Chapter 5

AM Modulation

Outline

- AM Modulation

AM Modulation

- In order to transfer signals we need to transfer the frequency to higher level
- One approach is using modulation
- Modulation:
  - Changing the amplitude of the carrier
- AM modulation is one type of modulation
  - Easy, cheap, low-quality
  - Used for AM receiver and CBs (citizen bands)
  - Generally high carrier frequency is used to modulate the voice signal (300 – 3000 Hz)

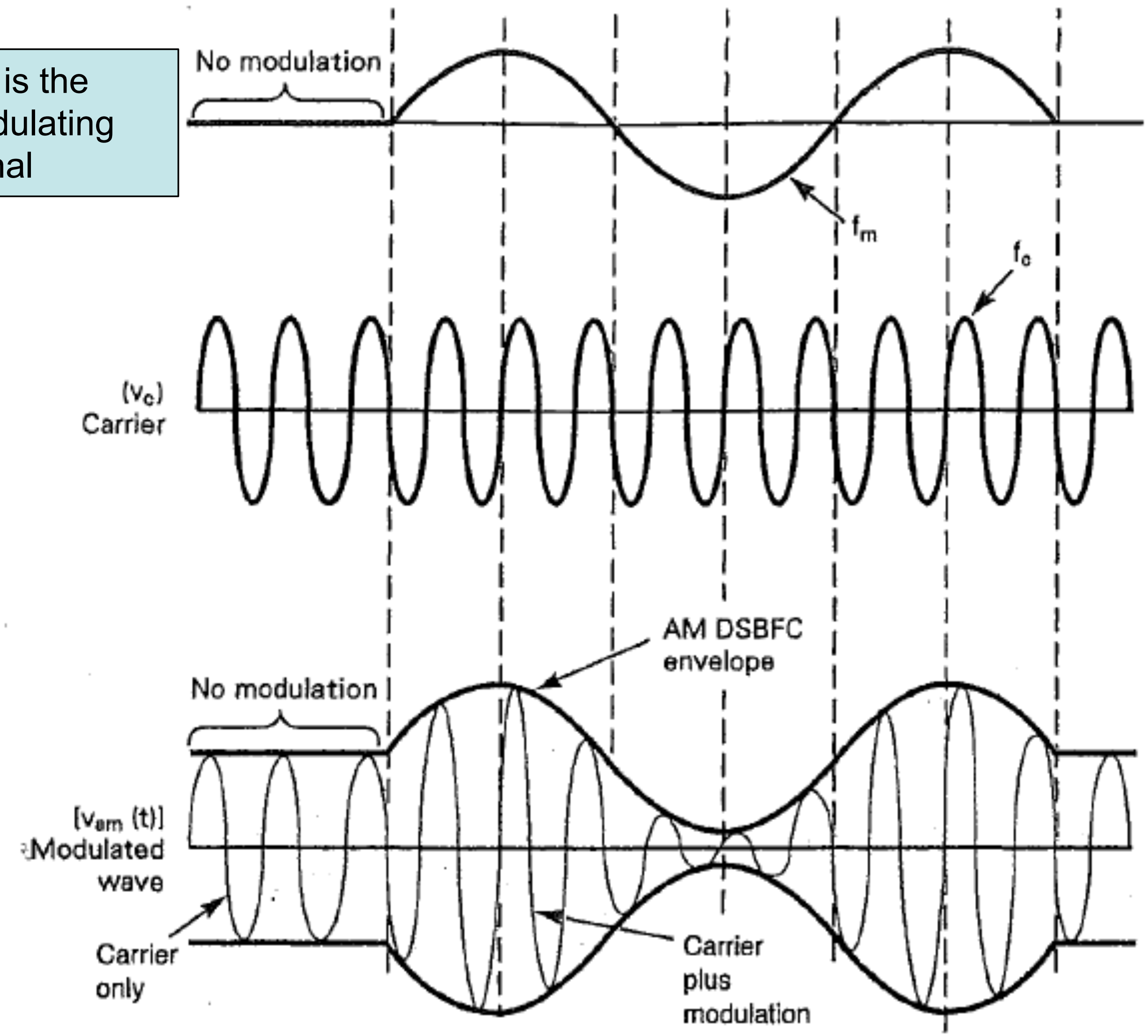
AM Modulation

- In AM modulation the carrier signal changes (almost) linearly according to the modulating signal - m(t)
- AM modulating has different schemes
  - Double-sideband suppressed carrier (DSB-SC)
  - Double-sideband Full Carrier (DSB-FC)
    - Also called the Ordinary AM Modulation (AM)
  - Single-sideband (SSB)
  - Vestigial Sideband (VSB) – Not covered here!

Assuming the Modulating Signal is Sinusoid

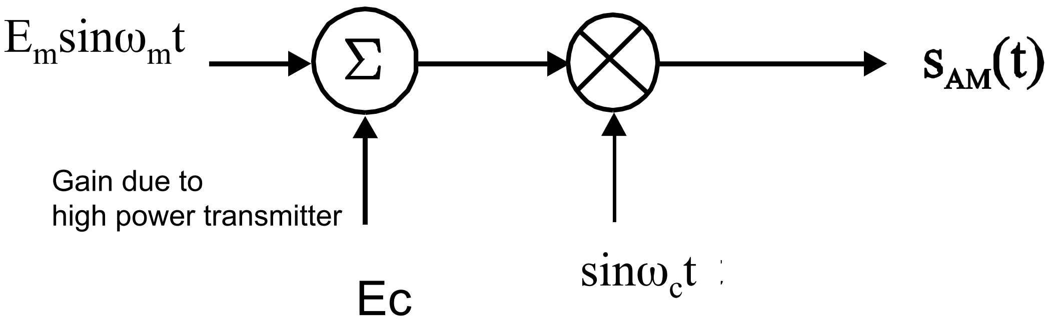
AM Modulation

Vm is the modulating signal



Ordinary AM Mathematical Expression

- In this case:
  - $V_c(t) = E_c \sin\omega_c t$
  - $V_m(t) = E_m \sin\omega_m t$
  - $V_{AM}(t) = E_c \sin\omega_c t + E_m \sin\omega_m t \, . \, \sin\omega_c t$



  - $V_{AM}(t) = [E_c + E_m \sin\omega_m t\,].\, \sin\omega_c t = [1 + m.\sin\omega_m t\,].\, E_c.\, \sin\omega_c t$

Amplitude of the modulated Wave

Constant + Modulated Signal

Modulated Carrier

Assume Em = mEc; where 0<m<1 → m is called the modulation index, or percentage modulation!

AM

- Rearranging the relationship:

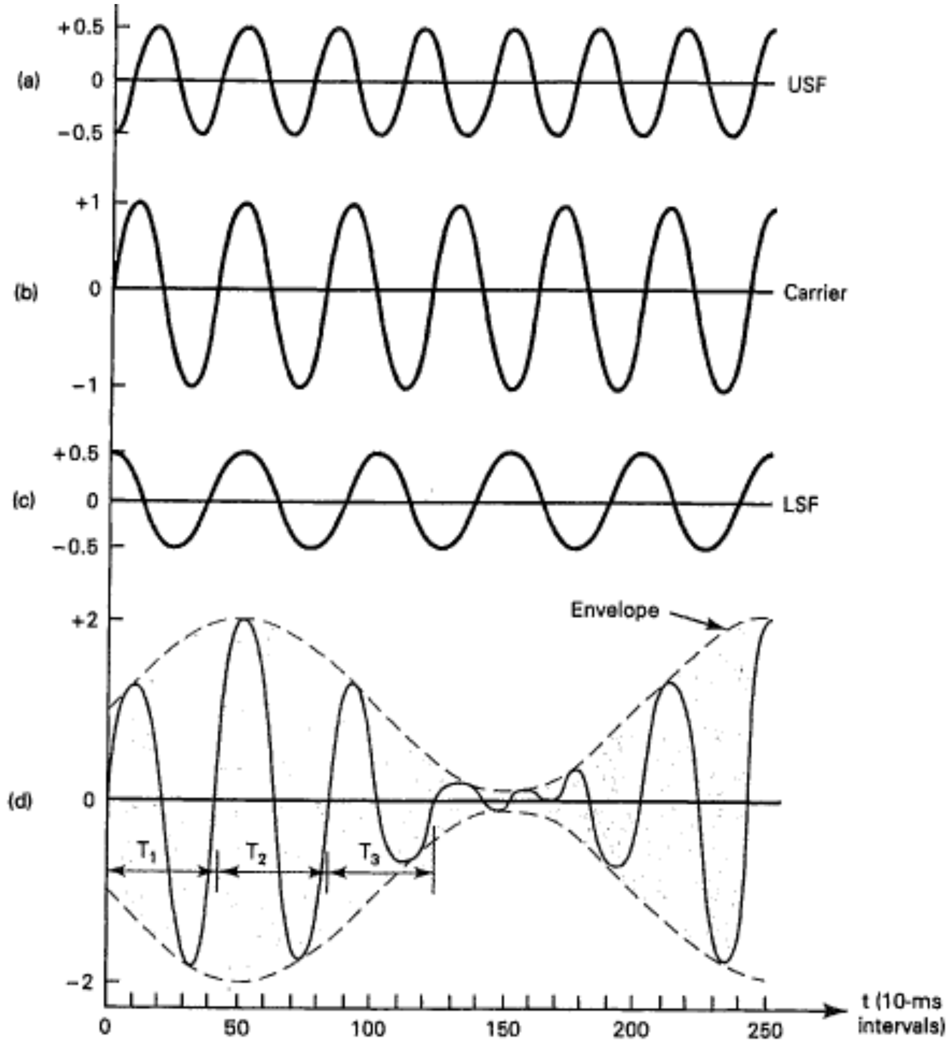
$$v_{am}(t) = E_c \sin(2\pi f_c t) + [mE_c \sin(2\pi f_m t)][\sin(2\pi f_c t)]$$

