1. In an effort to clear impurities from a fabricated nano-wire, a laser beam is swept repeatedly along the wire, in the presence of a parallel electric field. After one sweep, an impurity initially at $x = 0$ has the following probability density of being found at a new position x
$$p(x) = \begin{cases} p(x) = \frac{1}{3}\delta(x) + \frac{2}{3a}e^{-x/a} & x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$
where a is some characteristic length. Calculate the probability density for the total distance d, the impurity has moved along the wire after 36 sweeps.

2. Consider one cubic centimeter of a dilute gas of atoms of mass M in thermal equilibrium at temperature $T = 0°C$ and atmospheric pressure. The number of atoms (or molecules) in a cubic meter of an ideal gas at $T = 0°C$ and atmospheric pressure has the value $2.69 \times 10^{25} m^3$.

   a) For the kinetic energy of a single atom, find a numerical value for the ratio of standard deviation (the square root of the variance) to the mean.

   b) Find the same ratio for total energy of the gas, assumed to be all kinetic.

3. Consider a magnetic system which is at thermodynamic equilibrium at a temperature T. Let μ be the magnetic moment of each spin; and let M be the magnetization per spin, so that $-\mu < M < \mu$. The free energy per spin is given by $F(M)$.

   a) suppose $F(M) = \lambda\left[\left(\frac{M}{\mu}\right)^4 - \left(\frac{M}{\mu}\right)^2\right]$. Is this a valid expression for the free energy?

   b) what would go wrong if this is indeed a valid free energy?

4. Consider the velocity of a gas in one dimension $(-\infty < v < \infty)$

   a) Find the **unbiased probability density** $p_1(v)$, subject only to the constraint that the average speed is c, that $\langle |v| \rangle = c$

   b) Now find the probability density $p_2(v)$, given only the constraint of average kinetic energy, $\langle mv^2/2 \rangle = mc^2/2$.

   c) Which of the above statements provide more information on the velocity? Quantify the difference in information in terms of $l_2 - l_1 = (\langle \ln p_2 \rangle - \langle \ln p_1 \rangle)/2$

5. You are given several unusual three-sided dice which, when rolled, show either one, two, or three spots. There are three games played with these dice: *Distinguishable, Bosons, and Fermions*. In each turn in these games, the player rolls one die at a time, starting over if required by the rules, until a legal combination occurs. In *Distinguishable*, all rolls are legal. In *Bosons*, a roll is legal only if the new number is larger or equal to the preceding number. In *Fermions*, a roll is legal only if the new number is strictly larger than the preceding number. Let the sum of the spots as result of throwing n dices be S_n.

   a) construct a table showing the different values for S_2. Which of these correspond to *Fermions* and which of these are *Bosons*?

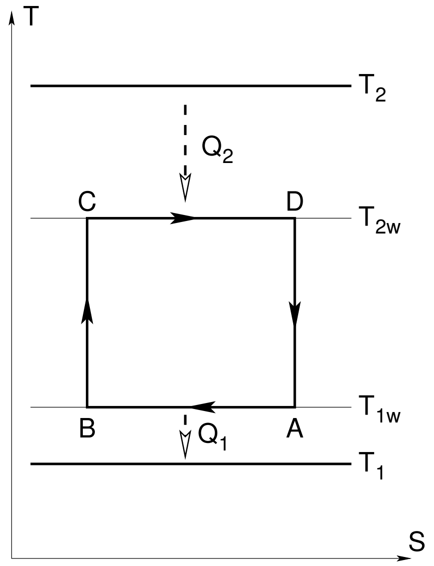
   b) Presume the dice are fair: each of the three numbers of dots shows up 1/3 of the time. For a legal turn rolling a die twice in Bosons, what is the probability of $S_2 = 4$ of rolling a 4? Similarly, among the legal Fermion turns rolling two dice, what is the probability of $S_2 = 4$?

c) For a legal turn rolling three three-sided dice in Fermions, what is the probability of $S_3 = 6$ of rolling a 6?

d) Consider rolling M dice with N sides each. How many legal turns for Bosons are there?

e) In a turn of three rolls, what is the factor by which the probability of getting triples in Bosons is enhanced over that in Distinguishable? In a turn of M rolls, what is the enhancement factor for generating an M-tuple (all rolls having the same number of dots showing) for a three sided dice? What happens to the same expression in the general case when you have a N faced dice?

6. The Carnot process is an idealized, quasistatic process with maximum efficiency. Taking a finite heat conduction between the working substance and the reservoirs into account, the duration of a cycle, however, is infinitely long and the power output is zero. To speed up the heat transfer, temperature differences between the working substance and the reservoirs are required. In the Curzon-Ahlborn heat engine, the working substance performs a Carnot cycle as depicted to the right. It alternates between temperatures $T_{1w} < T_{2w}$, where it is in contact with either of two reservoirs at T_1 and T_2 . Assume that the heat flux of the isothermal stages lasts for periods $\tau_i (i = 1, 2)$ and is described by $\dot{Q}_i = K\Delta T_i$ with heat conductivity K. Neglect the duration of the fast adiabatic stages and possible associated heat fluxes.



a) Determine the entropy changes of the working substance, S_{AB} and S_{CD} , and of the reservoirs, $\tilde{S}_{AB}$ and $\tilde{S}_{CD}$, during the isothermal stages. Calculate the rate of entropy production due to heat conduction.

b) Find the optimal temperatures T_{1w} ,T_{2w} for given T_1 ,T_2 by maximizing the power output, $P = (Q_2 Q_1)/(\tau_1 + \tau_2)$, with respect to the variables τ_1 ,τ_2 , T_{1w} ,T_{2w} under the constraint $S_{AB} = S_{CD}$.

c) Calculate the efficiency of the optimized engine in terms of T_1 and T_2 . Compare with the efficiency of the Carnot process and with existing engines, e.g., a Diesel engine or a coal-fired power plant.