A theoretical framework for LMS MIMO communication systems performance analysis

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***Abstract*— A statistical model for Land Mobile Satellite (LMS) channels, where transmitters and receivers are equipped with multiple antennas, is introduced. In addition to the probability density function (pdf) of the channel matrix grammian, spectral statistics are given. They allow the theoretical performance analysis of the newly proposed channel model from both a communication and an information-theoretic point of view. More specifically, relying on recent results on finite-dimensional random matrix theory, the statistics of an unordered eigenvalue and of the maximum eigenvalue of the channel matrix grammian are evaluated in closed form. The eigenanalysis paves the way for the calculus of the outage probability for adaptive transmission strategies. Moreover, it permits the computation of the ergodic mutual information in the coherent case. The results are illustrated through several examples, aimed at assessing the impact on the performance of the diversity order and/or the Line-of-Sight (LOS) fluctuations.**

I. Introduction

Due to the growing interest in satellite personal communications, several models for the statistics of the LMS channel have been proposed during the last years. Channel measurements have confirmed the presence in the received signal of a fluctuating LOS component whose first order statistical characterization has been studied in [1]. Due to the LOS randomness, such fading model has been referred to as Shadowed-Rice (SR). In [2], a thorough analysis has been conducted on real data, showing that LOS random fluctuations are conveniently modeled by the Gamma distribution. Relying on these results, the pdf and the moments of the received signal amplitude, averaged over the LOS fluctuations, have been also provided. A further step toward the study of LMS channels has been done in [3], where the sum of independent as well as correlated SR random variables has been characterized, allowing diversity analysis in LMS channels. In [4], the capacity improvement of a multiple antenna LMS link with single satellite over a Single Input Single Output (SISO) channel is observed through measurements, confirming the convenience of resorting to spatial diversity at both the ends of the link. Moreover, random fluctuations in the (matrix) LOS component have been observed as well.

The promising performance improvement in terms of transmission rate of Multiple Input Multiple Output (MIMO) LMS systems has then offered the rationale for the present work, which is aimed at providing a comprehensive statistical characterization of these channels. First of all, the pdf of the channel matrix grammian is derived under the assumption of a matrix-Gamma distribution [5] for the grammian of the LOS. Then, relying on recent results on spectral statistics of finite dimensional random matrices [6], the marginal density distribution of an unordered eigenvalue and the Cumulative Distribution Function (CDF) of the maximum eigenvalue of the grammian are derived. Ergodic mutual information and outage probability are also given.

The paper is organized as follows: Section II introduces the channel model, deals with the problem of evaluating the distribution of a Shadowed Rice matrix variate, and of its eigenvalues' statistics. Section III assesses the performance of communication systems on MIMO SR fading channels. Conclusions are given in Section IV.

Throughout the paper, matrices are denoted by uppercase boldface letters, vectors by lowercase boldface; $E[\cdot]$ denotes statistical expectation, $(\cdot)^\dagger$ indicates the conjugate transpose operator, $|\cdot|$ and $\mathrm{Tr}(\cdot)$, respectively, the determinant and the trace of a square matrix, $\mathbf{V}(\cdot)$ stands for the Vandermonde determinant [7], and $||\cdot||$ for the euclidean norm.

II. Signal and Channel Model

Denoting by n and m the number of transmit and receive antennas, we consider the complex frequency-flat linear channel model $\mathbf{y} = \sqrt{\frac{\mathrm{g}}{\delta}}\mathbf{H}\,\mathbf{x} + \mathbf{n}$ where $\mathbf{x}$ and $\mathbf{y}$ are the input and output vectors, $\mathbf{H}$ is the $(m\times n)$ channel matrix, g is the average channel gain, the normalization factor δ ensures that

$$\frac{E[\mathrm{Tr}\{\mathbf{H}\mathbf{H}^\dagger\}]}{\delta} = mn, \qquad (1)$$

and $\mathbf{n}$ is white Gaussian noise. We only consider the impairments arising from the presence of the thermal noise, whose single-sided spectral density can be expressed as $N_0 = \frac{E[\|\mathbf{n}\|^2]}{m}$.

