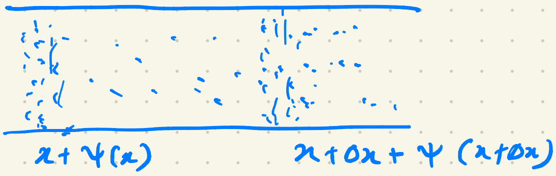
Longitudinal Waves

Horizontal displacement of a column at x is $\psi(x)$



$$P(x) = P_0 + \phi(x)$$
$$P(x+\Delta x) = P_0 + \phi(x+\Delta x)$$

For small forces

$$\Delta V = -kV\phi$$

$$\therefore \quad \frac{\Delta V}{V} = -k\phi$$

$$\frac{A\,\Delta\psi}{A\,\Delta x} = -k\phi \Rightarrow \frac{\partial\psi}{\partial x} = -k\phi$$

$$\boxed{\Rightarrow \frac{\partial^2\psi}{\partial x^2} = -k\frac{\partial\phi}{\partial x}} \quad - \quad (1)$$

$$\boxed{\begin{aligned} &\text{Since} \quad \phi(x+\Delta x) = \phi(x) + \frac{\partial\phi}{\partial x}\Delta x \\ &\Rightarrow \quad \Delta\phi = \frac{\partial\phi}{\partial x}\Delta x = -\frac{\Delta x}{k}\frac{\partial^2\psi}{\partial x^2} \end{aligned}}$$

Force on gas column between x and $x+\Delta x$

$$\Rightarrow \underset{\substack{\uparrow\\ m}}{\rho V}\ \underset{\substack{\uparrow\\ a}}{\frac{\partial^2\psi}{\partial t^2}} = -A\Delta\phi$$

$$= +\frac{A}{k}\frac{\partial^2\psi}{\partial x^2}\Delta x$$

$$\therefore \quad \frac{\partial^2\psi}{\partial t^2} = \frac{A}{kV\rho}\frac{\partial^2\psi}{\partial x^2}\Delta x = \frac{\cancel{A\Delta x}}{k\rho\cancel{V}}\frac{\partial^2\psi}{\partial x^2}$$

Thus the horizontal disturbance ψ satisfies a wave equation

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = \underbrace{\left(\frac{1}{K\rho}\right)}_{\to\, v^2} \frac{\partial^2 \psi}{\partial x^2}}$$

Travelling waves

We have the expression for a standing wave

$$y_n(x,t) = A_n \sin(k_n x)\cos(\omega_n t + \phi)$$

$$= \frac{A_n}{2}\left[\sin(k_n x + \omega_n t + \phi) + \sin(k_n x - \omega_n t - \phi)\right]$$

$$= \frac{A_n}{2}\underbrace{\sin(k_n x + \omega_n t + \phi)}_{y_n^{(1)}} + \frac{A_n}{2}\underbrace{\sin(k_n x - \omega_n t - \phi)}_{y_n^{(2)}}$$

$$\frac{\partial^2 y_n^{(1)}}{\partial t^2} = -\omega_n^2 y_n^{(1)} \; ; \quad \frac{\partial^2 y_n^{(1)}}{\partial x^2} = -k_n^2 y_n$$

Since $\omega_n^2 = k_n^2 v^2$

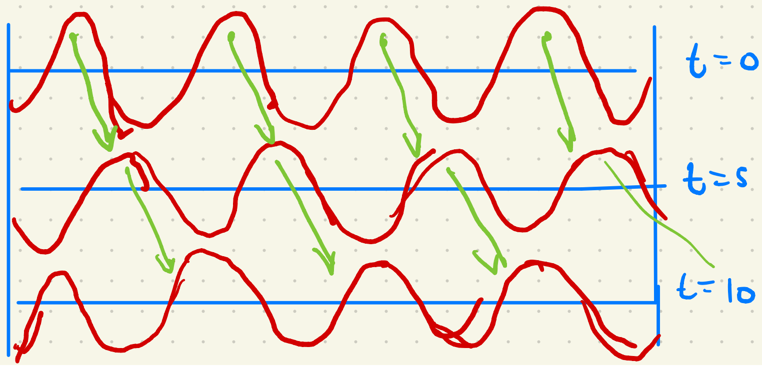
$\Rightarrow \quad \dfrac{\partial^2 y_n^{(1)}}{\partial t^2} = v^2 \dfrac{\partial^2 y_n^{(1)}}{\partial x^2}$ (Satisfies the wave eqn)

