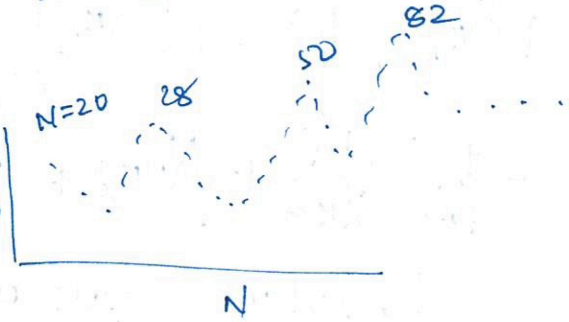
2, 8, 20, 28

①

ΔR

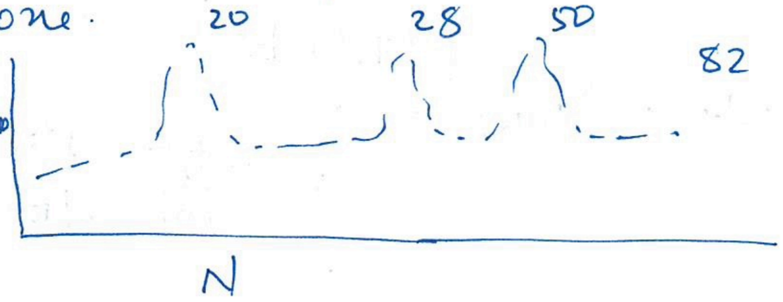
② (ΔR) ~~expt~~
(ΔR) calcula. } $\Delta R \rightarrow 2N$.



③ Number of stable isotone.

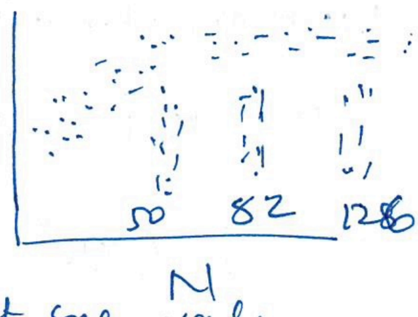
same proton → isotope
incr N

same neutron → isotone. isotone
incr. P



④

Probability of absorption
φ



1 MeV energy

Discontinu at one vale.

$N = 2, 8, 20, 28, 50, 82, 126$ Magic Number

How you understand this.

Let's understand this through the potential for single partcl

→ whole atom give a potential is given to each nucleon

→ Nuclei provides some kind poten.
↓ solve the energy level for particle potentia

what is the start point?
↳ what kind of potential has to be taken.

① ↓ infinite square-well potential. — Try-1.

$$V(r) = 0 \quad r < r_0$$
$$= \infty \quad r > r_0$$
} infin squarewell potn.

↳ central potential
it doesn't depend on θ & ϕ → no direc depend

→ wave fnc depend on θ, ϕ

$$\phi(r, \theta, \phi) = \left[\frac{u(r)}{r}\right] Y_l^m(\theta, \phi)$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2mr^2}\right]u(r) = Eu$$

↳ solve you get eign func & put in $\phi(r, \theta, \phi)$

$r < r_0$: $V(r) = 0$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{l(l+1)\hbar^2}{2mr^2}u = Eu.$$

solutn with boundary conditn will get a Bessel fundn.

$u = r\, j_l(kr)$ — $r = 0$ ~~should~~ u should be zero

$r = 0$ $\quad j_l(kr) = 0 \quad k = \sqrt{\frac{2mE}{\hbar^2}}$

if you know k you know E.

m is nuclear potential

Bessel fun differ ~~fund~~ ~~general~~ form for differ l. lu $l = 0$ sin fn

$l = 0$, $n = 1, 2, 3, 4 \ldots$

$l = 1$, $n = 1, 2, 3, 4 \ldots$

$l = 2$, $n = 1, 2, 3, 4 \ldots$

lets see the order of eng. Symbols are Sn.

s - sharp	p → principal	d - diffuse	f - fine	g	h
2	6	10	14	18	22
$n=1$	2	3	4	5	6.

