Week - 4
Practice Assignment Solution
Algebra of polynomials
Mathematics for Data Science - 1

1 Multiple Choice Questions (MCQ):

(Use the following data for the Question 1 and Question 2 only).

Let x be the number of years since the year 2000 (i.e., $x = 0$ denotes the year 2000). The total amount of profit (in ₹) on books in a shop is given by the function $T(x) = 5x^3 + 3x + 1$. The shop sells books of four languages English, Bengali, Hindi, and Tamil. The profits from selling English and Bengali books are given by $E(x) = 3x^3 - 5x^2 + x$ and $B(x) = x^2 + 4x + 5$ respectively. The profit from selling Hindi and Tamil books are found to be the same.

1. Which of the following polynomial functions represents the profit from selling Tamil books?
   - ○ $2x^3 + 4x^2 - 2x - 4$
   - ○ $x^3 - 2x^2 - x + 2$
   - ○ $\boldsymbol{x^3 + 2x^2 - x - 2}$
   - ○ $2x^3 - 4x^2 - 2x + 4$

2. In which year was the profit from Hindi books zero?
   - ○ **2001**
   - ○ 2002
   - ○ 2004
   - ○ 2010

   Solution:

   (a) The total profit from selling English and Bengali books is= $E(x) + B(x) = (3x^3 - 5x^2 + x) + (x^2 + 4x + 5) = 3x^3 - 4x^2 + 5x + 5$. Hence the total profit from selling Hindi and Tamil books is= $T(x) - (3x^3 - 4x^2 + 5x + 5) = 5x^3 + 3x + 1 - 3x^3 + 4x^2 - 5x - 5 = 2x^3 + 4x^2 - 2x - 4$.
   As the profit from selling Hindi and Tamil books are found to be the same, the profit from selling Tamil books is= $\frac{1}{2}(2x^3 + 4x^2 - 2x - 4) = x^3 + 2x^2 - x - 2$

   (b) Profit from selling Hindi books (which is same as the profit from selling Tamil books) is $x^3 + 2x^2 - x - 2$.

   $$x^3 + 2x^2 - x - 2 = x^2(x + 2) - 1(x + 2) = (x + 2)(x^2 - 1) = (x + 2)(x + 1)(x - 1)$$

So the profit will be zero if $(x+2)(x+1)(x-1) = 0$, i.e., at $x = -2, -1, 1$ the profit can be 0. But in this context, x cannot be negative. So $x = 1$ is the only possibility. Hence in the year 2001 the profit from Hindi books was zero.

3. Find the quadratic polynomial which when divided by x, $x-1$, and $x+1$ gives the remainders 7, 14, and 8 respectively.

   - ◯ $4x^2 - 3x + 7$
   - ◯ $x^2 + 7x + 7$
   - ◯ $7x^2 + x + 7$
   - ◯ $\mathbf{4x^2 + 3x + 7}$

   Solution: Let the quadratic polynomial which is satisfying the given condition be $p(x) = ax^2 + bx + c$.
   When it is divided by x the remainder is 7. It implies that if we substitute $x = 0$ in $p(x)$ we will get 7, i.e., $p(0) = 7$. Similarly we have $p(1) = 14$ and $p(-1) = 8$.
   Hence we have the following equations:

$$\begin{aligned} p(0) &= a(0)^2 + b(0) + c \\ &= c \\ &= 7 \\ p(1) &= a.(1)^2 + b.1 + c \\ &= a + b + c \\ &= 14 \\ p(-1) &= a(-1)^2 + b(-1) + c \\ &= a - b + c \\ &= 8 \end{aligned}$$

   So, we have $c = 7$, and substituting c in the second and third equation we get, $a+b = 7$, and $a - b = 1$. By solving these two equations we get $a = 4$ and $b = 3$.
   Hence the quadratic polynomial is $4x^2 + 3x + 7$.

4. Box A has length x unit, breadth $(x+1)$ unit, and height $(x+2)$ unit. Box B has length $(x+1)$ unit, breadth $(x+1)$ unit, and height $(x+2)$ unit. There are two more boxes C and D of cubic shape with side x unit. The total volume of A and B is y cubic unit more than the total volume of C and D. Find y in terms of x.

