Assignment 6

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Show that $\Phi(x,t) = (1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$ satisfy the following equation
$$(1-x^2)\frac{\partial^2 \Phi}{\partial x^2} - 2x\frac{\partial \Phi}{\partial x} + t\frac{\partial^2 (t\Phi)}{\partial t^2} = 0.$$

2. Prove that $P_{2n+1}(0) = 0$ and
$$P_{2n}(0) = (-1)^n \frac{1.3.5.\cdots\cdots(2n-1)}{2.4.6.\cdots\cdots.2n}.$$

3. Show that
$$\frac{1-t^2}{(1-2xt+t^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)P_n(x)t^n.$$

4. Show that $P_n(-1/2) = P_0(-1/2)P_{2n}(1/2) + P_1(-1/2)P_{2n-1}(1/2) + \cdots + P_{2n}(-1/2)P_0(1/2)$.

5. Prove that
$$1 + \frac{1}{2}P_1(\cos\theta) + \frac{1}{3}P_2(\cos\theta) + \frac{1}{4}P_3(\cos\theta) + \cdots\cdots = \log\frac{1+\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}.$$

6. In terms of δ-function, evaluate
$$\int_{-1}^{+1}(1-x^2)P'_m(x)P'_n(x)dx.$$

7. Evaluate
$$\int_{-1}^{+1}(x^2-1)P_{n+1}(x)P'_n(x)dx.$$

8. Evaluate
$$\int_{-1}^{+1} x^2\left[P_n(x)\right]^2 dx.$$

9. Show that $(x^2-1)P'_n(x) = (n+1)(P_{n+1}(x) - xP_n(x))$.

10. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ interms of Legendre Polynomials.

11. Let $P_n(x)$ be the Legendre Polynomial of degree n, show that for any function $f(x)$, for which the nth derivative is continuous
$$\int_{-1}^{+1} f(x)P_n(x)dx = \frac{(-1)^n}{2^n n!}\int_{-1}^{+1}(x^2-1)f^n(x)dx.$$

12. Evaluate
$$\int_{-1}^{+1} x P_n(x) dx.$$

13. Show that for $m > 0$
$$P_l^{-m}(x) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(x).$$

14. Show that
$$\int_{-1}^{+1} P_l^m(x) P_{l'}^m(x) dx = \frac{2(l+m)!}{(2l+1)(l-m)!} \delta_{ll'}.$$

15. Prove that
$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta Y_{l,m}(\theta,\phi) Y_{l',m'}(\theta,\phi) = \delta_{ll'}\delta_{mm'}.$$