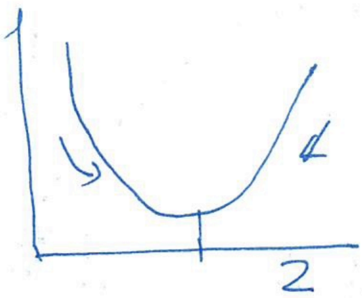
β- decay 3/10/24

(1) $^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}$ β⁻- decay.



A ⇒ Fixed

(2) $^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+} + \nu$ β⁻- decay.

start with

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(3) $^{A}_{Z}X + e \rightarrow {}^{A}_{Z-1}Y + \nu$ Electron capture.

calculate Q value. calculate without ν

(1) $Q = [m(X) - m(Y) - m(e) - m(\bar{\nu})]c^2$

$= \{m^a(X) - Zm_e - [m^a_{Z+1}(Y) - (Z+1)m_e] - m_e - m(\nu)\}$

$Q = [m^a(X) - m^a_{Z+1}(Y) - m(\nu)]$

(2) $\beta^+ Q = m(X) - m(Y_{Z+1}) - m(e) - m(\nu)$

$= m^a(X) - Zm_e - (m^a_{Z-1}Y - (Z-1)m(e)) - m(e) - m(\nu)$

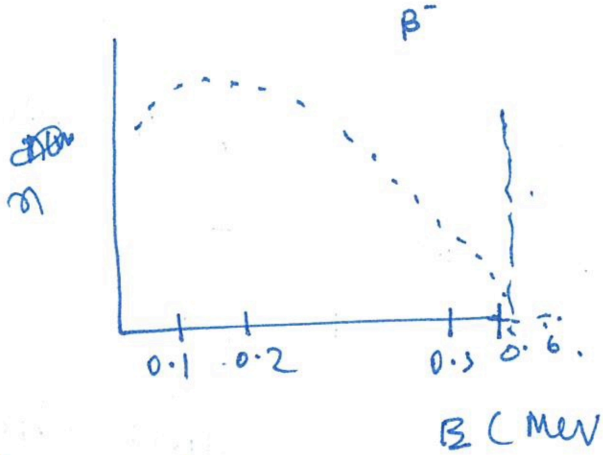
$= m^a(X) - m^a(Y) - \boxed{2m(e)} - m(\nu)$

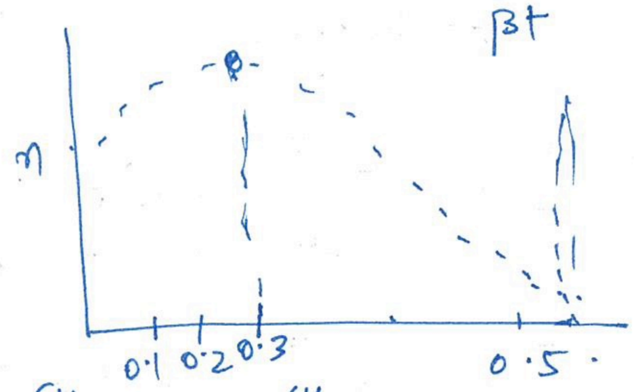
(3) $Q = m(X) + m(e) - m_{Z-1}Y$

$= m^a(X) - Zm_e + m_e - (m^a(Y) \mp (Z-1)m_e)$

$= m^a(X) - Z(m_e) + m_e - m^a(Y) - Zm_e - m_e - m_\nu$

$= m^a(X) - m(Y) - m_\nu$





$^{64}Cu \rightarrow ^{64}Ni + e^+ (+\nu)$

16

$^{64}Cu \rightarrow Zn + e^- (+\bar{\nu})$

Explain the missing energy.

① Energy.

② $^{12}_{5}B \rightarrow ^{12}_{6}C + e^-$

$^{12}C \leftarrow B \quad e^-$ } Back-to-Back due to conservation of linear momentum.

So it is violating linear conservation.

③ $n \longrightarrow p + e^-$ } violating the conservation of spin.

$\frac{1}{2} \qquad \frac{1}{2} \quad \frac{1}{2}$

- Pauli 1931
- E. Fermi 1934

$n \rightarrow p + e^- + \bar{\nu}$

Explain the energy curve.

missing energy = $E_{max} - E_B$

Discovered - 1956

Neutron.

↓

neutrino

↓

neutral little one.

(1) -ve β-decay.

$$^{12}_{5}B \longrightarrow {}^{12}_{6}C + e^- + \bar{\nu}_e$$

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + e^- + \bar{\nu}_e$$

Inverese decay

$$n + \nu_e \rightarrow p + e^-$$

inverese β-decay.

