**Computing Rule-Based Explanations by Leveraging Counterfactuals**

Zixuan Geng, Maximilian Schleich, Dan Suciu – Fatima Vahora, Haeun Suh, Prashant Ravic summary

**ABSTRACT**

We introduce a new method to efficiently compute rule-based explanations for automated high-stakes decisions, by leveraging counterfactual explanations, for which many systems are already in place. To validate our approach, we present a Duality Theorem that establishes a relationship between rule-based and counterfactual explanations. Through comprehensive experiments, we demonstrate that our system outperforms or matches the performance of previous systems like MinSetCover and Anchor.

**(Below section is uncertain to be included)**

1. **INTROCUTION**

Due to the increasing adoption of machine learning in high-stakes decisions, there is an urgent need for more explainable and debuggable models. As a result, explainable machine learning has become a crucial research topic.

The extensive literature on explanation techniques is well summarized in a book on interpretable machine learning [14]. While there are both local explanations (focusing on individual instances) and global explanations (addressing the model as a whole), this paper emphasizes local explanations.

The Counterfactual Explanation (also known as Actionable Recourse) is a form of local explanation. It suggests modifications to an "undesired" instance to achieve a "desired" outcome. Essentially, it informs users what features must change for a machine learning model to predict a positive outcome from a previously negative one.

Counterfactual explanations may be insufficient for high-stakes machine learning applications due to their potential to mislead by not reflecting all influential features. Rudin et al. [3, 22] advocate for rule-based explanations, which are conjunctions of predicates on features consistently leading to certain outcomes. Unlike prescriptive counterfactual explanations, rule-based explanations descriptively provide core reasons for decisions, making them preferred by financial institutions.

Black-box explanation systems derive explanations by probing the classifier using inputs from specific instances and large datasets, either from training data or historical decisions. Counterfactual explanations answer questions with an existential approach, identifying features that, when altered, lead to a positive outcome. In contrast, rule-based explanations use a universal approach, pointing out features whose current values always result in a negative outcome regardless of other features. Finding counterfactual explanations is easier, with systems like Mace[8], Geco[23], and Dice[15] providing efficient solutions. However, obtaining rule-based explanations is more challenging, often requiring complex solutions such as converting the issue into a minimum set-cover problem.

In the paper, we introduce a novel method for rule-based explanations by leveraging existing counterfactual systems. We demonstrate that counterfactual and rule-based explanations are duals, implying that every rule-based explanation must incorporate at least one feature from its counterfactual counterpart. This duality principle is foundational to our approach.

Using the duality theorem, we've developed a method to compute rule-based explanations by employing counterfactual explanations as a black box. Our base algorithm, GeneticRule, uses a genetic algorithm to find candidate rules for instances with bad outcomes. We propose two enhancements: GeneticRule with Counterfactual (GeneticRuleCF) and Greedy Algorithm with Counterfactual (GreedyRuleCF). GeneticRuleCF incorporates a counterfactual system to refine candidate rules. If a rule isn't globally consistent, it asks for a counterfactual explanation while ensuring features already in the rule remain unchanged. On the other hand, GreedyRuleCF applies the counterfactual approach solely to the top-performing candidate rule.

To validate a rule-based explanation, its global consistency must be checked, a task that's resource-intensive. The set-cover method in [22] conducts this test only on database instances. In contrast, our approach examines every possible combination of attribute values. To manage the vastness of this task, we employ a counterfactual explanation system. Specifically, a rule is considered globally consistent only if no counterfactual exists when keeping specific rule features unchanged.

In our experimental evaluation comparing our three algorithms with MinSetCover [22] and Anchor [21], we found the latter two too often return rules lacking global consistency. Specifically, MinSetCover had a 97.4% inconsistency rate for the Adult dataset, and Anchor produced rules with redundant predicates 87.0% of the time. Our GeneticRuleCF algorithm, on the other hand, always produced globally consistent rules with only 12.4% redundancy, while our GreedyRuleCF algorithm always generated globally consistent rules without any redundant predicates.

