NCA-6: Tutorial on Adaptive Filtering:

With applications for Active Control

Jeffrey S. Vipperman (jsv@pitt.edu)

Assoc. Professor of Mech. Engr.

Director, ***Sound, Systems, and Structures Laboratory***

University of Pittsburgh

Pittsburgh, PA 15228

IMECE '06

Nov. 9, 2006

Chicago, IL

SOUND, SYSTEMS & STRUCTURES LABORATORY

Outline

- Background on Active Control
- Linear Systems and Signal
- Digital Filters
- Adaptive Algorithms
- Some Practical Considerations

Active Control

- Add energy through a secondary source to cancel or dissipate energy from a primary or disturbance force
- Types of Active Control:
  - Active Noise Control (ANC) and Active Noise Reduction (ANR) headsets
  - Active Vibration Control (AVC)
  - Active Structural-Acoustic Control (ASAC)

Active Versus Passive Control

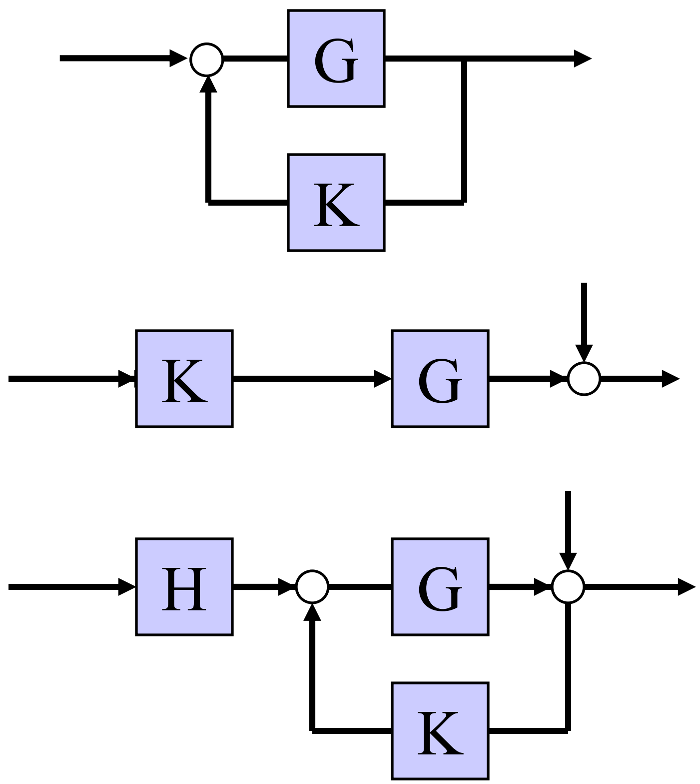
Passive

- (+) Works Well at High Frequencies (> 500 Hz)
- (+) No System Model Required
- (–) Typically heavier (CLD, Insulation, lead, etc.)
- Simple, but some knowledge of system/physics still required
- (+) Generally failsafe
- (+) Requires no power

Active

- (+) Works Well at Low Frequencies (< 500 Hz)
- (–) System Model may be required
- (+) Weight Savings can often be gained
- (–) Increased Complexity & Cost (transducers, computation, electronics)
- (–) May fail
- (–) Requires power

Control Topologies



- Feedback (FB) - Uses a measured system variable to create control signal
- Feedforward (FF) - Uses a coherent signal to create control signal
- "Hybrid" FB + FF Blends the best of both worlds

Feedback Versus Feedback Control

Feedback

- (–) System may become unstable
- (+) Controls unknown disturbances
- (+) Better for random or impulsive disturbances
- (n) Typically fixed gain; adaptive possible, but harder
- (–) Performance often limited to modest increases in damping
- (–) Often requires a System ID or model of plant

Feedfoward

- (+) System always Stable
- (–) Requires a priori, coherent reference signal
- (+) Excellent for harmonic and deterministic disturbances
- (+) Very amenable to adaptive methods
- (+) Complete attenuation possible (at point, plane waves)
- (–) Can also require System ID or model (for FXLMS)

Additional Features of Feedforward Control

- Most popular for adaptive control
- Potential for complete control of plane waves (duct) or at a point in space, since many signals are deterministic
- Global control in 2 or 3 dimensions is difficult
- Requires Coherent Reference Signal (deterministic or *a priori* knowledge)
- System is always stable, but adaptive algorithms can go unstable
- Represents the bulk of commercial applications:
  - (Active Headsets: Sony, NCT, BBN, aviation headsets);
  - Active mounts for vibration isolation (DC-9, car engines);
  - Active noise control (high end autos - sound system, and aircraft Saab 340b).
- Works by Superposition

Physics of Feedfoward Noise Control – 1-d Noise Fields

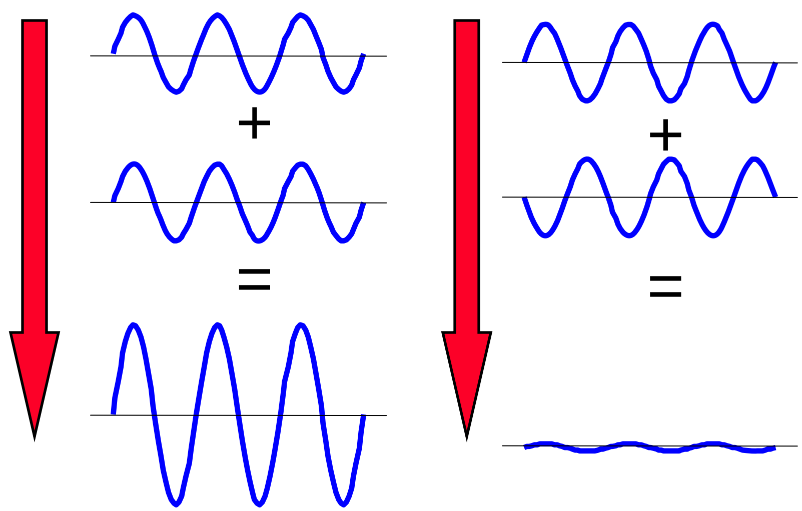
Cancellation of a plane wave or 1-d field in an ∞ duct

Constructive Interference | Destructive Interference

Primary Field:

Secondary Field:

Resulting Field:

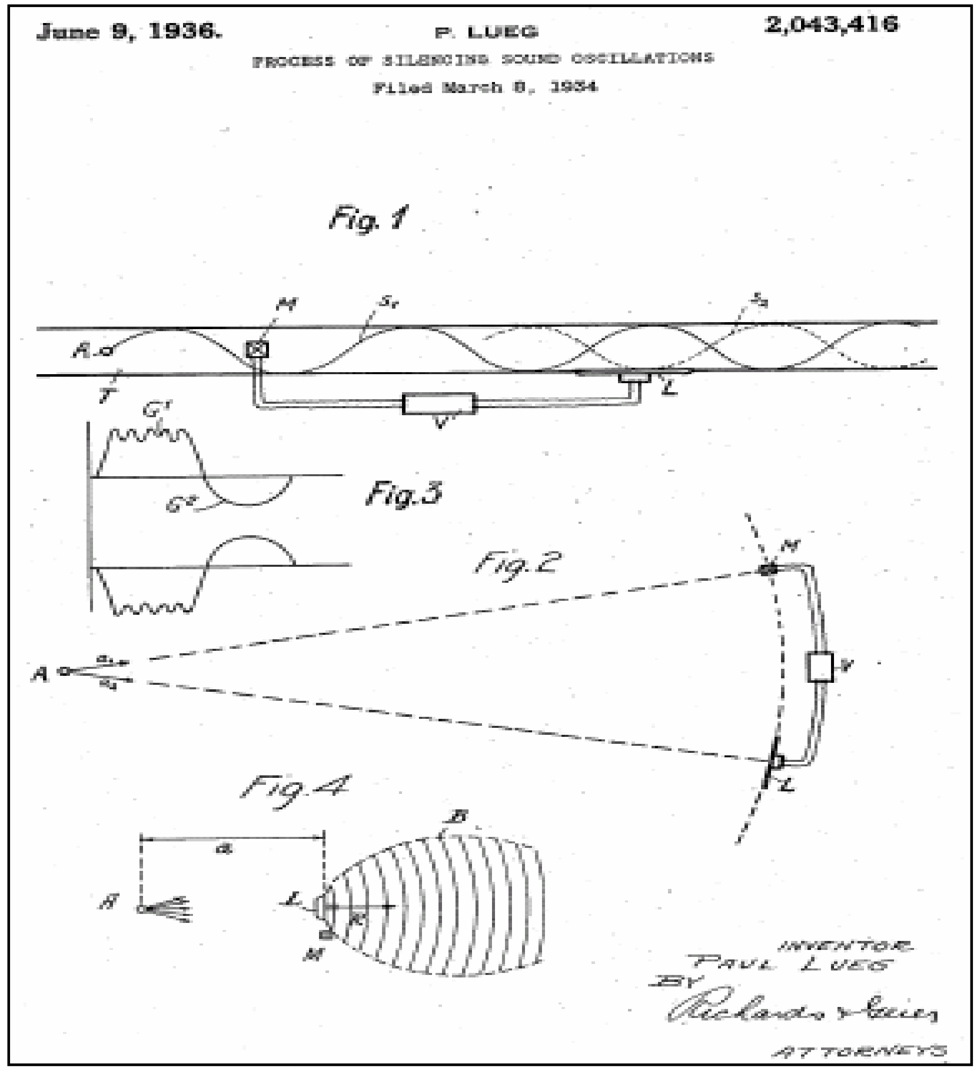


Requirements:

1. Need Coherent Reference Signal
2. Frequency must be below the cut-off frequency of the duct (1-d plane waves) roughly $c/(2*L)$, where L is the largest cross sectional dim.

