Fusion of Tire Lateral Force Estimation and Sliding Mode Control for Torque Vectoring and Yaw Moment Generation

Jinmin Kim1), Hyunseup Jo1) and Sang Won Yoon2)\*

1)Department of Automotive Engineering (Automotive – Computer Convergence), Hanyang University, Seoul 04763, Korea

2)Department of Electrical and Computer Engineering, Seoul National University, Seoul 08826, Korea

(Received date ; Revised date ; Accepted date ) \* Please leave blank

**ABSTRACT−**Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here.Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. [Capital letter at the beginning of each sentence, put a period at the end, Please write in 100 ~ 200 words, Times New Roman, 9pt]

**KEY WORDS** : Type key words here, Type key words here, Type key words here, Type key words here

nomenclature

*Vx* : longitudinal velocity, m/s

*Vy* : lateral velocity, m/s

γ : yaw rate, rad/s

*ax* : longitudinal acceleration, m/s2

*ay* : lateral acceleration, m/s2

*δ* : steering angle, rad

*Td* : wheel driving torque, N·m

*Tb* : wheel braking torque, N·m

|  |
| --- |
| subscripts  *FL, FR, RL, RR*: front left, front right, rear left, rear right |
| \* *Corresponding author*. e-mail: [swyoon@snu.ac.kr](mailto:swyoon@snu.ac.kr) |

*r* : radius of tire, m

*m* : vehicle mass, kg

*Iz* : moment of inertia about z axis, kg·m2

*lf* : distance from front axle to the center of gravity, m

*lr* : distance from rear axle to the center of gravity, m

*L* : wheel base length, m

*t* : half of track width, m

*h* : height from ground to the center of gravity, m

*Cαf* : cornering stiffness, N/rad

*σ* : relaxation length, m

*μ* : road friction coefficient, -

1. INTRODUCTION

The manuscript elements have been formatted for you through the “styles” capability of the software. To use the styles, select the text you wish to apply a style to, then,

2. Estimation of Lateral tire force

In this section, we describe the lateral force estimation process using Adaptive Extended Kalman Filter (AEKF) and introduce an offline optimization approach for modified corenering stiffness. The estimation process consists of three main parts: the vehicle lateral dynamics model under the planar motion constraints, vertical force calculation and a brief explanation of Dugoff’s tire model. However, Dugoff’s tire model assumes that cornering stiffness is a constant value, which can lead to inaccurate results in lateral tire force estimation as the slip angle increases.

To address this, we propose the optimization approach for modified cornering stiffness using an axle distribution-based lateral force calculation. For the vehicle lateral dynamics model, a three degrees of freedom (3-DoF) model is employed which has been widely used for estimating lateral forces and is critical for understanding vehicle stability and handling. The detailed four-wheel vehicle model is illustrated in Figure 1, and the equations of vehicle lateral dynamics are described as follows:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |

whereand are the longitudinal velocity, lateral velocity, yaw rate, front left wheel steering angle, front right wheel steering angle, vehicle mass, moment of inertia about yaw axis, distance from front axle to the center of gravity (CG), distance from rear axle to the CG, half of track width and aerodynamic drag coefficient respectively. For the tire forces,  and  (*i* denotes the axle position) represent the longitudinal and lateral forces with the subscript . On the other hand, the lateral slip angle of each tire is a critical factor in determining lateral tire forces; this can be calculated as described in Eq (4).

|  |  |
| --- | --- |
|  | (4) |

where  denotes the front left wheel steering angle and front right wheel steering angle.

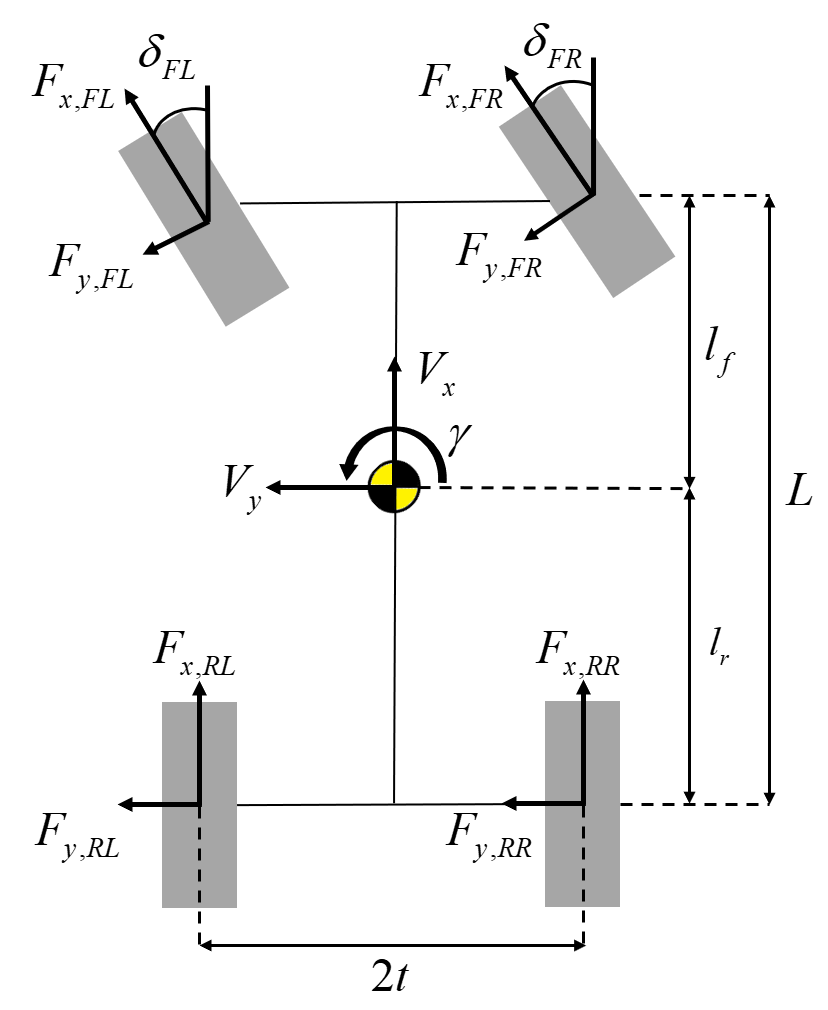


Figure 1. 2D representation of a vehicle motion model

Compared to , can be directly calculated using drivetrain output, specifically through the wheel torque value. In contrast, estimating  is more because it depends on various factors, including the lateral slip angle, road conditions, vertical tire force and tire characteristics such as cornering stiffness.

