PHY306 Advanced Quantum Mechanics Jan-April 2025: Assignment 4

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1. Use the virial theorem to obtain the expectation value of $1/r$ for the hydrogen atom.

2. Suppose the Hamiltonian H for a system is a function of some parameter λ with $E_n(\lambda), \psi_n(\lambda)$ being the eigenvalues and eigenfunctions of $H(\lambda)$. The Feynman-Hellmann theorem states that
$$\frac{\partial E_n}{\partial \lambda} = \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle$$
Assume that E_n is nondegenerate. Prove the Feynman-Hellmann theorem.

3. Use $\lambda = e$ and $\lambda = l$ in the Feynman-Hellmann theorem to obtain the expectation values of $1/r$ and $1/r^2$ for the hydrogen atom.

4. Consider $n = 2$ states of the Hydrogen atom and find the energy of each state under weak-field Zeeman splitting. Construct a table of energies and plot them as functions of the external field.

5. Consider the $n = 2$ states of the Hydrogen atom and find the energy of each state under strong-field Zeeman splitting. Construct a table of energies and plot them as functions of the external field.

6. Analyze the Stark effect (atom placed in electric field E_{ext}) for $n = 1$ and $n = 2$ states of Hydrogen. Let the field point in z direction so that potential energy of the electron is $H' = -eE_{ext}r\cos\theta$. Treat this as a perturbation on the Bohr Hamiltonian and ignore spin. Show that the ground state energy is not affected by this perturbation in the the first order. Find the first-order corrections to the energy for the first excited state (which is four-fold degenerate). What are the "good" wave functions for this case? Find the expectation value of the electric dipole moment $p = -er$ in each of these "good" states.

7. Analyze the Stark effect for the $n = 3$ states of hydrogen after turning on the electric field in the z direction. Write out the initial degenerate states and construct the perturbing Hamiltonian matrix. Find the eigenvalues and their degeneracies.