MTH101 (Symmetry)

Tutorial Sheet 05 / February 08, 2022 Spring 2022

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1. Convert the following matrices to a row echelon matrix and determine which of these are invertible.

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{pmatrix}, \quad \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \\ 12 & 2 & -3 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & -3 & 0 \end{pmatrix}$$

   In each case where the matrix is invertible find the inverse.

2. Use one of the matrices in Q1 to balance the following chemical reaction.

$$Al_2(SO_4)_3 + Ca(OH)_2 \rightarrow Al(OH)_3 + CaSO_4$$

3. Write the rotation matrix in Q1 as a product of elementary matrices.

4. Solve the following systems of linear equations.

   (a) $8x + y + 6z = 20$
   $3x + 5y + 7z = 40$
   $4x + 9y + 2z = 60$

   (b) $2x + 3y - z = 2$
   $x - y + z = 5$
   $x + 9y - 5z = 10$
