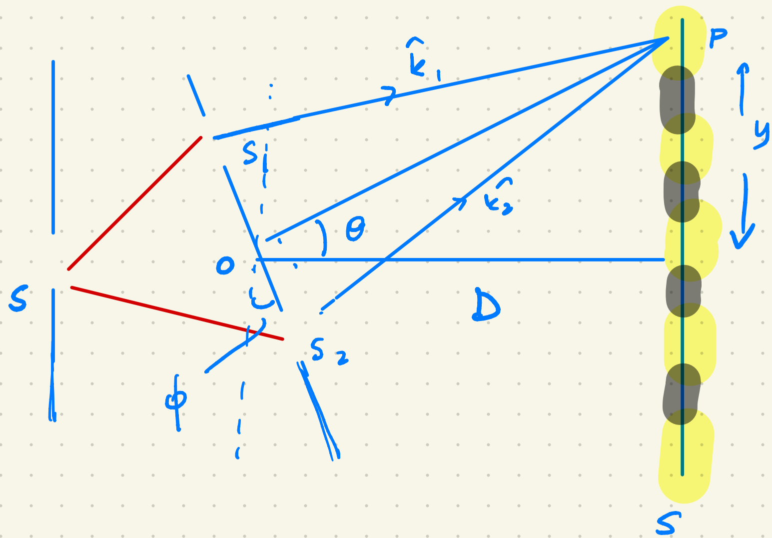
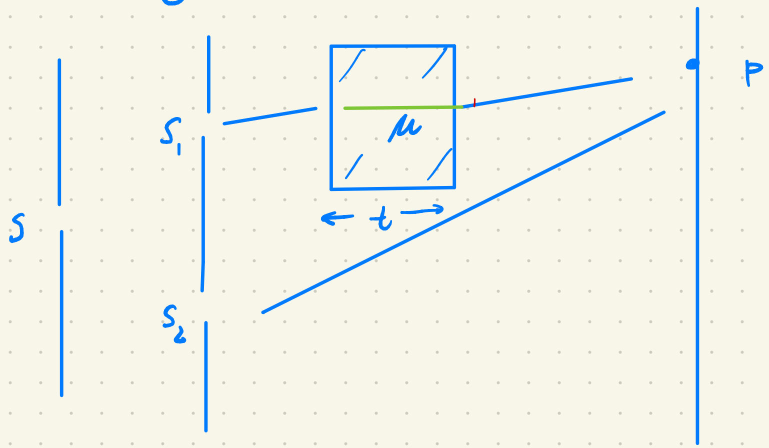
Exercise : Find out the width of central maxima and show that all bright fringes are of same width.

Exercise : Suppose the plane containing S_1, S_2 is rotated about the mid point by small angle ϕ. Find out the location of maxima/minima on the screen.



This time $\delta = k\,(SS_2 + S_2P - SS_1 - S_1P)$

Inserting a slab



* Effective path length from S_1 to P changes

$$\widetilde{S_1P} = (S_1P - t) + \mu t$$
$$= S_1P + (\mu - 1)t$$

{ In this we are assuming the $\hat{k}$ is roughly $\perp$ to the edges

$$\therefore \delta = k(S_2P - \widetilde{S_1P})$$
$$= k(S_2P - S_1P - (\mu-1)t)$$

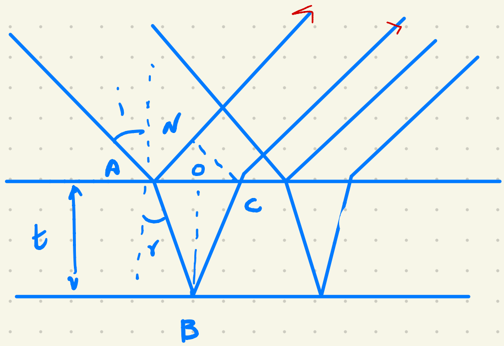
For maxima $k\left(\frac{dy}{D} - (\mu-1)t\right) = 2n\pi$

$$\Rightarrow \frac{dy}{D} = n\lambda + (\mu-1)t$$

* At $y=0$, $\delta = -(\mu-1)t\,\frac{2\pi}{\lambda}$ { Not maxima

Exercise : Find out width of bright dark fringes in this case.