(2) +ve. β-decay.

$$^{23}_{12}Mg \longrightarrow {}^{23}_{11}Na + e^+ + \nu_e$$

$$p \longrightarrow n + e^+ + \nu_e$$

$$p + \bar{\nu}_e \rightarrow n + e^+$$

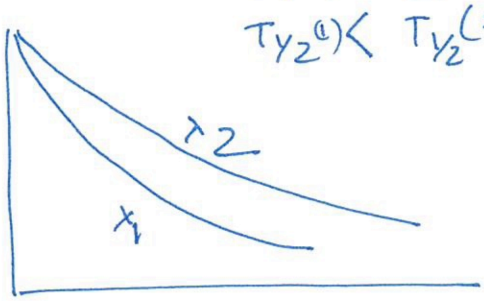
(3) Electron capt.

$$^{81}_{36}Kr + e^- \longrightarrow {}^{81}_{35}Br + \nu_e$$

$$p + e^- \rightarrow n + \nu_e$$

$\lambda_1 > \lambda_2$

$T_{1/2}^{(1)} < T_{1/2}^{(2)}$



$\boxed{\text{Rate.}}$

$T_{1/2} =$ ~~0.693~~

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1$$

$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_0 e^{-\lambda_1 t}$$

$e^{\lambda_2 t}$

$$\Rightarrow e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t}\lambda_2 N_2 = \lambda_1 N_0 e^{(\lambda_2-\lambda_1)t}$$

$$\Rightarrow \frac{d\left(e^{\lambda_2 t} + N_2\right)}{dt} = -\lambda N_0 e^{(\lambda_2-\lambda_1)t}$$

$$\boxed{N_2 = \left(\frac{\lambda_1}{\lambda_2-\lambda_1}\right) N_0 \left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]}$$

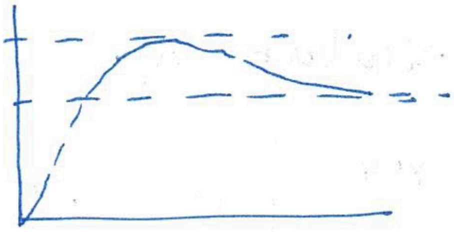
$$e^{\lambda_2 t} N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \, e^{(\lambda_2 - \lambda_1) t} + C$$

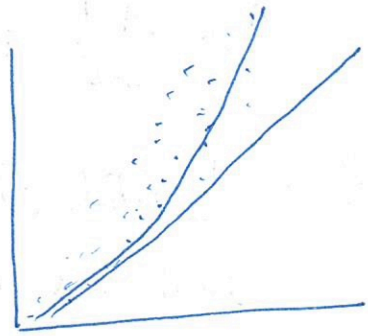
$$C = N_2^0 - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0$$

N_2

Nuclear Decay α- decay (1)

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sys} \frac{(N-Z)^2}{A} + \text{Pairing term}$$





if BE is not sufficient, nucleus breaks up:
if the nucleus breaks in comparable fragment are called nuclear fission
otherwise it's nuclear decay α, β, γ

$$X \longrightarrow Y + V + Q$$

if Q $Q < 0$ no breaking up.
$Q > 0$ it will break.

i.e. system prefers to lower down the ground state energy.

$$\Rightarrow Q = [m[X] - m[Y] - m[V]] c^2$$

$$= [-BE(X) + BE(Y) + BE(V)]$$

$$mA + mZ - mA' - mZ' - mA'' - mZ''$$

$$\Rightarrow {}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{A''}_{Z''}V$$

$$Q = BE(Y) + BE(V) - BE(X)$$

$$\left.\begin{aligned} A &= A' + A'' \\ Z &= Z' + Z'' \end{aligned}\right\}$$

α- decay.

$$ {}^{A}_{Z}X = {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q $$

$$ Q = [\, BE(Y) + BE(He) - BE(X) \,] $$

just use the formula & calculate Q.

From β- stability curve gives you which z value, nucleus is more stable.

i.e. $$ \boxed{Z = \frac{A/2}{1+0.0078\,A^{2/3}}} $$

if do calculation A > 150, Q becomes +ve. i.e. we should not have any nuclei, but we ~~see~~ upto 208 why?

$$ \left.\begin{matrix} 150 - 208 \\ \text{stable} \end{matrix}\right\} \text{exist.} $$

$$ {}^{A}X \rightarrow {}^{A-4}Y + {}^{4}_{2}He + Q $$

$$ {}^{A-4}Y \longleftarrow \boxed{X} \longrightarrow {}^{4}_{2}He $$

use conservation of linear momentum

$$ \left.\begin{matrix} |\vec{P}_y| = |\vec{P}_\alpha| = |\vec{P}| \\ P_y = P_\alpha = P \end{matrix}\right\} $$ same magnitude of momentum must be equal.

