INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 (Symmetry)

Tutorial Sheet 04 / February 01, 2022 Spring 2022

Let us not forget the **elementary matrices**: $S_{p,q}$, $M_p(\lambda)$ and $L_{p,q}(\lambda)$. Write them once more for your recollection. Multiplying these matrices on the left of a matrix A is called an **elementary row operation** on A.

Let us call a matrix to be a **row echelon matrix**[1] if it has the following three properties.

I. First nonzero entry in each row is 1. This entry is to be called the **pivot** of the row.

II. The pivot of a (not entirely 0) row is to the right of the pivot of the preceding row. If a row is entirely 0 then all the subsequent rows are also entirely 0.

III. All entries above pivots are zero. (or equivalently, the pivot element of a row is the only nonzero element of the column it belongs to).

[1] Different books will have a variation in this definition. We stick to the above definition in this course.

We may convert a matrix into a row echelon matrix through successive elementary row operations.

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1. Which of the following are row echelon matrices?

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 5 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In each of the cases when matrix is not row echelon, list the condition(s) I, II, III of the definition that it fails to satisfy.

2. Using 0, 1 and 2 make as many 2×2 row echelon matrices as you can.

3. Using 0 and 1 how many 3×3 row echelon matrices can you make? List all of them.

4. Is there a 3×3 rotation matrix which is a row echelon matrix?

5. Convert the following matrices into a row echelon matrix by suitable sequence of elementary row operations.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \begin{pmatrix} 1 & -1 & 4 & 3 \\ 2 & 1 & 0 & 3 \\ 2 & 1 & 5 & 0 \end{pmatrix}$$

6. Take a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ for which $ad - bc \neq 0$. Then multiply A by suitable elementary matrices to convert it to a row echelon matrix in each of the following cases.

   (a) When $a \neq 0$. (b) When $a = 0$ but $b \neq 0$.

   Keeping track of which elementary matrices were used in the process, find a 2×2 matrix B for which $AB = BA = I_2$? Can you write A itself as a product of elementary matrices?
