

# LTE Wireless Network Virtualization: Dynamic Slicing via Flexible Scheduling

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**Abstract**—The successful virtualization of wireless access networks is strongly affected by the way in which radio resources are managed. The Infrastructure Provider (InP) is required to deploy efficient and flexible scheduling techniques to dynamically allocate the resources for the users associated with different Service Providers (SPs). Service contracts with different SPs and fairness among their users are crucial to the success of the virtualization scheme deployed by the InP. In this paper we develop an efficient resource allocation scheme to allocate the radio resource blocks in LTE networks. The scheme keeps track of the service contracts with the SPs and also the fairness requirements between cell-center users and cell-edge users. Also the scheme allows the flexible definition of fairness requirements for different SPs. The performance of the proposed schemes is evaluated and the results show that the proposed low-complexity scheme is very efficient in terms of computation time and its performance in terms of sum rate is close to the results due to the relaxed solution and coordinate search algorithm.

## I. INTRODUCTION

Wireless network virtualization is a means by which an infrastructure provider (InP) can slice the wireless and physical resources to slices. These slices are assigned to service providers (SPs) so that they can serve their subscribers. The main motivation behind wireless network virtualization is cost saving of network roll-out, energy saving, maximization of revenue for InPs, and allowing small investors to enter the mobile communication business to improve the competition basis and hence the provided services to end-users [1]. The wireless virtualization of LTE networks has received an increasing attention in the academia, industry, and standardization bodies [1]–[3].

A successful wireless network virtualization scheme is benchmarked in terms of efficient resource utilization, inter-slice isolation, and customizable intra-slice resource allocation [8]. The proposed scheme in this paper dynamically allocates the slices for different SPs where the allocation scheme maximizes the efficiency of resources by maximizing the sum rate of the users of different SPs considering predefined fairness requirements. The fairness requirements are flexibly defined by the SPs as the rate ratio between the cell-center users and the cell-edge users. Also the slicing scheme is flexible to accommodate arbitrary fairness requirements' schemes as defined by the SPs.

The problem of wireless virtualization in LTE is addressed in many papers. The authors in [3]–[6] studied wireless virtu-

alization in LTE and they proposed a general framework for wireless virtualization in a multi-cell LTE system. They cast the term *hypervisor* to account for an entity that is responsible for collecting information from different virtual eNodeB stacks and allocating the radio resources to different SPs. In [7], NVS was proposed as a wireless resource slicing scheme that enables wireless virtualization by modifying the MAC schedulers within the base station. CellSlice in [8] presents the improvement over NVS to enable remote slicing implemented in gateways where the slicing scheme in CellSlice basically depends on remote traffic shaping and supports downlink and uplink slicing. VNTS [9] is another remote slicing scheme via traffic shaping, which supports downlink only. In [10], vBTS is a proposed virtualized base station to enable the sharing of radio access network components at the hardware level.

The rest of the paper is organized as follows. Section II provides a description of the system model. The proposed slicing schemes and solutions are discussed in Section III. The numerical results and their discussion are presented in Section IV followed by conclusions in Section V.

## II. SYSTEM MODEL

We consider the downlink of a single cell scenario in LTE systems as illustrated in Figure 1. This cell is managed by a single infrastructure provider (InP) who provides its service to a set of service providers (SPs). In general, the isolation between the slices of different SPs is required by any wireless virtualization scheme. In this paper, we assume a general isolation scheme with no restriction on the assigned resources to the users of different SPs, which guarantees the isolation by achieving a certain minimum share of the resources.

