

PROJECT-2
DC Motor Drive for an Electric Vehicle



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POWER ELECTRONICS PROJECT-2

DC MOTOR DRIVE FOR ELECTRICAL VEHICLE

INTRODUCTION:

With the world abruptly reacting to climate change problems, a lot of ways are being thought of reducing the carbon footprint from the globe by utilizing some cleaner source of energy, this alternative energy can be harnessed from two cleaner and efficient ways namely from solar and wind. Due to the ever-increasing number of solar and wind farms, lots of microgrids are formed, which has to be efficiently and reliably connected to the power grid in order to harness this theoretically infinite source of energy.

Problem with these sources is their very low conversion efficiency and very low voltage which can be solved by boosting this voltage to a higher level and maintaining a constant voltage at the DC bus from where power will be transformed to AC form and uploaded to the grid. Now to harness lots of energy from these greener alternatives we require lots of similar units connected in parallel which can then be fed to a DC-DC conversion stage, in order to handle such large amount of power we either require a single big converter stage which is economically not viable as it will require larger size of passives and will increase the size of the system along with the losses, an economical solution to this problem is connecting multiple converters stages in parallel which is exactly that is followed in **microgrids**.

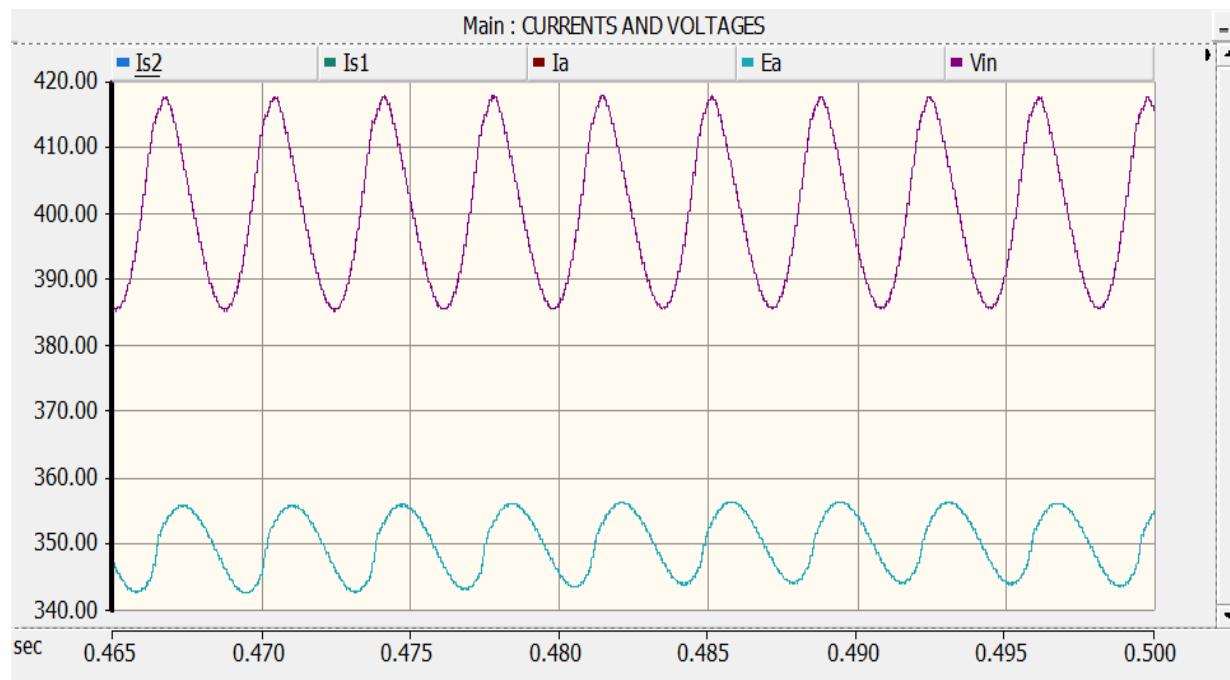
Another application where paralleling of converters helps is applications where reliability is the most important aspect like aeroplane power supplies.

Electrical vehicles employ these paralleling techniques to increase the life and performance of the battery systems by drawing currents from the batteries as per their SoC's.

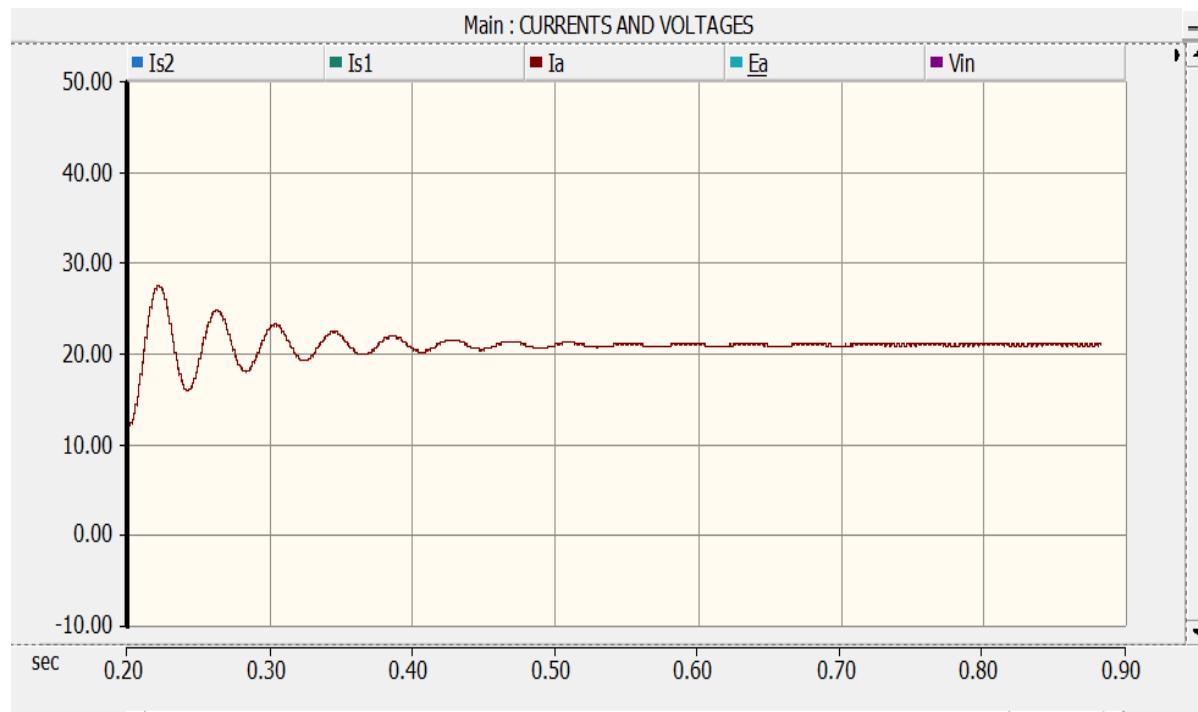
So, paralleling of DC-DC converters provides numerous advantages like reduction in switching losses, increase in efficiency, ease of maintenance and reliability.

CHALLENGES ASSOCIATED WITH CONNECTING MULTIPLE CONVERTERS IN PARALLEL

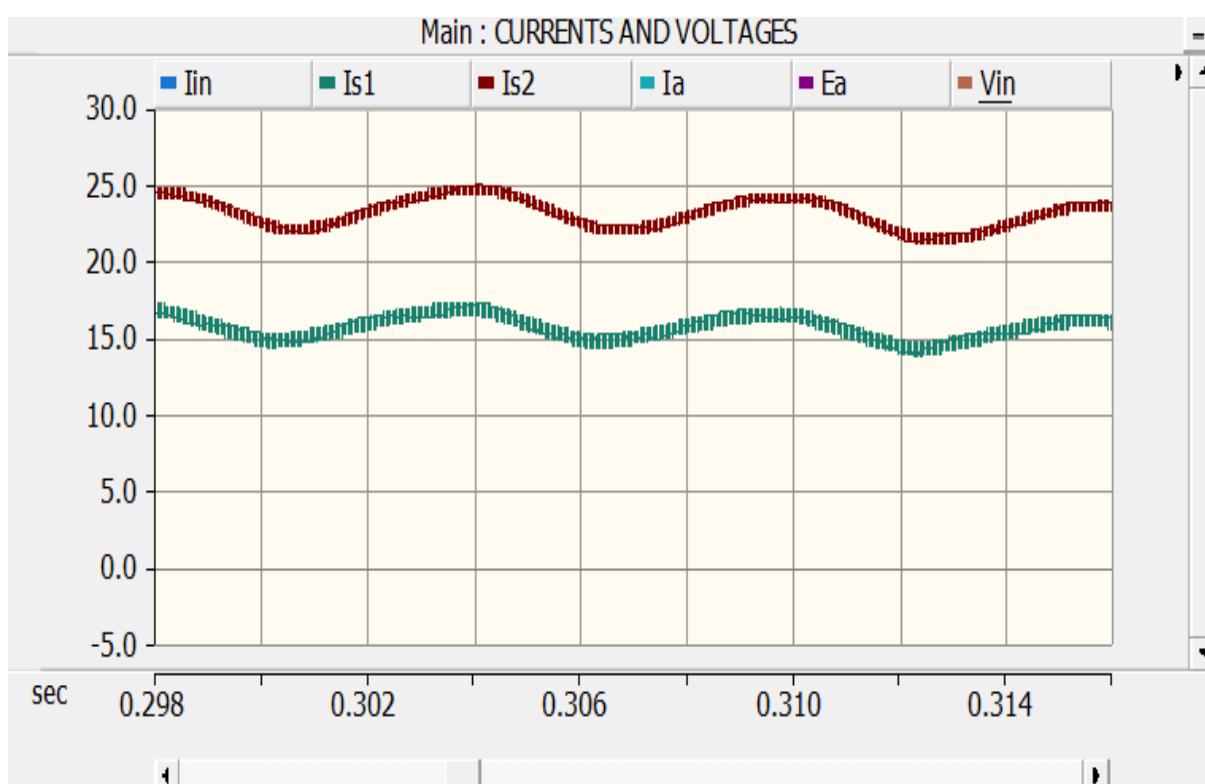
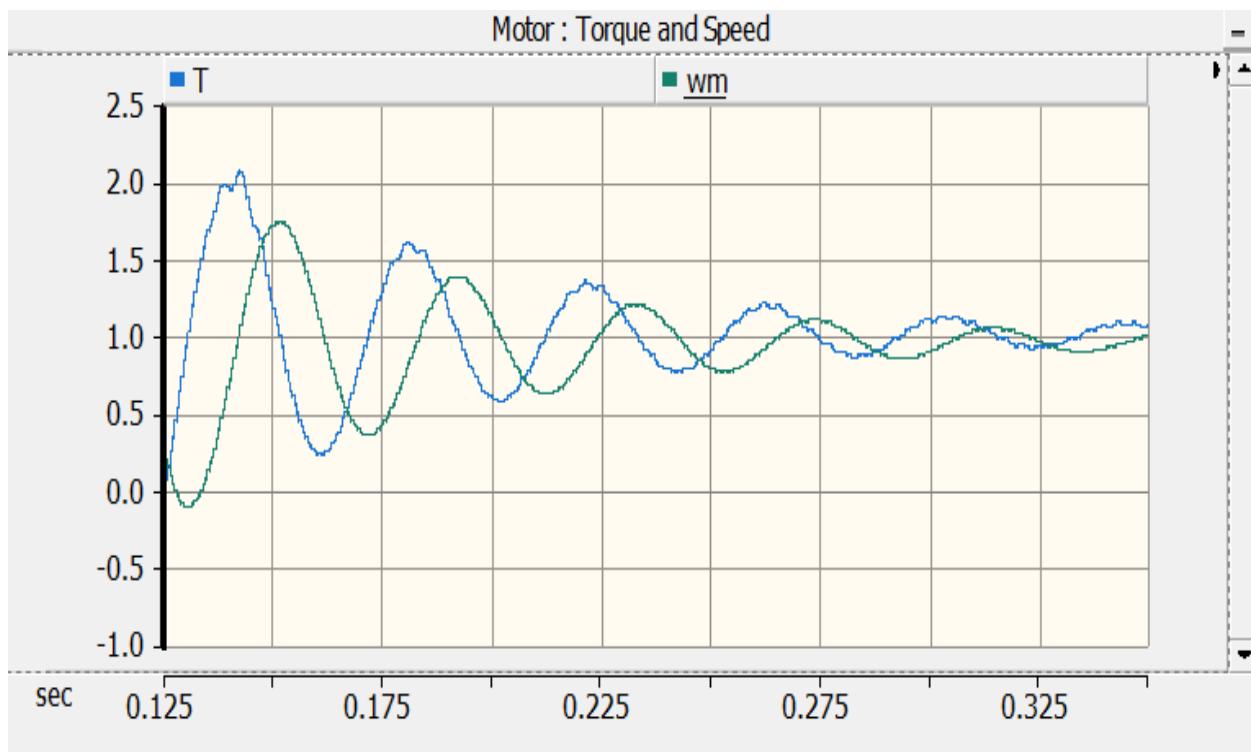
1. Uneven current sharing due to different values of components.
2. Poor transient response
3. Due to uneven current sharing it is difficult to operate all the converters in CCM mode due to which converters operating in DCM will exhibit different electrical characteristics.
4. Under light load conditions converters sharing less currents can enter DCM mode and exhibit large source current ripples.
5. Under stepping load conditions few converters may experience higher current stresses during transients because of non-identical components which reduces operational reliability in the long run.
6. Lower DC gain of the voltage control loop makes the system sluggish.

