IISER Mohali

[Session 2018-19, Even Semester]

PHY 304 (Statistical Mechanics)

Quiz # 4

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Name:

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Max. Marks: 10

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1. Consider a system of N identical particles of mass m distributed in single particle energy levels of energies $\epsilon, 2\epsilon, 3\epsilon, 4\epsilon, ..., \infty$. Let's assume the particles are *relativistic*. What is the chemical potential of the system at $T = 0$, if the particles are (i) bosons, (ii) fermions ? (2)

2. Calculate the Fermi energy (ϵ_F) of an ideal gas of N *electrons* in two dimensions. (2)

3. The quantity $A(T) = \int_0^\infty d^3r\ (e^{-\beta U(r)} - 1)$ contributes to the partition function of a real gas due to the interaction ($U(r)$) between the molecules. Consider following form of the potential

$$U(r) = \begin{cases} \infty & r < r_0 \\ -U_0\left(\frac{r_0}{r}\right)^n & r \geq r_0 \end{cases}$$

where, r_0 is the distance of closest approach and U_0 is the strength of the potential. What are the allowed values of n ? You can take $U_0 < k_BT$. (2)

4. Argue that the inequality $\beta(\epsilon - \mu) >> 1$ is consistent with the classical limit $\lambda^3(N/V) << 1$ in a system of quantum ideal gas of N identical particles. (2)

5. Given a system of quantum ideal gas, which measurement/s would you perform on it to learn if the gas contains bosons or fermions ? (2)

Useful expressions:

1. Thermal de Broglie wavelength

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$$

2. Mean occupation number in FD and BE statistics

$$\langle n_i \rangle = \frac{1}{e^{\beta(\epsilon_i - \mu)} \pm 1}$$