Similarly $\dfrac{\partial^2 y_n^{(2)}}{\partial t^2} = v^2 \dfrac{\partial^2 y_n^{(2)}}{\partial x^2}$ (satisfies the wave eq)

However for $y_n^{(1)}$, the maxima/antimaxima is obtained from $\dfrac{\partial y_n^{(1)}}{\partial x} = 0 \Rightarrow \dfrac{A_n}{2} k_n \cos(k_n x + \omega_n t + \phi) = 0$

$\therefore \quad k_n x + \omega_n t + \phi = (2n+1)\pi/2$

$\rightarrow \quad k_n x = (2n+1)\pi/2 - \phi - \omega_n t$ (locations shift !!)



Travelling Waves

For $y_n^{(1)} = \frac{A_n}{2} \sin(k_n x + \omega_n t + \phi)$

the extrema are at $k_n x = (2n+1)\frac{\pi}{2} - \phi - \omega_n t$

Thus, if x_* is an extrema at time t_*

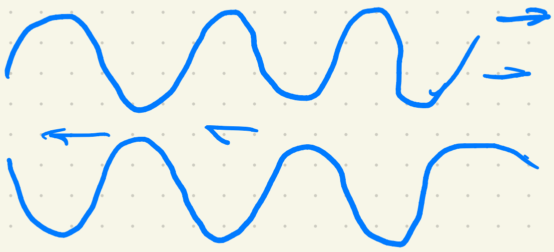
then at $t_* + \Delta t$, $x_* + \Delta x$ will be the new location of extrema s.t.

$$k_n \Delta x = -\omega_n \Delta t \quad (-\text{ve})$$

Hence, the extrema shift 'leftwards' in time!

$y_n^{(1)}$ is a left-moving wave!

Ex: $y_n^{(2)}$ is a right-moving wave



The left moving and the right moving waves in equal amplitude constitute a standing wave.

Rate of shift of extrema for $y_n^{(1)}$

$\frac{\Delta x}{\Delta t} = -\frac{\omega_n}{k_n} = -v$; Thus $y_n^{(1)}$ travels left with speed v.

Similarly $y_n^{(2)}$ travels right with speed v.

$$v^2 = \frac{T}{\rho} \quad (\text{string})$$

$$= \frac{S}{\sigma} \quad (\text{2-D membrane})$$

$$= \frac{1}{k_B \rho} \quad (\text{fluid})$$

In fact any function

$$f(x \pm vt) \Rightarrow \frac{\partial^2 f}{\partial x^2} = f''$$

$$\frac{\partial^2 f}{\partial t^2} = (\pm v)^2 f''$$

$$\Rightarrow \frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$; satisfies the wave eqn.

$f(x+vt)$ is left moving wave.

$f(x-vt)$ is right moving wave.

Thus functions of $x \pm vt$ are waves (still we needs to take care of boundary conditions)

Any $$\boxed{f(x+vt) = \sum_n A_n \sin(k_n(x+vt)+\phi)}$$

$$= \sum_n A_n \left[\sin(k_n x)\cos(\omega_n t+\phi) + \cos(k_n x)\sin(\omega_n t+\phi)\right]$$

Exercise: Find out A_2, A_3 !!

Waves in free space

Can be obtained by taking $L \to \infty$

$$k_{n+1} - k_n = \frac{\pi}{L} \to 0$$

Hence k will become continuous

$$y_k(x,t) = A \sin(kx) \cos(\omega t + \phi)$$

with $\omega = kv$

A general wave

$$\Rightarrow \quad y(x,t) = \int_{-\infty}^{\infty} dk \; A(k) \sin(kx) \cos(\omega t + \phi)$$

