

# Joint Clusterization and Power Allocation for Cloud Radio Access Networks

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**Abstract**—In this paper, the cloud radio access network (C-RAN) is considered to extend the transmission coverage via the distributed deployment of large-scale remote radio units (RRUs). However, this type of structure can induce considerable computational loadings due to the centralized management mechanisms. To reduce the complexity incurred in the C-RAN architecture, the clusterization technique is designed to categorize those RRUs into several groups. For the purpose of enhancing energy efficiency (EE) as well as the consideration of computational complexity, the joint clusterization and power allocation schemes are proposed to obtain the better tradeoff under the quality-of-service (QoS) requirement for each user equipment (UE). Simulation results show that the proposed algorithms can provide better performance gain than the existing method.

**Index Terms**—Cloud radio access networks (C-RAN), energy efficiency (EE), clusterization, power allocation.

## I. INTRODUCTION

To meet the ever growing traffic demand of wireless networks, the architecture of hyper-dense deployment is viewed as a key resolution. Recently, a distributed large-scale multi-input multi-output (DLS MIMO) system has drawn significant attention to improve both spectrum and energy efficiency (EE), namely, cloud radio access network (C-RAN). As shown in Fig.1, this paper focuses on the DLS C-RAN where a large number of spatially distributed remote radio units (RRUs) connect to the C-RAN controller via high speed optical fiber [1]. Apart from the conventional base stations, the RRUs can easily achieve green communications due to their lower power consumption. Moreover, the RRUs over fiber system can be used to extend the transmission coverage in the remote rural areas.

Triggered by the merits of C-RAN, the works in [2], [3] focus on the resource allocation issues to further boost performance gain. The centralized management in the C-RAN controller would be much easier to conduct. Since the computational loading of this structure would be considerable, it will greatly increase the complexity of management mechanisms. To reduce the complexity incurred in the C-RAN architecture, the RRU clusterization techniques must be investigated. The existing clusterization methods based on distance-similarity [4], [5], e.g.,  $K$ -means and affinity propagation clustering (AP-clustering) have been well-studied in the machine learning [6].

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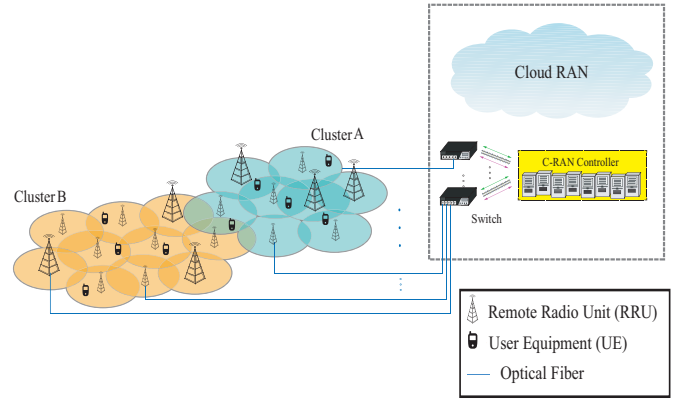


Fig. 1. Network scenario for the C-RAN.

There are many researches [4], [7]-[9] focus on clusterization and resource allocation schemes to boost spectrum efficiency as well as the consideration of computational complexity. In [7], spectral efficiency for different cluster layouts are evaluated. However, the clusterization mechanism is assumed to be predetermined. The works in [4], [8] proposed clustered-based resource allocation schemes to maximize achievable data rate for heterogeneous network (HetNet). Since the optimization approaches on clusterization and resource allocation are interacted on each other, the joint clusterization and resource allocation problem is designed in [9]. Although the above studies tend to propose cluster-based algorithms to enhance system performance gain for low-complexity management, the clusterization and resource allocation problems are divided into two independent sub-problems to be solved easily. Therefore, the optimal solution cannot be obtained. Furthermore, the energy-aware clusterization techniques should be investigated for the EE enhancement.

Motivated by the above observations, this paper focuses on the joint clusterization and resource allocation problem to maximize EE for C-RAN. The interference management and resource allocation are centralized processed by C-RAN controller. To solve the optimization problem efficiently, the joint clusterization and power allocation (JCPA) scheme is proposed to categorize RRUs into RRU clusters and perform power allocation to mitigate interference and enhance EE. Another separated clusterization and power allocation (SCPA) scheme is proposed to reduce the computation complexity

in JCPA scheme. Moreover, the complexity of the proposed algorithms has been analyzed. Performance evaluation shows that the proposed schemes outperform the existing method.