SIMULATION RESULTS:**CASE1: MOTOR WORKING IN RATED CONDITIONS**

* E_a = Rated armature voltage, * V_{in} = DC Link voltage

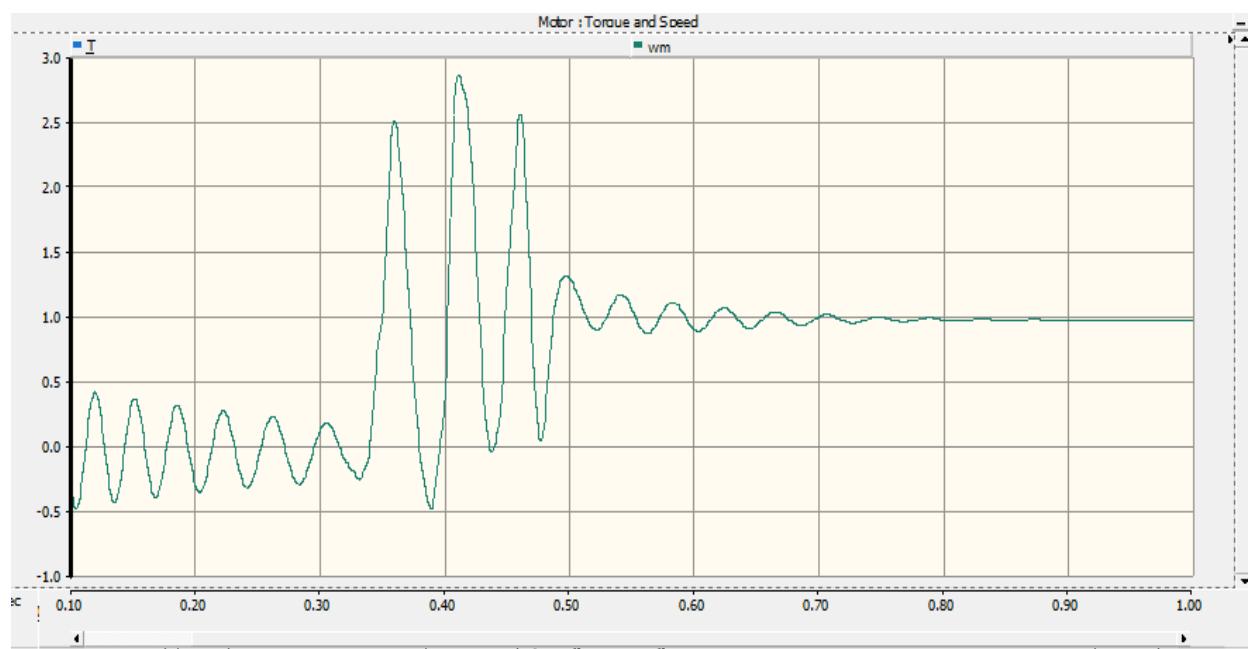
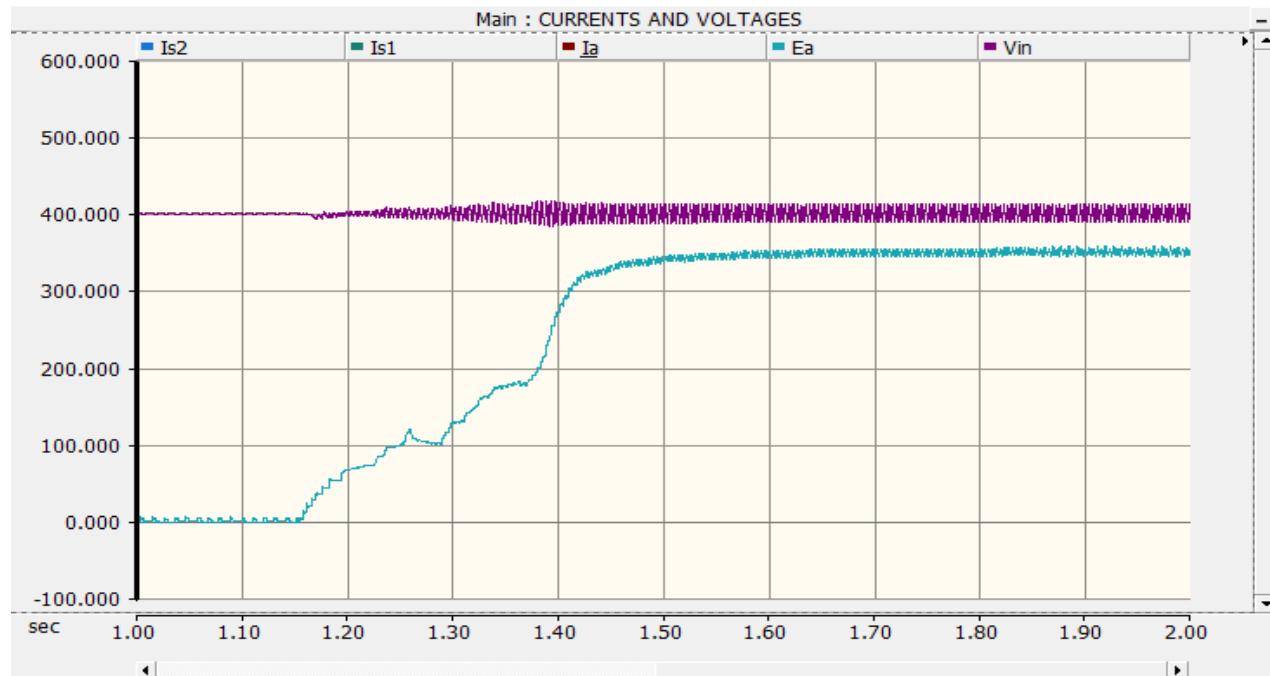


* I_a = Rated Armature current

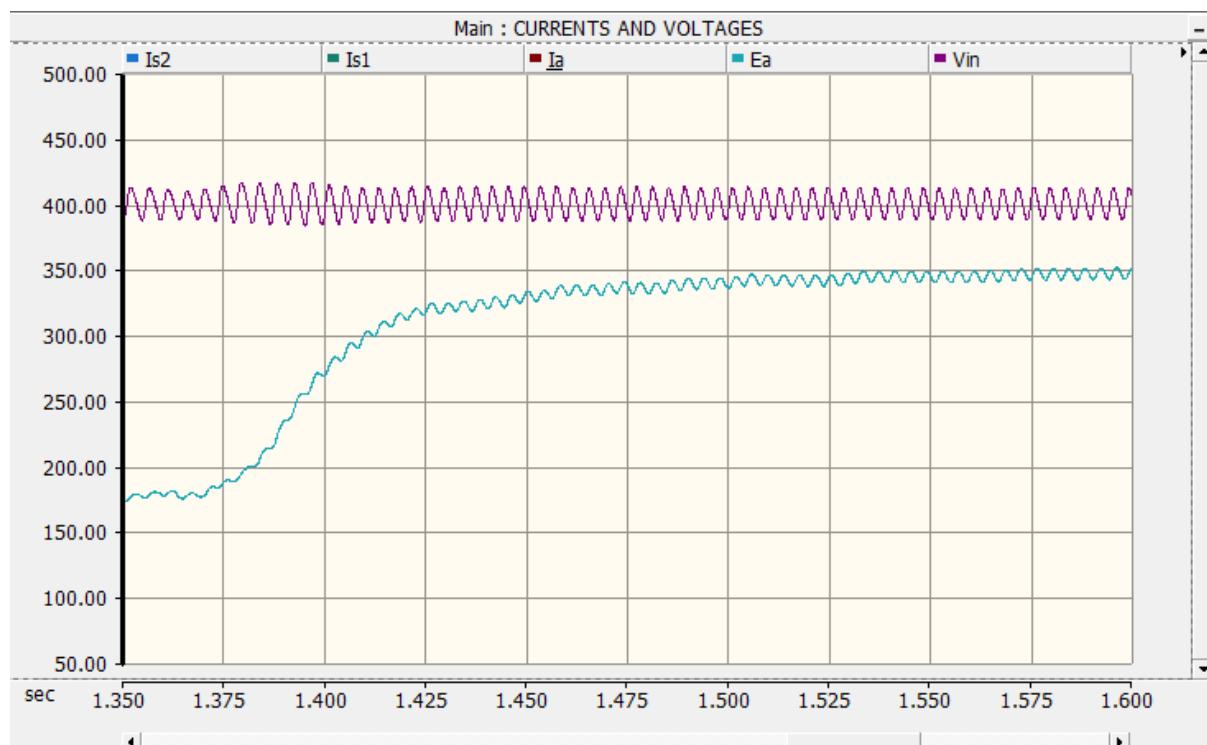


*SoC is 50% and 75% for battery 1 and 2 respectively.

