IDC102:Hands-on Electronics
Lecture - 5

Inductors

IISER

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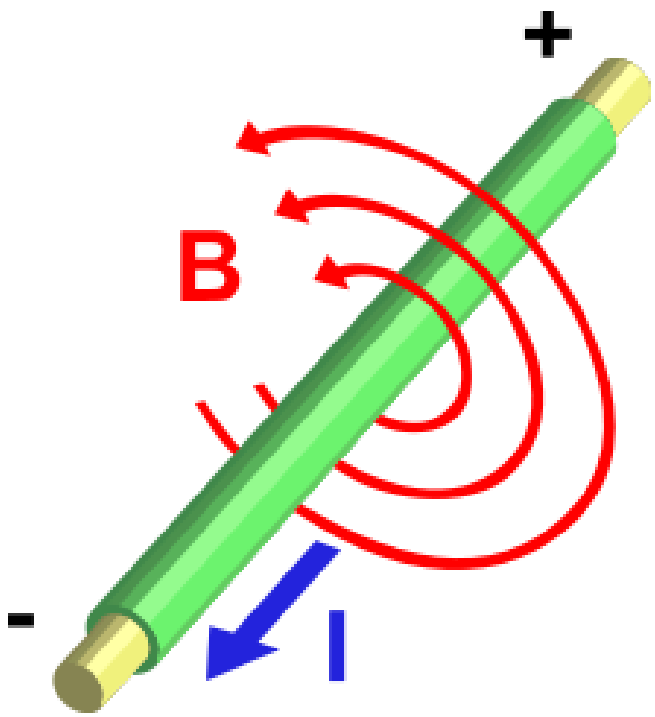
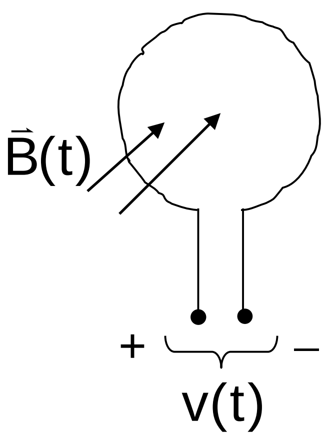
Induction

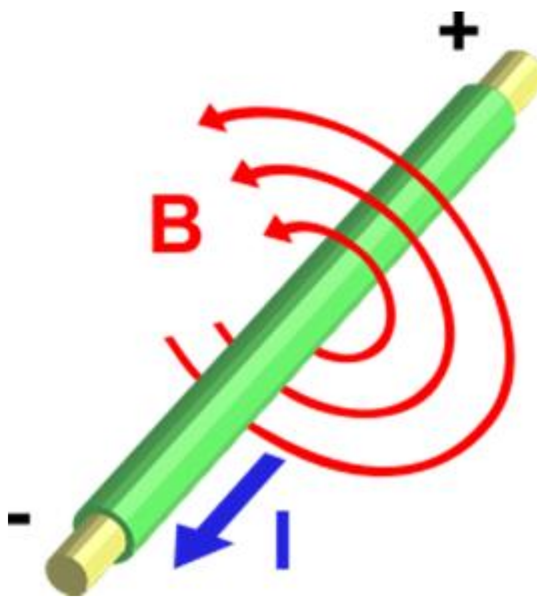
- The flow of current induces a magnetic field (Ampere's Law)
- A change in magnetic field through a loop of wire induces a voltage (Faraday's Law)

