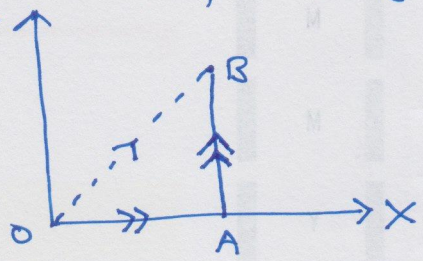
Assignment 10

Date - 10/11/2023
No Need To Submit.

1. Show that the integral $\int_{3+4i}^{4-3i} (4z^2 - 3iz)\,dz$ has the same value on the two paths: (a) the straight line connecting the integration limits, and (b) an arc on the circle $|z| = 5$.

2. Verify that $\int_0^{1+i} z^*\,dz$ depends on the path by evaluating the integral for two paths as shown in figure. Explain the outcome.



3. Show that $\oint_C \frac{dz}{z^2 + z} = 0$ in which the contour C is a circle defined by $|z| = R > 1$.

4. Evaluate $\oint_C \frac{dz}{z}$ on a square with vertices at $\pm 1 \pm i$.

5. Evaluate $\oint_C \frac{dz}{z^2 - 1}$, where C is a circle $|z-1| = 1$.

6. Evaluate $\oint_C \frac{e^{iz}}{z^3} dz$ for the contour a square with sides of length $a > 1$, centred at $z = 0$.

7. Evaluate $\oint_C \frac{\sin^2 z - z^2}{(z-a)^3} dz$ where the contour encircles the point $z = a$.

8. Obtain the Laurent expansion of

(a) $\frac{e^z}{z^2}$ about $z = 0$

(b) $\frac{ze^z}{z-1}$ about $z = 1$ and (c) $(z-1)e^{1/z}$ about $z = 0$.

9. Find the residue of $\frac{1}{\sin z}$ at $z=0$.

10. Find the residue of $\frac{z}{\sin^2 z}$ at $z=\pi$.

11. Determine the nature of the singularities of each of the following functions and evaluate the residues ($a>0$)

(a) $\frac{1}{z^2+a^2}$ (b) $\frac{\sin 1/z}{z^2+a^2}$ (c) $\frac{ze^{iz}}{z^2+a^2}$ (d) $\frac{z^{-k}}{z+1}$ $0<k<1$