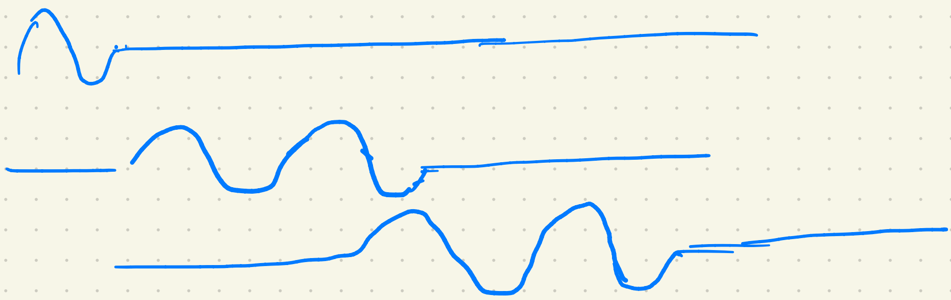
In order to find $A(k)$ we use

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dx \; e^{ik(x-x')} = \delta(x - x')$$

Dirac Delta function

Where $$\int_{-\infty}^{\infty} dx \; f(x) \delta(x-a) = f(a)$$

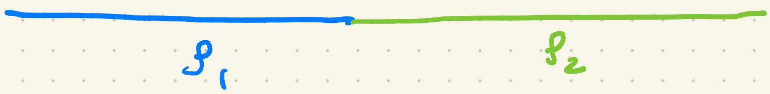
$$\int_{-\infty}^{\infty} dx \; \delta(x-a) = 1$$



Travelling waves : $f\left(k(x - vt)\right)$

$$k = \frac{\omega}{v} = \omega \sqrt{\frac{\rho}{T}}$$ depends on media through ρ.

If we have two media, joined at a junction

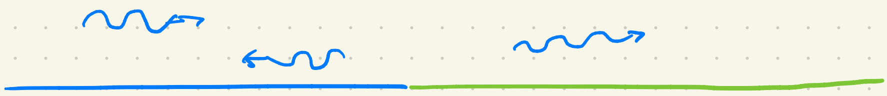


If same ω wave is sent, it will have two different k in the two media

At the junction the wave may pass into the new region with change in k, but to conserve energy

$$\therefore \quad E = \frac{1}{2} \int_0^L dx \left[\rho \dot{y}^2 + T y'^2 \right]$$

Some part should reflect back as well.



$$y_I = A_1 \sin(kx - \omega t) + A_2 \sin(kx + \omega t)$$

$x < 0$

$x = 0$

$$y_{II} = A_3 \sin(k'x - \omega t)$$

$x > 0$

At the junction they should match

$$y_I(0, t) = y_{II}(0, t)$$

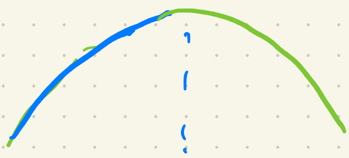
$$\Rightarrow -A_1 \sin \omega t + A_2 \sin \omega t = -A_3 \sin \omega t$$

$$\Rightarrow -A_1 + A_2 = -A_3$$

$$\boxed{\Rightarrow \quad A_1 = A_2 + A_3} \quad - \text{ (a)}$$

It should be a smooth matching

$$\left.\frac{\partial y_I}{\partial x}\right|_{x=0} = \left.\frac{\partial y_{II}}{\partial x}\right|_{x=0}$$



$x = 0$

$$\Rightarrow \left. A_1 k \cos(kx - \omega t) + A_2 k \cos(kx + \omega t) \right|_{x=0}$$

$$= \left. A_3 k' \cos(k'x - \omega t) \right|_{x=0}$$

$$\Rightarrow \quad A_1 k \cos \omega t + A_3 k \cos \omega t = A_3 k' \cos \omega t$$

$$\boxed{\Rightarrow \quad (A_1 + A_2) k = A_3 k'} \quad - \text{ (b)}$$

Exercise: From (a) and (b); show

$$\frac{A_2}{A_1} = \frac{k' - k}{k' + k}$$

$$\frac{A_3}{A_1} = \frac{2k}{k + k'}$$

Since $k = \frac{\omega}{v} = \frac{\omega}{\sqrt{T}} \sqrt{\rho}$

$$\frac{A_2}{A_1} = \frac{\omega\left(\frac{\sqrt{\rho'}}{\sqrt{T}} - \frac{\sqrt{\rho}}{\sqrt{T}}\right)}{\omega\left(\frac{\sqrt{\rho'}}{\sqrt{T}} + \sqrt{\frac{\rho}{T}}\right)} = \frac{\sqrt{T\rho'} - \sqrt{T\rho}}{\sqrt{T\rho'} + \sqrt{T\rho}}$$