Since the fading encountered by wireless systems on terrestrial links tends to be either Rayleigh or Ricean, the entries of $\mathbf{H}$ can be modeled as jointly Gaussian. Hence, the statistical characterization of $\mathbf{H}$ entails simply determining the mean and the correlation between its entries. However, as observed for the single antenna case, the fading which affects the LMS channel can be adequately described by a SR model [2], namely a Rice fading with random LOS component. It is thus of interest to generalize this model to the multivariate scenario.

Since we are mainly interested in performance analysis of MIMO systems on multiantenna SR channels, we characterize

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the grammian[1] of the channel matrix, whose spectrum plays an essential role in evaluating information-theoretic and communication performance indexes [8] and [9]. From now on, we assume, without loss of generality, $m \le n$ (the generalization to the case $n < m$ is straightforward), define $\tau = n - m$, and give the following

Definition 1. *The $m \times m$ random matrix $\mathbf{W}$ is a noncentral complex Wishart matrix with n degrees of freedom ($n \ge m$) and noncentrality matrix $\mathbf{M}$, ($\mathbf{W} \sim \mathcal{W}_m(n, \mathbf{M}, \mathbf{\Sigma})$), if the joint distribution of its entries can be written as [10]*

$$f_{\mathbf{W}}(\mathbf{B}) = \frac{e^{-\mathrm{Tr}\left\{\mathbf{\Sigma}^{-1}(\mathbf{M}+\mathbf{B})\right\}}|\mathbf{B}|^{n-m}}{|\mathbf{\Sigma}|^n \Gamma_m(n)} {}_0F_1\big(n; \mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{\Sigma}^{-1}\mathbf{B}\big), \quad (2)$$

where ${}_0F_1(\cdot;\cdot)$ is the Bessel hypergeometric function of matrix argument [10] and $\Gamma_p(q)$, $p \le q$, is the complex multivariate Gamma function [10]

$$\Gamma_p(q) = \pi^{\frac{p(p-1)}{2}} \prod_{\ell=1}^{p} \Gamma(q - \ell + 1).$$

Definition 2. *The $m \times m$ random matrix $\mathbf{M}$ is a complex Gamma matrix with scalar parameter α ($\alpha \ge m$) and matrix parameter $\mathbf{\Omega}$, ($\mathbf{M} \sim \Gamma_m(\alpha, \mathbf{\Omega})$), if the joint distribution of its entries can be written as [5]*

$$f_{\mathbf{M}}(\mathbf{A}) = \frac{|\mathbf{A}|^{\alpha-m}|\mathbf{\Omega}|^{\alpha}}{\Gamma_m(\alpha)} \exp\left[-\mathrm{Tr}\left\{\mathbf{\Omega}\mathbf{A}\right\}\right]. \quad (3)$$

Notice that, for integer α, it reduces to a central Wishart matrix [10], with α degrees of freedom and parameter matrix $\mathbf{\Omega}^{-1}$.

According to the multivariate SR model, the $m \times n$ matrix $\mathbf{H}$ can be written as

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{H}_w \quad (4)$$

where the columns of $\mathbf{H}_w$ are zero-mean independent[2] complex circular Gaussian vectors with common covariance matrix $\mathbf{\Sigma}$, while $\bar{\mathbf{H}}$ is the random $m \times n$ LOS matrix, statistically independent on $\mathbf{H}_w$, whose grammian $\bar{\mathbf{H}}\bar{\mathbf{H}}^{\dagger} \sim \Gamma_m(\alpha, \mathbf{\Omega})$. Under these assumptions, the statistical distribution of $\mathbf{H}\mathbf{H}^{\dagger}$ is given by the following

Proposition 1. *The statistical distribution of the grammian matrix $\mathbf{W} = \mathbf{H}\mathbf{H}^{\dagger}$, where $\mathbf{H}$ is a SR matrix variate (4), can be written as*

$$f_{\mathbf{W}}(\mathbf{B}) = \frac{e^{-\mathrm{Tr}\left\{\mathbf{\Sigma}^{-1}\mathbf{B}\right\}}|\mathbf{B}|^{n-m}}{|\mathbf{\Sigma}|^n \Gamma_m(n)} \frac{|\mathbf{\Omega}|^{\alpha}}{|\mathbf{\Sigma}^{-1} + \mathbf{\Omega}|^{\alpha}} \times {}_1F_1\Big(\alpha; n; \mathbf{\Sigma}^{-1}\mathbf{B}\mathbf{\Sigma}^{-1}\left(\mathbf{\Sigma}^{-1} + \mathbf{\Omega}\right)^{-1}\Big). \quad (5)$$

