IISER Mohali
Solid State Physics
PHY-402

Final exam, Date:Nov 26, 2024

Timing: 2 p.m to 5 p.m | Max mark:90

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As advised in class you are allowed 1 A4 sheet back to back with any handwritten notes. Submit your A4 formula/notes sheet.

1. (a) Pauli Paramagnetic Spin Susceptibility for free electrons in a band occurs due to Zeeman splitting of energy bands in strong magnetic fields. Draw a schematic band diagram to describe this. In a Band Ferromagnet what is the interaction that drives the system to retain spontaneous magnetic polarization. Can you give the Stoner criterion for this parameter with respect to some basic energy parameter for a simple metal. (5)

   (b) Diamond lattice like two interpenetrating FCC lattices. There is an extra atom between the face atoms that bonds tetrahedrally to neighbouring atoms. Assuming one tetrahedral atom as origin the lattice vector of 2 atoms bonded to it are described by $\vec{a_1} = \frac{a}{4}(-\hat{x}+\hat{y}+\hat{j})$ $\vec{a_2} = \frac{a}{4}(\hat{x}+\hat{y}-\hat{j})$. Find the bond angle of these two atoms with the third atom at the origin. (5)

   (c) Write down the Bravais lattice for a BCC crystal. Compute the reciprocal lattice. Does this match with any other lattice in real space ? (5)

   (d) What is the AC Josephson frequency produced by an estimated voltage bias of $1 \times 10^{-6} V$ across a Josephson junction. (5)

   (e) The Landau Theory of Phase transitions expands the free energy in-terms of an order parameter. For a Dielectric

$$F = \mathbf{P}.\mathbf{E} + +g2P^2 + G3P^3 + - - -\text{higher terms}$$

   What is the minimum order of P needed to have something beyond a simple para-electric. Can you give an analog for Magnetic susceptibility beyond a simple paramagnet. (5)

   (f) Consider a 1-D mono-atomic of mass m and di-atomic lattice masses m1, m2 forming unit cells . Which one of these will have a band gap. Sketch the response in ω vs K and explain the regimes

(g) Sketch a Graphene lattice with the Carbon atoms . Give the minimal basis vectors needed to describe graphene.

(h) Sketch a 1-D band dispersion . Shade or indicate the allowed states for a metal in 1st Zone . How do these states change with a voltage bias that is less than the Fermi energy. In a semiconductor what happens to the holes in valence band with applied electric field. (5)

(i) A 2-D metal with lattice constant a has a circular Fermi surface that occupies 1st Brillouin zone fully and 2nd and third zone partially . Draw the Brillouin zones showing the Fermi-circle as shaded regions. Label the 2nd Brillouin Zone as a,b,c ,d and make the appropriate transformation to make it in reduced zone scheme. Show 2nd zone as unshaded in reduced zone Shade electrons (5)

PART-B attempt any 3 thoroughly and mark the questions to be graded in the question paper

2. a) Derive the classical Langevin model of diamagnetic susceptibility.
b) Bohr and Van-Leeuwen proved that there is no classical magnetism especially diamagnetism . Can you give either Bohr's pictorial proof or Van-Leeuwen's proof based on partition function.
c) Write a Hamiltonian for free electrons in a B field and solve it in a plane to obtain the Landau theory of diamagnetism.
d) Why does the Landau theory work? extra 5 points (15)

3. a) A thin rectangular metal slab of area $A = L \times W$ has free electrons with carrier density n_e per unit area ignoring the thickness. A current is flowing in the x direction along the long side of the rectangle. A magnetic field is applied in the Z direction. Sketch this geometry. From the condition for equilibrium of charges in Y -axis derive $\mathbf{E}_y$ and calculate the Hall voltage and Hall coefficient $R_{Hall} = \frac{E_y}{J_x B_z}$. [5 pts]
b) Calculate the carrier mobility with $\mu_e = R_{Hall}/\rho$ where ρ is resistivity. (In 2D ρ and R have same units. If you are confused by this aspect you can choose to work in 3D with a slab of thickness t then convert current densities to appropriate scaled quantities for both parts) What does high mobility imply (3 pts)
c) A second material of same dimensions as in part a has two carriers i.e electrons and holes of densities n_e, p_h and mobilities μ_e, μ_p respectively for electrons and holes. With the previously developed balance conditions for charge carriers get an expression for Hall coefficient in terms of carrier densities and mobilities. (Hint: Express the total current density as a function of density and mobility. In Y-axis use the

