PHY306 Advanced Quantum Mechanics Jan-April 2024: Assignment 4

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1. Use the virial theorem to obtain the expectation value of $1/r$ for the hydrogen atom.

2. Find the lowest-order relativistic correction to the energy levels of the one-dimensional harmonic oscillator.

3. Suppose the Hamiltonian H for a system is a function of some parameter λ with $E_n(\lambda), \psi_n(\lambda)$ being the eigenvalues and eigenfunctions of $H(\lambda)$. The Feynman-Hellmann theorem states that

$$\frac{\partial E_n}{\partial \lambda} = \langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_n \rangle$$

   Assume that E_n is nondegenerate. Prove the Feynman-Hellmann theorem.

4. Use $\lambda = e$ and $\lambda = l$ in the Feynman-Hellmann theorem to obtain the expectation values of $1/r$ and $1/r^2$ for the hydrogen atom.

5. Derive the fine-structure formula for the hydrogen atom from the relativistic correction and the spin-orbit coupling.

6. Consider $n = 2$ states of the Hydrogen atom and find the energy of each state under weak-field Zeeman splitting. Construct a table of energies and plot them as functions of the external field.

7. Next find the energy of each state under strong-field Zeeman splitting. Construct a table of energies and plot them as functions of the external field.