OFDM for Optical Communications

Jean Armstrong, *Senior Member, IEEE*

(Invited Tutorial)

***Abstract*—Orthogonal frequency division multiplexing (OFDM) is a modulation technique which is now used in most new and emerging broadband wired and wireless communication systems because it is an effective solution to intersymbol interference caused by a dispersive channel. Very recently a number of researchers have shown that OFDM is also a promising technology for optical communications. This paper gives a tutorial overview of OFDM highlighting the aspects that are likely to be important in optical applications. To achieve good performance in optical systems OFDM must be adapted in various ways. The constraints imposed by single mode optical fiber, multimode optical fiber and optical wireless are discussed and the new forms of optical OFDM which have been developed are outlined. The main drawbacks of OFDM are its high peak to average power ratio and its sensitivity to phase noise and frequency offset. The impairments that these cause are described and their implications for optical systems discussed.**

***Index Terms*—Modulation, orthogonal frequency division multiplexing (OFDM), optical communication.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is used extensively in broadband wired and wireless communication systems because it is an effective solution to intersymbol interference (ISI) caused by a dispersive channel. This becomes increasingly important as data rates increase to the point where, when conventional serial modulation schemes like quadrature amplitude modulation (QAM) or NRZ are used, the received signal at any time depends on multiple transmitted symbols. In this case the complexity of equalization in serial schemes which use time domain equalization rises rapidly. In contrast, the complexity of OFDM, and of systems using serial modulation and frequency domain equalization, scale well as data rates and dispersion increase. [1]–[3]. A second major advantage of OFDM is that it transfers the complexity of transmitters and receivers from the analog to the digital domain. For example, while the precise design of analog filters can have a major impact on the performance of serial modulation systems, in OFDM any phase variation with frequency can be corrected at little or no cost in the digital parts of the receiver. Despite these important advantages of OFDM, it is only recently that it has been considered for optical communications.

While many details of OFDM systems are very complex, the basic concept of OFDM is quite simple [4]–[7]. Data is transmitted in parallel on a number of different frequencies, and as a result the symbol period is much longer than for a serial system with the same total data rate. Because the symbol period is longer, ISI affects at most one symbol, and equalization is simplified. In most OFDM implementations any residual ISI is removed by using a form of guard interval called a cyclic prefix.

When frequency division multiplexing (FDM) is used in conventional wireless systems, or wavelength division multiplexing (WDM) is used in optical systems, information is also transmitted on a number of different frequencies simultaneously. However there are a number of key theoretical and practical differences between OFDM and these conventional systems. In OFDM the subcarrier frequencies are chosen so that the signals are mathematically orthogonal over one OFDM symbol period. Both modulation and multiplexing are achieved digitally using an inverse fast Fourier transform (IFFT)[1] and as a result, the required orthogonal signals can be generated precisely and in a very computationally efficient way. In FDM/WDM there are frequency guard bands between the subcarriers. At the receiver the individual subcarriers are recovered using analog filtering techniques. Fig. 1 shows spectra for FDM/WDM and OFDM. In OFDM the spectra of individual subcarriers overlap, but because of the orthogonality property, as long as the channel is linear, the subcarriers can be demodulated without interference and without the need for analog filtering to separate the received subcarriers. Demodulation and demultiplexing is performed by a fast Fourier transform (FFT). The spectrum of an individual OFDM subcarrier has a $|\sin(x)/x|^2$ form, so each OFDM subcarrier has significant sidelobes over a frequency range which includes many other subcarriers. This is the cause of one of the major disadvantages of OFDM: that it is quite sensitive to frequency offset and phase noise.

This paper presents a tutorial overview of OFDM with particular emphasis on aspects that are likely to be important in optical applications. Section II outlines the history of OFDM. In Section III a typical OFDM system for wireless applications is presented, the signals at various points described and the function of each block described. Misconceptions that have in the past been common among OFDM researchers are explained, so that these can be avoided by new researchers in the field.

Manuscript received July 25, 2008; revised October 07, 2008. Current version published February 13, 2009. This work was supported by the Australian Research Council's Discovery funding scheme (DP0772937).

The author is with the Department of Electrical and Computer Systems Engineering, Monash University, Clayton, Vic. 3800, Australia (e-mail: jean.armstrong@eng.monash.edu.au).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JLT.2008.2010061

[1]Strictly speaking the mathematical operation is the discrete Fourier Transform (DFT) and the efficient algorithm for implementing it is the Fast Fourier Transform (FFT) but the terms DFT and FFT are often used interchangeably.

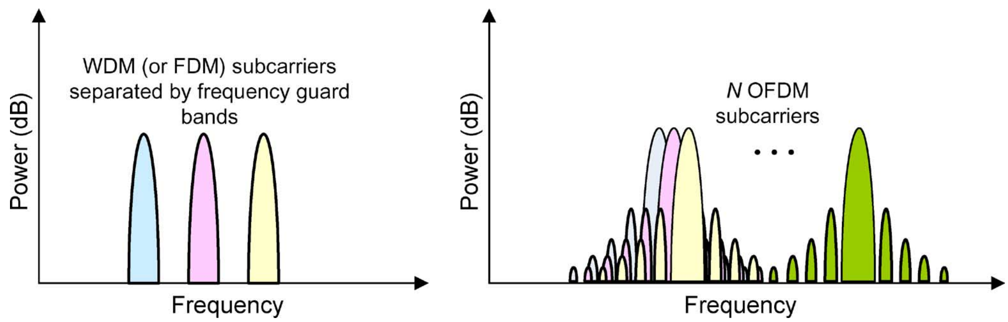

Fig. 1. Spectrum of (a) WDM or FDM signals (b) OFDM signal.

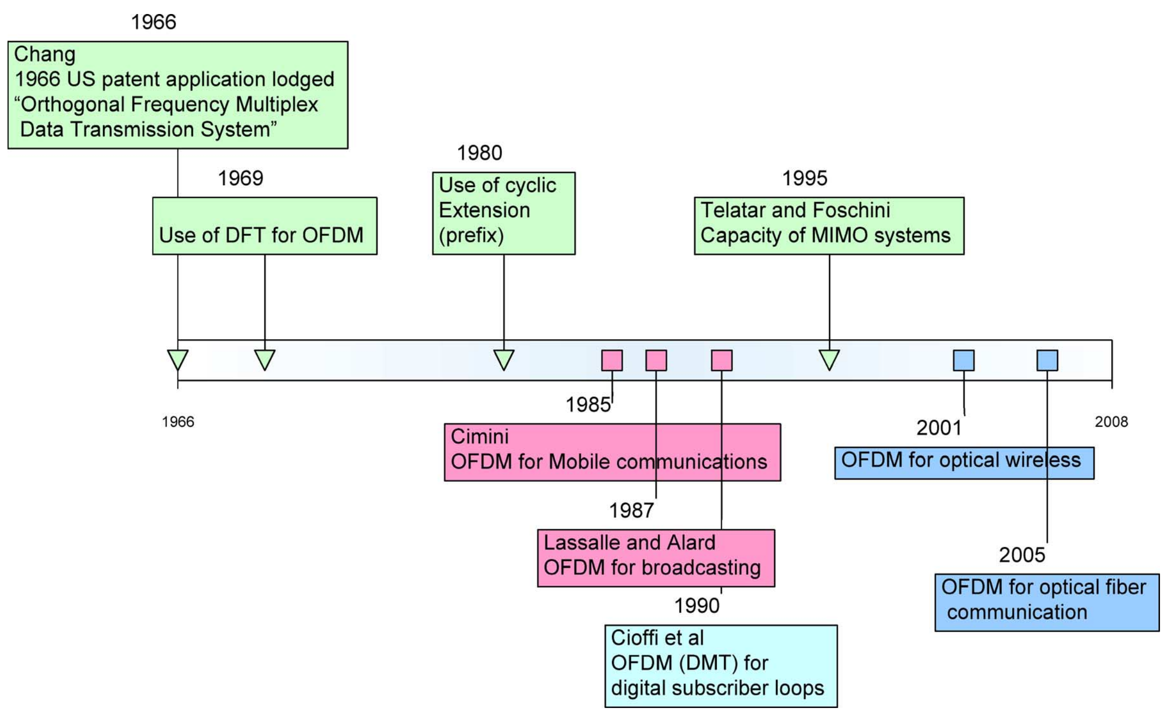

Fig. 2. Historical development of the underlying theory of OFDM and its practical implementation.

In Section IV the application of OFDM to optical communications is discussed. The special constraints that apply are explained and the new forms of OFDM for optical communications which have recently emerged are described. OFDM has a number of disadvantages. These are described in Section V. In Section VI, factors which will influence whether OFDM is used in future commercial optical applications are discussed. Finally in Section VII conclusions are presented.

II. History of OFDM

Fig. 2 shows the historical development of both the theoretical basis of OFDM and its practical application across a range of communication systems [8]. The first proposal to use orthogonal frequencies for transmission appears in a 1966 patent by Chang of Bell Labs [9]. The proposal to generate the orthogonal signals using an FFT came in 1969 [10]. The cyclic prefix (CP), which is an important aspect of almost all practical OFDM implementations, was proposed in 1980 [11]. These are the three key aspects that form the basis of most OFDM systems. The breakthrough papers by Telatar and Foschini on multiple antenna systems fuelled another wave of research in OFDM [12], [13]. Although the capacity gains of these multiple-input–multiple-output (MIMO) systems do not theoretically depend on any particular modulation scheme, the ability to combat dispersion and the good scalability of OFDM become even more important in this context.

OFDM began to be considered for practical wireless applications in the mid–1980s. Cimini of Bell Labs published a paper on OFDM for mobile communications in 1985 [14], while in 1987, Lassalle and Alard, [15] based in France considered the use of OFDM for radio broadcasting and noted the importance of combining forward error correction (FEC) with OFDM. Because of this interrelationship, OFDM is often called Coded OFDM (C-OFDM) by broadcast engineers. The application of OFDM for wireline communications was pioneered by Cioffi and others at Stanford who demonstrated its potential as a modulation technique for digital subscriber loop (DSL) applications [16]. OFDM is now the basis of many practical telecommunications standards including wireless local area networks (LAN), fixed wireless [17] and television and radio broadcasting in much of the world [18]. OFDM is also the basis of most DSL standards, though in DSL applications the

