PHY304: Statistical Mechanics

Assignment 7

February 25, 2025

1. The Hamiltonian for the system of N classical *distinguishable* harmonic oscillators with frequency ω is given by
$$H(q_i, p_i) = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 q_i^2$$
Calculate the thermodynamic properties in the *canonical ensemble.*

2. Set up the partition function of a classical relativistic ideal gas and obtain the free energy. Show that the chemical potential in the non-relativistic and the ultra-relativistic limits are given by
$$\text{Nonrelativistic :} \qquad \mu \approx mc^2 + kT\ln(n\lambda^3) \qquad (\lambda = \sqrt{2\pi\hbar^2/mkT})$$
$$\text{Ultra-relativistic:} \qquad \mu \approx kT\ln(nL^3) \qquad (L = \pi^{2/3}\hbar c/kT)$$

3. Consider a one-dimensional harmonic oscillator which is in equilibrium with a heat reservoir at absolute temperature T. The energy of such an oscillator is given by
$$E = \frac{p^2}{2m} + \frac{1}{2}k_0 x^2.$$
According to quantum mechanics the possible energy levels of the harmonic oscillator are given by
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (n = 0, 1, 2, \ldots),$$
where $\omega = \sqrt{k_0/m}$ is the classical angular frequency of oscillation of the oscillator.
   (a) Calculate the partition function for the quantum harmonic oscillator.
   (b) Calculate the mean energy $\langle E \rangle$ of the oscillator.
   (c) Show that at high temperatures $\hbar\omega \ll k_B T$, the mean energy of the oscillator is in agreement with the classical result.
   (d) Show that as $T \to 0$ it approaches the zero point energy of the ground state.

4. Using the grand canonical ensemble, evaluate the chemical potential $\mu(T, P)$ for an ultra-relativistic gas contained in a box of volume V.

5. A lattice gas consists of N sites, each of which may be occupied by at most one atom. The energy of a site is ϵ if occupied, and 0 if empty.

   (a) Calculate the grand partition function $\mathcal{G}(z, T)$ at fugacity z and temperature T.

   (b) What fraction of sites are occupied?

   (c) Find the heat capacity as a function of T at fixed z.