The rest of this paper is organized as follows. Section II describes the system model and problem formulation. The clusterization and power allocation algorithms are listed in section III. Performance evaluation of the proposed algorithms is illustrated in section IV. Section V draws the conclusions.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

Consider a downlink C-RAN consists of  $M$  RRUs aiming to serve  $K$  user equipments (UEs) with bandwidth  $B$ . All the RRUs are connected to C-RAN controller by optical fiber, thus the RRUs can perfectly synchronize with each other and the latency of message transmission can be neglected. The spatially distributed RRUs are clustered into  $C$  clusters. Each UE is equipped with a single antenna and only assigned into one cluster. Each UE will receive both intra- and inter-cluster interference. The former is caused by the mutual interference among UEs within the same cluster, and the latter is incurred by the lack of interference coordination among neighbouring clusters. For each cluster, the suitable linear precoding technique can be chosen to reduce the intra-cluster interference.

The received signal for UE  $k$  can be expressed as

$$y_k = \sum_{c=1}^C \sum_{m=1}^M \varphi_{c,m,k} h_{m,k} w_{m,k} \sqrt{p_k} x_k + \underbrace{\sum_{c=1}^C \sum_{m=1}^M \sum_{\substack{r=1 \\ r \neq k}}^K \varphi_{c,m,r} h_{m,k} w_{m,r} \sqrt{p_r} x_r}_{\text{Intra- and inter-cluster interferences}} + n_k, \quad (1)$$

where  $\varphi_{c,m,k} \in \{0, 1\}$  indicates whether RRU  $m$  and UE  $k$  are categorized into cluster  $c$ .  $h_{m,k}$  is the channel gain and  $w_{m,k}$  is the precoding weighting for the link from RRU  $m$  to UE  $k$ .  $p_k$  is the transmit symbol power and  $x_k$  is the transmit symbol for UE  $k$ .  $n_k \sim \mathcal{CN}(0, N_0)$  is the additive white Gaussian noise (AWGN), where  $N_0$  is the noise power and  $\mathcal{CN}(\mu, \sigma^2)$  is a complex Gaussian random variable with mean  $\mu$  and variance  $\sigma^2$ .

From (1), the received signal-to-interference-plus-noise ratio (SINR)  $\Gamma_k$  can be defined as

$$\Gamma_k = \frac{\sum_{c=1}^C \sum_{m=1}^M \varphi_{c,m,k} p_k |h_{m,k} w_{m,k}|^2}{I_k + B N_0}, \quad (2)$$

where  $I_k$  is the interference power which is denoted as

$$I_k = \sum_{c=1}^C \sum_{m=1}^M \sum_{\substack{r=1 \\ r \neq k}}^K \varphi_{c,m,r} p_r |h_{m,k} w_{m,r}|^2. \quad (3)$$

Therefore, the achievable data rate  $R_k$  of UE  $k$  can be

expressed as

$$R_k = B \log_2 (1 + \Gamma_k). \quad (4)$$

The total network power consumption is defined as

$$P_{total} = \sum_{c=1}^C \sum_{m=1}^M \sum_{k=1}^K \varphi_{c,m,k} p_k |w_{m,k}|^2. \quad (5)$$

The EE  $\eta$  of the considered system is the ratio of total data rate to the total power consumption. It can be written as

$$\eta = \frac{\sum_{k=1}^K R_k}{P_{total}}. \quad (6)$$

### B. Problem Formulation

The objective of this paper is to maximize EE through joint clusterization and power allocation under data rate requirement of each UE and maximum transmit power allowance of each RRU. The target of considered problem is to acquire the policies for clusterization  $\Phi = \{\varphi_{c,m,k} | 1 \leq c \leq C, 1 \leq m \leq M, 1 \leq k \leq K, c \in \mathbb{Z}^+, m \in \mathbb{Z}^+, k \in \mathbb{Z}^+\}$  and power allocation  $\mathbf{P} = \{p_k | 1 \leq k \leq K, k \in \mathbb{Z}^+\}$ . Therefore, the joint clusterization and resource allocation optimization problem can be formulated as follow

$$\max_{\Phi, \mathbf{P}} \eta \quad (7a)$$

$$\text{s.t.} \quad \sum_{c=1}^C \sum_{k=1}^K \varphi_{c,m,k} p_k |w_{m,k}|^2 \leq P_{max}^{RRU}, \quad \forall m, \quad (7b)$$

