

PHY 635 : Mid Semester Exam II

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Symbols have their usual meanings, unless specified otherwise.

1. For a spacetime described by a metric $\eta_{\mu\nu} + h_{\mu\nu}$ where $|h_{\mu\nu}| \ll 1$: find out the Ricci tensor at the leading order of $h_{\mu\nu}$. What is the Einstein tensor in the leading order? [2+2]

2. For the scalar field Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi).$$

obtain the expression of the gravitational stress energy tensor for the metric $g_{\mu\nu} = b^2(t)\eta_{\mu\nu}$. Express the 00 component of the Einstein's field equation in this set up. [3+3]

3. The famous Schwarzschild spacetime is described by the line element

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

How does the clock of a stationary observer at co-ordinate $(r \geq 4GM/c^2, \theta, \phi)$ run compared to that of another stationary observer at $(r/2, \theta, \phi)$? How much time according to an asymptotic observer will elapse if a photon moves from (r, θ, ϕ) to $(2GM/c^2, \theta, \phi)$? [2+3]

4. In the Schwarzschild spacetime, at the asymptotic region $r \gg 2GM/c^2$, compute the Newtonian force difference across the tips of a rod of length ℓ aligned radially on a geodesic. Find out the force difference close to $r \rightarrow 2GM/c^2$ [3+2]