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Assignment IIA

Q #1 Assume that the rod is initially placed along the x-axis and extends from $x=0$ to $x=L$. Let the temperature at $x=0$ be T_1, initially. Now, temperature at any position x is given by

$$T(x) = T_1 + \left(\frac{T_2 - T_1}{L}\right) x.$$

At $L/2$

$$T(L/2) = T_1 + \frac{T_2 - T_1}{L} \cdot \frac{L}{2}$$

$$= \frac{T_1 + T_2}{2}.$$

So ~~even~~ if $T_1 > T_2$ then $T(L/2) = \frac{T_1 + T_2}{2}$

if $T_2 > T_1$ then $T(L/2) = \frac{T_1 + T_2}{2}$

Hence there is a symmetry around the midpoint of the rod.

Now, suppose we consider a slice of length Δx at x.

Then it's mass is $\Delta M = \rho A \Delta x$

Now reversibly transfer an infinitesimal heat $|dQ|$ to the slice. This process will raise the temperature of the slice by ΔT.

$$\Rightarrow |dQ| = \Delta M C_v \, dT = C_v \Delta M \, dT.$$

so Change in entropy would be $dS = C_v \Delta M \frac{dT}{T}$

$$\Rightarrow \Delta S = C_V \Delta M \int \frac{dT}{T} = C_V \Delta M \ln \frac{T_{final}}{T_{initial}}$$

But $T_{initial}$ at x is $T(x) = T_1 + \frac{T_2 - T_1}{L} . x$

$$\Delta S = C_V \Delta M \ln \frac{T_{final}}{T_1 + \frac{T_2-T_1}{L}. x} = C_V \rho A \ln \frac{T_{final}}{T_1 + \frac{T_2-T_1}{L}. x} \delta x$$

Therefore total entropy change of the rod is

$$\Delta S_{Rod} = C_V \rho A \int dx \ln \frac{T_{final}}{T_1 + \frac{T_2-T_1}{L}. x}$$

$$= C_V \rho A \left[\int dx \ln T_{final} - \int dx \ln T_1 - \int dx \ln \left(1 + \frac{T_2 - T_1}{LT_1} x\right) \right]$$

$$= C_V \rho A \left[L \ln T_{final} - L \ln T_1 - D \right]$$

$$D = \int dx \ln (1 + Bx) \quad \text{with } B = \frac{T_2 - T_1}{LT_1}$$

$$1 + Bx = y \qquad dx = \frac{dy}{B}$$

$$D = \frac{1}{B} \int_1^{1+BL} dy \ln y = \frac{1}{B} \Big[y \ln y - y \Big]_1^{1+BL}$$

$$= \frac{1}{B} \Big[(1+BL) \ln (1+BL) - (1+BL) + 1 \Big]$$

$$1 + BL = 1 + \frac{T_2 - T_1}{T_1} = \frac{T_2}{T_1}$$

$$= \frac{1}{B} \Big[(1+BL) \ln (1+BL) - BL \Big] = \frac{LT_2}{T_2 - T_1} \ln \frac{T_2}{T_1} - L$$

$$\frac{1+BL}{B} = \frac{T_2}{T_1} \cdot \frac{LT_1}{(T_2 - T_1)}$$

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$$\Delta S = S_{eq} - S_0$$

$$= C_V A \rho L \left[1 + \ln T_{final} - \ln T_1 - \frac{T_2}{(T_2 - T_1)} \ln \frac{T_2}{T_1} \right].$$

Recall that I have symmetry with respect to the midpoint of the rod.

Hence $T_{final} = \frac{T_1 + T_2}{2}$.

$$\Delta S = C_V A \rho L \left[1 + \ln \frac{T_1 + T_2}{2} - \ln T_1 - \frac{T_2}{(T_2 - T_1)} \ln \frac{T_2}{T_1} \right].$$

Now $T_{final} = \frac{T_1 + T_2}{2}$ is one ~~possible~~ possibility.

If ~~the~~ I want $\Delta S = 0$ (operating a Carnot engine between T_1 & T_2).

then T_f is the solution of the equation

$$0 = 1 + \ln T_f - \ln T_1 - \frac{T_2}{T_2 - T_1} \ln \frac{T_2}{T_1}.$$

& $\Delta W_{max} = C_V A \rho L \left[\left(\frac{T_1 + T_2}{2} \right) - T_f \right].$

Assignment 1A

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#1.