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$

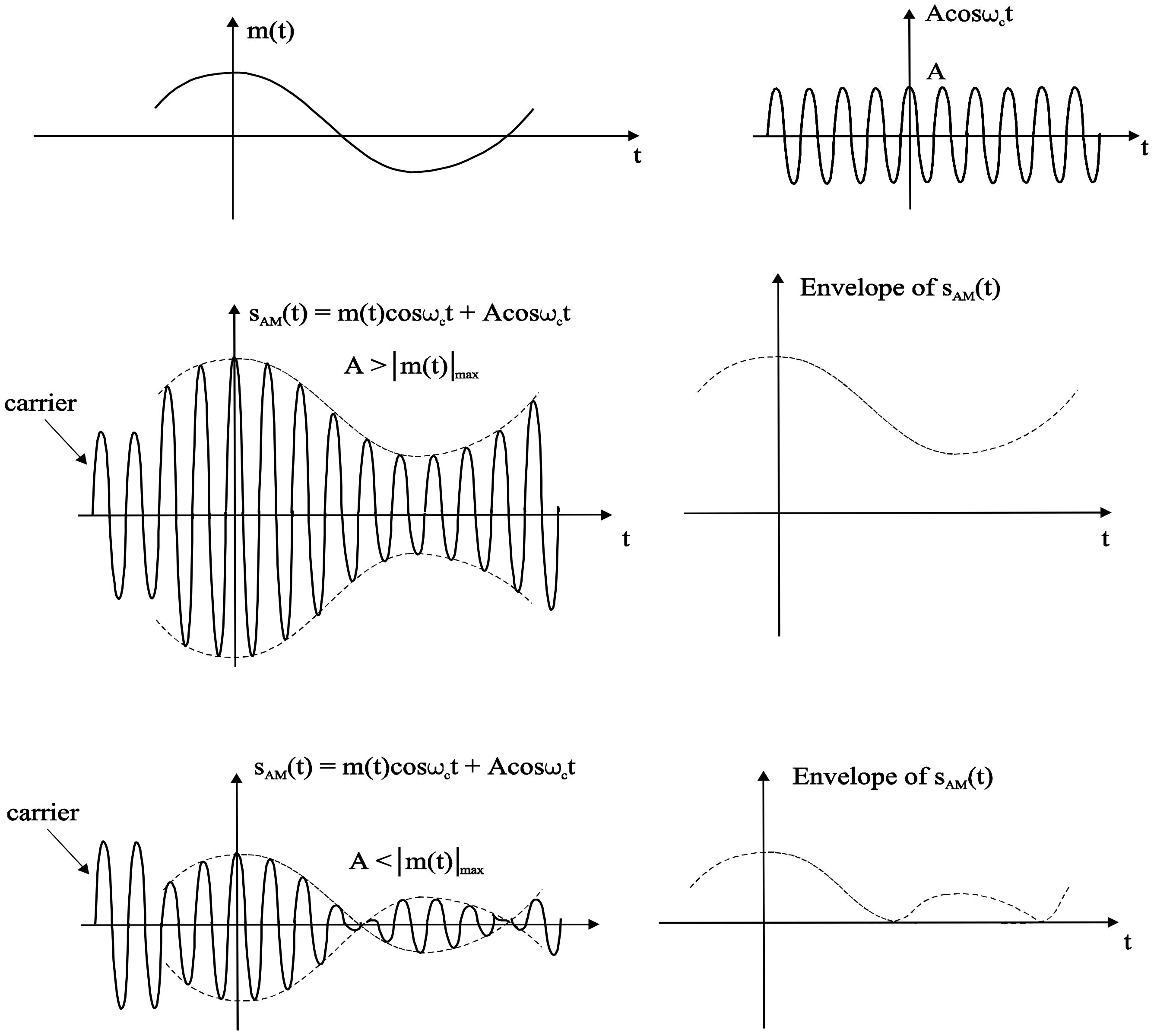
- This Carrier + LSB + USB
- Note that
  - Vam(max = Ec + mEc = 2Ec ; for m = 1
  - Vam(min = 0 ; for m=1

Phase Difference

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$



AM Modulation



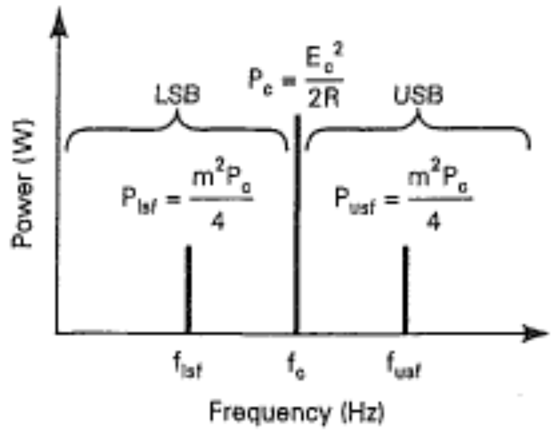
AM Power Distribution

- $P = E^2/2R = Vp^2/2R$; R = load resistance
- Remember: Pavg $Vrms^2/R$; where Vrms for sinusoidal is Vp/sqrt(2)

$$v_{am}(t) = E_c \sin(2\pi f_c t) - \frac{mE_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{mE_c}{2}\cos[2\pi(f_c - f_m)t]$$

- $P_{carrier\_average} = Ec^2/2R$
- $P_{usb\_average} = (mEc/2)^2/2R = (m^2/4)Pc$
- $P_{total} = P_{carrier\_average} + P_{usb\_average} + P_{lsb\_average}$

What happens as m increases?



Current Analysis

- Measuring output voltage may not be very practical
- $P = Vp^2/2R$ is difficult to measure in an antenna!
- However, measuring the current passing through an antenna may be more possible: Total Power is $P_T = I_T^2 R$

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \frac{I_t^2}{I_c^2} = 1 + \frac{m^2}{2}$$

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}$$

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

Note that we can obtain m if we measure currents!

Multiple Input Frequencies

- What if the modulating signal has multiple frequencies?

$$v_{am}(t) = \sin(2\pi f_c t) + \frac{1}{2}\cos[2\pi(f_c - f_{m1})t] - \frac{1}{2}\cos[2\pi(f_c + f_{m1})t]$$
$$+ \frac{1}{2}\cos[2\pi(f_c - f_{m2})t] - \frac{1}{2}\cos[2\pi(f_c + f_{m2})t]$$

- In this case:

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + m_n^2}$$

- All other power measurements will be the same!

Examples (5A, 5C)

General Case: m(t) can be any bandpass

Review: Bandpass Signal

- Remember for bandpass waveform we have

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\}$$

- The voltage (or current) spectrum of the bandpass signal is

$$S(f) = \tfrac{1}{2}[G(f-f_c) + G^*(-f-f_c)]$$

- The PSD will be

$$\mathcal{P}_s(f) = \frac{1}{4}[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)]$$

- In case of Ordinary AM (DSB – FC) modulation:

$$g(t) = A_c[1 + m(t)]$$

- In this case Ac is the power level of the carrier signal with no modulation;
- Therefore: $s(t) = A_c[1 + m(t)] \cos \omega_c t$

Make sure you know where these come from!

AM: Modulation Index

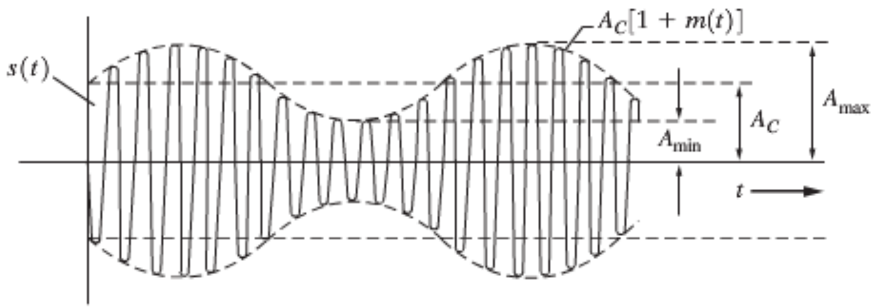
- Modulation Percentage (m)

$$\%\,\text{modulation} = \frac{A_{\max} - A_{\min}}{2A_c} \times 100 = \frac{\max[m(t)] - \min[m(t)]}{2} \times 100$$

- Note that m(t) has peak amplitude of $A_m = mE_m = mA_c$
- We note that for ordinary AM modulation,
  - if the modulation percentage > %100,
  - implying m(t) < -1
  - Then:

$$s(t) = \begin{cases} A_c[1 + m(t)]\cos w_c t, & \text{if } m(t) \geq -1 \\ 0, & \text{if } m(t) < -1 \end{cases}$$

(a) Sinusoidal Modulating Wave



(b) Resulting AM Signal

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

AM: MATLAB Model

- This is how we generate the ordinary AM using MATLAB

```
fc = 10;                 % carrier frequency
fa = 1;                  % modulating frequency
N  = 200;                % number of samples
To = 4;                  % observation time: To x periods
MI = 1;                  % Modulation Index (0.0-2.0 or 0 to 200 percent)
Ec = 1;                  % Ec is the level of the AM envelope in the
                         % absence of modulation, when m(t) = 0;

Ta = 1/fa;
dt = To*Ta/N;
wc = 2*pi*fc;
wa = 2*pi*fa;

t = 0:dt:To*Ta;     % simulation time

m = MI*cos(wa*t);   % modulating signal: m(t)
m = m(:);

y = zeros(length(t),1); % In this part we force [1+m] = 0 if
for (i = 1:1:length(t)) %
  if (m(i) > -1)        % in other words, we ensure [1+m(t)]=0 if
    y(i) = 1;           % m(t) < -1
  end;
end;
```

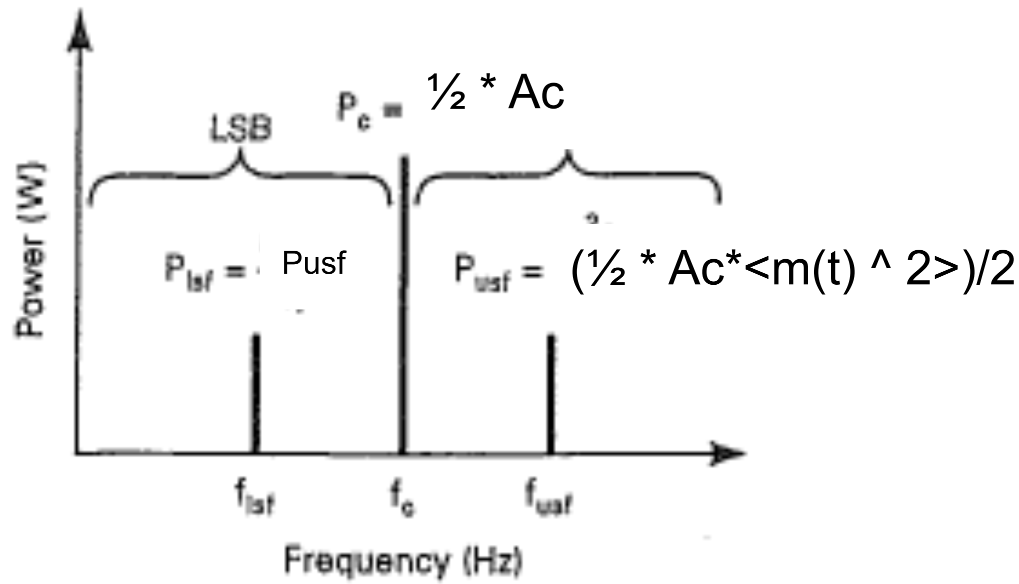
AM: Normalized Average Power

- Normalized Average Power (R=1)
- Note that

$$\langle s^2(t)\rangle = \tfrac{1}{2}\langle|g(t)|^2\rangle = \tfrac{1}{2}A_c^2\langle[1+m(t)]^2\rangle$$
$$= \tfrac{1}{2}A_c^2\langle 1 + 2m(t) + m^2(t)\rangle$$
$$= \tfrac{1}{2}A_c^2 + A_c^2\langle m(t)\rangle + \tfrac{1}{2}A_c^2\langle m^2(t)\rangle$$

$$\langle s^2(t)\rangle = \underbrace{\tfrac{1}{2}A_c^2}_{\substack{\text{discrete}\\ \text{carrier power}}} + \underbrace{\tfrac{1}{2}A_c^2\langle m^2(t)\rangle}_{\text{sideband power}}$$

