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43 YEARS JEE ADVANCED

(1978-2020) + JEE MAIN

Chapterwise & Topicwise Solved Papers

Physics

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PREFACE

Disha Publication feels proud to release its thoroughly Revised & Updated Edition of the Book "43 Years IIT-JEE Advanced + JEE Main Chapter-wise & Topic-wise Solved Paper PHYSICS". This is the 16th Edition which speaks volumes about the Quality & Hard Work that has gone in building this book.

It is an integrated book, which contains Chapter-wise & Topic-wise collection of past JEE Advanced (including 1978 - 2012 IIT-JEE & 2013 - 2020 JEE Advanced) questions from 1978 to 2020 and past JEE Main from 2013 – 19 (including all Online & Offline Papers).

- The unique feature of this new edition is the division of questions into 29 chapters as per NCERT. With this new feature this book has become the 1st to adopt NCERT Chapterisation among JEE Advanced Solved Papers.
- Each chapter divides the questions into 2 - 4 topics which are further divided into 10 categories of questions –

1. MCQ 1 option correct
2. Integer Answer
3. Numeric Answer
4. Fill in the Blanks
5. True/False
6. MCQ more than 1 option correct
7. Multiple Matching
8. Passage Based
9. Assertion-Reason
10. Subjective Questions.

- All the Screening and Mains papers of IIT-JEE have been incorporated in the book. All online papers of JEE Main including the 16 papers of 2020 & 16 papers of 2019 phase I & II have been incorporated in the book.

IIT-JEE/ JEE Advanced Papers	JEE Main (Online + Offline) Papers
• 1978 – 2012 IIT JEE Screening & Main • 2013 – 2020 JEE Advanced Papers 1 & 2	JEE Main 2013 – 5 (4 + 1) papers JEE Main 2014 – 4 (3 + 1) papers JEE Main 2015 – 3 (2 + 1) papers JEE Main 2016 – 3 (2 + 1) papers JEE Main 2017 – 3 (2 + 1) papers JEE Main 2018 – 4 (3 + 1) papers JEE Main 2019 – 16 Papers JEE Main 2020 – 16 Papers

- Detailed solution of each and every question has been provided for 100% conceptual clarity of the student. Well elaborated detailed solutions with user friendly language provided at the end of the book.
- Solutions have been given with enough diagrams, proper reasoning to bring conceptual clarity.
- The students are advised to attempt questions of a topic immediately after they complete a topic in their class/ school/ home. The book contains around 4000 Milestone Problems in Physics.

If utilised properly this book can take your JEE preparation to the next level.

Although all efforts are made to ensure quality but some errors might have crept in. We would invite our readers to send us these errors so that we can incorporate the corrections in the upcoming editions.

All the Best

Disha Experts

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1 Physical World, Units and Measurements

Topic-1 : Unit of Physical Quantities

1 MCQs with One Correct Answer

1. The density of a material in SI unit is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is: **[Main 10 Jan. 2019 (I)]**
(a) 40 (b) 16 (c) 640 (d) 410

2. A metal sample carrying a current along X-axis with density J is subjected to a magnetic field B_z (along z-axis). The electric field E_y developed along Y-axis is directly proportional to J_x as well as B_z. The constant of proportionality has SI unit **[Main Online April 25, 2013]**
(a) $\frac{m^2}{A}$ (b) $\frac{m^3}{As}$ (c) $\frac{m^2}{As}$ (d) $\frac{As}{m^3}$

3. he SI unit of inductance, the henry can be written as **[1998 - 2 Marks]**
(a) weber/ampere (b) volt-sec/amp
(c) Joule/(ampere)^2 (d) ohm-second

7 Match the Following

4.

Column I	Column II
(A) GM_eM_s , G – universal gravitational constant, M_e – mass of the eart , M_s – mass of the Su	(p) (volt) (coulomb)(metre)
(B) $\frac{3RT}{M}$, R – universal gas constant, T – absolute temperature, M – molar mass	(q) (kilogram) (metre)^3 (second)$^{-2}$
(C) $\frac{F^2}{q^2B^2}$, F – Force, q – charge, B – magnetic field	(r) (metre)^2 (second)$^{-2}$
(D) $\frac{GM_e}{R_e}$, G – universal M_e – mass of the earth, R_e – radius of the earth	(s) (farad) (volt)^2 (kg)$^{-1}$

[2007]

5.

Column I	Column II
(A) Capacitance	(i) ohm-second
(B) Inductance	(ii) coulomb^2–joule^{-1}
(C) Magnetic Induction	(iii) coulomb (volt)$^{-1}$
	(iv) newton (amp-metre)$^{-1}$
	(v) volt-second (ampere)$^{-1}$

[1990 - 3 Marks]

10 Subjective Problems

6. Give the MKS units for each of the following quantities.
(A) Young's modulus **[1980]**
(B) Magnetic Induction
(C) Power of a lens

Topic-2 : Dimensions of Physical Quantities

1 MCQs with One Correct Answer

1. The quantities $x = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{1}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ε_0, μ_0, - free space permittivity and permeability respectively. Then : **[Main Sep. 05, 2020 (II)]**
   (a) x, y and z have the same dimension.
   (b) Only x and z have the same dimension.
   (c) Only x and y have the same dimension.
   (d) Only y and z have the same dimension.

2. Dimensional formula for thermal conductivity is (here K denotes the temperature : **[Main Sep. 04, 2020 (I)]**
   (a) $MLT^{-2}K$ (b) $ML\ ^{-2}K^{-2}$
   (c) $MLT^{-3}K$ (d) $ML\ ^{-3}K^{-1}$

3. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of : **[Main Sep. 04, 2020 (II)]**
   (a) planck's constant (b) force constant
   (c) energy density (d) coefficient of viscosity

4. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is : **[Main Sep. 03, 2020 (II)]**
   (a) ML^2T^{-2} (b) ML^0T^{-3}
   (c) $M^2L^0T^{-1}$ (d) MLT^{-2}

5. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be : **[Main Sep. 02, 2020 (I)]**
   (a) FA^2V^{-1} (b) FA^2V^{-3} (c) FA^2V^{-2} (d) $FA^{-1}V^0$

6. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is : **[Main Sep. 02, 2020 (II)]**
   (a) $[P^2AT^{-2}]$ (b) $[PA^{-1}T^{-2}]$
   (c) $[PA^{1/2}T^{-1}]$ (d) $[P^{1/2}AT^{-1}]$

7. Which of the following combinations has the dimension of electrical resistance ($\in_0$ is the permittivity of vacuum and μ_0 is the permeability of vacuum)? **[Main 12 April 2019 (I)]**
   (a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

8. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ? **[Main 10 April 2019 (II)]**
   (a) $[M^{-3}L^{-2}\ ^{8}A^4]$ (b) $[M^{-1}L^{-2}\ ^{4}A^2]$
   (c) $[M^{-2}L^0\ ^{-4}A^{-2}]$ (d) $[M^{-2}L^{-2}\ ^{6}A^3]$

9. In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is: **[Main 8 April 2019 (I)]**
   (a) $A^{-1}TML^3$ (b) $AT^2M^{-1}L^{-1}$
   (c) $AT^{-3}ML^{3/2}$ (d) $A^2T^3M^{-1}L^{-2}$

10. Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be : **[Main 12 Jan. 2019 (II)]**
    (a) $[LA^{-2}]$ (b) $[A^{-1}]$ (c) $[LTA]$ (d) $[LT^2]$

11. he force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimensions of β is: **[Main 11 Jan. 2019 (I)]**
    (a) $M^0L^2T^{-4}$ (b) M^2LT^{-4}
    (c) MLT^{-2} (d) $M^2L^2\ ^{-2}$

12. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be : **[Main 11 Jan. 2019 (II)]**
    (a) $V^{-2}A^2\ ^{-2}$ (b) $V^{-2}A^2\ ^{2}$
    (c) $V^{-4}A^{-2}$ (d) $V^{-4}A^2$

13. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of : **[Main 9 Jan. 2019 (I)]**
    (a) area (b) energy
    (c) momentum (d) volume

14. Expression for time in terms of G (universal gravitational constant), h (Planck's constant) and c (speed of light) is roportional to: **[Main 9 Jan. 2019 (II)]**
    (a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

15. The dimensions of stopping potential V_0 in hotoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is: **[Main 8 Jan. 2019 (I)]**
    (a) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$ (b) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$
    (c) $h^{-2/3}e^{-1/3}G^{4/3}A^{-1}$ (d) $h^2G^{3/2}C^{1/3}A^{-1}$

16. e dimensions of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is: **[Main 8 Jan. 2019 (II)]**
    (a) MLT^{-2} (b) $ML^2\ ^{-1}$ (c) $ML^2\ ^{-2}$ (d) $ML^{-1}T^{-2}$

17. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, h and c. Which of the following correctly gives the Planck length? [Main Online April 15, 2018]

(a) G^2hc (b) $\left(\frac{Gh}{c^3}\right)^{\frac{1}{2}}$ (c) $G^{\frac{1}{2}}h^2c$ (d) Gh^2c^3

18. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be : [Main Online April 8, 2017]

(a) $[M]=[T^{-1}C^{-2}h]$ (b) $[M]=[T^{-1}C^{2}h]$
(c) $[M]=[T^{-1}C^{-2}h^{-1}]$ (d) $[M]=[TC^{-2}h]$

19. In the following 'I' refers to current and other symbols have their usual meaning, Choose the option that corresponds to the dimensions of electrical conductivity : [Main Online April 9, 2016]

(a) $M^{-1}L^{-3}T^3I$ (b) $M^{-1}L^{-3}T^3I^2$
(c) $M^{-1}L^3T^3I$ (d) $ML^{-3}T^{-3}I^2$

20. If electronic charge e, electron mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of : [Main Online April 11, 2015]

(a) $\left(\frac{h}{me^2}\right)$ (b) $\left(\frac{hc}{me^2}\right)$ (c) $\left(\frac{h}{ce^2}\right)$ (d) $\left(\frac{mc^2}{he^2}\right)$

21. If the capacitance of a nanocapacitor is measured in terms of a unit 'u' made by combining the electric charge 'e', Bohr radius 'a_0', Planck's constant 'h' and speed of light 'c' then: [Main Online April 10, 2015]

(a) $u=\frac{e^2h}{a_0}$ (b) $u=\frac{hc}{e^2a_0}$ (c) $u=\frac{e^2c}{ha_0}$ (d) $u=\frac{e^2a_0}{hc}$

22. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is: [Main Online April 12, 2014]

(a) $\frac{ch}{2\pi\varepsilon_o^2}$

(b) $\frac{e^2}{2\pi\varepsilon_o Gm_e^2}$ (m_e = mass of electron)

(c) $\frac{\mu_o\varepsilon_o}{c^2}\frac{G}{he^2}$

(d) $\frac{2\pi\sqrt{\mu_o\varepsilon_o}}{ce^2}\frac{h}{G}$

23. In terms of resistance R and time T, the dimensions of ratio $\frac{\mu}{\varepsilon}$ of the permeability μ and permittivity ε is: [Main Online April 11, 2014]

(a) $[RT^{-2}]$ (b) $[R^2T^{-1}]$ (c) $[R^2]$ (d) $[R^2T^2]$

24. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then: [Main 2013]

(a) $\epsilon_0=[M^{-1}L^{-3}T^2A]$ (b) $\epsilon_0=[M^{-1}L^{-3}T^4A^2]$
(c) $\epsilon_0=[M^1L^2T^1A^2]$ (d) $\epsilon_0=[M^1L^2T^1A]$

25. The dimensions of angular momentum, latent heat and capacitance are, respectively. [Main Online April 22, 2013]

(a) $ML^2T^1A^2$, L^2T^{-2}, $M^{-1}L^{-2}T^2$
(b) ML^2T^{-2}, L^2T^2, $M^{-1}L^{-2}T^4A^2$
(c) ML^2T^{-1}, L^2T^{-2}, ML^2TA^2
(d) ML^2T^{-1}, L^2T^{-2}, $M^{-1}L^{-2}T^4A^2$

26. Which of the following set have different dimensions?
(a) Pressure, Young's modulus , Stress [2005S]
(b) EMF, Potential difference, Electric potential
(c) Heat, Work done, Energy
(d) Dipole moment, Electric flux, Electric field

27. Pressure depends on distance as, $P=\frac{\alpha}{\beta}\exp\left(-\frac{\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzman's constant and θ is temperature. The dimension of β are [2004S]

(a) $M^0L^0T^0$ (b) $M^{-1}L^{-1}T^{-1}$
(c) $M^0L^2T^0$ (d) $M^{-1}L^1T^2$

28. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$ where ϵ_0 is the permittivity of the free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of [2001S]

(a) resistance (b) charge
(c) voltage (d) current

29. The dimension of $\left(\frac{1}{2}\right)\varepsilon_0E^2$ (ε_0 : permittivity of free space, E electric field) [2000S]

(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

2 Integer Value Answer

30. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways-engineer uses dimensions and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is [Adv. 2014]

4 Fill in the Blanks

31. The dimension of electrical conductivity is ________. [1997 - 1 Mark]

32. The equation of state for real gas is given by $\left(P+\frac{a}{V^2}\right)(V-b)=RT$. The dimensions of the constant a is ________. [1997 - 2 Marks]

33. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. The dimensions of Y in MKSQ sytem are __________, ________. **[1988 - 2 Marks]**

34. Planck's constant has dimension ________. **[1985 - 2 Marks]**

6 MCQs with One or More Than One Correct

35. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows:
[position] = [X^{α}]; [speed] = [X^{β}]; [acceleration] =[X^p]; [linear momentum] = [X^q]; [force] = [X^r]. Then **[Adv. 2020]**

(a) $\alpha + p = 2\beta$ (b) $p + q - r = \beta$

(c) $p - q + r = \alpha$ (d) $p + q + r = \beta$

36. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimensions of L, which of the following statement(s) is/are correct? **[Adv. 2019]**

(a) The dimension of force is L^{-2}

(b) The dimension of linear momentum is L^{-1}

(c) The dimension of energy is L^{-2}

(d) The dimension of power is L^{-5}

37. A length–scale (l) depends on the permittivity (ε) of a dielectric material. Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct? **[Adv. 2016]**

(a) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(c) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

38. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[Adv. 2015]**

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\mu_0 I = \mu_0 V$

(c) $I = \varepsilon_0 cV$ (d) $\mu_0 cI = \varepsilon_0 V$

39. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are) **[Adv. 2015]**

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$

(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

40. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum, and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, **[1998 - 2 Marks]**

(a) $[\varepsilon_0] = M^{-1} L^{-3} T^2 I$ (b) $[\varepsilon_0] = M^{-1} L^{-3} T^4 I^2$

(c) $[\mu_0] = M L T^{-2} I^{-2}$ (d) $[\mu_0] = M L^2 T^{-1} I$

41. The pairs of physical quantities that have the same dimensions is (are) : **[1995S]**

(a) Reynolds number and coefficient of friction
(b) Curie and frequency of a light wave
(c) Latent heat and gravitational potential
(d) Planck's constant and torque

42. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair (s) **[1986 - 2 Marks]**

(a) Torque and Work
(b) Angular momentum and Work
(c) Energy and Young's modulus
(d) Light year and Wavelength

7 Match the Following

43. Match List I with List II and select the correct answer using the codes given below the lists: **[Adv. 2013]**

List I		List II	
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

44. Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II and write the correct answer against the matched quantity in a tabular form in your answer book. **[1983 - 6 Marks]**

Column I	Column II
Angular momentum	ML^2T^{-2}
Latent heat	ML^2Q^{-2}
Torque	ML^2T^{-1}
Capacitance	$ML^3T^{-1}Q^{-2}$
Inductance	$M^{-1}L^{-2}T^2Q^2$
Resistivity	L^2T^{-2}

8 Comprehension/Passage Based Questions

Passage

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[E]$ and $[B]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. $[L]$ and $[T]$ are dimensions of length and time respectively. All the quantities are given in SI units. **[Adv. 2018]**

45. The relation between $[E]$ and $[B]$ is

(a) $[E] = [B][L][T]$ (b) $[E] = [B][L]^{-1}[T]$

(c) $[E] = [B][L][T]^{-1}$ (d) $[E] = [B][L]^{-1}[T]^{-1}$

46. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is
(a) $[\mu_0] = [\varepsilon_0] [L]^2 [T]^{-2}$ (b) $[\mu_0] = [\varepsilon_0] [L]^{-2} [T]^2$
(c) $[\mu_0] = [\varepsilon_0]^{-1} [L]^2 [T]^{-2}$ (d) $[\mu_0] = [\varepsilon_0]^{-1} [L]^{-2} [T]^2$

10 Subjective Problems

47. Write the dimensions of the following in terms of mass, time, length and charge **[1982 - 2 Marks]**
(i) magnetic flux
(ii) rigidity modulus

48. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$. Where 'P' is the static pressure, 'd' is the density of water and 'E' is the total energy of the explosion. Find the values of a, b and c. **[1981- 3 Marks]**

Topic-3 : Errors in Measurements & Experimental Physics

1 MCQs with One Correct Answer

1. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively : **[Main Sep. 06, 2020 (I)]**
(a) Negative, 2 μm (b) Positive, 10 μm
(c) Positive, 0.1 mm (d) Positive, 0.1 μm

2. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm, 5.65 mm, The average of these four reading is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as : **[Main Sep. 06, 2020 (II)]**
(a) (5.5375 ± 0.0739) mm (b) (5.5375 ± 0.0740) mm
(c) (5.538 ± 0.074) mm (d) (5.54 ± 0.07) mm

3. A physical quantity z depends on four observables a, b, c and d, as $z = \dfrac{a^2 b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is : **[Main Sep. 05, 2020 (I)]**
(a) 12.25% (b) 16.5% (c) 13.5% (d) 14.5%

4. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as: **[Main Sep. 03, 2020 (I)]**
(a) 2.121 cm (b) 2.124 cm (c) 2.125 cm (d) 2.123 cm

5. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale betwen 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is : (VSD is vernier scale division) **[Main Sep. 02, 2020 (I)]**
(a) 3.2 cm (b) 3.21 cm (c) 3.07 cm (d) 2.99 cm

6. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is: **[Main 9 Jan. 2020 (I)]**
(a) 0.001 cm (b) 0.02 mm (c) 0.01 cm (d) 0.001 mm

7. For the four sets of three measured physical quantities as given below. Which of the following options is correct? **[Main 9 Jan. 2020 (II)]**
(A) $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$
(B) $A_2 = 24.44$, $B_2 = 16.082$, $C_2 = 240.2$
(C) $A_3 = 25.2$, $B_3 = 19.2812$, $C_3 = 236.183$
(D) $A_4 = 25$, $B_4 = 236.191$, $C_4 = 19.5$
(a) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$
(b) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$
(c) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$
(d) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

8. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1 s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is: **[Main 8 Jan. 2020 (II)]**
(a) 5.40% (b) 3.40% (c) 4.40% (d) 2.40%

9. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is: **[Main 9 April 2019 (I)]**
(a) 0.01 kg/m^3 (b) 0.10 kg/m^3
(c) 0.013 kg/m^3 (d) 0.07 kg/m^3

10. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is: **[Main 9 April 2019 (II)]**
(a) 37 cm^2 (b) 37.030 cm^2
(c) 37.03 cm^2 (d) 37.0 cm^2

11. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to : **[Main 8 April 2019 (II)]**
(a) 0.7% (b) 0.2% (c) 3.5% (d) 6.8%

12. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is: **[Main 12 Jan. 2019 (I)]**
(a) 50 (b) 200 (c) 100 (d) 500

13. The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 division below the mean line.
The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of the sheet is: **[Main 9 Jan. 2019 (II)]**
(a) 5.755 mm (b) 5.950 mm
(c) 5.725 mm (d) 5.740 mm

14. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is: **[Main 2018]**
(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

15. The percentage errors in quantities P, Q, R and S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{RS}}$.
The maximum percentage error in the value of A will be **[Main Online April 16, 2018]**
(a) 8.5% (b) 6.0%
(c) 7.5% (d) 6.5%

16. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is: **[Main Online April 15, 2018]**
(a) 0.0430 cm (b) 0.3150 cm
(c) 0.4300 cm (d) 0.2150 cm

17. The following observations were taken for determining surface tensiton T of water by capillary method :
Diameter of capilary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$ m
Using $g = 9.80$ m/s^2 and the simplified relation
$T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to : **[Main 2017]**
(a) 2.4% (b) 10% (c) 0.15% (d) 1.5%

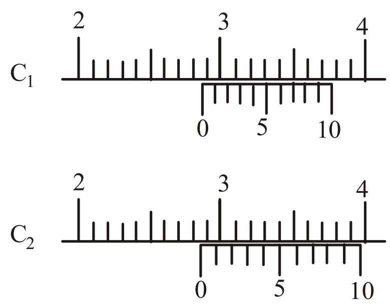
18. A physical quantity P is described by the relation
$P = a^{1/2} b^2 c^3 d^{-4}$
If the relative errors in the measurement of *a, b, c* and *d* respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be : **[Main Online April 9, 2017]**
(a) 8% (b) 12% (c) 32% (d) 25%

19. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity $g = 10$ ms^{-2} and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, $\delta L/L$, is closest to **[Adv. 2017]**
(a) 0.2% (b) 1% (c) 3% (d) 5%

20. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that wen the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? **[Main 2016]**
(a) 0.70 mm (b) 0.50 mm
(c) 0.75 mm (d) 0.80 mm

21. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s, and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be: **[Main 2016]**
(a) 92 ± 1.8 s (b) 92 ± 3s
(c) 92 ± 1.5 s (d) 92 ± 5.0 s

22. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively, are **[Adv. 2016]**



(a) 2.85 and 2.82 (b) 2.87 and 2.83
(c) 2.87 and 2.86 (d) 2.87 and 2.87

23. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is : **[Main 2015]**
(a) 1% (b) 5% (c) 2% (d) 3%

24. Diameter of a steel ball is measured using a Vernier callipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions of its vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as: **[Main Online April 10, 2015]**

S.No.	MS(cm)	VS divisions
1.	0.5	8
2.	0.5	4
3.	0.5	6

If the zero error is – 0.03 cm, then mean corrected diameter is:
(a) 0.52cm (b) 0.59 cm
(c) 0.56 cm (d) 0.53 cm

25. The current voltage relation of a diode is given by $I = \left(e^{1000V/T} - 1\right)$ mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **[Main 2014]**
(a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

26. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[Main 2014]**
(a) A meter scale.
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

27. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potential wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of volmeter will be: **[Main Online April 12, 2014]**
(a) – 0. 15 volt (b) 0.15 volt
(c) 0.5 volt (d) – 0.05 volt

28. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be: **[Main Online April 9, 2014]**
(a) 1.7% (b) 2.7% (c) 4.4% (d) 2.27%

29. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24[th] division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is **[Adv. 2013]**
(a) 5.112 cm (b) 5.124 cm
(c) 5.136 cm (d) 5.148 cm

30. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement **[2012]**
(a) due to the errors in the measurements of d and l are the same.
(b) due to the error in the measurement of d is twice that due to the error in the measurement of l.
(c) due to the error in the measurement of l is twice that due to the error in the measurement of d.
(d) due to the error in the measurement of d is four times that due to the error in the measurement of l.

31. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2 %, the relative percentage error in the density is **[2011]**
(a) 0.9 % (b) 2.4 % (c) 3.1 % (d) 4.2 %

32. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is **[2010]**
(a) 0.02 mm (b) 0.05 mm (c) 0.1 mm (d) 0.2 mm

33. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and /or record time for different number of oscillations. The observations are shown in the table. **[2008]**
Least count for length = 0.1 cm
Least count for time = 0.1 s

Student	Length of the pendulum (cm)	No. of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\dfrac{\Delta g}{g} \times 100\right)$ for students I, II and III, respectively, then
(a) $E_I = 0$ (b) E_I is minimum
(c) $E_I = E_{II}$ (d) E_{II} is maximum

34. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8\ \text{m/s}^2$ (exact). The Young's modulus obtained from the reading is **[2007]**

(a) $(2.0 \pm 0.3) \times 10^{11}\ \text{N/m}^2$ (b) $(2.0 \pm 0.2) \times 10^{11}\ \text{N/m}^2$

(c) $(2.0 \pm 0.1) \times 10^{11}\ \text{N/m}^2$ (d) $(2.0 \pm 0.05) \times 10^{11}\ \text{N/m}^2$

35. A student performs an experiment for determination of $g\left(=\dfrac{4\pi^2\ell}{T^2}\right)$. The error in length ℓ is $\Delta\ell$ and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for **[2006 - 3M, –1]**

	$\Delta\ell$	ΔT	n
(a)	5mm	0.2 sec	10
(b)	5mm	0.2 sec	20
(c)	5mm	0.1 sec.	10
(d)	1mm	0.1 sec	50

36. In a screw gauge, the zero of mainscale coincides with fifth division of circular scale in figure (i). The circular division of screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball in figure (ii) is **[2006 - 3M, –1]**

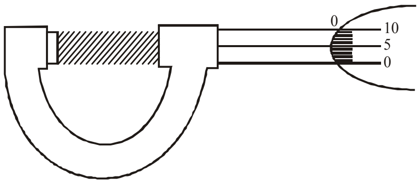


Figure (i)

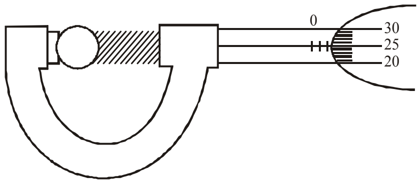


Figure (ii)

(a) 2.25 mm (b) 2.20 mm

(c) 1.20 mm (d) 1.25 mm

37. A wire of length $\ell = 6 \pm 0.06$ cm and radius $r = 0.5 \pm 0.005$ cm and mass $m = 0.3 \pm 0.003$ gm. Maximum percentage error in density is **[2004S]**

(a) 4 (b) 2 (c) 1 (d) 6.8

38. A cube has a side of length 1.2×10^{-2}m. Calculate its volume. **[2003S]**

(a) $1.7 \times 10^{-6}\ \text{m}^3$ (b) $1.73 \times 10^{-6}\ \text{m}^3$

(c) $1.70 \times 10^{-6}\ \text{m}^3$ (d) $1.732 \times 10^{-6}\ \text{m}^3$

2 Integer Value Answer

39. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t,)$ where $\alpha = 0.2\ \text{s}^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is **[Adv. 2015]**

40. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7}\ \text{m}^2$. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **[Adv. 2014]**

3 Numeric Answer

41. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\dfrac{x}{100}\right)$%. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is ______. **[Main Sep. 06, 2020 (I)]**

42. Two capacitors with capacitance values $C_1 = 2000 \pm 10$ pF and $C_2 = 3000 \pm 15$ pF are connected in series. The voltage applied across this combination is $V = 5.00 \pm 0.02$ V. The percentage error in the calculation of the energy stored in this combination of capacitors is _______. **[Adv. 2020]**

43. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ___ **[Adv. 2019]**

44. A steel wire of diameter 0.5 *mm* and Young's modulus $2 \times 10^{11}\ N\,m^{-2}$ carries a load of mass M. The length of the wire with the load is 1.0 *m*. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 *mm*, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 *kg*, the vernier scale division which coincides with a main scale division is __________. Take $g = 10\ m\,s^{-2}$ and $\pi = 3.2$. **[Adv. 2018]**

45. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. **[2005 - 2 Marks]**

46. In Searle's experiment, which is used to find Young's Modulus of elasticity, the diameter of experimental wire is $D = 0.05$ cm (measured by a scale of least count 0.001 cm) and length is $L = 110$ cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of X = 0.125 cm (measured by a micrometer of least count 0.001cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. **[2004 - 2 Marks]**

47. A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47th circular division coincides with the main scale. Find the curved surface area of wire in cm^2 to appropriate significant figure.

(use $\pi = \frac{22}{7}$). **[2004 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

48. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52s, 0.56s, 0.57s, 0.54s and 0.59s. The least count of the watch used for the measurement of time period is 0.01s. Which of the following statement(s) is (are) true? **[Adv. 2016]**

(a) The error in the measurement of r is 10%
(b) The error in the measurement of T is 3.75%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

49. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then : **[Adv. 2015]**

(a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm
(b) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm
(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm
(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm

50. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° **[Adv. 2013]**

(a) The absolute error in d remains constant
(b) The absolute error in d increases
(c) The fractional error in d remains constant
(d) The fractional error in d decreases

51. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true? **[2010]**

(a) Error ΔT in measuring T, the time period, is 0.05 seconds
(b) Error ΔT in measuring T, the time period, is 1 second
(c) Percentage error in the determination of g is 5%
(d) Percentage error in the determination of g is 2.5%

8 Comprehension/Passage Based Questions

Passage

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected. **[Adv. 2018]**

52. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$, then what is the error Δr in determining r?

(a) $\frac{\Delta a}{(1+a)^2}$ (b) $\frac{2\Delta a}{(1+a)^2}$ (c) $\frac{2\Delta a}{(1-a^2)}$ (d) $\frac{2a\Delta a}{(1-a^2)}$

53. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(a) 0.04 (b) 0.03 (c) 0.02 (d) 0.01

10 Subjective Problems

54. If n^{th} division of main scale coincides with $(n+1)^{th}$ divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier. **[2003 - 2 Marks]**

Topic-4 : Miscellaneous (Mixed Concepts) Problems

8 Comprehension/Passage Based Questions

Passage

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[2011]**

1. Taking the electronic charge as 'e' and the permittivity as 'ε_0'. Use dimensional analysis to determine the correct expression for ω_p.

   (a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$

2. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Taking $\varepsilon_0 = 10^{-11}$ and mass $m \approx 10^{-30}$, where these quantities are in proper SI units.

   (a) 800 nm (b) 600 nm (c) 300 nm (d) 200 nm

Answer Key

Topic-1 : Unit of Physical Quantities

1. (a)	**2.** (b)	**3.** (a, b, c, d)

Topic-2 : Dimensions of Physical Quantities

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (b)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (None)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (d)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (3)
31. $[M^{-1}L^{-3}T^3A^2]$		**32.** $[M^{-1}L^{-3}T^3A^2]$		**33.** $[M^{-3}L^{-2}T^4Q^4]$		**34.** $[ML^2T^{-1}]$		**35.** (a, b)	**36.** (a,b,c)
37. (b, d)	**38.** (a, c)	**39.** (a, c, d)	**40.** (b, c)	**41.** (a, b, c)		**42.** (a, d)	**43.** (c)	**45.** (c)	**46.** (d)

Topic-3 : Errors in Measurements & Experimental Physics

1. (b)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (Bonus)	**8.** (c)	**9.** (Bonus)	
10. (d)	**11.** (d)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (b)
20. (d)	**21.** (c)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (b)
30. (a)	**31.** (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (4)
40. (4)	**41.** (1050)	**42.** (1.30)	**43.** (1.39)	**44.** (3.00)	**45.** (2.66 g cm^{-3})		**46.** ($1.09 \times 10^{10}\ N/m^2$)		
47. (2.6 cm^2)		**48.** (a, b, d)	**49.** (b, c)	**50.** (d)	**51.** (a, c)	**52.** (b)	**53.** (c)	**54.** $\left(\dfrac{a}{n+1}\right)$	

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (c)	**2.** (b)

2 Motion in a Straight Line

Topic-1 : Distance, Displacement & Uniform Motion

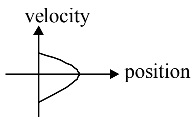
1 MCQs with One Correct Answer

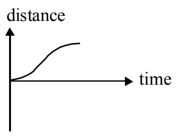
1. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0). **[Main 12 Apr. 2019 II]**

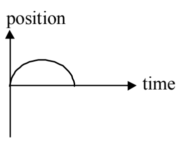
(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$

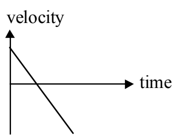
(c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[Main 2018]**

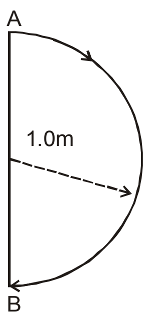
(a) 

(b) 

(c) 

(d) 

3. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m (see Figure). The magnitude of the average velocity **[1999S - 2 Marks]**



(a) 3.14 m/s

(b) 2.0 m/s

(c) 1.0 m/s

(d) Zero

4 Fill in the Blanks

4. A particle moves in a circle of radius R. In half the period of revolution its displacement is ____________ and distance covered is ________. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

5. A particle is moving eastwards with a velocity of 5 m/s. In 10s the velocity changes to 5 m/s northwards. The average acceleration in this time is/are **[1982 - 3 Marks]**

(a) zero

(b) $1/\sqrt{2}$ m/s^2 towards north-west

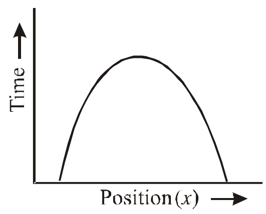
(c) $1/\sqrt{2}$ m/s^2 towards north-east

(d) $\frac{1}{2} m/s^2$ towards north-west

(e) $\frac{1}{2} m/s^2$ towards north

10 Subjective Problems

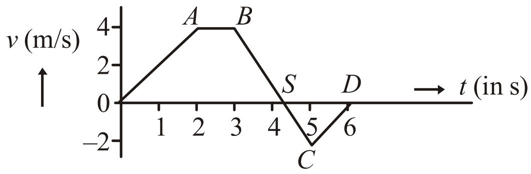
6. Answer the following giving reasons in brief : Is the time variation of position, shown in the figure observed in nature? **[1979]**



