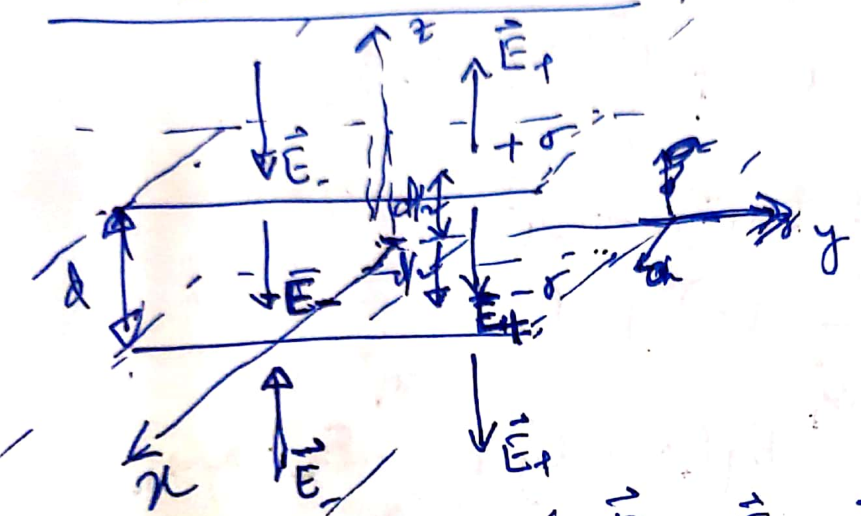
Assignment 3 sols.



$$\vec{E}_+ = \begin{cases} \dfrac{\sigma}{2\epsilon_0}\hat{k} & z > \dfrac{d}{2} \\ -\dfrac{\sigma}{2\epsilon_0}\hat{k} & z < \dfrac{d}{2} \end{cases}$$

$$\vec{E}_- = \begin{cases} -\dfrac{\sigma}{2\epsilon_0}\hat{k} & z > -\dfrac{d}{2} \\ +\dfrac{\sigma}{2\epsilon_0}\hat{k} & z < -\dfrac{d}{2} \end{cases}$$

$$\therefore \vec{E} = \vec{E}_+ + \vec{E}_- = \begin{cases} 0 & z > \dfrac{d}{2} \\ -\dfrac{\sigma}{\epsilon_0}\hat{k} & \dfrac{d}{2} > z > -\dfrac{d}{2} \\ 0 & z < -\dfrac{d}{2} \end{cases}$$

3. Since $z=0$ in all cases, we will ignore z component of argument.

Line integral along 1st path,

$$\int_{(0,0)}^{(x_1,y_1)} \vec{E}\cdot d\vec{s} = \int_0^{x_1} E_x(x,0)\,dx + \int_0^{y_1} E_y(x_1,y)\,dy$$

$$= 0 + \int_0^{y_1} (3x_1^2 - 3y^2)\,dy$$

$$= 3x_1^2 y_1 - y_1^3 .$$

Line integral along 2nd path,

$$\int_{(0,0)}^{(x_1,y_1)} \vec{E}\cdot d\vec{s} = \int_0^{y_1} E_y(0,y)\,dy + \int_0^{x_1} E_x(x,y_1)\,dx$$

$$= \int_0^{y_1} (0-3y^2)\,dy + \int_0^{x_1} 6xy_1\,dx$$

$$= -y_1^3 + 3x_1^2 y_1$$

These results ~~are~~ are the same. $\vec{E}$ may be an electrostatic field.

Let electric potential ϕ be zero at $(0,0)$.

