TUTORIAL-2

MTH202: SPRING 2023

[Notation: $P(AB) = P(A \cap B)$, i.e. $A \cap B = AB$, $A \cap B \cap C = ABC$ etc]

(1) Let A and B be any two events. Prove that

- (i) $P(A) = P(AB) + P(A\overline{B})$.
- (ii) $P(B) = P(\overline{A}B) + P(AB)$.
- (iii) $P(\bar{A}\bar{B}) = 1 - P(A) - P(B) + P(AB)$.

(2) Prove that the probability of occurance of exactly one of the events A and B is

$$P(A) + P(B) - 2P(AB).$$

(3) If A and B are two independent events, prove that

- (i) $\overline{A}$ and $\overline{B}$ are also independent.
- (ii) A and $\overline{B}$ are also independent.
- (iii) $\overline{A}$ and B are also independent.

(4) Prove that two events A and B having non-zero probabilities can not be simultaneously mutually exclusive and independent.

(5) Let A and B be two events such that $P(A) = P(B) = 1$. Prove that $P(A \cup B) = 1$ and $P(AB) = 1$.

(6) If A and B be two events such that $0 < P(B) < 1$, then prove that

$$P(A \mid \overline{B}) = \frac{P(A) - P(AB)}{1 - P(B)}.$$

(7) Let $P(A) = p$, $P(B) = q \neq 0$, $P(AB) = r$, find $P(\overline{A} \mid \overline{B})$ interms of p, q, r.

(8) Let A and B be two events such that $P(A) = \frac{3}{4}$, $P(B) = \frac{5}{8}$. Show that $\frac{3}{8} \leq P(AB) \leq \frac{5}{8}$.

(9) (*Polya's urn problem*) From a basket containing r red and s black balls, n, $n \leq r + s$, balls are drawn successively at random and each time a ball is drawn, one replaces it along with q balls of the same color. Find the probability of a complete run of n black balls.

(10) Let A and B be two events associated with the same experiment S and $P(A \cup B) = \frac{7}{8}$, $P(AB) = \frac{1}{4}$ and $P(\overline{A}) = \frac{5}{8}$. Find $P(A)$, $P(B)$ and $P(A\overline{B})$. Find whether the events A and B are independent to each other.

(11) A box contains 20 apples out of which 2 or 3 are defectives. The probability that the box contains exactly 2 rotten apples is 0.4, and the probability that it contains 3 rotten apples is 0.6. The apples are drawn one at a time at random and without replacements, and they are drawn untill all rotten ones are found. What is the probability that the testing procedure ends at the 12th draw?

(12) A bag contains 17 coins marked with the number 1 to 17. A coin is drawn and replaced (by an identical coin), a second drawing is then made. What is the probability that

(i) The first number drawn is even and second odd?

(ii) The first number is odd and the second even?

Suppose the replacement is not made. Then what would be the respective probabilities?

TUTORIAL-CONDITIONAL-PROBABLITY

MTH202: SPRING 2023

(1) In a campus job interview equal number of male and female students appeared. In the initial screening 5% females and 20% males are directly recruited based on their strong profile. If any directly selected person is chosen at random what is the probability that (a) it is a male. (b) it is a female?

(2) Four roads lead away from a jail. A prisoner escaping from the jail and selects a road at random. If road-I is selected the probability of escaping is 1/8, for road-II it is 1/6, for road III, it is 1/4 and for road-IV it is 9/10. What is the probability that the prisoner will succeed in escaping?

(3) A bag contains 5 balls and it is not known how many of these are white. Two balls are drawn and found to be white. What is the probability that all balls in the bag are while?

(4) There are two baskets A and B. Basket A contains n white and 2 black balls and basket B contains 2 white and n black balls. One of the two bags is selected at random and two balls are drawn from it without replacement. If both the balls drawn are white and the probability that the bag A was used to draw the ball is 6/7, find the value of n.

(5) A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?

(6) Let E_1, E_2, …, E_n are mutually exclusive events of a sample space $S = \cup_{i=1}^{n} E_i$, with $P(E_i) \neq 0$. Let E be an event of S such that $P(E) > 0$. Prove that the probability of the happening of another event C, given $P(C|E \cap E_i)$, $i = 1, 2, \ldots, n$, is given by

$$P(C|E) = \frac{\sum_{i=1}^{n} P(E_i)P(E|E_i)P(C|E \cap E_i)}{\sum P(E_i)P(E|E_i)}.$$

(7) Show that the probability of obtaining no head in an infinite sequence of independent tosses of an unbiased coin is zero.

(8) Let p be the probability that a man aged m years will get married in a year. Find the probability that out of n men $X_1, \ldots, X_n$ each of the same age m, X_1 will be married in a year and will be the first to marry.