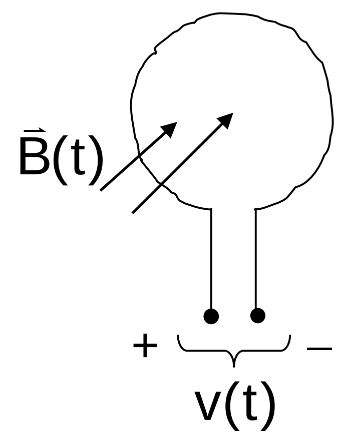


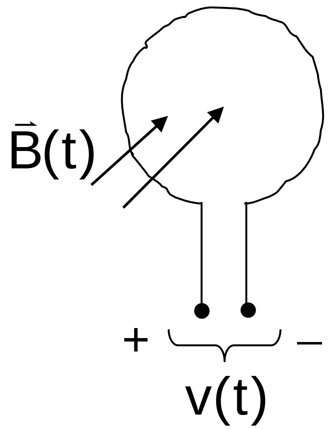


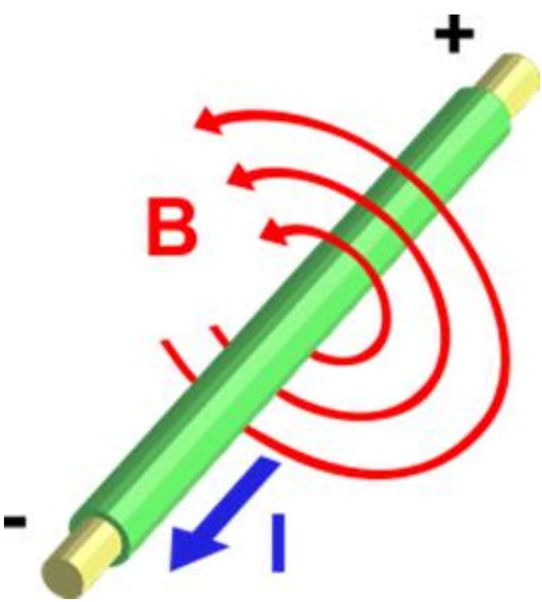
Induction

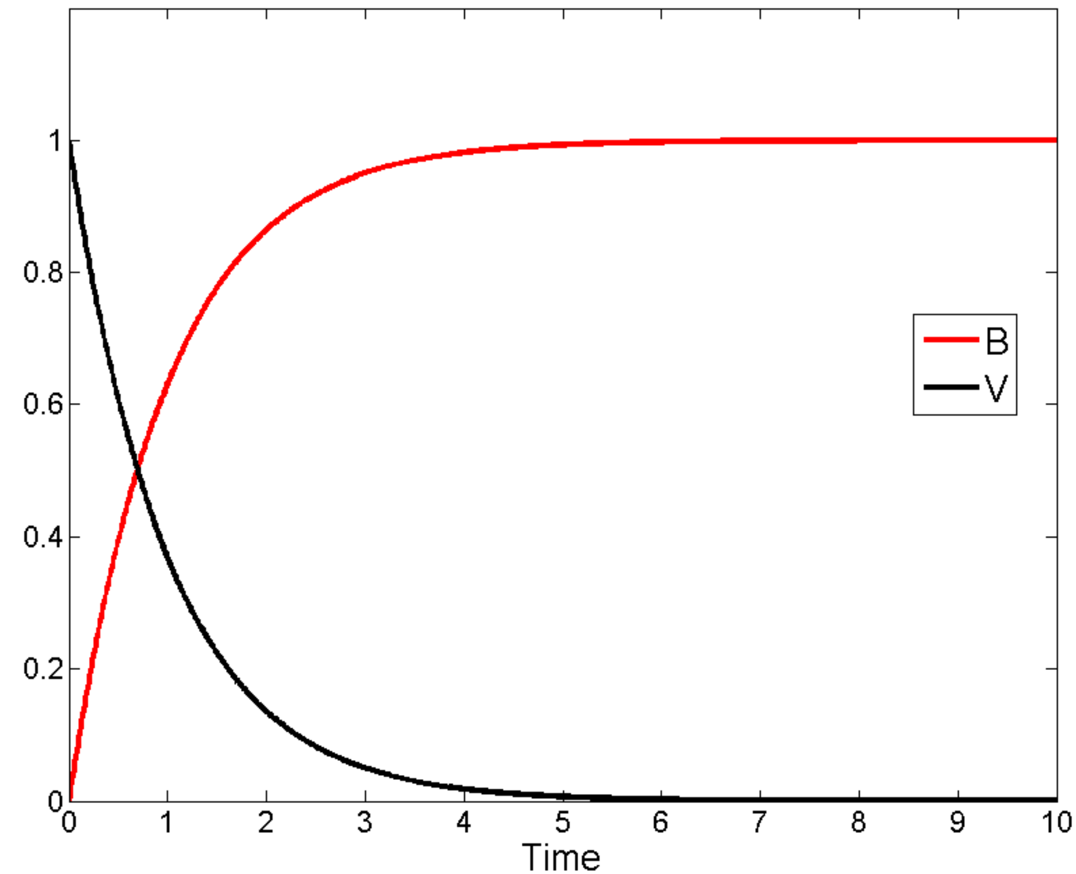
- When we connect a voltage source to a wire, current clearly takes a little time to get moving
- Thus, the magnetic field builds to some maximum strength over time









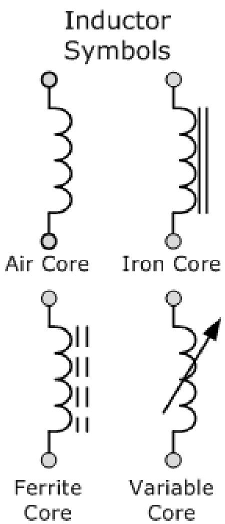


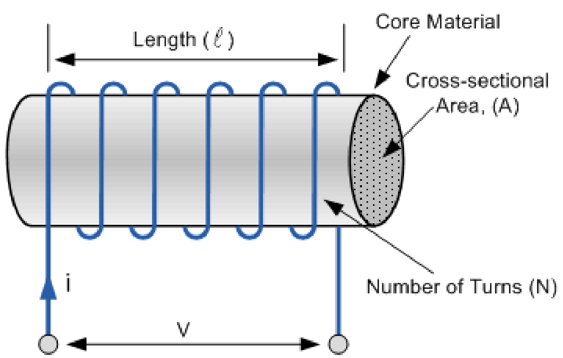
Current in a wire ~~causes~~ induces a voltage in any nearby circuit

Inductors

- Two fundamental principles of electromagnetics:
  1. Moving electrons create a magnetic field
  2. Change in a magnetic field causes electrons to move… it <u>induces a current</u>
- An inductor is a coil of wire through which electrons move, and energy is stored in the resulting magnetic field

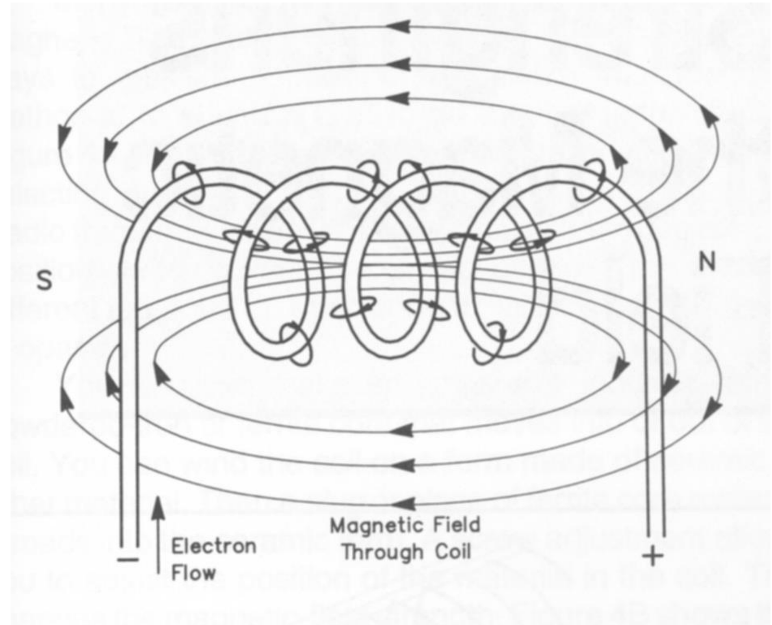
- Stores energy in an magnetic field created by the electric current flowing through it.
  - Inductor opposes change in current flowing through it.
    - Current through an inductor is continuous; voltage can be discontinuous.





