

TUTORIAL-RANDOM VARIABLE-II

MTH202: SPRING 2023

- (1) An urn contains 11 balls, 3 white, 3 red and 5 blue balls. Take out 3 balls at random, without replacement. You win Rs 1 for each red ball you select and lose a Rs 1 for each white ball you select. Determine the probability mass function of X , the amount you win. What is the average value of X ?
- (2) Let X be the number shown on a rolled dice. Compute the expectation and variance of X .
- (3) **(Binomial Random Variable)** A Binomial (n, p) random variable counts the number of success in n independent trials, each of which is a success with probability p . Prove the following.
 - (i) The probability mass function $P(X = i) = \binom{n}{i} p^i (1 - p)^{n-i}$, $i = 0, \dots, n$.
 - (ii) The expectation of X is $E(X) = np$.
 - (iii) The variance of X is $Var(X) = np(1 - p)$.
- (4) Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called the hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr , result in the phenotype of the dominant gene, while rr results in a recessive phenotype. Assuming that both parents are hybrid and have n children, what is the probability that at least two will have the recessive phenotype? Each child, independently, gets one of the genes at random from each parent.
- (5) Suppose that the probability that a person is killed by lightning in a year is, independently, $1/50,000,000$. Assume that the entire population is 300 million.
 - (i) Compute the probability that 3 or more people will be killed by lightning next year exactly.
 - (ii) Approximate the probability that two or more people are killed by lightning within the first 6 months of next year.
 - (iii) Compute the expected number of years, among the next 10, in which 2 or more people are killed by lightning.
- (6) You roll a die, your opponent tosses a coin. If you roll 6 you win; if you do not roll 6 and your opponent tosses Heads you lose; otherwise, this round ends and the game repeats. On the average, how many rounds does the game last?