$$p_k \geq 0, \quad \forall k, \quad (7c)$$

$$R_k \geq R_k^{th}, \quad \forall k, \quad (7d)$$

$$\sum_{\substack{t=1 \\ t \neq c}}^C \sum_{m=1}^M \varphi_{c,m,k} \sum_{m=1}^M \varphi_{t,m,k} \leq 0, \quad \forall c, \forall k, \quad (7e)$$

$$\sum_{\substack{t=1 \\ t \neq c}}^C \sum_{k=1}^K \varphi_{c,m,k} \sum_{k=1}^K \varphi_{t,m,k} \leq 0, \quad \forall c, \forall m, \quad (7f)$$

$$\sum_{k=1}^K \sum_{m=1}^M \varphi_{c,m,k} \geq 1, \quad \forall c, \quad (7g)$$

$$\varphi_{c,m,k} \in \{0, 1\}, \quad \forall c, \forall m, \forall k. \quad (7h)$$

The value of  $P_{max}^{RRU}$  in (7b) is the maximum transmit power constraint of each RRU. (7c) specifies that the power allocation parameters are non-negative. The condition in (7d) depicts that each UE is required to satisfy its target data rate  $R_k^{th}$  according to the quality-of-service (QoS) requirement. (7e) and (7f) restrict that each UE and RRU are only allowed to associate with exactly one cluster. (7g) indicates that each UE must be allocated into one cluster consisted at least one RRU. (7h) specifies  $\varphi_{c,m,k}$  is the binary integer variable for clusterization.

### III. CLUSTERIZATION AND POWER ALLOCATION SCHEMES

Since the original optimization problem contains nonlinear objective function, which is a ratio of two decision policies  $\Phi$  and  $\mathbf{P}$ , it is hard to solve this problem via conventional linear programming methods. In addition, the discrete variables and interference are considered in this problem, the optimization problem is non-convex with respect to  $\Phi$  and  $\mathbf{P}$ . To tackle this non-convex problem, the following JCPA and SCPA schemes are proposed.

#### A. Proposed Joint Clusterization and Power Allocation (JCPA) Scheme

The JCPA scheme is proposed to maximize network EE by categorizing RRUs into different clusters and performing power allocation. The optimization problem can be transformed into a convex problem by the following procedures.

1) *Problem Transformation:* The objective function in (7a) is viewed as a nonlinear fractional form. Using the fractional programming (FP) introduced in [10, Theorem 1], the original objective function in fractional form can be transformed into equivalent subtractive form, i.e.,  $\sum_{k=1}^K R_k - \eta^* P_{total}$ . With the optimal clusterization and power allocation policies, denoted as  $\Phi^*$  and  $\mathbf{P}^*$  respectively, the optimal energy efficiency  $\eta^*$  can be expressed as

$$\eta^*(\Phi^*, \mathbf{P}^*) = \max_{\Phi, \mathbf{P}} \frac{\sum_{k=1}^K R_k(\Phi^*, \mathbf{P}^*)}{P_{total}(\Phi^*, \mathbf{P}^*)}. \quad (8)$$

For a given  $\eta$ , the objective function in (7a) can be rewritten as

$$\max_{\Phi, \mathbf{P}} \sum_{k=1}^K R_k - \eta P_{total}. \quad (9)$$

As a result, the rest of this paper focuses on the equivalent expression of objective function as in (9).

To tackle the non-convexity due to the consideration of both intra- and inter-cluster interferences, the worst-case interference received by each UE is assumed. Hence, the maximal interference power from RRUs to UE  $k$  can be formulated as follow

$$\hat{I}_k = P_{max}^{RRU} \sum_{c=1}^C \sum_{m=1}^M \sum_{\substack{r=1 \\ r \neq k}}^K \varphi_{c,m,r} |h_{m,k} w_{m,r}|^2. \quad (10)$$

Furthermore, the constraint with discrete variable in (7h) is released to a continuous one by [11]. Therefore, the original form  $\varphi_{c,m,k} \in \{0, 1\}$  is replaced by  $\varphi_{c,m,k} \in [0, 1]$ .

