HY306 Advanced Quantum Mechanics Jan-Apr 2025: MidSemester Exam 1

Max Marks=10 Time=01 Hour Dated February 8, 2025

1. A particle of mass m is in a 1D potential well of width a given by

$$\begin{aligned} V(x) &= 0, \quad 0 \leq x \leq a \\ &= +\infty, \quad \text{elsewhere} \end{aligned}$$

The system is perturbed by a perturbation given by

$$W(x) = w_0 \delta(x - \frac{a}{2})$$

where w_0 is a real constant. Calculate to lowest non-vanishing order in perturbation theory, the corrected energy of the ground state.

[Marks=03]

2. The Hamiltonian of a two-state system is written as

$$H = \begin{pmatrix} A_1 + B_1\epsilon & B_2\epsilon \\ B_2\epsilon & A_2 \end{pmatrix}$$

where all the quantities are real and ϵ is a small parameter. Assuming $A_1 = A_2$, find the allowed energies to first order in ϵ.

[Marks=03]

3. A particle initially ($t \longrightarrow -\infty$) is in its ground state in an infinite potential well with walls at $x = 0$ and $x = a$. It is subjected at time $t = 0$ to a time-dependent perturbation $V(t) = \epsilon x e^{-t^2}$ where ϵ is a small real number. Calculate to first order the probability that the particle will be found in its first excited state after a very long time ($t \longrightarrow \infty$).

[Marks=04]

Useful Formulae

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$