Assignment-3. Date-01/09/2023.

No need to submit.

1. Solve the differential equation by using Frobenius method
$$x^4 y''(x) + 2x^2(1+x)y'(x) + y(x) = 0 \quad \text{at } x=\infty.$$

→ Set $x = 1/t$ → transform the given eq^n → Solve the transformed eq^n around zero-($t=0$) → replace t by $1/x$ in the final solution → Also known as Solution in descending power of x.

2. Solve the following DE: $x^4 y''(x) + 2x^3 y'(x) - y(x) = 0$ in descending powers of x.
$$y(x) = A\cosh(1/x) + B\sinh(1/x)$$

3. Solve the following D.E. by the Frobenius method
$$xy''(x) - (1-2x)y'(x) - (1-x)y(x) = 0$$
and show that $x=0$ is an apparent singularity of the DE.
$$y(x) = (A+Bx^2)e^x.$$

4. Solve the following D.E.s by the Frobenius method:

a) $2xy''(x) + (4x+1)y'(x) + (2x+1)y(x) = 0$
$$y(x) = e^{-x}(A+Bx^{1/2})$$

b) $xy''(x) + (1+2x)y'(x) + (1+x)y(x) = 0$
$$y(x) = e^{-x}[A + B\ln x]$$

c) $x^2(x^3-1)y''(x) - (x^2+3)xy'(x) + (x^2+3)y(x) = 0$
$$y(x) = Ax + B\left[x\ln x + \frac{1}{x} - \frac{1}{4x^3}\right]$$

d) $x(1+x)y''(x) - (1-3x)y'(x) + y(x) = 0$
$$\begin{cases} y_1(x) = 1\cdot 2\cdot x^2 - 2\cdot 3\cdot x^3 + 3\cdot 4\cdot x^4 - \cdots \\ y_2(x) = y_1\ln x - \left[1 + x - \left(1+\frac{1}{1}+\frac{1}{2}\right)1\cdot 2\cdot x^2 + \left(1+\frac{1}{2}+\frac{1}{3}\right)2\cdot 3\cdot x^3 \right. \\ \left. \quad - \left(1+\frac{1}{3}+\frac{1}{4}\right)3\cdot 4\cdot x^4 + \cdots\right] \end{cases}$$

5) Consider a differential equation

$$(1-x^2)y'' - 2xy' + \left\{(l+1)l - \frac{m^2}{1-x^2}\right\} y = 0$$

where m & l are constants. Show that $y = (1-x^2)^{m/2} u$ transforms the above equation to

$$(1-x^2)u'' - 2(m+1)xu' + [l(l+1) - m(m+1)]u = 0$$

→ Long straightforward calculation. Must do!

6) Consider a rectangular plate of length a and width b. Its edges at $x=0$ and $x=a$ are insulated. The edge at $y=b$ is kept at zero temperature, and the edge $y=0$ has a temperature distribution given by $T = \frac{10}{a}(a-x)$. Determine the steady state temperature distribution within the plate.

→ Temperature $T(x, y, t)$ follows $\nabla^2 T = K \frac{\partial T}{\partial t}$. In $d=2$, and steady state $\frac{\partial T}{\partial t} = 0$, $\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$. Apply separation of variables technique with B.Cs mentioned. Complete solution is

$$T(x,y) = \frac{5}{b}(b-y) + \frac{40}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos\frac{(2n-1)\pi x}{a} \sinh\frac{(2n-1)\pi}{a}(b-y)}{(2n-1)^2 \sinh\frac{(2n-1)\pi b}{a}}$$