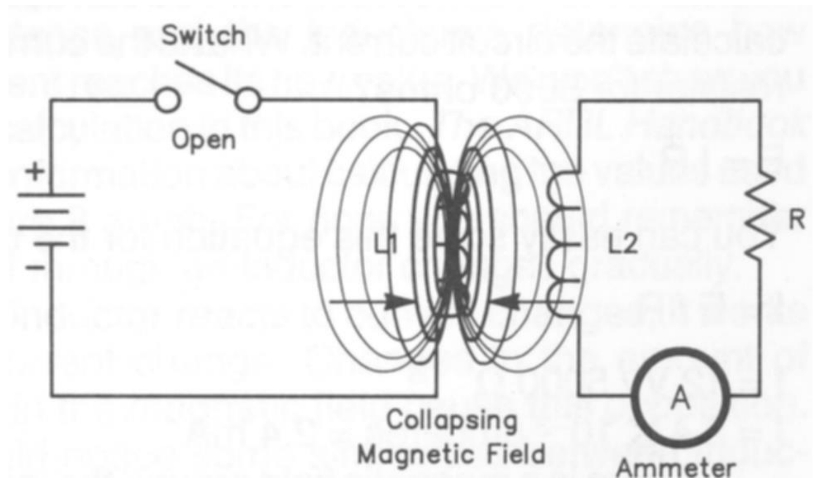
Inductors

- Like capacitors, inductors temporarily store energy
- Unlike capacitors:
  - Inductors store energy in a magnetic field, not an electric field
  - When the source of electrons is removed, the magnetic field collapses immediately



Transformer

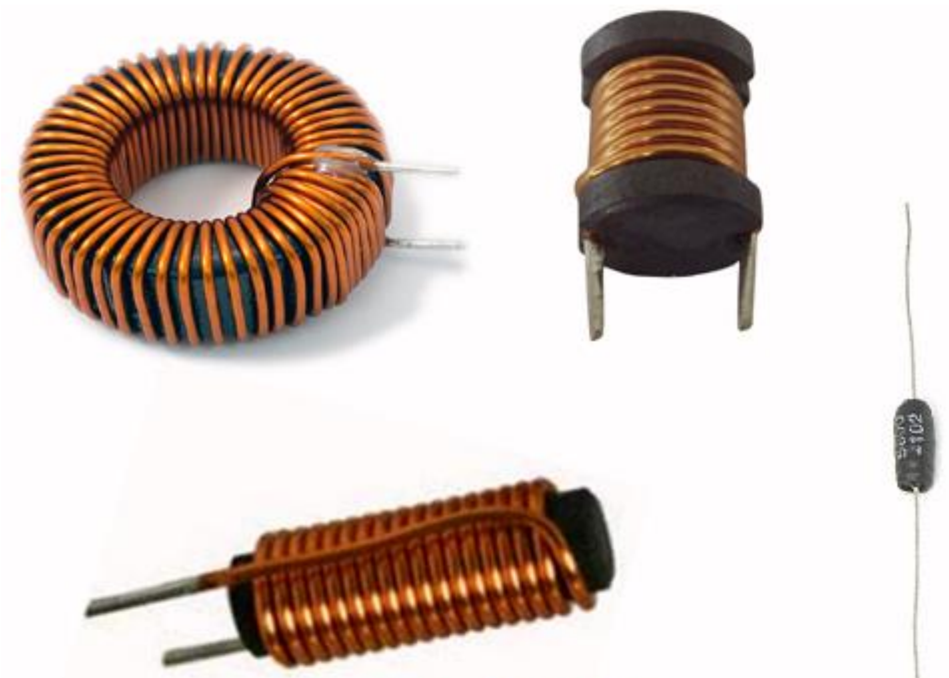
- Because the magnetic field surrounding an inductor can cut across another inductor in close proximity, the changing magnetic field in one can cause current to flow in the other.
- This is the basis of transformers



Inductors & Inductance

Inductance (L) refers to the capacity of a coil to develop a voltage in it as the result of a changing magnetic flux. It is customary to use the symbol **L** for inductance, in honour of the physicist Heinrich **L**enz.

In the metric system, the unit for inductance is the **henry (*H*)**, named in honor of Joseph Henry. The factors that determine the inductance include **the number of loops** of wire in the coil, the diameter or **area of the coil**, the **length of the coil** and the **permeability of the core material**.

