INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 : Linear Algebra (2023-24) **Tutorial 04 (October 05, 2023)**

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1. Take a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Multiply A, on left, by suitable elementary matrices to convert it to a row echelon matrix in each of the following cases.

   (a) When $a \neq 0$.

   (b) When $a = 0$ but $c \neq 0$.

   (c) When $a = c = 0$.

2. Examine your calculation in Exercise 1, and argue the following.

   (a) A 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible if and only if $ad - bc \neq 0$.

   (b) Every invertible 2×2 matrix is a product of at most four elementary matrices.

3. Convert the following matrices to a row echelon matrix and determine which of these are invertible.

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{pmatrix}, \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}, \begin{pmatrix} 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \\ 12 & 2 & -3 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & -3 & 0 \end{pmatrix}$$

   In each case where the matrix is invertible find the inverse.

4. Recall the puzzle from your childhood. "*An elephant costs ₹5, a horse costs ₹1, and 20 camels cost ₹1. You have ₹100 with you, and you need to purchase exactly 100 animals spending the entire money. How many elephants, horses and camels would you be purchasing?*"

   (a) Express this puzzle in the form of a system of linear equations.

   (b) Express this puzzle in the form of a matrix equation $Au = p$.

   (c) Use row operations to convert the augmented matrix $(A \mid p)$ into a row echelon matrix. Use this row echelon matrix to find a solution of the puzzle.

(d) Does this puzzle have more than one solution? Justify your answer.

5. A square matrix A is called an *idempotent matrix* if $A^2 = A$. Among all $n \times n$ matrix units e_{ij}, identify which ones are idempotent matrices. Are there idempotent matrices which are not matrix units?

6. Solve the following systems of linear equations.

(a)
$$\begin{aligned} 8x + y + 6z &= 20 \\ 3x + 5y + 7z &= 40 \\ 4x + 9y + 2z &= 60 \end{aligned}$$

(b)
$$\begin{aligned} 2x + 3y - z &= 2 \\ x - y + z &= 5 \\ x + 9y - 5z &= 10 \end{aligned}$$

7. Take three points in a plane: $(1, 2)$, $(2, 7)$ and $(-1, 4)$. If their x-coordinates and y-coordinates are related by $y = ax^2 + bx + c$. Find a, b and c. See the footnote[1].

8. Take three points in a plane: $(1, 2)$, $(2, 7)$ and $(-1, 4)$. Interpolate them through a suitable circle $(x - a)^2 + (y - b)^2 = c^2$. You need to see the footnote of Exercise 7 to understand this question.

9. Examine which of the following relations are (i). reflexive, (ii). symmetric, (iii). transitive

$\mathcal{R}1$. On the set $M_{m\times n}(\mathbb{R})$ consider the following relation: for two $m\times n$ matrices A and B, the matrix A is related to B if $A = EB$ for some $m \times m$ elementary matrix E.

$\mathcal{R}2$. On $\mathbb{N}$, the set of natural numbers, consider the following relation: a number k is related to a number ℓ if every divisor of k is also a divisor of ℓ.

$\mathcal{R}3$. On MS23, the set of students of BS-MS 2023 batch at IISER Mohali, consider the following relation: a student s_1 is related to a student s_2 if they have ever traveled together in the same transportation (cycle, car, bus, train, aeroplane, etc., with or without their knowledge).

Relation	**Reflexive**	**Symmetric**	**Transitive**
$\mathcal{R}1$			
$\mathcal{R}2$			
$\mathcal{R}3$			

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[1] An equation of the form $y = ax^2 + bx + c$ (where $a \neq 0$) is called a *parabola*. In this question, our attempt is to find a suitable parabola that passes through the three given points; that is, the x-coordinates and y-coordinates of these three points satisfy the equation $y = ax^2 + bx + c$. In other words, we have to *interpolate* the three points through a parabola.