Wireless Communications Seminar 04

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Contents

Review

Channel Capacity

- **Channel Capacity**
  - The **maximum data rates** that can be transmitted over the wireless channel with asymptotically small error probability

https://www.dictionary.com/browse/asymptotic

CHANNEL CAPACITY

- What does it mean?
  - **Channel Capacity** $(\boldsymbol{C})$ is the upper bound of Data Rate (R) that can be sent with negligible errors
  - In other words, if the transmission data rate (R) is higher than the Channel Capacity, the receiver cannot recover the message
  - We can think Channel as a Cup that can carry water

AWGN Channel Capacity

AWGN Channel Capacity
$C = W\log_2(1 + \frac{P}{WN_0})$ [bits/s]

FREQUENCY-FLAT/SLOW-FADING CHANNEL

Shannon Capacity
$C = W\log_2\left(1 + \frac{\lvert h\rvert^2 P}{WN_0}\right)$ [bits/s]

Frequency-Flat/Fast-Fading Channel

Ergodic Capacity
$C = \int_0^\infty W \log_2(1 + SNR) f(SNR) d_{SNR}$ [bits/s]

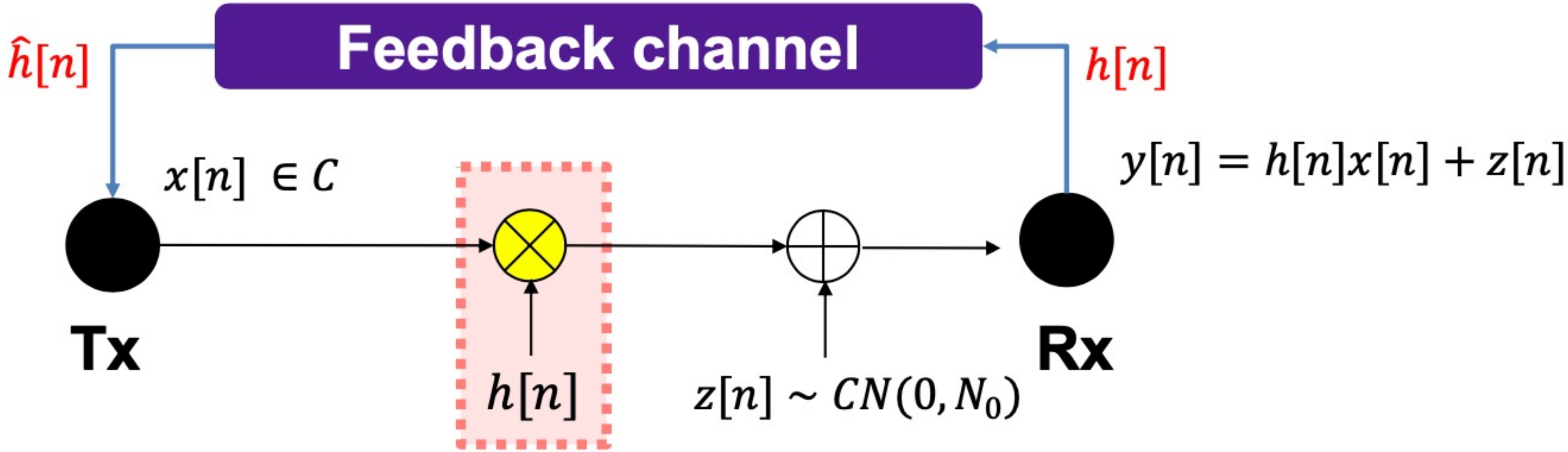
Outage Capacity

Outage Capacity
$C = (1 - P_{out}) W \log_2(1 + SNR_{min})$ [bits/s]

CASE III: FREQUENCY-FLAT/FAST FADING CHANNEL

Channel State Information (CSI) known at both Tx and RX

CSI AT BOTH TRANSMITTER AND RECEIVER



- Receiver always estimates the channel $h[n]$
- and sends back the signal $\hat{h}[n]$ to transmitter
- Assumptions
  - No Estimation & Feedback error

CSI AT BOTH TRANSMITTER AND RECEIVER

- Naïve Approach
  - Transmitter uses the same power $\bar{P}$ (average power) for every transmission, independent from CSI

naïve
〔형용사〕 소박한, 순진한, 천진 난만한, 고지식한 ; 우직(愚直)한.
2개 뜻 더보기 : 〔형용사〕 (전문적) 지식이 없는, 경험이 없는, 생무지의.

