Practice Assignment Solutions

Mathematics for Data Science - 1

NOTE: There are some questions which have functions with discrete valued domains (such as month or year). For simplicity, we treat them as continuous functions.

1 Multiple Choice Questions (MCQ):

1. If R is the set of all points which are 5 units away from the origin and are on the axes then R is:

   ◯ $R = \{(5,5), (-5,5), (-5,-5), (5,-5)\}$

   ◯ $R = \{(5,0),\ (5,-5),\ (5,5),\ (-5,0)\}$

   ◯ $R = \{(5,0),\ (0,5),\ (5,5),\ (0,-5)\}$

   ◯ $\boldsymbol{R = \{(5,0),\ (0,5),\ (-5,0),\ (0,-5)\}}$

   ◯ $R = \{(5,0),\ (0,5),\ (-5,0),\ (-5,5)\}$

   ◯ There is no such set.

   Solution:

   The points on the $x-$axis are represented by $(\pm a,\ 0)$, and on the $y-$axis are represented by $(0,\ \pm b)$, where a and b are the distances of the points $(\pm a,\ 0)$ and $(0,\ \pm b)$, respectively, from the origin. Therefore, the points (5, 0), (0, 5), (-5, 0), (0, -5) lie on the axes and are 5 units away from the origin. See Figure PS-2.1 for reference.

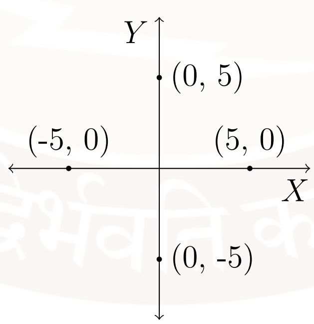


Figure PS-2.1

2. A point P divides the line segment MN such that $MP : PN = 2 : 1$. The coordinates of M and N are $(-2, 2)$ and $(1, -1)$ respectively. What will be the slope of the line passing through P and the point $(1, 1)$?

   - ◯ $\frac{4}{3}$
   - ◯ $\mathbf{1}$
   - ◯ Inadequate information.
   - ◯ $-\frac{4}{3}$
   - ◯ $\tan(\frac{4}{3})$
   - ◯ None of the above.

Solution:

By the sectional formula, the coordinates of a point (x, y) that divides a line segment defined by two points $(x_1, y_1), (x_2, y_2)$ in the ratio $m : n$ is given by

$$x = \frac{m \times x_2 + n \times x_1}{m + n}$$

$$y = \frac{m \times y_2 + n \times y_1}{m + n}$$

Since point P divides the line segment formed by the points $M(-2, 2)$ and $N(1, -1)$ in the ratio 2:1, we obtain the coordinates of point P denoted by, say (x_p, y_p), using the sectional formula as follows.

$$x_p = \frac{2 \times 1 + 1 \times (-2)}{2 + 1} = 0$$

$$y_p = \frac{2 \times (-1) + 1 \times 2}{2 + 1} = 0$$

Hence point $P = (0, 0)$ denotes the origin as shown in Figure PS-2.2

Now, we compute the slope of the line passing through P and (1,1) as,

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{1 - 0} = 1$$

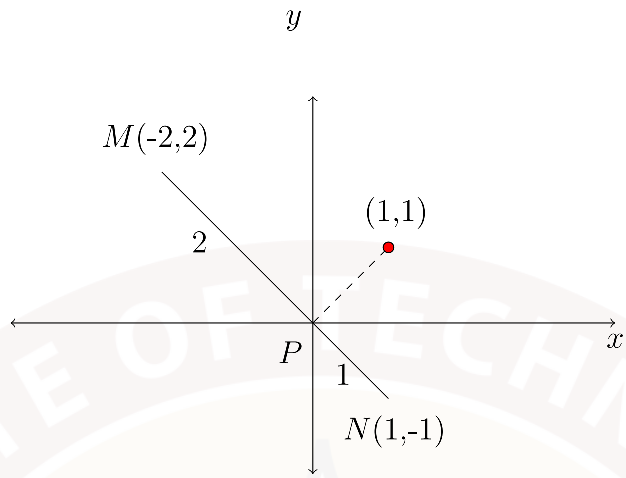


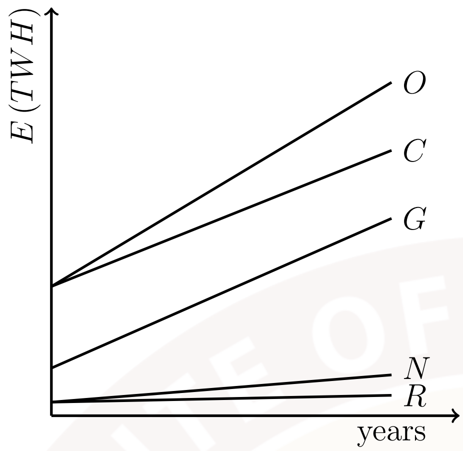
Figure PS-2.2

Use the following information to solve questions 3 and 4.
Table PS-2.1 shows the different types of energies consumed (approximate values) in years 1965 and 2015 across the world

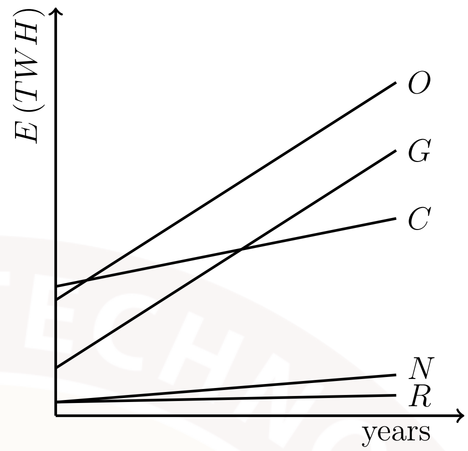
Energy type	Approximate energy used (TWH)	
	1965	2015
Oil (O)	19000	49000
Coal (C)	17000	38000
Gas (G)	7000	29000
Nuclear (N)	2000	6000
Renewable (R)	2000	3000

Table PS-2.1

3. A student assumes a linear relationship between energy consumed (E) and the number of years after 1965. Choose the option which best represents the linear relationships assumed by the student (from 1965 to 2015). [Ans: Option C]



A.