Proof. Observe that, given $\bar{\mathbf{H}}$, $\mathbf{W} = \mathbf{H}\mathbf{H}^{\dagger} \sim \mathcal{W}_m(n, \mathbf{M}, \mathbf{\Sigma})$, where $\mathbf{M} = \bar{\mathbf{H}}\bar{\mathbf{H}}^{\dagger}$. Hence, the unconditional distribution of $\mathbf{W}$ can be obtained averaging (2) over $\mathbf{M}$, i.e.

$$f_{\mathbf{W}}(\mathbf{B}) = \int_{\mathbf{A}>0} f_{\mathbf{W}|\mathbf{M}}(\mathbf{B}|\mathbf{M} = \mathbf{A}) f_{\mathbf{M}}(\mathbf{A}) d\mathbf{A},$$

where the integration is taken over the space of hermitian positive definite matrices of dimension m and is performed with the help of [12, Formula 115]. Notice that, if $\alpha = n$, $\mathbf{W} \sim \mathcal{W}_m(n, \mathbf{\Sigma} + \mathbf{\Omega}^{-1})$.

In order to get handy expressions, from now on we set $\mathbf{\Sigma} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. This is tantamount to assuming that the multipath component exhibits a shorter decorrelation distance than the LOS component.

Corollary 1. *The joint distribution of the ordered eigenvalues $\{\lambda_1 > \lambda_2 > \ldots > \lambda_m\}$ of $\mathbf{W}$ (5) can be expressed as*

$$f_{\mathbf{\Lambda}}(\mathbf{\Lambda}) = K e^{-\mathrm{Tr}\{\mathbf{\Lambda}\}}|\mathbf{\Lambda}|^{n-m}|\boldsymbol{F}| \prod_{k<\ell}^{m} (\lambda_k - \lambda_\ell) \quad (6)$$

with $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_m\}$, $\boldsymbol{F}_{i,j} = {}_1F_1\left(\alpha - m + 1; n - m + 1; \frac{\lambda_j}{1+\omega_i}\right)$,

$$K = \frac{\prod_{k=1}^{m-1}\left[\frac{(\alpha-m+k)}{k(n-m+k)}\right]^{k-m}}{\mathbf{V}\left((\mathbf{I}+\mathbf{\Omega})^{-1}\right)} \prod_{\ell=1}^{m} \frac{\omega_\ell^{\alpha}(1+\omega_\ell)^{-\alpha}}{(n-\ell)!(m-\ell)!},$$

and ω_i the i-th ordered eigenvalue of $\mathbf{\Omega}$.

Proof. The joint distribution of the ordered eigenvalues of a complex random matrix $\mathbf{W}$ is given by [10, Formula 93]

$$\prod_{k<\ell}^{m} (\lambda_k - \lambda_\ell)^2 \frac{\pi^{m(m-1)}}{\Gamma_m(m)} \int_{\mathcal{U}(m)} f_{\mathbf{W}}\left(\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\dagger}\right) d\mathbf{U}.$$

where $\mathcal{U}(m)$ stands for the unitary group of size m. The above integral can be evaluated applying the splitting formula [10, Formula 92]

$$\int_{\mathcal{U}(m)} {}_pF_q\left(a_1, \ldots, a_p; b_1, \ldots, b_q; \mathbf{A}\mathbf{U}\mathbf{B}\mathbf{U}^{\dagger}\right) d\mathbf{U} = {}_pF_q\left(a_1, \ldots, a_p; b_1, \ldots, b_q; \mathbf{A}, \mathbf{B}\right),$$

which yields

$$f_{\mathbf{\Lambda}}(\mathbf{\Lambda}) = \frac{{}_1F_1\big(\alpha; n; \mathbf{\Lambda}, (\mathbf{I}+\mathbf{\Omega})^{-1}\big)}{\Gamma_m(n)\Gamma_m(m) e^{\mathrm{Tr}\{\mathbf{\Lambda}\}}} \frac{\prod_{k<\ell}^{m}(\lambda_k - \lambda_\ell)^2 |\mathbf{\Lambda}|^{n-m}}{|\mathbf{I} + \mathbf{\Omega}^{-1}|^{\alpha}}.$$

We finally exploit the representation of hypergeometric functions of matrix arguments[3] in [13] to get (6).