Reflection by a thin film



$$\delta = k(\widetilde{AB} + \widetilde{BC} - AN)$$

$$= k\left(\frac{2\mu t}{\cos r} - AC \sin i\right)$$

$$AC = AO + OC$$

$$= 2t \tan r$$

$$\therefore \delta = k\left(\frac{2\mu t}{\cos r} - 2t \tan r \sin i\right.$$

$$= k\left(\frac{2\mu t}{\cos r} - 2t \frac{\sin r}{\cos r} \sin i\right)$$

Also, $\frac{\sin i}{\sin r} = \mu$

$$= k \frac{2\mu t}{\cos r}(1 - \sin^2 r) = k\, 2\mu t \cos r$$

Again, for maxima / minima

$$k\, 2\mu t \cos r = \pm 2n\pi$$

$$k\, 2\mu t \cos r = \pm (2n+1)\frac{\pi}{2}$$

Diffraction

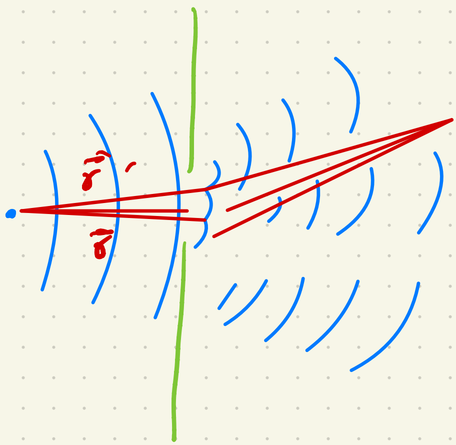
Ex: Prove the Green's theorem

$$\boxed{\int (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, d^3V = \int (\psi \nabla \phi - \phi \nabla \psi) \cdot d\vec{S}}$$

given $\int (\nabla \cdot \vec{A})\, d^3V = \int \vec{A} \cdot d\vec{S}$

Fresnel- Kirchoff diffraction theory

Each point of a wave front gives rise to secondary wavelets and at large distance we have a overlapping collection.



Starting with monochromatic wave

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

If $E = E_0 e^{i(kx-\omega t)}$

$$\Rightarrow \nabla^2 E + \frac{\omega^2}{c^2} E = 0$$

$$\boxed{\Rightarrow \nabla^2 E + k^2 E = 0}$$ Helmholtz eqn.

Let G be a solution to the Helmholtz eqn.

$$G = \frac{1}{r} + \int_0^k ds\, e^{isr}$$

$$\therefore \nabla^2 G = \sum_i \left(\frac{\partial^2}{\partial x_i^2} \frac{1}{r} + \int \frac{\partial^2}{\partial x_i^2} e^{isr} ds \right)$$

$$\frac{\partial^2}{\partial x_i^2} \frac{1}{r} = -\frac{1}{r^3} + \frac{3x_i^2}{r^5}$$

$$\frac{\partial^2}{\partial x_i^2} e^{isr} = -s^2 \frac{x_i^2}{r^2} e^{isr} + is \frac{\partial}{\partial x_i} \left(\frac{1}{r} - \frac{x_i^2}{r^3} \right) e^{isr}$$

$$\therefore \nabla^2 \frac{1}{r} = 0$$

$$\nabla^2 e^{isr} = -s^2 e^{isr} + 2is \frac{1}{r} e^{isr}$$

Using these we can show

H.W.: $\nabla^2 G = -k^2 G$

$$G = \frac{1}{r} e^{ikr}$$

For any U also satisfying

$$(\nabla^2 U + k^2 U) = 0$$

$$\int d^3v \underbrace{\left(U \nabla^2 G - G \nabla^2 U \right)}_{\to 0} = \int d\vec{S} \cdot (U \nabla G - G \nabla U)$$

$$\therefore \int ds' \left(U \frac{\partial G}{\partial n} - G \frac{\partial U}{\partial n} \right) = 0$$

$$\int_{S_1} ds \underbrace{\left(U \frac{\partial G}{\partial n} - G \frac{\partial U}{\partial n} \right)}_{A}$$

$$+ \int_{S_1} ds \underbrace{\left(U \frac{\partial G}{\partial n} - G \frac{\partial U}{\partial n} \right)}_{B} = 0$$

