PHY102 : Assignment 4

1. (a) Prove the identity
$$\nabla \cdot (\phi \mathbf{E}) = (\nabla \phi) \cdot \mathbf{E} + \phi \nabla \cdot \mathbf{E}$$
by explicitly calculating the various derivatives in Cartesian coordinates.

   (b) This identity holds for any scalar function ϕ and any vector function $\mathbf{E}$. In particular, it holds for the electric potential and field. Use this fact to show that
$$\frac{1}{2}\int_V \rho\phi dv = \frac{\epsilon_0}{2}\int_V E^2 dv.$$

2. (P&M Example) A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the center of one cavity and q_c at the center of the other. A considerable distance r away is another charge q_d. What force acts on each of the four objects, A, q_b, q_c, q_d ? Which answers, if any, are only approximate, and depend on r being relatively large ?

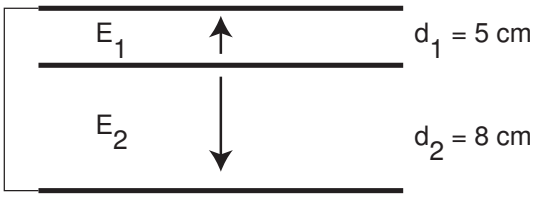
3. (P&M 3.9) A conducting spherical shell has charge Q and radius R_1. A larger concentric conducting spherical shell has charge $-Q$ and radius R_2. If the outer shell is grounded, explain why nothing happens to the charge on it. If instead the inner shell is grounded, find its final charge.

4. (P&M 3.12) A point charge q is located between two parallel infinite conducting planes, a distance d from one end and $l - d$ from the other. Where should image charges be located so that the electric field is everywhere perpendicular to the planes ?

5. (P&M 3.25) Given that the capacitance of an isolated conducting disk of radius a is $8\epsilon_0 a$, what is the energy stored in the electric field of such a disk when the net charge on the disk is Q ? Compare this with the energy in the field of a nonconducting disk of the same radius that has an equal charge Q distributed with uniform density over its surface. Which ought to be larger ? Why ?

6. (P&M 3.43) Imagine the xy plane, the xz plane, and the yz plane all made of metal and soldered together at the intersections. A single point charge Q is located a distance d from each of these planes. Describe by a sketch the configuration of image charges you need to satisfy the boundary conditions. What is the direction and magnitude of the force that acts on the charge Q ?

7. (P&M 3.54) Three conducting plates are placed parallel to one another, as shown in



the figure. The outer plates are connected by a wire. The inner plate is isolated and carries a net surface charge density σ (the combined value from the top and bottom

faces of the plate). In what proportion must this charge divide itself into a surface charge σ_1 on one face of the inner plate and a surface charge σ_2 on the other side of the same plate ?

8. (P&M 3.66) A 100-pF capacitor is charged to 100 volts. After the charging battery is disconnected, the capacitor is connected in parallel to another capacitor. If the final voltage is 30 volts, what is the capacitance of the second capacitor. How much energy was lost, and what happened to it ?