Long Quiz - 2

- A comet is revolving in an elliptical orbit around a star such that its farthest distance from the star is 12×10^{12} m while its nearest distance is 8×10^{12} m. Find out the mass of the star if the comet moves between these points in roughly 3 years ~ 10^{8} s. [You can use $G/4\pi \sim 5 \times 10^{-12}$ N m^2 Kg^{-2}]

- In a damped oscillator the kinetic energy decays by 1/4 in some time. How much would have the amplitude decayed in this time ? If the damped oscillator has a oscillation frequency which is half of its natural frequency the how much is the decay in amplitude in one cycle of oscillation ?

- In a unit mass forced oscillator driven by an external force f $\sin(\omega_0 t)$, find out the work done by the force in a half cycle of oscillation in the steady state. For what ϕ it is zero ?

Long Quiz - 2

(i) $r_> = a(1+e)$ $\qquad r_< = a(1-e)$

$$r_> + r_< = 2a = 20\times 10^{12}\ m$$

$$\Rightarrow a = 10^{13}\ m$$

$$\tau^2 = \frac{4\pi}{GM} a^3 \Rightarrow M = \frac{4\pi}{G}\frac{a^3}{\tau^2} \quad \pi$$

$$\tau = 2\times 10^8\ s$$

$$M = \frac{\pi}{5\times 10^{-12}} \times \frac{10^{39}}{4\times 10^{16}} = \frac{\pi}{20}\times 10^{35}\ kg$$

(ii) $$x(t) = c\,e^{-\beta t/2}\sin(\tilde{\omega}t+\phi)$$

$$\dot{x} = c\tilde{\omega}e^{-\beta t/2}\cos(\tilde{\omega}t+\phi) - \frac{c\beta}{2}e^{-\beta t/2}\sin(\tilde{\omega}t+\phi)$$

$$KE = \frac{1}{2}m\dot{x}^2 \sim \frac{m}{2}c^2 e^{-\beta t}\left[\tilde{\omega}\cos(\tilde{\omega}t+\phi) - \frac{\beta}{2}\sin(\tilde{\omega}t+\phi)\right]^2$$

$$\sim \frac{m}{2}c^2 e^{-\beta t}\left[D\sin(\tilde{\omega}t+\tilde{\phi})\right]^2 \qquad \left\{ \text{By choosing } \begin{array}{l} \tilde{\omega} = D\sin\tilde{\phi} \\ -\beta/2 = D\cos\tilde{\phi} \end{array}\right.$$

* This is periodic and decaying amplitude

In time KE amplitude decays as $e^{-\beta t} = \frac{KE(t_2)}{KE(t_1)}$

The oscillation amplitude decays as $e^{-\frac{\beta t}{2}} = \frac{A(t_2)}{A(t_1)}$

$$\Rightarrow \left(\frac{A_1}{A_2}\right) = \left(\frac{KE_1}{KE_2}\right)^{1/2} = \left(\frac{1}{1/4}\right)^{1/2} = 2$$

$\Rightarrow$ Amplitude becomes $1/2$

$$\tilde{\omega} = \omega/2, \qquad \beta = \sqrt{4(\omega^2-\tilde{\omega}^2)} = 2\sqrt{\omega^2-\tilde{\omega}^2}$$

$$\frac{\beta}{\tilde{\omega}} = 2\sqrt{\left(\frac{\omega}{\tilde{\omega}}\right)^2 - 1} = 2\sqrt{3}$$

In one oscillation cycle $t = 2\pi/\tilde{\omega}$

Amplitude decays by $e^{-\frac{\beta t}{2}} = e^{-\pi\beta/\tilde{\omega}}$

$$= e^{-2\pi\sqrt{3}} \text{ times}$$

(iii) For $f \sin \omega_0 t$ driving

$$x(t) = C \sin(\omega_0 t + \phi)$$

$$\dot{x}(t) = +C\omega_0 \cos(\omega_0 t + \phi)$$

$$= +C\omega_0[\cos\omega_0 t \cos\phi - \sin\omega_0 t \sin\phi]$$

$$dW = f\dot{x}\,dt = fC\omega_0 \left\{ \sin\omega_0 t (\cos\omega_0 t \cos\phi - \sin\omega_0 t \sin\phi) \right\} dt$$

In half cycle of oscillation $t = 2\pi/\omega_0$

$$\therefore W = fC\omega_0 \int_0^{\pi/\omega_0} dt \left(\cos\phi \sin\omega_0 t \cos\omega_0 t - \sin\phi \sin^2\omega_0 t \right)$$

$$= fC\omega_0 \int_0^{\pi/\omega_0} dt \left(\frac{\sin 2\omega_0 t}{2} \right) \cos\phi - \int_0^{\pi/\omega_0} dt \, \frac{(1-\cos 2\omega_0 t)}{2} \sin\phi$$

$$= fC\omega_0 \left[-\frac{\cos 2\omega_0 t}{4\omega_0} \Big|_0^{\pi/\omega_0} \cos\phi - \sin\phi \left[\frac{\pi}{2\omega_0} - \frac{\sin 2\omega_0 t}{4\omega_0} \Big|_0^{\pi/\omega_0} \right. \right.$$

$$= -\frac{fC\pi}{2} \sin\phi \quad \Rightarrow \text{ zero when } \phi = 0 \text{ or } \pi$$