### A. General Virtualization

We assume that the isolation between different SPs is achieved by proper allocation of physical resource blocks (PRBs) of the LTE resource grid to the users that belongs to different SPs. Each SP is assigned a minimum amount of the available PRBs based on a service contract with the InP where we denote by  $\rho_{min}^m$  the minimum portion of PRBs allocated to SP  $m$ . The system has  $M$  SPs where each SP  $m$  provides its service to  $K_m$  users (i.e., SP1 has  $K_1$  users and SP2 has  $K_2$  users and so on). Also there are  $C$  sub-channels in the frequency domain each of which has bandwidth  $B$ , and

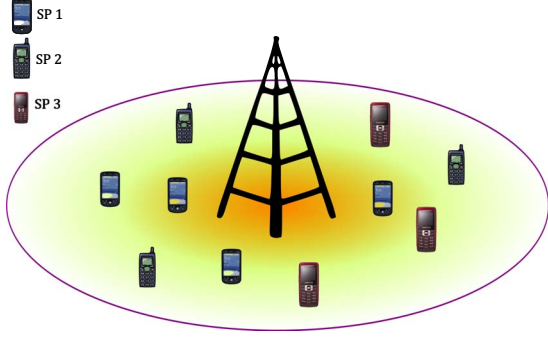


Fig. 1: System Model

we restrict our attention to  $T$  sub-frames in the time domain. Thus a total of  $TC$  PRBs are available for scheduling in each scheduling round with duration  $T$ . The resource allocation is then updated in each scheduling round. The base station in this cell has a total transmit power of  $P_{total}$  and let  $p_{tc}^{mk}$  denotes the power allocated to user  $(m, k)$  (i.e., user  $k$  of SP  $m$ ) in the sub-channel  $c$  at time  $t$ . We further assume that the base station receives perfect channel gain information from all users belonging to all SPs, where  $h_{tc}^{mk}$  is the channel gain for user  $(m, k)$  on a sub-channel  $c$  at time  $t$ . The rate associated to each PRB  $(t, c)$  for a given user  $(m, k)$  which is denoted by  $r_{tc}^{mk}$  can be calculated as

$$r_{tc}^{mk} = \frac{B}{T} \log_2 \left( 1 + \frac{p_{tc}^{mk} h_{tc}^{mk}}{N_o B} \right). \quad (1)$$

where  $N_o$  is the noise spectral density.

Denote by  $R_k^m$  the data rate achieved by user  $k$  that belongs to SP  $m$ , where  $m = 1, 2, \dots, M$  and  $k = 1, 2, \dots, K_m$ . Also denote by  $x_{tc}^{mk}$  an assignment binary variable, which indicates the assignment of the PRB  $(t, c)$  to the user  $(m, k)$  during a scheduling round where

$$x_{tc}^{mk} = \begin{cases} 1 & \text{if PRB}(t, c) \text{ is assigned to user } (m, k) \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

In addition, it is required that  $\sum_{m,k} x_{tc}^{mk} = 1$  meaning a PRB must be allocated to exactly one user belonging to one SP. Each SP is assigned a minimum number of PRBs  $N_{PRB}^m$  to guarantee a minimum acceptable service level for its users, such that  $N_{PRB}^m \geq \rho_{min}^m TC$ . Then the rate  $R_k^m$  is given by:

$$\begin{aligned} R_k^m &= \sum_{t=1}^T \sum_{c=1}^C x_{tc}^{mk} r_{tc}^{mk} \\ &= \sum_{t=1}^T \sum_{c=1}^C \frac{B}{T} x_{tc}^{mk} \log_2 \left( 1 + \frac{p_{tc}^{mk} h_{tc}^{mk}}{N_o B} \right), \quad \forall m, k. \end{aligned} \quad (3)$$

We propose to schedule the slices based on a proportional fairness rule with the objective to maximize the sum rate under the total power constraints, the service contract constraint, and the fairness constraint. Here, optimization variables are related to the PRBs and power allocation decisions, namely  $x_{tc}^{mk}$  and  $p_{tc}^{mk}$ , respectively. Hence, the resource allocation problem to

maximize the sum rate subject to a necessary set of constraints can be formulated as

$$\begin{aligned} \max_{x_{tc}^{mk}, p_{tc}^{mk}} \quad & \sum_{m=1}^M \sum_{k=1}^{K_m} R_k^m = \\ \max_{x_{tc}^{mk}, p_{tc}^{mk}} \quad & \sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{t=1}^T \sum_{c=1}^C \frac{B}{T} x_{tc}^{mk} \log_2 \left( 1 + \frac{p_{tc}^{mk} h_{tc}^{mk}}{N_o B} \right) \end{aligned} \quad (4)$$

subject to the following constraints:

C1 (Total power constraint):

$$\begin{aligned} \sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{c=1}^C p_{tc}^{mk} &\leq P_{total}, \quad \forall t \\ p_{tc}^{mk} &\geq 0, \quad \forall m, k, t, c \end{aligned} \quad (5)$$

C2 (Orthogonality constraint):

$$\begin{aligned} \sum_{m=1}^M \sum_{k=1}^{K_m} x_{tc}^{mk} &= 1, \quad \forall t, c \\ x_{tc}^{mk} &\in \{0, 1\}, \quad \forall m, k, t, c \end{aligned} \quad (6)$$

C3 (Service contract constraint):

$$N_{PRB}^m = \sum_{k,t,c} x_{tc}^{mk} \geq \rho_{min}^m TC, \quad \forall m \quad (7)$$

where  $\rho_{min}^m$  denotes the contracted resource share of SP  $m$ ,  $\rho_{min}^m \in [0, 1] \forall m$ , and  $\sum_{m=1}^M \rho_{min}^m \leq 1$

C4 (Fairness constraint):

$$(1 - \alpha) \frac{R_1^m}{\gamma_1^m} \leq \frac{R_k^m}{\gamma_k^m} \leq (1 + \alpha) \frac{R_1^m}{\gamma_1^m} \quad \forall m, k \quad (8)$$

where  $\{\gamma_k^m\} \forall m, k$  is a set of predetermined values to ensure the proportional fairness among users in each SP, and  $\alpha$  is a small number to relax the fairness constraints as desired. The proportional fairness is defined in the system by a means that allows the explicit control of the capacity ratios among users given sufficient available total transmit power [12].

### III. SLICE SCHEDULING ALGORITHMS

The problem in (4) with its constraints is a mixed binary integer nonlinear programming problem, the decision variables in the problem  $x_{tc}^{mk}$  and  $p_{tc}^{mk}$  are binary variables and continuous variables, respectively. Such an optimization problem is generally hard to solve, especially with the nonlinear fairness constraint in (8). The complexity of this problem comes from the fact that the possible PRBs allocations are  $(\sum_m K_m)^{TC}$ , and for each allocation there is an optimal power distribution to maximize the sum rate while maintaining the proportional fairness among the users of each SP. The complexity of solving this problem is high for a network of realistic size with fast varying channel conditions and very tight time-to-decide.

We propose to decompose the slicing problem in this paper into two distinct problems, namely the PRB allocation problem and the power allocation problem. Two different solution methods are provided. The first solution is the iterative coordinate search where we find an optimal solution

for the PRB allocation and the power allocation problems repeatedly. Moreover, the PRB allocation in each iteration is obtained by solving the underlying IP problem while the optimal power allocation solution is obtained by solving the transformed convex problem presented in section III-C. In the second solution, we employ uniform power allocation and the proposed PRB allocation heuristic presented in section III-B to find a suboptimal solution for the PRB allocation problem. For the sake of comparison, and to find an upper bound for the optimal solution, the integer PRB allocation variables (i.e.,  $x_{tc}^{mk}$ ) for problem (4)-(8) are relaxed to  $0 \leq x_{tc}^{mk} \leq 1$  so that the problem becomes a continuous nonlinear program that can be solved by standard techniques. This will be called “relaxed problem” in the numerical results.

#### A. Iterative Coordinate Search

The underlying optimization problem is typically required to be decomposed into two distinct problems to reduce its complexity, namely PRB allocation problem and power distribution problem. In particular, we can solve these problems separately and alternate between them until convergence. This solution is called the “coordinate search” in the sequel. In each iteration, for a given power distribution we try to find the optimal PRB allocation, and for the found PRB allocation we try to find the optimal power distribution and so forth. It can be observed that for a given optimal power distribution, the PRB allocation problem is a linear integer program (IP). In addition, for the given PRB allocation, the power distribution is a non-linear program that can be transformed into a convex problem as will be shown in section III-C.

In general, the iterative search of the optimal PRB allocation and the optimal power distribution is costly in terms of computation time, which is not suitable for real-time implementation. The computation cost is especially high for the integer PRB allocation problem. To address this challenge, we propose a fast heuristic to find an efficient PRB allocation, which returns a suboptimal but efficient PRB allocation for the users associated with different SPs.

