IDC205, Differential Equations for Scientists

Monsoon 2022

9/12/2022, 9:30AM-12:30 NOON.

INSTRUCTIONS

- The duration of the exam is 3 hours.
- No electronic gadgets are allowed. In particular, cellphones and calculators are not allowed inside the examination premise (LHC).
- Provide complete, coherent and detailed answers to each and every question. Full credits will be awarded only to "perfect solutions".

FINAL EXAMINATION

(1) (5 points) Find two linearly independent closed form solutions of the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0.$$

(2) (5 points) Find two linearly independent power solutions of the differential equation

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 2y = 0$$

about the singular point $x = 0$.

(3) (5 points) Find two linearly independent power series solutions of

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$$

about the ordinary point $x = 0$.

(4) (5 points) Use the method of Laplace transformation to solve the initial value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^{-x}; \qquad y(0) = 0, \quad y'(0) = 1.$$

(5) (5 points) Use the method of undetermined coefficients to solve the initial value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^{-x}; \qquad y(0) = 0, \quad y'(0) = 1.$$

(6) (5 points) Consider the equation

$$\left(\frac{1}{x^2y^2}+\frac{x}{y^3}\right)dx+N(x,y)dy=0$$

where N is a real valued differentiable function. Find the most general form of $N(x,y)$ so that the above equation is exact and solve the resulting exact differential equation.

(7) (5 points) Find a third order linear homogeneous differential equation with variable coefficients for which $\{1, x, \ln(x)\}$ is a fundamental set of solutions.

(8) (5 points) Find the orthogonal trajectory of the family

$$y = cx^2;\ \ c \text{ is a constant}$$

that passes through $(1,0)$.