PHY 303 : Assignment 4

Submit by 24 October 2024 midnight

1. Show that for any vector $\mathbf{v}$,

$$\int d^3\mathbf{x}\,(\nabla \times \mathbf{v}) = \int (\mathbf{dS} \times \mathbf{v})\,.$$

   Further show using this identity, that for a current distribution vanishing at spatial infinities, the Biot-Savart law can be written as

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int d^3\mathbf{x}'\,\frac{(\nabla_{\mathbf{x}'} \times \mathbf{J}(\mathbf{x}'))}{|\mathbf{x}-\mathbf{x}'|}.$$

2. If the divergence of the vector potential in some gauge is $g(\mathbf{x}) \equiv \nabla \cdot \mathbf{A}$, write down the gauge function $f(\mathbf{x})$ in terms of $g(\mathbf{x})$ which will take the vector potential to the Coulomb gauge. In an arbitrary gauge the vector potential is given as $\mathbf{A}(\mathbf{x}) = 2r^2(cos\theta)\hat{\theta}$ in spherical polar co-ordinates. write down the vector potential in the Coulomb gauge.

3. Suppose in an arbitrary gauge the divergence of the vector potential is some non zero function $g(\mathbf{x})$. If the free current density in the region is $\mathbf{J}(\mathbf{x})$, write down the vector potential in terms of $\mathbf{J}(\mathbf{x})$ and $g(\mathbf{x})$. Show that the magnetic field due to this $\mathbf{A}(\mathbf{x})$ is the same as in the Coulomb gauge (i.e. the extra term due to non zero divergence does not generate any magnetic field). [Hint : - Take help of Question 1]

4. A shell of uniform charge density ρ_0 and inner and outer radii a and b respectively is rotating about the $\mathbf{z}$ axis with an angular speed ω Find out the vector potential, magnetic moment and the magnetic field in the exterior region this set up.

5. Inside a long uniform cylindrical wire of radius R of permeability μ_1 the material inside a smaller radius r_0 from the center is replaced by another material of permeability μ_2. The two reasons carry uniform current densities $\mathbf{j_{out}}$, $\mathbf{j_{in}}$ respectively. Find out the magnetic fields in the region with radial distance $r < r_0$, $r_0 < r < R$ and $r > R$ respectively.