#### B. PRB Allocation Heuristic

To perform suboptimal PRB allocation, we assume uniform power distribution over the subchannels in each sub-frame  $t$ . With this power allocation, the rates achieved by each user on all PRBs are known and PRB allocation problem is now reduced to a standard IP problem. However, finding an optimal solution for this problem is very time consuming as we have discussed before. In this sub-section, we propose an algorithm that achieves a sub-optimal solution, this algorithm is inspired by the algorithm in [11] and it is not iterative, i.e. the PRB allocation is computed once for the uniform power distribution.

Define  $H_{tc}^{mk} = \frac{h_{tc}^{mk}}{N_o B}$  as the channel to noise ratio for user  $(m, k)$  in PRB  $(t, c)$  and  $\Omega_k^m$  is the set of PRBs  $\{(t, c)\}$  allocated to user  $(m, k)$  such that  $x_{tc}^{mk} = 1$ , also define  $N_{PRB}^m = \sum_{k,t,c} x_{tc}^{mk}$  as the number of PRBs allocated to the

slice corresponding to SP  $m$ . The algorithm can be described as

- 1) Initialization
  - a) Set  $R_k^m = 0$ ,  $\Omega_k^m = \phi$  for  $m = 1, 2, \dots, M$ ,  $k = 1, 2, \dots, K_m$ , and  $A = \{(1, 1), (1, 2), \dots, (t, c), \dots, (T, C)\}$
- 2) For  $m = 1$  to  $M$ ,  $k = 1$  to  $K_m$ ,
  - a) find  $(i, j)$  satisfying  $|H_{ij}^{mk}| \geq |H_{tc}^{mk}|$ ,  $\forall (t, c) \in A$
  - b) let  $\Omega_k^m = \Omega_k^m \cup \{(i, j)\}$ ,  $A = A - \{(i, j)\}$ , update  $R_k^m$  according to (3), and update  $N_{PRB}^m$ .
- 3) While  $A \neq \phi$ ,
  - a) find  $m$  satisfying  $N_{PRB}^m / \rho_{min}^m \leq N_{PRB}^n / \rho_{min}^n$ ,  $\forall n, 1 \leq n \leq M$
  - b) for the found  $m$ , find  $k$  satisfying  $R_k^m / \gamma_k^m \leq R_r^r / \gamma_r^r$ ,  $\forall r, 1 \leq r \leq K_m$
  - c) for the found  $m$  and  $k$ , find  $(i, j)$  satisfying  $|H_{ij}^{mk}| \geq |H_{tc}^{mk}|$ ,  $\forall (t, c) \in A$
  - d) for the found  $m$ ,  $k$ ,  $i$ , and  $j$ , let  $\Omega_k^m = \Omega_k^m \cup \{(i, j)\}$ ,  $A = A - \{(i, j)\}$ , update  $R_k^m$  according to (3), and update  $N_{PRB}^m$ .

The above algorithm tries to allocate the PRBs to different slices efficiently. Specifically, the PRB allocation heuristic lets each slice pick a PRB with high channel to noise ratio and assign it to one of its users with the conservation of the service contracts' restrictions and the proportional fairness inside each slice.

#### C. Optimal Power Distribution for a Given PRB Allocation

We now arrive to the next step in the decomposition of the problem in (4)-(8). Specifically, for a given PRB allocation, the optimal power distribution to this determined PRBs allocation can be formulated as

$$\max_{p_{tc}^{mk}} \sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{(t,c) \in \Omega_k^m} \frac{B}{T} \log_2(1 + p_{tc}^{mk} H_{tc}^{mk}) \quad (9)$$

subject to

$$\begin{aligned} & \sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{(t,c) \in \Omega_k^m} p_{tc}^{mk} \leq P_{total}, \quad \forall t \\ & p_{tc}^{mk} \geq 0, \quad \forall m, k, t, c \\ & (1 - \alpha) \frac{\tilde{R}_1^m}{\gamma_1^m} \leq \frac{\tilde{R}_k^m}{\gamma_k^m} \leq (1 + \alpha) \frac{\tilde{R}_1^m}{\gamma_1^m} \quad \forall m, k \end{aligned} \quad (10)$$