2) *Iterative Approach for JCPA Scheme:* In this section, the Lagrangian dual method [11] is used to solve the optimal clusterization policy, and power allocation policy with iterative method. By replacing (9) and (10) into (7), the original

optimization problem can be rewritten as

$$\max_{\Phi, \mathbf{P}} \sum_{k=1}^K \hat{R}_k - \eta P_{total} \quad (11a)$$

$$\text{s.t. } (7b), (7c), (7e), (7f), (7g),$$

$$\hat{R}_k \geq R_k^{th}, \quad \forall k, \quad (11b)$$

$$\varphi_{c,m,k} \in [0, 1], \quad \forall c, \forall m, \forall k, \quad (11c)$$

where

$$\hat{R}_k = B \log_2(1 + \hat{\Gamma}_k), \quad (12)$$

with

$$\hat{\Gamma}_k = \frac{\sum_{c=1}^C \sum_{m=1}^M \varphi_{c,m,k} p_k |h_{m,k} w_{m,k}|^2}{\hat{I}_k + B N_0}. \quad (13)$$

As the result, the original non-convex problem in (7) is transformed into a concave one with respect to  $\Phi$  and  $\mathbf{P}$ , and it can be solved by adopting Lagrangian dual method. The Lagrangian function can be stated as

$$\begin{aligned} L(\Phi, \mathbf{P}, \varepsilon, \theta, \mu, \psi, \beta, \sigma) = & \sum_{k=1}^K R_k \\ & - \eta \left( \sum_{c=1}^C \sum_{m=1}^M \sum_{k=1}^K \varphi_{c,m,k} p_k |w_{m,k}|^2 \right) + \sum_{k=1}^K \theta_k (R_k - R_k^{th}) \\ & - \sum_{m=1}^M \varepsilon_m \left( \sum_{c=1}^C \sum_{k=1}^K \varphi_{c,m,k} p_k |w_{m,k}|^2 - P_{max}^{RRU} \right) \\ & - \sum_{c=1}^C \sum_{k=1}^K \psi_{c,k} \left( \sum_{\substack{t=1 \\ t \neq c}}^C \sum_{m=1}^M \varphi_{c,m,k} \sum_{m=1}^M \varphi_{t,m,k} \right) \\ & - \sum_{c=1}^C \sum_{m=1}^M \mu_{c,m} \left( \sum_{\substack{t=1 \\ t \neq c}}^C \sum_{k=1}^K \varphi_{c,m,k} \sum_{k=1}^K \varphi_{t,m,k} \right) \\ & + \sum_{c=1}^C \beta_c \left( \sum_{m=1}^M \sum_{k=1}^K \varphi_{c,m,k} - 1 \right) \\ & - \sum_{c=1}^C \sum_{m=1}^M \sum_{k=1}^K \sigma_{c,m,k} (\varphi_{c,m,k} - 1), \end{aligned} \quad (14)$$

where  $\varepsilon_m$ ,  $\theta_k$ ,  $\mu_{c,m}$ ,  $\psi_{c,k}$ ,  $\beta_c$  and  $\sigma_{c,m,k}$  are the Lagrangian multipliers for the constraints (7b)-(7h), respectively. For simplification,  $\varepsilon$ ,  $\theta$ ,  $\mu$ ,  $\psi$ ,  $\beta$  and  $\sigma$  are used to denote the Lagrangian multiplier sets. Based on Karush-Kuhn-Tucker (KKT) conditions, the clusterization indicator for UE  $k$  and RRU  $m$  in cluster  $c$  is obtained as

$$\varphi_{c,m,k} = \left[ \frac{(1 + \theta_k) B}{[\eta (p_k |w_{m,k}|^2) + \zeta_{c,m,k}] \ln 2} - \frac{\hat{I}_k + B N_0}{p_k |h_{m,k} w_{m,k}|^2} - \frac{\sum_{\substack{(t,n) \neq (c,m)}} \varphi_{t,n,k} |h_{n,k} w_{n,k}|^2}{|h_{m,k} w_{m,k}|^2} \right]^+, \quad (15)$$

where  $[x]^+$  is the maximal value between 0 and  $x$ , and

$$\begin{aligned} \zeta_{c,m,k} = & -\varepsilon_m (p_k |w_{m,k}|^2) + \psi_{c,k} \left( \sum_{t=1}^C \sum_{m=1}^M \varphi_{t,m,k} \right) \\ & + \mu_{c,m} \left( \sum_{t=1}^C \sum_{k=1}^K \varphi_{t,m,k} \right) - \beta_c + \sigma_{c,m,k}. \end{aligned} \quad (16)$$