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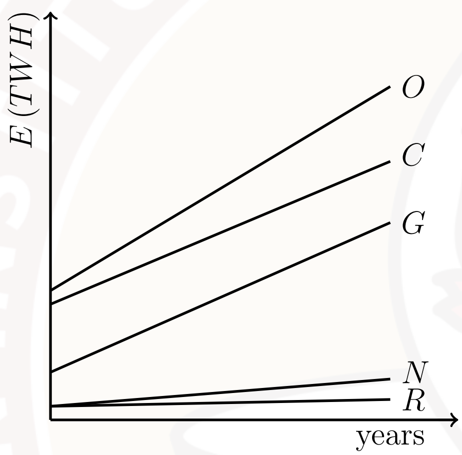
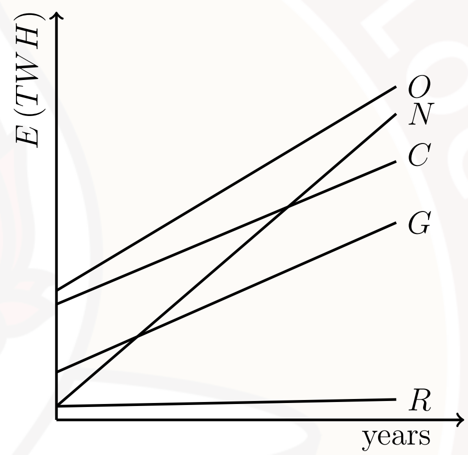


C.



D.

Solution:

Let x-axis and y-axis represent the years and the energy consumption respectively. The energy consumption in 2015 is in the order $O > C > G > N > R$, which is represented correctly in options (A) and (C). However, option (A) shows the energy consumption of O and C being same in the year 1965, which is not true. Hence, option (A) is not correct. Therefore, the correct answer is option (C).

4. The student estimated the energy consumption in 2025 and created Table PS-2.2. Choose the correct option.

Energy type	Approximate energy used (TWH)		
	1965	2015	2025
Oil (O)	19000	49000	o
Coal (C)	17000	38000	c
Gas (G)	7000	29000	g
Nuclear (N)	2000	6000	n
Renewable (R)	2000	3000	r

Table PS-2.2

- ◯ $o = 64000$
- ◯ $c = 48500$
- ◯ $g = 38500$
- ◯ $n = 8000$
- ◯ $r = 3500$
- ◯ **None of the above**.

Solution:

As earlier, let x-axis and y-axis represent the years and the energy consumption respectively. Using the data provided for two years, we can find the equation of the line in two-point form. Equation for the energy type *oil* (O) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 19000 = \frac{49000 - 19000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 19000 = \frac{49000 - 19000}{2015 - 1965}(2025 - 1965)$$

$$y = 55000$$

Equation for the energy type *coal* (C) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 17000 = \frac{38000 - 17000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$:

$$\implies y - 17000 = \frac{38000 - 17000}{2015 - 1965}(2025 - 1965)$$

$$y = 42200$$

Equation for the energy type *gas* (G) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 7000 = \frac{29000 - 7000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 7000 = \frac{29000 - 7000}{2015 - 1965}(2025 - 1965)$$

$$y = 33400$$

Equation for the energy type *nuclear* (N) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 2000 = \frac{6000 - 2000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 2000 = \frac{6000 - 2000}{2015 - 1965}(2025 - 1965)$$

$$y = 6800$$

Equation for the energy type *renewable* (R) will be:

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 2000 = \frac{3000 - 2000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$:

$$\implies y - 2000 = \frac{3000 - 2000}{2015 - 1965}(2025 - 1965)$$

$$y = 3200$$

Thus, none of the options given is correct.

2 Multiple Select Questions (MSQ):

5. The elements of a relation R are shown as points in the graph in Figure P-2.3. Choose the set of correct options:

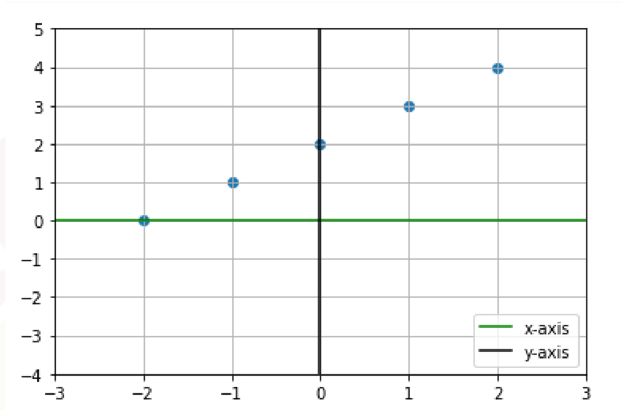


Figure PS-2.3

- ◯ **$\boldsymbol{R}$ can be represented as $\boldsymbol{R = \{(-2,0), (-1,1), (0,2), (1,3), (2,4)\}}$.**
- ◯ We can write R as $R = \{(a,b)|(a,b) \in X \times Y, b = a+2\}$, where X is the set of all values on the $x-axis$, and Y is the set of all values on the $y-axis$.
- ◯ R cannot be a function because it is a finite set.
- ◯ R is a symmetric relation.
- ◯ **$\boldsymbol{R}$ is a function because it has only one output for one input.**
- ◯ If R is a function then it is a bijective function on $X \times Y$, where X is the set of all values on the $x-axis$, and Y is the set of all values on the $y-axis$.
- ◯ **We can write $\boldsymbol{R}$ as $\boldsymbol{R = \{(a,b)|(a,b) \in X \times Y, b = a+2\}}$, where $\boldsymbol{X = \{-2,-1,0,1,2\}}$ and $\boldsymbol{Y = \{0,1,2,3,4\}}$.**

Solution:

- Option (a) is correct since the coordinates of the points in the Figure P-2.3 are as is defined by the function.
- Option (b) is incorrect. We can write R as $\{R = (a,b) \mid (a,b) \ in \ X \times Y, \ b = a+2\}$, where $X = \{-2,-1,0,1,2\}$ and $Y = \{0,1,2,3,4\}$. Here R is a finite set so we can not write for all values of x-axis or y-axis.
- Option (c) is incorrect since R can be a function of a finite set.
- Option (d) is incorrect since R is not a symmetric relation. For example, corresponding to the element (-2, 0), there is no element (0, -2) in R.

