ROLL NO : MS | | | | | | **NAME :**

PHY638 MidSem I (Part A) Date : Feb 7, 2025 Inst: Abhishek Chaudhuri

- **Time : 30 minutes, Max Marks : 10**
- **Attempt all questions. Please give your answers in the space provided.**

1. Show that $\hat{e}_i = \frac{1}{2}\varepsilon_{mni}(\hat{e}_m \times \hat{e}_n)$ [2]

2. Let a one-dimensional velocity field be $u_1 = u(x_1, t)$, with $u_2 = 0$ and $u_3 = 0$. The density varies as $\rho = \rho_0(2 - \cos\omega t)$. Find an expression for $u(x_1, t)$ if $u(0, t) = U$. [3]

3. What is $\epsilon_{pqr}\epsilon_{pqr} = ?$. [2]

4. Find the streamlines and pathlines for the simple plane flow: [3]

$$u_i = \frac{x_1}{1+t}, \quad u_2 = x_2, \quad u_3 = 0.$$

PHY638 MidSem I (Part B) Date : Feb 7, 2025 Inst: Abhishek Chaudhuri

- **Time : 30 minutes, Max Marks : 10**
- **Attempt all questions.**

1. Consider a plane Couette flow of a viscous fluid confined between two flat plates a distance b apart. At steady state, the velocity distribution is $u = Uy/b$ and $v = w = 0$, where the upper plate at $y = b$ is moving parallel to itself at speed U, and the lower plate is stationary. [2+2+2]
   (a) Find the components of the strain rate tensor.
   (b) Find the components of the rotation tensor.
   (c) Find the vorticity in this flow.

2. Reynold's transport theorem for the rate of change of function $\mathcal{F}(\mathbf{x}, t)$ over a volume V gives

$$\frac{D}{Dt}\int_{V(t)} \mathcal{F}(\mathbf{x},t)dV = \int_{V(t)} \left[\frac{\partial \mathcal{F}(\mathbf{x},t)}{\partial t} + \boldsymbol{\nabla}\cdot\{\mathcal{F}(\mathbf{x},t)\mathbf{u}\}\right] dV.$$

   Show that

$$\frac{D}{Dt}\int_{V(t)} \rho\mathcal{Q}dV = \int_{V(t)} \rho\frac{D\mathcal{Q}}{Dt}dV,$$

   where $\mathcal{Q}$ is a scalar field. [4]