An orthogonal approach to explanations involves the creation of interpretable machine learning models. Rule-based models, as described in [10], shouldn't be confused with rule-based explanations. While the former serves as a decision mechanism, the latter provides explanations for decisions typically made by uninterpretable models.

**Contributions.** In summary, in this paper we make the following contributions.

(1) We prove the Duality Theorem between counterfactual and rule-based explanations. Section 3.1.

(2) We show how to use the Duality Theorem to compute rule-based explanations by using a counterfactual-based explanation system. Section 3.2.

(3) We describe three algorithms: GeneticRule, GeneticRuleCF, and GreedyRuleCF for generating rule-based explanations. Section 4.

(4) We conduct an extensive experimental evaluation of GeneticRule, GeneticRuleCF, and GreedyRuleCF algorithms, and compare them with Anchor and MinSetCover. Section 5.

1. **DEFINITIONS**

|  |
| --- |
| * 𝐹1, . . ., 𝐹𝑛 be n features, with domains 𝑑𝑜𝑚(𝐹1), . . ., 𝑑𝑜𝑚(𝐹𝑛) [Ordered] * Define Inst = 𝑑𝑜𝑚(𝐹1) × · · · × 𝑑𝑜𝑚(𝐹𝑛) * Let an element 𝑥 ∈ Inst an instance. * Let C ais a black box classifier. * any instance 𝑥 ∈ Inst, returns a prediction 𝐶(𝑥) within range [0, 1] * If C(x) <= 0.5, it's classified as “undesired” or “bad”, [Binary classifier: 0] * If C(x) > 0.5, it's classified as “desired” or “good”, [Binary classifier: 1] * Let a database, D, consisting of m instances: D = {x1, …, xm}. * For every instance xi in D, its feature values are given by xi = (fi1, …, fin). |

* 1. **Rule-based Explanation**

For a given instance xi = (fi1, …, fin) ∈ 𝐷, a rule component, denoted as RC(x), is formulated by either 𝑓𝑗 ≤ 𝑓𝑖𝑗 or 𝑓𝑗 ≥ 𝑓𝑖𝑗. A rule relevant to 𝑥𝑖 is a set of rule components, represented as 𝑅 = {𝑅𝐶1, . . ., 𝑅𝐶𝑐} where R(x) = RC1 ^ … ^ -RCC. To indicate that a feature value 𝐹𝑗 = 𝑓𝑖𝑗, it is essential to incorporate both ≤ and ≥ rule components, meaning the cardinality constraint is 0 ≤ 𝑐 ≤ 2𝑛.

Instances with the "undesired" label have C(xi) <= 0.5. As suggested by Rudin and Shaposhnik [22], for a rule R to elucidate the undesired outcome of xi, it should satisfy:

1. Relevance: Ensure xi ∈ INSTR

\* The paper inherently satisfies this requirement as it only considers rules relevant to instance xi

1. Global Consistency: ∀𝑥 ∈ Inst𝑅, 𝐶(𝑥) ≤ 0.5.
2. Interpretability: The rule must be simple, favoring minimal cardinality.

The paper identifies a trivial rule, Rtriv, that guarantees global consistency. However, its cardinality, being 2n, compromises its interpretability. For computational feasibility, the study emphasizes the need to derive a minimal set of rule components that maintain global consistency. Recognizing the computational demands of exhaustive global consistency checks, the paper advocates "Data Consistency". This approach evaluates consistency relative to a predefined database D = {x1, …, xm}, ensuring ∀𝑥𝑘 ∈ 𝐷 ∩ Inst𝑅, 𝐶 (𝑥𝑘) ≤ 0.5.

It is noted that certain methods, such as MinSetCover and Anchor, might not always uphold global consistency.

