PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 6

Prof. Kavita Dorai, Department of Physics, IISERM, kavita@iisermohali.ac.in

1. Find the ground state energy of the delta-function potential using the variational method. Assume a Gaussian trial wave function.
$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

2. Find an upper bound on the ground-state energy of the one-dimensional infinite square well using the triangular trial wave function
$$\begin{aligned}\psi(x) &= Ax, \;\; 0 \leq x \leq a/2, \\ &= A(a-x), \;\; a/2 < x < a, \\ &= 0, \;\; \text{otherwise}\end{aligned}$$
Find A from the normalization.

3. Find the best bound on E_g for the delta-function potential $-\alpha\delta(x-a/2)$ using the triangular trial wave function given above, where a is an adjustable parameter.

4. Use a Gaussian trial function to obtain the lowest upper bound on the ground state energy of (a) the linear potential $V(x) = \alpha|x|$ and (b) the quartic potential $V(x) = \alpha x^4$.

5. Find the best bound on the ground state energy for a one-dimensional harmonic oscillator using a trial wave function of the form
$$\psi(x) = \frac{A}{x^2 + b^2}$$
where A is found by normalization and b is an adjustable parameter.

6. Assume a Yukawa potential of the form
$$V(r) = -\frac{e^2 e^{-\mu r}}{4\pi\epsilon_0 r}$$
where $\mu = m_\gamma c/\hbar$. Choose any trial wave function you want to and estimate the binding energy of a "hydrogen" atom with this potential. Assume $\mu a << 1$ and give your answer correct to order $(\mu a)^2$.

7. Consider a particle of mass m moving under a central force potential $V(r) = -\alpha e^{-2\mu r}$, where $\alpha, \mu > 0$. Use a variational trial wavefunction $\psi_\lambda(r) = Ce^{-\lambda r}$. Find C using the normalization of the wavefunction. Using the variational method find the lowest energy. A useful integral is $\int_0^\infty dx x^n e^{-x} = n!$.