PHY 201 : Waves and Optics
End Semester Exam
08^{th} December 2022, IISER Mohali

Full Marks : 40
All Problems carry equal weightage. Symbols have their usual meanings.

1. A damped harmonic oscillator with damping parameter β , mass m and natural frequency ω_0 is subjected to a periodic force s.t. it satisfies the equation

$$\ddot{x} + \beta\dot{x} + \omega_0^2 x = f_0(\cos\omega t + \sin\omega t).$$

If the displacement $x(t)$ in the steady state (i.e. time much larger than $2/\beta$) is given by $x(t) = D\cos(\omega t + \phi)$ **derive (not just quote)** the expressions for the ampllitude D and the phase ϕ. For the case $\beta \to 0$ what are their values when $\omega \to \omega_0$? [2 + 2 + 1]

2. A stretched **rectangular membrane** with sides ℓ and $\ell/2$ is fixed at the boundaries. Find out the expression for the normal modes of this system. What is the smallest frequency in this set up which is degenerate (i.e. shared by two different normal modes) ? What is the value of the kinetic energy of the membrane for these normal modes if the membrane has an areal (surface) mass density σ ? [1 + 2 + 2]

3. A wire segment is made of pieces of density ρ for $x < 0$, density $\alpha\rho$ (with a constant $\alpha > 0$) for $x > 0$. If a travelling wave $y_I(x,t) = A(\sin(\omega t - kx) + \cos(\omega t - kx))$ is sent from its left end, using the matching conditions at $x = 0$ **derive (not just quote)** the expressions for reflection to incident amplitude ratio as well as transmission to reflection amplitude ratio. Prove that for infinite density in the region $x > 0$ *(Hard wall condition)*, there is no transmission. [2 + 2 + 1]

4. For an electromagnetic wave described by the electric field $\mathbf{E} = E_0\cos\left(\frac{k(x-y)}{\sqrt{2}} - \omega t\right)(\hat{\mathbf{n}})$, find out the possible unit vectors $\hat{\mathbf{n}}$ for this electric field component. Find out the Poyning vector for the possible choices of $\hat{\mathbf{n}}$. What is the relation between the magnitude of the Poynting vector and the energy density of the wave ? [2 + 2 + 1]

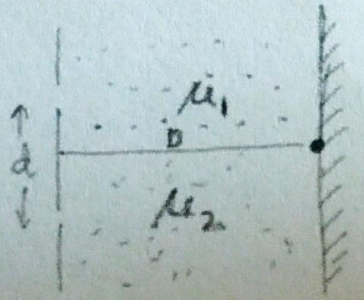
5. For an electromagnetic wave in free space described by the magnetic field

$$\mathbf{B} = 10(-\hat{\mathbf{j}})\cos\left(2\times 10^{-5} m^{-1} z - 6000 s^{-1} t\right)\text{Gauss},$$

what should be the energy density of another electromagnetic wave needed to be superposed with it to get the **polarization of the resultant electic field** at $z = 0$ as (i) Linear in $x - y$ plane at angle $\pi/3$ with the x-axis (ii) clockwise circular polarization. [1 + 2 + 2]

6. In the double slit experiment, a slab of refractive index μ and thickness t is introduced just immediately in front of one slit. For the separation of the slit d, find out the minimum wavelength possible for which the position $y = 0$ on the screen D distance away is a minima. What is theminimum wavelength for which $y = 0$ serves as a maxima ? If we choose the lowest allowed wavelengths in each of the above cases, which case will have the larger fringe width ? [2 + 2 + 1]

7. In the Young's double slit experiment with light of wavelenght λ, if the slit separation is d and one half of the region between the slit to screen (distance D apart) is filled with a fluid of refractive index μ_1 while the other half with a fluid of refractive index μ_2, fiind the relation between μ_1 and μ_2 such that the point $y = 0$ on the screen we have a minima. Maintaining the same relation between μ_1 and μ_2 now the amplitude of the wave emananting from one of the slit (S1) is halved with no phase change compared to the wave emananting from the other slit (S2). Find out the intensity at the center of the screen (y=0). [3 + 2]



8. For a single slit diffraction pattern on a screen put at a distance D away from a slit of width b, illuminated by light of wavelength λ, evaluate the width of the cetral bright fringe. Obtain the expression of the width of the second bright fringe. What is the condition for which the width of the central bright fringe and the second bright fringe are equal ? [2 + 2 + 1]

OR

or a rectangular slit of height a and width b in the $x - y$ plane at $z = 0$, obtain the expression for the ectric field at a point $P \equiv (X, Y, z = R)$ on a screen, due to an infintesimal element $dxdy$ at location $, y, z = 0)$ within the slit, if the electric field surface density emitted by that element is $\xi e^{i(\omega t - kr)}$ iere ξ is a constant electric field density and r is the distance between the point P on the screen and e infinitesimal element in the slit. Ignoring the quadratic appearances of $X^2/R^2, Y^2/R^2$ obtain the tensity of the net electric field at location P due to the whole slit. [2 + 3]