In-memory Incremental Maintenance of Provenance Sketches

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Abstract

Provenance-based data skipping [37] compactly over-approximates the provenance of a query using so-called provenance sketches and utilizes such sketches to speed-up the execution of subsequent queries by skipping irrelevant data. However, a sketch captured at some time in the past may become stale if the data has been updated subsequently. Thus, there is a need to maintain provenance sketches. In this work, we introduce **I**n-Memory incremental **M**aintenance of **P**rovenance sketches (IMP), a framework for maintaining sketches incrementally under updates. At the core of IMP is an incremental query engine for data annotated with sketches that exploits the coarse-grained nature of sketches to enable novel optimizations. We experimentally demonstrate that IMP significantly reduces the cost of sketch maintenance, thereby enabling the use of provenance sketches for a broad range of workloads that involve updates.

Q_{Top}

```
SELECT brand, SUM(price * numSold) AS rev
FROM sales
GROUP BY brand
HAVING SUM(price * numSold) > 5000
```

brand	rev
Apple	5074

sales

	sid	brand	productName	price	numSold	
s_1	1	Lenovo	ThinkPad T14s Gen 2	349	1	f_1
s_2	2	Lenovo	ThinkPad T14s Gen 2	449	2	f_1
s_3	3	Apple	MacBook Air 13-inch	1199	1	f_3
s_4	4	Apple	MacBook Pro 14-inch	3875	1	f_4
s_5	5	Dell	Dell XPS 13 Laptop	1345	1	f_3
s_6	6	HP	HP ProBook 450 G9	999	4	f_2
s_7	7	HP	HP ProBook 550 G9	899	1	f_2

Figure 1: Example query and relevant subsets of the database.

1 Introduction

Database engines take advantage of physical design such as index structures, zone maps [32] and partitioning to prune irrelevant data as early as possible during query evaluation. In order to prune data, database systems need to determine statically (at query compile time) what data is needed to answer a query and which physical design artifacts to use to skip irrelevant data. For instance, to answer a query with a `WHERE` clause condition `A = 3` filtering the rows of a table `R`, the optimizer may decide to use an index on `A` to filter out rows that do not fulfill the condition. However, as was demonstrated in [37], for important classes of queries like queries involving top-k and aggregation with `HAVING`, it is not possible to determine *statically* what data is needed, motivating the use of *dynamic relevance analysis* techniques that determine during query execution what data is relevant to answer a query. In [37] we introduced such a dynamic relevance analysis technique called provenance-based data skipping (PDBS). In PDBS, we encode what data is relevant for a query as a so-called *provenance sketch*. Given a range-partition of a table accessed by a query, a provenance sketch records which fragments of the partition contain provenance. That is, provenance sketches compactly encode an over-approximation of the provenance of a query. [37] presents safety conditions that ensure a sketch is *sufficient*, i.e., evaluating the query over the data represented by the sketch is guaranteed to produce the same result as evaluating the query over the full database. Thus, sketches are used to speed up queries by filtering data not in the sketch.

EXAMPLE 1.1. *Consider the database shown in Fig. 1 and query Q_{Top} that returns products whose total sale volume is greater than \$5000. The provenance of the single result tuple* (Apple, 5074) *are the two tuples (tuples s_3 and s_4 shown with purple background), as the group for Apple is the only group that fulfills the* `HAVING` *clause. To create a provenance sketch for this query, we select a range-partition of the* `sales` *table that optionally may correspond to the physical storage layout of this table. For instance, we may choose to partition on attribute* `price` *based on ranges ϕ_{price}:*

$$\{\rho_1 = [1, 600], \rho_2 = [601, 1000], \rho_3 = [1001, 1500], \rho_4 = [1501, 10000]\}$$

In Fig. 1, we show the fragment f_i for the range ρ_i each tuple belongs to. Two fragments (f_3 and f_4 highlighted in red) contain provenance and, thus, the provenance sketches for Q_{Top} wrt. $F_{sales,price}$ is $\mathcal{P} = \{\rho_3, \rho_4\}$. Evaluating the query over the sketch's data is guaranteed to produce the same result as evaluation on the full database.[1]

As demonstrated in [37], provenance-based data skipping can significantly improve query performance — we pay upfront for creating sketches for some of the queries of a workload and then amortize this cost by using sketches to answer future queries by skipping irrelevant data. To create, or *capture*, a sketch for a query Q we execute an instrumented version of Q. Similarly, to use a sketch for a query Q, this query is instrumented to filter out data that does not belong to the sketch. For instance, consider the sketch for Q_{Top} from Ex. 1.1 containing two ranges $\rho_3 = [1001, 1500]$ and $\rho_4 = [1501, 10000]$. To skip irrelevant data, we create a disjunction

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[1] In general, this is not the case for non-monotone queries. The safety check from [37] can be used to test whether a particular partition for a table is safe. The partition used here fulfills this condition.

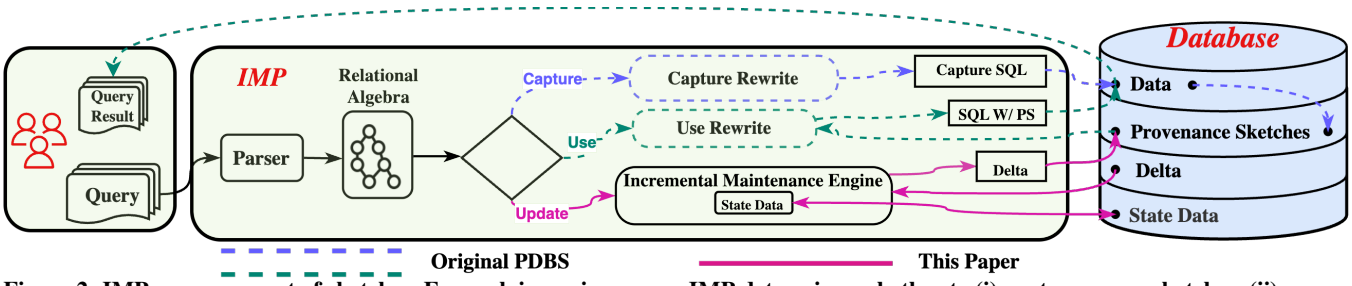


Figure 2: IMP manages a set of sketches. For each incoming query, IMP determines whether to (i) capture a new sketches, (ii) use an existing non-stale sketch, or (iii) incrementally maintain a stale sketch and then utilize the updated sketch to answer the query.

of conditions testing that each tuple passing the `WHERE` clause belongs to the sketch, i.e., has a price within ρ_3 or ρ_4:[2]

```
WHERE (price BETWEEN 1001 AND 1500)
   OR (price BETWEEN 1501 AND 10000)
```

PDBS enables databases to exploit physical design for new classes of queries, significantly improving the performance of aggregation queries with `HAVING` and top-k queries [37] and, more generally, any query where only a fraction of the database is relevant for answering the query. For instance, for a top-k query only tuples contributing to a result tuple in the top-k are relevant, but which tuples are in the top-k can only be determined at runtime. A counterexample are queries with selection conditions with low selectivity for which the database can effectively filter the data without PDBS.

However, just like materialized views, a sketch captured in the past may no longer correctly reflect what data is needed (has become *stale*) when the database is updated. The sketch then has to be maintained to be valid for the current version of the database.

EXAMPLE 1.2 (STALE SKETCHES). *Continuing with our running example, consider the effect of inserting a new tuple*

s_8 = `(8, HP, HP ProBook 650 G10, 1299, 1)`

into relation ***sales****. Running Q_{Top} over the updated table returns a second result tuple* `(HP, 6194)` *as the total revenue for HP is now above the threshold specified in the* `HAVING` *clause. For the updated database, the three tuples for HP also belong to the provenance. Thus, the sketch has become stale as it is missing the range ρ_2 which contains these tuples. Evaluating Q_{Top} over the outdated sketch leads to an incorrect result that misses the group for HP.*

Consider a partition F of a table R accessed by a query Q. We use $Q_{R,F}$ to denote the *capture query* for Q and F, generated using the rewrite rules from [37]. Such a query propagates coarse-grained provenance information and ultimately returns a sketch. A straightforward approach to maintain sketches under updates is *full maintenance* which means that we rerun the sketch's *capture query* $Q_{R,F}$ to regenerate the sketch. Typically, $Q_{R,F}$ is more expensive than Q. Thus, frequent execution of capture queries is not feasible. Alternatively, we could employ incremental view maintenance (IVM) techniques [23, 26] to maintain $Q_{R,F}$. However, capture queries use specialized data types and functions to efficiently implement common operations related to sketches. For instance, we use bitvectors to encode sketches compactly and utilize optimized (aggregate) functions and comparison operators for this encoding. To give two concrete examples, a function implementing binary search over the set of ranges for a sketch is used to determine which fragment an input tuple belongs to and an aggregation function that computes the bitwise-or of multiple bitvectors is used to implement the union of a set of partial sketches. To the best of our knowledge these operations are not supported by state-of-the-art IVM frameworks. Furthermore, sketches are compact over-approximations of the provenance of a query that are *sound*: evaluating the query over the sketch yields the same result as evaluating it over the full database. It is often possible to further over-approximate the sketch, trading improved maintenance performance for increased sketch size. Existing IVM methods do not support such trade-offs as they have to ensure that incremental maintenance yields the same result as full maintenance.

In this work, we study the problem of maintaining sketches under updates such that a sketch created in the past can be updated to be valid for the current state of the database. Towards this goal we develop an incremental maintenance framework for sketches that respects the approximate nature of sketches, has specialized data structures for representing data annotated with sketches, and maintenance rules tailored for data annotated with sketches.

We start by introducing a data model where each row is associated with a sketch and then develop incremental maintenance rules for operators over such annotated relations. We then present an implementation of these rules in an in-memory incremental engine called IMP (**I**ncremental **M**aintenance of **P**rovenance Sketches). The input to this engine is a set of annotated delta tuples (tuples that are inserted / deleted) that we extract from a backend DBMS. To maintain a sketch created by a capture query $Q_{R,F}$ at some point in the past, we extract the delta between the current version of the database and the database instance at the original time of capture (or the last time we maintained the sketch) and then feed this delta as input to our incremental engine to compute a delta for the sketch. IMP outsources some of the computation to the backend database. This is in particular useful for operations like joins where deltas from one side of the join have to be joined with the full table on the other side similar to the delta rule $\Delta R \bowtie S$ used in standard incremental view maintenance. Additionally, we present several optimizations of our approach: (i) filtering deltas determined by the database to prune delta tuples that are guaranteed to not affect the result of incremental maintenance and (ii) filtering deltas for joins using bloom filters. IMP is effective for any query that benefits from sketches, e.g., queries with `HAVING`, as long as the cost of maintaining sketches is amortized by using sketches for answering queries.

[2] Note that the conditions for adjacent ranges in a sketch can be merged. Thus, the actual instrumentation would be `WHERE price BETWEEN 1001 AND 10000`.

In summary, we present IMP, the first incremental engine for maintaining provenance sketches. Our main contributions are:

- We develop incremental versions of relational algebra operators for sketch-annotated data.
- We implement these operators in IMP, an in-memory engine for incremental sketch maintenance. IMP enables PDBS for any DBMS by acting as a middleware between the user and the database that manages and maintains sketches.
- We experimentally compare IMP against full maintenance and against a baseline that does not use PDBS using TPC-H, real world datasets and synthetic data. IMP outperforms full maintenance, often by several orders of magnitude. Furthermore, PDBS with IMP significantly improves the performance of mixed workloads including both queries and updates.

The remainder of this paper is organized as follows: Sec. 2 presents an overview of IMP. We discuss related work in Sec. 3. We formally define incremental maintenance of sketches and introduce our annotated data model in Sec. 4. In Sec. 5, we introduce incremental sketch maintenance rules for relational operators and prove their correctness in Sec. 6. We discuss IMP's implementation in Sec. 7, present experiments in Sec. 8, and conclude in Sec. 9.

2 Overview of IMP

Fig. 2 shows a overview of IMP that operates as a middleware between the user and a DBMS. We highlight parts of the system that utilize techniques from [37]. The dashed blue pipeline is for capture rewrite and dashed green pipeline is for use rewrite. Users send SQL queries and updates to IMP that are parsed using IMP's parser and translated into an intermediate representation (relational algebra with update operations). The system stores a set of provenance sketches in the database. For each sketch we store the sketch itself, the query it was captured for, the current state of incremental operators for this query, and the database version it was last maintained at or first captured at for sketches that have not been maintained yet. As sketches are small (100s of bytes), we treat sketches as immutable and retain some or all past versions of a sketch. This has the advantage that it avoids write conflicts (for updating the sketch) between concurrent transactions that need to update to different versions of the sketch. We assume that the DBMS uses snapshot isolation and we can use snapshot identifiers used by the database internally to identify versions of sketches and of the database. For systems that use other concurrency control mechanisms, IMP can maintain version identifiers. Furthermore, the system can persist the state that it maintains for its incremental operators in the database. This enables the system to continue incremental maintenance from a consistent state, e.g., when the database is restarted, or when we are running out of memory and need to evict the operator states for a query. IMP enables PDBS for workloads with updates on-top of any SQL databases.

IMP supports multiple incremental maintenance strategies. Under *eager* maintenance, the system incrementally maintains each sketch that may be affected by the update (based on which tables are referenced by the sketch's query) by processing the update, retrieving the delta from the database, and running the incremental maintenance. Eager maintenance can be configured to batch updates. If the operator states for a sketch's query are not currently in memory, they will be fetched from the database. The updates to the sketches determined by incremental maintenance are then directly applied. Under *lazy* maintenance, the system passes updates directly to the database. When a sketch is needed to answer a query, this triggers maintenance for the sketch. For that, IMP fetches the delta between the version of the database at the time of the last maintenance for the sketch and the current database state and incrementally maintains the sketch. The result is a sketch that is valid as of the current state of the database. More advanced strategies can be designed on top of these two primitives, e.g., triggering eager maintenance during times of low resource usage or eagerly maintaining sketches for queries with strict response time requirements to avoid slowing down such queries when maintenance is run for a large batch of updates.

For queries sent by the user, IMP first determines whether there exists a sketch that can be used to answer the query Q. For that, it applies the mechanism from [37] to determine whether a sketch captured for a query Q' in the past can be safely used to answer Q. If such a sketch $\mathcal{P}$ exists, we determine whether $\mathcal{P}$ is stale. If that is the case, then IMP incrementally maintains the sketch (solid red pipeline). Afterwards, the query Q is instrumented to filter input data based on sketch $\mathcal{P}$ and then the instrumented query is sent to the database and its results are forwarded to the user (dashed green pipeline)[37]. If no existing provenance sketch can be used to answer Q, then IMP creates a capture query for Q and evaluates this query to create a new sketch $\mathcal{P}$ (dashed blue pipeline) [37]. This sketch is then used to answer Q (dashed green pipeline) [37]. IMP is an in-memory engine, exploiting the fact that sketches are small and that deltas and state required for incremental maintenance are typically small enough to fit into main memory or can be split into multiple batches if this is not that case.

3 Related Work

Provenance. Provenance can be captured by annotating data and propagating these annotations using relational queries or by extending the database system [25] [39] [38]. Systems like GProM [7], Perm [19], Smoke [41], Smoked Duck [33], Links [16], ProvSQL[42] and DBNotes[8] capture provenance for SQL queries. In [37], we introduced provenance-based data skipping (PBDS). The approach captures sketches over-approximating the provenance of a query and utilizes these sketches to speed-up subsequent queries. We present the first approach for maintaining sketches under updates, thus, enabling efficient PBDS for databases that are subject to updates.

Incremental View Maintenance (IVM). View maintenance has been studied extensively [9, 11, 17, 23, 27, 43]. [22, 44] gives an overview of many techniques and applications of view maintenance. Early work on view maintenance, e.g., [9, 11], used set semantics. This was later expanded to bag semantics (e.g., [13, 20]). We consider bag semantics. Materialization has been studied for Datalog as well [21, 23, 34]. Incremental maintenance algorithms for iterative computations have been studied in [1, 10, 35, 36]. [26] proposed higher-order IVM. [45] maintains aggregate views in temporal databases. [40] proposes a general mechanism for aggregation functions. [2, 47] studied automated tuning of materialized views and indexes in databases. As mentioned before, existing view maintenance techniques can not be directly applied for provenance sketches

$D(\Delta D)$	a database (a delta database)
$\mathscr{D}(\Delta\mathscr{D})$	an annotated database (an annotated delta database)
$R(\mathscr{R}, \Delta\mathscr{R})$	a relation (an annotated relation, an annotated delta relation)
$\langle t, \mathcal{P}\rangle$	an annotated tuple: a tuple t associated with its provenance sketch $\mathcal{P}$
ρ, ϕ, Φ	a range, set of ranges for partitioning relation R (database D)
$\mathcal{S}$	State of an incremental relational algebra operator
$\mathcal{P}$	a provenance sketch
$\mathcal{I}$	an incremental maintenance procedure
Q	a query

Figure 3: Glossary

$$\sigma_\theta(R) = \{\!\{t^n | t^n \in R \wedge t \models \theta\}\!\} \quad \Pi_A(R) = \{\!\{t^n | n = \sum_{u.A=t} R(u)\}\!\}$$
$$\delta(R) = \{\!\{t^1 | t \in R\}\!\} \quad R \times S = \{\!\{(t \circ s)^{n*m} | t^n \in R \wedge s^m \in S\}\!\}$$
$$\gamma_{f(a);G}(R) = \{\!\{(t.G, f(G_t))^1 | t \in R\}\!\} \quad G_t = \{(t_1.a)^n | {t_1}^n \in R \wedge t_1.G = t.G\}$$
$$\tau_{k,O}(R) = \{\!\{t^m \mid \mathbf{pos}(t, R, O) < k \wedge m = min(R(t), k - \mathbf{pos}(t, R, O))\}\!\}$$

Figure 4: Bag Relational Algebra

maintenance, since [37] uses specialized data types and functions to efficiently handle sketches during capture, which are not supported in start-of-the-art IVM systems. Furthermore, classical IVMs solutions have no notion of over-approximating query results and, thus, can not trade sketch accuracy for performance. Several strategies have been studied for maintaining views eagerly and lazily. For instance, [14] presented algorithms for deferred maintenance and [9, 12, 23] studied immediate view maintenance. Our approach supports both cases: immediately maintaining sketches after each update or sketches can be updated lazily when needed.

Maintaining Provenance. [46] presents a system for maintenance of provenance in a distributed Datalog engine. In contrast to our work, [46] is concerned with efficient distributed computation and storage for provenance. Provenance maintenance has to deal with large provenance annotations that are generated by complex queries involving joins and operations like aggregation that compute a small number of result tuples based on a large number of inputs. [46] addresses this problem by splitting the storage of provenance annotations across intermediate query results requiring recursive reconstruction at query time. In contrast, provenance sketches are small and their size is determined upfront based on the partitioning that is used. Because of this and because of their coarse-grained nature, sketches enable new optimizations, including trading accuracy for performance.

4 Background and Problem Definition

In this section we introduce necessary background and introduce notation used in the following sections. Let $\mathbb{U}$ be a domain of values. An instance R of an n-ary relation schema $\text{SCH}(R) = (a_1, \ldots, a_n)$ is a function $\mathbb{U}^n \to \mathbb{N}$ mapping tuples to their multiplicity. We use $\{\!\{\cdot\}\!\}$ to denote bags and $t^n \in R$ to denote tuple t with multiplicity n in relation R, i.e., $R(t) = n$. A database D is a set of relations R_1 to R_m. The schema of a database $\text{SCH}(D)$ is the set of relation schemas $\text{SCH}(R_i)$ for $i \in [1, m]$. Fig. 4 shows the bag semantics relational algebra used in this work. We use $\text{SCH}(Q)$ to denote the schema of the query Q and $Q(D)$ to denote the result of evaluating query Q over database D. Selection $\sigma_\theta(R)$ returns all tuples from relation R which satisfy the condition θ. Projection $\Pi_A(R)$ projects all input tuples on a list of projection expressions. Here, A denotes a list of expressions with potential renaming (denoted by $e \to a$) and $t.A$ denotes applying these expressions to a tuple t. For example, $a+b \to c$ denotes renaming the result of $a+b$ as c. $R \times S$ is the cross product for bags. For convenience we also define join $R \bowtie_\theta S$ and natural join $R \bowtie S$ in the usual way. Aggregation $\gamma_{f(a);G}(R)$ groups tuples according to their values in attributes G and computes the aggregation function f over the bag of values of attribute a for each group. We also allow the attribute storing $f(a)$ to be named explicitly, e.g., $\gamma_{f(a)\to x;G}(R)$, renames $f(a)$ as x. Duplicate removal $\delta(R)$ removes duplicates (definable using aggregation). Top-K $\tau_{k,O}(R)$ returns the first k tuples from the relation R sorted on order-by attributes O. We use $<_O$ to denote the order induced by O. The position of a tuple in R ordered on O is denoted by $\mathbf{pos}(t, R, O)$ and defined as: $\mathbf{pos}(t, R, O) = \sum_{t' <_O t \wedge t^m \in R} m$. Fig. 3 shows an overview of the notations used in this work.

4.1 Range-based Provenance Sketches

We use provenance sketches to concisely represent a superset of the provenance of a query (a sufficient subset of the input) based on horizontal partitions of the input relations of the query.

4.1.1 *Range Partitioning.*

Given a set of intervals over the domains of a set of *partition attributes* $A \subset \text{SCH}(R)$, *range partitioning* determines membership of tuples to fragments based on their A values. For simplicity, we define partitioning for a single attribute a, but all of our techniques also apply when $|A| > 1$.

