PHY304: Statistical Mechanics

Assignment 4

January 29, 2025

1. Approximate the integral

$$I(\lambda) = \int_0^\infty \exp\left\{-\lambda\left(x + \frac{1}{x}\right)\right\} dx$$

for large values of $\lambda \gg 1$ using saddle point method.

2. Show that

$$\int_{-\infty}^{\infty} \exp(-ax^2)dx = \sqrt{\frac{\pi}{a}}$$

3. The probability that the particle doing one dimensional random walk ends up between x and $x + dx$ at time t is given by $W(x,t)dx$ where

$$W(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right).$$

Show that

(a) The average displacement $\langle x \rangle$ of the particle after time t is zero.

(b) The mean square distance travelled after time t is equal to $2Dt$. In this exercise you need to evaluate the integral $\int_{-\infty}^{\infty} x^2 \exp(-ax^2)dx$.
In general, by using the relation given in problem 1, one can evaluate the following family of integrals quite easily

$$\int_{-\infty}^{\infty} x^{2n} \exp\left(-ax^2\right).$$

(c) $\langle x^4 \rangle = 3\langle x^2 \rangle^2$

(d) $\langle x^6 \rangle = 15\langle x^2 \rangle^3$

4. Model the trajectory of a molecule in a gas as a random walk in 3D, due to the collisions. Give an order-of-magnitude estimate of the time it would take an air molecule in a room to traverse a distance of 1 cm. What about 1 meter? What about 6 meter?

5. Consider the random walk problem in one dimension and suppose that the probability of a single displacement between s and $s + ds$ is given by

$$w(s)ds = \frac{1}{\pi} \frac{b}{s^2 + b^2} ds$$

Calculate the probability $P(x)dx$ that the total displacement after N steps lies between x and $x + dx$. Does $P(x)$ become Gaussian when N becomes large? If not, does this violate the central limit theorem.