Similarly, the power allocated to UE  $k$  is obtained as

$$p_k = \left[ \frac{(1 + \theta_k) B}{\left[ \eta \left( \sum_{c=1}^C \sum_{m=1}^M \varphi_{c,m,k} |w_{m,k}|^2 \right) + \Im_k \right] \ln 2} - \frac{1}{\omega_k} \right]^+, \quad (17)$$

where  $\Im_k = \sum_{m=1}^M \varepsilon_m \left( \sum_{c=1}^C \varphi_{c,m,k} |w_{m,k}|^2 \right)$  and  $\omega_k = \sum_{c=1}^C \sum_{m=1}^M \varphi_{c,m,k} |h_{m,k} w_{m,k}|^2 / (\hat{I}_k + B N_0)$  is the channel-to-interference-noise ratio (CINR) of UE  $k$  [12]. The optimal power allocation in (17) will allocate more power to UE  $k$  with the higher CINR. Furthermore, the multiplier sets can be updated by subgradient method as in [13].

Moreover, for the sake of obtaining the discrete solution of the optimal  $\varphi_{c,m,k}$ , denoted as  $\varphi_{c,m,k}^*$ , the largest value obtained in (15) for each cluster  $c$  is selected as one [14], which provides higher benefit to the C-RAN system by clusterization RRU  $m$  and UE  $k$  into cluster  $c$ , that is,

$$\varphi_{c,m,k}^* = \begin{cases} 1, & \text{if } \varphi_{c,m,k}^* = \max \{ \varphi_{c,m,k} \mid \forall m, \forall k \}, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

According to (15) and (17), the optimal solutions of clusterization indicator and power allocation variable can be obtained iteratively until convergence.

#### B. Proposed Separated Clusterization and Power Allocation (SCPA) Scheme

In order to reduce the computational complexity in JCPA scheme, the original problem is divided into two sub-problems to reduce the computational iterations, namely, the clusterization sub-problem and power allocation sub-problem.

First, let  $\mathbf{P}$  be a fixed value, i.e., the equal power allocation is assumed, the clusterization sub-problem can be written as follow

$$\begin{aligned} \max_{\Phi} \quad & \sum_{k=1}^K R_k \\ \text{s.t.} \quad & (7d), (7e), (7f), (7g), (7h). \end{aligned} \quad (19)$$

After getting the cluster information, the power allocation sub-problem can be solved as follow

$$\begin{aligned} \max_{\mathbf{P}} \quad & \eta \\ \text{s.t.} \quad & (7b), (7c), (7d). \end{aligned} \quad (20)$$

Note that these two sub-problems can be solved by the

TABLE I  
MAIN SYSTEM PARAMETERS

Parameter	Value
Number of clusters $C$	4
System bandwidth $B$	10 MHz
Carrier frequency	2 GHz
Noise power $N_0$	-174 dBm
Maximum transmit power per RRU $P_{max}^{RRU}$	23 dBm
Minimum data rate requirement $R_k^{th}$	1 Mbits/sec
Pathloss factor $\varsigma$	2.5

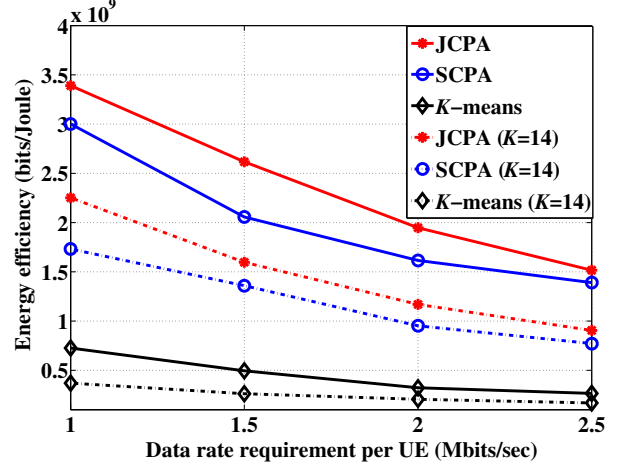


Fig. 2. Performance comparison between  $K$ -means scheme and proposed algorithms: energy efficiency versus minimum data rate requirement under  $K = \{10, 14\}$  and  $M = 24$ .

same manner as in JCPA. The difference between JCPA and SCPA scheme is the joint operation between clusterization and power allocation. Hence, the performance of JCPA scheme is expected to be better than SCPA scheme while the higher computational complexity incurred.

#### IV. PERFORMANCE EVALUATION

In this section, the system performance of the proposed clusterization and power allocation algorithms is provided. Consider a C-RAN system, the RRUs are grid distributed and each UE is uniformly distributed within the square coverage area of 0.2 km $\times$ 0.2 km. The channel model for the C-RAN system is referred to [3]. Note that the zero-forcing beamforming (ZFBF) precoding is adopted here to eliminate intra-cluster interference. It can also be replaced by other feasible linear precoding techniques. Table I. shows the main system parameters.

