# HW1

1) a) Show that the kinetic energy of an electron gas is 3-D with $N$ electrons at 0k is

$$U_0 = \frac{3}{5} N E_f$$

b) Derive a PV relation for an electron gas in 3-D.

c) Find the bulk modulus $B = -V\left(\frac{\partial P}{\partial V}\right)$ of an electron gas. You can ignore the role of lattice

2) Kinetic Inductance:

Write down the expression for AC conductivity of an electron gas in complex form.

In the limit AC frequency $\omega << \frac{1}{\tau}$ where $\tau$ is collision time. & in the limit

$\omega >> \frac{1}{\tau}$ or $\tau \to \infty$. what happens.

Specifically show the kinetic energy acquired by electrons acts like an inductive term in the complex conductivity.

## 3) Complex Refractive index & dielectric constants

We showed in class that the model for a dielectric is a charge that is bound to a harmonic potential. We showed that

$$\vec{P}(t) = \mathrm{Re}\left[\chi(\omega)\,\vec{E}(\omega)\,e^{-i\omega t}\right].$$

$$\chi = \frac{n_b e^2}{m}\,\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

Use $\vec{J}_{pol} = \frac{\partial \vec{P}}{\partial t}$ in Maxwell's equations

Assume a harmonic electric field $\vec{E} = \vec{E_0}\,e^{i(\omega t - \vec{k}\cdot\vec{r})}$ is incident. Arrive at a dispersion relation for $(k, \omega)$ using the complex susceptibility.

a) Arrive at a complex refractive index.

b) Similarly arrive at a complex Dielectric constant for a D. electric.

4) Why metals shine etc.

The model for a metal is similar to a dielectric ~~but~~ but has no harmonic restoring force that bind the electrons.

a) Arrive at a complex refractive index for metals.

b) Rewrite your equations in terms of a plasma frequency $\omega_p = \frac{N e^2}{\varepsilon_0 m}$

use this $\omega_p$ as a cut off with appropriate equations to describe at what wavelengths metals are shiny and at what wavelengths they are transparent.

c) Arrive at an expression for skin depth $\delta = \sqrt{\frac{2\varepsilon_0 c^2}{\sigma \omega}}$ the characteristic depth that an wave penetrate in a metal.

d) Justify the behaviour of metals in part B & C for an example like copper or gold.

e) Indium tin oxide is a transparent metal film. Do a literature survey to find similar metals.

5) 2-D Metals.

a) Show that density of states in a 2D metal is a constant.

b) Show that the fermi energy $E_f \propto \sqrt{n}$ where $n$ is density of electrons per unit area.

6) Landau Levels:

An electron with velocity $v$ is confined to $x$-$y$ plane. A magnetic field is applied in $z$ direction.

a) In the Landau gauge where $\vec{A} = B \times \vec{y}$. show that the problem can be reduced to a harmonic oscillator with

$$\omega_0 = \frac{eB}{m^*}$$ $m^*$ is effective mass of electron.

b) If the vector potential is changed but still $\vec{B} = B\hat{z}$. As in case (a) will the behaviour of electrons change.