1g (22)

2p (6)

4f (18) — (34) — no of magic no.

2s (2)

1d (10) } 20

1p (6) — (8) $l = 1$ $n = 1$

1s (2) — (2) $l = 0$, $n = 1$

3s (2)

1h (22)

2d (10)

atomic ~~sta~~ phy 1p is not 1.

Not working : second harmonic potential:

$V(r) = \frac{1}{2} m \omega^2 r^2$

$$\phi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$$

$$-\frac{\hbar^2}{2m}\frac{d^2 u}{dr^2} + \left[\frac{1}{2} m\omega^2 r^2 + \frac{l(l+1)\hbar^2}{2mr^2}\right] u = E u$$

↳ This is to be solved using boundy cn.

should not be

$$E = (2n' + l + 3/2)\hbar\omega$$

Energy depend on n & l.

$n' = \cancel{\quad} n-1$

$n' = 0, 1, 2, 3, 4$ ✓

$l = 0, 1, 2, 3$ ✓

1s

$$E = \left(N + \frac{3}{2}\right)\hbar\omega.$$

$\boxed{n = 1, 2, 3}$

$\boxed{N = 2n' + l}$

$N = 0, 1, 2, 3, \ldots$

$N = 0$, $3/2$, $n = 0$, $l = 0$. 1s.

$N = 1$ $\frac{5}{2}\hbar\omega$, $n = 0$, $l = 1$

$N = 2$, $\frac{7}{2}\hbar\omega$, $n = 1$, $l = 0$ 2s ; $n = 0$, $l = 2$ 1 1d d

$N = 3$, $\frac{9}{2}\hbar\omega$, $n = 1$, $l = 1$ 2p. ; $n = 0$ $l = 3$ ↳ 1f

$N = 4$, $\frac{11}{2}\hbar\omega$ $n = 2$, $l = 0$ 2g ; $n = 1$ $l = 2$ 3s ; $n = 0$ $l = 4$ ↳ 1g

3p 2f 1h ⑥ + ⑭ + ㉒ ㊷

3s 2d 1g ② + ⑩ + ⑱ = 30

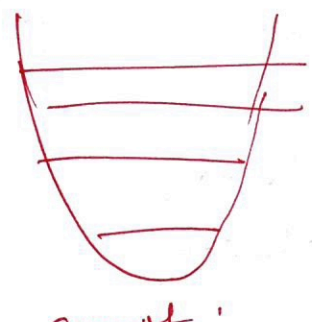
1f, 2p ⑭ + ⑥ = 20

1d 2s ② + ⑩

1p ⑥ ⑧

$\} \hbar\omega$

1s ② ②

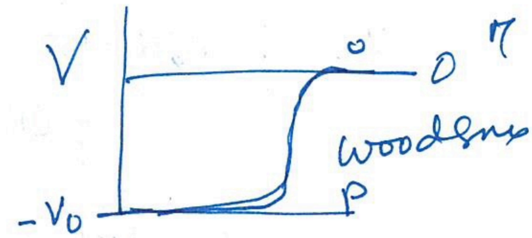


Cannot.

Finite potentn. NO - infinite potential

$$V = \frac{V_0}{1+e^{(r-R)/a}}$$

$R = r_0 A^{1/3}$ $a = 0.52$ fm.



Tried $V_0 = 50$ MeV

but it didnot work

spin - orbit interaction

$f(r) \cdot \vec{l} \cdot \vec{s}$ felt by one portn.

Fine-structures

⇓ H- α lines are several lns.

$H = H_0 + f(r)\,\vec{l}\cdot\vec{s}$

$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$ → Central potn.