- Pc is the normalized carrier power(1/2)Ac^2 (when R= 1, Ac = Ec, and m is the modulation index)
- The rest is the power of each side band
- Thus:



AM: Modulation Efficiency

- Defined as the percentage of the total power of the modulated signal that conveys information

$$s(t) = A_c[1 + m(t)] \cos \omega_c t$$

- Defined as:

$$E = \frac{\langle m^2(t) \rangle}{1 + \langle m^2(t) \rangle} \times 100\%$$

- Normalized Peak Envelop Power is defined as

  $P_{PEP} = (A_c^2 / 2) * (1 + A_{max})^2 =$

  (when load resistance R=1)

- We use P_{PEP} to express transmitter output power.
- In general, Normalized Peak Envelop Power, P_{PEP} ,can be expressed as follow:

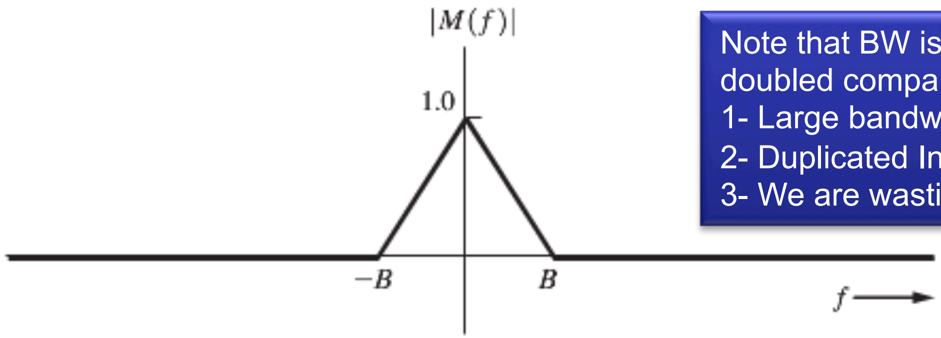
$$\frac{1}{2} \max \{|g(t)|^2\}$$

AM: Voltage and Current Spectrum

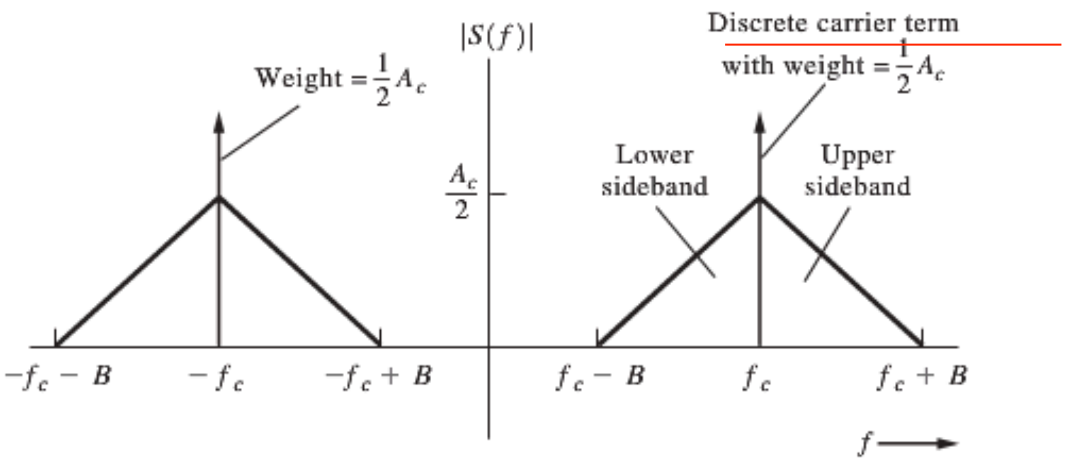
- We know for AM: $s(t) = A_c[1 + m(t)] \cos \omega_c t$
- The voltage or Current Spectrum will be

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)]$$

Note that BW is 2B –
doubled compared to M(f) →
1- Large bandwidth requirement
2- Duplicated Information in Upper and Lower Sides
3- We are wasting power to send the discrete carrier power



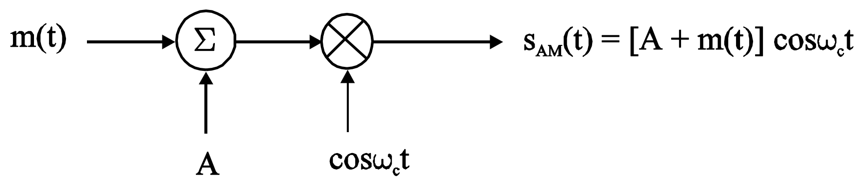
(a) Magnitude Spectrum of Modulation



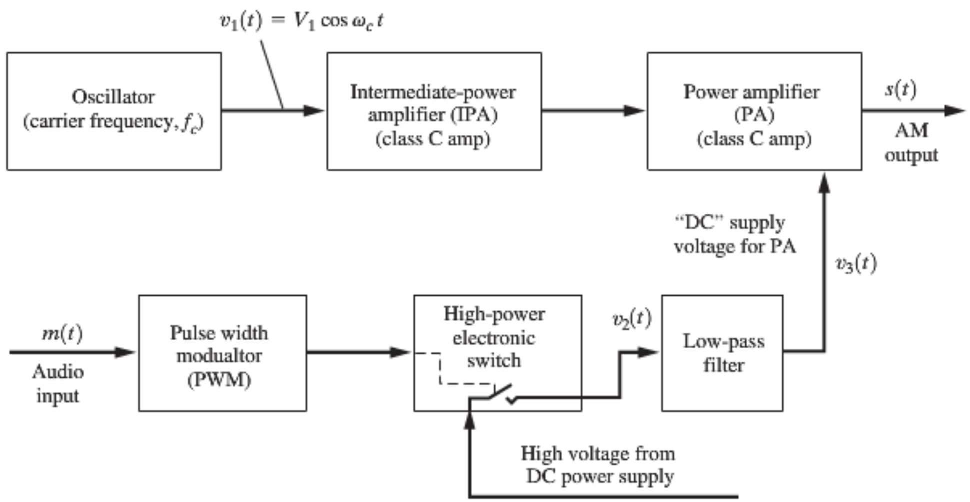
(b) Magnitude Spectrum of AM Signal

Building an Ordinary AM Modulator

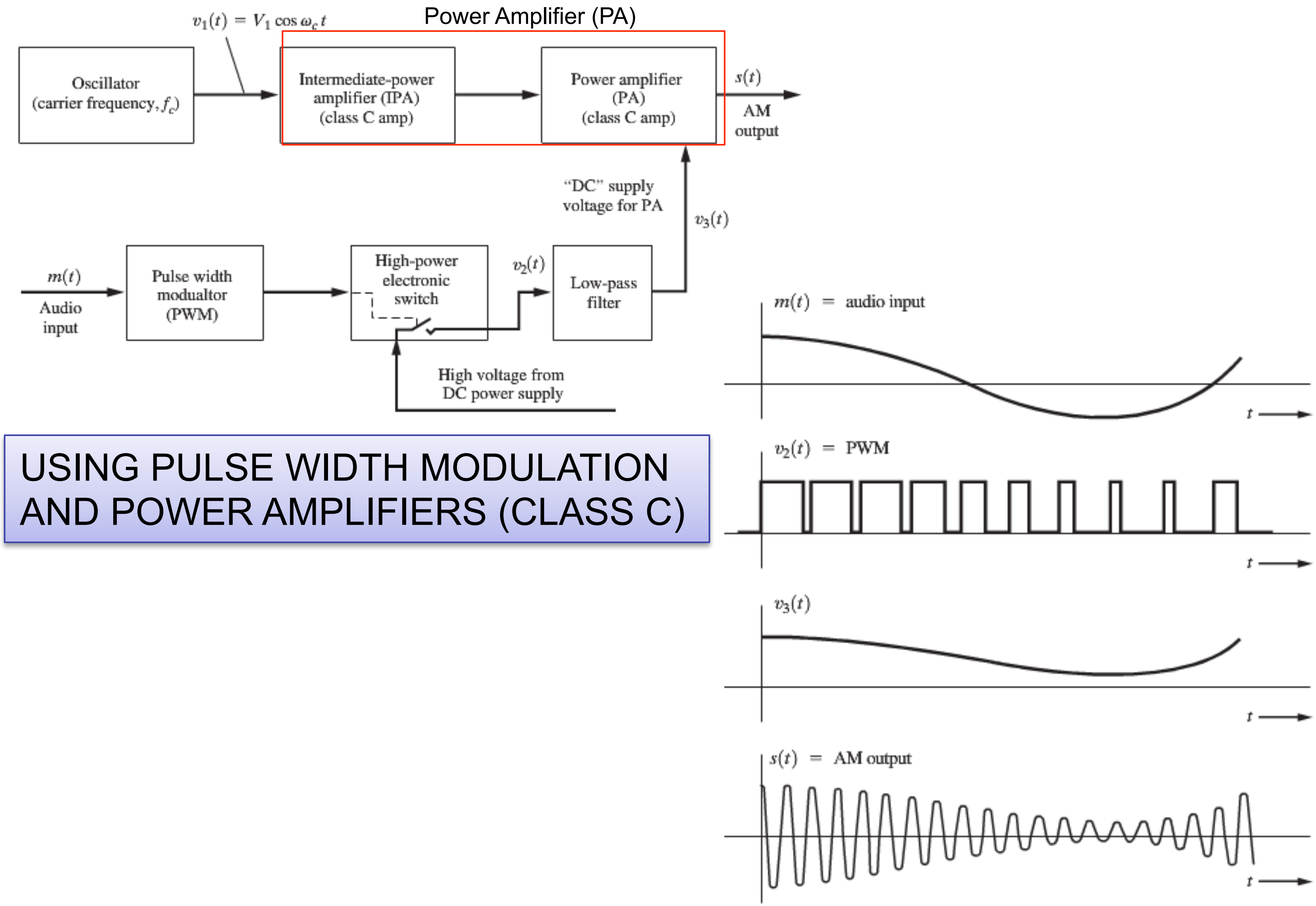
- Transferring AC power to RF power!
- Two general types
  - Low power modulators
  - High power modulators
- Low Power Modulators
  - Using multipliers and amplifiers
  - Issue: Linear amplifiers must be used; however not so efficient when it comes to high power transfer



- High Power Modulators
  - Using PWM



Building an Ordinary AM Modulator



USING PULSE WIDTH MODULATION AND POWER AMPLIFIERS (CLASS C)

Example (5B)

- Assume Pc_avg = 5000 W for a radio station (un-modulated carrier signal); If m=1 (100 percent modulation) with modulated frequency of 1KHz sinusoid find the following:
  - Peak Voltage across the load (Ac)
  - Total normalized power ($\langle s(t)^2 \rangle$)
  - Total Average (actual) Power
  - Normalized PEP
  - Average PEP
  - Modulation Efficiency – Is it good?

