CS 480 – Introduction to Artificial Intelligence

Topic: Search
Chapter: 3

Mustafa Bilgic

http://www.cs.iit.edu/~mbilgic

https://twitter.com/bilgicm

Outline

- Problem solving agent
- The search process
- Tree search
- Graph search
- Complexity analysis

Motivation

- We have discussed several types of agents
  - Reflex agents, goal-based agents, utility-based agents, etc.
- Reflex agents act purely based on current percept, which is limited
- We now discuss, in detail, the goal-based agents, where a sequence of actions are necessary to reach a desired world state
- Actions have associated costs with them and we'd like to minimize the total cost to reach the goal state

Representation

- The world is represented as an atomic state
- An action takes us from one world state to another
- The environment is
  - Observable
  - Discrete
  - Deterministic
  - Known

AN EXAMPLE

- We are now in Chicago
- We'd like to drive to Pittsburgh
- We represent the world as "In X" where X is a member of a predefined set of major cities in US
- An action takes us from being in one major city (a world state) to being in a neighboring major city (another world state)
- The cost of the action is the distance traveled
- We would like to travel from Chicago to Pittsburgh while minimizing the distance traveled

The Search Process

- We will start at an initial state
- At each state, there is a set of actions we can perform
  - The action that can be performed can be different depending on the current world state
- We have a test that can specify whether a given state is a goal state
- Each action has a cost
- We want to find the optimal solution, i.e., the minimum-cost sequence to reach the goal state

MORE FORMALLY

- The **initial state**
  - *In(Chicago)*
- A set of **actions**
  - *Go(Milwaukee)*, *Go(Madison)*, *Go(Bloomington)*, etc.
- A **transition model**: *Result(s, a)*: $S \rightarrow S$
  - *Result(In(Chicago), Go(Madison))* = *In(Madison)*
- The **goal test**
  - {*In(Pittsburgh)*}
- A **path cost**: Sum of step costs *c(s,a,s')*
  - *c(In(Chicago), Go(Madison), In(Madison))* = 150

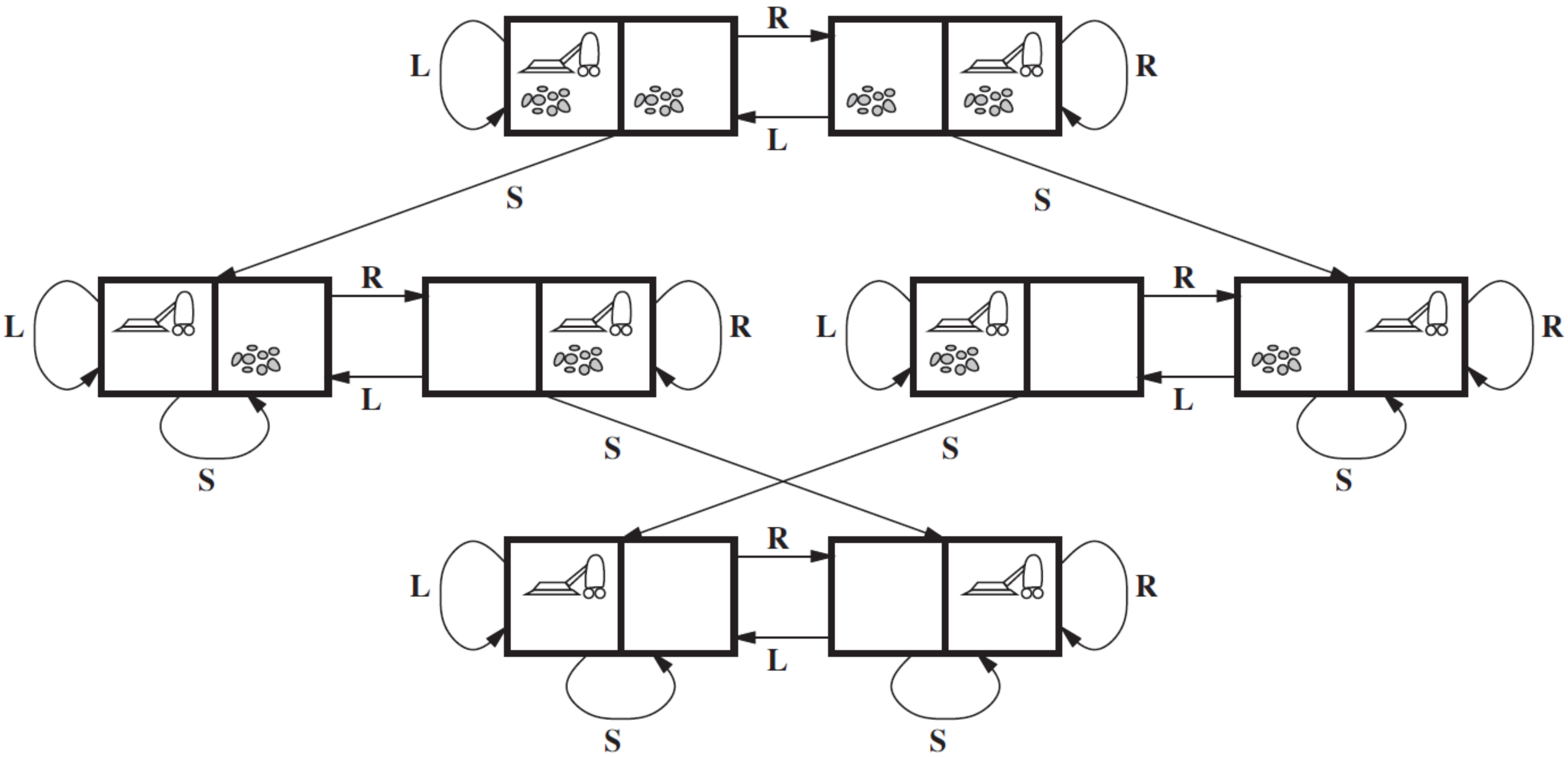
Assumptions and Simplifications

- For the Chicago to Pittsburgh problem, we made several assumptions and simplifications
- What are some of these?