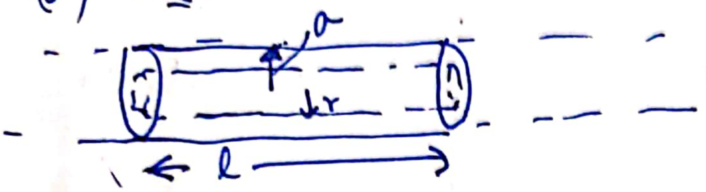
Then, $\phi = -\int \vec{E}\cdot d\vec{s} = y^3 - 3x^2 y .$

$$-\vec{\nabla}\phi = -\hat{x}\frac{\partial \phi}{\partial x} - \hat{y}\frac{\partial \phi}{\partial y} - \hat{z}\frac{\partial \phi}{\partial z}$$

$$= +6xy\,\hat{x} + (3x^2 - 3y^2)\,\hat{y} + 0 .$$

$$= 6xy\,\hat{x} + (3x^2 - 3y^2)\,\hat{y}$$

(a) $r \leq a$



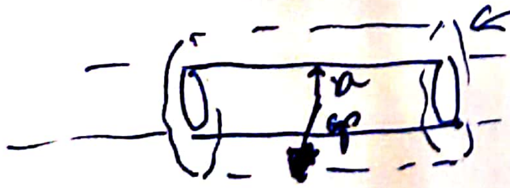
Consider the Gaussian ~~surface~~ surface as the coaxial cylinder of radius $r<a$ & length l. Charge enclosed $= \rho.(\pi r^2 l)$.

$\vec{E}$ is $\perp^r$ to the curved surface by symmetry. & therefore $\parallel^e$ to the area vector. The other 2 surfaces do not contribute. Therefore, Gauss's law gives,

$$\oint_S \vec{E}.d\vec{a} = \frac{q}{\epsilon_0} \Rightarrow E.2\pi r l = \frac{\pi r^2 l \rho}{\epsilon_0}$$

$$\therefore E = \frac{\rho r}{2\epsilon_0} \text{ radially outward}$$

(b) $r \geq a$.



Gaussian surface of radius $r \geq a$ & length l.

$$\oint \vec{E}.d\vec{a} = \frac{q}{\epsilon_0} \Rightarrow E.2\pi r l = \frac{\pi a^2 l \rho}{\epsilon_0}$$

$(\because \text{charge enclosed} = (\pi a^2 l)\rho)$.

$$\therefore E = \frac{\rho a^2}{2\epsilon_0 r}$$

(b) $\phi = 0$ at $r = 0$.

$\therefore$ For $r \leq a$: $\phi(r) = -\int_0^r E\,dr = -\int \frac{\rho r}{2\epsilon_0}\,dr$

$$= -\frac{\rho r^2}{4\epsilon_0}.$$

For $r \geq a$: $\phi(r) = -\int_0^a E\,dr - \int_a^r E\,dr$

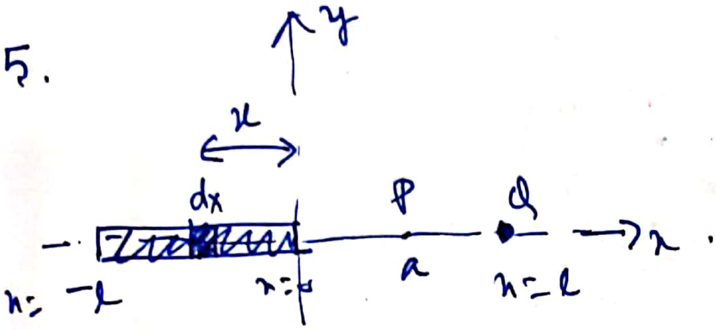
$$= -\int_0^a \frac{\rho r}{2\epsilon_0}\,dr - \int_a^r \frac{\rho a^2}{2\epsilon_0 r}\,dr$$

$$= -\frac{\rho a^2}{4\epsilon_0} - \frac{\rho a^2}{2\epsilon_0}\int_a^r dr$$

$$= -\frac{\rho a^2}{4\epsilon_0} - \frac{\rho a^2}{2\epsilon_0}\ln(r/a).$$

[2]

5.



(a) Consider an element of length dx on the stick. The electric field at P due to the element is

$$k \frac{\lambda\, dx}{(a - x)^2}.$$ Note : x is −ve.