Double Sideband Suppressed Carrier

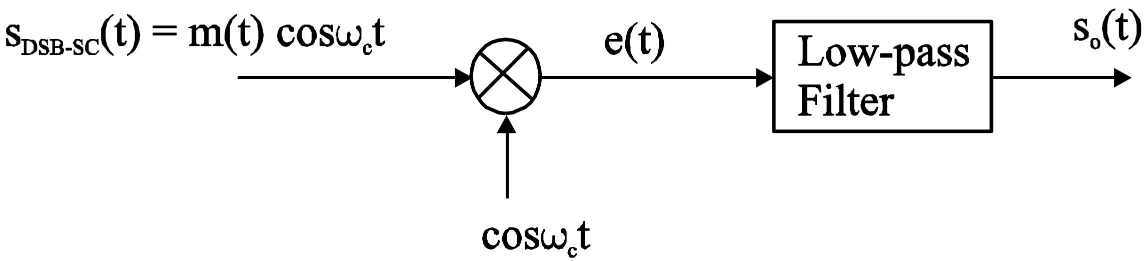
- DSB-SC is useful to ensure the discrete carrier signal is suppressed:

$$s(t) = A_c m(t) \cos \omega_c t$$

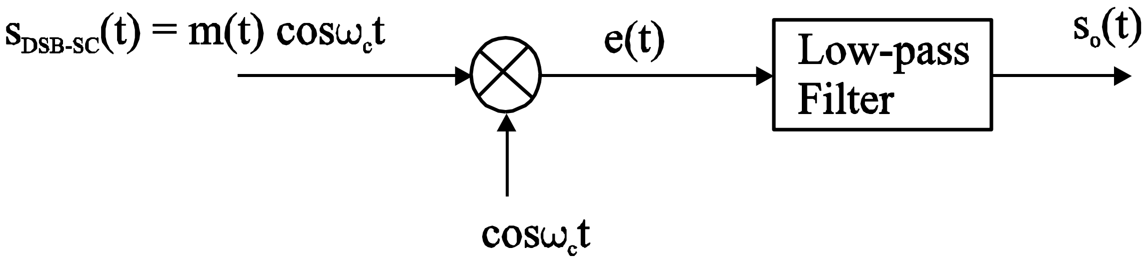
- The voltage or current spectrum of DSB-SC will be

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

- Therefore no waste of power for discrete carrier component !
- What is the modulation efficiency? → 100 Percent!
  - Effic = $\langle m(t)^2 \rangle / \langle m(t)^2 \rangle$
- Generating DSB-SC



DSB-SC – Coherent Demodulation



Multiplying the signal $m(t)\cos\omega_c t$ by a local carrier wave $\cos\omega_c t$

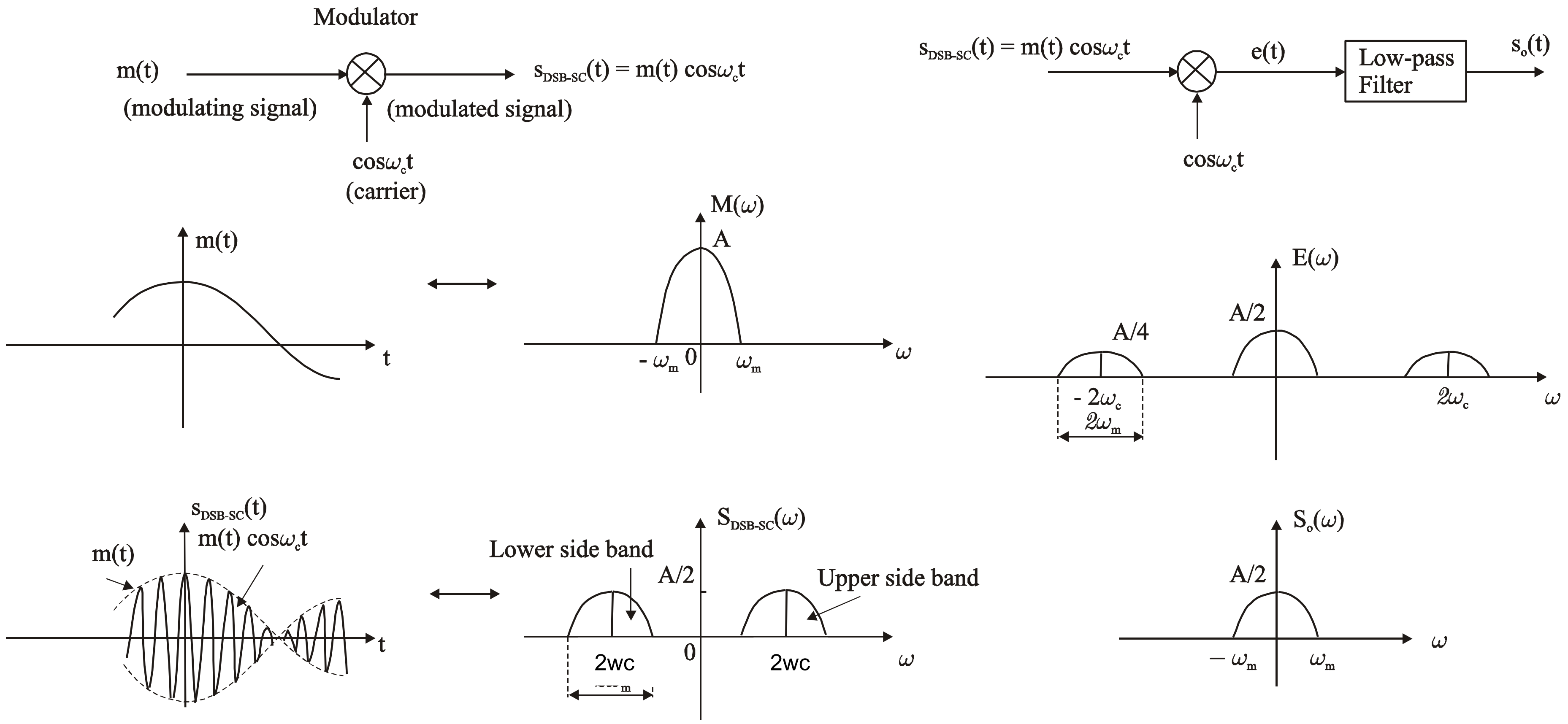
$$e(t) = m(t)\cos^2\omega_c t = (1/2)[m(t) + m(t)\cos 2\omega_c t]$$

$$E(\omega) = (1/2)M(\omega) + (1/4)[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

Passing through a low pass filter: $S_o(\omega) = (1/2)M(\omega)$

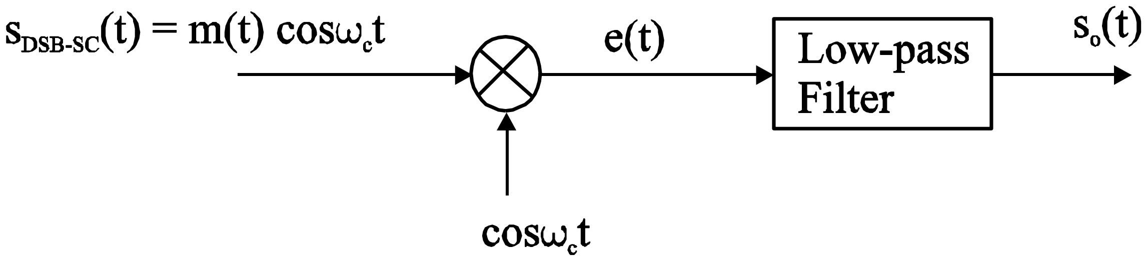
The output signal: $s_o(t) = (1/2)m(t)$

DSB-SC


Modulator
m(t)
(modulating signal)
$s_{DSB\text{-}SC}(t) = m(t)\cos\omega_c t$
(modulated signal)
$\cos\omega_c t$
(carrier)
$s_{DSB\text{-}SC}(t) = m(t)\cos\omega_c t$
e(t)
Low-pass Filter
$s_o(t)$
$\cos\omega_c t$
m(t)
t
$M(\omega)$
A
$-\omega_m$
0
ω_m
ω
$E(\omega)$
A/2
A/4
$-2\omega_c$
$2\omega_m$
$2\omega_c$
ω
$s_{DSB\text{-}SC}(t)$
$m(t)\cos\omega_c t$
m(t)
t
$S_{DSB\text{-}SC}(\omega)$
Lower side band
A/2
Upper side band
2wc
0
2wc
ω
$S_o(\omega)$
A/2
$-\omega_m$
ω_m
ω

DSB-SC – Coherent Demodulation Issues

So what if the Local oscillator frequency is a bit off with the center frequency ($\Delta\omega$)?



Multiplying the signal $m(t)\cos\omega_c t$ by a local carrier wave $\cos[(\omega_c+\Delta\omega)t]$

$$
\begin{aligned}
e(t) &= m(t)\cos\omega_c t \,.\, \cos[(\omega_c+\Delta\omega)t] \\
&= (1/2)[m(t)] \,.\, \{\cos[\omega_c t - (\omega_c+\Delta\omega)t] + \cos[\omega_c t + (\omega_c+\Delta\omega)t]\} \\
&= (1/2)[m(t)] \,.\, \{\cos(\Delta\omega t) + \cos(2\omega_c+\Delta\omega)t\} \\
&= m(t)/2 \,.\, \cos(\Delta\omega t) \leftarrow \text{The beating factor (being distorted)}
\end{aligned}
$$

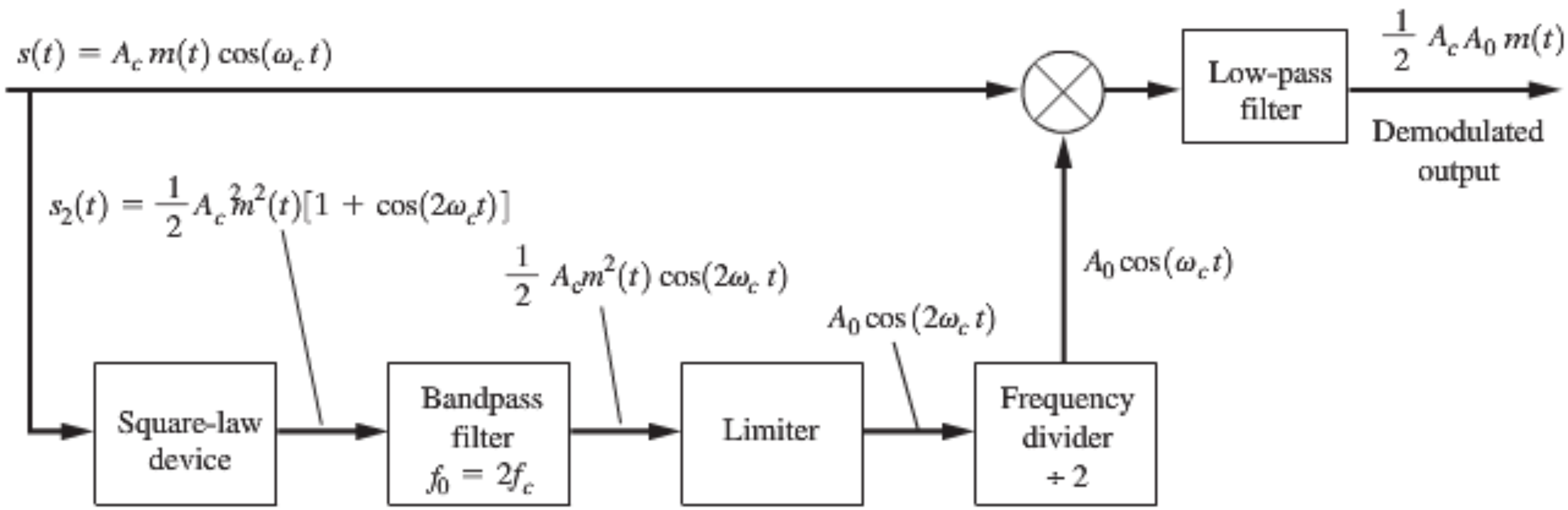
The coherent demodulator must be synchronized with the modulator both in frequency and phase!