   - ◯ $x^3 + 7x^2 + 7x + 2$
   - ◯ $\mathbf{7x^2 + 7x + 2}$
   - ◯ $7x^2 - 7x - 2$
   - ◯ $x^3 + 7x^2 - 7x - 2$

   Solution: The volume of box A is $x(x+1)(x+2) = x^3 + 3x^2 + 2x$ cubic unit.
   The volume of box B is $(x+1)(x+1)(x+2) = (x^2+2x+1)(x+2) = x^3 + 4x^2 + 5x + 2$ cubic unit.
   The volume of box C and D is x^3 cubic unit each. So the total volume of A and B is $2x^3 + 7x^2 + 7x + 2$ and the total volume of C and D is $2x^3$.
   Hence $y = (2x^3 + 7x^2 + 7x + 2) - 2x^3 = 7x^2 + 7x + 2$.

5. The population of a bacteria culture in laboratory conditions is known to be a function of time of the form $p(t) = at^5 + bt^2 + c$, where p represents the population (in lakhs) and t represents the time (in minutes). Suppose a student conducts an experiment to determine the coefficients a, b, and c in the formula and obtains the following data:

   - $p(0) = 3$
   - $p(1) = 5$
   - $p(2) = 39$

   Which of the following options is correct?

   - ○ $p(t) = 3t^5 - t^2 + 3$
   - ○ $p(t) = 4t^5 - 2t^2 + 3$
   - ○ $\boldsymbol{p(t) = t^5 + t^2 + 3}$
   - ○ $p(t) = 39t^5 + 5t^2 + 3$

   Solution: Given that, $p(t) = at^5 + bt^2 + c$.
   $p(0) = c = 3$
   $p(1) = a + b + c = 5$, putting $c = 3$, we get $a + b = 2$.
   $p(2) = a(2)^5 + b(2)^2 + c = 32a + 4b + c = 39$, substituting $c = 3$, we get $32a + 4b = 36$, implies, $8a + b = 9$ (cancelling 4 from both sides)
   By solving these two equations we get $a = 1$, and $b = 1$.
   Hence, $p(t) = t^5 + t^2 + 3$.

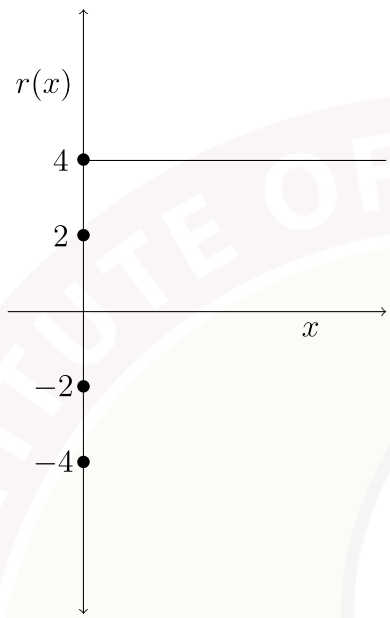
6. If the polynomials $x^3 + ax^2 + 5x + 7$ and $x^3 + 2x^2 + 3x + 2a$ leave the same remainder when divided by $(x - 2)$, then the value of a is:

   - ○ $\frac{3}{2}$
   - ○ $\mathbf{-\frac{3}{2}}$
   - ○ $\frac{5}{2}$
   - ○ $-\frac{5}{2}$

   Solution: Given that both the polynomials leave same remainder when divided by $(x - 2)$. By substituting $x = 2$ both the polynomial should have same value.
   By substituting $x = 2$ in $x^3 + ax^2 + 5x + 7$, we get $8 + 4a + 10 + 7 = 4a + 25$.
   By substituting $x = 2$ in $x^3 + 2x^2 + 3x + 2a$, we get $8 + 8 + 6 + 2a = 2a + 22$.
   So we have,

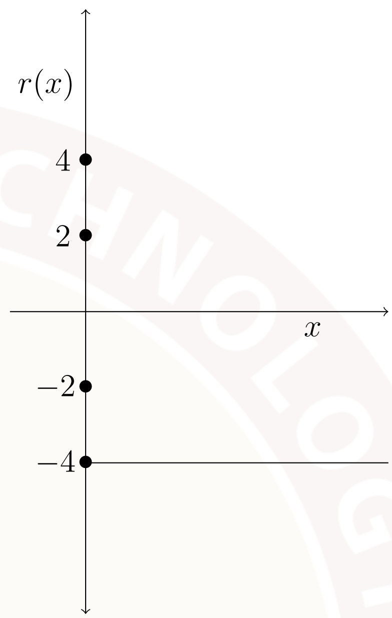
$$\begin{aligned} 4a + 25 &= 2a + 22 \\ 2a &= -3 \\ a &= -\frac{3}{2} \end{aligned}$$

7. Let $r(x)$ be a polynomial function which is obtained as the remainder after dividing the polynomial $2x^3 + x^2 - 5$ by the polynomial $2x - 3$. Choose the correct option which represents the polynomial $r(x)$ most appropriately.