First Active Control Patent 1936

Paul Leug (1936). Process of Silencing Sound Oscillations, US Patent No. 2,043,416



SOUND, SYSTEMS & STRUCTURES LABORATORY

Physics of Feedfoward Noise Control – 2d Noise Fields

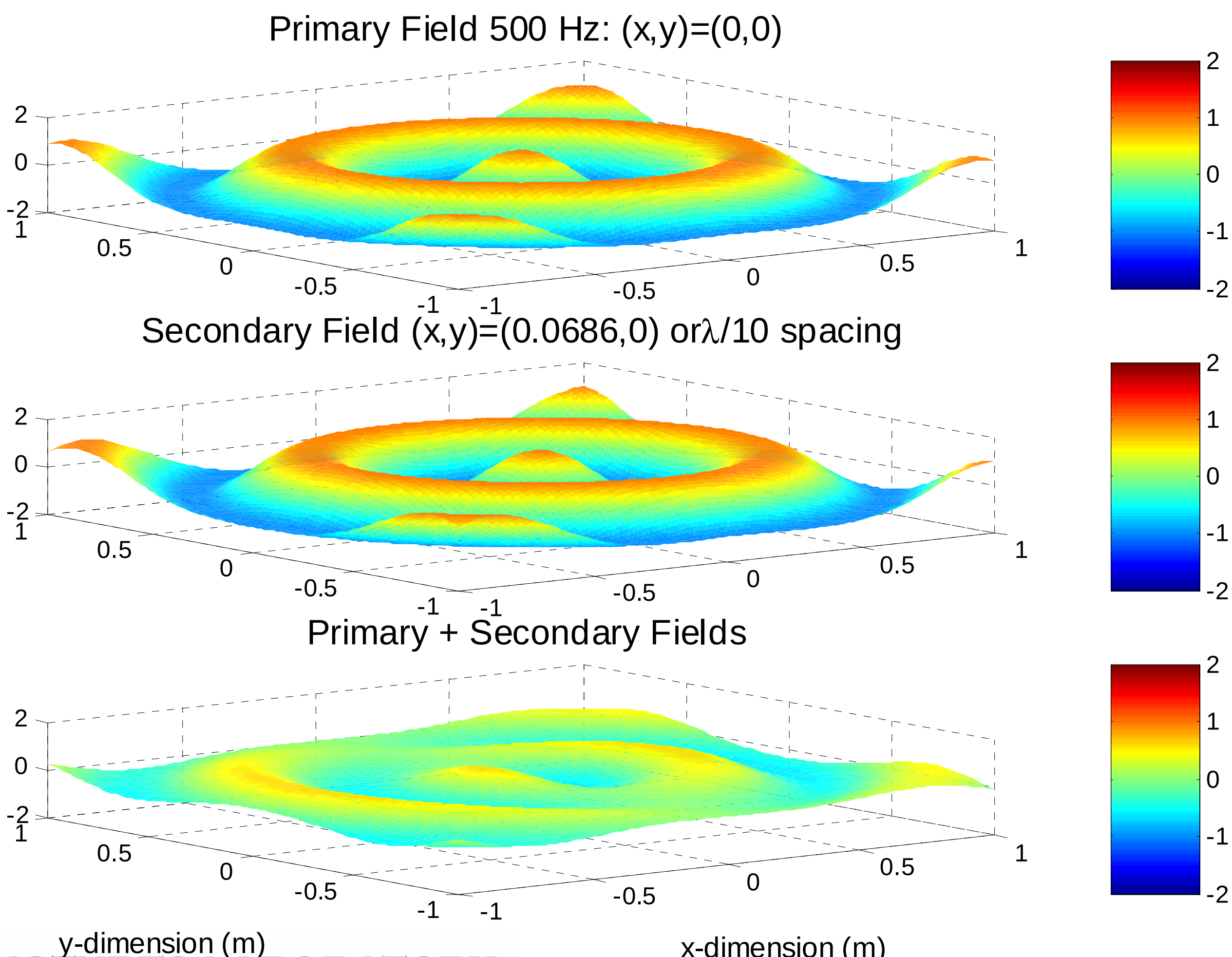
Primary Field

+

Secondary Field (separated by $\lambda/10$)

||

Total Field



SOUND, SYSTEMS & STRUCTURES LABORATORY

Physics of Feedfoward Noise Control – 2d Noise Fields

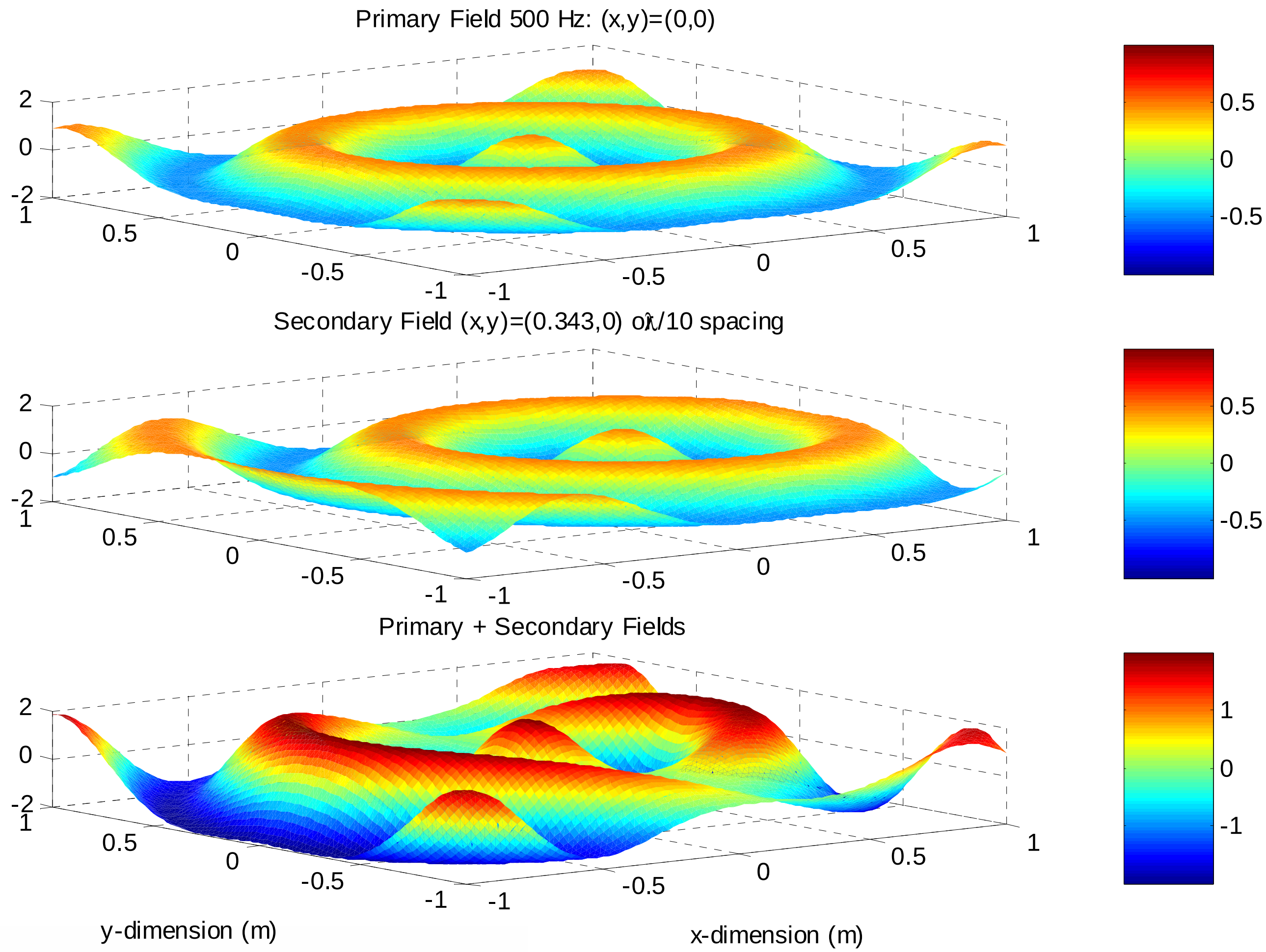
Primary Field

+

Secondary Field (separated by $\lambda/2$)

||

Total Field

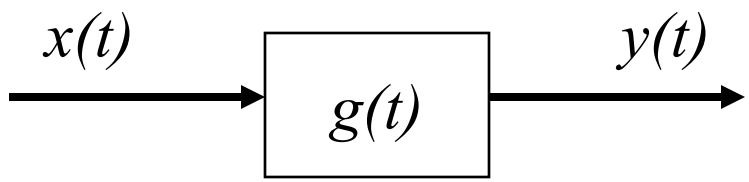


SOUND, SYSTEMS & STRUCTURES LABORATORY

Feedforward Control Requirements

- Spacing between primary and secondary sources must be $\leq$ 1/3-1/6 of a wavelength apart for global control
- Thus, physics limits performance to low frequency (<500 Hz)
- Control of 3-d spaces become more difficult – at best, notch out zones of silence (e.g. around the head of a passenger)
- Coherent, *apriori* reference signal

