# Notes on Localized random sampling for robust compressive beam alignment

### **Background:**

Compressed sensing (CS) techniques have been extensively studied for millimeter wave (mmWave) channel estimation or beam alignment. CS-based algorithms exploit the sparse nature of mmWave channels to recover them with fewer channel measurements. In our work [1], we focus on CS robust to carrier frequency offset (CFO). CFO is one of the important challenges that needs to be addressed in translating CS technology to practical mmWave systems.

## Challenge:

Most of the standard CS-based algorithms that use the common random phase shift-based training are sensitive to residual CFO. CFO corrupts the phase of the channel measurements in the standard linear CS model. An interesting problem is to design a class of hardware compatible CS matrices such that standard CS under a CFO error is robust for any matrix in this class. Another question that naturally arises out of this problem is ``Under what conditions do the CS matrices in the new class satisfy the restricted isometry property?". We have answered both these questions.

#### **Prior work:**

One approach to solve the phase perturbed CS model is using phase retrieval-based techniques [2]. Phase retrieval-based techniques ignore the phase of the channel measurements. As a result, they do not exploit the structure in the unknown phase errors due to CFO.

A second approach is to model the unknown and structured phase errors due to CFO, and solve the joint CFO and channel estimation problem. Then, the joint estimation problem can be solved using ideas from sparse self-calibration [3], tensors [4] or bilinear message passing [5]. Unfortunately, all the three techniques introduce a new dimension to the underlying linear CS-based channel estimation problem.

In this work, we do not focus on CS algorithm design for robustness to CFO. Instead, we design the CS matrix in a clever way such that the phase perturbed channel measurements, when fed to any standard CS algorithm, result in a ``minimally" perturbed solution for the channel.

#### **Solution:**

In our ICASSP paper [1], we have designed a class of CS matrices that achieve robustness to CFO. The key takeaway from our paper [1] is that restricting the randomness in CS matrices in an appropriate manner can result in robustness to structured errors.

- [1] N. J. Myers, and R. W. Heath Jr., `Localized random sampling for robust compressive beam alignment", to appear in proc. of IEEE ICASSP 2019
- [2] O. Abari, H. Hassanieh, M. Rodreguez, and D. Katabi, ``Millimeter wave communications: From point-to-point links to agile network connections," in Proc. ACM HotNets 2016
- [3] S. Ling and T. Strohmer, ``Self-calibration and biconvex compressive sensing," Inverse Problems, vol. 31, no. 11, p. 115002, 2015
- [4] N. J. Myers, and R. W. Heath Jr., ``A compressive channel estimation technique robust to synchronization impairments," in Proc. of IEEE SPAWC 2017
- [5] N. J. Myers, and R. W. Heath Jr., "Message passing-based joint CFO and channel estimation in millimeter wave systems with one-bit ADCs", submitted to IEEE Trans. on Wireless Commun., available at arxiv: 1803.09012