DEFINITION 4.1 (RANGE PARTITION). *Consider a relation R and $a \in \text{SCH}(R)$. Let $\mathbb{D}(a)$ denote the domain of a and $\phi = \{\rho_1, \ldots, \rho_n\}$ be a set of intervals $[l, u] \subseteq \mathbb{D}(a)$ such that $\bigcup_{i=0}^{n} \rho_i = \mathbb{D}(a)$ and $\rho_i \cap \rho_j = \emptyset$ for $i \neq j$. The* range-partition *of R on a according to ϕ denoted as $F_{\phi,a}(R)$ is defined as:*

$$F_{\phi,a}(R) = \{R_{\rho_1}, \ldots, R_{\rho_n}\} \quad \textbf{\textit{where}} \quad R_\rho = \{\!\{t^n \mid t^n \in R \wedge t.a \in \rho\}\!\}$$

We will use F instead of $F_{\phi,a}$ if ϕ and a are clear from the context and f, f', f_i, etc. to denote fragments. We also extend range partitioning to databases. For a database $D = \{R_1, \ldots, R_n\}$, we use Φ to denote a set of range - attribute pairs $\{(\phi_1, a_1), \ldots, (\phi_n, a_n)\}$ such that F_{ϕ_i,a_i} is a partition for R_i. Relations R_i that do not have a sketch can be modeled by setting $\phi_i = \{[min(\mathbb{D}(a_i)), max(\mathbb{D}(a_i))]\}$, a single range covering all domain values.

4.1.2 *Provenance Sketches.*

Consider a database D, query Q, and a range partition F_Φ of D. We use $P(Q, D) \subseteq D$ to denote the provenance of Q wrt. D. For the purpose of PDBS, any provenance model that represents the provenance of Q as a subset of D can be used as long as the model guarantees *sufficiency*[3] [18]: $Q(P(Q, D)) = Q(D)$. A provenance sketch $\mathcal{P}$ for Q according to Φ is a subset of the ranges ϕ_i for each $\phi_i \in \Phi$ such that the fragments corresponding to the ranges in $\mathcal{P}$ fully cover Q's provenance within each R_i in D, i.e.,$P(Q, D) \cap R_i$. We will write $\rho \in \Phi$ to denote that $\rho \in \phi_i$ for some $\phi_i \in \Phi$ and D_ρ for ρ from ϕ_i to denote the subsets of the database where all relations are empty except for R_i which is set to $R_{i,\rho}$, the

[3] Note that our notion of sufficiency aligns with the one from [24] which differs slightly from the one used in [18] that is defined for a single result tuple of Q.

fragment for ρ. We use $\mathcal{P}_\Phi(D, \Phi, Q) \subseteq \Phi$ to denote the set of ranges whose fragments overlap with the provenance $P(Q, D)$:

$$\mathcal{P}_\Phi(D, \Phi, Q) = \{\rho \mid \rho \in \phi_i \wedge \exists t \in P(Q, D) : t \in R_{i,\rho}\}$$

Definition 4.2 (Provenance Sketch). *Let Q be a query, D a database, R a relation accessed by Q, and Φ a partition of D. We call a subset $\mathcal{P}$ of Φ a **provenance sketch** iff $\mathcal{P} \supseteq \mathcal{P}_\Phi(D, \Phi, Q)$. A sketch is **accurate** if $\mathcal{P} = \mathcal{P}_\Phi(D, \Phi, Q)$. The **instance** $D_\mathcal{P}$ of $\mathcal{P}$ is defined as $D_\mathcal{P} = \bigcup_{\rho \in \mathcal{P}} D_\rho$. A sketch is **safe** if $Q(D_\mathcal{P}) = Q(D)$.*

Consider the database consisting of a single relation (`sales`) from our running example shown in Fig. 1. According to the partition $\Phi = \{(\phi_{price}, price)\}$, the accurate provenance sketch $\mathcal{P}$ for the query Q_{Top} according to Φ consists of the set of ranges $\{\rho_3, \rho_4\}$ (the two tuples in the provenance of this query highlighted in Fig. 1 belong to the fragments f_3 and f_4 corresponding to these ranges). The instance $D_\mathcal{P}$, i.e., the data covered by the sketch, consists of all tuples contained in fragments f_3 and f_4 which are: $\{s_3, s_4, s_5\}$. This sketch is safe. We use the method from [37] to determine for an attribute a and query Q whether a sketch build on any partition of R on a will be safe.

4.2 Updates, Histories, and Deltas

For the purpose of incremental maintenance we are interested in the difference between database states. Given two databases D_1 and D_2 we define the *delta* between D_1 and D_2 to be the symmetric difference between D_1 and D_2 where tuples t that have to be inserted into D_1 to generate D_2 are tagged as $\mathbb{\Delta}t$ and tuples that have to be deleted to derive D_2 from D_1 are tagged as Δt:

$$\Delta(D_1, D_2) = \{\!\{\Delta t \mid t \in D_1 - D_2\}\!\} \cup \{\!\{\mathbb{\Delta} t \mid t \in D_2 - D_1\}\!\}$$

For a given delta ΔD, we use ΔD ($\mathbb{\Delta} D$) to denote $\{\!\{\Delta t \mid \Delta t \in \Delta D\}\!\}$ ($\{\!\{\mathbb{\Delta} t \mid \mathbb{\Delta} t \in \Delta D\}\!\}$). We use $D \uplus \Delta D$ to denote *applying* delta ΔD to database D:

$$D \uplus \Delta D = D - \{\!\{\Delta t \mid \Delta t \in \Delta D\}\!\} \cup \{\!\{\mathbb{\Delta} t \mid \mathbb{\Delta} t \in \Delta D\}\!\}$$

Example 4.1. *Reconsider the insertion of tuple s_8 (also shown below) into **sales** as shown in Ex. 1.2.*

s_8 = `(8, HP, HP ProBook 650 G10, 1299, 1)`

Let us assume that the database before (after) the insertion of this tuple is D_1 (D_2), then we get: $\Delta D_2 = \{\!\{\mathbb{\Delta}\ s_8\}\!\}$

We use the same delta notation for sketches, e.g., for two sketch versions $\mathcal{P}_1$ and $\mathcal{P}_2$, $\Delta\mathcal{P}$ is their delta if $\mathcal{P}_2 = \mathcal{P}_1 \uplus \Delta\mathcal{P}$, where $\uplus$ on sketches is defined as expected by inserting $\mathbb{\Delta}\mathcal{P}$ and deleting $\Delta\mathcal{P}$.

4.3 Sketch-Annotated Databases And Deltas

Our incremental maintenance approach utilizes relations whose tuples are annotated with sketches. We define an incremental semantics for maintaining the results of operators over such annotated relations and demonstrate that this semantics correctly maintains sketches.

Definition 4.3 (Sketch Annotated Relation). *A sketch annotated relation $\mathcal{R}$ of arity m for a given set of ranges ϕ over the domain of some attribute $a \in \text{Sch}(R)$, is a bag of pairs $\langle t, \mathcal{P}\rangle$ such that t is an m-ary tuple and $\mathcal{P} \subseteq \phi$.*

We next define an operator $\textbf{annotate}(R, \Phi)$ that annotates each tuple with the singleton set containing the range its value in attribute a belongs to. This operator will be used to generate inputs for incremental relational algebra operators over annotated relations.

Definition 4.4 (Annotating Relations). *Given a relation R, attribute $a \in \text{Sch}(R)$ and ranges $\Phi = \{\ldots, (\phi, a), \ldots\}$, i.e., (ϕ, a) is the partition for R in Φ, the operator* **annotate** *returns a sketch-annotated relation $\mathcal{R}$ with the same schema as R:*

$$\textbf{annotate}(R, \Phi) = \{\!\{\langle t, \{\rho\}\rangle \mid t \in R \wedge t.a \in \rho \wedge \rho \in \phi\}\!\}$$

We define annotated deltas as deltas where each tuple is annotated using the **annotate** operator. Consider a delta ΔR between two versions R_1 and R_2 of relation R. Given ranges ϕ for attribute $a \in \text{Sch}(R)$, we define $\Delta\mathcal{R}$ as: $\Delta\mathcal{R} = \textbf{annotate}(\Delta R, \Phi)$. $\Delta\mathcal{R}$ contains all tuples from R that differ between R_1 and R_2 tagged with $\mathbb{\Delta}$ or Δ depending on whether they got inserted or deleted. Each tuple t is annotated with the range $\rho \in \phi$ that $t.a$ belongs to. Analog we use $\mathcal{D}$ to denote the annotated version of database D and use $\Delta\mathcal{D}$ to denote the annotated version of delta database ΔD.

Example 4.2. *Continuing with Ex. 4.1, the annotated version of ΔD_2 according to ϕ_{price} is $\{\!\{\langle\mathbb{\Delta}\ s_8, \{\rho_3\}\rangle\}\!\}$, because $s_8.price$ belongs to $\rho_3 = [1001, 1500] \in \phi_{price}$.*

4.4 Problem Definition

We are now ready to define *incremental maintenance procedures (IMs)* that maintain provenance sketches. An IM takes as input a query Q and an annotated delta $\Delta\mathcal{D}$ for the ranges Φ of a provenance sketch $\mathcal{P}$ and produces a delta $\Delta\mathcal{P}$ for the sketch. Note that we assume that all attributes used in Φ are *safe*. An attribute a is safe for a query Q if every sketch based on some range partition on a is safe. We use the safety test from [37] to determine safe attributes. IMs are allowed to store some state $\mathcal{S}$, e.g., information about groups produced by an aggregation operator, to allow for more efficient maintenance. Given the current state and $\Delta\mathcal{D}$, the IM should return a delta $\Delta\mathcal{P}$ for the sketch $\mathcal{P}$ and an updated state $\mathcal{S}'$ such that $\mathcal{P} \uplus \Delta\mathcal{P}$ over-approximates an accurate sketch for the updated database.

Definition 4.5 (Incremental Maintenance Procedure). *Given a query Q, a database D and a delta ΔD. Let $\mathcal{P}$ be a provenance sketch over D for Q wrt. some partition Φ. An* **incremental maintenance procedure** *$\mathcal{I}$ takes as input a state $\mathcal{S}$, the annotated delta $\Delta\mathcal{D}$, and returns an updated state $\mathcal{S}'$ and a provenance sketch delta $\Delta\mathcal{P}$:*

$$\mathcal{I}(Q, \Phi, \mathcal{S}, \Delta\mathcal{D}) = (\Delta\mathcal{P}, \mathcal{S}')$$

Let $\mathcal{P}[Q, \Phi, D]$ denote an accurate sketch for Q over D wrt. Φ. Niu et al. [37] demonstrated that any over-approximation of a safe sketch is also safe, i.e., evaluating the query over the over-approximated sketch yields the same result as evaluating the query over the full database. Thus, for a IM $\mathcal{I}$ to be correct, the following condition has to hold: for every sketch $\mathcal{P}$ that is valid for D and delta ΔD, $\mathcal{I}$, if provided with the state $\mathcal{S}$ for D and the annotated version $\Delta\mathcal{D}$ of ΔD, returns an over-approximation of the accurate sketch $\mathcal{P}[Q, \Phi, D \uplus \Delta D]$:

$$\mathcal{P}[Q, \Phi, D \uplus \Delta D] \subseteq \mathcal{P} \uplus \mathcal{I}(Q, \Phi, \mathcal{S}, \Delta\mathcal{D})$$

5 Incremental Annotated Semantics

We now introduce an IM that maintains sketches using annotated and incremental semantics for relational algebra operators. Each operator takes as input an annotated delta produced by its inputs (or passed to the IM in case of the table access operator), updates its internal state, and outputs an annotated delta. Together, the states of all such incremental operators in a query make up the state of our IM. For an operator O (or query Q) we use $\mathcal{I}(O, \Phi, \Delta\mathscr{D}, \mathcal{S})$ ($\mathcal{I}(Q, \Phi, \Delta\mathscr{D}, \mathcal{S})$) to denote the result of evaluating O (Q) over the annotated delta $\Delta\mathscr{D}$ using the state $\mathcal{S}$. We will often drop $\mathcal{S}$ and Φ. Our IM evaluates a query Q expressed in relational algebra producing an updated state and outputting a delta where each row is annotated with a partial sketch delta. These partial sketch deltas are then combined into a final result $\Delta\mathcal{P}$.

Example 5.1. *Fig. 5 shows annotated tables $\mathscr{R}$ and $\mathscr{S}$, ranges ϕ_a and ϕ_c for attribute a (table R) and c (table S), the delta ΔR and the sketches before the delta has been applied: $\mathcal{P}_R$ and $\mathcal{P}_S$. Consider the following query over R and S:*

```
SELECT a, sum(c) as sc
FROM (SELECT a, b FROM R WHERE a > 3) JOIN S on (b = d)
GROUP BY a HAVING SUM(c) > 5
```

Fig. 5 (right table) shows each operator's output. We will further discuss these in the following when introducing the incremental semantics for individual operators. In this example, a new tuple is inserted into R. Leading to a sketch deltas $\Delta\mathcal{P}_R = \{\!\{f_1\}\!\}$ and $\Delta\mathcal{P}_S = \{\!\{g_2\}\!\}$. The tuple inserted into R results in the generation of a new group for the aggregation subquery which passes the `HAVING` *condition and in turn causes the two fragments from the tuple belonging to this group to be added to the sketches.*

5.1 Merging Sketch Deltas

Each incremental algebra operator returns an annotated relation where each tuple is associated with a sketch that is sufficient to produce it. To generate the sketch for a query Q we evaluate the query under our incremental annotated semantics to produce the tuples of $Q(D)$ each annotated with a partial sketch. We then combine these partial sketches into a single sketch for Q. We now discuss the operator μ that implements this final merging step. To determine whether a change to the annotated query result will result in a change to the current sketch, this operator maintains as state a map $\mathcal{S} : \Phi \to \mathbb{N}$ that records for each range $\rho \in \Phi$ the number of result tuples for which ρ is in their sketch. If the counter for a fragment ρ reaches 0 (due to the deletion of tuples), then the fragment needs to be removed from the sketch. If the counter for a fragment ρ changes from 0 to a non-zero value, then the fragment now belongs to the sketch for the query (we have to add a delta inserting this fragment to the sketch).

$$\mathcal{I}(\mu(Q), \Delta\mathscr{D}, \mathcal{S}) = (\Delta\mathcal{P}, \mathcal{S}')$$

We first explain how $\mathcal{S}'$, the updated state for the operator, is computed and then explain how to compute $\Delta\mathcal{P}$ using $\mathcal{S}$. We define $\mathcal{S}'$ pointwise for a fragment ρ. Any newly inserted (deleted) tuple whose sketch includes ρ increases (decreases) the count for ρ. That is the total cardinality of such inserted tuples (of bag $\mathbb{\Delta}\ \mathscr{D}$ and $\Delta\ \mathscr{D}$, respectively) has to be added (subtracted) from the current count for ρ. Depending on the change of the count for ρ between $\mathcal{S}$ and $\mathcal{S}'$, the operator μ has to output a delta for $\mathcal{P}$. Specifically, if $\mathcal{S}[\rho] = 0 \neq \mathcal{S}'[\rho]$ then the fragment has to be inserted into the sketch and if $\mathcal{S}[\rho] \neq 0 = \mathcal{S}'[\rho]$ then the fragment was part of the sketch, but no longer contributes and needs to be removed.

$$\mathcal{S}'[\rho] = \mathcal{S}[\rho] + |\mathbb{\Delta}\ \mathscr{D}_\rho| - |\Delta\ \mathscr{D}_\rho|$$
$$\mathbb{\Delta}\ \mathscr{D}_\rho = \{\!\{\mathbb{\Delta}\ \langle t, \mathcal{P}\rangle^n \mid \mathbb{\Delta}\ \langle t, \mathcal{P}\rangle^n \in \mathcal{I}(Q, \Delta\mathscr{D}) \wedge \rho \in \mathcal{P}\}\!\}$$
$$\Delta\ \mathscr{D}_\rho = \{\!\{\Delta\ \langle t, \mathcal{P}\rangle^n \mid \Delta\ \langle t, \mathcal{P}\rangle^n \in \mathcal{I}(Q, \Delta\mathscr{D}) \wedge \rho \in \mathcal{P}\}\!\}$$
$$\Delta\mathcal{P} = \bigcup_{\rho:\, \mathcal{S}[\rho]=0 \wedge \mathcal{S}'[\rho]\neq 0} \{\mathbb{\Delta}\ \rho\} \cup \bigcup_{\rho:\, \mathcal{S}[\rho]\neq 0 \wedge \mathcal{S}'[\rho]=0} \{\Delta\ \rho\}$$

Example 5.2. *Reconsider our running example from Ex. 1.1 that partitions based on ϕ_{price}. Assume that there are two result tuples t_1 and t_2 of a query Q that have $\rho_2 = [601, 1000]$ in their sketch and one result tuple t_3 that has ρ_1 and ρ_2 in its sketch. Then the current sketch for the query is $\mathcal{P} = \{\rho_1, \rho_2\}$ and the state of μ is as shown below. If we are processing a delta $\Delta\ \langle t_3, \{\rho_1, \rho_2\}\rangle$ deleting tuple t_3, the updated counts $\mathcal{S}'$ are:*

$$\mathcal{S}[\rho_1] = 1 \quad \mathcal{S}[\rho_2] = 3 \qquad \mathcal{S}'[\rho_1] = 0 \quad \mathcal{S}'[\rho_2] = 2$$

As there is no longer any justification for ρ_1 to belong to the sketch (its count changed to 0), μ returns a delta: $\{\Delta\ \rho_1\}$

Consider the merge operator μ in Ex. 5.1. The state data before maintenance contains ranges f_2 and g_1. A single tuple annotated with f_1 and g_2 is added to the input of this operator. Both ranges were not present in $\mathcal{S}$ and, thus, in addition adding them to $\mathcal{S}'$ the merge operator returns a sketch delta $\mathbb{\Delta}\ \{f_1, g_2\}$.

5.2 Incremental Relational Algebra

5.2.1 Table Access Operator. The incremental version of the table access operator R returns the annotated delta $\Delta\mathscr{R}$ for R passed as part of $\Delta\mathscr{D}$ to the IM unmodified. This operator has no state.

$$\mathcal{I}(R, \Delta\mathscr{D}) = \Delta\mathscr{R}$$

Fig. 5 (top) shows the result of annotating the relation ΔR from Ex. 5.1.

5.2.2 Projection. The projection operator does not maintain any state as each output tuple is produced independently from an input tuple if we consider multiple duplicates of the same tuple as separate tuples. For each annotated delta tuple $\Delta\langle t, \mathcal{P}\rangle$, we project t on the project expressions A and propagate $\mathcal{P}$ unmodified as $t.A$ in the result depends on the same input tuples as t.

$$\mathcal{I}(\Pi_A(Q), \Delta\mathscr{D}) = \{\!\{\Delta\langle t.A, \mathcal{P}\rangle^n \mid \Delta\langle t, \mathcal{P}\rangle^n \in \mathcal{I}(Q, \Delta\mathscr{D})\}\!\}$$

5.2.3 Selection. The incremental selection operator is stateless and the sketch of an input tuple is sufficient for producing the same tuple in the output of selection. Thus, selection returns all input delta tuples that fulfill the selection condition unmodified and filters out all other delta tuples. In our running example (Fig. 5), the single input delta tuple fulfills the condition of selection $\sigma_{a>3}$.

$$\mathcal{I}(\sigma_\theta(Q), \Delta\mathscr{D}) = \{\!\{\Delta\langle t, \mathcal{P}\rangle^n \mid \Delta\langle t, \mathcal{P}\rangle^n \in \mathcal{I}(Q, \Delta\mathscr{D}) \wedge t \models \theta\}\!\}$$

5.2.4 Cross Product. The incremental version of a cross product (and join) $Q_1 \times Q_2$ combines three sets of deltas: (i) joining the delta of Q_1 with the current annotated state of Q_2 ($Q_2(\mathscr{D})$), (ii) joining the delta of the Q_2 with $Q_1(\mathscr{D})$, (iii) joining the deltas of Q_1 and Q_2. For (iii) there are four possible cases depending on which of the

Table, Ranges and Delta

$\mathcal{R}$

a	b	$\mathcal{P}$
1	7	$\{f_1\}$
9	9	$\{f_2\}$

$\mathcal{S}$

c	d	$\mathcal{P}$
6	9	$\{g_1\}$
7	8	$\{g_2\}$

$\phi_a = \{f_1 = [1,5], f_2 = [6,10]\}$
$\phi_c = \{g_1 = [1,6], g_2 = [7,15]\}$

$\mathcal{P}_R = \{f_2\}$ $\mathcal{P}_S = \{g_1\}$

$\Delta R = \{\!\{\triangle\ (5,8)\}\!\}$

Output for each incremental operator

Table access R	$\{\!\{\triangle\ \langle(5,8), \{f_1\}\rangle\}\!\}$
Selection $\sigma_{a>3}$	$\{\!\{\triangle\ \langle(5,8), \{f_1\}\rangle\}\!\}$
Join $\bowtie_{b=d}$	$\{\!\{\triangle\ \langle(5,8,7,8), \{f_1,g_2\}\rangle\}\!\}$
Aggregation $\gamma_{\text{sum}(c);a}$	$\mathcal{S}[9] = (\text{SUM} = 6, \text{CNT} = 1, \mathcal{P} = \{f_2, g_1\}, \mathcal{F}_9 = \{f_2 : 1, g_1 : 1\})$
	$\mathcal{S}'[9] = (\text{SUM} = 6, \text{CNT} = 1, \mathcal{P} = \{f_2, g_1\}, \mathcal{F}_9 = \{f_2 : 1, g_1 : 1\})$
	$\mathcal{S}'[5] = (\text{SUM} = 7, \text{CNT} = 1, \mathcal{P} = \{f_1, g_2\}, \mathcal{F}_5 = \{f_1 : 1, g_2 : 1\})$
	$\{\!\{\triangle\ \langle(5,7), \{f_1,g_2\}\rangle\}\!\}$
Having $\sigma_{\text{sum}(c)>5}$	$\{\!\{\triangle\ \langle(5,7), \{f_1,g_2\}\rangle\}\!\}$
Merging μ	$\mathcal{S} : \{f_2 : 1, g_1 : 1\}$
	$\mathcal{S}' : \{f_1 : 1, f_2 : 1, g_1 : 1, g_2 : 1\}$
Sketch delta	$\triangle\ \{f_1, g_2\}$

Figure 5: Using our IM to evaluate a query under incremental annotated semantics.

two delta tuples being joined is an insertion or a deletion. For two inserted tuples that join, the joined tuple $s \circ t$ is inserted into the result of the cross product. For two deleted tuples, we also have to insert the joined tuple $s \circ t$ into the result. For a deleted tuple joining an inserted tuple, we should delete the tuple $s \circ t$. The non-annotated version of these rules have been discussed in [14, 20, 26, 31]. We use ΔQ_i to denote $\mathcal{I}(Q_i, \Delta\mathcal{D})$ for $i \in \{1,2\}$ below.

$$\begin{aligned}
&\mathcal{I}(Q_1 \times Q_2, \Delta\mathcal{D}) = \\
&\{\!\{\triangle\ \langle s \circ t, \mathcal{P}_1 \uplus \mathcal{P}_2\rangle^{n\cdot m} \mid (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2) \\
&\qquad \vee (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2) \\
&\qquad \vee (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \langle t,\mathcal{P}_2\rangle^m \in Q_2(\mathcal{D})) \\
&\qquad \vee (\langle s,\mathcal{P}_1\rangle^n \in Q_1(\mathcal{D}) \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2)\}\!\} \\
&\cup \\
&\{\!\{\triangle\ \langle s \circ t, \mathcal{P}_1 \uplus \mathcal{P}_2\rangle^{n\cdot m} \mid (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2) \\
&\qquad \vee (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2) \\
&\qquad \vee (\triangle\ \langle s,\mathcal{P}_1\rangle^n \in \Delta Q_1 \wedge \langle t,\mathcal{P}_2\rangle^m \in Q_2(\mathcal{D})) \\
&\qquad \vee (\langle s,\mathcal{P}_1\rangle^n \in Q_1(\mathcal{D}) \wedge \triangle\ \langle t,\mathcal{P}_2\rangle^m \in \Delta Q_2)\}\!\}
\end{aligned}$$

Continuing with Ex. 5.1, as $\Delta\mathcal{S} = \emptyset$ and $\Delta\mathcal{R} = \{\!\{\triangle\ \langle(5,8), \{f_1\}\rangle\}\!\}$ only contains insertions, only $\Delta\mathcal{R} \bowtie_{b=d} \mathcal{S}$ returns a non-empty result (the third case above). As $(5,8)$ only joins with tuple $(7,8)$, a single delta tuple $\triangle\ \langle(5,8,7,8), \{f_1,g_2\}\rangle$ is returned.

