IDC204 - Theory of Computation
2nd Mid-semester examination

4th of March, 2023

Except for question 2, you MUST give complete, clear, and precise justifications for your answers. Answers without justificaion will **not** earn any credit.

1. For each of these languages over the alphabet $\Sigma := \{0, 1, 2\}$, answer the following questions and explain your reason completely.
   (a) (2 points) Can the language $L := \{0^n \mid n \in \mathbb{N}\}$ be described by a regular expression?
   (b) (2 points) Can the language $L := \{0^n 1^n \mid n \in \mathbb{N}\}$ be described by a context free grammar?
   (c) (3 points) Can the language $L := \{0^n 1^n 2^n \mid n \in \mathbb{N}\}$ be described by a context free grammar?

2. Answer the following questions without explanations.
   (a) (2 points) Find a regular expression to define the language consisting of all even numbers expressed in the decimal number system. i.e. $\Sigma := \{0, 1, \ldots, 9\}$, and $L := \{2, 4, 6, \ldots, 10, 12, \ldots\}$. 008X
   (b) (2 points) If the alphabet is $\Sigma = \{0, 1, \ldots, 9\}$, what language does the regular expression $(1 \cup \ldots \cup 9)\Sigma\Sigma\Sigma^*$ define?
   (c) (1 point) Which of these is true: A. *Every* regular language is also context free B. *Every* context free language is also regular C. Both D. None of the above
   (d) (1 point) Which of these is true: A. *Every* finite state automaton is also a push-down automaton. B. *Every* deterministic push-down automaton with a finite stack is a finite state automaton. C. Both D. None of the above
   (e) (1 point) If a language is a finite set, is it context free?

3. (3 points) If a regular expression R_1 defines a language L_1 and a regular expression R_2 defines a language L_2, does there exist a regular expression to define $R_1 \cap R_2$? Explain the details clearly and completely. (*Hint:* Remember that languages that can be described by regular expressions are precisely the same languages that can be recognized by finite state automata. Also remember deMorgan's law!)

4. (3 points) Let L_1 be the language defined by a context free grammar $(V_1, \Sigma_1, R_1, S_1)$ and L_2 be the language defined by a context free grammar $(V_2, \Sigma_2, R_2, S_2)$. Recall that for each $i = 1, 2$, V_i denotes the set of variables, Σ_i denotes the alphabet (which is also known as the set of terminals), R_i denotes the set of rules, and S_i denotes the start variable. Is $L_1 \cup L_2$ always context free? If yes, you must clearly state what the new variables, rules, and start variable are in terms of the old ones. If no, give examples of two context free languages whose union is not context free.