MTH101 (Symmetry)

Tutorial Sheet 02 / January 18, 2022 Spring 2022

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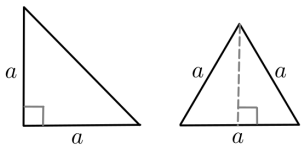
1. Look at the triangles below and conclude the following (using Pythagoras theorem).



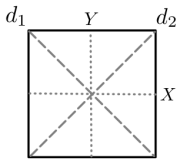
(a) $\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ (b) $\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (c) $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Also, calculate $\sin\left(\frac{5\pi}{12}\right)$.

2. Why is $\sin(-\theta) = -\sin(\theta)$ but $\cos(-\theta) = \cos(\theta)$? Substantiate your argument with the help of an example involving rotation in a plane.

3. A point $P = (1, 1)$ is rotated by an angle $\frac{5\pi}{12}$ in a plane so that it moves to point Q. Now the point Q is reflected about y-axis so that it moves to the point Q'. What are the coordinates of Q and Q'?

4. Match the following in connection with the symmetries of a square.



Matrix	Symmetry
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	f_{d_2}
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	f_Y
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$r_{3\pi/2}$
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$r_{\pi/2}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	1
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	f_{d_1}
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	f_X
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	r_π

Try making a similar exercise for symmetries of a triangle.

5. Write 3×3 rotation matrices $R_{x,\theta}$, $R_{y,\theta}$ and $R_{z,\theta}$. Consider a point $P = (a, b, c)$ and calculate the following.

   (a) $R_{x,\theta}(R_{y,\theta}(P))$ and $R_{y,\theta}(R_{x,\theta}(P))$ when $\theta = \pi$.

   (b) $R_{x,\theta}(R_{y,\theta}(P))$ and $R_{y,\theta}(R_{x,\theta}(P))$ when $\theta = \frac{\pi}{2}$.

   Do you connect these computations with an ongoing discussion over moodle forum?