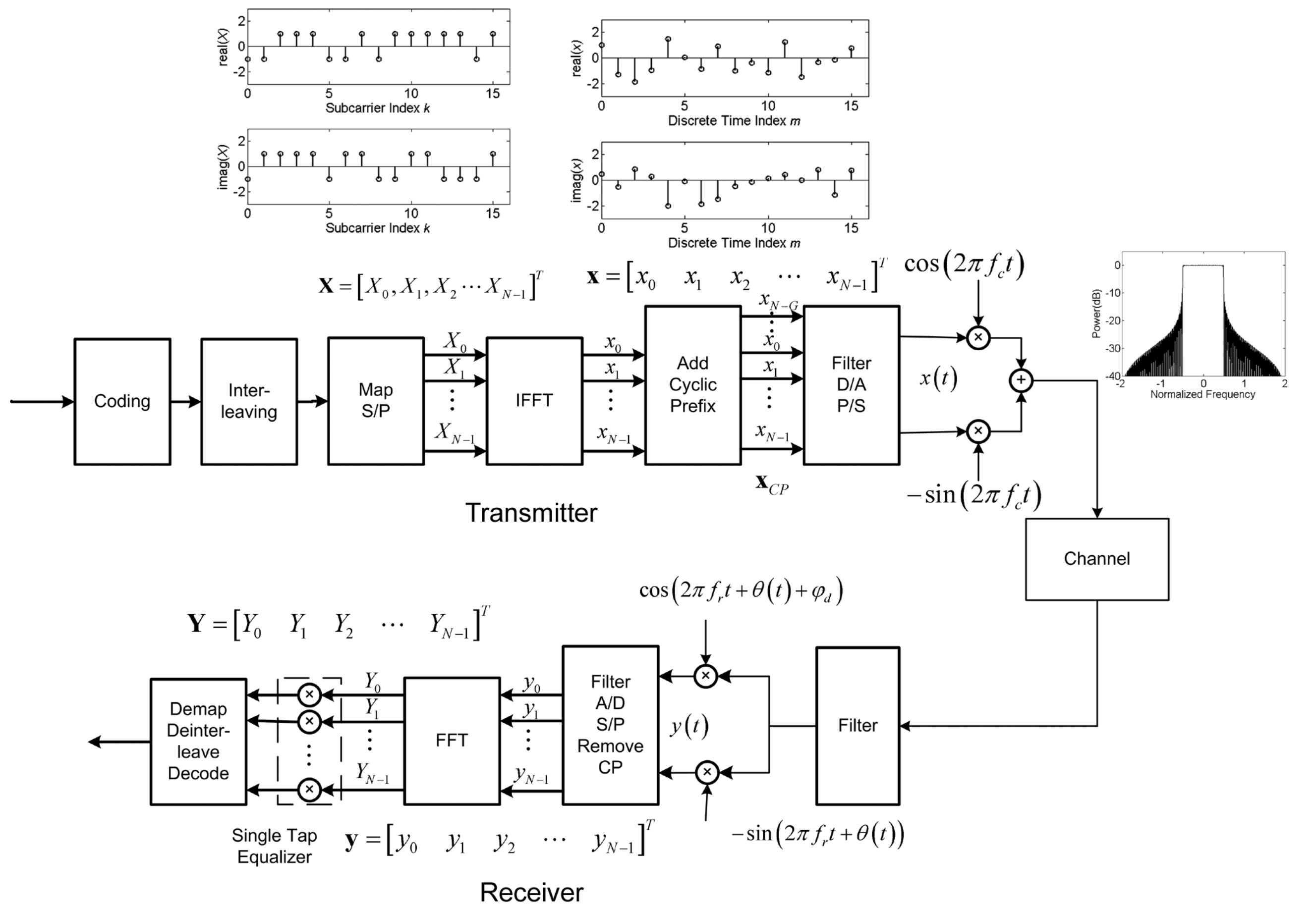

Fig. 3. Block diagram of an OFDM communication system for RF wireless applications.

baseband signal is not modulated onto a carrier frequency and in this context OFDM is usually called discrete multitone (DMT).

The application of OFDM to optical communications has only occurred very recently, but there are an increasing number of papers on the theoretical and practical performance of OFDM in many optical systems including optical wireless [19], [20], single mode optical fiber [21]–[24], multimode optical fiber [25]–[27] and plastic optical fiber [28].

III. OFDM System Description

In this section the basic functions of a typical OFDM system for wireless applications are described. Fig. 3 shows the block diagram of the transmitter and receiver of a typical OFDM wireless system.

A. FFT and IFFT

As the IFFT block is the main component in the transmitter and the FFT in the receiver, and these are the functions which distinguish OFDM from single carrier systems we will start by considering the signals at the input and the output of the IFFT and FFT and consider the other blocks later.

The input to the IFFT is the complex vector $\mathbf{X} = [X_0\ X_1\ X_2\ \cdots\ X_{N-1}]^T$, the vector has length N where N is the size of the IFFT. Each of the elements of $\mathbf{X}$ represents the data to be carried on the corresponding subcarrier, so for example X_k represents the data to be carried on the kth subcarrier.[2] Usually QAM modulation is used in OFDM, so each of the elements of $\mathbf{X}$ is a complex number representing a particular QAM constellation point. Throughout this paper we will use upper case to represent frequency or discrete frequency domain variables, and lower case for time domain. Bold font will be used for vectors. The output of the IFFT is the complex vector $\mathbf{x} = [x_0\ x_1\ x_2\ \cdots\ x_{N-1}]^T$. Using the definition of the inverse discrete Fourier transform which will be used in this paper

$$x_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(\frac{j2\pi km}{N}\right) \quad \text{for } 0 \le m \le N-1. \tag{1}$$

Note that the forward and inverse discrete Fourier transforms are defined in slightly different ways in different publications. The forward FFT corresponding to (1) is

$$X_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m \exp\left(\frac{-j2\pi km}{N}\right) \quad \text{for } 0 \le k \le N-1. \tag{2}$$

This form of the IFFT/FFT transform pair has the important advantage that the discrete signals at the input and the output of the transform for each symbol have the same total energy and

[2]Most of the literature for OFDM for wireless communication uses the term 'subcarrier' but the literature on OFDM for wired communication uses the term 'tone'.

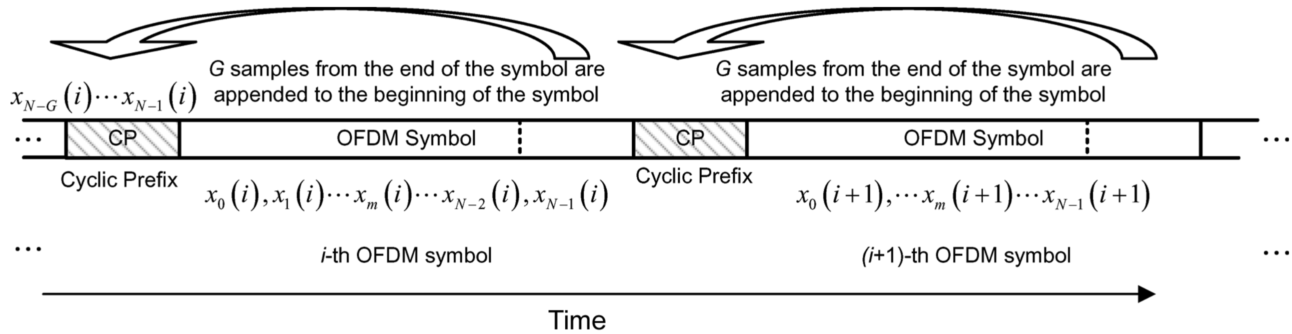

Fig. 4. Time domain sequence of OFDM symbols showing the cyclic prefix.

the same average power. This simplifies the analysis of many OFDM functions. The insets in Fig. 3 show an example of the signals at the input and the output of the IFFT for 4 QAM modulation and $N = 16$. The input to the IFFT is $\mathbf{X}$ a vector of random values from the 4 QAM constellation $\{1+j, 1-j, -1+j, -1-j,\}$. The output is the corresponding time domain vector $\mathbf{x}$. While the components of $\mathbf{X}$ take only a few discrete values, the probability distribution of $\mathbf{x}$ is not obvious from the diagram. In fact for $N \geq 64$ the real and imaginary components of an OFDM time domain signal are approximately Gaussian. For wireless OFDM systems which have already been standardized, values of N ranging from 64 in wireless LAN systems to 8096 in digital television systems have been used. The terminology throughout the OFDM literature is not consistent. In this paper the term 'symbol' is used to describe the time domain or frequency domain sequence associated with one IFFT operation. (In some papers this is described as block or frame.)

At the receiver the FFT performs a forward transform on the received sampled data for each symbol

$$Y_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_m \exp\left(\frac{-j2\pi km}{N}\right) \quad \text{for } 0 \leq k \leq N-1 \tag{3}$$

where $\mathbf{y} = [y_0\ y_1\ y_2\ \cdots\ y_{N-1}]^T$ is the vector representing the sampled time domain signal at the input to the receiver FFT and $\mathbf{Y} = [Y_0\ Y_1\ Y_2\ \cdots\ Y_{N-1}]^T$ is the discrete frequency domain vector at the FFT output. Note that only N samples are required per OFDM symbol (excluding CP). To understand the function of the IFFT, first consider what would happen if there were no noise or distortion in the channel or the transmitter and receiver front ends, then because the FFT and IFFT are transform pairs, $\mathbf{Y} = \mathbf{X}$.

If additive white Gaussian noise (AWGN) is added to the signal, but the signal is not distorted then

$$y_m = x_m + w_m \tag{4}$$

where w_m is a sample of white Gaussian noise, substituting (4) in (3) and rearranging gives

$$\begin{aligned} Y_k &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_m \exp\left(\frac{-j2\pi km}{N}\right) \\ &= X_k + W_k \end{aligned} \tag{5}$$

where

$$W_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} w_m \exp\left(\frac{-j2\pi km}{N}\right) \quad \text{for } 0 \leq k \leq N-1. \tag{6}$$

W_k is the noise component of the kth output of the receiver FFT. Because each value of W_k is the summation of N independent white Gaussian noise samples, w_m, it too is an independent white Gaussian noise process. Even if the time domain noise, w_m, does not have a Gaussian distribution, in most cases, because of the central limit theorem, the frequency domain noise W_k will be Gaussian. This, combined with the use of FEC, means that usually the performance of OFDM systems depend on the average noise power, unlike conventional serial optical systems where it is the peak values of the noise which often limit performance.

B. Sequences of Symbols and the Cyclic Prefix

The description above showed how the IFFT generates each OFDM symbol. The transmitted signal consists of a sequence of these OFDM symbols. To denote different OFDM symbols when a sequence of symbols rather than a single symbol is being considered we need to extend the notation to include a time index. Let $\mathbf{x}(i) = [x_0(i)\ x_1(i)\ x_2(i)\ \cdots\ x_{N-1}(i)]^T$ be the output of the IFFT in the ith symbol period. In most OFDM systems, a CP is added to the start of each time domain OFDM symbol before transmission. In other words a number of samples from the end of the symbol is appended to the start of the symbol. So instead of transmitting $\mathbf{x}(i) = [x_0(i)\ x_1(i)\ x_2(i)\ \cdots\ x_{N-1}(i)]^T$ the sequence

$$\mathbf{x}_{CP}(i) = [x_{N-G}(i) \cdots x_{N-1}(i), x_0(i) \cdots x_{N-1}(i)]^T \tag{7}$$

is transmitted; where G is the length of the cyclic prefix. Although the CP introduces some redundancy, and reduces the overall data rate, we will show that the use of the CP eliminates both ISI and intercarrier interference (ICI) from the received signal and is the key to simple equalization in OFDM. Fig. 4. shows the time domain sequence of OFDM symbols.

C. Individual OFDM Subcarriers

Considerable insight into the operation of an OFDM system can be obtained by considering what happens to individual subcarriers as they pass through the system. However, it is also important to note that in an OFDM system because the IFFT

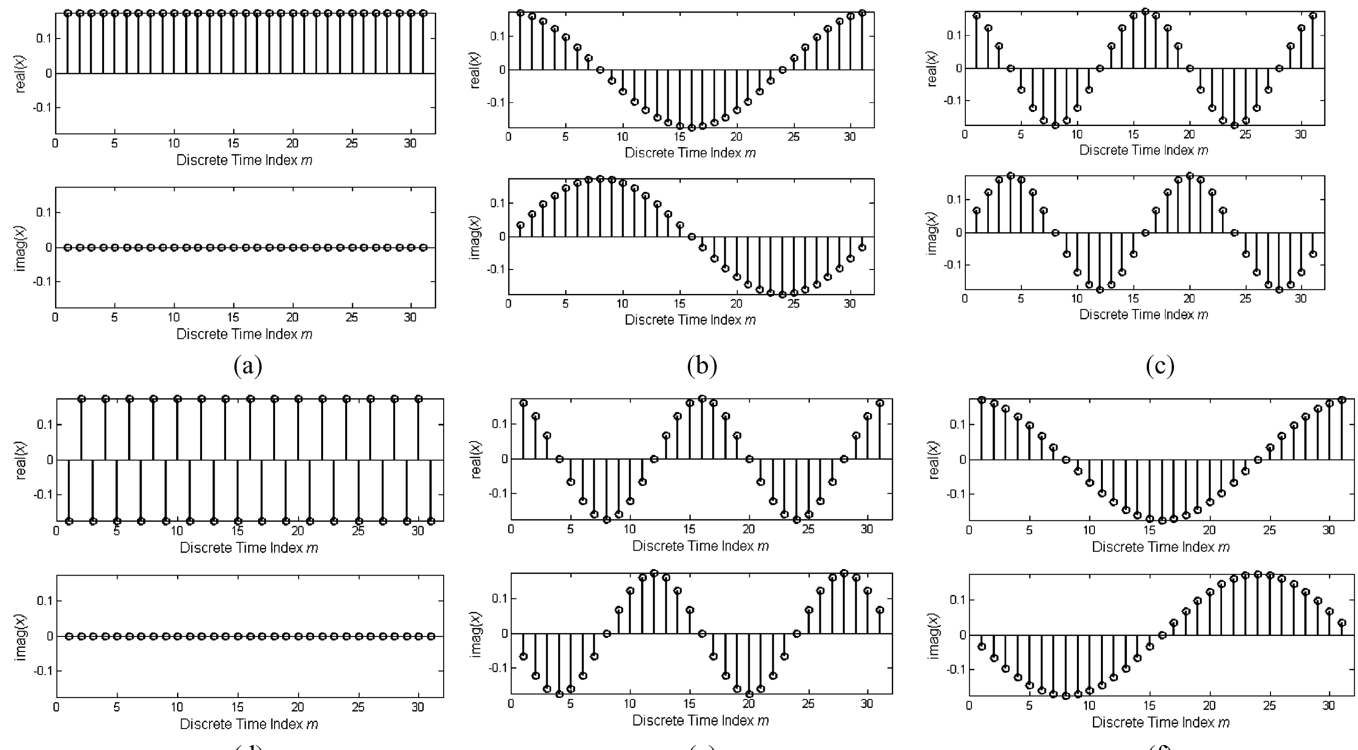

Fig. 5 Discrete time domain signal for individual OFDM subcarriers for $N = 32$. (a) $k = 0$, dc component, (b) $k = 1$, (c) $k = 2$, (d) $k = N/2$, Nyquist term, (e) $k = N/2 - 2$, and (f) $k = N/2 - 1$.

