Assignment-8

Date-16/10/2023
No Need to Submit.

1. Write in Sturm-Liouville form and identify $p(x)$, $q(x)$ and $w(x)$ in each of the following equations-

a) $(1-x^2)y'' - xy' + \lambda y(x) = 0$; $-1 \le x \le 1$

b) $xy'' + (1-x)y' + \lambda y(x) = 0$; $0 \le x < \infty$

c) $(1-x^2)y'' - 2xy' + \left[\lambda - \frac{m^2}{1-x^2}\right] y(x) = 0$; $-1 \le x \le 1$

2. Show that the eigenvalues and the corresponding eigenfunctions of the Sturm-Liouville problem

$$x^2u'' + xu' + \lambda u = 0 ; \quad 1 \le x \le e^2$$

$u(1) = 0$, $u(e^2) = 0$ are given by

$\lambda_n = \left(\frac{n\pi}{2}\right)^2$ and $u_n(x) = C_n \sin\left(\frac{n\pi}{2}\ln x\right)$, $n = 1, 2, 3, \ldots$

3. Show that the eigenvalues and the eigenfunctions of the Sturm-Liouville problem

$x^4u'' - 2x^3u' + \lambda u = 0$; $1 \le x \le 2$; $u(1) = 0$, $u(2) = 0$ are given by $\lambda_n = (2n\pi)^2$ and $u_n(x) = C_n \sin \frac{2\pi n}{x}$.

4. Use Schmidt orthogonalization procedure to obtain the 1st three normalized orthogonal polynomials $\psi_n(x)$ from a non-orthogonal set of linearly independent functions $u_n(x)$ given by $u_n(x) = x^n$; $n = 0, 1, 2, 3, \ldots$

a) $-1 \le x \le 1$, $w(x) = 1$ b) $-\infty < x < \infty$, $w(x) = e^{-x^2}$

5. Which of the following boundary conditions do not satisfy the orthonormality conditions?

a) $p(x) = 1$, $0 \le x \le 1$; $u(0) = u(1) = 2$, $u'(0) = u'(1)$

b) $p(x) = x^2$, $1 \le x \le 2$; $u(0) = u(2)$, $u'(0) = u'(2)$.

6. Which of the following boundary conditions ensure that all the eigenvalues will be non-negative, if $q(x) \geq 0$.

a) $p(x) = 2$, $0 \leq x \leq 1$, $u(0) = 0$, $u'(1) = 0$

b) $p(x) = e^x$, $-10 \leq x \leq 10$ $u(10) = u(-10)$, $u'(10) = u'(-10)$

c) $p(x) = \sin x$, $0 \leq x \leq \pi$, $u(0) = u'(\pi)$, $u(\pi) = u'(0)$.