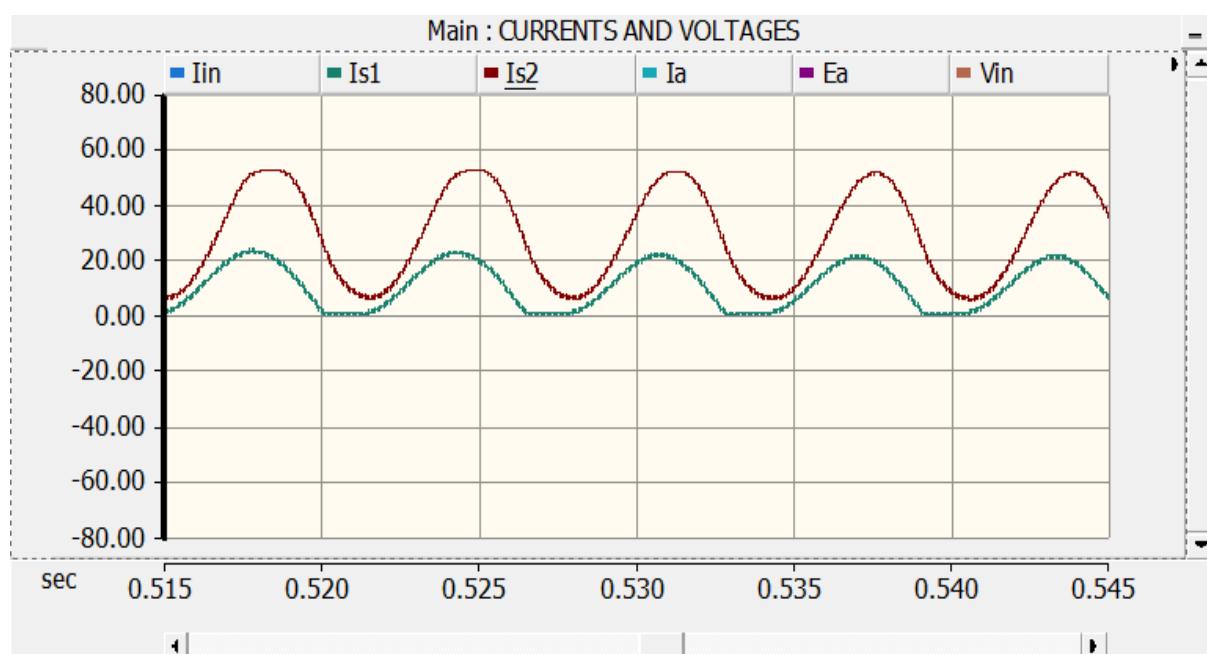
CASE2: MOTOR STARTING FROM STANDSTILL CONDITION AND GOES TO RATED CONDITIONS



Speed changing from 0 to rated as armature voltage is slowly increased

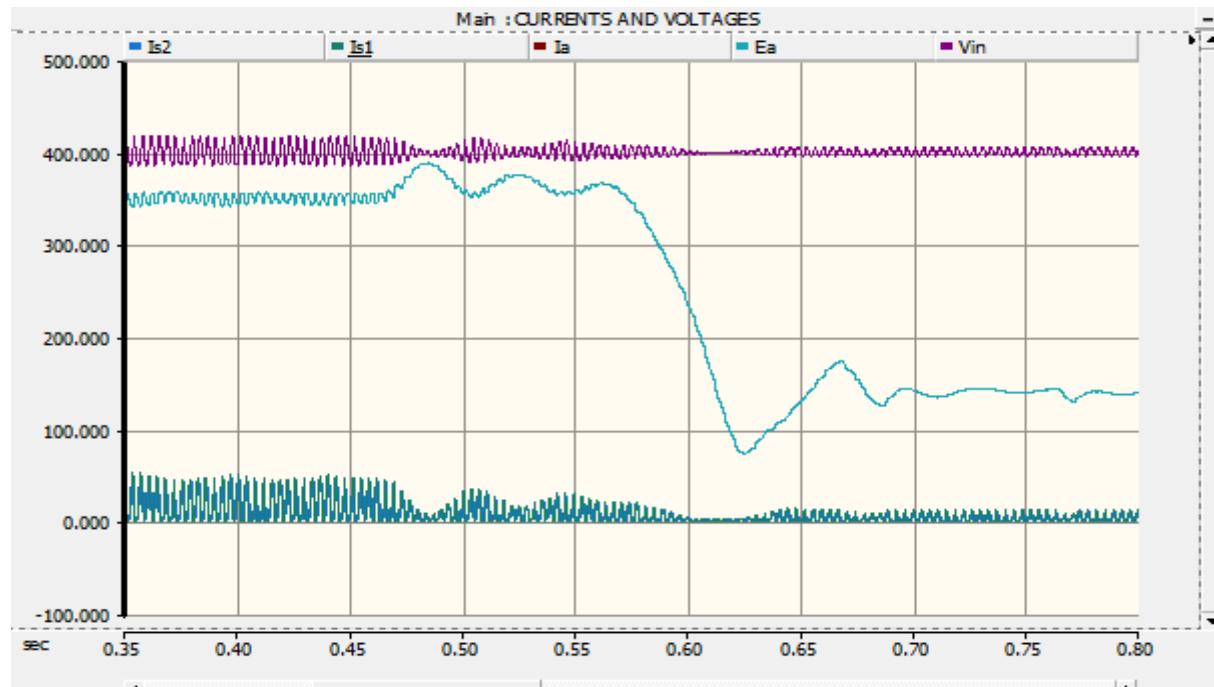


Controller action justifying the gradual increase in armature voltage to its rated value

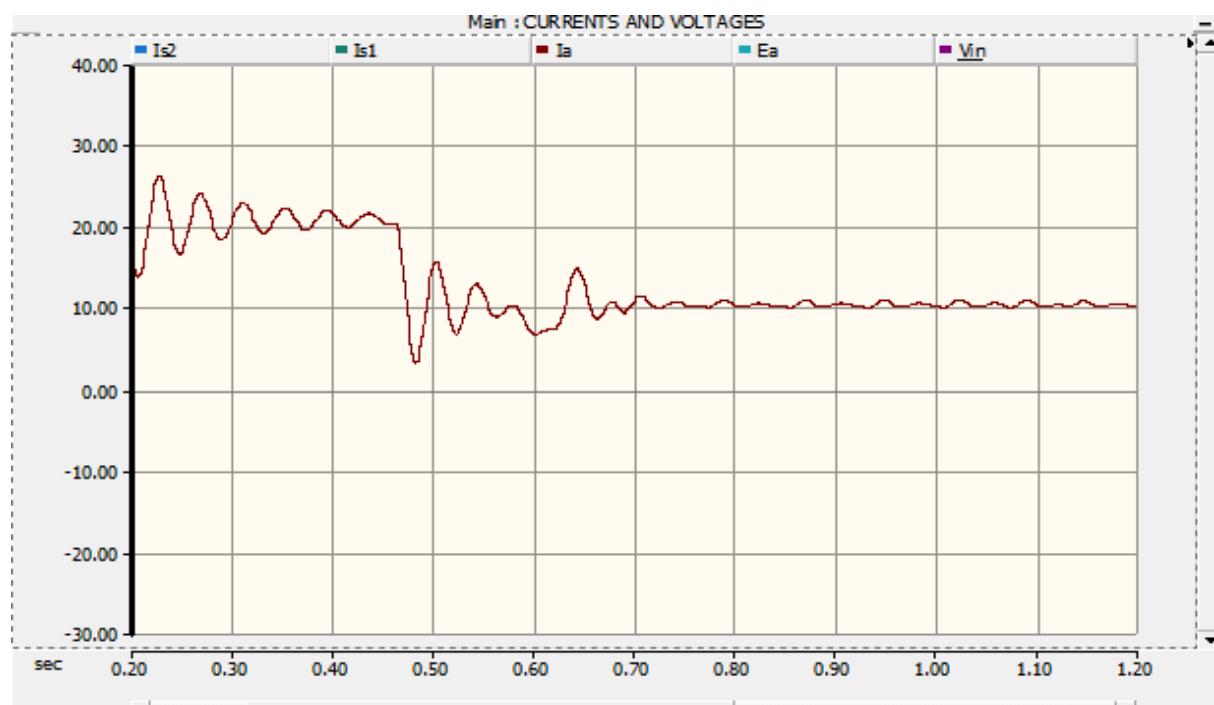


SoC of Battery 1 and 2 are 25% and 75% respectively

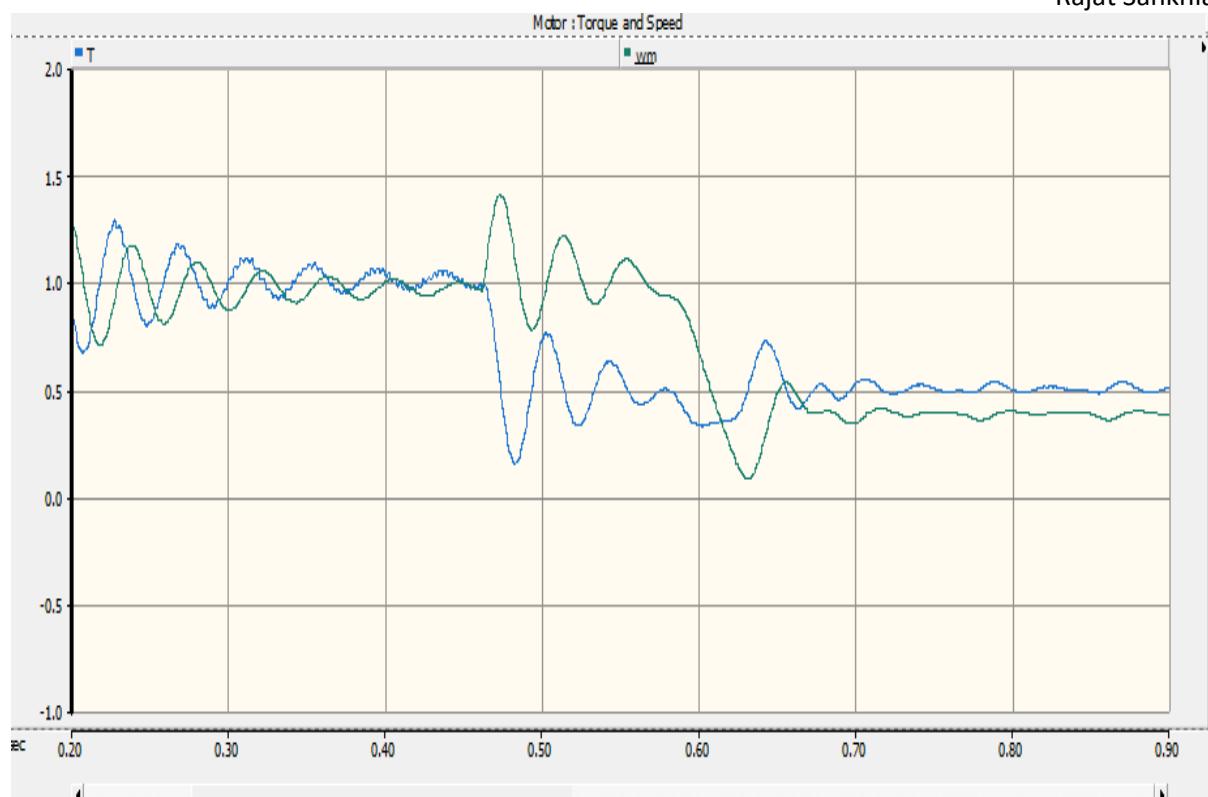
CASE 3: MOTOR OPERATING AT 20% RATED POWER AND 50% RATED TORQUE:



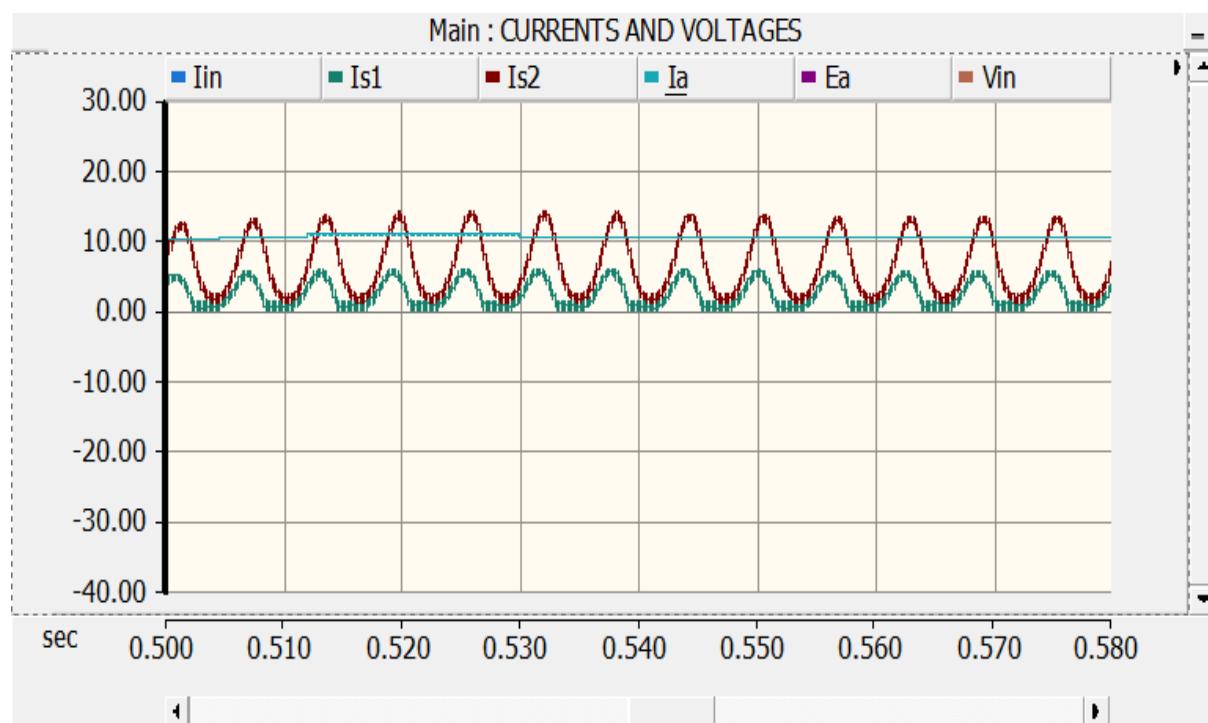
Armature voltage and source currents settles to their new values when speed and load torque changes to 40% and 50% of their rated values respectively.