$$ KE = \frac{P_\alpha^2}{2m_y} + \frac{P_\alpha^2}{2m_\alpha} = Q $$

$$ \Rightarrow \frac{P^2}{2}\left[\frac{m_\alpha + m_y}{m_\alpha m_y}\right] = Q \quad \Rightarrow \quad \frac{P^2}{2} = \frac{Q\, m_\alpha m_y}{m_\alpha + m_y} $$

$$ \Rightarrow KE_\alpha = \frac{P^2}{2m_\alpha} = \left(\frac{Q\, m_\alpha m_y}{m_\alpha + m_y}\right)\frac{1}{m_\alpha} = Q\left(\frac{m_y}{m_\alpha + m_y}\right) = Q\left(\frac{A-4}{A}\right) = Q\left(1 - \frac{4}{A}\right) $$

$$ K_y = \frac{P^2}{2m_y} = Q\left(\frac{m_\alpha}{m_\alpha + m_y}\right) = Q\,\frac{4}{4 + A - 4} = Q\,\frac{4}{A} $$

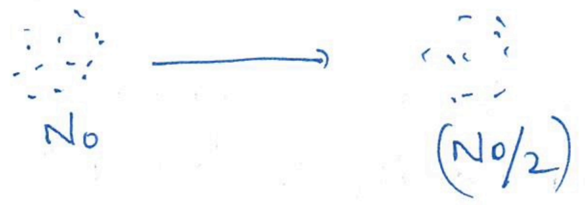
Q-value

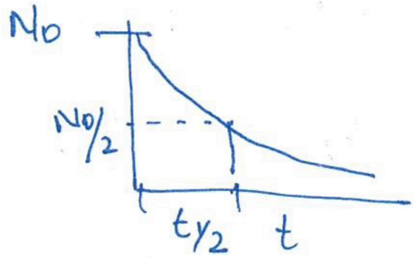
$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q $$

A > 150, Q is positive.

This is energetically possible to have α-decay A > 150

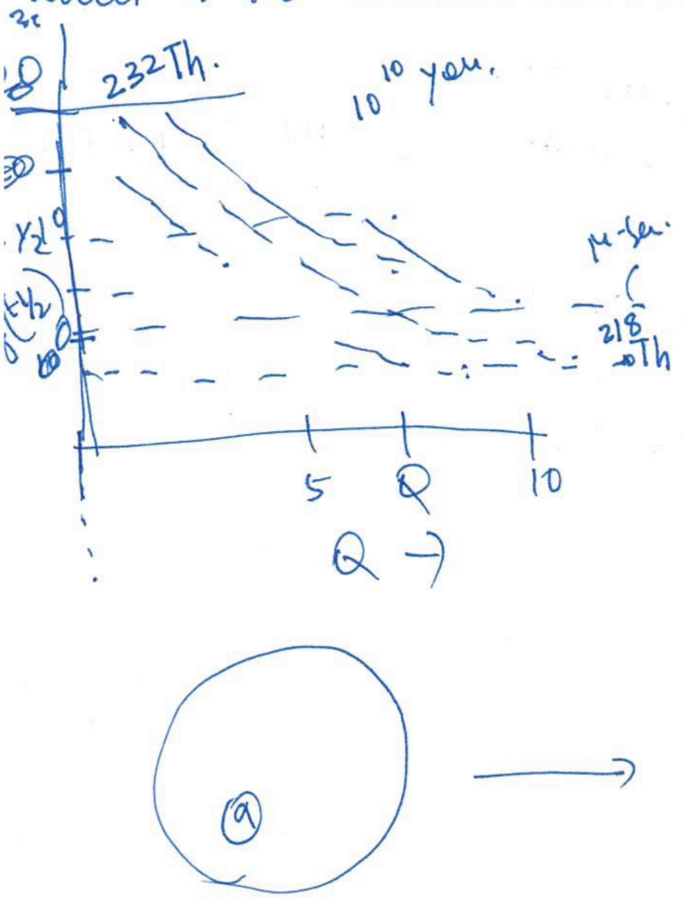
But we also know that ^{208}Pb is very stable.

