Maths-1

Week-1

- Natural numbers (N),
  - N = {0, 1, 2, 3, - - - - }
- Integers (Z),
  - Z = {- - -, -2, -1, 0, 1, 2, 3, - - -}
  - Integers are discrete
- a divides b if b mod a = 0
  - a|b => 4|20, 7|63
  - **a** is a factor of **b** if a|b
- Rational numbers (Q),
  - p/q are rationals, p & q are integers
  - Reduced form of p/q is if gcd(p,q) = 1
  - Rationals are dense
- Irrational numbers
  - Square root of any number is irrational
  - $\sqrt{2}$, $\sqrt{3}$, - - - -
  - π, e are also irrational
- Real numbers (R),
  - All rational and irrational numbers are Real
  - Real numbers are also dense
- Complex numbers
  - Square root of any negative number is complex
  - $\sqrt{-1}$, $\sqrt{-2}$ - - - - -
- Cardinality
  - No. of items in a set
  - Items in a set are called elements
- Subset
  - $X \subseteq Y$

- Proper subset
  - $X \subseteq Y$ but $X \neq Y$, hence $X \subset Y$ or $X \subsetneq Y$
- Empty set (ϕ),
  - $\phi = \{\}$
- Power set
  - A set of subsets of a set
  - X = {a, b}
  - Power set of X = {ϕ, {a}, {b}, {a, b}}
  - Set with n elements has 2^n subsets
- Cartesian product ($A \times B$)
  - $A = \{0, 1\}$, $B = \{2, 3\}$
  - $A \times B = \{(0, 2), (0, 3), (1, 2), (1, 3)\}$
  - Order is important
- Identity relation
  - (a, b) if a = b
- Reflexive relation,
  - (a, b) if and only if $a \leq b \text{ and } a, b > 0$
- Symmetric relation
  - $(a, b) \in R$ if and only if $(b, a) \in R$
- Transitive relation
  - If $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$
- Antisymmetric relation
  - If $(a, b) \in R$ and $a \neq b$, then $(b, a) \notin R$
- Equivalence relation
  - Combination of Reflexive, Symmetric and Transitive
- Types of functions
  - Injective (one-to-one)
    - If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$
  - Surjective (onto)
    - Range is the codomain

- ➔ Bijective
  - ➔ If a function is both injective and surjective, then it is bijective

Week-2

- ➔ Distance between two points (x_1, y_1) & (x_2, y_2) is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- ➔ Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$$

- ➔ Area of triangle using coordinates,

$$Area = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

- ➔ Slope of a line,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = tan\theta$$

  - ➔ Slope of lines parallel to x-axis is always '0' as $tan(0) = 0$
  - ➔ Inclination of vertical line is 90^0, slope is undefined

$$m = tan(180 - \theta) = -tan\theta = -\left[\frac{y_2 - y_1}{x_2 - x_1}\right] = \left[\frac{y_1 - y_2}{x_2 - x_1}\right]$$

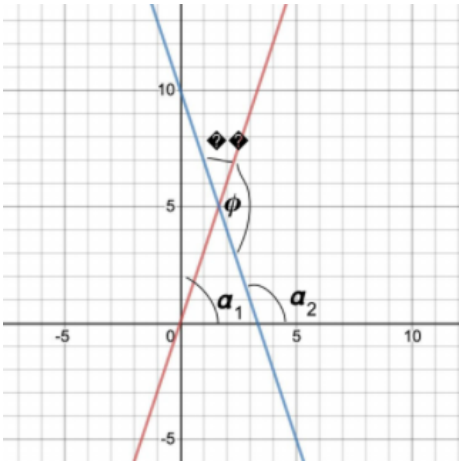
- ➔ For parallel lines, $m_1 = m_2$
- ➔ For perpendicular lines, $m_1 \times m_2 = -1$
- ➔ For two intersecting lines,

$$\theta = \alpha_2 - \alpha_1$$

$$tan\theta = tan(\alpha_2 - \alpha_1)$$

$$tan\theta = \frac{tan\alpha_2 - tan\alpha_1}{1 + tan\alpha_1 . tan\alpha_2} = \frac{m_2 - m_1}{1 + m_1 . m_2}$$

$$tan\phi = tan(180 - \theta) = -tan\theta = \frac{m_1 - m_2}{1 + m_1 . m_2}$$



- ➔ Horizontal line (parallel to x-axis)

$$y = a$$

➔ Vertical line (parallel to y-axis)

$$x = a$$

➔ Point-slope form,

$$y - y_1 = m(x - x_1)$$

➔ Two-point form,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

➔ Slope-intercept form,

$y = mx + c$ for $(0, c)$ (c = y-intercept)

$y = m(x - d)$ for $(d, 0)$ (d = x-intercept)

➔ Intercept form,

$$\frac{x}{a} + \frac{y}{b} = 1$$

➔ For general form, $Ax + By + C = 0$

$$slope = \frac{-A}{B},\ x - intercept = \frac{-C}{A},\ y - intercept = \frac{-C}{B}$$

Week-3

➔ If two lines are parallel, $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

$$m_1 = \frac{-a_1}{b_1},\ m_2 = \frac{-a_2}{b_2}$$

$$m_1 = m_2 \Rightarrow a_1b_2 = a_2b_1$$

➔ If two lines are perpendicular, $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

$$m_1.m_2 = -1$$

$$\frac{-a_1}{b_1} \cdot \frac{-a_2}{b_2} = -1$$

$$a_1a_2 + b_1b_2 = 0$$

➔ Distance of a point from line,

Point (x_1, y_1), line $Ax + By + C = 0$

$$Distance = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

➔ Distance between two parallel lines,

$$d = \frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

➔ Sum of Squared Errors (SSE),

$$SSE = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

Week-4

➔