IISER Mohali

[Session 2018-19, Even Semester]

PHY 304 (Statistical Mechanics)

Problem Set 02: Review of Probability

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1. We can define generating function for an infinite sequence $\{a_0, a_1, a_2....\}$ as

$$G(s) = \sum_{n=0}^{n=\infty} a_n s^n$$

so that,

$$a_n = \frac{1}{n!}\frac{\partial^n G(s)}{\partial s^n}\Big|_{s=0}.$$

Find the generating functions for following sequences

(a) $a_n(\lambda) = \frac{\lambda^n}{n!}e^{-\lambda}$

(b) $a_n = n^2$

2. Show that the Binomial distribution reduces to Poisson distribution in the limit when number of trials is large ($n >> 1$), probability of success in each trial is small ($p << 1$) and np remains constant.

3. *Stirling's Approximation:* The integral representation of $N!$ is given by

$$N! = \int_0^\infty dx\ e^{-x} x^N.$$

(a) Show that,

$$\lim_{N\to\infty} \ln N! \approx N \ln N - N + \frac{1}{2}\ln(2\pi N)$$

(b) Plot $\delta_N = \ln N!/(N\ln N - N) - 1$ for increasing values of $N(> 1)$. What is the (rough) minimum value of N for which δ_N falls below 1%.

[**Hint:** Argue that for large N the integral receives most of its contribution where the integrand is maximum.]

4. Show that in large n limit the Binomial distribution,

$$P(n, k) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

with mean $\langle k\rangle$ and variance σ_k^2 reduces to a Gaussian distribution with same mean and variance.

[**Hint:** Argue that for large n, the binomial distribution develops a peak around $k = \langle k\rangle$ and k can be treated as a continuous variable.]

5. *Breit-Wigner distribution:*

$$p(\omega) = \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}$$

In particle physics, this distribution is used to describe the energy profile of a resonance. In mathematics,

(a) Calculate mean and variance for this distribution.

(b) Find its characteristic function.

(c) Find the distribution for $y = \sum_{i=1}^{N} \omega_i$, where ω_i follows Breit-Wigner distribution.