Topic-2 : Non-uniform Motion

1 MCQs with One Correct Answer

1. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is: **[Main 05 Sep. 2020 (II)]**



(a) $\frac{37}{3}$ m (b) 12m (c) 11m (d) $\frac{49}{4}$ m

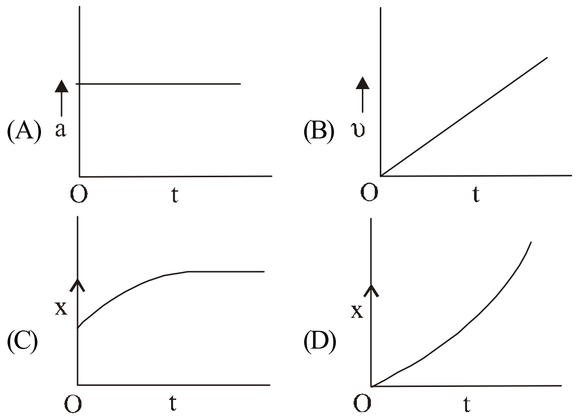
2. A bullet of mass 20g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to : **[Main 10 Apr. 2019 (II)]**

(a) 0.1 ms^{-1} (b) 0.7 ms^{-1}
(c) 0.3 ms^{-1} (d) 0.4 ms^{-1}

3. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$
where, a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **[Main 9 Apr. 2019 (II)]**

(a) $a+\frac{b^2}{4c}$ (b) $a+\frac{b^2}{3c}$
(c) $a+\frac{b^2}{c}$ (d) $a+\frac{b^2}{2c}$

4. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represents the motion qualitatively (a = acceleration, v = velocity, x = displacement, t = time) **[Main 8 Apr. 2019 (II)]**

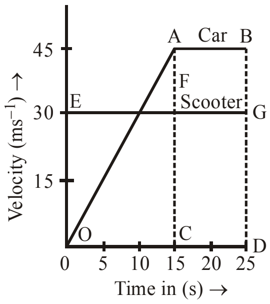


(a) (B), (C) (b) (A)
(c) (A), (B), (C) (d) (A), (B), (D)

5. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to: **[Main 9 Jan. 2019 (II)]**

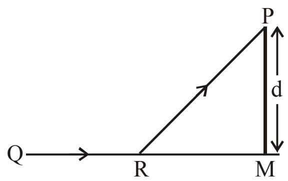
(a) $\frac{2a_1 a_2}{a_1 + a_2}t$ (b) $\sqrt{2a_1 a_2}\ t$
(c) $\sqrt{a_1 a_2}\ t$ (d) $\frac{a_1 + a_2}{2}t$

6. The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively **[Main Online April 15, 2018]**



(a) 337.5m and 25s
(b) 225.5m and 10s
(c) 112.5m and 22.5s
(d) 11.2.5m and 15s

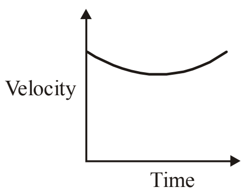
7. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum? **[Main Online April 15, 2018]**



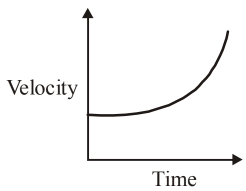
(a) $\frac{d}{\sqrt{3}}$ (b) $\frac{d}{2}$ (c) $\frac{d}{\sqrt{2}}$ (d) d

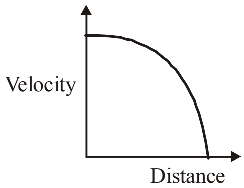
8. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ? **[Main Online April 8, 2017]**

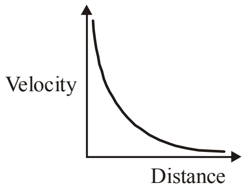
(a)



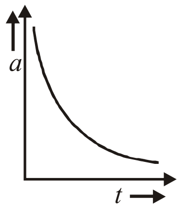
(b)

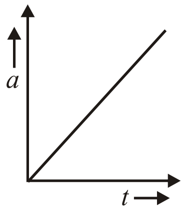


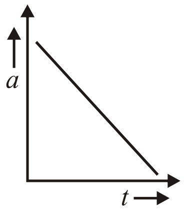
(c) 

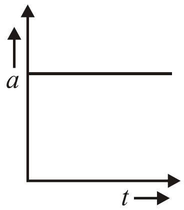
(d) 

9. The distance travelled by a body moving along a line in time t is proportional to t^3.
The acceleration-time (a, t) graph for the motion of the body will be **[Main Online May 12, 2012]**

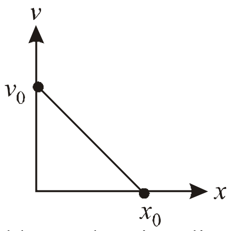
(a) 

(b) 

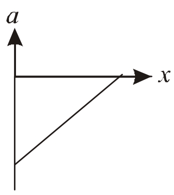
(c) 

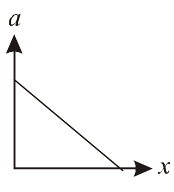
(d) 

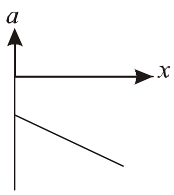
10. The velocity-displacement graph of a particle moving along a straight line is shown **[2005S]**

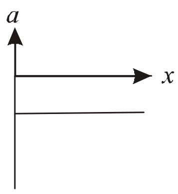


The most suitable acceleration-displacement graph will be

(a) 

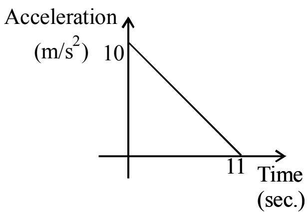
(b) 

(c) 

(d) 

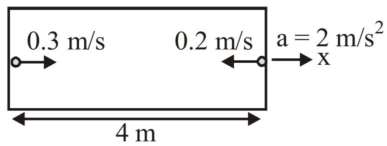
11. A body starts from rest at time $t = 0$, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be **[2004S]**

(a) 110 m/s
(b) 55 m/s
(c) 650 m/s
(d) 550 m/s



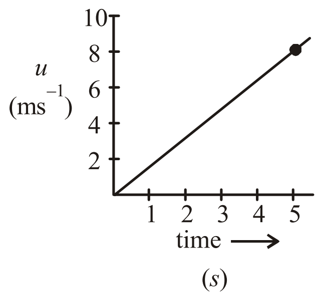
2 Integer Value Answer

12. A rocket is moving in a gravity free space with a constant acceleration of 2 m/s^2 along +x direction (see figure). The length of a chamber inside the rocket is 4 m. *A* ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[Adv. 2014]**



3 Numeric Answer

13. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be __________. **[Main 4 Sep. 2020 (II)]**



14. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ______. **[Main 9 Jan 2020 (I)]**

6 MCQs with One or More than One Correct Answer

15. A particle of mass m moves on the x-axis as follows : it starts from rest at $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. NO other information is available about its motion at intermediate times $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle, then: **[1993-2 Marks]**

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$.
(b) $|\alpha|$ cannot exceed 2 at any point in its path.
(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.
(d) α must change sign during the motion, but no other assertion can be made with the information given.

10 Subjective Problems

16. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time lapse is t seconds, evaluate. **[1978]**

(i) maximum velocity reached, and
(ii) the total distance travelled.

Topic-3 : Relative Velocity in One Dimension

1 MCQs with One Correct Answer

1. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible) **[Main 2 Sep. 2020 (I)]**

(a) 29.5 ms^{-1} (b) 28.5 ms^{-1} (c) 31.5 ms^{-1q} (d) 30.5 ms^{-1}

2. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60º with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is: **[Main 12 Jan. 2019 (II)]**

(a) $\frac{\sqrt{3}}{2}v$ (b) $\frac{2v}{\sqrt{3}}$ (c) v (d) $\frac{v}{2}$

3. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of : **[Main Online April 9, 2017]**

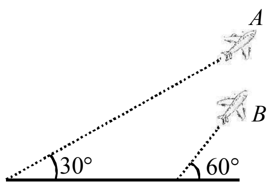
(a) $\sqrt{110}$ s (b) $\sqrt{120}$ s (c) $10\sqrt{2}$ s (d) 15 s

4. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator? **[Main Online April 12, 2014]**

(a) 37 s (b) 27 s (c) 24 s (d) 45 s

2 Integer Value Answer

5. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ m/s. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **[Adv. 2014]**



4 Fill in the Blanks

6. Four persons K, L, M, N are initially at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N, and N directly towards K. The four persons will meet at a time .... **[1984- 2 Marks]**

5 True / False

7. Two identical trains are moving on rails along the equator on the earth in opposite directions with the same speed. They will exert the same pressure on the rails. **[1985 - 3 Marks]**

9 Assertion and Reason Type Questions

8. **STATEMENT-1 :** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

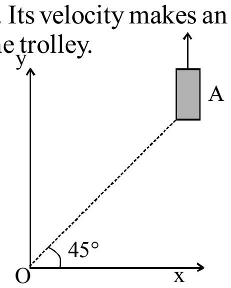
STATEMENT-2 : If the observer and the object are moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v}_2 - \vec{v}_1$. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

10 Subjective Problems

9. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.
(b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/4$. **[2002 - 5 Marks]**



Topic-4 : Motion Under Gravity

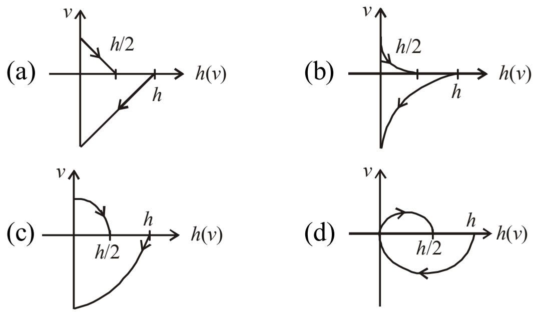
1 MCQs with One Correct Answer

1. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the accelertion due to gravity] : **[Main 5 Sep. 2020 (I)]**

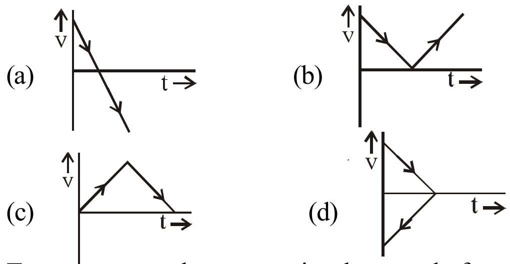
(a) $t=\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (b) $t=1.8\sqrt{\frac{h}{g}}$

(c) $t=3.4\sqrt{\left(\frac{h}{g}\right)}$ (d) $t=\sqrt{\frac{2h}{3g}}$

2. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :
(graph are drawn schematically and on not to scale)
[Main 4 Sep. 2020 (I)]

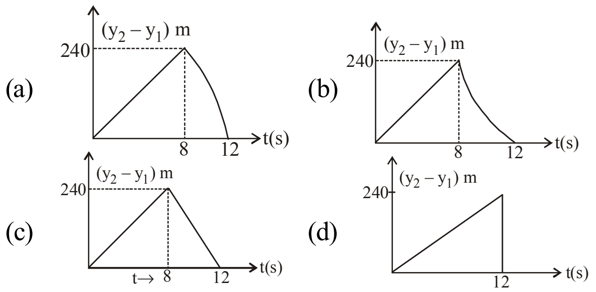


3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?
[Main 2017]



4. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/ s^2) **[Main 2015]**

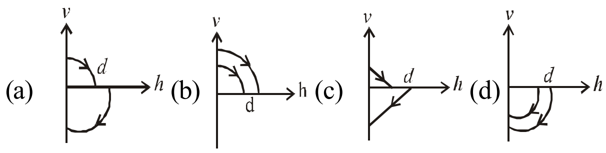
(*The figures are schematic and not drawn to scale*)



5. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: **[Main 2014]**

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2 u^2d$
(c) $2gH = nu^2 (n-2)$ (d) $gH = (n-2)u^2$

6. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as **[2000S]**



3 Numeric Answer

7. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is ________. **[Main 8 Jan. 2020 (II)]**

5 True / False

8. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglect air resistance). **[1983 - 2 Marks]**

Answer Key

Topic-1 : Distance, Displacement & Uniform Motion

1. (b) **2.** (b) **3.** (b) **4.** (2R, πR) **5.** (b)

Topic-2 : Non-uniform Motion

1. (a) **2.** (b) **3.** (b) **4.** (d) **5.** (c) **6.** (c) **7.** (a) **8.** (c) **9.** (b) **10.** (a)
11. (b) **12.** (8) **13.** (20) **14.** (3) **15.** (a, c, d)

Topic-3 : Relative Velocity in one Dimension

1. (a) **2.** (d) **3.** (c) **4.** (c) **5.** (5) **6.** $\left(\frac{d}{v}\right)$ **7.** (False) **8.** (b)

Topic-4 : Motion Under Gravity

1. (c) **2.** (c) **3.** (a) **4.** (b) **5.** (c) **6.** (a) **7.** (8.00) **8.** (True)

3 Motion in a Plane

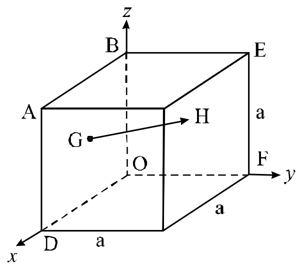
Topic-1 : Vectors

1 MCQs with One Correct Answer

1. Let $|\overrightarrow{A_1}| = 3$, $|\overrightarrow{A_2}| = 5$ and $|\overrightarrow{A_1} + \overrightarrow{A_2}| = 5$. The value of $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \bullet (3\overrightarrow{A_1} - 2\overrightarrow{A_2})$ is : **[Main 8 April 2019 (II)]**

(a) -106.5 (b) -99.5 (c) -112.5 (d) -118.5

2. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be: **[Main 10 Jan. 2019 (I)]**



(a) $\frac{1}{2}a(\hat{k} - \hat{i})$ (b) $\frac{1}{2}a(\hat{i} - \hat{k})$

(c) $\frac{1}{2}a(\hat{j} - \hat{i})$ (d) $\frac{1}{2}a(\hat{j} - \hat{k})$

3. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is: **[Main 10 Jan. 2019 (II)]**

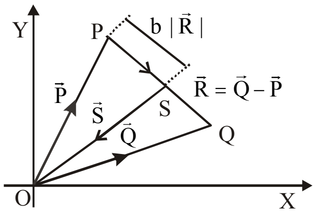
(a) $\cos^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (b) $\cos^{-1}\left[\frac{n - 1}{n + 1}\right]$

(c) $\sin^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (d) $\sin^{-1}\left[\frac{n - 1}{n + 1}\right]$

4. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by **[Main Online April 16, 2018]**

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$ (c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

5. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is **[Adv. 2017]**



(a) $\vec{S} = (1 - b)\vec{P} + b\vec{Q}$ (b) $\vec{S} = (b - 1)\vec{P} + b\vec{Q}$

(c) $\vec{S} = (1 - b^2)\vec{P} + b\vec{Q}$ (d) $\vec{S} = (1 - b)\vec{P} + b^2\vec{Q}$

3 Numeric Answer

6. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of x is ______. **[Main Sep. 05, 2020 (I)]**

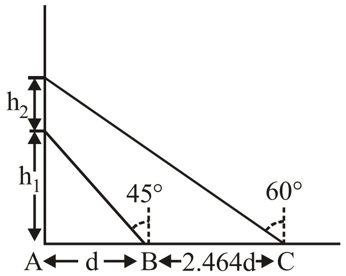
7. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of 2 $\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is______. **[Main 7 Jan. 2020 II]**

8. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in *seconds*, is __________. **[Adv. 2018]**

Topic-2 : Motion in a Plane with Constant Acceleration

1 MCQs with One Correct Answer

1. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45º with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60º with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30º = 0.5774): **[Main Sep. 05, 2020 (I)]**



(a) 1.464 d (b) 0.732 d (c) 0.464 d (d) d

2. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms^{-1}, a particle moves in the x–y plane with a constant acceleration of $(10\hat{i}+4\hat{j})$ ms^{-2}. At time t, its coordiantes are (20 m, y_0 m). The values of t and y_0 are, respectively : **[Main Sep. 04, 2020 (I)]**

(a) 2 s and 18 m (b) 4 s and 52 m
(c) 2 s and 24 m (d) 5 s and 25 m

3. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i}+4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is: **[Main 9 Jan. 2020 (II)]**

(a) 32 (b) 50 (c) 60 (d) 40

4. A particle moves such that its position vector $\vec{r}(t) = \cos \omega t\,\hat{i} + \sin \omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle: **[Main 8 Jan. 2020 (II)]**

(a) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
(b) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
(c) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

5. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4-20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[Main 9 April 2019 (II)]**

(a) 40 (b) 25 (c) 100 (d) 50

6. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$ m, at t = 0, with an initial velocity $(5.0\hat{i}+4.0\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})$ ms^{-2}. What is the distance of the particle from the origin at time 2s? **[Main 11 Jan. 2019 (II)]**

