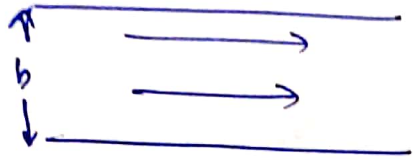
1.



$u = \frac{Uy}{b}$, $v = w = 0$.

(a) Strain rate : $S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

$$\underline{\underline{S}} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 & U/2b \\ U/2b & 0 \end{pmatrix}$$

(b) Rotation tensor : $R_{ij} = \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}$

$$\underline{\underline{R}} = \begin{pmatrix} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 \end{pmatrix} = \begin{pmatrix} 0 & U/b \\ -U/b & 0 \end{pmatrix}$$

(c) $\underline{\omega} = \underline{\nabla} \times \underline{u} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{Uy}{b} & 0 & 0 \end{vmatrix}$

$$= \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}\left(0 - \frac{U}{b}\right)$$

$$= -\frac{U}{b}\hat{z}.$$

2. $$\frac{D}{Dt}\int_{V(t)} \rho Q \, dV = \int_{V(t)} \left[\frac{\partial}{\partial t}(\rho Q) + \underline{\nabla}\cdot(\rho Q \underline{u})\right] dV$$

(By Reynold's transport theorem)

$$= \int_{V(t)} \left[\rho \frac{\partial Q}{\partial t} + Q \frac{\partial \rho}{\partial t} + \rho Q \, \underline{\nabla}\cdot\underline{u} + (\underline{u}\cdot\underline{\nabla})(\rho Q)\right] dV$$

$$= \int_{V(t)} \left[\rho\left\{\frac{\partial Q}{\partial t} + (\underline{u}\cdot\underline{\nabla})Q\right\} + Q\left\{\frac{\partial \rho}{\partial t} + (\underline{u}\cdot\underline{\nabla})\rho + \rho(\underline{\nabla}\cdot\underline{u})\right\}\right] dV$$

... What is $\epsilon_{pqr}\epsilon_{pqr} = ?$.

[2]

4. Find the streamlines and pathlines for the simple plane flow: [3]

$$u_i = \frac{x_1}{1+t}, \quad u_2 = x_2, \quad u_3 = 0.$$

$$= \int_{V(t)} \left[\rho \frac{D\phi}{Dt} + \phi \left\{ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) \right\} \right] dV.$$

$$= \int_{V(t)} \rho \frac{D\phi}{Dt} \, dV.$$

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PHY638 MidSem I (Part A) **Date : Feb 7, 2025** **Inst: Abhishek Chaudhuri**

- **Time : 30 minutes, Max Marks : 10**
- **Attempt all questions. Please give your answers in the space provided.**

1. Show that $\hat{e}_i = \frac{1}{2}\varepsilon_{mni}(\hat{e}_m \times \hat{e}_n)$ [2]

$$\varepsilon_{mni}(\hat{e}_m \times \hat{e}_n) = \varepsilon_{mni}\varepsilon_{mnj}\hat{e}_j$$
$$= \varepsilon_{nim}\varepsilon_{mnj}\hat{e}_j \quad \text{(even permutation)}$$
$$= (\delta_{nn}\delta_{ij} - \delta_{nj}\delta_{in})\hat{e}_j \quad (\because \varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})$$
$$= (3\delta_{ij} - \delta_{nj}\delta_{in})\hat{e}_j$$
$$= (3\delta_{ij} - \delta_{ij})\hat{e}_j = 2\delta_{ij}\hat{e}_j = 2\hat{e}_i.$$

2. Let a one-dimensional velocity field be $u_1 = u(x_1, t)$, with $u_2 = 0$ and $u_3 = 0$. The density varies as $\rho = \rho_0(2 - \cos\omega t)$. Find an expression for $u(x_1, t)$ if $u(0, t) = U$. [3]

$$\underline{u} = u(x_1, t)\hat{e}_1$$

Eqn. of continuity : $\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{u}) = 0.$

$$\Rightarrow \frac{\partial\rho}{\partial t} + u_1\frac{\partial\rho}{\partial x_1} + \rho\frac{\partial u_1}{\partial x_1} = 0.$$

