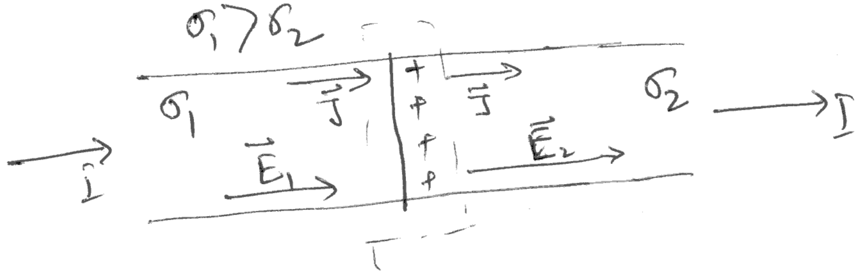
Assignment 5 (Sols.)

1. $\sigma_1 > \sigma_2$



- Normal component of $\vec{J}$ same on left & right of junction.

$\therefore \; |\vec{E}_1| = \dfrac{J}{\sigma_1}$, $\quad |\vec{E}_2| = \dfrac{J}{\sigma_2}$. $\left(\because \sigma_1 > \sigma_2 \; \therefore E_2 > E_1\right)$.

By Gauss's law, the difference in the electric fields gives the surface charge density, σ' as,

$$E_2 - E_1 = \frac{\sigma'}{\epsilon_0}$$

$$\Rightarrow \quad \frac{J}{\sigma_2} - \frac{J}{\sigma_1} = \frac{\sigma'}{\epsilon_0} \Rightarrow \sigma' = J\epsilon_0\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right).$$

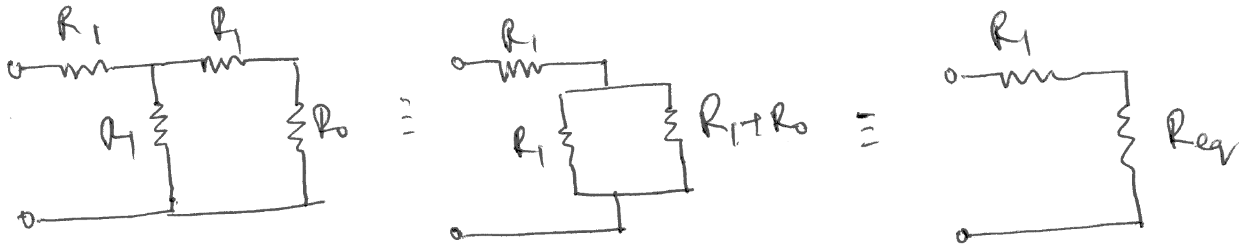
If A is the area of the interface, then,

$$Q = \sigma' A \quad \& \quad I = JA = \frac{JQ}{\sigma'}.$$

$$\therefore \; Q = \frac{I\sigma'}{J} = \frac{I}{J} \cdot J\epsilon_0\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

$$\therefore \boxed{Q = I\epsilon_0\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)}$$

3.



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_1 + R_o}} = \frac{R_1 (R_1 + R_o)}{R_1 + (R_1 + R_o)}.$$

Finally, $R = R_1 + R_{eq} = R_1 + \dfrac{R_1(R_1+R_o)}{R_1 + R_1 + R_o}$

We require, $R = R_o$.

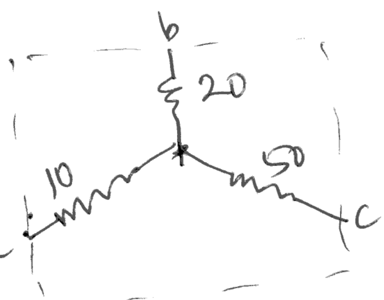
$$\therefore R_0 = R_1 + \frac{R_1(R_1+R_0)}{2R_1+R_0} = \frac{R_1(2R_1+R_0)+R_1(R_1+R_0)}{2R_1+R_0}.$$

$$\Rightarrow R_0(2R_1+R_0) = R_1(2R_1+R_0+R_1+R_0) = R_1(3R_1+2R_0)$$

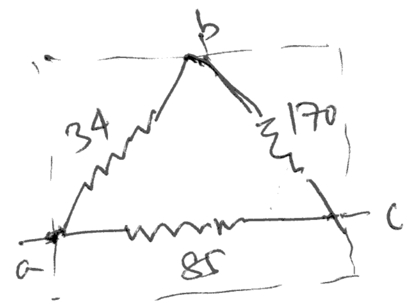
$$\Rightarrow \cancel{2R_0R_1}+R_0^2 = 3R_1^2+\cancel{2R_0R_1} \Rightarrow R_0^2 = 3R_1^2$$

$$\therefore \boxed{R_1 = \frac{R_0}{\sqrt{3}}}$$

4.



