MTH201 - Curves and Surfaces
End-semester examination

6th of December, 2022

1. Consider the circle of radius r
   (a) (1 point) Find a parametrization γ for a part of the circle.
   (b) (2 points) Compute the curvature of γ and show that it is constant.

2. (3 points) Consider a curve parametrized by $\gamma : (\alpha, \beta) \to \mathbb{R}^2$. Show that if the curvature is a constant and 0, then it lies on a straight line.

3. Consider a surface patch $\sigma : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $\sigma(x, y) = (x, \sqrt{1 - x^2}, y)$ and the curve $\gamma(t) = \cancel{(0, 1, t)}$ $(0,1,t)$
   (a) (3 points) Compute the unit tangent $\mathbf{T}(t)$, unit normal $\mathbf{N}(t)$, and unit binormal, $\mathbf{B}(t)$ of γ.
   (b) (2 points) Compute the usual curvature, $\kappa(t)$, of γ.
   (c) (3 points) Compute the normal curvature, $\kappa_n(t)$, of γ on the surface.
   (d) (3 points) Compute the geodesic curvature, $\kappa_g(t)$, of γ on the surface. $\gamma''.(N \times \gamma')$

4. (3 points) Consider a curve parametrized by γ lying on a surface patch given by some $\sigma : U \to S \subset \mathbb{R}^3$. Show that if the unit normal $\mathbf{N}(t_0)$ of the curve at the point $\gamma(t_0)$ is parallel to the normal $\hat{\mathbf{n}}$ of the surface at the point $\gamma(t_0)$, then the geodesic curvature is 0.

5. Consider a surface patch defined by $\sigma(x, y) = (x, \sqrt{1 - x^2}, y)$.
   (a) (2 points) What surface is this surface patch a part of?
   (b) (2 points) Compute the first fundamental form, E, F, G.
   (c) (2 points) Compute the second fundamental form, L, M, N.
   (d) (2 points) Compute the Gaussian Curvature

6. (3 points) Show that if $\gamma : (\alpha, \beta) \to S$ parametrizes a straight line, then it is a geodesic.

7. Consider a surface patch σ, with the terms of the first fundamental form, E, F, G and tems of its second fundamental form, L, M, and N.
   (a) (2 points) Express $\sigma_{xx}.\sigma_x$ in terms of E, F, G, and / or their partial derivatives.
   (b) (2 points) Express $\sigma_{xx}.\sigma_y$ in terms of E, F, G, and / or their partial derivatives.
   (c) (2 points) Compute the Christoffel symbol Γ^1_{11}. Recall the Christoffel symbol appears as a coefficient in

$$\sigma_{xx} = \Gamma^1_{11}\sigma_x + \Gamma^2_{11}\sigma_y + L\hat{\mathbf{n}}$$

8. (3 points) Consider a tangent vector field $\mathbf{v}$ along a curve parametrized by γ on a part of a surface covered by the surface patch $\sigma : U \to S$. Show that its covariant derivative along γ, i.e. $\nabla_\gamma \mathbf{v}$, is 0 if $\dot{\mathbf{v}}.\sigma_x = 0$ and $\dot{\mathbf{v}}.\sigma_y = 0$.