Based on this finding, we can now characterize both the marginal density distribution of an unordered eigenvalue and the CDF of the maximum eigenvalue of a SR matrix grammian.

[1] We just recall that given a $m \times n$ matrix $\mathbf{A}$, its grammian is defined as the square matrix $\mathbf{A}\mathbf{A}^{\dagger}$.

[2] We implicitly adhere to the *separable* or Kronecker correlation model, (see e.g. [11] for a comprehensive presentation of widely used correlation models in multiantenna scenario), and assume the link to be correlated only at the more constrained end. It would adequately model a downlink scenario where the receiving mobile terminal is a handheld device. Indeed, due to space limitations, the number of receiving antennas is less than the number of transmit antennas. Moreover the closeness between the receiving sensors may cause a nonnegligible correlation.

[3] We consider the case where $\mathbf{\Omega}$ has distinct eigenvalues. However, such a requirement can be relaxed resorting to a proper perturbation technique [13]. For sake of brevity, we omit here the analysis.

Proposition 2. *The marginal density distribution of an unordered eigenvalue of* $\mathbf{W}$ *(5) is given by*

$$f_\lambda(\lambda) = \widetilde{K} \sum_{i,j=1}^{m} \mathcal{D}(i,j) \lambda^{n-m+j-1} e^{-\lambda} \tag{7}$$
$$\times {}_1F_1\left(\alpha - m + 1; n - m + 1; \frac{\lambda}{1+\omega_i}\right),$$

where $\widetilde{K} = \frac{K}{m}$ *and* $\mathcal{D}(i,j)$ *is the* (i,j)*-cofactor of the* $(m \times m)$ *matrix* $\mathbf{A}$ *whose* (k,ℓ)*-th entry equals*

$$\frac{{}_2F_1\left(n-m+\ell, \alpha-m+1; n-m+1; (1+\omega_k)^{-1}\right)}{[(n-m+\ell-1)!]^{-1}}.$$

Proof. The joint distribution of the unordered eigenvalues of $\mathbf{W}$, following the symmetry argument [14], can be obtained dividing (6) by $m!$ As a consequence, it can be expressed as

$$f_{\mathbf{\Lambda_u}}(\mathbf{\Lambda}) = K^* e^{-\mathrm{Tr}\{\mathbf{\Lambda}\}} |\mathbf{\Lambda}|^{n-m} |\boldsymbol{F}| \prod_{k<\ell}^{m} (\lambda_k - \lambda_\ell) \tag{8}$$

with $K^* = \frac{K}{m!}$. Let then $\mathbf{G}$ be the matrix whose (i,j)-th entry is $\lambda_i^{n-m+j-1}$, namely, the matrix whose determinant coincides with $|\mathbf{\Lambda}|^{n-m} \prod_{k<\ell}^{m} (\lambda_k - \lambda_\ell)$. We now substitute $|\mathbf{G}|$ in (8) and then exploit the Laplace determinant expansion, obtaining

$$\begin{aligned} f_{\mathbf{\Lambda_u}}(\mathbf{\Lambda}) = K^* &\sum_{i,j=1}^{m} (-1)^{i+j} \frac{\lambda_1^{\tau+j-1}}{e^{\lambda_1}} \\ &\times {}_1F_1\left(\alpha - m + 1; \tau + 1; \frac{\lambda_1}{1+\omega_i}\right) \\ &\times \prod_{k=2}^{m} e^{-\lambda_k} \, |\mathbf{G}_i| \, |\boldsymbol{F}_j| \end{aligned} \tag{9}$$

with $\boldsymbol{F}_j$ the $(m-1)\times(m-1)$ matrix obtained deleting the first row and the j-th column from $\boldsymbol{F}$, and $\mathbf{G}_i$ the $(m-1)\times(m-1)$ matrix obtained deleting the first row and the i-th column from $\mathbf{G}$. Eq. (7) stems from integration of (9) over $\lambda_2, \ldots, \lambda_m$ via Cauchy-Binet Lemma, and the entries of $\mathbf{A}$ are evaluated through [10, Formula 28]. Finally, we remark that the choice of λ_1 in (9) has no effect on the result since we started from an unordered distribution.