Signals and Linear Systems: Time Domain

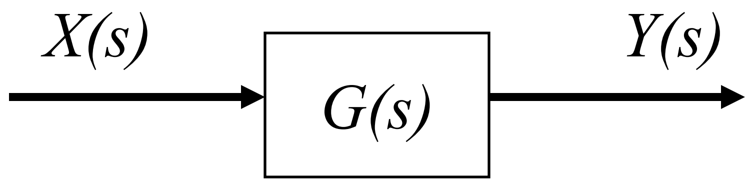


x(t) – input signal
g(t) – impulse response of linear system
y(t) – output of system

$$y(t) = g(t) * x(t) = \int_0^t g(t-\tau)h(\tau)d\tau \quad \text{(time domain)}$$

Convolution integral – continuous time filtering process

Signals and Linear Systems: Laplace Domain



$$X(s) = L[x(t)] \quad \text{Laplace transform}$$

$$Y(s) = L[y(t)]$$

$$G(s) = \frac{ouput}{input} = \frac{Y(s)}{X(s)} \quad \text{(Transfer Function)}$$

$$\text{or} \quad G(s) = L[g(t)]$$

$$\text{Also} \quad G(\omega) = \Im[g(t)] = G(s)\big|_{s=j\omega} \quad \text{(FRF)}$$

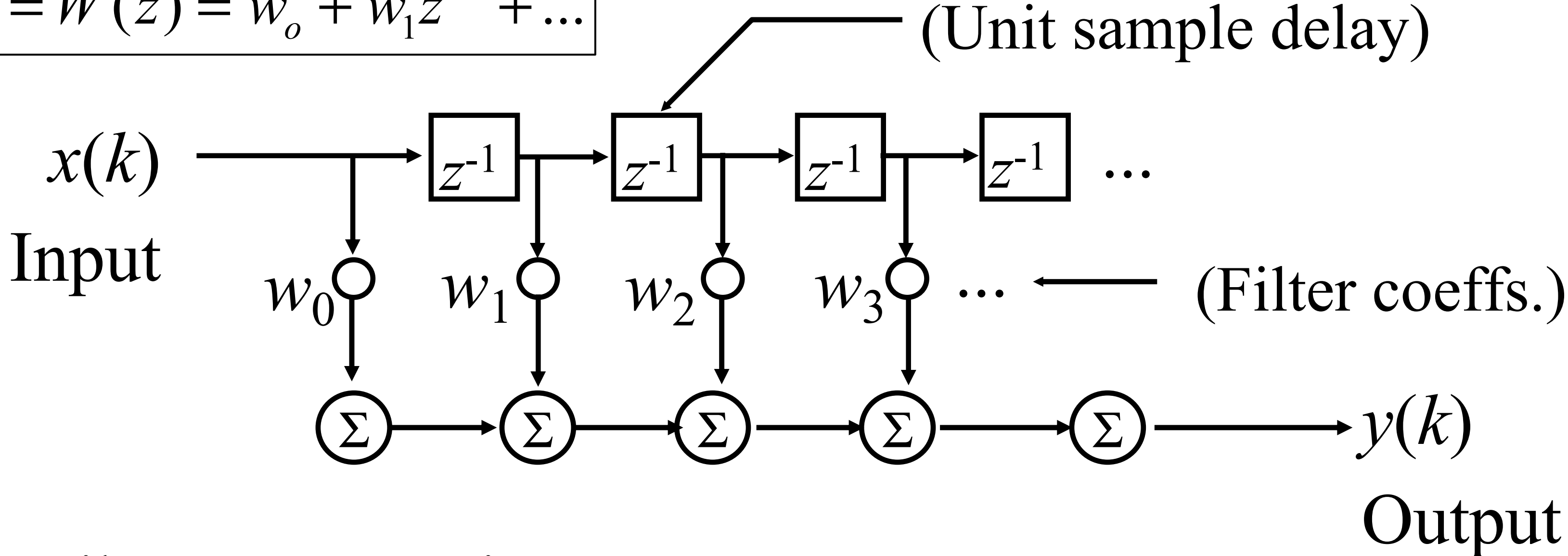
Relationship between Discrete and Continuous Time Signals

	Time Domain	"Frequency" Domain
Continuous	$x(t)$ —Laplace Transform→	$X(s)$
Discrete	$x(k)$ ——z-transform——→	$X(z)$

z-transform: $X(z) = Z[x(k)] = \sum_{k=0}^{M} x(k) z^{-k}$

Finite Impulse Response (FIR) Digital Filter a.k.a. Moving Average (MA)

$$\boxed{G(z) = W(z) = w_o + w_1 z^{-1} + \ldots}$$



Filter representations:

$W(z) = w_0 + w_1 z^{-1} + w_2 z^{-2} + \ldots$ (z-domain)

$\{W\} = \{w_0 \;\; w_1 \;\; w_2 \ldots\}$ (vector rep.)

FIRs, Cont.

Filter Output Representations:

$y(k) = w(k) * x(k)$ (time-domain, convolution)

$Y(z) = W(z)X(z)$ (z-domain, multiplication)

$y(k) = \{\boldsymbol{W}\}\{\boldsymbol{X}\}^T$ (vector multiplication)

Unit Impulse Response, $h(k)$

$h(k) = w(k)$, Impulse response is finite and equal to the number of filter coefficients, $w(k)$

Frequency Response of FIR

- Discrete Fourier Transform (DFT)

$$\mathrm{DFT}[k^{-1}] = e^{-j\omega T}$$

where:

k^{-1} is unit sample delay

T is sample period

ω is frequency of excitation

j is $(-1)^{1/2}$

Frequency Response, cont

- Example, 2-coefficient FIR:

$$W(z) = w_0 + w_1 z^{-1}$$

$$W(\omega) = w_0 + w_1 e^{-j\omega T}$$

$$= [w_0 + w_1\cos(\omega T)] + j\,[w_1 \sin(\omega T)]$$

$$= \mathrm{Re}[W(\omega)] + j\,\mathrm{Im}[W(\omega)]$$

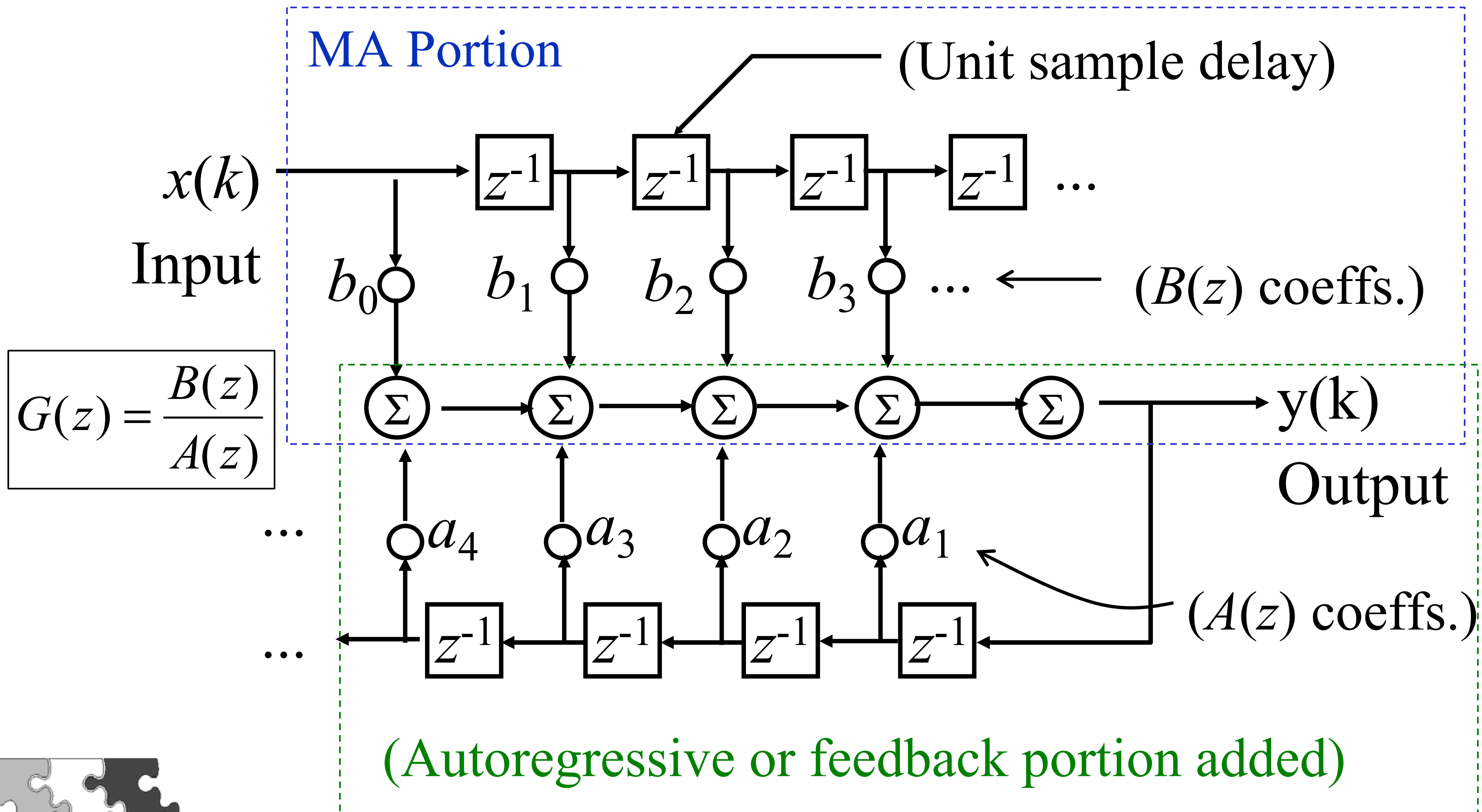
- From complex math:

$$\mathrm{Mag} = [\mathrm{Re}^2 + \mathrm{Im}^2]^{½}$$

$$\mathrm{Phase} = -\tan^{-1}[\mathrm{Im}/\mathrm{Re}]$$ (lin. phase – prop. delays)