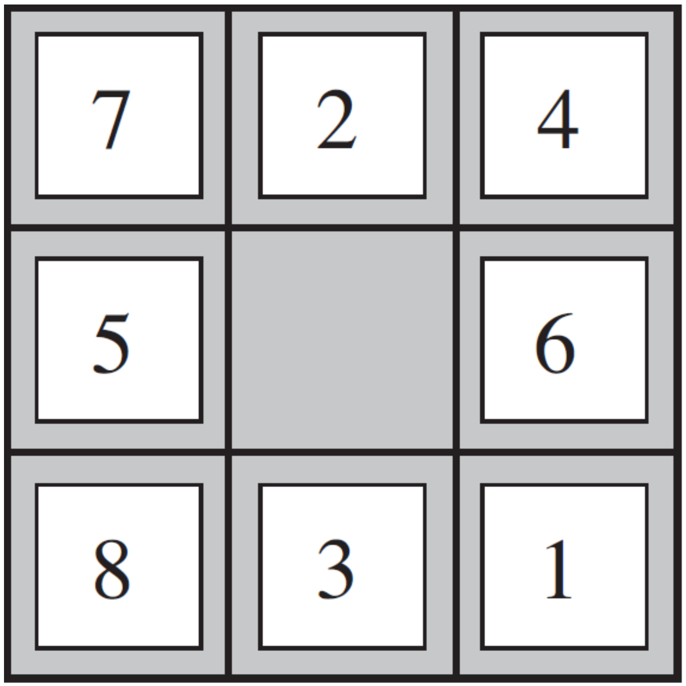
The Vacuum World

- **States:** The cross-product of the agent's location and dirt locations
  - How many possible states?
- **Initial state:** Any state
- **Actions:** Left, Right, and Suck
- **Transition model:** Expected effects. Except Left on the left square, Right on the right square, and Suck on a clean square has no effect
- **Goal test:** Test whether all squares are clean
- **Path cost:** Each step costs 1; the path cost is then the total number of steps

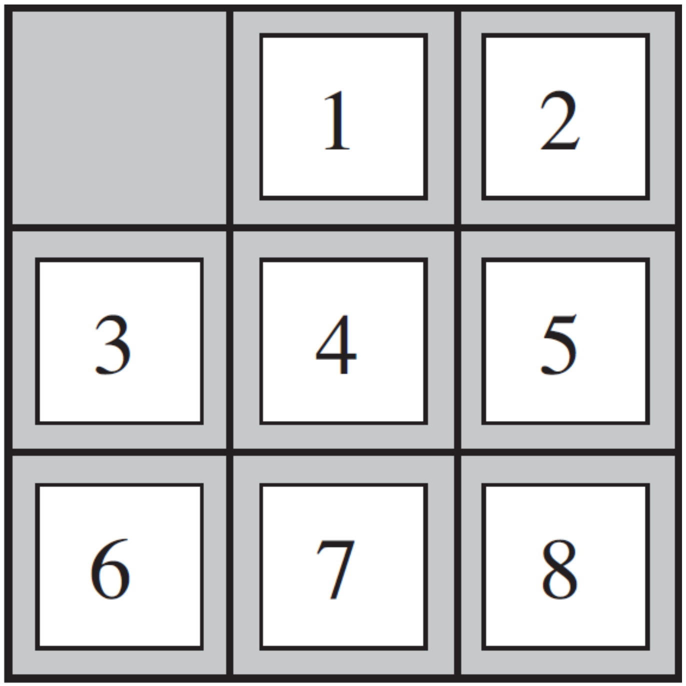
The Vacuum State Space



The 8-Puzzle



Start State

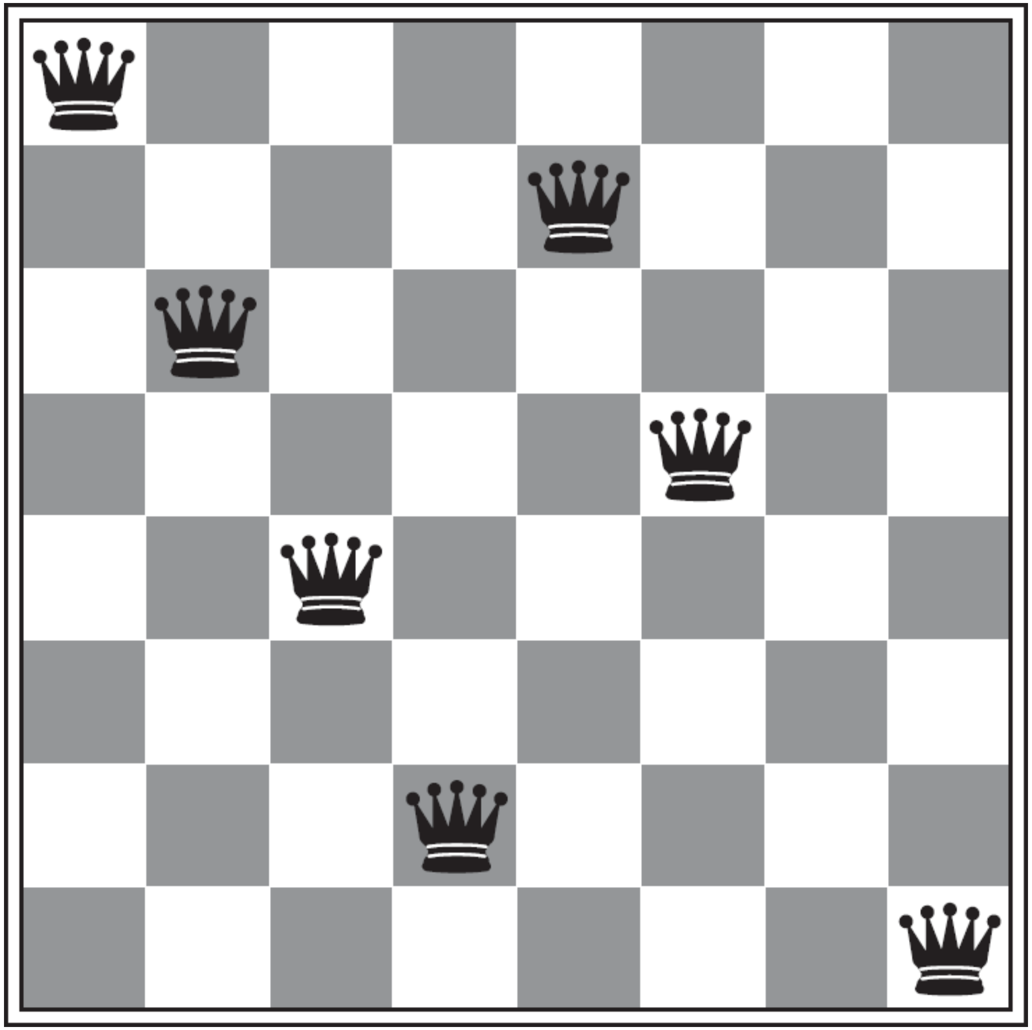


Goal State

THE 8-PUZZLE

- **States:** All possible locations of the eight tiles and the blank tile
  - How many?
- **Initial state:** Any state
- **Actions:** Left, Right, Up, Down, except on the edges
- **Transition model:** The state with the blank the numbered tile switched
- **Goal test:** Blank on the top left, others are ordered in increasing order
- **Path cost:** Each step costs 1; the path cost is then the total number of steps

