Extra connectin:

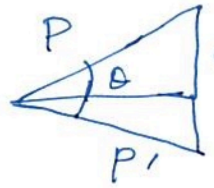
Before we procced further I should menten that the classical energy loss equatin can be derive directly from the crosssectin calculatin. This is due to the fact that, charge particles like, $\alpha$-particles, protons, pions & muons etc scatter electrons according to the Rutherford scattering [which we have done in detailed derivatins]. Thus the scattering of charge particles by electrons are the same process;

we know that the Rutherford scattering formula is improved by the Mott's scatting formula for electrons of momentum $p$ and velocity $v$ and particle with charge $ze$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{z\alpha\hbar c}{2pv}\right)^2 \operatorname{cosec}^4\left(\frac{\theta}{2}\right)\left[1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right]$$

converting the scattering angle $\theta$ into a momentum transfer $q$:

$$\left(\frac{q}{2}\right) = p\sin\left(\frac{\theta}{2}\right)$$



$$\sin\left(\frac{\theta}{2}\right) = \frac{q}{2p} \Rightarrow \sin^2\left(\frac{\theta}{2}\right) = \frac{q^2}{(2p)^2}$$

using these with $\operatorname{cosec}^4(\theta/2)$ an $\sin^2(\theta/2)$ terms in Moth's formula we can get

$$\left(\frac{d\sigma}{d\Omega}\right)_{mott} = \left(\frac{z\alpha\hbar c}{2pv}\right)^2 \frac{16p^2}{q^4}\left[1-\frac{v^2}{c^2}\frac{q^2}{4p^2}\right]$$

Now convert the diffential cross-section, $\frac{d\sigma}{d\Omega}$ into $\frac{d\sigma}{dq^2}$ by replaceing the differential ~~cross-sectin~~ $\cancel{\frac{d\sigma}{d\Omega}}$ ~~into~~ $\cancel{\frac{d\sigma}{dq^2}}$ solid angle $d\Omega$ using $d\Omega = 4\pi\sin\theta\, d\theta$.

$$\frac{d\sigma}{\sin\theta\, d\theta} = 4\pi\left(\frac{z\alpha\hbar c}{2pv}\right)^2 \frac{16p^4}{q^4}\left[1-\frac{v^2}{c^2}\frac{q^2}{4p^2}\right]$$

The operator $\sin\theta\, d\theta$ can be obtained by diffentiatin

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{q^2}{(2p)^2} \Rightarrow 4p^2\sin^2\left(\frac{\theta}{2}\right) = q^2$$

$$d(q^2) = 4p^2 \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) d\theta.$$
$$= 4p^2\left[\frac{1}{2}\sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right)\right]d\theta = 2p^2 \sin\theta\, d\theta.$$

$$\Rightarrow \frac{dq^2}{2p^2} = \sin\theta\, d\theta.$$

by substituting this into the cross-section

$$\frac{d\sigma}{\sin\theta\, d\theta} = \left(\frac{d\sigma}{dq^2}\right)2P^2 = 4\pi\left(\frac{Z\alpha\hbar c}{2pv}\right)^2 \frac{16p^4}{q^4}\left[1-\frac{v^2}{c^2}\frac{q^2}{4p^2}\right]$$

$$\Rightarrow \left(\frac{d\sigma}{dq^2}\right) = 8\pi\left(\frac{Z\alpha\hbar c}{q^2 v}\right)^2\left[1-\frac{v^2}{c^2}\frac{q^2}{4p^2}\right]$$

Now transfere this to the frame where electron is at rest & charge particle moving towards it.

$$P_e \rightarrow P_{nuc} = m_e \gamma v$$

$$\Rightarrow \left(\frac{d\sigma}{dq^2}\right) = 8\pi\left(\frac{Z\alpha\hbar c}{q^2 v}\right)^2\left[1-\frac{v^2}{c^2}\frac{q^2}{4(m_e\gamma v)^2}\right]$$

The momentum transfer squared is the same in both frame and is related to the energy transfer $\eta$ as follows

$$q^2 = 2m_e\eta \rightarrow dq^2 = 2m_e d\eta$$

$$\Rightarrow \frac{d\sigma}{2m_e d\eta} = 8\pi\left(\frac{Z\alpha\hbar c}{2m_e\eta v}\right)^2\left[1-\frac{v^2}{c^2}\frac{2m_e\eta}{4(m_e\gamma v)^2}\right]$$

Hence the cross-section for energy loss in the range $\eta - \eta + d\eta$ can be given as

$$\frac{d\sigma}{d\eta} = \left(\frac{4\pi}{m_e}\right)\left(\frac{Z\alpha\hbar c}{\eta v}\right)^2\left[1-\frac{\eta}{2m_e c^2}\left(1-\frac{v^2}{c^2}\right)\right]$$

if the charge particle loses energy $-dE$ in a depth $dx$ in the material containing $n_a$ atoms of atomic number $Z$ per unit volume

$$-dE = \left(\begin{matrix}\text{number of electron}\\ \text{per unit volume}\end{matrix}\right) \times \left(\begin{matrix}\text{energy loss}\\ \text{per unit volume}\end{matrix}\right)$$

$-dE = n_e Z \times dx$ (energy loss per unit area)

$$\Rightarrow -\frac{dE}{dx} = n_e Z \int_{\eta_{min}}^{\eta_{max}} \eta \frac{d\sigma}{d\eta} d\eta$$

by substituting $\frac{d\sigma}{d\eta}$ in $\frac{dE}{dx}$ equation we get.

$$-\frac{dE}{dx} = n_e Z \int \left(\frac{4\pi}{m_e}\right)\left(\frac{z\alpha\hbar c}{v}\right)^2 \left[1 - \frac{\eta}{2m_e c^2}\left(1-\frac{v^2}{c^2}\right)\right] d\eta$$

$$= n_e Z \frac{4\pi}{m_e}\left(\frac{z\alpha\hbar c}{v}\right)^2 \int_{\eta_{min}}^{max}\left[\frac{1}{\eta} - \frac{1}{2m_e c^2}\left(1-\frac{v^2}{c^2}\right)\right] d\eta$$

$$= n_e Z \frac{4\pi}{m_e}\left(\frac{z\alpha\hbar c}{v}\right)^2 \left[\ln\left(\frac{\eta_{max}}{\eta_{min}}\right) - \frac{\eta_{max}-\eta_{min}}{2m_e c^2}\left(1-\frac{v^2}{c^2}\right)\right]$$

