PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 8

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1. Estimate the approximate values of the ground state energy of a particle of mass m moving in a potential $V(x) = V_0|x|$, $V_0 > 0$, using the WKB approximation.

2. Use the WKB approximation to find the allowed energies of the general power law potential $V(x) = \alpha|x|^\nu$ where ν is a positive number.

3. Use the WKB method to estimate the ground state energy of a particle of mass m that moves in a 3D potential $V(r) = kr$, where k is a constant having the dimensions of a force.

4. The WKB approximation can be applied to the radial equation for spherically symmetric potentials. Consider the case $l = 0$ and use the formula $\int_0^{r_0} p(r)dr = (n - 1/4)\pi\hbar$ (r_0 is the turning point) to find the allowed energies of a particle in the logarithmic potential $V(r) = V_0 ln(r/a)$, V_0, a being constants.

5. Use the WKB approximation in the form

$$\int_{r_1}^{r_2} p(r)dr = (n - 1/2)\pi\hbar$$

to estimate the bound state energies for hydrogen atom. Retain the centrifugal term in the effective potential. A helpful integral is

$$\int_a^b \frac{1}{x}\sqrt{(x-a)(b-x)} = \frac{\pi}{2}(\sqrt{b} - \sqrt{a})^2$$

6. Consider a symmetric double well potential with turning points x_1, x_2 and consider only the bound states with $E < V(0)$. Write down the WKB wave functions in the regions (i) $x > x_2$, (ii) $x_1 < x < x_2$, (iii) $0 < x < x_1$. Impose the connection formulae and write down the wave function $\psi(x)$ in terms of a constant D and θ where

$$\theta \equiv \frac{1}{\hbar}\int_{x_1}^{x_2} p(x)dx$$

Since $V(x)$ is symmetric consider only even and odd wave functions and show that this leads to the quantization condition $\tan\theta = \pm 2e^{\phi}$, where

$$\phi \equiv \frac{1}{\hbar}\int_{-x_1}^{x_1} |p(x')|dx'$$

7. Use the WKB approximation to estimate the transmission coefficient of a particle of mass m and energy E moving in the potential barrier

$$\begin{aligned} V(x) &= V_0(x/a+1), \ -a<x<0 \\ &= V_0(1-x/a), \ 0<x<a \\ &= 0, \ \text{elsewhere} \end{aligned}$$

with $0 < E < V_0$.

8. Use the WKB approximation to estimate the transmission coefficient of a particle of mass m and energy E moving in the potential barrier

$$\begin{aligned} V(x) &= V_0(1-x^2/a^2), \ |x|<a \\ &= 0, \ |x|>a \end{aligned}$$

with $0 < E \leq V_0$.