PHY306 Advanced Quantum Mechanics Jan-April 2025: Assignment 6

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1. Find the Clebsch-Gordan coefficients for the addition of two angular momenta $j_1 = 1$ and $j_2 = 2$.

2. Find the Clebsch-Gordan coefficients for the addition of and three angular momenta each of $j = 1/2$.

3. Consider two nonidentical particles each with angular momentum 1, with the Hamiltonian given by

$$H = \frac{\epsilon_1}{\hbar^2}(L_1 + L_2).L_2 + \frac{\epsilon_2}{\hbar^2}(L_{1z} + L_{2z})^2$$

where ϵ_1, ϵ_2 are constants with dimensions of energy. Find the energy levels and degeneracies for those states of the system whose total angular momentum is equal to $2\hbar$.

4. A deuteron has spin 1. What are the possible spin and total angular momentum states of two deuterons in an arbitrary angular momentum state L.

5. A particle of spin 1/2 is in a state with orbital angular momentum $l = 2$. What are its possible states of total angular momentum. If its Hamiltonian is given by $H = a + bL.S + cL^2$ where a, b, c are numbers, find the values of the energy for each of the different states of total angular momentum. Write the answer in terms fo a, b, c.

6. The spin-dependent Hamiltonian of an electron-positron system in the presence of a uniform magnetic field in the z-direction ($B = B\hat{k}$) can be written as

$$H = \lambda S_1.S_2 + \left(\frac{eB}{mc}\right)(S_{1z} - S_{2z})$$

where λ is a real number and S_1, S_2 are the spin operators for the electron and positron, respectively. If the spin function is given by $|1/2, -1/2\rangle$, find the energy eigenvalues and their corresponding eigenvectors.