Assignment 12

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Prove that
$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^n} = \frac{\pi}{2^{2n-2}} \frac{(2n-2)!}{[(n-1)!]^2}$$

2. Evaluate the integral
$$\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta}$$
by the method of residues when $-1 < a < 1$.

3. For $n = 0, 1, 2, \cdots \infty$, show that
$$\int_0^{\pi} \cos^{2n}\theta d\theta = \pi \frac{(2n)!}{2^{2n} n!}.$$

4. Using method of residues, show that
$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}.$$

5. Prove that
$$\int_0^{\infty} e^{-x^2}\cos(2ax)dx = \frac{\sqrt{\pi}}{2} e^{-a^2}$$
by integrating e^{-z^2} around a rectangular contour whose vertices are $(0,0)$, $(R,0)$, (R,ia), and $(0,ia)$.

6. For $a > 1$, show that
$$\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = \frac{\pi a}{(a^2-1)^{3/2}}.$$

7. For $|t| < 1$, show that
$$\int_0^{2\pi} \frac{d\theta}{1-2t\cos\theta+t^2} = \frac{2\pi}{1-t^2}.$$

8. Using method of Residues, evaluate
$$\int_0^{\infty} \frac{\cos x}{x^2+1} dx.$$

9. Using method of Residues, show that
$$\int_0^{\pi} \frac{d\theta}{1+\sin^2\theta} = \frac{\pi}{\sqrt{2}}.$$

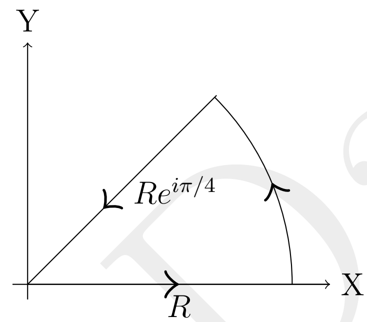
10. Using method of Residues, show that

$$\int_0^\infty \frac{x \sin x}{x^2+1} dx = \frac{\pi}{2e}.$$

11. Prove that

$$\int_0^\infty \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$

Hint: One can choose the following complex function: $f(z) = \exp(iz^2)$ and the contour shown on the right.



12. Using method of residues, show that

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}.$$

13. Using method of residues, show that

$$\int_0^\infty \frac{\sin^3 x}{x^3} dx = \frac{3\pi}{8}.$$

14. Using method of residues, calculate

$$\int_{-\infty}^{+\infty} \frac{\cos ax}{x^2+x+1} dx.$$

15. Using method of residues, show that

$$\int_0^\infty \frac{\sin mx}{x(x^2+a^2)} dx = \frac{\pi}{2a^2}(1-e^{-ma}).$$