Disadvantages:

1. **It transmits both sidebands which contain <u>identical information</u> and thus waste the channel bandwidth resources;**
2. **It requires a fairly complicated (expensive) circuitry at a remotely located receiver in order to avoid phase errors.**

Demodulation DSB-SC

- One common approach is using Squaring Loop:



Note that in this case the initial phase must be known!

Single Sideband AM (SSB)

- Is there anyway to reduce the bandwidth in ordinary AM?
- The complex envelop of SSB AM is defined by

$$g(t) = A_c[m(t) \pm j\hat{m}(t)]$$

- Thus, we will have

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$

- In this case the (+) → USSB and (-) → LSSB
- We define (~m(t) is the Hilbert Transfer of m(t))

$$\hat{m}(t) \triangleq m(t) * h(t)$$

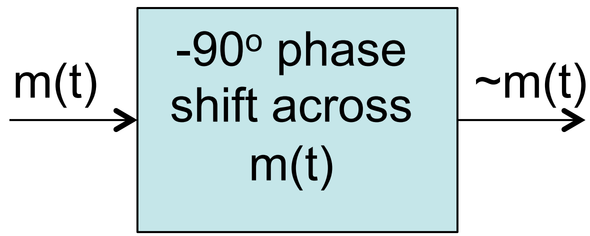
- Where:

$$h(t) = \frac{1}{\pi t}$$

- With

$$H(f) = \begin{Bmatrix} -j, & f > 0 \\ j, & f < 0 \end{Bmatrix}$$

- Thus:



$$G(f) = A_c\{M(f) \pm j\mathscr{F}[\hat{m}(t)]\} \longrightarrow G(f) = A_c M(f)[1 \pm jH(f)]$$

See Notes

Frequency Spectrum of SSB-AM - USSB

For Upper SSB use (+) $\boxed{G(f) = A_c M(f)[1 \pm jH(f)]}$

$$H(f)=\begin{Bmatrix} -j, & f>0 \\ j, & f<0 \end{Bmatrix} \longrightarrow G(f)=\begin{Bmatrix} 2A_c M(f), & f>0 \\ 0, & f<0 \end{Bmatrix}$$

Therefore:

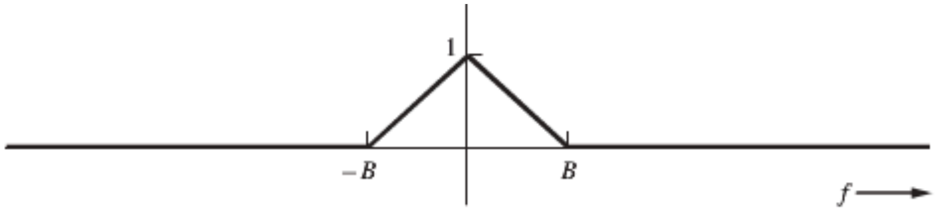
$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$

$$S(f)=A_c\begin{Bmatrix} M(f-f_c), & \text{f} > f_c \\ 0, & f<f_c \end{Bmatrix}+A_c\begin{Bmatrix} 0, & f>-f_c \\ M(f+f_c), & f<-f_c \end{Bmatrix}$$

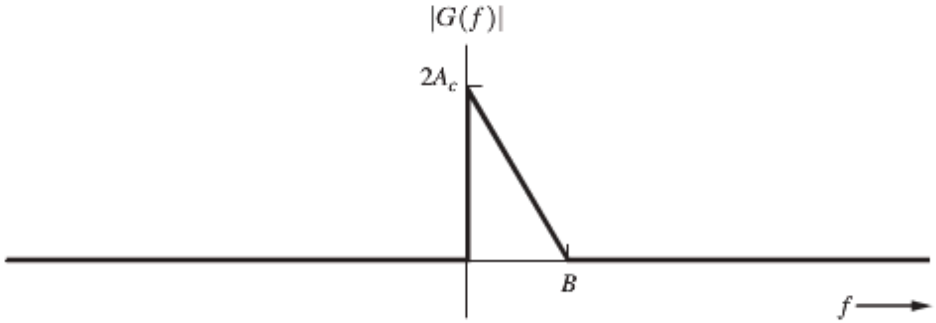
Normalized Average Power:

$$\langle s^2(t)\rangle = \tfrac{1}{2}\langle |g(t)|^2\rangle = \tfrac{1}{2}A_c^2\langle m^2(t) + [\hat{m}(t)]^2\rangle \qquad \langle \hat{m}(t)^2\rangle = \langle m^2(t)\rangle$$

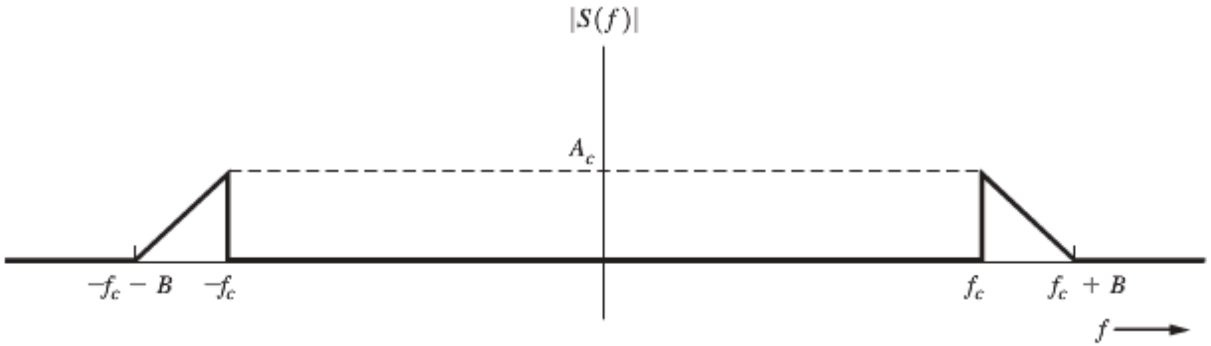
$$\boxed{\langle s^2(t)\rangle = A_c^2\langle m^2(t)\rangle}$$



(a) Baseband Magnitude Spectrum



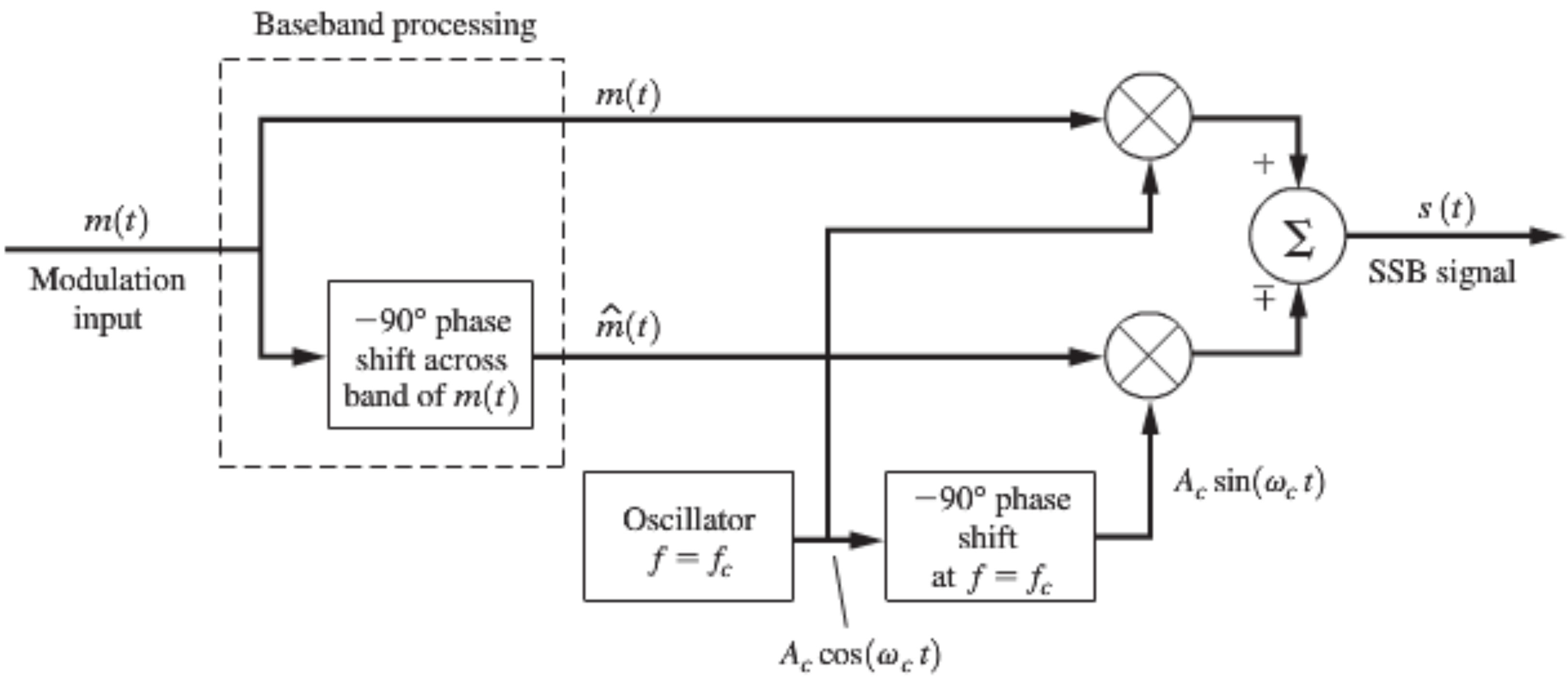
(b) Magnitude of Corresponding Spectrum of the Complex Envelope for USSB