(a) 15m (b) $20\sqrt{2}$m (c) 5m (d) $10\sqrt{2}$m

7. A particle is moving with a velocity $\vec{v}$ = K $(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[Main 9 Jan. 2019 (I)]**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $y^2 = x^2$ + constant (d) xy = constant

3 Numeric Answer

8. A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is____ **[Main 8 Jan. 2020 (I)]**

6 MCQs with One or More than One Correct Answer

9. Starting at time $t = 0$ from the origin with speed 1 ms^{-1}, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then **[Adv 2020]**

(a) a_x = 1 ms^{-2} implies that when the particle is at the origin, a_y = 1 ms^{-2}
(b) a_x = 0 implies a_y = 1 ms^{-2} at all times
(c) at $t = 0$, the particle's velocity points in the x-direction
(d) a_x = 0 implies that at t = 1 s, the angle between the particle's velocity and the x axis is 45°

10. The coordinates of a particle moving in a plane are given by $x(t) = a \cos(pt)$ and $y(t) = b \sin(pt)$ where $a, b\,(< a)$ and p are positive constants of appropriate dimensions. Then **[1999S - 3 Marks]**

(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$
(c) the acceleration of the particle is always directed towards a focus
(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/(2p)$ is a

Topic-3 : Projectile Motion

1 MCQs with One Correct Answer

1. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: **[Main 9 Jan. 2020 (II)]**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

2. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ \text{ms}^{-2}$): **[Main 12 April 2019 (I)]**

(a) $\theta_0 = \sin^{-1}\frac{1}{\sqrt{5}}$ and $v_0 = \frac{5}{3}\ \text{ms}^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ \text{ms}^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{9}{3}\ \text{ms}^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ \text{ms}^{-1}$

3. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is : **[Main 12 April 2019 (I)]**

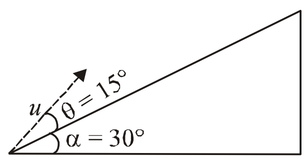
(a) R/4g (b) R/g (c) R/2g (d) 2R/g

4. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **[Main 12 April 2019 (II)]**

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$

(c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

5. A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\ \text{ms}^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to: (Take $g = 10\ \text{ms}^{-2}$) **[Main 10 April 2019 (II)]**



(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

6. A body is projected at t = 0 with a velocity $10\ \text{ms}^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ \text{ms}^{-2}$, the value of R is: **[Main 11 Jan. 2019 (I)]**

(a) 10.3 m (b) 2.8 m (c) 2.5 m (d) 5.1 m

7. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is: **[Main 10 Jan. 2019 (I)]**

(a) 1 : 16 (b) 1 : 2 (c) 1 : 4 (d) 1 : 8

8. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = (40\hat{i} + 50\hat{j})$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is (take $g = 10\ \text{ms}^{-2}$) **[Main Online April 9, 2014]**

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$ (c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

9. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ \text{m/s}^2$, the equation of its trajectory is **[Main 2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

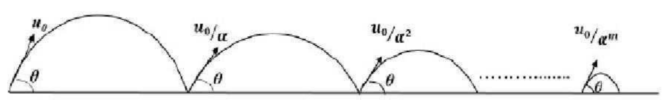
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

10. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is : **[Main Online April 22, 2013]**

(a) 4.4 m (b) 2.4 m (c) 3.6 m (d) 1.6 m

2 Integer Value Answer

11. A ball is thrown from ground at angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\ V_1$, the value of α is ____ **[Adv. 2019]**



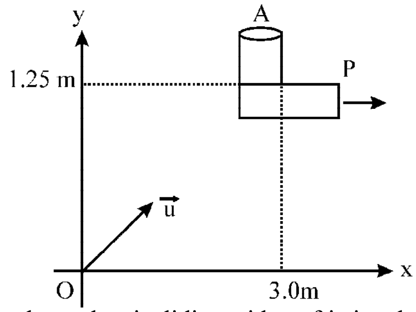
12. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s², is **[Adv. 2011]**

5 True / False

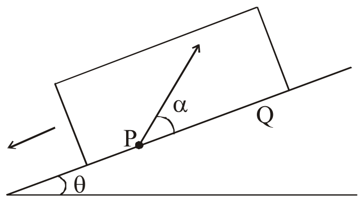
13. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. **[1984 - 2 Marks]**

10 Subjective Problems

14. An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity $\vec{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the X–Y plane. Find $\vec{u}$ and the time after which the stone hits the object. Take $g = 10$ m/s **[2000 - 10 Marks]**



15. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in Figure. **[1998 - 8 Marks]**



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

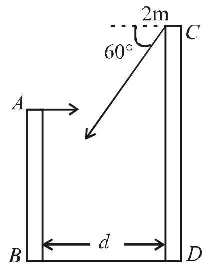
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

16. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m s^{-1} at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the firings, and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. **[1996 - 5 Marks]**

17. Two towers AB and CD are situated a distance d apart as shown in figure.

AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. **[1994 - 6 Marks]**

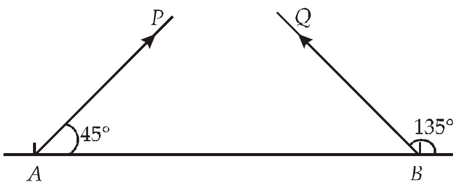
Simultaneously another object of mass 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.



(i) Calculate the distance 'd' between the towers and,

(ii) Find the position where the objects hit the ground.

18. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. **[1982 - 8 Marks]**



Both particle travel in the same vertical plane and undergo a collision After the collision, P retraces its path, Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g = 9.8$ m/s^2.

Topic 4 : Relative Velocity in Two Dimensions & Uniform Circular Motion

1 MCQs with One Correct Answer

1. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of: **[Main Sep. 06, 2020 (I)]**

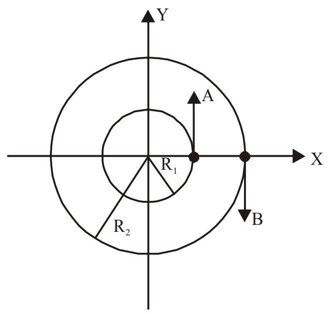
(a) 10^{-3} (b) 10^{-4} (c) 10^{-2} (d) 10^{-1}

2. When a car sit at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60º from the horizontal. On furter increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to: **[Main Sep. 06, 2020 (II)]**
(a) 0.50 (b) 0.41 (c) 0.37 (d) 0.73

3. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[Main 9 April 2019 (I)]**
(a) 90º (b) 150º (c) 120º (d) 60º

4. Ship A is sailing towards north-east with velocity km/hr where points east and , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: **[Main 8 April 2019 (I)]**
(a) 4.2 hrs. (b) 2.6 hrs. (c) 3.2 hrs. (d) 2.2 hrs.

5. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure : $\left(t = \frac{\pi}{2\omega}\right)$ **[Main 12 Jan. 2019 (II)]**

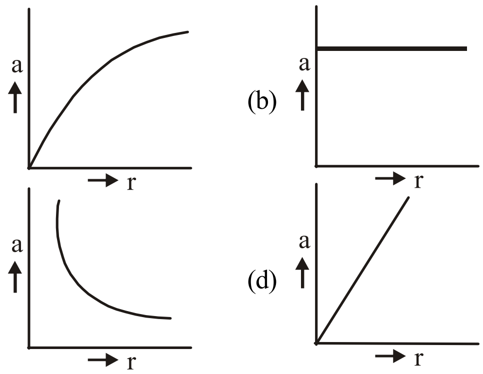


The relative velocity $\vec{v_A} - \vec{v_B}$ is given by:
(a) $\omega(R_1 + R_2)\hat{i}$ (b) $-\omega(R_1 + R_2)\hat{i}$
(c) $\omega(R_2 - R_1)\hat{i}$ (d) $\omega(R_1 - R_2)\hat{i}$

6. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[Main Online April 10, 2015]**
(a) $10\sqrt{3}$m/s (b) zero
(c) $10\sqrt{2}$m/s (d) 10 m/s

7. If a body moving in circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between acceleration and radius? **[Main Online April 10, 2015]**

(a) (b) (c) (d)



8. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is **[1988 - 1 Mark]**
(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

9. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction **[1983 - 1 Mark]**
(a) due north (b) 30° east of north
(c) 30° west of north (d) 60° east of north

Answer Key

Topic-1 : Vectors

1. (d) **2.** (c) **3.** (a) **4.** (a) **5.** (a) **6.** (195) **7.** (90) **8.** (2.00)

Topic-2 : Motion in a Plane with Constant Acceleration

1. (d) **2.** (a) **3.** (c) **4.** (d) **5.** (d) **6.** (b) **7.** (c) **8.** (580) **9.** (a, b, c, d)
10. (a, b, c)

Topic-3 : Projectile Motion

1. (a) **2.** (c) **3.** (d) **4.** (b) **5.** (a) **6.** (b) **7.** (a) **8.** (c) **9.** (b) **10.** (b)
11. (4.00) **12.** (5) **13.** (True)

Topic-4 : Relative Velocity in two Dimensions & Uniform Circular Motion

1. (a) **2.** (d) **3.** (c) **4.** (b) **5.** (c) **6.** (d) **7.** (c) **8.** (b) **9.** (a)

4 Laws of Motion

Topic-1 : Ist, IInd & IIIrd Laws of Motion

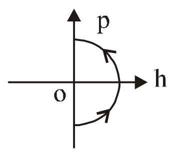
1 MCQs with One Correct Answer

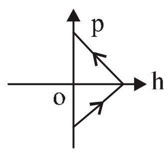
1. A particle moving in the *xy* plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$, where v_x and v_y are *x* and *y* components of its velocity $\vec{v}$. if $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle? **[Main Sep. 06, 2020 (II)]**
   (a) quantity $\vec{v} \times \vec{a}$ is constant in time
   (b) $\vec{F}$ arises due to a magnetic field
   (c) kinetic energy of particle is constant in time
   (d) quantity $\vec{v} \cdot \vec{a}$ is constant in time

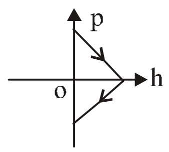
2. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is : **[Main Sep. 05, 2020 (II)]**
   (a) $-bv^3(t)$
   (b) $-\frac{bv^3}{M(t)}$
   (c) $-\frac{2bv^3}{M(t)}$
   (d) $-\frac{bv^3}{2M(t)}$

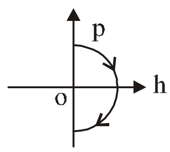
3. A small ball of mass *m* is thrown upward with velocity *u* from the ground. The ball experiences a resistive force mkv^2 where *v* is its speed. The maximum height attained by the ball is : **[Main Sep. 04, 2020 (II)]**
   (a) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$
   (b) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$
   (c) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$
   (d) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

4. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is : **[Main 10 April 2019 I]**
   (a) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
   (b) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
   (c) $\frac{1}{\sqrt{\gamma g}}ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$
   (d) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

5. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is: **[Main 9 April 2019 I]**
   (a)


   (b)


   (c)


   (d)


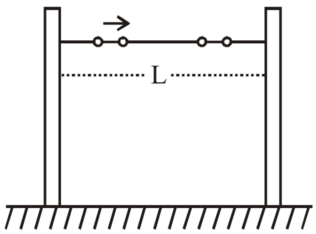
6. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = *k*t acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here *k* is a constant. The value of T is : **[Main 11 Jan. 2019 II]**
   (a) $2\sqrt{\frac{k}{p}}$
   (b) $2\sqrt{\frac{p}{k}}$
   (c) $\sqrt{\frac{2k}{p}}$
   (d) $\sqrt{\frac{2p}{k}}$

7. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2} v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot : **[Main Online April 10, 2016]**

(a) log v(t) against $\frac{1}{t}$ (b) v(t) against t^2

(c) log v(t) against $\frac{1}{t^2}$ (d) log v(t) against t

8. A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length L (L >> r) and are at rest at random positions. The rod is mounted between two rigid supports (see figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic): **[Main Online April 11, 2015]**



(a) $\frac{mv^2}{2(L-nr)}$ (b) $\frac{mv^2}{L-2nr}$

(c) $\frac{mv^2}{L-nr}$ (d) zero

9. A body of mass 5 kg under the action of constant force $\vec{F} = F_x\hat{i} + F_y\hat{j}$ has velocity at t = 0 s as $\vec{v} = (6\hat{i} - 2\hat{j})$ m/s and at t = 10s as $\vec{v} = +6\hat{j}$ m/s. The force $\vec{F}$ is: **[Main Online April 11, 2014]**

(a) $(-3\hat{j} + 4\hat{j})$N (b) $\left(-\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$N

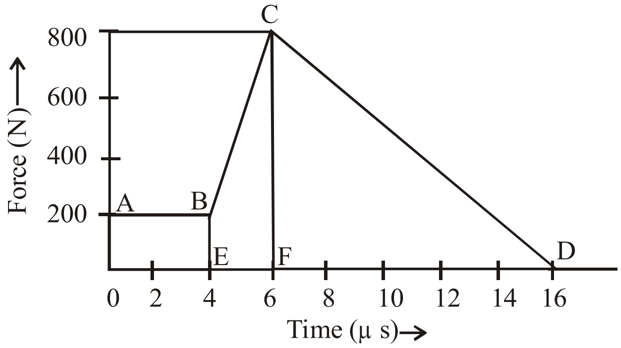
(c) $(3\hat{i} - 4\hat{j})$N (d) $\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$N

10. A particle moves in the *X-Y* plane under the influence of a force such that its linear momentum $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$, where *A* and *k* are constants. The angle between the force and the momentum is **[2007]**

(a) 0° (b) 30°
(c) 45° (d) 90°

4 Fill in the Blanks

11. The magnitude of the force (in newtons) acting on a body varies with time *t* (in micro seconds) as shown in the fig *AB*, *BC* and *CD* are straight line segments. The magnitude of the total impulse of the force on the body from *t* = 4 μs to *t* = 16μs is ....Ns. **[1994 - 2 Marks]**



5 True / False

12. A rocket moves forward by pushing the surrounding air backwards. **[1980]**

6 MCQs with One or More than One Correct Answer

13. A reference frame attached to the earth **[1986 - 2 Marks]**

(a) is an inertial frame by definition.
(b) cannot be an inertial frame because the earth is revolving round the sun.
(c) is an inertial frame because Newton's laws are applicable in this frame.
(d) cannot be an inertial frame because the earth is rotating about its own axis.

