PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 1

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1. Consider the unperturbed wave functions and energies for an infinite 1-dimensional square well. Perturb the system by raising the floor of the well by a constant amount V_0, with the perturbation extending only halfway across the well. Calculate to first order the perturbed energies and wavefunctions.

2. Suppose a delta-function bump is put in the center of the infinite square well: $H' = \alpha\delta(x - a/2)$ where α is a constant. Find the first-order correction to the allowed energies.

3. Consider a harmonic oscillator with a slightly increased spring constant $k' = (1 + \epsilon)k$. Calculate the first order perturbation in the energy.

4. Two identical bosons are placed in an infinite square well. They interact weakly with one another via the potential $V(x_1, x_2) = -aV_0\delta(x_1 - x_2)$ where V_0 is a constant and a is the width of the well. Use first-order perturbation theory to calculate the effect of the particle-particle interaction on the ground and first excited state energies.

5. Consider a charged particle in a one-dimensional harmonic oscillator potential. Suppose a weak electric field is turned on so that the energy is shifted by an amount $H' = -qEx$. Show that there is no first order change in the energy levels and calculate the second order correction. The Schrodinger equation can be solved exactly for this case by a change of variables $x' = x - (qE/m\omega^2)$. Find the exact energies and show that they are consistent with the perturbation theory expressions.

6. Consider an isotropic harmonic oscillator in two dimensions with the Hamiltonian
$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2)$$
(a) What are the energies (and degeneracies) of the three lowest states? (b) Now apply a perturbation $H_p = \delta m\omega^2 xy$, where δ is a dimensionless real number $<< 1$. Find the energy upto first order for the ground state. (c) Solve the $H_0 + H_p$ problem exactly and compare with the perturbation results.

7. Find the exact energy and wave function of the ground state for an infinite cubic potential well of dimension L along x, y, z. Now add the perturbation:

$$H_p = V_0 L^3 \delta(x - \frac{L}{4})\delta(y - \frac{L}{4})\delta(z - \frac{L}{4})$$

Calculate the energy of the ground state using first-order perturbation theory.

8. Find the effect of a weak uniform electric field along the positive z axis $E = E_0\hat{z}$ on the ground state of a hydrogen atom. Ignore spin. Find an approximate value for the polarizability of the hydrogen atom.