where  $\tilde{R}_k^m = \sum_{(t,c) \in \Omega_k^m} \frac{B}{T} \log_2(1 + p_{tc}^{mk} H_{tc}^{mk})$ . We can easily see (9)-(10) is a convex problem by a simple change of variables as follows:

$$z_{tc}^{mk} = \log_2(1 + p_{tc}^{mk} H_{tc}^{mk}) \quad (11)$$

and so

$$p_{tc}^{mk} = \frac{2^{z_{tc}^{mk}} - 1}{H_{tc}^{mk}} \quad (12)$$

From this, we can rewrite the problem as

$$\max_{z_{tc}^{mk}} \sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{(t,c) \in \Omega_k^m} \frac{B}{T} z_{tc}^{mk} \quad (13)$$

subject to

$$\sum_{m=1}^M \sum_{k=1}^{K_m} \sum_{(t,c) \in \Omega_k^m} \frac{2z_{tc}^{mk} - 1}{H_{tc}^{mk}} \leq P_{total}, \quad \forall t \quad (14)$$

$$z_{tc}^{mk} \geq 0, \quad \forall m, k, t, c \quad (15)$$

$$\frac{(1-\alpha)}{\gamma_1^m} \sum_{(t,c) \in \Omega_1^m} z_{tc}^{m1} \leq \frac{1}{\gamma_k^m} \sum_{(t,c) \in \Omega_k^m} z_{tc}^{mk} \leq \frac{(1+\alpha)}{\gamma_1^m} \sum_{(t,c) \in \Omega_1^m} z_{tc}^{m1} \quad \forall m, k. \quad (16)$$

This problem is convex because all functions are linear except (14), which are inequality constraints with the convex constraint functions. We can solve this problem by using a nonlinear solver to find the optimal power allocation.

#### IV. PERFORMANCE EVALUATION

In this section, the performance evaluation for the proposed slicing algorithms will be presented. The evaluated scenario is tabulated in Table I. The considered system deploys a single base station (BS) centered in a cell coverage area with radius of 500 m. The InP serves 3 service providers, each with a certain minimum contracted resources. Each service provider serves 5 subscribers in which 2 are cell-center users and 3 are cell-edge users with a rate ratio between the cell-center users and cell-edge users is 10:1 respectively. Here, cell-center and cell-edge users are randomly located in the cell where cell-center users are at most 250 m away from the BS and the distances between the BS and any cell-edge users are between 250 m and 500 m.

The channel gains are generated based on a simple channel model  $\beta d^{-\alpha}$  where  $d$  is the distance from the user and the BS in meters,  $\alpha$  is the path loss exponent and is assumed 5, and  $\beta$  is the multipath fading parameter and follow an exponential distribution with average value of 1. Also, the noise power per subchannel of 180 KHz is set equal to  $10^{-13}$  watt. The relaxed optimization problem is solved using SNOPT to find an upper bound for the sum rate, and the iterative coordinate search is solved using CPLEX and SNOPT. A Matlab based simulation is implemented to study the performance of the PRB allocation based on the slicing algorithm heuristic. The solvers and the Matlab simulator run on i7 core 3400 MHz with 12 GB of memory.

Parameter	Value
Carrier Bandwidth	5 MHz.
Number of Sub-channels	25
Number of Sub-frames	10
Service Providers	3 each with 5 subscribers
Service Contract Vector	[0.5 0.3 0.2]
Fairness Ratios for all SPs	[10 10 1 1 1]

TABLE I: System parameters for evaluated scenario

The results presented in Figure 2 show that the rate achieved by the proposed PRB allocation heuristic is slightly less than what is achieved by the coordinate search, and the upper bound of the relaxed problem which are computed using optimization solvers. However, the time required to compute the solution using the PRB allocation heuristic is much less than that required by the solvers to find the solutions for other methods as tabulated in Table II.