Armature current settling to 50% of its rated value



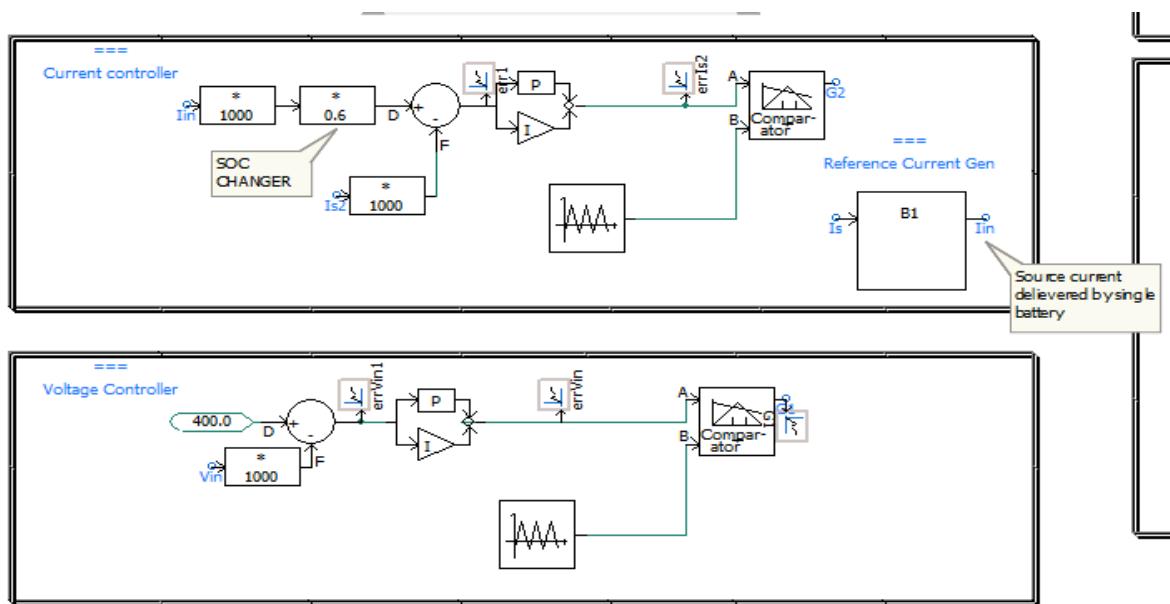
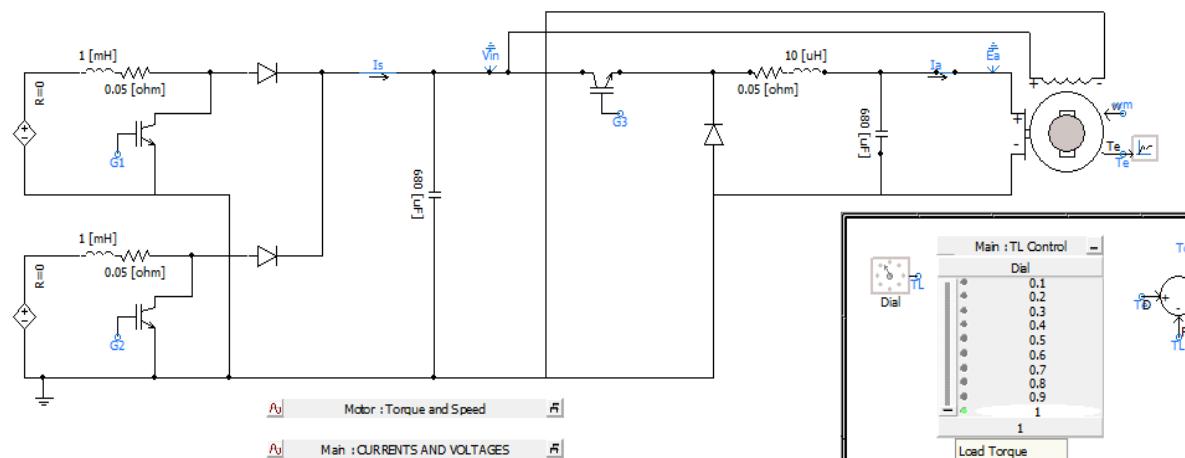
Speed and torque adjust to 40% and 50% of their rated values.

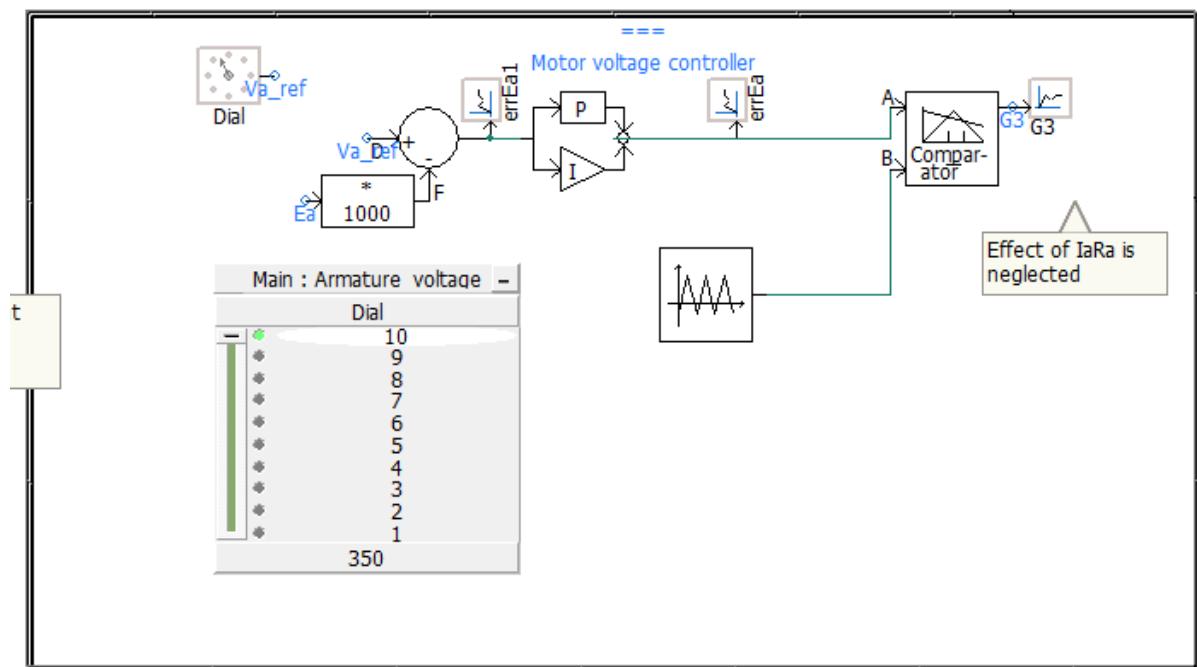
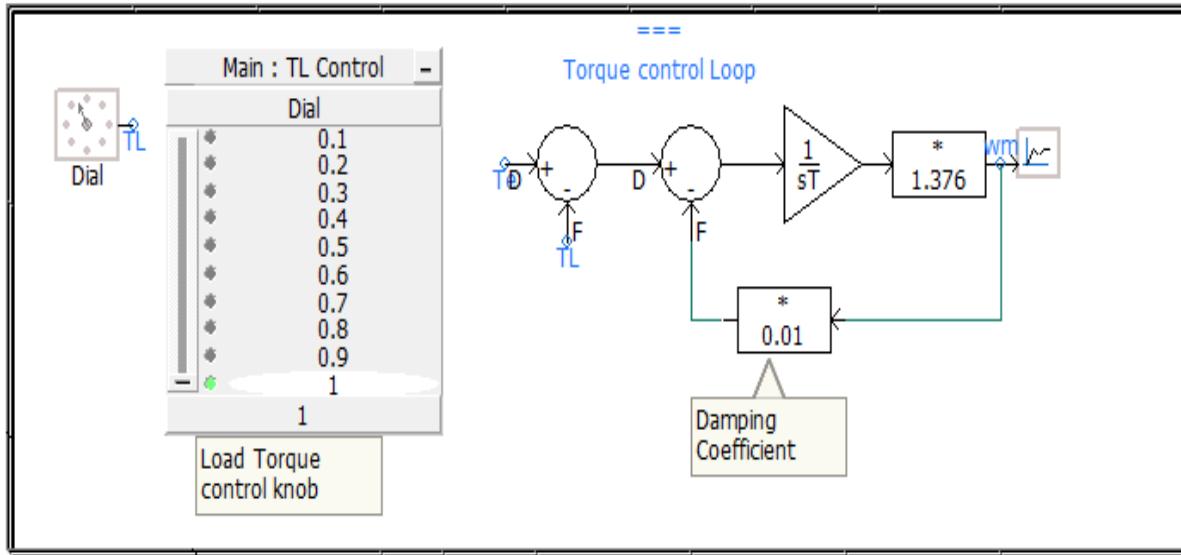


SoC of battery 1 and Battery 2 is 50% respectively

CONCLUSION:

1. Under light load conditions converters sharing less currents can enter DCM mode and exhibit large source current ripples.
2. Under dynamic load changing situations one converter may experience higher current stresses during transients because of non-identical components which reduces operational reliability in the long run.
3. Due to the very high value of switching frequency, accurate results are obtained at the solution time step of 0.01MicroSec and the plot step of 1MicroSec.
4. DC gain of the PI controller of voltage loop is very small due to which the system takes more time to settle.

Interface design for simulating control of armature voltage and load torque.

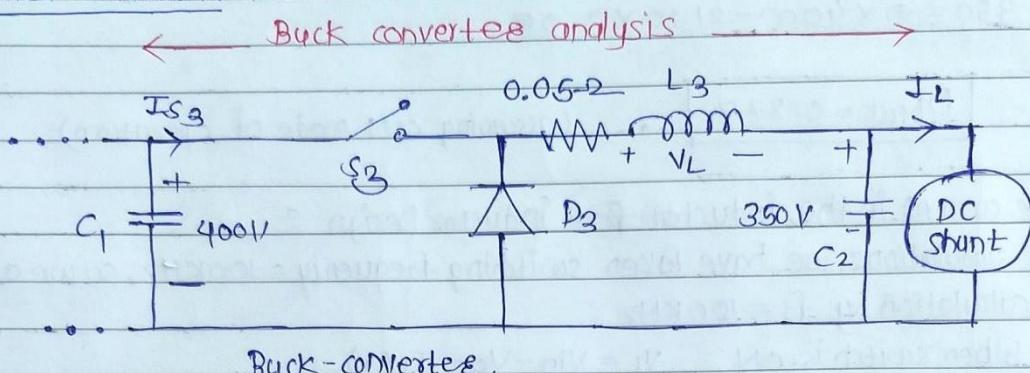


ANALYSIS OF CONVERTERS:CASE-I ANALYSIS

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(ii) Theoretical Analysis :

case I: SOC of $B_1 = 50\%$, SOC of $B_2 = 75\%$ and motor working in rated condition.



By power balance,

$$350 \times I_L = 7000 + \left(\frac{350}{700}\right)^2 + \left(I_L - \frac{350}{700}\right)^2 \times 0.5$$

(rated output power)

$$350 I_L = 7000 + 175 + 0.5 \left(I_L^2 - I_L + 0.25 \right)$$

$$0.5 I_L^2 - 350.5 I_L + 7175.125 = 0.$$

$$I_L = \frac{350 \pm \sqrt{(350.5)^2 - 4 \times 0.5 \times 7175.125}}{2 \times 0.5}$$

$$I_L = 21.10 \text{ A} = I_{\text{field}} + (I_q)_{\text{rated}} \quad \therefore I_{\text{field}} = 0.5 \text{ A}$$

$$(I_q)_{\text{rated}} = 20.6 \text{ A}$$

@ calculating duty cycle of buck converterapplying volt-sec balance to voltage across L_3

$$[V_{in} - V_o - I_{Lr}] DT + [-V_o - I_{Lr}] (1-D) T = 0$$

$$[(400 - 350) - (I_{Lr})] DT + [-350 - I_{Lr}] [1-D] T = 0.$$

$$50DT - I_{Lr}/DT - 350T + 350DT - I_{Lr}T + I_{Lr}/DT = 0$$

$$350 = 400D - I_{Lr} \quad \textcircled{1}$$

for Buck converter $I_L(\text{avg}) = I_o(\text{avg})$

$$I_L(\text{avg}) = 21.10 \text{ A.}$$

$$350 = D \times 400 - 21.10 \times 0.05$$

$$D_{\text{buck}} = 0.877 \quad \dots \text{ (assuming ccm mode of operation)}$$

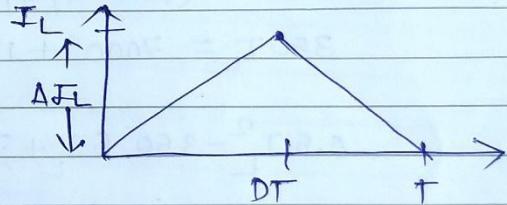
(b) Ripple current in the inductor for inductor Design :

Note: In simulations, we have taken switching frequency = 100 KHz, so we are doing all calculation by $f_s = 100 \text{ KHz}$.

