IISER Mohali

[Session 2018-19, Even Semester]

PHY 304 (Statistical Mechanics)

Problem Set 03: Microstates, Macrostates and Ensembles

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1. Consider a system of three spin-1/2 particles. Under a magnetic field H (along $+z$-axis), each particle can be in a state with energy $-\mu H$ (spin-up) or $+\mu H$ (spin-down) respectively. What is the total number of possible microstates ? Write down the ensemble of microstates with energy $E = \mu H$. What is the probability of each state in this ensemble ?

2. Calculate F, U, C_v and S for a two-level system whose energy is either 0 or ϵ. Consider low and high temperature limits for U, C_v and S and plot them. Explain the plots.

3. Compute the entropy of a set of N one-dimensional harmonic oscillators, the energy eigenvalues of the oscillators being $(n + 1/2)\hbar\omega; = 0, 1, 2, ...$, as a function of the total energy E.

4. Consider a classical system of energy,

$$E = \sum_{i=1}^{n} \alpha_i x_i^2$$

where α_i are constants and x_i are some variables which can take any value between $-\infty$ and $+\infty$. If the system is in contact with a heat reservoir at temperature T, show that the mean energy of the system is given by $n \times \frac{1}{2} k_B T$. Should there be any constraint on α_i ? Calculate the average energy if the energy of the system follows $E = \alpha|x|$.

5. Calculate the partition function of Ideal gas in Grand Canonical Ensemble and show that the chemical potential is given by,

$$\mu = k_B T \, \ln\Big(\frac{\lambda^3 \, \langle N \rangle}{V}\Big),$$

where, λ is thermal de Broglie wavelength. Plot $\mu(T)$. Calculate the temperature at which it vanishes.

6. Derive the equation of state for a gas for which the N particle partition function is given by,

$$Z_N = \frac{1}{N!}\Big(\frac{V - \bar{b}}{\lambda^3}\Big)^N e^{\beta \bar{a}/V}$$

7. Calculate the density of states $g(p)dp$ in the phase-space for a free particle in D-dimensions. Calculate $g(E)dE$ if the particle is (a) non-relativistic (b) relativistic (c) ultra-relativistic.