9 Assertion and Reason Type Questions

14. **Statement-1 :** It is easier to pull a heavy object than to push it on a level ground and

Statement-2 : The magnitude of frictional force depends on the nature of the two surfaces in contact. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

15. **STATEMENT-1 :** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

STATEMENT-2 : For every action there is an equal and opposite reaction. **[2007]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

Topic-2 : Motion of Connected Bodies, Pulley & Equilibrium of Forces

1 MCQs with One Correct Answer

1. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take $g = 10\ ms^{-2}$ and the rope to be massless) **[Main 7 Jan. 2020 II]**

(a) 100 N (b) 90 N
(c) 70 N (d) 75 N

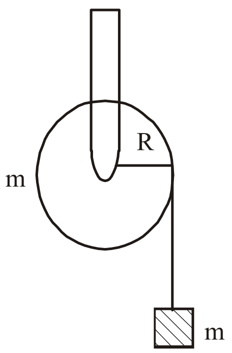
2. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least: **[Main 7 Jan. 2020 II]**

(a) 56300 W (b) 62360 W
(c) 48000 W (d) 66000 W

3. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$) **[Main 9 Jan. 2019 II]**

(a) 200 N (b) 140 N
(c) 70 N (d) 100 N

4. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall or release? **[Main 2014]**



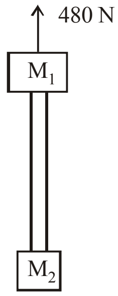
(a) $\frac{2g}{3}$

(b) $\frac{g}{2}$

(c) $\frac{5g}{6}$

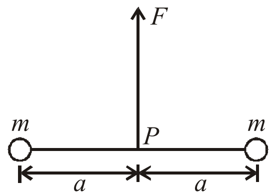
(d) g

5. Two blocks of mass $M_1 = 20$ kg and $M_2 = 12$ kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is : **[Main Online April 22, 2013]**



(a) 144 N
(b) 96 N
(c) 240 N
(d) 192 N

6. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the centre P (as shown in the figure).
Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is **[2007]**



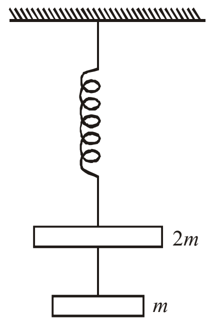
(a) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$

(b) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(c) $\frac{F}{2m}\frac{x}{a}$

(d) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

7. The string between blocks of mass m and $2m$ is massless and inextensible. The system is suspended by a massless spring as shown. If the string is cut find the magnitudes of accelerations of mass $2m$ and m (immediately after cutting) **[2006 - 3M, –1]**



(a) g, g (b) g, $\frac{g}{2}$

(c) $\frac{g}{2}$, g (d) $\frac{g}{2}$, $\frac{g}{2}$

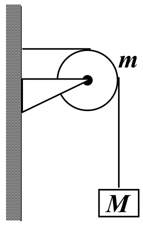
8. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by **[2001S]**

(a) $\sqrt{2}$ Mg

(b) $\sqrt{2}$ mg

(c) $\sqrt{(M+m)^2+m^2}\,g$

(d) $\sqrt{(M+m)^2+M^2}\,g$



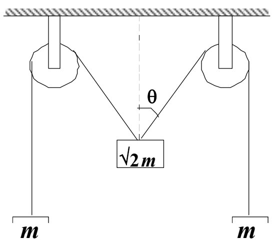
9. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **[2001S]**

(a) 0°

(b) 30°

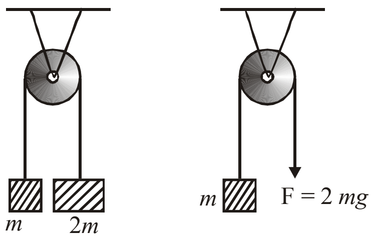
(c) 45°

(d) 60°



5 True / False

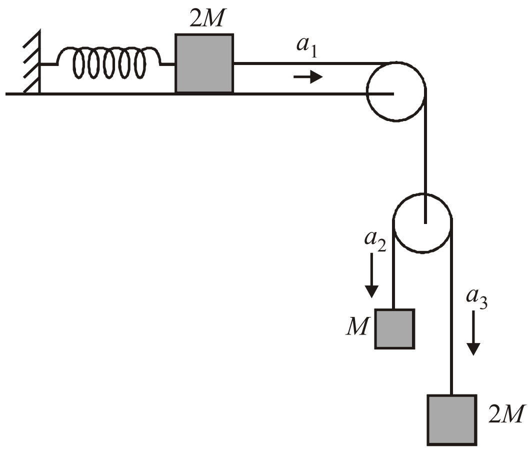
10. The pulley arrangements of Figs. (a) and (b) are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b) , m is lifted up by pulling the other end of the rope with a constant downward force $F = 2\ mg$. The acceleration of m is the same in both cases **[1984 - 2 Marks]**



(a) (b)

6 MCQs with One or More than One Correct Answer

11. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The acceleration of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? [g is the acceleration due to gravity. Neglect friction] **[Adv. 2019]**



(a) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(b) $x_0 = \frac{4Mg}{k}$

(c) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(d) $a_2 - a_1 = a_1 - a_3$

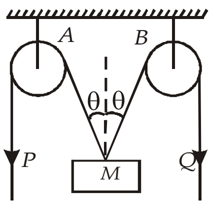
12. In the arrangement shown in the Fig, the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. **[1982 -3 Marks]** Mass M moves upwards with a speed

(a) 2Ucos θ

(b) U/ cos θ

(c) 2U / cos θ

(d) U cos θ

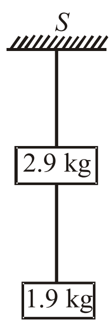


10 Subjective Problems

13. Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 meter, see fig. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks wires and support have an upward acceleration of 0.2 m/s^2. Acceleration due to gravity is 9.8 m/s^2. **[1989 - 6 Marks]**

(i) Find the tension at the mid-point of the lower wire.

(ii) Find the tension at the mid-point of the upper wire.



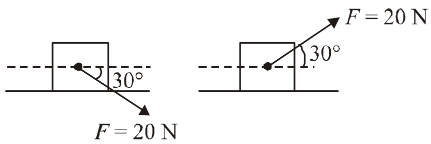
Topic-3 : Friction

1 MCQs with One Correct Answer

1. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is : ($g = 10\ ms^{-2}$) **[Main Sep. 06, 2020 (I)]**

   (a) 0.20m (b) 0.45m
   (c) 0.60m (d) 0.80m

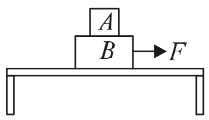
2. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F=20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be : ($g = 10\ ms^{-2}$) **[Main 12 April 2019 (II)]**



(A) (B)

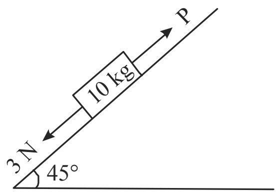
   (a) $0.4\ ms^{-2}$ (b) $3.2\ ms^{-2}$
   (c) $0.8\ ms^{-2}$ (d) $0\ ms^{-2}$

3. Two blocks A and B masses m_A=1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is : [Take g = 10 m/s²] **[Main 10 April 2019 (II)]**



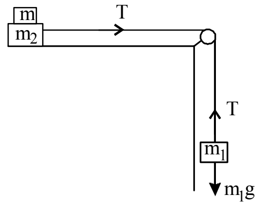
   (a) 8 N (b) 16 N (c) 40 N (d) 12 N

4. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward? (take $g = 10\ ms^{-2}$) **[Main 9 Jan. 2019 (I)]**



   (a) 32 N (b) 18 N (c) 23 N (d) 25 N

5. Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is: **[Main 2018]**



   (a) 18.3 kg
   (b) 27.3 kg
   (c) 43.3 kg
   (d) 10.3 kg

6. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is : **[Main Online April 15, 2018]**

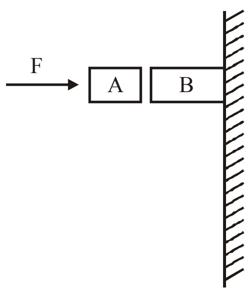
   (a) $\sqrt{1-\dfrac{1}{n^2}}$ (b) $1-\dfrac{1}{n^2}$

   (c) $\dfrac{1}{2-n^2}$ (d) $\sqrt{\dfrac{1}{1-n^2}}$

7. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is : **[Main Online April 9, 2016]**

   (a) tan2θ (b) tanθ
   (c) 3 tanθ (d) 2 tan θ

8. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is: **[Main 2015]**



   (a) 120 N (b) 150 N
   (c) 100 N (d) 80 N

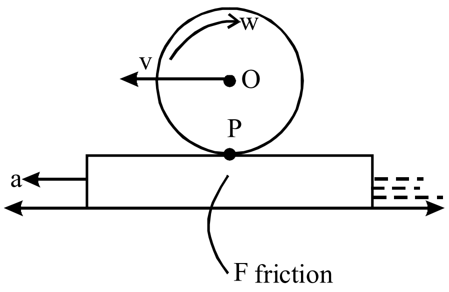
9. A block of mass m = 10 kg rests on a horizontal table. The coefficient of friction between the block and the table is 0.05. When hit by a bullet of mass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of 2 m on the table. If a freely falling object were to acquire speed $\frac{v}{10}$ after being dropped from height H, then neglecting energy losses and taking g = 10 ms^{-2}, the value of H is close to: **[Main Online April 10, 2015]**

(a) 0.05 km (b) 0.02 km
(c) 0.03 km (d) 0.04 km

10. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: **[Main 2014]**

(a) $\frac{1}{6}$m (b) $\frac{2}{3}$m (c) $\frac{1}{3}$m (d) $\frac{1}{2}$m

11. Consider a cylinder of mass M resting on a rough horizontal rug that is pulled out from under it with acceleration 'a' perpendicular to the axis of the cylinder. What is $F_{friction}$ at point P? It is assumed that the cylinder does not slip. **[Main Online April 19, 2014]**

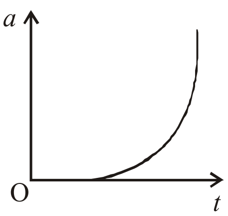


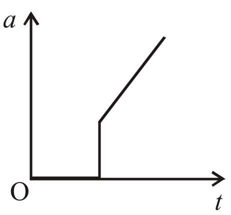
(a) Mg (b) Ma (c) $\frac{Ma}{2}$ (d) $\frac{Ma}{3}$

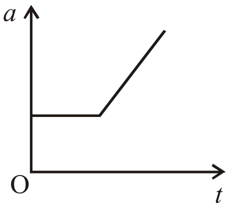
12. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied to B for the blocks to move together will be: **[Main Online April 9, 2014]**

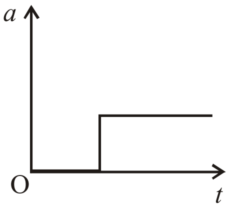
(a) 30 N (b) 25 N (c) 27 N (d) 48 N

13. A block is placed on a rough horizontal plane. A time dependent horizontal force F = kt acts on the block, where k is a positive constant. The acceleration - time graph of the block is : **[Main Online April 25, 2013]**

(a)


(b)


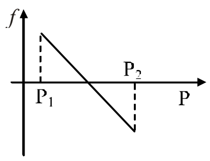
(c)


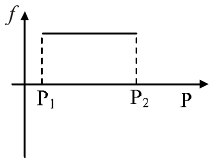
(d)


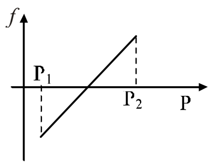
14. A body starts from rest on a long inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3\,x$, where x is distance travelled down the plane. The body will have maximum speed (for $g = 10$ m/s^2) when x = **[Main Online April 22, 2013]**

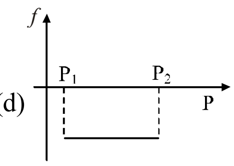
(a) 9.8 m (b) 27 m (c) 12 m (d) 3.33 m

15. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tan θ > μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like **[2010]**

(a)


(b)


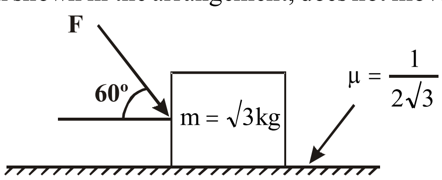
(c)


(d)


16. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then **[2009]**

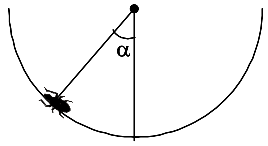
(a) at θ = 30°, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles
(d) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ.

17. What is the maximum value of the force F such that the block shown in the arrangement, does not move? **[2003S]**



(a) 20N (b) 10N (c) 12N (d) 15N

18. An insect crawls up a hemispherical surface very slowly (see fig.). The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by **[2001S]**



(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$
(c) $\sec\alpha = 3$ (d) $\text{cosec}\,\alpha = 3$

19. A block of mass 0.1 is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is : **[1994 - 1 Mark]**

(a) 2.5 N (b) 0.98 N (c) 4.9 N (d) 0.49 N

20. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is **[1980]**

(a) 9.8 N (b) $0.7 \times 9.8 \times \sqrt{3}$N
(c) $9.8 \times \sqrt{3}$N (d) 0.7×9.8 N

2 Integer Value Answer

21. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is **[2011]**

22. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ..... newtons. **[1984 - 2 Marks]**

3 Numeric Answer

23. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The value of I is _________. **[Main Sep. 03, 2020 (II)]**

5 True / False

24. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. **[1981 - 2 Marks]**

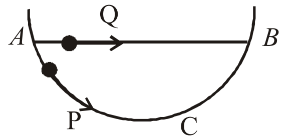
6 MCQs with One or More than One Correct Answer

25. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. **[2012]**

The block remains stationary if (take $g = 10$ m/s^2)

(a) θ = 45°
(b) θ > 45° and a frictional force acts on the block towards P.
(c) θ > 45° and a frictional force acts on the block towards Q.
(d) θ < 45° and a frictional force acts on the block towards Q.

26. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then : **[1993-2 Marks]**

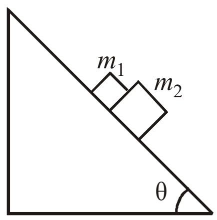


(a) $t_P < t_Q$
(b) $t_P = t_Q$
(c) $t_P > t_Q$
(d) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of arc } AB}$

7 Match the Following

27. A block of mass m_1 = 1 kg another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List-I. The coefficient of friction between the block m_1 and plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List-II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g.
[**Useful information:** tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3 **[Adv. 2014]**

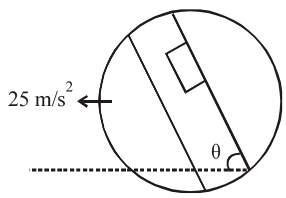
	List-I		List-II
P.	θ = 5°	**1.**	$m_2 g \sin\theta$
Q.	θ = 10°	**2.**	$(m_1 + m_2) g \sin\theta$
R.	θ = 15°	**3.**	$\mu m_2 g \cos\theta$
S.	θ = 20°	**4.**	$\mu(m_1 + m_2) g \cos\theta$



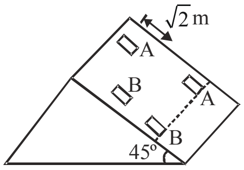
Code:
(a) P-1, Q-1, R-1, S-3 (b) P-2, Q-2, R-2, S-3
(c) P-2, Q-2, R-2, S-4 (d) P-2, Q-2, R-3, S-3

10 Subjective Problems

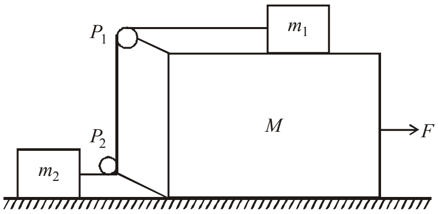
28. A circular disc with a groove along its diameter is placed horizontally on a rough surface. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove and horizontal surface in contact is $\mu = \frac{2}{5}$. The disc has an acceleration of 25 m/s^2 towards left. Find the acceleration of the block with respect to disc. Given $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. **[2006 - 6M]**



29. Two block A and B of equal masses are placed on rough inclined plane as shown in figure. When and where will the two blocks come on the same line on the inclined plane if they are released simultaneously? Initially the block A is $\sqrt{2}$ m behind the block B. Co-efficient of kinetic friction for the blocks A and B are 0.2 and 0.3 respectively (g=10 m/s^2). **[2004 - Marks]**



30. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m$_1$ and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. Take $g = 10$ m/s^2. **[2000 - 10 Marks]**

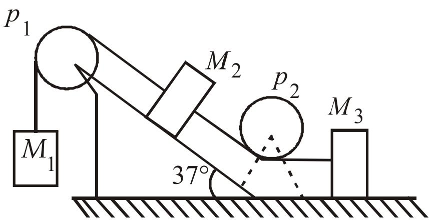


(a) Draw a free body diagram for mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write equations of motion of all the masses. Find F, tension in the string and acceleration of the masses.

