PHY 201 : Assignment-2

All Problems carry equal weightage. Symbols have their usual meanings.

1. On a circular elastic sheet of radius a, find out the solution to the wave equation. What is the kinetic energy contained in an annular region between radii r_1 and r_2.

2. A D-shaped membrane with radius a of the semicircular portion is fixed along its boundaries. Write down the equation of motion a wave satisifies in polar (r, θ) co-ordinates and find out the condition for the normal modes of this membrane. Can you write down the normal modes for this set-up ?

3. On a string of length L a wave of the form $x \sin \pi x/L$ is set up with zero velocity at time $t = 0$. Find out the contribution of $n-$th normal mode in this configuration and find out the wave at a time t.

4. If two right-moving cosine waves are combined with slightly different frequencies and wave numbers $(\omega, k; \omega + \delta\omega, k + \delta k)$, show that the envelope of the resultant profile moves with a velocity $\delta\omega/\delta k$ (*This is called the group velocity*). Does the answer change if there is a relative phase between the two waves ?

5. Find out the expression for kinetic energy of a right-moving longitudinal wave in a cylinder of gas, if the gas density is ρ, cylinder cross section is A and the length of the gas column is L.

6. For a transverse wave on a string, find out the expression for the acceleration of an infintesimal segment. Find out the average aceeleration of the string and its variance.

7. For a wave configuration on a string of length ℓ which is started with zero velocity at $t = 0$, can one find out the coefficients of the normal modes by a time integration ? If so, establish the orthogonality relation in time in analogy to

$$\int_0^L y_n(x, 0) y_m(x, 0) = \frac{\ell}{2} \delta_{nm}$$

   and suggest a strategy.

8. If a right moving sine wave and a left moving sine wave (both without any phase) of equal frequencies are superposed with different amplitudes find out the locations where the resultant amplitudes are maximum. Find out the places where the resultant amplitude is minimum. What is the maximum to minimum amplitude ratio ?