INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

MTH101 : Linear Algebra (2023-24) **Tutorial 06 (November 02, 2023)**

Recall the following four properties that a binary operation $* : G \times G \to G$ may have: (I). Associativity, (II). Existence of a neutral element, (III). Existence of an inverse for every element of G, and, (IV). Commutativity.

A pair $(G, *)$, where G is a set and $* : G \times G \to G$ is a binary operation on the set G, is called a

- **semigroup**, if (I) holds.
- **monoid**, if (I) and (II) hold.
- **group**, if (I), (II) and (III) hold.
- **Abelian group**, if (I), (II), (III) and (IV) hold.

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1. Determine which of the above four algebraic structure do the following binary operations from?

(A). $\mathbb{Z}$, under the operation $*$, where $*$ denotes the multiplication of integers.

(B). $\mathbb{R}$, under the operation $*$, where $*$ denotes the multiplication of real numbers.

(C). The collection of irrational numbers under addition.

(D). The set of clock hours $\{12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ under the addition of clock hours. (Therefore $10 + 3 = 1$ under this operation).

(E). The set $\{1, 3, 7, 9\}$ under the operation "*rightmost digit in the multiplication of numbers*."

(F). The collection $\{a, b, c\}$ of three alphabets with the operation given by the following composition table:

	a	b	c
a	b	a	c
b	c	b	a
c	a	c	b

(G). The collection {□, ⬡, •, ∘} of four symbols with the operation given by the following composition table:

	□	⬡	•	∘
□	•	□	∘	⬡
⬡	□	⬡	•	∘
•	∘	•	⬡	□
∘	⬡	∘	□	•

(H). The collection of 2×2 matrices having nonzero determinant and entries in $\mathbb{Z}$, under the operation of matrix multiplication; i.e. $\{A \in M_2(\mathbb{Z}) : D(A) \neq 0\}$, under multiplication of matrices.

(I). $\mathrm{GL}_n(\mathbb{R}) := \{A \in M_n(\mathbb{R}) : D(A) \neq 0\}$, under multiplication of matrices. invertible matrices

(J). $\mathrm{SL}_n(\mathbb{R}) := \{A \in M_n(\mathbb{R}) : D(A) = 1\}$, under multiplication of matrices.

(K). $\mathrm{Sym}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = A\}$, under multiplication of matrices. symmetric matrices

(L). $\mathrm{Sym}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = A\}$, under addition of matrices.

(M). $\mathrm{Skew}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = -A\}$, under addition of matrices. skew-symmetric matrices

(N). $\mathrm{Sym}_3(\mathbb{R}) \cap \mathrm{GL}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible and } A^t = A\}$, under multiplication of matrices.

(O). $\mathrm{O}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible and } A^t = A^{-1}\}$, under multiplication of matrices.

(P). $\mathrm{SO}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible, } A^t = A^{-1} \text{ and } D(A) = 1\}$, under multiplication of matrices.

(Q). The collection of 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where a is a nonzero element in $\mathbb{Q}$, under the operation of matrix multiplication.

(R). The collection of 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where a is a nonzero element in $\mathbb{Q}$, under the operation of matrix addition.

(S). Collection of all polynomials in one variable with coefficients in $\mathbb{R}$, under the addition of polynomials.

(T). Collection of all polynomials in one variable with coefficients in $\mathbb{R}$, under the multiplication of polynomials.