balance condition) [7pts] d) It is not possible to sustain a net dipole like field in a metal under static conditions . By Gauss's law the entire metal can have a uniform static charge density. However in Hall effect there is a field in X axis which drives a current and also a field in Y axis with a charge build up. Can you explain this. [extra 5 points]

4. a) Explain the Meissner effect by sketching a superconducting sphere and the experimental situation. What is the magnetic susceptibility of the sphere when it is in a superconnducting state [3pts]
b) Derive the relation for London Penetration depth from Maxwell's equation for a perfect diamgentic system. Find an coherence length § for a supercondcutor with gap Δ . [5pts] (show how B is screened)
c) A Ring shaped superconductor has 2 Josephson Junctions. Assume the wave functions for Cooper pairs in both sections of the ring are ψ_1 and Psi_2 . Write down a coupled Schroedinger equation for ψ_1 and ψ_2. With appropriate choice of wave functions show that the current density has a constant phase difference. [5pts]
d) A dimensionless Ginzburg Landau Parameter $\kappa = \frac{\lambda_{London}}{\Sigma}$. Can you tell how this classifies different types of superconductors and key features of type-II superconductors. [extra 5pts]

5. a) A 1-D periodic lattice has minima in potential at $x = 0, \pm na$ where a is the lattice spacing and n some integer. The solutions chosen for this are

$$\psi_A \sim (e^{iGx/2} + \bar{e}^{iGx/2})$$

and

$$\psi_B \sim (e^{iGx/2} - \bar{e}^{iGx/2})$$

where G is a reciprocal lattice Sketch the probability density of these wave functions along the lattice. Which wave function has a minimum energy. (7 pts)
b) Assume V_G is the potential in reciprocal space . Assuming only 2 states can you calculate the gap in this 1-D band using perturbation interms of V_G (Hint : use Secular equation). If you are unable to do it ise a potential $V = V_o Cos\left(\frac{2\pi x}{a}\right)$. (8pt)
c) What is the correct formula for effective mass for a Band with linear dispersion (extra 5 points)

6. (a) Assume that the band-gap energy $Eg >> k_B T$ in an intrinsic semiconductor. Write down the Density of states for electrons and holes in-terms of effective mass m_e, m_h and E_c, E_v energy E and Fermi energy μ Find the density of n and p What is the product np ? (15 pts)
(b) Why is this product constant even if a semiconductor gets doped uniformly? (extra 5 points)

7. a) A tight binding solution for a cubic lattice wit lattice constant a is of the form

$$E(K) = E_0 - \alpha - 2\gamma(cosK_x a + cosk_y a + cosk_z a)$$

(7points)

Find the bandwidth and effective mass at $k = 0$

b) Discuss Bohr-Sommerfeld Quantization for electron orbits in a Fermi surface in presence of B fields . Discuss how you will get the area of Fermi surface from the de-Haas Van-Alphen or Shubnikov Dehaas effect or alteast show how the density of states look like . (8points)

c) Quantized Electron orbits have a unique commutation relation . Write them down (extra 5 points)

Some constants

$K_B \sim 1.3806488(13) \times 10^{-23}$ J/K
$e \sim -1.602176565(35) \times 10^{-19}$ C
$h \sim 6.62606957(29) \times 10^{-34}$ J $\cdot$ s
$m_e \sim 9.10938291(40) \times 10^{-31}$kg
Useful Integral

$$\int_0^\infty \sqrt{x}e^{-x}dx = \frac{\sqrt{\pi}}{2}$$