PHY622/Quiz 1

Date: February 2, 2018 **[Total Maximum Marks: 20]**

Enrol. No.: — **Name:** SOLUTION

Instructions:

- Marks: $+\mathbf{2}$ for each **correct** answer and $-\mathbf{1}$ for each **incorrect** answer.
- For multiple choice type questions, mark your answer neatly. Answers with more than one selection will not be taken into account.
- For other questions write **only** the final answer in a space given in the paper.

Questions

1. Which of the following subsets of $\mathbb{R}^4$ is a subspace of $\mathbb{R}^4$?

   (A) $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1x_2x_3x_4 = 0\}$
   (B) $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid ax_1 + bx_2 + cx_3x_4 = 0; \quad a, b, c \in \mathbb{R}\}$
   (C) $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid ax_3 + bx_4 = 0 \;\&\; x_1 = 0; \quad a, b \in \mathbb{R}\}$
   (D) None of the above

2. A linear transformation $T : \mathbb{C}^4 \to \mathbb{C}^4$ is given as
   $T(z_1,\ z_2,\ z_3,\ z_4) = (z_1 + iz_2 - z_3 + iz_4,\ iz_1 + z_2 - iz_3 - z_4,\ z_1 + z_2 - z_3 + iz_4,\ iz_1 - iz_3 + z_4)$. The rank of T is

   (A) 1 (B) 2 (C) 3 (D) 4

3. The operation $T : \mathbb{C}^2 \to \mathbb{C};\ T(z_1,\ z_2) = z_1 + z_2^*$ is

   (A) linear if real numbers are used as scalars and $z_{1,2}$ are complex.
   (B) linear if complex numbers are used as scalars and $z_{1,2}$ are reals.
   (C) always linear.
   (D) never linear.

4. The number of independent real parameters in a general $n \times n$ complex hermitian matrix are

   (A) n^2 (B) $2n^2$ (C) $n^2 - n$ (D) $2(n^2 - n)$

5. A vector product on $\mathbb{R}^2$ defined by $(x_1, x_2) \cdot (y_1, y_2) = (x_1 - y_2, x_2 + y_1)$

   (A) forms an associative but not commutative algebra.
   (B) forms a commutative but not associative algebra.
   (C) forms both associative and commutative algebra.
   (D) is not an algebra.

6. Find an orthonormal set of vectors out of $\{(1,1,0),\ (1,0,1),\ (1,1,1)\}$.

Answer:

$$\left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \right\}$$

7. What is a matrix representation of an operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(x,\ y,\ z) = (z,\ x,\ y)$ in the basis $\{(-1,1,0),\ (-1,-1,2),\ (1,1,1)\}$?

Answer:

$$T = \begin{pmatrix} -\frac{1}{2} & -3/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

8. $T: \mathbb{C}^3 \to \mathbb{C}^3$ is given by $T(\alpha_1, \alpha_2, \alpha_3) = (\alpha_1 + i\alpha_2, \alpha_2 - i\alpha_3, i\alpha_3 - i\alpha_1)$. What is $T^\dagger(\alpha_1, \alpha_2, \alpha_3)$?

Answer:

$$T^{+}(\alpha_1, \alpha_2, \alpha_3) = (\alpha_1 + i\alpha_3,\ -i\alpha_1 + \alpha_2,\ i\alpha_2 - i\alpha_3)$$

9. $\mathcal{P}_n(x)$ is polynomial of order n in x with complex coefficients. An operator $D: \mathcal{P}_4(x) \to \mathcal{P}_5(x)$ is $D = x^3 \frac{\partial^2}{\partial x^2}$. Find matrix representations of D in the basis $\{1, x, x^2, ..., x^n\}$ for $\mathcal{P}_n(x)$.

Answer:

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 12 \end{pmatrix}$$

[6x5 matrix]

10. Find the solution to the operator differential $\frac{dU(t)}{dt} = t\,H\,U(t)$, where H is time independent operator. Use $U(0) = \mathbf{1}$.

Answer:

$$U(t) = e^{\frac{t^2}{2} H}$$