Assignment – 4

1. Determine by inspection one particular solution of the following D.E.'s –

   a) $y'' - (x-1)y'(x) + (x-2)y(x) = 0$

   b) $x^2 y''(x) + 4x y'(x) + 2y(x) = 0$

2. By changing the dependent variable, solve the DE – $y''(x) + 2xy'(x) + x^2 y(x) = 0$ and show that the general solution is

   $$y(x) = \left[A e^x + B e^{-x}\right] \exp\left(-\frac{x^2}{2}\right)$$

3. Determine one solution of the DE $xy''(x) - y'(x) - (x-1)y(x) = 0$ by inspection and hence show that the general solution is $y(x) = A e^x + B e^2(2x+1)$

4. verify that $y = \cos ax$ is one of the solutions of

   $$y''(x) + 2a(\cot ax) y'(x) + 3a^2 y(x) = 0$$

   and show that the general solution of

   $$y''(x) + 2a(\cot ax)y'(x) + 3a^2 y(x) = \csc ax \text{ is given}$$

   by $y(x) = C_1 \cos ax + C_2 \dfrac{\cos 2ax}{\sin ax} + \dfrac{1}{4a^2} \csc ax.$

5. By change of the dependent variable solve the DE

   $$y''(x) + 2y'(x) + \left(1 - \frac{12}{x^2}\right) y(x) = x^2 + 4x - 10$$

   and show that the general solution is

   $$y(x) = e^{-x}\left(Ax^4 + \frac{B}{x^3}\right) + x^2$$

6. Find by inspection one solution of the following D.E.

   $$(x^2-1)x^2 y''(x) - (x^2+3)xy'(x) + (x^2+3)y(x) = 0$$

   and show that the general solution is given by

   $$y = Ax + B\left(x\ln x + \frac{1}{x} - \frac{1}{4x^3}\right)$$