$\eta_{max}$ is the highest energy transfer where electron will be almost free i.e.

$$\eta_{max} = 2 m_e v^2 \gamma^2$$

whereas this is not true at small $\eta$. At low $\eta$ the momentum & energy transfer can lead to both ionization & excitation: it is therefore necessary to integrate over $\eta$ and $q$ with cross-section that depends on the atomic structure of the material. Thus the $\eta_{min}$ can be written in terms of mean ionization potential. Thus the final equation can be obtained by putting $n_e$, & $r_e$ as done earlier

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \rho \frac{1}{\beta^2}\left[\ln\left(\frac{4\pi\epsilon_0 \gamma^2 m_e v^2}{I^2}\right) - \beta^2\right]$$

$\alpha = \left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)$ : $r_0 = \frac{4\pi\epsilon_0\hbar}{m_e e^2}$ $\quad r_e = \alpha^2 r_0$.

Rest of the derivation remain same by adjusting some term & apply all effects & correction that we have already described.

observatin and understanding of Bethe-Block formula.

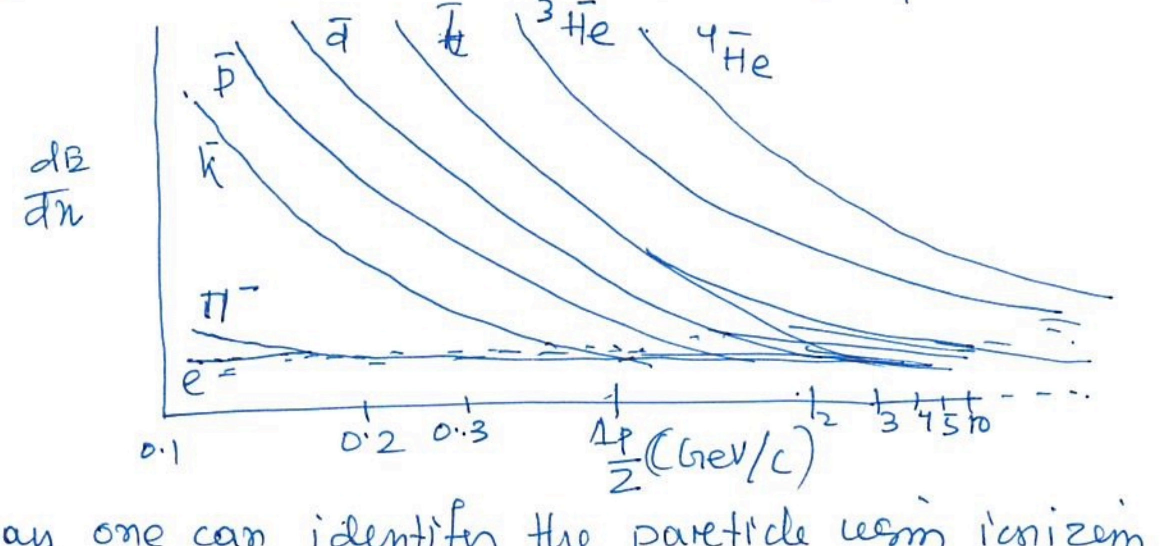
① it is almost independent of the mass of the incident particle

② It directly depends on the charge of the particle (incident, $z^2$

③ Treating the slowly varying logarithmic term as a constant, it can be seen that $\frac{dE}{dx} \propto \frac{1}{\beta^2} \Rightarrow \frac{dE}{dx} \propto \frac{c^2}{v^2} \Rightarrow \frac{dE}{dx} \propto \left(\frac{m^2}{p^2}\right)$

incase of heavy particle the loss depends on Mass A charge & Z of the target nucleus. i.e. $-\left(\frac{dE}{dx}\right) \propto \left(\frac{Z}{A}\right)$ & $\left(\frac{m^2}{p^2}\right)$

Thus one can utilize this ~~effect~~ fact to characterize particles with diffent mass & can be identified. by knowing the momentum of the particle & measuring $\frac{dE}{dx}$ (infer ~~its~~ mass. But there will be a limit to this effect. as this will work at low momentum range where $1/\beta^2$ strongly depends on the mass & higher the momenta there will not be separater




This the way one can identify the particle using ionizing ~~gas~~ detector that uses purely the $-\frac{dE}{dx}$ formula. One such detector name is TPC (Time projection chamber)

<u>δ-Electrons:</u> These are the secondary electrons from the target knocked out by the incident ionizing particle which have enough energy to further ionize.

$$\frac{d^2N}{dT\,dx} = \frac{1}{2} K z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2} \qquad I \ll T \ll W_{max}.$$

The factor $F(T)$ is about unit to $T << W_{max}$ and it depends on the spin of the incident particle $F(T) = 1 - \beta^2 \frac{T}{T_{max}}$ (for spin 0). The angle of emission $\cos\theta = \left(\frac{T_e}{P_e}\right)\left(\frac{P_{max}}{W_{max}}\right)$ with $P_e$, $T_e$ momentum & energy of the emitted photon, $P_{max}$ momentum of an electron emitted with maximum energy transfer $W_{max}$.