31. A particle of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the maguitude of F and the direction in which it has to be applied. **[1987 - 7 Marks]**

32. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and friction less pulleys P_1 and P_2 as shown in fig The masses move such that the portion of the string between P_1 and P_2 in parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. **[1981- 6 Marks]**



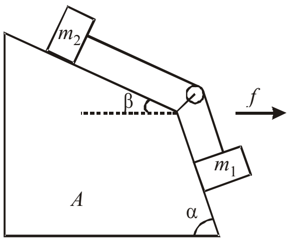
If the mass M_1 moves downwards with a uniform velocity, find

(i) the mass of M_1

(ii) The tension in the horizontal portion of the string ($g = 9.8$ m/sec^2, sin 37° $\simeq$ 3/5)

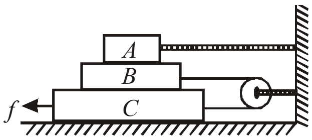
33. A horizontal uniform rope of length L, resting on a frictionless horizontal surface, is pulled at one end by force F. What is the tension in the rope at a distance l from the end where the force is applied? **[1978]**

34. Two cubes of masses m_1 and m_2 be on two frictionless slopes of block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in the figure. To what horizontal acceleration



f should the whole system (that is blocks and cubes) be subjected so that the cubes do not slide down the planes. What is the tension of the string in this situation? **[1978]**

35. In the diagram shown, the blocks A, B and C weighs, 3 kg, 4 kg and 5 kg respectively. The coefficient of sliding friction between any two surface is 0.25. A is held at rest by a massless rigid rod fixed to the wall while B and C are connected by a light flexible cord passing around a frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, B on C and A on B, is maintained all through. (g = 9.8 m/s^2) **[1978]**

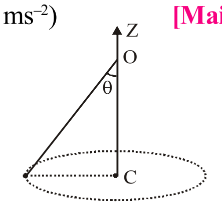


Topic-4 : Circular Motion & Banking of Road

1 MCQs with One Correct Answer

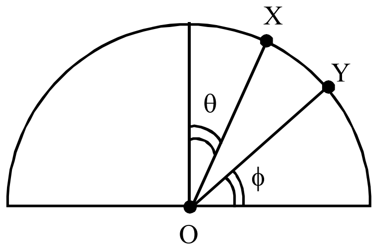
1. A disc rotates about its axis of symmetry in a hoizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is ($g = 10 m/s^2$)
[Main Online April 15, 2018]
(a) 0.5 (b) 0.7 (c) 0.3 (d) 0.6

2. A conical pendulum of length 1 m makes an angle $\theta = 45°$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4 m and its centre is vertically below O. The speed of the pendulum, in its circular path, will be :
(Take $g = 10\ ms^{-2}$) **[Main Online April 9, 2017]**



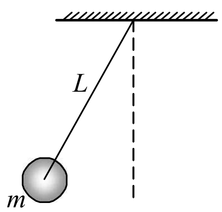
(a) 0.4 m/s
(b) 4 m/s
(c) 0.2 m/s
(d) 2 m/s

3. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle ϕ with the horizontal. Then: **[Main Online April 19, 2014]**



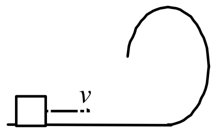
(a) $\sin\phi = \cos\phi$ (b) $\sin\phi = \frac{1}{2}\cos\theta$
(c) $\sin\phi = \frac{2}{3}\cos\theta$ (d) $\sin\phi = \frac{3}{4}\cos\theta$

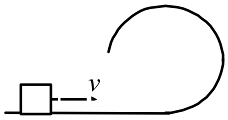
4. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of anguar velocity of ball (in radian/s) is **[2011]**

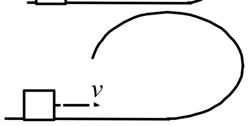


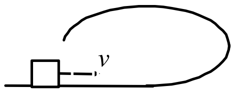
(a) 9 (b) 18 (c) 27 (d) 36

5. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in **[2001S]**

(a) 

(b) 

(c) 

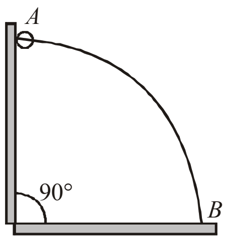
(d) 

5 True / False

6. A simple pendulum with a bob of mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is greater than $mg \cos 20°$. **[1984 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

7. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

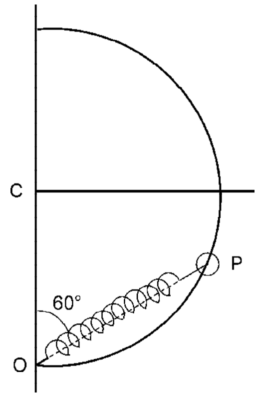


[Adv. 2014]
(a) always radially outwards
(b) always radially inwards
(c) radially outwards initially and radially inwards later
(d) radially inwards initially and radially outwards later

8. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limit $-\phi$ and $+\phi$. For an angular displacement $\theta\ (|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and V respectively. The following relations hold good under the above conditions : **[1986 - 2 Marks]**
(a) $T \cos\theta = Mg$.
(b) $T - Mg\cos\theta = \frac{MV^2}{L}$
(c) The magnitude of the tangenial acceleration of the bob $|a_T| = g \sin\theta$
(d) $T = Mg\cos\theta$

10 Subject Problems

9. A smooth semicircular wire-track of radius R is fixed in a vertical plane. One end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire-track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $K = mg/R$. Consider the instant when the ring is released, and (i) draw the free body diagram of the ring, (ii) determine the tangential acceleration of the ring and the normal reaction. **[1996 - 5 Marks]**



Answer Key

Topic-1 : Ist, IInd & IIIrd Laws of Motion

1. (a)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (d)
11. (0.005)	**12.** False	**13.** (b, d)	**14.** (b)	**15.** (b)					

Topic-2 : Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** False
11. (d)	**12.** (b)								

Topic-3 : Friction

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (a)
11. (d)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)
21. (5)	**22.** (5N)	**23.** (346)	**24.** False	**25.** (a, c)	**26.** (a)	**27.** (d)			

Topic-4 : Circular Motion & Banking of Road

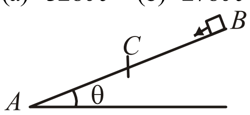
1. (d)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** False	**7.** (d)	**8.** (b, c)

5 Work, Energy and Power

Topic-1 : Work Done by Constant & Variable Force

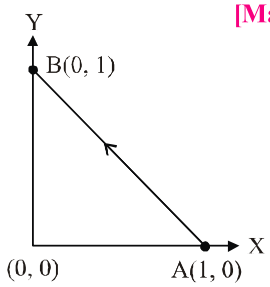
1 MCQs with One Correct Answer

1. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ? **[Main 4 Sep. 2020 (II)]**
   (a) 3280 J (b) 2780 J (c) 5690 J (d) 5250 J

2. 

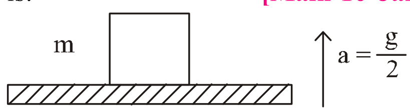
   A small block starts slipping down from a point *B* on an inclined plane *AB*, which is making an angle θ with the horizontal section *BC* is smooth and the remaining section *CA* is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point *A*) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k\tan\theta$. The value of *k* is **[Main 2 Sep. 2020 (I)]**
   (a) 1 (b) 2 (c) 3 (d) 4

3. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units) **[Main 9 Jan. 2020 (I)]**



   (a) 2J (b) $\frac{1}{2}$J (c) 1J (d) $\frac{3}{2}$J

4. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. work done by normal reaction on block in time t is: **[Main 10 Jan. 2019 (I)]**



   (a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$
   (c) 0 (d) $\frac{3mg^2t^2}{8}$

5. A body of mass starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant s and is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start of the motion is: **[Main Online April 16, 2018]**
   (a) $\frac{1}{8}ma^4t^2$ (b) $4ma^4t^2$ (c) $8ma^4t^2$ (d) $\frac{1}{4}ma^4t^2$

6. When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is: **[Main 2014]**
   (a) $aL^2 + bL^3$ (b) $\frac{1}{2}\left(aL^2 + bL^3\right)$
   (c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

7. The work done on a particle of mass m by a force,

$$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the $x-y$ plane is **[Adv. 2013]**

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$ (c) $\frac{K\pi}{2a}$ (d) 0

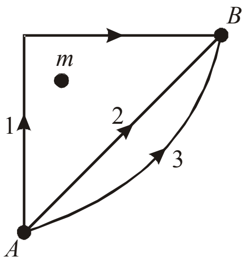
8. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3 **[2003S]**

(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$

(d) $W_2 > W_1 > W_3$



9. A force $F = -K(y\hat{i}+x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x axis to the point $(a, 0)$, and then parallel to the y axis to the point (a, a), The total work done by the force F on the particle is **[1998S - 2 Marks]**

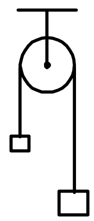
(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

10. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is **[1985 - 2 Marks]**

(a) MgL (b) $MgL/3$ (c) $MgL/9$ (d) $MgL/18$

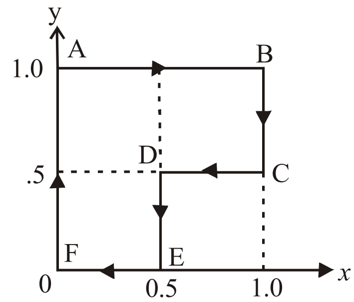
2 Integer Value Answer

11. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m/s^2, find the work done **(in joules)** by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[2009]**



3 Numeric Answer

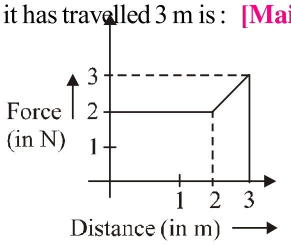
12. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a forced $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$, where x and y are in meter and $\alpha = -1Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ___ Joule. **[Adv. 2019]**



Topic-2 : Energy

1 MCQs with One Correct Answer

1. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is : **[Main 7 Jan. 2020 II]**



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

2. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be: **[Main 12 April 2019 (II)]**

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

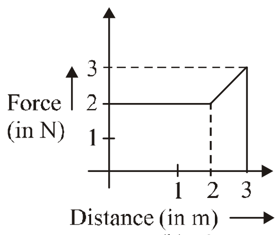
3. A body of mass 1 kg falls freely from a height of 100m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10$ ms^{-2}, the value of x will be close to : **[Main 11 April 2019 (I)]**

(a) 40 cm (b) 4 cm (c) 80 cm (d) 8 cm

4. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be: **[Main 9 April 2019 (I)]**

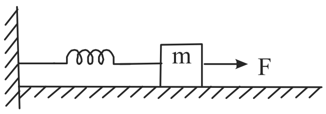
(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$ (c) $\frac{2MgL}{n^2}$ (d) nMgL

5. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is : **[Main 8 April 2019 (I)]**



(a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

6. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **[Main 9 Jan. 2019 (I)]**



(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$ (c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$

7. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is: **[Main 2018]**

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$
(c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

8. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is v_0 = 10 ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: **[Main 2017]**

(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1} s^{-1}
(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

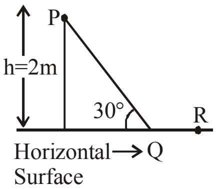
9. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as $t \to \infty$ is **[Main Online April 8, 2017]**

(a) 3h (b) ∞ (c) $\frac{5}{3}$h (d) $\frac{8}{3}$h

10. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be **[Main 2017]**

(a) 9 J (b) 18 J (c) 4.5 J (d) 22 J

11. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.
The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to : **[Main 2016]**



(a) 0.29 and 3.5 m (b) 0.29 and 6.5 m
(c) 0.2 and 6.5 m (d) 0.2 and 3.5 m

12. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} : **[Main 2016]**

(a) 9.89×10^{-3} kg (b) 12.89×10^{-3} kg
(c) 2.45×10^{-3} kg (d) 6.45×10^{-3} kg

13. A particle is moving in a circle of radius r under the action of a force F = αr^2 which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for r = 0) : **[Main Online April 11, 2015]**

(a) $\frac{1}{2}\alpha r^3$ (b) $\frac{5}{6}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) αr^3

14. A block of mass *m* = 0.1 kg is connected to a spring of unknown spring constant k. It is compressed to a distance *x* from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 ms^{-1}.
The total initial energy of the spring is : **[Main Online April 10, 2015]**

(a) 0.3 J (b) 0.6 J
(c) 0.8 J (d) 1.5 J

15. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be: **[Main Online April 19, 2014]**

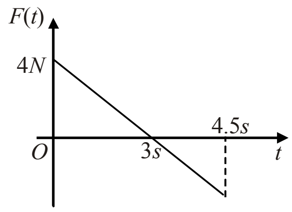
(a) $\frac{n^2}{2n-1}$ (b) $\frac{2n^2}{n-1}$ (c) infinite (d) n

16. Two springs of force constants 300 N/m (Spring A) and 400 N/m (Spring B) are joined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is $\frac{E_A}{E_B}$. Then $\frac{E_A}{E_B}$ is equal to : **[Main Online April 9, 2013]**

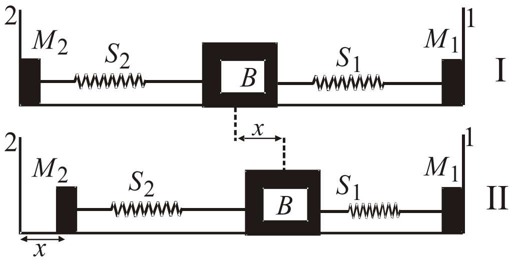
(a) $\frac{4}{3}$ (b) $\frac{16}{9}$ (c) $\frac{3}{4}$ (d) $\frac{9}{16}$

17. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is **[2010]**

(a) 4.50 J
(b) 7.50 J
(c) 5.06 J
(d) 14.06 J

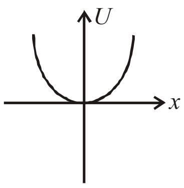


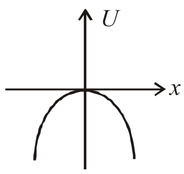
18. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively (see fig. I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is – **[2008]**

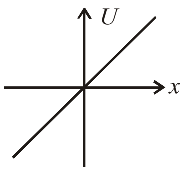


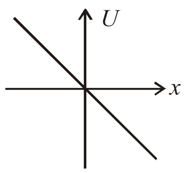
(a) 4 (b) 2 (c) 1/2 (d) 1/4

19. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x **[2004S]**

(a) 

(b) 

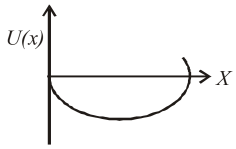
(c) 

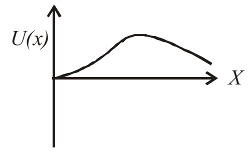
(d) 

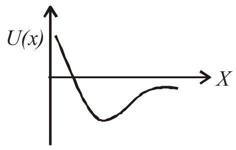
20. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is **[2002S]**

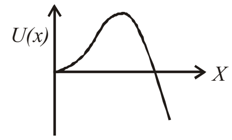
(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

21. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is **[2002S]**

(a) 

(b) 

(c) 

(d) 

22. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of **[1999S - 2 Marks]**

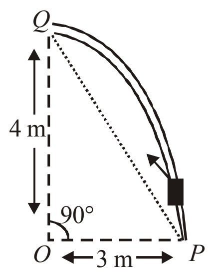
(a) $(2/3)k$ (b) $(3/2)k$ (c) $3k$ (d) $6k$

23. Two masses of 1 gm and 4 gm are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is **[1980]**

(a) 4 : 1 (b) $\sqrt{2}:1$ (c) 1 : 2 (d) 1 : 16

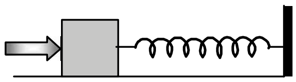
2 Integer Value Answer

24. Consider an elliptical shaped rail PQ in the vertical plane with $OP = 3$ m and $OQ = 4$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictionless losses, the kinetic energy of the block when it reaches Q is ($n \times 10$) joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}) **[Adv. 2014]**



25. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block

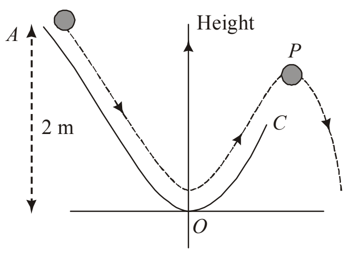
as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is **[2011]**



3 Numeric Answer

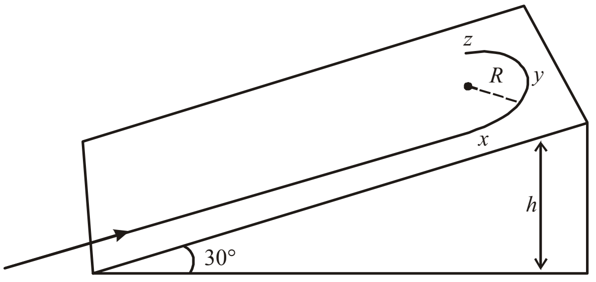
26. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is ($g = 10\ ms^{-2}$) ________. **[Main 3 Sep. 2020 (I)]**

27. A particle ($m = 1$ kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale; take $g = 10\ ms^{-2}$) ______. **[Main 7 Jan. 2020 (I)]**



6 MCQs with One or More than One Correct Answer

28. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity) **[Adv. 2020]**



(a) $v_0^2 - 2gh = \frac{1}{2}gR$

(b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(c) the centripetal force required at points x and z is zero

(d) the centripetal force required is maximum at points x and z

29. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **[Adv. 2018]**

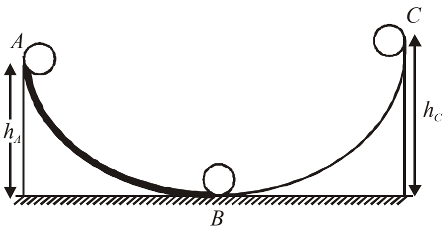
(a) The force applied on the particle is constant

(b) The speed of the particle is proportional to time

(c) The distance of the particle from the origin increases linearly with time

(d) The force is conservative

30. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then **[2006 - 5M, –1]**



(a) $h_A > h_C;\ K_B > K_C$ (b) $h_A > h_C;\ K_C > K_A$

(c) $h_A = h_C;\ K_B = K_C$ (d) $h_A < h_C;\ K_B > K_C$

31. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is **[1998S - 2 Marks]**

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

32. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that : **[1987 - 2 Marks]**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant.

(d) it moves in a circular path.

7 Match the Following

33. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constants). Match the potential energies in column I to the corresponding statement(s) in column II.

