HW 7

1. Let $f(x) = \sqrt{1 - x^{2/3}}$
   Show that f is continuous on $[0, 1]$

2. Suppose $f: (0, 1] \rightarrow \mathbb{R}$ is a bounded continuous function. Show that
   $$g(x) = \begin{cases} x f(x) & x \in (0, 1] \\ 0 & x = 0 \end{cases}$$
   is continuous on $[0, 1]$.

   Conclude that $h(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

   is continuous on $\mathbb{R}$.

3. Use the fact that $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ and verify if $h(x)$ as in (2) is uniformly continuous on $\mathbb{R}$.

4. If $f: [0, \infty) \rightarrow \mathbb{R}$ is continuous then show that f is uniformly continuous on $[0, \infty)$ iff $\exists K \in (0, \infty)$ such that f is uniformly continuous on $[K, \infty)$.

5. Check if the following functions are uniformly continuous on the given intervals:
   (i) $f(x) = \frac{1}{x}$ on $[2, \infty)$
   (ii) $f(x) = \frac{x}{x+1}$ on $[0, \infty)$

(iii) $f(x) = \sqrt{x}$ on $[0, \infty)$

6. Solve the Ex 19.12 from Ross

7.※ Solve the problems mentioned in class and complete the proofs left for you as exercises.