## Mean Energy loss distribution (Fluctuation in Energy loss)

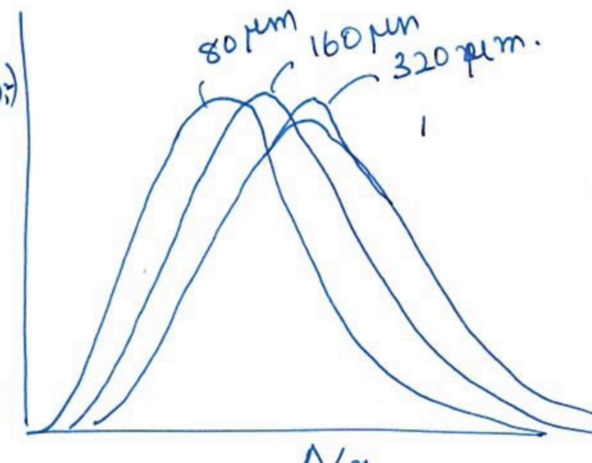
The ionization energy loss is in principle a stochastic process and one should note that the Bethe-Bloch equation describes only the average energy loss. If one shoots a single charged particle on a material, will observe that the energy loss follows a highly scewed distribution, where the most probable value is far lower than the average predicted by Bethe-Bloch equation.

Thus the detector with a moderate thickness $x$, the energy loss probability distribution $f(\Delta, \beta\gamma, x)$ is adequately described by highly scewed Landau (or landau-Vavilov) distribution

$$\Delta_p = \xi \left[ \ln \frac{2mc^2\beta^2\gamma^2}{I} + \ln\frac{\xi}{I} + j - \beta^2 - \delta \right]$$

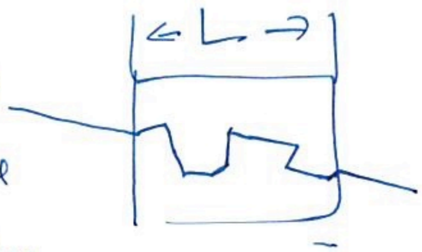
[we will discuss later during detector discuss-]

where $\xi = (K/2) \langle Z/A \rangle z^2 (x/\beta^2)$ MeV




## Multiple Colomb scattering:

charged particle with moderate mass traversing a medium, apart from the loss in energy to the electrons in the medium, interact also with the nuclei in the medium.

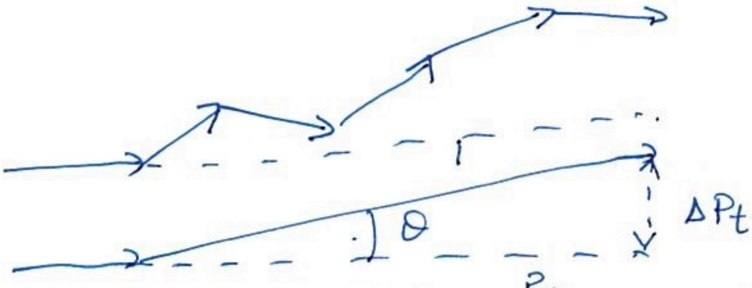


They cannot transfer sufficient energy to them, because the nuclei are much heavier than the electrons and the energy transfer, as we have seen, is inversely proportional to the target mass. However they do "feel" the columb field of the nuclei are a lot heavier than them they scatter transversely in the field of the nuclei. The elementary process for one such a scatter is well known and it is described by the Rutherford formula. However, one has to take in to account that a charged particle moving in a medium will undergo a large number of collisions in a process which is stochastic in nature and it is called Multiple Columb scattering.

Due to the multiple columb scattering the incoming particle undergoes ~~an~~ deflection trajectory, which becomes more significant because of th factor of Z.

After K collisions.

$$\langle \theta_K^2 \rangle = \sum_{m=1}^{K} \theta_m^2 = K \langle \theta^2 \rangle$$




For single collision we can utilize Rutherford scattering formula to estimate

$$\theta \simeq \frac{\Delta P_{\perp}}{P_{\parallel}} \approx \frac{\Delta P_{\perp}}{P} = \frac{2zZe^2}{b}\frac{1}{pv}$$

but, for many collision (>20) we can utilize statistical treatment. However, if the number of collision is less than 20 & greater than a, it becomes extremely difficult to calculate. starting from the single particle interaction

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{zZ\alpha}{\beta P}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

After passing through distance L and as a result of multiple scattering on nuclei, the incident particle will

will experience some typical displacements & diflection. These will have approximately Gaussian distributions, whose average are zero and sigmas are given as follows.

$$\theta_0 = \frac{14\,MeV}{\beta P} z \sqrt{\frac{L}{X_0}} \qquad r_0 = \frac{1}{\sqrt{3}} L \theta_0$$

The Gaussian shape describes well the bulk of scattering (98%) while the tail exhibit $\sim (1/\sin^4(\theta/2))$ dependence coming from the Rutherford formula. $X_0$ is characteristic of media called radiation length & we will discuss this little later.

## Range:

It is the main length that is the mean distance a particle travels within the material before it comes to stop. A theoretical approach to the determination of charged particle range utilizes stopping power expression. The value found is often called the continuous slowing down approximation (CSDA) range. Since the range is an average value, fluctuations can be ignored and losses are assumed to be continuous.

The range can be easily obtained integrating the stopping power curve

$$R = S(T_0) = \int_0^{T_0} \left( \frac{dE}{dx} \right)^{-1} dE \qquad T = \text{kinetic Energy}$$

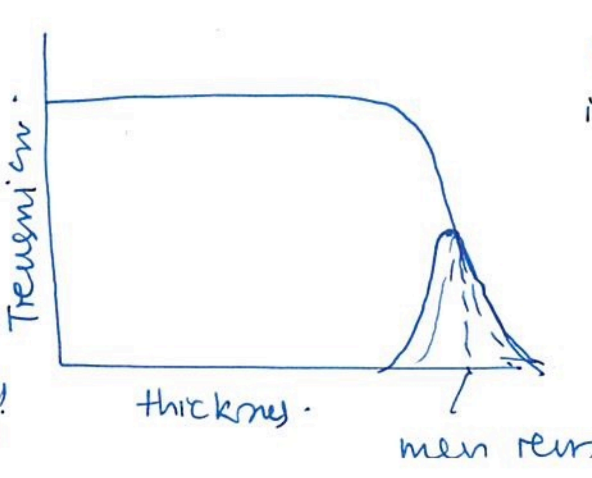
ignoring the fact that particle doesnot go straight, but it bounces around in a zig-zag path.

$$R = \int_E^0 \frac{dE}{(dE/dx)}$$

integrate over energy loss from initial energy E to zero

The smearing observed at the end of the curve comes from the fact that the energy loss in a statistical process.

