PHY 310 - Mathematical Methods for Physicists I

Odd Term 2019, IISER Mohali

Instructor: Dr. Anosh Joseph

Homework 2

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1. Show that Bessel function $J_p(x)$ is an even function when p is even and is an odd function when p is odd.

2. Show that, for a positive integer p, $J_{-p}(x) = (-1)^p J_p(x)$.

3. Show that
$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

4. Express $J_5(x)$ in terms of $J_1(x)$ and $J_2(x)$.

5. Show that
$$\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x).$$

6. Prove that
$$4J_0'''(x) - 3J_1(x) + J_3(x) = 0.$$

7. Prove that
$$\cos x = J_0(x) - 2J_2(x) + 2J_4(x) - \cdots$$

8. Prove that
$$J_0(x)^2 + 2J_1(x)^2 + 2J_2(x)^2 + \cdots = 1.$$