

# Indian Institute of Science Education and Research, Mohali

MS-II Year, 4<sup>th</sup> Semester

Second Mid-semester Examination : Probability and Statistic: MTH202

Time: 09.30 AM - 10.30 AM      Date: March 04, 2023      Total Marks: 15

Answer all FOUR questions.

1. Let  $X$  be the number of Heads in 50 tosses of a fair coin. Determine the expectation of  $X$ , the variance of  $X$  and  $P(X \leq 10)$ ? [1+1+1]
  
2. Each of the 50 students in class belongs to exactly one of the four groups  $A$ ,  $B$ ,  $C$ , or  $D$ . The numbers of students for the four groups are as follows:  $A : 5$ ,  $B : 10$ ,  $C : 15$ ,  $D : 20$ . First, choose one of the 50 students at random and let  $X$  be the size of that students group. Next, choose one of the four groups at random and let  $Y$  be its size. Here all random choices are with equal probability.
  - (i) Write down the probability mass functions for  $X$  and  $Y$ .
  - (ii) Compute the expectations of  $X$  and  $Y$ . [2+2]
  
3. Let  $X$  be a random variable with probability density function  $f_X(x) = cx^2$  for  $x \in [0, 1]$  (zero for other  $x \in \mathbb{R}$ ). [1+2+1]
  - (a) Find the value of  $c$ .
  - (b) Find the cumulative distribution function  $F_X$ .
  - (c) Find the expectation  $E(X)$ .
  
4. Let  $X$  be the random variable with probability density function  $f_X(x) = 7e^{-7x}, \forall x \geq 0$ . [1+3]
  - (a) Find the conditional probability  $P(X \geq 6 | X \geq 4)$ .
  - (b) Find the variance  $Var(X)$ .