simultaneously performs modulation and multiplexing there is no point in the transmitter or receiver where an individual time domain subcarrier can be observed. Individual subcarriers are present only in the frequency domain. Nevertheless consideration of the time domain components due to individual subcarriers is important, and if the channel is linear the performance of the overall system can be derived in this way. To simplify the discussion we will not at first consider the CP and will consider only one symbol.

From (1), the discrete time domain component associated with the kth subcarrier of a given OFDM symbol is

$$x_m = \frac{1}{\sqrt{N}} X_k \exp\left(\frac{j2\pi km}{N}\right) \quad \text{for } 0 \le m \le N-1. \quad (8)$$

Fig. 5 plots the discrete signal for $N = 32$, $X_k = 1$ and $k = 0, 1, 2, N/2, N-2, N-1$. For $k = 0$, the samples have a constant value. This represents the DC term in the baseband signal and the component at the carrier frequency in wireless (or optical) systems where the OFDM baseband signal is upconverted to a higher frequency. For $k = 1$, the sequence of $\mathbf{x} = [x_0\, x_1\, x_2\, \cdots\, x_{N-1}]^T$ represents the samples of one cycle of a sinusoid of frequency $1/T$, where T is the symbol period (without the CP). For $k = 2$ the (baseband) frequency has doubled and the samples now give two cycles of a sinusoid. Fig. 5(d) shows the $N/2$th term. This is called the Nyquist term and for this subcarrier the baseband signal is critically sampled. Most OFDM systems do not use a number of band-edge subcarriers so that the Nyquist frequency and other frequencies close to the Nyquist frequency are not used, as this simplifies the analog filtering requirements at the transmitter and receiver. Fig. 5(f) shows the sequence for $k = N-1$. Because of the circular property of the FFT and IFFT, the sequence has one cycle (not $N-1$ cycles). This is important when the samples are transformed to the continuous time domain and is the source of many errors in the literature on OFDM.

D. OFDM in a Dispersive Environment: the Cyclic Prefix, Frequency Selective Fading and the Single Tap Equalizer

OFDM is so widely used because, when a CP is used, any distortion caused by a linear dispersive channel can be corrected simply using a 'single-tap' equalizer. To understand why this is true, consider a simple case where there is perfect upconversion and downconversion, but where the received baseband signal is the sum of two versions of the transmitted signal with different gains and delay.

$$y(t) = g_1 x(t+\tau_1) + g_2 x(t+\tau_2). \quad (9)$$

For the case where OFDM transmission is at passband, the gains and the signals will be complex; for the case of baseband transmission the gains and signals are real. Fig. 6 shows two delayed versions of an OFDM signal and the time window for the receiver FFT. For each OFDM symbol the receiver FFT has as input N samples from the signal within the time period shown. From Fig. 6. it can be seen that as long as the start of the receiver time window is aligned with the start of the "main" OFDM symbol of the first arriving signal, and if the delay spread (in this case $\tau_2 - \tau_1$) is less than the length of the CP, there is no intersymbol interference. The signal received in the ith time window depends only on the ith transmitted symbol.

Intersymbol interference could also be eliminated by preceding each OFDM symbol with a guard interval in which no

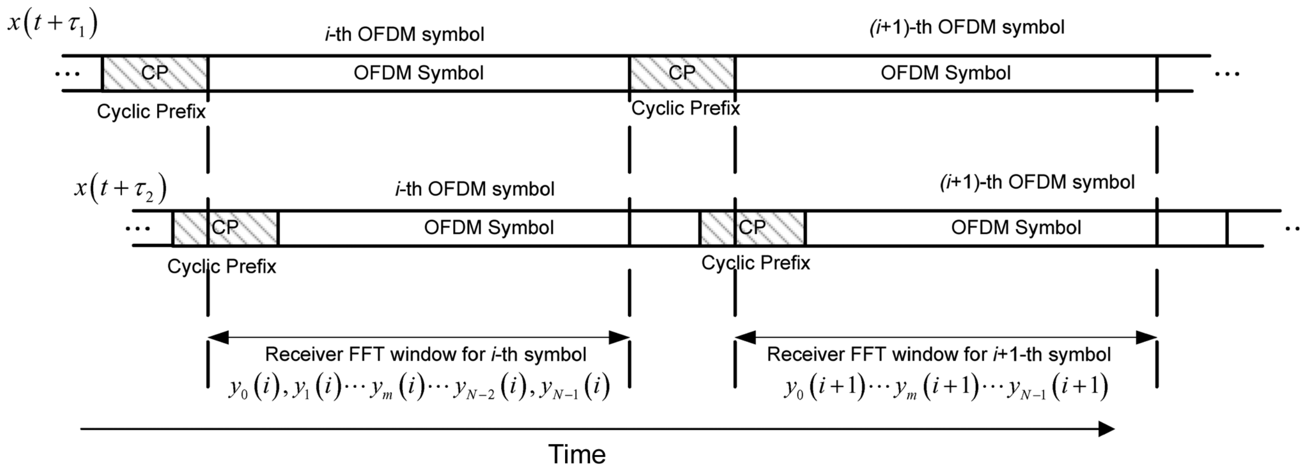

Fig. 6. OFDM symbols in a multipath channel: two components of the received signal with different delays.

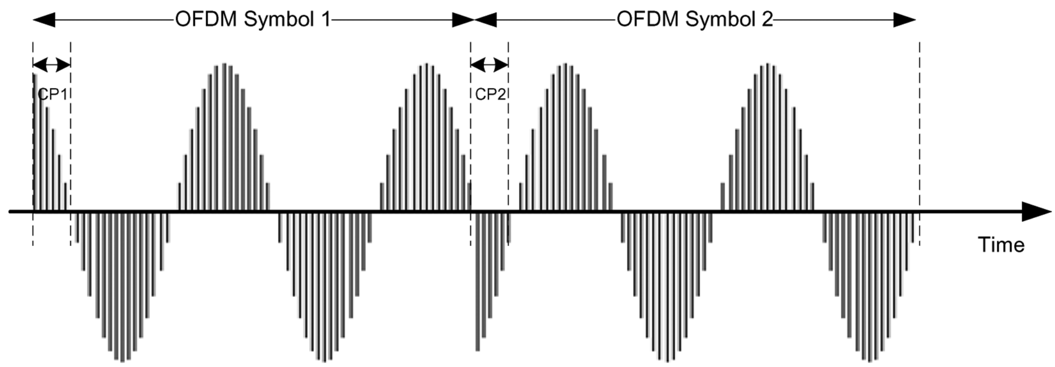

Fig. 7. Time domain components of one subcarrier for two symbols showing cyclic prefix.

signal was transmitted, however this would result in a phenomenon called intercarrier interference (ICI). Each value of Y_k would depend on input values $\mathbf{X}$ other than X_k. When a CP is used, each OFDM subcarrier is represented by a continuous sinusoid of the appropriate frequency throughout the main symbol period and the associated CP. This is shown in Fig. 7. So long as the delay spread does not exceed the CP, and the receiver FFT window is aligned with the start of the main symbol period of the first arriving signal, then no ISI or ICI occurs.

Now consider analytically the effect of a dispersive channel on a single subcarrier. To simplify the discussion we will consider a subcarrier in the range $0 \le k \le N/2 - 1$ and ignore the effect of noise.

Let the continuous baseband signal at the transmitter associated with the kth subcarrier of a given OFDM symbol (including the CP) be

$$x(k,t) = \frac{1}{\sqrt{N}} X_k \exp\left(\frac{j2\pi kt}{T}\right) \quad \text{for } 0 \le k \le \frac{N}{2} - 1. \quad (10)$$

Then the received continuous time domain signal for the two path channel described in (9) is

$$y(k,t) = \frac{1}{\sqrt{N}} g_1 X_k \exp\left(\frac{j2\pi k(t-\tau_1)}{T}\right) + \frac{1}{\sqrt{N}} g_2 X_k \exp\left(\frac{j2\pi k(t-\tau_2)}{T}\right) \quad \text{for } 0 \le k \le \frac{N}{2} - 1. \quad (11)$$

Ideally the receiver should be synchronized so that the FFT window is aligned with the start of the main symbol period for the first arriving version of the transmitted signal. So for this case the receiver FFT window should be offset by τ_1. In this case

$$\begin{aligned} y(k,t) &= \frac{1}{\sqrt{N}} g_1 X_k \exp\left(\frac{j2\pi kt}{T}\right) \\ &\quad + \frac{1}{\sqrt{N}} g_2 X_k \exp\left(\frac{j2\pi k\left(t-(\tau_2-\tau_1)\right)}{T}\right) \\ &= \frac{1}{\sqrt{N}} X_k \exp\left(\frac{j2\pi kt}{T}\right) \\ &\quad \times \left(g_1 + g_2 \exp\left(\frac{-j2\pi k(\tau_2-\tau_1)}{T}\right)\right). \end{aligned} \quad (12)$$

So after demodulation by the FFT and including the effect of noise

$$\begin{aligned} Y_k &= X_k \left(g_1 + g_2 \exp\left(\frac{-j2\pi k(\tau_2-\tau_1)}{T}\right)\right) + W_k \\ &= H_k X_k + W_k \end{aligned} \quad (13)$$

where

$$H_k = g_1 + g_2 \exp\left(\frac{-j2\pi k(\tau_2-\tau_1)}{T}\right). \quad (14)$$

The transmitted data can be recovered from the received signal by multiplying by X_k by $1/H_k$. That is each subcarrier