The performance analysis of adaptive transmission strategies requires the CDF of the maximum eigenvalue of the channel matrix grammian. In this concern, we prove the following

Proposition 3. *The CDF of the maximum eigenvalue of* $\mathbf{W}$ *(5) can be written as*

$$F_{\lambda_{max}}(x) = K |\mathbf{F}(x)|, \tag{10}$$

where the entries of the $m \times m$ *matrix* $\mathbf{F}(x)$ *are given by*

$$\begin{aligned} \mathbf{F}(x)_{i,j} = \Bigg[&\frac{{}_2F_1(\tau+j, \alpha-m+1; \tau+1; (1+\omega_i)^{-1})}{[(\tau+j-1)!]^{-1}} - \\ &(\tau+j-1)! \sum_{\ell=0}^{\tau+j-1} \frac{e^{-x} x^\ell}{\ell!} \times \\ \Phi_1 &\left(\alpha-m+1, \tau+j-\ell; \tau+1; (1+\omega_i)^{-1}; \frac{x}{(1+\omega_i)}\right) \Bigg], \end{aligned}$$

when $\alpha < n$, *with* $\Phi_1(\cdot)$ *the confluent hypergeometric function of two variables [15, 9.261.1]. When* $\alpha > n$,

$$\begin{aligned} \mathbf{F}(x)_{i,j} = \Bigg[&\frac{{}_2F_1\left(\tau+j, \alpha-m+1; \tau+1; (1+\omega_i)^{-1}\right)}{[(\tau+j-1)!]^{-1}} - \\ &\sum_{t=0}^{\infty} \frac{(\tau+j+t-1)!(\alpha-m+1)_t}{(\tau+1)_t t! (1+\omega_i)^t} \sum_{\ell=0}^{\tau+j+t-1} \frac{e^{-x} x^\ell}{\ell!} \Bigg]. \end{aligned}$$

Finally, for $\alpha = n$, $F_{\lambda_{max}}(x)$ *coincides with the CDF of the maximum eigenvalue of a complex central Wishart matrix* $\mathbf{W} \sim \mathcal{W}_m(n, \mathbf{\Sigma} + \mathbf{I})$.

Proof. The CDF of the maximum eigenvalue can be obtained from the joint density of the ordered eigenvalues following the procedure outlined in [16, Theorem I]

$$F_{\lambda_{max}}(x) = \int_D f_{\mathbf{\Lambda}}(\mathbf{\Lambda}) d\mathbf{\Lambda} = K \left| \int_0^x f_i(\omega_j, \lambda) d\lambda \right|, \tag{11}$$

where $D = \{0 < \lambda_m < \ldots < \lambda_1 < x\}$, $f_i(\omega_j, \lambda) = \lambda^{n-m+j-1} e^{-\lambda} {}_1F_1\left(\alpha - m + 1; n - m + 1; \lambda(1+\omega_i)^{-1}\right)$.

In order to evaluate the integral at the right hand side of (11) we decompose it as

$$\int_0^x f_i(\omega_j, \lambda) d\lambda = \int_0^\infty f_i(\omega_j, \lambda) d\lambda - \int_x^\infty f_i(\omega_j, \lambda) d\lambda \tag{12}$$

where the first term at the right hand side can be evaluated via [10, Formula 28]. As to the second term, when $\alpha < n$, we substitute [17, Formula 13.2.1] in the integral. Exchanging the integration order and using the definition of the Incomplete Gamma Function [15, Formula 8.350.2] we get

$$\begin{aligned} &\frac{(\tau+j-1)!\tau!}{\Gamma(\alpha-m+1)\Gamma(n-\alpha)} \sum_{\ell=0}^{n-m+j-1} \frac{e^{-x} x^\ell}{\ell!} \\ &\times \int_0^1 e^{tx/(1+\omega_i)} t^{\alpha-m} \frac{(1-t)^{n-\alpha-1}}{\left(1 - \frac{t}{1+\omega_i}\right)^{n-m+j-\ell}} dt. \end{aligned}$$

It can be finally evaluated in closed form via [15, Formula 3.385]. As to the case of $\alpha > n$, it can be handled exploiting the series expression [17, Formula ...] in the second term at the right hand side of (12). Integrating term by term, and still using [15, Formula 8.350.2], we get the result in (10).

