Ans. 1

Solution   The situations in frames F and F' are shown in Fig. 5.12. Let's first find the surface density σ'. The γ factor associated with $v = 0.6c$ is $\gamma = 5/4$, so in going from F to F', longitudinal distances are decreased by 4/5. The distance between points A' and B' in F' is therefore shorter than the distance between points A and B in F. With the distance ℓ shown, the latter distance is $\sqrt{2}\ell$, while the former is $\sqrt{1+(4/5)^2}\,\ell$. Since charge is invariant, the same amount of charge is contained between A' and B' as between A and B. Therefore, $\sigma'\sqrt{1+(4/5)^2}\ell = \sigma\sqrt{2}\ell \Longrightarrow \sigma' = (1.1043)\sigma$.

Now let's find the electric field $\mathbf{E}'$. The magnitude of the field $\mathbf{E}$ in F is simply $E = \sigma/2\epsilon_0$ (by the standard Gauss's law argument), and it points at a 45° angle. So Eq. (5.7) gives the field components in F' as

$$E'_{\parallel} = E_{\parallel} = E/\sqrt{2} \quad \text{and} \quad E'_{\perp} = \gamma E_{\perp} = \gamma E/\sqrt{2}. \tag{5.8}$$

$\mathbf{E}'$ is shown in Fig. 5.12(b). Its magnitude is $E' = (E/\sqrt{2})\sqrt{1+(5/4)^2} = (1.1319)E$, and its slope is (negative) $E'_{\perp}/E'_{\parallel} = \gamma$. The sheet's slope is also (positive) $\ell/(\ell/\gamma) = \gamma$. So the angles θ shown are all equal to $\tan^{-1}\gamma = 51.34°$. This means that $\mathbf{E}'$ points at an angle of 2θ with respect to the sheet. Equivalently, it points at an angle of $2\theta - 90° \approx 12.68°$ with respect to the normal to the sheet. The normal component of E' is then

$$E'_{\rm n} = E'\cos 12.68° = (1.1319E)\cos 12.68° = (1.1043)E. \tag{5.9}$$

Therefore, since the same numerical factor appears in the two equations, $E'_{\rm n} = (1.1043)E$ and $\sigma' = (1.1043)\sigma$, we can multiply both sides of the relation $E = \sigma/2\epsilon_0$ by 1.1043 to obtain $E'_{\rm n} = \sigma'/2\epsilon_0$. In other words, Gauss's law holds in frame F'. There is also an electric field component parallel to the sheet in F' (unlike in F), but this doesn't affect the flux through a Gaussian surface. You can check that it is no coincidence that the numbers worked out here, by solving the problem symbolically in terms of γ; see Exercise 5.12.

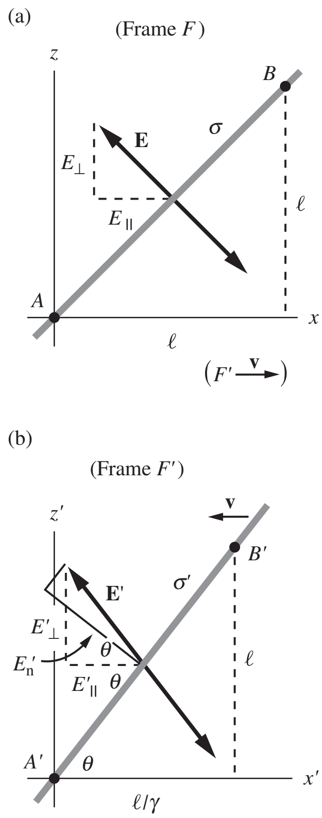

Figure 5.12.
The setup as viewed in frames F and F'. The sheet moves to the left in F'.

Ans. 2

Excess charge, $Q = 5\times10^{8} \times 1.6\times10^{-19}\,C = 8\times10^{-11}\,C$.

~~Charge~~ Charge density, $\lambda = \frac{Q}{l} = \frac{8\times10^{-11}\,C}{0.04\,m}$

$= 2\times10^{-9}\,C/m$.

(a) Electric field in the rest frame,

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \simeq \frac{2\times10^{-9}\,C/m}{\left(\frac{0.0001}{2}\right)m} \times 18\times10^{9}\,\frac{Nm^2}{C^2}$$

$$\simeq 72\times10^{4}\,V/m. \quad \left(\because \frac{1}{4\pi\epsilon_0} \simeq 9\times10^{9}\,\frac{Nm^2}{C^2} \;\therefore \frac{1}{2\pi\epsilon_0} \simeq 18\times10^{9}\,\frac{Nm^2}{C^2}\right).$$

Direction : radial.