↳ commutes with L^2, L_z, S^2, S_z

↳ you can have definite L^2, L_z, S^2, S_z values in energy eigen functn.

you can have one single unique value, for which it commut.

But This H doesnot comute with L_z & S_z

does commutes with L^2, S^2

However J^2 & J_z it cmmutes.

$\vec{J} = \vec{L} + \vec{S}$

↙

spin angular mmt couplg.

$\boxed{J_x = L_x + S_x}$

$\boxed{J_z = l_z + S_z}$

⇓

p, d, f stats are there difu. we can't com

m_l or m_s

you can. ↓ ↑↓

$l = m_l, 0$ we can't talk

mlcm of all abou spin lep.

What is the value ③

Total angular momentum of that one nucleus.

J^2 : $j(j+1)\hbar^2$

Z-component → J_z $m_j\hbar$: $m_j : +j, -j$ in steps of 1

↓ This can have different eigen values.

What is the change in the energy.

$m_j = (m_l + m_s)$ → mixed.

l, s, j, m_j are all definite

m will be then often.

$$\langle \vec{l}\cdot\vec{s} \rangle = \left\langle \frac{J^2 - L^2 - S^2}{2} \right\rangle \qquad \left[j(j+1) - l(l+1) - s(s+1) \right]\frac{\hbar^2}{2}$$

$\vec{J} = \vec{L} + \vec{s}$ $\qquad \vec{s} = \frac{1}{2}$ is always 1/2

if $l = 0$: $\quad j = \frac{1}{2}$

$l \neq 0 \quad j = l + \frac{1}{2}$ or $l - \frac{1}{2}$

If $J = l + \frac{1}{2}$ $\quad \langle \vec{l}\cdot\vec{s} \rangle = \dfrac{\left(l+\frac{1}{2}\right)\left(l+\frac{3}{2}\right)\hbar^2 - l(l+1)\hbar^2 - \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2}{2}$

$$= \frac{\hbar^2}{2}\left[l^2 + 2l + \frac{3}{4} - l^2 - l - \frac{3}{4} \right]$$

$$= \frac{\hbar^2}{2} l \quad \downarrow$$

$j = l - \frac{1}{2}$; $\quad \langle \vec{l}\cdot\vec{s} \rangle = \dfrac{\left(l - \frac{1}{2}\right)\left(l+\frac{1}{2}\right)\hbar^2 - l(l+1)\hbar^2 - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2}$

$$= \left[l^2 - \frac{1}{2} - l^2 - l - \frac{3}{4} \right]\frac{\hbar^2}{2}$$

$$= -\frac{\hbar^2}{2}(l+1) \quad \uparrow$$

Splitting $= \frac{\hbar^2}{2}(2l+1)$ $\qquad$ larger is l effect is less f(r) gives a negative sign.

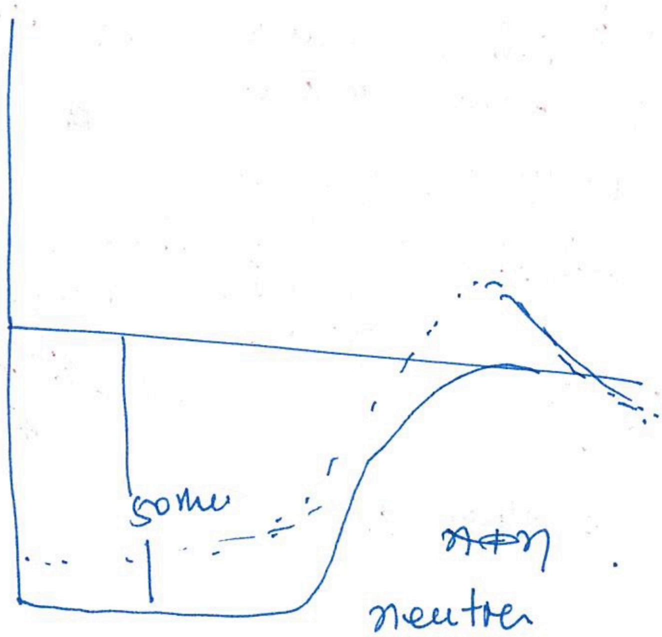
$J = l + 1/2$ energy will go down.