5.2.5 Aggregation: Sum, Count, and Average. For the aggregation operator, we need to maintain the current aggregation result for each individual group and record the contribution of fragments from a provenance sketch towards the aggregation result to be able to efficiently maintain the operator's result. Consider an aggregation operator $\gamma_{\mathbf{f}(a);G}(R)$ where $\mathbf{f}$ is an aggregation function and G are the group by attributes ($G = \emptyset$ for aggregation without group-by). Given an version R of the input of the aggregation operator, we use $\mathcal{G} = \{t.G | t \in R\}$ to denote the set of distinct group-by values.

The state data needed for aggregation depends on what aggregation function we have to maintain. However, for all aggregation functions the state maintained for aggregation is a map $\mathcal{S}$ from groups to a per-group state storing aggregation function results for this group, the sketch for the group, and a map $\mathcal{F}_g$ recording for each range ρ of Φ the number of input tuples belonging to the group with ρ in their provenance sketch. Intuitively, $\mathcal{F}_g$ is used in a similar fashion as for operator μ to determine when a range has to be added to or removed from a sketch for the group. We will discuss aggregation functions **sum**, **count**, and **avg** that share the same state.

Sum. Consider an aggregation $\gamma_{\text{sum}(a);G}(Q)$. To be able to incrementally maintain the aggregation result and provenance sketch for a group g, we store the following state:

$$\mathcal{S}[g] = (\text{SUM}, \text{CNT}, \mathcal{P}, \mathcal{F}_g)$$

SUM and CNT store the sum and count for the group, $\mathcal{P}$ stores the group's sketch, and $\mathcal{F}_g : \Phi \to \mathbb{N}$ introduced above tracks for each range $\rho \in \Phi$ how many input tuples from $Q(D)$ belonging to the group have ρ in their sketch. State $\mathcal{S}$ is initialized to $\emptyset$.

Incremental Maintenance. The operator processes an annotated delta as explained in the following. Consider an annotated delta $\Delta\mathcal{D}$. Let ΔQ denote $\mathcal{I}(Q, \Delta\mathcal{D})$, i.e., the delta produced by incremental evaluation for Q using $\Delta\mathcal{D}$. We use $\mathcal{G}_{\Delta Q}$ to denote the set of groups present in ΔQ and ΔQ_g to denote the subset of ΔQ including all annotated delta tuples $\Delta\langle t, \mathcal{P}\rangle$ where $t.G = g$. We now explain how to produce the output for one such group. The result of the incremental aggregation operators is then just the union of these results. We first discuss the case where the group already exists and still exists after applying the input delta.

Updating an existing group. Assume the current and updated state for g as shown below:

$$\mathcal{S}[g] = (\text{SUM}, \text{CNT}, \mathcal{P}, \mathcal{F}_g) \qquad \mathcal{S}'[g] = (\text{SUM}', \text{CNT}', \mathcal{P}', \mathcal{F}'_g)$$

The updated sum is produced by adding $t.a \cdot n$ for each inserted input tuple with multiplicity n: $\triangle\ \langle t,\mathcal{P}\rangle^n \in \Delta Q_g$ and subtracting this amount for each deleted tuple: $\triangle\ \langle t,\mathcal{P}\rangle^n \in \Delta Q_g$. For instance, if the delta contains the insertion of 3 duplicates of a tuple with a value 5, then the SUM will be increased by $3 \cdot 5$.

$$\text{SUM}' = \text{SUM} \quad + \sum_{\triangle\langle t,\mathcal{P}\rangle^n \in \Delta Q_g} t.a \cdot n \quad - \sum_{\triangle\langle t,\mathcal{P}\rangle^n \in \Delta Q_g} t.a \cdot n$$

The update for CNT is computed in the same fashion using n instead of $t.a \cdot n$. The updated count in $\mathcal{F}'_g$ is computed for each $\rho \in \Phi$ as:

$$\mathcal{F}'_g[\rho] = \mathcal{F}_g[\rho] + \sum_{\triangle\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge \rho \in \mathcal{P}} n \quad - \sum_{\triangle\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge \rho \in \mathcal{P}} n$$

Based on $\mathcal{F}'_g$ we then determine the updated sketch for the group:

$$\mathcal{P}' = \{\rho \mid \mathcal{F}'_g[\rho] > 0\}$$

We then output a pair of annotated delta tuples that deletes the previous result for the group and inserts the updated result:

$$\text{🔺}\ \langle g \circ (\text{SUM}), \mathcal{P}\rangle \qquad \text{🔺}\ \langle g \circ (\text{SUM}'), \mathcal{P}'\rangle$$

Creating and Deleting Groups. For groups g that are not in $\mathcal{S}$, we initialize the state for g as shown below: $\mathcal{S}'[g] = (0, 0, \emptyset, \emptyset)$ and only output $\text{🔺}\ \langle g \circ (\text{SUM}'), \mathcal{P}'\rangle$. An existing group gets deleted if $\text{CNT} \neq 0$ and $\text{CNT}' = 0$. In this case we only output $\text{🔺}\ \langle g \circ (\text{SUM}), \mathcal{P}\rangle$.

Average and Count. For average we maintain the same state as for sum. The only difference is that the updated average is computed as $\frac{\text{SUM}'}{\text{CNT}'}$. For count we only maintain the count and output CNT'.

Continuing with Ex. 5.1, the output of the join (single delta tuple with group 5) is fed into the aggregation operator using **sum**. As no such group is in $\mathcal{S}$ we create new entry $\mathcal{S}[5]$. After maintaining the state, the output delta produced for this group is $\{\!\{\text{🔺}\ \langle(5, 7), \{f_1, g_2\}\rangle\}\!\}$. This result satisfies `HAVING` condition (selection $\sigma_{\text{sum}(c)>5}$) and is passed on to the merge operator.

5.2.6 Aggregation: minimum and maximum. The aggregation functions **min** and **max** share the same state. To be able to efficiently determine the minimum (maximum) value of the aggregation function **min** (**max**), we use a data structure like balanced search trees that can provide efficiently access to the value in sort order.

Min. Consider an aggregation $\gamma_{\text{min}(a);G}(Q)$. To be able to maintain the aggregation result and provenance sketch incrementally for a group g, we store the following state:

$$\mathcal{S}[g] = (\text{CNT}, \mathcal{P}, \mathcal{F}_g)$$

$\mathcal{P}$ and $\mathcal{F}_g$ are the same as described in aggregation function **sum**. $\mathcal{P}$ stores the groups' sketch and $\mathcal{F}_g$ stores for each range how many tuples in this group have this range in their sketch. CNT is an balanced search tree that record all values of aggregate attribute in sort order, and for each node in CNT, we store the multiplicity of this aggregate value.

Incremental Maintenance. Consider an aggregation $\gamma_{\text{min}(a);G}(Q)$ and an annotated delta $\Delta\mathcal{D}$. Recall that ΔQ denotes the $\mathcal{I}(Q, \Delta\mathcal{D})$ and ΔQ_g to denote the subset of ΔQ including all annotated delta tuples $\Delta\langle t, \mathcal{P}\rangle$ where $t.G = g$. We now discuss how to produce the output for one such group and how the incremental maintenance works for the case where the group already exists and still exits after maintenance.

Updating an existing group. Assume the current and updated state for g as shown below:

$$\mathcal{S}[g] = (\text{CNT}, \mathcal{P}, \mathcal{F}_g) \qquad \mathcal{S}'[g] = (\text{CNT}', \mathcal{P}', \mathcal{F}'_g)$$

The CNT is updated as follow: for each annotated tuple in ΔQ_g, the multiplicity of aggregate attribute value (a) will be increased by 1 if it is an inserted annotated tuple ($\text{🔺}\ \langle t, \mathcal{P}\rangle$) and $t.a = a$. If this value is new to the tree, we just initialize for this value with a multiplicity 1. Otherwise, the multiplicity will be decreased by one if the annotated tuple is a deletion and $t.a = a$. We remove a node from the tree if the multiplicity becomes 0.

$$\text{CNT}[a]' = \text{CNT}[a] \ + \sum_{\text{🔺}\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge t.a=a} n \ - \sum_{\text{🔻}\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge t.a=a} n$$

Here, the use of $\text{CNT}[a]$ and $\text{CNT}[a]'$ do not imply CNT is a map but they indicate the multiplicity of a in CNT.

The updated count in $\mathcal{F}'_g$ is computed for each $\rho \in \Phi$ as:

$$\mathcal{F}'_g[\rho] = \mathcal{F}_g[\rho] + \sum_{\text{🔺}\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge \rho \in \mathcal{P}} n \ - \sum_{\text{🔻}\langle t,\mathcal{P}\rangle^n \in \Delta Q_g \wedge \rho \in \mathcal{P}} n$$

Based on $\mathcal{F}'_g$ we then determine the updated sketch for the group:

$$\mathcal{P}' = \{\rho \mid \mathcal{F}'_g[\rho] > 0\}$$

We then output a pair of annotated delta tuples that deletes the previous result for the group and inserts the updated result:

$$\text{🔺}\ \langle g \circ (min(\text{CNT})), \mathcal{P}\rangle \qquad \text{🔺}\ \langle g \circ (min(\text{CNT}')), \mathcal{P}'\rangle$$

For the group g, we need to output the minimum value in the balanced search tree.

Creating and Deleting Groups. For groups g that are not in $\mathcal{S}$, we initialize the state for g as shown below: $\mathcal{S}'[g] = (\emptyset, \emptyset, \emptyset)$ and only output $\text{🔺}\ \langle g \circ (min(\text{CNT}')), \mathcal{P}'\rangle$. An existing group gets deleted if size of the tree becomes zero from a non-zero value such that: $|\text{CNT}| \neq 0 = |\text{CNT}'|$. In this case we only output $\text{🔺}\ \langle g \circ (\min \text{CNT}), \mathcal{P}\rangle$.

Max. For max, we maintain the same state as for min. The only difference is that we output the maximum value from tree CNT and CNT'.

5.2.7 Top-k. The top-k operator $\tau_{k,O}$ returns the first k tuples sorted on O. As we are dealing with bag semantics, the top-k tuples may contain a tuple with multiplicity larger than 1. As before, we use ΔQ to denote $\mathcal{I}(Q, \Delta\mathcal{D})$.

State Data. To be able to efficiently determine updates to the top-k tuples with sketch annotations we maintain a nested map. The outer map $\mathcal{S}$ is order and should be implemented using a data structure like balanced search trees (BSTs) that provide efficient access to entries in sort order on O, maps order-by values o to another map CNT which stores multiplicities for each annotated tuple $\langle t, \mathcal{P}\rangle$ for which $t.O = o$.

$$\mathcal{S}[o] = (\text{CNT})$$

and for any $\langle t, \mathcal{P}\rangle$ with $t.O = o$ with $\langle t, \mathcal{P}\rangle^n \in \Delta Q$ we store

$$\text{CNT}[\langle t, \mathcal{P}\rangle] = n$$

This data structure allows efficient updates to the multiplicity of any annotated tuple based on the input delta as shown below. Consider such a tuple $\langle t, \mathcal{P}\rangle$ with $t.O = o$ with $\text{🔺}\ \langle t, \mathcal{P}\rangle^n \in \Delta Q$ and $\text{🔻}\ \langle t, \mathcal{P}\rangle^m \in \Delta Q$.

$$\mathcal{S}'[o][\langle t, \mathcal{P}\rangle] = \mathcal{S}[o][\langle t, \mathcal{P}\rangle] + n - m$$

Computing Deltas. As k is typically relatively small, we select a simple approach for computing deltas by deleting the previous top-k and then inserting the updated top-k. Should the need arise to handle large k, we can use a balanced search tree and mark nodes in the tree as modified when updating the multiplicity of annotated tuples based on the input delta and use data structures which enable efficient positional access under updates, e.g., order-statistic trees [15]. Our simpler technique just fetches the first tuples in sort order from $\mathcal{S}$

and $\mathcal{S}'$ by accessing the keys stored in the outer map $\mathcal{S}$ in sort order. For each o we then iterate through the tuples in $\mathcal{S}[o]$ (in an arbitrary, but deterministic order since they are incomparable) keeping track of the total multiplicity m of tuples we have processed so far. As long as $m \leq k$ we output the current tuple and proceed to the next tuple (or order-by key once we have processed all tuples in $\mathcal{S}[o]$). Once $m \geq k$, we terminate. If the last tuple's multiplicity exceeds the threshold we output this tuple with the remaining multiplicity. Applied to $\mathcal{S}$ this approach produces the tuples to delete and applied to $\mathcal{S}'$ it produces the tuples to insert:

$$\mathbb{\Delta}\ \tau_{k,O}(\mathcal{S}) \qquad\qquad \mathbb{\Delta}\ \tau_{k,O}(\mathcal{S}')$$

5.3 Complexity Analysis

We now analyze the runtime complexity of operators. Let n denote the input delta tuple size and p denote the number of ranges of the partition on which the sketch is build on. For table access, selection, and projection, we need to iterate over these n annotated tuples to generate the output. As for these operations we do not modify the sketches of tuples, the complexity is $O(n)$. For aggregation, for each aggregation function we maintain a hashmap that tracks the current aggregation result for each group and a count for each fragment that occurs in a sketch for each tuple in the group. For each input delta tuple, we can update this information in $O(1)$ if we assume that the number of aggregation functions used in an aggregation operator is constant. Thus, the overall runtime for aggregation is $O(n \cdot p)$. For join operator, we store the bloom filter to pre-filter the input data. Suppose the right input table has m tuples. Building such a filter incurs a one-time cost of $O(m)$ for scanning the table once. Consider the part where we join a delta of size n for the left input with the right table producing o output tuples. The cost this join depends on what join algorithm is used ranging from $O(n + m + o)$ for a hash join to $O(n \cdot m + o)$ for a nested loop join (in both cases assuming the worst case where no tuples are filtered using the bloom filter). For the top-k operator, we assume there are l nodes stored in the balanced search tree. Building this tree will cost $O(l \cdot \log l)$ (only built once). An insertion, deletion, or lookup will take $O(\log l)$ time. Thus, the runtime complexity of the top-k operator is $O(n \cdot \log l)$ to complete the top-k operator. Regarding space complexity, the selection and projection only require constant space. For aggregation, the space is linear in the number of groups and in p. For join, the bloom filter's size is linear in m, but for a small constant factor. For top-k operators, we store $l \geq k$ entries in the search tree, each requiring $O(p)$ space. Thus, the overall space complexity for this operator is $O(l \cdot p)$.

6 Correctness Proof

We are now ready to state the main result of this paper, i.e., the incremental operator semantics we have defined is an incremental maintenance procedure. That is, it outputs valid sketch deltas that applied to the safe sketch for the database D before the update yield an over-approximation of an accurate sketch for the database $D \uplus \Delta D$.

THEOREM 6.1 (CORRECTNESS). *$\mathcal{I}$ as defined in Sec. 4.4 is an incremental maintenance procedure such that it takes as input a state $\mathcal{S}$, the annotated delta $\Delta\mathcal{D}$, the ranges Φ, a query Q and returns an updated state $\mathcal{S}'$ and a provenance sketch delta $\Delta\mathcal{P}$: $\mathcal{I}(Q, \Phi, \mathcal{S}, \Delta\mathcal{D}) = (\Delta\mathcal{P}, \mathcal{S}')$. For any query Q, sketch $\mathcal{P}$ that is valid for D, and state $\mathcal{S}$ corresponding to D we have:*

$$\mathcal{P}[Q, \Phi, D \uplus \Delta D] \subseteq \mathcal{P} \uplus \mathcal{I}(Q, \Phi, \mathcal{S}, \Delta\mathcal{D})$$

In this section, we will demonstrate the correctness of Theorem 6.1. Specifically, we introduce two auxiliary notions that two aspects: 1. Tuple correctness (**tuple correctness**): the incremental maintenance procedure can always generate the correct bag of tuples for the operators it maintains. 2. Fragment correctness (**fragment correctness**): the maintenance procedure can output the correct delta sketches as well for the operators it maintains. Before we present the proof of the theorem, we will establish several lemmas that used in the proof, and propose the criteria of **tuple correctness** and **fragment correctness**.

6.1 Tuple Correctness, Fragment Correctness, and Auxiliary Results

In this section, we will introduce two functions: tuple extract $\mathbb{T}(\cdot)$ and fragment extract $\mathbb{F}(\cdot)$ where the function $\mathbb{T}(\cdot)$ function specifies the procedure for handing tuples from annotated relations (database) and the function $\mathbb{F}(\cdot)$ specifies that for handing fragments from annotated relations (database). To demonstrate the **tuple correctness** and **fragment correctness**, we will introduce a series of auxiliary lemmas which present properties of the two function $\mathbb{T}(\cdot)$ and $\mathbb{F}(\cdot)$ and are used during the proof for operators when present the correctness of tuple and fragment. Then, we will define the **tuple correctness** and **fragment correctness** as tools for the proof of Theorem 6.1. Next, we introduce a lemma that for each operator, the Theorem 6.1 holds if we can demonstrate both tuple correctness and fragment correctness hold.

We denote Q^n to be a query having at most n operators and $\mathbb{Q}^i$ to be the class of all queries with i operators such $Q^i \in \mathbb{Q}^i$.