For spherical surface

$$\frac{\partial G}{\partial n} = \frac{\partial G}{\partial r}\frac{\partial r}{\partial n} = \frac{1}{r}\left(ik - \frac{1}{r}\right)e^{ikr}$$

$$\therefore \quad B = \int_{S_\varepsilon} ds \left(\frac{U}{r} e^{ikr}\left(ik - \frac{1}{r}\right) - \frac{1}{r} e^{ikr} \frac{\partial U}{\partial n} \right] ds$$

Since S_ε is sphere of radius ε

$$B = \int \varepsilon^2 d\Omega \left(\frac{U}{\varepsilon} e^{ik\varepsilon}\left(ik - \frac{1}{\varepsilon}\right) - \frac{1}{\varepsilon} e^{ik\varepsilon} \frac{\partial U}{\partial r} \right)$$

$$= \int d\Omega \left[\varepsilon U e^{ik\varepsilon} ik - U e^{ik\varepsilon} - \varepsilon e^{ik\varepsilon} \frac{\partial U}{\partial r} \right)$$

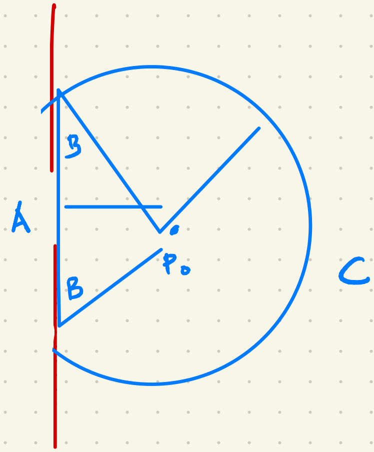
$$=$$

$$\lim_{\varepsilon \to 0} B \rightarrow -4\pi U(\varepsilon = 0)$$

$$= -4\pi U(P_0)$$

Thus,

$$U(P_0) = \frac{1}{4\pi} \int_S ds \left[\frac{1}{r} e^{ikr} \left[U\left(ik - \frac{1}{r}\right) - \frac{\partial U}{\partial n} \right] \right.$$



The contribution to the surface integrals

C : Radius R, angle from θ_0 to $-\theta_0$

$$\int R^2 d\Omega \left[\frac{e^{ikR}}{R}\left(ik - \frac{1}{R}\right) U - \frac{1}{R} e^{ikR} \frac{\partial U}{\partial n} \right]$$

U is expected to fall as $R \to \infty$

$\Rightarrow$ If we take P_0 to sufficiently far, $C \to 0$ (negligibly small)

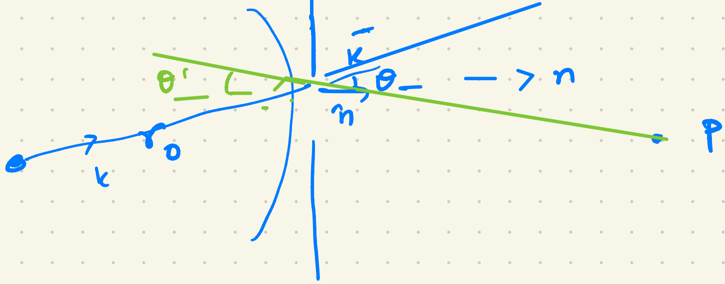
Then we are left with A and B.

For B : (We assume $U, \frac{\partial U}{\partial n} = 0$)

It is in geometric shadow regime !

Beware : We are doing wave optics and not ray optics. So, strictly we can not have $U, \frac{\partial U}{\partial n} = 0$ particularly close to the edge. But for $k r_0 >> 1$, we are making small error.

—



If the source is away from the aperture

$$U \sim \frac{e^{ikr_0}}{r_0} \Rightarrow \frac{\partial U}{\partial n} = \left(-i\vec{k}\cdot\hat{n} - \frac{1}{r_0}\right)\frac{e^{ikr_0}}{r_0}$$

$$\sim -i\vec{k}\cdot\hat{n}\,\frac{e^{ikr_0}}{r_0}$$

$$\sim +ik\cos\theta\,\frac{e^{ikr_0}}{r_0}$$

Similarly for P_0, $G \sim \frac{e^{ikr}}{r}$

$$\frac{\partial G}{\partial n} \sim -ik\cos\theta'\,\frac{e^{ikr}}{r}$$

$$\boxed{\therefore \quad U(P_0) = \frac{ik}{4\pi}\int ds\,(\cos\theta + \cos\theta')\frac{e^{ik(r_0+r)}}{r_0 r}.}$$

Fresnel Kirchoff integral Formula