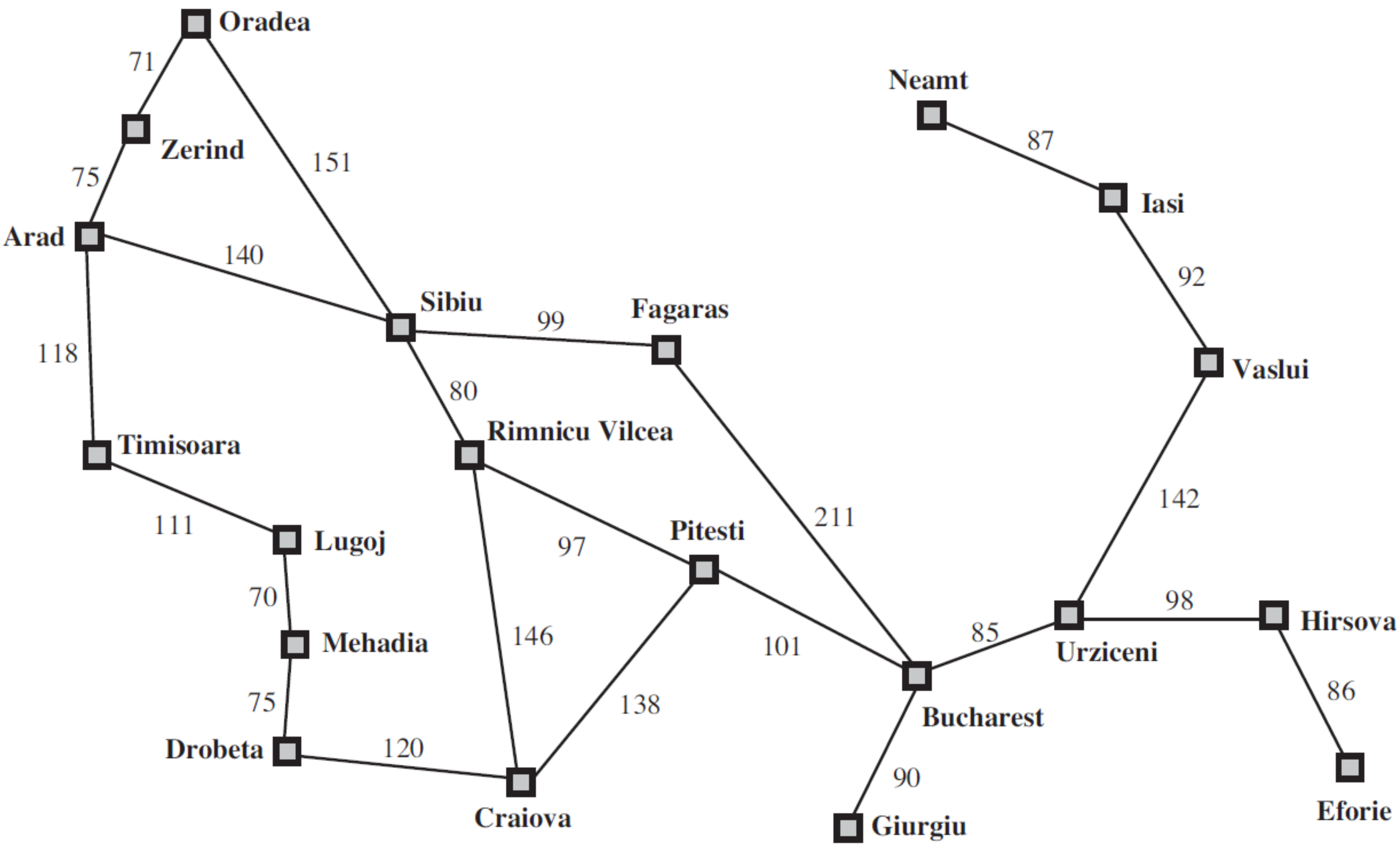
The 8-Queens Problem



THE 8-QUEENS PROBLEM

- **States:** Any arrangement of 0 to 8 queens on the board
  - How many?
- **Initial state:** No queens on the board
- **Actions:** Add a queen to an empty square
- **Transition model:** The board with the added queen
- **Goal test:** 8 queens on the board, none attacked
- **Path cost:** Each step costs 1; the path cost is then the total number of steps

Romania



Oradea
71
Zerind
151
75
Arad
140
Neamt
87
Iasi
92
Sibiu
99
Fagaras
118
80
Vaslui
Rimnicu Vilcea
Timisoara
211
142
111
Lugoj
97
Pitesti
70
98
Hirsova
146
85
Urziceni
Mehadia
101
86
75
138
Bucharest
Drobeta
120
90
Craiova
Giurgiu
Eforie

Romania

- **States:** Any city
- **Initial state:** Arad
- **Actions:** Go to a neighboring city
- **Transition model:** The city that was just traveled
- **Goal test:** Bucharest
- **Path cost:** Step costs are distances between the neighboring cities; path cost is the total distance traveled

Can you think of any real-world problems?

Let's Solve Toy Versions

- The 4-Queens problem
  - Initial state: Empty board
  - Action: Place a queen on the left most empty column
- The 3-Puzzle problem
  - Initial state: First row: 2 – Blank, Second row: 3 – 1
  - Action: Move the blank tile

SEARCH

- The key algorithm of the goal-based agent is *search*

1. Start with the initial state
2. Expand that state through possible actions
3. Pick an unexplored state
4. If goal, return solution; otherwise, go to step 2

Tree Search

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

The key question is which leaf node to pick for expansion.