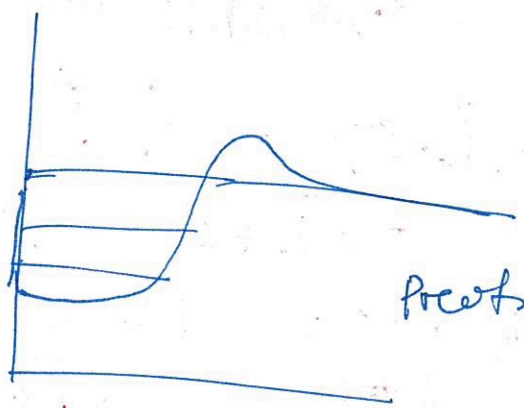
$J = l - 1/2$ energy will go up.

$\frac{1}{R}$

$$V_c(r) = \frac{Ze^2}{4\pi\epsilon_0 R}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right]$$

coulomb potential





N > Z

Spin - Parity.

spin of a nuclear → total angular momentum

J^{π} — π → parity, J → spin.

l is even π is +ve.

l is odd π is -ve

even - even nuclei.

Z = even 3 zero
N = even 3 zero } Pair is zero.

$J = 0$ $\quad J^{[(-1)^l]}$

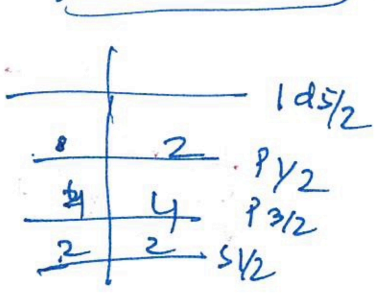
parity will be +ve,

$\boxed{\text{odd - A}}$

either Z is odd
or N is odd

extreme single particle.