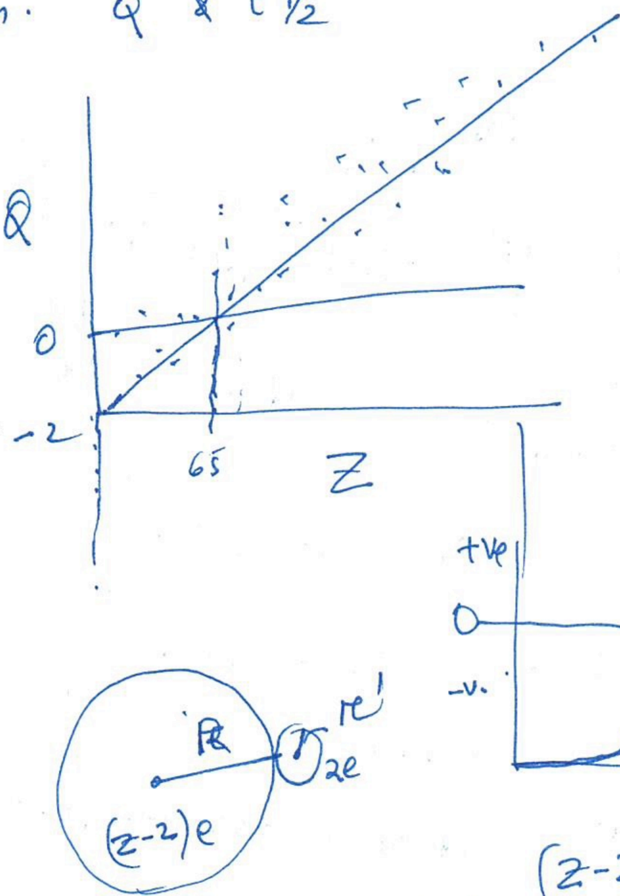


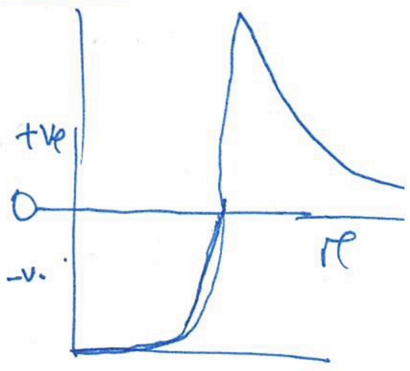


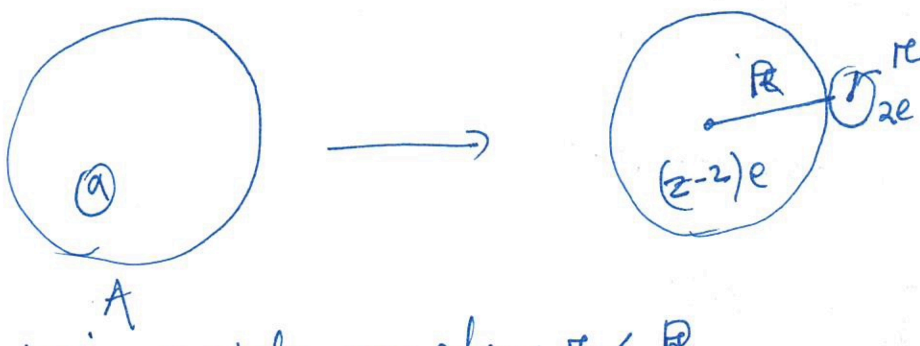
Q is observable & $t_{1/2}$ also experimentally observable.

What is the relation between. Q & $t_{1/2}$









$$ V(r) = \frac{(Z-2)e \cdot 2e}{4\pi \epsilon_0 r}. $$

if within inside nucleus. r < R

↳ nuclear force dominate.

V(r) = Nuclear attractive potential r < R
= Coulomb repulsion potential r > R

Two

α-decay.

KE is findouts.

$\boxed{\frac{dN}{dt} =}$

$t = 0 \quad N_0$

$t: \quad N = N_0 e^{-\lambda t}$

at what time $N \to N/2$

$\frac{N_0}{2} = N_0 e^{-\lambda t/2}$ $\Rightarrow \frac{1}{2} = e^{-\lambda t/2}$ $\Rightarrow \ln(2) = \lambda t/2$

$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$

$\Rightarrow \boxed{\lambda = \frac{0.693}{t_{1/2}}}$

$\lambda \to$ is decay constant.

↳ Probability of decay per ~~un~~ unit time of a particular nucleus.

$\frac{dN}{dt} \propto N$ undecayed nuclei at time t.

$\frac{dN}{dt}$ is Rate.

$\Rightarrow \frac{dN}{dt} = -\lambda N$ $\lambda \to$ is decay const.

$\frac{dN}{N} = -\lambda\, dt$

$\int \frac{dN}{N} = -\lambda \int dt$

$\Rightarrow \ln N = -\lambda t + C$ } use boundary cond.

$t = 0 \quad N = N_0$

$\ln N_0 = C$

Thus $\ln N = -\lambda t + \ln N_0$

$\Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$

$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$

$\Rightarrow N(t) = N_0 e^{-\lambda t}$

what is the height of this potential from zero?
coulumb Barrier height.

$$V_c = \frac{2(z-2)e^2}{4\pi\epsilon_0 r}$$

$$r = R + r'$$

$Z = 82$, $N = 208$

$R = 1.18 \times A^{1/3}$

$= 1.18 \times (208)^{1/3}$ → 6.062

$R \approx 7.27$ fm.

