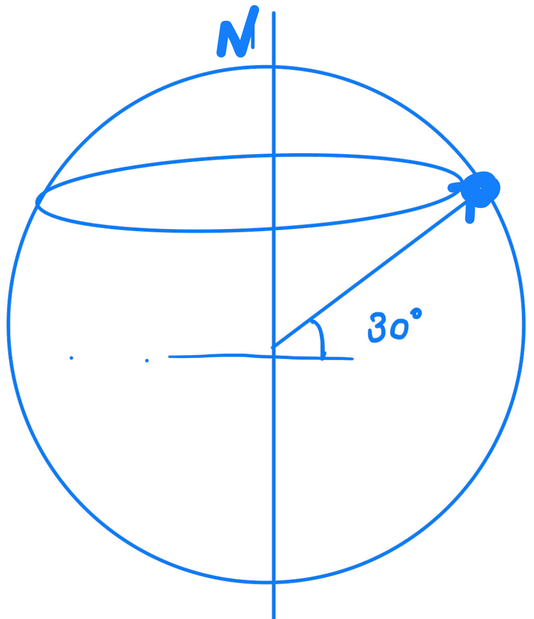
QUIZ 3

- A particle moves on a trajectory r = β θ trajectory in a plane for some constant parameter β. Find out the torque experienced by the particle at location (r, θ) moving under a force according to Newton's law.

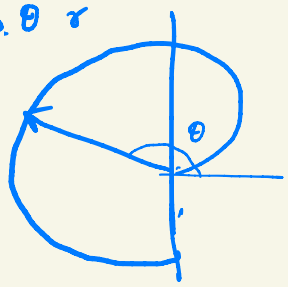
- Mohali is situated at a latitude of 30^{o} north of equator. Find the magnitude of acceleration and its direction in spherical polar basis a stationary particle in Mohali will feel due to its rotation around the axis in a circular trajectory.



Quiz - 03

1. $\vec{r} = \beta r \hat{r}$

$= \beta \cdot \theta \hat{r}$ or $r = \beta \cdot \theta \hat{r}$



$$\frac{d\vec{r}}{dt} = \beta \dot{\theta} \hat{r} + \beta \theta \dot{\theta} \hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = \beta \ddot{\theta} \hat{r} + \beta \dot{\theta}^2 \hat{\theta} + \beta \dot{\theta}^2 \hat{\theta} + \beta \ddot{\theta} \theta \hat{\theta} - \beta \theta \dot{\theta}^2 \hat{r}$$

$$= \beta (\ddot{\theta} - \theta \dot{\theta}^2) \hat{r} + \beta (2\dot{\theta}^2 + \theta \ddot{\theta}) \hat{\theta}$$

$$\tau = \vec{r} \times \vec{F} = m \left[\beta \theta \hat{r} \times \left\{ \beta (\ddot{\theta} - \theta \dot{\theta}^2) \hat{r} + \beta (2\dot{\theta}^2 + \theta \ddot{\theta}) \hat{\theta} \right\} \right]$$

$$= -m \beta^2 \theta (2\dot{\theta}^2 + \theta \ddot{\theta}) \hat{k}$$

2. $\vec{r} = r \hat{r}$

$$\frac{d\vec{r}}{dt} = \dot{r} \hat{r} + \frac{d\hat{r}}{dt}$$

$$= \dot{r} \hat{r} + r \dot{\theta} \hat{\theta} + r \sin\theta \, \dot{\phi} \hat{\phi}$$

Since $\dot{r} = 0$, $\dot{\theta} = 0$

$$\frac{d\vec{r}}{dt} = r \sin\theta \, \dot{\phi} \hat{\phi}$$

$$\frac{d^2\vec{r}}{dt^2} = r \sin\theta \, \ddot{\phi} \hat{\phi} + r \sin\theta \, \dot{\phi} \frac{d\hat{\phi}}{dt}$$

$$= -r \sin\theta \, \dot{\phi}^2 (\sin\theta \hat{r} + \cos\theta \hat{\theta})$$

$$= -\sqrt{3} \frac{r}{4} \dot{\phi}^2 (\sqrt{3} \hat{r} + \hat{\phi})$$