Advanced QM HW1

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Q1) Construct a suitable wave packet to describe a free particle in momentum space and real space Why do wave packets spread?

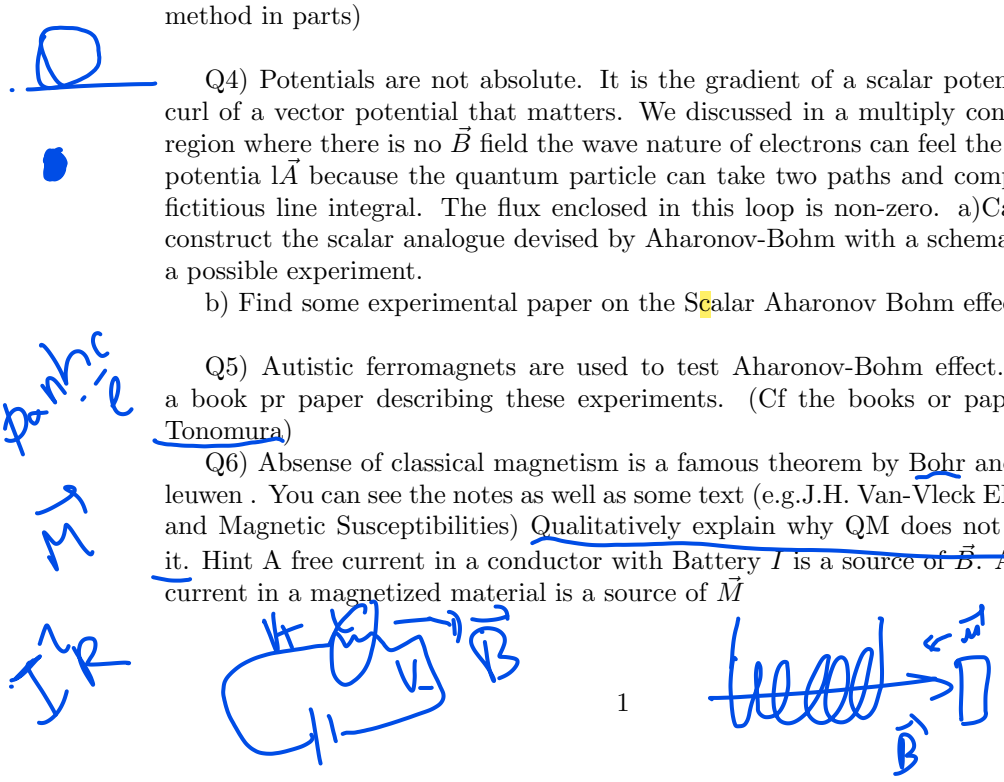
Q2) An infinite square well between $0 \to a$ was solved in class with a step of $0.1E_0$ from $a/2 \to a$ where E_0 is the energy level for n=1 for the unperturbed well. a) Repeat the same problem with the well located between $-a/2 \to +a/2$ find the correction to first order and wave function b) You are given the solutions for the well $0 \to a$ with the step from $0 \to a/2$. You may solve it or change the solutions given in class appropriately. Now assume the well's co-ordinates are labelled from $-a/2 \to a/2$. Without solving can you use some operator to get the solution for unperturbed and perturbed well in new co=ordinates .
Q3)a) Write down the Lagrangian for a 1-d Harmonic oscillator.
b) Write down the action c) Assume some solution for Harmonic oscillator as a combination of sine and cosine. You can keep the initial phase zero and evaluate the action integral using the solution . (Hint: Integrate the action by UV method in parts)

Q4) Potentials are not absolute. It is the gradient of a scalar potential or curl of a vector potential that matters. We discussed in a multiply connected region where there is no $\vec{B}$ field the wave nature of electrons can feel the vector potentia l$\vec{A}$ because the quantum particle can take two paths and complete a fictitious line integral. The flux enclosed in this loop is non-zero. a)Can you construct the scalar analogue devised by Aharonov-Bohm with a schematic for a possible experiment.

b) Find some experimental paper on the Scalar Aharonov Bohm effect.

Q5) Autistic ferromagnets are used to test Aharonov-Bohm effect. Find a book pr paper describing these experiments. (Cf the books or papers by Tonomura)

Q6) Absense of classical magnetism is a famous theorem by Bohr and Van-leuwen . You can see the notes as well as some text (e.g.J.H. Van-Vleck ELectric and Magnetic Susceptibilities) Qualitatively explain why QM does not forbid it. Hint A free current in a conductor with Battery I is a source of $\vec{B}$. A body current in a magnetized material is a source of $\vec{M}$



Q7) Consider a Harmonic oscillator with a cubic restoring force added to it. Write down the Hamiltonian. Solve the extra term as a perturbation to the least order to the ground state wave function. Solve the same problem using ladder operators.