(c) Magnitude of Corresponding Spectrum of the USSB Signal

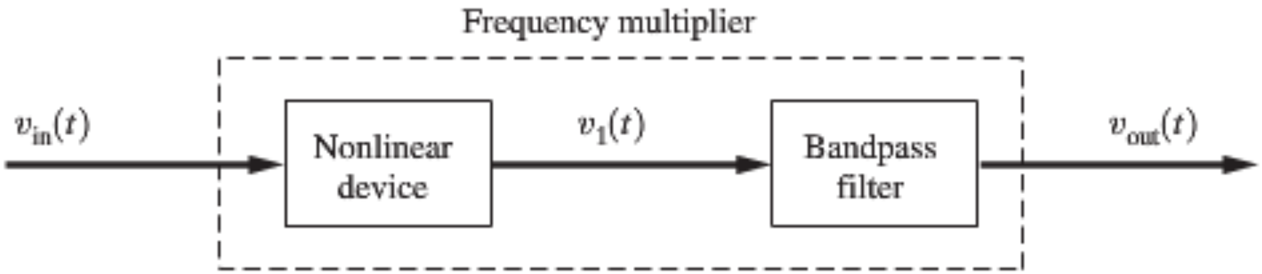
Phasic Method

$$s(t) = A_c[m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t]$$

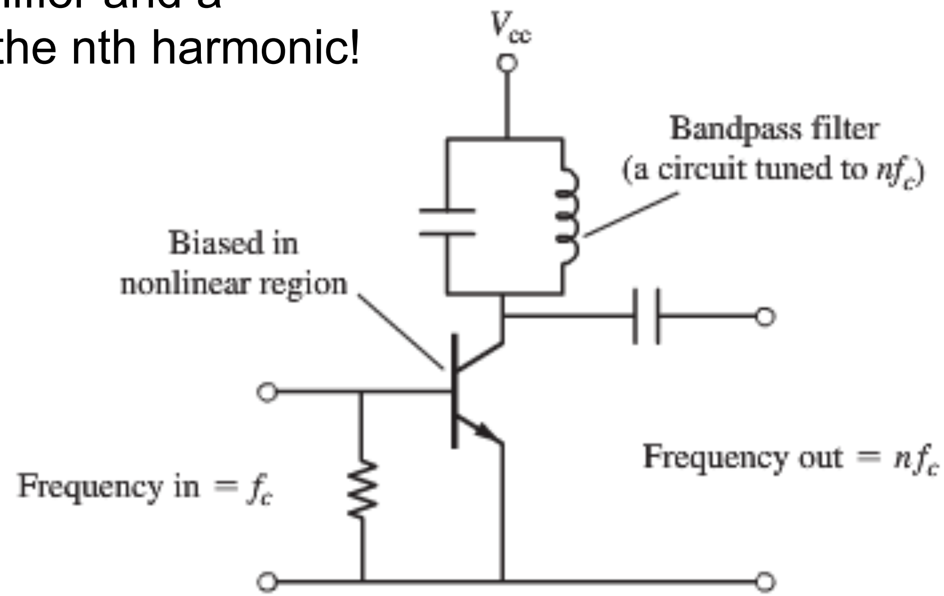


This is also called Quadrature AM (QAM) modulator with **I** and **Q** channels

AM Modulators: Frequency Multiplier



Nonlinear amplifier and a
filter to extract the nth harmonic!



Building AM Modulators

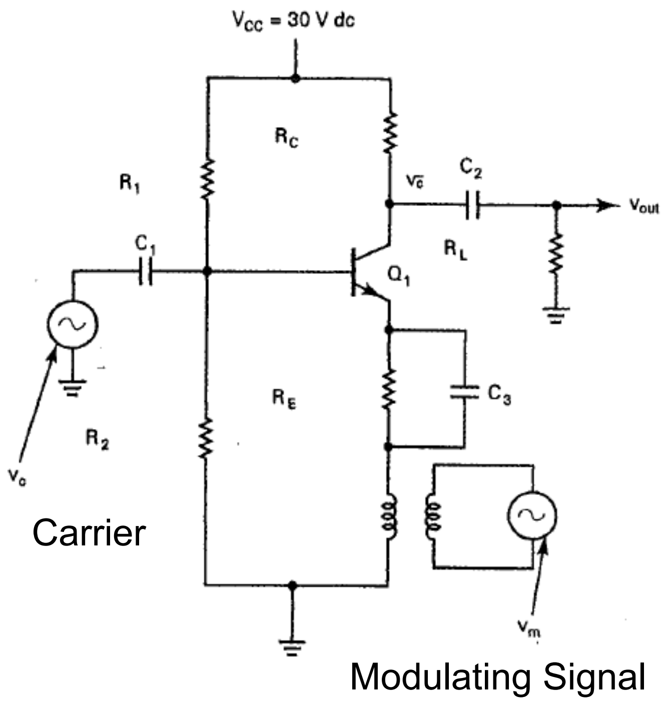
- AM Modulating Circuits are categorized as
  - Low-level Transmitters
  - Medium-level Transmitters
  - High-level Transmitters

Other Key Components

- Mixers
- Phase shifter
  - RC
  - Inverters
- Amplifiers
  - Linear
  - Nonlinear

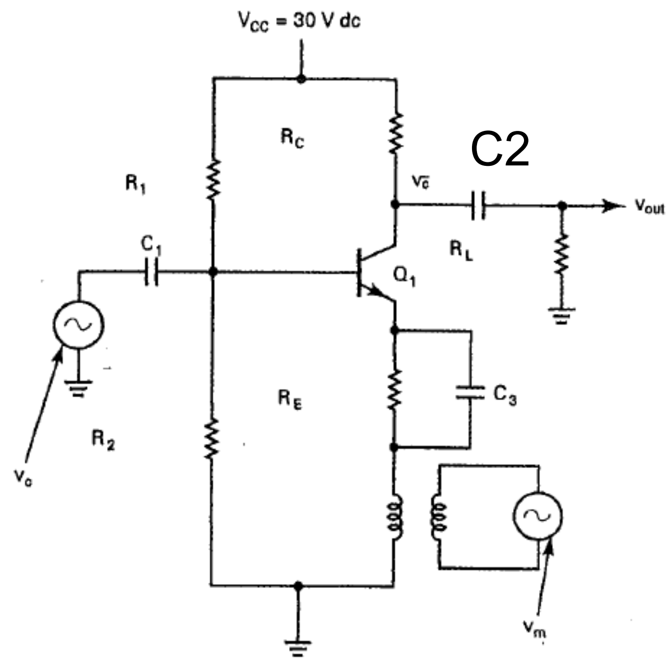
Low-Level AM Modulators

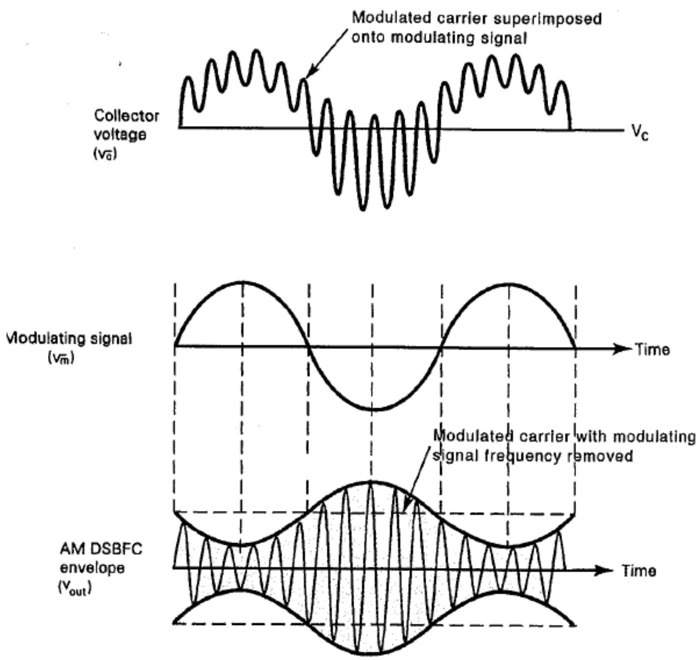
- Requires less modulating signal power to achieve high *m*
- Mainly for low-power applications
- Uses an Emitter Modulator (low power)
  - Incapable of providing high-power
- The amplifier has two inputs: *Vc(t)* and *Vm(t)*
- The amplifier operates in both linear and nonlinear modes



Low-Level AM Modulators – Circuit Operation

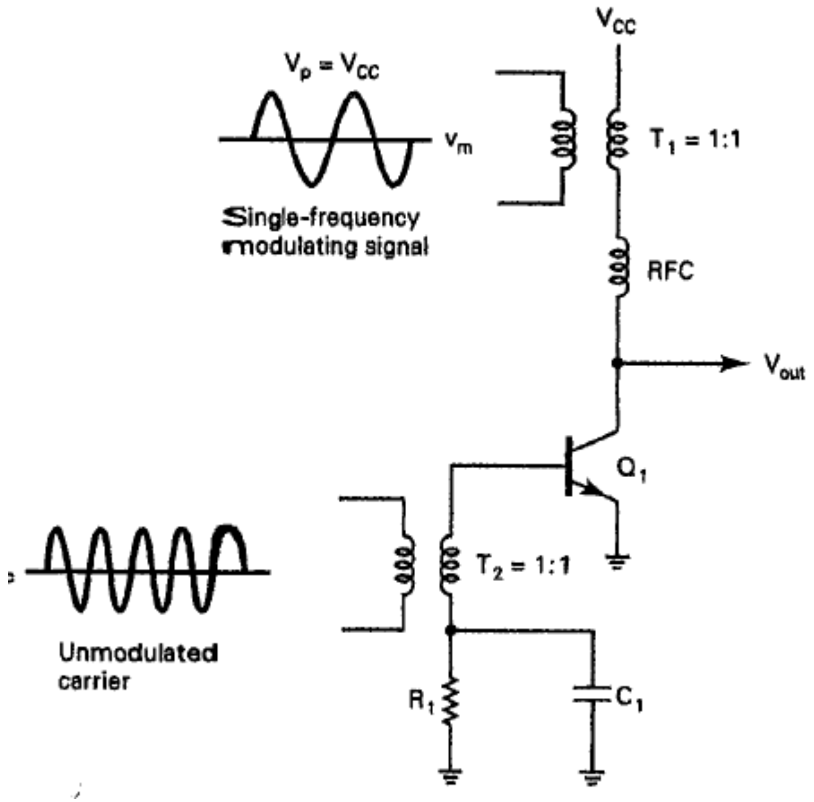
- If Vm(t) =0 → amplifier will be in **linear** mode
  - → $Aout=V_c\cos(w_c t)$; Vc is voltage gain (unit less)
- If Vm(t) >0 → amplifier will be in **nonlinear** mode
  - → $Aout=[V_c + V_m\cos(w_c t)]\cos(w_c t)$
- Vm(t) is isolated using T1
  - The value of Vm(t) results in Q1 to go into cutoff or saturation modes
- C2 is used for coupling
  - Removes modulating frequency from AM waveform





