PHY 201 : Waves and Optics
End Semester Exam
06^{th} December 2021, IISER Mohali

All Problems carry equal weightage. Symbols have their usual meanings.

1. A damped harmonic oscillator with damping parameter β , mass m and natural frequency ω_0 is subjected to a periodic force $F(t) = F_0 \sin \omega t$. Find out the phase difference between the applied force and the displacement in the steady state (i.e. time much larger than $2/\beta$). Derive the expression for the amplitude in the steady state. At what times the dispacements are maximum as $\omega \to \omega_0$?

2. In the previous problem, find out the expression for power used once the oscillator achieves the steady state. Derive the expression for the average power needed over one oscillation cycle in the steady state. What is its resonance value ?

3. A wire segment is made of three pieces of density ρ_1 for $x < 0$, density ρ_2 for $0 < x < a$ and density ρ_3 for $x > a$. Using the junction conditons of continuity and slope matching, derive the expression for transmission coefficient (ratio of transmission wave amplitude to the amplitude of the incident wave) at the two junctions. From this obtain the expression for net transmission coefficient (ratio of transmission amplitude in the region $(x > a)$ to the amplitude of the incident wave from $(x < a)$) post the second junction.

4. On a string (length ℓ, mass density ρ, tension T) a standing wave is generated by superposing first and the second normal modes. Obtain the expression for kinetic energy and the potential energy of the string. From that show that the net energy is just the sum of net energy of the individual modes.

5. Starting from the Maxwell's equations demonstrate that a field defined as a linear combination of the electric and the magnetic field $\mathbf{\Phi} = \alpha\mathbf{E} + \beta\mathbf{B}$ satisfies the wave equation in a charge and current free space, where α, β are constants. Is the wave identified by $\mathbf{\Phi}$ a transverse wave ? What is $\hat{\mathbf{k}} \times \mathbf{\Phi}$ in terms of $\mathbf{E}$ and $\mathbf{B}$, if $\hat{\mathbf{k}}$ is the propagation direction ? [vector triple product : $\mathbf{a} \times \mathbf{b} \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$]

6. For an electromagnetic wave described by the electric field $\mathbf{E} = E_0 \cos\left(\frac{k(x+y)}{\sqrt{2}} - \omega t\right)(\hat{\mathbf{i}} - \hat{\mathbf{j}})$, find out the associated magnetic field. Obtain the expressions of the energy density of the wave and its Poynting vector.

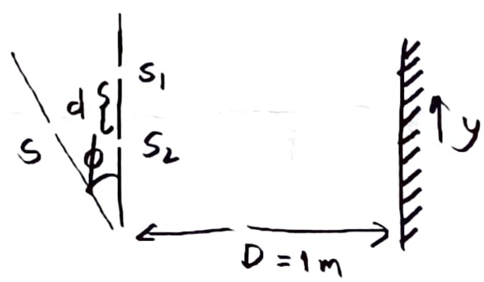
7. For an electromagnetic wave described by the magnetic field

$$\mathbf{B} = 10(-\hat{\mathbf{j}})\cos\left(2\times 10^{-5}m^{-1}z - 6000s^{-1}t\right)\text{Gauss}.$$

what should be the magnetic field of another electromagnetic wave superposed to get the **polarization of the resultant electic field** as (i) Linear in $x-y$ plane at angle $\pi/3$ with the x-axis (ii) circular in the $x-y$ plane with anticlockwise propagation.

8. In the double slit experiment with slit separation d, a slab of refractive index μ and thickness t is introduced just infront of one slit. Find out the thickness of the ~~slit~~ slab such that the position $y=0$ on the screen D distance away is order 1 maxima. What are the wavelengths for which $y=0$ serves as a minima ?

9. The plane containing the original source in the double slit experiment is rotated by a small angle ϕ (as shown in the figure) with respect to the plane of the double slit. How does the the width of a dark fringe depend on the angle of rotation ? For what angles of roation, the point $y=0$ on the screen $1m$ away for $d=6mm$ and wavelength $30\mu m$, is a maxima ?



10. In the Young's double slit experiment, the amplitude of the wave emananting from one of the slit (S1) is halved with no phase change compared to the wave emananting from the other slit (S2). Find out the intensity at the center of the screen (y=0) , which is placed distance D apart, if the slit separation is d. Find out the intensity if the plane containing the double slit is rotated by small angle ϕ w.r.t the plane of the screen, pivoted from the center of the slits (as shown below) while the amplitude from slit S1 remains half of that of S2.