* 1. **Counterfactual Explanation**

Counterfactual explanations identify changes in specific features that could lead to a desired outcome. Given an instance xi with and undesired outcome, a counterfactual explanation xcf specifies how xi could be modified to achieve a desired outcome where C(xi) > 0.5.

xcf must satisfy two main properties:

(1) Feasibility and Plausibility:

- Feasibility sets limits on potential feature values (e.g., income constraints).

- Plausibility dictates how new values in xcf can differ from the values in xi (e.g., gender shouldn't change).

- These criteria are encompassed by the PLAF (plausibility/feasibility) predicates:

P(xcf) = Φ1 ∧ · · · ∧ Φ𝑚 ⇒ Φ0, where Φi is a predicate over the features of xi and xcf.  
- Examples:

e.g., Feasibility: genderCF = genderi

e.g., Plausibility: education𝐶𝐹 > education𝑖 ⇒ age𝐶𝐹 ≥ age𝑖 +4

(2) Magnitude of Changes:

- Changes between xi and xcf can be quantified using a distance function dist(x, x’)

* A counterfactual explanation system takes as input an instance xi, a PLAF constraint P(x), and a distance function dist(x, x’). It outputs a ranked list of counterfactuals based on their proximity to xi.
  1. **Discussion**

Different explanations offer varied insights:

SHAP Score assigns significance percentages to features, helping rank them but not providing actionable guidance. In the case of Counterfactual Explanation, it suggests specific feature changes for desired outcomes but might raise fairness questions. In the other hand, rule-based Explanation outlines decision-making criteria, ensuring consistency and fairness but without actionable advice.

1. **DUALITY**

Rule-based explanations and counterfactual explanations offer distinct information. Though both prioritize brevity — a relevant rule should have few components, and a counterfactual should modify minimal features of 𝑥𝑖. Efficient counterfactual systems are available, but current rule-based systems compromise global consistency for speed.

When comparing counterfactual and rule-based explanations, differing complexities arise due to the nature of their constructions. Using a small set of features, F, counterfactual explanations require 𝑁 𝑘 calls to the oracle, assuming domains of size N, to ascertain possible changes. Rule-based explanations, on the other hand, necessitate N n−k calls to the classifier to verify global consistency, which can become extensive given the difference in feature counts. An innovative approach proposes using a counterfactual explanation system as a black box to compute rule-based explanations, tapping into an advantageous duality that exists between the two types of explanations.

* 1. **The Duality Theorem**

Lemma 3.1. If 𝑅 is a globally consistent rule, and 𝑥𝑐𝑓 is any counterfactual, then 𝑅 (𝑥𝑐𝑓) is false.

Proof. Given Q: ∀𝑥’(𝑅(𝑥′) = T -> 𝐶(𝑥′) ≤ 0.5). Also Given R(xcf) = ⌝𝑅(𝑥′).

Contrapositive of Q’: ∀𝑥𝐶(𝑥′)> 0.5) = F -> ⌝ 𝑅(𝑥′) = R(xcf)

Dual rules: Given two instances xi and x, the dual rule for x (denoted as Rx) consists of a union (logical OR) of rule components that represent the disparities or “conflicts” between feature values of x and xi.

e.g., Given xi = (F1 = 10, F2 = 20, F3 = 30) and x = (F1 = 5, F2 = 90, F3 = 30).

Then dual rule Rx = (F1 >= 10) V (F2 <= 20)

Theorem 3.2 (Duality). Fix a globally consistent rule 𝑅 relevant to 𝑥𝑖, let 𝑥𝑐𝑓 ,1, . . ., 𝑥𝑐𝑓, 𝑘 be counterfactual instances, and let 𝑅𝑥𝑐𝑓 ,1, . . ., 𝑅𝑥𝑐𝑓, 𝑘 be their duals. Then 𝑅 is a set cover of {𝑅𝑥𝑐 𝑓 ,1, . . ., 𝑅𝑥𝑐𝑓, 𝑘}. In other words, for every counterfactual 𝑥𝑐f, 𝑚 the rule 𝑅 contains at least one rule component that conflicts with 𝑥𝑐𝑓, 𝑚. Conversely, fix any counterfactual 𝑥𝑐𝑓, and let 𝑅1, . . ., 𝑅𝑘 be globally consistent rules. Then the dual 𝑅𝑥𝑐 𝑓 is a set cover of {𝑅1, . . ., 𝑅𝑘}.