- Adaptive Power Control
  - For each transmission n, transmit a signal with power P_i according to h_i
  - Subject to "Average power constraint":

$$\sum P_i p(P_i) \underset{=}{\leq} \bar{P}$$

CSI AT BOTH TRANSMITTER AND RECEIVER

- Naïve Approach vs. Adaptive Power Control
  - Both approaches emply **SAME average power**
  - How much **GAIN** with adaptive power control?
  - How to **“ASSIGN A POWER”** for transmission?

NAÏVE APPROACH

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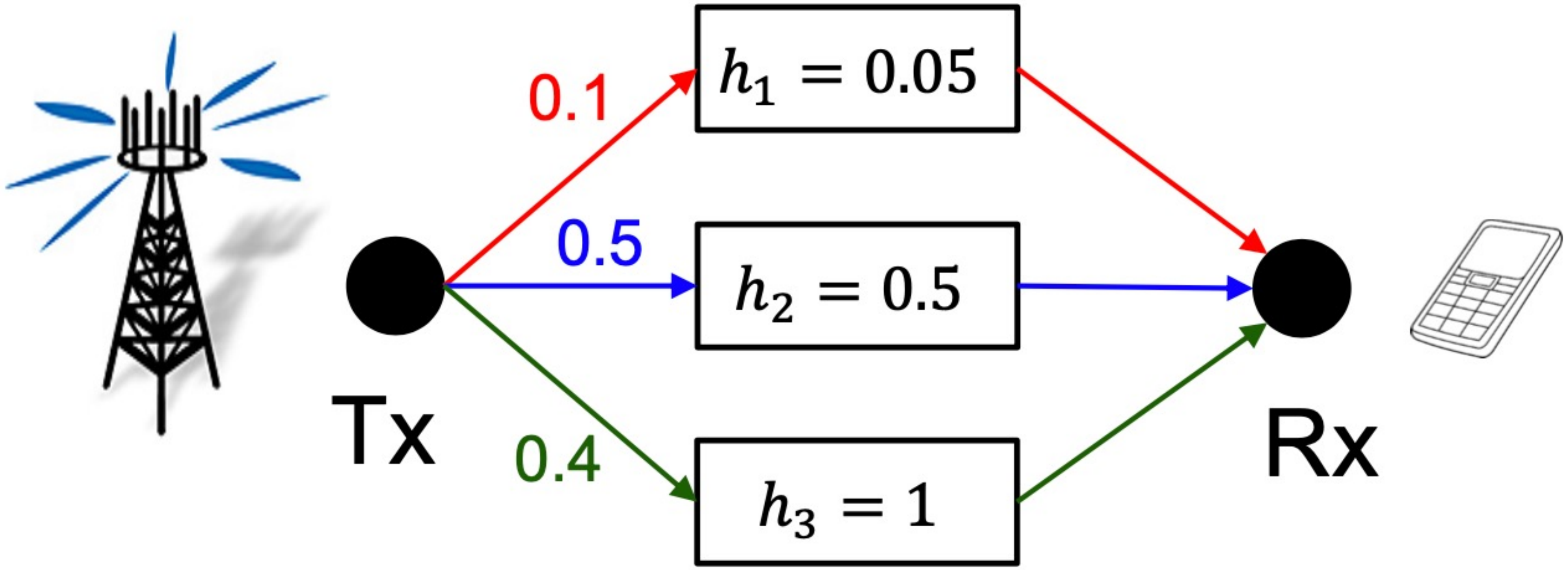
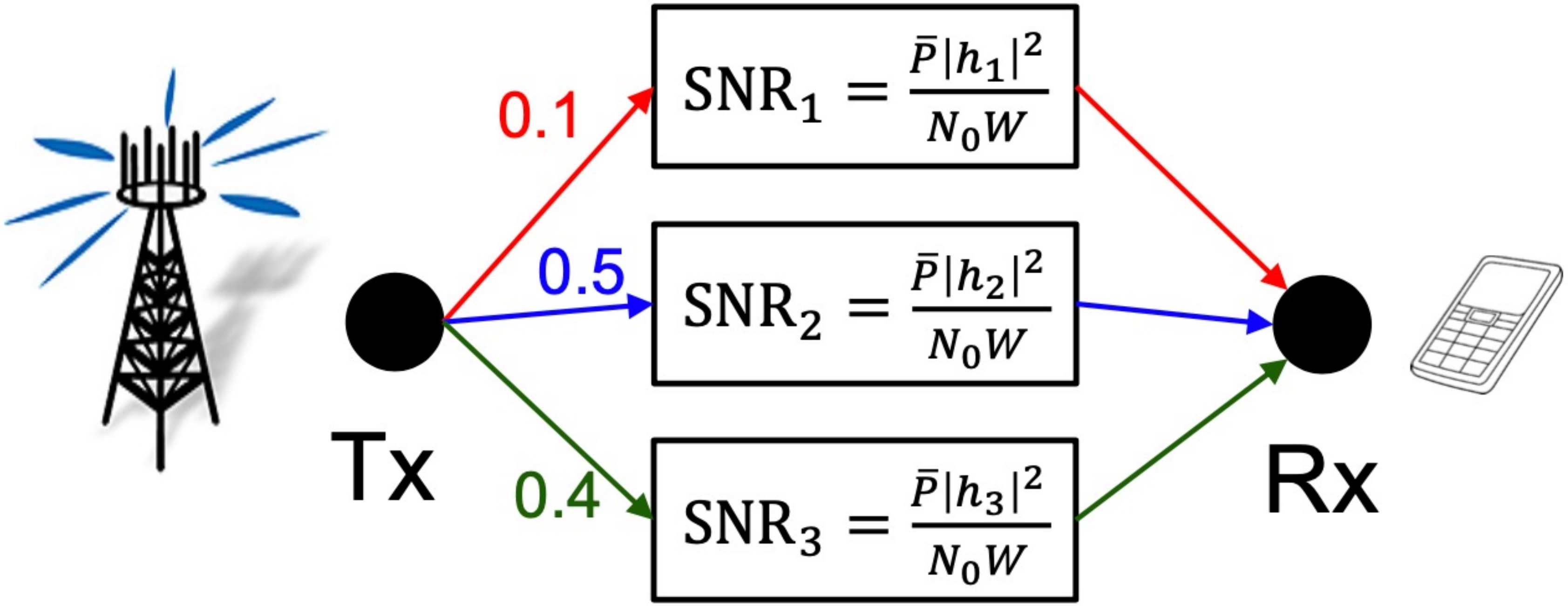
- Regardless of channels, transmitter uses the same transmit power $\bar{P}$



