Assignment 11

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Assuming that $f(z)$ is analytic on and within a closed contour C and that the point z_0 is within C, show that

$$\oint_C \frac{f'(z)}{z-z_0}dz = \oint_C \frac{f(z)}{(z-z_0)^2}dz.$$

2. For a square shaped contour with sides $a > 1$ and centred at $z = 0$, evaluate

$$\oint \frac{e^{iz}}{z^3}dz.$$

3. For a contour that encircles the point $z = a$, evaluate

$$\oint \frac{\sin^2 z - z^2}{(z-a)^3}dz.$$

4. For the contour the unit circle, evaluate

$$\text{(a)} \quad \oint \frac{dz}{z(2z+1)} \quad \text{and} \quad \text{(b)} \quad \oint \frac{f(z)}{z(2z+1)^2}dz.$$

5. Develope the Taylor expansion of $\ln(1+z)$ around $z = 0$.

6. Obtain the Laurent Series expansion of

$$\text{(a)} \quad f(z) = \frac{1}{z(z-1)} \quad \text{around} \quad z = 0, \quad \text{(b)} \quad f(z) = \frac{e^z}{z^2} \quad \text{around} \quad z = 0,$$

$$\text{(c)} \quad f(z) = \frac{ze^z}{z-1} \quad \text{around} \quad z = 1, \quad \text{(d)} \quad f(z) = (z-1)e^{1/z} \quad \text{around} \quad z = 0.$$

7. Find the residue of

$$\text{(a)} \quad f(z) = \frac{1}{\sin z} \quad \text{at} \quad z = 0, \quad \text{(b)} \quad f(z) = \frac{z}{\sin^2 z} \quad \text{at} \quad z = \pi,$$

$$\text{(c)} \quad f(z) = \frac{\ln z}{z^2+4} \quad \text{at} \quad z = 2e^{i\pi}, \quad \text{(d)} \quad f(z) = \frac{\cot \pi z}{z(z+2)} \quad \text{at} \quad z = 0.$$

8. Evaluate the residues at $z = 0$ and $z = -1$ of

$$\frac{\pi \cot \pi z}{z(z+1)}.$$