→ "Range straggling"



$$R = \int_{E_0}^{0} \frac{dE}{dE/dx} \Rightarrow R \approx \int_{E_0}^{0} \frac{1}{nZ} \frac{m_e}{4\pi} \left(\frac{v}{z\alpha\hbar c}\right)^2 dE$$

Bethe $dE/dx$ ignoring slow varying logarithmic term.

$$R \propto \int_{E_0}^{0} \frac{v^2}{z^2} dE.$$

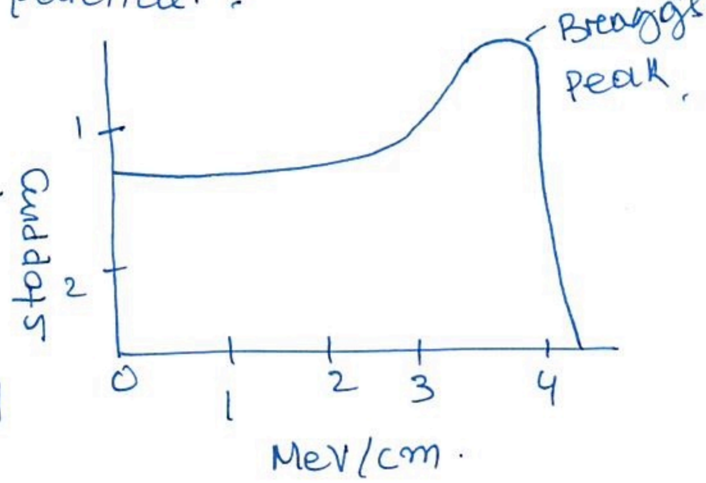
$$\Rightarrow R \propto \int_{E_0}^{0} \frac{E}{mz^2} dE \qquad E = \frac{1}{2}mv^2$$

When a particle is travelling through the matter, the last part of the path length is usually not accurately calculated when an analytical expression is used for the stopping power. Most such expressions are inaccurate at low energies. Because of this, a finite lower limit can be utilized. The Range is usually expressed in terms of $g/cm^2$, i.e. the range in cm multiplied by the density of the material.

Range Straggling: when heavy particle lose energy by ionization all of the particles do not come to the end of their energy at the range and stop after ~~trans~~ traversing the thickness of the material. & this variation is known as range straggling

The increase of ionization near the end of the path occurs because of the ionization loss is inversely proportional to the square of speed of incoming particle.

Thus the ionization density rises as the particle slows, of course, the ionization must drop to zero when the energy of the incident particles has been dissipated and near the end of the track, the charge is reduced through electron pickup and the curve falls off. And the Peak is called Bragg's peak & curve is called Braggs curve.



Electron & positron "$e^\pm$" energy loss in matter.
First of the electron (positron) mass is exact same as target electrons in material. Thus the calculation becomes more complicated as (a) they are spin ½ particles (b) small & identical masses of projectile & target, and (c) ~~identical~~ identical particle in final & initial particle. (c) the multiple scattering becomes severe, (d) QM implies a indistinguishable of particles.

Electron as incident particle gets an repulsion than the positron as incident particle, thus the energy loss is for electrons and positrons are different.

| Electron | | Positron |
| --- | --- | --- |
| small mass of electron → diffection becomes more important | ⟷ | same here. |
| incident and target electron have the same mass $m_e$ ($W_{max} = T/2$) | ⟷ | same here positron have same mass as $e^-$ |
| QM: after scattering, the incoming electron and the one from the ionization are indistinguishable | ⟷ | Positron is not indistinguishable |
| low energy the energy loss is smaller than $e^+$ | ⟷ | low energy positron have larger energy loss because of annihilation |
| | | At same β, the diffence is within 10%. |

Thus the energy of $e^\pm$ is ~~divided~~ constitute contribution from two distint categories.

$$\left.\frac{dE}{dx}\right|_{tot} = \left.\frac{dE}{dx}\right|_{rad} + \left.\frac{dE}{dx}\right|_{coll}$$

i.e, on top of the collisional energy loss electrons also loses energy through Bremsstrahlung. At low energies $e^\pm$

primarily lose the energy by ionization, and while ionization energy loss rise logarithimically with energy, radiative losses rises slowly, neally linearly, and dominates above the critical energy; where the energy loss by collisions is equal to the energy loss by bremsstrahlung: critical energy.

$$\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll} \quad \text{for } E = E_c.$$

Collision Energy loss for electron:

The stopping power differs somewhat for electron & positrons. due to the kinematics, spin, charge etc. & somewhat differ from the Bethe-Bloch calculatin for heavy particle. Thus Bethe-Bloch formula needs modificatin

$$-\frac{dE}{dx} = k\frac{2}{A}\frac{1}{\beta^2}\left[\ln\frac{m_e\beta^2c^2\gamma^2T}{2I^2} + F(\gamma) - \delta - \frac{2C}{Z}\right]$$

$T$ = kinetic energy of electron.

$W_{max} = \frac{1}{2}T$.

