IDC204 - Theory of Computation
1st Mid-semester examination

1st of February, 2023

Except for question 4, you MUST give complete, clear, and precise justifications for your answers. Answers without justificaion will **not** earn any credit.

1. Consider the Boolean function with two arguments, which outputs a True if *exactly* one of its arguments is True and outputs a False in all other cases.
   (a) (1 point) Write a truth table for this function
   (b) (2 points) Represent this function as an expression involving only the $\wedge$, $\vee$, and $\neg$ operators.

2. (3 points) Design a finite state automaton to recognize all strings over the alphabet $\Sigma = \{0, 1\}$ that end in 1.

3. (3 points) Define the language L over the alphabet $\Sigma := \{a, b, c\}$ as the set of those strings with fewer c's than b's or a's. Show that L cannot be regular.

4. Only write the correct answer for each of the following sub-questions. You need not give explanations.
   (a) (1 point) Express the NAND operator in terms of only $\neg$ and $\vee$ (note, you cannot use $\wedge$).
   (b) (1 point) How many *distinct* n argument boolean functions are there?
   (c) (1 point) Write the negation of $\forall x(P(x) \wedge Q(x))$.
   (d) (1 point) Is the expression "$\forall x(P(x) \wedge Q(x))$" a predicate, or a proposition, or neither?
   (e) (1 point) Consider a finite state automaton with only one state. List the languages over the alphabet $\{0, 1, 2\}$ that it can recognize.
   (f) (1 point) If a language L is finite and can be recognized by a finite state automaton with a *minimum* of N states, then can the infinite set $\Sigma^* \setminus L$ be recognized by a finite state automaton with only n states? If you feel that the language is not even regular, just write "not regular".

5. (2 points) If a language L is **not** regular, then is any superset of L, i.e. a language L' so that $L \subset L'$, also not regular? Give complete justifications, i.e. either prove that all supersets of a non-regular language are not regular or give an example to show that it is not true.

6. (3 points) Consider the language consisting of each string over the English alphabet that is **not** a 6 letter word in English. Is it possible to design a finite state automaton *with only 5 states* to recognize this language? Justify your answer. Only a clear and complete justification will earn credit.