When α is an integer, and $\alpha > n$, (10) further simplifies by virtue of Kummer transform [17]

$${}_1\mathcal{F}_1(a, b; x) = e^x {}_1\mathcal{F}_1(b-a, b; -x) = \sum_{n=0}^{a-b} \frac{e^x (a-b)! x^n}{(a-b-n)! n! (b)_n}.$$

Specifically, in this last case the entries of the $m \times m$ matrix $\mathbf{F}(x)$ are given by

$$\begin{aligned} \mathbf{F}(x)_{i,j} = &\frac{{}_2F_1\left(\tau+j, \alpha-m+1; \tau+1; (1+\omega_i)^{-1}\right)}{[(\tau+j-1)!]^{-1}} - \\ &(1+\omega_i)^{\tau+j} \sum_{t=0}^{\alpha-n} \frac{(\alpha-n)!(\tau+j+t-1)!}{(\alpha-n-t)!(\tau+1)_t t! \omega_i^{\tau+j+t}} \\ &\times e^{-\frac{x\omega_i}{(1+\omega_i)}} \sum_{\ell=0}^{\tau+j+t-1} \frac{\left(\frac{x\omega_i}{(1+\omega_i)}\right)^\ell}{\ell!}. \end{aligned}$$

If $\alpha = n$, then $\mathbf{W}$ reduces to a complex central Wishart matrix and the CDF of the maximum eigenvalue can be found in [18].

III. Performance analysis

This section is devoted to the performance analysis of communication systems on a MIMO SR fading channel. Exploiting the eigenanalysis results of the previous Section, we first present the ergodic mutual information characterization. Then, we provide the expression of the outage probability in the presence of adaptive transmission strategies as a direct application of Theorem 3.

A. Mutual information characterization

We consider at the transmit array a signaling strategy which complies with $E[\mathbf{x}\mathbf{x}^\dagger] = \frac{E[\|\mathbf{x}\|^2]}{n}\mathbf{I}$. This *isotropic* signaling is known to be optimal in the canonical case when the transmitter has no knowledge of the channel realization [8], and for other symmetric situations. As a consequence, the average Signal to Noise Ratio ($\overline{\mathsf{SNR}}$) can be computed as $\overline{\mathsf{SNR}} = \mathrm{g}\frac{E[\|\mathbf{x}\|^2]}{N_0}$. The average mutual information in the MIMO ergodic framework can be expressed as [8]

$$I(\overline{\mathsf{SNR}}) = m\,E\left[\log_2\left(1 + \frac{\overline{\mathsf{SNR}}}{\delta n}\lambda(\mathbf{W})\right)\right] \qquad (13)$$

where λ denotes an arbitrary nonzero eigenvalue of $\mathbf{W}$ and the expectation is over its distribution. By virtue of Theorem 2, we get

$$I(\overline{\mathsf{SNR}}) = \frac{m\widetilde{K}e^{n\delta/\overline{\mathsf{SNR}}}}{\ln 2}\sum_{i,j=1}^{m}\mathcal{D}(i,j)\sum_{t=0}^{\infty}\frac{(\alpha-m+1)_t}{(1+\omega_i)^t t!(\tau+1)_t} \times\frac{(\tau+t+j-1)!}{(\overline{\mathsf{SNR}}/n\delta)^{\tau+t+j}}\sum_{k=1}^{\tau+t+j}\frac{\Gamma\left(k-\tau-t-j, n\delta/\overline{\mathsf{SNR}}\right)}{(\overline{\mathsf{SNR}}/n\delta)^{-k}},$$

where $\Gamma(\cdot,\cdot)$ is the Incomplete Gamma Function [15, 8.350.2].