$$\begin{aligned} \text{When switch is ON, } V_L &= V_{\text{in}} - V_o - I_L(r) \\ &= (400) - 350 - (21.10)(0.05) \\ &= 48.945 \text{ V.} \end{aligned}$$

$$V_L = L \frac{dI}{dt}. \quad (\frac{V_L}{L} \rightarrow +ve \text{ slope}) \quad I_L \text{ charging (+ve slope)}$$

$$48.945 = L \frac{\Delta I}{DT}$$



$$\text{Now for critical conduction mode.} \quad I_L(\text{min}) = 0.$$

$$I_L(\text{avg}) = \frac{\Delta I_L}{2}$$

$$I_o(\text{avg}) = \frac{48.945 DT}{2 L_c}$$

$$(21.10) = 48.945 \times 0.877$$

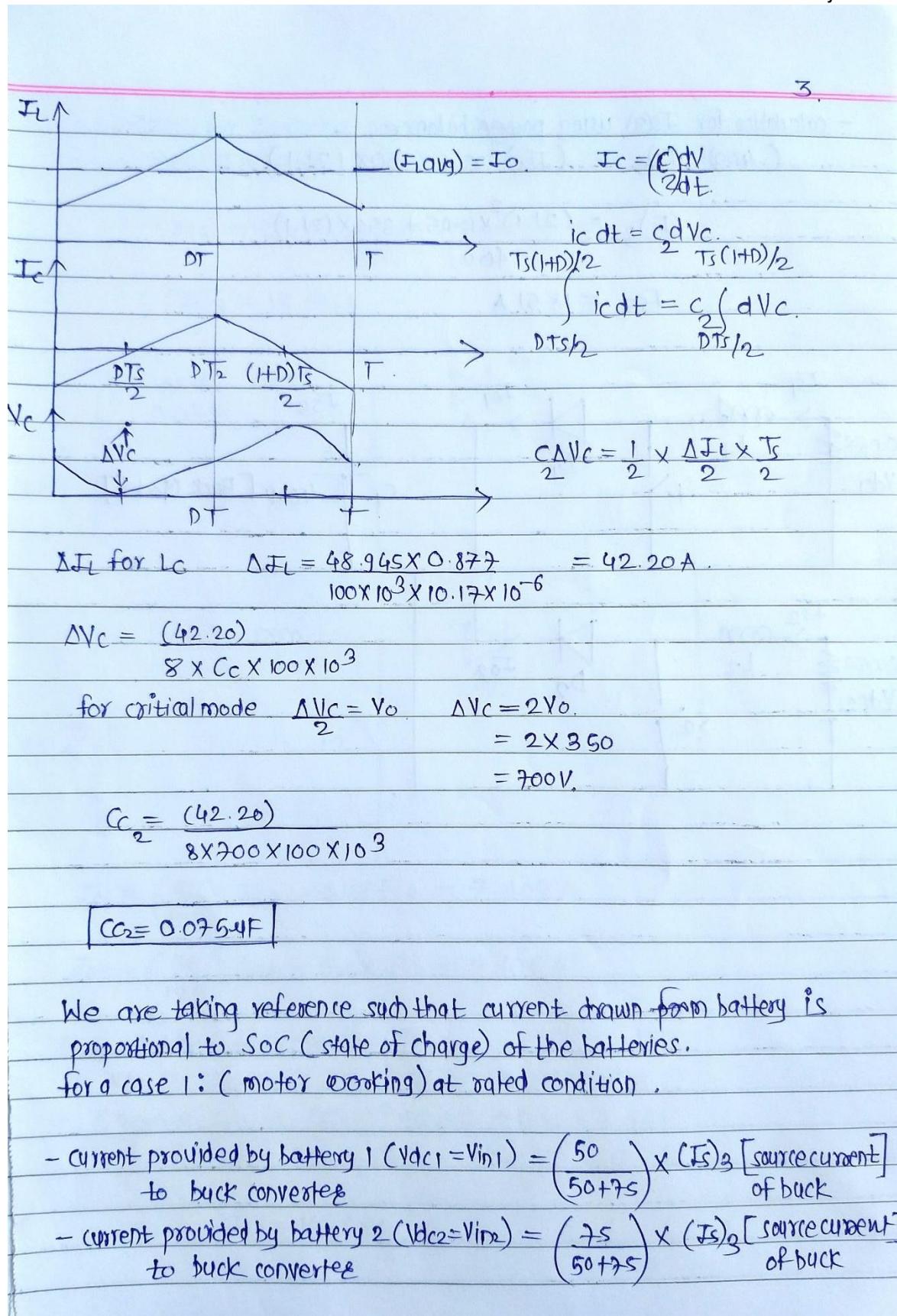
$$2 \times L_c \times 100 \times 10^{-3}$$

$$L_c = 10.17 \text{ mH}$$

(c) Voltage ripple at output and capacitor value design :

$$i_C = C \frac{dv}{dt}$$

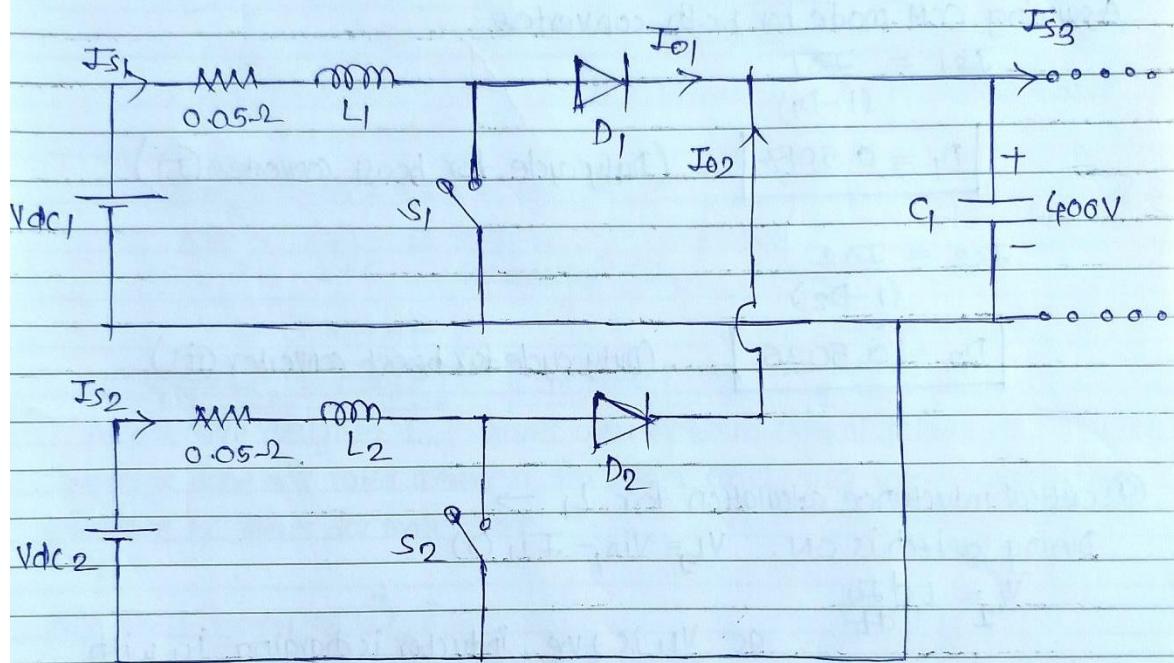
$$i_C = I_L - I_o$$



- calculation for I_{S3} using power balance,
 $(400) I_{S3} = (I_{S3})^2 \times 0.05 + 350 \times (21.10)$

$$(I_{S3}) = \frac{(21.10)^2 \times 0.05 + 350 \times (21.10)}{400}$$

$$(I_{S3}) = 18.51 \text{ A}$$



$$I_{01} = \left(\frac{50}{125} \right) I_{S3} = 0.4 \times I_{S3} = 7.404 \text{ A.}$$

$$I_{02} = \left(\frac{75}{125} \right) I_{S3} = 0.6 \times I_{S3} = 11.106 \text{ A.}$$

Power balance for converter ①

$$V_{dc1} \times I_{S1} = (I_{S1})^2 \times 0.05 + 400 \times I_{01}$$

$$(200) \times I_{S1} = (I_{S1})^2 \times 0.05 + 400 \times 7.404$$

$$0.05 I_{S1}^2 - 200 I_{S1} + 2961.6 = 0.$$

$$\boxed{I_{S1} = 14.86 \text{ A}}$$

Power balance for converter (2).

$$V_{DC2} \times I_{S2} = (I_{S2})^2 \times 0.05 + 400 \times 11.106$$

$$(200) \times I_{S2} = (I_{S2})^2 \times 0.05 + 4442.4$$

$$\therefore 0.05 I_{S2}^2 - 200 I_{S2} + 4442.4 = 0$$

$$I_{S2} = 22.33 \text{ A}$$

Assuming CCM mode for both converters.

$$I_{S1} = \frac{I_{O1}}{(1-D_1)}$$

$$D_1 = 0.5017 \quad \text{... (Duty cycle for boost converter (I))}$$

$$I_{S2} = \frac{I_{O2}}{(1-D_2)}$$

$$D_2 = 0.5026 \quad \text{... (Duty cycle for boost converter (II))}$$

④ critical inductance calculation for $L_1 \rightarrow$

During switch is ON, $V_L = V_{in} - I_{L1} (s)$

$$V_L = L_1 \frac{dI_{L1}}{dt}$$

as V_L is +ve, inductor is charging, i_{L1} with +ve slope.

$$\text{for critical mode } \frac{\Delta I_{L1}}{2} = I_{L1}(\text{avg})$$

$$\Delta I_{L1} = 2 \times I_{L1}(\text{avg}) = 2 \times 14.86 = 29.72 \text{ A.}$$

$$(200) - (14.86)(0.05) = L_{C1} \times \frac{(29.72) \times 100 \times 10^3}{0.5017}$$

$$L_{C1} = \frac{199.25 \times 0.5017}{29.72 \times 100 \times 10^3}$$

$$L_{C1} = \frac{0.067 \text{ mH}}{2}$$

$$L_{C1} = 0.034 \text{ mH}$$

(b) critical inductance calculation for $L_2 \rightarrow$

During switch S_2 is ON $V_{L2} = V_{in2} - I_{L2}(t)$

$$V_{L2} = i_2 \frac{d i_2}{dt}$$

i_2 with +ve slope, inductor charges.

$$(200) - (22.33) \times 0.05 = L_2 \frac{\Delta I_{L2}}{D_2 T}, \text{ for critical conduction mode}$$

$$\Delta I_L = I_L(\text{avg})$$

$$198.88 = L_2 \times \frac{(44.66) \times 100 \times 10^3}{0.5026}$$

$$\Delta I_L = 2 \times I_L(\text{avg}) = 2 \times 22.33 \\ = 44.66 \text{ A}$$

$$L_2 = 0.022 \text{ mH}$$

(c) critical capacitance calculation for common capacitor $C_1 \rightarrow$

$$\Delta V_C = V_0 \quad \Delta V_C = 2 \times 400 = 800 \text{ V}$$

taking ² critical conduction mode,

When switches are ON

$$-(I_{O1} + I_{O2}) = C_1 \frac{dV_C}{DT} \quad C \text{ discharging } \frac{dV_C}{dt} \text{ with -ve slope}$$

$$dV_C = V_{C(\text{min})} - V_{C(\text{max})}$$

$$-(18.61) = C_1 \times \frac{-800 \times 100 \times 10^3}{(0.5)} \quad (\text{final} - \text{initial})$$

$$dV_C = -\Delta V_C = V_{C(\text{max})} - V_{C(\text{min})}$$

$$C_1 = \frac{(18.61) \times 0.5}{800 \times 100 \times 10^3}$$

$$C_1 = 0.115 \mu\text{F}$$

Conclusion \rightarrow As DC motor is working at rated condition, to operate converter in com, design value should be greater than critical values and we can keep design parameters of buck for rest cases as it has calculated for rated load.

Q1

③ conduction mode verification by comparing L, C values with critical values.
 We are taking general ripple references,

$$\frac{\Delta I_L}{I_{L(\text{avg})}} = 20\% \quad \frac{\Delta V_C}{V_0} = 5\%$$

(i) Buck converter :

$$\Delta I_{L_3} = 0.2 \times 21.10 = 4.22 \text{ A}$$

$$\frac{\Delta I_{L_3}}{3} = \frac{(V_L) D T}{L_3} \quad , \quad L_3 = \frac{(V_L)(D T)}{\Delta I_L}$$

$$L_3 = \frac{(48.945)(0.877)}{4.22 \times 100 \times 10^3} = 0.1017 \text{ mH} , \quad L_C = \frac{0.01017}{3} \text{ mH}$$

$$\Delta V_{C_3} = 0.05 \times 350 = 17.5 \text{ V.}$$

$$\Delta V_C = \frac{\Delta I_L}{8 f_C} = \frac{4.22}{8 \times 100 \times 10^3 \times C_3} = 17.5$$

$$C = 0.3014 \text{ nF}$$

$$C_{C_3} = 0.0754 \text{ F.}$$

$$[L_3 > L_{C_3}, C_3 > C_{C_3}]$$

CCM MODE

As we have designed L, C values greater than critical values and as this design is done for rated condition, for other cases also we can take it as reference L, C values for mode check.