NAÏVE APPROACH

- Regardless of channels, transmitter uses the same transmit power $\bar{P}$



NAÏVE APPROACH

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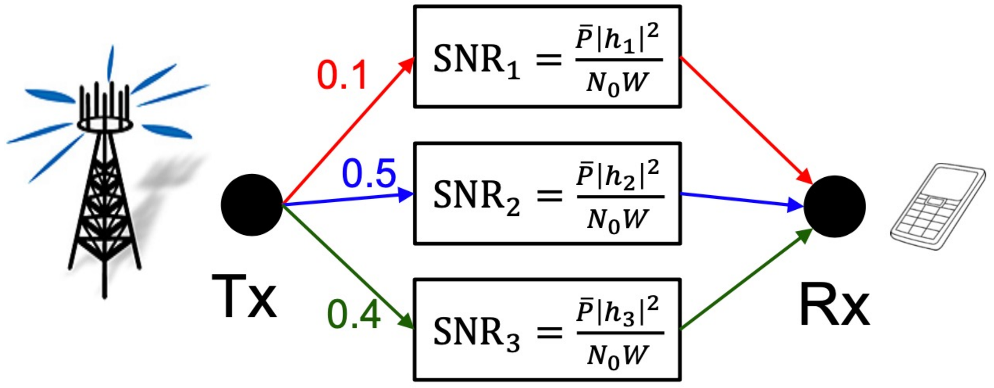
- Regardless of channels, transmitter uses the same transmit power $\bar{P}$



$$C = 0.1 \times \log_2(1 + SNR_1) + 0.5 \times \log_2(1 + SNR_2) + 0.4 \times \log_2(1 + SNR_3)$$

- In General

$$C = \int W \log_2(1 + SNR) P(SNR) d_{SNR}$$

Ergodic Capacity
$C = \int_0^\infty W \log_2(1 + SNR) f(SNR) d_{SNR}$ [bits/s]

