

Quiz 8

- Due to length contraction the length of a moving rod appears shrunk by 25% of its stationary value to an inertial observer. How long 1 second of a clock which is moving along with the rod would appear to the same observer ?
- Two supernovae (burst of some special stars) are observed to happen from earth, one after another - 3 years ($\sim 10^8$ s) apart in two different galaxies which are 10 light years ($\sim 9 \times 10^{16}$ m) apart and are on diametrically opposite sides of the earth. To an astronaut launched towards one of them in a uniformly speeding rocket, the two supernovae appear to have happened **simultaneously**. How fast the rocket was moving ? [Treat earth as an inertial frame for this problem]

Quiz - 08

1. Let L' and T' be length of rod and time period of a clock in frame S' which is moving at speed v w.r.t. S . Then measurement in S will yield

$$L = \frac{1}{\gamma} L' \quad \text{and} \quad T = \gamma T'$$

$$\Rightarrow \frac{L}{L'} = \frac{1}{\gamma} \quad \text{and} \quad \frac{T}{T'} = \gamma$$

$$\Rightarrow \text{Since } L < L' \Rightarrow \frac{\Delta L'}{L'} = \frac{L' - L}{L'} = 1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} = \frac{25}{100} = \frac{1}{4}$$

$$\gamma = 4/3$$

$$\text{But } T > T' \Rightarrow T = \gamma T' = \frac{4}{3} \times 1 \text{ s} = \frac{4}{3} \text{ s}$$

2.

Given - from earth's frame the two supernovae happen with : $\Delta t = 10^8 \text{ s}$, $\Delta x = 9 \times 10^{16} \text{ m}$

For the moving rocket frame $\Delta t' = 0$

$$\text{From, } \Delta t' = \gamma \left(\Delta t - \frac{v}{c^2} \Delta x \right)$$

$$0 = \gamma \left(10^8 \text{ s} - \frac{v}{c} \times \frac{9 \times 10^{16} \text{ m}}{3 \times 10^8 \text{ m s}^{-1}} \right)$$

$$10^8 \text{ s} - \frac{v}{c} \times 3 \times 10^8 \text{ s} = 0$$

$$\Rightarrow v/c = 1/3 \quad \Rightarrow v = 10^8 \text{ m s}^{-1}$$