Given a database $D = \{\!\{t_1, \ldots, t_n\}\!\}$ and the annotated database $\mathcal{D} = \{\!\{\langle t_1, \mathcal{P}_1\rangle, \ldots, \langle t_n, \mathcal{P}_n\rangle\}\!\}$, the results of running query Q over the database D and running query over the annotated database are:

$$Q(D) = \{\!\{t_{o_1}, \ldots, t_{o_m}\}\!\} \quad Q(\mathcal{D}) = \{\!\{\langle t_{o_1}, \mathcal{P}_{o_1}\rangle, \ldots, \langle t_{o_m}, \mathcal{P}_{o_m}\rangle\}\!\}$$

Given the delta database $\Delta D = \mathbb{\Delta} D \cup \mathbb{\Delta} D$ where $\mathbb{\Delta} D = \{\!\{t_{i_1}, \ldots, t_{i_k}\}\!\}$ and $\mathbb{\Delta} D = \{\!\{t_{d_1}, \ldots, t_{d_x}\}\!\}$. Let D' to be the updated database such that: $D' = D \uplus \Delta D$. Suppose the results of running query Q over the database and annotated database after updating are:

$$Q(D') = \{\!\{t_{o_1}, \ldots, t_{o_l}\}\!\} \quad Q(\mathcal{D}') = \{\!\{\langle t_{o_1}, \mathcal{P}_{o_1}\rangle, \ldots, \langle t_{o_l}, \mathcal{P}_{o_l}\rangle\}\!\}$$

We define $\mathbb{T}(\cdot)$ (**extract tuples**), a function that takes as input a bag of annotated tuples and returns all the tuples from the bag such that:

$$\mathbb{T}(\{\!\{\langle t_1, \mathcal{P}_1\rangle, \ldots, \langle t_y, \mathcal{P}_y\rangle\}\!\}) = \{\!\{t_1, \ldots, t_y\}\!\}$$

LEMMA 6.1. *Let $\mathbb{T}(\cdot)$ be the **extract tuples** function, $\mathcal{D}_1$ and $\mathcal{D}_2$ be two annotated databases with the same schema. The following holds:*

$$\mathbb{T}(\mathcal{D}_1 \cup \mathcal{D}_2) = \mathbb{T}(\mathcal{D}_1) \cup \mathbb{T}(\mathcal{D}_2)$$

PROOF. Suppose $\mathcal{D}_1 = \{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_m, \mathcal{P}_{t_m}\rangle\}\!\}$ and $\mathcal{D}_2 = \{\!\{\langle s_1, \mathcal{P}_{s_n}\rangle, \ldots, \langle s_n, \mathcal{P}_{s_n}\rangle\}\!\}$. Then $\mathcal{D}_1 \cup \mathcal{D}_2$, $\mathbb{T}(\mathcal{D}_1)$ and

$\mathbb{T}(\mathscr{D}_2)$ are:

$$\mathscr{D}_1 \cup \mathscr{D}_2 = \{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_n, \mathcal{P}_{t_n}\rangle, \langle s_1, \mathcal{P}_{s_1}\rangle, \ldots, \langle s_n, \mathcal{P}_{s_n}\rangle\}\!\}$$
$$\mathbb{T}(\mathscr{D}_1) = \{\!\{t_1, \ldots, t_m\}\!\} \quad \mathbb{T}(\mathscr{D}_2) = \{\!\{s_2, \ldots, s_n\}\!\}$$

We can get that $\mathbb{T}(\mathscr{D}_1 \cup \mathscr{D}_2)$ is:

$$\mathbb{T}(\mathscr{D}_1 \cup \mathscr{D}_2) = \{\!\{t_1, \ldots, t_m, s_1, \ldots, s_n\}\!\}$$

and $\mathbb{T}(\mathscr{D}_1) \cup \mathbb{T}(\mathscr{D}_2)$ is:

$$\mathbb{T}(\mathscr{D}_1) \cup \mathbb{T}(\mathscr{D}_2) = \{\!\{t_1, \ldots, t_m\}\!\} \cup \{\!\{t_m, s_1, \ldots, s_n\}\!\} = \{\!\{t_1, \ldots, t_m, s_1, \ldots, s_n\}\!\}$$

Therefore, $\mathbb{T}(\mathscr{D}_1 \cup \mathscr{D}_2) = \mathbb{T}(\mathscr{D}_1) \cup \mathbb{T}(\mathscr{D}_2)$ □

Analog we can know that $\mathbb{T}(\Delta\mathscr{D}) = \mathbb{T}(\Delta D) \cup \mathbb{T}(\Delta D)$, since $\Delta\mathscr{D} = \triangle\, \mathscr{D} \cup \triangle\, \mathscr{D}$

LEMMA 6.2. *Let $\mathscr{D}_1$ and $\mathscr{D}_2$ be two annotated databases over the same schema. We have:*

$$\mathbb{T}(\mathscr{D}_1 - \mathscr{D}_2) = \mathbb{T}(\mathscr{D}_1) - \mathbb{T}(\mathscr{D}_2)$$

PROOF. The proof is analog to the proof for Lemma 6.1. □

LEMMA 6.3. *Let $\mathbb{T}(\cdot)$ be the **extract tuples** function, $\mathscr{D}$ and $\Delta\mathscr{D}$ be an annotated database and an annotated database delta. The following property holds:*

$$\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D}) = \mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\Delta\mathscr{D})$$

PROOF. Suppose $\mathscr{D}$ and $\Delta\mathscr{D}$ are:

$$\mathscr{D} = \{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_m, \mathcal{P}_{t_m}\rangle\}\!\}$$
$$\Delta\mathscr{D} = \triangle\, \mathscr{D} \cup \triangle\, \mathscr{D} = \{\!\{\triangle\, \langle t_{d_1}, \mathcal{P}_{d_1}\rangle, \ldots, \triangle\, \langle t_{d_j}, \mathcal{P}_{d_j}\rangle\}\!\} \cup \{\!\{\triangle\, \langle t_{i_1}, \mathcal{P}_{i_1}\rangle, \ldots, \triangle\, \langle t_{i_i}, \mathcal{P}_{i_i}\rangle\}\!\}$$

Then $\mathscr{D} \uplus \Delta\mathscr{D}$ is:

$$\begin{aligned}
&\mathscr{D} \uplus \Delta\mathscr{D} \\
=&\mathscr{D} - \triangle\, \mathscr{D} \cup \triangle\, \mathscr{D} \\
=&\{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \mathscr{D}\}\!\} - \{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \triangle\, \mathscr{D}\}\!\} \\
&\cup \{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \triangle\, \mathscr{D}\}\!\} \\
=&\{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_m, \mathcal{P}_{t_m}\rangle\}\!\} \\
&- \{\!\{\triangle\, \langle t_{d_1}, \mathcal{P}_{d_1}\rangle, \ldots, \triangle\, \langle t_{d_j}, \mathcal{P}_{d_j}\rangle\}\!\} \\
&\cup \{\!\{\triangle\, \langle t_{i_1}, \mathcal{P}_{i_1}\rangle, \ldots, \triangle\, \langle t_{i_i}, \mathcal{P}_{i_i}\rangle\}\!\}
\end{aligned}$$

Then $\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D})$:

$$\begin{aligned}
&\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D}) \\
=&\{\!\{t_1, \ldots, t_m\}\!\} \\
&- \{\!\{\triangle\, t_{d_1}, \ldots, \triangle\, t_{d_j}\}\!\} \\
&\cup \{\!\{\triangle\, t_{i_1}, \ldots, \triangle\, t_{i_i}\}\!\}
\end{aligned}$$

$\mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\Delta\mathscr{D})$ are:

$$\begin{aligned}
&\mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\Delta\mathscr{D}) \\
=&\mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\triangle\, \mathscr{D} \cup \triangle\, \mathscr{D}) \\
=&\mathbb{T}(\mathscr{D}) - \mathbb{T}(\triangle\, \mathscr{D}) \cup \mathbb{T}(\triangle\, \mathscr{D}) \\
=&\{\!\{t_1, \ldots, t_m\}\!\} \\
&- \{\!\{\triangle\, t_{d_1}, \ldots, \triangle\, t_{d_j}\}\!\} \\
&\cup \{\!\{\triangle\, t_{i_1}, \ldots, \triangle\, t_{i_i}\}\!\}
\end{aligned}$$

Therefore, $\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D}) = \mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\Delta\mathscr{D})$ □

DEFINITION 6.1 (**TUPLE CORRECTNESS**). *Consider a query Q, a database D and a delta database ΔD, Let D' be the updated database such that $D' = D \uplus \Delta D$. Let $\mathcal{I}$ be an incremental maintenance procedure takes as input a state $\mathcal{S}$, the query Q, the annotated delta $\Delta\mathscr{D}$ and the range partition Φ. The tuples in the result of the query running over the updated database are equivalent to tuples in the result of applying query running over the database to incremental maintenance procedure such that:*

$$\mathbb{T}(Q(\mathscr{D}) \uplus \mathcal{I}(Q, \Phi, \Delta\mathscr{D}, \mathcal{S})) = Q(D')$$

Recall $\mathcal{P}[Q, \Phi, D]$ defines an accurate provenance sketch $\mathcal{P}$ for Q wrt. to D, and ranges Φ, and $D_{\mathcal{P}}$ is an instance of $\mathcal{P}$ which is the data covered by the sketch.

Now we define a function $\mathbb{F}(\cdot)$ (**fragments extracting**) that takes as input a bag of annotated tuples and return all the sketches from this bag such that:

$$\mathbb{F}(\{\!\{\langle t_1, \mathcal{P}_1\rangle, \ldots, \langle t_y, \mathcal{P}_y\rangle\}\!\}) = \{\!\{\mathcal{P}_1, \ldots, \mathcal{P}_y\}\!\}$$

And the $\mathbb{F}(\cdot)$ function has the following property:

LEMMA 6.4. $\mathbb{F}(\mathscr{D}_1 \cup \mathscr{D}_2) = \mathbb{F}(\mathscr{D}_1) \cup \mathbb{F}(\mathscr{D}_2)$

The proof of this property is similar to lemma 6.1 where for $\mathbb{F}(\cdot)$, we focus on fragments instead of tuples. Therefore, $\mathbb{F}(\Delta\mathscr{D}) = \mathbb{F}(\triangle\, \mathscr{D}) \cup \mathbb{F}(\triangle\, \mathscr{D})$, since $\Delta D = \triangle\, D \cup \triangle\, D$.

Like Lemma 6.3, the **extract fragments** function has the following properties as well.

LEMMA 6.5. $\mathbb{F}(\mathscr{D} \uplus \Delta\mathscr{D}) = \mathbb{F}(\mathscr{D}) \uplus \mathbb{F}(\Delta\mathscr{D})$

PROOF. Suppose $\mathscr{D}$ and $\Delta\mathscr{D}$ are:

$$\mathscr{D} = \{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_m, \mathcal{P}_{t_m}\rangle\}\!\}$$
$$\Delta\mathscr{D} = \triangle\, \mathscr{D} \cup \triangle\, \mathscr{D} = \{\!\{\triangle\, \langle t_{d_1}, \mathcal{P}_{d_1}\rangle, \ldots, \triangle\, \langle t_{d_j}, \mathcal{P}_{d_j}\rangle\}\!\} \cup \{\!\{\triangle\, \langle t_{i_1}, \mathcal{P}_{i_1}\rangle, \ldots, \triangle\, \langle t_{i_i}, \mathcal{P}_{i_i}\rangle\}\!\}$$

Then $\mathscr{D} \uplus \Delta\mathscr{D}$ is:

$$\begin{aligned}
&\mathscr{D} \uplus \Delta\mathscr{D} \\
=&\mathscr{D} - \triangle\, \mathscr{D} \cup \triangle\, \mathscr{D} \\
=&\{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \mathscr{D}\}\!\} - \{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \triangle\, \mathscr{D}\}\!\} \\
&\cup \{\!\{\langle t, \mathcal{P}\rangle \mid \langle t, \mathcal{P}\rangle \in \triangle\, \mathscr{D}\}\!\} \\
=&\{\!\{\langle t_1, \mathcal{P}_{t_1}\rangle, \ldots, \langle t_m, \mathcal{P}_{t_m}\rangle\}\!\} \\
&- \{\!\{\triangle\, \langle t_{d_1}, \mathcal{P}_{d_1}\rangle, \ldots, \triangle\, \langle t_{d_j}, \mathcal{P}_{d_j}\rangle\}\!\} \\
&\cup \{\!\{\triangle\, \langle t_{i_1}, \mathcal{P}_{i_1}\rangle, \ldots, \triangle\, \langle t_{i_i}, \mathcal{P}_{i_i}\rangle\}\!\}
\end{aligned}$$

Then $\mathbb{F}(\mathscr{D} \uplus \Delta\mathscr{D})$:

$$\begin{aligned}
&\mathbb{F}(\mathscr{D} \uplus \Delta\mathscr{D}) \\
=&\{\!\{\mathcal{P}_1, \ldots, \mathcal{P}_m\}\!\} \\
&- \{\!\{\triangle\, \mathcal{P}_{d_1}, \ldots, \triangle\, \mathcal{P}_{d_j}\}\!\} \\
&\cup \{\!\{\triangle\, \mathcal{P}_{i_1}, \ldots, \triangle\, \mathcal{P}_{i_i}\}\!\}
\end{aligned}$$

$\mathbb{F}(\mathscr{D}) \uplus \mathbb{F}(\Delta\mathscr{D})$ are:

$$\begin{aligned}&\mathbb{F}(\mathscr{D}) \uplus \mathbb{F}(\Delta\mathscr{D})\\ =&\mathbb{F}(\mathscr{D}) \uplus \mathbb{F}(\triangle\ \mathscr{D} \cup \triangleq\ \mathscr{D})\\ =&\mathbb{F}(\mathscr{D}) - \mathbb{F}(\triangle\ \mathscr{D}) \cup \mathbb{F}(\triangleq\ \mathscr{D})\\ =&\{\!\{\mathcal{P}_1, \ldots, \mathcal{P}_m\}\!\}\\ &- \{\!\{\triangle\ \mathcal{P}_{d_1}, \ldots, \triangle\ \mathcal{P}_{d_j}\}\!\}\\ &\cup \{\!\{\triangleq\ \mathcal{P}_{i_1}, \ldots, \triangleq\ \mathcal{P}_{i_i}\}\!\}\end{aligned}$$

Therefore, $\mathbb{F}(\mathscr{D} \uplus \Delta\mathscr{D}) = \mathbb{F}(\mathscr{D}) \uplus \mathbb{F}(\Delta\mathscr{D})$ □

We now define $\mathbb{S}(\cdot)$ as a function that takes as input a bag of fragments where each fragment has a multiplicity at least 1, and returns a set of fragments such that the multiplicity of each fragment in input bag is 1 in the output set. For a query Q running over a annotated database $\mathscr{D}$, the annotated result is $Q(\mathscr{D})$. The fragments in the annotated result are $\mathbb{F}(Q(\mathscr{D}))$. Suppose the provenance sketch captured for this query given the ranges Φ is $\mathcal{P}[Q, \Phi, D]$, then we can get that:

$$\mathcal{P}[Q, \Phi, D] = \mathbb{S}(\mathbb{F}(Q(\mathscr{D})))$$

A provenance sketch covers relevant data of the database to answer a query such that:

$$Q(D) = Q(D_{\mathcal{P}[Q,\Phi,D]}) = Q(D_{\mathbb{S}(\mathbb{F}(Q(\mathscr{D})))})$$

For a query running over a set of fragments, it is the same as running a bag of fragments where the ranges are exactly the same as in the set but each one having different multiplicity. The reason is for a fragment appearing multiple times in the bag, when using this fragment to answer, they will be translate into the same expression in the `WHERE` clause multiple times concatenating with `OR`. And this expression appearing multiple times will be treated as a single one when the database engine evaluates the `WHERE`. For example, `WHERE (a BETWEEN 10 AND 20) OR (a BETWEEN 10 AND 20)` has the same effect as `WHERE (a BETWEEN 10 AND 20)` for a query. Thus, the following holds:

$$Q(D) = Q(D_{\mathbb{S}(\mathbb{F}(Q(\mathscr{D})))} = Q(D_{\mathbb{F}(Q(\mathscr{D}))})$$

DEFINITION 6.2 (**FRAGMENT CORRECTNESS**). *Consider a query Q, a database D and a delta database ΔD, Let D' be the updated database such that $D' = D \uplus \Delta D$. Let $\mathcal{I}$ be an incremental maintenance procedure takes as input a state $\mathcal{S}$, the query Q, the annotated delta $\Delta\mathscr{D}$ and the range partition Φ. The result of the running query over the updated database is equivalent to the result of running query over the data of updated database covered by applying the fragments in current provenance sketch to fragments generated from the incremental maintenance procedure such that:*

$$Q(D') = Q(D'_{\mathbb{S}(\mathbb{F}(Q(\mathscr{D})) \uplus \mathbb{F}(\mathcal{I}(Q,\Phi,\Delta\mathscr{D},\mathcal{S})))})$$

LEMMA 6.6. *For an operator that the sketch is maintained by the incremental procedure, if both **tuple correctness** and **fragment correctness** hold, then the Theorem 6.1 holds for this operator.*

In the following, we will prove Theorem 6.1 by induction over the structure of queries starting with the base case which is the correctness of query consisting of a single table access operator followed by inductive steps for other operators distinguishing the cases of tuple and fragment correctness to show the Lemma 6.6.

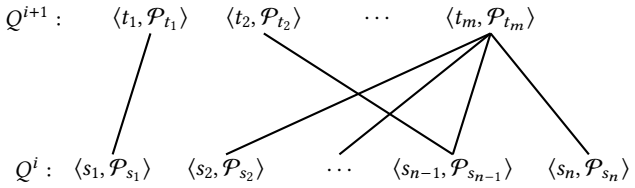


Figure 6: Annotated tuples' relation between Q^i and Q^{i+1}

6.2 Proof of Theorem 6.1

Having the properties **tuple correctness** and **fragment correctness** defined, we are ready to prove Theorem 6.1 by induction over the structure of a query showing that the tuple correctness and fragment correctness properties hold for supported query which implies the theorem.

6.2.1 Base Case. We start with Q^1, which is a single table R. We will show that for any $Q^1 \in \mathbb{Q}^1$, the **tuple correctness** and **fragment correctness** properties hold for table access operator.

Suppose the relation and its annotated relation are:

$$R = \{\!\{t_1, \ldots, t_n\}\!\} \quad \mathscr{R} = \{\!\{\langle t_1, \mathcal{P}_1\rangle, \ldots, \langle t_n, \mathcal{P}_n\rangle\}\!\}$$

and delta relation and annotated delta relation are:

$$\triangleq\ R = \{\!\{t_{i_1}, \ldots, t_{i_i}\}\!\} \quad \triangle\ R = \{\!\{t_{d_1}, \ldots, t_{d_j}\}\!\}$$

$$\triangleq\ \mathscr{R} = \{\!\{\langle t_{i_1}, \mathcal{P}_{i_1}\rangle, \ldots, \langle t_{i_i}, \mathcal{P}_{i_i}\rangle\}\!\} \quad \triangle\ \mathscr{R} = \{\!\{\langle t_{d_1}, \mathcal{P}_{d_1}\rangle, \ldots, \langle t_{d_j}, \mathcal{P}_{d_j}\rangle\}\!\}$$

Tuple Correctness.

$$\begin{aligned}&\mathbb{T}(Q^1(\mathscr{D}) \uplus \mathcal{I}(Q^1, \Phi, \Delta\mathscr{D}))\\ =&\mathbb{T}(Q^1(\mathscr{R}) \uplus \mathcal{I}(Q^1, \Phi, \Delta\mathscr{R}))\\ =&\mathbb{T}(\mathscr{R} \uplus \Delta\mathscr{R}) && (\text{Lemma 6.3})\\ =&\mathbb{T}(\mathscr{R} - \triangle\ \mathscr{R} \cup \triangleq\ \mathscr{R}) && (\text{Applying delta})\\ =&\mathbb{T}(\mathscr{R}) - \mathbb{T}(\triangle\ \mathscr{R}) \cup \mathbb{T}(\triangleq\ \mathscr{R}) && (\text{Lemma 6.1 and Lemma 6.2})\\ =&R - \triangle\ R \cup \triangleq\ R && (\mathbb{T}(\cdot))\\ =&Q^1(R - \triangle\ R \cup \triangleq\ R)\\ =&Q^1(R \uplus \Delta R)\\ =&Q^1(R')\end{aligned}$$

Fragment Correctness. First, determine the fragments after the incremental maintenance:

$$\begin{aligned}&\mathbb{F}(Q^1(\mathscr{D})) \uplus \mathbb{F}(\mathcal{I}(Q^1, \Phi, \Delta\mathscr{D}))\\ =&\mathbb{F}(\mathscr{R}) \uplus \mathbb{F}(\Delta\mathscr{R})\\ =&\mathbb{F}(\mathscr{R}) \uplus \mathbb{F}(\triangle\ \mathscr{R} \cup \triangleq\ \mathscr{R})\\ =&\mathbb{F}(\mathscr{R}) - \mathbb{F}(\triangle\ \mathscr{R}) \cup \triangleq\ \mathscr{R}\\ =&\{\!\{\mathcal{P}_1, \ldots, \mathcal{P}_n\}\!\} - \triangle\ \{\!\{\mathcal{P}_{d_1}, \ldots, \mathcal{P}_{d_j}\}\!\} \cup \triangleq\ \{\!\{\mathcal{P}_{i_1}, \ldots, \mathcal{P}_{i_i}\}\!\}\end{aligned}$$

Since for every sketch in $\{\!\{\mathcal{P}_1,\ldots,\mathcal{P}_n\}\!\}$ $\uplus_\Delta$ $\{\!\{\mathcal{P}_{d_1},\ldots,\mathcal{P}_{d_j}\}\!\}$ $\uplus_\Delta$ $\{\!\{\mathcal{P}_{i_1},\ldots,\mathcal{P}_{i_i}\}\!\}$, it associates a tuple, and the associated tuple will be inserted or deleted as the sketch does. Thus:

$$D'_{\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^1,\Phi,\Delta\mathscr{D}))} = \{\!\{t_1,\ldots,t_n\}\!\} \uplus_\Delta \{\!\{t_{i_1},\ldots,t_{i_i}\}\!\} \uplus_\Delta \{\!\{t_{d_1},\ldots,t_{d_j}\}\!\}$$

Thus,

$$\begin{aligned}&Q^1(D'_{\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q,\Phi,\Delta\mathscr{D},\mathcal{S}))})\\ =&\{\!\{t_1,\ldots,t_n\}\!\} \uplus_\Delta \{\!\{t_{i_1},\ldots,t_{i_i}\}\!\} \uplus_\Delta \{\!\{t_{d_1},\ldots,t_{d_j}\}\!\}\\ =&R\cup_\Delta R-_\Delta R\end{aligned}$$

Since $Q^1(R') = Q^1(R \uplus \Delta R) = R-_\Delta R\cup_\Delta R$, then we get for $Q^1 \in \mathbb{Q}^1$ that: $Q^1(D') = Q^1(D'_{\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^1,\Phi,\Delta\mathscr{D},\mathcal{S}))}) = Q^1(D'_{\mathbb{S}(\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^1,\Phi,\Delta\mathscr{D},\mathcal{S})))})$.

6.2.2 Inductive Step. Assume for Theorem 6.1, the two correctness properties hold for $Q^i \in \mathbb{Q}^i$ such that the incremental maintenance procedure can correctly produce the tuples and provenance sketches:

$$Q^i(D') = \mathbb{T}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})) \quad \textbf{(tuple correctness)}$$

$$Q^i(D') = Q^i(D'_{\mathbb{S}(\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})))}) \quad \textbf{(fragment correctness)}$$

Next we will show that for $Q^{i+1} \in \mathbb{Q}^{i+1}$, where the $(i+1)$'s operator is on top of Q^i, both properties hold such that:

$$Q^{i+1}(D') = \mathbb{T}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})) \quad \textbf{(tuple correctness)}$$

$$Q^{i+1}(D') = Q^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))}) \quad \textbf{(fragment correctness)}$$

In the following parts, we will demonstrate the proof for operators distinguishing cases for correctness of tuples and fragments.