$\rho = \rho_0(2 - \cos\omega t) = f(t) \quad \therefore \frac{\partial\rho}{\partial x_1} = 0.$

$$\therefore \frac{\partial\rho}{\partial t} + \rho\frac{\partial u_1}{\partial x_1} = 0 \Rightarrow \frac{\partial u_1}{\partial x_1} = -\frac{1}{\rho}\frac{\partial\rho}{\partial t}$$
$$= -\frac{\cancel{\rho_0}\sin\omega t\cdot\omega}{\cancel{\rho_0}(2 - \cos\omega t)}$$

Integrating, $u_1 = -\frac{\omega\sin\omega t}{2 - \cos\omega t}x_1 + c(y, z, t)$

From the initial condn : $u(0, t) = U \Rightarrow C = U.$

$$\therefore u_1 = U - \omega\frac{\sin\omega t}{2 - \cos\omega t}x_1$$
$$\Rightarrow u(x_1, t) = U - \omega\frac{\sin\omega t}{2 - \cos\omega t}x_1$$

3. **What is $\epsilon_{pqr}\epsilon_{pqr}$ =?.** [2]

$\varepsilon_{pqr}\varepsilon_{pqr} = \varepsilon_{pqr}\varepsilon_{rpq}$ (even permutation)

$= \delta_{pp}\delta_{qq} - \delta_{pq}\delta_{pq}$ $\quad (\because \varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})$

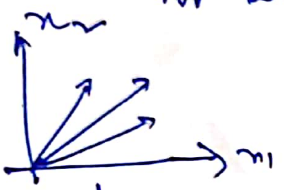
$= 3(3) - \delta_{pp}$

$= 9 - 3 = 6.$

4. **Find the streamlines and pathlines for the simple plane flow:** [3]

$$u_i = \frac{x_1}{1+t}, \quad u_2 = x_2, \quad u_3 = 0.$$

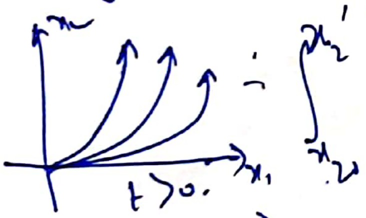
For streamlines, $\dfrac{dx_1}{u_1} = \dfrac{dx_2}{u_2} \Rightarrow \dfrac{dx_1}{x_1/1+t} = \dfrac{dx_2}{x_2}$



$\Rightarrow (1+t)\dfrac{dx_1}{x_1} = \dfrac{dx_2}{x_2} \Rightarrow \int \dfrac{dx_2}{x_2} = (1+t)\int \dfrac{dx_1}{x_1}.$

Assume $x_1 = x_{10}$ & $x_2 = x_{20}$ at $t=0$

& $x_2 = x_2'$ & $x_1 = x_1'$ at $t = t'$

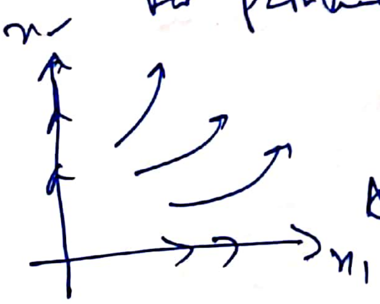


$\therefore \displaystyle\int_{x_{20}}^{x_2'} \frac{dx_2}{x_2} = (1+t)\int_{x_{10}}^{x_1'} \frac{dx_1}{x_1} \Rightarrow \ln\left(\frac{x_2'}{x_{20}}\right) = \ln\left(\frac{x_1'}{x_{10}}\right)^{1+t}$

$\Rightarrow \dfrac{x_2}{x_{20}} = \left(\dfrac{x_1}{x_{10}}\right)^{1+t}$

$\Rightarrow x_2 = x_{20}e^{t}$

For pathlines, $u_1 = \dfrac{dx_1}{dt} = \dfrac{x_1}{1+t}$, $u_2 = \dfrac{dx_2}{dt} = x_2$, $u_3 = \dfrac{dx_3}{dt} = 0$



$\Rightarrow \dfrac{dx_1}{x_1} = \dfrac{dt}{1+t} \Rightarrow \ln x_1 = \ln(1+t) + \ln c.$

At $t=0$, $x_1 = x_{10} \Rightarrow c = x_{10}. \Rightarrow x_1 = x_{10}(1+t)$

$t = \left(\dfrac{x_1}{x_{10}} - 1\right).$

$\therefore$ Curves in the x-y plane : $x_2 = x_{20}e^{(x_1 - x_{10})/x_{10}}$