The quantity $\sqrt{T\rho}$ is called impedance (Z) of the medium

$\therefore \frac{A_2}{A_1} = \frac{z' - z}{z + z}$; Reflectivity : $r = \left(\frac{A_2}{A_1}\right)^2$

$$= \frac{(z' - z)^2}{(z + z')^2}$$

Similarly $\frac{A_3}{A_1} = \frac{2z'}{(z + z')}$ $\Rightarrow$ Transmitivity

$$t = \frac{z'}{z}\left(\frac{A_3}{A_1}\right)^2 = \frac{4zz'}{(z + z')^2}$$

$$\boxed{\text{Hence, } \quad r + t = 1}$$

Energy carried in a wave

$$E = \frac{1}{2}\int dx\,[\rho\dot{y}^2 + Ty'^2]$$

Further, the wave eqn.

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

$\Rightarrow$ Multiplying through $\rho\frac{\partial y}{\partial t}$

$$\rho\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial t^2} = \rho\frac{\partial y}{\partial t}v^2\frac{\partial^2 y}{\partial x^2} = T\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial x^2}$$

Adding $T\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial t\,\partial x}$

$$\Rightarrow \rho\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial t^2} + T\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial t\,\partial x} = T\left(\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial x^2} + \frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial t\,\partial x}\right)$$

$$\Rightarrow \frac{1}{2}\frac{\partial}{\partial t}\left[\rho\left(\frac{\partial y}{\partial t}\right)^2 + T\left(\frac{\partial y}{\partial x}\right)^2\right]$$

$$= \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial t}\frac{\partial y}{\partial x}\right)$$

$$\int_{x_1}^{x_2} dx\,\frac{\partial}{\partial t}\left[\frac{1}{2}(\rho\dot{y}^2 + Ty'^2)\right] = \int_{x_1}^{x_2} dx\,\frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial t}\frac{\partial y}{\partial x}\right)$$

$$\Rightarrow \frac{\partial}{\partial t}E = T\frac{\partial y}{\partial t}\frac{\partial y}{\partial x}\bigg|_{x_2} - T\frac{\partial y}{\partial t}\frac{\partial y}{\partial x}\bigg|_{x_1}$$

$$\Rightarrow \frac{\partial E}{\partial t} + \mathcal{L}(x_2) - \mathcal{L}(x_1) = 0$$

where $\mathcal{L}(x) = -T\frac{\partial y}{\partial t}\frac{\partial y}{\partial x}$

$\mathcal{Z}(x)$ is the flux of energy carried out !

Exercise: Calculate the flux of energy for

(i) A stationary wave $A \sin kx \cos(\omega t + \phi)$

(ii) A right-moving wave $A \sin(\omega t - kx + \phi)$

Mean flux carried out for one cycle

For $y = A \cos(\omega t + kx + \phi)$

$$\Rightarrow \mathcal{Z}(x) = -T A^2 \omega k \sin^2(\omega t + kx + \phi)$$

$$\langle \mathcal{Z}(x) \rangle = \frac{\int_0^{2\pi/\omega} dt\, \mathcal{Z}(x)}{\frac{2\pi}{\omega}}$$

$$= \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \left(-T A^2 \omega k\right) \sin^2(\omega t + kx + \phi)$$

$$= -\frac{T A^2 \omega k}{2} = -\frac{T A^2 \omega^2}{2v} = -\frac{\sqrt{T\rho}\, A^2 \omega^2}{2}$$

$$= -\frac{Z A^2 \omega^2}{2}$$

$Z = \sqrt{T\rho}$ is the impedance

Exercise : Find out the mean flux carried out for

$$y = A\cos(\omega t - kx + \phi) + A\cos(\omega t + kx + \phi)$$

Electromagnetic Waves

Maxwell's equations

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$\xrightarrow[\text{Form}]{\text{Integral}}$

$$\int \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$
$$\int \vec{B}\cdot d\vec{s} = 0$$
$$\int \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$$
$$\int \vec{B}\cdot d\vec{l} = \mu_0 i + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