- A 2-coefficient FIR filter can change the phase and magnitude at a single frequency!

Infinite Impulse Response (IIR) Digital Filter
a.k.a. Auto-regressive, Moving Average (ARMA)



IIRs, cont.

Filter, W(z)

$$W(z) = \frac{B(z)}{A(z)}$$

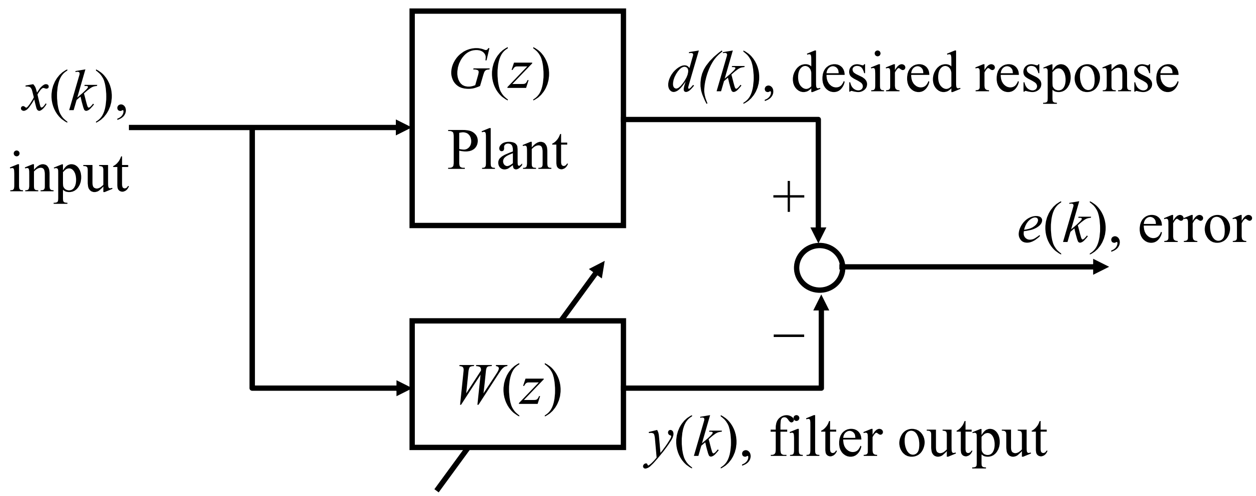
$$W(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}}{1 + a_1^{z-1} + a_2 z^{-2} + \ldots a_n z^{-n}}$$

Notes

- Poles introduce stability requirement
- Impulse response, $h(k)$ is infinite in duration due to FB
- An infinite length FIR has an infinite impulse response, but numerical effects limit the practical length to a few hundred coefficients
- Most adaptive feedforward control systems use FIRs since they are inherently stable

Example: Finding the Optimal Filter Solution, W_{opt}

System Identification Problem:



$$\text{As } \; e(k) \rightarrow 0, \quad W(z) \rightarrow G(z)$$

$$e(k) = d(k) - y(k)$$

$$e(k) = d(k) - w(k) * x(k) = d(k) - \{W\}\{X\}^T$$

Finding the Optimal Filter Solution, W_{opt}, cont.

Consider:

- The filter output, $y(k)=\{X\}\{W\}^T$, is linear combination of the filter weight vector $\{W\}$
- Assuming stationarity, the error is characterized statistically
- A "cost function from the mean-square error (MSE) as:

$$C=\mathrm{E}[e(k)^2]$$

where

E[·] represents the expected value, which for N samples is:

$$E\left[e(k)^2\right]=\sum_{k=1}^{N} e(k)^2$$

Finding the Optimal Filter Solution, W_{opt}, cont.

- The MSE, C=E[(d(k)-{X}{W}^{T})^2], is a quadratic function of the filter weights, {W}
- There is a unique minimum of C corresponding to $W_{\mathrm{opt}}(k)$
- The minimum is found through differentiation:

$$\frac{\partial C}{\partial\{W\}} = \{0\}$$

- Solve the above equation for $\{W\}_{\mathrm{opt}}$
- In real life, we don't have the luxury of the equation for C

SOUND, SYSTEMS & STRUCTURES LABORATORY

Finding the Optimal Filter Solution, W_{opt}, cont.

$$\frac{\partial C}{\partial\{W\}} = \frac{\partial E\left[\left(d(k) - \{W\}\{x\}^T\right)^2\right]}{\partial\{W\}} = 0$$

$$\frac{\partial C}{\partial\{W\}} = 2E\left[-\left(d(k)\{X\}^T\right) + \{W\}\{X\}^T\{X\}\right] = 0$$

$$\frac{\partial C}{\partial\{W\}} = -2\{P\} + 2\{W\}[R] = \{0\} \quad \text{(gradient)}$$

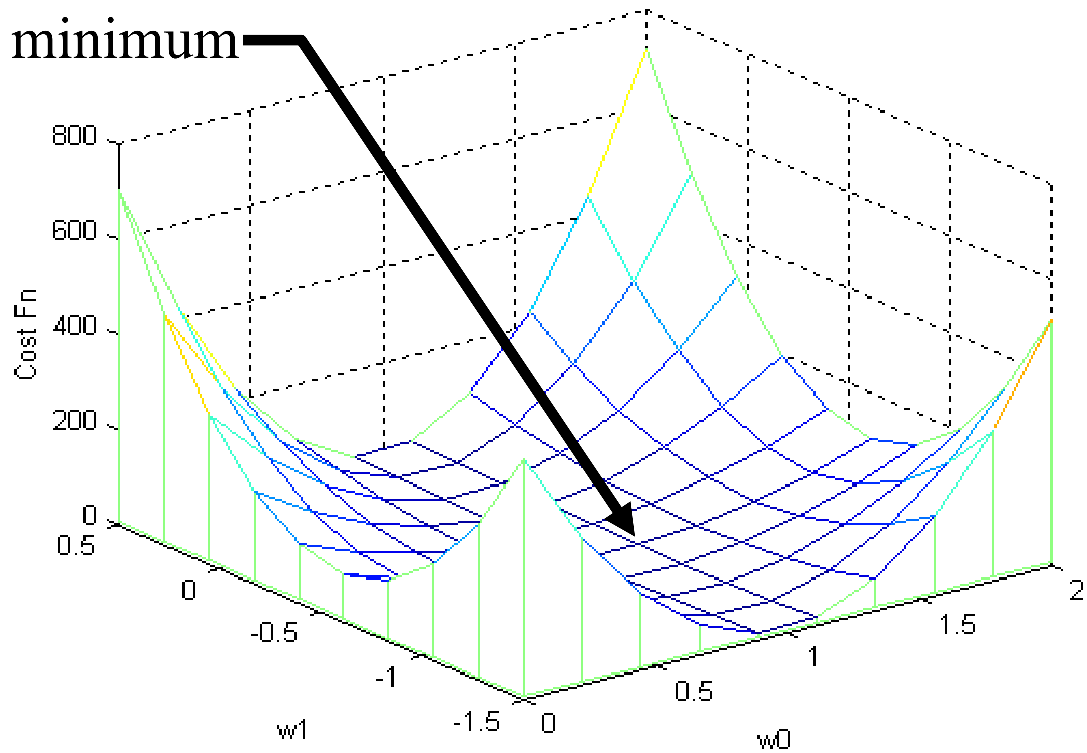
Solving for $\{W_{opt}\}$, $\boxed{\{\mathrm{W}_{\mathrm{opt}}\} = [R]^{-1}\ \{P\}}$

- $[R]$ is the input autocorrelation matrix (symmetric Toeplitz, positive definite)
- $\{P\}$ is the cross correlation vector

Performance or Error Surface

A 2-Coefficient FIR Filter Perf. Surf. is 3-d:

{Wopt} is at the minimum



Adaptive algorithms
search along this surface

Adaptive Minimization Methods (Finding $\{W_{opt}\}$)

1. Random Search
2. Adaptive Newton's Method
3. Time Averaged Gradient (TAG)
4. RLS
5. Steepest Descent
6. Least Mean Square (LMS)
7. Variations of LMS (FXLMS, PC-LMS)

1. Random Search

- Perturb each filter coefficient, w_i, and see if the measured cost, $C(\{W\})=E[e(k)^2]$, increases or decreases
- If $C(\{W\})$ decreases, keep the change
- Remember the direction of perturbation and the result

Adaptive Minimization Methods, cont.