Proof. If R and Rxcf, m does not share any common rule component, then Rxcf, m is would be true, contradicting the Lemma 3.1

e.g., Given xi = (F1 = 10, F2 = 20, F3 = 30) [Good] and x = (F1 = 15, F2 = 19, F3 = 30) [BAD]

And dual rule Rx have been (F1 >= 10) V (F2 <= 20)

If there is no conflict, then x should be Good => Contradictory!

Therefore, at least one confliction is needed.

* 1. **Using the Duality Theorem**

Theorem 3.2 provides a naive algorithm. However, this can't be used because counterfactual systems often don't produce a comprehensive list of counterfactuals. Our approach builds the rule R incrementally, starting from R = ∅, adding one rule component at a time until achieving global consistency. The steps are as follows:

Step 1: Construct the predicate 𝑅(𝑥 ′) associated with the rule 𝑅.

Step 2: Utilize the counterfactual explanation system to identify a list of counterfactuals 𝑥𝑐𝑓 ,1, . . ., 𝑥𝑐𝑓, 𝑘 for 𝑥𝑖 that meet the PLAF predicate. If no counterfactual matches, then R is deemed globally consistent, and the process stops.

Step 3: For each 𝑗 = 1 to 𝑘, compute the dual Rxcf, j for each counterfactual 𝑥𝑐𝑓, 𝑗.

Step 4: For every minimal set that encompasses 𝑅0 of 𝑆, create the augmented rule 𝑅 ∪ 𝑅0 and start again from Step 1.

e.g., xi = (Age = 50, AccNum = 4, Income = 500, Debt = 10k) => “DENIED”

Step1. (Current) R = (Age <= 50) ^ (AccNum >= 4)

Step2. 𝑥𝑐𝑓 ,1 = (Age = 50, **AccNum = 5, Income = 900**, Debt = 10𝑘) => “APPROVE”

𝑥𝑐𝑓 ,2 = (Age = 50, AccNum = 4, **Income = 600**, **Debt = 2𝑘**) => “APPROVE”

Step3. Rxcf, 1 = (AccNum ≤ 4) ∨ (Income ≤ 500)

Rxcf, 2 = (Income ≤ 500) ∨ (Debt ≥ 10𝑘)

Step4. R1 =(Age ≤ 50) ∧ (AccNum ≥ 4) ∧ (Income ≤ 500) Cardinality = 3

𝑅2 =(Age ≤ 50) ∧ (AccNum = 4) ∧ (Debt ≥ 10𝑘) Cardinality = 4

\* ‘=’ represents two rule component AccNum≤4 and AccNum≥4

1. ALGORITHMS

GeneticRule: This is a base-line algorithm, which explores the space of rule-based explanations using a genetic algorithm. It does not use counterfactuals.

GeneticRuleCF: This algorithm extends GeneticRule by using an oracle call to a counterfactual explanation system in order to generate and validate the rule-based explanations.

GreedyRuleCF This algorithm replaces the genetic search with a greedy search: we greedily expand only the rule with the smallest cardinality in the population, using the counterfactual explanation system as an oracle.

**(Below part is uncertain to be included)**

For GeneticRule and GeneticRuleCF we have chosen a genetic algorithm, which is a meta-heuristic for constraint optimization based on the process of natural selection. First, it defines an initial population of candidates. Then, it repeatedly selects the fittest candidate in the population and generates new candidates by changing and combining the selected candidates (called mutation and crossover). It stops when a certain criterion is met, e.g., it finds a specified number of solutions.