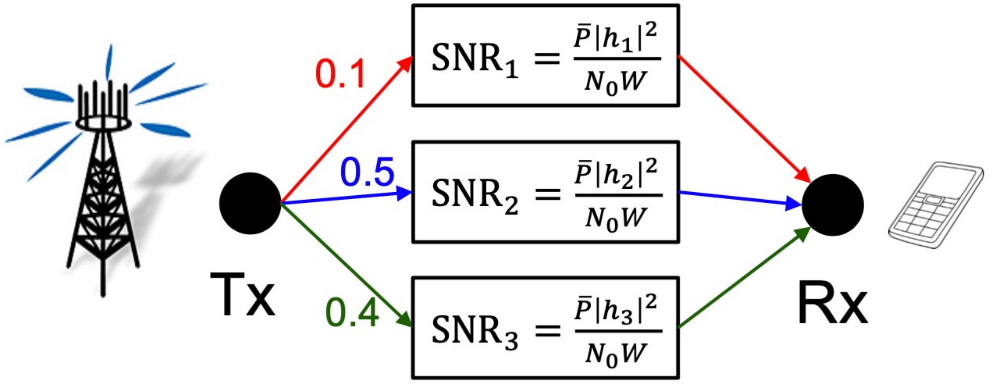
Naïve Approach

- Naïve Approach has no advantage over ergodic capacity

- As a result, We need to adapt transmission powers using the knowledge of the Channel Side Information (i.e., the channel loss of the time slot)

ADAPTIVE POWER CONTROL

- Tx can optimize a transmit power P_i according to h_i subject to average power constraint $\bar{P}$



- Average Power Constraint

$$0.1 \times P_1 + 0.5 \times P_2 + 0.3 \times P_3 = \bar{P}$$

Adaptive Power Control

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- Maximizing the Capacity (CSI at both Tx and Rx)

$$\max_{P_1,P_2,P_3} 0.1\times W\log_2\left(1+\frac{P_1|h_1|^2}{WN_0}\right)+0.5\times W\log_2\left(1+\frac{P_2|h_2|^2}{WN_0}\right)+0.4\times W\log_2\left(1+\frac{P_3|h_3|^2}{WN_0}\right)$$

$$\text{subject to,}\qquad 0.1\times P_1+0.5\times P_2+0.4\times P_3=\bar{\mathrm{P}}$$

- In General (Discrete Case)

$$C=\max_{P_i\ s.t.,i\in I}\sum_i p_i W\log_2\left(1+\frac{P_i|h_i|^2}{WN_0}\right)$$

$$\text{subject to,}\qquad \sum_i p_i\times P_i=\bar{P}$$

Change of Variables

$$\boldsymbol{Q_i}\leftarrow\boldsymbol{P_i}/\bar{\boldsymbol{P}}$$

$$C=\max_{Q_i\ s.t.,i\in I}\sum_i p_i W\log_2\left(1+\frac{\overset{\boldsymbol{P_i}}{\boxed{Q_i\bar{P}}}|h_i|^2}{WN_0}\right)$$

$$\text{subject to,}\qquad \sum_i p_i\times Q_i=1$$

ADAPTIVE POWER CONTROL

- Capacity (CSI at both transmitter and receiver)

$$C = \max_{Q_i\ s.t.,i\in I} \sum_i p_i W \log_2\left(1 + \frac{Q_i \bar{P}|h_i|^2}{W N_0}\right)$$

$$\text{subject to,} \quad \sum_i p_i \times Q_i = 1$$

- The Lagrangian Method

$$\boxed{\mathcal{L}(\mathbf{Q},\lambda)} \triangleq -\sum_i p_i W \log_2\left(1 + \frac{Q_i \bar{P}|h_i|^2}{W N_0}\right) + \lambda\left(\sum_i (p_i Q_i) - 1\right)$$

Always convex

Adaptive Power Control

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- Next we differentiate the function and set the derivative zero:

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\frac{p_i W}{\ln 2} \frac{\bar{P}|h_i|^2}{W N_0 + P_i |h_i|} + \lambda p_i = 0$$

$$\Leftrightarrow \quad \left(\lambda - \frac{W}{\ln 2} \frac{\bar{P}|h_i|^2}{W N_0 + P_i |h_i|}\right) p_i = 0$$

$$\Leftrightarrow \quad P_i = \left[\frac{W}{\ln 2} \frac{1}{\bar{P}\lambda} - \frac{W N_0}{\bar{P}|h_i|^2}\right]^+ \qquad (\because P_i \geq 0)$$

$$\Leftrightarrow \quad P_i = \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right]^+$$

Adaptive Power Control

- Finding the optimal transmit powers
  - Finding the cut-off SNR γ_0 to satisfy

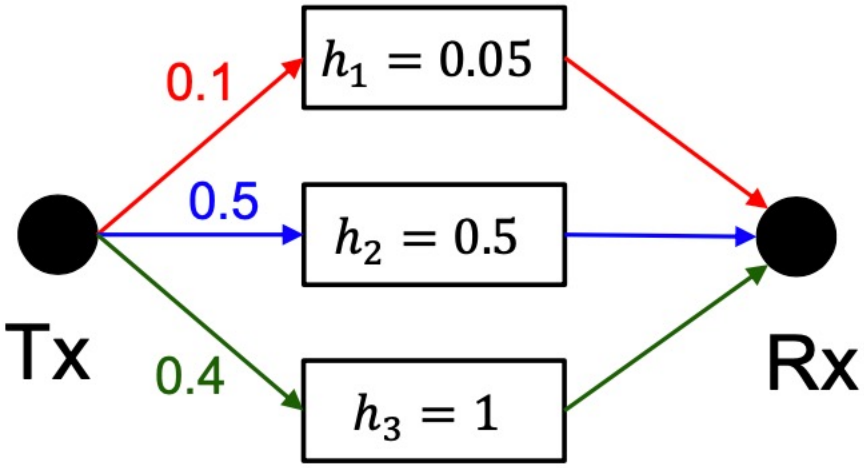
$$\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p_i = 1$$

GOOD Channel Condition
→ More power and a higher data rate are sent over the channel

Shannon Channel Capacity (Perfect CSI at Tx and Rx)
$C = \sum_{\gamma_i \geq \gamma_0} p_i W \log_2\left(\frac{\gamma_i}{\gamma_0}\right)$ [bits/s]

ADAPTIVE POWER CONTROL

Example



- $\bar{P} = 0.01W,\ N_0 = 10^{-9}W/Hz,\ W = 30kHz$
- What is the capacity when CSI is available at both transmitter and receiver?

ADAPTIVE POWER CONTROL

1) Find the cutoff SNR γ_0 s.t. $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p_i = 1$
   - $\gamma_1 = 0.8333,\ \gamma_2 = 83.333,\ \gamma_3 = 333.33$

   A. All Channel states are used ($\gamma_0 \leq \min_i \gamma_i$)

$$\sum_{i=1}^{3} \frac{p_i}{\gamma_0} - \sum_{i=1}^{3} \frac{p_i}{\gamma_i} = 1 \quad \rightarrow \quad \gamma_0 = \frac{1}{1 + \sum_{i=1}^{3} \frac{p_i}{\gamma_i}} = 0.885$$

ADAPTIVE POWER CONTROL

1) Find the cutoff SNR γ_0 s.t. $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p_i = 1$

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Adaptive Power Control

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Adaptive Power Control

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1) Find the cutoff SNR γ_0 s.t. $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p_i = 1$

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B. The weakest Channel is NOT Used ($\gamma_1 < \gamma_0 \leq \min\{\gamma_2, \gamma_3\}$)

$$\sum_{i=2}^{3} \frac{p_i}{\gamma_0} - \sum_{i=2}^{3} \frac{p_i}{\gamma_i} = 1 \quad \rightarrow \quad \gamma_0 = \frac{\sum_{i=2}^{3} p_i}{1 + \sum_{i=2}^{3} \frac{p_i}{\gamma_i}} = \mathbf{0.894}$$

Adaptive Power Control

1) Find the cutoff SNR γ_0 s.t. $\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p_i = 1$

- $\boldsymbol{\gamma_1 = 0.8333}$, $\boldsymbol{\gamma_2 = 83.333}$, $\gamma_3 = 333.33$