For electron:, large energy transfers to atomic electrons & the maximum energy transfer in a single collisin should be the entire kinetic energy ⇒ Molar cross section:

$$W_{max} = m_ec^2(\gamma - 1)$$

But the particle is identical so maximum is $W_{max}/2$

$$-\frac{dE}{dx} = \frac{1}{2}K\frac{2}{A}\frac{1}{\beta^2}\left[\ln\frac{m_ec^2\beta^2\gamma^2\{m_ec^2(\gamma-1)/2\}}{I^2} + (1-\beta^2) - \frac{2\gamma-1}{\gamma^2}\ln 2 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 - \delta - \frac{2C}{Z}\right]$$

for positron: the scattering can be described by the fairely complicated Bhaba cross-section. In this case there is no identical problem.

$$\left(-\frac{dE}{dx}\right) = \frac{1}{2}K\frac{2}{A}\frac{1}{\beta^2}\left[\ln\frac{m_ec^2\beta^2\gamma^2\{m_ec^2(\gamma-1)\}}{2I^2} + 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\gamma+1} + \frac{10}{(\gamma+1)^2} + \frac{4}{(\gamma+1)^3}\right) - \delta - \frac{2C}{Z}\right]$$

For both electrons & positron $F(\gamma)$ becomes a constant at very high incident energies. Comparison of electron & Heavy particle at $\beta = 1$.

$$\left.\frac{dE}{dx}\right|_c \propto \left[2 \ln \frac{2 m_e c^2}{I} + A \ln \gamma - B\right]$$

| | A | B |
|---|---|---|
| electron | 3 | 1.95 |
| Heavy particle | 4 | 2 |

## Radiation loss for $e^{\pm}$ : Bremsstrahlung

Relativistic charge particles, as they propagate through matter and wiggle due to multiple scattering on nuclei, experience acceleration/deacceleration, and therefore must be radiating the electromagnetic waves - emission of such photons is called breaking radiation, or bremsstrahlung. This is a QED process.

The only particle for which radiation loss is not negligible upto several hundreds of GeV is the electron. Already for muons the contribution is 40000 time lower $\sim (100\,\text{MeV}/0.5\,\text{MeV})^2$

Bremsstrahlung depends on the electric field seen by the incident electron: the screening of the electrons around the nucleus play an important role.

The quantum mechanical calculation was first performed by the Bethe- and Heitler cross section for bremsstrahlung emission. However we will simple take the approximated equation somewhat the final one.

$$-\frac{dE}{dx} = 4 \alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2}\right)^2 E \ln \frac{183}{Z^{1/3}}$$

$\propto \frac{E}{m^2}$ Relavent for electron & positrons in the range upto few hundred GeV/c

$$\Rightarrow -\frac{dE}{dx} = 4 \alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}}$$

$$\Rightarrow -\frac{dE}{dx} = \frac{E}{X_0} \quad \Rightarrow X_0 = \frac{A}{4 \alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

~~React~~ $X_0$ is the radiation length. This is the thickness of materia over which the charged particle's energy is reduce by a facto $e$:

$$\Rightarrow \quad \frac{dE}{dx} = -\frac{E}{X_0}$$

$$\Rightarrow \quad \boxed{E = E_0\, e^{-x/X_0}}$$

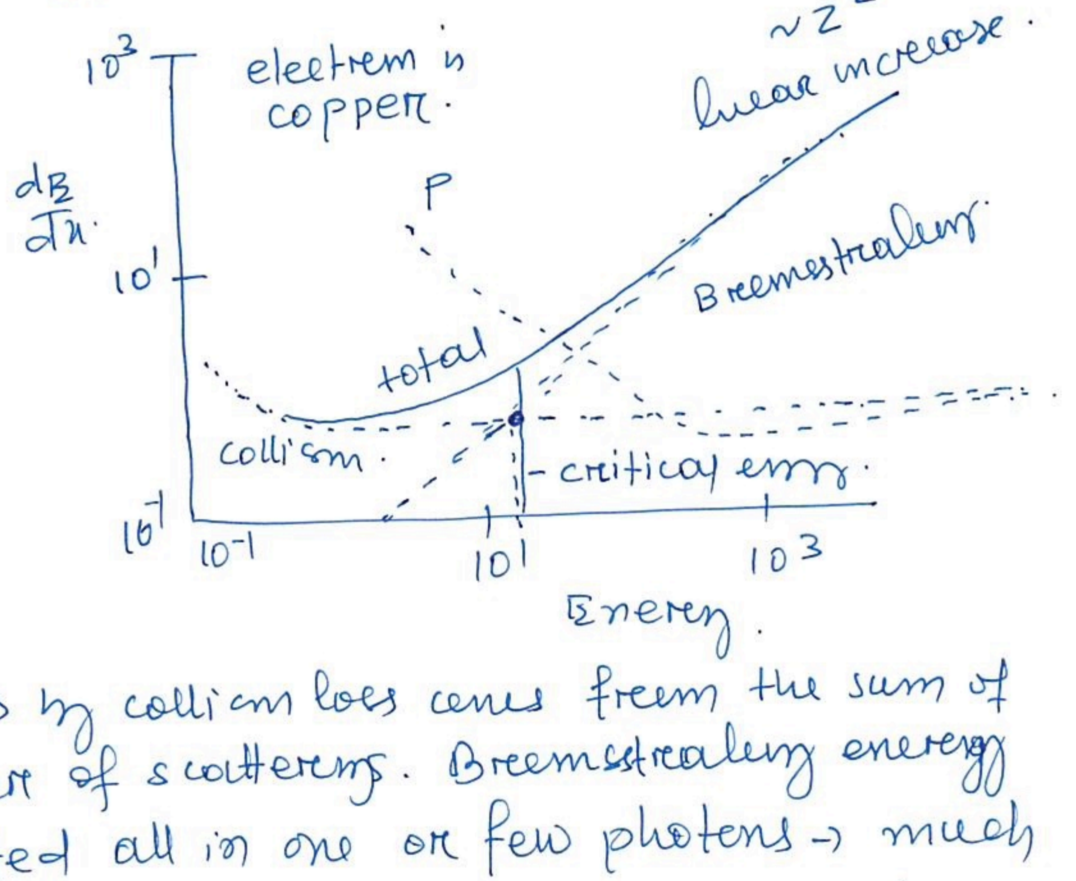
that is after passage of one $X_0$ electron has lost all but $(1/e)^{th}$ of its energy. i.e. 63%.