- Option (e) is correct since for every value of X there is single corresponding value in Y.
- Option (f) is incorrect since R as a function is not defined for all values on the x-axis, and Y is not the set of all values on the y-axis, whereas $X = \{-2, -1, 0, 1, 2\}$ and $Y = \{0, 1, 2, 3, 4\}$.
- Option (g) is correct, and explained in accordance with definition of function.

6. Find the values of a for which the triangle ΔABC is an isosceles triangle, where A, B, and C have the coordinates $(-1, 1)$, $(1, 3)$, and $(3, a)$ respectively.

- ◯ **If $\boldsymbol{AB = BC}$, then $\boldsymbol{a = 1}$.**
- ◯ If $AB = BC$, then $a = -1$ or -5.
- ◯ **If $\boldsymbol{BC = CA}$, then $\boldsymbol{a = -1}$.**
- ◯ If $BC = CA$, then $a = 1$.

Solution:

As we know, for an isosceles triangle two of its sides are equal. According to the question the vertices of C is $(3, a)$ therefore, depending on the value of a we can have length of $AB = BC$ or $BC = CA$

Since the vertices of triangle are given, we can find the length of each side using distance formula.

Value of a when length of $AB = BC$:

Length of any side of triangle is given by

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\implies \sqrt{(3-1)^2 + (1-(-1))^2} \; = \; \sqrt{(a-3)^2 + (3-1)^2}$$

$$\implies \sqrt{8} \; = \; \sqrt{4 + (a-3)^2}$$

Squaring them on both sides, we have

$$\implies (a-3)^2 \; = \; 4 \implies a - 3 \; = \; \pm 2$$

Therefore,

$$a \; = \; 5, \; 1$$

But, if $a = 5$ then the three points will be co-linear therefore,

$$\boldsymbol{a = 1}$$

Value of a when length of $BC = CA$:

$$\sqrt{(a-3)^2 + (3-1)^2} \; = \; \sqrt{(a-1)^2 + (3-(-1))^2}$$

$$\implies \sqrt{4 + (a-3)^2} \; = \; \sqrt{16 + (a-1)^2}$$

Squaring on both sides of the equation, we get

$$\implies 4 + (a-3)^2 \; = \; 16 + (a-1)^2 \implies (a-3)^2 - (a-1)^2 \; = \; 12$$

$$\implies (2a-4)(-2) \; = \; \implies 12 \implies a = -1$$

Therefore,

$$\boldsymbol{a \; = \; -1}$$

7. A plane begins to land when it is at a height of 1500 metre above the ground. It follows a straight line path and lands at a point which is at a horizontal distance of 2700 metre away. There are two towers which are at horizontal distances of 900 metre and 1800 metre away in the same direction as the landing point. Choose the correct option(s) regarding the plane's trajectory and safe landing.

   - ◯ The trajectory of the path could be $\frac{y}{27} + \frac{x}{15} = 100$ if $x - axis$ and $y - axis$ are horizontal and vertical respectively.
   - ◯ The maximum safe height of the towers are 1000 metre and 1500 metre respectively.
   - ◯ **The trajectory of the path could be $\frac{y}{15} + \frac{x}{27} = 100$ if $x - axis$ and $y - axis$ are horizontal and vertical respectively.**
   - ◯ The maximum safe height of the towers are 1500 metre and 500 metre respectively.
   - ◯ **The maximum safe height of the towers are 1000 metre and 500 metre respectively.**
   - ◯ None of the above.

Solution:

Let us consider the height of plane from ground as $y-$axis and horizontal distance on ground as $x-$axis as shown in Figure PS-2.4

Then, the point P(0,1500) represents the position of the airplane when it began its descent and point Q(2700,0) represents the point where the plane landed.

The two towers which are 900m and 1800m away from the $y-$axis are represented by A and B respectively.

The equation of a straight line path traced by plane from $P(0, 1500)$ to $Q(2700, 0)$ can be obtained using the intercept-form.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\implies \frac{x}{2700} + \frac{y}{1500} = 1$$

On rearranging:

$$\frac{y}{15} + \frac{x}{27} = 100$$

Now, to check the maximum safe height of towers:

For tower A at $X-$coordinate $= 900m$, the maximum safe height will be:

$$\frac{y}{15} + \frac{900}{27} = 100$$

$$\implies y = 1000m$$

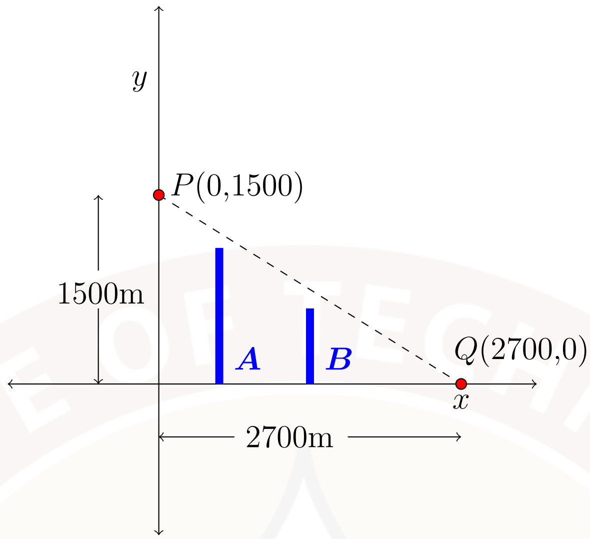


Figure PS-2.4

For tower B at $X-$coordinate $= 1800m$, the maximum safe height will be:

$$\frac{y}{15} + \frac{1800}{27} = 100$$

$$\implies y = 500m$$

3 Numerical Answer Type (NAT):

Use the following information to solve the question 1-2.
The coordinates of points A, B, C and E are shown in the figure PS-2.5 below. Points D and F are the midpoints of lines BC and AD respectively. Using the data given and Figure PS-2.5, answer the questions below.