∴ Electric field at P due to the stick,

$$E_{stick} = k \int_{x=-l}^{0} \frac{\lambda\, dx}{(a-x)^2} = k\lambda \left[\frac{1}{a-x}\right]_{x=-l}^{0}$$

$$= k\lambda\left[\frac{1}{a} - \frac{1}{a+l}\right] = k\frac{\lambda l}{a(a+l)} = \frac{kQ}{a(a+l)}$$

$(\because Q = \lambda l)$

Field due to charge Q at P.

$$E_Q = k \frac{Q}{(l-a)^2}.$$

$$\therefore E_{stick} = E_Q \Rightarrow \frac{\cancel{k}Q}{a(a+l)} = \frac{Q}{(l-a)^2}.$$

$$\Rightarrow (l-a)^2 = a(a+l)$$

$$\Rightarrow a = l/3.$$

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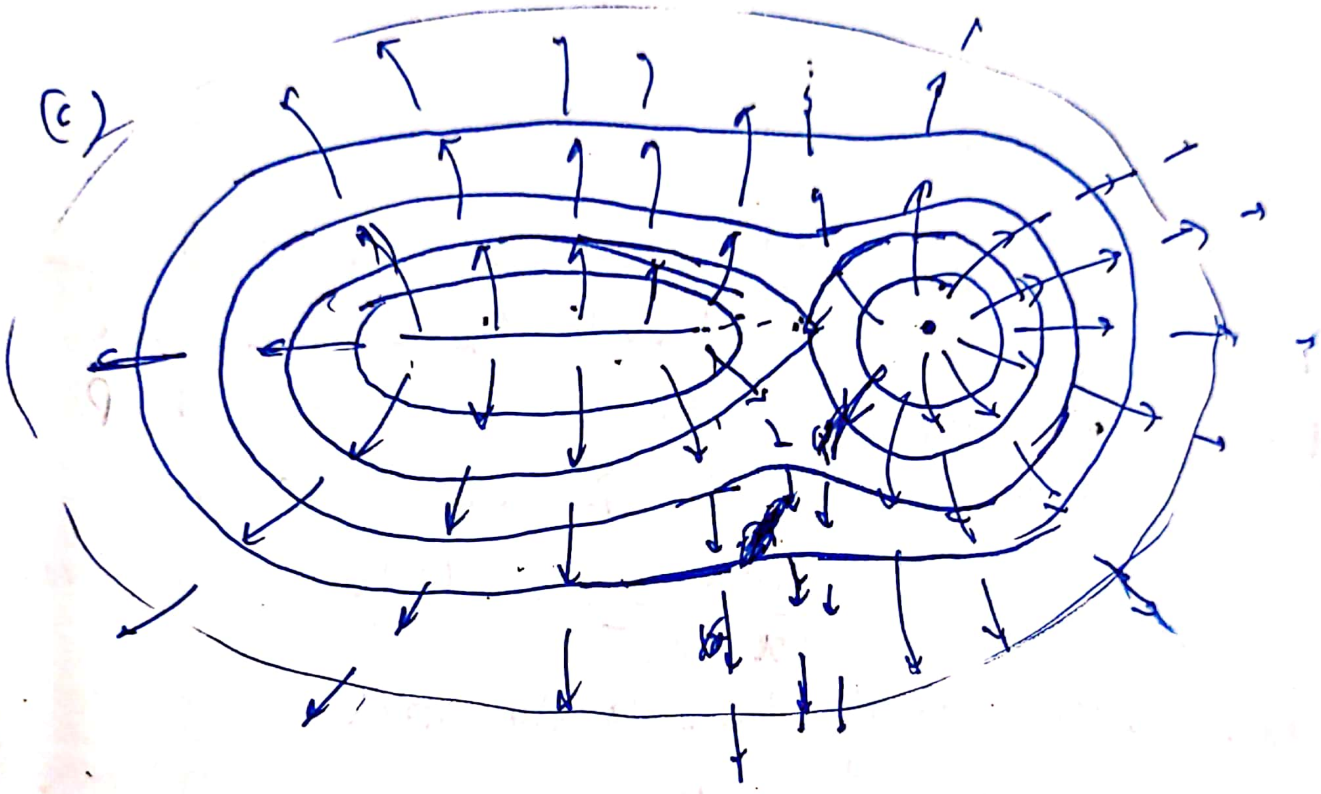
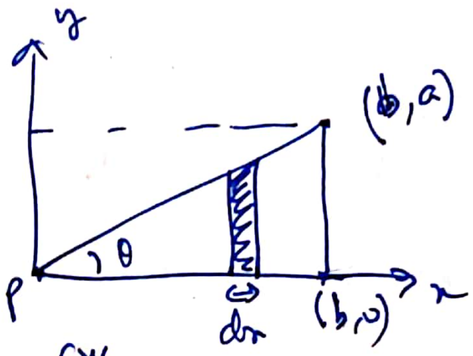
(b) There are no other points. Argue why!