* $\int (\vec{\nabla}\cdot\vec{A})\,dv = \int \vec{A}\cdot d\vec{s}$ (Divergence Theorem)
* $\int (\vec{\nabla}\times\vec{A})\cdot d\vec{s} = \int \vec{A}\cdot d\vec{l}$ (Stroke's theorem)

Taking $\vec{\nabla}\times$ of $(\vec{\nabla}\times\vec{E})$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\left[\vec{\nabla}\times\left(\frac{\partial \vec{B}}{\partial t}\right)\right] = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

Exercise : Prove that for any vector $\vec{A}$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

$\therefore$ LHS

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\nabla(\rho/\varepsilon_0) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\right]$$

For a charge-free and current free region

$$-\nabla^2\vec{E} = -\frac{\partial^2}{\partial t^2}(\mu_0\varepsilon_0\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

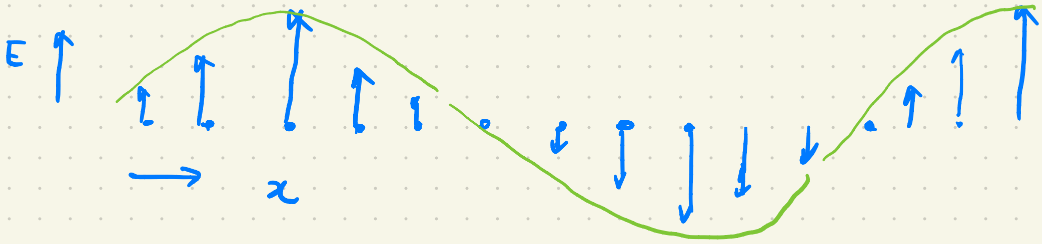
$$\therefore \quad \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \vec{E}$$

$$\Rightarrow \quad \boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E}}$$

Wave eqn. with $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

$\Rightarrow$ A source free region may still support electromagnetic wave.

$\Rightarrow$ For this wave Electric field is the variable (rising and coming down) (analogue of y)



Exercise: Prove that magnetic field also satisfies the same wave eqn.

$$\frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}$$

Solutions of wave eqn.

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \left(\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} \right)$$

Writing for x- component

$$\frac{\partial^2 E_1}{\partial t^2} = c^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E_1$$

$$E_1(t, x, y, z) = T_1(t) X_1(x) Y_1(y) Z_1(z)$$

$$\Rightarrow X Y Z \frac{\partial^2 T}{\partial t^2} = c^2 \left[Y Z \partial_x^2 X + Z X \partial_y^2 Y + X Y \partial_z^2 Z \right]$$

$$\Rightarrow \frac{1}{T} \frac{\partial^2 T}{\partial t^2} = c^2 \left[\frac{\partial_x^2 X}{X} + \frac{\partial_y^2 Y}{Y} + \frac{\partial_z^2 Z}{Z} \right]$$

LHS is function of t only and RHS is function of x, y, z.

$$\frac{1}{T} \frac{\partial^2 T}{\partial t^2} = -\omega^2 = c^2 \left[\frac{\partial_x^2 X}{X} + \frac{\partial_y^2 Y}{Y} + \frac{\partial_z^2 Z}{Z} \right]$$

$$\Rightarrow \frac{\partial^2 T}{\partial t^2} = -\omega^2 T \Rightarrow T = T_0 e^{i\omega t} \text{ or } T_0 e^{-i\omega t}$$

$$\left[\frac{\partial_x^2 X}{X} + \frac{\partial_y^2 Y}{Y} + \frac{\partial_z^2 Z}{Z} \right] = \frac{-\omega^2}{c^2} = -k^2$$

$$\frac{\partial_x^2 X}{X} = -k^2 - \frac{\partial_y^2 Y}{Y} - \frac{\partial_z^2 Z}{Z}$$