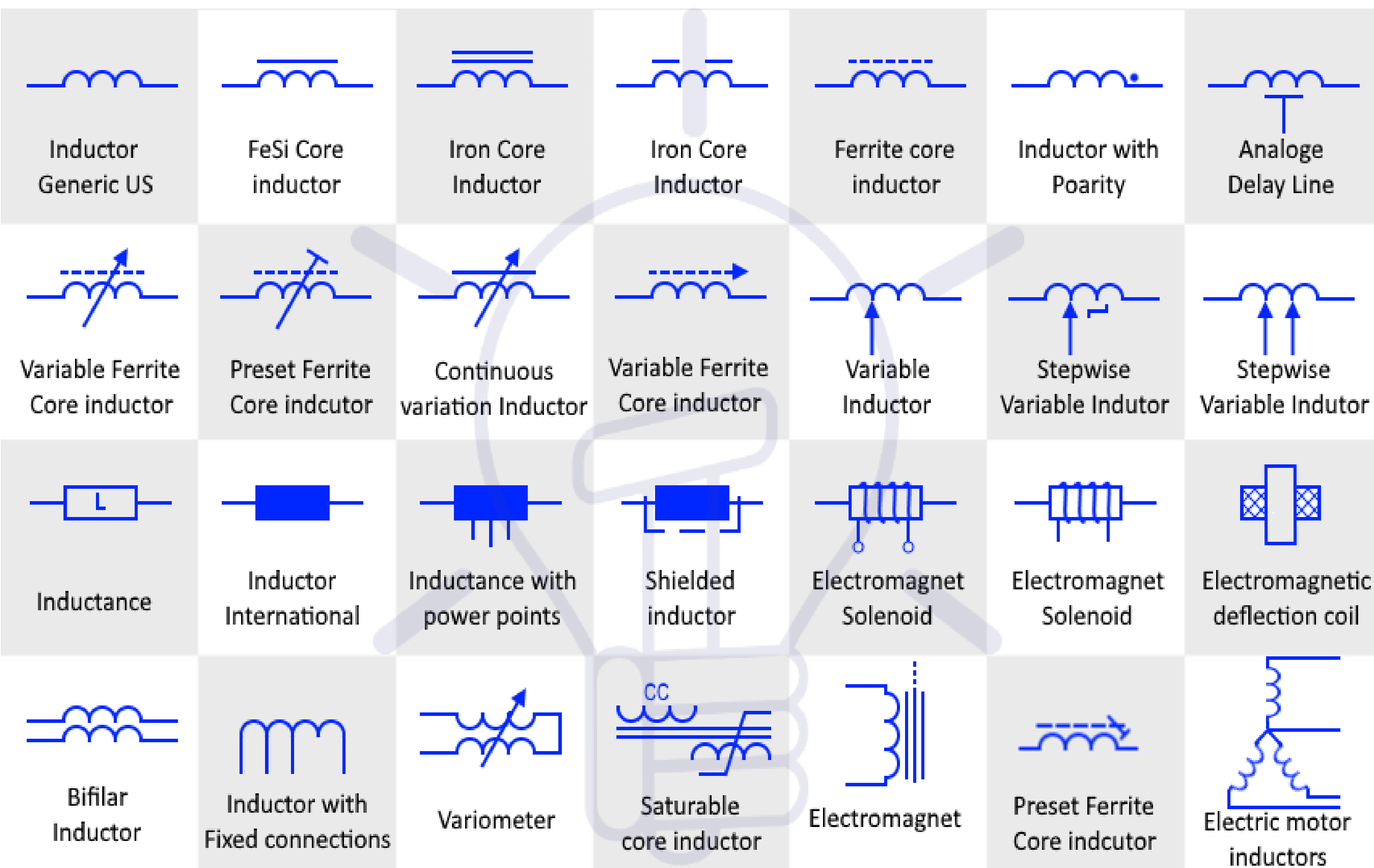


Practical Inductors

Inductor

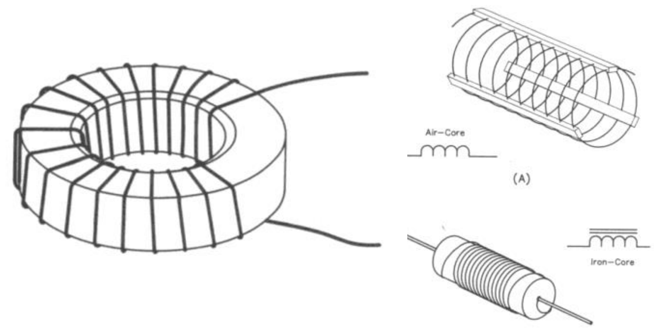
Inductor	Fixed	Variable	Pre-set	Shape
Air Core				
Iron Core				
Ferrite Core				

Inductor Symbols

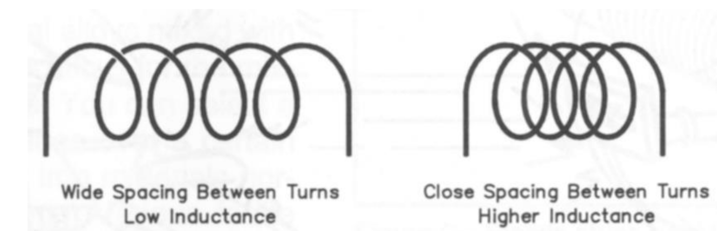


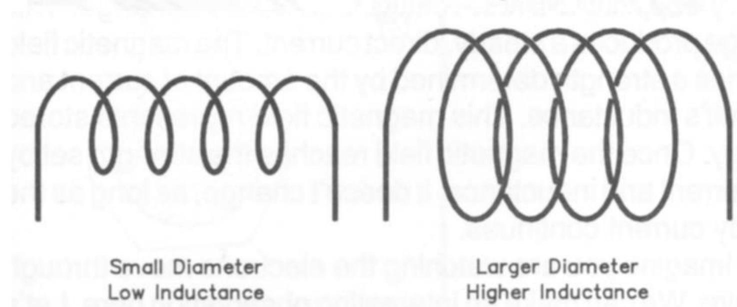
Inductor

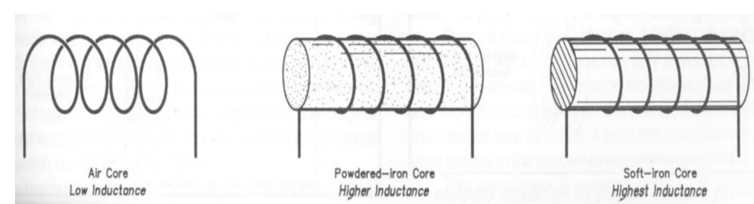
- Inductors are simply coils of wire
  - Can be air wound (just air in the middle of the coil)
  - Can be wound around a permeable material (material that concentrates magnetic fields)
  - Can be wound around a circular form (toroid)



- The amount of inductance is influenced by a number of factors:
  - Number of coil turns
  - Diameter of coil
  - Spacing between turns
  - Size of the wire used
  - Type of material inside the coil







Inductor Performance: With DC Currents

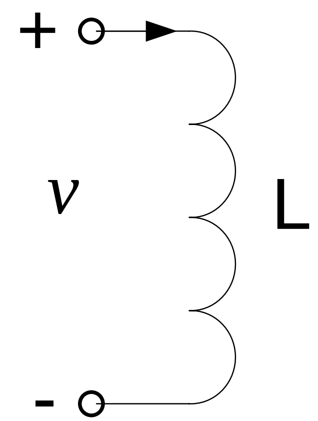
- When a DC current is first applied to an inductor, the increasing magnetic field opposes current flow - current flow is at a minimum
- Eventually (milliseconds) the magnetic field achieves its maximum and current flows, maintaining the field
- As soon as the current source is removed, the magnetic field begins to collapse and creates a rush of current in the other direction, sometimes at very high voltage

Inductor Performance: With AC Currents

- When AC current is applied to an inductor, during the first half of the cycle, the magnetic field builds as if it were a DC current
- During the next half of the cycle, the current is reversed and the magnetic field decreases the reverse polarity in step with the changing current
- These forces can work against each other resulting in a lower current flow

An inductor is a passive element that stores energy in its magnetic field. Generally. An inductor consists of a coil of conducting wire wound around a core. For the inductor

$$v(t) = L\frac{di(t)}{dt}$$



where <u>*L* is the inductance in henrys (H),</u> and 1 H = 1 volt second/ampere.

