PHY 101 : Quiz 1

1. In the spherical polar co-ordinates find out the expression for $d\hat{\theta}/dt$.

2. For a particle moving in a circular orbit in the $x - y$ plane, find out the condition for which $\mathbf{v} \cdot \mathbf{a} = 0$, where $\mathbf{v}$ is the velocity in the circular trajectory and $\mathbf{a}$ its acceleration.

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Q. 1. $\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\sin\theta\cos\phi\,\hat{i} - \dot{\phi}\cos\theta\sin\phi\,\hat{i} - \dot{\theta}\sin\theta\sin\phi\,\hat{j} + \dot{\phi}\cos\theta\cos\phi\,\hat{j} - \dot{\theta}\cos\theta\,\hat{k}$$

$$= -\dot{\theta}\left(\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}\right) + \dot{\phi}\cos\theta\left(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}\right)$$

$$= -\dot{\theta}\hat{r} + \cos\theta\,\dot{\phi}\,\hat{\phi}$$

Q. 2. $\vec{r}\cdot\vec{a}$ [Let us do this in cylindrical polar co-ordinates]

$$= \left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{k}\right)\cdot\left((\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} + \ddot{z}\hat{k}\right)$$

$$= \dot{r}(\ddot{r} - r\dot{\theta}^2) + r\dot{\theta}(2\dot{r}\dot{\theta} + r\ddot{\theta}) + \dot{z}\ddot{z}$$

For constant r and z $\Rightarrow$ $\dot{r} = 0$, $\dot{z} = 0$

$$\vec{v}\cdot\vec{a} = r^2\dot{\theta}\ddot{\theta} = 0 \quad \text{(demanded)}$$

$r = 0$, $\dot{\theta} = 0$ (not moving) ruled out

$\ddot{\theta} = 0$!!