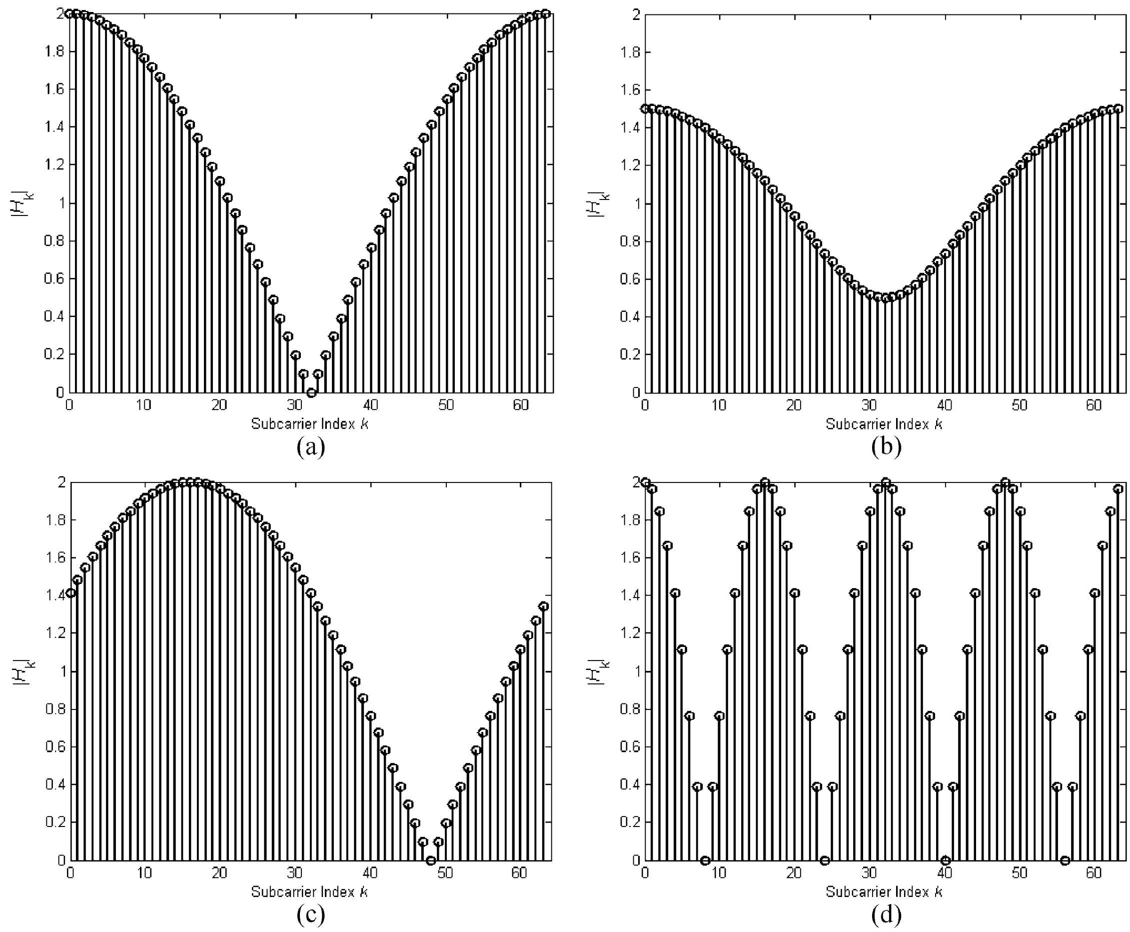

Fig. 8. Channel response $|H_k|$ for each subcarrier for a two path channel and $N = 64$. (a) $g_1 = g_2 = 1$ and $\tau_2 - \tau_1 = T/N$, (b) $g_1 = 1$, $g_2 = 0.5$ and $\tau_2 - \tau_1 = T/N$, (c) $g_1 = 1$, $g_2 = j$ and $\tau_2 - \tau_1 = T/N$, and (d) $g_1 = g_2 = 1$ and $\tau_2 - \tau_1 = 4T/N$.

can be recovered using one complex multiplication. This is the role of the single tap equalizer

$$\hat{X}_k = \frac{Y_k}{H_k} = X_k + \frac{W_k}{H_k}. \tag{15}$$

A disadvantage of single tap equalization is demonstrated in (15). If H_k is very small, the noise is enhanced.

Fig. 8. shows $|H_k|$ as a function of k for a number of two path channels. For all cases $N = 64$ and the delay spread is less than the length of the cyclic prefix. Fig. 8(a) shows the effect of two equal gain paths with $g_1 = g_2 = 1$ and $\tau_2 - \tau_1 = T/N$, in other words the delay between the paths is equal to one sampling interval. Remembering that the high frequency subcarriers correspond to subcarrier indices around $N/2$, this channel has a low pass characteristic with a very deep null at the highest frequencies. The position of the null depends on the difference in delay between the two paths. This case does not occur in practice in wireless systems, because the gains are in general complex, and it would be rare to have two paths with equal gain, but it is an important limitation for single mode (SM) optical systems. For example, if double side band OFDM is used in SM systems, chromatic dispersion can result in the signals from the two sidebands canceling in this way. Fig. 8(b) shows the results for $g_1 = 1$, $g_2 = 0.5$ and $\tau_2 - \tau_1 = T/N$. The position of the null has not changed, but the null is now not nearly so deep (note the linear scale). The low pass characteristic occurs only when $\arg(g_1) = \arg(g_2)$. This occurs in optical wireless systems and multimode systems [29]. Fig. 8(c) shows how the position of the null changes when this condition is no longer satisfied and $g_1 = 1$, $g_2 = j$. This type of characteristic can occur when coherent optical OFDM (CO-OFDM) is used in single mode systems because $\arg(g)$ depends on the phase of the optical carrier as well as on the differential delay. Finally Fig. 8(d) shows what happens when the delay between the two paths is increased to $\tau_2 - \tau_1 = 4T/N$. Now the first null occurs at a lower frequency and there are multiple nulls within the bandwidth of the OFDM signal.

E. Coding Interleaving and Mapping

The first blocks in the transmitter are interleaving and coding. All OFDM systems use some form of error correction or detection because, if there is frequency selective fading in the channel, some of the parallel data streams will experience deep fading. The coding is usually preceded by interleaving because, as shown in Fig. 8, a number of adjacent OFDM subcarriers may fall within the frequencies which are experiencing fading. In most broadcast applications of OFDM such as digital audio broadcasting (DAB) and digital video broadcasting (DVB) there are two layers of interleaving and coding so that a very low overall bit error rate (BER) can be achieved even over a very

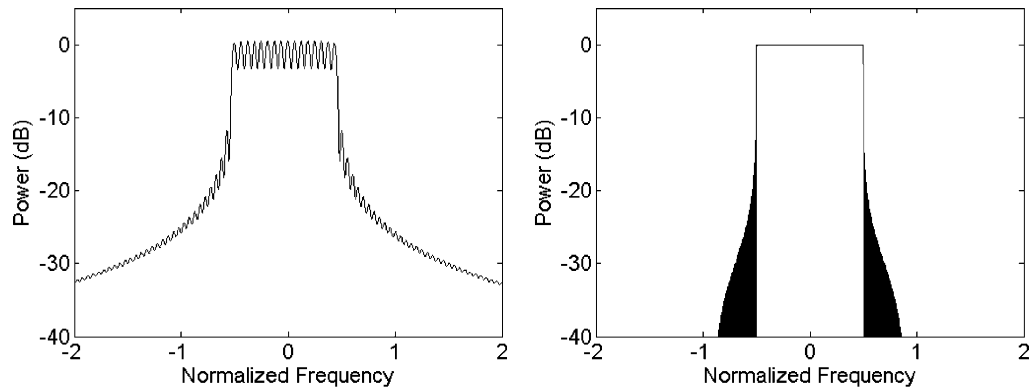

Fig. 9. Spectra of OFDM transmitted signals. (a) $N = 16$, Cyclic prefix length, $G = 4$. (b) $N = 256$, $G = 0$ (no cyclic prefix).

noisy channel. After coding, the data is mapped onto complex numbers representing the QAM constellation being used for transmission. Constellation sizes from 4 QAM to 64 QAM are typically used. While phase shift keying (PSK) is compatible with OFDM, it is rarely used. PSK in OFDM, unlike PSK in single carrier systems, does not have a constant signal envelope and, for large constellations, has smaller distance between constellation points and so is more susceptible to noise. The sequence of complex numbers output from the constellation mapping are then serial-to-parallel (S/P) converted to form a vector suitable for input to the IFFT.

F. Transmitter and Receiver Front End

The remaining section of the transmitter is the front end. Fig. 3. shows a block combining filtering, parallel-to-serial conversion (P/S) and digital-to-analog conversion (D/A) because in practice there is some choice about the order of these processes. For example, OFDM symbols are often windowed (a form of time variant filtering) to reduce the sidelobes, sometimes the digital signal is upsampled before D/A conversion to simplify the analog filtering, and filtering can be in the analog or digital domain. However after this process the signal $x(t)$ is an approximately bandlimited signal consisting of sinusoids of the baseband subcarrier frequencies. In wireless OFDM systems $x(t)$ is a complex signal which forms the input to an IQ modulator for upconversion to the carrier frequency. In this case the transmitted signal is given by

$$\begin{aligned} s(t) &= \Re\{x(t)\}\cos(2\pi f_c t) - \Im\{x(t)\}\sin(2\pi f_c t) \\ &= \Re\left\{x(t)e^{2\pi f_c t}\right\} \end{aligned} \tag{16}$$

where $\Re\{\ \}$ represents the real component and $\Im\{\ \}$ represents the imaginary component. In baseband systems such as ADSL, $x(t)$ is a real signal. In these systems, $\mathbf{X}$, the input to the transmitter IFFT is constrained to have Hermitian symmetry; $X_{N-k} = X_k^*$ where $*$ denote complex conjugation. This results in the imaginary components of the IFFT outputs canceling. This symmetry can be seen when the kth and $(N-k)$th subcarriers in Fig. 5.

Fig. 9. shows the spectra of the baseband signal for two combinations of CP length, and N. When there is no CP, the in-band spectrum is flat. The CP results in ripple in the in-band spectrum,

TABLE I
COMPARISON OF TYPICAL OFDM SYSTEM AND TYPICAL OPTICAL SYSTEM

Typical OFDM System	Bipolar	Information carried on electrical field	Local Oscillator at receiver	Coherent Reception
Typical Optical System	Unipolar	Information carried on optical intensity	No Local Oscillator (laser) at receiver	Direct Detection

but this does not cause any practical problems. If no windowing is used, the first sidelobe of an OFDM spectrum is always 13 dB below the in-band power. The rate with which the out-of-band power falls off depends on the number of subcarriers. So for DVB systems, where the FFT size is 2048 or 8196, the spectrum falls of very rapidly, but for wireless LANs, where $N = 64$, the out-of-band power would create problems, so time domain windowing, or filtering is used to reduce the sidelobes.

At the receiver, in wireless systems the signal is downconverted by mixing with in-phase and quadrature components of a locally generated carrier, $\cos(2\pi f_r t + \theta(t) + \varphi_d)$ and $-\sin(2\pi f_r t + \theta(t))$. Ideally the frequency of the local carrier, f_r, is identical to the carrier frequency of the received signal, but in practice there may be some difference. This can be caused by error in the carrier recovery at the receiver, or in wireless systems, by Doppler effects due to moving transmitter, receiver or reflectors. Any constant error in the absolute phase $\theta(t)$ is unimportant, as it is compensated for automatically by the single tap equalizer, however any frequency error or phase noise can cause problems as discussed in Section V.

IV. OFDM FOR OPTICAL COMMUNICATIONS

Despite the many advantages of OFDM, and its widespread use in wireless communications, OFDM has only recently been applied to optical communications. This is partly because of the recent demand for increased data rates across dispersive optical media and partly because developments in digital signal processing (DSP) technology make processing at optical data rates feasible. However another important obstacle has been the fundamental differences between conventional OFDM systems and conventional optical systems. Table I summarizes these differences.

In typical (nonoptical) OFDM systems, the information is carried on the electrical field and the signal can have both positive and negative values (bipolar). At the receiver there is a local oscillator and coherent detection is used. In contrast in a typical intensity-modulated direct-detection optical system, the information is carried on the intensity of the optical signal and therefore can only be positive (unipolar). There is no laser at the receiver acting as a local oscillator and direct detection rather than coherent detection is used.

A variety of optical OFDM solutions have been proposed for different applications. To understand these different techniques, it is useful to realize what is fundamental in each domain. For an OFDM system to work successfully the system must be (approximately) linear between the transmitter IFFT input and the receiver FFT output. In other words, $Y_k \approx H_k X_k$ where H_k is either a constant or is slowly varying so that it can be tracked at that the receiver. In the optical domain, optical receivers use square-law detectors.

Optical OFDM solutions can be broadly divided into two groups. The first group comprises techniques for systems where many different optical modes are received, for example, optical wireless, multimode fiber systems and plastic optical fiber systems. For these the OFDM signal should be represented by the intensity of the optical signal. The second group includes techniques for single mode fiber, where only one mode of the signal is received and for these the OFDM signal should be represented by the optical field.