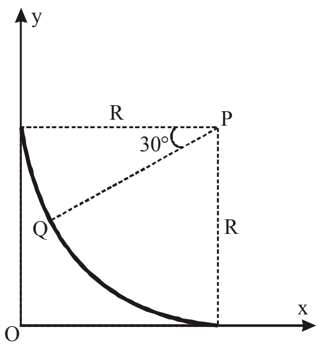
Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x = a$
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x = 0$
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at $x = -a$
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x\rvert < a$
	(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$

8 Comprehension/Passage Based Questions

PASSAGE

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J.
(Take the acceleration due to gravity, $g = 10\ ms^{-2}$)

[Adv. 2013]



34. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N
(c) 11.5 N (d) 22.5 N

35. The speed of the block when it reaches the point Q is

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$
(c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

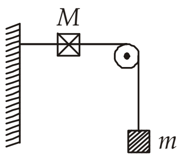
9 Assertion and Reason Type Questions

36. **STATEMENT–1 :** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.
STATEMENT–2 : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **[2007]**

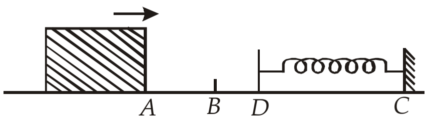
(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

10 Subjective Problems

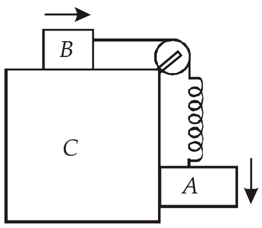
37. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? **[1985 - 6 Marks]**



38. A 0.5 kg block slides from the point A (see Fig) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0 2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely.
(Take $g = 10 m/s^2$) **[1983 - 7 Marks]**

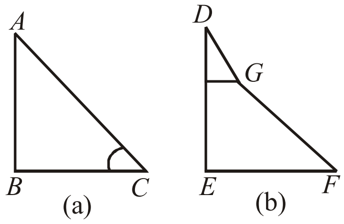


39. Two blocks A and B are connected to each other by a string and a spring; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed.



The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 newtons/m. If mass of block A is 2 Kg., calculate the mass of block B and the energy stored in the spring. **[1982 - 5 Marks]**

40.



In the figures (a) and (b) AC, DG and GF are fixed inclined planes, $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_c. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic frictions between the block and both the surface AC and DGF are μ. **[1980]**
Calculate V_C and V_F.

41. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? **[1979]**

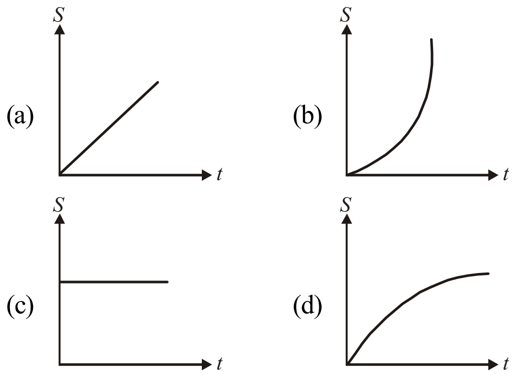
42. A spring of force constant k is cut into three equal parts. What is force constant of each part? **[1978]**

43. A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal or less than that of the bullet. **[1978]**

Topic-3 : Power

1 MCQs with One Correct Answer

1. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) : **[Main 3 Sep. 2020 (II)]**



2. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, $g = 10\ ms^{-2}$) **[Main 7 Jan. 2020 I]**

(a) $1.7\ ms^{-1}$ (b) $1.9\ ms^{-1}$
(c) $1.5\ ms^{-1}$ (d) $2.0\ ms^{-1}$

3. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by $n^2 R t^2$ where n is a constant. The power delivered to the particle by the force acting on it, is : **[Main Online April 10, 2016]**

(a) $\frac{1}{2} M n^2 R^2 t^2$ (b) $M n^2 R^2 t$
(c) $M n R^2 t^2$ (d) $M n R^2 t$

4. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 Km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of $10\ ms^{-1}$, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is: **[Main Online April 9, 2016]**

(a) $20\ ms^{-1}$ (b) $5\ ms^{-1}$ (c) $15\ ms^{-1}$ (d) $10\ ms^{-1}$

5. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to **[Main Online April 25, 2013]**
(a) v^4 (b) v^2 (c) v (d) v

6. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The center of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is : (Take $g = 10\ ms^{-2}$) **[Main Online April 23, 2013]**
(a) 6.26×10^3 Watts at the start
(b) 6.26×10^3 Watts at take off
(c) 6.26×10^4 Watts at the start
(d) 6.26×10^4 Watts at take off

7. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to **[2000S]**
(a) v (b) v^2 (c) v^3 (d) v^4

8. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is: **[1994 - 1 Mark]**
(a) $2\pi\ mk^2r^2\ t$ (b) mk^2r^2t
(c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

9. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to **[1984- 2 Marks]**
(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

10. If a machine is lubricated with oil **[1980]**
(a) the mechanical advantage of the machine increases.
(b) the mechanical efficiency of the machine increases.
(c) both its mechanical advantage and efficiency increase.
(d) its efficiency increases, but its mechanical advantage decreases.

2 Integer Value Answer

11. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is **[Adv. 2013]**

3 Numeric Answer

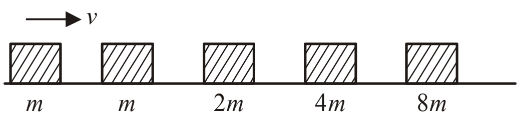
12. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ______. **[Main 5 Sep. 2020 (II)]**

Topic-4 : Collisions

1 MCQs with One Correct Answer

1. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i} + \hat{j})\ ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})\ ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is : **[Main 6 Sep. 2020 (II)]**
(a) 15º (b) 60º
(c) –45º (d) 105º

2. Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving the total energy loss is p% of the original energy. Value of 'p' is close to : **[4 Sep. 2020 (I)]**



(a) 77 (b) 94 (c) 37 (d) 87

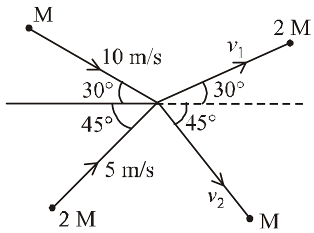
3. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10\ m/s^2$. Assume there is no rotational motion and losss of energy after the collision is negligiable.] **[Main 3 Sep. 2020 (II)]**
(a) 20 J (b) 21 J (c) 19 J (d) 23 J

4. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by : **[Main 2 Sep. 2020 (I)]**
(a) $v = \sqrt{\frac{2}{3}}u$ (b) $v = \frac{u}{\sqrt{3}}$
(c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{1}{\sqrt{6}}u$

5. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is: **[Main 9 Jan. 2020 I]**
(a) $\frac{1}{3}mu^2$ (b) $\frac{1}{8}mu^2$

(c) $\frac{3}{4}$ mu^2 (d) $\sqrt{\frac{2}{3}}$ mu^2

6. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly : **[Main 10 April 2019 I]**



(a) 6.5 m/s and 6.3 m/s (b) 3.2 m/s and 6.3 m/s
(c) 6.5 m/s and 3.2 m/s (d) 3.2 m/s and 12.6 m/s

7. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.
If $m_2 = 0.5\, m_1$ and $v_3 = 0.5\, v_1$, then v_1 is : **[Main 8 April 2019 II]**

(a) $v_4 - \frac{v_2}{2}$ (b) $v_4 - v_2$ (c) $v_4 - \frac{v_2}{4}$ (d) $v_4 + v_2$

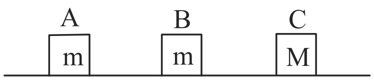
8. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is : **[Main 12 Jan. 2019 II]**

(a) 2m (b) 3.5m (c) 1.5m (d) 4m

9. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: (g = 10 ms^{-2}) **[Main 10 Jan. 2019 I]**

(a) 20m (b) 30m (c) 40m (d) 10m

10. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m? **[Main 9 Jan. 2019 I]**



(a) 5 (b) 2 (c) 4 (d) 3

11. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: **[Main 2018]**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

12. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: **[Main 2018]**

(a) 2.35×10^3 N/m^2 (b) 4.70×10^3 N/m^2
(c) 2.35×10^2 N/m^2 (d) 4.70×10^2 N/m^2

13. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy isp_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively: **[Main 2018]**

(a) $(\cdot 89, \cdot 28)$ (b) $(\cdot 28, \cdot 89)$ (c) (0, 0) (d) (0, 1)

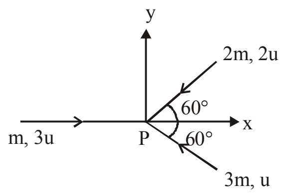
14. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is : **[Main Online April 10, 2016]**

(a) 20.4 eV (b) 10.2 eV (c) 12.1 eV (d) 16.8 eV

15. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed **v**. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[Main 2015]**

(a) 56% (b) 62% (c) 44% (d) 50%

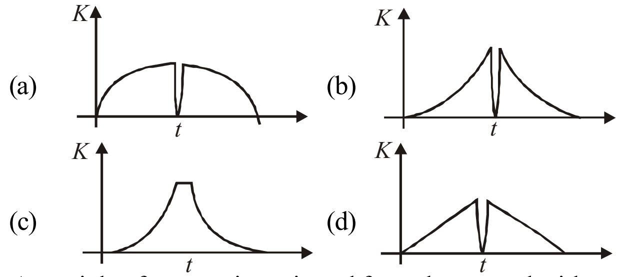
16. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be: **[Main Online April 12, 2014]**



(a) $\frac{u}{12}(\hat{i} + \sqrt{3}\hat{j})$ (b) $\frac{u}{12}(\hat{i} - \sqrt{3}\hat{j})$

(c) $\frac{u}{12}(-\hat{i} + \sqrt{3}\hat{j})$ (d) $\frac{u}{12}(-\hat{i} - \sqrt{3}\hat{j})$

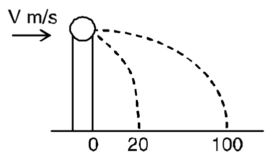
17. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figure are only illustrative and not to the scale. **[Adv. 2014]**



18. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **[Adv. 2013]**

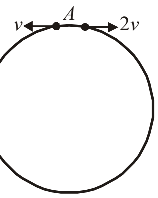
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$ (c) $\frac{\pi}{2}-\alpha$ (d) $\frac{\pi}{2}$

19. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is **[2011]**



(a) 250 m/s (b) $250\sqrt{2}$ m/s
(c) 400 m/s (d) 500 m/s

20. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[2009]**



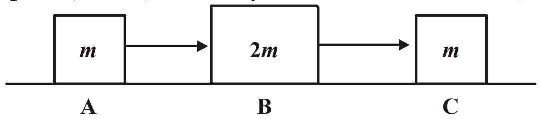
(a) 4 (b) 3 (c) 2 (d) 1

21. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_o. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_o$ while still moving in air. The value of $\left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$ is **[2001S]**

(a) zero (b) $(m_1 + m_2)gt_o$
(c) $\frac{1}{2}(m_1 + m_2)gt_o$ (d) $2(m_1 + m_2)gt_o$

2 Integer Value Answer

22. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. There after, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C. **[2009]**

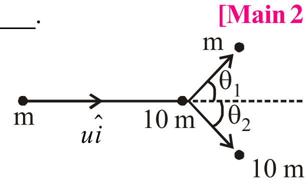


23. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is **[Adv. 2013]**

3 Numeric Answer

24. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is ______. **[Main 6 Sep. 2020 (I)]**

25. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is ___________. **[Main 2 Sep. 2020 (II)]**

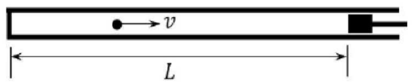


26. A body A, of mass $m = 0.1$ kg has an initial velocity of $3\,\hat{i}$ ms^{-1}. It collides elastically with another body, B of the same mass which has an initial velocity of $5\,\hat{j}$ ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is _______. **[Main 8 Jan. 2020 I]**

27. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is ___________. **[Adv. 2018]**

6 MCQs with One or More than One Correct Answer

28. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[Adv. 2019]**



(a) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(c) The rate at which the particle strikes the piston is v/L

(d) After each collision with the piston, the particle speed increases by 2 V.

29. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules.
Which of the following options is/are true? **[Adv. 2017]**

(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(b) The resistive force experienced by the plate is proportional to v

(c) The plate will continue to move with constant non-zero acceleration, at all times

(d) At a later time the external force F balances the resistive force

30. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses? **[2010]**

(a) Total momentum of the system is 3 kg ms^{-1}

(b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(c) Kinetic energy of the centre of mass is 0.75 J

(d) Total kinetic energy of the system is 4J

31. The balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2. **[2008]**

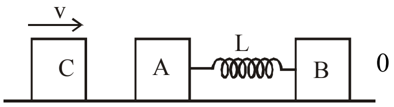
(a) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$

(b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$

(c) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(d) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

32. Two blocks A and B, each of mass m, are connected by a massless spring of natural length L and spring constant K. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in fig.. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then **[1993-2 Marks]**



(a) the kinetic energy of the A-B system, at maximum compression of the spring, is zero.

(b) the kinetic energy of the A-B system, at maximum compression of the spring, is $mv^2/4$.

(c) the maximum compression of the spring is $v\sqrt{(m/K)}$

(d) the maximum compression of the spring is $v\sqrt{(m/2K)}$

33. A shell is fired from a cannon with a velocity v (m/sec.) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec.) of the other piece immediately after the explosion is **[1986 - 2 Marks]**

(a) $3v\cos\theta$ (b) $2v\cos\theta$

(c) $\frac{3}{2}v\cos\theta$ (d) $\sqrt{\frac{3}{2}}v\cos\theta$

34. A ball hits the floor and rebounds after an inelastic collision. In this case **[1986 - 2 Marks]**

(a) the momentum of the ball just after the collision is the same as that just before the collision.

(b) the mechanical energy of the ball remains the same in the collision

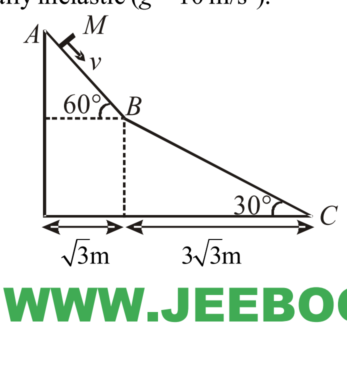
(c) the total momentum of the ball and the earth is conserved

(d) the total energy of the ball and the earth is conserved

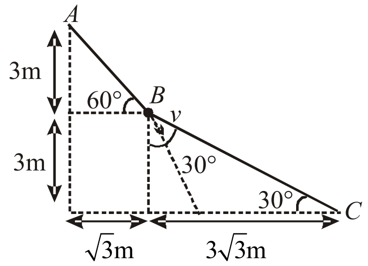
8 Comprehension/Passage Based Questions

Passage

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10$ m/s^2). **[2008]**



35. The speed of the block at point B immediately after it strikes the second incline is –



(a) $\sqrt{60}$ m/s (b) $\sqrt{45}$ m/s

(c) $\sqrt{30}$ m/s (d) $\sqrt{15}$ m/s

36. The speed of the block at point C, immediately before it leaves the second incline is

(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s

(c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

37. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is –

(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s

(c) 0 (d) $-\sqrt{15}$ m/s

9 Assertion and Reason Type Questions

38. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - I: A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

[Main 2013]

(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.

(b) Statement-I is true, Statment - II is true, Statement - II is not the correct explanation of Statement - II.

(c) Statement - I is true, Statment - II is false.

(d) Statement - I is false, Statment - II is true.

39. **STATEMENT–1 :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. **[2007]**

STATEMENT–2 : In an elastic collision, the linear momentum of the system is conserved.

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.

(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False.

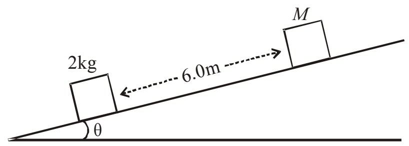
(d) Statement–1 is False, Statement–2 is True.

10 Subjective Problems

40. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown in Figure. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.

[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10 m/s^2$.]

[1999 - 10 Marks]

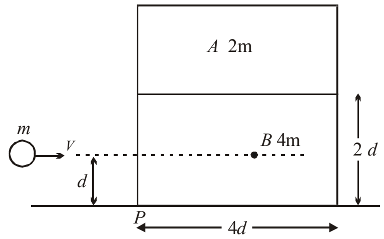


41. A cart is moving along $+x$ direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine : **[1997 - 5 Marks]**

(i) The speed of the combined mass immediately after the collision with respect to an observer on the ground,

(ii) The length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

42. A block 'A' of mass 2m is placed on another block 'B' of mass 4m which in turn is placed on a fixed table. The two blocks have a same length 4d and they are placed as shown in fig. The coefficient of friction (both static and kinetic) between the block 'B' and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (cm.) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table. **[1991 - 4 + 4 Marks]**

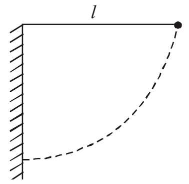


(a) What is the minimum value of v (call it v_0) required to make the block A topple ?

(b) If $v = 2v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision).

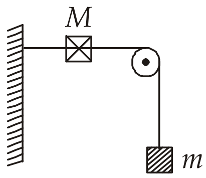
43. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$.

[1987 - 7 Marks]



What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ?