We chose a genetic algorithm because:

(1) it is easily customizable to our problem of finding rule explanations,

(2) it seamlessly integrates counterfactual explanations to generate and verify rules,

(3) it does not require any restrictions on the underlying classifier and data, and thus is able to provide black-box explanations, and

(4) it returns a diverse set of explanations, which may provide different rules that can give the user more information.

* 1. A white paper with black text

     Description automatically generatedGeneticRule

GeneticRule is a “naïve” algorithm that generates rules by a genetic algorithm, shown in Algorithm 1.

* Input: Instance x, classifier C, dataset D.
* Output: a set of rules explaining instance x for classifier C.
* Five integer hyperparameters
  + q > 0: The number of rules that kept after each iteration.
  + k <= q: The number of rules that the algorithm returns to the user.
  + s: The number of samples taken from INST to check for global consistency.
  + m: The number of new candidates that are generated during mutation.
  + c: The number of new candidates that are generated during crossover.

e.g., q = 50, k = 5, s = 1000, m = 3, c = 2. Used in most of the experiments. (Why? See sec. 5)

1. Computes the initial population of rule candidates.
   1. Initial population has all possible rule candidates with exactly one rule component.
   2. Initial candidates are likely not valid and consistent rules.
2. Repeatedly applies below:
   1. Mutate and crossover to generate new candidates.
   2. Compute the fitness score vis selectFittest for each candidate.
   3. Selects the q fittest candidates for the next generation.
   4. Repeat until finding K candidates that are consistent on both the dataset D and s samples from the more general INST space.
3. Check the top-k candidates are not in the set of new generated candidates CAND to guarantee their stability for at least one generation of the algorithm.

* Mutate operator: Generates m new rule candidates for each candidate 𝑅 ∈ POP.
  + Find the set S of all rule components that are not part of R.
  + Generates each new candidate by sampling without replacing a single component from S and adding it to R. (For keeping the low cardinality of the rules and avoiding redundant rule components as possible)
* Crossover operator: Generates c new candidates for each pair of candidates, Ri and Rj.
  + Compute the set 𝑆 = 𝑅𝑖 ∪𝑅𝑗 of all rule components in Ri and Rj.
  + Randomly sample t components from S to form a new candidate, as 𝑡 = 𝑚𝑎𝑥 (|𝑅𝑖|, |𝑅𝑗|) + 1 to keeping the low cardinality of the rules.
  + Repeat the sampling process c times to generate c candidates for every pair of candidates in the population.
* The selectFittest operator: Calculate the fitness score of each candidate in POP, sorts the candidates by in descending order of their fitness scores, and returns the top q candidates.
  + See sec 4.4 for detail.
* GeneticRule only guarantees of the returned rules are global consistency for a sample of INST, thus only guaranteed the rules are data consistent.
  1. A white text with black text

     Description automatically generatedGeneticRuleCF

GeneticRuleCF applies a counterfactual explanation to GeneticRule to generate and verify rule candidates, shown in Algorithm 2.

* CFRules function: This function takes as input a set of rule candidates and generates new candidates by computing the counterfactual explanations for each input candidate.
  + Consisted as multiple processes like as below (Also in Section 3.2):
    - Compute the PLAF predicates for a given input candidate.
    - Compute the counterfactual explanation for this candidate.
    - Return the dual of the counterfactual explanation as a new candidate.
  + It extends both initial population and the candidate pool in the main loop of the genetic algorithm.
* consistentCF function: This function verifies global consistency of the top rule candidates using a counterfactual model.
  + By checking if any counterfactual example exists for a given candidate, this function guarantees higher global consistency than GeneticRule.
* Performance optimizations: To maximize performance at minimal effects:
  + Reduced the calling of the counterfactual explanation model, by only running CFRules once for every three iteration or when all top-k candidates are marked as data consistent.
  + Cache to check availability of generating counterfactuals for each rule candidate, to ensure the single run of the counterfactual model per candidate.
* An optional post process stage – not shown in the pseudo code: Ensures that the returned rules have no redundant components.
  + Remove one rule component at a time for each returned rule.
  + Check if the rule without this component is still verified by consistentCF.
  + If so, remove that redundant component.
  + Repeat the above processes until the returned rules do not have any redundant component.
  1. A screenshot of a computer code

     Description automatically generatedGreedyRuleCF

GreedyRuleCF greedily finds rule candidates with small cardinality by repeatedly utilizing the underlying counterfactual explanation, shown in Algorithm 3.

