# PHY306 Advanced Quantum Mechanics Jan-April 2024: Assignment 5

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1. Find the Clebsch-Gordan coefficients for the addition of (a) two angular momenta $j_1 = 1$ and $j_2 = 2$ and (b) three angular momenta each of $j = 1/2$.

2. Consider two nonidentical particles each with angular momentum 1, with the Hamiltonian given by

$$H = \frac{\epsilon_1}{\hbar^2}(L_1 + L_2).L_2 + \frac{\epsilon_2}{\hbar^2}(L_{1z} + L_{2z})^2$$

   where $\epsilon_1, \epsilon_2$ are constants with dimensions of energy. Find the energy levels and degeneracies for those states of the system whose total angular momentum is equal to $2\hbar$.

3. Analyze the Zeeman effect for $n = 2$ states of the Hydrogen atom in the intermediate field regime. As the basis for degenerate perturbation theory choose the states characterized by $l, j, m_j$ and use the Clebsch-Gordan coefficients to express $|jm_j\rangle$ as a linear combination of $|lm_l sm_s\rangle$. Construct a table of energies and plot them as functions of the external field and check that the in the two limiting cases the intermediate field results reduce properly to the weak Zeeman and the strong Zeeman splitting.

4. Analyze the Zeeman effect for $n = 3$ states of the Hydrogen atom in the intermediate field regime and as in the above problem, construct the table of energies, plot as a function of external field and check the limiting cases of weak and strong Zeeman splittings.

5. Analyze the Stark effect (atom placed in electric field $E_{ext}$) for $n = 1$ and $n = 2$ states of Hydrogen. Let the field point in $z$ direction so that potential energy of the electron is $H' = -eE_{ext}r\cos\theta$. Treat this as a perturbation on the Bohr Hamiltonian and ignore spin. Show that the ground state energy is not affected by this perturbation in the the first order. Find the first-order corrections to the energy for the first excited state (which is four-fold degenerate). What are the "good" wave functions for this case? Find the expectation value of the electric dipole moment $p = -er$ in each of these "good" states.

6. Analyze the Stark effect for the $n = 3$ states of hydrogen after turning on the electric field in the $z$ direction. Write out the initial degenerate states and construct the perturbing Hamiltonian matrix. Find the eigenvalues and their degeneracies.

7. Considering the proton itself as a magnetic dipole, write out the Hamiltonian $H'_{hf}$ ($hf$ denoting hyperfine) of the electron in the magnetic field due to the proton's dipole moment. Find the first order correction to the energy $E^1_{hf}$ for the ground state of the hydrogen atom. Find the energy gap between the singlet and triplet states, the frequency and corresponding wavelength of the photon emitted in a transition between the triplet and singlet states.

8. Find the hyperfine splitting in the ground state of (a) muonic hydrogen where a muon substitutes for the electron, (b) positronium where a positron substitutes for the proton, and (c) muonium where an antimuon substitutes for the proton.