PHY 635: Gravitation & Cosmology : Problem Sheet 5

Submit before 23 April 2025

1. In the Kruskal double null co-ordinate system, identify the points $r = 0$ and $r = 2GM/c^2$. Inside the horizon of a black hole, show that the constant spatial co-ordinate trajectories will also meet the singularity at $r = 0$ in finite proper time.

2. A neutrino is expected to not interact with matter and only move along geodesics provided by the geometry. Inside a star of uniform density, governed by the TOV equation, write down the trajectory of neutrino with energy and angular momentum parmeters E and L respectively, and identify the effective potential it experiences.

3. For a two fluid model of a star, where constant density ρ fluid occupies interior core $r < R_c$ and the remaining portion $R_c < r < R_*$ is occupied by a second fluid of a different constant density ρ^*, develop the TOV equations. What is the solution for a shell (whose core is empty) ?

4. If the linear perturbation in a diffeomorphism along ζ_μ changes as $h_{\mu\nu} \to h_{\mu\nu} + 2\epsilon\partial_{(\mu}\zeta_{\nu)}$, prove that the linearized Riemann tensor remains invariant under the diffeomorphism. Obtain the equation of motion for $h_{\mu\nu}$.

5. Suppose initially in some co-ordinate system,

$$\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h = f_\mu.$$

Show that using the diffeomorphism we can **always** make a transformation such that in the new frame (gaugue)

$$\partial^\nu h'_{\mu\nu} - \frac{1}{2}\partial_\mu h' = 0.$$

Find out the condition on the diffeoemorphism generating vector ζ_μ for such a gauge choice.