PHY306 Advanced Quantum Mechanics Jan-Apr 2024: EndSemester Exam

Max Marks=50 Time=03 Hours Dated April 27, 2024

1. Consider a 1D harmonic oscillator with mass m and angular frequency ω with a slightly increased spring constant $k' = (1+\epsilon)k$. Calculate the first order perturbation in the energy.

   [Marks=05]

2. Two protons are located on the z-axis and separated by a distance d ($\hat{r} = d\hat{k}$), are subject to a z-direction magnetic field $B = B\hat{k}$. Treat the dipole-dipole magnetic interaction between the protons as a perturbation

$$H_p = \frac{1}{r^3}\left[\mu_1.\mu_2 - 3\frac{(\mu_1.r)(\mu_2.r)}{r^2}\right]$$

   Calculate the energy using first-order perturbation theory.

   [Marks=05]

3. A particle of mass m is initially in the ground state of a one-dimensional infinite square well. At time $t = 0$ a brick is dropped into the well so that the potential becomes

$$\begin{aligned} V(x) &= V_0, \ 0 \le x \le a/2 \\ &= 0, \ a/2 < x \le a \\ &= \infty, \ \text{otherwise} \end{aligned}$$

   where $V_0 << E_1$. After a time T, the brick is removed and the energy of the particle is measured. Find the probability (upto first order) that the energy is now E_2.

   [Marks=10]

4. Consider the $n = 2$ and $n = 3$ levels of a hydrogen atom and in the linear Stark effect, find their splitting. Draw a diagram showing the splitting of each level.

   [Marks=05]

5. Consider two spin-1 particles A and B in a box. Write out the Clebsch-Gordan coefficients.

   [Marks=05]

6. (a) Estimate the ground state energy of the hydrogen atom by means of the variational method using the trial wave function $\psi = Ae^{-\alpha r^2}$ where α is an adjustable parameter and A is a normalization constant.

   [Marks=02]

   (b) Consider a 1D harmonic oscillator and use the variational method to estimate the energy of the first excited state. Choose your trial wave function judiciously.

   [Marks=03]

7. Use the WKB approximation in the form

$$\int_{r_1}^{r_2} p(r)dr = (n - 1/2)\pi\hbar$$

to estimate the bound state energies for hydrogen atom. Retain the centrifugal term in the effective potential. A helpful integral is

$$\int_a^b \frac{1}{x}\sqrt{(x-a)(b-x)} = \frac{\pi}{2}(\sqrt{b} - \sqrt{a})^2$$

[Marks=05]

OR

Consider scattering from the potential $V(r) = V_0 e^{-r^2/a^2}$. Find the differential cross section in the first Born approximation and the total cross section.

[Marks=05]

8. Use the WKB approximation to estimate the transmission coefficient of a particle of mass m and energy E moving in the potential barrier

$$\begin{aligned} V(x) &= V_0(x/a + 1), \quad -a < x < 0 \\ &= V_0(1 - x/a), \quad 0 < x < a \\ &= 0, \quad \text{elsewhere} \end{aligned}$$

with $0 < E < V_0$.

[Marks=05]

OR

Use the Einstein relation $H = (c^2p^2 + m^2c^4)^{1/2}$ in the Schroedinger equation, expand the momentum in a series in the momentum basis and obtain the Klein-Gordan equation by transforming back to the coordinate basis.

[Marks=05]

9. Consider scattering of a particle by an attractive square well potential $V(r) = -V_0, r < a$ and $V(r) = 0, r > a$. Find the differential cross section in the first Born approximation.

[Marks=05]

OR

Use the Einstein relation $H = (c^2p^2 + m^2c^4)^{1/2}$ in the Schroedinger equation, write the quantity in the square root as a perfect square of a quantity which is linear in the momentum. Match coefficients and obtain the 4x4 α and β matrices and hence write out the Dirac equation in terms of these quantities.

[Marks=05]

Useful Formulae

$$R_{10}(r) = 2(\frac{1}{a_0})^{3/2} e^{-r/a_0};$$

$$R_{20}(r) = (2 - r/a_0)(\frac{1}{2a_0})^{3/2} e^{-r/2a_0};$$

$$R_{21}(r) = (\frac{1}{2a_0})^{3/2} \frac{r}{\sqrt{3}a_0} e^{-r/2a_0};$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$Y_0^0(\theta, \phi) = \sqrt{\frac{1}{4\pi}}; \;\; Y_1^0(\theta, \phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cos\theta = \frac{1}{2}\sqrt{\frac{3}{\pi}} \frac{z}{r}$$

$$Y_1^{\pm 1}(\theta, \phi) = \mp \frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin\theta e^{\pm i\phi} = \mp \frac{1}{2}\sqrt{\frac{3}{2\pi}} \frac{x \pm iy}{r};$$

$$\vec{L} = \frac{\hbar}{i}\left(\hat{\phi}\frac{\partial}{\partial\theta} - \hat{\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right)$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial f}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \frac{\partial f}{\partial\theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$