Scheduling and Fairness

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System

Consider a TDMA system where three users share the channel in time. The peak data rate of user 1 is 1Mbps while users 2 and 3 achieve 2Mbps peak rate. In this case we have the following:

$r_1 = 1Mbps$
$r_2 = 2Mbps$
$r_3 = 2Mbps$

a) Max-min fair scheduling:

In this approach the minimum is being maximized, then we have that $\alpha \to \infty$ and $w_j = 1$ (since $w_j > 0$), with this considerations we can find the function:

$\phi_i = \dfrac{r_i^{-1}}{\sum_{j=1}^{N} r_j^{-1}}$ .... $where \quad N = 3$, now we find the fraction of time to be allocated for each user, i.e. we find the ϕ for every user:

$$\phi_1 = \frac{r_1^{-1}}{\sum_{j=1}^{3} r_j^{-1}} = \frac{1^{-1}}{1^{-1} + 2^{-1} + 2^{-1}} = 0.5$$

$$\phi_2 = \frac{r_2^{-1}}{\sum_{j=1}^{3} r_j^{-1}} = \frac{2^{-1}}{1^{-1} + 2^{-1} + 2^{-1}} = 0.25$$

$$\phi_3 = \frac{r_2^{-1}}{\sum_{j=1}^{3} r_j^{-1}} = \frac{2^{-1}}{1^{-1} + 2^{-1} + 2^{-1}} = 0.25$$

- Now we calculate the average throughput for every user:

  $Avg\_throughput_1 = \phi_1 r_1 = 0.5x1 = 0.5Mbps$
  $Avg\_throughput_2 = \phi_2 r_2 = 0.25x2 = 0.5Mbps$
  $Avg\_throughput_3 = \phi_3 r_3 = 0.25x2 = 0.5Mbps$

- Now we calculate the aggregate throughput in the cell:

$$Aggregate\_throughput = \sum_{i=1}^{3} \phi_i r_i = 0.5 + 0.5 + 0.5 = 1.5Mbps$$

b) Proportional fair scheduling:

In this approach $w_j = 1$ and $\alpha = 1$ which means that the fraction of time for each user is inversely proportional to the amount of users, then our ϕ function will be:

$$\phi_i = \frac{1}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots where \quad N = 3$$
$$\phi_1 = 1/3 = 0.33$$
$$\phi_2 = 1/3 = 0.33$$
$$\phi_3 = 1/3 = 0.33$$

- Now we calculate the average throughput for every user:

$$Avg\_throughput_1 = \phi_1 r_1 = 0.33x1 = 0.33Mbps$$
$$Avg\_throughput_2 = \phi_2 r_2 = 0.33x2 = 0.66Mbps$$
$$Avg\_throughput_3 = \phi_3 r_3 = 0.33x2 = 0.66Mbps$$

- Now we calculate the aggregate throughput in the cell:

$$Aggregate\_throughput = \sum_{i=1}^{3} \phi_i r_i = 0.33 + 0.66 + 0.66 = 1.65Mbps$$

c) Nash Bargaining scheduling:

In this case the fraction of time allocated for each user is given by the following function:

$\phi_i = \dfrac{d_i}{r_i} + \dfrac{1 - \sum_{i=1}^{N} \frac{d_i}{r_i}}{N}$, where $N = 3$ and $d_i = 250Kbps = 0.25Mbps$, then we have the following fraction of time allocated for each user:

$$\phi_1 = \frac{d_1}{r_1} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.25}{1} + \frac{1 - \left(\frac{0.25}{1} + \frac{0.25}{2} + \frac{0.25}{2} \right)}{3} = 0.4166$$

$$\phi_2 = \frac{d_2}{r_2} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.25}{2} + \frac{1 - \left(\frac{0.25}{1} + \frac{0.25}{2} + \frac{0.25}{2} \right)}{3} = 0.29166$$

$$\phi_3 = \frac{d_3}{r_3} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.25}{2} + \frac{1 - \left(\frac{0.25}{1} + \frac{0.25}{2} + \frac{0.25}{2} \right)}{3} = 0.29166$$

