Assignment 3

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. By changing the dependent variable of the DE $y''(x) + 2xy'(x) + x^2y(x) = 0$, show that the general solution is
$$y(x) = \left[Ae^{x} + Be^{-x}\right] \exp\left(-\frac{x^2}{2}\right).$$

2. Given $y = e^x$ be the one solution of the DE $xy''(x) - y'(x) - (x-1)y(x) = 0$, show that the general solution is
$$y(x) = Ae^{x} + Be^{-x}(2x+1).$$

3. Verify that $y_1(x) = \cos(ax)$ is one of the solutions of
$$y''(x) + 2a\cot(ax)y'(x) + 3a^2y(x) = 0$$
and show that the general solution of $y''(x) + 2a\cot(ax)y'(x) + 3a^2y(x) = \operatorname{cosec}(ax)$ is given by
$$y(x) = c_1\cos(ax) + c_2\frac{\cos(2ax)}{\sin(ax)} + \frac{1}{4a^2}\operatorname{cosec}(ax).$$

4. By changing the dependent variable $y(x) = x^2w(x)$, solve the differential equation
$$y''(x) + 2y'(x) + \left(1 - \frac{12}{x^2}\right)y(x) = x^2 + 4x - 10$$
and show that the general solution is
$$y(x) = e^{-x}\left(Ax^4 + \frac{B}{x^3}\right) + x^2.$$

5. Given that $y = x$ is one of the solutions of the homogeneous part of the differential equation $x^2y''(x) + xy'(x) - y(x) = x^2$, show that the general solution of the above equation is
$$y(x) = Ax + \frac{B}{x} + \frac{x^2}{3}.$$

6. Given $y(x) = (3x^2 - 1)/2$ be the 1st solution of the differential equation
$$(1 - x^2)y''(x) - 2xy'(x) + 6y(x) = 0,$$
obtain it's 2nd solution.

7. Given $y(x) = 1 - x$ be the 1st solution of the differential equation

$$xy''(x) + (1 - x)y'(x) + y(x) = 0,$$

obtain it's 2nd solution.

8. Determine by changing independent variable the general solution of the following differential equations

   (a)
   $$y''(x) + \frac{2}{x}\left(1 - \frac{1}{x}\right)y'(x) + \frac{1}{x^4}y(x) = 0.$$

   (b)
   $$y''(x) + \frac{2}{x}y'(x) + \frac{a^4}{x^4}y(x) = 0.$$

   (c)
   $$x^2y''(x) + xy'(x) + 4y(x) = 0.$$

9. If $y_1 = x$ and $y_2 = xe^x$ are two solutions of the homogeneous differential equation associated with $x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3$, find the general solution of the differential equation.

10. Using Wronskian/Variation of parameters, solve the following differential equation:

   (a)
   $$y''(x) + 3y'(x) + 2y(x) = \frac{1}{1 + e^{-x}}.$$

   (b)
   $$x^2y''(x) - 2xy'(x) + 2y(x) = x \ln x.$$