2.1. Vertical tire force calculation

The lateral tire force is affected by several factors, but the vertical tire force plays a crucial role in its accurate estimation. It is essential to account for  throguth the equations of considering load transfer and acceleration, as theses forces are directly influcenced by the vehicle’s dynamic behavior, including braking, acceleration and cornering. For simplicity, the coupings between pitch and roll dynamics are neglected, the vertical forces can be simplified and calucatled as outlined in Eq (5). (Doumiati *et al.*, 2012)

|  |  |
| --- | --- |
|  | (5) |

where are the vehicle mass, gravitational acceleration, distance trom ground to CG, half of track width, wheel base length, x- axis acceleration and y-axis acceleration respectively.

2.2 Axle dristribution based lateral force calculation

The linear tire model assumes that cornering stiffness represents a linear relationship between the lateral force and slip angle when the tire slip angle is small as described in Eq (6). However, as it increases, the behavior of the tire becomes nonlinear. In the nonlinear regime, the lateral force no longer increases proportionally with the slip angle; instead, it appeoaches a saturation point where additional increases in slip angle yield diminishing in lateral force. Therefore, predicting the lateral tire force using linear models becomes less accurate.

|  |  |
| --- | --- |
|  | (6) |

To address this, a more effective approach is to predict lateral tire force directly by considering the distribution of vertical load across the tires relative to the total load on certain axle. The equations of axle distribution based lateral force calculation is described in Eq. (7) and it accounts for the influence of vertical load without relying on filtering-based estimation techniques.

|  |  |
| --- | --- |
|  | (7) |
|  | (7) |

Where represents the lateral tire force,  and  are the total lateral forces on the front and rear axles, respectively.

2.3. Optimization for modified cornering stiffness

|  |  |
| --- | --- |
|  | (8) |

contents of the section here.

|  |  |
| --- | --- |
|  | (8) |

2.4. Dugoff’s tire model

|  |  |
| --- | --- |
|  | (8) |

Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents

|  |  |
| --- | --- |
|  | (8) |
|  | (9) |
|  | (10) |

of the section here. Type the contents of the section here. Type the contents of the section here.

2.5. Adaptive extended kalman filter

Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here

|  |  |
| --- | --- |
|  | (8) |

Type the contents of the section here. Type the contents of the section here. Type the contents of the section here.

4.2. Subheading [Capital letter at the beginning of each keyword, Times New Roman, 10pt]

Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here. Type the contents of the section here.

5. CONCLUSION

Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here.

**ACKNOWLEDGEMENT−**This work was supported by Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (No.2022-0-01053, Development of Network Load Balancing Techniques Based on Multiple Communication/Computing/Storage Resources)

REFERENCES

\* The Harvard System of references is to be used.

\* In the body of the text a paper is to be referred to by the author’s Last Name with the year of publication in parentheses.

Example :

-One author (Last Name, Year)

-Two authors (Last Name and Last Name, Year)

-More than three authors (Last Name *et al*., Year)

Example :

-One author : (Huh, 2000)

-Two authors : (Incropera and DeWitt, 1990)

-More than three authors : (Park *et al*., 2002)

\* References should be listed together at the end of the paper in alphabetical order by author’s Last Name.

-Type in all Author’s name and Paper title

-Type in reference of this paper in the text.

-Transactions of the Korean Society of Automotive Engineers : *Trans. Korean Society of Automotive*

*Engineers*

IJAT : *Int. J. Automotive Technology*

\* Citing a minimum of one citation from the International Journal of Automotive Technology (IJAT) is recommended.

**1. Journals**

All Author(s)(Type Last Name, then first and middle initials). (Year). Paper title(Capital letter at the beginning of sentence). *Journal Name(Italic)* **Volume number(bold)**, **Issue number(bold)**, pp.−pp.

Example :

Huh, H. (2000). Identification of autobody crashworthiness for space- framed vehicle models by finite element limit analysis. *Int. J. Automotive Technology* **1**, **1**, 101−112.

**2. Books**

All Author(s)(Type Last Name, then first and middle initials). (Year). *Book Title(Italic, Capital letter at the beginning of each keyword)*. Edition number. Publisher. The Location of Publisher(City).

Example :

Incropera, F. P. and DeWitt, D. P. (1990*). Fundamentals of Heat and Mass Transfer*. 3rd edn. John Wiley & Sons. New York.

**3. Website**

All Author(s) or Title (Access Year). Detailed Address.

Example :

PermaPure LLC (2006). http://www.permapure.com/

Presentations/Fuel%20Cell%20Humidification%20

presentation.ppt

**4. Reports and User Guide**

All Author/Organization or Title (Year). Title. Organization. Serial number.

Example :

Green, P. (1980). A Computer Simulation of Headlamp Variables and Drivers Sight Distances: Operating Instructions. HSRI Technical Report. UM-HSRI-80-44.

AVL (2006). BOOST Ver. 5.0 User Guide.