PHY102 : Assignment 9

1. (P&M 9.18) If the electric field in free space is $\mathbf{E} = E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin[(2\pi/\lambda)(z + ct)]$ with $E_0 = 20$ volts/m, then the magnetic field, not including any static magnetic field, must be what?

2. (P&M 9.26) Here is a particular electromagnetic field in free space :

$$E_x = 0 \quad E_y = E_0 \sin(kx + \omega t) \quad E_z = 0$$
$$B_x = 0 \quad B_y = 0 \quad B_z = -\frac{E_0}{c}\sin(kx + \omega t)$$

   Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

3. (P&M 9.23) Show that the electromagnetic field described by

$$\begin{aligned} \mathbf{E} &= E_0\hat{\mathbf{z}}\cos kx \cos ky \cos \omega t \\ \mathbf{B} &= B_0(\hat{\mathbf{x}}\cos kx \sin ky - \hat{\mathbf{y}}\sin kx \cos ky)\sin \omega t \end{aligned}$$

   will satisfy Maxwell's equations in vacuum if $E_0 = \sqrt{2}cB_0$ and $\omega = \sqrt{2}ck$.

4. (P&M 9.32) Starting from the field transformations, show that the scalar quantity $E^2 - c^2B^2$ is invariant under the transformation. In other words, show that $E^2 - c^2B^2 = E'^2 - c^2B'^2$. You can do this using only vector algebra, without writing out x, y, z components of anything. (The resolution into parallel and perpendicular vectors is convenient for this, since $\mathbf{E}_\perp \cdot \mathbf{E}_\parallel = 0$, $\mathbf{B}_\parallel \times \mathbf{E}_\parallel = 0$, etc.)

5. Imagine two concentric metal spherical shells (Fig.). The inner one (radius a) carries a charge $Q(t)$, and the outer one (radius b) an opposite charge $-Q(t)$. The space between them is filled with Ohmic material of conductivity σ, so a radial current flows:

$$\mathbf{J} = \sigma\mathbf{E} = \sigma\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}; \quad I = -\dot{Q} = \int \mathbf{J}\cdot d\mathbf{a} = \frac{\sigma Q}{\epsilon_0}.$$

   This configuration is spherically symmetrical, so the magnetic field has to be zero (the only direction it could possibly point is radial, and $\nabla \cdot \mathbf{B} = 0$ should give $\oint \mathbf{B} \cdot d\mathbf{a} = B(4\pi r^2) = 0$, so $\mathbf{B} = 0$). What? I thought currents produce magnetic fields! Isn't that what Biot-Savart and Ampère taught us? How can there be a $\mathbf{J}$ with no accompanying $\mathbf{B}$?