2. Discrete form of Newton's Method

$$w_i(k+1) = w_i(k) - \frac{1}{2}[R]^{-1} C'(k)$$

$$w_i(k+1) = w_i(k) - \frac{C'(k)}{C''(k)}$$

- One Iteration to the Optimal Solution
- Don't know $C(k)$ explicitly, so must estimate $C'(k)$ and $C''(k)$ in practice
- Could compute C' and C'' from expected value definitions, but they are noisy
- Must ensure that C'' *is* always positive-valued
- Forms the basis of Time Averaged Gradient (TAG) algorithm

Adaptive Minimization Methods, cont.

3. Time Averaged Gradient (TAG)

$$w_i(k+1) = w_i(k) - \mu \frac{\delta w_i \left[C(w_i + \delta w_i) - C(w_i - \delta w_i) \right]}{2\left[C(w_i + \delta w_i) - 2C(w_i) + C(w_i - \delta w_i) \right]}$$

- Measure cost function, *C*, at three points:

  $\{w_i - \delta w_i,\ \ w_i,\ \ w_i + \delta w_i\}$

- μ controls convergence rate and stability
- δw_i must be large enough for accuracy but small enough not to excessively excite the plant
- This is an approximation to Newton's Method
- No System ID required!
- Typically much slower to converge though - particularly for MIMO

Adaptive Minimization Methods, cont.

Recursive Least Squares (RLS)

- Complex
- Fast Convergence
- Computational intensive (L^2 calculations for filter with L coefficients

Adaptive Minimization Methods, cont.

5. Steepest Descent

$$w_i(k+1) = w_i(k) - \frac{\mu}{2}\frac{\partial C(k)}{\partial W}$$

$$\{W\}(k+1) = \{W\}(k) - \mu\left([R]\{W\}(k) - \{P\}\right)$$

$$\{W\}(k+1) = \left([I] - \mu[R]\right)\{W\}(k) - \mu\{P\}$$

- μ is again a learning rate parameter that controls convergence rate and stability of the adaptive algorithm
- March down the gradient towards the optimal solution $\{W_{opt}\}$

Adaptive Minimization Methods, cont.

6. LMS Algorithm

$$w_i(k+1) = w_i(k) - 2\mu e(k)x(k)$$

- This is actually Steepest Decent with a stochastic approximation to the gradient
- μ controls convergence rate and stability
- $0 < \mu < 2/\lambda_{max}$, λ_{max} is max eig.value of $[R]$
- March down the gradient (in a very noisy manner) towards the optimal solution $\{W_{opt}\}$
- Converges ("noisily") to the optimal Wiener filter solution
- Excess MSE as a result of noise
- Very simple and robust

Adaptive Control Using PC-LMS

7. Filter-*x* LMS (FXLMS) Algorithm for System Control:

- Will examine in more detail soon
- The plant dynamics in the control path, $G_c(z)$, affect weight update

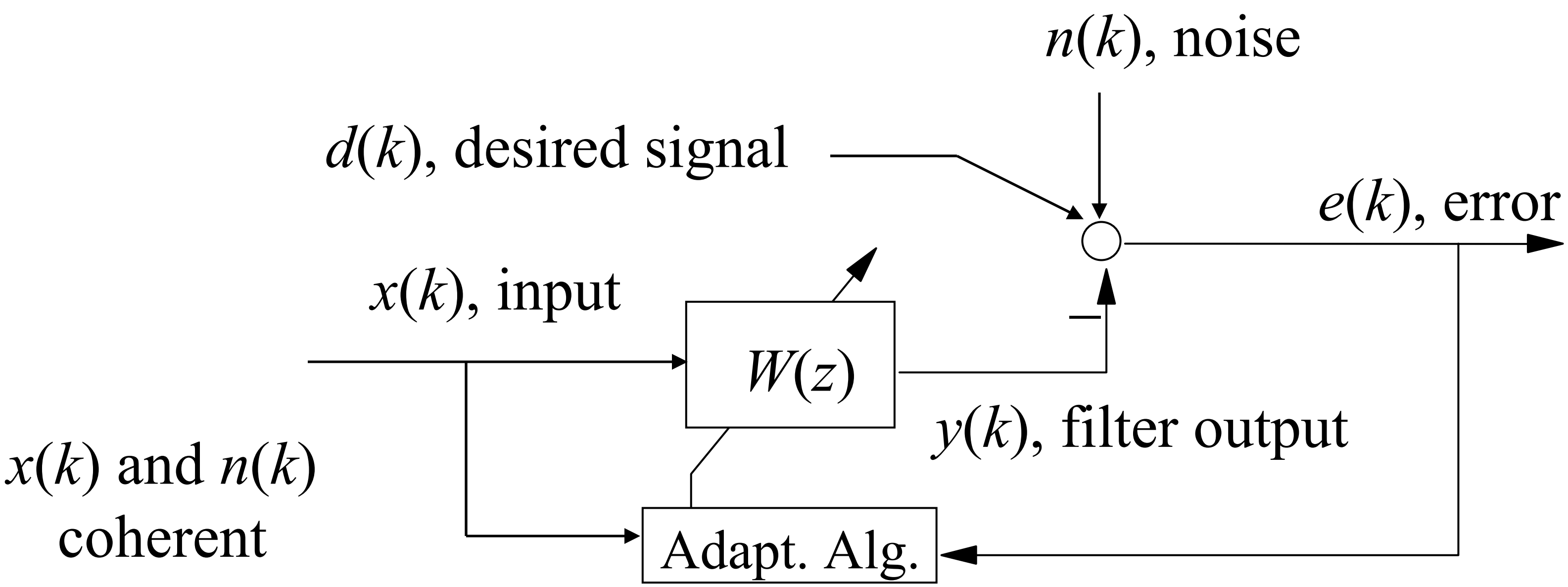
$$w_i(k+1) = w_i(k) + \mu 2 e(k) x_{fx}(k)$$

$$x_{fx}(k) = g_c(k) * x(k)$$

- Sign change in update
- $x_{fx}(k)$ is the "Filtered-*x*" (filtered ref.) signal

Uses for Adaptive Signal Processing, cont.

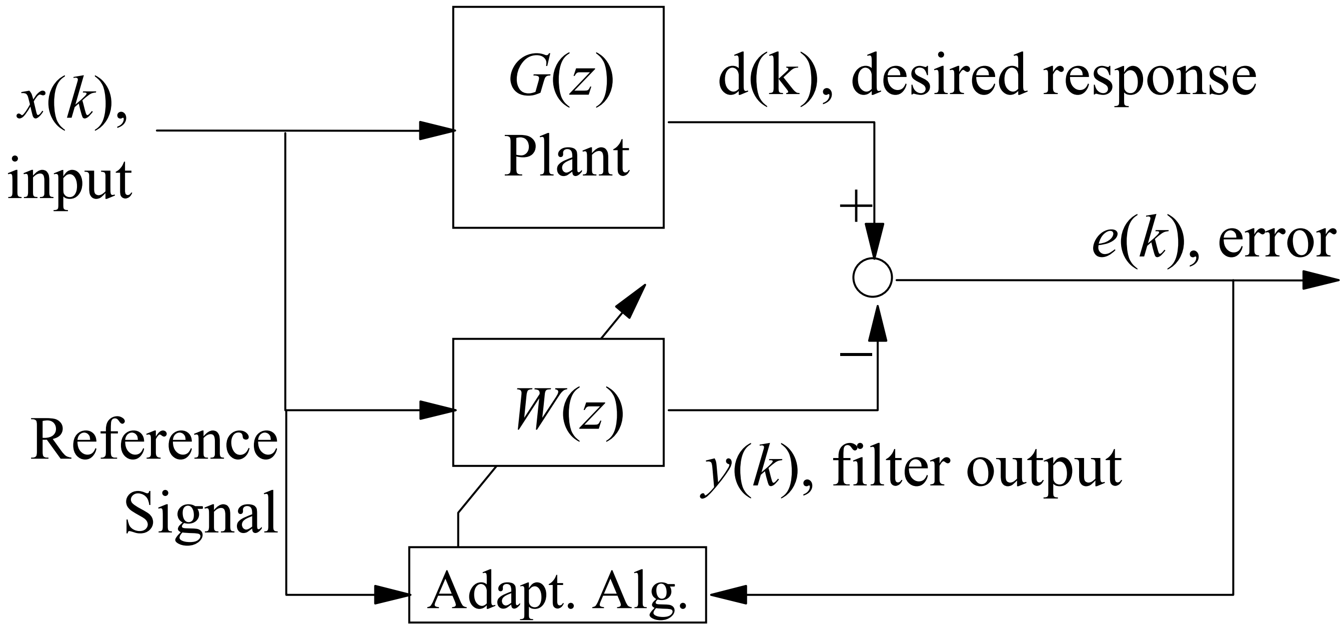
Disturbance/Echo cancellation



$$\text{As minimize } e(k), \quad e(k) \rightarrow d(k)$$

Uses for Adaptive Signal Processing, cont.

