PHY 635 : Mid Semester Exam I

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Symbols have their usual meanings, unless specified otherwise.

1. If the position of one particle in cartesian co-ordinate system $x^\mu \equiv (x^0, x^1, x^2, x^3)$ is being monitored w.r.t. the position of another particle $y^\mu \equiv (y^0, y^1, y^2, y^3)$, find out how do dx^μ/dy^ν and $\partial x^\mu/\partial y^\nu$ transform under Lorentz transformation? Give your reasons. **[2+2]**

2. Under the Lorentz transformation write down the transformation matrix which transforms $(dt, dr, d\theta, d\phi)$ for a boost along the x-axis. If that is the transformation of a Lorentz vector in this co-ordinate basis, for what condition is $d^2r/d\tau^2$ a Lorentz vector ? State your logic. **[3+1+1]**

3. For the geodesic equation in some frame :

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

some observer in the same frame decided to measure velocity, acceleration etc. in terms of a parameter $\tau' = \mathcal{B}\tau^2$, for a constant $\mathcal{B} > 0$. Find out the expression of the geodesic equation in terms of new clock parameter. In this case if metric seen in this frame was $\eta_{\mu\nu} + h_{\mu\nu}$ for weak $h_{\mu\nu}$. s.t. $h_{00} = 2\phi_N/c^2$, what is the modification in Newton's law of gravity generated by the Newtonian potential ϕ_N, under the usual assumptions of static spacetime and slowly moving particles? **[3+2]**

4. A spacetime is decribed by the line element

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2r}\right)dt^2 + a^2(t)\left[\left(1 + \frac{2GM}{c^2r}\right)dr^2 + r^2d\Omega^2\right],$$

for some time dependent scale factor $a(t)$ in a particular frame. Write down the geodesic equation in this spacetime for radially moving particles in this frame. How will the Newton's law of gravity be modified in this setting, with leading order velocity as well as dynamics effect ? **[3+3]**