MTH101 (Symmetry)

Tutorial Sheet 06 / March 01, 2022 Spring 2022

Notation

- $\mathbb{N}$: natural numbers, $\mathbb{Z}$: integers, $\mathbb{Q}$: rational numbers, $\mathbb{R}$: real numbers.

Which of the following are groups? In case not, find which condition(s) is/are not satisfied.

(A). $\mathbb{Z}$, under the operation $*$, where $*$ denotes the multiplication of integers.

(B). $\mathbb{R}$, under the operation $*$, where $*$ denotes the multiplication of real numbers.

(C). The collection of irrational numbers under addition.

(D). The set of clock hours $\{12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ under the addition of clock hours. (Therefore $10 + 3 = 1$ under this operation).

(E). The set $\{1, 3, 7, 9\}$ under the operation "*rightmost digit in the multiplication of numbers*."

(F). Symmetries of an amoeba.

(G). The collection $\{a, b, c\}$ of three alphabets with the operation given by the following composition table:

	a	b	c
a	b	a	c
b	c	b	a
c	a	c	b

(H). The collection $\{\square, \hexagon, \bullet, \circ\}$ of four symbols with the operation given by the following composition table:

	$\square$	$\hexagon$	$\bullet$	$\circ$
$\square$	$\bullet$	$\square$	$\circ$	$\hexagon$
$\hexagon$	$\square$	$\hexagon$	$\bullet$	$\circ$
$\bullet$	$\circ$	$\bullet$	$\hexagon$	$\square$
$\circ$	$\hexagon$	$\circ$	$\square$	$\bullet$

(I). Collection S_4 of all permutations of $1, 2, 3, 4$ under the composition of permutations.

(J). The subset $\{1, (1\ 2), (1\ 3), (1\ 4), (2\ 3), (2\ 4), (3\ 4)\}$ of S_4 under the composition of permutations.

(K). The subset even permutations

$\{1, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3), (1\ 2\ 3), (1\ 3\ 2), (1\ 2\ 4), (1\ 4\ 2), (1\ 3\ 4), (1\ 4\ 3), (2\ 3\ 4), (2\ 4\ 3)\}$

of S_4 under the composition of permutations.

(L). The collection of 2×2 matrices having nonzero determinant and entries in $\mathbb{Z}$, under the operation of matrix multiplication; i.e. $\{A \in M_2(\mathbb{Z}) : \det(A) \neq 0\}$, under multiplication of matrices.

(M). $\mathrm{GL}_n(\mathbb{R}) := \{A \in M_n(\mathbb{R}) : \det(A) \neq 0\}$, under multiplication of matrices. invertible matrices

(N). $\mathrm{SL}_n(\mathbb{R}) := \{A \in M_n(\mathbb{R}) : \det(A) = 1\}$, under multiplication of matrices.

(O). $\mathrm{Sym}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = A\}$, under multiplication of matrices. symmetric matrices

(P). $\mathrm{Sym}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = A\}$, under addition of matrices.

(Q). $\mathrm{Skew}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A^t = -A\}$, under addition of matrices. skew-symmetric matrices

(R). $\mathrm{Sym}_3(\mathbb{R}) \cap \mathrm{GL}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible and } A^t = A\}$, under multiplication of matrices.

(S). $\mathrm{O}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible and } A^t = A^{-1}\}$, under multiplication of matrices.

(T). $\mathrm{SO}_3(\mathbb{R}) := \{A \in M_3(\mathbb{R}) : A \text{ is invertible, } A^t = A^{-1} \text{ and } \det(A) = 1\}$, under multiplication of matrices.

(U). The collection of rotations R_θ of a circular disc, under composition of symmetries.

(V). The collection of reflections f_θ of a circular disc, under composition of symmetries.

(W). The collection of 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where a is a nonzero element in $\mathbb{Q}$, under the operation of matrix multiplication.

(X). The collection of 2×2 matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$, where a is a nonzero element in $\mathbb{Q}$, under the operation of matrix addition.

(Y). Collection of all polynomials in one variable with coefficients in $\mathbb{R}$, under the addition of polynomials.

(Z). Collection of all polynomials in one variable with coefficients in $\mathbb{R}$, under the multiplication of polynomials.