LHS is a function of x only while RHS of y, z

$$\frac{\partial_x^2 X}{X} = -k_x^2 = -k^2 - \frac{\partial_y^2 Y}{Y} - \frac{\partial_z^2 Z}{Z}$$

$$\Rightarrow \quad \partial^2 X - k_x^2 X = 0$$

$$\Rightarrow \quad X = X_0 e^{ik_x x} \quad \text{or} \quad X_0 e^{-ik_x x}$$

$$-k^2 - \frac{\partial_y^2 Y}{Y} - \frac{\partial_z^2 Z}{Z} = -k_x^2$$

$$\Rightarrow \quad \underbrace{\frac{\partial_y^2 Y}{Y}}_{y\text{-dep}} = \underbrace{-(k^2 - k_x^2) - \frac{\partial_z^2 Z}{Z}}_{z\text{-dep}}$$

$$\therefore \quad \frac{\partial_y^2 Y}{Y} = -k_y^2 = -(k^2 - k_x^2) - \frac{\partial_z^2 Z}{Z}$$

$$\Rightarrow \quad Y = Y_0 e^{+ik_y y} \quad \text{or} \quad Y_0 e^{-ik_y y}$$

$$\therefore \quad -(k^2 - k_x^2) - \frac{\partial_z^2 Z}{Z} = -k_y^2$$

$$\Rightarrow \quad \frac{\partial_z^2 Z}{Z} = -(k^2 - k_x^2 - k_y^2) = -k_z^2$$

$$Z = Z_0 e^{ik_z z} \quad \text{or} \quad Z_0 e^{-ik_z z}$$

$$\therefore \quad E_1 = T_0 X_0 Y_0 Z_0 e^{i\omega t} e^{i(k_1 x + k_2 y + k_3 z)}$$

$$\text{or} \quad T_0 X_0 Y_0 Z_0 e^{-i\omega t} e^{i(k_1 x + k_2 y + k_3 z)}$$

$$= e^{\pm i\omega t + i\vec{k}\cdot\vec{r}}$$

Similarly for other components E_2, E_3

Ultimately we have

$$\vec{E} = \vec{E}_0 \, e^{i(\omega t - \vec{k}\cdot\vec{r})}$$

or $\vec{E}_0 \, e^{-i(\omega t + \vec{k}\cdot\vec{r})}$

For an electromagnetic wave of given $\vec{k}$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\Rightarrow \; -i\vec{k}\cdot\vec{E} = 0$$

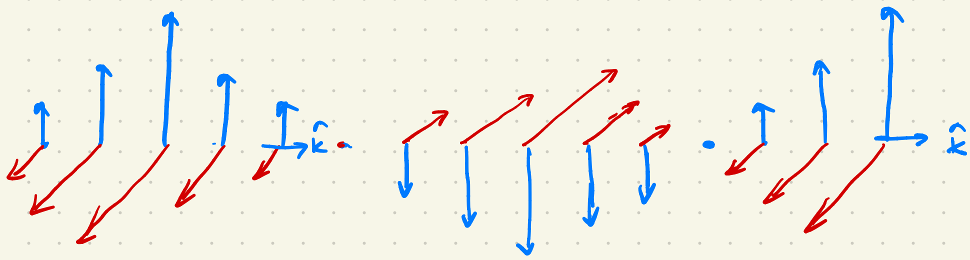
(The oscillation is orthogonal to the propagation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

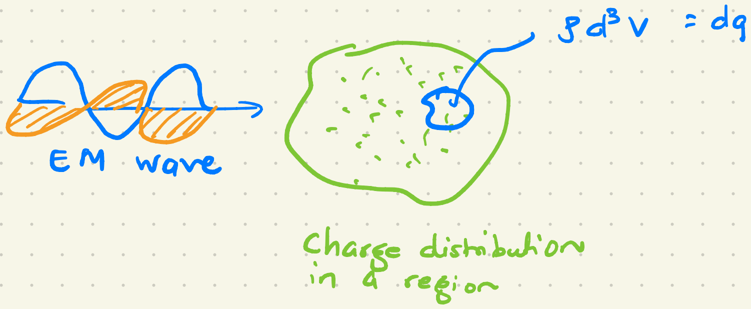
$$\Rightarrow \; i\vec{k}\times\vec{E} = i\omega\vec{B}$$