Two Famous and Important Search Strategies

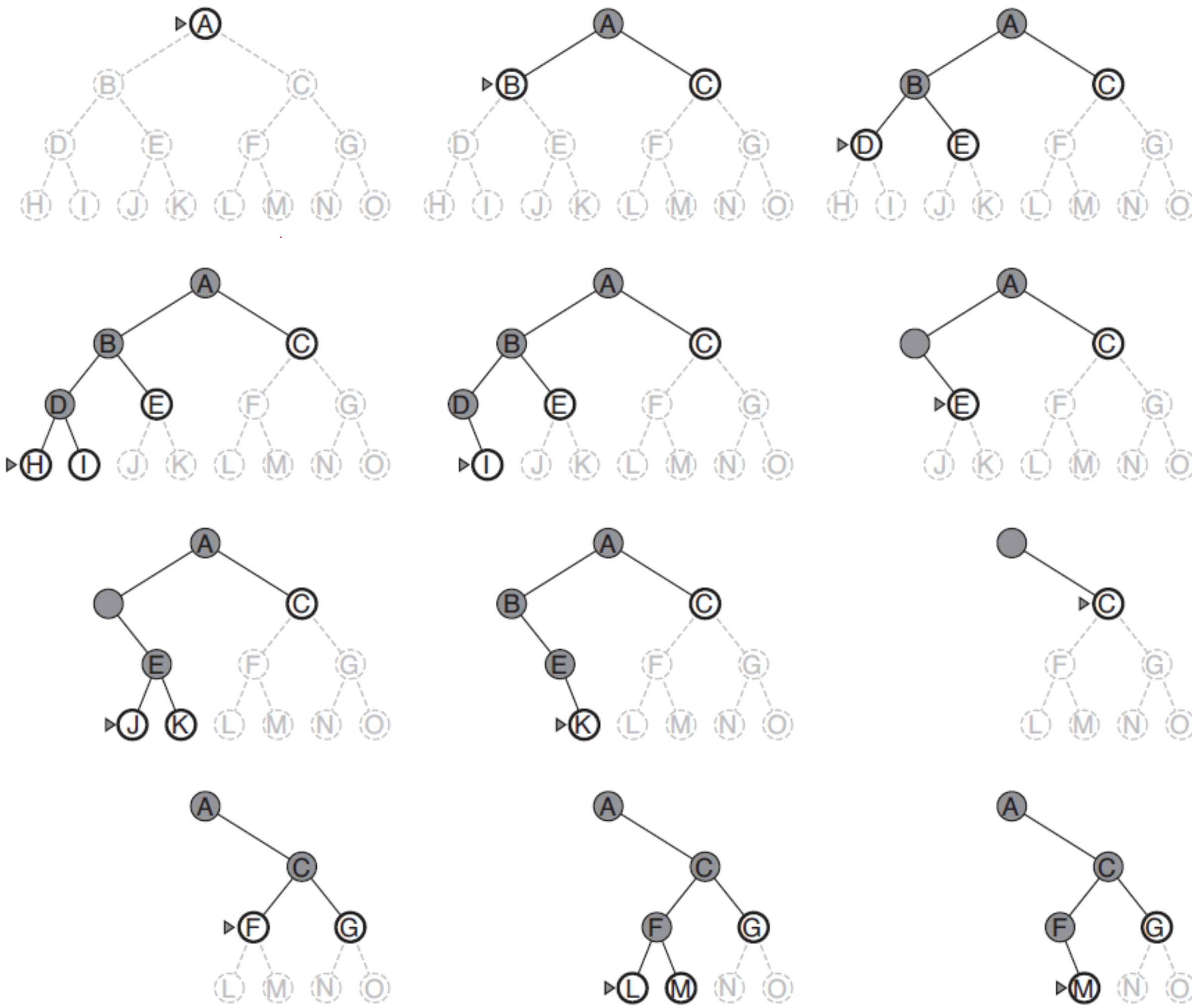
- Depth-first search
  - i.e., go as deep as possible
  - **frontier** = LIFO queue (stack)
- Breadth-first search
  - i.e., go broad; expand all the nodes at level i before expanding the nodes at level i+1
  - **frontier** = FIFO queue

Rist most

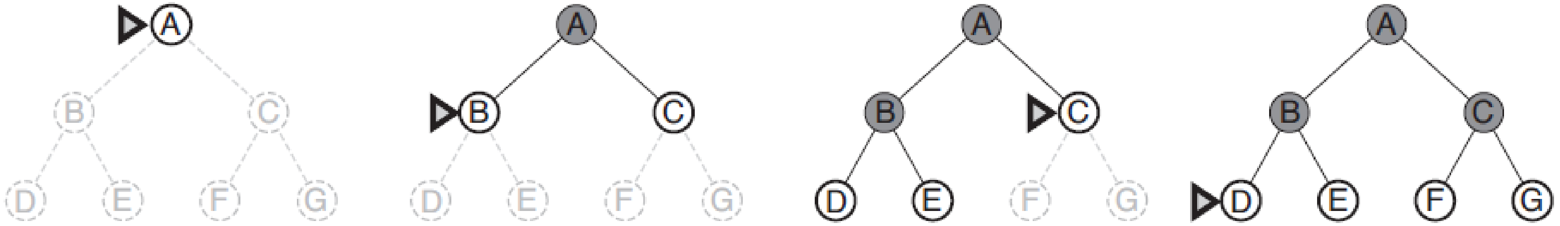
Is this LIFO or recursive?

Left most

DEPTH-FIRST SEARCH



Breadth-first Search



Let's See Them in Action

- 4-Queens
- 3-Puzzle
- Travel
  - A → B, C
  - B → D, E
  - C → F, G
  - D → H, I
  - E → J, K
  - F → L, M
  - G → N, O
  - H, I, J, K, L, M, N, O are terminal states
  - Start: A, Goal: O

COMPLEXITY

- We're concerned with
  - Time complexity
  - Space complexity
  - Completeness (if there is a solution, can it find it?)
  - Optimality (is the found solution the optimal one?)
- Three relevant quantities
  - The number of nodes expanded
  - The maximum size of the **frontier**
  - The cost of the solution

Analysis – tree Search

Criterion	Depth-first	Breadth-first
Complete?		
Time		
Space		
Optimal?		

Memory is a Big Problem

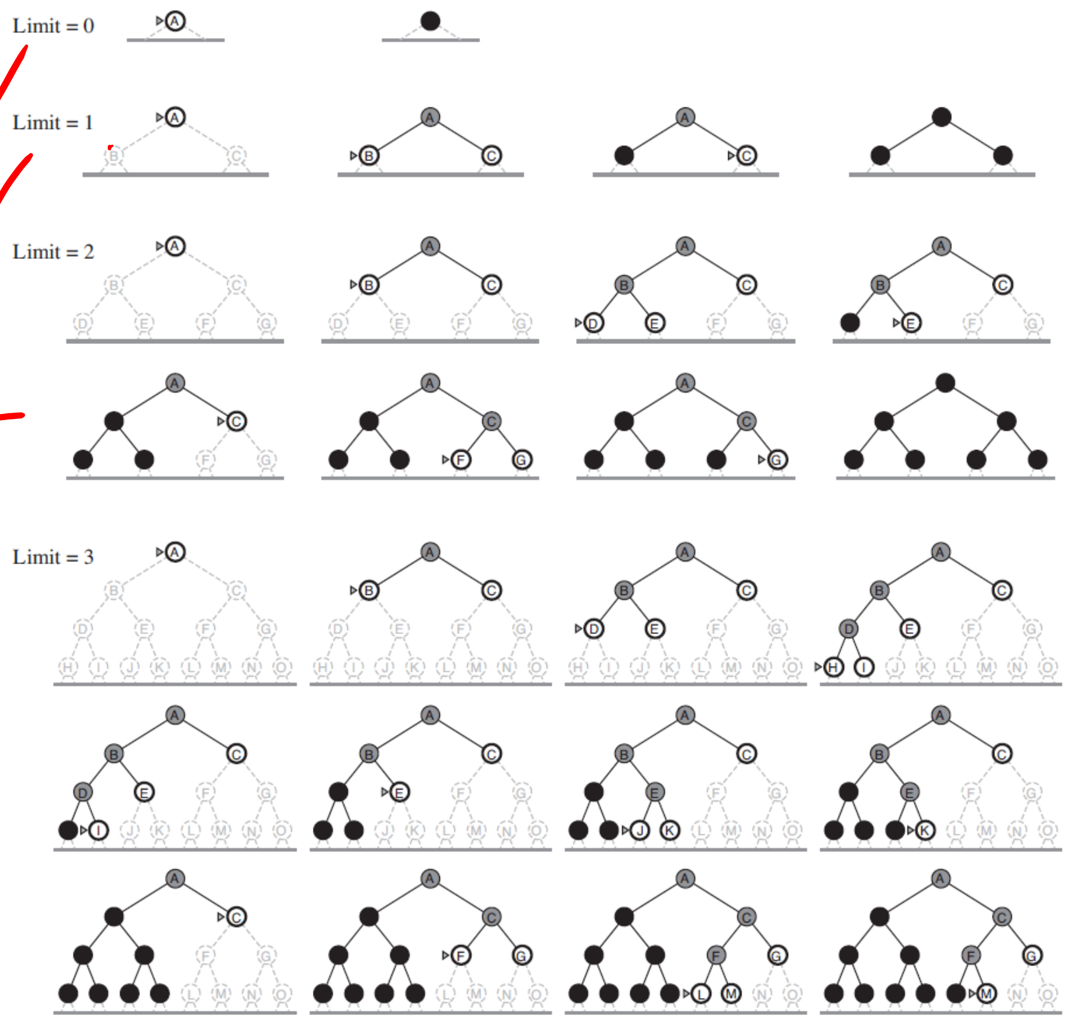
- Breadth-first search has an exponential memory requirement
- Depth-first search requires polynomial size memory, but it is not complete and it is not optimal
- What can we do?
  - Depth-limited search
    - Impose a limit on the depth
    - Is it complete? Optimal?
  - Iterative-deepening search
    - Start with a depth limit of 0, and run depth-limited search; if no goal found, increase the depth limit and repeat the process
    - Is it complete? Optimal?

