PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 2

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1. Using first-order degenerate perturbation theory, calculate the energy eigenvalues of the $n = 2$ states of a hydrogen atom placed in an external uniform weak electric field along the positive z-axis.

2. Consider an infinite cubic potential well of dimension L along x, y, z. Now add the perturbation:

$$H_p = V_0 L^3 \delta(x - \frac{L}{4})\delta(y - \frac{L}{4})\delta(z - \frac{L}{4})$$

   Calculate the energy of the first excited state using first-order degenerate perturbation theory.

3. Consider a hydrogen atom which is subject to two weak static fields: an electric field in the xy plane $E = E_0(\hat{i} + \hat{j})$ and a magnetic field along the z-axis $B = B_0\hat{k}$, where E_0, B_0 are constant. Neglecting the spin–orbit interaction, calculate the energies of the $n = 2$ states to first-order perturbation.

4. Consider a particle of mass m free to move on a circular wire of circumference L, with stationary states

$$\psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x/L}, -L/2 < x < L/2$$

   where $n = 0, \pm 1, \pm 2, ...$ and energies are

$$E_n = \frac{2}{m}\left(\frac{n\pi\hbar}{L}\right)^2$$

   . Introduce the perturbation $H_p = -V_0 e^{-x^2/a^2}$ where $a << L$. (i) Find the first-order correction to E_n. (ii) What are the good linear combinations of ψ_n and ψ_{-n} (iii) Find a Hermitian operator A that fits the requirements of the theorem $[A, H_p]$ and show that the simultaneous eigenstates of H_0 and A are the same as the ones found in (ii).

5. A system that has three unperturbed states is represented by the perturbed Hamiltonian matrix

$$\begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix}$$

where $E_2 > E_1$. The quantities a and b are to be regarded as perturbations that are of the same order and are small compared with $E_2 - E_1$. Use second-order nondegenerate perturbation theory to calculate the perturbed eigenvalues. Is this procedure correct? Then diagonalize the matrix to find the exact eigenvalues. Compare the results.

6. Consider a spinless particle in a 2D infinite square well

$$V = 0, 0 \leq x \leq a, 0 \leq y \leq a$$

and $V = \infty$ otherwise. What are the energy eigenvalues for the three lowest states? Is there any degeneracy? Now add a potential as a weak perturbation:

$$V_1 = \lambda xy, 0 \leq x \leq a, 0 \leq y \leq a$$

Is the energy shift due to the perturbation linear or quadratic in λ for each of the three states? Find expressions for the energy shifts of the three lowest states accurate to order λ. You need not evaluate any integrals that may appear. Draw an energy diagram with and without the perturbation for the three states and specify which unperturbed state is connected to which perturbed state.

7. Suppose the Hamiltonian of a rigid rotator in a magnetic field perpendicular to its axis is of the form

$$AL^2 + BL_z + CL_y$$

Assume $B >> C$ and use perturbation theory to lowest nonvanishing order to get approximate energy eigenvalues.