TUTORIAL-RANDOM VARIABLE

MTH202: SPRING 2023

(1) Let the probability mass function of a discrete random variable $X : \mathbb{N} \to \mathbb{R}$ is given by
$$p(x) = \begin{cases} \frac{x}{15} & x = 1, 2, 3, 4, 5; \\ 0 & otherwise. \end{cases}$$
Find (i) $P(\{X = 1 \text{ or } 2\})$, and (ii) $P\big(\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\}\big)$.

(2) Two unbiased dice are rolled simultaneously and independently. Let X denote the random variable that counts the total number of points on the unturned faces. Find out the probability mass function of X and also find the distribution function of X.

(3) A random variable X on a smple space (S, P) has the following probability mass function.
$$p(0) = 0;\ p(1) = k;\ p(2) = 2k;\ p(3) = 2k;$$
$$p(4) = 3k;\ p(5) = k^2;\ p(6) = 2k^2;\ p(7) = 7k^2 + k.$$
  (i) Find k.
  (ii) Evaluate $P(X \geq 6)$ and $P(0 < X < 5)$.
  (iii) Determine the distribution function of X.

(4) A random variable X assumes the values $-3, -2, -1, 0, 1, 2, 3$ such that
$$P(X = -3) = P(X = -2) = P(X = -1),$$
$$P(X = 3) = P(X = 2) = P(X = 1),$$
and $P(X = 0) = P(X > 0) = P(X < 0)$. Obtain the probability mass function of X and the distribution function of X.

(5) Let X be the randaom variable given in the above exercise. Suppose $Y = 2X^2+3X+4$. Find the probability mass function of Y.

(6) Let two dices are rolled. Let X be the random variable that counts the total number of points on the faces that point upwards. Find the probability mass function and the distribution function of X.

(7) If a dice is thrown n times, what is the probability that (i) the greatest number, (ii) the least number obtained will be a given value k.