This is the textbook version; we do the DFS part differently.

What are the time and space complexities for IDS?

ITERATIVE-DEEPENING SEARCH

IDS
complete ✓
optimal ✓



What if the Step Costs are not Identical?

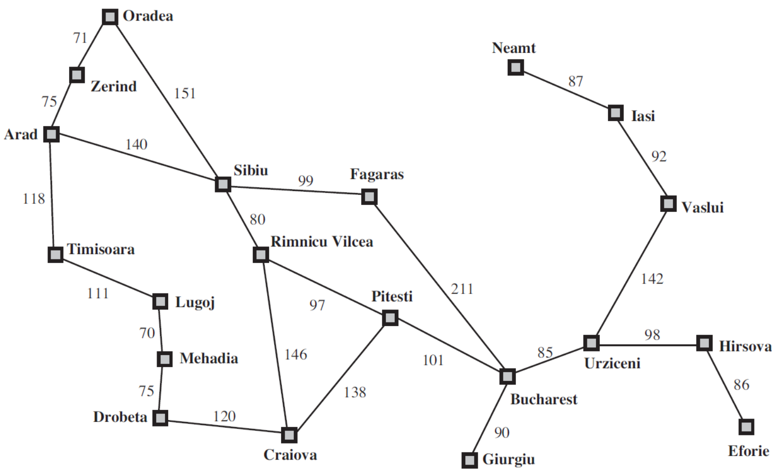
- BFS and Iterative Deepening are not guaranteed to be optimal anymore
- Define $f(n) = h(n) + g(n)$
  - $h(n)$: Cost estimate from n to the goal
  - $g(n)$: Cost from the initial state to n

1. Uniform-cost search
   - Expand using $g(n)$
2. Greedy best-first search
   - Expand using $h(n)$
3. A* search
   - Expand using $f(n)$

The Running Example

Edge costs

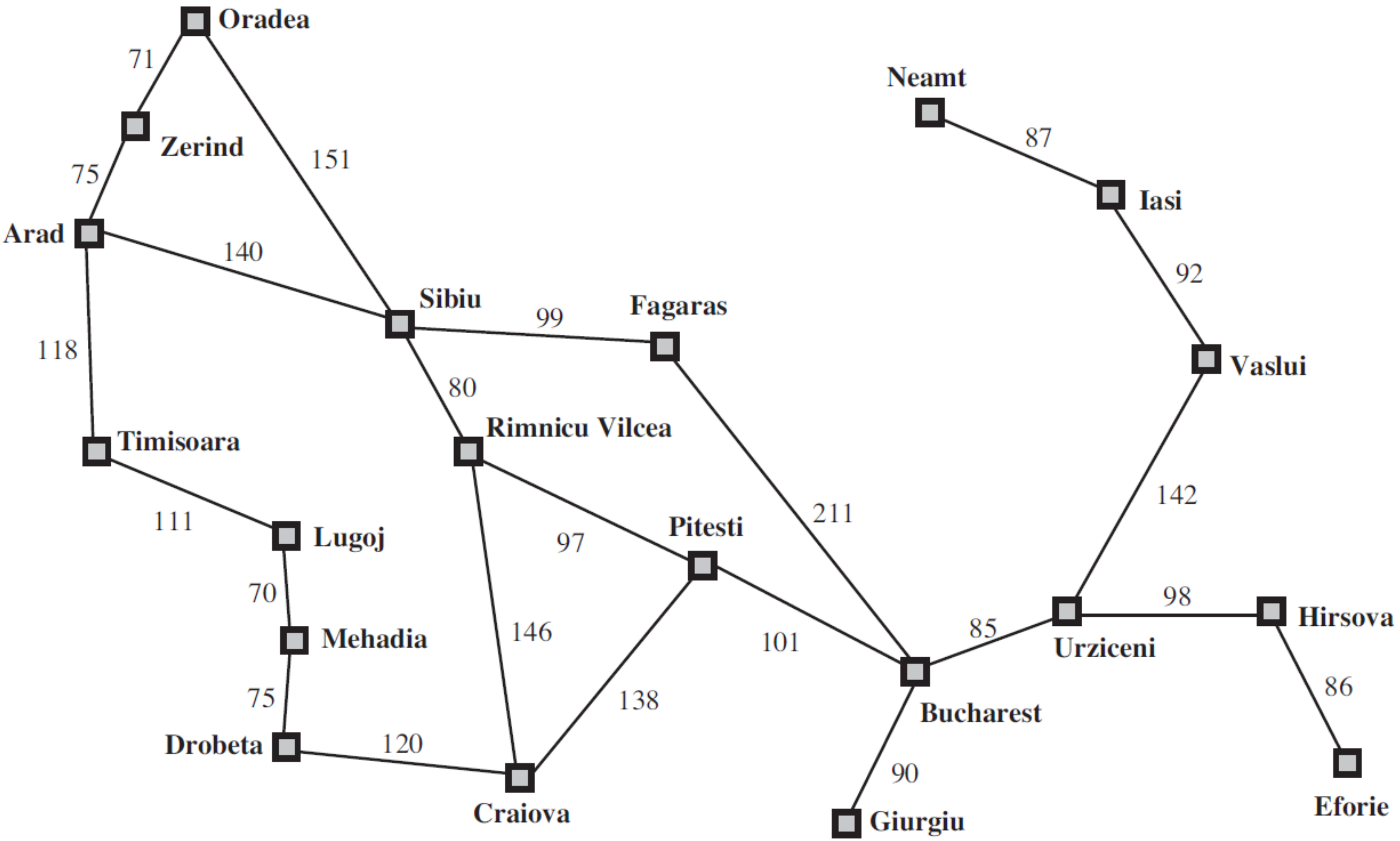
h(n)