6.2.3 Selection. Suppose the operator at level $i+1$ is an selection operator. Then we know that

$$Q^{i+1}(D) = \sigma_\theta(Q^i(D))$$

For a selection operator, the annotated tuples' relation between Q^{i+1} and Q^i is like $\langle t_1,\mathcal{P}_1\rangle$ and $\langle s_1,\mathcal{P}_{s_1}\rangle$ in Fig. 6 if the annotated tuple $\langle s_1,\mathcal{P}_{s_1}\rangle$ can satisfy the selection condition of Q^{i+1}. Otherwise, there is no output for an input annotated tuple.

Tuple Correctness. Before we demonstrate the tuple correctness, we first show two properties that will be used for selection operator:

LEMMA 6.7. *The following properties hold:*

$$\sigma_\theta(\mathbb{T}(\mathscr{D})) = \mathbb{T}(\sigma_\theta(\mathscr{D}))$$

PROOF. For $\sigma_\theta(\mathbb{T}(\mathscr{D}))$:

$$\begin{aligned}&\sigma_\theta(\mathbb{T}(\mathscr{D}))\\ =&\sigma_\theta(D) && (\mathbb{T}(\mathscr{D}) = D)\\ =&\{\!\{t \mid t\in D\wedge t\models\theta\}\!\} && (\sigma_\theta)\end{aligned}$$

For $\mathbb{T}(\sigma_\theta(\mathscr{D}))$:

$$\begin{aligned}&\mathbb{T}(\sigma_\theta(\mathscr{D}))\\ =&\mathbb{T}(\{\!\{\langle t,\mathcal{P}\rangle \mid \langle t,\mathcal{P}\rangle\in\mathscr{D}\wedge t\models\theta\}\!\}) && (\sigma_\theta)\\ =&\{\!\{t \mid t\in D\wedge t\models\theta\}\!\} && (\mathbb{T}(\mathscr{D}) = D)\end{aligned}$$

Therefore, $\sigma_\theta(\mathbb{T}(\mathscr{D})) = \mathbb{T}(\sigma_\theta(\mathscr{D}))$: □

LEMMA 6.8. *The following properties hold:*

$$\sigma_\theta(\mathbb{T}(\mathscr{D}\uplus\Delta\mathscr{D})) = \sigma_\theta(\mathbb{T}(\mathscr{D}))\uplus\sigma_\theta(\mathbb{T}(\Delta\mathscr{D}))$$

PROOF. For $\sigma_\theta(\mathbb{T}(\mathscr{D}\uplus\Delta\mathscr{D}))$:

$$\begin{aligned}&\sigma_\theta(\mathbb{T}(\mathscr{D}\uplus\Delta\mathscr{D}))\\ =&\sigma_\theta(\mathbb{T}(\mathscr{D})\uplus\mathbb{T}(\Delta\mathscr{D})) && \text{(lemma 6.3)}\\ =&\sigma_\theta(D\uplus\Delta D) && (\mathbb{T}(\mathscr{D}) = D)\\ =&\sigma_\theta(D-_\Delta D\cup_\Delta D) && (D\uplus\Delta D = D-_\Delta D\cup_\Delta D)\\ =&\sigma_\theta(D)-\sigma_\theta(\Delta D)\cup\sigma_\theta(\Delta D) && (\sigma_\theta \text{ and } \cup,-)\end{aligned}$$

For $\sigma_\theta(\mathbb{T}(\mathscr{D}))\uplus\sigma_\theta(\mathbb{T}(\Delta\mathscr{D}))$

$$\begin{aligned}&\sigma_\theta(\mathbb{T}(\mathscr{D}))\uplus\sigma_\theta(\mathbb{T}(\Delta\mathscr{D}))\\ =&\mathbb{T}(\sigma_\theta(\mathscr{D}))\uplus\mathbb{T}(\sigma_\theta(\Delta\mathscr{D})) && \text{(lemma 6.7)}\\ =&\sigma_\theta(\mathscr{D})\uplus\sigma_\theta(\Delta D) && (\mathbb{T}(\mathscr{D}) = D)\\ =&\sigma_\theta(\mathscr{D})\uplus\sigma_\theta(\Delta D\cup_\Delta D)\\ =&\sigma_\theta(\mathscr{D})\uplus(\sigma_\theta(\Delta D)\cup\sigma_\theta(\Delta D))\\ =&\sigma_\theta(\mathscr{D})-\sigma_\theta(\Delta D)\cup\sigma_\theta(\Delta D) && (\uplus)\end{aligned}$$

Therefore, $\sigma_\theta(\mathbb{T}(\mathscr{D}\uplus\Delta\mathscr{D})) = \sigma_\theta(\mathbb{T}(\mathscr{D}))\uplus\sigma_\theta(\mathbb{T}(\Delta\mathscr{D}))$ □

Therefore, the **tuple correctness** is as following:

PROOF.

$$\begin{aligned}&Q^{i+1}(D')\\ =&\sigma_\theta(Q^i(D')) && (Q^{i+1} = \sigma_\theta)\\ =&\sigma_\theta\left(\mathbb{T}(Q^i(\mathscr{D})\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && (Q^i \text{ holds the property})\\ =&\sigma_\theta\left(\mathbb{T}(Q^i(\mathscr{D}))\uplus\mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && \text{(lemma 6.3)}\\ =&\sigma_\theta\left(\mathbb{T}(Q^i(\mathscr{D}))\right)\uplus\sigma_\theta\left(\mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && \text{(lemma 6.8)}\\ =&\mathbb{T}(\sigma_\theta(Q^i(\mathscr{D})))\uplus\mathbb{T}(\sigma_\theta(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))) && \text{(lemma 6.7)}\\ =&\mathbb{T}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{T}(\sigma_\theta(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))) && (\sigma_\theta(Q^i) = Q^{i+1})\\ =&\mathbb{T}(Q^{i+1}(\mathscr{D}))\uplus\{\!\{t \mid t\in\mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\wedge t\models\theta\}\!\} && (\sigma_\theta)\\ =&\mathbb{T}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{T}(\mathcal{I}(\sigma_\theta(Q^i),\Phi,\Delta\mathscr{D},\mathcal{S})) && (\mathcal{I}(\sigma_\theta) \text{ rule})\\ =&\mathbb{T}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{T}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})) && (\sigma_\theta(Q^i) = Q^{i+1})\\ =&\mathbb{T}(Q^{i+1}(\mathscr{D})\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})) && \text{(lemma 6.3)}\end{aligned}$$

□

Fragment Correctness. We have the assumption that:

$$Q^i(D') = Q^i(D'_{\mathbb{S}(\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})))})$$

Then:

$$
\begin{aligned}
& Q^{i+1}(D') \\
=& \sigma_\theta(Q^i(D')) && (Q^{i+1} = \sigma_\theta(Q^i)) \\
=& \sigma_\theta(Q^i(D'_{\mathbb{S}(\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})))})) \\
& && (Q^i \text{ holds fragments correctness}) \\
=& \sigma_\theta(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})) && (D_{\mathbb{S}(\mathbb{F}(\cdot))} = D_{\mathbb{F}(\cdot)}) \\
=& \sigma_\theta(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})) && (\text{Lemma 6.5}) \\
=& \sigma_\theta(\mathbb{T}(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}))) && (\mathbb{T}(\mathscr{D}) = D) \\
=& \mathbb{T}(\sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}))) && (\text{Lemma 6.7})
\end{aligned}
$$

From above, we can get that

$$Q^{i+1}(D') = \mathbb{T}(\sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})))$$

We now focus on an annotated tuples. For an annotated tuple $\langle t, \mathcal{P}\rangle \in Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})$, we can get that: 1. $t \in Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})$, 2. $\mathcal{P} \in \mathbb{F}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))$ and 3. tuple t can be obtain by $Q^i(D'_{\mathcal{P}})$. For this annotated tuple, if tuple t satisfies the selection condition,$t \models \theta$, of an selection operator on top of Q^i, then $t \in \sigma_\theta(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}))$, $\mathcal{P} \in \mathbb{F}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))$, and tuple t can be obtain by $\sigma_\theta(Q^i(D'_{\mathcal{P}}))$. Now the tuple t is in the result of selection operator, and it can be obtained by $\sigma_\theta(Q^i(D'_{\mathcal{P}}))$. Every tuple t associates with its sketch $\mathcal{P}$, and according to the selection semantics rule, $\mathcal{P}$ is in $\mathbb{F}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))$, which is $\langle t, \mathcal{P}\rangle \in \sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))}))$. Then:

$$
\begin{aligned}
& \langle t, \mathcal{P}\rangle \in \sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})) \\
\Leftrightarrow & \langle t, \mathcal{P}\rangle \in Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}) \wedge t \models \theta && (\sigma_\theta \text{ definition}) \\
\Leftrightarrow & \langle t, \mathcal{P}\rangle \in \sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})})) && (\mathcal{I}(\sigma_\theta)) \\
\Leftrightarrow & \langle t, \mathcal{P}\rangle \in Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})}) && (Q^{i+1} = \sigma_\theta)
\end{aligned}
$$

Thus, for all annotated tuples in $\sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}))$, they are in $Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})})$. Therefore,

$$
\begin{aligned}
& Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})}) \\
=& \sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})})) \\
=& \mathbb{T}(\sigma_\theta(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})}))) \\
=& Q^{i+1}(D')
\end{aligned}
$$

6.2.4 Projection. Suppose the operator at level $i+1$ is an projection operator. Then we have the following:

$$Q^{i+1}(D) = \Pi_A(Q^i(D))$$

For projection operator, the annotated tuples' relation between Q^{i+1} and Q^i is like $\langle t_1, \mathcal{P}_1\rangle$ and $\langle s_1, \mathcal{P}_{s_1}\rangle$ in Fig. 6 such that:

$$\langle t_1, \mathcal{P}_{t_1}\rangle = \Pi_A(\langle s_1, \mathcal{P}_{s_1}\rangle)$$

Before we demonstrate the tuple correctness, we first show two properties hold for projection operator

LEMMA 6.9. *The following properties hold:*

$$\Pi_A(\mathbb{T}(\mathscr{D})) = \mathbb{T}(\Pi_A(\mathscr{D}))$$

PROOF. For $\Pi_A(\mathbb{T}(\mathscr{D}))$:

$$
\begin{aligned}
& \Pi_A(\mathbb{T}(\mathscr{D})) \\
=& \Pi_A(D) && (\mathbb{T}(\mathscr{D}) = D) \\
=& \{\!\{t \mid t' \in D \wedge t'.A = t\}\!\} && (\Pi_A)
\end{aligned}
$$

For $\mathbb{T}(\Pi_A(\mathscr{D}))$:

$$
\begin{aligned}
& \mathbb{T}(\Pi_A(\mathscr{D})) \\
=& \mathbb{T}(\{\!\{\langle t, \mathcal{P}\rangle \mid \langle t', \mathcal{P}\rangle \in \mathscr{D} \wedge t'.A = t\}\!\}) && (\Pi_A) \\
=& \{\!\{t \mid t' \in D \wedge t'.A = t\}\!\} && (\mathbb{T}(\mathscr{D}) = D)
\end{aligned}
$$

Therefore, $\Pi_A(\mathbb{T}(\mathscr{D})) = \mathbb{T}(\Pi_A(\mathscr{D}))$: □

LEMMA 6.10. *The following properties hold:*

$$\Pi_A(\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D})) = \Pi_A(\mathbb{T}(\mathscr{D})) \uplus \Pi_A(\mathbb{T}(\Delta\mathscr{D}))$$

PROOF. For $\Pi_A(\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D}))$:

$$
\begin{aligned}
& \Pi_A(\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D})) \\
=& \Pi_A(\mathbb{T}(\mathscr{D}) \uplus \mathbb{T}(\Delta\mathscr{D})) && (\text{lemma 6.3}) \\
=& \Pi_A(D \uplus \Delta D) && (\mathbb{T}(\mathscr{D}) = D) \\
=& \Pi_A(D - \triangle\, D \,\triangleq\, D) && (D \uplus \Delta D = D - \triangle\, D \cup \triangleq\, D) \\
=& \Pi_A(D) - \Pi_A(\triangle\, D) \cup \Pi_A(\triangleq\, D) && (\Pi_A \text{ and } \cup, -)
\end{aligned}
$$

For $\Pi_A(\mathbb{T}(\mathscr{D})) \uplus \Pi_A(\mathbb{T}(\Delta\mathscr{D}))$

$$
\begin{aligned}
& \Pi_A(\mathbb{T}(\mathscr{D})) \uplus \Pi_A(\mathbb{T}(\Delta\mathscr{D})) \\
=& \mathbb{T}(\Pi_A(\mathscr{D})) \uplus \mathbb{T}(\Pi_A(\Delta\mathscr{D})) && (\text{lemma 6.7}) \\
=& \Pi_A(\mathscr{D}) \uplus \Pi_A(\Delta D) && (\mathbb{T}(\mathscr{D}) = D) \\
=& \Pi_A(\mathscr{D}) \uplus \Pi_A(\triangle\, D \cup \triangleq\, D) \\
=& \Pi_A(\mathscr{D}) \uplus (\Pi_A(\triangle\, D) \cup \Pi_A(\triangleq\, D)) \\
=& \Pi_A(\mathscr{D}) - \Pi_A(\triangle\, D) \cup \Pi_A(\triangleq\, D) && (\uplus)
\end{aligned}
$$

Therefore, $\Pi_A(\mathbb{T}(\mathscr{D} \uplus \Delta\mathscr{D})) = \Pi_A(\mathbb{T}(\mathscr{D})) \uplus \Pi_A(\mathbb{T}(\Delta\mathscr{D}))$ □

Tuple Correctness.

PROOF.

$$
\begin{aligned}
& Q^{i+1}(D') \\
=& \Pi_A(Q^i(D')) && (Q^{i+1} = \Pi_A) \\
=& \Pi_A\left(\mathbb{T}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && (Q^i \text{ holds the property}) \\
=& \Pi_A\left(\mathbb{T}(Q^i(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && (\text{lemma 6.3}) \\
=& \Pi_A\left(\mathbb{T}(Q^i(\mathscr{D}))\right) \uplus \Pi_A\left(\mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))\right) && (\text{lemma 6.10}) \\
=& \mathbb{T}(\Pi_A(Q^i(\mathscr{D})) \uplus \mathbb{T}(\Pi_A(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})) && (\text{lemma 6.9}) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\Pi_A(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})) && (\Pi_A(Q^i) = Q^{i+1}) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \{\!\{t \mid t' \in \mathbb{T}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})) \wedge t'.A = t\}\!\} && (\Pi_A) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(\Pi_A(Q^i),\Phi,\Delta\mathscr{D},\mathcal{S})) && (\mathcal{I}(\Pi_A) \text{ rule}) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})) && (\Pi_A(Q^i) = Q^{i+1}(Q^i)) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})) && (\text{lemma 6.3})
\end{aligned}
$$

□

Fragment Correctness. We have the assumption that:

$$Q^i(D') = Q^i(D'_{\mathbb{S}(\mathbb{F}(Q^1(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})))})$$

Then for a projection operator above Q^i, we have the following proof:

Proof.

$$\begin{aligned}
&Q^{i+1}(D')\\
=&\Pi_A(Q^i(D')) && (Q^{i+1} = \Pi_A(Q^i))\\
=&\Pi_A(Q^i(D'_{\mathbb{S}(\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S})))})) && (Q^i \text{ holds fragments correctness})\\
=&\Pi_A(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})) && (D_{\mathbb{S}(\mathbb{F}(\cdot))} = D_{\mathbb{F}(\cdot)})\\
=&\Pi_A(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})) && (\text{Lemma 6.5})\\
=&\Pi_A(\mathbb{T}(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))) && (\mathbb{T}(\mathscr{D}) = D)\\
=&\mathbb{T}(\Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))) && (\text{Lemma 6.7})
\end{aligned}$$

From above, we can get that

$$Q^{i+1}(D') = \mathbb{T}(\Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})))$$

For an annotated tuple $\langle t,\mathcal{P}\rangle \in Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})$, the following holds: 1. $t \in Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})$, 2. $\mathcal{P} \in \mathbb{F}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))$ and 3. tuple t can be obtain by $Q^i(D'_{\mathcal{P}})$. For this annotated tuple, if expressions in A are projected from tuple t of an projection operator on top of Q^i, then

$$t.A \in \Pi_A(Q^i(D'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))$$

, $\mathcal{P} \in \mathbb{F}(Q^i(\mathscr{D}) \uplus \mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))$, and $t.A$ can be obtain by $\Pi_A(Q^i(D'_{\mathcal{P}}))$. Now the $t.A$ is in the result of projection operator, and it can be obtained by $\Pi_A(Q^i(D'_{\mathcal{P}}))$. Every $t.A$ associates with its sketch $\mathcal{P}$, and according to the projection semantics rule, $\mathcal{P}$ is in $\mathbb{F}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))$, which is

$$\langle t,\mathcal{P}\rangle \in \Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))}))$$

Then:

$$\begin{aligned}
&\langle t,\mathcal{P}\rangle \in \Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))\\
\Leftrightarrow&\langle t,\mathcal{P}\rangle \in t' \in Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}) \wedge t'.A = t && (\Pi_A \text{ definition})\\
\Leftrightarrow&\langle t,\mathcal{P}\rangle \in \Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))})) && (\mathcal{I}(\Pi_A))\\
\Leftrightarrow&\langle t,\mathcal{P}\rangle \in Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))}) && (Q^{i+1} = \Pi_A(Q^i))
\end{aligned}$$

Thus, for all annotated tuples in $\Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))$, they are in $Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))})$.

Therefore,

$$\begin{aligned}
&Q^{i+1}(\mathscr{D}'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))})\\
=&\Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))}))\\
=&\mathbb{T}(\Pi_A(Q^i(\mathscr{D}'_{\mathbb{F}(Q^i(\mathscr{D}))\uplus\mathcal{I}(Q^i,\Phi,\Delta\mathscr{D},\mathcal{S}))})))\\
=&Q^{i+1}(D')
\end{aligned}$$

□

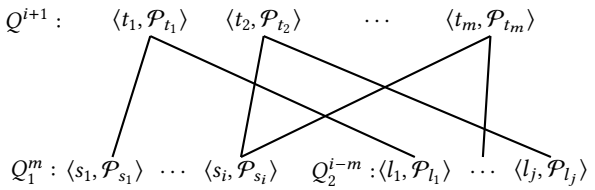


Figure 7: Annotated tuples' relation between Q^i, Q_1^m and Q_2^{i-m}

6.2.5 Cross Product. Suppose the operator at level $i+1$ is a cross product (join) operator, then we have the following:

$$Q^{i+1}(D) = Q_1^m(D) \times Q_2^{i-m}(D)$$

For cross product, the annotated tuples' relation between Q^{i+1} and Q_1^i and Q_2^{i-m} is like $\langle t_2,\mathcal{P}_2\rangle$ and $\langle s_i,\mathcal{P}_{s_i}\rangle$ and $\langle l_j,\mathcal{P}_{l_j}\rangle$ in Fig. 7, such that:

$$\langle t_2,\mathcal{P}_2\rangle = \langle (s_i \circ l_j), \{\!\{\mathcal{P}_{s_i},\mathcal{P}_{l_j}\}\!\}\rangle$$

Each annotated tuple in the result of Q^{i+1} is the product of two annotated tuples, each one from one side.

