INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 : Linear Algebra (2023-24) **Tutorial 05 (October 13, 2023)**

Recall the three rule of $A \rightsquigarrow D(A)$: (I). $D(I_n)$, (II). linearity in each row, and, (III). $D(A) = 0$ if A has repeated rows.

1. Show that
$$D\left(\begin{pmatrix} 0 & 1 & 0 \\ a & r & b \\ c & s & d \end{pmatrix}\right) = -(ad - bc)$$
and
$$D\left(\begin{pmatrix} 0 & 0 & 1 \\ a & b & r \\ c & d & s \end{pmatrix}\right) = ad - bc$$

2. Show that for two $n \times n$ matrices A and B, we have $(AB)^t = B^t A^t$, where A^t and B^t denote the transpose matrices of A and B, respectively. What if A and B are not square matrices?

3. Consider the following system of linear equations.
$$\begin{aligned} a_{11}x + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$
Let
$$A := \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad A_1 := \begin{pmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{pmatrix} \quad X := \begin{pmatrix} x_1 & 0 & 0 \\ x_2 & 1 & 0 \\ x_3 & 0 & 1 \end{pmatrix}$$
Then, show that $AX = A_1$. Assume that A is an invertible matrix, and argue that $D(AX) = D(A)D(X)$. Use it to show that $x_1 = D(A_1)D(A)^{-1}$.

4. Using Rules I, II, III, compute $D(A)$ for the following matrix
$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$