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln\frac{183}{Z^{1/3}}}$$

Critical Energy: The energy where Bethe-Bloch contribution is equal to the Bremsstrahlung contribution

$$\left[\frac{dE}{dx}(E_c)\right]_{Brem} = \left[\frac{dE}{dx}(E_c)\right]_{ion}$$

$$E_c^{Gas} = \frac{710\ \text{MeV}}{Z+0.92} \qquad E_c^{sol/liq} = \frac{610\ \text{MeV}}{Z+1.24}$$




Energy loss by collision loss comes from the sum of large number of scatterings. Bremsstrahlung energy can be emitted all in one or few photons → much larger fluctuation the energy in e± electron beams

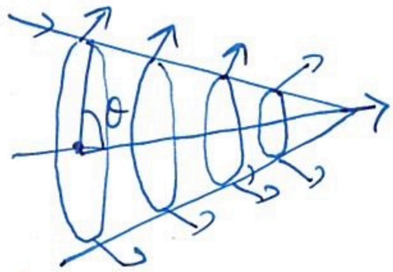
Note: For a compound material the radiation length is calculated as

$$\frac{1}{X_0} = w_1\left(\frac{1}{X_0}\right)_1 + w_2\left(\frac{1}{X_0}\right)_2 + \cdots$$

## Cherenkov Radiation:

charged particles undergoes another form of energy loss in the media if they travel faster than ~~light~~ the local speed of light ($v = c/n$ for medium with refractive index $n$) they emit a radiation in characteristic angle $\theta_c$ called ~~cheren~~ Cerenkov Radiation.

As a charge particle traverse matter, it produces a wake of polarized molecules along its path. As the molecules get polarised and then get depolarized they emit radiation in all direction - Cerenkov Radiation




- → charge particle polarize medium generating an electrical dipole varying in time
- → Every point in trajectory emits a spherical EM wave, waves constructively interfere.

The direction of radiation becomes coherent and results in significant amount of light produced. The direction in which waves would add up coherently can be easily constructed using Huygens ~~teqte~~ technique of circular front. A charge particle mass M and velocity $\beta = v/c$ travels in a medium with refractive index $n$

$$n^2 = \epsilon_1 = \left(\frac{c}{c_m}\right)^2$$

$\epsilon_1$ = real part of the medium of dielectric coeffnt.

if $v > c_m$ ~~namely $\beta >$~~

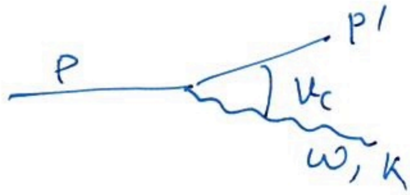
namely $\beta > \beta_{th} = \frac{1}{n}$

$c_m$ = speed of light in medium $\frac{c}{n}$

real photons are emitted.

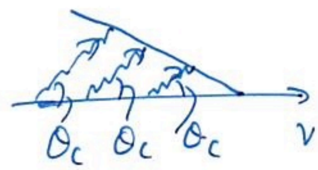
photons are "soft" $|P| \approx |P'|$

$\omega << \gamma Mc^2$



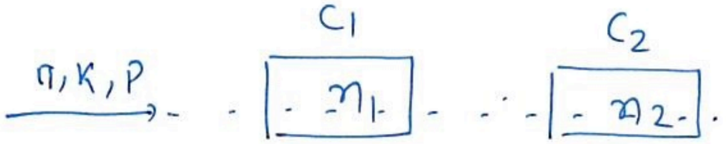

Thus the charateristic emission angle

$$\cos\theta_c = \frac{\omega}{k \cdot v} = \frac{1}{n\beta}$$




$\theta_c$ = cerenkov angle.

A detector can be construcected using the cerenkov threeshold,: use different matereials (refractive indexed) such that particles of different masses at equal momentum P, produces cherenkov readiation or not.




choose $n_1$ & $n_2$ such that for a given $P$ ($\beta = P/E$)

$\beta_\pi > \frac{1}{n_1}$ $\quad \beta_K, \beta_P < \frac{1}{n_1}$

$\beta_\pi, \beta_K > \frac{1}{n_2}$ $\quad \beta_P < \frac{1}{n_2}$

$\Rightarrow$ light in $C_1$ and $C_2$ $\rightarrow$ Pion ($\pi$)

light in $C_1$ and not $C_2$ $\rightarrow$ kaon ($K$)

light in not $C_1$ & $C_2$ $\rightarrow$ Proton ($P$)

The number of photon can be calculated using followin formula:

wave lngth

$$\frac{dN}{dx\,d\lambda} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2(\theta_c)$$

integreate over total sensitive reange of a good detector.

$$\frac{dN}{dx} = \int d\lambda \frac{d^2N}{dx\,d\lambda} = 750\, z^2 \sin^2\theta_c \text{ photons/cm}.$$

Energy

$$\frac{d^2N}{dE\,dx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin\theta_c$$

one can see that below threeshored minimum velocity $v = \frac{1}{n}$ there is no readiatin.

Passage of photons through matter:

① photons are electromagnetic radiation with zero mass, zero charge, velocity c.

② Electrical neutral thus no colombic interaction

③ photons travel considerable distances before undergoing the energy loss via electromagnetic interactions. There three very distint processes of photon interaction with media:

$$\Rightarrow \gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

$$\Rightarrow \gamma + e^{-} \rightarrow \gamma + e^{-}$$

$$\Rightarrow \gamma + \text{nucleus} \rightarrow e^{+} + e^{-} + \text{nucleus}.$$

④ Eventually photon deposits energy, but it is far more penetrating than charged particles with similar energies.

photon interacts in matter basically in three principle as mensioned in ③ ⓐ photo electric effect, ⓑ compton scattering, and ⓒ pair production

$$\sigma(E) = \sigma_{PE} + \sigma_{CS} + \sigma_{pair}.$$

photon interaction probabilities ocan be estimated from the linear attenuation coefficient, $\mu$. The exponential absorption of a beam of photons ~~may~~ ~~by~~ is described by.

$$N(x) = N_0 e^{-\mu x}$$

The coeffient $\mu$ depends on photon energy and on the material being traversed. ~~Mass cof coeffint~~

The mass attenuation coeffient $\mu/\rho$

$$\Rightarrow N = N_0 e^{-\left(\frac{\mu}{\rho}\right)(\rho x)}$$

Here $N(x)$ is the number of photons after distance $x$, with $N_0$ the initial number of photons at $x=0$, $\mu$ is the linear attenuation coefficient. If each atomic absorbing centre is given a cross-section $\sigma$ and they have a number density $n$ then $\mu = n\sigma$. Sometime the mass attenuation coefficient is used $\mu/\rho$.