(ii) Boost converter ① :

$$\Delta I_{L_1} = 0.2 \times (14.86) = 2.972 \text{ A}$$

$$\frac{\Delta I_{L_1}}{L_1} = \frac{(V_L) D T}{L_1} \quad L_1 = \frac{(V_L)(D T)}{\Delta I_{L_1}}$$

$$L_1 = \frac{(199.25) \times (0.5017)}{2.972 \times 100 \times 10^3}$$

$$L_1 = 0.336 \text{ mH}$$

$$L_C = 0.034 \text{ mH.}$$

$$\Delta V_{C_1} = 0.05 \times 400 = 20 \text{ V.}$$

$$(f_{o1} + f_{o2}) = C_1 \frac{\Delta V_C}{D T}$$

$$C_1 = \frac{(18.51) \times 0.5}{20 \times 100 \times 10^3}$$

6.2

$$C_1 = 4.62 \mu F$$

$$C_2 = 0.115 \mu F$$

$$\boxed{L_1 > L_{c1}} \quad \& \quad \boxed{C_1 > C_{c1}} \quad \boxed{\text{CCM MODE}}$$

- it is observed that for the considered ripple values, designed, L, C values are greater than its critical values so, boost converter operates in CCM mode for this case

- As L_1, C_1 designed for rated case, we can take its reference for the remaining cases.

(ii) Boost converter (II) :

$$\Delta I_{L2} = 0.2 \times I_{L2(\text{avg})} = 0.2 \times (22.33) = 4.466 A$$

$$V_{L2} = \frac{L_2 d i_{L2}}{dt} \quad L_2 = \frac{(V_{L2})(D_2)T}{\Delta I_{L2}}$$

$$I_2 = \frac{(198.88)(0.5026)}{4.466 \times 100 \times 10^3}$$

$$L_2 = 0.2238 mH$$

$$L_{c2} = 0.022 mH$$

$$\boxed{L_2 > L_{c2}} \quad \boxed{\text{CCM MODE}}$$

- for Designed I_2 with 20% current ripple, we observed that converter operate in CCM for rated condition.

- We can take L_2 value reference for other cases as well.

Calculated Results for case I & rated condition $SOC(1) = 50\%$, $SOC(2) = 75\%$

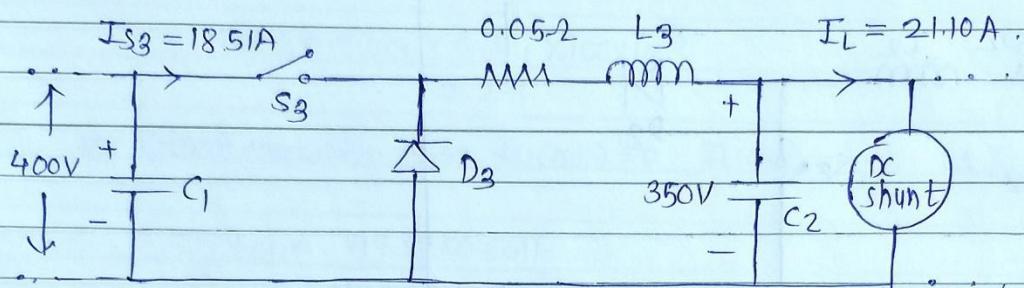
- (i) total current drawn by DC motor = 21.10 A.
- (ii) input current drawn by buck converter = $I_{S3} = 18.51$ A.
- (iii) Duty cycle ratio of buck converter = $D_3 = 0.877$.
- (iv) critical inductance for $L_3 = 10.17 \mu H$.
- (v) critical capacitance for C_2 (cap of buck) = $0.075 \mu F$.
- (vi) source current of boost converter (I) = $I_{S1} = 14.86$ A
- (vii) source current of boost converter (II) = $I_{S2} = 22.33$ A
- (viii) Duty cycle for boost converter (I) = 0.5017
- (ix) Duty cycle for " " (II) = 0.5026
- (x) critical inductance for $L_1 = 0.034 mH$
- (xi) critical inductance for $L_2 = 0.022 mH$
- (xii) critical capacitance for output capacitor $C_1 = 0.115 \mu F$.

CASE-II ANALYSIS

8.

case II : SOC of $B_1 = 25\%$, SOC of $B_2 = 75\%$, motor starts from standstill condition and goes to rated condition with rated load torque.

∴ As we are doing analysis in steady state, Buck converter analysis will be same as case I.



$$D_{\text{buck}} = 0.877$$

$$L_{\text{critical}3} = 10.17 \mu\text{H}$$

$$C_{\text{critical}2} = 0.075 \mu\text{F}$$

∴ As proportional to the SOC of batteries, both boost converters will share output current with reference to SOC.

- current shared by boost converter I = $\left(\frac{25}{100}\right) \times (I_{S3})$

$$I_{O1} = 0.25 \times (18.51)$$

$$I_{O1} = 4.627 \text{ A}$$

- current shared by boost converter II = $\left(\frac{75}{100}\right) \times (I_{S3})$

$$I_{O2} = 0.75 \times (18.51)$$

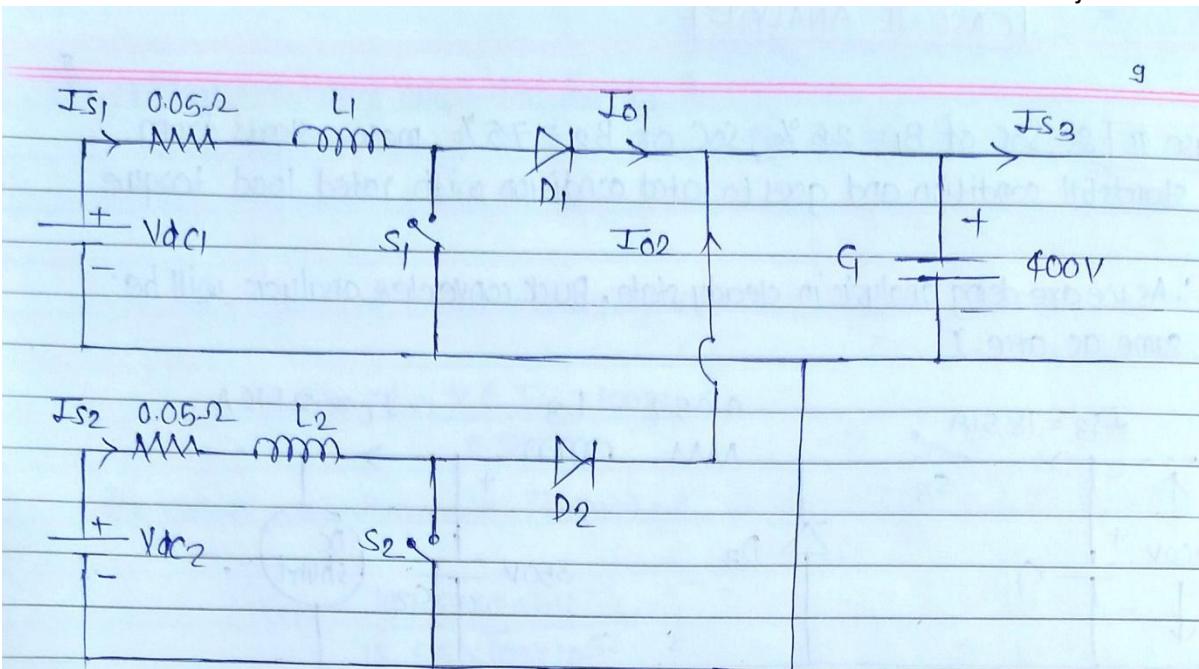
$$I_{O2} = 13.88 \text{ A}$$

- Power balance for boost converter I :

$$V_{DC1} \times I_{S1} = (I_{S1})^2 \times 0.05 + 400 \times I_{O1}$$

$$(200) \times I_{S1} = (I_{S1})^2 \times 0.05 + 400 \times 4.627$$

$$\boxed{I_{S1} = 9.275 \text{ A}}$$



- Power balance for boost converter (II) :

$$V_{dc2} \times I_{S2} = (I_{S2})^2 \times 0.05 + 400 \times I_{O2}$$

$$(200) \times I_{S2} = (I_{S2})^2 \times 0.05 + 400 \times 13.88.$$

$$0.05 I_{S2}^2 - 200 I_{S2} + 5552 = 0$$

$$I_{S2} = 27.95A$$

Assuming CCM mode for both converters,

$$I_{S1} = \frac{I_{O1}}{(1-D_1)}$$

$$9.275 = \frac{4.627}{(1-D_1)}$$

$$D_1 = 0.5011 \quad \dots \text{(Duty cycle for boost converter (I))}$$

$$\therefore I_{S2} = \frac{I_{O2}}{(1-D_2)}$$

$$27.95 = \frac{13.88}{(1-D_2)}$$

$$D_2 = 0.5033 \quad \dots \text{(Duty cycle for boost converter (II))}$$

(A) critical inductance calculation for L_1

When switch s_1 is ON $V_{L1} = V_{inj} - J_{L1}(t)$ & $\frac{dV_{L1}}{dt} = L_1 \frac{di_1}{dt}$
 $V_{L1} = (200) - 9.275 \times 0.05$
 $= 199.53V.$

V_{L1}/L_1 positive, inductor charging with +ve slope of J_{L1}

$$199.53 = L_{C1} \times \Delta J_{L1} \times 100 \times 10^3 / 0.5011$$

for critical conduction mode, $J_{L1(\min)} = 0$ $J_{L1(\text{avg})} = \frac{\Delta J_{L1}}{2}$ $\Delta J_{L1} = 18.55A$.

$$L_{C1} = \frac{199.53 \times 0.5011}{18.55 \times 100 \times 10^3}$$

$$\boxed{L_{C1} = 0.054 \text{ mH}}$$

(B) critical inductance calculation for L_2

When switch s_2 is ON $V_{L2} = V_{dc2} - J_{L2}(t)$

for CCM $J_{L2(\min)} = 0$, $J_{L2(\text{avg})} = \frac{\Delta J_{L2}}{2}$ $\Delta J_{L2} = 55.9 A$.

$$V_{L2} = (200) - (27.95) \times 0.05 = 198.60V.$$

V_{L2}/L_2 slope is +ve L_2 charging

$$V_{L2} = L_2 \frac{di_2}{dt} \quad L_2 = \frac{V_{L2} \times D_2 T}{\Delta J_{L2}}$$

$$L_{C2} = \frac{198.60 \times 0.5033}{55.9 \times 100 \times 10^3}$$

$$\boxed{L_{C2} = 0.0178 \text{ mH}}$$

(C) critical capacitance calculation for common capacitor

As total current ($J_{01} + J_{02}$) is same as the previous case and duty ratios are also almost matching for both converters with same output voltage requirement.

$$\boxed{C_{C1} = 0.1154 \text{ F}}$$

10.1

④ conduction mode verification \Rightarrow

(i) Buck converter :

In case ① we are getting rated motor output so again like case ① Buck converter will operate in ccm.

(ii) Boost converter ① :

$$V_L = L \frac{dI_L}{dt}$$

$$\Delta I_{L1} = \frac{(V_{L1})(D_1)(T)}{L} = \frac{(199.53)(0.5011)}{0.336 \times 10^{-3} \times 100 \times 10^{-3}}$$

$$\Delta I_{L1} = 2.975 \text{ A}$$

$$I_{L1(\text{avg})} = 9.275$$

$$I_{L1(\text{avg})} > \frac{\Delta I_{L1}}{2}$$

CCM MODE

boost converter ① operates in ccm mode.

(iii) Boost converter ② :

$$V_{L2} = L_2 \frac{dI_{L2}}{dt}$$

$$\Delta I_{L2} = \frac{(V_{L2})(D_2)T}{L_2} = \frac{(198.60)(0.5033)}{0.2238 \times 10^{-3} \times 100 \times 10^{-3}}$$

$$\Delta I_{L2} = 4.466 \text{ A}$$

$$I_{L2(\text{avg})} > \frac{\Delta I_{L2}}{2}$$

CCM MODE

boost converter ② operate in ccm mode.

Conclusion : if we are using inductance values more than critical values in our calculation, it's 10-time greater, then it is obvious that inductor current will become continuous.

