Q.1 A 2-D membrane has size L along x direction as 2L in the y-direction. What is the minimum frequency of standing waves on this system? List frequencies of first five harmonics.

Q.2 For $z_{n,m} = A \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}y\right) \cos\left(\omega_{nm}t+\phi\right)$

find out the kinetic energy of the membrane.

Q.3 A right travelling wave on a 1-D string is $y(x,t) = A \sin(k_n x - \omega_n t + \phi)$.

Find out the contribution of m^{th} standing wave in this.

Q.4 Find out the energy carried by a left travelling wave of the kind

$$y(x,t) = A \cos(k_n x + \omega_n t + \phi)$$

Q. 1 2-D Wave eqn.

$$\partial_t^2 z = v^2 \left(\partial_x^2 z + \partial_y^2 z \right)$$

$$z = A \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{2L} y\right) \cos\left(\omega_{nm} t + \phi\right)$$

$$\omega^2 = v^2 \left[\left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{2L}\right)^2 \right]$$

$$\omega_{11}^2 = v^2 \left[\left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{2L}\right)^2 \right] = v^2 \left(\frac{\pi}{L}\right)^2 \frac{5}{4}$$

$$\omega_{12}^2 = v^2 \left[\left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{L}\right)^2 \right] = v^2 (\pi/L)^2 \, 2$$

$$\omega_{21}^2 = v^2 \left[\left(\frac{2\pi}{L}\right)^2 + \left(\frac{\pi}{2L}\right)^2 \right] = v^2 \left(\frac{\pi}{L}\right)^2 \left(\frac{17}{4}\right)$$

$$\omega_{22}^2 = v^2 \left[\left(\frac{2\pi}{L}\right)^2 + \left(\frac{\pi}{L}\right)^2 \right] = v^2 \left(\frac{\pi}{L}\right)^2 5$$

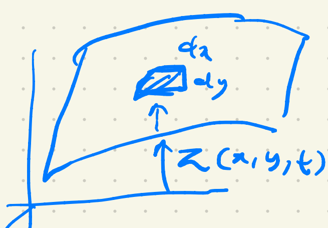
$$= \omega_{14}^2$$

Q. 2

$$z_{nm} = A \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{L} y\right) \cos\left(\omega_{nm} t + \phi\right)$$

$$\dot{z}_{nm} = -A\omega_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{L} y\right) \sin\left(\omega_{nm} t + \phi\right)$$

KE of an element dx dy



$$dk = \frac{1}{2} \sigma \, dx \, dy \left(\dot{z}_{nm}\right)^2$$

$$K = \frac{1}{2} \int_0^L \int_0^L dx \, dy \, A^2 \omega_{nm}^2 \sin^2\left(\frac{n\pi}{L} x\right) \sin^2\left(\frac{m\pi}{L} y\right) \sin^2\left(\omega_{nm} t + \phi\right)$$

$$\int_0^L dx \sin^2\left(\frac{n\pi}{L} x\right) = \frac{L}{2} = \int_0^L dy \sin^2\left(\frac{n\pi}{L} y\right)$$

$$\therefore \quad K = \frac{A^2 6\, \omega_{nm}^2 \sin^2(\omega_{nm} t + \phi)\, L^2}{2 \times 4}$$

Q·3

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

$$= A\left[\sin kx \cos(\omega t - \phi) - \cos kx \sin(\omega t - \phi)\right]$$

$$= A\left[\sin\left(\frac{n\pi}{L} x\right) \cos(\omega_n t - \phi) - \cos\left(\frac{n\pi}{L} x\right) \sin(\omega_n t - \phi)\right]$$

$$\frac{2}{L} \int_0^L \sin \frac{m\pi}{L} x \; y(x,t)\, dx \; \cos(\omega_m t + \tilde{\phi}) =$$

$$A\left[\frac{2}{L} \int_0^L dx \sin\left(\frac{m\pi}{L} x\right) \sin\left(\frac{n\pi}{L} x\right) \cos(\omega_n t - \phi) \cos(\omega_m t + \tilde{\phi})\right.$$

$$\left. - \frac{2}{L} \int_0^L dx \sin\left(\frac{m\pi}{L} x\right) \cos\left(\frac{n\pi}{L} x\right) \sin(\omega_n t - \phi) \cos(\omega_m t + \tilde{\phi})\right]$$

Since $\frac{2}{L} \int_0^L dx \sin\left(\frac{m\pi}{L} x\right) \sin\left(\frac{n\pi}{L} x\right) = \delta_{nm}$

and

$$\frac{2}{L}\int_0^L dx \, \sin\left(\frac{m\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)$$

$$= \frac{1}{L}\int_0^L dx \left[\sin\left(\left(\frac{m\pi}{L}+\frac{n\pi}{L}\right)x\right) + \sin\left(\frac{m\pi}{L}-\frac{n\pi}{L}\right)x \right]$$

$$= -\frac{1}{L}\left[\frac{\cos\left(\frac{(m+n)}{L}\cdot\pi x\right)}{\frac{(m+n)\pi}{L}} + \frac{\cos\left((m-n)\frac{\pi}{L}x\right)}{\frac{(m-n)\pi}{L}} \right]_0^L$$

$$= -\frac{1}{\pi}\left[\frac{\left[(-1)^{n+m}-1\right]}{(n+m)} + \frac{\left[(-1)^{m-n}-1\right]}{(m-n)} \right]$$

$$\therefore \quad A_{nm} = A\Big[\delta_{nm}\cos(\omega_n t-\phi)\cos(\omega_m t+\tilde{\phi})$$
$$+ \frac{1}{\pi}\left(\frac{(-1)^{n+m}-1}{(n+m)} + \frac{(-1)^{m-n}-1}{(m-n)} \right)$$
$$\sin(\omega_n t-\phi)\cos(\omega_m t+\tilde{\phi})\Big]$$

Q. 4. The energy of a wave on a string is.

$$E = \frac{1}{2}\left[\int_0^x dx\left(\rho\dot{y}^2 + T(y')^2\right)\right]$$

$$y = A\cos(\omega_n t + k_n x + \phi)$$

$$\dot{y} = -A\omega_n \sin(\omega_n t + k_n x + \phi)$$

$$y' = -A k_n \sin(\omega_n t + k_n x + \phi)$$

$$\therefore E = \frac{1}{2}\int_0^L dx \left[\rho A^2\omega_n^2 + T A^2 k_n^2\right] \sin^2(\omega_n t + k_n x + \phi)$$

$$= \frac{1}{2}\left[\rho A^2\omega_n^2 + T A^2 k_n^2\right] \times \frac{L}{2}$$

$$= \frac{1}{4}\left[\rho A^2\omega_n^2 + T A^2 k_n^2\right] L$$

Since $\displaystyle\int_0^L dx \ \sin^2(\omega_n t + k_n x + \phi)$

$$= \frac{1}{2}\int_0^L dx \left[1 - \cos(2(\omega_n t + k_n x + \phi))\right]$$

$$= \frac{L}{2} - \left.\frac{\sin 2(\omega_n t + k_n x + \phi)}{2k_n}\right|_0^L$$

$$= \frac{L}{2} - \frac{L}{2n\pi}\Big(\sin(2(\omega_n t + n\pi + \phi)) - \sin(2(\omega_n t + \phi))\Big)$$

$$= \frac{L}{2}$$