1. Using the first of law of thermodynamics, show that the relation between the specific heats C_P and C_V for a hydrostatic system with fixed particle number is given by $C_P = C_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P$.

2. Irreversible expansion of a gas - A container with volume V which is insulated from its surroundings is divided into two sub-volumes V_1 and V_2. Initially, the volume V_1 contains a gas at temperature T, while V_2 is evacuated. The partition is then removed and the gas is allowed to flow rapidly into V_2. The purpose of the experiment is to determine the dependence of the internal energy U on the volume V.

   a) What is the relevant quantity that you should calculate and related to a measurable quantity from the experiment?

   b) Show that this quantity can be related to the specific heat at constant volume C_V and $\left(\frac{\partial T}{\partial V}\right)_U$

   c) Using the first law of thermodynamics derive an expression for $\left(\frac{\partial T}{\partial V}\right)_U$. What is the value of this for an ideal gas? What can you infer from this result?

   d) We now intend to show that the process is irreversible. From the first law show that the entropy production is positive $\mathrm{d}S > 0$ and the process obeys the inequality $T\mathrm{d}S > \delta Q$.

   e) Recall the expression of the entropy we derived in the class when discussing Gibbs-Durhem relations. Using that show that the entropy production is positive.

3. Consider the expression for the Gibbs free energy G. Show that $G = \mu N$. Hence show that $\left(\frac{\partial P}{\partial \mu}\right)_T = N/V$.

4. Consider a thermodynamic system whose fundamental equation of state is given by $S = S(U, V, N)$. Now imagine I have two such systems which are brought together, and subsequently the whole system reach equilibrium, with the constraint that $U_0 = U_1 + U_2$, $V_0 + V_1 + V_2$ and $N = N_1 + N_2$ remains fixed, where U_0, V_0, N are the total energy, total volume and total particle number of the system.

   a) What is the total entropy of the system at any instant?

   b) Now you allow for a variation in the energy, volume and particle number and using the constraints, show that at equilibrium the condition $\delta S = 0$ implies that the temperature, pressure and the chemical potential must be equal at equilibrium.

5. Now consider a system made of n-components which can exist in r-phases. We assume that no chemical reaction can happen between the phases. We want to find out how many phases can coexist in equilibrium.

   a) Determine how many variables you need to specify the state of the thermodynamic system (consider how many intensive variables you need).

   b) Generalize the conditions for thermodynamic equilibrium that you derived earlier. How many conditions do you get?

   c) Hence show that the number of quantities which can be varied (degree of freedom) f is given by $f = 2 + n - r$.

   d) Consider now a singe component system which can have 1,2, and 3 phases. Find out the degree of freedom in each case and their physical significance (you can think of water).

6. Derive the following relations: a) $\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V}\left(\frac{\partial P}{\partial T}\right)_V$ and (b) $\left(\frac{\partial T}{\partial P}\right)_S = \frac{T}{C_P}\left(\frac{\partial V}{\partial T}\right)_P$

7. A paramagnetic system in an uniform magnetic field H is thermally insulated from the surroundings. It has an induced magnetization $M = aH/T$ and a heat capacity $C_H = b/T^2$ constant H, where a and b are constants and T is the temperature.

   a) Write down the Maxwell's relation for this system.

   b) How will the temperature of the system change when H is quasi-statically reduced to zero?

   c) In order to have the final temperature change by a factor of 2 from the initial temperature, how strong should be the initial H ?