1.

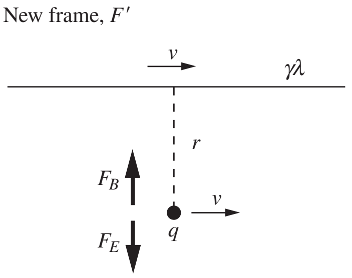


Figure 6.29.
The electric and magnetic forces in the new frame.

(a) The force on a particle is always largest in the rest frame of the particle. It is smaller in any other frame by the γ factor associated with the speed v of the particle. The force in the particle frame (the lab frame) is $q\lambda/2\pi\epsilon_0 r$, so the force in the new frame is $q\lambda/2\gamma\pi\epsilon_0 r$.

(b) In the new frame (call it F'), the linear charge density in the rod is increased to $\gamma\lambda$, due to length contraction. So the electric field is $E' = \gamma\lambda/2\pi\epsilon_0 r$. This field produces a repulsive electric force of $F_E = \gamma q\lambda/2\pi\epsilon_0 r$.

In F' the current produced by the rod is the density times the speed, so $I = (\gamma\lambda)v$. The magnetic field is then $B' = \mu_0 I/2\pi r = \mu_0\gamma\lambda v/2\pi r$, directed into the page in Fig. 6.29 (assuming λ is positive). The magnetic force is therefore attractive and has magnitude (using $\mu_0 = 1/\epsilon_0 c^2$)

$$F_B = qvB' = qv \cdot \frac{\mu_0\gamma\lambda v}{2\pi r} = \frac{\gamma q\lambda v^2}{2\pi\epsilon_0 rc^2}. \tag{6.78}$$

The net repulsive force acting on the charge q in the new frame is therefore

$$F_E - F_B = \frac{\gamma q\lambda}{2\pi\epsilon_0 r} - \frac{\gamma q\lambda v^2}{2\pi\epsilon_0 rc^2} = \frac{\gamma q\lambda}{2\pi\epsilon_0 r}\left(1 - \frac{v^2}{c^2}\right) = \frac{q\lambda}{2\gamma\pi\epsilon_0 r}, \tag{6.79}$$

where we have used $1 - v^2/c^2 \equiv 1/\gamma^2$. This net force agrees with the result in part (a).

(c) In the lab frame, the charges in the rod aren't moving, so $\mathbf{E}_\perp$ is the only nonzero field in the Lorentz transformations in Eq. (6.76). It is directed away from the rod with magnitude $\lambda/2\pi\epsilon_0 r$. Equation (6.76) immediately gives the electric field in the new frame as $\mathbf{E}'_\perp = \gamma\mathbf{E}_\perp$. So $\mathbf{E}'_\perp$ has magnitude $E'_\perp = \gamma\lambda/2\pi\epsilon_0 r$ and is directed away from the rod, in agreement with the electric field we found in part (b).

Equation (6.76) gives the magnetic field in the new frame as $\mathbf{B}'_\perp = -\gamma(\mathbf{v}/c^2) \times \mathbf{E}_\perp$. The velocity $\mathbf{v}$ of F' with respect to the lab frame F points to the *left* with magnitude v. We therefore find that $\mathbf{B}'_\perp$ points into the page with magnitude $B'_\perp = \gamma(v/c^2)(\lambda/2\pi\epsilon_0 r)$. In terms of $\mu_0 = 1/\epsilon_0 c^2$, this can be written as $B'_\perp = \mu_0\gamma\lambda v/2\pi r$, in agreement with the magnetic field we found in part (b). We therefore arrive at the same net force, $F_E - F_B$, as in part (b).

2. $P = IV$ (P: Power).

$\therefore I = \frac{P}{V} = \frac{10^7}{5\times10^4}$ A $= 200$ A.

Field due to one wire, $B = \frac{\mu_0 I}{2\pi r}$.

$\therefore B = \frac{4\pi\times10^{-7}\times200}{2\pi\times1} = 4\times10^{-5}$ T.

Other wire causes equal field in same direction.

$\therefore$ Total $B = 8\times10^{-5}$ T.

3. $B_x = 0$, $B_y = 0$, $B_z = B_0$.

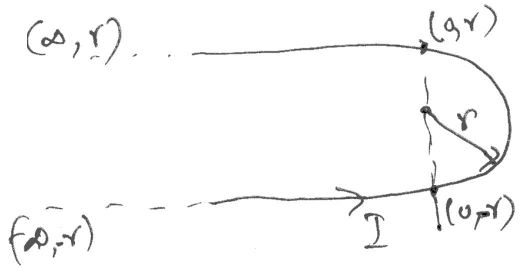
$\vec{B} = \vec{\nabla}\times\vec{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

$\therefore \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0$; $\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0$; $\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_0$.

One possible choice : $A_x = A_z = 0$; $A_y = B_0 x$.

What are other choices?

4.