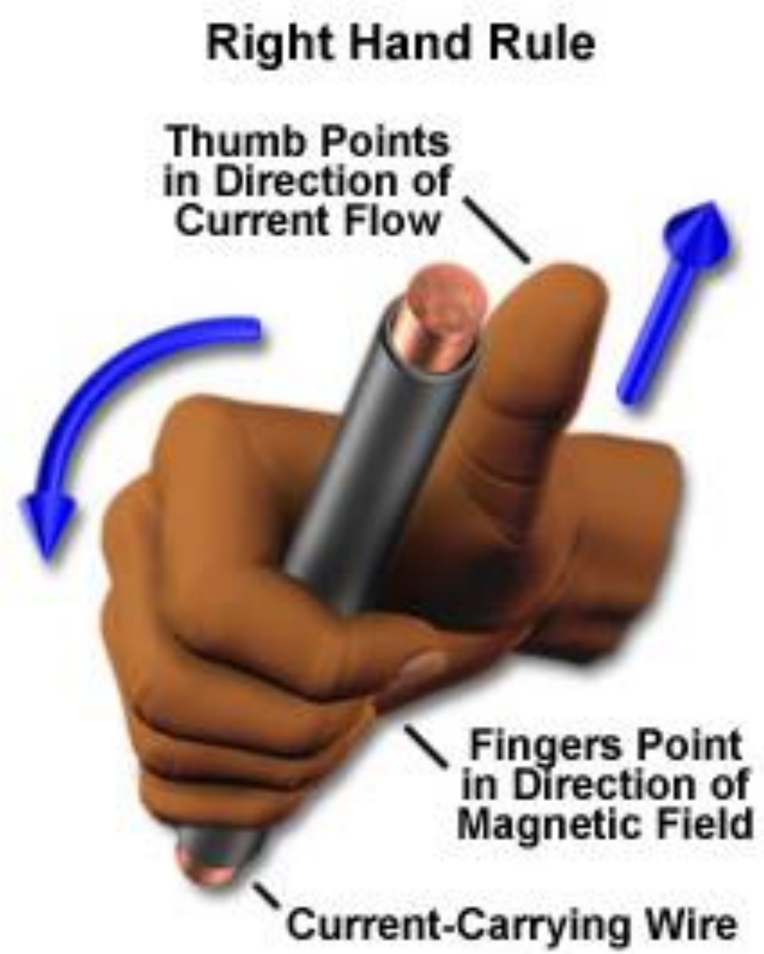
Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.

What happens in a wire?

Unfortunately, even bare wire has inductance.

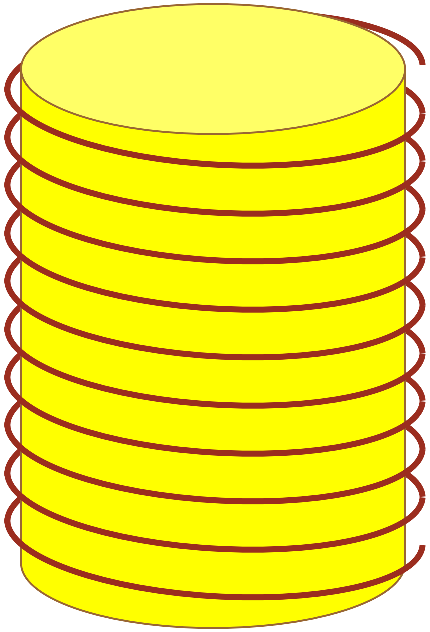
$$L = \ell\left[\ln\left(4\frac{\ell}{d}\right)-1\right]\left(2x10^{-7}\right)H$$

d is the diameter of the wire in meters.



Material Inside Inductor

- For capacitors, we gained significantly by putting materials inside
- Can we gain any benefit by putting something in inductors?
- It gives you something to wrap around

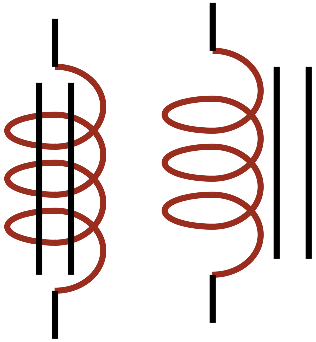


Can materials increase the inductance?

- Most materials have negligible magnetic properties
- A few materials, like iron are *ferromagnetic*
  - They can enhance inductance enormously
- Many inductors (and similar devices, like transformers) have iron cores

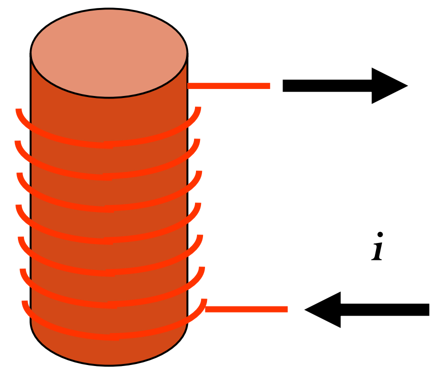
Symbol for iron core inductor:

