PHY 310 - Mathematical Methods for Physicists I

Odd Term 2019, IISER Mohali

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Homework 7

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1. Consider a string stretched under unit tension between fixed endpoints $x = 0$ and $x = L$, and subject to a force distribution $\phi(x)$ Newtons per unit length. The static deflection of of the string, $y(x)$, solves the following boundary value problem

$$y'' = \phi(x), \quad y(0) = y(L) = 0. \tag{1}$$

   (i.) Find the Green's function of this system by applying the boundary conditions and the constraints in the continuity and the jump in the derivative.

   (ii.) Find the solution to Eq. (1) using the method of direct integration.

2. Find the Green's function for the one-dimensional Helmholtz problem

$$\frac{d}{dx^2}\phi(x) - k^2\phi(x) = h(x), \quad \lim_{|x|\to\infty} \phi(x) = 0, \tag{2}$$

   upon using the boundary conditions and constraints on continuity and jump in the derivative.

3. Find the Green's function for the Laplacian in two dimensions,

$$\nabla^2\phi(r,\theta) = h(r,\theta), \tag{3}$$

   with the boundary conditions $\lim_{r\to\infty}(\phi(r,\theta) - \phi_r(r,\theta) r \ln r) = 0$.

   (*Hint: Use Gauss' theorem to get the normalization condition*

$$1 = \int_{\partial D} \frac{\partial_x}{\partial n} G(\mathbf{x}, \mathbf{x}_0)\ dx, \tag{4}$$

   where D is the unit disk, $\vec{n}$ is the outward normal, and the derivative is with respect to x in the notation $r = |\mathbf{x} - \mathbf{x}_0|$.)

4. Show that the Green's function for the system

$$y''(x) + 3y'(x) + 2y(x) = f(x), \tag{5}$$

with the boundary conditions $y(0) = 0 = y'(0)$ is

$$G(x, s) = \begin{cases} 0 & : x < s, \\ e^{2(s-x)} + e^{s-x} & : x > s. \end{cases} \tag{6}$$

5. Construct the Green's function for the equation

$$\frac{d}{dx^2} y(x) - a^2 y(x) = 0, \quad \lim_{|x|\to\infty} y(x) = 0, \quad a > 0, \tag{7}$$

using the Fourier transform method.

6. Express the following equations in their respective Sturm-Liouville form

(i.) Bessel's equation

$$x^2 y'' + xy' + (a^2x^2 - n^2)y(x) = 0. \tag{8}$$

(ii.) Chebyshev's equation

$$(1 - x^2)y'' - xy' + n^2 y(x) = 0. \tag{9}$$