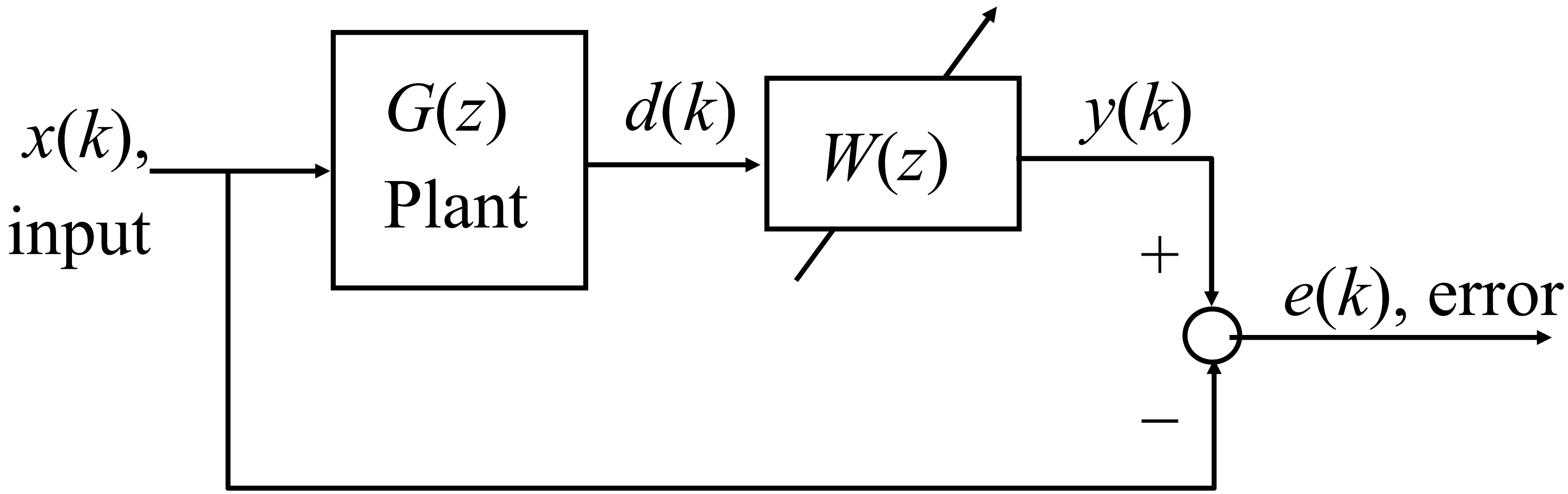
System Identification:



$$As \quad e(k) \rightarrow 0, \qquad W(z) \rightarrow W_{opt}(z) = G(z)$$

Uses for Adaptive Signal Processing, cont.

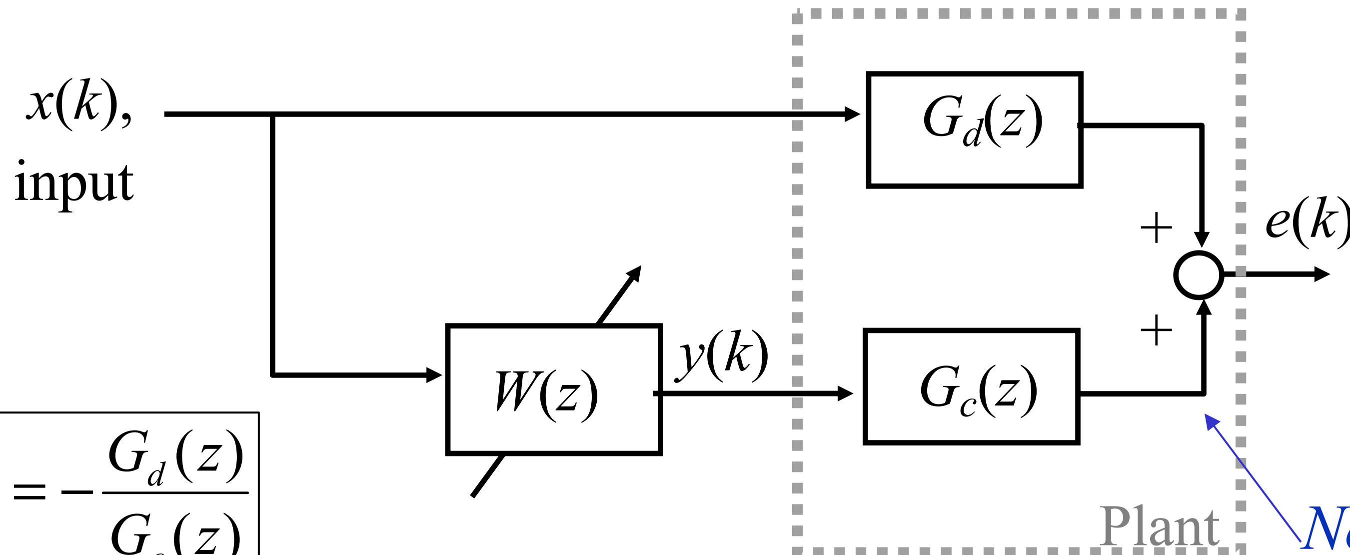
Inverse Modeling:



$$\text{As } e(k) \to 0, \;\; W(z) \to W_{opt}(z) = G(z)^{-1}$$

Adaptive Control Using FX-LMS

FXLMS Algorithm for Adaptive Control:



$$Wopt(z) = -\frac{G_d(z)}{G_c(z)}$$

Need for sign change in LMS update

$$E(z) = G_d(z)X(z) + G_c(z)W(z)X(z)$$

$$\frac{\partial E(z)}{\partial W(z)} = G_c(z)X(z) \longleftarrow$$ *Need for filtered-x signal*

SOUND, SYSTEMS & STRUCTURES LABORATORY

Adaptive Control Using FXLMS, cont.

- LMS update equation becomes:

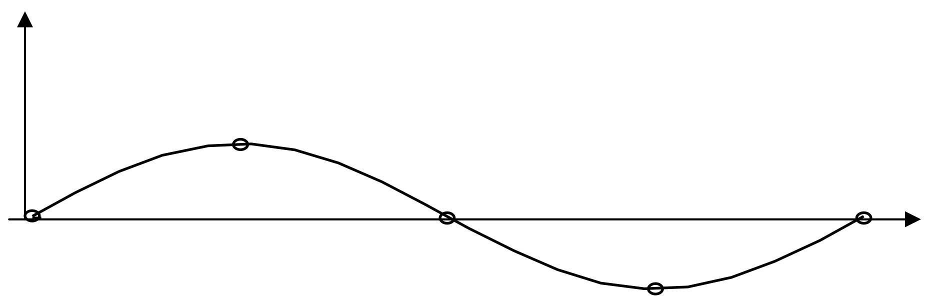
$$w_i(k+1) = w_i(k) + \mu 2 e(k) x_{fx}(k)$$

where x_{fx}(k) = $g_c(k) * x(k)$ is the so-called "filtered-x" signal

- Note the gradient based on control path transfer function, $G_c(z)$
- Therefore a model or system ID (SID) of the control path is required to implement
- SID can be *a priori*, or simultaneous with control
- Model dependent, but really only need the correct phase of the $Gc(z)$ path

Controlling Harmonic Signals

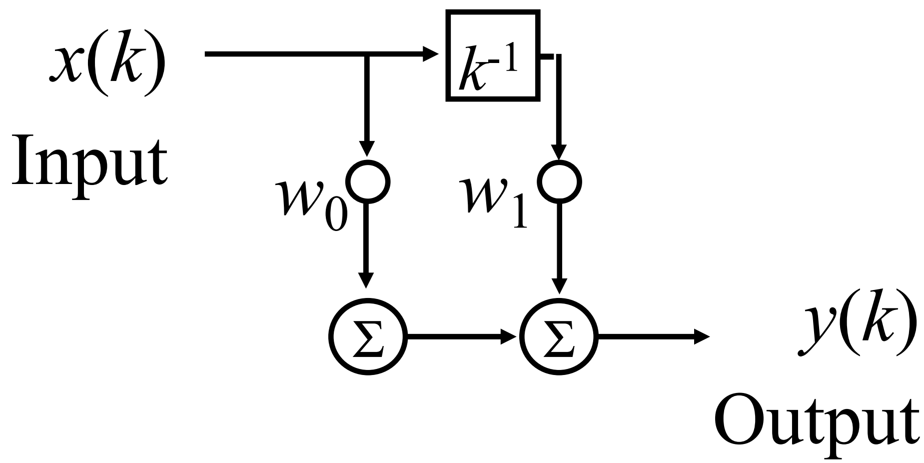
- A 2-coefficient FIR filter can change the magnitude and phase of a sine wave
- If 4x oversampling or separate sine and cosine ref. signals are used, then the two filter coeffs are orthogonal

