

PHY304 - Statistical Mechanics

Spring 2021, IISER Mohali

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PHY304: Homework 2 Solutions

Due: Thursday, January 28, 2021 at 11:00pm.

(Upload your solutions to Moodle as a single .pdf file.)

1. For an electromagnetic cavity at temperature T we can write down the equations of state

$$U = bVT^4, \quad (1)$$

the so-called Stefan-Boltzmann law, and

$$p = \frac{U}{3V}. \quad (2)$$

In the above b is a dimensionfull constant. Note that these empirical equations of state are functions of U and V , but not of N . This is coming from the fact that the particle number (photons here) is not conserved in the system. The electromagnetic radiation in the cavity is governed by the fundamental relation of the form

$$S \equiv S(U, V), \quad (3)$$

with only two independent extensive variables.

Upon using the information provided above find the right hand side of the fundamental relation given in Eq. (3).

Solution:

1. We have the Euler relation for the system

$$S = \frac{1}{T}U + \frac{p}{V}V. \quad (4)$$

From the given equations of state

$$\frac{1}{T} = b^{1/4} \left(\frac{V}{U} \right)^{1/4}. \quad (5)$$

and

$$\frac{p}{T} = \frac{1}{3} b^{1/4} \left(\frac{U}{V} \right)^{3/4}. \quad (6)$$

Substituting the above two equations in the Euler equation we have

$$S = \frac{4}{3} b^{1/4} U^{3/4} V^{1/4}. \quad (7)$$

This is the required fundamental relation.

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