- Now we calculate the average throughput for every user:

$$Avg\_throughput_1 = \phi_1 r_1 = 0.4166x1 = 0.4166 Mbps$$
$$Avg\_throughput_2 = \phi_2 r_2 = 0.29166x2 = 0.5833 Mbps$$
$$Avg\_throughput_3 = \phi_3 r_3 = 0.29166x2 = 0.5833 Mbps$$

- Now we calculate the aggregate throughput in the cell:

$$Aggregate\_throughput = \sum_{i=1}^{3} \phi_i r_i = 0.4166 + 0.5833 + 0.5833 = 1.5832 Mbps$$

d) Plot fraction of time, average throughput and aggregate throughput:

According to the paper and the theory in the slides, the requirement is that $w_j > 0$, then we can consider $w_j = 1$ which yields to have the following function for our fraction of time allocated for every user as a function of α in the max-min scheduling approach:

$$\phi_i = \frac{w_i^{\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-1}}{\sum_{j=1}^{N} w_j^{\frac{1}{\alpha}} r_j^{\frac{1}{\alpha}-1}}, where \quad w = 1, N = 3$$

$$\phi_i = \frac{r_i^{\frac{1}{\alpha}-1}}{\sum_{j=1}^{3} r_j^{\frac{1}{\alpha}-1}}$$

And from the values obtained in 'b' and 'c' we observe that the value of ϕ does not depend on α, therefore the plot in these cases will be a constant line.

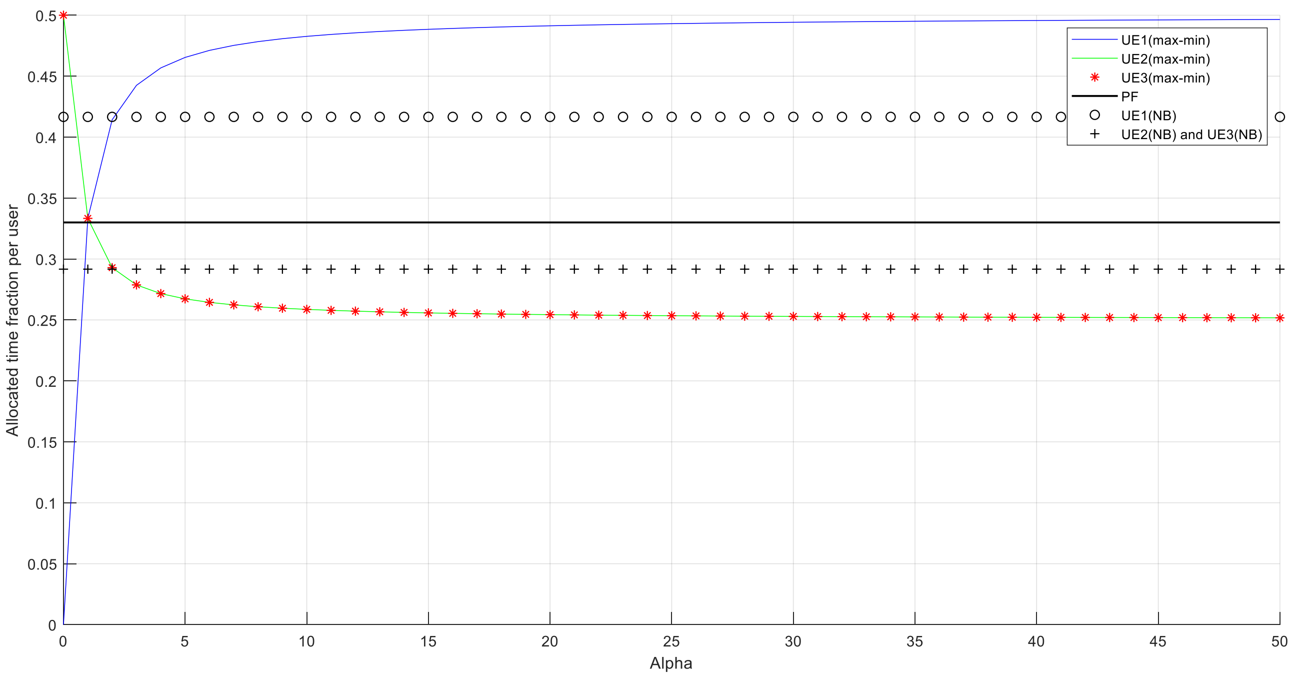

Figure 1. Plot with the values for ϕ obtained in the items 'a', 'b' and 'c'