Mechanism of energy loss: ~~phot~~

photo electric effect:

This is the process of photo absorption leading to ionization of an atom. If photon energy is sufficient, an electron from a shell will eject out.

→ The photoelectric effect dominates at low energy

→ The photoelectric effects leads to the emission of electrons from matter after it is illuminated by light of a sufficiently high frequency.

→ Since light is quantised into photons of energy $hf$ the maximum energy an electron can receive in any one interaction is $hf$, therefore no matter how intense the radiation is, if it is not of sufficient frequency no ~~photon will be emitted~~. photoelectron will be emitted.

→ Typically, in metal, there are free electrons in the electron sea, and bound

bound electrons in atomic orbital. As a result of need to conserve both energy and momentum in any given reaction, a free electron cannot wholly absorb a photon instead it is the bound atomic electrons which partake in the photoelectric effect; transferring excess energy and momentum to the nucleus.

- The $n = 1$ or K-shell electrons are tightly bound, and hence, are well coupled to the atomic nucleus making it easier for them to transfer excess momentum to the nucleus during a photoelectric interaction. For this reason, the cross-section for the emission of K-shell photoelectrons is bigger than that of L-shell photo-electrons and so forth.

The minimum energy an incident photon must have to liberate an electron from the surface of a particular material is the work function of that material.

$$E_{min} = \phi = hf_0$$

Any access energy an incident photon might have result in KE of liberated photoelectron:

$$E_{photon} = \phi + E_{kin}$$

$$\Rightarrow \quad E_{kin} = h\nu - E_0$$

The crosssection due to the photoelectric effect is a function of the atomic number of the target atom Z and the energy of incident photon E,

$$\sigma_{PE} \propto \frac{Z^n}{E^3} \qquad n \approx 4, 5$$

compton scattering:

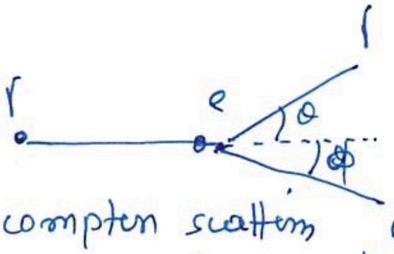
This is the process of scattering of photons on atomic electrons. It is most often the predominent interaction mechanism for gamma-ray energies typically of radio isotope sources.

- compton scattering dominates at mid energies
- compton scattering is the scattering of photon by atomic electrons
- compairing the photon before and after compton scattering reveals that they undergo a wave-length shift, which turns out to be independent of both the initial wavelength and indeed the material they scattered off.
- The photon transfer energy to electron, which is then known as recoil electron or compten electron
- All angle of scattering are possible
- Energy transfer to the electron can vary from zero to a large fraction of gamma-ray energy.
- The compton process is most important for energy obsorption for soft tissues in the range from 100 keV to 10 MeV.

compton shift formula:

conservation of four vectors in lab frame

$$P_{\gamma,i}^{\mu} + P_{e,i} = P_{\gamma,f} + P_{e,f}$$



compton scatterin in lab frame before & after.

rearrange in a way that it ~~enorm~~

$$\Rightarrow (P_{\gamma,i} - P_{\gamma,f})^2 = (P_{e,f} - P_{e,i})^2$$

$$\Rightarrow P_{\gamma,i}^2 + P_{\gamma,f}^2 - 2P_{\gamma,i} \cdot P_{\gamma,f} = P_{e,f}^2 + P_{e,i}^2 - 2P_{e,i} \cdot P_{e,f}$$

$$\Rightarrow 2P_{\gamma,i} \cdot P_{\gamma,f} = 2P_{e,i} \cdot P_{e,f} - 2m_e^2c^2 \qquad P_\gamma^2 = 0 \quad P_e^2 = m_e^2c^2$$

$$\Rightarrow P_{\gamma,i} \cdot P_{\gamma,f} = P_{e,i} \cdot P_{e,f} - m_e^2c^2$$

This yeilds the angle between the initial photon's path and the final photon's path, $\theta$

$$\frac{E_{\gamma,i} E_{\gamma,f}}{c^2} - p_{\gamma,i} \cdot p_{\gamma,f} \cos\theta = \frac{E_{e,i} E_{e,f}}{c^2} - p_{e,i}\, p_{e,f} \cos\phi - m_e^2c^2$$

since the initial electron is in statenary $P_{e,i} = 0$ & $E_{e,i}$ is the reest mass enerergy of electrum $m_ec^2$

$$\Rightarrow E_{\gamma,i} E_{\gamma,f} (1 - \cos\theta) = m_ec^2 E_{e,f} - m_e^2c^4 \qquad \begin{array}{c} E = pc \\ \text{fore photon} \end{array}$$

Now look into the consereration of the enerergy'

$$E_{\gamma,i} E_{\gamma,f} (1 - \cos\theta) = m_ec^2 \left[ E_{\gamma,i} + m_ec^2 - E_{\gamma,f} \right] - m_e^2c^4$$

$$\Rightarrow (1 - \cos\theta) = m_ec^2 \left[ \frac{1}{E_{\gamma,f}} - \frac{1}{E_{\gamma,i}} \right]$$

Finally converet the photon enerergy $E_\gamma$ into wavelegths using $E_\gamma = hf = hc/\lambda$

$$\Rightarrow (1 - \cos\theta) = m_ec^2 \left( \frac{\lambda_f}{hc} - \frac{\lambda_i}{hc} \right)$$

$$\Rightarrow \Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_ec}(1 - \cos\theta)$$

The compton shift foremula is ucually simplifeed with the compton wavelength $\lambda_c$

$$\lambda_c = \frac{h}{m_ec}. \qquad \Rightarrow \Delta\lambda = \lambda_c (1 - \cos\theta)$$

similarly one can easily find out the compton scattering energies. The energy of scattered photon $hf'$ and compton electron $E_e$ can be written as

$$hf' = hf \frac{1}{1+\alpha(1-\cos\theta)}$$ $f' = f_f$, $f = f_i$

$$E_e = hf_i \frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$ $\alpha = \frac{hf_i}{m_e c^2}$

$m_e c^2 = 0.511 MeV$ rest mass of electron.