A. All Channel states are used ($\gamma_0 \leq \min_i \gamma_i$)

$$\sum_{i=1}^{3} \frac{p_i}{\gamma_0} - \sum_{i=1}^{3} \frac{p_i}{\gamma_i} = 1 \quad \rightarrow \quad \gamma_0 = \frac{1}{1 + \sum_{i=1}^{3} \frac{p_i}{\gamma_i}} = 0.885$$

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$$\sum_{i=2}^{3} \frac{p_i}{\gamma_0} - \sum_{i=2}^{3} \frac{p_i}{\gamma_i} = 1 \quad \rightarrow \quad \gamma_0 = \frac{\sum_{i=2}^{3} p_i}{1 + \sum_{i=2}^{3} \frac{p_i}{\gamma_i}} = \mathbf{0.894}$$

Adaptive Power Control

The weakest Channel is NOT Used

$$C = 30000 \times \left(0.5 \times \log_2\left(\frac{83.33}{0.89}\right) + 0.4 \times \log_2\left(\frac{333.33}{0.89}\right)\right)$$

$$= 200.82 \text{ kbps}$$

- Transmission Power?
  - $P_1 = 0$ (Not Transmitted)
  - $\boxed{P_2 = \bar{P} \times 1.11} \left(\because \log_2\left(1 + \frac{P_2|h_2|^2}{WN_0}\right) = \log_2\left(\frac{83.33}{0.89}\right)\right)$, $\boxed{P_3 = \bar{P} \times 1.12}$

ADAPTIVE POWER CONTROL

- Using the Optimal Power Control Method

Shannon Channel Capacity (Perfect CSI at Tx and Rx)
$C = \sum_{\gamma_i \geq \gamma_0} p_i W \log_2 \left(\frac{\gamma_i}{\gamma_0}\right)$ [bits/s]

Practical Issues remaining!

Channel Inversion

- h_i (Channel State) is known at the transmitter
- Approach that ensures the receiver signal power is constant as P

$$\boxed{P_i = \frac{P}{|h_i|^2} \quad \text{and} \quad \sum_i p_i \times P_i = \bar{P}}$$

- Advantage
  - Maintaining a **fixed transmission rate**
  - Enabling a **low-complexity hardware**

Channel Inversion

- Is there a channel outage?

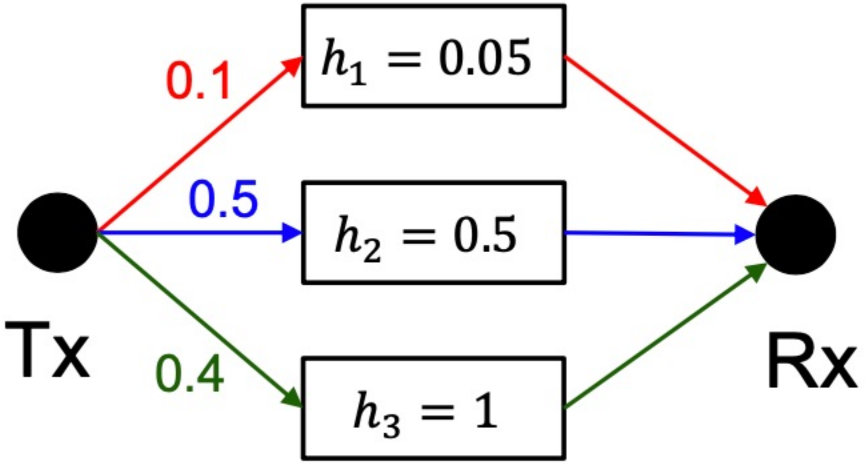
  No

- The data rate achieved by channel inversion is called zero-outage capacity, given by

Constant Received Power

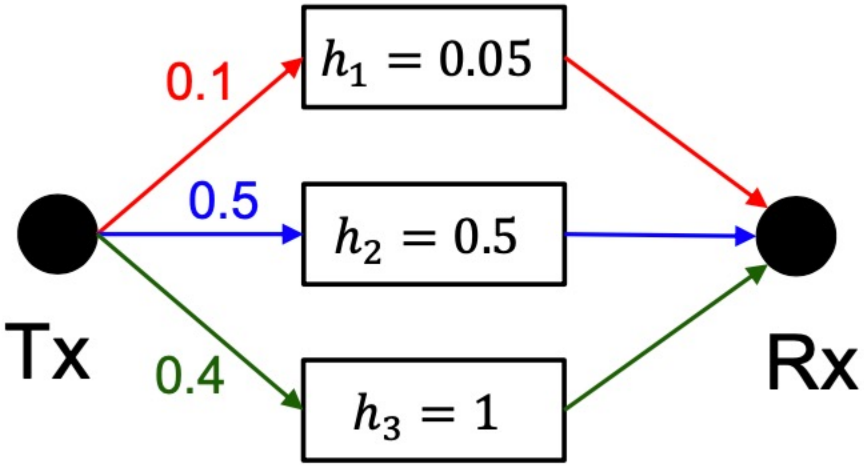
$$C = W \log_2\left(1 + \frac{P}{W N_0}\right)$$