$V_A = V_0 \quad | \quad V_B = 2V_0.$

a) At constant temperature T_0, the work done is

$$\Delta W = -\int_A^B p\,dV = -Nk_BT_0\int_A^B \frac{dV}{V} = -Nk_BT_0 \ln\left(\frac{V_B}{V_A}\right) = -Nk_BT_0 \ln 2.$$

$\Delta Q = \Delta U - \Delta W = Nk_BT_0 \ln 2$ since $\Delta U = 0$ for an isothermal process.

b) At constant pressure.

$$\Delta W = -\int_A^B p\,dV = -p(V_B - V_A) = -pV_0 = -Nk_BT_0.$$

$\Delta Q = \Delta U - \Delta W.$

$$\Delta U = \frac{3}{2}Nk_B\,\Delta T = \frac{3}{2}Nk_BT_0.$$

$$T_A = \frac{pV_0}{Nk_B} \equiv T_0.$$

$$T_B = 2\frac{pV_0}{Nk_B} = 2T_A.$$

$$\therefore \Delta Q = \frac{3}{2}Nk_BT_0 + Nk_BT_0 = \frac{5}{2}Nk_BT_0.$$

#2 For expansion at constant pressure.

$$\Delta W = -p(V_2 - V_1) = -(pV_2 - pV_1) = -Nk_B\,\Delta T$$

$\Delta T = T_2 - T_1$

$$\Delta Q = \cancel{\Delta U + \Delta W} \; \Delta U + Nk_B\Delta T$$

$$C_V = 5/2 . Nk_B$$

$$\Delta U = C_V\,\Delta T = \frac{5}{2}Nk_B\,\Delta T.$$

$$\Delta Q = \frac{7}{2}Nk_B\,\Delta T. \quad \therefore \frac{|\Delta W|}{\Delta Q} = \frac{Nk_B\Delta T}{\frac{7}{2}Nk_B\Delta T} = \frac{2}{7}.$$

(b) For an isothermal process $\Delta U = 0$

$$\Rightarrow \frac{\Delta W}{\Delta Q} = 1 \longrightarrow \text{Does it violate Kelvin's Statement?}$$

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#3 For an adiabatic process $pV^{\gamma} = \text{Const.}$

and $pV = Nk_BT$.

$$\Rightarrow TV^{\gamma-1} = \text{Const.}$$

$$\therefore T_i V_1^{\gamma-1} = T_f V_2^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Let's say the gas is monoatomic. Then $\gamma = 5/3$. $V_2 = 10V_0$, $V_1 = V_0$.

$$\Rightarrow T_f = T_i \frac{1}{10^{2/3}}$$

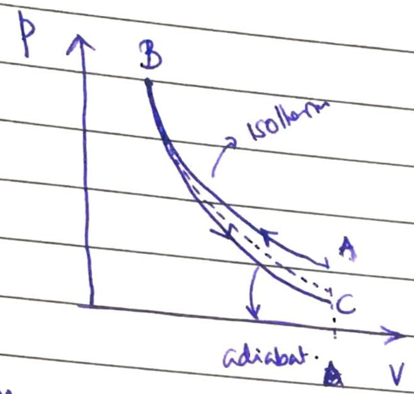
$$T_f \simeq 59\,K \quad \text{or} \quad -214^\circ C.$$

#5. Isothermally compressed and then adiabatically expanded.

$pV = Nk_BT$. $\Rightarrow \left.\frac{\partial p}{\partial V}\right)_T = -\frac{Nk_BT}{V^2} = -\frac{p}{V}$

$pV^{\gamma} = \text{Const.}$ $\left.\frac{\partial p}{\partial V}\right)_S = -\gamma\frac{\text{Const}}{V^{\gamma+1}} = -\gamma\frac{p}{V}.$

So the slope of an adiabat is γ times more than that of an isotherm. !!



AB → Isotherm
AC (Solid line) Monoatomic
AC (Dashed line) Diatomic

Why?

To understand this

Isotherm

$P_A V_A = P_B V_B$

$P_B = P_A \dfrac{V_A}{V_B} = 10 P_A.$

$P_A = 1\text{ atm.} \Rightarrow P_B = 10\text{atm.}$

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$V_B = 1$
$V_C = 10.$

$P_C = P_B \left(\dfrac{V_B}{V_C}\right)^{\gamma}$

$= P_B 10^{-\gamma} \Rightarrow P_C = 10^{1-\gamma}\text{ atm.}$

For monoatomic $\gamma = 5/3 \Rightarrow P_C^M = 10^{-2/3}\text{ atm.}$

For Diatomic $\gamma = 7/5 \Rightarrow P_C^D = 10^{-2/5}\text{ atm.}$

$P_C^D > P_C^M.$

Hence Dashed line for Diatomic.

In each case since the curve for compression is above the curve for expansion, net work is done on the system.

As P_C (monoatomic) $<$ P_C (diatomic) work on the monoatomic gas is more than on the diatomic gas.

