PHY 310 - Mathematical Methods for Physicists I

Odd Term 2019, IISER Mohali

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Quiz 2 - Solutions

In class - Tuesday, 24th September, 2019

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1. We have the Taylor series expansion of the function $f(z) = (1-z)^{-1}$ about the origin

$$f(z) = \frac{1}{1-z} = 1 + z + z^2 + \cdots . \tag{1}$$

Obtain the Taylor series expansion of the same function about the point $z = -2$.

Solution:

We have

$$\begin{aligned} f(z) &= \frac{1}{z} = \frac{1}{3-(z+2)} \\ &= \frac{1}{3}\frac{1}{1-\frac{z+2}{3}} \\ &= \frac{1}{3}\left[1 + \left(\frac{z+2}{3}\right) + \left(\frac{z+2}{3}\right)^2 + \cdots\right]. \end{aligned} \tag{2}$$

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2. Consider the function

$$f(z) = e^{1/z}. \tag{3}$$

Does this function have a singularity? Give a brief explanation on how you arrived at your answer.

Solution:

Yes, this function has a singularity. It has an essential singularity at $z = 0$.

The function

$$f(z) = e^{1/z} \tag{4}$$

may be expanded in any annular region enclosing the origin in the form

$$e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2} + \frac{1}{3!}\frac{1}{z^3} + \cdots, \quad z \neq 0. \tag{5}$$

We see that no finite value of n can be found such that

$$\lim_{z \to z_0} \left[(z - z_0)^n f(z)\right] = a, \tag{6}$$

where a is a finite and non-zero complex number, is satisfied. In other words, the principal part of the Laurent series expansion is an infinite sum.

Thus the function $e^{1/z}$ has essential singularity at $z = 0$.

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