1. Let X be the number of Heads in 50 tosses of a fair coin. Determine the expectation of X, the variance of X and $P(X \leq 10)$? [1+1+1]

2. Each of the 50 students in class belongs to exactly one of the four groups A, B, C, or D. The numbers of students for the four groups are as follows: $A : 5$, $B : 10$, $C : 15$, $D : 20$. First, choose one of the 50 students at random and let X be the size of that students group. Next, choose one of the four groups at random and let Y be its size. Here all random choices are with equal probability.

   (i) Write down the probability mass functions for X and Y.

   (ii) Compute the expectations of X and Y. [2+2]

3. Let X be a random variable with probability density function $f_X(x) = cx^2$ for $x \in [0, 1]$ (zero for other $x \in \mathbb{R}$). [1+2+1]

   (a) Find the value of c.

   (b) Find the cumulative distribution function F_X.

   (c) Find the expectation $E(X)$.

4. Let X be the random variable with probability density function $f_X(x) = 7e^{-7x}, \forall x \geq 0$. [1+3]

   (a) Find the conditional probability $P(X \geq 6 | X \geq 4)$.

   (b) Find the variance $Var(X)$.