Break wire into 3 segments:
2 line segments (infinite)
1 semicircular segment

1 segment goes from $(-\infty, -r)$ to $(0,-r)$

$$B_1 = \frac{\mu_0 I}{4\pi r}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi r}\left(\cos 0^\circ - \cos\frac{\pi}{2}\right)$$

$$= \frac{\mu_0 I}{4\pi r}(1-0) = \frac{\mu_0 I}{4\pi r} = B_3$$ (contribution from other line segment)

Contribution from semicircular arc,

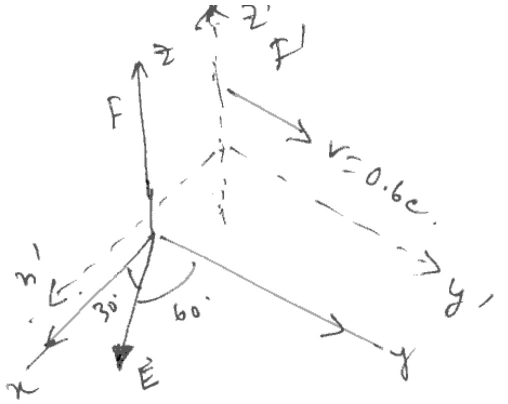
$$B_2 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^\pi \frac{r\,d\theta}{r^2} = \frac{\mu_0 I}{4r}.$$

Note that direction of all 3 magnetic fields are out of page. If that is the $+\hat{z}$ direction, then,

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \left(2\cdot\frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}\right)\hat{z}$$

$$= \frac{\mu_0 I}{4\pi r}(2+\pi)\,\hat{z}.$$

5.



Frame F : $\vec{E}$: $\begin{cases} E_x = 100 \cos 30^\circ \\ E_y = 100 \sin 30^\circ \\ E_z = 0. \end{cases}$

$\vec{B} = 0.$

Transformation eqns: $\vec{E}'_{\parallel} = \vec{E}_{\parallel}$; $\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp})$

$\vec{B}'_{\parallel} = \vec{B}_{\parallel}$; $\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}_{\perp})$

Here, $\vec{v} = 0.6c\,\hat{y}$.

The component $\parallel^l$ to direction of relative motion is the $\hat{y}$ component.

$\therefore E'_y = E_y$. & since, $\vec{E}'_{\perp} = \gamma \vec{E}_{\perp}$ $(\because \vec{B} = 0)$.

$\therefore E'_x = \gamma E_x$; $E'_z = \gamma E_z = 0$ $(\because E_z$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25.$$

$\therefore E'_x = 1.25 E_x$; $E'_y = E_y$.

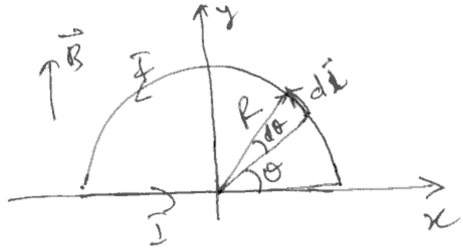
$\therefore \tan\phi' = \frac{E'_y}{E'_x} = \frac{1}{1.25}\frac{E_y}{E_x} \Rightarrow \phi' = \tan^{-1}\left(\frac{E_y}{1.25 E_x}\right.$

$|\vec{E}'| = \sqrt{E_x'^2 + E_y'^2 + E_z'^2} = \sqrt{E_x'^2 + E_y'^2}$.

Now, since, $\vec{B} = 0$ in F, $\therefore B'_y = B_y = 0$.

$$\vec{B}'_{\perp} = -\frac{\gamma \vec{v} \times \vec{E}_{\perp}}{c^2} = -\frac{\gamma}{c^2}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0.6c & 0 \\ E_x & E_y & 0 \end{vmatrix} = -\frac{\gamma}{c^2}\hat{z}(0 - 0.6c \times E_x$$

$$= +\frac{\gamma}{c^{\cancel{2}}}\, 0.6\cancel{c}\, E_x \hat{z} = \frac{1.25}{c} E_x \hat{z}.$$

6. 

$\vec{B} = B\hat{y}$.

Length of straight segment $= 2R$

∴ Magnetic force on straight segment,

$\vec{F}_1 = I(2R\hat{x}) \times B\hat{y}$ (recall, $\vec{F} = I\int(d\vec{l}\times\vec{B})$)

$= 2IRB\hat{z}$. (out of page).

For the semicircular segment, take an infinitesimal length $d\vec{l}$ on the segment (fig.)

Then, $d\vec{l} = -R\sin\theta\, d\theta\, \hat{x} + R\cos\theta\, d\theta\, \hat{y}$.

∴ $d\vec{F}_2 = I(d\vec{l}\times\vec{B}) = I(-R\sin\theta\, d\theta\, \hat{x} + R\cos\theta\, d\theta\, \hat{y}) \times B\hat{y}$.