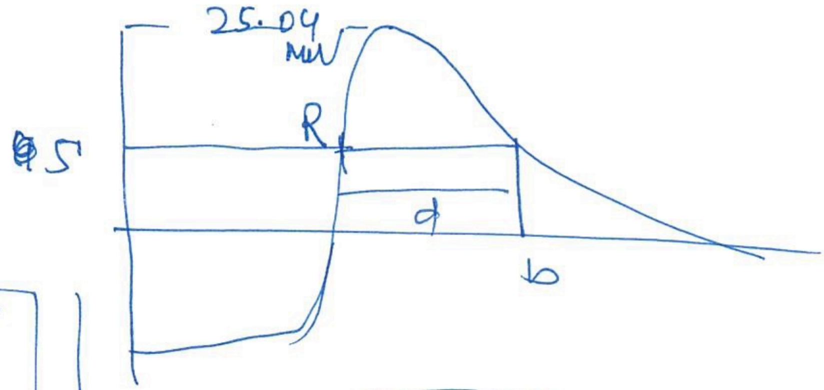
$r' = 1.904$ fm.

⇒ 9.2 fm.

$$\Rightarrow V_c = \frac{2 \times 80}{9.2}\left(\frac{e^2}{4\pi\epsilon_0}\right)$$

$$= \frac{2 \times 80}{9.2\text{ fm}} \times 1.44\text{ MeV fm}$$

$= 25.04$ MeV } But Q is ~ 4 - 9 MeV.



probability.

$P = e^{-\kappa d}$.

$$\boxed{\kappa = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}}$$

$$\boxed{\kappa \cdot d = \int \kappa(r)\,dr}$$

$$\boxed{P = e^{-2G}}$$

$$G = \frac{z_1 Z_2 e^2}{4\epsilon_0 \hbar v}$$ → speed of α-particle.

$$t = \frac{0.693}{\lambda}$$

$\lambda = f P$ → probability of attempt

number of attempt per unit time

$${}^{238}_{92}U \longrightarrow {}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa \xrightarrow{\beta} {}^{234}_{92}U \rightarrow {}^{230}_{90}Th$$

04/10/2024

Nucleour Fisen

$${}^{A}_{Z}X \longrightarrow {}^{A_1}_{Z_1}Y + {}^{A_2}_{Z_2}V \quad \text{Fissen reacfn}$$

$$\left.\begin{aligned} A &= A_1 + A_2 \\ Z &= Z_1 + Z_2 \\ N &= N_1 + N_2 \end{aligned}\right\}$$

Tell the history.

After theory of β-decay by Feremi 1934, it was established that. β-decay can take down or up the atomic number by 1. (one).

So Feremi | Began bombarding n to the nucles, trying to create new heavier nucleus

$$n + {}^{238}U \longrightarrow {}^{239}U$$

U → is heaviest naturally occurring elm.

He belived created a Transuranium element,

But he faced unusual radioactivity & couldn't understand what exactly happens

- $\boxed{1938}$ Hahn and Strassmann

$$n + {}^{238}U \longrightarrow \cancel{X} \; \cancel{+} \; \cancel{X}$$

$${}^{138}_{56}Ba.$$

nucls by.

$$n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 2n$$

This was puzzling

$\boxed{1939}$ Lise Meitner & Otto Robert Frisch.

nephew

↳ Fled Nazi Germany.

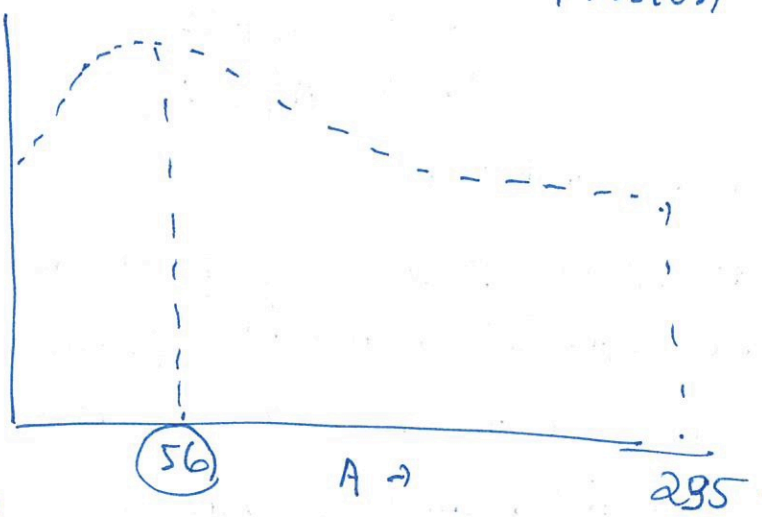
They theoretically explained it by observing lot of energy out come.

