(1)

$$r = \frac{r_0}{1 + e\cos\phi}$$

$$r_0 = J^2/GM\mu^2$$

$$e^2 = 1 - 2\frac{J^2|E|}{G^2M^2\mu^3}$$

$$2a = r_{min} + r_{max}$$

$$= \frac{r_0}{1-e} + \frac{r_0}{1+e}$$

$$= \frac{2r_0}{1-e^2} = \frac{2J^2}{GM\mu^2}\cdot\frac{G^2M^2\mu^3}{2J^2|E|}$$

$$2a = \frac{GM\mu}{2|E|} \Rightarrow a = \frac{GM\mu}{2|E|}$$

$$|E| = \frac{GM\mu}{2a} \qquad J^2 = GM\mu^2 r_0$$

If $\mu \equiv m_1 << m_2$, $\mu \simeq m_1$, $M \simeq m_2$

In our case, $m_1 = 1 \Rightarrow \mu \simeq 1$

$m_2 = M_\odot \Rightarrow M \simeq M_\odot$

$$\Rightarrow |E| = \frac{GM_\odot}{2a}, \quad J^2 = GM_\odot r_0$$

$$= GM_\odot a$$

for circular orbit.

$$\Rightarrow |E| = \frac{6.7\times10^{-11}\times 2\times10^{30}}{2\times 1.5\times10^{11}}$$

$$= 4.5\times10^{8}\ J$$

$$J^2 = 6.7\times10^{-11}\times 2\times10^{30}\times 1.5\times10^{11}$$

$$= 6.7\times3\times10^{30}$$

$$= 20.1\times10^{30}$$

$$\Rightarrow J \simeq 4.5\times10^{15}\ Kg\ m^2\ s^{-1}$$

Transfer orbit

$$2a = (1.5 + 2.3) \times 10^{11}\ m = 3.8 \times 10^{11}\ m$$

$$a = 1.9 \times 10^{11}\ m$$

$$r_{min} = a(1-e) = 1.5 \times 10^{11}\ m$$

$$\Rightarrow \quad 1 - e = \frac{1.5}{1.9} = 0.79$$

$$\Rightarrow e = 0.21$$

$$\Rightarrow |E| = \frac{GM}{2a} = 3.55 \times 10^{8}\ J$$

$$\Delta E = 9.5 \times 10^{7}\ J$$

$$J^2 = GM\mu^2 r_0 = GM_\odot a (1-e^2)$$

$$\Rightarrow J = 4.9 \times 10^{15}\ kg m^2 s^{-1}$$

$J = m_1 v r$ at perihelion,

$$\Rightarrow \Delta J = m_1 r \Delta v \Rightarrow \Delta v = \frac{\Delta J}{m_1 r}$$

$$\Delta v = \frac{0.4 \times 10^{15}}{1.5 \times 10^{11}} = 2.67\ km/s$$

Mars orbit $a = 2.3 \times 10^{11}$ m

$$|E| = \frac{GM_0}{2a} = 2.9 \times 10^{8} \text{ KJ m}^2 \text{s}^{-1}$$

$$J^2 = 5.5 \times 10^{15} \text{ kg m}^2 \text{s}^{-1}$$

$$\Delta v = \frac{\Delta J}{m_1 a_{max}} = \frac{0.6 \times 10^{15}}{2.3 \times 10^{11}}$$

$$= 2.6 \text{ km s}^{-1}$$

In both cases velocity needs to be increased.

$$\Delta E = 6.5 \times 10^{7} \text{ J}$$

$$\Delta J = 6 \times 10^{14} \text{ kg m}^2 \text{s}^{-1}$$