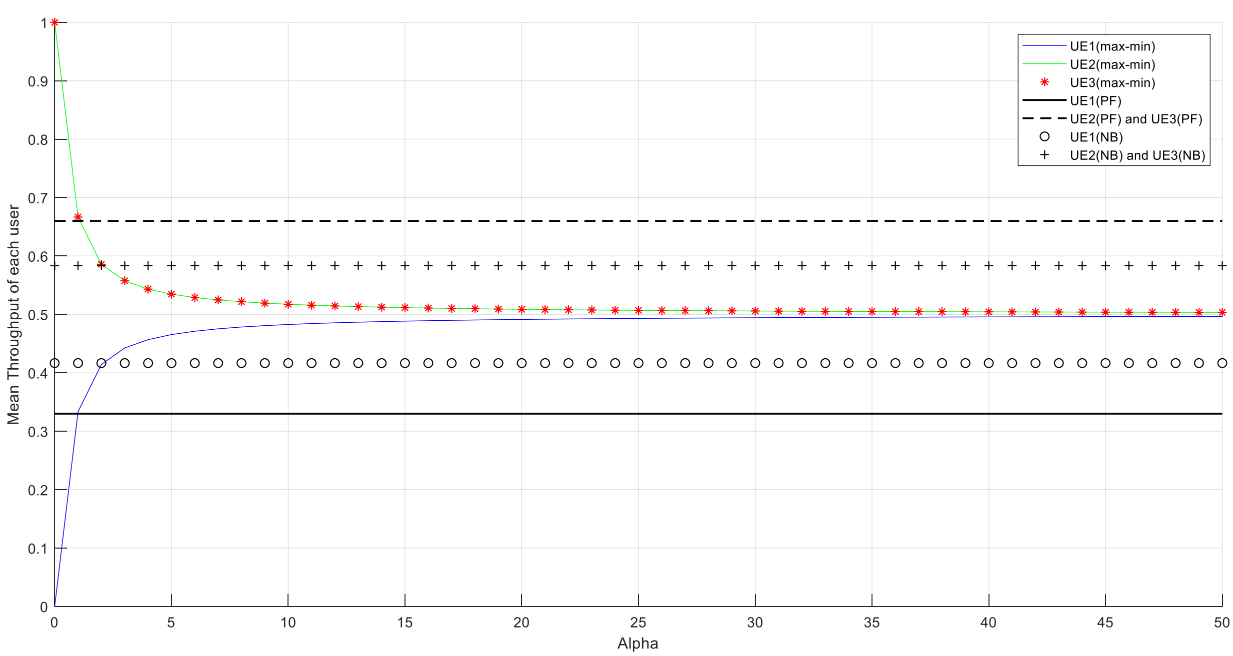

Figure 2. Plot with the values for mean throughput obtained in the items 'a', 'b' and 'c'

