Assignment 4

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Convert the following polynomial in to Hermite polynomial:

$$64x^4 + 8x^3 - 32x^2 + 40x + 10.$$

2. Calculate $H_{2n}(0)$, where n is a positive integer.

3. Prove that [Use: Mathematical Induction]

$$\left(2x - \frac{d}{dx}\right)^n 1 = H_n(x).$$

4. Prove that for $m < n$,

$$\frac{d^m}{dx^m} H_n(x) = \frac{2^n n!}{(n-m)!} H_{n-m}(x).$$

5. The 1-D Schrödinger wave equation for a particle in a potential field $V(x) = \frac{1}{2}kx^2$ is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = E\psi.$$

(a) Defining

$$a = \left(\frac{mk}{\hbar^2}\right)^{1/4}, \qquad \lambda = \frac{2E}{\hbar}\left(\frac{m}{k}\right)^{1/2},$$

and setting $\xi = ax$, show that

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\psi(\xi) = 0.$$

(b) Substituting

$$\psi(\xi) = y(\xi)e^{-\xi^2/2},$$

show that $y(\xi)$ satisfies the Hermite differential equation.

6. Show that

$$\int_{-\infty}^{+\infty} H_n(x) \exp\left[-\frac{x^2}{2}\right] dx = 2\pi n!/(n/2)! \text{ for even } n \text{ and zero other wise.}$$

7. Show that for m an integer and $0 \le m \le n-1$,

$$\int_{-\infty}^{+\infty} x^m H_n(x) \exp\left[-\frac{x^2}{2}\right] dx = 0.$$

8. Show that

$$\int_{-\infty}^{+\infty} x^2 H_n(x) H_n(x) \exp\left[-\frac{x^2}{2}\right] dx = \sqrt{\pi} 2^n n! \left(n + \frac{1}{2}\right).$$

Optional

9. Starting from the Rodrigus formula - Show that

$$H_n(x) = \frac{2^n(-i)^n}{\sqrt{\pi}} e^{x^2} \int_{-\infty}^{+\infty} t^n e^{-t^2+i2xt} dt.$$

This is known as integral representation of $H_n(x)$. Hint:

$$e^{-x^2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-t^2+i2xt} dt$$

10. For n, p, and r nonnegative integers, show that

$$\int_{-\infty}^{+\infty} x^r H_n(x) H_{n+p}(x) \exp\left[-\frac{x^2}{2}\right] dx = 2^n \sqrt{\pi} (n+r)! \text{ for } p = r \text{ and } = 0 \text{ for } p > r.$$