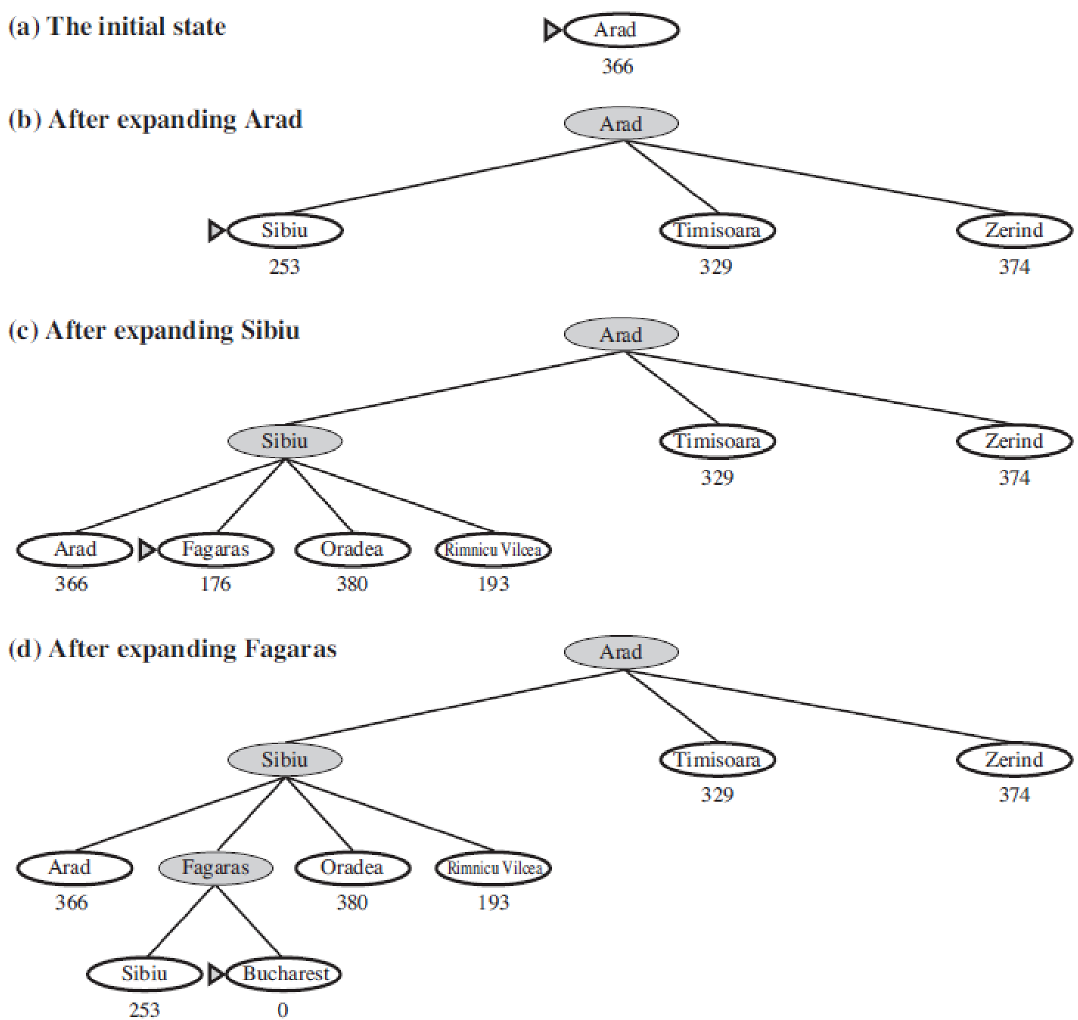
Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

