PHY403 Atomic and Molecular Physics Aug-Dec 2019: Tutorial 2

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1. Write down the expression for the commutator $[\sigma_i, \sigma_j]$ of two Pauli matrices. Show that the anticommutator of two Pauli matrices is $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$. For a spin-1/2 particle the rotation operator J is equivalent to the spin operator S. Use the commutation and anti-commutation relations of the Pauli matrices to show that the rotation operator $U(\alpha) \equiv exp(-i\alpha.J)$ is given by

$$U(\alpha) \;=\; \cos\alpha/2I - i\sin\alpha/2(\alpha.\sigma)$$

   where I is identity, and α is the unit vector parallel to α.

2. Write $\hbar^2 L^2 = (\mathbf{x}\times\mathbf{p}).(\mathbf{x}\times\mathbf{p}) = \sum_{ijklm}\epsilon_{ijk}x_j p_k \epsilon_{ilm} x_l p_m$ and show that

$$p^2 \;=\; \frac{\hbar^2 L^2}{r^2} + \frac{1}{r^2}((\mathbf{r}.\mathbf{p})^2 - i\hbar\mathbf{r}.\mathbf{p})$$

   Show that $\mathbf{p}.\hat{\mathbf{r}} - \hat{\mathbf{r}}.\mathbf{p} = -2i\hbar/r$ and hence show that $p^2 = p_r^2 + \frac{\hbar L^2}{r^2}$

3. Calculate $\langle(1/|r_1 - r_2|)\rangle$ for the ground state of helium $\psi_0(r_1, r_2) = \psi_{100}(r_1)\psi_{100}(r_2) = \frac{8}{\pi a^3}e^{-2(r_1+r_2)/a}$ where a is the Bohr radius and we have taken the noninteracting electron approximation. First do the d^3r integral using spherical polar coordinates and setting the polar axis along r_1 so that $|r_1 - r_2| = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta_2}$. Break up the r_2 integral into two parts, ranging from 0 to r_1, the other from r_1 to ∞.

4. Consider three particles in each of the orthonormal states $\psi_a(x), \psi_b(x), \psi_c(x)$. Construct three-particles states for distinguishable particles, for bosons and for fermions.

5. Consider two noninteracting particles each of mass m in an infinite square well. If one is in state ψ_n and other is in state ψ_m (orthogonal to ψ_n), calculate the quantity $\langle(x_1 - x_2)^2\rangle$, for when they are distinguishable, identical bosons and identical fermions.