$$\hat{k}\times\vec{E} = c\vec{B}$$

(Magnetic field is orthogonal to both $\hat{k}$ and $\vec{E}$,



Work done by an EM wave



Let us consider a case when an EM wave enters a charge region

On an infinitesimal segment the work done

$$dW = \vec{F} \cdot d\vec{l}$$
$$= dq\,(\vec{v} \times \vec{B} + \vec{E}) \cdot \vec{v}\,dt$$
$$= dq\,dt\,(\vec{E} \cdot \vec{v}) \quad \Big\{ \text{since } \vec{v} \cdot (\vec{v} \times \vec{B}) = 0$$
$$= \rho d^3V\,dt\,(\vec{E} \cdot \vec{v})$$

$$\therefore \frac{dW}{dt} = (\rho \vec{v} \cdot \vec{E})\,d^3V$$
$$= \vec{J} \cdot \vec{E}\,d^3V \quad (\text{as } \vec{J} = \rho \vec{v}$$

Now from Maxwell's eqn

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J} = \frac{1}{\mu_0}\left(\vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \vec{J} \cdot \vec{E} = \frac{1}{\mu_0}\left(\vec{E} \cdot \vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{B})$$

$$\vec{E}\cdot(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

$$= -\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

$$\therefore \vec{J}\cdot\vec{E} = \frac{1}{\mu_0}\left[-\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\cdot(\vec{E}\times\vec{B})\right] - \mu_0\varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$= \frac{1}{2}\left[-\frac{1}{\mu_0}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) - \varepsilon_0\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E})\right] - \vec{\nabla}\cdot\left[\frac{1}{\mu_0}(\vec{E}\times\vec{B})\right]$$

$$\therefore \frac{dW}{dt} = -\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\frac{1}{\mu_0}\vec{B}\cdot\vec{B} + \varepsilon_0\vec{E}\cdot\vec{E}\right)\right]d^3V - \vec{\nabla}\cdot\left[\frac{1}{\mu_0}(\vec{E}\times\vec{B})\right]d^3V$$

For the full distribution

$$\frac{dW}{dt} = -\frac{\partial}{\partial t}\left[\int d^3V\,\frac{1}{2}\left(\frac{1}{\mu_0}\vec{B}\cdot\vec{B} + \varepsilon_0\vec{E}\cdot\vec{E}\right)\right] - \int\vec{\nabla}\cdot\left[\frac{1}{\mu_0}(\vec{E}\times\vec{B})\right]d^3V$$

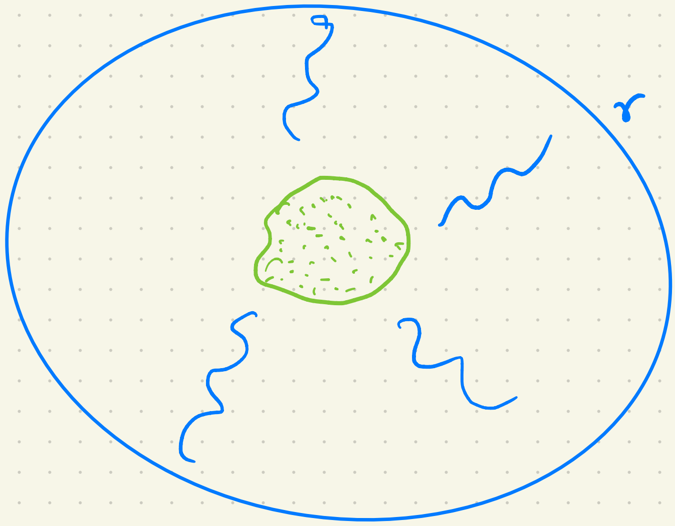
$$= \underbrace{-\frac{\partial}{\partial t}\int d^3V\,\rho}_{\text{internal energy change}} - \underbrace{\oint d\vec{S}\cdot\frac{1}{\mu_0}(\vec{E}\times\vec{B})}_{\text{Flux carried through the boundary}}$$

Energy density of EM - wave

$$\rho = \frac{1}{2}\left[\frac{1}{\mu_0}\vec{B}\cdot\vec{B} + \varepsilon_0 \vec{E}\cdot\vec{E}\right]$$

Flux of EM wave (Poynting vector)

$$\mathcal{F} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$



$\mathcal{F}(r)$ measures flux on energy carried outwards at r

★ Light is a wave

Wave aspects
(i) Superposition
(ii) Polarization
(iii) Interference
(iv) Diffraction
(v) Coherence