PHY403 Atomic & Molecular Physics Aug-Dec 2019: Assignment 1

1. Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$. Show that the Hamiltonian $H = (p^2/2m) + V$ commutes with all three components of angular momentum L, provided that V depends only on r.

2. Prove that for a particle in a potential $V(r)$, the rate of change of expectation of orbital angular momentum L is equal to the expectation value of the torque:
$$\frac{d}{dt}\langle L \rangle = \langle N \rangle$$
where torque $N = r \times (-\nabla V)$. Show that angular momentum is conserved i.e $d\langle L \rangle / dt = 0$ for any spherically symmetric potential.

3. Consider a 3D rigid quantum rotor: two particles each of mass m attached to the ends of a massless rigid rod of length a. The system is free to rotate in three dimensions about the center, but the center point is kept fixed. Find the allowed energies. What are the normalized eigenfunctions for this system? What is the degeneracy of the nth energy level. Before tackling this problem, refresh your intuition by solving the Schrodinger equation for a 2D rotor (two masses attached by a rod and free to rotate about the center of mass in their x-y plane).

4. A particle of mass m is placed in a finite spherical well
$$\begin{aligned} V(r) &= 0, \quad r \leq a \\ V(r) &= V_0, \quad r > a \end{aligned}$$
Find the ground state by solving the radial equation with $l = 0$. Find the condition when there will be no bound state.

5. Show that $\Theta(\theta) = A ln[\tan\theta/2]$ satisfies the θ equation of the angular part of the Schroedinger equation for $l = m = 0$. Why is it not an acceptable solution?

6. Find the momentum-space wave function $\phi(p)$ for the ground state of the hydrogen atom. Use spherical coordinates and set the polar axis along the direction of p. Use $\phi(p)$ to calculate $\langle p^2 \rangle$. What is the expectation value of the kinetic energy in this state?