(c)



6.



$$\phi_P = k\int \frac{\sigma\, da}{r}$$

$$= k\sigma \int_0^b dx \int_0^{ax/b} \frac{dy}{\sqrt{x^2+y^2}}.$$

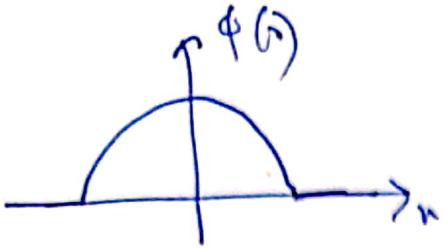
$$\int_0^{ax/b} \frac{dy}{\sqrt{x^2+y^2}} = \ln\left(y+\sqrt{x^2+y^2}\right)\Big|_0^{ax/b}$$

$$= \ln\left[\sqrt{1+\frac{a^2}{b^2}} + \frac{a}{b}\right] \rightarrow \text{independent of } x\,!$$

$$\therefore\ \phi_P = k\sigma b \ln\left[\sqrt{1+\frac{a^2}{b^2}} + \frac{a}{b}\right].$$

$$\frac{a}{b} = \tan\theta = \frac{\sin\theta}{\cos\theta}\ ; \quad \cos\theta = \frac{b}{\sqrt{a^2+b^2}}$$

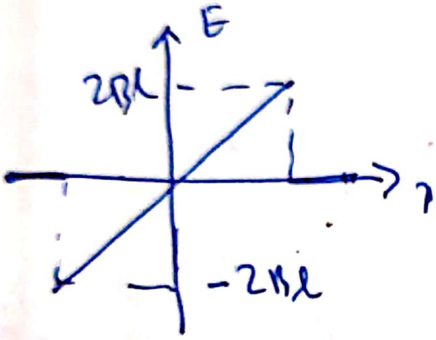
$$\therefore\ \phi_P = \frac{\sigma b}{4\pi\epsilon_0} \ln\left(\frac{1+\sin\theta}{\cos\theta}\right).$$

7. 

The potential $\phi(x) = \begin{cases} B(l^2-x^2) & |x|\le l \\ 0 & |x|>l \end{cases}$ is as shown.

Now, $\vec{E} = 0$ for $|x| > l$ as potential is const.

For $|x| \le l$, $\vec{E} = -\vec{\nabla}\phi \Rightarrow E_x = -\frac{d\phi}{dx} = 2Bx$



→ discontinuous at $x = \pm l$.

→ surface charge density on the planes at $x = \pm l$.

$$\rho = -\epsilon_0 \nabla^2 \phi = \epsilon_0 \vec{\nabla}\cdot\vec{E}.$$

$\vec{E} = 0$ for $|x| > l$ $\therefore \rho = 0$

$$\rho = -\epsilon_0 \nabla^2\phi \Rightarrow \rho(x) = -\epsilon_0 \frac{d^2\phi}{dx^2} = 2\epsilon_0 B.$$