PHY622/Assignment 01

Date: January 20, 2018

Note: Solve the following problems. The submission of assignment is NOT required. You are encouraged to discuss with each other and/or contact the instructor if you have any difficulty in solving the problems.

Problem 1. Check if the given vectors in each of the following sets are linearly independent. If they are not then find a subset of linearly independent vectors and express the remaining as linear combinations of the independent vectors.

- $(1, 4, 2)$, $(3, 5, -1)$, $(-1, 0, 5)$, $(6, 14, 5)$.
- $(1, 0, 2)$, $(0, 1, 1)$, $(-1, 5, 3)$, $(2, -15, 1)$.
- $(1, 1, 1, 1)$, $(1, 1, 1, 0)$, $(1, 1, 0, 0)$, $(1, 0, 0, 0)$.

Problem 2. Find out orthonormal set of vectors for the given vectors: $(0, 0, 5, 0)$, $(2, 0, 3, 0)$ and $(7, 1, -5, 3)$.

Problem 3. Prove the triangle inequality: $||a + b|| \leq ||a|| + ||b||$, where $||a|| = \sqrt{\langle a|a\rangle}$.

Problem 4. Let $\mathcal{P}^n(x)$ is polynomial of order n in a real variable x and $D = \frac{\partial}{\partial x}$ is a derivative operator which acts on $\mathcal{P}^n(x)$. Are the operators D^2, $D^2 + 2D + 1$ and $x^2D^2 - 2xD + 7$ linear?

Problem 5. For $T : V \to V$, are the following transformations linear?

- For vector space $V = \mathbb{C}$ over $\mathbb{R}$ and $T|z\rangle = |z^*\rangle$ for $z \in \mathbb{C}$.
- V is a vector space of polynomials of order n, defined as $\mathcal{P}^c_n(t)$, over $\mathbb{C}$ and $T|x(t)\rangle = |x(t + 1)\rangle - |x(t)\rangle$.

Problem 6. Verify the Dimension Theorem for the transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$T(x_1, x_2, x_3, x_4) = (x_2 - x_4 - 3x_1, 2x_2 + x_3 + x_4, x_2 + x_3).$$

Problem 7. Consider linear maps (operators) $T : V \to U$ and $S : U \to W$. Show that the product $S \cdot T : V \to W$ is also a linear map.