The maximum energy transfer to recoil electron
- angle of electron recoil is foreward at 0°, $\phi = 0°$

→ the scattered photon will be scattered straight back $\theta = 180°$ : $\cos 180° = -1$

$$\Rightarrow E_e^{max} = hf_i \frac{2\alpha}{1+2\alpha}$$

$$(hf_f)^{min} = hf_i \frac{1}{1+2\alpha}$$

let's calculate for photon energy 5.11 keV & 5.11 MeV.

$$\alpha = \frac{5.11\ keV}{0.511\ MeV} = 0.01 \Rightarrow E_e^{max} = 5.11\ keV\ 2 \times \frac{0.1}{1.02} = $$

$$= 0.1\ keV.$$

$$\Rightarrow hf_f = 5.11\ keV \frac{1}{1.02} = 5.01 keV$$

Energy transfered is only 2%.

for 5.11 MeV. $\alpha = \frac{5.11\ MeV}{0.511\ MeV} = 10$

$$E_e^{max} = 5.11\ MeV \left(2 * \frac{10}{21}\right) = 4.87\ MeV.$$

$$hf_f = 5.11\ MeV \frac{1}{21} = 0.24\ MeV$$

energy transfer is 95%.

Higher the energy the transfer is really large.

The crossection is for compten scattering is estimated by Klein-Nishima formula.

$$\sigma_{cs} \propto \frac{\ln E}{E}$$

and the differenitial cross-section is given by Klein-Nishima distribution function

$$\frac{d\sigma_c}{d\Omega} = \frac{r_e^2}{2} \frac{1}{(1+E(1-\cos\theta))^2}\left[1+\cos\theta \frac{E(1-\cos\theta)^2}{1+E(1-\cos\theta)}\right]$$

$$E = \frac{E_\gamma}{m_e c^2}$$

Pair produection: if the photone enters matter with an energy in excess of 1.022 MeV, it may interact by a process called pair productin. photo passing near the nucleus of an atom, is subjected to strong field effects from the nucleus and may disappear as a photon and reappear as a positive and negative electron pair. The two electrons produced, $e^+$ & $e^-$, are not scattered orbital electrons, rather they are newly created, in the energy/mass conversion of disappearing photons.

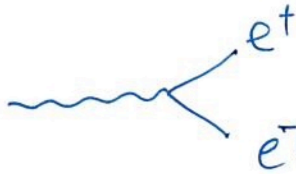
→ pair productin dominates at high energies

→ The process is effectively gamma-ray absoption by the process & productin of $e^+ e^-$ pair.

→ Pair productin canot occur until the threshold energy of two electron masses is reached

$$E_{th} = 2 m_e c^2 = 2 \times 0.511 \text{ MeV} = 1.022 \text{ MeV}$$

The pair productin cannot take place in free space, because it is impossible to conserve momentum in free space pair productin conserevation of momentem.




free space pair productn.

$$P_\gamma = \frac{E_\gamma}{c} = 2 p_e \cos\theta$$

conserevatin of enerergy

$$E_\gamma = 2 E_e$$

equatin $E_\gamma$ from both the equati $E_e = p_e \cos\theta$.

The invariant applied to electreen

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

$$\Rightarrow \quad p_e^2 c^2 \cos^2\theta = p_e^2 c^2 + m_e^2 c^4$$

The biggest value that the left side can take is $p_e^2 c^2$ when $\cos^2\theta = 1$, so unless $m_e \to 0$ momentem cannot be ever consereveved for pair productin in free space.

Thus the kinetic Enerergy may utilize little bit in the medium, hence the kinetic energy of electrons produced will be difference between the energy of the incoming photon and the energy equivalent of two electreen masses

$$E_{e^+} + E_{e^-} = hf - 1.022\text{ MeV}$$

~~Rather~~ This like bremsstrahlung, pair productin mus takes place in the vicinity of an atomic nucleus to absorb the recoil momenteem.

The positrons nearly always annihilate to two photons istead of simply reversing the productin process ($e^+e^- - \gamma$) and anhilating to one photon. This is because this is also forbidden in free space, since it is simplify the reverse of the forbidden free space pair productin

Like pair production, it can take place in the E-field of an ~~electron~~ atom, but this rarely happens because of the ease with which free space annihilation to two photons can occur.

pair production cross section: This raises above the threshold but eventually saturates at large $E_\gamma$ because of the screening effects of the nuclear charge.

$$\sigma_{pair} \propto \frac{Z^2 \alpha^3}{m_e^2}$$

An approximated calculation shows

$$\sigma_{pair} = 4Z^2 \alpha r_e^2 \left( \frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right)$$

$$\approx 4Z^2 \alpha r_e^2 \left( \frac{7}{9} \ln \frac{183}{Z^{1/3}} \right)$$

$$\approx \frac{7}{9} \frac{A}{N_A} X_0$$

Absorption coeffnt $\mu = n\sigma$  $n$ is particle density.

$$\Rightarrow \quad \mu_{pair} = \rho \frac{N_A}{A} \sigma_{pair}$$

$$\approx \frac{7}{9} \frac{1}{X_0}$$

Photo electric effect — Dominates below ~~$E_\gamma \approx$~~ $E_\gamma \approx$ few 100 keV

Compton scattering — dominating at $E_\gamma \approx 1$ MeV

Pair production — dominates $\sim 2$ MeV