In Figure 3, the sum rate and the fairness index at different fairness ratios are plotted by using the proposed PRB allocation heuristic. The fairness ratio is the rate ratio between the cell-center users and the cell-edge users. The results show that the maximum fairness index is achieved at rate ratio of 1, this rate ratio corresponds to max-min fairness where all users are allocated resources so as to achieve the same rate. However, at rate ratio of 1, the sum rate is the worst as the allocation of more PRBs to cell-edge users affects the efficiency of these resources due to the bad channel conditions for those users. Based on this result, a SP can make a trade off between the efficiency of the resources in its slice and the fairness achieved among its users by proper choice of the rate ratio between users.

Method	CPU time
PRB Allocation Heuristic	0.0207 secs.
Relaxed Problem	728.319 secs.
Coordinate Search	1820.312 secs.

TABLE II: CPU time

The service contracts are employed to determine the allocation of a certain number of PRBs for each SP. However, the resource based slicing does not guarantee the allocated sum rate for the SP, as the achieved sum rate depends on the channel conditions of the users corresponding to this SP, and the designated fairness between its users. Figure 4 shows how the resource ratio is compared to the actual achieved sum rate ratio. In particular, the sum rate ratio for SP1 is almost the same as the resource ratio. However, the sum rate ratio may be less than the contracted resource ratio as in the case of SP2, or greater than the contracted resource ratio as in the case of SP3. In general, the sum rate of each SP vary with the time depending on the channel conditions of its users. In Figure 5 the normalized sum rate of the 3 SPs is plotted with the time, the normalized sum rate is varying around the contracted ratio of each SP.

#### V. CONCLUSION

Wireless network virtualization is a promising technique in which the wireless and the physical resources of an InP may be shared by several SPs. In this paper, we propose a slicing scheme to efficiently allocate the PRBs of LTE system resource grid to different SPs. The slicing scheme is dynamic and flexible where the flexibility of the scheme is emerged in the arbitrary definition of fairness requirements of different SPs. Numerical results show the efficiency of the proposed low-complexity slicing scheme compared to other higher complexity algorithms in terms of the sum rate. Also

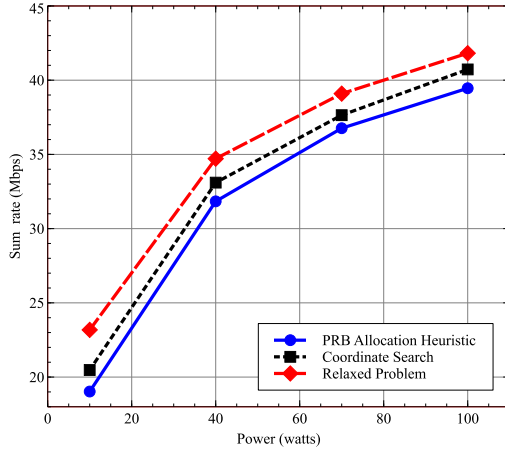


Fig. 2: The data rate achieved by allocating the PRBs using coordinate search and PRB allocation heuristic