Fission

$\downarrow$

nuclei - split roughly equal } process

Nuclear 'Fission'



$^{235}_{92}U$

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sy} \frac{(A-2Z)^2}{A} \pm \delta(A,Z)$$

$\Rightarrow$ $BE = 1328.09$ MeV.

$$\left.\begin{aligned} (BE/n) &= 5.6 \text{ MeV}/n \\ (BE/n)_{Ba} &= 7.35 \text{ MeV}/n \\ (BE/n)_{Kr} &= (6.88 \text{ MeV/nucl}) \end{aligned}\right\}$$

Let's calculate the theoretical value.

$$^{A}_{Z}X \longrightarrow {}^{A_1}_{Z_1}Y + {}^{A_2}_{Z_2}V$$

$$Q = m(X) - m(Y) - m(V)$$

$\Rightarrow$ ~~$m^a(X) = BE$~~

$Q = \cancel{A m_n} \cancel{N m_n}$

$$\left(N m_n + Z m_p - BE(X)\right) - \left\{N_1 m_n + Z_1 m_p - BE(Y)\right\}$$

$$- \left\{N_2 m_n + Z_2 m_p - BE V\right\}.$$

$$= - BE(X) + BE(Y) + BE(V)$$

$$= -5.6 \times A + 7.35 \times A_1 + 6.88 \times A_2$$

$$= -5.6 \times 235 + 7.35 \times 141 + 6.88 \times 92$$

$$= -1316 + \underline{1036.35 + 632.96}$$
$$1669.31$$

$$= 353.31 \text{ MeV}.$$

---

what is min A for fission to happen.

$A_1 = A_2 = A/2$ symmetric f.

$$M = \left(Z m_p + N m_n - BE\right)$$

$$BE = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sys} \frac{(A-2Z)^2}{A} \pm \delta(\text{pair})$$

→ no chg

$$\left[a_v\left(\frac{A}{2}\right) - a_s\left(\frac{A}{2}\right)^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} \downarrow - \right.$$

$\uparrow$

$$\boxed{2\left(\frac{A}{2}\right)^{2/3} > (A)^{2/3}}$$

$$\frac{2}{1.5872}$$

$$\frac{(A-2Z)^2}{A}$$

$$2\,\frac{\left(\frac{A}{2} - \frac{Z}{2}\right)^2}{A/2} = \left(\frac{A-2Z}{A}\right)$$

