PHY306 Advanced Quantum Mechanics Jan-Apr 2025: MidSemester Exam 2

Max Marks=10 Time=01 Hour Dated March 8, 2025

1. The spin-dependent Hamiltonian of two spin-1/2 particles in the presence of a uniform magnetic field in the z-direction is written as

$$H = AS^1.S^2 + \left(\frac{eB}{mc}\right)(S_z^1 - S_z^2)$$

Let the spin function of the system be given by $\chi_+^1\chi_-^2$.

   (a) Find the energy eigenvalues and the corresponding eigenvectors.

   [Marks=02]

   (b) Is this spin function an eigenfunction of H in the limit $A \to 0, eB/mc \neq 0$? If it is, what is the energy eigenvalue? And if it is not, what is the expectation value of H?

   [Marks=02]

2. Consider two particles with individual angular momenta $j_1 = 1$ and $j_2 = 1$.

   (a) Write out the dimensionality of the total Hilbert space as a direct product in the individual angular momentum spaces and as a direct sum in the total angular momentum space and show that they match.

   [Marks=01]

   (b) Add the angular momenta and express the $|j, m\rangle$ eigenkets in terms of the $|j_1, j_2; m_1, m_2\rangle$ eigenkets, only for the highest value of the total angular momentum. (Note:-Since $j_1 = j_2 = 1$, you can save time and compactify the notation and write the product kets as $|m_1, m_2\rangle$).

   [Marks=05]

Useful Formulae

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$