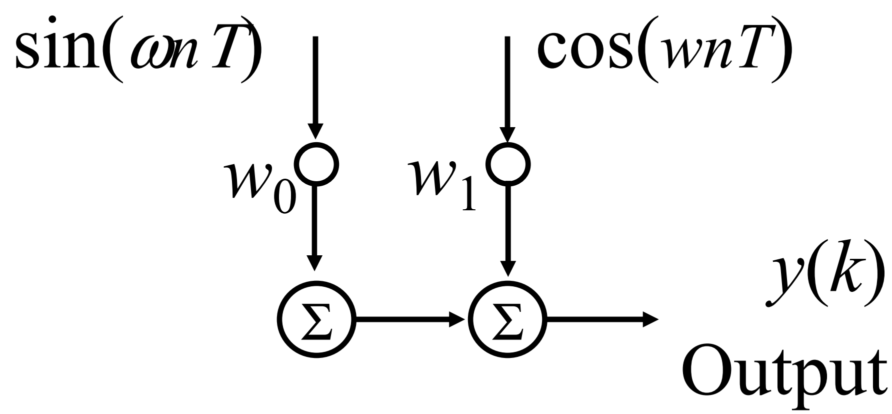


- Can use a Hilbert filter to perform quadrature on a signal

$$A\cos(\omega t) + B\sin(\omega t) = M\sin(\omega t + \phi)$$

Orthogonal filter





Controlling Multiple Frequencies

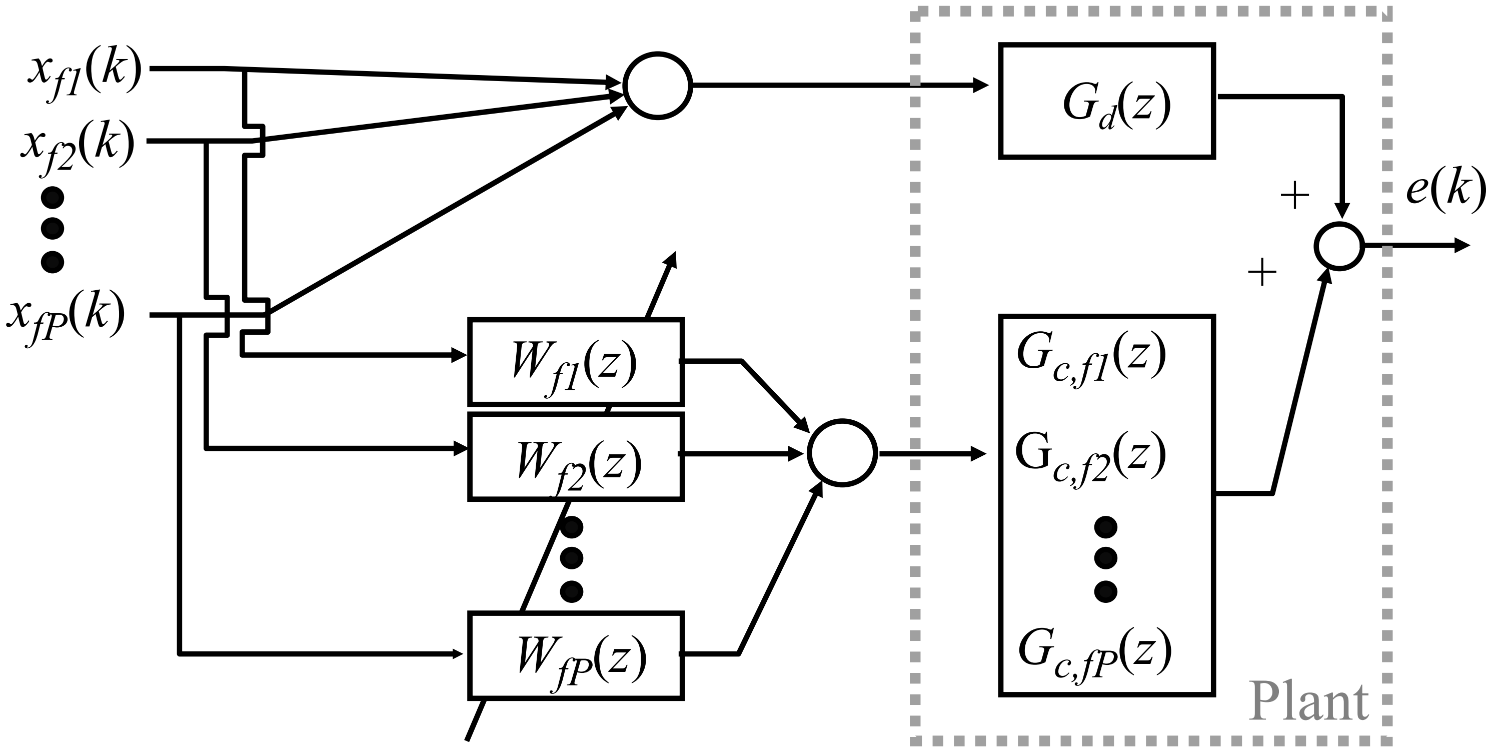
- Two distinct ways to perform multi-frequency control:
  (a) with large FIR filters or
  (b) with parallel 2-coefficient filters

(a) Large FIR filters (easier but not as good)

- Use an FIR filter having twice the number of coefficients as there are frequencies
- Careful! This only works for higher-harmonic control when the sampling rate is twice the frequency of the highest harmonic (bad idea - 4x oversampling is ideal)
- Otherwise, there are Time-varying parts to the LMS solution which will oscillate about the Wiener solution
- Use more filter coefficients to compensate
- Sum all frequencies to make the reference signal

Multi-frequency Adaptive Control Using FX-LMS, cont.

Using *P*-Parrallel 2-Coefficient FIR filters (Best way)

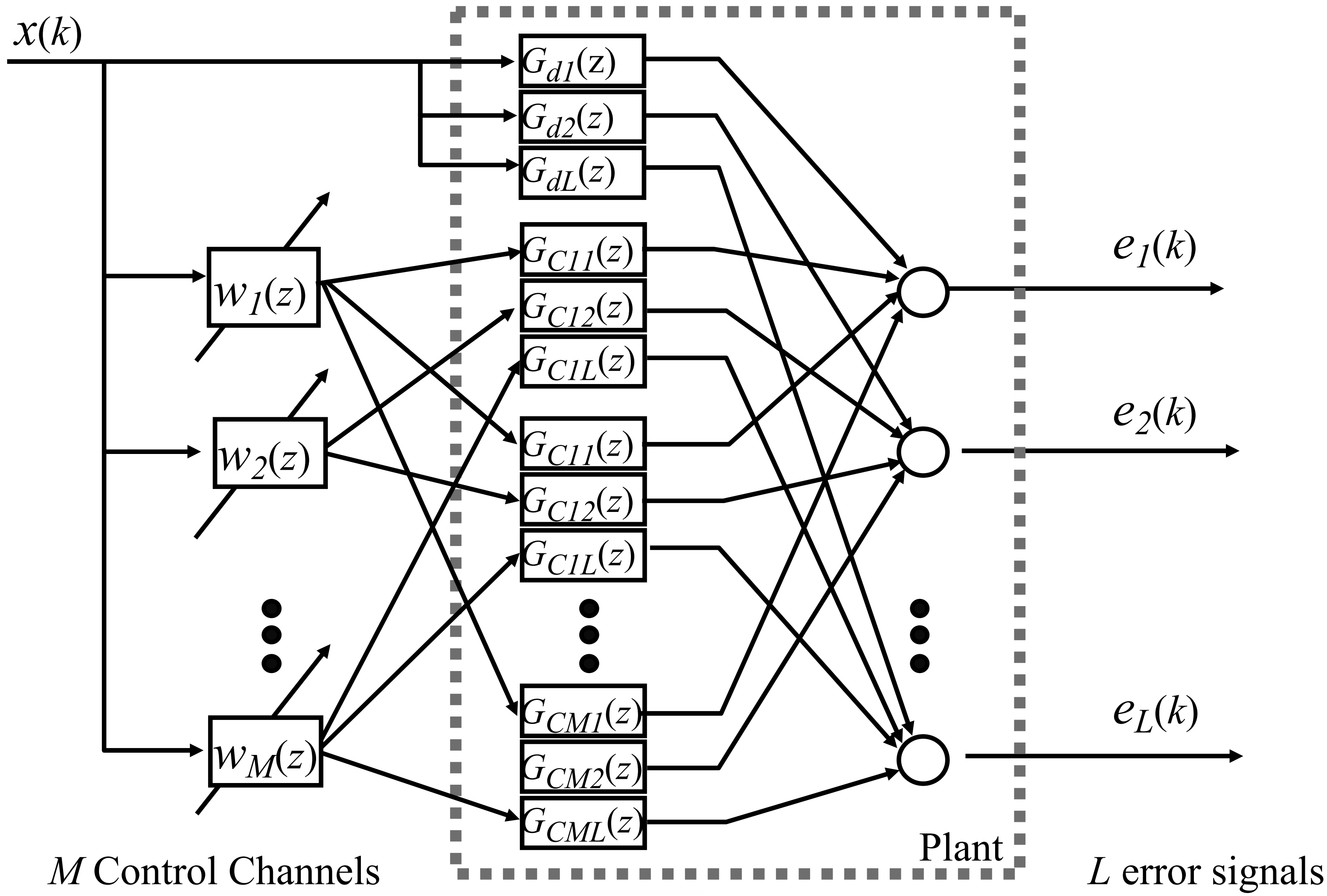


(Controlling *P* harmonics of the fundamental frequency)

Controlling Broadband (Random) Signals

- Use a large (20-200 coefficient) FIR filter for the compensator, $W(z)$
- If the plant is non-reverberant (e.g. high-damping, acoustic free-field, etc.) an FIR can be used to model $G_c(z)$ since it will essentially be a pure delay
- For reverberant systems, an IIR is required to model the control path, $G_c(z)$
- May require IIR for control filter
- The reference signal must be known ahead of time (i.e. the Wiener solution must be "causal")
- Otherwise, control is limited to the deterministic portion of the plant response
- Can also get transient control (training an issue)

Multichannel FX-LMS Control



Multichannel FX-LMS, cont.