$^{15}_{7}N_{8}$

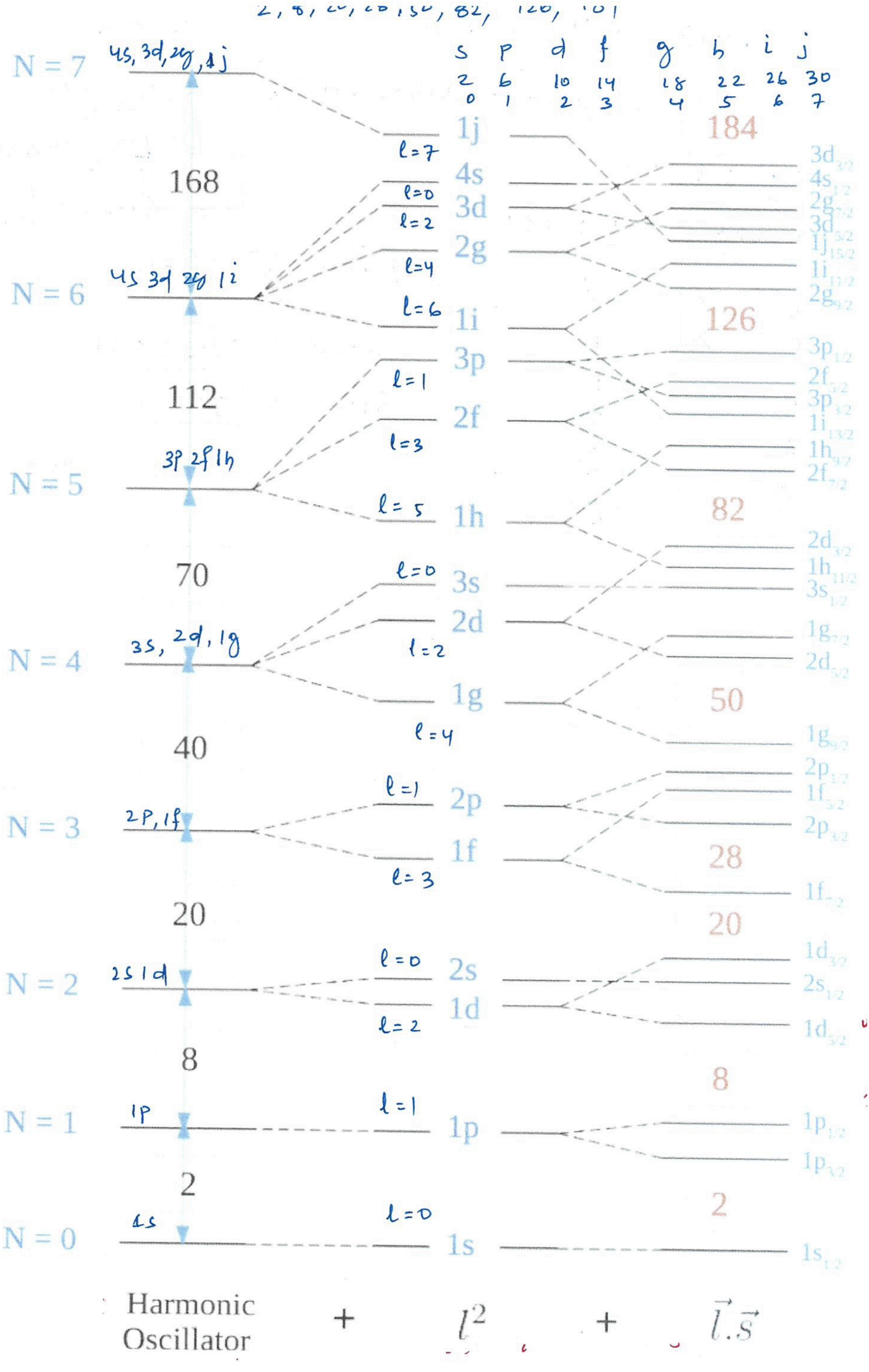


$\boxed{\begin{matrix} N & Z \\ \text{odd} & \text{odd} \end{matrix}}$ $^{14}_{7}N_{7}$, $^{2}_{1}D_{1}$

l = 1, parity -

$\bar{1/2}$

l = even +ve



N = 7
4s, 3d, 2g, 1j
168
N = 6
4s 3d 2g 1i
112
N = 5
3p 2f 1h
70
N = 4
3s, 2d, 1g
40
N = 3
2p, 1f
20
N = 2
2s 1d
8
N = 1
1p
2
N = 0
1s
Harmonic Oscillator
+
l^2
+
$\vec{l}.\vec{s}$
s p d f g h i j
2 6 10 14 18 22 26 30
0 1 2 3 4 5 6 7
1j
$l=7$
4s
$l=0$
3d
$l=2$
2g
$l=4$
1i
$l=6$
3p
$l=1$
2f
$l=3$
1h
$l=5$
3s
$l=0$
2d
$l=2$
1g
$l=4$
2p
$l=1$
1f
$l=3$
2s
$l=0$
1d
$l=2$
1p
$l=1$
1s
$l=0$
184
$3d_{3/2}$
$4s_{1/2}$
$2g_{7/2}$
$3d_{5/2}$
$1j_{15/2}$
$1i_{11/2}$
$2g_{9/2}$
126
$3p_{1/2}$
$2f_{5/2}$
$3p_{3/2}$
$1i_{13/2}$
$1h_{9/2}$
$2f_{7/2}$
82
$2d_{3/2}$
$1h_{11/2}$
$3s_{1/2}$
$1g_{7/2}$
$2d_{5/2}$
50
$1g_{9/2}$
$2p_{1/2}$
$1f_{5/2}$
$2p_{3/2}$
28
$1f_{7/2}$
20
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
8
$1p_{1/2}$
$1p_{3/2}$
2
$1s_{1/2}$