* Generate the initial population by running CFRules, keep them sorted as ascending order of their cardinalities.
* Choose the candidate with the smallest cardinality, generate the new candidate towards this candidate by CFRules, and replace the secondary candidate with the new candidate from the population.
* Repeat until the candidate with the smallest cardinality is found to be consistent by consistentCF.
* The cardinality of the rules in the population are monotonically increasing while repeatedly replacing the inconsistent rule candidate with the smallest cardinality.
* The algorithm is guaranteed to terminate with a consistent rule by having at most 2n rule components, where n is the number of variables in D.
  1. Fitness Score Function
* selectFittest function: Computes the fitness score for ranking the rule candidates.
  + The fitness score is based on its degree of consistency and its cardinality.
  + Three degrees of consistency:
    - The rule failed data consistency (𝐹𝐷𝐶): it violates instances in the database 𝐷.
    - The rule failed global consistency (FGC): it is data consistent (satisfies all instances in the dataset 𝐷), but fails for some instances in Inst.
    - The rule is globally consistent (𝐺𝐶): The rule is consistent for both the dataset 𝐷 as well as the instances from Inst.
  + The fitness score of a Rule R):
    - Define m as the cardinality of D (a.k.a |D|).
    - Define n is the number of features in D.
    - Define VD the number of instances in D that violates R.
    - If VD = 0, use sample s instances from INST, and define VS the number of samples the violates R.
    - A math equations with numbers

      Description automatically generatedThe fitness score - score(R):
    - The expressions were chosen to ensure that the score function always ranks candidates in each level higher than the candidates in the levels below, to prioritize candidates having better consistency.

e.g., GC > FGC > FDC for different level, min(cardinality) for the same level.

1. EXPERIMENTS
   1. Experiment Setup
   2. Quality in terms of Consistency and Interpretability
   3. Runtime Comparison
   4. Microbenchmarks
2. LIMITATIONS AND FUTURE WORK

**Bound of Rule Components.** To simplify the search space, we've confined rule components to directly relate to input instance values. Thus, they can only be larger, smaller, or equal to the feature value, even if the actual rule might be broader. While this narrows the rule, it's effective for analyzing classifier behavior. We aim to generalize rules in future work.

**Realistic Feature Value Distributions**. While GeCo and other advanced Counterfactual Explanation models utilize perturbation distributions as the instance search space for interpretability, estimating these distributions accurately remains challenging, especially when representing causal dependencies between features. Improving this will benefit various explanation methods.

**Underlying Counterfactual Explanation System.** Our algorithm's stability and run time are greatly influenced by the underlying counterfactual system, GeCo. While GeCo is among the best we've found, it can be costly and unstable in extreme scenarios, impacting our system's performance. A more efficient counterfactual system would significantly enhance our systems.

**Better Counterfactual Explanation Model.** Our paper uses a counterfactual explanation model to generate rule-based explanations. A more advanced rule-based model could enhance our approach and potentially improve the counterfactual explanation process.

**Static Data and Classifier.** Our algorithms currently assume a constant data and classifier setup. Given their susceptibility to changes in either, we aim to develop explanations that can adapt to minor shifts in data or classifiers, aligning with the broader challenge of robust machine learning.

1. **CONCLUSION**

We've developed a method using counterfactual explanations to generate and ensure global consistency of rule-based explanations. We introduced a base algorithm, GeneticRule, and two advanced ones, GeneticRuleCF and GreedyRuleCF. Extensive tests validate our system's effectiveness.