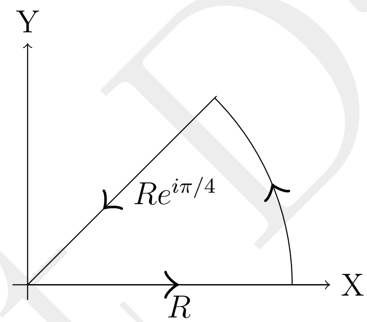
Assignment 11

PHY310: Mathematical Methods for Physicists I

Instructor:Dr. Prasenjit Das **No Need to Submit**

1. Prove that
$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^n} = \frac{\pi}{2^{2n-2}} \frac{(2n-2)!}{[(n-1)!]^2}$$

2. Prove that
$$\int_0^{\infty} \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$
Hint: One can choose the following complex function: $f(z) = \exp(iz^2)$ and the contour shown on the right.



3. Evaluate the integral
$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$$
by the method of residues when $-1 < a < 1$.

4. For $n = 0, 1, 2, \cdots \infty$, show that
$$\int_0^{\pi} \cos^{2n}\theta d\theta = \pi \frac{(2n)!}{2^{2n} n!}.$$

5. Using method of residues, show that
$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta = \frac{\pi}{6}.$$

6. Prove that
$$\int_0^{\infty} e^{-x^2} \cos(2ax) dx = \frac{\sqrt{\pi}}{2} e^{-a^2}$$
by integrating e^{-z^2} around a rectangle whose vertices are $(0, 0)$, $(R, 0)$, (R, ia), and $(0, R + ia)$.

7. Using method of Residues, evaluate
$$\int_0^{\infty} \frac{\cos x}{x^2+1} dx.$$

8. For $a > 1$, show that
$$\int_0^{\pi} \frac{d\theta}{(a + \cos\theta)^2} = \frac{\pi a}{(a^2-1)^{3/2}}.$$

9. For $|t| < 1$, show that

$$\int_0^{2\pi} \frac{d\theta}{1 - 2t\cos\theta + t^2} = \frac{2\pi}{1 - t^2}.$$

10. Using method of Residues, show that

$$\int_0^{\infty} \frac{x \sin x}{x^2 + 1} dx = \frac{\pi}{2e}.$$