Assignment 2

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Check whether a series solution is possible for the equations below around $x = 0$.
   (a) $y''(x) + xy'(x) + (x^2 + 2)y = 0$.
   (b) $y''(x) + \frac{y(x)}{x^3} = 0$.
   (c) $y''(x) + \frac{1}{x}y'(x) - \frac{1}{x^2}y(x) = 0$.

2. Find the nature of singularities at $x = \infty$ for $y''(x) - 2xy'(x) + 2\alpha y(x) = 0$.

3. Find the nature of singularities at $x = +1$, -1, and ∞:
   (a) $(1 - x^2)y''(x) - 2xy'(x) + l(l + 1)y(x) = 0$ with $l > 0$, integer
   (b) $(1 - x^2)y''(x) - xy'(x) + n^2y(x) = 0$

4. Solve the following differential equations using the Frobenius method:
   (a) $(1 - x^2)y'' - xy' + 4y = 0$
   (b) $(1 + x^2)y'' + xy' - y = 0$
   (c) $2xy''(x) + (4x + 1)y'(x) + (2x + 1)y(x) = 0$
   (d) $xy''(x) + (1 + 2x)y'(x) + (1 + x)y(x) = 0$
   (e) $x^2(x^2 - 1)y''(x) - (x^2 + 3)xy'(x) + (x^2 + 3)y(x) = 0$
   (f) $x(1 + x)y''(x) - (1 - 3x)y'(x) + y(x) = 0$

5. Find two linearly independent series of ascending powers of x which satisfy the following differential equation:
   (a) $y''(x) + xy(x) = 0$
   (b) $x(1 - x)y'' - (1 + 3x)y' + y = 0$
   (c) $4xy'' + 2(1 - x)y' - y = 0$

6. Solve the following differential equations by the power series method.
   (a) $xy'' + y' + xy = 0$
   (b) $x(1 - x)y'' - (1 + 3x)y' - y = 0$

7. Obtain two linearly independent solutions of the equation
$$x^2y''(x) + 2x^2y'(x) - 2y(x) = 0,$$
which is valid near $x = 0$.

8. Show that $x = \infty$ is not a regular singular point of the equation

$$y''(x) + ay'(x) + by(x) = 0,$$

where a and b are nonzero constants.

9. Using the Frobenius method, solve the differential equation at $x = \infty$.

$$x^4 y''(x) + 2x^2(1+x)y'(x) + y(x) = 0$$

10. Solve the following differential equation in descending powers of x ($x = \infty$).

$$x^4 y''(x) + 2x^3 y'(x) - y(x) = 0$$

11. Solve the following differential equation by the Frobenius method

$$xy''(x) - (1-2x)y'(x) - (1-x)y(x) = 0$$

and show that $x = 0$ is an appeerent singularity of the differential equation.

12. **A must do!** Consider the following differential equation:

$$(1-x^2)y''(x) - 2xy'(x) + \left\{ l(l+1) - \frac{m^2}{1-x^2} \right\} y(x) = 0,$$

where l and m are constants. Show that $y = (1-x^2)^{m/2}u(x)$ transforms the above equation to

$$(1-x^2)u''(x) - 2(m+1)xu'(x) + \{l(l+1) - m(m+1)\}\, u(x) = 0.$$