$= -IBR\sin\theta\, d\theta\, \hat{z}$

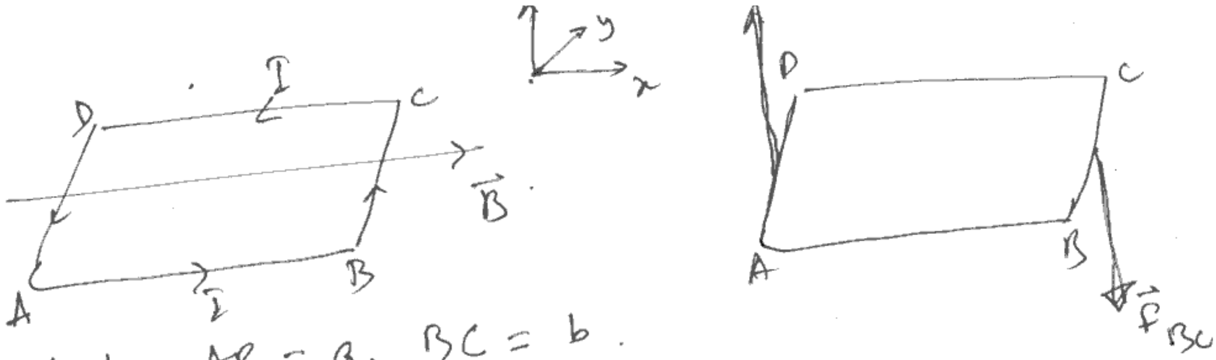
Integrating, $\vec{F}_2 = -IBR\int_0^{\pi}\sin\theta\, d\theta\, \hat{z}$

$= -2IBR\hat{z}$. (~~out~~ into page)

∴ Net force on the wire, $\vec{F} = \vec{F}_1 + \vec{F}_2 = 0$.

→ consistent with the fact that net magnetic force acting on a closed current carrying loop must be zero.

7.



Let, $AB = a$, $BC = b$.

Magnetic forces acting on line segments AB & CD are zero since they are $\parallel^{l}$ & antiparallel to $\vec{B}$.

$\therefore$ Cross products $= 0$.

On segment BC :

$$\vec{F}_{BC} = I\,(+b\,\hat{y}) \times B\,\hat{x} = -IbB\,\hat{z}.$$

On segment DA :

$$\vec{F}_{DA} = I\,(-b\hat{y}) \times B\hat{x} = +IbB\,\hat{z} = -\vec{F}_{BC}$$

Net force on loop:

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{DA} = 0 - IbB\hat{z} + 0 + IbB\hat{z} = 0.$$

Although net force is zero, $\vec{F}_{BC}$ & $\vec{F}_{DA}$ produce a torque leading to a rotation of loop about the y axis. $\therefore$ Torque w.r.t center of loop:

$$\vec{\tau} = \left(\frac{a}{2}\right)\hat{x} \times \vec{F}_{BC} + \left(-\frac{a}{2}\hat{x}\right) \times \vec{F}_{DA}$$

$$= \frac{a}{2}\hat{x} \times (-IbB\hat{z}) - \frac{a}{2}\hat{x} \times (IbB\hat{z})$$

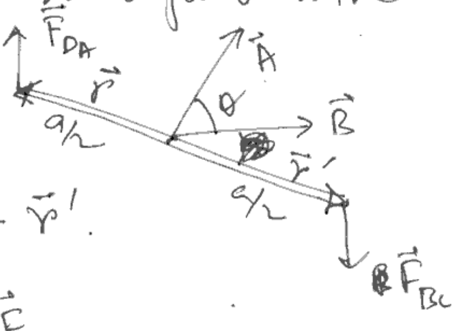
$$= \frac{IabB}{2}\hat{y} + \frac{IabB}{2}\hat{y} = IabB\hat{y}$$

$$= IAB\hat{y} \quad \text{where, } A = ab \text{ is the area of loop.}$$

→ rotation is clockwise.

$\therefore \vec{\tau} = I(\vec{A} \times \vec{B})$ where $\vec{A} = A\hat{n}$ ($\hat{n}$: unit vector normal to plane of loop)

For the general case when loop makes an angle θ with the field $\vec{B}$:



The lever arm,

$$\vec{r} = \frac{a}{2}\left(-\sin\theta\,\hat{x} + \cos\theta\,\hat{z}\right) = -\vec{r}\,'$$

$$\therefore \vec{\tau} = \vec{r} \times \vec{F}_{DA} + \vec{r}\,' \times \vec{F}_{BC} = 2\vec{r} \times \vec{F}_{DA}$$

$$= \cancel{2}\frac{a}{\cancel{2}}\left(-\sin\theta\,\hat{x} + \cos\theta\,\hat{z}\right) \times IbB\hat{z}$$

$$= IabB\sin\theta\,\hat{y} = I\vec{A} \times \vec{B}$$

For a loop of N turns, $|\vec{\tau}| = \underbrace{NIA}_{\text{magnetic moment}(\mu)}B\sin\theta$

$$\vec{\mu} = NI\vec{A}. \qquad \therefore \boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$