ASSINGNMENT- 6

Date - 22-Sept-2023
No Need to Submit.

1. Show that $g(x,t) = (1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$, satisfies the following eq^n $(1-x^2)\dfrac{\partial^2 \phi}{\partial x^2} - 2x\dfrac{\partial \phi}{\partial x} + t\dfrac{\partial^2}{\partial t^2}(t\phi) = 0$.

2. Prove that $P_{2n}(0) = (-1)^n \dfrac{1\cdot 3\cdot 5 \cdots (2n-1)}{2\cdot 4\cdot 6 \cdots 2n}$, $P_{2n+1}(0) = 0$.

3. Show that $\dfrac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)P_n(x)\, z^n$.

4. Prove that $P_n(-\tfrac{1}{2}) = P_0(-\tfrac{1}{2})P_{2n}(\tfrac{1}{2}) + P_1(-\tfrac{1}{2})P_{2n-1}(\tfrac{1}{2}) + \cdots + P_{2n}(-\tfrac{1}{2})P_0(\tfrac{1}{2})$.

5. Show that $1 + \frac{1}{2}P_1(\cos\theta) + \frac{1}{3}P_2(\cos\theta) + \frac{1}{4}P_3(\cos\theta) + \cdots = \log\dfrac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}$.

6. Show that
$$P_l^{-m}(x) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(x) \qquad m>0$$

7. Show that
$$\int_{-1}^{+1} P_l^m(x)\, P_{l'}^m(x)\, dx = \frac{2(l+m)!}{(2l+1)(l-m)!}\,\delta_{ll'}$$

8. Show that
$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, Y_{l,m}(\theta,\phi)\, Y_{l'm'}(\theta,\phi)\, d\theta = \delta_{ll'}\,\delta_{mm'}$$

9. Calculate - $\int_{-1}^{+1}(1-x^2)\,P_m'(x)\,P_n'(x)\,dx$.

10. Evaluate - $\int_{-1}^{+1}(x^2-1)\,P_{n+1}P_n'\,dx$.

11. Show that - $(x^2-1)P_n' = (n+1)(P_{n+1} - xP_n)$.

12. Evaluate - $\int_{-1}^{+1} x^2\,[P_n(x)]^2\,dx$.

For More Examples - See.
Riley, Hobson and Bence.