- We won't make this distinction



Self-inductance of a solenoid

- Solenoid of cross-sectional area $\boldsymbol{A}$, length $\boldsymbol{l}$, total number of turns $\boldsymbol{N}$, turns per unit length $\boldsymbol{n}$
- Field inside solenoid = $\boldsymbol{\mu_0\, n\, i}$
- Field outside ~ 0



$$\Phi_B = NAB = NA\mu_0 ni = Li$$

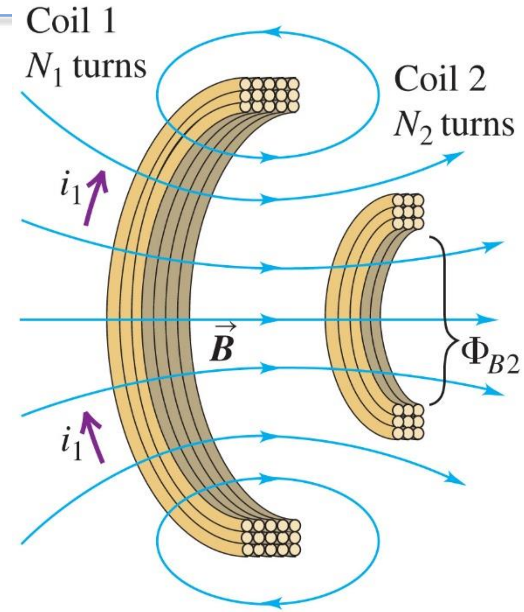
$$\boxed{\text{L = "inductance"} \quad = \mu_0 NAn = \mu_0 \frac{N^2}{l} A}$$

Inductance

Mutual Inductance of two coils:

Some of the magnetic flux through one coil also passes through the other coil, inducing a voltage.

Inductance is magnetic flux/current.



Mutual inductance

The mutual inductance M of two coils is given by

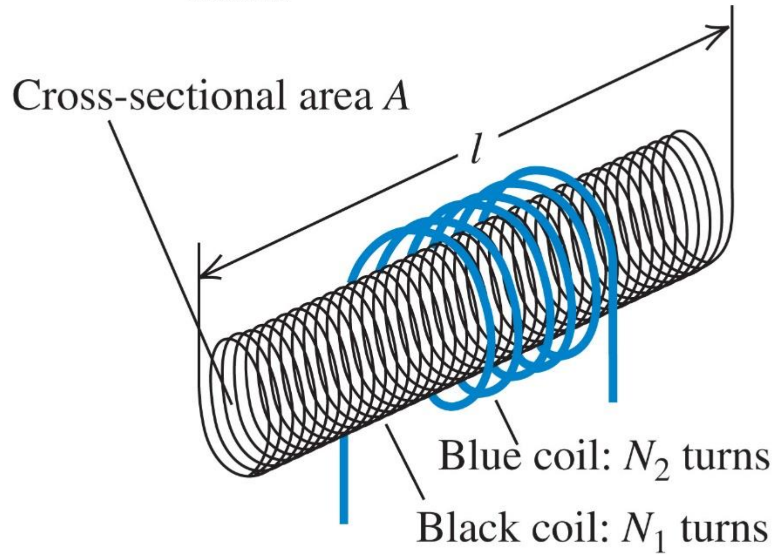
$$M = M_{21} = M_{12} = \left|\frac{N_2\Phi_{B2}}{i_1}\right| = \left|\frac{N_1\Phi_{B1}}{i_2}\right|. \qquad (21.11)$$

From the preceding analysis, we can also write

$$\mathcal{E}_2 = M\left|\frac{\Delta i_1}{\Delta t}\right| \quad \text{and} \quad \mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right|. \qquad (21.12)$$

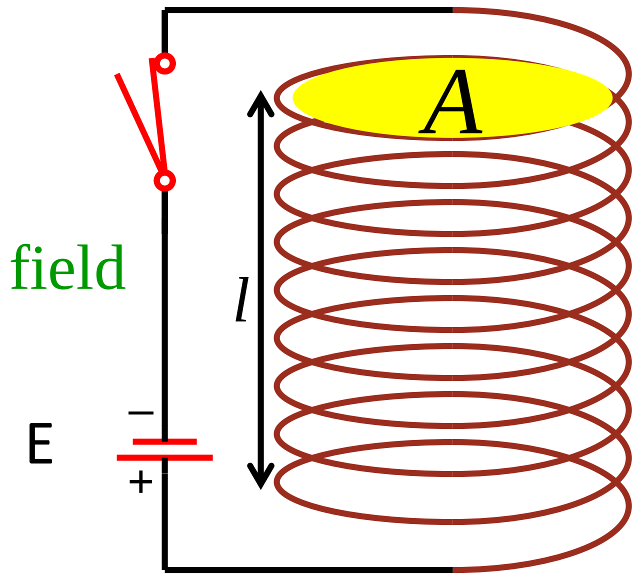
Unit: The SI unit of mutual inductance is called the **henry** (1 H), in honor of Joseph Henry (1797–1878), one of the discoverers of electromagnetic induction. From Equation 21.11, one henry is equal to *one weber per ampere.* Other equivalent units, obtained by reference to Equation 21.10, are *one volt-second per ampere* and *one ohm-second:*

$$1\ \text{H} = 1\ \text{Wb/A} = 1\ \text{V}\cdot\text{s/A} = 1\ \Omega\cdot\text{s}.$$



Self Inductance

- Consider a solenoid *L*, connect it to a battery
  - Area *A*, length *l*, *N* turns
- What happens as you close the switch?
- Lenz's law – loop resists change in magnetic field
- Magnetic field is caused by the current
- "Inductor" resists change in current



$$B = \frac{\mu_0 NI}{\ell} \qquad \Phi_{B1} = \frac{\mu_0 NIA}{\ell}$$

$$\mathcal{E}_L = -\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\Phi_{B1} = -\frac{d}{dt}\left(\frac{\mu_0 N^2 IA}{\ell}\right) = -\frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt}$$

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}} \qquad \boxed{L = \frac{\mu_0 N^2 A}{\ell}}$$

Self-Inductance

Changing current through the wires in a coil will induce a voltage that opposes the CHANGE in the current.