Tuple Correctness. Before we demonstrate the tuple correctness, we first show the property hold for cross product operator

Lemma 6.11. *The following properties hold:*

$$\mathbb{T}(\mathscr{D}_1 \times \mathscr{D}_2) = \mathbb{T}(\mathscr{D}_1) \times \mathbb{T}(\mathscr{D}_2)$$

Proof. For $\mathbb{T}(\mathscr{D}_1 \times \mathscr{D}_2)$:

$$\begin{aligned}
&\mathbb{T}(\mathscr{D}_1 \times \mathscr{D}_2)\\
=&\mathbb{T}(\{\!\{(\langle t,\mathcal{P}_t\rangle \circ \langle s,\mathcal{P}_s\rangle)^{m*n} \mid \langle t,\mathcal{P}_t\rangle^m \in \mathscr{D}_1 \wedge \langle s,\mathcal{P}_s\rangle^n \in \mathscr{D}_2\}\!\})\\
=&\{\!\{(t \circ s)^{m*n} \mid t^m \in D_1 \wedge s^n \in D_2\}\!\}\\
=&D_1 \times D_2
\end{aligned}$$

For $\mathbb{T}(\mathscr{D}_1) \times \mathbb{T}(\mathscr{D}_2)$:

$$\begin{aligned}
&\mathbb{T}(\mathscr{D}_1) \times \mathbb{T}(\mathscr{D}_2)\\
=&D_1 \times D_2
\end{aligned}$$

Therefore, $\mathbb{T}(\mathscr{D}_1 \times \mathscr{D}_2) = \mathbb{T}(\mathscr{D}_1) \times \mathbb{T}(\mathscr{D}_2)$ □

Thus the following is the **tuple correctness**:

PROOF.

$$
\begin{aligned}
& Q^{i+1}(D') \\
=& Q_1^m(D') \times Q_2^{i-m}(D') && (Q^{i+1}(D) = Q_1^m(D) \times Q_2^{i-m}(D)) \\
=& \mathbb{T}(Q_1^m(\mathscr{D}) \uplus \mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S})) \\
& \times \mathbb{T}(Q_2^{i-m}(\mathscr{D}) \uplus \mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S})) \\
& && \text{(property hold for } Q_1^m(D') \text{ and } Q_2^{i-m}(D')) \\
=& (\mathbb{T}(Q_1^m(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}))) \\
& \times (\mathbb{T}(Q_2^{i-m}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S}))) && \text{(lemma 6.3)} \\
=& (\mathbb{T}(Q_1^m(\mathscr{D})) \times \mathbb{T}(Q_2^{i-m}(\mathscr{D}))) \\
& \uplus (\mathbb{T}(Q_1^m(\mathscr{D})) \times \mathbb{T}(\mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S}))) \\
& \uplus (\mathbb{T}(\mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}) \times \mathbb{T}(Q_2^{i-m}(\mathscr{D})))) \\
& \uplus (\mathbb{T}(\mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S})) \times \mathbb{T}(\mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S}))) \\
& && \text{(lemma 6.11)} \\
=& \mathbb{T}(Q_1^m(\mathscr{D}) \times Q_2^{i-m}(\mathscr{D})) \\
& \uplus \mathbb{T}(Q_1^m(\mathscr{D}) \times \mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S})) \\
& \uplus \mathbb{T}(\mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}) \times Q_2^{i-m}(\mathscr{D})) \\
& \uplus \mathbb{T}(\mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}) \times \mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S})) && \text{(lemma 6.3)} \\
=& \mathbb{T}(Q_1^m(\mathscr{D}) \times Q_2^{i-m}(\mathscr{D})) \\
& \uplus \mathbb{T}\big(Q_1^m(\mathscr{D}) \times \mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S}) \\
& \quad \cup \mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}) \times Q_2^{i-m}(\mathscr{D}) \\
& \quad \cup \mathcal{I}(Q_1^m, \Phi, \Delta\mathscr{D}, \mathcal{S}) \times \mathcal{I}(Q_2^{i-m}, \Phi, \Delta\mathscr{D}, \mathcal{S})\big) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})) && (\mathcal{I}(\times) \text{ rule}) \\
=& \mathbb{T}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})) && \text{(lemma 6.3)}
\end{aligned}
$$

□

Fragment Correctness. From the **tuple correctness**, we get that

$$
\begin{aligned}
& Q^{i+1}(D') \\
=& Q_1^m(D') \times Q_2^{i-m}(D') && (Q^{i+1} = Q_1^m \times Q_2^{i-m}) \\
=& Q_1^m(D'_{\mathbb{S}(\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathbb{F}(\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S})))}) \\
& \times Q_2^{i-m}(D'_{\mathbb{S}(\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathbb{F}(\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S})))}) \\
& && (Q_1^m \text{ and } Q_2^{i-m} \text{ hold the fragments correctness}) \\
=& Q_1^m(D'_{\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathbb{F}(\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S}))}) \\
& \times Q_2^{i-m}(D'_{\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathbb{F}(\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S}))}) && (D_{\mathbb{S}(\mathbb{F}(\cdot))=D_{\mathbb{F}(\cdot)}}) \\
=& Q_1^m(D'_{\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S}))}) \\
& \times Q_2^{i-m}(D'_{\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S}))}) && \text{(Lemma 6.5)} \\
=& \mathbb{T}(Q_1^m(\mathscr{D}'_{\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S}))})) \\
& \times \mathbb{T}(Q_2^{i-m}(\mathscr{D}'_{\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S}))})) && (\mathbb{T}(\mathscr{D} = D))
\end{aligned}
$$

From above, we know that:

$$
\begin{cases}
Q_1^m(D') = \mathbb{T}(Q_1^m(\mathscr{D}'_{\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S}))})) \\
Q_2^{i-m}(D') = \mathbb{T}(Q_2^{i-m}(\mathscr{D}'_{\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S}))}))
\end{cases}
$$

We now focus two annotated tuples $\langle t, \mathcal{P}_t\rangle$ and $\langle s, \mathcal{P}_s\rangle$ such that:

$$
\begin{cases}
\langle t, \mathcal{P}_t\rangle \in \mathbb{T}(Q_1^m(\mathscr{D}'_{\mathbb{F}(Q_1^m(\mathscr{D}'))\uplus\mathcal{I}(Q_1^m,\Phi,\Delta\mathscr{D},\mathcal{S}))})) \\
\langle s, \mathcal{P}_s\rangle \in \mathbb{T}(Q_2^{i-m}(\mathscr{D}'_{\mathbb{F}(Q_2^{i-m}(\mathscr{D}'))\uplus\mathcal{I}(Q_2^{i-m},\Phi,\Delta\mathscr{D},\mathcal{S}))}))
\end{cases}
$$

If $\langle t, \mathcal{P}_t\rangle$ is a non-delta annotated tuple and $\langle s, \mathcal{P}_s\rangle$ is a non-delta annotated from , $\langle (t \circ s), \{\!\{\mathcal{P}_t, \mathcal{P}_s\}\!\}\rangle$ is a non-delta annotated tuple in $Q^{i+1}(\mathscr{D}')$, and $(t \circ s)$ can be obtain by $Q_1^m(D'_{\mathcal{P}_t}) \times Q_2^{i-m}(D'_{\mathcal{P}_s})$. Thus $\mathcal{P}_t$and $\mathcal{P}_s$ are in $\mathbb{F}(Q^{i+1}(\mathscr{D}'))$. If one of $\langle t, \mathcal{P}_t\rangle$ and $\langle s, \mathcal{P}_s\rangle$ is a delta annotated tuple or both are annotated tuples, then $(t \circ s)$ in $\Delta Q^{i+1}(D)$, and the fragments $\mathcal{P}_t$and $\mathcal{P}_s$ are in $\Delta\mathbb{F}(Q^{i+1}(\mathscr{D}'))$. Therefore, for any $\langle t, \mathcal{P}_t\rangle$ and $\langle s, \mathcal{P}_s\rangle$, $(t \circ s)$ in $\Delta Q^{i+1}(D')$, and $\mathcal{P}_t$ and $\mathcal{P}_s$ are in $\mathbb{F}(Q^{i+1}(\mathscr{D}')) \uplus \Delta\mathbb{F}(Q^{i+1}(\mathscr{D}'))$. Then for all annotated tuples from $Q_1^m(\mathscr{D}')$ and $Q_2^{i-m}(\mathscr{D}')$, the cross product result is a bag of annotated tuples that is the same as $Q^{i+1}(\mathscr{D}')$ and all the tuples of $Q^{i+1}(D')$ are the same as $Q^{i+1}(D'_{\mathbb{F}(Q^{i+1}(\mathscr{D}'))\uplus\Delta\mathbb{F}(Q^{i+1}(\mathscr{D}'))})$. From the incremental semantics $\Delta\mathbb{F}(Q^{i+1}(\mathscr{D}')) = \mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})$. Then, $Q^{i+1}(D') = Q^{i+1}(D'_{\mathbb{F}(Q^{i+1}(\mathscr{D}'))\uplus\Delta\mathbb{F}(Q^{i+1}(\mathscr{D}'))})$.

6.2.6 Aggregation. Suppose the operator at level $i+1$ is an aggregation function (any one of **sum**, **count**, **avg**, **min** and **max**), Then we have the following:

$$Q^{i+1}(D) = \gamma_{f(a);G}(Q^i(D))$$

We have the assumption that for $Q^i \in \mathbb{Q}^i$, the **tuple correctness** and **fragment correctness** hold. We will show that these properties still hold for $Q^{i+1} \in \mathbb{Q}^{i+1}$ when Q^{i+1} is an aggregation function.

To show the correctness of tuples and fragments, we focus on one group g. Let $Q_g^{i+1}(Q(D))$ to be an aggregation function works on $Q^i(D)$ and only focus on groups g, $\forall t \in Q^i(D) : t.G = g$. Thus, the two properties for g will be:

$$
\begin{aligned}
& Q_g^{i+1}(D') = \mathbb{T}(Q_g^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})) \\
& && \textbf{(tuple correctness)} \\
& Q_g^{i+1}(D') = Q_g^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q_g^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))}) \\
& && \textbf{(fragment correctness)}
\end{aligned}
$$

Tuple Correctness. For one group g, based on our rule, the annotated tuple before and after applying delta annotated tuples are:

$$
\begin{aligned}
Q_g^{i+1}(\mathscr{D}) &= \{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \\
Q_g^{i+1}(\mathscr{D}') &= \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}
\end{aligned}
$$

Since $Q_g^{i+1}(\mathscr{D}) = \{\!\{\langle \mathbb{A}\ g \circ (f(a)), \mathcal{P}\rangle\}\!\}$, then, we can get that:

$$\mathbb{T}(Q_g^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\}) = \emptyset \qquad (\emptyset_g)$$

Therefore, the tuple correctness for $Q_g^{i+1}(D')$ is shows:

$$
\begin{aligned}
& Q_g^{i+1}(D') \\
=& \mathbb{T}(Q_g^{i+1}(\mathscr{D}')) && (\mathbb{T}(\mathscr{D}) = D) \\
=& \mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}) \\
=& \emptyset \uplus \mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}) \\
=& \mathbb{T}(Q_g^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\}) \uplus \mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}) && (\emptyset_g) \\
=& \mathbb{T}(Q_g^{i+1}(\mathscr{D})) \uplus \left(\mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \uplus \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\})\right) && (\text{lemma } 6.3)
\end{aligned}
$$

Based on the incremental rule of aggregation, it will delete the current group's $(g \circ (f(a)))$ and insert an tuple $(g \circ (\widehat{f(a)}))$. If we do not apply the $\uplus$ but keep them as two independent tuples, which is $\mathbb{T}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \cup \{\!\{\mathbb{A}\ \langle g \circ (f(a)), \widehat{\mathcal{P}}\rangle\}\!\})$. And it is the output of incremental procedure which is $\mathbb{T}(\mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))$

Therefore, for one group g, the tuple correctness hold such that:

$$
\begin{aligned}
& Q_g^{i+1}(D') \\
=& \mathbb{T}(Q_g^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})) \\
=& \mathbb{T}(Q_g^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))
\end{aligned}
$$

Creating or deleting a group. If we create a new group, then $Q_g^{i+1}(\mathscr{D}) = \{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} = \emptyset$. Then $\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}$ is the only output of $\mathcal{I}(Q^i, \Phi, \Delta\mathscr{D}, \mathcal{S})$.

Therefore, $Q_g^{i+1}(D') = \mathbb{T}(Q_g^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))$. If we delete a group, then $Q_g^{i+1}(D') = \emptyset$. From the incremental semantics, we just output $\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \mathcal{P}\rangle\}\!\}$, then from $\emptyset_g$, then the result is empty as well. Thus, for deleting a group, the results are empty. Therefore, for a group g, the property holds such that:

$$
Q_g^{i+1}(D') = \mathbb{T}(Q_g^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))
$$

Fragment Correctness. For one group g, based on the rule, the annotated tuple before and after applying delta annotated tuples are:

$$
\begin{aligned}
Q_g^{i+1}(\mathscr{D}) &= \{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \\
Q_g^{i+1}(\mathscr{D}') &= \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}
\end{aligned}
$$

Since $Q_g^{i+1}(\mathscr{D}) = \{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\}$, then, we can get that:

$$
\mathbb{F}(Q_g^{i+1}(\mathscr{D})) \uplus \mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\}) = \emptyset \qquad (\emptyset_g)
$$

Since $Q_g^{i+1}(\mathscr{D}') = \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\}$, then $Q_g^{i+1}(D') = Q_g^{i+1}(D'_{\widehat{\mathcal{P}}}) = Q_g^{i+1}(D'_{\mathbb{S}(\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\}))})$. Therefore, the fragment correctness for $Q_g^{i+1}(D)$ is shows:

$$
\begin{aligned}
& Q_g^{i+1}(D') \\
=& Q_g^{i+1}(D'_{\mathbb{S}(\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\}))}) \\
=& Q_g^{i+1}(D'_{\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\})}) \\
=& Q_g^{i+1}(D'_{\emptyset\uplus\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\})}) && (\emptyset \uplus \mathbb{F}(\cdot) = \mathbb{F}(\cdot)) \\
=& Q_g^{i+1}(D'_{\mathbb{F}(Q_g^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(f(a)),\mathcal{P}\rangle\}\!\})\uplus\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\})}) && (\emptyset_g) \\
=& Q_g^{i+1}(D'_{\mathbb{F}(Q_g^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(f(a)),\mathcal{P}\rangle\}\!\}\uplus\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\})}) && (\text{Lemma } 6.5)
\end{aligned}
$$

Based on the incremental semantics of aggregation, it will delete the current group's fragments $\mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\})$ and insert fragments $\mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\})$. As tuple correctness, we do not apply the $\uplus$ but keep then as two fragments bags which is

$$
\mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \cup \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\})
$$

And $\mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (f(a)), \mathcal{P}\rangle\}\!\} \cup \{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\})$ is the fragments output from incremental maintenance of aggregation for group g which is $\mathbb{F}(\mathcal{I}(Q_g^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))$

$$
\begin{aligned}
& Q_g^{i+1}(D') \\
=& Q_g^{i+1}(D'_{\mathbb{F}(Q_g^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q_g^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))}) \\
=& Q_g^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q_g^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q_g^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))})
\end{aligned}
$$

Create or deleting a group. If the group g is newly created, then there is no previous sketch for this group, and the $\mathbb{F}(Q_g^{i+1}(\mathscr{D}')) = \mathbb{F}(\{\!\{\mathbb{A}\ \langle g \circ (\widehat{f(a)}), \widehat{\mathcal{P}}\rangle\}\!\})$. Thus $Q_g^{i+1}(D') = Q_g^{i+1}(D'_{\mathbb{F}(\{\!\{\mathbb{A}\langle g\circ(\widehat{f(a)}),\widehat{\mathcal{P}}\rangle\}\!\})})$. If current group g is deleted. Then after maintenance, this group does not exist anymore, and from the incremental semantics, there is no fragments related to this group.

So for a group g, the property holds such that:

$$
Q_g^{i+1}(D') = Q_g^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q_g^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))}) \qquad (\textbf{fragment correctness})
$$

All groups. We have shown that for one group g, the correctness of Theorem 6.1 holds, therefore, for all groups the theorem holds for group-by aggregation query $Q^{i+1} \in \mathbb{Q}^{i+1}$ such that:

$$
Q^{i+1}(D') = \mathbb{T}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S})) \qquad (\textbf{tuple correctness})
$$

$$
Q^{i+1}(D') = Q^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))}) \qquad (\textbf{fragment correctness})
$$

6.2.7 Top-K. Suppose the operator at level $i+1$ is a top-k operator, we have

Tuple Correctness. Based on the rule, the annotated tuple before and after applying delta annotated tuples are:

$$
\begin{aligned}
Q^{i+1}(\mathscr{D}) &= \mathbb{A}\ \tau_{k,O}(\mathcal{S}) \\
Q^{i+1}(\mathscr{D}') &= \mathbb{A}\ \tau_{k,O}(\mathcal{S}')
\end{aligned}
$$

Since $Q^{i+1}(\mathscr{D}) = \tau_{k,O}(\mathcal{S})$, then, we can get that:

$$\mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\triangle\ \tau_{k,O}(\mathcal{S})) = \emptyset \qquad (\emptyset_{\tau_{k,O}})$$

Therefore, the tuple correctness for $Q^{i+1}(D')$ is shown:

$$\begin{aligned}
&Q^{i}(D')\\
=&\mathbb{T}(Q^{i+1}(\mathscr{D}')) && (\mathbb{T}(\mathscr{D}) = D)\\
=&\mathbb{T}(\tau_{k,O}(\mathcal{S}'))\\
=&\emptyset \uplus \mathbb{T}(\tau_{k,O}(\mathcal{S}'))\\
=&\mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\triangle\ \tau_{k,O}(\mathcal{S})) \uplus \mathbb{T}(\triangle\ \tau_{k,O}(\mathcal{S}')) && (\emptyset_{\tau_{k,O}})\\
=&\mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\triangle\ \tau_{k,O}(\mathcal{S}) \uplus\triangle\ \tau_{k,O}(\mathcal{S}')) && (\text{lemma } 6.3)
\end{aligned}$$

From the $\tau_{k,O}$ incremental rule, it will delete a bag of k annotated tuples which is $\triangle\ \tau_{k,O}(\mathcal{S})$, and insert a bag of k updated annotated tuples which is $\triangle\ \tau_{k,O}(\mathcal{S}')$. Like tuple correctness of aggregation, we can keep they as two independent bags of annotated tuples. Then, they are output of incremental procedure which is: $\mathbb{T}(\mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))$ Therefore:

$$\begin{aligned}
&Q^{i}(D')\\
=&\mathbb{T}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{T}(\mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))\\
=&\mathbb{T}(Q^{i+1}(\mathscr{D}) \uplus \mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))
\end{aligned}$$

Fragment Correctness. Based on the rule, the annotated tuple before and after applying delta annotated tuples are:

$$\begin{aligned}
Q^{i+1}(\mathscr{D}) =&\triangle\ \tau_{k,O}(\mathcal{S})\\
Q^{i+1}(\mathscr{D}') =&\triangle\ \tau_{k,O}(\mathcal{S}')
\end{aligned}$$

For $\mathbb{F}(\triangle\ \tau_{k,O}(\mathcal{S}))$, the fragments are the same as from $\mathbb{F}(Q^{i+1}(\mathscr{D}))$. Then

$$\mathbb{F}(Q^{i+1}(\mathscr{D})) \uplus \mathbb{F}(\triangle\ \tau_{k,O}(\mathcal{S})) = \emptyset$$

For $\mathbb{F}(\triangle\ \tau_{k,O}(\mathcal{S}'))$, since the state $\mathcal{S}'$ contains all annotated tuples corresponding to $\mathscr{D}'$, then, the $\mathbb{S}(\mathbb{F}(\tau_{k,O}(\mathcal{S}')))$ contains all fragments of D' to get $Q^{i+1}(D')$ due to the association of tuple and its provenance sketch. Therefore:

$$\begin{aligned}
&Q^{i+1}(D')\\
=&Q^{i+1}(D'_{\mathbb{S}(\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S}')))})\\
=&Q^{i+1}(D'_{\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S}'))})\\
=&Q^{i+1}(D'_{\emptyset\uplus\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S}'))})\\
=&Q^{i+1}(D'_{\mathbb{F}(\tau_{k,O}(Q^{i}(\mathscr{D})))\uplus\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S}))\uplus\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S}'))})\\
=&Q^{i+1}(D'_{\mathbb{F}(\tau_{k,O}(Q^{i}(\mathscr{D})))\uplus\mathbb{F}(\triangle\tau_{k,O}(\mathcal{S})\uplus\triangle\tau_{k,O}(\mathcal{S}'))})
\end{aligned}$$

For $\mathbb{F}(\triangle\ \tau_{k,O}(\mathcal{S}) \uplus\triangle\ \tau_{k,O}(\mathcal{S}'))$, we do not apply $\uplus$ but keep them two independent bags of annotated tuples. Then they are the output of incremental procedure which is: $\mathbb{F}(\mathcal{I}(Q^{i+1}, \Phi, \Delta\mathscr{D}, \mathcal{S}))$. Therefore:

$$\begin{aligned}
&Q^{i+1}(D')\\
=&Q^{i+1}(D'_{\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S}))})\\
=&Q^{i+1}(D'_{\mathbb{S}(\mathbb{F}(Q^{i+1}(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q^{i+1},\Phi,\Delta\mathscr{D},\mathcal{S})))})
\end{aligned}$$

6.3 Conclusion

We have shown that both **tuple correctness** and **fragment correctness** hold for every operator the IMP supports such that the the following holds:

$$Q(D') = \mathbb{T}(Q(\mathscr{D}) \uplus \mathcal{I}(Q, \Phi, \Delta\mathscr{D}, \mathcal{S})) \qquad \textbf{(tuple correctness)}$$

$$Q(D') = Q(D'_{\mathbb{S}(\mathbb{F}(Q(\mathscr{D}))\uplus\mathbb{F}(\mathcal{I}(Q,\Phi,\Delta\mathscr{D},\mathcal{S})))}) \qquad \textbf{(fragment correctness)}$$

Then for every operator the IMP supports to maintain its sketches, the Theorem 6.1 holds.

7 The IMP System

While the semantics from Sec. 5 can be implemented in SQL, an in-memory implementation can be significantly more efficient as we can utilize data structures not available in a SQL-based implementation (see the discussion in Sec. B). As deltas are typically small, having a purely in-memory engine is sufficient. IMP is implemented as a stand-alone in-memory engine that uses a backend database for fetching deltas and for evaluating operations (joins) that require access to large amounts of data. In Fig. 2, IMP's incremental engine is the pipeline shown in red. IMP executes $\mathcal{I}(Q, \mathcal{S}, \Delta\mathscr{D})$ to generate delta sketches. For joins (and cross products), $\Delta\mathscr{R} \bowtie \mathcal{S}$ and $\mathscr{R} \bowtie \Delta\mathcal{S}$ are executed by sending $\Delta\mathscr{R}$ ($\Delta\mathcal{S}$) to the database and evaluating the join in the database.

7.1 Storage Layout & State Data

We store data in a columnar representation for horizontal chunks of a table (*data chunks*). Annotated inserted / deleted tuples are stored in separate chunks. The annotations (provenance sketches) of the rows in a data chunk are stored in a separate column as bit sets.

Sketch & State Data. IMP stores sketches in a hash-table where the key is a query template for which the sketch was created and the value is the sketch and the state of the incremental operators for this query. Here a query template refers to a version of a query Q where constants in selection conditions are replaced with placeholders such that two queries that only differ in these constants have the same key. This is done to be able to efficiently prefilter candidate sketches to be used for a query as the techniques from [37] can determine whether a sketch for query Q_1 can be used to answer a query Q_2 if these queries share the same template. Furthermore, for each sketch we store a version identifier to record which database version the sketch corresponds to. IMP can persist its state in the database.