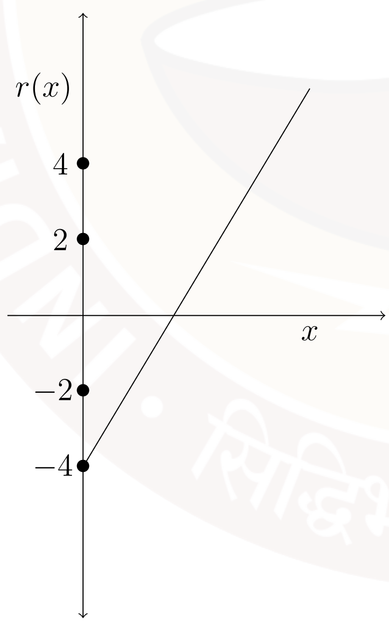
◯ **Option A**

Fig P-6.2



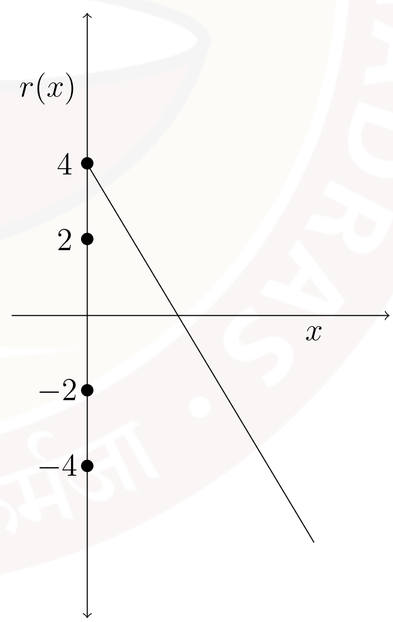
◯ Option B

Fig P-6.3



◯ Option C

Fig P-6.4



◯ Option D

Fig P-6.5

Solution We get 4 as the remainder if $2x^3 + x^2 - 5$ is divided by the polynomial $2x - 3$.

$$2x^3 + x^2 - 5 = (2x - 3)(x^2 + 2x + 3) + 4$$

Hence $r(x) = 4$, which is a constant polynomial. Hence, the first option is the correct.

2 Multiple Select Questions (MSQ):

(Use the following data for the Question 8 and Question 9 only).
By dividing a polynomial $p(x)$ with another polynomial $q(x)$ we get $h(x)$ as the quotient and $r(x)$ as the remainder.

8. The maximum degree of $r(x)$ can be,

   ○ $deg\ p(x)$

   ○ $deg\ (p(x)) - 1$

   ○ $deg\ q(x)$

   ○ $\boldsymbol{deg\ (q(x)) - 1}$

9. If $deg\ p(x) < deg\ q(x)$, then choose the set of correct answers:

   ○ $\boldsymbol{h(x) = 0}$

   ○ $deg\ h(x) = deg\ q(x)$

   ○ $deg\ r(x) = deg\ q(x)$

   ○ $\boldsymbol{deg\ r(x) = deg\ p(x)}$

Solution:

(a) The degree of the remainder $r(x)$ should be strictly less than the degree of the polynomial $q(x)$. So the maximum degree of $r(x)$ is $deg\ (q(x)) - 1$.

(b) If $deg\ p(x) < deg\ q(x)$, then quotient will be zero polynomial, hence $deg\ h(x) = 0$. The remainder will be $p(x)$ itself. So $deg\ r(x) = deg\ p(x)$.

3 Numerical Answer Type (NAT):

(Use the following data for the Question 10 and Question 11 only).
An open box can be made from a piece of cardboard of length $7x$ unit and breadth $5x$ unit, by cutting squares of side x unit out of the corners of the rectangular cardboard, then folding up the sides as shown in the Figure P-6.1.

