PHY638: Quiz 1 Date : March 3, 2025 Inst: Abhishek Chaudhuri

ROLL NO : MS **NAME :**

1. Suppose wind is blowing over a hut which looks like a half cylinder of radius a and length L. The wind and pressure at far distances are U_∞ and p_∞, respectively. Let the pressure inside the hut be p_∞. Assuming the flow is ideal and can be modelled using the potential for flow past a cylinder without circulation, what is the expression for the upward force on the hut due to the difference in pressures? Assume the flow is two-dimensional.

[Complex potential for uniform flow past a cylinder is

$$w(z) = U_\infty \left(z + \frac{a^2}{z}\right) \; ; \; z = re^{i\theta}.$$

This gives. $\phi = U_\infty\left(r + \frac{a^2}{r}\right)\cos\theta$ & $\psi = U_\infty\left(r - \frac{a^2}{r}\right)\sin\theta$.

& vel. components: $u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty\left(1 - \frac{a^2}{r^2}\right)\cos\theta$

& $u_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty\left(1 + \frac{a^2}{r^2}\right)\sin\theta$.]

On the surface $r = a$, $u_\theta = -U_\infty\left(1 + \frac{a^2}{a^2}\right)\sin\theta = -2U_\infty \sin\theta$. 1

This result for our case is valid for $0 \leq \theta \leq \pi$ (upper half)

From Bernoulli's eqn: 1

$$\frac{1}{2}\rho U_\infty^2 + p_\infty = \frac{1}{2}\rho u_\theta^2 + p(\theta) \Rightarrow p_\theta = p_\infty + \frac{1}{2}\rho\left(U_\infty^2 - u_\theta^2\right)$$

Putting $u_\theta = -2U_\infty \sin\theta \Rightarrow p_\theta = p_\infty + \frac{1}{2}\rho U_\infty^2\left(1 - 4\sin^2\theta\right)$ 1

$\therefore$ Upward force/length, $F_y = -\int_0^\pi p_\theta \sin\theta \, a\, d\theta$.

$$\Rightarrow F_y = -a\int_0^\pi \left[p_\infty + \frac{1}{2}\rho U_\infty^2 \left(1 - 4\sin^2\theta\right)\right] \sin\theta\, d\theta$$

$$= -a\left[p_\infty \int_0^\pi \sin\theta\, d\theta + \frac{1}{2}\rho U_\infty^2 \int_0^\pi \sin\theta\, d\theta - 2\rho U_\infty^2 \int_0^\pi \sin^3\theta\, d\theta\right].$$

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$\int_0^\pi \sin\theta \, d\theta = 2$; $\int_0^\pi \sin^3\theta \, d\theta = \frac{4}{3}$

$$\therefore F_y = -a\left[2p_\infty + \rho U_\infty^2 - \frac{8}{3}\rho U_\infty^2\right]$$

$$= -a\left[2p_\infty - \frac{5}{3}\rho U_\infty^2\right].$$

Note: pressure inside the hut is p_∞. We only need net upward force due to outer pressure relative to inside.

$$\therefore F_y = -a\int_0^\pi (p_s - p_\infty)\sin\theta \, d\theta$$

$$= -a\int_0^\pi \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)\sin\theta \, d\theta$$

$$= -a\left[\frac{1}{2}\rho U_\infty^2 \cdot 2 - 2\rho U_\infty^2 \cdot \frac{4}{3}\right] \quad (2)$$

$$= \frac{5}{3} a \rho U_\infty^2.$$

PHY638: Quiz 2 Date : April 7, 2025 Inst: Abhishek Chaudhuri

ROLL NO : MS | | | | | | **NAME :**

[2+3]

1. For capillary waves with the dispersion relation:

$$\omega = \sqrt{k\left(g + \frac{\sigma k^2}{\rho}\right)\tanh(kH)}$$

where ρ is the liquid density and σ is the surface tension, determine the following:

(a) At what water depth H are the waves non-dispersive (wave speed c is independent of wavelength λ)?

(b) In deep water, if the gravitational effects are negligible, what is the relation between the group velocity c_g and the phase velocity c?