A. Optical OFDM Using Intensity Modulation

In [30], Kahn and Barry explain why the many optical modes that are present at the receiver result in optical wireless systems being linear in intensity. So, for optical wireless systems and other systems where many modes are received, the OFDM signal must be represented as intensity. This means that the modulating signal must be both real and positive, whereas baseband OFDM signals are generally complex and bipolar. As noted in Section III, a real baseband signal OFDM signal can be generated by constraining $\mathbf{X}$ to have Hermitian symmetry. Two forms of unipolar OFDM have been proposed: dc-biased optical OFDM (DCO-OFDM) [31], [32] and asymmetrically clipped OFDM (ACO-OFDM) [33], [34]. In dc-biased OFDM, a DC bias is added to the signal, however because of the large peak-to-average power ratio of OFDM, even with a large bias some negative peaks of the signal will be clipped and the resulting distortion limits performance [34]. In ACO-OFDM the bipolar OFDM signal is clipped at the zero level: all negative going signals are removed. If only the odd frequency OFDM subcarriers are non zero at the IFFT input, all of the clipping noise falls on the even subcarriers, and the data carrying odd subcarriers are not impaired [33].

In [34] it was shown that except for extremely large constellations ACO-OFDM requires a lower average optical power for a given BER and data rate than DCO-OFDM. ACO-OFDM has also been shown to be efficient from an information theoretic perspective [35]. The use of DCO-OFDM has been demonstrated experimentally for optical wireless [36], multimode fiber [27] and plastic optical fiber [28]. A number of simulation studies examine the performance of DCO-OFDM in more detail [37], and how adaptive modulation can be used to improve performance [26].

B. Optical OFDM Using Linear Field Modulation

In single mode optical fiber systems the best way to achieve linearity between the transmitter IFFT input and the receiver FFT output is to map each discrete OFDM subcarrier frequency in the baseband electrical domain to a single discrete frequency in the optical domain. This is achieved by using linear field modulation, so that there is a linear relationship between the optical field of the transmitted signal and the OFDM baseband signal [21]. At the receiver the OFDM signal is mixed with a component at the optical carrier frequency and the signal detected from the carrier $\times$ signal mixing products. The component at the optical carrier frequency can either be transmitted with the OFDM signal as in direct-detection optical OFDM (DD-OOFDM) [38] or coherent detection can be used where the received signal is mixed with a locally generated carrier signal as in coherent optical OFDM (CO-OFDM) [22].

Both techniques have advantages. DD-OOFDM has a simple receiver, but some optical frequencies must be unused if unwanted mixing products are not to cause interference. This is usually achieved by inserting a guard band between the optical carrier and the OFDM subcarriers. This reduces spectral efficiency. DD-OOFDM also requires more transmitted optical power, as some power is required for the transmitted carrier. CO-OFDM requires a laser at the receiver to generate the carrier locally, and is more sensitive to phase noise [23], [39]. There is currently extensive research into the performance of both systems and on techniques to mitigate the disadvantages of each [23], [40]–[47].

It is useful to understand the problems which arise in single mode systems if an OFDM subcarrier is mapped to more than one optical frequency. If double-sideband modulation is used, each OFDM subcarrier is represented by two optical frequencies, one on either side of the optical carrier, chromatic dispersion (CD) results in two components with equal amplitude and different phases. Subcarriers for which these two components cancel, experience deep fades. If intensity modulation is used, but one sideband is suppressed, the combination of the nonlinear effect of intensity modulation and dispersion in the channel results in ICI in the received signal.

C. MIMO-OFDM for Optical Communications

In wireless communications, MIMO OFDM has very quickly moved from theoretical concept to commercial application. In the literature on RF wireless systems, the term "MIMO" is used to describe a range of systems with multiple transmit and/or receive antennas. Depending on the relationship between the signals transmitted from different antennas MIMO schemes can be used to either increase the overall capacity of the system, or to reduce the probability of outage [48], [49]. Because wireless channels usually introduce significant multipath dispersion, MIMO is often combined with OFDM.

MIMO techniques have also been shown to give significant benefit across a range of optical systems. In indoor optical wireless, multiple transmitters and or receivers can be used to increase the probability of line of sight between transmitter and

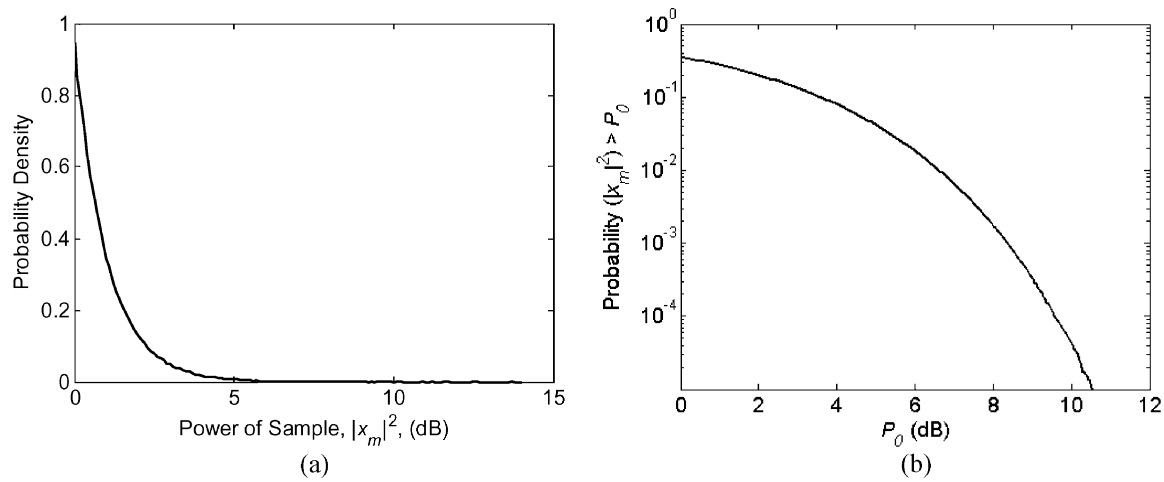

Fig. 10. Distribution of power of OFDM signal (a) probability density function and (b) cumulative distribution.

receiver [50]. In this application MIMO OFDM combines the advantages of MIMO with tolerance to delay spread [51].

MIMO techniques have also been applied to free space optical systems [52]–[54] but none of these have used OFDM. As signal dispersion is relatively unimportant in these applications, the dispersion tolerance of OFDM is not a significant advantage, though the power efficiency of ACO-OFDM has potential benefit.

MIMO techniques have also been applied to a range of optical fiber applications. A number of authors have noted the potential of MIMO techniques in multimode fiber [55]–[57]. Intermodal dispersion is usually considered to be a problem in multimode systems, however when MIMO techniques are applied, it can be used to increase the information capacity of the fiber [57]. So far there do not appear to be any papers considering the combination of OFDM with MIMO in multimode systems, despite the significant potential advantages.

MIMO, both with and without OFDM, has been applied very successfully in single-mode fiber applications by transmitting and receiving signals on both polarizations. In this context, MIMO is also called polarization multiplexing. MIMO in single-mode fiber systems has very different characteristics from MIMO in wireless applications and may well give even greater benefits. With polarization multiplexing, all of the received signal power is divided between the two received polarizations, whereas in wireless systems, the signals at different receive antennas are at best uncorrelated, and there is always some probability of outage when no antenna is receiving a good signal. It has been shown experimentally that by using MIMO/polarization multiplexing very high data rate transmission can be achieved both in systems using OFDM [58]–[61] and systems using single carrier formats [62], [63].

V. Disadvantages of OFDM

As well as its many advantages, OFDM has a number of disadvantages, of these the most important in wireless communications are the high peak-to-average power ratio (PAPR) and the sensitivity to phase noise and frequency offset.

A. Peak-to-Average Power Ratio

The high PAPR of OFDM means that if the signal is not to be distorted, many of components in the transmitter and receiver must have a wide dynamic range. In particular the output amplifier of the transmitter must be very linear over a wide range of signal levels. In wireless systems the expense and power consumption of these amplifiers is often an important design constraint. Intermodulation resulting from any nonlinearity results in two major impairments: out-of-band (OOB) power and in-band distortion. In wireless communications OOB power is usually the more important, because of the near-far problem; interference from the OOB power of a close transmitter may swamp reception from a distant transmitter. For this reason the specifications on OOB power in wireless are very stringent. OOB power caused by transmitter nonlinearities may be much less of a problem in optical applications of OFDM. As we will show, in-band distortion is a relatively small effect and becomes important only for large signal constellations.

1) Statistics of OFDM Signals: We will now consider the statistics of the signals at various points within the transmitter. The real and imaginary components of x_m, the signal samples at the output of the receiver IFFT, have approximately Gaussian distributions. This is because the IFFT operation results in summing many independently modulated subcarriers and the central limit theorem applies. This results in a probability density function for $|x_m|^2$, the power of the samples, of the form shown in Fig. 10(a). Because it is the tail of the amplitude distribution that is important, the literature on PAPR on OFDM usually presents the amplitude statistics in terms of cumulative density. Fig. 10(b) shows the distribution of an OFDM signal in this form. Although OFDM has high signal peaks, these peaks occur relatively rarely. For example only one in a thousand values is more than 8 dB above the mean.[3] Despite the relatively infrequent occurrence of peaks, they can cause significant OOB power when the output amplifier is even slightly nonlinear or when the amplifier or other components saturate.

[3]Much of the literature on PAPR on OFDM presents results in terms of the cumulative density per OFDM *symbol*, rather than per *sample*. The author prefers the per sample form, as this relates much more closely to the OOB and in-band distortion and is not a function of N for $N \geq 64$.

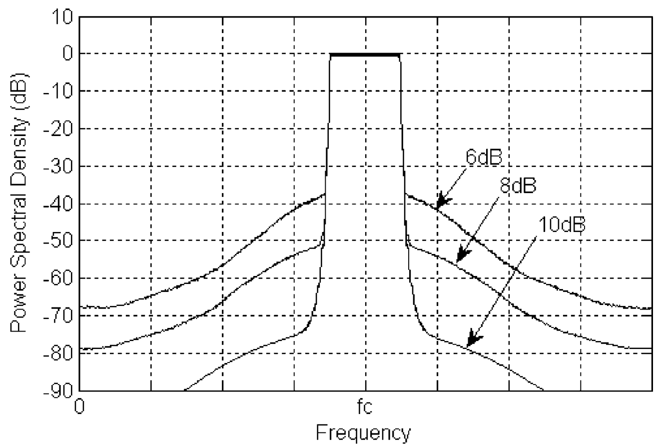

Fig. 11. OFDM spectrum when signal is clipped at 6, 8, and 10 dB.

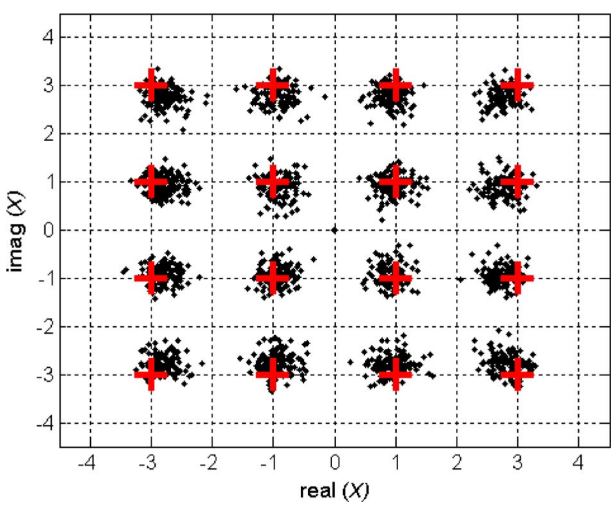

Fig. 12. Signal constellation before (red crosses) and after clipping (black dots).

In the following, we consider the case where the nonlinearity has the form of amplitude clipping of the complex analog baseband signal in the transmitter.

$$x_{\text{clip}}(t) = \begin{cases} x(t) & |x(t)| \le A \\ Ae^{j\arg(x(t))} & |x(t)| > A. \end{cases} \tag{17}$$

The clipping ratio (CR) is defined as

$$CR = 20\log_{10}\left(\frac{A}{\sigma}\right) \text{ dB} \tag{18}$$

where σ^2 is the power of $x(t)$.