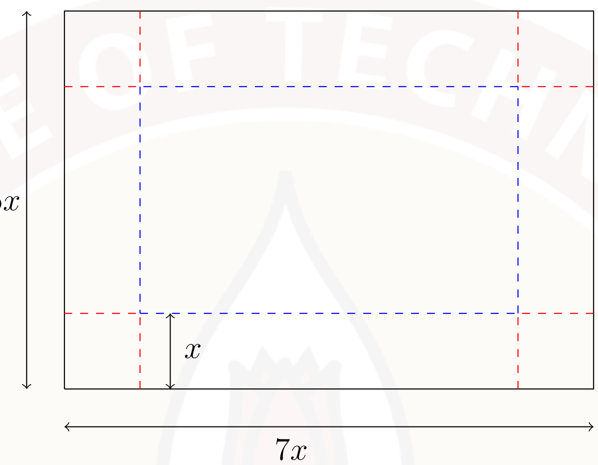


Figure P-6.1

10. What will be the coefficient of x^3 in the polynomial representing the volume of the box? [Answer:15]

11. What will be the coefficient of x^2 in the polynomial representing the volume of the box? [Answer:0]

    Solution: As the sides of the piece of the cardboard has been cut out, the length of the box made will be $7x - (x + x) = 5x$ unit and the breadth of the box made will be $5x - (x + x) = 3x$ unit, and the height will be x unit.
    Hence the volume of the box will be $5x \times 3x \times x = 15x^3$ cubic unit.

    (a) The coefficient of x^3 in the polynomial representing the volume of the box is 15.
    (b) The coefficient of x^2 in the polynomial representing the volume of the box is 0.

12. What should be subtracted from the polynomial $P(x) = 6x^4 + 5x^3 + 4x - 4$ to make it divisible by $2x^2 + x - 1$?

    ◯ $4x$
    ◯ $\mathbf{4x - 3}$
    ◯ $6x - 3$
    ◯ $2x - 3$

Solution:

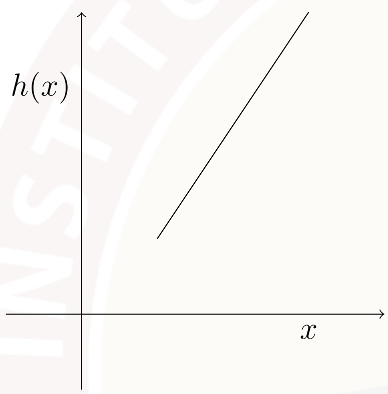
Using 4 step division algorithm, we find the remainder when $P(x)$ is divided by $2x^2+x-1$. If we subtract the obtained remainder from $P(x)$ then the resultant polynomial will be divisible by $2x^2 + x - 1$.
Now,

$6\text{x}^4 + 5x^3 + 0x^2 + 4x - 42x^2 + x - 1$ Therefore, when $P(x)$ is divided by $2x^2 + x - 1$, we get $4x - 3$ as the remainder.
Hence, $\boldsymbol{4x - 3}$ should be subtracted from the polynomial $P(x) = 6x^4 + 5x^3 + 4x - 4$ to make it divisible by $2x^2 + x - 1$.

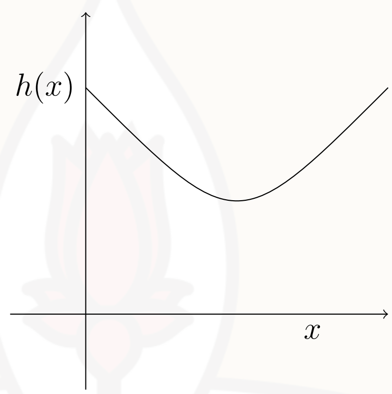
13. Table 1 provides the information regarding some polynomials. Which is the most suitable (not exact) representation of $h(x)$ where $h(x)$ is known to be a polynomial in x, and if $h(x) = \frac{P(x)Q(x) - R(x)S(x) + S(x)P(x)}{P(x) + P(x)Q^2(x)}$? [Ans: option D]

Polynomial	Degree	Condition
$P(x)$	m	$m > 0$
$Q(x)$	n	$m > 2n > 0$
$R(x)$	k	$k = m - n$
$S(x)$	t	$t = 2n$

Table 1



A.



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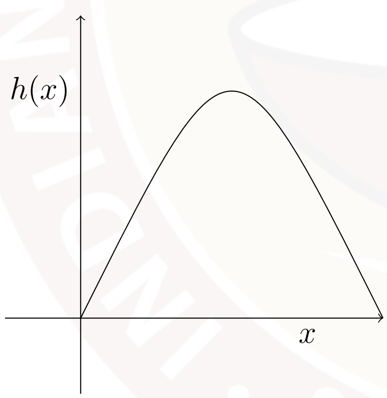

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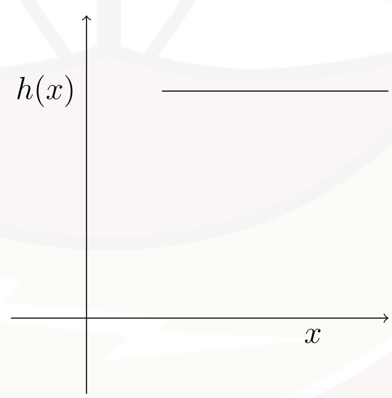


D.