In Fig. 2, the EE versus the minimum data rate requirement of proposed algorithms is compared with  $K$ -means scheme for  $K = \{10, 14\}$  and  $M = 24$ . The  $K$ -means scheme is the clusterization method proposed in [4], which only considers distance similarity. Assuming that the proposed power allocation algorithm is adopted in  $K$ -means scheme to achieve fair comparison. It can be observed that the proposed schemes



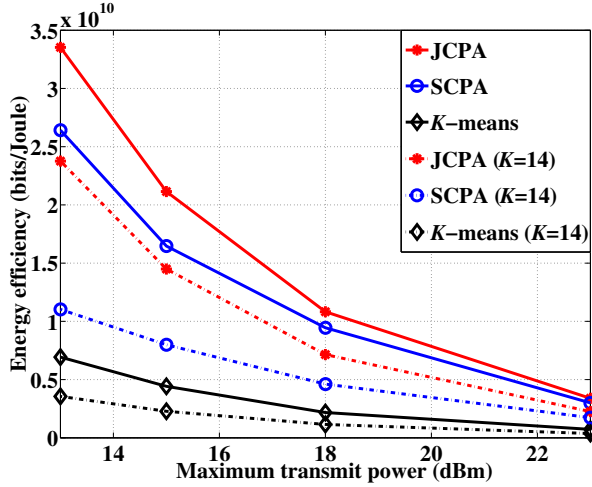


Fig. 3. Performance comparison between  $K$ -means scheme and proposed algorithms: energy efficiency versus maximum transmit power per RRU under  $K = \{10, 14\}$  and  $M = 24$ .

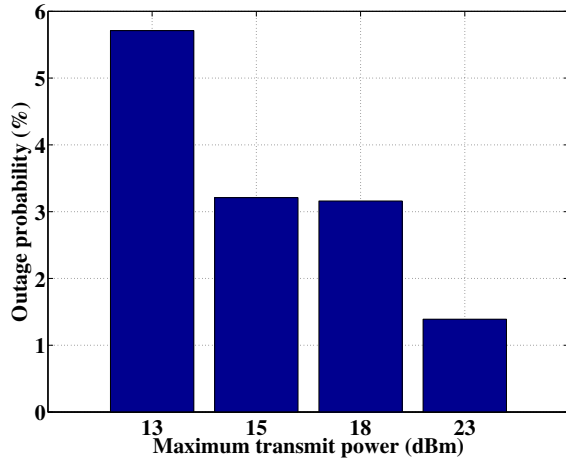


Fig. 4. Outage probability for SCPA scheme.

can achieve better performance than  $K$ -means scheme due to the superior clusterization scheme. Also, the EE of JCPA scheme is better than SCPA scheme due to its joint operation of clusterization and power allocation. Furthermore, the EE decreases as minimize data rate requirement increases, because more power consumption is needed to satisfy the data rate requirement.

From the computational complexity viewpoint [15], the complexity of JCPA scheme grows as  $\mathcal{O}(M^4)$ , whereas the complexity of SCPA scheme is  $\mathcal{O}(M^3)$ . Moreover, the complexity of  $K$ -means clusterization with power allocation scheme is  $\mathcal{O}(M^3)$ . It is worth mentioning that there exists a trade-off between performance and computational complexity.

Fig. 3 shows that the maximum transmit power per RRU versus EE for the proposed algorithms and  $K$ -means scheme. Simulation result shows that the EE increases as the maximum transmit power constraint decreases. Although the lower allow-

able power constraint leads to more energy-aware communications, the QoS requirement for each UE might be unsatisfied. The outage probability of data rate assignment is illustrated in Fig. 4. Since the degree of freedom for power allocation will decrease with lower power constraint, it can be seen that the outage probability will increase.

## V. CONCLUSIONS

In this paper, the joint clusterization and power allocation problem is formulated for the distributed large-scale MIMO cloud radio access network (DLS MIMO C-RAN) with the presence of interference. The joint clusterization and power allocation (JCPA) scheme has been proposed to maximize energy efficiency (EE) and reduce computational complexity of management mechanism under quality-of-service (QoS) requirement. Also, the separated clusterization and power allocation (SCPA) scheme has been proposed to reduce the complexity of JCPA scheme. Simulation results show that the proposed algorithms have better network EE with the consideration of computational complexity.

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