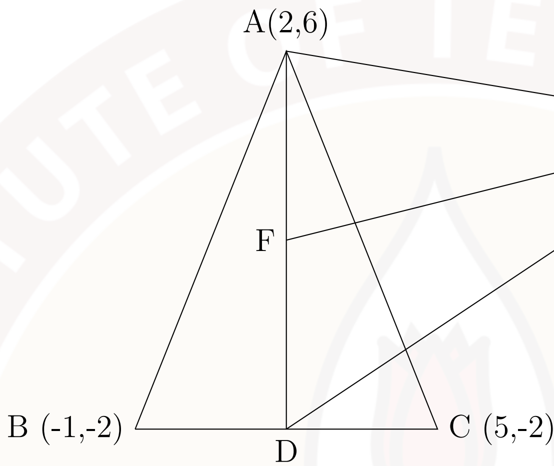


Figure PS-2.5

8. Find the area of triangle ADE. [Ans: 32]

Solution:

By the sectional formula, the coordinates of a point (x, y) that divides a line segment defined by two points $(x_1, y_1), (x_2, y_2)$ in the ratio $m : n$ is given by

$$x = \frac{m \times x_2 + n \times x_1}{m + n}$$

$$y = \frac{m \times y_2 + n \times y_1}{m + n}$$

Since point D is the midpoint of the line segment BC formed by the points $B(-1, -2)$ and $C(5, -2)$ so they are in the ratio 1:1. Thus, we can obtain the coordinates of the point D denoted by, say (x_d, y_d), using the sectional formula as follows.

$$x_d = \frac{1 \times 5 + 1 \times (-1)}{1 + 1} = 2$$

$$y_d = \frac{1 \times (-2) + 1 \times (-2)}{1 + 1} = -2$$

Therefore,

$$\implies D(2, -2)$$

Now, area of triangle ADE with vertices A(2, 6), D(2, -2) and E(10, 5) can be obtained as:

$$= \frac{1}{2}|\ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\ |$$

$$= \frac{1}{2}|\ 2(-2 - 5) + 2(5 - 6) + 10(6 - (-2))\ |$$

$$= 32$$

9. Let the slope of a line F G be 2 and the coordinate of the point G be (a, 9). Then, what is the value of a? [Ans: 5.5]

Solution:

As seen earlier, the point F is the midpoint of the line segment AD formed by the points $A(2, 6)$ and $D(2, -2)$ so they are in the ratio 1:1. Thus we can obtain the coordinates of the point F denoted by, say (x_f, y_f), using the sectional formula as follows.

$$x_f = \frac{1 \times 2 + 1 \times 2}{1 + 1} = 2$$

$$y_f = \frac{1 \times (-2) + 1 \times 6}{1 + 1} = 2$$

Therefore,

$$\implies F(2, 2)$$

Now, the slope of FG will be $= \frac{9\ -\ 2}{a\ -\ 2} = 2$
On solving the above equation, we get $\boldsymbol{a\ =\ 5.5}$

10. Leo rents a motorcycle for 2 days. Hence, the rental company provides the motorcycle at Rs. 500 per day with 100 km free per day. The additional charges after 100 km are Rs. 2 per km Leo drives the motorcycle for a total of 500 km. How much (Rs.) will he have to pay to the rental company? [Ans: 1600]

Solution:

Leo has rented a motorcycle for 2 days, thus he has to pay Rs. 1,000 for free 200 km ride. Thereafter, he has to pay Rs. 2 per km. for rest of 300km, which accounts for Rs. 600. Thus, in total he has to pay Rs. 1,600.

– Multiple Choice Questions (MCQ):

11. A vehicle is travelling on a straight line path and it passes through the points A(4, 2), $B(-1,3)$, and $C(2,\mu)$. The value of μ is:

- ◯ 2
- ◯ **4**
- ◯ -2
- ◯ 10

Solution:

Since the vehicle is travelling on a straight line path and passes through the points A, B, and C, it follows that A, B, and C are collinear. Hence the slope of the straight line path joining A and B will be equal to the slope of the straight line path joining B and C. Using the slope formula for two points, we have

$$\frac{3-2}{-1+4} = \frac{\mu-3}{2+1}$$

$$\implies \boldsymbol{\mu = 4}.$$

Suppose two boats are starting their journey from the ferry ghat A (considered as the origin), one towards ferry ghat B along the straight line $y = -2x$ and the other towards the ferry ghat C along a straight line perpendicular to the path followed by B. The river is 1 km wide uniformly and parallel to the X-axis. Suppose Rahul wants to go to the exact opposite point of A along the river.

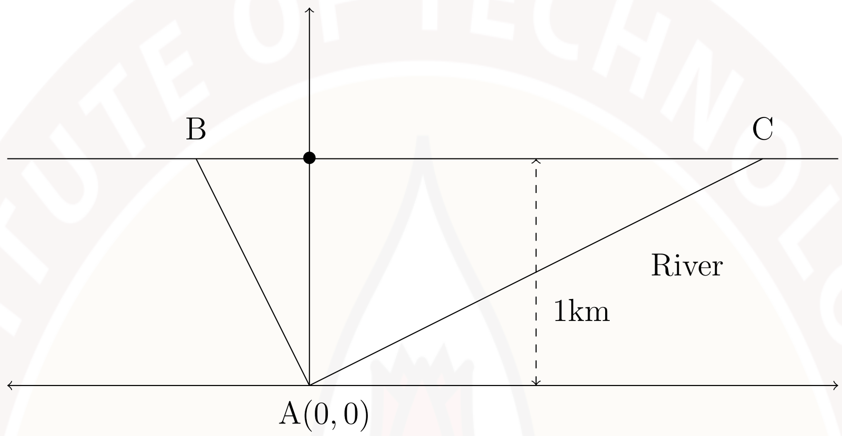


Figure PS-3.1

Then, answer the following questions.

12. How much total distance does Rahul have to travel to reach his destination if he takes the boat towards ferry ghat B?

