PHY306 Advanced Quantum Mechanics Jan-April 2025: Assignment 5

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1. Analyze the Zeeman effect for $n = 2$ states of the Hydrogen atom in the intermediate field regime. As the basis for degenerate perturbation theory choose the states characterized by l, j, m_j and use the Clebsch-Gordan coefficients to express $|jm_j\rangle$ as a linear combination of $|lm_l sm_s\rangle$. Construct a table of energies and plot them as functions of the external field and check that the in the two limiting cases the intermediate field results reduce properly to the weak Zeeman and the strong Zeeman splitting.

2. Analyze the Zeeman effect for $n = 3$ states of the Hydrogen atom in the intermediate field regime and as in the above problem, construct the table of energies, plot as a function of external field and check the limiting cases of weak and strong Zeeman splittings.

3. Considering the proton itself as a magnetic dipole, write out the Hamiltonian H'_{hf} (hf denoting hyperfine) of the electron in the magnetic field due to the proton's dipole moment. Find the first order correction to the energy E^1_{hf} for the ground state of the hydrogen atom. Find the energy gap between the singlet and triplet states, the frequency and corresponding wavelength of the photon emitted in a transition between the triplet and singlet states.

4. Find the hyperfine splitting in the ground state of (a) muonic hydrogen where a muon substitutes for the electron, (b) positronium where a positron substitutes for the proton, and (c) muonium where an antimuon substitutes for the proton.

5. Find the lowest-order relativistic correction to the energy levels of the one-dimensional harmonic oscillator.

6. Derive the fine-structure formula for the hydrogen atom from the relativistic correction and the spin-orbit coupling.