Assignment - 9
PHY 310

Date - 03-Nov-2023
No Need to Submit.

1. Find the analytic function $f(z) = u(x,y) + i v(x,y)$ ~~when~~ (a) if $u(x,y) = x^3 - 3xy^2$ and (b) if $v(x,y) = e^{-y} \sin x$.

2. If there is some common region in which $w_1 = u(x,y) + i v(x,y)$ and $w_2 = w_1^*$ are both analytic prove that $u(x,y)$ and $v(x,y)$ are constants.

3. Show that $f(z) = 1/z$ is an analytic function in the entire $z$ plane except at the point $z = 0$.

4. Using $f(z) = f(re^{i\theta}) = R(r,\theta) e^{i\phi(r,\theta)}$ in which $R(r,\theta)$ and $\phi(r,\theta)$ are differentiable real functions of $r$ and $\theta$, Show that the Cauchy-Riemann conditions in polar coordinates becomes

(a) $\dfrac{\partial R}{\partial r} = \dfrac{R}{r} \dfrac{\partial \phi}{\partial \theta}$ and (b) $\dfrac{1}{r} \dfrac{\partial R}{\partial \theta} = -R \dfrac{\partial \phi}{\partial r}$.

5. For each of the following functions $f(z)$, find $f'(z)$ and identify the maximal region within which $f(z)$ is analytic.

(a) $f(z) = \dfrac{\sin z}{z}$

(b) $f(z) = z^2 - 3z + 2$

(c) $f(z) = \dfrac{1}{z(z+1)}$

(d) $f(z) = \tanh(z)$

6. The functions $f(z)$ have a derivative for what complex values -

(a) $f(z) = z^{3/2}$

(b) $f(z) = \tan^{-1}(z)$

(c) $f(z) = z^{-3/2}$

(d) $f(z) = \tanh^{-1}(z)$.