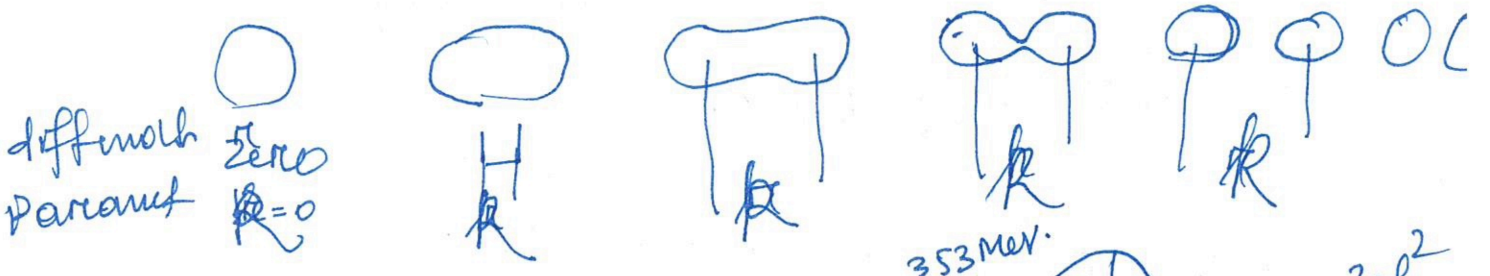
$\cancel{Mc^2 = M_0c^2 + A}$

$$M = M_0 + A - \left(-a_s A^{2/3} - a_s \frac{Z^2}{A^{1/3}}\right)$$

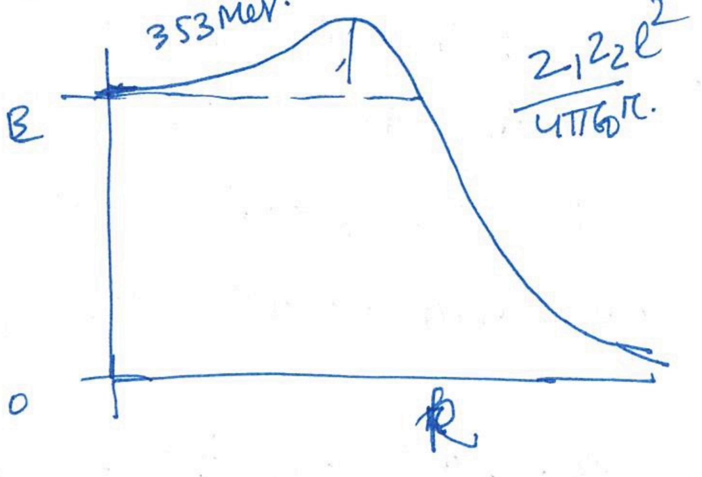
$$\boxed{Q = M - 2M_1}$$

$$= \left. \begin{array}{l} a_s\left[A^{2/3} - 2\left(A/2\right)^{2/3}\right] \\ a_c\left[\dfrac{Z^2}{A^{1/3}} - 2\dfrac{(Z/2)^2}{(A/2)^{1/3}}\right] \end{array} \right\} > 0$$

$A > 150$



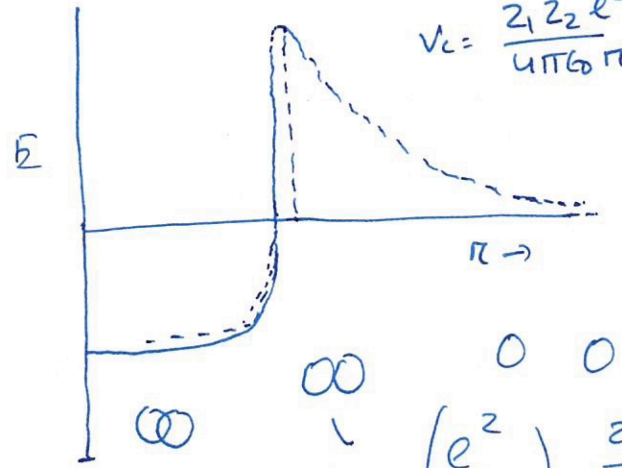
activatin Energy
Barrier height

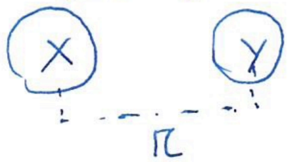


Fusion 8/10/2024

How fusion happens - conditions for nuclear fusion.
Actual issue is the coulumb barrier.

$$V_c = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} = \left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{Z_1 Z_2}{r}\right)$$





$$V_0 = \left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{Z_1 Z_2}{r_1 + r_2}$$

${}^{12}_{6}C + {}^{12}_{6}C$: $V_0 = 1.44 \text{ MeV.fm} \dfrac{6 \cdot 6}{2 r_0 \, 12^{1/3}} = 1.44 \times 4.367$

$2 \times 1.8 \times 2.29 \qquad = 6.288 \text{ MeV}$

${}^{1}_{1}H + {}^{1}_{1}H$: $V_0 = \dfrac{1.44}{2 \times 0.8} 1\times1 \text{MeV} = \dfrac{1.44}{1.6} \text{MeV} = 0.9 \text{ MeV}.$

${}^{40}_{20}Ca + {}^{40}_{20}Ca$: $\cancel{1.44 \dfrac{20 \times 20}{2 \times 1.18 \, 40^{1/2}}}$ $\quad \dfrac{1.44 \times 20 \times 20}{2 \times 1.18 \times 40^{1/3}}$

$$= \frac{1.44 \times 20 \times 20}{2 \times 1.18 \times 3.42} = 72.36485 \text{ MeV}.$$

Do the discussion how to get this energy to cross the Barrier.

→ Kinetic energy ↗ accelerator.
↘ random thermal motion

↓

Take it to high temperature

↓ create a fusion

↓ There will be random motion

↓

They will have kinetic energy.

↓

use this to create fusion react. ← These are thermal kinetic energy

↑

Thermo-nuclear Fusion

↑

↙ stars

Room Temperu.