Solution:

Given, the degree of $P(x)$ is 'm' where $m > 0$, the degree of $Q(x)$ is 'n' where $m > 2n > 0$, the degree of $R(x)$ is 'k' where $k = m - n$, and the degree of $S(x)$ is 't' where $t = 2n$.

Also, $h(x) = \frac{P(x)Q(x) - R(x)S(x) + S(x)P(x)}{P(x) + P(x)Q^2(x)}$ and $h(x)$ is known to be a polynomial. The degree of $h(x)$ will be the difference between the degree of the numerator and the degree of the denominator. The degree of the numerator will be the degree of the term which has the highest degree in the numerator. Similarly, the degree of the denominator will be the degree of the term which has the highest degree in the denominator.

Now, the degree of the polynomial $P(x)Q(x)$ will be '$m + n$', the degree of $R(x)S(x)$ will be '$k+t = m-n+2n = m+n$', and the degree of $S(x)P(x)$ will be '$t+m = 2n+m$'.

Therefore, the degree of the numerator (polynomial $P(x)Q(x) - R(x)S(x) + S(x)P(x)$) will be '$m + 2n$'.
Similarly, the degree of the denominator (polynomial $P(x) + P(x)Q^2(x)$) will be $m + 2n$.

As $h(x)$ is given to be a polynomial and also the degrees of the polynomials in the numerator and the denominator are same, we can conclude that the degree of $h(x)$ is zero i.e. $h(x)$ should be a constant.
So, option D is the most suitable representation of $h(x)$.

Use the following information to solve questions 14-16.

A manufacturing company produces three type of products A, B, and C from one raw material in a single continuous process. This process generates total solid wastes (W) (in kg) as $W(r) = -0.0001r^3 + 0.1r^2 + r$, where r is the amount of raw material used in kg. If instead, the company uses three different batch-processes (one batch process for one product) to produce the above products, then the amount of waste generated because of products A, B, and C are given as $W_A = -0.00001r^4 + 0.015r^3$, $W_B = -0.005r^3 + 0.05r^2$ and $W_C = 0.05r^2$ respectively.

(See the Figure A-5.1 for the reference.)

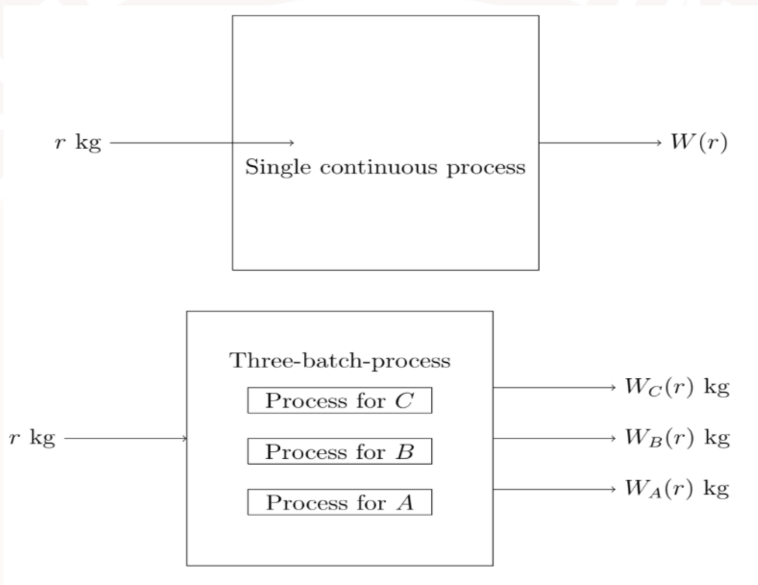


Figure A-5.1

14. What is the total amount of waste generated because of the three different batch-processes?

   - ◯ $-0.00001r^4 + 0.01r^3 + 1.5r^2$
   - ◯ $-0.00001r^4 + 0.015r^3 + 1.5r^2$
   - ◯ $\mathbf{-0.00001r^4 + 0.01r^3 + 0.1r^2}$
   - ◯ $-0.00001r^4 + 0.01r^3 + 0.5r^2$
   - ◯ $-0.00001r^4 + r^3 + 1.5r^2$
   - ◯ $0.0001r^4 + 0.01r^3 + 1.5r^2$