6 Incomplete question given!!

$$\left.\frac{\partial p}{\partial v}\right)_T = -\frac{\kappa_T^{-1}}{v} = \frac{2a}{v^4} - \frac{RT}{(v-b)^3}$$

$$p(v,T) = 2a \int dv\, v^{-4} - RT \int dv\, (v-b)^{-3} + f(T).$$

$$= 2a \frac{v^{-4+1}}{-4+1} - RT \frac{(v-b)^{-3+1}}{-3+1} + f(T).$$

$$= -\frac{2a}{3} \frac{1}{v^3} + \frac{RT}{2} \frac{1}{(v-b)^2} + f(T).$$

$$\left.\frac{\partial v}{\partial T}\right)_p = ?$$

$$0 = -\frac{2a}{3} \frac{\partial}{\partial T}\left(\frac{1}{v^3}\right) + \frac{\partial}{\partial T}\left[\frac{RT}{2} \frac{1}{(v-b)^2}\right] + \underbrace{\left.\frac{\partial f}{\partial T}\right)_p}_{\equiv \frac{df}{dT}}$$

$$= -\frac{2a}{3}(-3)\, v^{-3-1} \left.\frac{\partial v}{\partial T}\right)_p + \frac{R}{2} \frac{1}{(v-b)^2} + \frac{RT}{2} \frac{\partial}{\partial T}\left(\frac{1}{v-b}\right)^2 + \frac{df}{dT}$$

$$= \boxed{\frac{2a}{v^4} \left.\frac{\partial v}{\partial T}\right)_p} + \frac{R}{2} \frac{1}{(v-b)^2} + \boxed{\frac{RT}{2}(-2)(v-b)^{-2-1} \left.\frac{\partial v}{\partial T}\right)_p} + \frac{df}{dT}$$

$$= \left[\frac{2a}{v^4} - \frac{RT}{(v-b)^3}\right] \left.\frac{\partial v}{\partial T}\right)_p + \frac{R}{2} \frac{1}{(v-b)^2} + \frac{df}{dT}$$

$$\frac{df}{dT} + \frac{R}{2} \frac{1}{(v-b)^2} = \frac{v}{v}\left[\frac{RT}{(v-b)^3} - \frac{2a}{v^4}\right] \left.\frac{\partial v}{\partial T}\right)_p$$

$$= \underbrace{\left[\frac{RTv}{(v-b)^3} - \frac{2a}{v^3}\right]}_{\kappa_T^{-1}} \underbrace{\frac{1}{v} \left.\frac{\partial v}{\partial T}\right)_p}_{\alpha_p}$$

$$= \alpha_p \kappa_T^{-1} = \frac{R}{(v-b)} \cdot \kappa_T \kappa_T^{-1} = \frac{R}{(v-b)}$$

$$\frac{df}{dT} = \frac{R}{(v-b)} - \frac{R}{2} \frac{1}{(v-b)^2} \Rightarrow f(T) = \frac{RT}{(v-b)} - \frac{RT}{2} \frac{1}{(v-b)^2}.$$

$$\Rightarrow \cancel{p =}\ p(v,T) = \frac{RT}{2}\frac{1}{(v-b)^2} - \frac{2a}{3}\frac{1}{v^3} + \frac{RT}{(v-b)} - \frac{RT}{2}\frac{1}{(v-b)^2}$$

$$\boxed{p(v,T) = \frac{RT}{(v-b)} - \frac{2a}{3}\frac{1}{v^3}.}$$

When you press the ball down, the volume decreases. Since the container is insulated, the process is adiabatic. So that we have

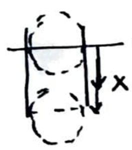
$$pV^{\gamma} = \text{Const.}$$

$$\Rightarrow \ln p + \gamma \ln V = \text{const.}$$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0.$$

$$\Rightarrow dp = -\gamma \frac{p}{V} dV$$

So any change in volume results in a pressure change according to the above equation. Now force on the ball is $F = A\,dp$.



$$\therefore F = -\gamma A \frac{p}{V} dV$$

Now $dV = A \cancel{dx}\, x$ $\Rightarrow$ $F = -\frac{\gamma A^2 p}{V} \cancel{dx}\, x$

$$\therefore K = \frac{\gamma A^2 p}{V} \qquad \omega^2 = \frac{K}{M} = \frac{\gamma A^2 p}{MV}$$

$$\omega^2 = \frac{\gamma A^2 (p_0 + Mg/A)}{MV_0}$$

atmospheric pressure.

$$p = p_0 + \frac{Mg}{A}$$

$V = V_0$ is the original volume of the gas.

$$\omega = \sqrt{\frac{\gamma A^2 (p_0 + Mg/A)}{MV_0}}.$$