Assignment 10

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Find the real and imaginary parts $u(x,y)$ and $v(x,y)$ of the following functions: (a) e^{iz} and (b) $\sqrt{1+z^2}$.

2. Show whether or not the functions (a) $f(z) = x$ and (b) $f(z) = z^*$ is analytic.

3. If $f(z) = u(x,y) + iv(x,y)$ is analytic, show that $u(x,y)$ and $v(x,y)$ cannot both have either a maximum or a minimum in the interior of any region in which $f(z)$ is analytic.

4. Find the analytic function $f(z) = u(x,y) + iv(x,y)$ if (a) $u(x,y) = x^3 - 3xy^2$ and (b) $v(x,y) = e^{-y}\sin x$.

5. If there is some common region in which $w_1 = u(x,y) + iv(x,y)$ and $w_2 = w_1^*$ are both analytic, prove that $u(x,y)$ and $v(x,y)$ are constants.

6. The function $f(z) = u(x,y) + iv(x,y)$ is analytic. Show that $f^*(z^*)$ is also analytic.

7. Using $f(z) = f(re^{i\theta}) = R(r,\theta)e^{i\phi(r,\theta)}$ in which $R(r,\theta)$ and $\phi(r,\theta)$ are differentiable real functions of r and θ, show that the *Cauchy-Riemann* conditions in polar coordinates becomes
$$\frac{\partial R}{\partial r} = \frac{R}{r}\frac{\partial \phi}{\partial \theta} \quad \text{and} \quad \frac{1}{r}\frac{\partial R}{\partial \theta} = -R\frac{\partial \phi}{\partial r}.$$
Applying these conditions, study the analyticity of $|z|$.

8. The function $f(z)$ is analytic. Show that the derivative of $f(z)$ with respect to z^* does not exist unless $f(z)$ is a constant.

9. For each of the following functions $f(z)$, find $f'(z)$ and identify the maximal region within which $f(z)$ is analytic (a) $f(z) = \sin z/z$, (b) $f(z) = z^2 - 3z + 2$, (c) $f(z) = 1/z(z+1)$, and (d) $f(z) = \tanh(z)$.

10. The functions $f(z)$ have a derivative for what complex values (a) $f(z) = z^{3/2}$, (b) $f(z) = \tan^{-1} z$, (c) $f(z) = z^{-3/2}$, and (d) $f(z) = \tanh^{-1}(z)$.

11. Show that the integral
$$\int_{3+i4}^{4-i3} (4z^2 - i3z)dz$$
has the same value on the two paths: (a) the straight line connecting the integration limits and (b) an arc on the circle $|z| = 5$.

12. Verify that $\int_0^{1+i} z^* dz$ depends on the path by evaluating the integral for two paths (a) a straight line connecting (0,0) and (1,1) (b) two straight lines (0,0) to (1,0) and (1,0) to (1,1). Explain the outcome.

13. Show that

$$\oint \frac{dz}{z^2 + z} = 0$$

in which the contour C is a circle defined by $|z| = R > 1$.

14. For a square shaped contour with vertices located at $\pm 1 \pm i$, evaluate

$$\oint \frac{dz}{z}.$$

15. For a circular contour C defined by $|z - 1| = 1$, evaluate

$$\oint \frac{dz}{z^2 - 1}.$$

16. Show that the following integral is a representation of the Kronecker $\delta_{m,n}$:

$$\frac{1}{2\pi i} \oint z^{m-n-1} dz,$$

where m, n are integers.