$$\langle l\cdot s\rangle = \left\langle \frac{j^2 - l^2 - s^2}{2} \right\rangle$$

$$= \left[j(j+1) - l(l+1) - s(s+1)\right]\frac{\hbar^2}{2}$$

$$\left[\begin{array}{l} \vec{j} = \vec{l}\cdot\vec{s} \\ J^2 = [L+s]^2 \\ \quad = \underline{L}^2 + \underline{s}^2 + \underline{2\, l\cdot s} \end{array}\right.$$

$$\Rightarrow \langle l\cdot s\rangle = \frac{1}{2}\left[J^2 - L^2 - s^2\right]$$

$j = \frac{1}{2}$; $l = 0$.

$j = l + \frac{1}{2}$, $l - \frac{1}{2}$; $l > 0$

l	s	j
0	1/2	1/2
1		1/2
		3/2
2		3/2
		5/2
3		5/2
		7/2

splitting $\frac{\hbar^2}{2}(2l+1)$

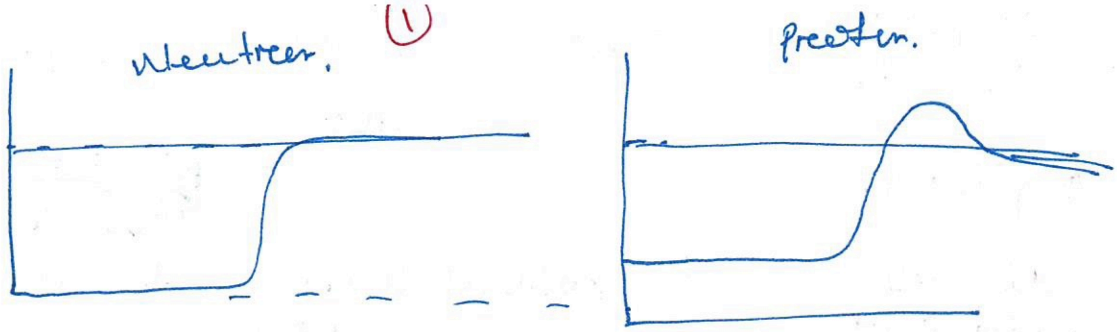
$\langle l\cdot s\rangle = -\frac{\hbar^2}{2}(l+1)$ $\quad j = l - \frac{1}{2}$

$\langle l\cdot s\rangle = \frac{\hbar^2}{2} l$ $\quad j = l + \frac{1}{2}$

m_l & m_s are not required

23/9/2024

(1)



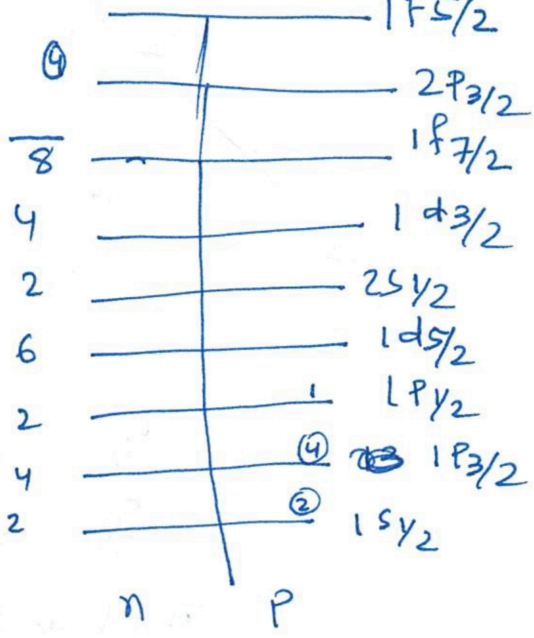
Spin & Parity.