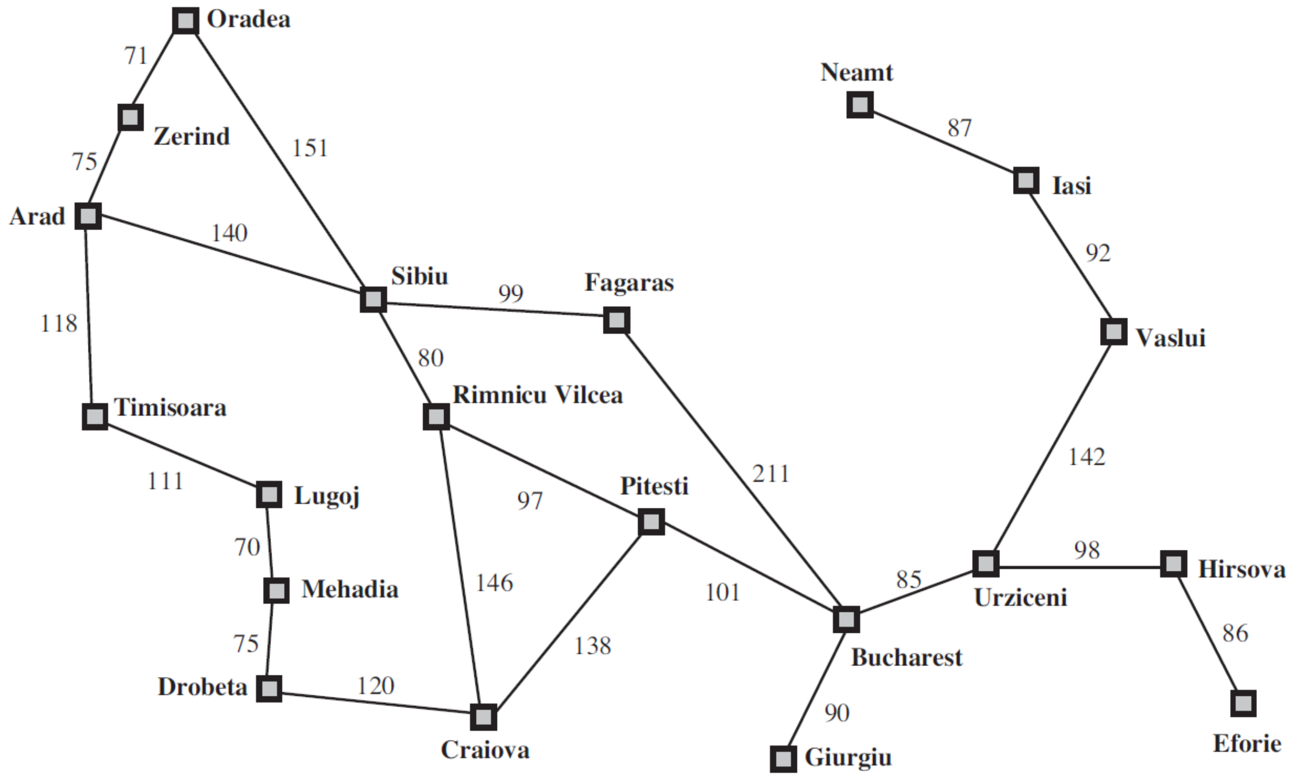
Uniform-Cost Search

- Arad to Bucharest


Oradea
71
Neamt
Zerind
87
151
75
Iasi
Arad
140
92
Sibiu
99
Fagaras
118
Vaslui
80
Rimnicu Vilcea
Timisoara
142
211
111
Lugoj
97
Pitesti
70
98
Hirsova
85
Urziceni
Mehadia
146
101
86
75
138
Bucharest
Drobeta
120
90
Craiova
Giurgiu
Eforie

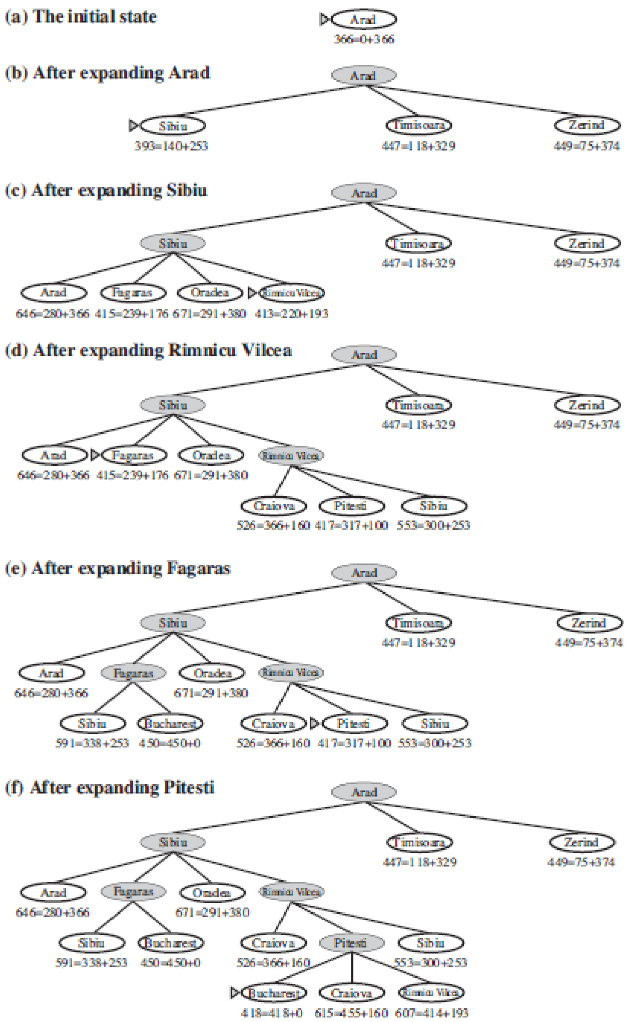
Greedy Best-First Search





Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

A* Search


(a) The initial state
Arad
366=0+366
(b) After expanding Arad
Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374
(c) After expanding Sibiu
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193
(d) After expanding Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253
(e) After expanding Fagaras
Sibiu
591=338+253
Bucharest
450=450+0
(f) After expanding Pitesti
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

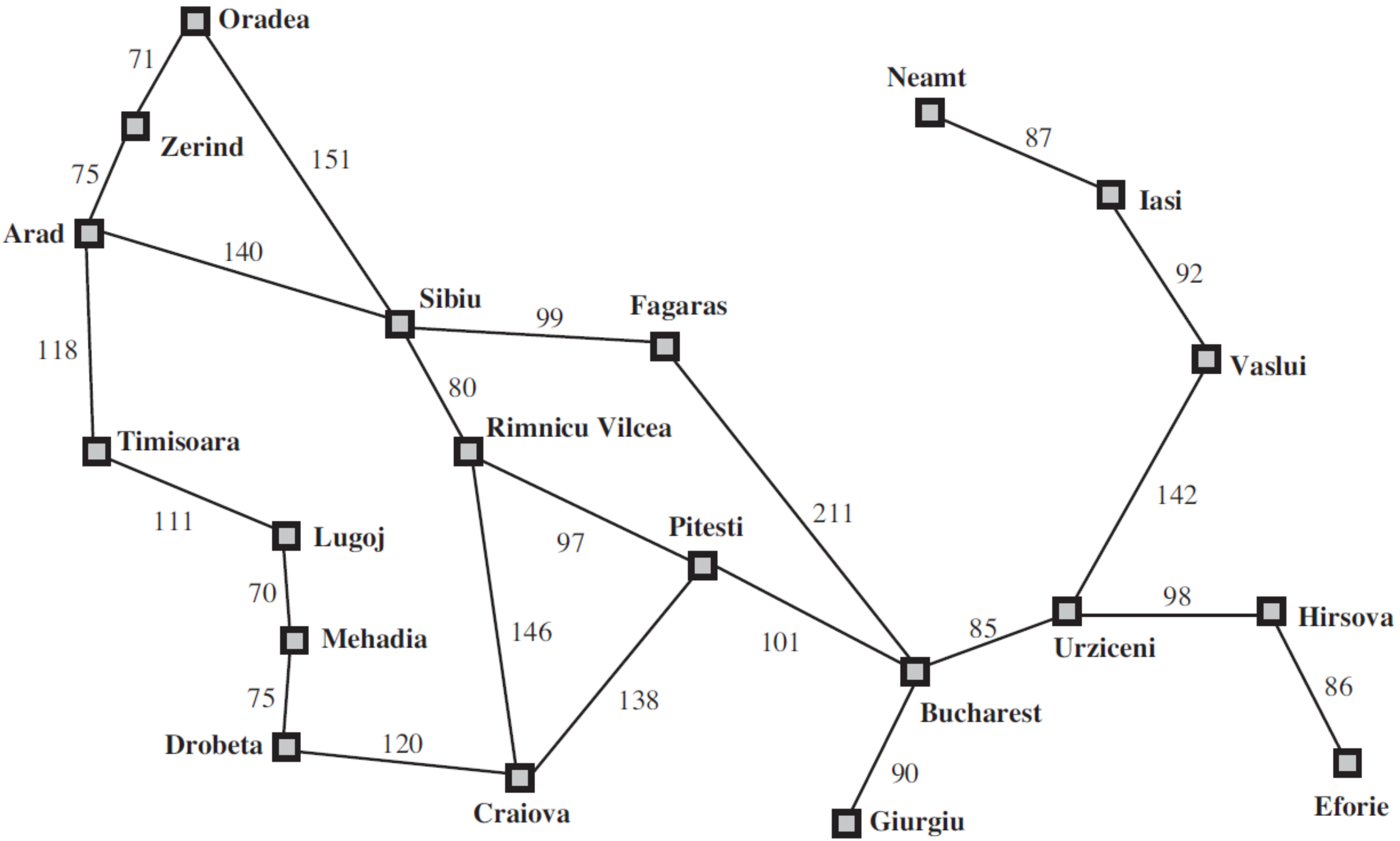
Repeated States

- Repeated states can be a major problem
- Repeated states are especially common for problems where actions are reversible
  - But, they also occur for problems where there are multiple distinct paths between two states
- One way to avoid this problem is to remember the set of explored and generated states