Aggregation and Top-k. For aggregation, we use hashmaps to implement the state for **sum**, **count**, and **avg**. For **min** and **max** we use red-black trees. The aggregation result for a single group may be updated multiple times when processing a delta with multiple tuples. To avoid producing multiple delta tuples per group we maintain copies of the previous states of groups before an incremental

maintenance operations that are created lazily when a group is updated for the first time when processing a delta $\Delta\mathcal{D}$. Once a delta has been processed, we use the per batch data structure to determine which groups have been updated and output deltas for the group as described in Sec. 5.2.5 and 5.2.6. We follow the same approach for the top-k operator. We use red-black trees for maintaining state for top-k operators (and for **min** and **max**).

7.2 Optimizations

Data transfer between IMP and the DBMS can become a bottleneck. We now introduce several optimizations that reduce communication.

Bloom Filters For Join. For join operators, IMP uses the DBMS to compute the result of $\mathcal{R} \bowtie \Delta\mathcal{S}$ and $\Delta\mathcal{R} \bowtie \mathcal{S}$ which requires sending $\Delta\mathcal{R}$ (or $\Delta\mathcal{S}$) to the database. IMP maintains bloom filters on the join attributes for both sides of equi-joins that are used to filter out rows from $\Delta\mathcal{R}$ (and $\Delta\mathcal{S}$) that do not have any join partners in the other table. If according to bloom filter no rows from the delta have join partners then we can avoid the round trip to the database completely.

Filtering Deltas Based On Selections. If a query involves a selection and all operators in the subtree rooted at a selection are stateless, then we can avoid fetching delta tuples from the database that do not fulfill the selection's condition as such tuples will neither affect the state of operators downstream from the selection nor will they impact the final maintenance result as their decedents will be filtered by the incremental selection. That is, we can push the selection conditions into the query that retrieves the delta.

Optimizing Minimum, Maximum, and Top-k. If the input to an aggregation with **min** or **max** or the top-k is large, then maintaining the sorted map can become a bottleneck. Instead of storing the full input, we can only store the top / bottom m tuples. By keeping a record of the first m tuples, it is safe-guaranteed to delete m tuples from its input. This is useful when dealing with deletion. For example, if we keep first 20 minimum values for a group when we build the state for this group. The state data can support deletions that removes at least than 20 tuples. To achieve this, we pass an parameter to IMP engine when building the state data for these operators, the engine will only get a certain number of tuples to build the state. This optimization will help for deletion, because it is necessary to know the next minimum/maximum, while for insertion, only one tuple per group stored can make incremental maintenance work, because the only tuple is always the current minimum/maximum and the new minimum/maximum will always be in the state data if it comes from inputs.

7.3 Concurrency Control & Sketch Versions

So far we have assumed that the database backend uses snapshot isolation and that sketch versions are identified by snapshot identifiers. However, in snapshot isolation, each transaction sees data committed before it started (identified by a snapshot identifier) and its own changes. Thus, for a transaction that wants to use a sketch after updating a table accessed by the query for the sketch, we have to include the transaction's updates when maintaining the sketch. We can track these updates using standard audit logging mechanisms supported nativly in databases like Oracle or implemented through extensibility mechanisms like triggers to keep a history of row versions. For statement-level snapshot isolation (isolation level `READ COMMITTED` in systems like Postgres or Oracle), we face the challenge that even if we run the queries for incremental maintenance in the same transaction as the query that uses the updated sketch, these queries may see different versions of the database. Thus, supporting statement-level snapshot isolation requires either deeper integration into the database to run the maintenance query as of the same snapshot as the query that uses the sketch or use techniques like reenactment [5, 6] to reconstruct such database states.

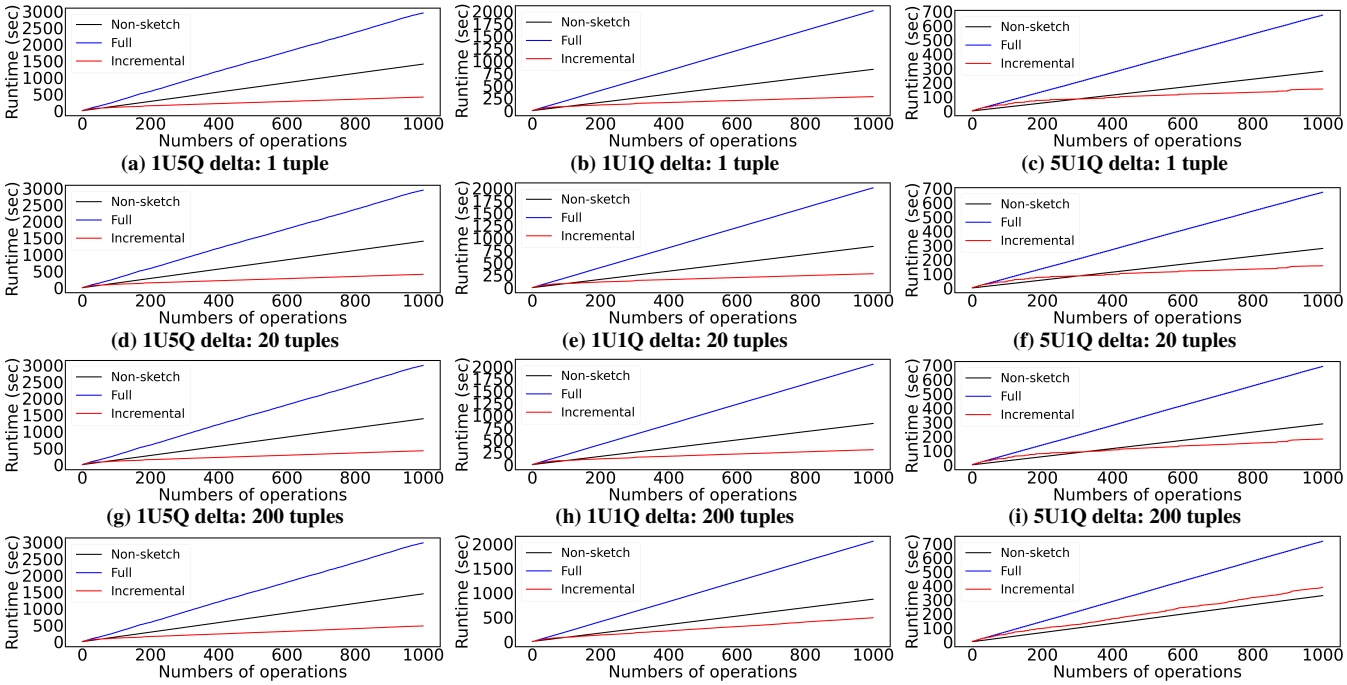
7.4 Selecting Sketch Attributes and Ranges

Both the choice of attribute for a sketch and the choice of ranges can affect the amount of data covered by a sketch. Regarding the choice of attribute, we first identify which attributes are safe using techniques from [37]. In [30] we studied cost-based selection of attributes for sketches. For IMP we employ a heuristic to select attributes based on the insights from [30]. We select attributes that are important for a query such as group-by attributes or attributes for which efficient access methods, e.g., an index, are available. Regarding the choice of ranges, as long as ranges are fine-granular enough and data is roughly evenly distributed across ranges, the exact choice of ranges has typically neglectable effect on sketch size. We use the bounds of equi-depth histograms maintained by many DBMS as statistics as ranges. Note that we generate ranges to cover the whole domain of an attribute instead of only its active domain. If a significant fraction of the data in a relation is updated, then this can lead to an imbalance in the amount of data per range and in turn to a degradation of the performance of sketches over time. As a significant change in distribution is unlikely to occur frequently, we can simply update the ranges and recapture sketches. If more frequent changes to distribution are expected, then we could track estimates of the number of tuples per range and split or merge ranges that under- or overflow. If a range ρ is split into two ranges ρ_1 and ρ_2 then any sketch containing ρ would then be updated to contain ρ_1 and ρ_2. If two ranges ρ_1 and ρ_2 are merged into a single range ρ, then any sketch containing either ρ_1 and ρ_2 is updated to contain ρ instead.

8 Experiments

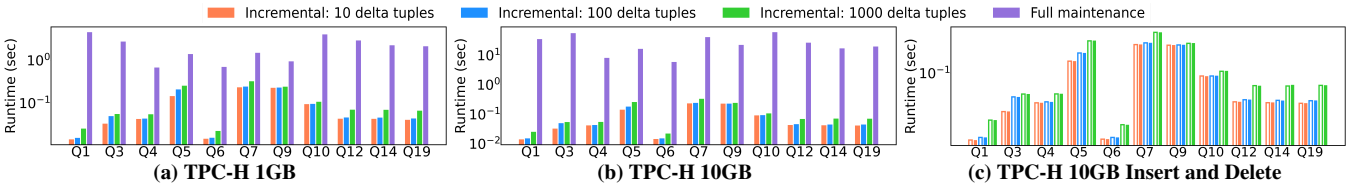
We evaluate IMP against two baselines: *full maintenance* and an approach that does not use PDBS to answer the following questions: Q1: Does PDBS with IMP improve the performance of workloads that mix queries and updates? Q2: How does incremental maintenance with IMP compare against full maintenance? Q3: Which parameters affect IMP's performance? Q4: How effective are our optimizations? Q5: How do eager and lazy maintenance compare? All experiments use a machine with 2 x 3.3Ghz AMD Opteron 4238 CPUs (12 cores) and 128GB RAM running Ubuntu 20.04 (linux kernel 5.4.0-96-generic) and Postgres 16.2. IMP's source code and the experimental setup are available at [29]. Experiments are repeated at least 10 times. We report median runtimes. The maximum variance was $< 5\%$ and $< 1\%$ in most experiments.

Datasets and Workloads. We use the *TPC-H* benchmark, a real world *Crime* dataset and synthetic data (tables with 10M rows with at least 11 attributes). Each synthetic table has a key attribute `id`. For the other attributes, the values of one attribute (a) are chosen



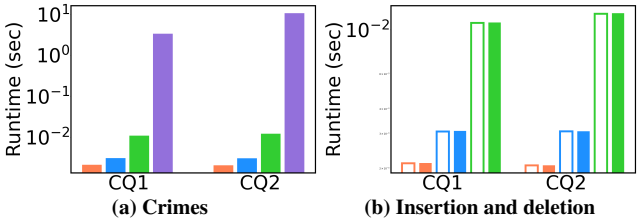
(a) 1U5Q delta: 1 tuple (b) 1U1Q delta: 1 tuple (c) 5U1Q delta: 1 tuple
(d) 1U5Q delta: 20 tuples (e) 1U1Q delta: 20 tuples (f) 5U1Q delta: 20 tuples
(g) 1U5Q delta: 200 tuples (h) 1U1Q delta: 200 tuples (i) 5U1Q delta: 200 tuples
(j) 1U5Q delta: 2000 tuples (k) 1U1Q delta: 2000 tuples (l) 5U1Q delta: 2000 tuples

Figure 8: Varying delta size to check the performance of non-sketch, full and incremental approaches



(a) TPC-H 1GB (b) TPC-H 10GB (c) TPC-H 10GB Insert and Delete

Figure 9: Maintaining provenance sketches: incremental versus full maintenance on the TPC-H.



(a) Crimes (b) Insertion and deletion

Figure 10: Maintaining provenance sketches: incremental versus full maintenance on the Crime dataset.

uniform at random. The remaining attributes are linearly correlated with a subject to Gaussian noise to create partially correlated values.

8.1 Mixed Workload Performance

In this experiment, we measure the end-to-end runtime of *IMP*, full maintenance (*FM*), and non-sketch (*NS*) on mixed workloads consisting of queries and updates. Both IMP and FM start without any sketches. The cost of maintaining and creating sketches is included in the runtime. Each workload consists of 1000 operations (each operation is either a query or an update). We refer to the ratio between queries and updates (the *query-update ratio*). We use the technique from [37] to determine whether an existing sketch for a query Q' can be used to answer the current query Q. If an existing sketch can be reused, we maintain the sketch if necessary. Otherwise, we create a new sketch. When an update on relation R is executed, we determine the delta and append it to the delta table for R, associating each tuple with a snapshot identifier. This enables us to fetch only delta tuples of updates that were executed after the sketch was last maintained.

For this experiment, we utilize a query template $Q_{endtoend}$ (see Sec. A) which is a group-by-aggregation-having query evaluated over the synthetic dataset and delta sizes 1, 20, 200 and 2000. We consider three query-update ratios: *1U1Q* (one update per one query), *1U5Q* (one update per five queries) and *5U1Q* (five updates per one query). Fig. 8 shows the runtime for several combinations of query-update ratio and delta size (for this experiment the number of tuples affected by each update in the workload). FM has the highest cost, as the cost incurred by recapturing sketches frequently outweighs the benefit of using these sketches. IMP outperforms both baselines, except for the extreme case 5U1Q with delta size 2000 (per update) where 5 updates, in total affecting at least 10000 tuples are executed

between two adjacent queries, since the adjacent queries may not use the same sketch.

Insights: IMP significantly improves the performance of mixed workloads using PDBS.

8.2 Incremental Versus Full maintenance

We now compare IMP against FM. We vary the *delta size* focusing on realistic delta sizes: 10, 50, 100, 500 and 1000.

8.2.1 TPC-H. The results for TPC-H (www.https://www.tpc.org/tpch/) at SF1 (~ 1 GB) and SF10 (~ 10GB) are shown in Fig. 9a and 9b. We selected queries that benefit from sketches [37] and are sufficiently complex (multiple joins, aggregation with HAVING or top-k). We turn on the selection push down and join bloom filter optimizations (see Sec. 7.2). The runtime of FM only depends on the current size of the database. Thus, we do not include results for different delta sizes for this method. IMP outperforms FM by at least a factor of 3.9 and up to a factor of ~2497, demonstrating the effectiveness of incremental maintenance. Importantly, the runtime of IMP, while depending on delta size, is mostly unaffected database size as join is the only incremental operator accessing the database.

Fig. 9c shows the incremental maintenance runtime for both insertion and deletion for certain amount of delta for 10 GB database size.

8.2.2 Crime Dataset. The Crime[4] dataset records incidents in Chicago and consists of a single 1.87GB table with $7.3M$ rows. We use two queries (SQL code for all queries is shown in Sec. A): `CQ1`: The numbers crime of each year in each beat (geographical location). `CQ2`: Areas with more than 1000 crimes. We use realistic delta sizes (10 to 1000). As shown in Fig. 10a incremental maintenance outperform full maintenance by at least 2 orders of magnitude (OOM). Fig. 10b show the incremental maintenance runtime for both insertion and deletion under given delta size.

Insights: For deltas up to 1000 tuples, IMP outperforms FM by several OOM.

8.3 Microbenchmarks

Next, we evaluate in detail how IMP's performance is affected by various workload parameters using the synthetic dataset. We compare IMP against FM. The database size is kept constant, i.e., for FM, the runtime is not affected by varying the delta size.

8.3.1 Number of groups. We use a query Q_{groups} (SQL code shown in Sec. A) that is group-by aggregation with HAVING over a single table and vary the number of groups: 50, $1K$, $5K$ and $500K$. As the data structures maintained by our approach for aggregation stores an entry for each group and the number of groups affected by a delta also depends on the number of groups, we expect that runtime will increase when increasing the number of groups. As shown in Fig. 11b, for delta sizes up to 1000 tuples, incremental maintenance outperforms FM by 2 (500k groups) to 3 (50 groups) OOM. Fig. 12b shows that the break even point (where FM starts to outperform incremental maintenance) lies at delta sizes between ~3.5% (for 50 group) and ~ 5.5% for (500k groups). While the runtime of IMP increases when increasing the number of groups, the effect is more pronounced for FM that calculates results for all groups.

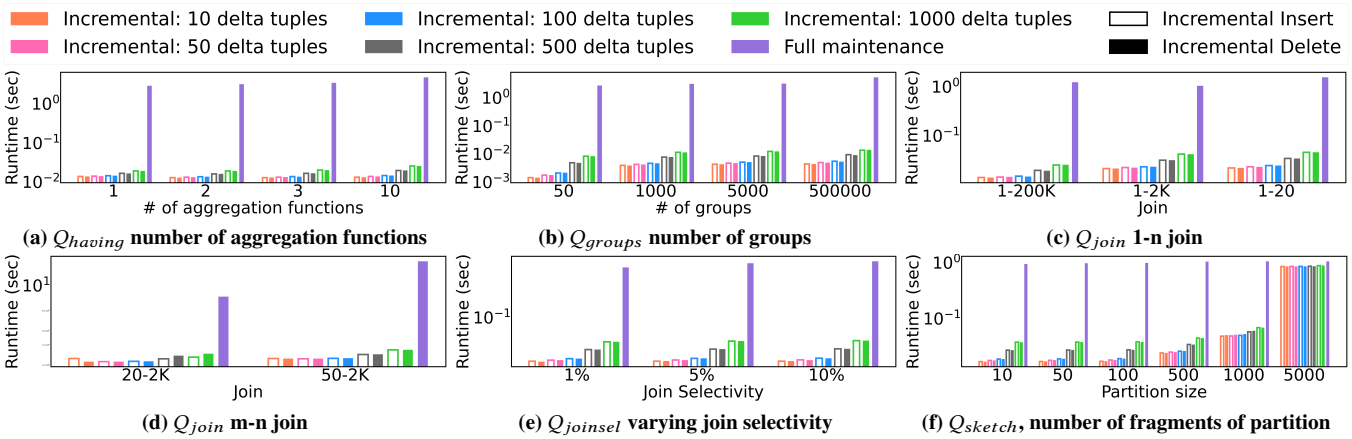
8.3.2 Number of aggregation functions. In this experiment, we use a query Q_{having} (see Sec. A) which is an group-by aggregation on a single table with filtering in the HAVING clause on the aggregation function result. The total number of groups is set to 5000 in this experiment. We vary the number of aggregation functions used in the HAVING condition as our approach has to maintain the results for these aggregation. Fig. 11a and Fig. 12a show the runtime of incremental vs. FM using both realistic delta sizes and large deltas. The runtime of IMP is linear in the size of the delta for this query with a coefficient that depends on the number of aggregation functions. For realistic delta sizes, IMP outperforms FM by ~ 2 OOM. As shown in Fig. 12a, IMP is faster than FM for deltas of up to ~ 5% of the database.

8.3.3 Joins. We evaluate the performance of group-by aggregation queries with HAVING over the result of an equi-join using query template Q_{join} (see Sec. A). Both input tables have $10M$ rows. The synthetic tables are designed as the follows: for an $m-n$ join $R \bowtie S$, the selectivity is 100% for table S, and there are $10^8/n$ distinct join attribute values with a multiplicity of n; for the other table R, there are m tuples that join with each distinct join attribute value in S. For instance, the result size for $2-2k$ as well as for $2-200k$ is $2 \cdot 10M = 20M$ tuples. Fig. 11c and Fig. 12c show the runtime of IMP vs. FM for $1-n$ joins, and Fig. 11d and Fig. 12d show the results for $m-2K$ joins. In the 1-n join experiment, the 1-20 join is more expensive than the 1-20k and 1-200k joins because even through the 1-20 join produces less results, there are more groups for the aggregation functions above the join operator as the join attribute of table S the group-by attribute. There are $10^8/20$ distinct groups and each group has a multiplicity of 20. For the m-n join, the queries all have the same number of groups. For 50-2k more join results have to be produced and, thus, 50-2k is slower than 20-2k. Recall from Sec. 8.2.1, that IMP computes $\Delta R \bowtie S$ by running a SQL query. Thus, incrementally maintaining joins requires sending delta tuples for the join inputs to the DBMS, resulting in a lower break even point for Q_{join} than for Q_{having}.

8.3.4 Join selectivity. To evaluate performance of queries with more selective joins, we use query template group-by-aggregation over join: $Q_{joinsel}$ (R join S) (see Sec. A), to evaluate different join selectivity: 1%, 5%, and 10%. We control the join attribute values in table S (this attribute joins with another attribute in table R) such that only certain percentage of tuples join. Fig. 11e and Fig. 12e show the runtime of incremental and full maintenance using both realistic delta size and large deltas. For small deltas, the selectivity of the join has less of an impact on IMP than for large delta sizes. This is due to the fact that for small deltas we are joining a small table ($\Delta\mathcal{R}$) with a large table ($\mathcal{S}$), i.e., the bottleneck is scanning the large table.

