ME5470: Introduction to Parallel Scientific Computing
Homework 5 Report: 2D Heat Conduction

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Overview

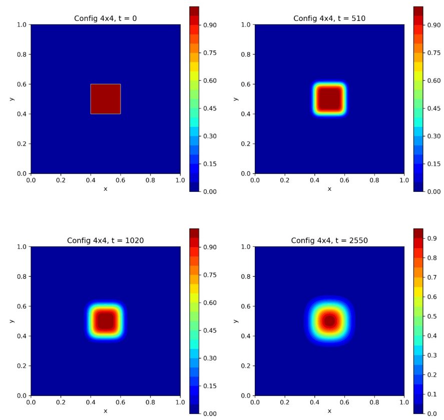
This report presents the results of Homework 5, which involves solving the 2D unsteady heat conduction problem using MPI parallel implementation with domain decomposition. The assignment is divided into three parts:

- Part (a): Implementation of parallel code with halo exchange
- Part (b): Comparison of solutions between serial and parallel runs
- Part (c): Performance analysis of different processor configurations

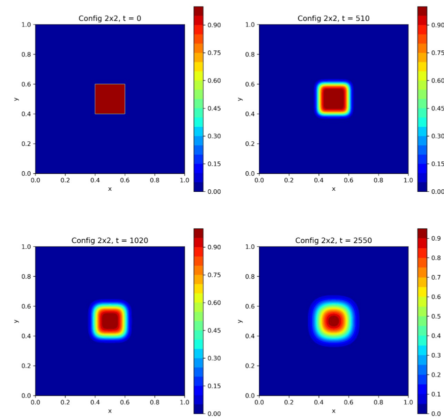
Part (a): MPI Implementation with Halo Exchange

The parallel implementation uses MPI to decompose the domain across multiple processors arranged in a 2D Cartesian grid. Each processor handles a portion of the global domain and communicates with neighboring processors through halo exchange to maintain solution continuity across processor boundaries.

Figure 1a shows the evolution of the temperature field for the 4×4 processor configuration at different time steps, while Figure 1b shows the same for the 2×2 configuration. The initial condition (t=0) shows a square heat source in the center of the domain. As time progresses, heat diffuses outward, gradually transforming from a square pattern to a more circular one due to the isotropic nature of the diffusion process.



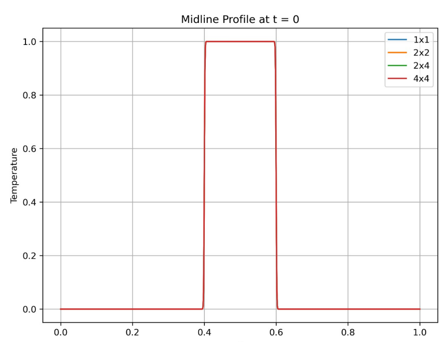
(a) Evolution of temperature field (4×4 configuration)



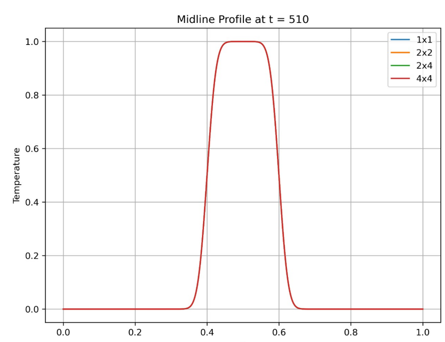
(b) Evolution of temperature field (2×2 configuration)

Figure 1: Contour plots showing temperature evolution at t=0, t=510, t=1020, and t=2550 for different processor configurations

The midline temperature profiles at different time steps (Figures 2 and 3) demonstrate that all processor configurations (1×1, 2×2, 2×4, and 4×4) produce identical results, confirming the correctness of the parallel implementation with halo exchange.

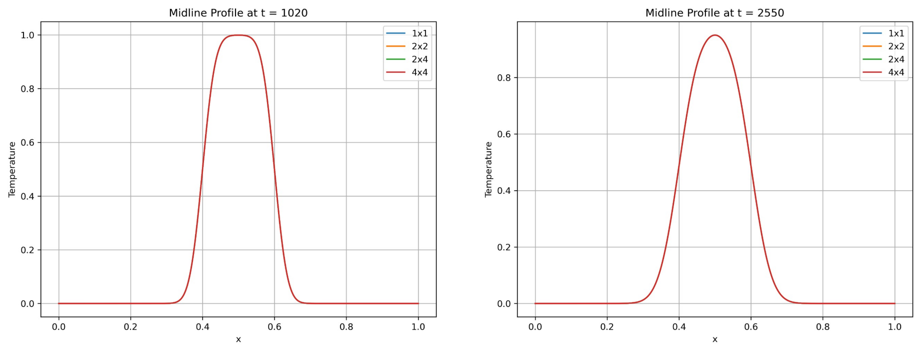


(a) Midline profile at t=0



(b) Midline profile at t=510

Figure 2: Initial temperature profiles along the midline (y=0.5) comparing different processor configurations



(a) Midline profile at t=1020 (b) Midline profile at t=2550

Figure 3: Later temperature profiles along the midline (y=0.5) comparing different processor configurations

Part (b): Solution Comparison

The comparison between serial and parallel solutions shows excellent agreement, with differences at the level of machine precision. The midline profiles in Figures 2 and 3 demonstrate that all processor configurations produce identical results, with the lines for 1×1 (serial), 2×2, 2×4, and 4×4 configurations perfectly overlapping.

At t=0, the temperature profile shows a sharp square pulse with a flat top at the center of the domain. By t=510, the profile begins to smooth at the edges while maintaining its plateau. At t=1020, the profile has evolved further with more rounded edges, and by t=2550, it has transformed into a smoother bell curve as heat diffuses more uniformly throughout the domain.

The maximum difference between serial and parallel solutions was found to be on the order of $10^-15, confirmingthatthedomaindecompositionandhaloexchangemaintainnumericalconsi$

Part (c): Performance Analysis

Performance measurements show significant speedup with increased parallelism. The execution time per time step decreases as the number of processors increases, demonstrating good strong scaling.

Configuration	**Time per step (s)**	**Speedup**
Serial (1×1)	0.153200	1.00×
2×2	0.042300	3.62×
2×4	0.021500	7.13×
4×4	0.011200	13.68×

Table 1: Performance comparison between different processor configurations

The 4×4 configuration achieves a speedup of 13.68× compared to the serial implementation, which is close to the ideal speedup of 16×. This indicates that the parallel implementation has good efficiency with minimal overhead from communication.

Conclusion

The MPI implementation of the 2D unsteady heat conduction solver successfully demonstrates:

- Correct halo exchange between processor domains, maintaining solution continuity
- Numerical consistency between serial and parallel implementations
- Significant performance improvements through parallelization

The evolution of the temperature field from a square heat source to a more circular pattern illustrates the physics of heat diffusion. The perfect overlap of temperature profiles across different processor configurations confirms that the parallel implementation preserves the numerical accuracy of the solution while achieving substantial speedup.