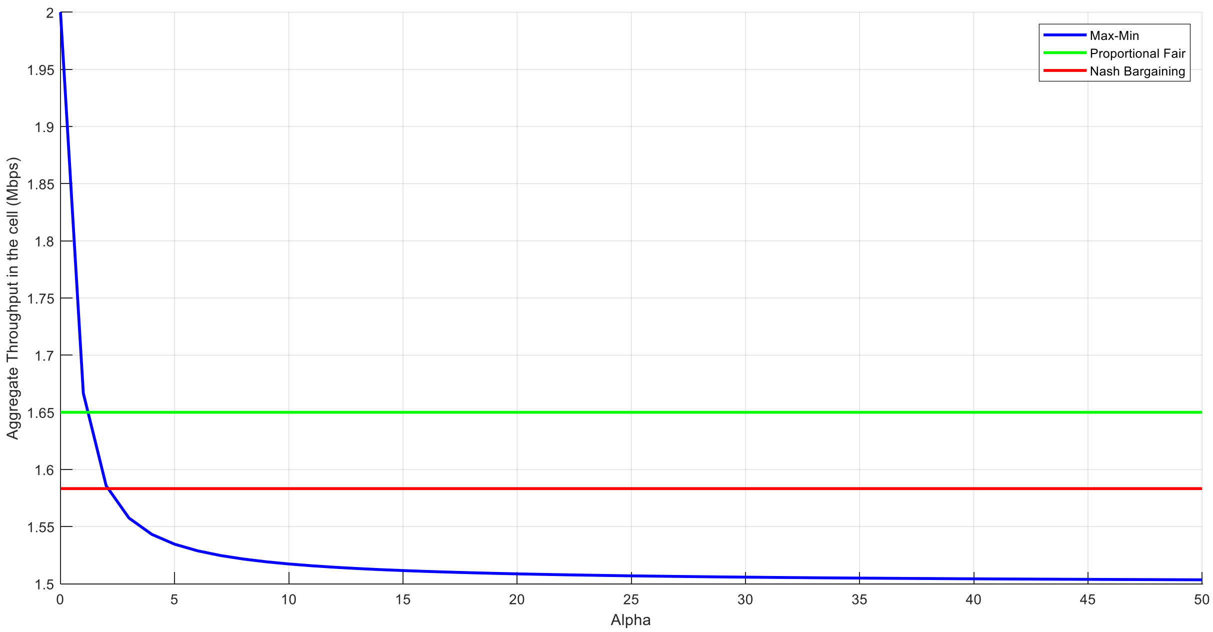

Figure 3. Plot with the values for aggregate throughput obtained in the items 'a', 'b' and 'c'

In the figure 1 we can observe how the value of ϕ increases as the value of α also increases (in the max-min scheduling model) for the UE1, which is a desirable behavior considering that the UE1 is the one with the lowest peak data rate and the system is intended to be fair with all users compensating this lower data peak rate with a longer fraction of time for the resource allocation, on the other hand, the value of ϕ decreases for UE2 and UE3 because these users have the higher peak data rate. In the max-min model ϕ depends on α, this is why the value of ϕ tends to a maximum of the minimum when α grows to a big value, i.e. when $\alpha \to \infty$ the values of ϕ_i are constant and are the ones obtained in the part 'a' of the exercise. In the case of the proportional fair and Nash Bargaining models, the fraction of time ϕ does not depend on α, this is why the plots are a constant line, in these approaches the fairness for each user will be determined by other parameters like the number of users (in the proportional fair model) and the peak rates and the minimum tolerable rate (in the Nash Bargaining model). Similarly, when we observe the average throughput in figure 2 and the aggregate throughput in figure 3, the behavior is similar to the observed in the figure 1, the system is trying to be fairer with the user with lower peak data rate (in the max-min model) converging the average throughput for all the users of the system to a fairest value when $\alpha \to \infty$. On the other hand, the values for the average throughput and aggregate throughput are constant for the other approaches.

e) <u>Nash Bargaining scheduling with d = 0.5Mbps:</u>

In this case the fraction of time allocated for each user is given by the following function:

$\phi_i = \frac{d_i}{r_i} + \frac{1 - \sum_{i=1}^{N} \frac{d_i}{r_i}}{N}$, where $N = 3$ and $d_i = 0.5Mbps$, then we have the following fraction of time allocated for each user:

$$\phi_1 = \frac{d_1}{r_1} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.5}{1} + \frac{1 - \left(\frac{0.5}{1} + \frac{0.5}{2} + \frac{0.5}{2} \right)}{3} = 0.5$$

$$\phi_2 = \frac{d_2}{r_2} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.5}{2} + \frac{1 - \left(\frac{0.5}{1} + \frac{0.5}{2} + \frac{0.5}{2} \right)}{3} = 0.25$$

$$\phi_3 = \frac{d_3}{r_3} + \frac{1 - \sum_{i=1}^{3} \frac{d_i}{r_i}}{3} = \frac{0.5}{2} + \frac{1 - \left(\frac{0.5}{1} + \frac{0.5}{2} + \frac{0.5}{2} \right)}{3} = 0.25$$

- Now we calculate the average throughput for every user:

  $$Avg\_throughput_1 = \phi_1 r_1 = 0.5x1 = 0.5Mbps$$
  $$Avg\_throughput_2 = \phi_2 r_2 = 0.25x2 = 0.5Mbps$$
  $$Avg\_throughput_3 = \phi_3 r_3 = 0.25x2 = 0.5Mbps$$

