direction of $\mathcal{E}$ as counterclockwise when viewed from the top. Current in the loop,

$$I_\ell = \frac{\mathcal{E}}{R} = \frac{\pi r^2 \mu_0 n I_0 \omega}{R} \sin\omega t$$

(b) Force on a small element of the ring,

$$d\vec{F}(t) = I_\ell \, d\vec{l} \times \vec{B}$$

I : counterclockwise, $\vec{B}$: upward

$\therefore$ Force is radial

$$|d\vec{F}| = \frac{\pi r^2 \mu_0 n I_0 \omega}{R} \sin\omega t \,.\, dl \,.\, \mu_0 n I_0 \cos\omega t$$

$$= \frac{\pi r^2 \mu_0^2 n^2 I_0^2 \omega \, dl}{R} \sin\omega t \cos\omega t.$$

$\rightarrow$ radially outward if +ve

" " inward if −ve.

$\sin\omega t \cos\omega t = \frac{1}{2}\sin 2\omega t$:

$\therefore |d\vec{F}|$ max^m outward when $\omega t = \frac{\pi}{4}$

" " inward " " $\omega t = 3\pi/4$.

(c) Force in horizontal plane $\rightarrow$ can only stretch or shrink.

5.

Solution At the center of C_1, with I_1 flowing, the field B_1 is given by Eq. (6.54) as

$$B_1 = \frac{\mu_0 I_1}{2R_1}. \tag{7.40}$$

Since we are assuming $R_2 \ll R_1$, we can neglect the variation of B_1 over the interior of the small ring. The flux through the small ring is then

$$\Phi_{21} = (\pi R_2^2)\frac{\mu_0 I_1}{2R_1} = \frac{\mu_0 \pi I_1 R_2^2}{2R_1}. \tag{7.41}$$

The mutual inductance M_{21} in Eq. (7.37) is therefore

$$M_{21} = \frac{\Phi_{21}}{I_1} = \frac{\mu_0 \pi R_2^2}{2R_1}, \tag{7.42}$$

and the electromotive force induced in C_2 is

$$\mathcal{E}_{21} = -M_{21}\frac{dI_1}{dt} = -\frac{\mu_0 \pi R_2^2}{2R_1}\frac{dI_1}{dt}. \tag{7.43}$$

Since $\mu_0 = 4\pi \cdot 10^{-7}\,\text{kg m/C}^2$, we can write M_{21} alternatively as

$$M_{21} = \frac{(2\pi^2 \cdot 10^{-7}\,\text{kg m/C}^2)R_2^2}{R_1}. \tag{7.44}$$

The numerical value of this expression gives M_{21} in henrys. In Gaussian units, you can show that the relation corresponding to Eq. (7.43) is

$$\mathcal{E}_{21} = -\frac{1}{c}\frac{2\pi^2 R_2^2}{cR_1}\frac{dI_1}{dt}, \tag{7.45}$$

with $\mathcal{E}_{21}$ in statvolts, the R's in cm, and I_1 in esu/second. M_{21} is the coefficient of the dI_1/dt term, namely $2\pi^2 R_2^2/c^2 R_1$ (in second^2/cm). Appendix C states, and derives, the conversion factor from henry to second^2/cm.

Incidentally, the minus sign we have been carrying along doesn't tell us much at this stage. If you want to be sure which way the electromotive force will tend to drive current in C_2, Lenz's law is your most reliable guide.

6. (a) $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$.

Magnetic field due to a current carrying wire at a distance 'r' is, $B = \frac{\mu_0 I}{2\pi r}$.

Magnetic flux,

$$\Phi = \int \vec{B} \cdot d\vec{A} = \frac{\mu_0 I l}{2\pi} \int_s^{s+w} \frac{dr}{r} \quad (dA = l\,dr)$$

$$= \frac{\mu_0 I l}{2\pi} \ln\left(\frac{s+w}{s}\right).$$

Area vector points into the page $\therefore \Phi > 0$.

(b) $$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I l}{2\pi} \ln\left(\frac{s+w}{s}\right)\right].$$

$$= -\frac{\mu_0 l}{2\pi} \ln\left(\frac{s+w}{s}\right) \frac{dI}{dt}.$$

$I(t) = a + bt \quad \therefore \frac{dI}{dt} = b.$

$$\therefore \mathcal{E} = -\frac{\mu_0 b l}{2\pi} \ln\left(\frac{s+w}{s}\right).$$

The wire carrying current produces magnetic flux into the page. By Lenz's law, the induced current in the loop must be counterclockwise to produce a magnetic field out of page to counteract the increase in inward flux.