PHY306 Advanced Quantum Mechanics Jan-Apr 2025: Assignment 2

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1. Consider a hydrogen atom at the origin, surround by six point charges. first pair $(q1, q1)$ separated by distance $2d_1$ along x-axis, second pair $(q2, q2)$ separated by distance $2d_2$ along y-axis, third pair $(q3, q3)$ separated by distance $2d_3$ along z-axis. Ignore spin. Assume a perturbation

$$H_p = V_0 + 3(\beta_1 x^2 + \beta_2 y^2 + \beta_3 z^2) - (\beta_1 + \beta_2 + \beta_3) r^2$$

where $\beta_1 = -\frac{eq_i}{4\pi\epsilon_0 d_i^3}$ and $V_0 = 2(\beta_1 d_1^2 + \beta_2 d_2^2 + \beta_3 d_3^2)$. Assume that $r << d_1$, $r << d_2$, $r << d_3$. (a) Find lowest order correction to the ground state energy. (b) Calculate the first order corrections to energy of the first excited states $n = 2$. (c) Into how many levels does this fourfold degenerate system split if $\beta_1 = \beta_2 = \beta_3$ and if $\beta_1 = \beta_2 \neq \beta_3$ and if all three β are different?

2. Consider an isotropic 3D harmonic oscillator subjected to a perturbation $V_p = -\lambda xy$ where λ is a small real number. Find the energy of the first excited state to first-order degenerate time-independent perturbation theory. You may write x, y, z in terms of creation and annihilation operators $a, a^\dagger$.

3. Consider a positronium particle subjected to a weak static magnetic field in the xz plane $B = B_0(i + k)$ where B_0 is a small constant. Neglect spin-orbit interaction and calculate the energy levels of the $n = 2$ states to first-order perturbation.

4. Consider a particle of mass m free to move on a circular wire of circumference L, with stationary states

$$\psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x/L}, -L/2 < x < L/2$$

where $n = 0, \pm 1, \pm 2, ...$ and energies are

$$E_n = \frac{2}{m} \left(\frac{n\pi\hbar}{L} \right)^2$$

. Introduce the perturbation $H_p = -V_0 e^{-x^2/a^2}$ where $a << L$. (i) Find the first-order correction to E_n. (ii) What are the good linear combinations of ψ_n and ψ_{-n} (iii) Find a Hermitian operator A that fits the requirements of the theorem $[A, H_p]$ and show that the simultaneous eigenstates of H_0 and A are the same as the ones found in (ii).

5. Suppose the Hamiltonian of a rigid rotator in a magnetic field perpendicular to its axis is of the form

$$AL^2 + BL_z + CL_y$$

Assume $B >> C$ and use perturbation theory to lowest nonvanishing order to get approximate energy eigenvalues.

6. Three distinguishable particles of equal mass m are in a one-dimensional harmonic oscillator potential $H_0 = \sum_{i=1}^{3}()p_i^2/2m + 1/2m\omega^2 x_i^2$. The three particles are subject to a weak, short-range attractive potential

$$H_p = -V_0(\delta(x_1 - x_2) + \delta(x_2 - x_3) + \delta(x_3 - x_1))$$

Use first-order perturbation theory to calculate the system's energy levels of (a) the ground state and (b) the first-excited state.