INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 (Symmetry)

Tutorial Sheet 07 / March 08, 2022 Spring 2022

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0. Revise the definition of a group and a group action.

1. Let $G = \{1, f_X, f_Y, f_{d_1}, f_{d_2}, r_{\pi/2}, r_\pi, r_{3\pi/2}\}$ be the group of symmetries of a square. Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x| \leq 1, |y| \leq 1\}$$

It is a square centred at origin, with each side of length 2 units. For $g \in G$ and $P \in S$ let $g.P$ denote the point after operating the symmetry g on P. (Observe that $g.P \in S$, else g would not be a symmetry!).

   (a) Take some explicit $P \in S$ and $g, h \in G$, and show that $h.(g.P) = (hg).P$.

   (b) Fix a point $P = \left(\frac{1}{2}, \frac{1}{3}\right) \in S$. As g varies, mark $g.P$ on the square S. What you have marked is called the *orbit* of P.

   (c) Fix a point $P = \left(\frac{1}{2}, \frac{1}{2}\right) \in S$. Mark the orbit of P. Identify all $g \in G$ which do not move P, *i.e.*, find the set $\{g \in G : g.P = P\}$. This set will be called the *stabilizer* of P.

2. Take the Klein 4-group $V_4 := \{1, a, b, c\}$. The composition table of this group is given by $a^2 = b^2 = c^2 = 1$, $ab = ba = c$, $ac = ca = b$, $bc = cb = a$. Consider the set

$$S = \{(1, 1, 1), (1, 1, -1), (1, -1, 1), (1, -1, -1), (-1, 1, 1), (-1, 1, -1), (-1, -1, 1), (-1, -1, -1)\}.$$

For each $P = (x, y, z) \in S$, define

$$1.P := (x, y, z), \quad a.P := (x, -y, -z), \quad b.P := (-x, y, -z), \quad c.P := (-x, -y, z).$$

Show that the association of a $g \in V_4$ and $P \in S$ to $g.P \in S$ as defined above, is a group action.