8.3.5 Varying Partition Granularity. In this experiment, we vary #frag, the number of fragments of the partition Φ from 10 to $5,000$. We use query template Q_{sketch} (SQL code shown in Sec. A) which is a group-by aggregation query with HAVING over the results of a join. Fig. 11f and Fig. 12f show the runtime for IMP and FM. While the cost of FM is impacted by #frag, the dominating cost is evaluating the full capture query, resulting in an insignificant runtime increase

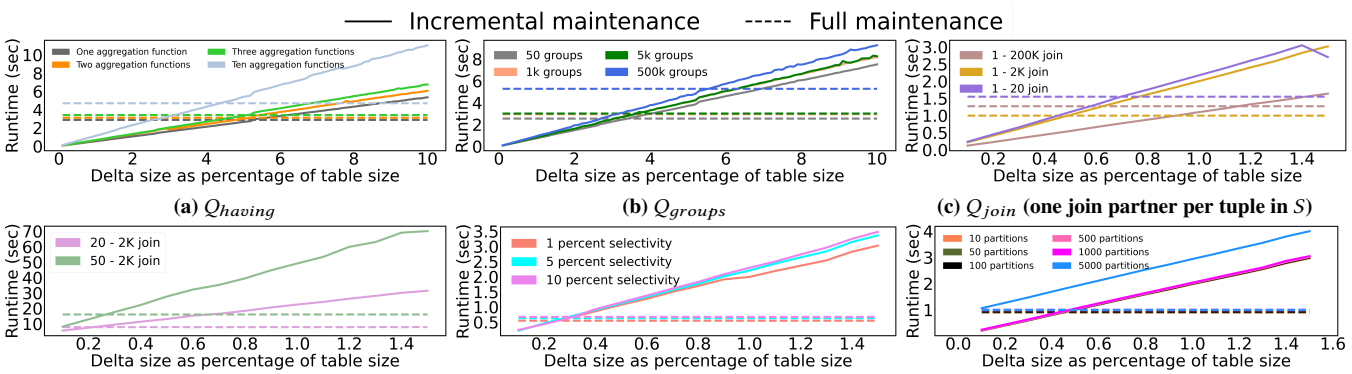
[4] https://data.cityofchicago.org/Public-Safety/Crimes-2001-to-Present/ijzp-q8t2



(a) Q_{having} number of aggregation functions (b) Q_{groups} number of groups (c) Q_{join} 1-n join

(d) Q_{join} m-n join (e) $Q_{joinsel}$ varying join selectivity (f) Q_{sketch}, number of fragments of partition

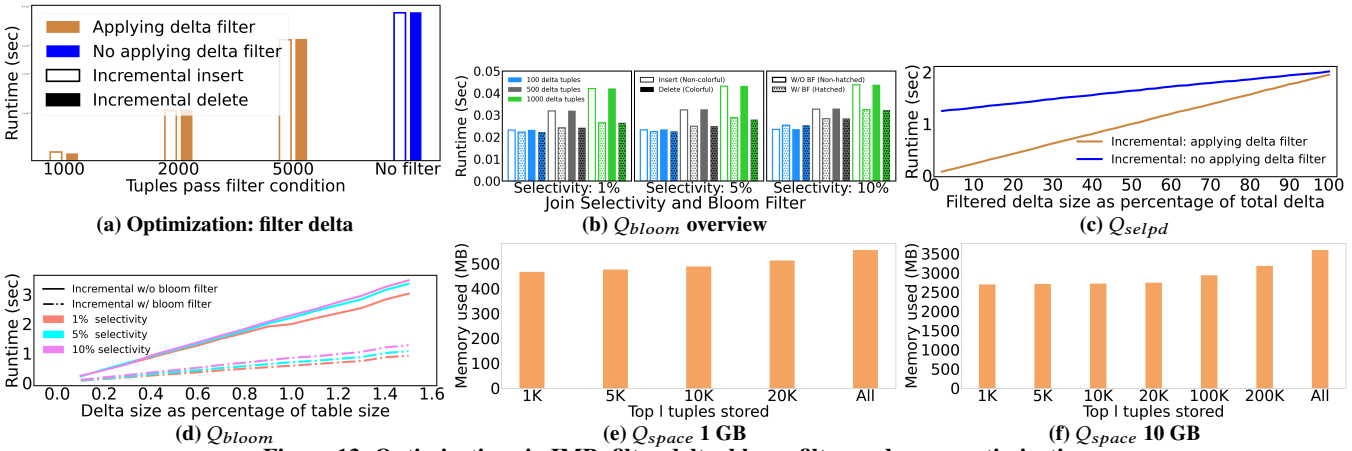
Figure 11: Microbenchmarks ("realistic" delta size): varying delta size from 10 tuples to 1000 tuples.



(a) Q_{having} (b) Q_{groups} (c) Q_{join} (one join partner per tuple in S)

(d) Q_{join} (varying #join partners per tuple in S) (e) $Q_{joinsel}$ (f) Q_{sketch}

Figure 12: Microbenchmarks: varying delta size to determine the "break even point" where FM outperforms IMP.



(a) Optimization: filter delta (b) Q_{bloom} overview (c) Q_{selpd}

(d) Q_{bloom} (e) Q_{space} 1 GB (f) Q_{space} 10 GB

Figure 13: Optimizations in IMP: filter delta, bloom filter and space optimization

when #frag is increased. In contrast, IMP maintenance cost increases linearly in the delta size and the cost paid per tuple is roughly linear in #frag.

8.4 Optimizations

8.4.1 Selection push-down for deltas. In this experiment, we evaluate the effectiveness of our delta selection push-down optimization that pre-filters the delta based on selection conditions in the query. We use the query Q_{selpd} which is a group-by aggregation query without joins (SQL code is shown in Sec. A). We evaluate

the performance of IMP with and without delta filtering, varying the selectivity of the query's selection condition (`WHERE` clause). We fix the delta size to 2.5% of the table. Then we gradually increase the fraction of the delta that fulfills the selection conditions from 2% to 100%. Fig. 13c shows results of this experiment. The results show that the runtime of filtering deltas increases linearly in the selectivity of the query's selection condition. Even for larger selectivities, the cost of filtering delta tuples is amortized by reducing the cost of IMP (and communication between IMP and that database). Thus, this optimization should be applied whenever possible.

8.4.2 Join optimization using bloom filters. IMP uses bloom filters for each join to track which tuples potentially have join partners. This enables us to avoid evaluating a join $\Delta\mathcal{R} \bowtie \mathcal{S}$ ($\mathcal{R} \bowtie \Delta\mathcal{S}$) when we can determine that no tuples in the delta have any join partners or at least to reduce delta size. We again use $Q_{joinsel}$ (group-by aggregation with `HAVING` over a join result). Fig. 13b shows the runtime of IMP applying bloom filter for joins varying selectivity and delta size. Fig. 13d shows runtime of IMP for large delta sizes. The result shows that filtering the delta using bloom filters is effective due to (i) the reduction in data transfer between IMP and the database, (ii) the reduction of the input size for the query evaluating $\Delta\mathcal{R} \bowtie \mathcal{S}$ by reducing the size of $\Delta\mathcal{R}$, and (iii) reducing the input size for incremental operators. As show in Fig. 13d, bloom filter optimization is effective for both low and high selectivity and across all delta sizes we have tested.

Insights: IMP's performance is mainly impacted by delta size. As join requires a round trip to the databases, queries with join are typically more expensive. Our bloom filter optimization reduces this cost. Nonetheless, IMP significantly outperforms FM.

8.4.3 Top-K operator. According to Sec. 5.2.7, for a top-k operator we store all of its inputs in an ordered map to be able to deal with deletions that remove a tuple from the current top-k. In practice this may be overkill as keeping a buffer of top-l for $l > k$ tuples is often sufficient. The potential drawback is that if all tuples from the buffer are deleted, we have to recapture the sketch. In this experiment, we evaluate the performance of IMP engine for top-k operator by storing certain amount of top-k. The query we use is a top-10 query Q_{top-k} (see Sec. A). We control the number of how many top items stored in the state data: 20, 50 and 100. Then we delete data (20 tuples per update) from the table (table contains 50000 tuples and 5000 group by values with each group has roughly 10 tuples). For deletion, we have several strategies: 1. always delete the first 2 minimal groups, 2. always delete randomly tuples, 3. control the ratio between random deletion and deleting minimal groups called R-M ratio where like query-update ratio: R updates containing randomly deleted tuples followed by M updates including tuples of deleting 2 minimal groups each update. In our setting, the ratio are 2:1 and 4:1. For strategy 1 and 3, we have 500 updates in total while for strategy 2, we delete data till the table is empty since table and updates are all uniformly distributed and only deleting all tuples can see the effect of how the IMP engine performs. If there are less than k groups stored in the state, our IMP will fully maintain the sketches. Fig. 14 shows the runtime of different top items stored in the state under different deleting strategies. We can observe the following: 1. The more data items stored in the top-k state data, the less frequent to fully maintain the sketches. 2. The more randomly the delta is generated, the less frequently the sketches are maintained. Fig. 15 demonstrates the memory consumption of maintaining the Q_{top-k} in IMP (the y value is the memory consumption for each the x-th operation). These figures show that: 1. The more data items stored in the state, the more the memory consumes. 2. Memory consumption will gradually decrease before sketches fully maintained. 3. At the point the memory consumption sudden increasing, it means a full maintenance will increase size of the state data of top-k to the size as it should be of storing top 20, 50 or 100, while the total memory size still decreases due to the size decreasing of aggregation state (less number of groups than before).

For top-k optimization, we evaluate one TPC-H query Q_{space} (TPC-H Q10, see Sec. A) as well to examine the state memory consumption under storing different top number of data items coming into top-k operator. For 1 GB dataset, the total tuples number for top-k is 37293 and for 10 GB dataset, the number is 371104. We vary the number for m to examine memory used (MB). For TPC-H 1GB, we vary the number of l (top l data items stored in top-k state data) in $1K$, $5K$, $10K$, $20K$ and all tuples. For TPC-H 10GB, these number values are $1K$, $5K$, $10K$, $20K$, $100K$, $200K$ and all. Fig. 13e and Fig. 13f show the memory used varying the stored tuple number l. A knowledge learned from the experiment is that memory saving can be achieved by reducing the number of tuples kept in the state data.

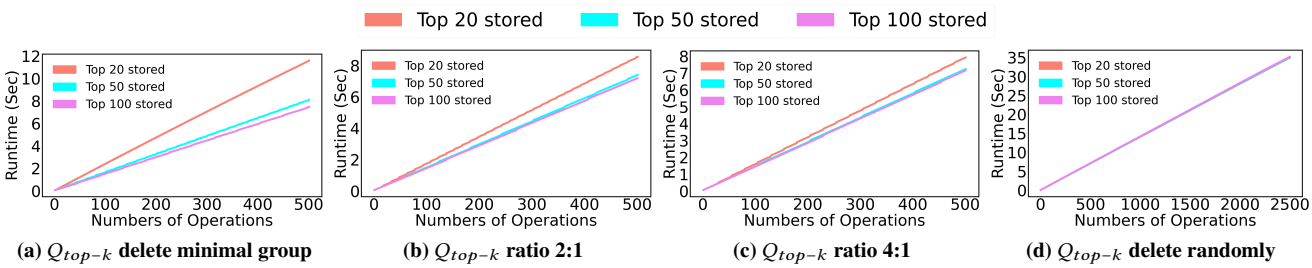
8.5 Maintenance Strategies

Recall the we support two strategies for maintaining sketches: (i) *eager* which batches deltas and maintains sketches proactively once a number of delta tuples equal or exceeding the batch size have been updated and (ii) *lazy* which only maintains stale sketches when a new version is needed to answer a query. We evaluating the impact of batch size on maintenance cost, by measuring the total maintenance cost for 1000 updates that are processed in batches of varying sizes using the eager strategy. For this setting, we have two queries: first is a aggregation with `HAVING` query template $Q_{endtoend}$ (see Sec. A) and the other is join-aggregation with `HAVING` $Q_{joinsel}$ query template with 5% selectivity (see Sec. A). Fig. 16 shows the total runtime of this setting. The net result is that batch sizes below 50 should be avoided for eager maintenance as they significantly increase the cost of maintenance. Another take-away is that lazy is typically preferable as (i) it leads to larger delta sizes as we delay maintenance as long as possible and (ii) we avoid maintaining provenance sketches that are not used.

8.6 Additional Memory Consumption

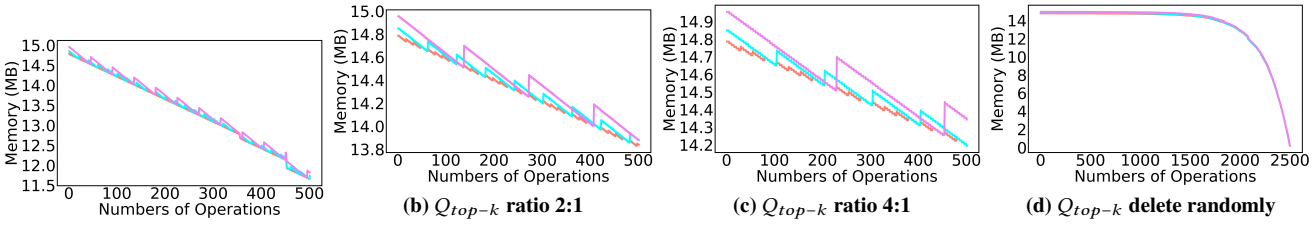
8.6.1 Memory Usage of Aggregation Function and Join. In this experiment, we present the memory usage of queries $Q_{joinsel}$ (see Sec. A) and Q_{groups} (see Sec. A). We have exactly the same setting as evaluating for runtime performance. Fig. 17 shows the memory consumption. For Q_{groups}, for fixed number of groups, the state data size is stable, and the memory consumption increases due to the increasing of delta data size. The is the same for $Q_{joinsel}$.

8.6.2 Memory usage of Sketches and Ranges. In the Fig. 18, we show the actual sizes of sketches and ranges. We encode each sketch as a bitvector. Typically, the size is pretty small. For ranges, we store the boundaries of each range in a list. For n ranges, we



(a) Q_{top-k} **delete minimal group** **(b)** Q_{top-k} **ratio 2:1** **(c)** Q_{top-k} **ratio 4:1** **(d)** Q_{top-k} **delete randomly**

Figure 14: Top-K: runtime of varying maintenance strategies



(a) Q_{top-k} **delete minimal group** **(b)** Q_{top-k} **ratio 2:1** **(c)** Q_{top-k} **ratio 4:1** **(d)** Q_{top-k} **delete randomly**

Figure 15: Top-K: memory usage of varying maintenance strategies

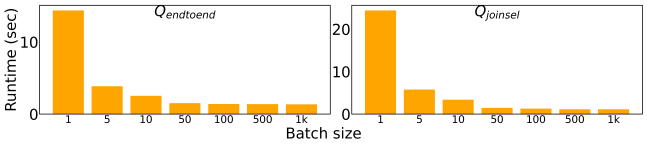
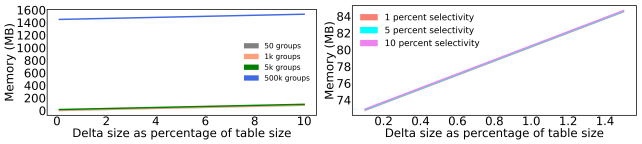


Figure 16: Cost of maintaining 1000 updates using eager maintenance, varying batch size.



(a) Q_{groups} **Memory usage** **(b)** $Q_{joinsel}$ **Memory usage**

Figure 17: Memory usage of Q_{groups}

record $n+1$ values in the list i.e., assuming the ranges are $[1, 4)$, $[4, 9)$, the list will be: $(1, 4, 9)$.

9 Conclusions And Future Work

We present the first approach for in-memory incremental maintenance of provenance sketches. Our IMP system implement incremental maintenance rules for sketch-annotated data. Using bloom filters and selection-push-down, we further improve the performance of the incremental maintenance process. Our experimental results demonstrate the effectiveness of our approach and optimizations, outperforming full maintenance by several orders of magnitude. In future work, in addition to extending IMP with support for more operators, e.g., set difference and nested or recursive queries, we will investigate how to integrate provenance-based data skipping and incremental maintenance of sketches with cost-based query optimization and self-tuning. Another open research question is how IMP can be extended as a general IVM engine for provenance information. While many of the incremental operator semantics could be extended to support that, optimizations that rely on approximation may no longer be acceptable and, in contrast to sketches which are of a fixed size, provenance information may be large. Furthermore, we conjecture that IMP could be extended for maintaining summarizes of provenance [3, 4, 28] which, like sketches, can tolerate approximation and are typically small. For example, consider a data integration scenario where we want to track the set of data sources a query result depends on.

Artifacts

Supplementary material, IMP's source code (folder `IMP engine` and `postgreSQL`), and experiment scripts and data (folder `dataset`) are available at [29].

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Number of sketches or ranges	100	200	500	1000	2000	5000	10000	20000	100000
Memory of Sketches (MB)	0.000040	0.000056	0.000088	0.000152	0.000280	0.000656	0.001280	0.002528	0.012528
Memory of Ranges (MB)	0.004508	0.008908	0.022108	0.044108	0.088108	0.220108	0.440108	0.880108	4.400108

Figure 18: Sizes different sketches and ranges in memory

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A Appendix I: Query List

A.1 Synthetic dataset query list

A.1.1 Different number of aggregation functions Q_{having}.

One aggregation function.

```
SELECT a, avg(b) AS ab
FROM r500
GROUP BY a
```

Two aggregation functions.

```
SELECT a, avg(b) AS ab
FROM r500
GROUP BY a
HAVING avg(c) < 1000
```

Three aggregation functions.

```
SELECT a, avg(b) AS ab
FROM r500
GROUP BY a
HAVING avg(c) < 1000 and avg(d) < 1200
```

Ten aggregation functions.

```
SELECT a, avg(b) AS ab
FROM r500
GROUP BY a
HAVING avg(c) < 1000 and avg(d) < 1200 and avg(e) > 0
    and avg(f) > 0 and avg(g) > 0 and avg(h) > 0
    and avg(i) > 0 and avg(j) > 0
```

A.1.2 Number of groups Q_{groups}.

50 groups.

```
SELECT a, avg(b) AS ab
FROM t1gb50g
GROUP BY a
HAVING avg(c) < 3
```

1K groups.

```
SELECT a, avg(b) AS ab
FROM t1gb1000g
GROUP BY a
HAVING avg(c) < 320
```

5K groups.

```
SELECT a, avg(b) AS ab
FROM t1gb5000g
GROUP BY a
HAVING avg(c) < 1600
```

500K groups.

```
SELECT a, avg(b) AS ab
FROM t1gb500000g
GROUP BY a
HAVING avg(c) < 1600
```

A.1.3 Join.

1-200K joins.

```
SELECT a, avg(b) AS ab
FROM (
    SELECT a AS a, b AS b, c AS c
    FROM t1gb50g
    WHERE b < 10
) tt JOIN tjoinhelp ON (a = ttid)
GROUP BY a
HAVING avg(c) < 10
```

1-2K joins.

```
SELECT a, avg(b) AS ab
FROM(
    SELECT a AS a, b AS b, c AS c
    FROM t1gbjoin WHERE b < 1000
) tt JOIN tjoinhelp ON (a = ttid)
GROUP BY a
HAVING avg(c) < 1000
```

1-20 joins.

```
SELECT a, avg(b) AS ab
FROM (
    SELECT a AS a, b AS b, c AS c
    FROM t1gb500000g
    WHERE b < 100000
) tt JOIN tjoinhelp ON (a = ttid)
GROUP BY a
HAVING avg(c) < 100000
```

A.1.4 Join selectivity: $Q_{joinsel}$.

1% selectivity.

```
SELECT a, avg(b) AS ab
FROM t1gbjoin JOIN tjoinhelppercent1 ON (a = ttid)
WHERE b < 1000
GROUP BY a
HAVING avg(c) < 1000
```

5% selectivity.

```
SELECT a, avg(b) AS ab
FROM t1gbjoin JOIN tjoinhelppercent5 ON (a = ttid)
WHERE b < 1000
GROUP BY a
HAVING avg(c) < 1000
```

10% selectivity.

```
SELECT a, avg(b) AS ab
FROM t1gbjoin JOIN tjoinhelppercent10 ON (a = ttid)
WHERE b < 1000
GROUP BY a
HAVING avg(c) < 1000
```

A.1.5 Fragment number: Q_{sketch}.

```
SELECT a, avg(b) as ab
FROM (
      SELECT a as a, b as b, c as c
      FROM t1gbjoin
      WHERE b < 1000) tt
JOIN tjoinhelp on (a = ttid)
GROUP BY a
HAVING avg(c) < 1000
```

A.1.6 Delta filter by selection push down Q_{selpd}.

```
SELECT a, avg(b) AS ab
FROM t1gb1000g
WHERE b < 1000
GROUP BY a
HAVING avg(c) < 300
```

A.1.7 End-to-end $Q_{endtoend}$.

```
SELECT a, avg(c) AS ac
FROM edb1
GROUP BY a
HAVING avg(c)> 1684845 AND avg(c) < 1686014;
```

A.2 Crimes dataset query list

Q1.

```
SELECT beat, year, count(id) AS crime_count
FROM crimes
GROUP BY beat, year
```

Q2.

```
SELECT district, community_area, ward, beat,
    count(beat) AS crime_count
FROM crimes
GROUP BY district, community_area, ward, beat
HAVING count(id) > 1000
```

A.3 Top-K query Q_{top-k}

```
SELECT a, avg(b) AS ab
FROM R
GROUP BY a
ORDER BY a
LIMIT 10
```

A.4 TPC-H Top-K query Q_{space}

```
SELECT c_custkey, c_name,
   sum(l_extendedprice * (1 - l_discount)) AS revenue,
   c_acctbal, n_name, c_address, c_phone, c_comment
FROM customer, orders, lineitem, nation
WHERE  c_custkey = o_custkey and l_orderkey = o_orderkey
   AND o_orderdate >= to_date('1994-12-01', 'YYYY-MM-DD')
   AND o_orderdate < to_date('1995-03-01', 'YYYY-MM-DD')
   AND l_returnflag = 'R'
   AND c_nationkey = n_nationkey
GROUP BY c_custkey, c_name, c_acctbal, c_phone,
   n_name, c_address, c_comment
ORDER BY revenue
LIMIT 20
```

B SQL-based Strategy

A pure SQL-based strategy is an option to update the provenance sketches incrementally. Accordingly, the state data required for operators is persisted as database tables. Then the incremental maintenance is modeled as running a series of queries over tables. For each operator of a query, queries are executed to process its input and to handle state data if required for both utilizing and updating the data.

This SQL-based approach has the advantages: first, there is no need to transfer data between the client and DBMS. What is more, DBMS can offer good plans and apply optimizations for query executions. Furthermore, data-heavy operations are executed inside the database, which reduces potential bottlenecks arising from data transfer across systems. All these advantages can enhance the efficiency of the incremental procedure.

However, this approach introduces challenges: first, it lacks flexibility to use specialized data structures to store operator state. All state data will be maintained in tables. But different operators' state data can be kept in different data structures in-memory: all groups' average can be stored in a map and order and limit operators together can use binary search trees to fast access Top-K elements. In addition, state data storing in tables is less efficient compared to in in-memory data structures. For example, to update the average value of a group, the state data table should be scanned twice: accessing and updating data. While, these two operations can be done more faster if the state data is keep in a map in-memory. Moreover, incremental operations cannot always be efficiently expressed in SQL.