Standard properties of the confluent hypergeometric function imply that for integer values of α such expression simplifies. Specifically, for integer $\alpha > n$, by virtue of Kummer transform

$$I(\overline{\mathsf{SNR}}) = \frac{m\widetilde{K}}{\ln 2}\sum_{i,j=1}^{m}\mathcal{D}(i,j)e^{\left[\frac{n\delta\omega_i}{\overline{\mathsf{SNR}}(1+\omega_i)}\right]} \times\sum_{t=0}^{\alpha-n}\frac{(\alpha-n)!(1+\omega_i)^{-t}}{(\alpha-n-t)!t!(\tau+1)_t}\frac{(\tau+t+j-1)!}{(\overline{\mathsf{SNR}}/n\delta)^{\tau+t+j}} \times\sum_{k=1}^{\tau+t+j}\left[\frac{\overline{\mathsf{SNR}}(1+\omega_i)}{n\delta\omega_i}\right]^k\Gamma\left(k-\tau-t-j, \frac{n\delta\omega_i}{\overline{\mathsf{SNR}}(1+\omega_i)}\right),$$

while for $\alpha = n$, [19]

$$I(\overline{\mathsf{SNR}}) = \frac{m\,\widetilde{K}}{\ln 2}\sum_{i,j=1}^{m}\mathcal{D}(i,j)\frac{(\tau+j-1)!}{e^{-n\delta\omega_i/[\overline{\mathsf{SNR}}(1+\omega_i)]}}\left(\frac{1+\omega_i}{\omega_i}\right)^{\tau+j} \times\sum_{k=1}^{\tau+j}E_i\left(k-\tau-j, \frac{\omega_i}{1+\omega_i}\frac{n\delta}{\overline{\mathsf{SNR}}}\right),$$

with $E_i(\cdot)$ the exponential integral [17].

The above listed expressions allow for the evaluation of the ergodic mutual information over all the $\overline{\mathsf{SNR}}$ range and for an arbitrary number of antennas. More compact albeit approximated expressions can be also obtained in the low and high-$\overline{\mathsf{SNR}}$ ranges, following [20] and [21] respectively. However they are not reported here for the lack of space.

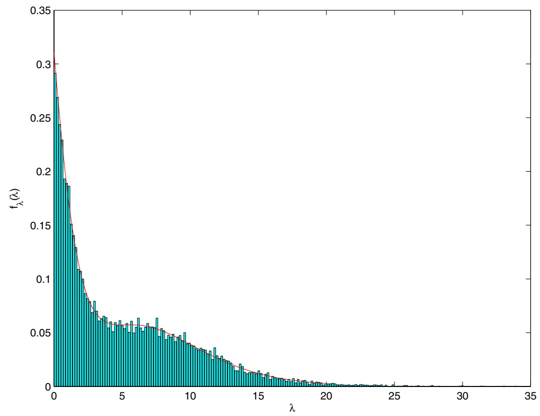

Figure 1: Marginal density distribution (solid curve), and corresponding hystogram of an unordered eigenvalue for a 2×2 LMS channel with $\alpha = 3$ and $\rho = 0.1$.

B. Outage probability

Another important statistical measure, crucial to the performance assessment of a wireless system is the outage probability. It is defined as the probability of failing to achieve a specified SNR value, say μ_{th}, sufficient for a successful reception. We assume that $\mathbf{x} = s_D\mathbf{w}_t$, where s_D is the useful transmit signal (without loss of generality, with unit average average power) and $\mathbf{w}_t$ is the weight vector at the transmitter. The optimum combining vector at the receiver, given the transmit weight, is given by $\gamma\mathbf{H}\mathbf{w}_t$ where γ is a constant that does not affect the output SNR.

The maximum value of the output SNR is achieved when $\mathbf{w}_t$ is parallel to the unit norm eigenvector $\mathbf{u}_{max}$ of the channel matrix grammian corresponding to the largest eigenvalue λ_{max}. Otherwise stated,

$$\mathsf{SNR}_{max} = \frac{\mathrm{g}\|\mathbf{w}_t\|^2}{\delta\mathcal{N}_0}\lambda_{max},$$

and the outage probability at the output of the optimal combiner can be evaluated as

$$P_{out} = P(\mathsf{SNR}_{max} < \mu_{th}) = K\left|\mathbf{F}\left(\frac{\delta\mathcal{N}_0\mu_{th}}{g\|\mathbf{w}_t\|^2}\right)\right|,$$

where in the last equality we have exploited (10).