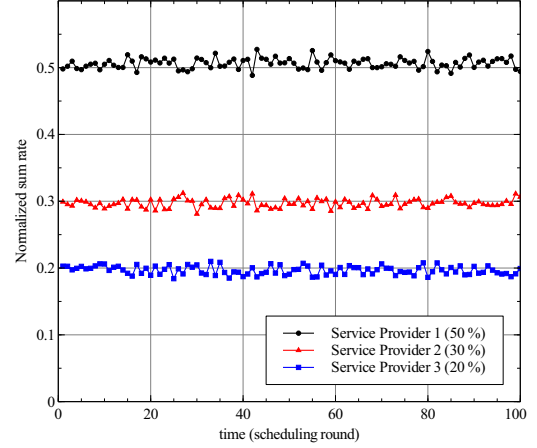


Fig. 5: Normalized sum rate for different SPs

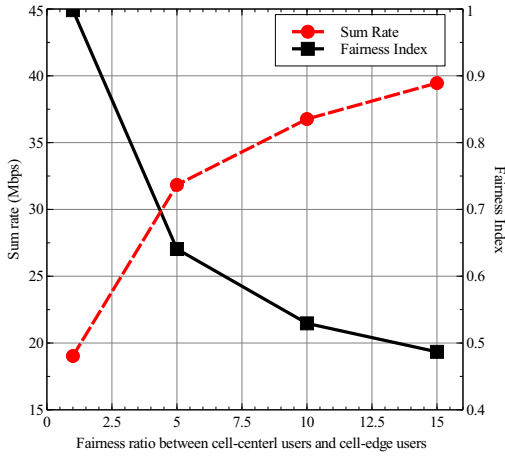


Fig. 3: Sum rate and fairness index for different center to edge rate ratios

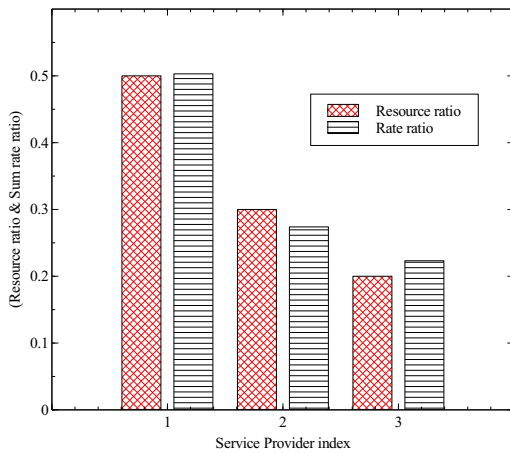


Fig. 4: Service contract resource ratio and the sum rate ratio for different SPs

the computation time of the scheme is compared to off-line solvers, where it is apparent that the scheme requires much less computation time.

## REFERENCES

- [1] X. Costa-Perez, J. Swetina, T. Guo, R. Mahindra, and S. Rangarajan, "Radio access network virtualization for future mobile carrier networks," *IEEE Commun. Mag.*, vol. 51, no. 7, pp. 27–35, Jul. 2013.
- [2] 3GPP TR 22.852, "Study on Radio Access Network (RAN) sharing enhancements," Jun. 2013.
- [3] L. Zhao, M. Li, Y. Zaki, A. Timm-Giel and C. Gorg, "LTE virtualization: From theoretical gain to practical solution," in *Proc. International Teletraffic Congress (ITC)*, pp. 71–78, Sep. 2011.
- [4] Y. Zaki, L. Zhao, C. Gorg, and A. Timm-Giel, "A novel LTE wireless virtualization framework," *Mobile Networks and Management*, pp. 245–257. Springer Berlin Heidelberg, 2011.
- [5] Y. Zaki, L. Zhao, C. Gorg, and A. Timm-Giel, "LTE mobile network virtualization: Exploiting multiplexing and multi-user diversity gain," in *Mobile Networks and Applications*, no. 4, pp. 424–432, 2011.
- [6] Y. Zaki, L. Zhao, C. Gorg, and A. Timm-Giel, "LTE wireless virtualization and spectrum management," in *Proc. Wireless and Mobile Network Conference (WMNC)*, pp. 1–6, 2010.
- [7] R. Kokku, R. Mahindra, H. Zhang, and S. Rangarajan, "NVS: A substrate for virtualizing WiMAX networks," in *Proc. ACM MobiCom*, Sep. 2010.
- [8] R. Kokku, R. Mahindra, H. Zhang, and S. Rangarajan, "CellSlice: Cellular wireless resource slicing for active RAN sharing," *Communication Systems and Networks (COMSNETS)*, pp. 1–10, Jan. 2013.
- [9] G. Bhanage, R. Daya, I. Seskar, and D. Raychaudhuri, "VNTS: A virtual network traffic shaper for air time fairness in 802.16 e systems," in *Proc. IEEE International Conference on Communications (ICC)*, pp. 1–6, 2010.
- [10] G. Bhanage, I. Seskar, R. Mahindra, and D. Raychaudhuri, "Virtual basestation: Architecture for an open shared wimax framework," in *Proc. ACM SIGCOMM VISA Workshop*, 2010.
- [11] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE VTC'2000-Spring*, vol. 2, pp. 1085–1089, 2000.
- [12] Z. Shen and J.G. Andrews, and B.L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints" *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.