PHY304: Statistical Mechanics

1st Mid Semester Examination 2025

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Instructor: Rajeev Kapri **Max. Marks 20**

1. The fundamental relation for the system is given by

$$S = \left(\frac{R}{\theta}\right)^{1/2}\left(NU + \frac{R\theta V^2}{v_0^2}\right)^{1/2},$$

where R, θ and v_0 are constants.

   (a) Calculate the fundamental relation in Gibbs representation. [3]

   (b) Calculate the molar heat capacity $c_p(T,P)$ and the isothermal compressibility $\kappa_T(T,P)$ by differentiation of G. [3]

2. The adiabatic bulk modulus is defined by

$$\beta_S = -v\left(\frac{\partial P}{\partial v}\right)_s = -V\left(\frac{\partial P}{\partial V}\right)_{S,N}$$

Express this quantity in terms of c_p, c_v, α, and κ_T. [4]

3. Consider a random walker in one dimension. Assuming that in each step the displacement of the walker is always positive and uniformly distributed in the range between $\ell - b$ and $\ell + b$, where $b < \ell$. The walker takes a total of N steps.

   (a) Obtain the characteristic function of the walker. [3]

   (b) What is walker's mean displacement? [2]

   (c) What is walker's dispersion (i.e., the variance)? [2]

   (d) What is the probability $P(x)dx$ that the total displacement of the walker lies between x and $x + dx$ after large number of steps? [3]

Some important mathematical identities:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P; \quad \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T; \quad c_p = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_P; \quad c_v = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_V$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}; \qquad \left(\frac{\partial x}{\partial y}\right)_z = \frac{\left(\frac{\partial x}{\partial w}\right)_z}{\left(\frac{\partial y}{\partial w}\right)_z}; \qquad \left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x\left(\frac{\partial z}{\partial x}\right)_y = -1$$

For large N, $(\sin kb/kb)^N$ becomes very small unless kb is very small. For $kb \ll 1$, $(\sin kb/kb)^N$ can be approximated as

$$\left(\frac{\sin kb}{kb}\right)^N = \left(1 - \frac{k^2b^2}{6}\right)^N \approx \exp\left(-\frac{Nk^2b^2}{6}\right)$$

$$\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$$