PHY302 - Tutorial 7

Instructor: Manabendra Nath Bera
(Dated: 5 November 2020)

1. (a) Starting with the canonical commutation relations for position and momentum, work out the following commutators:

$$[L_z, x] = i\hbar y, \quad [L_z, y] = -i\hbar x, \quad [L_z, z] = 0,$$
$$[L_z, p_x] = i\hbar p_y, \quad [L_z, p_y] = -i\hbar p_x, \quad [L_z, p_z] = 0.$$

   (b) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$, where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

2. Express the L_x, L_y, L_z, and $\mathbf{L^2}$ in spherical coordinate (see Section 4.3.2 in QM by Griffiths).

3. Show that the Laplacian operator in spherical coordinate is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \tag{1}$$
$$= \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}. \tag{2}$$

4. Consider a (spinless) particle represented by the wave function

$$\psi = K(x + y + 2z)e^{-\alpha r}, \tag{3}$$

   where $r = \sqrt{x^2 + y^2 + z^2}$, and K and α are real constants.

   (a) What is the total angular momentum of the particle?

   (b) What is the expectation value of the z-component of angular momentum?

   (c) If the z-component of angular momentum, L_z, were measured, what is the probability that the result would be $L_z = +\hbar$?

   You may find the following expressions for the first few spherical harmonics Y_l^m useful:

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}, \quad Y_1^{\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\phi}, \tag{4}$$
$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta, \quad Y_2^{\pm 1} = \mp \sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{\pm i\phi}. \tag{5}$$