Fig. 11 shows the typical form of the spectra for CRs of 6, 8, and 10 dB above the mean power. The limiting causes 'shoulders' on the spectra which increase as the limiting level falls.

The second problem that nonlinearities can cause is in-band distortion. This is much less of a problem than some early papers on the topic suggest. The main effect of a memoryless nonlinearity is to shrink the constellation, not to cause interference. Fig. 12 shows this effect. The crosses show the original 16 QAM constellation points, while the dots show the constellation after clipping. In this case, clipping was very severe, with CR only 3 dB above the mean power of the unclipped signal. Clipping causes the constellation to shrink and also adds a noise like distortion

$$x_{clip}(t) = \alpha x(t) + d(t) \tag{19}$$

where $d(t)$ is the "clipping" noise, which is uncorrelated with the signal and α is a constant which depends on the nonlinearity.[4] In this example

$$\alpha = \left(1 - e^{-\frac{A^2}{\sigma^2}}\right) + \sqrt{\frac{\pi A^2}{4\sigma^2}}\text{erfc}\left(\frac{A}{\sigma}\right) \tag{20}$$

where $\text{erfc}(x) = 2/\sqrt{\pi}\int_x^\infty e^{-y^2}dy$ [64].

Fig. 13 shows the BER for 4 QAM and 16 QAM when the signal is clipped in the transmitter. For 4 QAM, even extreme clipping with a clipping ratio of 3 dB causes little degradation as long as the shrinkage of the constellation is corrected before detection.

2) Solutions to the PAPR Problem: There are numerous papers describing different solutions to the PAPR problem. These can be broadly classified into techniques involving coding, techniques involving multiple signal representation (MSR), and techniques involving non linear distortion, such as clipping. Coding techniques aim to apply coding to the input vector $\mathbf{X}$ so that OFDM symbols which have high PAPR are not used. Despite extensive research, effective codes have not been developed. MSR involves generating a number of possible transmit signals for each input data sequence and using the one with the lowest PAPR. These techniques are probably too computationally intensive to be useful in most optical applications. Because OOB power will probably be less of a problem in optical applications, if PAPR reduction is required simple nonlinear techniques, possibly in combination with some form of predistortion, may be the most appropriate.

3) Clipping to Reduce PAPR: For clipping to be an effective solution to PAPR, clipping must be performed on either the analog signal, or an upsampled version of the digital signal with an oversampling factor of at least two. This is because once the signal is D/A converted the peaks of the signal may occur between the discrete samples. Fig. 14 shows this effect.

B. Sensitivity to Frequency Offset, Phase Noise, and I/Q Imbalance

Differences in the frequency and phase of the receiver local oscillator and the carrier of the received signal can degrade system performance. In the existing OFDM literature these impairments are usually classified in terms of their source, for example, frequency offset between transmitter and receiver local oscillator [65], Doppler spread in the channel [66], and a variety of phase noise models with characteristics that depend on the mechanisms of carrier recovery in the receiver [67]–[70]. These results are in general not directly applicable to optical applications of OFDM. Instead in this section we discuss the

[4]Note that the noise is not the difference between the clipped and unclipped signal. Another common source of error in the OFDM literature, is to consider the difference between the clipped and unclipped signal as a form of impulsive noise. Impulsive noise is normally completing uncorrelated with the signal, whereas clipping only occurs for large signal amplitudes and so is highly correlated with the signal.

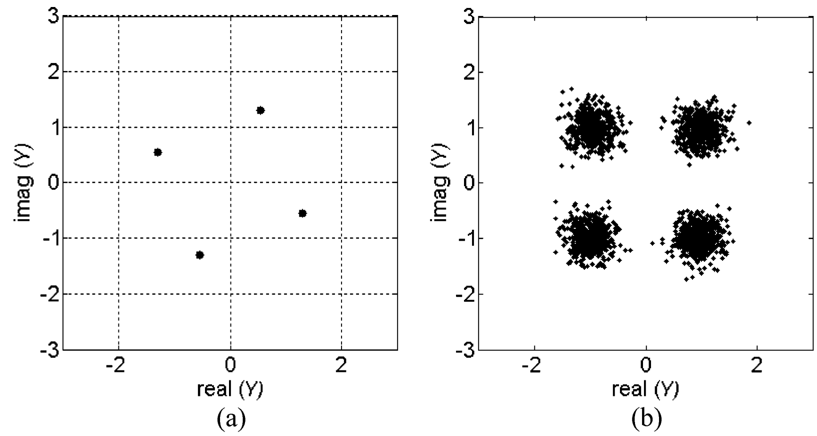

Fig. 15. Effect of phase error on received OFDM constellation (a) when the phase error is constant $\theta_m = \theta_0 = \pi/8$ and (b) when phase noise is uncorrelated and $E\{\theta_m^2\} = 0.05\ \mathrm{rad}^2$.

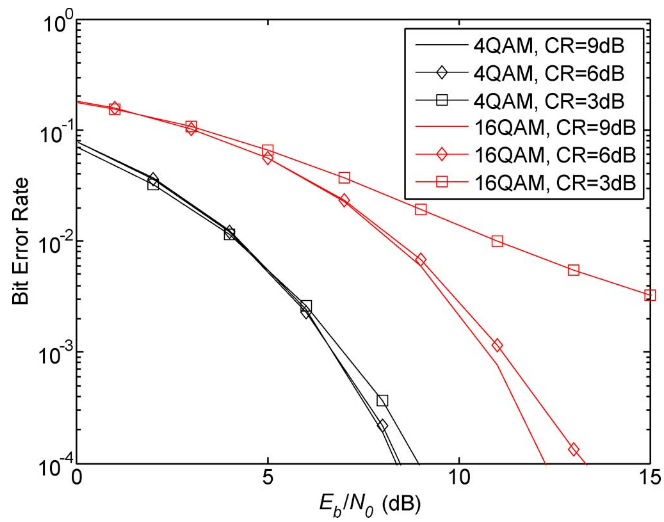

Fig. 13. Effect of clipping on BER.

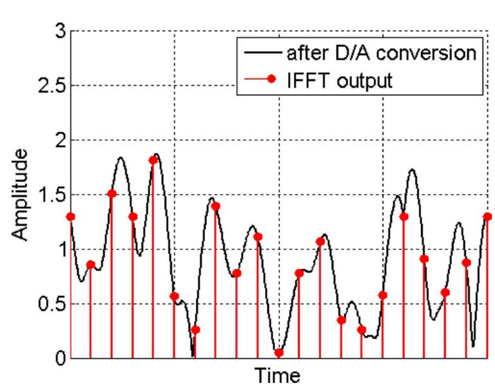

Fig. 14. Amplitude of signal samples at the output of the IFFT and the continuous signal after D/A conversion.

effect of phase and frequency errors in terms of the relationship between the time variation of phase and its effect on the received constellation. We then discuss how this affects how easily these errors can be corrected in the digital domain.

First consider the case where there is no noise or distortion in the channel, and $\varphi_d = 0$ so that there is no I/Q phase imbalance. Then using the formulae for sums and products of angles and with simple manipulation, the time domain received signal samples are given by

$$y_m = x_m \exp(j\theta_m) \tag{21}$$

where θ_m is the phase error at the receiver for the mth sample of the OFDM symbol under consideration.

For the case where there is a constant phase error, $\theta_m = \theta_0$. Then $y_m = x_m \exp(j\theta_0)$ and it is simple to show that

$$Y_k = \exp(j\theta_0)X_k.$$

The constellation is simply rotated by angle θ_0. An example is shown in Fig. 15(a). In an OFDM system, this would be automatically corrected in the single tap equalizer.

Now consider the opposite extreme, where the phase noise is zero mean, and there is no correlation between phase noise samples $E\{\theta_m\theta_n\} = 0\, m \neq n$, where $E\{\,\}$ denotes the expectation operator.

Then taking the FFT gives

$$\begin{aligned} Y_k &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_m \exp\left(\frac{-j2\pi km}{N}\right) \\ &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m \exp(j\theta_m) \exp\left(\frac{-j2\pi km}{N}\right). \end{aligned} \tag{22}$$

If the phase error is small, then using the small angle approximation $\exp(j\theta_m) \approx 1 + j\theta_m$

$$\begin{aligned} Y_k &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m(1+j\theta_m) \exp\left(\frac{-j2\pi km}{N}\right) \\ &= X_k + \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} j\theta_m x_m \exp\left(\frac{-j2\pi km}{N}\right) \\ &= X_k + \mathcal{N}_k. \end{aligned} \tag{23}$$

The demodulated subcarrier Y_k is equal to the transmitted subcarrier plus a noise like term, $\mathcal{N}_k$, which depends on all of the transmitted subcarriers. The power of $\mathcal{N}_k$ can be calculated by using the fact that θ_m and x_m are statistically independent. See Fig. 15(b).

For θ_m with characteristics between these extremes, phase error results in a combination of the noise like ICI and constellation rotation (usually called "common phase error" in the literature). For example, if there is a fixed frequency offset between transmitter and receiver so that $f_r = f_c + \Delta f$, then the phase varies linearly with time $\theta(t) = \Delta f t$. This results in both a noise-like term and a continuously rotating constellation. The resulting BER of the system depends on how the receiver tracks this changing phase. If the receiver tracks the phase so that $\theta(t) = 0$ at the beginning of each symbol period, then the received constellation has the form shown in Fig. 16(a). If the receiver can predict the phase change, then the constellation has the form shown in Fig. 16(b). While if there is no tracking or correction, the constellation continuously rotates and no useful information can be transferred. See Fig. 16(c). It has been found that by inserting pilot tones within the OFDM symbol and using these to track the variation in laser phase noise, the effect of phase noise in CO-OFDM systems can be reduced [24].

I/Q imbalance also results in ICI in OFDM systems. Liu shows that a phase imbalance φ_d of less than 6.2° and an amplitude imbalance less than 0.54 dB is required if the signal to ICI power ratio is to be greater than 30 dB [71].

VI. The Future of Optical OFDM

In the last few years, there has been an explosion of interest in OFDM for optical applications and an increasing number of papers have been published showing that both theoretically and experimentally, OFDM can be used in optical systems. However there are, to the knowledge of this author, so far no commercial products using optical OFDM. Whether optical OFDM will in the future dominate high speed optical communication as it now dominates broadband wireless, or whether it finds application only in particular niches will depend on a combination of the fundamental advantages and disadvantages of OFDM in various optical channels, and the cost of implementing OFDM transmitters and receivers relative to competing technologies. This is currently the subject of heated debate among experts in the field [72]–[77].

A. Fundamental Advantages and Disadvantages of OFDM in Optical Applications

An important advantage of OFDM is its ability to precisely tailor the transmitted signal to the frequency characteristics of the channel, for example by avoiding frequencies with low SNR, and increasing the constellation size on 'good' subcarriers. Techniques of this type have already been proposed for optical systems [29], [78], [79].

In long haul single mode optical fiber, the high peak-to-average power of OFDM may be a fundamental disadvantage, but the difference is only likely to be significant in systems with dispersion compensation [80]. In systems without dispersion compensation, signal dispersion will cause the power distribution of OFDM and other modulation schemes to become similar at points distant from the transmitter. For systems with dispersion compensation, clipping of the OFDM signal before transmission may significantly reduce the effect of fiber nonlinearity.

B. Difficulty and Cost of OFDM Implementation

One drawback of OFDM may be the cost of implementing optical OFDM transmitters and receivers relative to other modulation formats. We will now briefly discuss the implementation of three aspects of a transceiver: the DSP required for the digital sections, the analog-to-digital convertors (ADCs) and digital-to-analog converters (DACs), and the optical components required for the transmitter and receiver front ends.

The DSP requirements depend on both the number of arithmetic operations (e.g. multiplications and additions) per bit and on the number of bits required to represent the signal at various points within the transmitter and receiver. A number of papers have compared the number of arithmetic operations in OFDM and single carrier optical systems [1]–[3] and found that the computational requirements in terms of arithmetic operations are similar for the most computationally efficient system of each type, but there have been no papers considering the second factor.