- ◯ $\sqrt{5}$
- ◯ $\sqrt{5} + 2$
- ◯ $\frac{\sqrt{5}}{2}$
- ◯ $\frac{\sqrt{5}+1}{2}$

Solution:

See the Figure PS-3.2 for reference:

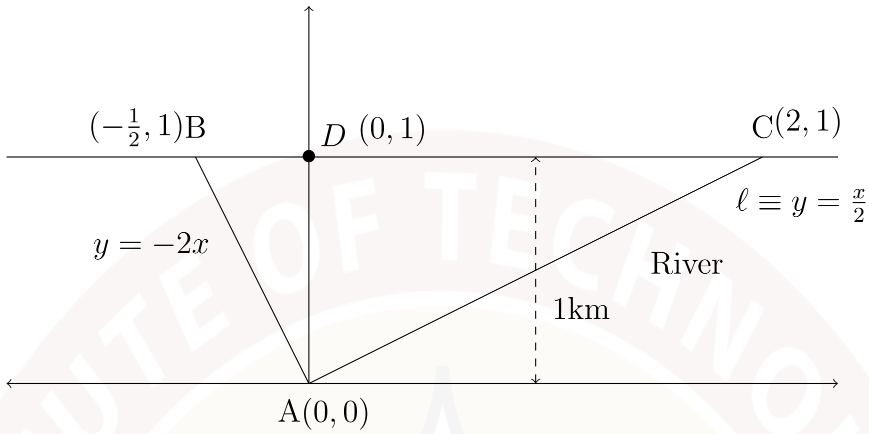


Figure PS-3.2

Since the point A is assumed to be the origin, the side of the river from which Rahul is starting his journey is considered to be the X-axis. The path towards Rahul's destination, which is perpendicular to the X-axis, is hence the Y-axis. Let D be Rahul's destination, which is 1 km away from the point A and is on the opposite side of the river. It follows that the point D is $(0, 1)$.

Hence, the equation of the line representing the opposite side of the river is $y = 1$. Solution of the equations $y = 1$ and $y = -2x$ gives the location of ferry ghat B which is the point $(-\frac{1}{2}, 1)$.

Using the distance formula between two points, the distance between ferry ghat A and ferry ghat B is given by

$$\sqrt{(-\frac{1}{2} - 0)^2 + (1 - 0)^2} = \frac{\sqrt{5}}{2}\text{units}$$

Similarly, the distance between ferry ghat B and the point D is $\frac{1}{2}$ units.
Hence, the total distance that Rahul has to travel to reach his destination D if he takes the boat toward ferry ghat B is given by

$$\frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5}+1}{2} \text{ units}$$

13. How much total distance does Rahul have to travel to reach his destination if he takes the boat towards ferry ghat C?
   - ◯ $\sqrt{5}$
   - ◯ $\boldsymbol{\sqrt{5}+2}$
   - ◯ $\frac{\sqrt{5}}{2}$
   - ◯ $\frac{\sqrt{5}+1}{2}$

Solution: Let ℓ denote the path towards ferry ghat C from A. The equation of path ℓ will be $y = mx$ since it passes through the origin. Since ℓ is perpendicular to the line $y = -2x$, which has a slope $m_1 = -2$, it follows that

$$m = -\frac{1}{m_1} = \frac{1}{2}$$

$\implies$ the equation of ℓ is $y = \frac{x}{2}$.

Solution of the equations $y = \frac{x}{2}$ and $y = 1$ gives the location of ferry ghat C which is (2,1).

Using the distance formula between two points, the distance between ferry ghat A and ferry ghat C is

$$\sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5} \text{ units}$$

Similarly, the distance between ferry ghat C and the destination point D is 2 units.
Hence, the total distance that Rahul has to travel to reach his destination D if he takes the boat towards ferry ghat C is $\sqrt{5} + 2$ units.

14. Suppose a bird is flying along the straight line 4x $-5y=20$ on the plane formed by the path of the flying bird and the line of eye point view of a person who shoots an arrow which passes through the origin and the point $(10,8)$. What is the point on the co-ordinate plane where the arrow hits the bird?

- ◯ $(20,12)$
- ◯ $(25,16)$
- ◯ **The arrow will miss the bird.**
- ◯ Inadequate information.

Solution:
Using the two point form of a line, the equation of the path of arrow passing through the origin and the point (10,8) is

$$(y-0)=\frac{8-0}{10-0}(x-0) \implies 8x-10y=0$$

The slope intercept form of the above line is given by

$$y=\frac{8}{10}x$$

From the above line, we obtain the slope as

$$m_1=\ \frac{8}{10}=\frac{4}{5}$$

Similarly, for the path of the bird along the straight line $4x-5y=20$, we get the slope

$$m_2=\frac{4}{5}$$

Here, $m_1=m_2$,
That is, the lines $8x-10y=0$ and $4x-5y=20$ have the same slope. Therefore, the path of flying bird and the path of the arrow are parallel to each other. Hence, the arrow will miss the bird.

15. We plot the displacement (S) versus time (t) for different velocities as it follows the equation $S = vt$, where v is the velocity. Identify the best possible straight lines in the Figure P-3.2 for the given set of velocities.

Table PS-3.1

v_1	v_2	v_3	v_4
1	-2	0.5	-1

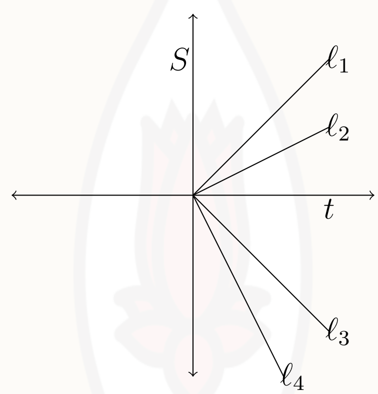


Figure PS-3.3

- ○ $v_1 \rightarrow \ell_1$, $v_2 \rightarrow \ell_2$, $v_3 \rightarrow \ell_3$, and $v_4 \rightarrow \ell_4$.
- ○ $v_1 \rightarrow \ell_1$, $v_2 \rightarrow \ell_4$, $v_3 \rightarrow \ell_3$, and $v_4 \rightarrow \ell_2$.
- ○ **$v_1 \rightarrow \ell_1$,$v_2 \rightarrow \ell_4$, $v_3 \rightarrow \ell_2$, and $v_4 \rightarrow \ell_3$.**
- ○ $v_1 \rightarrow \ell_2$, $v_2 \rightarrow \ell_4$, $v_3 \rightarrow \ell_1$, and $v_4 \rightarrow \ell_3$.