Channel Inversion



- $\bar{P} = 0.01\,W,\ N_0 = 10^{-9}\,W/Hz, W = 30kHz$
- Find a **zero-outage capacity**

CHANNEL INVERSION



- $\bar{P} = 0.01\,W,\ N_0 = 10^{-9}\,W/Hz, W = 30kHz$

- Find a **zero-outage capacity**

$$\bar{P} = \left(0.1\times\frac{P}{|h_1|^2} + 0.5\times\frac{P}{|h_2|^2} + 0.4\times\frac{P}{|h_3|^2}\right) \quad \rightarrow \quad P = 2.3585\times10^{-4}W$$

$$C = 30000\times\log_2\left(1 + \frac{2.3585\times10^{-4}}{30000\times10^{-9}}\right) = 94.43\text{kbps}$$

COMPARISON

- Naïve approach (Using the Same transmit power)

$$C = 199.26 \text{ kbps}$$

- Adaptive power control

$$C = 200.82 \text{ kbps}$$

- Channel Inversion (Using the Same Receiving power & fixed transmission rate)

$$C = 94.43 \text{ kbps}$$

(Channel inversion is the simplest scheme to implement)

Truncated Channel Inversion

- Maximizing the capacity (by Truncated channel inversion)

$$\boxed{C = \max_{\gamma_0} W \log_2(1 + SNR)p(SNR \geq \gamma_0)}$$

"SNR is chosen as a function of γ_0"

as in channel inversion method,

SNR should be chosen by considering average power constraint

- If γ_0 is chosen lower than weakest channel, then this scheme is equivalent to "Channel Inversion"

TRUNCATED CHANNEL INVERSION

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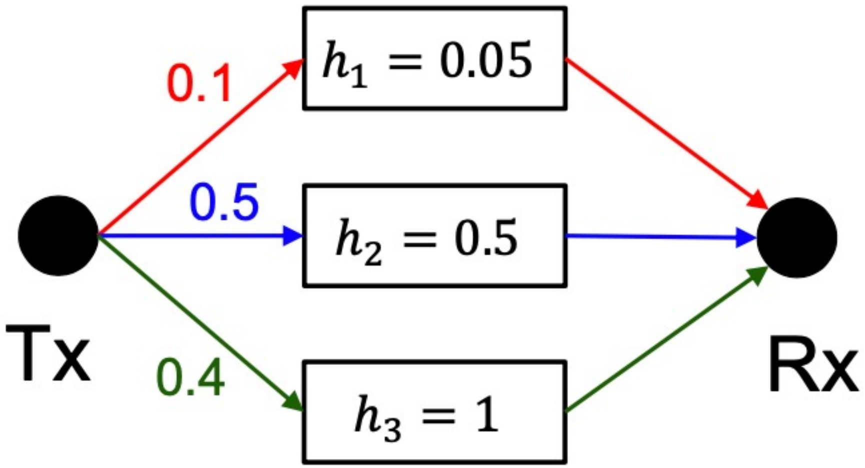
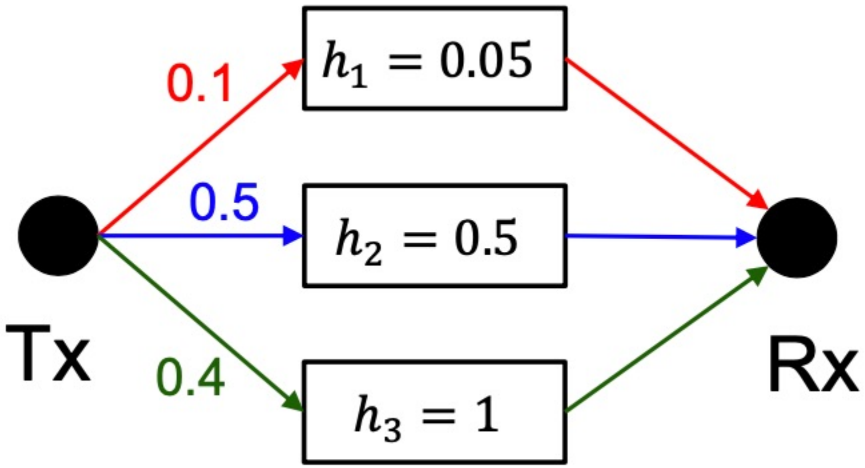