Very fast DSP implementations will use fixed point, rather than floating point arithmetic. For signals with a wide dynamic range such as OFDM, the choice in a fixed point system of the number of bits, and the signal levels which they represent, is a trade-off between being able to represent the largest signal values with low probability of numerical overflow, and the quantization noise and rounding errors which result from using only a few bits [81]. This is particularly important within the IFFT and FFT, where the additions that occur at each butterfly stage may cause overflow. A simple solution for the IFFT/FFT is to ensure that numeric overflow results in saturation at the maximum positive or negative level not 'wrap around' [82].

The design of the DACs and ADCs may well be the most critical factor for OFDM. An advantage of single carrier systems may be that the DAC is required to represent only the few discrete levels of the QAM modulation. So for example for a 4QAM system, each of the I and Q analog outputs from the DAC require only two levels (one bit resolution), for 16 QAM only four levels (two bit resolution) are required for each. This may mean that the circuitry is very simple. For OFDM depending on the required SNR a DAC with six or seven bit resolution may be required [81]. However other probably equally important aspects of DAC and ADC design will be the maximum allowable timing jitter, and the linearity and accuracy of the conversion and how errors in these interact with the modulation format. In general because of the averaging effect of the FFT the performance of OFDM depends on the mean power of impairments, whereas for single carrier systems, the peak value may be more important. Timing jitter has received relatively little attention in the OFDM literature and even at the highest data rates used in broadband wireless systems causes very little degradation [83]. In contrast timing jitter has historically been one of the main limitations of high data rate optical systems.

Sampling rate will also be an important factor. For OFDM, the overhead of the cyclic prefix, unused band-edge subcarriers, and any pilot tones mean that some oversampling is required [84]. The exact value will depend on the detailed system design but will probably be in the range of 10–30% oversampling. Most single carrier systems use an oversampling rate of 2, but Ip and Kahn [85] have shown that a factor of only 1.5 may be required.

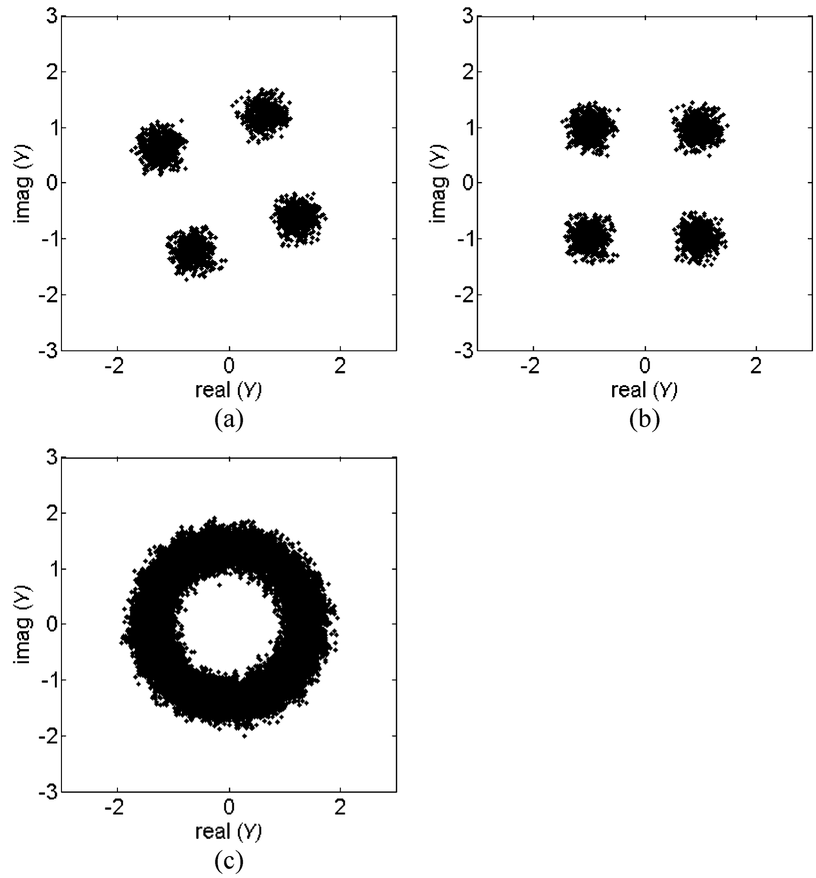

Fig. 16. Effect of frequency offset $\Delta f T = 0.2$ on received constellation. (a) Phase offset zero at start of each OFDM symbol, (b) phase offset zero in the middle of each OFDM symbol, and (c) no phase offset correction.

Finally, the tolerance on the optical components may be different in OFDM and single carrier systems. For coherent optical systems, the sensitivity of OFDM to phase noise and frequency offset will set stringent tolerances on the linewidth of lasers, but in OFDM it is also possible to digitally compensate for some of these effects in the digital domain [84]. The tolerance of OFDM to the impairments introduced by optical components and signal processing algorithms to mitigate them are likely to provide many interesting future research topics.

VII. Conclusions

This paper presents a tutorial introduction to OFDM. A typical OFDM transmitter and receiver are described and the roles of the main signal processing blocks explained. The time and frequency domain signals at various points in the system are described. It is shown that if a cyclic prefix is added to each OFDM symbol, any linear distortion introduced by the channel can be equalized by a single tap equalizer. This process is explained by considering the effect of a simple two-path channel on the component of the transmitted signal due to one subcarrier frequency. Throughout the description particular emphasis is given to those aspects of an OFDM system that are often misunderstood.

Although the theoretical basis for OFDM was laid several decades ago, and OFDM became the basis of many communications standards for wireless and wired applications in the 1990's, it is only very recently that the application of OFDM to optical communication has been considered. This is partly because of the apparent incompatibility between OFDM modulation and conventional optical systems. A number of forms of OFDM which overcome these incompatibilities in various ways have been developed for a variety of optical applications. These are classified into those appropriate for optical wireless and multimode fiber applications and those for single mode fiber. For the former, intensity modulation should be used, while for the latter the OFDM signal should be carried on the field of optical signal.

OFDM has a number of drawbacks including its high peak-to-average power ratio and sensitivity to frequency offset and phase noise. These are described and their likely implications for optical communications discussed.

In conclusion, OFDM is a very promising technology for optical communications, but the very different constraints introduced open up many new interesting avenues for research.

Acknowledgment

The author would like to thank the anonymous reviewers for their helpful comments and suggestions.

References

[1] H. Bülow, "Electronic dispersion compensation," presented at the Proc. OFC/NFOEC 2007, Anaheim, CA, Mar. 25–29, 2007, Tutorial, OMG5.

[2] S. J. Savory, "Digital signal processing options in long haul transmission," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OTuO3.

[3] B. Spinnler, "Recent advances on polarization multiplexing," presented at the Proc. IEEE Summer Topicals, 2008, TuD2.3.

[4] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5–14, 1990.
[5] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Boston: Artech House, 2000.
[6] W. Y. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadcasting*, vol. 41, pp. 1–8, 1995.
[7] J. H. Stott, "The how and why of COFDM," *EBU Tech. Rev.*, pp. 43–50, 1998.
[8] J. Armstrong, "OFDM: From copper and wireless to optical," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Tutorial, OMM1.
[9] R. W. Chang, "Orthogonal Frequency Multiplex Data Transmission System," USA U.S. Patent 3,488,445, 1966.
[10] J. Salz and S. B. Weinstein, "Fourier transform communication system," in *Proc. ACM Symp. Problems Optim. Data Commun. Syst.*, Pine Mountain, GA, USA, 1969.
[11] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proc. ICASSP 80*, Denver, CO, USA, 1980, vol. III, pp. 964–967, IEEE.
[12] I. E. Telatar, Capacity of Multi-Antenna Gaussian Channels Tech. Memo., Bell Laboratories, Lucent Technologies, (Published in *European Trans. Telecommun.*, Vol. 10, No. 6, pp. 585–595, Nov./Dec. 1999), October 1995.
[13] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, pp. 311–335, 1998.
[14] L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. CM-33, pp. 665–675, 1985.
[15] R. Lassalle and M. Alard, "Principles of modulation and channel coding for digital broadcasting for mobile receivers," *EBU Tech. Rev.*, pp. 168–190, 1987.
[16] J. S. Chow, J. C. Tu, and J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE J. Sel. Areas Commun.*, vol. 9, pp. 895–908, 1991.
[17] I. Koffman and V. Roman, "Broadband wireless access solutions based on OFDM access in IEEE 802.16," *IEEE Commun. Mag.*, vol. 40, pp. 96–103, 2002.
[18] U. Reimers, "Digital video broadcasting," *IEEE Commun. Mag.*, vol. 36, pp. 104–110, 1998.
[19] O. Gonzalez, R. Perez-Jimenez, S. Rodriguez, J. Rabadan, and A. Ayala, "Adaptive OFDM system for communications over the indoor wireless optical channel," *IEE Proc.–Optoelectron.*, vol. 153, pp. 139–144, 2006.
[20] J. Grubor, V. Jungnickel, and K.-D. Langer, "Adaptive optical wireless OFDM system with controlled asymmetric clipping," in *Proc. ACSSC*, 2007.
[21] B. J. C. Schmidt, A. J. Lowery, and J. Armstrong, "Experimental demonstrations of electronic dispersion compensation for long haul transmission using direct-detection optical OFDM," *J. Lightw. Technol.*, pp. 196–203, 2008.
[22] W. Shieh and C. Athaudage, "Coherent optical orthogonal frequency division multiplexing," *Electron. Lett.*, vol. 42, pp. 587–588, 2006.
[23] S. L. Jansen, I. Morita, N. Takeda, and H. Tanaka, "20-Gb/s OFDM transmission over 4,160-km SSMF enabled by RF-pilot tone phase noise compensation," presented at the Proc. OFC/NFOEC 2007, Anaheim, CA, Mar. 25–29, 2007, Paper PDP15.
[24] S. L. Jansen, I. Morita, T. C. W. Schenk, N. Takeda, and H. Tanaka, "Coherent optical 25.8-Gb/s OFDM transmission over 4160-km SSMF," *J. Lightw. Technol.*, vol. 26, pp. 6–15, 2008.
[25] A. J. Lowery and J. Armstrong, "10 Gbit/s multimode fiber link using power-efficient orthogonal-frequency-division multiplexing," *Optics Expr.*, vol. 13, pp. 10003–10009, 2005.
[26] J. M. Tang, P. M. Lane, and K. A. Shore, "Transmission performance of adaptively modulated optical OFDM signals in multimode fiber links," *IEEE Photon. Technol. Lett.*, vol. 18, pp. 205–207, 2006.
[27] S. C. J. Lee, F. Breyer, S. Randel, M. Schuster, J. Zeng, F. Huiskens, H. P. A. van den Boom, A. M. J. Koonen, and N. Hanik, "24-Gb/s transmission over 730 m of multimode fiber by direct modulation of 850-nm VCSEL using discrete multi-tone modulation," presented at the Proc. OFC/NFOEC 2007, Anaheim, CA, Mar. 25–29, 2007, Paper PDP6.
[28] S. C. J. Lee, F. Breyer, S. Randel, O. Ziemann, H. P. A. van den Boom, and A. M. J. Koonen, "Low-cost and robust 1-Gbit/s plastic optical fiber link based on light-emitting diode technology," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OWB3.
[29] S. K. Wilson and J. Armstrong, "Digital modulation techniques for optical asymmetrically-clipped OFDM," in *Proc IEEE Wireless Commun. Netw. Conf.*, Las Vegas, NV, USA, 2008, pp. 538–542.
[30] J. M. Kahn and J. R. Barry, "Wireless infrared communications," *Proc. IEEE*, vol. 85, pp. 265–298, 1997.
[31] J. B. Carruthers and J. M. Kahn, "Multiple-subcarrier modulation for nondirected wireless infrared communication," *IEEE J. Sel. Areas Commun.*, vol. 14, pp. 538–546, 1996.
[32] O. Gonzalez, R. Perez-Jimenez, S. Rodriguez, J. Rabadan, and A. Ayala, "OFDM over indoor wireless optical channel," *IEE Proc.—Optoelectronics*, vol. 152, pp. 199–204, 2005.
[33] J. Armstrong and A. J. Lowery, "Power efficient optical OFDM," *Electron. Lett.*, vol. 42, pp. 370–371, 2006.
[34] J. Armstrong and B. J. C. Schmidt, "Comparison of asymmetrically clipped optical OFDM and DC-biased optical OFDM in AWGN," *IEEE Commun. Lett.*, vol. 12, pp. 343–345, 2008.
[35] X. Li, R. Mardling, and J. Armstrong, "Channel capacity of IM/DD optical communication systems and of ACO-OFDM," in *Proc. ICC '07*, 2007, pp. 2128–2133.
[36] N. Cvijetic, D. Qian, and T. Wang, "10 Gb/s free-space optical transmission using OFDM," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OTHD2.
[37] N. Cvijetic and W. Ting, "WiMAX over free-space optics—Evaluating OFDM multi-subcarrier modulation in optical wireless channels," in *Proc. IEEE Sarnoff Symp.*, Princeton, NJ, USA, 2006.
[38] A. J. Lowery and J. Armstrong, "Orthogonal-frequency-division multiplexing for dispersion compensation of long-haul optical systems," *Opt. Expr.*, vol. 14, pp. 2079–2084, 2006.
[39] S. L. Jansen, I. Morita, T. C. W. Schenk, D. van den Borne, and H. Tanaka, "Optical OFDM—A candidate for future long-haul optical transmission systems," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OMU3.
[40] W. Shieh, W. Chen, and R. S. Tucker, "Polarisation mode dispersion mitigation in coherent optical orthogonal frequency division multiplexed systems," *Electron. Lett.*, vol. 42, pp. 996–997, 2006.
[41] A. J. Lowery, "Fiber nonlinearity mitigation in optical links that use OFDM for dispersion compensation," *IEEE Photon. Technol. Lett.*, vol. 19, pp. 1556–1558, 2007.
[42] X. Yi, W. Shieh, and T. Yan, "Phase estimation for coherent optical OFDM," *IEEE Photonics Technology Letters*, vol. 19, pp. 919–921, 2007.
[43] R. Dischler and F. Buchali, "Experimental assessment of a direct detection optical OFDM system targeting 10 Gb/s and beyond," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OM12.
[44] D. F. Hewitt, "Orthogonal frequency division multiplexing using baseband optical single sideband for simpler adaptive dispersion compensation," presented at the Proc. OFC/NFOEC 2007, Anaheim, CA, Mar. 25–29, 2007, Paper, OME7.
[45] B. Goebel, B. Fesl, L. D. Coelho, and N. Hanik, "On the effect of FWM in coherent optical OFDM systems," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, JWA58.
[46] W.-R. Peng, X. Wu, V. R. Arbab, B. Shamee, J.-Y. Yang, L. C. Christen, K.-M. Feng, A. E. Willner, and S. Chi, "Experimental demonstration of 340 km SSMF transmission using a virtual single sideband OFDM signal that employs carrier suppressed and iterative detection techniques," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OMU1.
[47] C. Xie, "PMD insensitive direct-detection optical OFDM systems using self-polarization diversity," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, OMM2.
[48] A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microw. Mag.*, vol. 5, pp. 46–56, 2004.
[49] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications—A key to gigabit wireless," *Proc. IEEE*, vol. 92, pp. 198–218, 2004.
[50] S. Jivkova and M. Kavehrad, "Shadowing and blockage in indoor optical wireless communications," in *Proc. GLOBECOM 2003*, pp. 3269–3273.
[51] O. Gonzalez, S. Rodriguez, R. Perez-Jimenez, B. R. Mendoza, and F. Delgado, "Adaptive OFDM system for multi-user communications over the indoor wireless optical channel," in *Proc. SPIE*, 2007, vol. 6593.
[52] S. G. Wilson, M. Brandt-Pearce, C. Qianling, and J. H. Leveque, III, "Free-space optical MIMO transmission with Q-ary PPM," *IEEE Trans. Commun.*, vol. 53, pp. 1402–1412, 2005.

