Problem Sheet 2

1. In a spacetime described by the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

find out the time taken by a photon in traveling dx unit along the x-axis. If the outgoing ray takes more amount of time than in-moving rays for the same dx what can we conclude about the metric components ?

2. If expanding universe can be explained by a spacetime

$$ds^2 = -c^2 dt^2 + a^2(t) d\mathbf{x}^2$$

Show that if $a(t) = (Ht)^{1/2}$ for a constant H, the distance traveled by photons since the beginning of the universe $t = 0$ is finite. Then find out the condition on n for which the distance traveled will be infinite.

3. In the Newtonian gravity description, the spacetime line element is

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + r^2 d\Omega^2$$

If a photon is moving radially, what is the co-ordinate velocity it is having in this set up ? Find out the co-ordinate set-up in which the co-ordinate velocity is c.

4. In the Newtonian gravity set up, given above, find out the angle between the radially ingoing ray and out-going ray in the $t - r$ plane. What is expected about this opening angle between the two null rays for the limits $r \to \infty$ and $r \to GM/c^2 r$?

5. We can not use photon's proper time for describing its velocity etc. . But we can use someone else's proper time to analyse the movement of photons. Using such a clock τ write down the **null** geodesic equation in a static metric. What is the Newtonian acceleration of photons in static spacetime ?