Solution:

The total amount of waste generated because of the three different batch-processes is

$$
\begin{aligned}
W_A + W_B + W_C &= -0.00001r^4 + 0.015r^3 - 0.005r^3 + 0.05r^2 + 0.05r^2 \\
&= \boldsymbol{-0.00001r^4 + 0.01r^3 + 0.1r^2}
\end{aligned}
$$

15. What is the ratio of the total waste generated by the three-batch-processes with respect to the single continuous process?

- ◯ $-0.001r^2$
- ◯ $-0.001r$
- ◯ $-0.01r$
- ◯ $-0.1r$
- ◯ $\mathbf{0.1}\boldsymbol{r}$
- ◯ $0.01r$

Solution:

The total waste generated by the three-batch-processes is
$W_A + W_B + W_C = -0.00001r^4 + 0.01r^3 + 0.1r^2$.
The waste generated in the single continuous process is $W(r) = -0.0001r^3 + 0.1r^2 + r$.
The ratio of the total waste generated by the three-batch-processes with respect to the single continuous process is

$$\begin{aligned}\frac{W_A + W_B + W_C}{W(r)} &= \frac{-0.00001r^4 + 0.01r^3 + 0.1r^2}{-0.0001r^3 + 0.1r^2 + r} \\ &= \frac{(0.1r)(-0.0001r^3 + 0.1r^2 + r)}{-0.0001r^3 + 0.1r^2 + r} \\ &= \mathbf{0.1}\boldsymbol{r}\end{aligned}$$

16. Let the company wastes Rs. 5,000 in waste treatment when it uses the single continuous process by consuming 100 kg of raw material. If instead of continuous process the company uses the three-batch-processes, then how much extra amount (in Rs.) will the company have to pay for waste treatment with respect to the continuous process?

- ◯ 50,000
- ◯ 500
- ◯ **45,000**
- ◯ 5,000
- ◯ 4,000

Solution:

As the ratio for waste generation (continuous to batch) is 10 we can calculate cost for waste management from batch process will be ten times of the continuous process.
Therefore the cost for waste management from the batch process will be $5,000 \times 10 = 50,000$.
So the the extra amount required is $50,000 - 5,000 = 45,000$.

4 Multiple Select Questions (MSQ):

17. Given $P(x)$ and $Q(x)$ be two non zero polynomials of degrees m and n respectively. If $f(x) = P(x) + Q(x)$, $g(x) = P(x)Q(x)$, and $h(x) = P(x)\{P(x)Q(x) + \frac{P(x)}{Q(x)}\}$, If $h(x)$ is known to be a polynomial in x,then choose the set of correct options.

   - ○ The degree of $f(x)$ is $m + n$.
   - ○ **The degree of $\boldsymbol{g(x)}$ is $\boldsymbol{m + n}$.**
   - ○ **The degree of $\boldsymbol{f(x)}$ is $\boldsymbol{max\{m, n\}}$ if $\boldsymbol{m \neq n}$, where $\boldsymbol{max\{m, n\}}$ represents the maximum value from $\boldsymbol{m}$ and $\boldsymbol{n}$.**
   - ○ The degree of $h(x)$ is m^3.
   - ○ The degree of $h(x)$ is n^3.
   - ○ **The degree of $\boldsymbol{h(x)}$ is $\boldsymbol{2m + n}$.**

Solution:

Given, $P(x)$ and $Q(x)$ are two non zero polynomials of degree m and n respectively.
Also, $f(x) = P(x) + Q(x)$.
If $m > n$, then the degree of the polynomial $f(x)$ will be m, else if $m < n$, then the degree of the polynomial $f(x)$ will be n, else if $m = n$, then the degree of the polynomial will be less than or equal to m(or n).
Therefore, we can conclude that the degree of the polynomial $f(x)$ is $max\{m, n\}$ if $m \neq n$, where $max\{m, n\}$ represents the maximum value from m and n.
Hence, option 1 is incorrect, and option 3 is correct.

Now, $g(x) = P(x)Q(x)$, the degree of the polynomial $g(x)$ will be the sum of the degrees of the polynomials $P(x)$ and $Q(x)$.
Therefore, the degree of $g(x)$ is $m + n$. Hence, option 2 is correct.

