PHY306 Advanced Quantum Mechanics Jan-Apr 2025: Assignment 3

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1. A particle at time $t = 0$ starts out in the Nth state of an infinite square well. Now water leaks into the well and then drains out again, so that the bottom is at a uniform potential $V_0(t)$ with $V_0(0) = V_0(T) = 0$. (i) Solve the exact equation and show that the wave function changes phase but no transitions to other states occur. Find the phase change $\phi(T)$ in terms of the function $V_0(t)$. (ii) Solve the problem using first-order perturbation theory and compare the solutions.

2. A particle of mass m is initially in the ground state of a one-dimensional infinite square well. At time $t = 0$ a brick is dropped into the well so that the potential becomes

$$\begin{aligned} V(x) &= V_0, \;\; 0 \leq x \leq a/2 \\ &= 0, \;\; a/2 < x \leq a \\ &= \infty, \;\; \text{otherwise} \end{aligned}$$

   where $V_0 << E_1$. After a time T, the brick is removed and the energy of the particle is measured. Find the probability (upto first order) that the energy is now E_2.

3. An electron is at rest at the origin, in the presence of a magnetic field whose magnitude B_0 is constant but whose direction rides around at a constant angular velocity ω on the lip of a cone of opening angle α such that

$$B(t) = B_0[\sin\alpha\cos\omega t\hat{i} + \sin\alpha\sin\omega t\hat{j} + \cos\alpha\hat{k}]$$

   Use time-dependent perturbation theory (to first order) to calculate the probability of a transition from spin up (initial state) to spin down, as a function of time.

4. A particle initially ($t \longrightarrow -\infty$) is in its ground state in an infinite potential well with walls at $x = 0$ and $x = a$. It is subjected at time $t = 0$ to a time-dependent perturbation $V(t) = V_0 x^2 e^{-t^2}$ where V_0 is a

small real parameter. Calculate to first order the probability that the particle will be found in its second excited state after a very long time $(t \longrightarrow \infty)$.

5. A hydrogen atom initially $(t \longrightarrow -\infty)$ in its ground state, is placed at time $t = 0$ in a time-dependent electric field pointing along the z-axis $E(t) = E_0 \tau \hat{k}/(\tau^2 + t^2)$ where τ is a constant having the dimension of time. Calculate the probability that the atom will be found in the $2p$ state after a sufficiently long time $(t \longrightarrow \infty)$.

6. A particle initially $(t \longrightarrow -\infty)$ is in its ground state in a one-dimensional harmonic oscillator potential. It is subjected at time $t = 0$ to a time-dependent perturbation $V(t) = V_0 x^2 e^{-t/\tau}$ where V_0 is a small real parameter. Calculate to first order, the probability that the system will have made a transition to a given excited state (consider all final states) after a very long time $(t \longrightarrow \infty)$.