Assignment I

1. Show that for a single particle with constant mass the equation of motion implies that

$$\frac{dT}{dt} = \vec{F}.\vec{v}$$

2. Prove that the magnitude R of the center of mass from any arbitrary origin is given by an equation

$$M^2 R^2 = M \sum m_i r_i^2 - \frac{1}{2} \sum m_i m_j r_{ij}^2$$

3. A particle moves in the x-y plane under the constraint that its velocity vector is always directed towards a point on the x axis whose abscissa is a function of time : $f(t)$. Show that for $f(t)$ differentiable, but otherwise arbitrary, the constraint is non-holonomic.

4. Two points of mass m are joined by a rigid weightless rod of length l, the center of which is constrained to move on a circle of radius a. Set up the kinetic energy of the system in the generalised coordinates.

5. Show that the Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

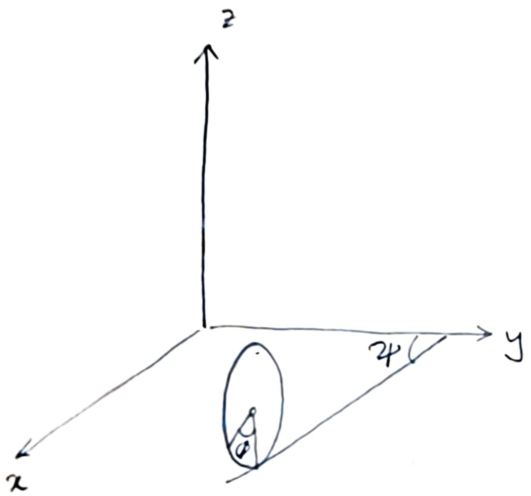
Can also be written down as $\dfrac{\partial \dot{T}}{\partial \dot{q}_j} - 2\dfrac{\partial T}{\partial q_j} = Q_j$

6. If L is a Lagrangian for a system with N degrees of freedom satisfying the Lagrange equations, show ~~by~~ that

$$L' = L + \frac{dF}{dt} \quad \text{where} \quad F = F(q_1, q_2, \ldots q_N, t)$$

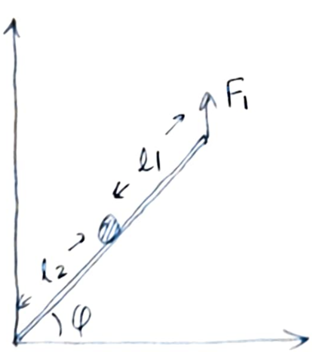
(F is arbitrary but differentiable), also satisfies Lagrange equations of motion.

7.



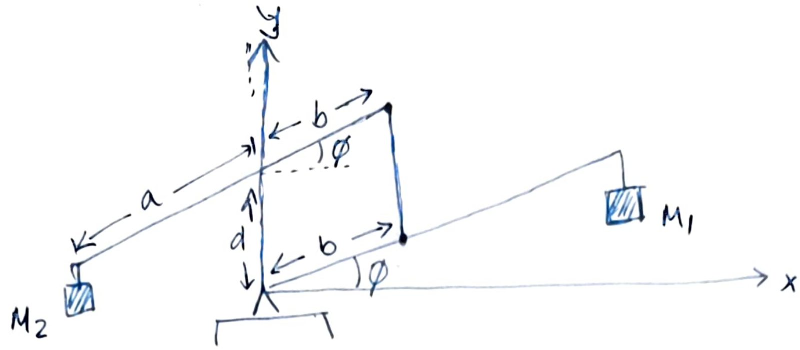
Consider a wheel that rolls on a plane without gliding. The wheel can not fall over. The radius of the wheel is a. Find out the equation(s) of the constraint.

8.



Find the equilibrium condition for the lever of length l_1 with a mass m at a distance of l_2 from the bearing point, and with a force F_1 acting vertically upward at its end as shown.

9.



Find out the equilibrium condition for the above.

10. A sphere moves in a tube that rotates in the x-y plane about the z axis with constant angular velocity ω. Determine the equation of motion for r and solve it.

11. Find the vibration frequencies of a linear three atom symmetric molecule A-B-A.