Finally, $h(x) = P(x)\{P(x)Q(x) + \frac{P(x)}{Q(x)}\} = (P(x))^2Q(x) + \frac{(P(x))^2}{Q(x)}$.
The degree of the polynomial $(P(x))^2Q(x)$ will be $2m + n$ and as given that $h(x)$ is a polynomial implies $Q(x)$ divides $(P(x))^2$, so the degree of the polynomial $\frac{(P(x))^2}{Q(x)}$ will be $2m - n$.
Since $2m + n > 2m - n$, the degree of the polynomial $h(x)$ is $2m + n$. Hence, options 4 and 5 are incorrect, and option 6 is correct.

18. Given a polynomial $P(x) = (2x+5)(1-3x)(x^2+3x+1)$, then choose the set of correct options.

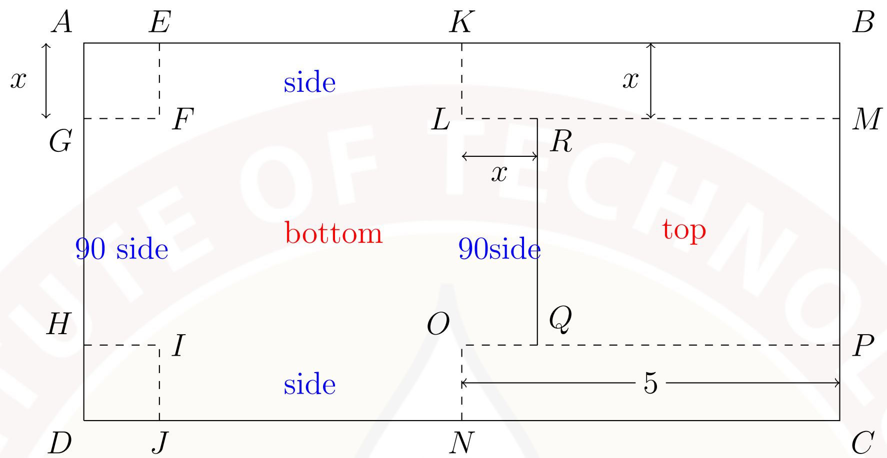
- ◯ **Coefficient of $\boldsymbol{x^5}$ is 0**.
- ◯ Coefficient of x^3 is -18.
- ◯ **Degree of $\boldsymbol{P}$ is 4.**
- ◯ Coefficient of x^3 is -13.
- ◯ Degree of P is 7.
- ◯ All of the above.

Solution:

$$\begin{aligned}
\text{Given, } P(x) &= (2x+5)(1-3x)(x^2+3x+1) \\
&= (2x+5-6x^2-15x)(x^2+3x+1) \\
&= (5-6x^2-13x)(x^2+3x+1) \\
&= 5x^2-6x^4-13x^3+15x-18x^3-39x^2+5-6x^2-13x \\
&= -6x^4-31x^3-40x^2+2x+5
\end{aligned}$$

Option 1 is correct, because there is no x^5 term in the polynomial $P(x)$. So, the coefficient of x^5 is 0.
The degree of the polynomial $P(x)$ is 4. Hence, option 3 is correct and option 5 is incorrect.
The coefficient of x^3 is -31. Hence, options 2 and 4 are incorrect.

19. A sheet $ABCD$ of dimensions 10 ft x 3 ft is shown in Figure 7. A box is made by removing two squares of equal dimensions $AEFG$ and $DHIJ$ and two rectangles of equal dimensions $BKLM$ and $CNOP$ respectively.



- ◯ The volume of the box is $2x^2 - 23x + 30$.
- ◯ **The volume of the box is $\boldsymbol{2x^3 - 13x^2 + 15x}$.**
- ◯ If $x = 0.5$, then the volume of the box is 5.625 cubic ft.
- ◯ To create the box, value of x should always be greater than 0 but less than 1.5.

Solution:
From Figure A-6.1, the length of the box will be $EK = AB - KB - AE = 10 - 5 - x = 5 - x$, the breadth of the box will be $GH = AD - AG - HD = 3 - x - x = 3 - 2x$, and the height of the box will be $AE = x$.
Therefore, the volume of the box V given by length $\times$ breadth $\times$ height will be

$$V = (5 - x)(3 - 2x)(x)$$
$$V = (15 - 3x - 10x + 2x^2)(x)$$
$$V = 2x^3 - 13x^2 + 15x$$

Hence, options 1 is incorrect, and option 2 is correct.