GRAPH SEARCH

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node
      for child in children of the chosen node:
        if child is not in explored:
          add child to frontier only if
            child is not in frontier or
            child is better than the one in frontier

Graph Search Examples


Oradea
71
Zerind
151
75
Arad
140
Neamt
87
Iasi
92
Sibiu
99
Fagaras
118
80
Vaslui
Rimnicu Vilcea
Timisoara
111
Lugoj
97
Pitesti
211
142
70
Mehadia
146
101
85
Urziceni
98
Hirsova
75
138
Bucharest
86
Drobeta
120
Craiova
90
Giurgiu
Eforie

Questions to think about

How does graph search affect

- Completeness,
- Time complexity,
- Space complexity, and
- Optimality

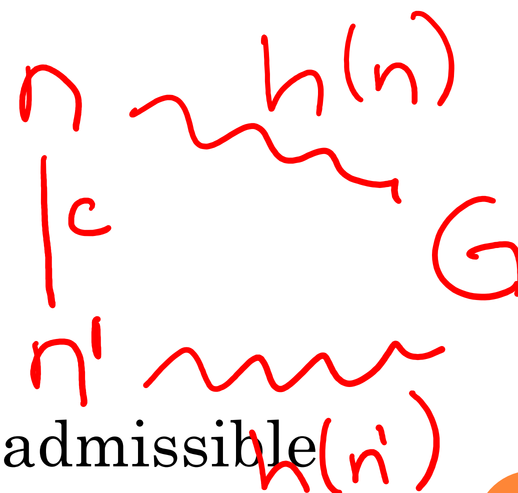
of various search algorithms?

A* Optimal?

- Tree version is optimal if
  - *h(n)* is admissible
- Graph version is optimal if
  - *h(n)* is consistent

ADMISSIBLE? CONSISTENT?

- **Admissible** if
  - *h(n)* never overestimates the optimal cost
  - That is *h(n)* is always optimistic
  - E.g., straight line distance between two cities
- **Consistent** if
  - If $h(n) \leq c(n,n') + h(n')$
  - n' is the successor of n
  - Triangle inequality
- If a heuristic is consistent, it is also admissible (the other way around is not guaranteed)



A* Optimality – Graph Search – Proof

1. If $h(n)$ is consistent, then $f(n)$ along any path is non-decreasing
2. When n is expanded, the optimal path to it has been found

A* VS UNIFORM-COST SEARCH

- Compare and contrast A* and UCS
  - Which one is better and why?
- Given the same heuristic function, can you design a complete and optimal algorithm that expands fewer nodes than A*?

8-Puzzle Heuristics

- h_1 = The number of misplaced tiles
- h_2 = Manhattan distance

Let's see an example

5	6	1
2	4	8
3	7	

Effective Branching Factor

- If the total number of nodes generated is N and the solution depth is d, then
  - b^* is the branching factor that a uniform tree of depth d would need to have in order to contain N+1 nodes
- $N+1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$
- If A* finds a solution at depth 4 using 40 nodes, what is b*?
  - ≈ 2.182
- A good heuristic function achieves $b^* \approx 1$

$1 + b^* + b^{*2} \dots - (b^*)^{28}$

BFS vs A*

	Search Cost (nodes generated)			Effective Branching Factor		
d	BFS	A*(h_1)	A*(h_2)	BFS	A*(h_1)	A*(h_2)
6	128	24	19	2.01	1.42	1.34
8	368	48	31	1.91	1.40	1.30
10	1033	116	48	1.85	1.43	1.27
12	2672	279	84	1.80	1.45	1.28
14	6783	678	174	1.77	1.47	1.31
16	17270	1683	364	1.74	1.48	1.32
18	41558	4102	751	1.72	1.49	1.34
20	91493	9905	1318	1.69	1.50	1.34
22	175921	22955	2548	1.66	1.50	1.34
24	290082	53039	5733	1.62	1.50	1.36
26	395355	110372	10080	1.58	1.50	1.35
28	463234	202565	22055	1.53	1.49	1.36

Figure 3.26 Comparison of the search costs and effective branching factors for 8-puzzle problems using breadth-first search, A* with h_1 (misplaced tiles), and A* with h_2 (Manhattan distance). Data are averaged over 100 puzzles for each solution length d from 6 to 28.

Heuristic Functions

- Let $h(n)$=0 for all n
  - Is it admissible?
  - Is it consistent?
  - Is it any good?
- Can we say an admissible heuristic function h_i is *always* better than another admissible heuristic function h_j?
- How can we find good heuristics?
- A heuristic: run uniform-cost search, find the solution, return the cost of the solution as the heuristic value. Can you beat it?
- Let $h(n) = \max(h_1(n), h_2(n), \ldots, h_m(n))$, where $h_i(n)$, are admissible
  - Is $h(n)$ admissible?
  - Can we say $h(n)$ is better than any of $h_i(n)$?
  - What's the catch?

Summary

- Depth-first search
- Breadth-first search
- Depth-limited depth-first search
- Iterative deepening
- Uniform cost search: $g(n)$
- Greedy best-first search: $h(n)$
- A* search: $g(n) + h(n)$

Which one is better under what conditions?

OTHERS

- Bidirectional search
  - Expands two frontiers: one around the start state and one around the goal state
- Beam search
  - Puts a limit on the size of the frontier
- Modified versions of A*
  - Bidirectional A*
  - Iterative deepening A*
  - Weighted A*: $f = w*h+g$
    - Which algorithms do we get if w=0, w=1, w=inf?
    - Is the algorithm still optimal if w>1?

Historical notes from the book

- BFS – 1959
- Dijkstra's algorithm – 1959
  - Modified to be UCS – 1971
- Iterative deepening used for timed chess – 1977
- Heuristic search – 1965
- A* – 1968
  - Variations are still under research
    - Bidirectional A* applied to maps and landmarks in 2006
      - 24-million point on the graph
      - An optimal path between any two points searches only 0.1% of the graph
- Search is still an active area of research
  - https://aaai.org/Conferences/AAAI/aaai.php

NEXT

- Skip Chapter 4
- Chapter 5: Adversarial search and games