PHY304: Statistical Mechanics

Assignment 9

April 08, 2025

1. Calculate the Fermi energy of N spin-$\frac{1}{2}$ particles in a one dimensional square well of length L.

2. Show that the asymptotic expansion for the internal energy of a spinless ideal Fermi gas is given by
$$U = \frac{3}{5} N \epsilon_F \left[1 + \frac{5}{12} \pi^2 \left(\frac{kT}{\epsilon_F} \right)^2 + \cdots \right]$$

3. The electrons in a metallic solid may be considered to be a three-dimensional free electron gas. For this case:

   (a) Obtain the allowed values of k, sketch the appropriate Fermi sphere in k-space (Use periodic boundary conditions with length L).

   (b) Obtain the maximum value of k for a system of N electrons, and hence an expression for the Fermi energy at $T = 0$K.

   (c) Use a simple argument to show that the electronic contribution to the specific heat at constant volume is proportional to the temperature T.

4. The wave vector corresponding to the Fermi energy is called the Fermi wave vector k_F. Calculate k_F for one-, two- and three-dimensional systems.

5. Consider a relativistic gas of N particles of spin 1/2 obeying Fermi statistics, enclosed in volume V, at absolute zero. The energy-momentum relation is $E = \sqrt{(pc)^2 + (mc^2)^2}$, where m is the rest mass.

   (a) Find the Fermi energy at density n.

   (b) Define the internal energy U as the average of $E - mc^2$, and the pressure P as the average force per unit area exerted on a perfectly-reflecting wall of the container. Set up expressions for these quantities in the form of integrals, but you need not evaluate them.

   (c) Show that $PV = 2U/3$ at low densities, and $PV = U/3$ at high densities, State the criteria for low and high densities.