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Indian Institute of Science Education and Research, Mohali

Integrated MSc, Semester: IV *Probability and Statistics: MTH 202*

End Semester Examination (9 AM-12 NOON, April 21, 2022)

Answere **ALL** the question with neat and clean explanation, any unfair means will attract severe penalty

Maximum Mark: 50

1. What is the probability that two natural numbers chosen at random will be prime to each other? [5]

2. A bag contains 17 coins marked with the number 1 to 17. A coin is drawn and replaced (by an identical coin), a second drawing is then made. What is the probability that the first number drawn is even and the second odd? [5]

3. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA? [5]

4. A box contains 15 white balls and 10 black balls. Suppose 5 balls are drawn at random. Find the expected value of the number of white balls drawn. [5]

5. Let X be random variable having exponential distribution with parameter 3. Then [1.5 × 4]

   (a) Compute the conditional probability $P(X \geq 5|X \geq 4)$.

   (b) Find the value of hazard rate $\lambda(x)$ at $x = 5$.

   (c) Find the moment generating function $\phi(t)$ of $2X$ at $t = -6$.

   (d) Find the density of X^2.

6. Consider the annual rainfall of Chandigarh in centimeter and let denote this random variable by X. Suppose average rainfall $E(X) = 80\ cm$ and variance $Var(X) = 10$. By Chebyshev's inequality give an estimate of lower bound on the probability that rainfall will be in between 70 cm to 90 cm. [4]

7. Suppose a point $Z = (X, Y)$ is chosen randomly in $\mathbb{R}^2$. Let the joint density of continuous random variable $Z = (X, Y\ )$ is given by

$$f_{X,Y}(x, y) = \begin{cases} 12xy(1-x) & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

[3+1+1]

   Then

   (a) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$.

   (b) Are X, Y independent?

   (c) Find expectation of X.

[Please turn over]

8. Let $Z = (X, Y)$ be a continuous bi-variate random variable with joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{e^{-\frac{x}{y}}e^{-y}}{y} & \text{if } 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

For $y > 0$, answer the following [3+2]

(a) Find the conditional probability density functions $f_{X|Y}(x|y)$ of X given $Y = y$.

(b) Find conditional expectation of $E[X|Y = y]$ of X given $Y = y$.

9. Let X and Y are independent random variables, distributed uniformly on $(0, 1)$. Then [4+1]

(a) Find the joint density $f_{S,T}(s,t)$ for the bi-variate random variable (S, T) where $S = X + Y$ and $T = (X - Y)$.

(b) Find the co-variance between random variables S and T.

10. Suppose during winter season of a year, daily income (in rupees) of a tourist guide in Shimla is independent and identically distributed with mean $1,000$ and variance 225. Using central limit theorem approximate the the probability that the guide will earn more than one lakh three thundred rupees in 100 days. You may assume that the cumulative probability distribution function F of standard normal random variable takes value $F(2) = 0.977$. [5]