PHY622/Assignment 02

Date: February 2, 2018

Note: Solve the following problems. The submission of assignment is NOT required. You are encouraged to discuss with each other and/or contact the instructor if you have any difficulty in solving the problems.

Problem 1. If $[[A, B], A] = 0$ then show that for any positive integer k,

$$[A^k, B] = kA^{k-1}[A, B] \ .$$

Problem 2. If $[[A, B], A] = [[A, B], B] = 0$ and $\alpha \in \mathbb{R}$ then show that,

$$[A, \exp(\alpha B)] = \alpha[A, B] \exp(\alpha B) \ .$$

Problem 3. Given H and $U(t)$ as operators, solve

$$\frac{dU}{dt} = t^n HU(t)$$

Problem 4. Prove that a linear transformation H on a complex inner product space is hermitian if and only if $\langle a|H|a\rangle$ is real for all $|a\rangle$.

Problem 5. Find $T^\dagger$ for each of the following operators.
(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x, y, z) = (x + 2y - z, 3x - y + 2z, -x + 2y + 3z)$$

(a) $T : \mathbb{C}^3 \to \mathbb{C}^3$ given by

$$T(\alpha, \beta, \gamma) = (\alpha + i\beta - 2i\gamma, -2i\alpha + \beta + i\gamma, i\alpha - 2i\beta + \gamma)$$

Problem 6. Let $P^{(M)} \equiv \sum_{i=1}^{M} |e_i\rangle\langle e_i|$ is a projection operator constructed out of the first m orthonormal vectors of the basis $B = \{|e_i\rangle\}_{i=1}^{N}$ of V. Show that $P^{(M)}$ projects into the subspace spanned by the first M vectors in B.

Problem 7. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by

$$T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, 2x_1 - x_3, x_1 + 2x_2) \ .$$

Find the matrix representation of T in the basis $B = \{(1, 1, 0),\ (1, 0, -1),\ (0, 2, 3)\}$.