J^{π} — l is even Parity +ve, $\pi = (-1)^{l}$.

l is odd parity −ve

<u>even-even nuclei.</u> A ev

A even
Z even
n even
} Zero, $\pi = +$
$J = 0^{+}$



<u>odd-A.</u> odd nuclei.

${}^{15}_{7}N_{8}$, ${}^{13}_{7}N_{6}$, $\boxed{{}^{14}_{7}N_{7}}$

What are spin & parity.

${}^{15}_{7}N_{8} = \left(\frac{1}{2}\right)^{-}$ ${}^{15}_{8}O_{7} = \left(\frac{1}{2}\right)^{-}$

${}^{15}_{7}N_{6} = \left(\frac{1}{2}\right)^{-}$

$\cancel{{}^{14}_{7}N_{7}}$

Exception. ${}^{61}_{28}Ni_{33}$ $J^{\pi} = (5/2)^{-}$

observed value is $(3/2)^{-}$ }

if ~~nn~~ 1f5/2, nnn 3/2

excited state.

${}^{17}_{8}O_{9}$ $\left(\frac{1}{2}\right)^{-}$, $\left(\frac{1}{2}\right)^{+}$, $\left(\frac{5}{2}\right)^{+}$

$1/2^{-}$

$1/2^{+}$ 1st excited.

$5/2^{+}$ ground

2nd e. nn $d_{5/2}$

n $p_{1/2}$

5/2, nnn d5/2, p1/2, (6) p3/2

Even-even
Z N } $J = 0$ any of of Z-projection also zero

Any level.

PP nn
addin to zero

if paired then. $J_1 \quad J_2 \rightarrow$ $J_1 + J_2 = 0$
they chosed to be $\rightarrow 0$ at 0.

$P_{3/2}$ P P

$\frac{3}{2} \oplus \frac{3}{2} \rightarrow 3, 2, 1, 0$ in general

That shell model will predict even-even nuclei.

$\langle \mu_z \rangle = 0$ at ground state.

Prediction & Expt.

odd-A

$$\mu_z = \left(g_\ell \cdot l_z + g_s S_z\right) \frac{\mu_n}{\hbar}$$ operator.

μ_N = nuclear magneton

$\hookrightarrow \left(\frac{e\hbar}{2m_p}\right)$ unit

[atom mostly come from electron we neglect them.

		Proton	neutron
g_ℓ	=	1	5.586.
g_s	=	0	-3.826

called g-factor.

For nucleon. each nucleon.
$\vec{l} + \vec{s} = \vec{J}$ Z-component of l_z & S_z are not defing but $J_z = L_z + S_z$ is defined. $m_j = m_\ell + m_s$

~~not~~ $J_z = L_z + S_z$
$m_j = m_\ell + m_s$

l, s, j, m_j ($\frac{1}{2}$)

$S_z = \pm\frac{1}{2}$
$l = +l \ldots -l$

highest value of m_j are left for n.

$$\langle \mu_z \rangle = \langle j, m_j = j | \left(g_\ell l_z + g_s S_z\right) \frac{\mu_N}{\hbar} | j, m_j = 1 \rangle$$

L projection is not ~~not~~

$S_z = m_s \hbar$

$J_z = m_j \hbar$

$j = l + \frac{1}{2} <^{p}_{n}$

$j = l - \frac{1}{2} <^{p}_{n}$

$j = l + \frac{1}{2}$

$l + \frac{1}{2} = m_\ell + m_s \longrightarrow \frac{1}{2}, -\frac{1}{2}$

$m_\ell = -l, -(l-1) \ldots l-1, l$ $\quad l, l-1, l-2, -l$

23/9/24 ② $m_l = l$ & $m_s = \frac{1}{2}$

$\langle L_z \rangle = l\hbar$ — $\langle S_z \rangle = \frac{1}{2}\hbar$ —

$$\left.\begin{aligned}\langle \mu_z \rangle &= \left(g_l l\hbar + g_s \frac{\hbar}{2}\right) \cdot \frac{\mu_N}{\hbar} \\ &= \left(g_l l + \frac{g_s}{2}\right) \mu_N\end{aligned}\right\}$$ Calculate for p & N