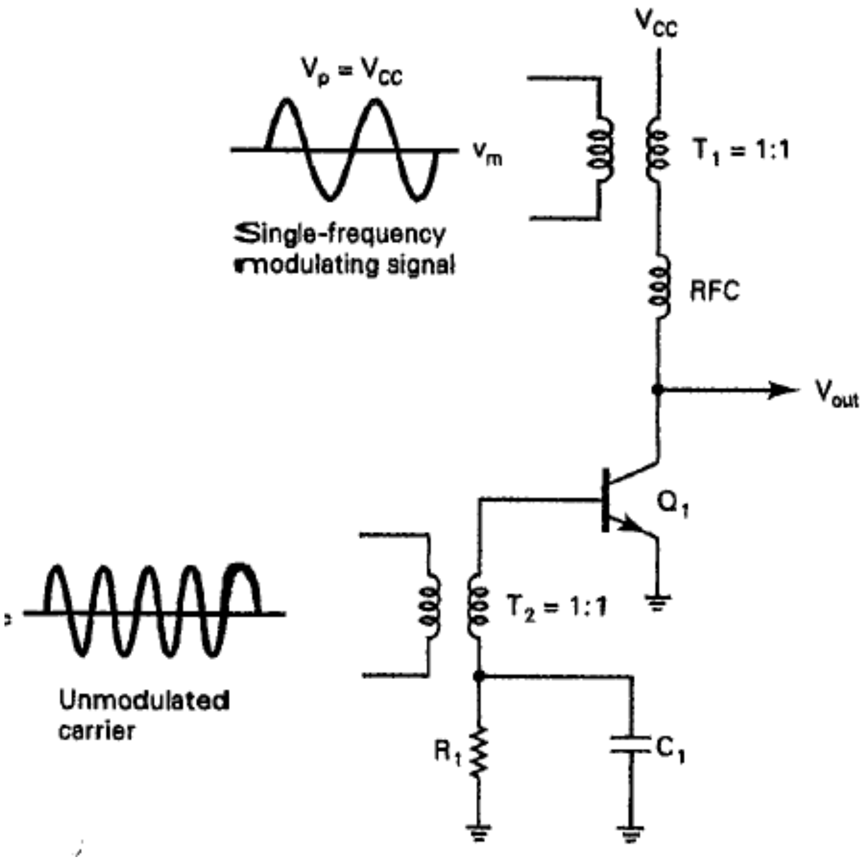
High-Level AM Modulators – Circuit Operation

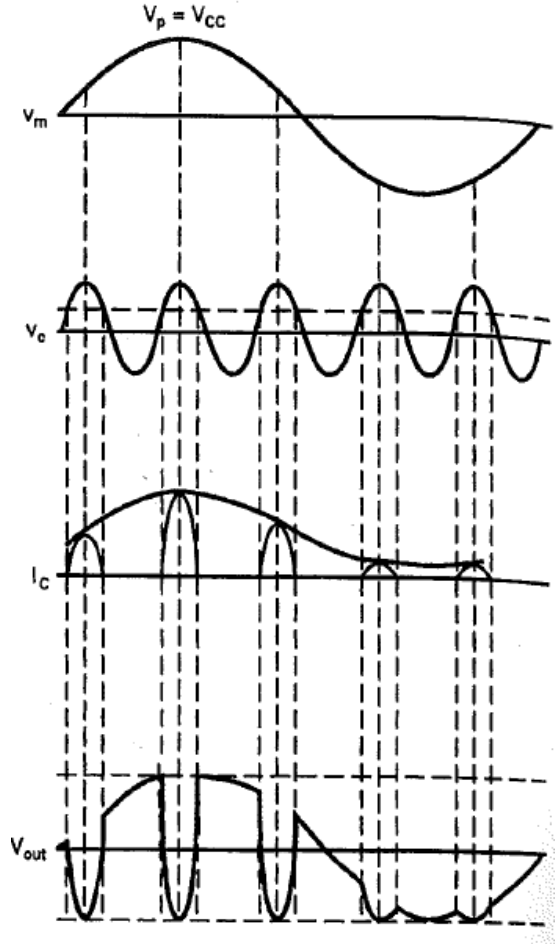
- Used for high-power transmission
- Uses an Collector Modulator (high power)
  - Nonlinear modulator
- The amplifier has two inputs: *Vc(t)* and *Vm(t)*
- RFC is radio frequency choke – blocks RF



High-Level AM Modulators – Circuit Operation

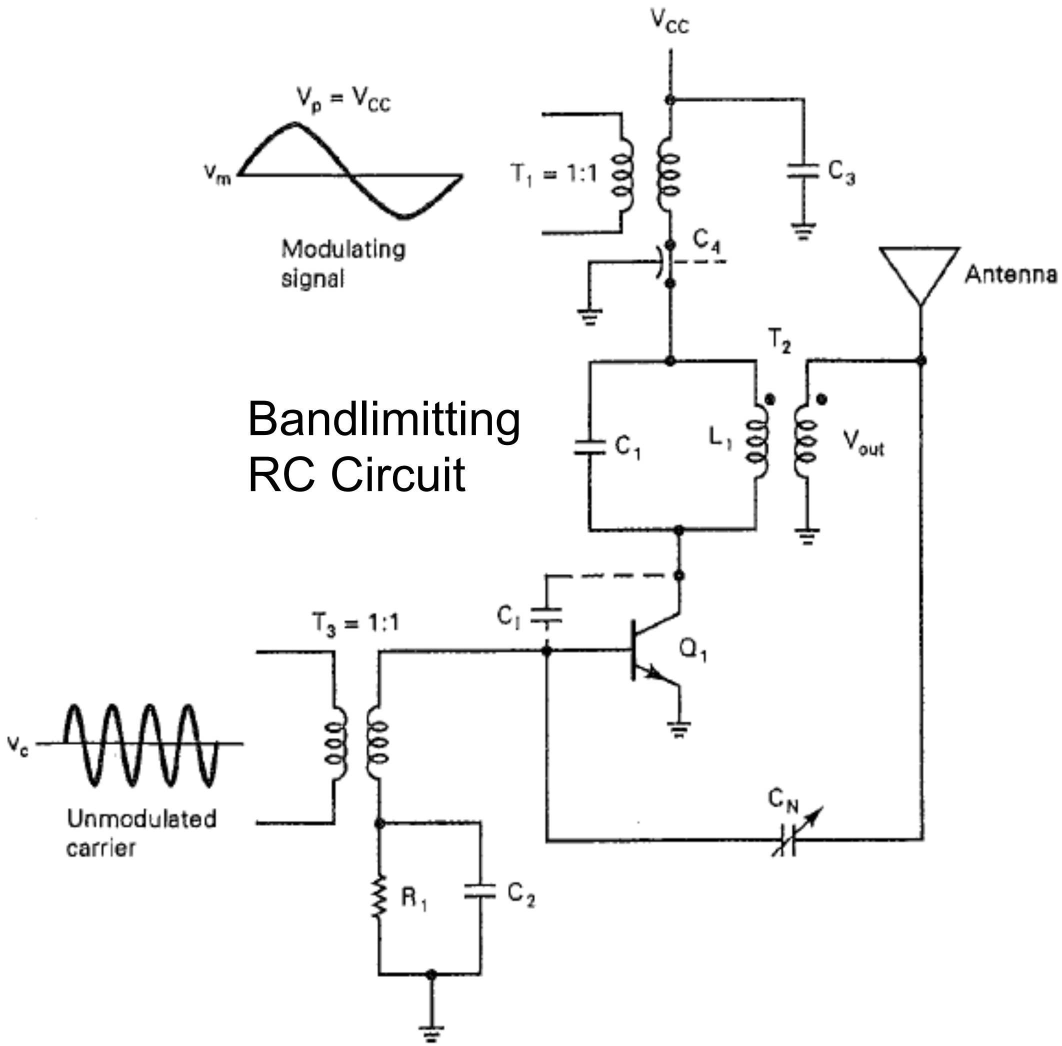
- General operation:
  - If Base Voltage > 0.7 → Q1 is ON → Ic != 0 → Saturation
  - If Base Voltage < 0.7 → Q1 is OFF → Ic = 0 → Cutoff
  - The Transistor changes between Saturation and Cutoff
- When in nonlinear → high harmonics are generated→ Vout must be bandlimited





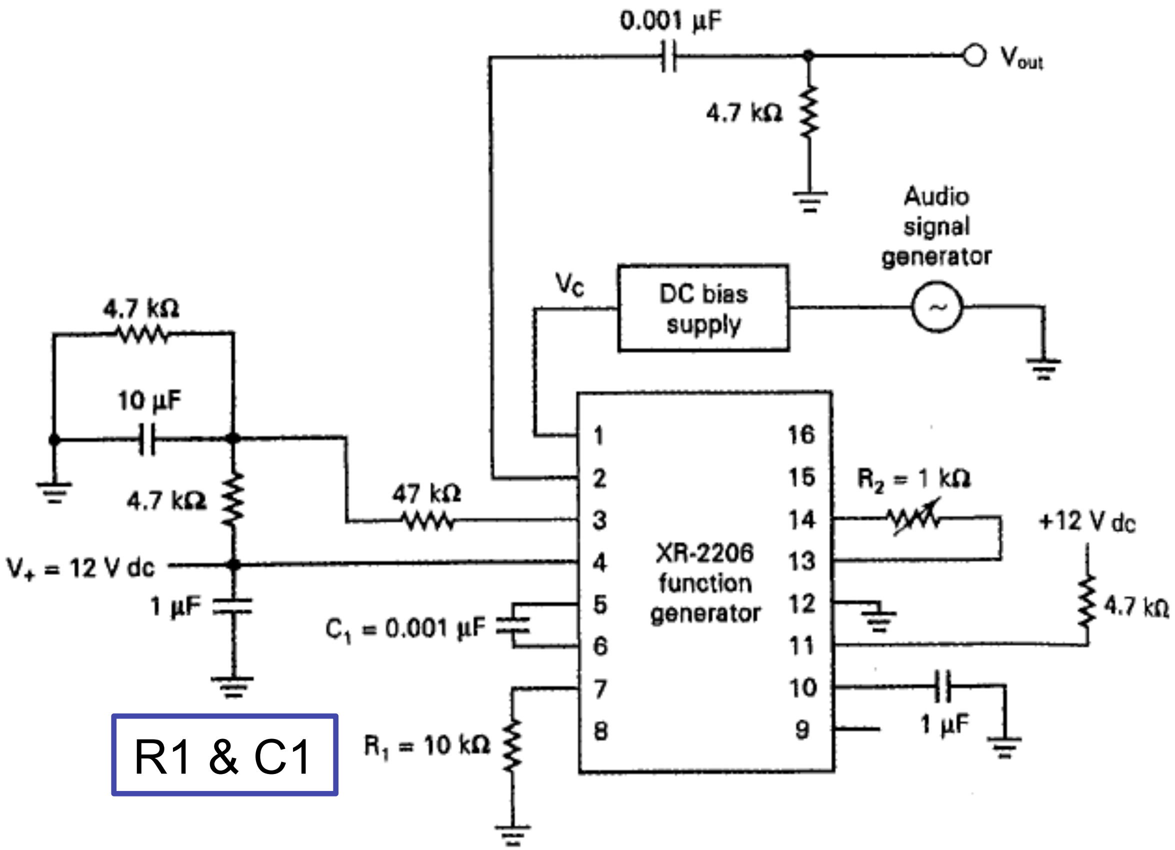
High-Level AM Modulators – Circuit Operation

- C_L and L_L tank can be added to act as Bandlimited
  - Only fc + fm and fc – fm can be transmitted



AM Modulators – Using Integrated Devices

- XR-2206 is an integrated circuit function generator
- In this case $fc=1/R_1C_1$ Hz
- For example in this case: if fm = 4kHz; fc = 100kHz