[53] N. Letzepis, I. Holland, and W. Cowley, "The Gaussian free space optical MIMO channel with Q-ary pulse position modulation," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1744–1753, 2008.

[54] N. Cvijetic, S. G. Wilson, and M. Brandt-Pearce, "Performance bounds for free-space optical MIMO systems with APD receivers in atmospheric turbulence," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 3–12, 2008.

[55] M. Greenberg, M. Nazarathy, and M. Orenstein, "Data parallelization by optical MIMO transmission over multimode fiber with intermodal coupling," *J. Lightw. Technol.*, vol. 25, pp. 1503–1514, 2007.

[56] A. Tarighat, R. C. J. Hsu, A. Shah, A. H. Sayed, and B. Jalali, "Fundamentals and challenges of optical multiple-input multiple-output multimode fiber links," *IEEE Commun. Mag.*, vol. 45, pp. 57–63, 2007.

[57] A. P. T. Lau, X. Lei, and W. Ting, "Performance of receivers and detection algorithms for modal multiplexing in multimode fiber systems," *IEEE Photon. Technol. Lett.*, vol. 19, pp. 1087–1089, 2007.

[58] W. Shieh, "PMD-supported coherent optical OFDM systems," *IEEE Photon. Technol. Lett.*, vol. 19, pp. 134–136, 2007.

[59] S. L. Jansen, I. Morita, and H. Tanaka, "16 × 52.5-Gb/s, 50-GHz spaced,POLYMUX-CO-OFDM transmission over 4,160 km of SSMF enabled by MIMO processing," presented at the Proc. ECOC 2007, 2007, Paper PD1.3.

[60] Y. Ma, W. Shieh, and Q. Yang, "Bandwidth-efficient 21.4 Gb/s coherent optical 2 × 2 MIMO OFDM transmission," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, Paper, JWA59.

[61] A. Sano, H. Masuda, T. K. E. Yoshida, E. Yamada, Y. Miyamoto, F. Inuzuka, Y. Hibino, Y. Takatori, K. Hagimoto, T. Yamada, and Y. Sakamaki, "30 × 100-Gb/s all-optical OFDM transmission over 1300 km SMF with 10 ROADM nodes," presented at the ECOC 2007, 2007, Paper, PDS1.7.

[62] S. Chandrasekhar, X. Liu, E. C. Burrows, and L. L. Buhl, "Hybrid 107-Gb/s polarization-multiplexed DQPSK and 42.7-Gb/s DQPSK transmission at 1.4-bits/s/Hz spectral efficiency over 1280 km of SSMF and 4 bandwidth-managed ROADMs," presented at the ECOC, 2007.

[63] G. Charlet, J. Renaudier, H. Mardoyan, O. B. Pardo, F. Cerou, P. Tran, and S. Bigo, "12.8 Tbit/s transmission of 160 PDM-QPSK $(160 \times 2 \times 40\ \mathrm{Gbit/s})$ channels with coherent detection over 2,550 km," presented at the ECOC, 2007, PD1.6.

[64] H. A. Suraweera, K. R. Panta, M. Feramez, and J. Armstrong, "OFDM peak-to-average power reduction scheme with spectral masking," in *Proc Commun. Syst., Netw. Digital Signal Processing CSNDSP 2004*, Newcastle upon Tyne, U.K., Jul. 20–22, 2004.

[65] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, pp. 365–369, 1999.

[66] Y. Li and L. J. Cimini, Jr., "Bounds on the interchannel interference of OFDM in time-varying impairments," *IEEE Trans. Commun.*, vol. 49, pp. 401–404, 2001.

[67] T. Pollet, M. Van Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Trans. Commun.*, vol. 43, pp. 191–193, 1995.

[68] J. Stott, "The effects of phase noise in COFDM," *EBU Tech. Rev.*, pp. 12–25, 1998.

[69] L. Tomba, "On the effect of Wiener phase noise in OFDM systems," *IEEE Trans. Commun.*, vol. 46, pp. 580–583, 1998.

[70] A. Garcia Armada, "Understanding the effects of phase noise in orthogonal frequency division multiplexing (OFDM)," *IEEE Trans. Broadcasting*, vol. 47, pp. 153–159, 2001.

[71] C.-L. Liu, "Impacts of I/Q imbalance on QPSK-OFDM-QAM detection," *IEEE Trans. Consumer Electron.*, vol. 44, pp. 984–989, 1998.

[72] S. L. Jansen, I. Morita, T. C. W. Schenk, D. van den Borne, and H. Tanaka, "Optical OFDM—A candidate for future long-haul optical transmission systems," presented at the Proc. OFC/NFOEC 2008, San Diego, CA, 2008, paper, OMU3.

[73] S. L. Jansen, "Optical OFDM, a hype or is it for real?," presented at the ECOC, 2008.

[74] S. Randel, M. Schuster, J. Lee, and F. Breyer, "Myths and truths about optical OFDM," presented at the 2007 IEEE/LEOS Summer Topical Meetings, 2007.

[75] W. Shieh, Y. Xingwen, M. Yiran, and Y. Qi, "Coherent optical OFDM: Has its time come? [Invited]," *J. Opt. Netw.*, vol. 7, pp. 234–255, 2008.

[76] A. J. Lowery, "Optical OFDM," in *Proc. 2008 Conf. Lasers Electro-Optics (CLEO)*, San Jose, CA, USA, 2008.

[77] S. L. Jansen, I. Morita, and H. Tanaka, "Narrowband filtering tolerance and spectral efficiency of 100 GbE PDM-OFDM," presented at the IEEE/LEOS Summer Topical Meetings, 2008.

[78] J. M. Tang, P. M. Lane, and K. A. Shore, "High-speed transmission of adatively modulated optical OFDM signals over multimode fibers using directly modulated DFBs," *J. Lightw. Technol.*, vol. 24, pp. 429–441, 2006.

[79] Q. Yang, W. Shieh, and Y. Ma, "Bit and power loading for coherent optical OFDM," *IEEE Photon. Technol. Lett.*, vol. 20, pp. 1305–1307, 2008.

[80] K. Forozesh, S. L. Jansen, S. Randel, I. Morita, and H. Tanaka, "The influence of the dispersion map in coherent optical OFDM transmission systems," presented at the IEEE/LEOS Summer Topical Meetings, 2008.

[81] D. Dardari, "Joint clip and quantization effects characterization in OFDM receivers," *IEEE Trans. Circuits Syst. I: Fundamental Theory Applicat.*, vol. 53, pp. 1741–1748, 2006.

[82] J. Armstrong, H. A. Suraweera, S. Brewer, and R. Slaviero, "Effect of rounding and saturation in fixed-point DSP implementation of IFFT and FFT for OFDM applications," presented at the Embedded Signal Processing Conference (GSPx 2004), Santa Clara, CA, USA, 2004, unpublished.

[83] U. Onunkwo, L. Ye, and A. Swami, "Effect of timing jitter on OFDM-based UWB systems," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 787–793, 2006.

[84] X. Yi, W. Shieh, and Y. Ma, "Phase noise effects on high spectral efficiency coherent optical OFDM transmission," *J. Lightw. Technol.*, vol. 26, pp. 1309–1316, 2008.

[85] E. Ip and J. M. Kahn, "Digital equalization of chromatic dispersion and polarization mode dispersion," *J. Lightw. Technol.*, vol. 25, pp. 2033–2043, 2007.

Jean Armstrong (M'89–SM'06) received the B.Sc. degree (First Class Honors) in electrical engineering from the University of Edinburgh, U.K., in 1974, the M.Sc. degree in Digital Techniques from Heriot-Watt University, Edinburgh, Scotland, in 1980, and the Ph.D. degree in digital communications from Monash University, Melbourne, Australia, in 1993.

From 1974 to 1977, she worked as a Design Engineer at Hewlett-Packard Ltd., Scotland. In 1977, she was appointed Lecturer in Electrical Engineering at the University of Melbourne, Australia. Since 1977, she has held a range of academic positions at the University of Melbourne, Monash University and La Trobe University. She is currently a Professor at Monash University. Her research interests include digital communications, engineering education, and women in engineering. She has published over 100 papers including over 70 on aspects of OFDM. Her work on OFDM has also led to a number of patents.

In June 2006, Dr. Armstrong and Arthur Lowery were awarded the Peter Doherty Prize for Innovation for their work on optical OFDM. She is Fellow of the Institution of Engineers Australia