Calculated Results for case II : motor operating from standstill condition to rated condition. $SOC(1) = 25\%$ $SOC(2) = 75\%$

- (i) total current drawn by DC motor = $21.10A$
- (ii) input current of buck converter = $I_{S3} = 18.51A$
- (iii) Duty ratio of buck converter = $D_3 = 0.877$
- (iv) $L_{C3} = 10.19 \mu H$ (v) C_{C3} (cap. of output of buck) = $0.075 \mu F$.
- (vi) converter (boost) (II) :

$$\text{source current } I_{S1} = 9.275A$$

$$\text{Duty cycle } D_1 = 0.5011$$

$$\text{critical inductance for } L_1 = 0.054 \mu H$$

- (vii) boost converter (II) :

$$\text{source current } I_{S2} = 27.95 A$$

$$\text{Duty cycle } D_2 = 0.5033$$

$$\text{critical inductance for } L_2 = 0.0178 \mu H$$

- (viii) critical capacitance for $C_1 = C_{C1} = 0.115 \mu F$

CASE-3 ANALYSIS

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case 3rd | SOC(1) = 75%, SOC(2) = 75%, rated power decreased to 20% and torque = 50% of rated torque.

- calculating machine constant with rated DC conditions,

$$E_b = (K\phi)(\omega)$$

$$V_d - (J_q)_{\text{rated}}(\gamma_d) = (K\phi) \left(\frac{2\pi N}{60} \right)$$

$$350 - (20.6) \times 0.5 = (K\phi) \frac{2\pi \times 1500}{60}$$

$$K\phi = 2.16$$

for 3rd case, $P = 0.2 \times P_{\text{rated}}$, $T = 0.5 \times T_{\text{rated}}$, $\omega = x \times \omega_{\text{rated}}$.
 $0.2 = 0.5 \times x$.

$$x = 0.4$$

speed for 3rd case = 40% of rated speed!

$$V_d = E_b + J_q \gamma_d$$

$$= (2.16) \left(\frac{2\pi \times 0.4 \times 1500}{60} \right)$$

$$+ (10.3) \times (0.5)$$

$$T \propto J_q$$

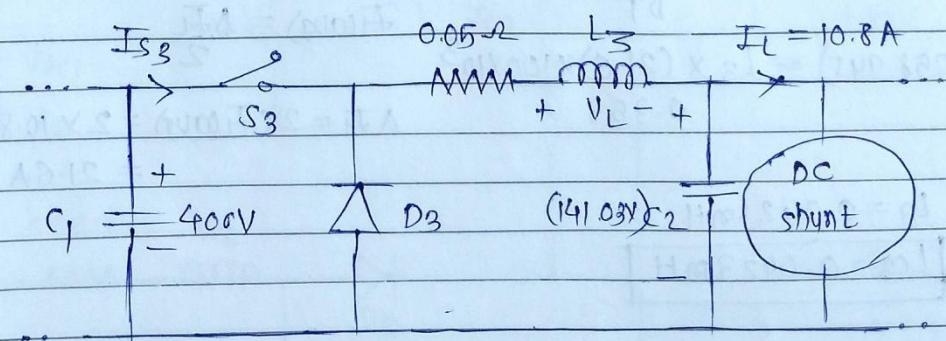
for 50% torque ($K\phi$) = constant,

$$J_q = 0.5 \times J_q(\text{rated})$$

$$= 0.5 \times 20.6 = 10.3 \text{ A}$$

$$V_d = 141.03 \text{ V}$$

$$I_L = (J_q) + (J_f) = 10.3 + 0.5 = 10.8 \text{ A}$$



(a) Calculation of duty cycle for buck converter :

$$I_L(\text{avg}) = 10.8 \text{ A}$$

Applying volt-sec balance to V across L₃ assuming CCM,

$$\int V_{L3} dt = 0$$

$$[V_{in} - V_0 - I_{L3}(\gamma)DT] + [-V_0 - I_{L3}\gamma(1-D)T] = 0$$

$$141.03 = 400D - I_{L3}\gamma$$

$$141.03 = 400D - 10.8 \times 0.05$$

$$\boxed{D_{\text{back}} = 0.353}$$

Calculation for I_{S3} using power balance to buck converter,

$$(400)(I_{S3}) = (I_{L3})^2 \times 0.05 + (141.03) \times (10.8)$$

$$= (10.8)^2 \times 0.05 + (141.03) \times (10.8)$$

$$\boxed{I_{S3} = 3.822 \text{ A}}$$

(b) Critical inductance calculation :

$$V_{L3} = (V_{in}) - V_0 - I_{L3}(\gamma) = (400 - 141.03) - (10.8)(0.05)$$

$$= 258.43 \text{ V}$$

$$V_L = L_3 \frac{di_{L3}}{dt} \quad I_{L3} \text{ charging.}$$

$$258.945 = L_3 \times \frac{\Delta I_{L3}}{DT}$$

for critical mode,

$$I_{L(\text{avg})} = \frac{\Delta I_L}{2}$$

$$(258.945) = L_3 \times (21.6) \times 100 \times 10^3$$

$$0.353$$

$$\Delta I_L = 2 \times I_{L(\text{avg})} = 2 \times 10.8$$

$$= 21.6 \text{ A}$$

$$L_3 = 0.0423 \text{ mH}$$

$$\boxed{L_{C3} = 0.0423 \text{ mH}}$$

(c) Critical capacitance calculation :

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$$C_2 \Delta V_C = \frac{1}{2} \times \frac{\Delta I_L}{2} \times \frac{T_S}{2}$$

$$C_{C_2} = \frac{\Delta I_L}{8 f(\Delta V_C)}$$

$$= (21.6)$$

$$\frac{8 \times 100 \times 10^3}{282.06} \times 282.06$$

$$\begin{aligned}\Delta V_C &= 2 \times V_0 \\ &= 2 \times (141.03) \\ &= 282.06V.\end{aligned}$$

$$C_{C_2} = 0.095 \mu F$$

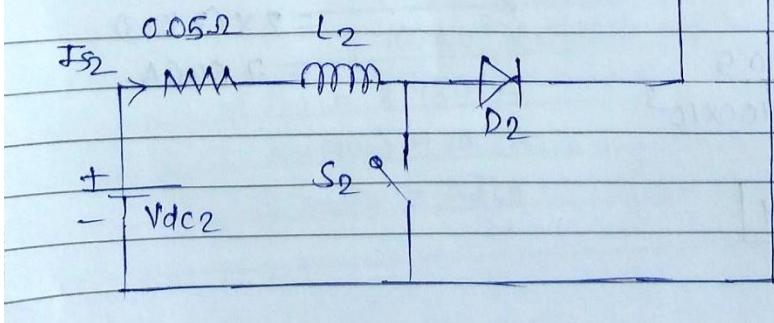
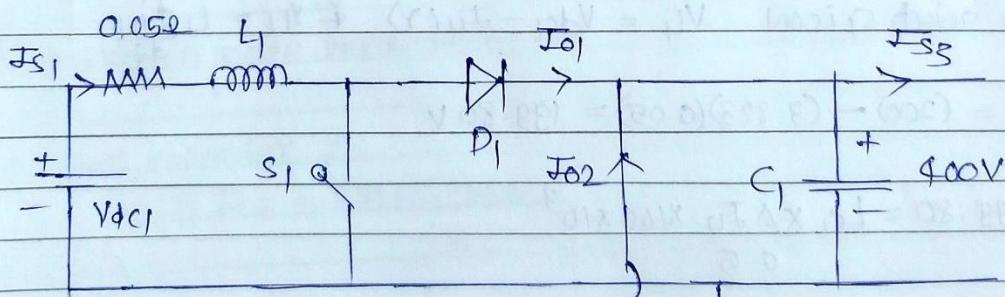
- Both boost converters will share their output current with reference to SOC.

$$\text{- current shared by boost converter I} = \frac{1}{2} (I_{S3}) = \frac{1}{2} \times (3.822)$$

$$I_{O1} = 1.91A.$$

$$\text{- current shared by boost converter II} = \frac{1}{2} (I_{S3}) = \frac{1}{2} \times (3.822)$$

$$I_{O2} = 1.91A.$$



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Power balance for boost converter (I) :

$$(V_{dc1}) \times I_{S1} = (I_{S1})^2 \times 0.05 + 400 \times J_{01}$$

$$\therefore 0.05 I_{S1}^2 - 200 I_{S1} + 764 = 0.$$

$$I_{S1} = 3.823 \text{ A}$$

Power balance for boost converter (II) :

$$(V_{dc2}) \times I_{S2} = (I_{S2})^2 \times 0.05 + 400 \times J_{02}$$

$$0.05 I_{S2}^2 - 200 I_{S2} + 764 = 0$$

$$I_{S2} = 3.823 \text{ A}$$

- Assuming CCM mode for both converters.

$$I_{S1} = \frac{J_{01}}{(1-D_1)}$$

$$D_1 = 0.50$$

similarly

$$D_2 = 0.50$$

(a) critical inductance calculation for L_1 :

$$\text{When switch } S_1 \text{ is on } V_L = V_{dc1} - J_{L1}(t) \quad \& \quad V_L = L_1 \frac{dI_L}{dt}$$

$$V_L = (200) - (3.823)(0.05) = 199.80 \text{ V}$$

$$199.80 = L_1 \times \Delta J_{L1} \times 100 \times 10^3$$

0.5

$$\text{for critical conduction mode } J_{L1(\text{avg})} = \frac{\Delta J_{L1}}{2} \quad \Delta J_{L1} = 2 \times J_{L1(\text{avg})} \\ = 2 \times (3.823) \\ = 7.646 \text{ A.}$$

$$L_1 = \frac{(199.80) \times 0.5}{(7.646) \times 100 \times 10^3}$$

$$L_1 = 0.1306 \text{ mH}$$

(b) critical inductance calculation for L_2 :

$$\text{When switch } s_2 \text{ is ON} \quad V_{L2} = (V_{DC2}) - \Delta I_{L2}(t) \quad V_{L2} = L_2 \frac{di_{L2}}{dt}$$

$$V_{L2} = (200) - (3.82)(0.05) = 199.80V.$$

$$\Delta I_{L2} = 2 \times I_{L2}(\text{avg}) = 2 \times (3.823) = 7.646A$$

$$L_{C2} = \frac{(199.80) \times 0.5}{(7.646) \times 100 \times 10^3}$$

$$L_{C2} = 0.1306 \text{ mH}$$

(c) critical capacitance calculation for C_1 :

$$\frac{\Delta V_{C1}}{2} = V_0 \quad \Delta V_{C1} = 2 \times 400 = 800V.$$

When any switch s_1 or s_2 is ON,

$$\therefore -(\Delta I_1 + \Delta I_2) = C_1 \frac{(-\Delta V_C)}{DT}$$

$$\therefore - (3.822) = C_1 \frac{(-800) \times 100 \times 10^3}{0.5}$$

$$C_{C1} = 0.02384F$$

(d) conduction mode verifications \Rightarrow

(i) Buck converter:

$$\Delta I_{L3} = \frac{(V_{L3})(P_{Buck})}{L_3} T$$

$$= \frac{(258.945)(0.353)}{100 \times 10^3 \times 0.1017 \times 10^{-3}}$$

$$\Delta I_{L3} = 8.987A$$

$$I_{L3}(\text{avg}) = 10.8A.$$

$$I_{L3}(\text{avg}) > \frac{\Delta I_{L3}}{2}$$

CCM MODE

(ii) Boost converter (I) :

$$\Delta I_{L1} = \frac{(V_{L1})(D_1)T}{L_1}$$

$$= (199.80) \times 0.5 \\ 100 \times 10^{-3} \times 0.836 \times 10^{-3}$$

$$\Delta I_{L1} = 2.97 \text{ A}$$

$$I_{L1(\text{avg})} = 3.823 \text{ A}$$

$$I_{L1(\text{avg})} > \frac{\Delta I_{L1}}{2}$$

CCM MODE

(iii) Boost converter (II) :

$$\Delta I_{L2} = \frac{(V_{L2})(D_2)T}{L_2}$$

$$= (199.80) \times 0.5 \\ 0.336 \times 10^{-3} \times 100 \times 10^{-3}$$

$$\Delta I_{L2} = 2.97 \text{ A}$$

$$I_{L2(\text{avg})} = 3.823 \text{ A}$$

$$I_{L2(\text{avg})} > \frac{\Delta I_{L2}}{2}$$

CCM MODECalculation Results for case III 8 $SOC(1)=75\%$, $SOC(2)=75\%$, $P=0.2 P_{\text{rated}}$.

$$T = 0.5 \times T_{\text{rated}}$$

(i) total current drawn by DC motor = 10.8 A.