$\boxed{E = kT}$ Bo

$kT = 0.027\,eV$ average kinetic energy.

$$\boxed{\langle E \rangle = \frac{3}{2} kT} \Rightarrow T \Rightarrow$$

$$= \frac{\cancel{(0.027 \times 3}}{\cancel{2}}$$

$\frac{3}{2}\cancel{k_B}T = 8.617 \times 10^{-5} eV/k \times T$

900 KeV =

$$T = \frac{900.\,keV}{8.617\,eV} \times 10^{+5}\,k = \frac{900{,}000}{8.617} \times 10^{5}\,k$$

$$= 1.045 \times 10^{10}\,k.$$

Nuclear fusion. 7/10/24 (1)

$$Z_1 m_p + A_1 m_n - BB \qquad Z_2 m_p + N_2 m_N - BB \qquad - Z m_p - A m_n + BB$$

$${}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \longrightarrow {}^{A}_{Z}V \qquad Q = m(X) + m(Y) - m(V)$$

$$\Rightarrow \quad A_1 + A_2 = A \qquad = -BE^X - BE^Y + BE^V.$$

$$Z_1 + Z_2 = Z \qquad \Rightarrow Q = BE^V - BE^X - BE^Y$$

$$N_1 + N_2 = N \qquad = (BE/n)A - (BE/n)A_1 - (BE/n)A_2$$

$${}^2H + {}^3H \rightarrow {}^4He + n + Q$$

${}^2H$: $\Delta m = 0.001839$ $\qquad m({}^2H) = 2.0141024$

$\Delta B = 1.714$ MeV $\qquad m_p = 1.0072764$

$BE/n = 0.857$ MeV. $\qquad m_n = 1.0086654$

${}^3H$: $\Delta m = 0.008557$ $\qquad m({}^3H) = 3.0160494$

$\Delta B = 7.973$

$(BE/n) = 2.658$ MeV.

${}^4He$:- $\Delta m = 0.029279$ $\qquad m({}^4He) = 4.0026034$

$\Delta B = 27.41$ MeV.

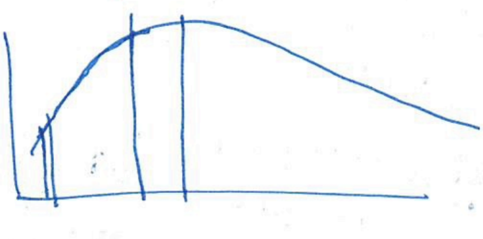
$(BE)/n = 6.85$ MeV.

$$Q = 6.85 \times 4 - 0.857 \times 2 - 2.658 \times 3 \text{ MeV.}$$

$$= 27.41 - \underline{1.714 - 7.974}$$

$$= 27.41 - 9.688 = 17.722 \text{ MeV.}$$

$Q > 0$ ∴ There will be release of energy.



CNO - cycling.

Carbon - Nitrogen - Oxygen.

$$H + H + H + H \longrightarrow {}^4He + \underbrace{2e^+}_{n-P} + 2\nu_e + Q$$

$$\boxed{\Delta M = 4\times m_p - m({}^4He)} - 2\times m_e - 2 m_{\nu_e}$$

$$\Delta M = (4.031300 - 4.002603)\,u - 2m_e - 2m_{\nu_e}$$

$$2m_{\nu_e} \to 0$$

$$= 0.028697 \times 931.5 \text{ MeV}.$$

$$= 26.72 - \cancel{1.022 \text{ MeV}}$$

$$= \cancel{25.699 \text{ MeV}}$$

interesting CNO been catalyst for this reaction

$${}^{12}C + {}^1H \rightarrow {}^{13}N + \gamma$$ — Proton capture.

$${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$$ — β^+ decay

$${}^{13}C + H \longrightarrow {}^{14}N + \gamma$$ → Proton capture.

$${}^{14}N + H \longrightarrow {}^{15}O + \gamma$$ — Proton capture

$${}^{15}O \longrightarrow {}^{15}N + e^+ + \nu_e.$$ → Beta decay.

$${}^{15}N + H \longrightarrow {}^{12}C + {}^4He$$ — Proton capture.

$$4\,{}^1H \rightarrow {}^{12}C + {}^4He + 2e^+ + 2\nu_e + \gamma + 26.72 \text{ MeV}.$$

$\frac{3}{2}kT = 1\text{ keV}$

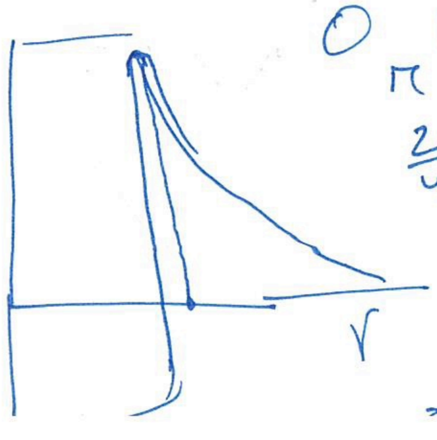
$1.5\times10^7 K$

Columb barrier

$$V_0 = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 (R_1+R_2)} \quad 0.9 \text{ MeV}$$

900 keV

$$\frac{2 \times 1.44 \text{ MeV fm}}{(0.8+0.8)} = 1.8 \approx 1 \text{ MeV}$$



$$\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

$$\left(\frac{3}{2}kT\right) =$$

150 keV