Definition of self-inductance
The self-inductance L of a circuit is the magnitude of the self-induced emf $\mathcal{E}$ per unit rate of change of current, so that:

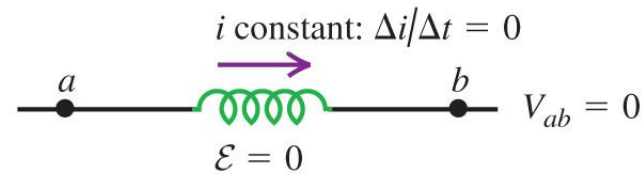
$$\mathcal{E} = L\left|\frac{\Delta i}{\Delta t}\right|. \qquad (21.14)$$

From the definition, the units of self-inductance are the same as those of mutual inductance; the SI unit of self-inductance is *one henry.*

Energy Stored in an Inductor

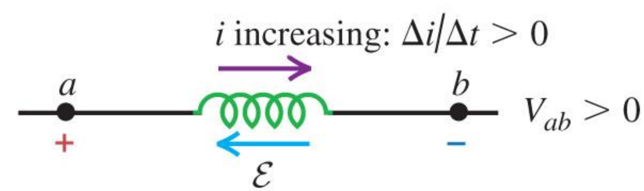
$$U = \frac{1}{2}LI^2$$

SI unit: joule, J

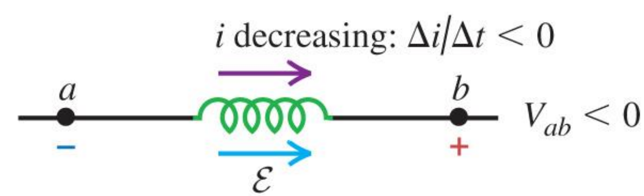


(a) When the current is constant, there is no self-induced emf, so the voltage across the inductor is zero.

(b) and (c) show that a changing current induces an emf and hence a voltage across the inductor.



(b) If the current is *increasing*, then, by Lenz's law, the emf points opposite to i.



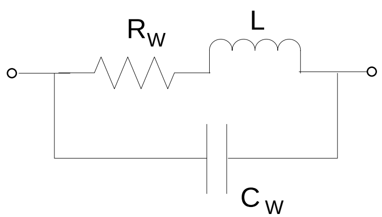
(c) If the current is *decreasing*, then, by Lenz's law, the emf points in the same direction as i.

Properties of Inductor

- **An inductor acts like a short circuit to dc**, since from ($v = 0$ when $i =$ a constant.)

$$v(t) = L\frac{di(t)}{dt}$$

- **The current through an inductor cannot change instantaneously**, since an instantaneous change in current would require an infinite voltage, which is not physically possible.
- Like the ideal capacitor, the ideal inductor does not dissipate energy.
- A real inductor has a significant resistance due to the resistance of the coil, as well as a "winding capacitance". Thus, the model for a real inductor is shown below.



Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero
- Thus, an inductor acts like a short for DC
- The current through an inductor cannot change instantaneously
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage

Properties of Inductors

- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it.
- There is also a small winding capacitance due to the closeness of the windings
- These two characteristics are typically small, though at high frequencies, the capacitance may matter.

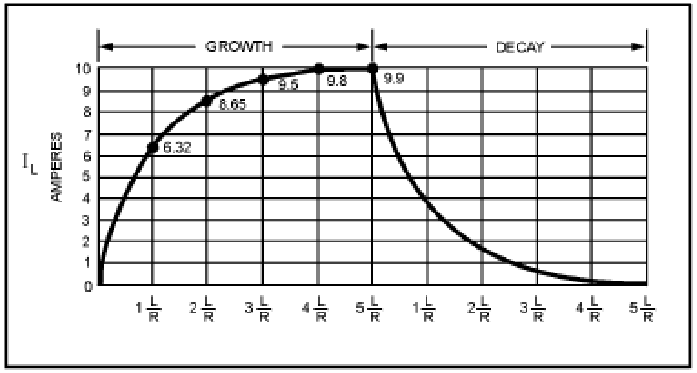
Time constant

- Voltage reaches its maximum right away
- **Current takes time to build up** as magnetic field is established
- Time constant for inductors is defined as:
  - The time it takes the circuit to achieve 63% of the total expected current
- The time constant is calculated as:

$$T = \frac{L}{R}$$

Values must be in base units (ie. H and Ω)

63% of expected current after 1 time constant

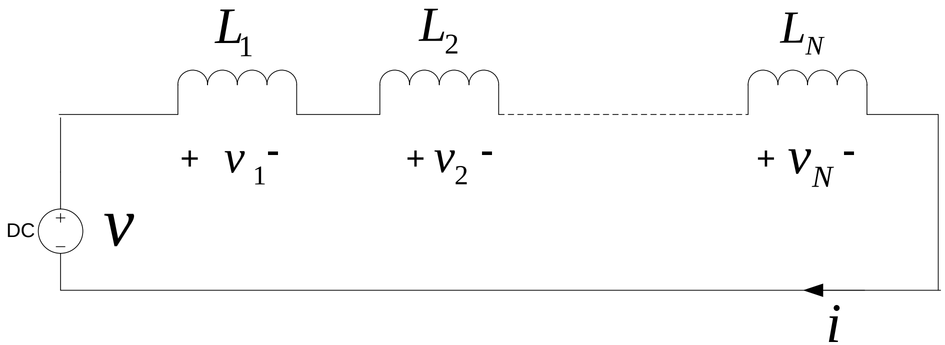


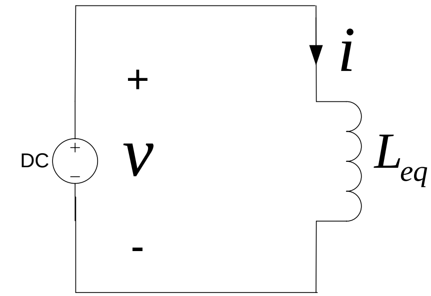
It's built into the mathematics of exponential decay associated with first-order systems. If the response starts at unity at t=0, then after one "unit of time", the response is $e^{-1}=0.36788$. When you're looking at a risetime, you subtract this from unity, giving 0.63212 or 63.2%.

