Assignment - 5

Date - 15/09/2023

No Need to Submit.

1. Show that $\phi_n(x) = e^{-\frac{x^2}{2}} H_n(x)$ satisfies

$$\frac{d^2\phi_n}{dx^2} + (2n+1-x^2)\phi_n(x) = 0$$

2. Starting from the Rodrigues formula - Show that

$$H_n(x) = \frac{2^n (-i)^n}{\sqrt{\pi}} e^{x^2} \int_{-\infty}^{\infty} t^n e^{-t^2 + 2ixt} dt$$

Hint - $e^{-x^2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2 + 2ixt} dt$. This is known as integral representation of $H_n(x)$.

3. Convert the following polynomial into Hermite Polynomial: $64x^4 + 8x^3 - 32x^2 + 40x + 10$.

Ans: $4H_4(x) + H_3(x) + 40H_2(x) + 26H_1(x) + 42H_0(x)$.

4. Prove that $H_{2n}(0) = (-1)^n 2^{2n} \left(\frac{1}{2}\right)_n$.

5. Prove that $\frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^n n!}{(n-m)!} H_{n-m} \quad m < n$.

6. Show that if m is an integer

$$\int_{-\infty}^{\infty} x^m e^{-x^2} H_n(x)\, dx = 0$$

- Search Internet for similar problems.