$\omega^2 = gk + \frac{\sigma}{\rho} k^3 \tanh(kH)$

(a) Waves are non-dispersive when wave speed ~~is~~ is independent of wavelength λ (or wavenumber k)

Phase speed $c = \frac{\omega}{k} \Rightarrow c^2 = \frac{\omega^2}{k^2} = \frac{g}{k} + \frac{\sigma k^3}{\rho}\tanh(kH)$

$\Rightarrow c^2 = g\frac{2\pi}{\lambda} + \frac{\sigma}{\rho}\left(\frac{2\pi}{\lambda}\right)^3 \tanh\left(\frac{2\pi H}{\lambda}\right)$ 1

For non-dispersive: $\frac{dc}{d\lambda} = 0$ or $\frac{dc}{dk} = 0$,

Rather than solving for this, recall:

in shallow water $kH \ll 1 \Rightarrow \tanh(kH) \simeq kH$

in deep water $kH \gg 1 \Rightarrow \tanh(kH) \simeq 1$

In shallow water, $\omega^2 \simeq gk + \frac{\sigma}{\rho} k^3 . kH$

$= gk + \frac{\sigma}{\rho} H k^4.$

→ still dispersive due to k^4 term.

For very shallow water & negligible surface tension ($\sigma = 0$): $\omega^2 = gk \Rightarrow c = \frac{\omega}{k} = \sqrt{gH}$

→ non-dispersive   1

(b) In deep water, when gravity is negligible,

$$\omega^2 \approx \frac{\sigma}{\rho} k^3 \Rightarrow \omega = \sqrt{\frac{\sigma}{\rho}}\, k^{3/2} \qquad 1$$

$$c = \frac{\omega}{k} = \sqrt{\frac{\sigma}{\rho}}\, k^{1/2}$$

Group velocity, $c_g = \dfrac{d\omega}{dk} = \dfrac{3}{2}\sqrt{\dfrac{\sigma}{\rho}}\, k^{1/2}$ 1

$$\therefore \frac{c_g}{c} = \frac{\frac{3}{2}\sqrt{\frac{\sigma}{\rho}}\, k^{1/2}}{\sqrt{\frac{\sigma}{\rho}}\, k^{1/2}} = \frac{3}{2}.$$

$$c_g = \underline{\frac{3}{2}c} \qquad 1$$

PHY638: Quiz 3 Date : April 17, 2025 Inst: Abhishek Chaudhuri

ROLL NO : MS **NAME :**

1. Using the velocity field

$$u_r = U\cos\theta\left(1 - \frac{3a}{2r} + \frac{a^3}{2r^3}\right), \text{ and } u_\theta = -U\sin\theta\left(1 - \frac{3a}{4r} - \frac{a^3}{4r^3}\right)$$

and the pressure $p - p_\infty = -\frac{3\mu a U\cos\theta}{2r^2}$, determine the drag on Stokes' sphere from the surface pressure and the viscous surface stresses σ_{rr} and $\sigma_{r\theta}$:

$$\sigma_{r\theta} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), \text{ and } \sigma_{rr} = 2\mu\frac{\partial u_r}{\partial r}.$$

Component of drag force per unit area in the direction of uniform stream is:

$$\left[-p\cos\theta + \sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta\right]_{r=a}$$

$$\sigma_{rr} = 2\mu\frac{\partial u_r}{\partial r} = 2\mu U\cos\theta\left[\frac{3a}{2r^2} - \frac{3a^3}{2r^4}\right]. \quad (1)$$

$$\sigma_{r\theta} = \mu\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right]$$

$$= \mu\left[-\frac{U}{r}\sin\theta\left(1 - \frac{3a}{2r} + \frac{a^3}{2r^3}\right) - U\sin\theta\left(\frac{3a}{4r^2} + \frac{3a^3}{4r^4}\right) + \frac{U\sin\theta}{r}\left(1 - \frac{3a}{4r} - \frac{a^3}{4r^3}\right)\right]$$

$$= \mu\left[\frac{U\sin\theta}{r}\left\{-\cancel{1} + \cancel{\frac{3a}{2r}} - \frac{a^3}{2r^3} - \cancel{\frac{3a}{4r}} - \frac{3a^3}{4r^3} + \cancel{1} - \cancel{\frac{3a}{4r}} - \frac{a^3}{4r^3}\right\}\right].$$

$$= -\frac{3\mu U a^3}{2r^4}\sin\theta \quad (1)$$

$$\therefore \left(\sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta\right)\Big|_{r=a} = \cancel{\quad} 0 + \frac{3\mu U}{2a}\sin^2\theta. \quad (1)$$

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Pressure contribution $= \frac{3\mu U}{2a}\cos^2\theta$. (1)

$\therefore \left[-p\cos\theta + \sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta\right]_{r=a} = \frac{3\mu U}{2a}\cos^2\theta + \frac{3\mu U}{2a}\sin^2\theta$

$= \frac{3\mu U}{2a}$.

$\therefore$ Drag force $= \frac{3\mu U}{2a} \times 4\pi a^2 = 6\pi\mu a U$. (1)

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