INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 (Symmetry)

Tutorial Sheet 03 / January 25, 2022 Spring 2022

---

1. Consider the following 3×3 matrices

$$S_{2,3} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad M_2(\lambda) := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad L_{1,2}(\lambda) := \begin{pmatrix} 1 & \lambda & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A := \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

   (a) Compute $S_{2,3}A$, $M_2(\lambda)A$ and $L_{1,2}(\lambda)A$.

   (b) Compute $AS_{2,3}$, $AM_2(\lambda)$ and $AL_{1,2}(\lambda)$.

   (c) Matrix $S_{2,3}$ is a **swapper**, $M_2(\lambda)$ is a **multiplier** and $L_{1,2}(\lambda)$ is a **product adder**. Why do you think we should call them by these names?

2. Find all angles θ for which $R_{x,\theta}R_{y,\theta} = R_{y,\theta}R_{x,\theta}$, where $R_{x,\theta}$ and $R_{y,\theta}$ are rotation matrices by θ about x and y axes, respectively.

3. Take three 2×2 matrices A, B, C of your choice and show that $A(BC) = (AB)C$. Do you think that for every choice of 2×2 matrices this equality will hold? What about 3×3 matrices?

4. Consider the following system of linear equations:

$$\begin{aligned} x + y - z &= 4 \\ 3x - 2z &= 6 \\ x + 2y - z &= 7 \end{aligned}$$

   and express it in the matrix form. Now, compute

$$\begin{pmatrix} -4 & 1 & 2 \\ -1 & 0 & 1 \\ -6 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 3 & 0 & -2 \\ 1 & 2 & -1 \end{pmatrix}.$$

   Can you use this computation to obtain values of x, y, z that satisfy the above system of equations?

5. Resultant of multiplying a matrix A with itself is called the **square** of A. It is written as A^2. So $A^2 := AA$.

   (a) Can you find a 2×2 matrix A such that none of the entries of A is zero, but $A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$?

   (b) Can you find a 2×2 matrix A such that $A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$?