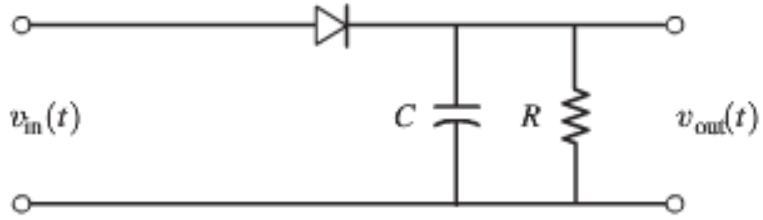


R1 & C1

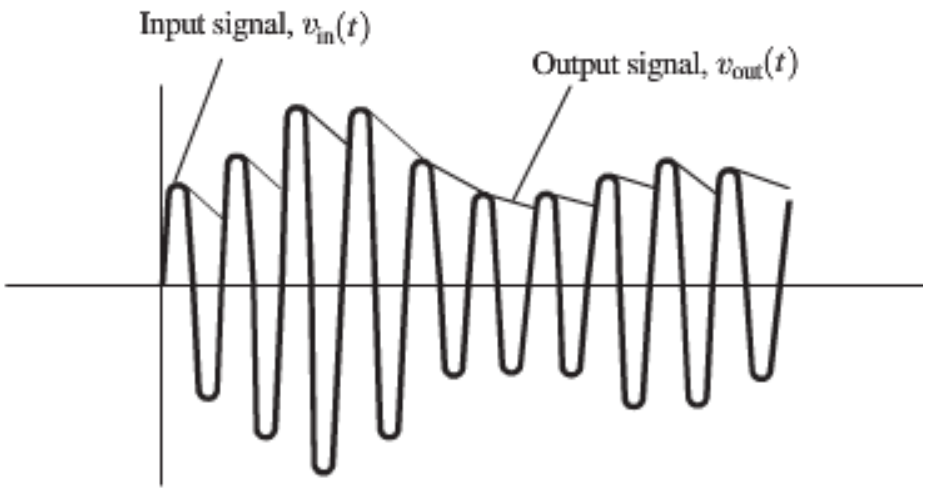
AM Demodulators: Envelope Detector

- Considered as **non-coherent** demodulators
- The diode acts as a **nonlinear** mixer
- Other **names**
  - Diode Detector
  - Peak Detector (Positive)
  - Envelope Detector
- Basic operation: Assume fc = 300 KHz and fm = 2KHz
  - Then there will be frequencies 298, 300, 302 KHz
  - The detector will detect many different frequencies
  - AM frequencies + AM harmonics + SUM of AM frequencies + DIFF of AM frequencies
  - The RC LPF is set to pass only DIFF frequencies

$$B \ll \frac{1}{2\pi RC} \ll f_c$$



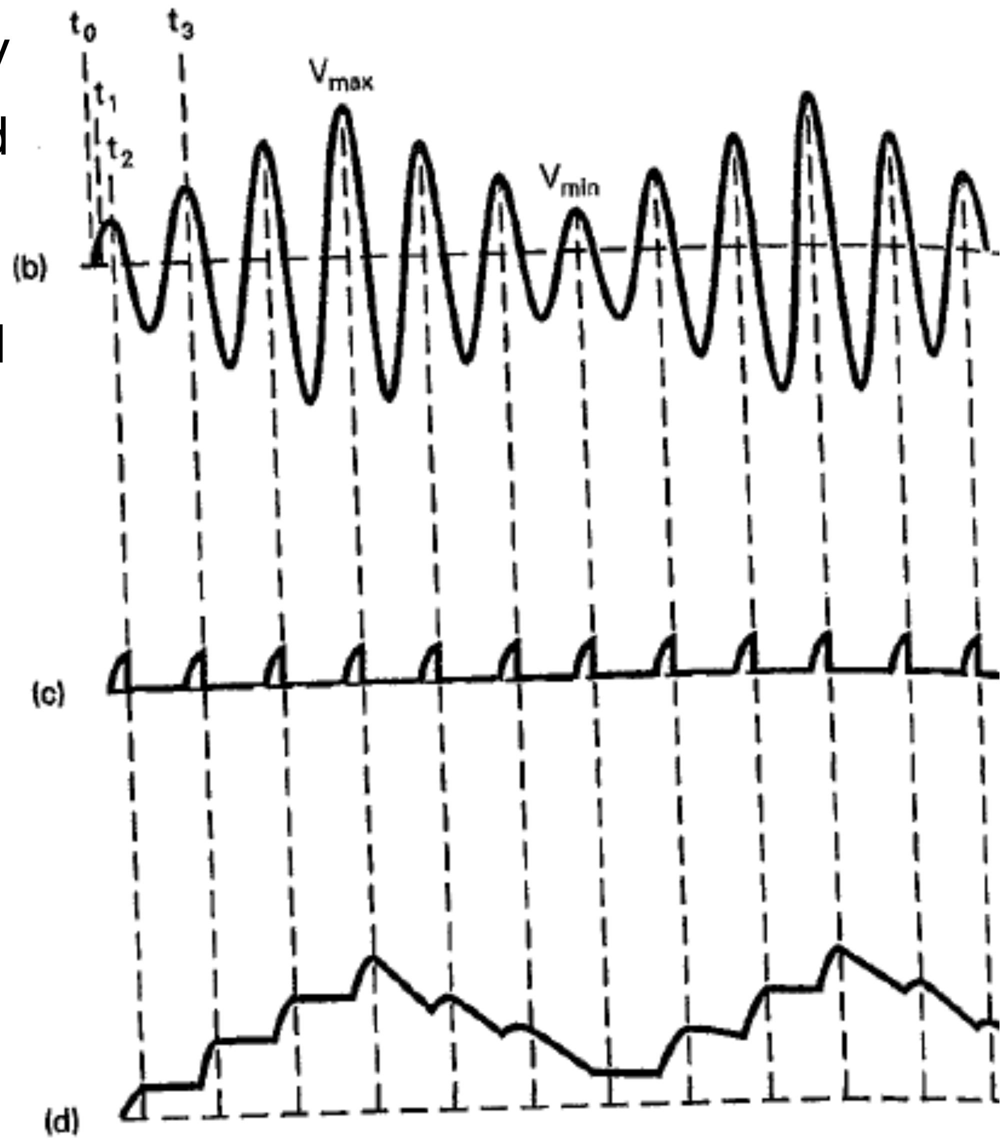
(a) A Diode Envelope Detector



(b) Waveforms Associated with the Diode Envelope Detector

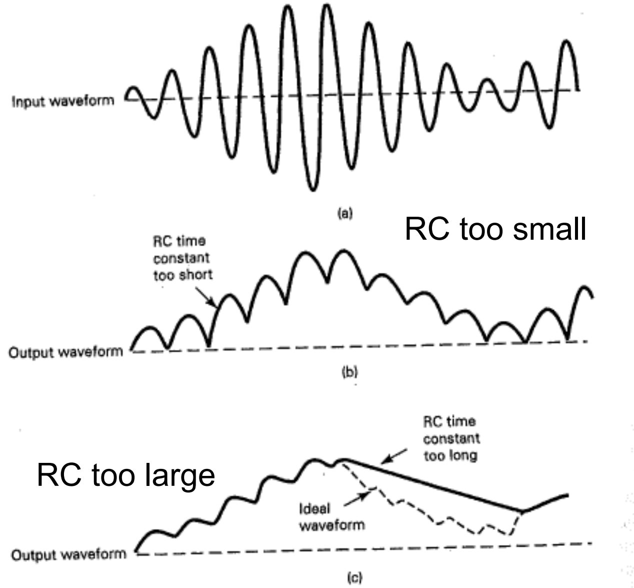
Envelope Detector – Basic Operation

- The diode has $V_{barrier} = V_b = 0.3V$
- When $V_{in} < V_b$ → Reverse Biased → DIODE is OFF
  - → $i_d = 0$ → $V_{cap} = 0$
- When $V_{in} > V_b$ → Forward Biased → DIODE is ON
  - → $i_d > 0$ → $V_{cap} = V_{in} - 0.3$



Envelope Detector – Distortion

- What should be the value of RC?
  - If too low then discharges too fast
  - If too high the envelope will be distorted
  - The highest modulating signal:



$$f_{m(\max)} = \frac{\sqrt{(1/m^2)-1}}{2\pi RC}$$

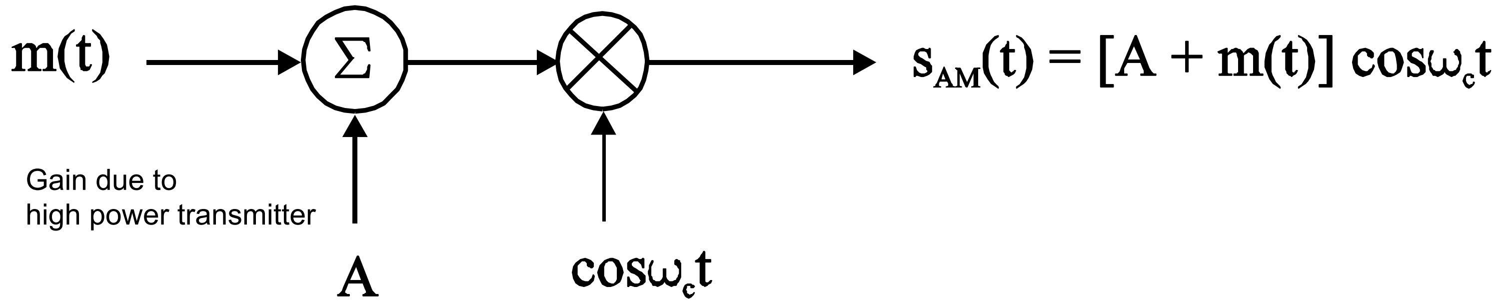
  - Note that in most cases m=0.70 or 70 percent of modulation →

$$f_{m(\max)} = \frac{1}{2\pi RC}$$

$$B \ll \frac{1}{2\pi RC} \ll f_c$$

Standard (Ordinary) AM

AM signal generation



Waveform :

$$s_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t = [A + m(t)]\cos\omega_c t$$

Spectrum :

$$S_{AM}(\omega) = (1/2)[M(\omega + \omega_c) + M(\omega - \omega_c)] + \pi A[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)]$$

Standard (Ordinary) AM

- The disadvantage of high cost receiver circuit of the DSB-SC system can be solved by use of AM, but at the price of a less <u>efficient transmitter</u>

- An AM system transmits a **large power carrier** wave, $A\cos\omega_c t$, along with the modulated signal, $m(t)\cos\omega_c t$, so that there is no need to generate a carrier at the receiver.
  - Advantage : simple and low cost receiver

- *In a broadcast system, the transmitter is associated with a large number of low cost receivers. The AM system is therefore preferred for this type of application.*

References

- Leon W. Couch II, Digital and Analog Communication Systems, 8th edition, Pearson / Prentice, Chapter 5
- Electronic Communications System: Fundamentals Through Advanced, Fifth Edition by Wayne Tomasi – Chapter 4 & 5

  (https://www.goodreads.com/book/show/209442.Electronic_Communications_System)

See
Notes