$$\boxed{p \div \left(l + \frac{5.586}{2}\right) \mu_N}$$

$\langle \mu_z \rangle_N = -\frac{3.8260}{2} \mu_N$

$= -1.9130\ \mu_N$

$j = l - \frac{1}{2}$ $m_j = j = l - \frac{1}{2}$.

$m_l + m_s = l - \frac{1}{2}$

$m_j = j = l - \frac{1}{2}$ m_l $m_s = -\frac{1}{2}$

$m_l + m_s = l - \frac{1}{2}$

l $\frac{1}{2}$

$l-1$

$l-2$ $-\frac{1}{2}$.

$-l$

$$\left.\begin{aligned} m_l &= \tfrac{1}{2} & m_s &= -\tfrac{1}{2} \\ m_l &= l-1 & m_s &= +\tfrac{1}{2}\end{aligned}\right\}$$ They will mixed up.

$|j, m_j = j\rangle = a\,|m_l = l, m_s = -\frac{1}{2}\rangle \to \psi_1$

$+ b\,|m_l = l-1, m_s = \frac{1}{2}\rangle \to \psi_2$

$= a\psi_1 + b\psi_2$

$\langle a\psi_1 + b\psi_2 | \mu_z | a\psi_1 + b\psi_2 \rangle$

$= \langle a\psi_1 | \mu_z | a\psi_1 \rangle + \langle b\psi_2 | \mu_z | b\psi_2 \rangle$

$+ \underbrace{\langle a\psi_1 | \mu_z | b\psi_2 \rangle}_{\text{zero}} + \underbrace{\langle b\psi_2 | \mu_z | a\psi_1 \rangle}_{\text{zero.}}$

$= |a|^2 \langle \psi_1 | \mu_z | \psi_2 \rangle + |b|^2 \langle \psi_2 | \mu_z | \psi_2 \rangle$

$$\langle \psi_1 | \mu_z | \psi_1 \rangle = \langle m_l = l, m_s = -\tfrac{1}{2} | g_l l_z + g_s S_z | m_l = l, m_s = -\tfrac{1}{2} \rangle$$

$$\langle m_l = l, m_s = -\tfrac{1}{2} | g_l l \hbar | m_l = l, m_s = -\tfrac{1}{2} \rangle +$$

$$\langle m_l = l, m_s = -\tfrac{1}{2} | g_s (-\tfrac{\hbar}{2}) | m_l = l, m_s = -\tfrac{1}{2} \rangle$$

$$\langle \psi_1 | \mu_z | \psi_1 \rangle = g_l l \hbar - g_s \frac{\hbar}{2}$$

$$\langle \psi_2 | \mu_z | \psi_2 \rangle = \langle m_l = l-1, m_s = \tfrac{1}{2} | g_l l_z + g_s S_z | m_l = l-1, m_s = \tfrac{1}{2} \rangle$$

$$= g_l (l-1) \hbar + g_s \frac{\hbar}{2}$$

~~$|d, m_j\rangle =$~~ $\therefore \psi = a\psi_1 + b\psi_2$

$$a + b\sqrt{2l} = 0, \quad a = -b\sqrt{2l}.$$

$$|a|^2 = |b|^2 \, 2l = \frac{2l}{2l+1}$$

N $\langle \mu_z \rangle$ ~~$= -1.913\, \mu_N$~~ $\left(\frac{j}{j+1}\right) 1.9130\, \mu_N$

P $\langle \mu_z \rangle = \left(\frac{j}{j+1}\right) [\, j - 1.2928\,]\, \mu_N$