(b) In the moving frame,

$$E_r' = \gamma E_r \quad \text{where, } \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.9)^2}} \simeq 2.29}$$

$$= 2.29 \times 72\times10^{4}\,V/m$$

$$\simeq 1.65\times10^{6}\,V/m.$$

direction: radial.

Ans. 4

Since the charged particle feels the force due to the electric field only in the y-direction, therefore

$$f_x = \frac{dp_x}{dt} = 0 \Rightarrow p_x \text{ is conserved.}$$

However, p is relativistic momentum and is given as, $\vec{p} = \gamma m_0 \vec{u}$ where m_0 : rest mass

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

Since p_x is conserved,

$$\therefore (p_x)_{\text{before entering field}} = (p_x)_{\text{after entering field}}.$$

$$(p_x)_b = \gamma_b m_0 (u_x)_b = \frac{m_0 (u_x)_b}{\sqrt{1-\frac{(u_x)_b^2}{c^2}}}$$

$$(p_x)_a = \gamma_a m_0 (u_x)_a = \frac{m_0 (u_x)_a}{\sqrt{1-\frac{(u_x)_a^2+(u_y)_a^2}{c^2}}}$$

Since, the particle entering the region of the electric field has a y-component of velocity as well.

$$\therefore \frac{\cancel{m_0}(u_x)_b}{\sqrt{1-\frac{(u_x)_b^2}{c^2}}} = \frac{\cancel{m_0}(u_x)_a}{\sqrt{1-\frac{(u_x)_a^2+(u_y)_a^2}{c^2}}}$$

$$\Rightarrow (u_x)_b^2\left(1-\frac{(u_x)_a^2+(u_y)_a^2}{c^2}\right) = (u_x)_a^2\left(1-\frac{(u_x)_b^2}{c^2}\right)$$

$$\Rightarrow (u_x)_b^2 - \cancel{\frac{(u_x)_b^2(u_x)_a^2}{c^2}} - \frac{(u_x)_b^2(u_y)_a^2}{c^2} = (u_x)_a^2 - \cancel{\frac{(u_x)_a^2(u_x)_b^2}{c^2}}$$

$$\Rightarrow (u_x)_a^2 = (u_x)_b^2\left[1-\frac{(u_y)_a^2}{c^2}\right].$$

$$\Rightarrow (u_x)_a = (u_x)_b\sqrt{1-\frac{(u_y)_a^2}{c^2}}.$$

Since, $(u_y)_a \neq 0$, $\therefore (u_x)_a < (u_x)_b$

$\therefore$ x component of velocity decreases.

Solution In the rest frame of the two protons, the force of repulsion is simply $e^2/4\pi\epsilon_0 r^2$. The force in the lab frame is therefore $(1/\gamma)(e^2/4\pi\epsilon_0 r^2)$. (Remember, the force is always largest in the rest frame of the particle on which it acts.) This *is* the correct total force in the lab frame. But, as mentioned above, the repulsive electrical force eE in the lab frame is $\gamma e^2/4\pi\epsilon_0 r^2$, because Eq. (5.15) tells us that the electric field due to a moving charge is larger by a factor γ in the transverse direction. Apparently this must not be the whole force. There must be an extra attractive force that partially cancels the repulsive electric force $\gamma e^2/4\pi\epsilon_0 r^2$, bringing it down to the correct value of $e^2/\gamma 4\pi\epsilon_0 r^2$. This extra attractive force must therefore have magnitude (using $1/\gamma^2 = 1 - \beta^2$)

$$\begin{aligned} \frac{\gamma e^2}{4\pi\epsilon_0 r^2} - \frac{e^2}{\gamma 4\pi\epsilon_0 r^2} &= \gamma\left(1 - \frac{1}{\gamma^2}\right)\frac{e^2}{4\pi\epsilon_0 r^2} \\ &= \gamma\beta^2\frac{e^2}{4\pi\epsilon_0 r^2} = e(\beta c)\left(\frac{\beta}{c}\frac{\gamma e}{4\pi\epsilon_0 r^2}\right). \end{aligned} \tag{5.29}$$