(A)



(B)

For (A), resistance between any 2 terminals ~~is~~ is basically the series combo of the 2 resistors which connect the terminals. The third resistance does not feature. Therefore, the resistance between terminals (a) & (b) would be,

$$R_{ab}^{A} = (10+20)\,\Omega = 30\,\Omega.$$

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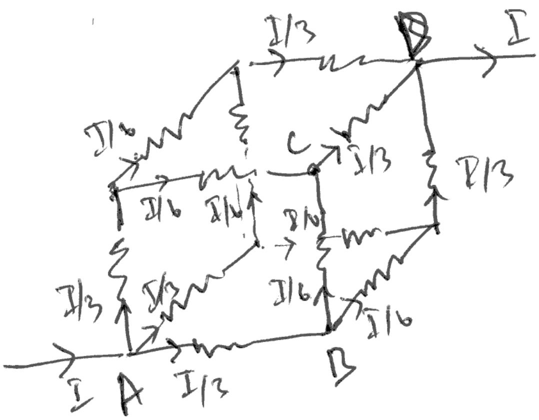
For (B), resistance between 2 terminals would involve a resistor in $\|$ with 2 others being in series. For example, to find the resistance between terminals (a) & (b), 34 Ω is in $\|$ to the series combination of $170\,\Omega$ and $85\,\Omega$.

$$\therefore R_{ab}^{B} = \left(\frac{1}{\frac{1}{34}+\frac{1}{170+85}}\right)\Omega = \frac{1}{\frac{255+34}{34\times 255}}$$

$$= \frac{\cancel{255}^{15}\times\cancel{34}^{2}}{\cancel{289}_{\cancel{17}}} = 30\,\Omega. = R_{AB}^{A}$$

You can similarly show for the other 2.

These are the only 2 possible configurations.

5. 

Resistors at the edges of the cube all have same value. R_0.

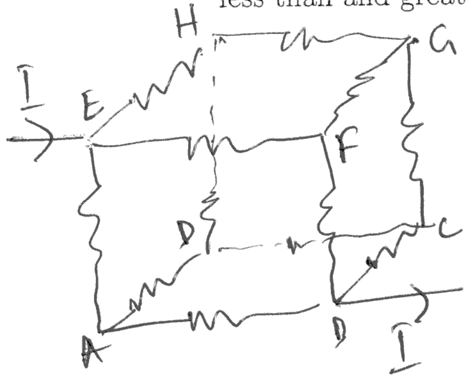
A current I entering at A will get equally divided in the three sides and so on.

Now take any path say ABCD & for that path,

$$V_{AD} = \underbrace{\frac{I}{3}R_0}_{AB} + \underbrace{\frac{I}{6}R_0}_{BC} + \underbrace{\frac{I}{3}R_0}_{CD.} = \frac{5}{6}R_0 I$$

$$\therefore \boxed{R_{AD} = \frac{V_{AD}}{I} = \frac{5}{6}R_0}$$

less than and greater than R? [2.5]



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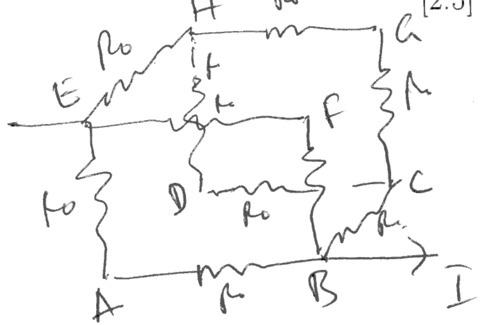
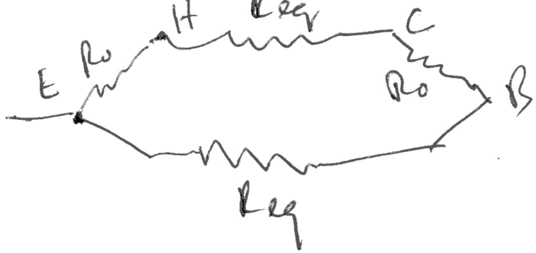


By symmetry, A & F & D & G are equivalent.
Current in the branches AD & FG must be zero.

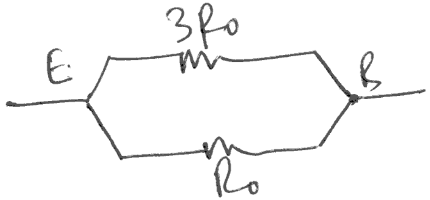


$$\frac{1}{R_{eq}} = \frac{1}{R_0+R_0} + \frac{1}{R_0+R_0}$$

$$= \frac{1}{R_0}.$$

$$\therefore R_{eq} = R_0.$$

$\equiv$



$$\therefore R_{EB} = \frac{1}{\frac{1}{3R_0} + \frac{1}{R_0}} = \frac{3}{4} R_0.$$