Assignment 5

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Show that

   (a) $L_n(0) = 1$

   (b) $L'_n(0) = -n$

   (c) $L''_n(0) = n(n+1)/2$

2. Show that
$$\int_0^x L_n(t)dt = L_n(x) - L_{n+1}(x).$$

3. Evaluate interms of delta functions
$$\int_0^\infty xe^{-x} L'_m(x) L_n(x) dx.$$

4. The wave equation for the three-dimensional harmonic oscillator is
$$-\frac{\hbar^2}{2m}\nabla^2\psi + \frac{1}{2}m\omega^2 r^2\psi = E\psi.$$
Here ω is the angular frequency of the corresponding classical oscillator. Show that the radial part of ψ in spherical polar coordinates may be written in terms of associated Laguerre functions of argument βr^2, where $\beta = m\omega/\hbar$.

5. Obtain the second solution of the Laguerre equation for arbitrary n.

6. The generating function for the associated Laguerre polynomials is
$$g(x,t) = \frac{1}{(1-t)^{k+1}} \exp -\frac{xt}{1-t} = \sum_{n=0}^{\infty} L_n^k(x) t^n.$$

   (a) Prove the identity
$$(1-t)\frac{\partial g}{\partial t} + [x - (1-t)(1+k)]\, g = 0$$
   and then derive the recurrence relation
$$(n+1)L_{n+1}^k(x) - (2n+1+k-x)L_n^k(x) + (n+k)L_{n-1}^k(x)$$
   with $n \geq 1$.

   (b) Prove the identity
$$(1-t)\frac{\partial g}{\partial x} + tg(x,t) = 0$$
   and then derive the following relation
$$\frac{dL_n^k(x)}{dx} - \frac{dL_{n-1}^k(x)}{dx} + L_{n-1}^k(x) = 0$$
   with $n = 1, 2, \cdots$.