PHY304: Statistical Mechanics

Assignment 8

March 26, 2025

1. Consider a system of two atoms, each having only 3 quantum states of energy 0, ϵ and 2ϵ. The system is in contact with a heat reservoir at temperature T. Write down the partition function Z for the system if the particles obey

   (a) Classical statistics and are distinguishable.

   (b) Classical statistics and are indistinguishable.

   (c) Fermi-Dirac statistics.

   (d) Bose-Einstein statistics.

2. Consider two identical particles which are to be placed in four single-particle states. Two of these states have energy 0, one has energy ϵ, the last has energy 2ϵ. Calculate the partition function given that the particles are (a) fermions and (b) bosons.

3. Given a system of N identical non-interacting magnetic ions of spin-$\frac{1}{2}$, magnetic moment μ_0 in a crystal at absolute temperature T in a magnetic field B. For this system calculate:

   (a) The partition function.

   (b) The entropy.

   (c) the average energy.

   (d) The average magnetic moment $\overline{M}$, and the fluctuation in the magnetic moment, $\Delta M = \sqrt{\overline{(M - \overline{M})^2}}$.

4. (a) Find the density matrix ρ of a beam of spin-$\frac{1}{2}$ atoms prepared in the state

$$| \uparrow_y \rangle = \frac{1}{\sqrt{2}} \left(| \uparrow_z \rangle + i| \downarrow_z \rangle \right).$$

   (b) Show that $| \uparrow_y \rangle$ is a pure state.

   (c) Calculate $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$.

5. (a) Find the density matrix ρ of a beam of spin-$\frac{1}{2}$ atoms containing a mixture of 75% of $| \uparrow_z \rangle$ and 25% of $| \downarrow_z \rangle$.

(b) Show that this is a mixed state.

(c) Calculate $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ for this system.

6. (a) Find the density matrix ρ of a partially polarized beam of spin-$\frac{1}{2}$ atoms containing a mixture of 75% of $| \uparrow_z \rangle$ and 25% of $| \uparrow_x \rangle$.

(b) Check whether it is a pure or a mixed state?

(c) Calculate $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ for this system.

7. Suppose we have a system with total angular momentum 1. Pick a basis corresponding to the three eigenvectors of the z-component of angular momentum, J_z, with eigenvalues +1, 0, -1, respectively. We are given an ensemble described by density matrix:

$$\frac{1}{4} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(a) Is ρ a permissible density matrix? Give your reasoning. For the remainder of this problem, assume that it is permissible. Does it describe a pure or mixed state? Give your reasoning.

(b) Given the ensemble described by ρ, what is the average value of J_z ?

(c) What is the spread (standard deviation) in measured values of J_z ?