- Define $C=\mathrm{E}[e_1(k)^2 + e_2(k)^2 + \ldots + e_L(k)^2]$
- $L \geq M$, otherwise an overdetermined solution results
- Choose L (number of error sensors) very large, especially for acoustic control in order to get representative energy of system
- Must identify $L \cdot M$ control path transfer functions
- Weight update for each of the M filters depends on all L error signals and L filtered-x signals as

$$w_i(k+1) = w_i(k) - 2\mu \sum_{j=1}^{L} e_j(k) x(k) * g_{c,ij}(k), \quad i = 1,2,\ldots M$$

Principle Component LMS (PCLMS)

<u>Principal Component LMS (PC-LMS) Algorithm for Control:</u>

- Perform SVD to decouple each control channel

$$Gc = V\, S\, U^H$$

- Compute control signals and update compensator, $\nu(k)$, in PC's and convert back to physical coordinates before sending out of dig. sig. proc. (DSP) board

$Gc = U\, S\, V^H$	(SVD)
$\zeta(k) = U^H\, e(k)$	(PC - error)
$\nu(k) = V^H\, w(k)$	(PC-filter coefficient)

Adaptive Control Using PC-LMS, cont.

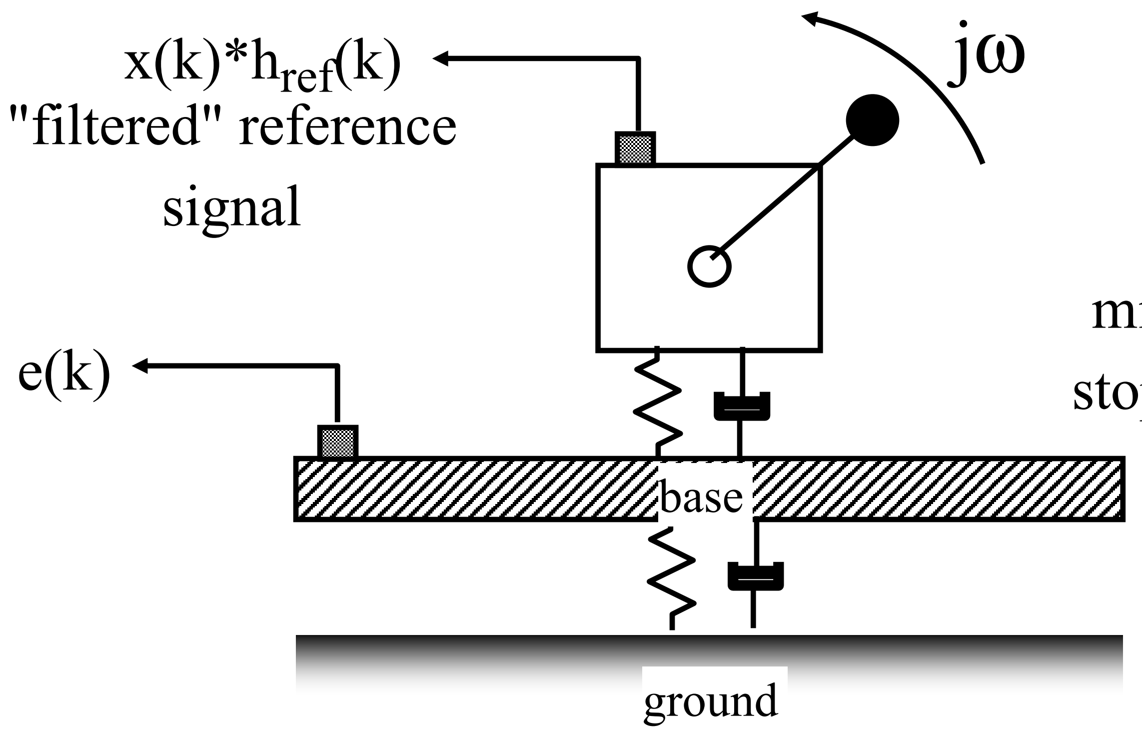
- Limited to single and multifrequency control, since $Gc(z)$ is represented as a matrix of complex numbers at each frequency
- After SVD, find 2-coefficient FIR filters for V and U^H
- Less computationally intensive than FXLMS
- Can easily incorporate control effort penalties into the weight update (like LQR control)

Obtaining the Coherent Reference Signal

- Use optical encoded disk
- Tachometer
- Directional mics (duct)
  - FB removal techniques (filtering, phase-locked loop devices)
  - Sometimes reference is filtered by other dynamics, but still coherent with disturbance
  - Internally generate deterministic signals
  - Use an “upstream” sensor with little feedback

Obtaining the Coherent Reference Signal

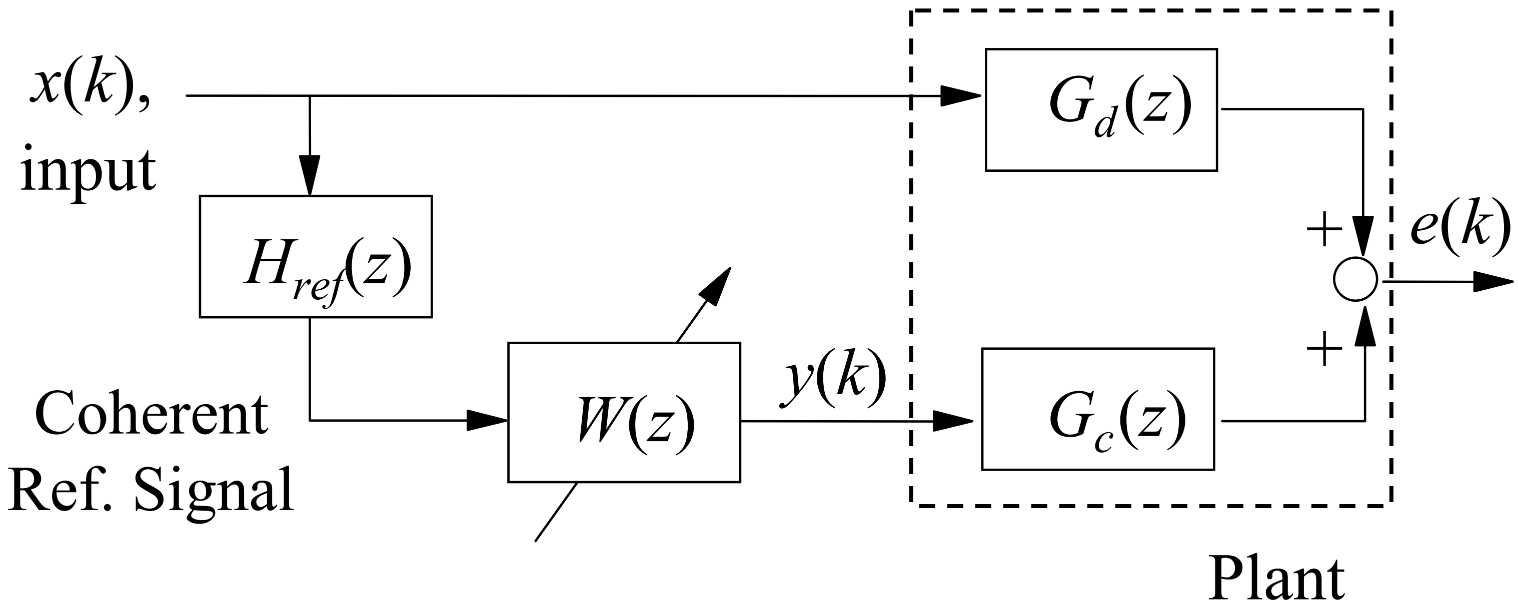
Active Vibration Isolation Problem:



minimize e(k) to stop the base from moving

Uses for Adaptive Signal Processing, cont.

Feedforward System Control with filtered ref. sig.



$$\text{As } e(k) \to 0, W(z) \to W_{opt}(z) = \frac{G_d(z)}{H_{ref}(z) G_c(z)}$$

•Requires knowledge of $G_c(z)$ to update adaptive filter, $W(z)$

Other Issues for Active Control

- Stability of adaptive algorithms
- Robust stability of controlled plant
- Robust performance and reliability
- Cost/benefit
- Take advantage of existing infrastructure (e.g. car audio systems)?
- Control is often only as good as your model and your algorithm and your transducers and ...