If $x = 0.5$, then the volume of the box

$$V = 2x^3 - 13x^2 + 15x$$
$$V = 2(0.5)^3 - 13(0.5)^2 + 15(0.5)$$
$$V = 2(0.625) - 13(0.25) + 7.5$$
$$V = 1.25 - 3.25 + 7.5 = 5.5$$

Hence, option 3 is incorrect.

Now, given the volume of the cubical bar of soap is $(5 - x)$ cubic ft. and we know the volume of the box is $2x^3 - 13x^2 + 15x$.
So, the maximum bars of soap that can be packed in the box $= \frac{2x^3-13x^2+15x}{5-x}$

2x$^3 - 13x^2 + 15x5 - x$ Therefore, at most $-2x^2 + 3x = \boldsymbol{2x(1.5 - x)}$ bars of soap can be packed in the box. Hence, option 4 is correct.

5 Numerical Answer Type (NAT):

20. A curious student created a performance profile of his favourite cricketer as $R = -x^5 + 6x^4 - 30x^3 + 80x^2 + 70x + c$, where R is the total cumulative runs scored by the cricketer in x matches. He picked three starting values shown in Table 2 and tried to find the value of c. If he uses Sum Squared Error method, then what will be the value of c?[Ans: -2]

No. of matches	Total score
1	120
2	285
3	361

Table 2

Solution:

Let us calculate the predicted cumulative runs scored by the player in the first three matches.
Substituting $x = 1, 2, 3$ in the given function, we get

$$\begin{aligned}
R(1) &= -(1)^5 + 6(1)^4 - 30(1)^3 + 80(1)^2 + 70(1) + c \\
&= -1 + 6 - 30 + 80 + 70 + c \\
&= 125 + c \\
R(2) &= -(2)^5 + 6(2)^4 - 30(2)^3 + 80(2)^2 + 70(2) + c \\
&= -32 + 96 - 240 + 320 + 140 + c \\
&= 284 + c \\
R(3) &= -(3)^5 + 6(3)^4 - 30(3)^3 + 80(3)^2 + 70(3) + c \\
&= -243 + 486 - 810 + 720 + 210 + c \\
&= 363 + c
\end{aligned}$$

Now, let us find the sum squared error of cumulative score for these three matches.

$$\begin{aligned}
\text{SSE} &= \sum_{n=1}^{3}(R(n) - y_n)^2, \text{where } y_n \text{ is the total cumulative score in } n \text{ matches.}\\
&= (R(1) - y_1)^2 + (R(2) - y_2)^2 + (R(3) - y_3)^2\\
&= (125 + c - 120)^2 + (284 + c - 285)^2 + (363 + c - 361)^2\\
&= (5 + c)^2 + (c - 1)^2 + (2 + c)^2\\
&= 25 + 10c + c^2 + c^2 - 2c + 1 + 4 + 4c + c^2\\
&= 3c^2 + 12c + 30
\end{aligned}$$

We have to find the value of c such that SSE becomes minimum, this is equal to the minimum value of the quadratic equation $3c^2 + 12c + 30$.
We know that the minimum value of any quadratic function of form $f(x) = Ax^2+Bx+D$, occurs at $x = \frac{-B}{2A}$. Here, $A = 3, B = 12$
So, the minimum value of the quadratic equation $3c^2 + 12c + 30$, occurs at
$c = \frac{-B}{2A} = \frac{-12}{2(3)} = -2$
Therefore, the minimum SSE is obtained when the value of c is $\mathbf{-2}$.

21. What is the minimum value of $x-$coordinate for the points of intersection of functions $f(x) = -x^5 + 5x^4 - 7x - 2$ and $g(x) = -x^5 + 5x^4 - x^2 - 2$?

Solution:

At the points of intersection, observe that $f(x) = g(x)$.
Here, $f(x) = -x^5 + 5x^4 - 7x - 2$ and $g(x) = -x^5 + 5x^4 - x^2 - 2$.
Equating the functions we get,

$$\begin{aligned}
-x^5 + 5x^4 - 7x - 2 &= -x^5 + 5x^4 - x^2 - 2\\
-7x &= -x^2\\
x^2 - 7x &= 0\\
x(x - 7) &= 0\\
\implies x &= 0 \text{ (or) } x = 7
\end{aligned}$$

Therefore, the minimum value of x- coordinate for the points of intersection of functions $f(x)$ and $g(x)$ is $\mathbf{0}$.