IISER Mohali

[Session 2018-19, Even Semester]

PHY 304 (Statistical Mechanics)

Mid-Semester Exam

Instructor: Ambresh Shivaji (**email**: ashivaji)
TA: Subhadip Ghosh (**email**: subhadipg)
Max. Marks: 20

Name:
Reg. No.:

---

1. Consider a system of two particles each of which can be in one of the three states with energies 0, ϵ and 2ϵ. Write down the partition function of the system, if the particles follow:

   (a) Classical statistics (MB)

   (b) Quantum statistics of *identical bosons* (BE)

   (c) Quantum statistics of *identical fermions* (FD).

   What is the statistical probability that system is in a state with energy 4ϵ among all possible allowed states, in each case ? Calculate the average energy of the system in three cases. Compare them in $T \to \infty$ limit and justify the outcome.

2. Consider an *ultrarelativistic* ideal gas ($p >> mc$) of N *identical* monoatomic molecules.

   (a) Calculate the partition function of the gas.

   (b) Show that the entropy of the gas is given by

   $$S = Nk_B\Big[4 - \ln(N\Lambda^3/V)\Big],$$

   where $\Lambda = \hbar c \pi^{2/3}/(k_B T)$.

   (c) Show that the adiabatic expansion of such a gas is governed by

   $$PV^{4/3} = \text{constt.}$$

3. The equation of state for a real gas is given by,

   $$\Big(P + \frac{\alpha}{V^2}\Big)(V - \beta) = Nk_BT,$$

   where, α and β are phenomenological parameters. Show that, at a given temperature, the specific heat at constant volume (C_V) for a real gas with fixed N does not depend on the volume.

Useful expressions:

$$\int_0^\infty x^n e^{-x}\, dx = n!$$