Solution:
From Figure PS-3.3, ℓ_1 and ℓ_2 have positive slope and the slope of ℓ_1 is greater than the slope of ℓ_2. Similarly the slopes of ℓ_3 and ℓ_4 are negative and the slope of line ℓ_3 is greater than the slope of line ℓ_4.
Substituting the value of v in equation $s = vt$, we get the equations

$$s = t, s = -2t, s = 0.5t, s = -t$$

By comparing the above equations of lines and the lines in Figure PS-3.3, we conclude that v_1 corresponds to the line ℓ_1, v_2 corresponds to the line ℓ_4, v_3 corresponds to the line ℓ_2, and v_4 corresponds to the line ℓ_3.

2 Multiple Select Questions (MSQ):

16. A constructor is asked to construct a road which is at a distance of $\sqrt{2}$ km from the municipality office and perpendicular to a road which can be defined by the equation of the straight line $x-y=8$ (considering the municipality office to be the origin). Find out the possible equations of the straight lines to represent the new road to be constructed.

   - ○ $x-y-2=0$
   - ○ $\boldsymbol{x+y+2=0}$
   - ○ $x-y+2=0$
   - ○ $\boldsymbol{x+y-2=0}$

   Solution:

   See the Figure PS-3.4 for reference:

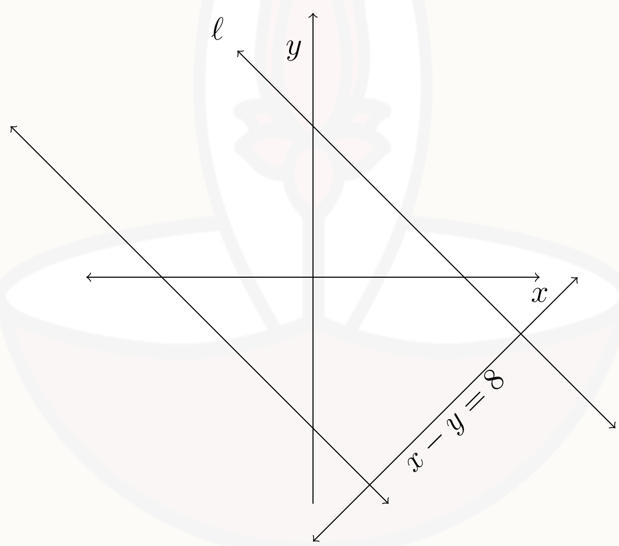


Figure PS-3.4

Let the new road constructed be denoted by ℓ. Given, ℓ is perpendicular to the straight line $x-y=8$. That is, ℓ is perpendicular to the line $y=x-8$ whose slope is $m_1=1$. Therefore, the slope of ℓ is

$$m_2 = -\frac{1}{m_1} = -1$$

By the slope intercept form, the equation of ℓ is

$$y = m_2\ x\ +\ c$$

$$\implies y = -x + c \text{ , where } c \text{ is a constant}$$

That is, ℓ is the line given by

$$x + y - c = 0$$

It is given that the distance of ℓ from the municipality office is $\sqrt{2}$.
The distance formula of a point (x_1, y_1) from a line $(Ax + By + C = 0)$ is given by $\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$. Substituting $x_1 = 0, y_1 = 0, A = 1, B = 1, C = -c$ in formula, we will get the distance of the point(0,0) from the line ℓ,

$$\frac{|1 \times 0 + 1 \times 0 - c|}{\sqrt{1^2 + 1^2}}$$

which is equal to $\sqrt{2}$. So,

$$\frac{|0 + 0 - c|}{\sqrt{1 + 1}} = \sqrt{2}$$

$$\implies \frac{|c|}{\sqrt{2}} = \sqrt{2}$$

$$\implies |c| = 2$$

$\implies c = +2$ or $c = -2$.
Hence, the equation of the new road ℓ is

$$x + y + 2 = 0$$

or

$$x + y - 2 = 0.$$

17. Suppose there are two roads perpendicular to each other which are both at the same distance from Priya's house (considered as the origin). The meeting point of the two roads is on the x-axis and at a distance of 5 units from Priya's house.
Choose the correct possible equations representing the roads.

- ◯ Inadequate information.
- ◯ $y = \frac{1}{2}x + 5, y = -2x - 5$
- ◯ $\boldsymbol{y = -x - 5, y = x + 5}$
- ◯ $y = 2x - 10, y = -2x - 10$
- ◯ $y = 2x - 5, y = -\frac{1}{2}x - 5$
- ◯ $\boldsymbol{y = -x + 5, y = x - 5}$
- ◯ $x = 5, x = -5$

Solution:
Denote the two roads by ℓ_1 and ℓ_2. The meeting point of ℓ_1 and ℓ_2 are on the X-axis and at a distance of 5 units from Priya's house (origin) i.e x-intercepts of the roads are 5 or -5 and passing through the points (5,0) or (-5,0) respectively.

Case 1: when x-intercept is 5 and passes through (5,0)
Using intercept form of a line on the axes, the equation of line ℓ_1 is

$$\frac{x}{5} + \frac{y}{b} = 1$$

where b is a constant.

That is, ℓ_1 is

$$bx + 5y - 5b = 0 \tag{1}$$

See Figure PS-3.5 for reference.

The slope of the road ℓ_1 is $m_1 = -\frac{b}{5}$.
Since the road ℓ_2 is perpendicular to ℓ_1, the slope of road ℓ_2 is

$$m_2 = -\frac{1}{m_1} = \frac{5}{b}$$

Using the slope intercept form, the equation of the road ℓ_2 is

$$y = \frac{5}{b}x + c \Longrightarrow by - 5x - bc = 0 \text{ where } b \text{ and } c \text{ are constant}$$

The roads ℓ_1 and ℓ_2 are at the same distance from Priya's house (origin).
Using distance formula of a line from a point, we get

$$\frac{|-5b|}{\sqrt{b^2+25}} = \frac{|-bc|}{\sqrt{b^2+25}} \Longrightarrow |c| = |5| \Longrightarrow c = 5 \ \ or - 5$$