We have chosen to write the force in this way, because we can then interpret it as the $q\mathbf{v} \times \mathbf{B}$ magnetic force in Eq. (5.1), provided that the magnitude of $\mathbf{B}$ is $(\beta/c)(\gamma e/4\pi\epsilon_0 r^2)$, which is β/c times the magnitude of the electric field in the lab frame, and provided that $\mathbf{B}$ points out of (or into) the page at the location of the top (or bottom) proton in Fig. 5.24. The cross product $\mathbf{v} \times \mathbf{B}$ then points in the proper (attractive) direction. Each proton creates a magnetic field at the location of the other proton. The relative factor of β/c between the magnetic and electric fields is consistent with the Lorentz transformations we will derive in Section 6.7.

We see that the magnetic force, through its partial cancelation of the electric force, allows the following two statements to be consistent: (1) the transverse *electric field* due to a charge is *smallest* in the frame of that charge (by a factor of γ compared with any other frame), and (2) the transverse *force* on a particle is *largest* in the frame of that particle (by a factor of γ compared with any other frame). These two statements imply, respectively, that the *electric* force is larger in the lab frame than in the protons' frame, but the *total* force is smaller in the lab frame than in the protons' frame. These two facts are consistent because the existence of the magnetic force means that the total force isn't equal to just the electric force.

You might be tempted to argue that the proton is not "moving through" the $\mathbf{B}$ field of the other proton because that field is "moving right along with it." That would be incorrect. In the force law that is the fundamental definition of $\mathbf{B}$, namely $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$, $\mathbf{B}$ is the field at the position of the charge q at an instant in time, with both position and time measured in the frame in which we are measuring the force on q. What the "source" of $\mathbf{B}$ may be doing at that instant is irrelevant.

Note that the structure of the reasoning in this example is the same as the reasoning in the charge-and-wire example above in the text. In both cases we first found the force in the rest frame of a given point charge. (This was simple in the present example, but involved a detailed length-contraction argument in the charge-and-wire example.) We then transformed the force to the lab frame by a quick division by γ. And finally we determined what the extra (magnetic) force must be to make the sum of the electric and magnetic forces in the lab frame be correct. (The lab-frame electric force was trivially zero in the charge-and-wire example, but not in the present example.) See Exercise 5.29 for more practice with this type of problem.

Ans. 6

In frame F, $I_k = \lambda_k \beta_k c$.

In the frame F', $\beta_k' = \dfrac{\beta_k + \beta}{1 + \beta_k \beta}$ since F' moves $\parallel^{l}$ to line with velocity $(-\beta c)$.

Now, $\lambda_k' = \dfrac{\lambda_k \gamma_k'}{\gamma_k}$ (recall the argument from what I did in the ~~last~~ class. $F \longrightarrow$ rest frame $\longrightarrow F'$).

$$\gamma_k = \frac{1}{\sqrt{1-\beta_k^2}} \; ; \quad \gamma_k' = \frac{1}{\sqrt{1-\beta_k'^2}} \; , \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} .$$

$$\therefore \gamma_k' = \frac{1}{\sqrt{1 - \left(\dfrac{\beta_k + \beta}{1+\beta_k \beta}\right)^2}} = \gamma \gamma_k (1 + \beta \beta_k) .$$

$$\therefore I_k' = \lambda_k' \beta_k' c = \lambda_k \cdot \frac{\gamma \cancel{\gamma_k} \cancel{(1+\beta\beta_k)}}{\cancel{\gamma_k}} \left(\frac{\beta_k + \beta}{\cancel{1+\beta\beta_k}} \right) \cdot c$$

$$= \lambda_k \gamma (\beta_k + \beta) c = \gamma (\lambda_k \beta_k c + \lambda_k \beta c)$$

$$= \gamma (I_k + \beta c \lambda_k) .$$

$$\lambda_k' = \lambda_k \cdot \frac{\gamma \cancel{\gamma_k} (1 + \beta\beta_k)}{\cancel{\gamma_k}} = \gamma \left(\lambda_k + \frac{\beta I_k}{c} \right)$$

Total, $\lambda = \sum_k \lambda_k$ & $I = \sum_k I_k$

$$\therefore \lambda' = \gamma \left(\lambda + \frac{\beta I}{c} \right) \; ; \quad I' = \gamma (I + \beta c \lambda) .$$