(ii) input current of buck (I_{S3}) = 3.822 A(iii) Duty ratio of buck = 0.353 (iv) $L_{C3} = 0.042 \text{ mH}$ (v) $C_3 = 0.095 \mu\text{F}$

(vi) Boost converter (I)

- source current $I_{S1} = 3.823 \text{ A}$

$$- D_1 = 0.5$$

- critical inductance $L_{C1} = 0.1306 \text{ mH}$ - critical inductance $L_{C2} = 0.1306 \text{ mH}$ (vii) critical capacitance for C_1 $C_1 = 0.0238 \mu\text{F}$

(viii) Boost converter (II)

- source current $I_{S2} = 3.823 \text{ A}$

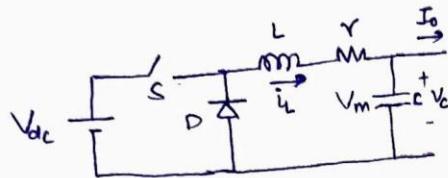
$$- D_2 = 0.5$$

CALCULATION FOR CONTROLLER PARAMETERS:

Transfer function of Buck Converter -

State variables - \hat{i}_L, \hat{v}_c input - V_{dc} Output - i_o

①

When switch is ON ($s=1$)

$$L \frac{di_L}{dt} = V_{dc} - v_c - i_L r \quad \text{---(1)}$$

$$C \frac{dv_c}{dt} = \hat{i}_L - I_o \quad \text{---(2)}$$

When switch is off ($s=0$)

$$L \frac{di_L}{dt} = -v_c - i_L r \quad \text{---(3)}$$

$$C \frac{dv_c}{dt} = \hat{i}_L - I_o \quad \text{---(4)}$$

Volt-Sec. Balance and Amp-Sec. Balance is applied -

$$L \frac{di_L}{dt} = (V_{dc} - v_c - i_L r) D + (-v_c - i_L r)(1-D) \quad \text{---(5)}$$

$$C \frac{dv_c}{dt} = \hat{i}_L - I_o \quad \text{---(6)}$$

small perturbation is given in duty cycle 'D' is \hat{D} , \rightarrow so variation in i_L and v_c is also observed \hat{i}_L and \hat{v}_c

$$\therefore L \frac{d(\hat{i}_L + \hat{i}_L)}{dt} = [V_{dc} - (\hat{v}_c + \hat{v}_c)] - (\hat{i}_L + \hat{i}_L)(0 + \hat{D}) + [-v_c - \hat{i}_L r - \hat{v}_c - \hat{i}_L r](1 - D - \hat{D})$$

$$\Rightarrow L \frac{d\hat{i}_L}{dt} + L \frac{d\hat{i}_L}{dt} = \cancel{[V_{dc} D - \hat{v}_c D - \hat{v}_c(1-D) - \hat{i}_L r(1-D) - \hat{v}_c D - \hat{i}_L r D + V_{dc} \hat{D} - \hat{v}_c \hat{D} - \hat{i}_L \hat{r} \hat{D}]} + \cancel{\hat{v}_c D} + \cancel{\hat{i}_L r D} + \cancel{\hat{v}_c \hat{D}} + \cancel{\hat{i}_L \hat{r} \hat{D}}$$

$$\Rightarrow L \frac{d\hat{i}_L}{dt} = \hat{i}_L [-rD - r(1-D)] + \hat{v}_c [-D - (1-D)] + \hat{D} [V_{dc} - \hat{v}_c - \hat{i}_L r + \hat{v}_c + \hat{i}_L r]$$

$$\therefore L \frac{d\hat{i}_L}{dt} = \hat{i}_L (-r) + \hat{v}_c (-1) + \hat{D} [V_{dc}]$$

$$\Rightarrow \frac{d\hat{i}_L}{dt} = \left(\frac{-r}{L} \right) \hat{i}_L + \left(\frac{-1}{L} \right) \hat{v}_c + \left(\frac{\hat{D}}{L} \right) V_{dc} \quad \text{---(7)}$$

and

$$C \frac{d(\hat{U}_c \hat{A}_c)}{dt} = \hat{i}_L + \hat{i}_R - I_0$$

2

$$\Rightarrow \frac{cdy_c}{dx} + \frac{cd\hat{y}_c}{dx} = jx + \hat{j}_c - j_0$$

$$\Rightarrow \frac{d\hat{v}_c}{dt} = \left(\frac{1}{c}\right) \hat{u} \quad - (8)$$

Output $y = \sum_i$ - (9)

state space model -

$$\begin{aligned} \dot{x} &= Ax + BU \\ y &= Cx + DU \end{aligned} \quad \left\{ \begin{array}{l} \dot{x} = \begin{bmatrix} \hat{d}_1 \\ \frac{\partial \hat{u}}{\partial t} \\ \vdots \\ \hat{d}_c \end{bmatrix}, \quad x = \begin{bmatrix} \hat{u} \\ \vdots \\ \hat{v}_c \end{bmatrix}, \quad u = \hat{D} \end{array} \right.$$

from ⑦ ⑧ and ⑨

$$\begin{bmatrix} \hat{\frac{di_L}{dt}} \\ \hat{\frac{d^2i_C}{dt^2}} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{r}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} \begin{bmatrix} \hat{i_L} \\ \hat{v_C} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{V_{dc}}{L} \\ 0 \end{bmatrix}}_{B}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = [0]$$

from this Transfer function of Buck Converter -

$$TF = C(SI - A)^{-1}B + D$$

$$[SI - A]^{-1} = \begin{bmatrix} s + \frac{Y}{L} & \frac{1}{L} \\ -\frac{1}{c} & s \end{bmatrix}^{-1} = \frac{1}{s(s + \frac{Y}{L}) + \frac{1}{Lc}} \begin{bmatrix} s & -\frac{1}{L} \\ \frac{1}{c} & s + \frac{Y}{L} \end{bmatrix}$$

$$TF = \frac{V_{dc}/LC}{s(s + \frac{Y}{L}) + \frac{1}{LC}} = \frac{\hat{v}_c}{\hat{D}}$$

(3)

Transfer function of PI controller -

$$= K_p + \frac{K_I}{s}$$

$$= \left[\frac{K_p s + K_I}{s} \right]$$

$$\therefore \left[\frac{K_p s + K_I}{s} \right] \left[\frac{V_{dc}/LC}{s(s + \frac{Y}{L}) + \frac{1}{LC}} \right] \text{ for this, } \cancel{\text{should be}} \text{ Phase margin should be positive.}$$

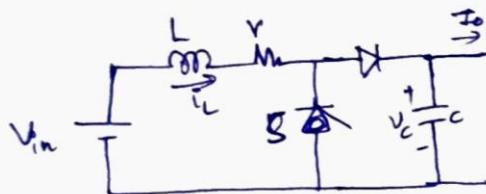
Transfer function of Boost Converter -

State Variable - \hat{i}_L, \hat{v}_c

input - \hat{v}_in

output - \hat{v}_c (for Voltage Controlled Converter)

\hat{i}_L (for current controlled converter)



When switch is ON ($s=1$)

$$L \frac{di_L}{dt} = V_{in} - i_L r - ①$$

$$C \frac{dv_c}{dt} = -I_o - ②$$

When switch 's' is off ($s=0$)

$$L \frac{di_L}{dt} = V_{in} - V_c - i_L r - ③$$

$$C \frac{dv_c}{dt} = i_L - I_o - ④$$

Volt - sec. Balance and Amps sec. Balance is applied -

$$L \frac{di_L}{dt} = (V_{in} - i_L r) D + (V_{in} - V_c - i_L r)(1-D) - ⑤$$

$$C \frac{dv_c}{dt} = (-I_o) D + (i_L - I_o)(1-D) - ⑥$$

small perturbation is given in duty cycle 'D' is \hat{D} \rightarrow so variation in i_L and v_c is also observed \hat{i}_L and \hat{v}_c

$$L \frac{di_L + \hat{i}_L}{dt} = (V_{in} - i_L r - \hat{i}_L r)(D + \hat{D}) + (V_{in} - V_c - \hat{v}_c - i_L r - \hat{i}_L r)(1-D - \hat{D})$$

$$\Rightarrow L \frac{di_L}{dt} + L \frac{\hat{i}_L}{dt} = (V_{in} - i_L r) D + (V_{in} - V_c - i_L r)(1-D) - \hat{i}_L r D + V_{in} \hat{D} - \hat{i}_L r \hat{D} - \hat{i}_L (1-D) - \hat{i}_L r (1-D) + V_{in} \hat{D} + V_c \hat{D} + I_o \hat{D}$$

$$\Rightarrow \frac{L \hat{i}_L}{dt} = i_L (-rD - r(1-D)) + \hat{i}_L (1-D) + V_c \hat{D}$$

$$\Rightarrow \frac{d \hat{i}_L}{dt} = \left(\frac{-r}{L}\right) \hat{i}_L + \left(\frac{1-D}{L}\right) \hat{v}_c + \left(\frac{V_c}{L}\right) \hat{D} - ⑦$$

Also,

$$C \frac{d(\hat{v}_c + \hat{v}_i)}{dt} = -I_o(D + \hat{D}) + (i_L + \hat{i}_L - I_o)(1 - D - \hat{D})$$

$$\Rightarrow C \frac{d\hat{v}_c}{dt} + C \frac{d\hat{v}_i}{dt} = -I_o D + (i_L - I_o)(1 - D) - \hat{i}_L \hat{D} + \hat{i}_L D$$

$$\Rightarrow \frac{d\hat{v}_c}{dt} = \left(\frac{1-D}{C} \right) \hat{i}_L + \left(\frac{i_L}{C} \right) \hat{D} \quad \text{--- (8)}$$

from (7) & (8)

$$\begin{bmatrix} \frac{d\hat{i}_L}{dt} \\ \frac{d\hat{v}_c}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{r}{L} & \frac{(1-D)}{L} \\ \frac{(1-D)}{C} & 0 \end{bmatrix}}_A \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{v_c}{L} \\ -\frac{i_L}{C} \end{bmatrix}}_B \hat{D}$$

for Voltage Controlled Converter $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

$$\therefore \text{Transfer function } \frac{\hat{v}_c(s)}{\hat{D}(s)} = C[SI - A]^{-1} B = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \frac{r}{L} & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & s \end{bmatrix}^{-1} \begin{bmatrix} \frac{v_c}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$\frac{\hat{v}_c(s)}{\hat{D}(s)} = \frac{1}{s(s + \frac{r}{L}) - \frac{(1-D)^2}{LC}} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + \frac{r}{L} & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & s + \frac{r}{L} \end{bmatrix}^{-1} \begin{bmatrix} \frac{v_c}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$\frac{\hat{v}_c(s)}{\hat{D}(s)} = \frac{1}{s(s + \frac{r}{L}) - \frac{(1-D)^2}{LC}} \begin{bmatrix} \frac{(1-D)}{C} & \left(s + \frac{r}{L} \right) \\ \frac{(1-D)}{C} & s + \frac{r}{L} \end{bmatrix} \begin{bmatrix} \frac{v_c}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$\boxed{\frac{\hat{v}_c(s)}{\hat{D}(s)} = \frac{\frac{v_c(1-D)}{LC} - \frac{i_L(s+r/L)}{C}}{s(s+r/L) - \frac{(1-D)^2}{LC}}}$$

for PI controller parameters
 $\Rightarrow \left(\frac{K_p s + K_I}{s} \right) \times \frac{\hat{v}_c}{\hat{D}}$ should have phase margin in positive.

for current controlled Converter-

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\therefore \text{Transfer function } \frac{\hat{i}_L(s)}{\hat{D}(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s + \frac{r}{L} & -\left(\frac{1-D}{L}\right) \\ -\left(\frac{1-D}{C}\right) & s \end{bmatrix}^{-1} \begin{bmatrix} \frac{v_C}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$\frac{\hat{i}_L(s)}{\hat{D}(s)} = \frac{1}{s(s + \frac{r}{L}) - \frac{(1-D)^2}{LC}} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & \frac{1-D}{L} \\ \frac{1-D}{C} & s + \frac{r}{L} \end{bmatrix} \begin{bmatrix} \frac{v_C}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$= \frac{1}{s(s + \frac{r}{L}) - \frac{(1-D)^2}{LC}} \begin{bmatrix} s & \frac{1-D}{L} \\ \frac{1-D}{C} & s + \frac{r}{L} \end{bmatrix} \begin{bmatrix} \frac{v_C}{L} \\ -\frac{i_L}{C} \end{bmatrix}$$

$$\boxed{\frac{\hat{i}_L(s)}{\hat{D}(s)} = \frac{s\left(\frac{v_C}{L}\right) - \frac{\hat{i}_L(1-D)}{LC}}{s\left(s + \frac{r}{L}\right) - \frac{(1-D)^2}{LC}}}$$

for PI Controller -

$$\Rightarrow \left(\frac{K_P s + K_I}{s} \right) \left(\frac{\hat{i}_L(s)}{\hat{D}(s)} \right)$$

Should have positive phase and gain margin.

Group 2

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