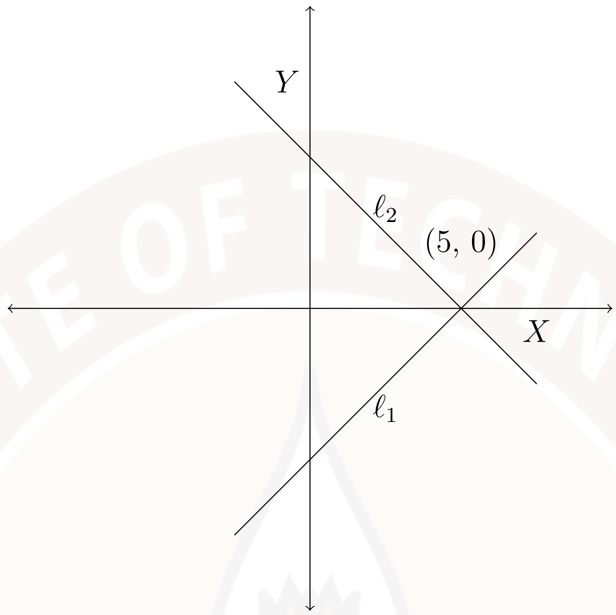


Figure PS-3.5

When $c = 5$, the equation of road ℓ_2 becomes $by - 5x - 5b = 0$. Since ℓ_2 passes through $(5, 0)$, we get $b = -5$.
Therefore, the equation of the road ℓ_2 is $y = -x + 5$.
Substituting $b = -5$ in Equation (1), we will get the equation of the road ℓ_1 as $y = x - 5$.
When $c = -5$, we will get the same equation alternatively.

Case 2: when x-intercept is -5 and passing through (-5,0)
We follow the same process as in Case 1 and we get the equation of the road ℓ_2 as $y = x + 5$ and the equation of the road ℓ_1 as $y = -x - 5$.
See Figure PS-3.6 for reference.

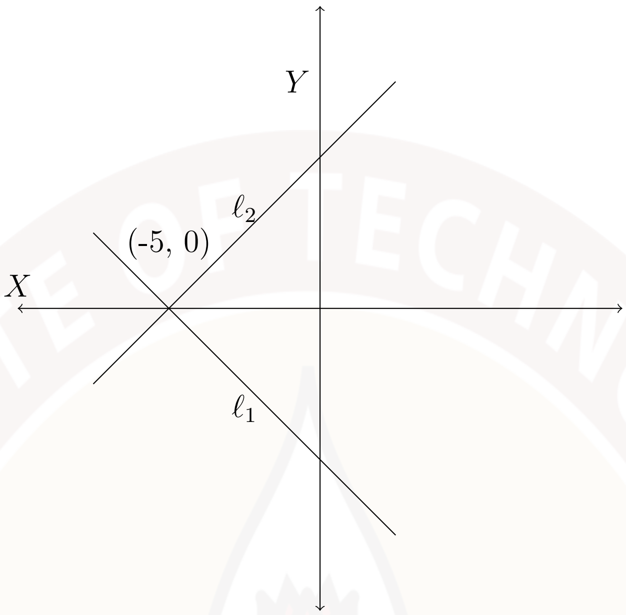


Figure PS-3.6

18. Consider the following two diagrams.

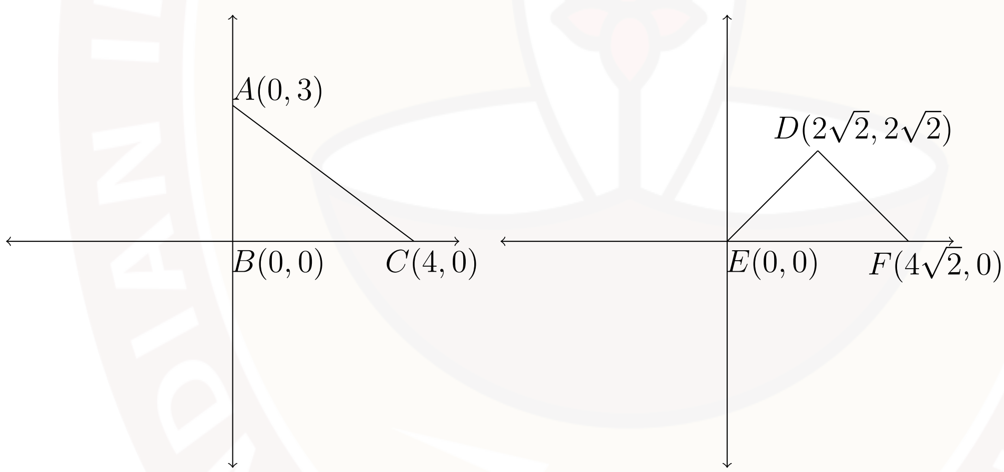


Figure PS-3.7

Which of the following option(s) is(are) true about the triangles ΔABC and ΔDEF given in Figure PS-3.7?

- ◯ Only ΔABC is a right angled triangle while ΔDEF is not.
- ◯ **Both ΔABC and ΔDEF are right angled triangles.**
- ◯ The area of ΔABC is greater than the area of ΔDEF.
- ◯ Both the triangles have the same area.

◯ **The area of ΔDEF is 8 sq.unit.**

Solution:

In Figure PS-3.7, vertices A and C are on $Y-$axis and $X-$ axis respectively and the vertex B is at the origin itself.
Therefore, $\triangle ABC$ is a right angle triangle.
The distance formula between two points $(x_1, y_1), (x_2, y_2)$ is given by

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Using the above formula, in $\triangle DEF$, the length of side DE is

$$\sqrt{(2\sqrt{2} - 0)^2 + (2\sqrt{2} - 0)^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4$$

Similarly, the length of side DF is

$$\sqrt{(4\sqrt{2} - 2\sqrt{2})^2 + (0 - 2\sqrt{2})^2} = 4$$

The length of side EF is $4\sqrt{2}$. We have

$$DE^2 + DF^2 = 16 + 16 = 32 = (4\sqrt{2})^2 = EF^2$$

Hence, by the Pythagoras theorem, $\triangle DEF$ is also a right angled triangle.
Area of the right angled $\triangle ABC = \frac{1}{2} \times 4 \times 3 = 6$ sq. unit.
Area of the right angled $\triangle DEF = \frac{1}{2} \times 4 \times 4 = 8$ sq. unit.

