PHY622/Assignment 5

Date: April 8, 2018

Note: Solve the following problems. The submission of assignment is NOT required. You are encouraged to discuss with each other and/or contact the instructor if you have any difficulty in solving the problems.

Problem 1. SO(2):

1. Show that the rotation matrix $R(\theta) = e^{-i\theta J}$, where $J = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, is an orthogonal matrix and prove that every such matrix represents a rotation in 2D Euclidean space.
2. Find the eigenvalues and eigenvectors of J.
3. Find a unitary matrix U such that $\tilde{J} = UJU^\dagger$, where $\tilde{J} = \text{Diag.}(1, -1)$ is diagonal matrix.
4. Find the elements of matrix $\tilde{R}(\theta) = e^{-i\theta\tilde{J}}$.

Problem 2. SO(3):

1. Find out continuous subgroups of SO(3).
2. Construct the irreducible matrix representation of SO(3) generators for $j = 1$ and $j = 3/2$.
3. For $j = 1$, using the matrix representation of generators, obtain the matrix representation of rotation matrix $R(\theta_1, \theta_2, \theta_3) = R_{12}(\theta_3)R_{13}(\theta_2)R_{23}(\theta_1)$, where $R_{ij}(\theta)$ is rotation in i-j^{th} plane by an angle θ.

Problem 3. SU(2):

1. Find out the most general 2×2 unitary matrix which has determinant equal to 1. Using it, derive the generators. Show that the generators respect Lie algebra.
2. Show that the group parameter space of SU(2) is surface of a unit sphere in 4D Euclidean space.

Problem 4. SO(1,1):

1. Consider one space and one time dimension. Show that an invariance of spacetime interval under the SO(1,1) transformation implies *finite* velocity for exchange of information.

Problem 5. Lorentz transformations in higher spacetime dimensions:

1. Consider 2 time and 3 space dimensions. Find out the number of generators for homogeneous Lorentz transformation which leaves the spacetime interval unchanged.
2. Repeat the above exercise for 1 time and 4 space dimensions.

Problem 6. SO(4) and SU(2) $\otimes$ SU(2):

1. The SO(4) is a group of special orthogonal matrices with determinant +1 which leaves a length of vector unchanged when acted on a four dimensional Euclidean space. Find the number of generators and their Lie algebra.
2. Show that the Lie algebra of SO(4) group is same as that of SU(2) $\otimes$ SU(2).