- $\bar{P} = 0.01W,\ N_0 = 10^{-9}\, W/Hz\ , W = 30kHz$

- Maximize the Capacity by **Truncated Channel Inversion**

Truncated Channel Inversion



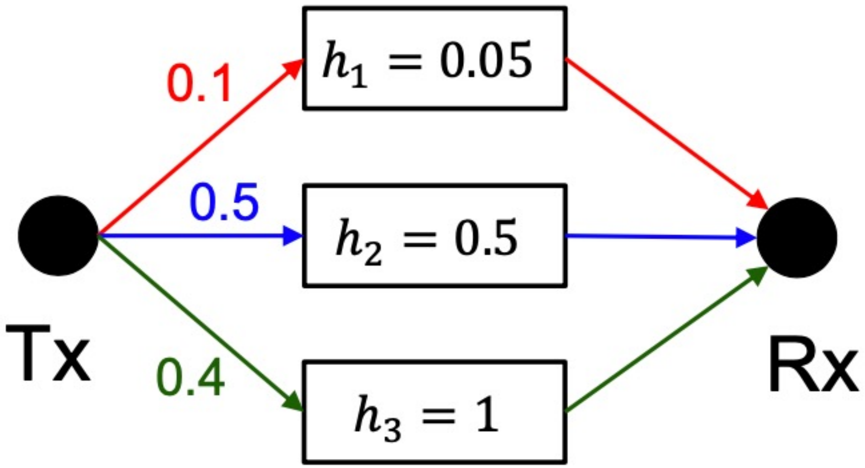
- $\bar{P} = 0.01W,\ N_0 = 10^{-9}\ W/Hz\ ,W = 30kHz$

- Maximize the Capacity by **Truncated Channel Inversion**
- CASE1) Assume “no transmission” at channel h_1

$$C = (W\log_2(1 + \mathrm{SNR}))\times 0.9 = 192.457\ \mathrm{kbps}$$

$$\bar{P} = \left(0.5\times\frac{P}{|h_2|^2} + 0.4\times\frac{P}{|h_3|^2}\right) \rightarrow P = 0.0042\ \mathrm{W}$$

TRUNCATED CHANNEL INVERSION



- $\bar{P} = 0.01W,\ N_0 = 10^{-9}\, W/Hz\, , W = 30kHz$

- Maximize the Capacity by **Truncated Channel Inversion**
- CASE2) Assume "no transmission" at channel h_1 and h_2

$$C = (W \log_2(1 + \mathrm{SNR}))\times 0.4 = 116.45\ \mathrm{kbps}$$

$$\bar{P} = \left(0.4\times \frac{P}{|h_3|^2}\right) \rightarrow P = 0.0250\ \mathrm{W}$$

COMPARISON

- Channel Inversion (Case 0) :

$$C = 94.43\text{ kbps}$$

- Truncated Channel Inversion
  - Case 1: "No transmission at weakes channel h_1"

$$C = 192.48\text{ kbps}$$

  - Case 2: "No transmission ate both h_1 and h_2"

$$C = 116.45\text{ kbps}$$

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Final Comparison

- Naïve Approach (Fixed Tx Power, Various Tranmission Rates)

$$C = 199.26\ \text{kbps}$$

- Adaptive Power Control (Various Powers, Various Transmission Rates)

$$\boxed{C = 200.82\ \text{kbps}}$$

"Optimal Performance"

- Channel Inversion (Various Powers, Fixed Transmission rate)

$$C = 94.43\ \text{kbps}$$

- Truncated Channel Inversion (Various Powers, Fixed Transmission Rates)

$$\boxed{C = 192.48\ \text{kbps}}$$

"Good Practical Approach"

Thank You