IV. Numerical results

We consider a LMS wireless MIMO channel with uncorrelated transmit and correlated receive antennas and assume $(\mathbf{\Omega})_{i,j} = \rho^{|i-j|}$. Figure 1 provides a comparison between the theoretical pdf of an unordered eigenvalue of $\mathbf{W}$ given in (7) and a Monte Carlo simulation with $m = n = 2$, $\alpha = 3$, and $\rho = 0.1$. A good match can be observed between the theoretical curve and the histogram. The effect of an increasing number of sensors at the transmit end is analyzed in Figure 2 where the theoretical and the Monte Carlo simulated CDF of the maximum eigenvalue of a SR matrix grammian are plotted, for $\rho = 0.1$ and $m = 2$. The value of n is taken as a parameter and $\alpha > n$. The curves highlight that, increasing the number of transmit antennas, the probability

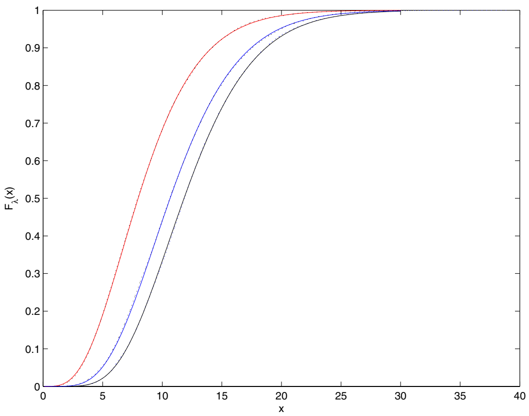

Figure 2: Outage probability for a 2×2 LMS channel with $\alpha = 3$ (red curve), for a 2×3 LMS channel with $\alpha = 4$ (blue curve), and for a 2×4 LMS channel with $\alpha = 4$ (dark curve), with $\rho = 0.1$. Theoretical results (solid curves). Simulated results (dotted curves).

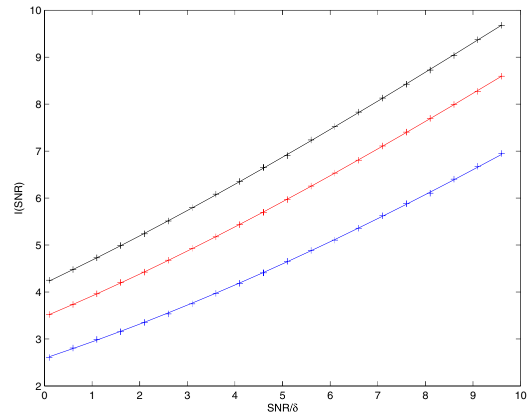

Figure 3: Coherent mutual information versus $\overline{\mathsf{SNR}}/\delta$ for a 2×2 LMS channel with $\alpha = 2$ (blue curve), 2×3 with $\alpha = 3$ (red curve) and 2×4 (black curve) with $\alpha = 4$. Theoretical results (solid curves). Simulated results (cross-marked points).

that the maximum eigenvalue of the channel grammian falls below a certain value decreases, since more degrees of freedom are available to perform beamforming. The dependence of the mutual information on the channel statistics is analyzed in Figure 3. Both Monte Carlo simulated, as well as analytical values of the ergodic mutual information are plotted. The curves are obtained setting $\rho = 0.1$, $m = 2$, and varying both the number of transmit antennas and the LOS fluctuation parameter α.

V. Conclusions

In this paper the multivariate SR model for MIMO LMS channels has been introduced. Both the channel matrix grammian pdf and the spectral statistics are given, allowing the theoretical performance analysis of the newly proposed channel model from both a communication and an information-theoretic point of view. Specifically, the outage probability for the case of adaptive transmission strategies and the ergodic mutual information in the coherent case have been evaluated, mostly in closed form. Finally, numerical examples are provided in order to assess the impact on the performance of the diversity order and/or the LOS fluctuations.

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