PHY306 Advanced Quantum Mechanics Jan-Apr 2025: Assignment 1

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1. Suppose a delta-function bump is put in the center of the infinite square well: $H' = \alpha\delta(x - a/2)$ where α is a constant. Find the first-order correction to the allowed energies.

2. Consider a harmonic oscillator with a slightly increased spring constant $k' = (1 + \epsilon)k$. Calculate the first order perturbation in the energy.

3. Consider a charged particle in a one-dimensional harmonic oscillator potential. Suppose a weak electric field is turned on so that the energy is shifted by an amount $H' = -qEx$. Show that there is no first order change in the energy levels and calculate the second order correction. The Schrodinger equation can be solved exactly for this case by a change of variables $x' = x - (qE/m\omega^2)$. Find the exact energies and show that they are consistent with the perturbation theory expressions.

4. Consider an isotropic harmonic oscillator in two dimensions with the Hamiltonian
$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2)$$
(a) Now apply a perturbation $H_p = \delta m\omega^2 xy$, where δ is a dimensionless real number $<< 1$. Find the energy upto first order for the ground state. (b) Solve the $H_0 + H_p$ problem exactly and compare with the perturbation results.

5. Calculate the energy of the ground state using first-order perturbation theory for a particle moving in a 1D box potential of length L with walls at $x = 0$ and $x = L$, when a weak potential $H_p = \lambda x^2$ is added,where $\lambda << 1$.

6. Consider two identical spin-1/2 particles confined in an isotropic 3D harmonic oscillator potential of frequency ω. (a) Find the ground state energy and the corresponding wave function of this system when the two particles do not interact. (b) Consider now that there exists a weakly attractive spin-dependent potential between the two particles, $V(r_1, r_2) = -kr_1r_2 - \lambda S_{1z}S_{2z}$, where k and λ are two small positive real numbers. Find the ground state to first-order time-independent perturbation theory.