PHY304: Statistical Mechanics

Assignment 6

February 20, 2025

1. The elasticity of a rubber band can be described in terms of a one-dimensional model of polymer involving N molecules linked together end-to-end. The angle between successive links is equally likely to be $0°$ or $180°$.

   (a) Obtain the number of conformations $g(N, m)$ of the rubber band that give an overall length $L = 2md$, where $m > 0$, and d is the length of one link.

   (b) Obtain the expression $g(N, m)$ for $m \ll N$ and find the entropy of the system as a function of L for $N \gg 1, L \ll Nd$.

   (c) Find the force required to maintain the length L for $L \ll Nd$.

   (d) Find the relationship between the force and the length, without using the condition in (c), i.e., for any possible value of L but $N \gg 1$.

2. Consider an idealization of a crystal which has N lattice points and the same number of interstitial positions (places between the lattice points where atoms can reside). Let E be the energy necessary to remove an atom from a lattice site to an interstitial position and let n be the number of atoms occupying interstitial sites in equilibrium.

   (a) What is the internal energy U of the system?

   (b) What is the entropy S? Give an asymptotic formula valid when $n \gg 1$.

   (c) In equilibrium at temperature T, how may such defects are there in the solid, i.e., what is n? (Assume $n \gg 1$)

3. The energy of individual atoms in a gas fluctuates about an average value $\frac{3}{2}k_BT$ because of collisions.

   (a) Verify this by calculating $\bar{\epsilon}$, the mean of $\epsilon = \mathbf{p}^2/2m$ with respect to the Maxwell-Boltzmann distribution.

   (b) Show $\overline{\epsilon^2} - \bar{\epsilon}^2 = \frac{3}{2}(k_BT)^2$.

4. The Maxwell-Boltzmann distribution for a relativistic gas is

$$f(\mathbf{p}) = C \exp\left(-\frac{\sqrt{p^2 + m^2}}{k_BT}\right)$$

   where we use units in which the velocity of light is $c = 1$.

(a) Find the most probable velocity. Obtain its non-relativistic ($k_BT \ll m$), and ultra-relativistic ($k_BT \gg m$) limits, both with first order corrections.

(b) Set up an expression for the pressure. Show that $PV = U/3$ in the ultra-relativistic limit, where U is the internal energy.

(c) Find the velocity distribution function $f(\mathbf{v})$, such that $f(\mathbf{v})d^3v$ is the density of particles whose velocity lies in the volume element d^3v. Find the non-relativistic limit to first order in v/c.

(d) At what temperatures would relativistic effects be important for a gas of H_2 molecules?

5. Suppose a surface of the container of a gas absorbs all molecules striking it with a normal velocity greater than v_0. Find the absorption rate W per unit area.