PHY306 Advanced Quantum Mechanics Jan-Apr 2024: Assignment 3

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1. Consider a two-level system. Let a perturbation act on it which is a delta function in time $H_p = U\delta(t - t_0)$. Assume that $U_{aa} = U_{bb} = 0$ and let $U_{ab} = \alpha$. Calculate the probability P_{ab} that a transition occurs from state $|a\rangle$ to state $|b\rangle$.

2. Consider a multilevel system. Assume a constant perturbation (which was switched on at time $t = 0$ and switched off again at a later time t). Find the probability of a transition (to first order) from a state N to a state m ($m \neq N$) as a function of t.

3. Find the transition probability (to first order) if the perturbation is a sinusoidal function of time $H_p = V\cos(\omega t)$.

4. A particle at time $t = 0$ starts out in the Nth state of an infinite square well. Now water leaks into the well and then drains out again, so that the bottom is at a uniform potential $V_0(t)$ with $V_0(0) = V_0(T) = 0$. (i) Solve the exact equation and show that the wave function changes phase but no transitions to other states occur. Find the phase change $\phi(T)$ in terms of the function $V_0(t)$. (ii) Solve the problem using first-order perturbation theory and compare the solutions.

5. A particle of mass m is initially in the ground state of a one-dimensional infinite square well. At time $t = 0$ a brick is dropped into the well so that the potential becomes

$$\begin{aligned} V(x) &= V_0, \;\; 0 \leq x \leq a/2 \\ &= 0, \;\; a/2 < x \leq a \\ &= \infty, \;\; \text{otherwise} \end{aligned}$$

   where $V_0 << E_1$. After a time T, the brick is removed and the energy of the particle is measured. Find the probability (upto first order) that the energy is now E_2.

6. An electron is at rest at the origin, in the presence of a magnetic field whose magnitude B_0 is constant but whose direction rides around at

a constant angular velocity ω on the lip of a cone of opening angle α such that

$$B(t) = B_0[\sin\alpha\cos\omega t\hat{i} + \sin\alpha\sin\omega t\hat{j} + \cos\alpha\hat{k}]$$

Use time-dependent perturbation theory (to first order) to calculate the probability of a transition from spin up (initial state) to spin down, as a function of time.

7. A particle initially ($t \longrightarrow -\infty$) is in its ground state in an infinite potential well with walls at $x = 0$ and $x = a$. It is subjected at time $t = 0$ to a time-dependent perturbation $V(t) = \epsilon x e^{-t^2}$ where ϵ is a small real number. Calculate the probability that the particle will be found in its first excited state after a very long time ($t \longrightarrow \infty$).

8. A hydrogen atom initially ($t \longrightarrow -\infty$) in its ground state, is placed at time $t = 0$ in a time-dependent electric field pointing along the z-axis $E(t) = E_0\tau\hat{k}/(\tau^2 + t^2)$ where τ is a constant having the dimension of time. Calculate the probability that the atom will be found in the $2p$ state after a sufficiently long time ($t \longrightarrow \infty$).