IISER Mohali

[August 2024 Session]

PHY 403 (Atomic and molecular physics)

Exercises

Instructor: Ambresh Shivaji (**email**: ashivaji)

2024-08-19 00:08:58+05:30

Review of H-atom and one electron atoms

1. In classical mechanics, for $\frac{1}{r}$ potential, apart from orbital angular momentum ($\vec{L}$), there exists another conserved quantity known as Laplace-Runge-Lunz vector,

$$\vec{N} = \frac{\vec{p} \times \vec{L}}{m} - \frac{c}{r}\vec{r} \tag{1}$$

where c is a dimensionful constant. Argue that the correct generalization of this quantity to quantum mechanics is given by the operator,

$$\vec{N} = \frac{1}{2m}(\vec{p} \times \vec{L} - \vec{L} \times \vec{p}) - \frac{c}{r}\vec{r} \tag{2}$$

Further, show that it commutes with the Hamiltonian for the H-atom. Take $c = \frac{e^2}{4\pi\epsilon_0}$.

2. Show that, for a general eigenstate $\psi_{n\ell m}$, the expectation values of kinetic energy T and potential energy V, satisfy

$$2\langle T \rangle = -\langle V \rangle . \tag{3}$$

3. Compare the expectation value of the radial coordinate and the most probable radial location of the electron in the ground state of the H-atom.

4. Estimate the binding energy of a pionic atom (a system of π^- and nucleus).

5. Show that the parity of the eigenstate $\psi_{n\ell m}$ is determined by $(-1)^\ell$.

6. Go through the Appendix 7 of B & J for the non-relativistic limit of the Dirac equation which leads to relativistic corrections, H_1', H_2' and H_3' discussed in the class. [**This exercise is optional. Any difficulty in following the derivation can be discussed outside the regular class schedule.**]