19. Let the diagonals of a quadrilateral with one vertex at $(0, 0)$ bisect each other perpendicularly at the point $(1, 2)$. Further, let one of the diagonals be on the straight line $y = 2x$. Then, which of the following is (are) correct statements?

- ◯ **The diagonally opposite vertex of $(0, 0)$ is $(2, 4)$.**
- ◯ The other diagonal is on the straight line $y = -\frac{1}{2}x$.
- ◯ **The other diagonal is on the straight line $y = -\frac{1}{2}x + \frac{5}{2}$.**
- ◯ The diagonally opposite vertex of $(0, 0)$ is $(\frac{3}{2}, 3)$.

Solution:

Figure PS-3.8 shows a sketch of the quadrilateral.

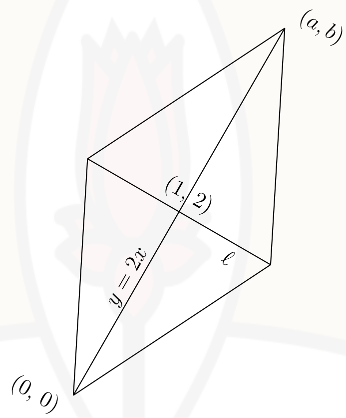


Figure PS-3.8

The diagonal $y = 2x$ has slope $m_1 = 2$.
Let the other diagonal, perpendicular to the line $y = 2x$, be on the line ℓ.
So, the slope of the line ℓ is

$$m_2 = -\frac{1}{m_1} = -\frac{1}{2}$$

From the slope intercept form, the equation of the line ℓ is $y = -\frac{x}{2} + c$, where c is a constant.
Since both the diagonals intersect at the point (1,2) and one diagonal is on line ℓ, the point (1,2) belongs to ℓ and hence $c = \frac{5}{2}$.
Hence, the equation of the line ℓ is $\boldsymbol{y = -\frac{1}{2}x + \frac{5}{2}}$.

Let the opposite vertex of $(0, 0)$ be (a, b).
Since the point (1, 2) is the bisection point of the both diagonals, it follows that the point $(1, 2)$ is mid-point of the line segment joining the points $(0, 0)$ and (a, b).
Using the section formula of a line segment,

$$\frac{a}{2} = 1 \implies a = 2$$

$$\frac{b}{2} = 2 \implies b = 4$$

Hence the diagonally opposite vertex of $(0, 0)$ is $(2, 4)$.

A woman is reported missing in a locality. The police department finds a human femur bone during their investigation. They estimate the height H of a female adult (in cm) using the relationship $H = 1.8f + 70$, where f is the length (in cm) of the femur bone. The length of the femur found is 35 cm, and the missing woman is known to be 130 cm tall. In the particular locality, maximum height of a female is 195 cm and the minimum length of a female femur bone is 15 cm. Based on the given data answer the following questions.

20. Choose the set of correct options.

- ○ If an error of 1 cm is allowed, bone could belong to missing female.
- ○ **If an error of 3 cm is allowed, bone could belong to missing female.**
- ○ If the height as a function of femur length is known to be accurate, the range of the function is [70, 195].
- ○ **If the height as a function of femur length is known to be accurate, the range of the function is [97, 195].**
- ○ **If the height as a function of femur length is known to be accurate, the domain of the function is $[15, \frac{625}{9}]$.**

Solution:

The relationship between height of a woman H and the length of her femur bone f is given by

$$H = 1.8f + 70. \tag{2}$$

Since the length of the femur bone found during the investigation is 35 cm, we have

$$H = 1.8 \times 35 + 70 = 133 \text{ cm}$$

The height of missing woman is known to be 130cm. Since $133 - 130 = 3 \leq 3$ and by our assumption, an error of 3 cm is allowed, it is possible that the femur bone found during the investigation belongs to the missing woman.

Given that the maximum height of a female in that location is 195 cm.
Substituting $H = 195$ in Equation (2) , we get the maximum length of female femur bone in that location i.e maximum $f = \frac{625}{9}$ cm.
Since the minimum length of of femur bone known in that location is 15 cm and if height as a function of femur length is known to be accurate then the domain of the function is $[15, \frac{625}{9}]$.

Given that the minimum length of the female femur bone in that location is 15 cm.
The minimum height of a female in that location is $H = 1.8 \times 15 + 70 = 97$ cm.
Since the maximum height of a female in that location is 195 cm, the range of the height function is $[97, 195]$

21. A new detective agency came up with a relationship H = mf + 70, where H is the height of a male adult (in cm) and f is the length (in cm) of the femur bone. They have used the following sample set given below in the Table P-3.2 , such that the sum squared error is minimum.

height(H) (in cm)	150	160	170	180
length of femur bone(f) (in cm)	40	42	48	56

Table PS-3.2

Choose the correct option (only one option is correct).

- ◯ $m = 1$
- ◯ $m = 1.5$
- ◯ $\boldsymbol{m = 2}$
- ◯ $m = 2.5$

Solution:

From Table PS-3.3, we can see that the minimum SSE is for $m = 2$.

H (in cm)	f (in cm)	$(H - mf - 70)^2$			
		$m = 1$	$m = 2$	$m = 1.5$	$m = 2.5$
150	40	1600	0	400	400
160	42	2304	36	729	225
170	48	2704	16	784	400
180	56	2916	4	676	900
SSE		$\sum = 9524$	$\sum = 56$	$\sum = 2589$	$\sum = 1925$

Table PS-3.3

3 Numerical Answer Type (NAT):

22. What will be the slopes of the straight lines perpendicular to the following lines?

a) $2x + 5y - 9 = 0$

Answer:2.5

Solution:

Using the slope intercept form, the slope of the line $2x + 5y - 9 = 0$ is $m_1 = -\frac{2}{5}$.
Let the slope of the perpendicular line be m_2. Then

$$m_1.m_2 = -1$$

$$\implies m_2 = \frac{5}{2} = 2.5$$

23.$-5x + 25y + 28 = 0$

Answer:-5

Solution:

Using the slope intercept form, the slope of the line $-5x + 25y + 28 = 0$ is $m_1 = \frac{1}{5}$.
Let the slope of the perpendicular line be m_2, then

$$m_1.m_2 = -1$$

$$\implies m_2 = -5.$$