Where does the time constant value 63% come from?

$$\tau_n = 1 - e^{-n}$$

Inductors in Series





$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_N = L_N \frac{di}{dt}$$

$$v = v_1 + v_2 + \cdots + v_N = \left(L_1 + L_2 + \cdots + L_N\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = \sum_{k=1}^{N} L_k$$

The equivalent inductance of series connected inductors is the sum of the individual inductances. Thus, inductances in series combine in the same way as resistors in series.

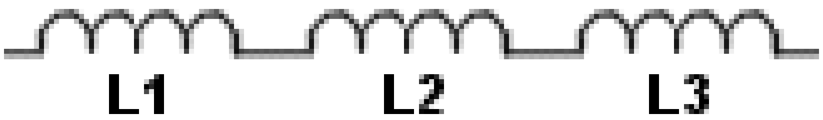
Inductors in Series: Example

- You have three inductors in series and their values are:

$L_1 = 25\text{mH}$

$L_2 = 15\text{mH}$

$L_3 = 100\mu\text{H}$



What is the total inductance of this simple circuit?

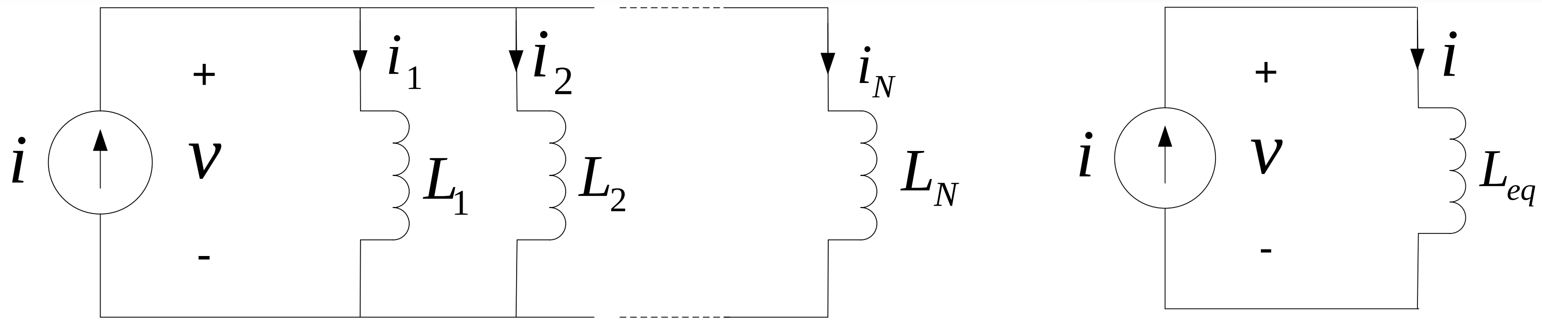
$$L_t = L_1 + L_2 + \ldots L_n \quad = L_1 + L_2 + L_3$$

$$= 25\text{mH} + 15\text{mH} + 100\mu\text{H}$$

$$= 40\text{mH} + 0.1\text{mH}$$

$$= 40.1\text{mH}$$

Inductors in parallel



$$i_1 = \frac{1}{L_1}\int v dt \qquad i_2 = \frac{1}{L_2}\int v dt \qquad i_N = \frac{1}{L_N}\int v dt$$

$$i = i_1 + i_2 + \cdots + i_N = \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int v dt = \frac{1}{L_{eq}}\int v dt$$

$$\frac{1}{L_{eq}} = \sum_{k=1}^{N} \frac{1}{L_k}$$

The equivalent inductance of parallel connected inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

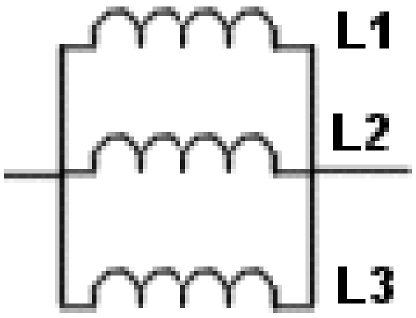
Inductors in parallel: Example

- Now you have three inductors in parallel and their values are:

$L_1 = 40\text{mH}$

$L_2 = 80\text{mH}$

$L_3 = 40\text{mH}$



$$\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{40} + \frac{1}{80} + \frac{1}{40}$$

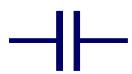
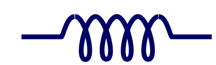
$$= \frac{1}{16\text{mH}}$$

$$L_t = 16\text{mH}$$

A common pit-fall here is forgetting that the formula is based on $1/L_t$

Be sure that you recall this fact when you do the calculations on your own!

INDUCTORS vs CAPACITORS

CAPACITOR	INDUCTOR
Has a capacity to “hold” charge, called capacitance… $C = \dfrac{Q}{\Delta V}$ [$^{C}/_{V}$ = farad, F] …determined by its geometry: $C = \varepsilon_0 \dfrac{A(N-1)}{d}$	Has an ability to “hold” flux, called inductance… $L = \dfrac{\Phi_m}{I}$ [$^{Wb}/_{A} = {}^{T\,m^2}/_{A}$ = henry, H] …determined by its geometry: $L = \mu_0 \dfrac{AN^2}{\ell}$
Circuit diagram symbol: 	Circuit diagram symbol: 
Stores energy in its electric field: $U_C = \frac{1}{2} C(\Delta V)^2$ $u_E = \dfrac{\varepsilon_0}{2} E^2$	Stores energy in its magnetic field: $U_L = \frac{1}{2} LI^2$ $u_B = \dfrac{1}{2\mu_0} B^2$
Regulates current through a series resistor…	
…at switch on: $I = \dfrac{\mathcal{E}}{R} e^{-t/\tau}$ …and switch off: $I = I_0 e^{-t/\tau}$	…at switch on: $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$ …and switch off: $I = I_0 e^{-t/\tau_L}$