1. Consider a double pendulum composed of two identical pendula of massless rods of length l, and masses m, attached along the vertical direction. Obtain the Hamiltonian of this system, and derive Hamilton's equations of motion.

2. The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2+y^2},$$

where a, b, c, f, g, and k are constants. What is the Hamiltonian? What quantities are conserved?

3. A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a+bq_1^2} + k_1 q_1^2 + k_2\dot{q}_1\dot{q}_2,$$

where a, b, k_1, and k_2 are constants. Find the equations of motion in the Hamiltonian formalism.

4. A Hamiltonian of one degree of freedom has the form

$$H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{ba}{2}q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2},$$

where a, b, α, and k are constants.

   (a) Find a Lagrangian corresponding to this Hamiltonian
   (b) Is it possible to find an equivalent Lagrangian that is not explicitly dependent on time?
   (c) If you are able to solve part (b), what is the Hamiltonian corresponding the new Lagrangian, and what is the relationship between the two Hamiltonians?

5. (a) The Lagrangian for a system of one degree of freedom can be written as

$$L = \frac{m}{2}\left(\dot{q}^2\sin^2\omega t + \dot{q}q\omega\sin 2\omega t + q^2\omega^2\right).$$

   What is the corresponding Hamiltonian? Is it conserved?

   (b) Introduce a new coordinate defined by

$$Q = q\sin\omega t.$$

   Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is H conserved?