- Now we calculate the aggregate throughput in the cell:

$$Aggregate\_throughput = \sum_{i=1}^{3} \phi_i r_i = 0.5 + 0.5 + 0.5 = 1.5Mbps$$

Following a table summarizing the values of the throughput and fraction of time to compare the results in each case:

User	Peak rate (Mbps)	Nash Bargaining (d = 0.25 Mbps)		Nash Bargaining (d = 0.5 Mbps)		Max-Min scheduling		Proportional fair	
		Fraction of time allocated	Avg Throughput (Mbps)	Fraction of time allocated	Avg Throughput (Mbps)	Fraction of time	Avg Throughput	Fraction of time	Avg Throughput
UE 1	1	0.4166	0.4166	0.5	0.5	0.5	0.5	0.33	0.33
UE 2	2	0.29166	0.5833	0.25	0.5	0.25	0.5	0.33	0.66
UE 3	2	0.29166	0.5833	0.25	0.5	0.25	0.5	0.33	0.66

Table 1. Comparison of the scheduling approaches obtained in items 'a', 'b', 'c' and 'e'

In the table 1 we can see that the system aims to balance the resources between the users with a higher fairness value, like in the case of Max-min model where the value of $\alpha \to \infty$ and consequently the average throughput for every user is the same, unlike the case of the proportional fair model where $\alpha = 1$ and the UE2 and UE3 are favored in terms of average throughput. In the case of the Nash Bargaining model, when the minimum tolerable rate was increased this model obtained fairness results similar to the Max-min scheduling in terms of average throughput.

How NOMA technology improves the fairness of the considered system

NOMA (Nonorthogonal Multiple Access) is intended to achieve massive connectivity of users in 5G by implementing the superposition coding and the Successive Interference Cancellation (SIC), in this context multiple users will be served with the available resources of the network opening two dimensions of resource management in NOMA networks: the power control/allocation and the user/resource allocation because in NOMA the weak users needs more power due to its poor channel condition and less power needs to be allocated for users with a good channel state, guaranteeing this way the proper fairness of the users in the system. In the system analyzed in the previous section, the fairness of the system was tackled by implementing max-min fair scheduling, proportional fair scheduling and Nash Bargaining scheduling.

In NOMA networks it is also considered the α utility function, which is a generalized form of the max-min ($\alpha \to \infty$) and proportional fair ($\alpha = 1$) approaches, to achieve the PA (Power Allocation) and fair network resource allocation for the users, but also the weighted sum rate that will provide a certain grade of fairness in the Media-access control (MAC) layer and the Jain's fairness index comparison that will strike a tradeoff between the sum rate and the fairness of the system, are models implemented in NOMA that could improve the fairness of the system in the problem 1. In addition, there are more sophisticated and intelligent approaches proposed in NOMA to implement a fair PA among the users in the system, which are the Cognitive-Radio-Inspired Power Control, the User Scheduling in Dynamic Cluster/Pair-Based Hybrid MA Networks and the Software-Defined NOMA Network Architecture.

In the case of the CR-Power Control scheme, we can identify the following advantages to improve the fairness of our system: the QoS is guaranteed for the requirements of the weak users, there is a fairness/throughput tradeoff where the targeted data rate of the weak users is satisfied with adequate PA, high flexibility and low complexity. Second, the user scheduling in dynamic cluster will help the fairness in the problem 1 by proposing efficient user allocation algorithms such as combinatorial relaxation which convert to convex problem, monotonic optimization that has the advantage of optimal solution, matching theory which achieve near-optimal performance and heuristic algorithms which are a flexible tradeoff between complexity and performance. Finally, the novel Software-Defined NOMA approach is intended to globally control the software defined NOMA elements via the SDN controller in order to manage the resource allocation for the users and the power optimization, interference management, user association, and dynamic user clustering/pairing.

References

Y. Liu, Z. Qin, M. Elkashlan, Z. Ding, A. Nallanathan and L. Hanzo, "Nonorthogonal Multiple Access for 5G and Beyond," in *Proceedings of the IEEE*, vol. 105, no. 12, pp. 2347-2381, Dec. 2017.