PHY304: Statistical Mechanics

Assignment 1

January 8, 2025

1. (a) Show that the relation (R, θ, v_0 are positive constants)

   $$u = \left(\frac{v_0\theta}{R}\right)\frac{s^2}{v}\, e^{s/R},$$

   where, $u = U/N$, $v = V/N$, and $s = S/N$ obeyed by a system is a fundamental relation.

   (b) Find the three equations of state for the above fundamental relation.

   (c) Show that the equations of state obtained in part (b) is are homogeneous zero order (i.e., T, P, and μ are intensive parameters).

2. (a) Find the three equations of state for a system with the fundamental equation

   $$U = \left(\frac{v_0\theta}{R^2}\right)\frac{S^3}{NV}.$$

   (b) Find μ as a function of T, V and N for the above system.

3. Find the three equations of state in the *entropy representation* for a system with the fundamental relation

   $$u = \left(\frac{v_0^{1/2}\theta}{R^{3/2}}\right)\frac{s^{5/2}}{v^{1/2}}.$$

4. Find the relation among T, P, and μ for the system with the fundamental equation

   $$U = \left(\frac{v_0^2\theta}{R^3}\right)\frac{S^4}{NV^2}.$$

5. It is found that a particular system obeys the relations

   $$U = PV \qquad \text{and} \qquad P = BT^2,$$

   where B is a constant. Find the fundamental equation of this system.

6. Calculate the three equations of state of a simple paramagnetic model system with the fundamental equation $U = U(S, I, N)$

$$U = \frac{\mu}{2N\chi} I^2 + N\epsilon \exp\left(\frac{2S}{NR}\right),$$

where χ and ϵ are positive constants. Note I is the magnetic moment which is an extensive parameter. The intensive parameter conjugate to I is B_e, the external magnetic field that would exist in the absence of the system

$$B_e = \left(\frac{\partial U}{\partial I}\right)_{S,N}.$$

   (a) Calculate the three equations of state i.e., $T(S, I, N)$, $B_e(S, I, N)$ and $\mu(S, I, N)$.

   (b) Show that the three equations of state satisfy the Euler relation.

7. (a) The two equations of state (EoS) for the "ideal van der Waals fluid" are

$$P = \frac{RT}{v-b} - \frac{a}{v^2}; \qquad \frac{1}{T} = \frac{cR}{u + a/v}.$$

   Using the above two EoS, find the fundamental equation for the "ideal van der Waals fluid".

   (b) Find the fundamental equation of the ideal van der Waals fluid in the Helmholtz representation.

   (c) Perform an inverse Legendre transform on the Helmholtz potential and show that the fundamental equation in the energy representation is recovered.

8. The enthalpy of a particular system is

$$H = A\frac{S^2}{N} \ln\left(\frac{P}{P_0}\right),$$

where A is a positive constant. Calculate the molar heat capacity at constant volume $c_v = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_V$ as a function of T and P.