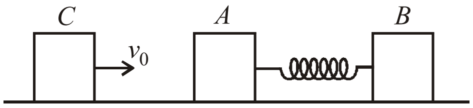
44. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released ?



[1985 - 6 Marks]

45. A ball of mass 100 gm is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **[(1985 - 8 Marks]**

46. Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig.



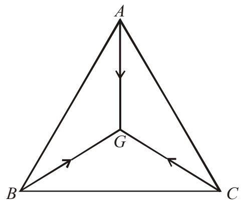
At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of A and B at time t_0; and (ii) the spring constant. **[(1984- 6 Marks]**

47. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3 metres and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find

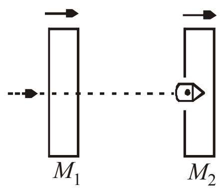
(i) the angle θ ;

(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. Take $g = 10$ m /s^2

48. Three particles A, B and C of equal mass move with equal speed V along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed V. What is the velocity of C ? **[1982 - 2 Marks]**



49. A 20 gm bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 = 2.98$ kg. as shown. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of the bullet. **[1979]**



50. A body of mass m moving with velocity V in the X-direction collides with another body of mass M moving in Y-direction with velocity v. They coalesce into one body during collision. Calculate : **[1978]**

(i) the direction and magnitude of the momentum of the final body.

(ii) the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses.

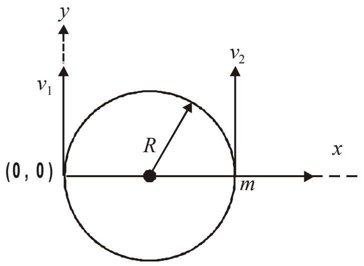
Topic-5 : Miscellaneous (Mixed Concepts) Problems

3 Numeric Answer

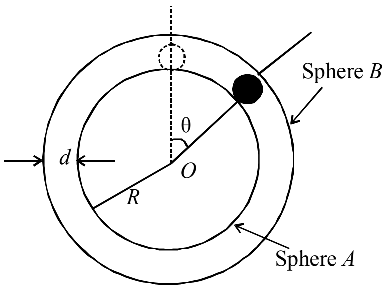
1. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ kg is at rest on this surface. An impulse of 1.0 Ns is applied to the block at time $t = 0$ so that it starts moving along the x -axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4s$. The displacement of the block, in $metres$, at $t = \tau$ is __________. Take $e^{-1} = 0.37$. **[Adv. 2017]**

10 Subjective Problems

2. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time $t = 0$ and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[2003 - 2 Marks]**



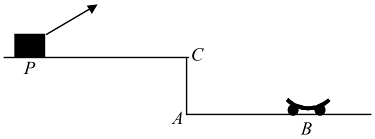
3. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). **[2002 - 5 Marks]**



(a) Express the total normal reaction force exerted by the sphere on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. **Sketch** the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two **separate** graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

4. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30^o with the horizontal. The first cannon ball hits the stationary carriage after a time t_o and sticks to it. Determine t_o. **[2001 - 10 Marks]**



At t_o, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

Answer Key

Topic-1 : Work Done by Constant & Variable Force

1. (d)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (d)
11. (8)	**12.** (0.75)								

Topic-2 : Energy

1. (c)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (b)
21. (d)	**22.** (b)	**23.** (c)	**24.** (5)	**25.** (4)	**26.** (150.00)	**27.** (10.00)	**28.** (a,d)	**29.** (a,b,c)	**30.** (a,b)
31. (d)	**32.** (c, d)	**33.** A $\to$ p, r, t; B $\to$ q, s; C $\to$ p, q, r, s; D $\to$ p, r, t					**34.** (a)	**35.** (b)	

Topic-3 : Power

1. (b)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)
11. (5)	**12.** (18)								

Topic-4 : Collisions

1. (d)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (c)
11. (b)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (c)
21. (d)	**22.** (4)	**23.** (5)	**24.** (120)	**25.** (10.00)	**26.** (a) (1.00)		**27.** (30.00)	**28.** (a, d)	**29.** (a, b, d)
30. (a, c)	**31.** (a, d)	**32.** (b, d)	**33.** (a)	**34.** (c, d)	**35.** (b)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (d)

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (6.30)

6 System of Particles and Rotational Motion

Topic-1 : Centre of Mass, Centre of Gravity & Principle of Moments

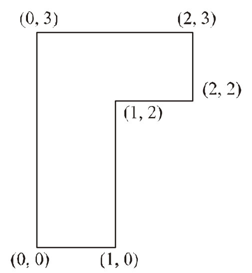
1 MCQs with One Correct Answer

1. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at: **[Main 9 Jan. 2020 II]**

(a) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (b) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

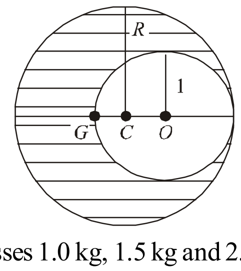
(c) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (d) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

2. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plale) of mass 4 kg. (The coordinates of the same are shown in figure) are: **[Main 8 Jan. 2020 I]**



(a) (1.25 m, 1.50 m) (b) (0.75 m, 1.75 m)
(c) (0.75 m, 0.75 m) (d) (1 m, 1.75 m)

3. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation: **[Main 8 Jan. 2020 II]**



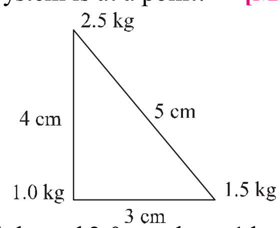
(a) $(R^2 + R + 1)(2 - R) = 1$

(b) $(R^2 - R - 1)(2 - R) = 1$

(c) $(R^2 - R + 1)(2 - R) = 1$

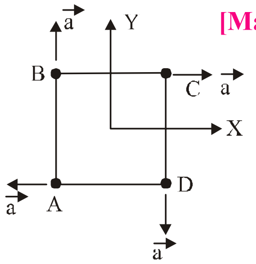
(d) $(R^2 + R - 1)(2 - R) = 1$

4. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: **[Main 7 Jan. 2020 I]**



(a) 0.6 cm right and 2.0 cm above 1 kg mass
(b) 1.5 cm right and 1.2 cm above 1 kg mass
(c) 2.0 cm right and 0.9 cm above 1 kg mass
(d) 0.9 cm right and 2.0 cm above 1 kg mass

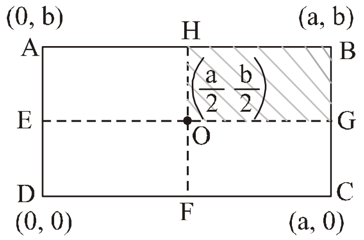
5. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : **[Main 8 April 2019 I]**



(a) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$ (b) a

(c) Zero (d) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$

6. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be : **[Main 8 Apr. 2019 II]**



(a) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

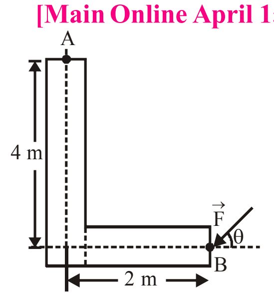
7. A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by: **[Main Online April 15, 2018]**

(a) $\tan\theta = \frac{1}{4}$

(b) $\tan\theta = 2$

(c) $\tan\theta = \frac{1}{2}$

(d) $\tan\theta = 4$



8. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct? **[Main Online April 8, 2017]**

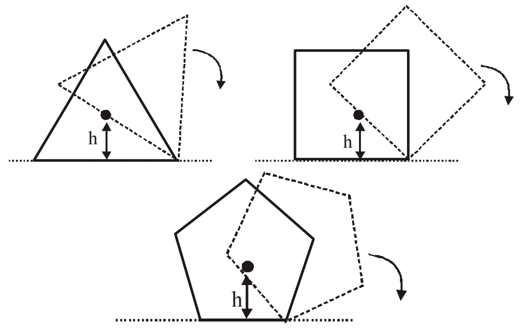
(a) Left arm is longer than the right arm

(b) Both the arms are of same length

(c) Left arm is shorter than the right arm

(d) Every object that is weighed using this balance appears lighter than its actual weight.

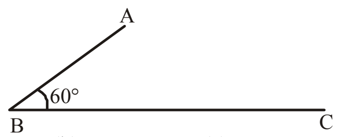
9. Consider regular polygons with number of sides n = 3, 4, 5.... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as **[Adv. 2017]**



(a) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (b) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

(c) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (d) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

10. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to : **[Main Online April 10, 2016]**



(a) 1.85 (b) 1.5 (c) 1.37 (d) 3

11. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[Main 2015]**

(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$ (c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$

12. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as : **[Main Online April 11, 2015]**

(a) $a = 2b$ (b) $2a = b$

(c) $a = b$ (d) $3a = 2b$

13. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_0, then the distance of the centre of mass from the lighter end is: **[Main Online April 11, 2014]**

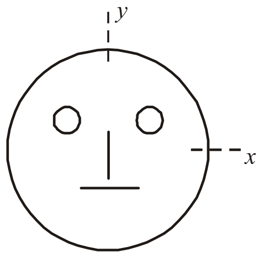
(a) $\frac{L}{2} - \frac{\lambda_0 L^2}{4M}$ (b) $\frac{L}{3} + \frac{\lambda_0 L^2}{8M}$

(c) $\frac{L}{3} + \frac{\lambda_0 L^2}{4M}$ (d) $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

14. A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be **[Main Online April 23, 2013]**

(a) 15 m (b) 12.5 m (c) 15.5 m (d) 17 m

15. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6 m.
The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle $(-a, a)$, right inner circle (a, a), vertical line (0, 0) and horizontal line $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is **[2009]**



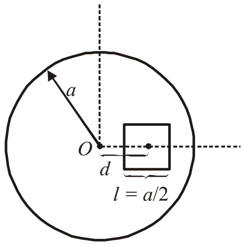
(a) $\frac{a}{10}$
(b) $\frac{a}{8}$
(c) $\frac{a}{12}$
(d) $\frac{a}{3}$

16. Two particles A and B initially at rest, move towards each other under mutual force of attraction. At the instant when the speed of A is V and the speed of B is $2V$, the speed of the centre of mass of the system is **(1982 - 3 Marks)**
(a) $3V$ (b) V (c) $1.5V$ (d) zero

3 Numeric Answer

17. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is ______. **[Main Sep. 06, 2020 (II)]**

18. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is ___________. **[Main Sep. 02, 2020 (II)]**



4 Fill in the Blanks

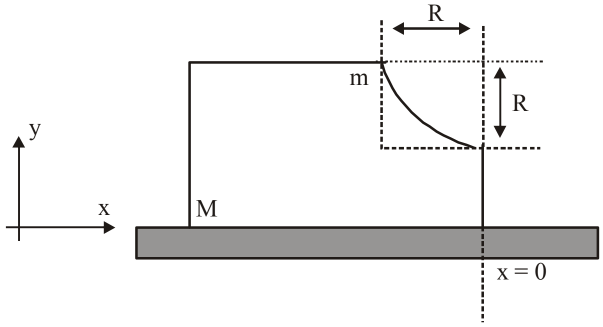
19. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is... and on B is..... **[1997 - 2 Marks]**

5 True / False

20. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

21. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down.
When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct? **[Adv. 2017]**



(a) The position of the point mass m is: $x = -\sqrt{2}\frac{mR}{M+m}$
(b) The velocity of the point mass m is: $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$
(c) The x component of displacement of the center of mass of the block M is: $-\frac{mR}{M+m}$
(d) The velocity of the block M is: $V = -\frac{m}{M}\sqrt{2gR}$

22. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts **[1990 - 2 Marks]**
(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.
(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.
(c) in the backward direction on both the front and the rear wheels.
(d) in the forward direction on both the front the rear wheels.

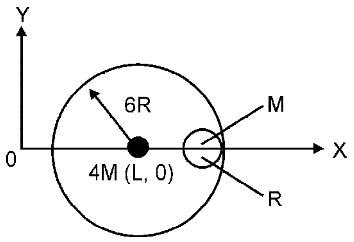
9 Assertion and Reason Type Questions

23. **STATEMENT-1:** If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.
STATEMENT-2: The linear momentum of an isolated system remains constant. **[2007]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

10 Subjective Problems

24. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$ (Fig. P-3). The masses of large and small spheres are $4M$ and M, respectively, This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position.

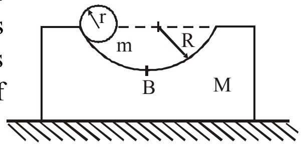


[1996 - 3 Marks]

25. A uniform thin rod of mass M and length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling. **[1993-1+5 Marks]**

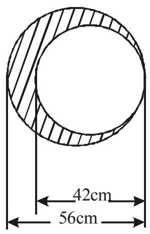
(i) What is the path followed by the centre of mass of the rod during its fall?
(ii) Find the equation to the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point?

26. A block of mass M with a semicircular of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ?



[1983 - 7 Marks]

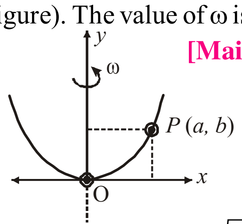
27. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure.
Find the position of the centre of mass of the remaining portion. **[1980]**



Topic-2 : Angular Displacement, Velocity and Aceleration

1 MCQs with One Correct Answer

1. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) : **[Main Sep. 02, 2020 (I)]**



(a) $2\sqrt{2gC}$ (b) $2\sqrt{gC}$ (c) $\sqrt{\frac{2gC}{ab}}$ (d) $\sqrt{\frac{2g}{C}}$

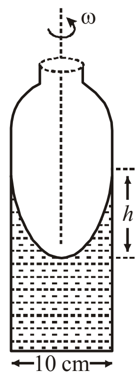
2. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the side will be : **[Main Sep. 02, 2020 (I)]**

(a) $\frac{2\omega^2}{25g}$

(b) $\frac{5\omega^2}{2g}$

(c) $\frac{25\omega^2}{2g}$

(d) $\frac{2\omega^2}{5g}$



3. A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option: **[Main 9 Jan. 2020 II]**

(a) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (b) $\frac{2m\omega^2}{k}$

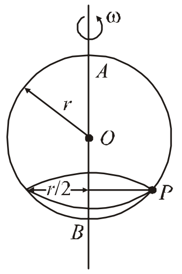
(c) $\frac{m\omega^2}{k}$ (d) $\frac{m\omega^2}{3k}$

4. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is: **[Main 8 Jan. 2020 I]**

(a) $\frac{ml\omega^2}{k-\omega m}$ (b) $\frac{ml\omega^2}{k-m\omega^2}$

(c) $\frac{ml\omega^2}{k+m\omega^2}$ (d) $\frac{ml\omega^2}{k+m\omega}$

5. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to : **[Main 12 Apr. 2019 II]**



(a) $\frac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$

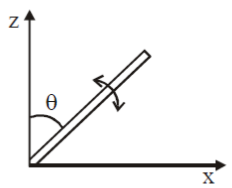
(c) $(g\sqrt{3})/r$ (d) $2g/r$

6. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then: **[Main 2018]**

(a) $T \propto R^{3/2}$ for any n. (b) $T \propto R^{n/2+1}$

(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$

7. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[Main 2017]**



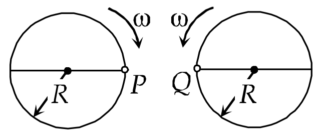
(a) $\frac{3g}{2\ell}\cos\theta$ (b) $\frac{2g}{3\ell}\cos\theta$

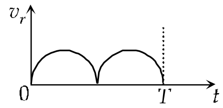
(c) $\frac{3g}{2\ell}\sin\theta$ (d) $\frac{2g}{2\ell}\sin\theta$

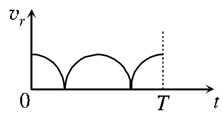
8. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to :
(Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[Main Online April 10, 2016]**

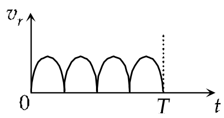
(a) 27.0 (b) 0.4 (c) 1.3 (d) 8.0

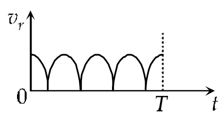
9. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[2012]**



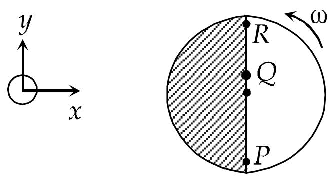
(a) 

(b) 

(c) 

(d) 

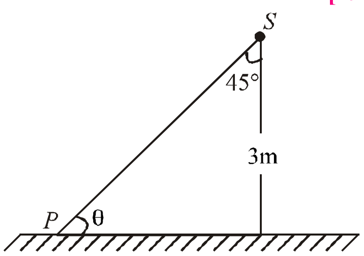
10. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **[2012]**



(a) P lands in the shaded region and Q in the unshaded region.
(b) P lands in the unshaded region and Q in the shaded region.
(c) Both P and Q land in the unshaded region.
(d) Both P and Q land in the shaded region.

4 Fill in the Blanks

11. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45°$ (see fig.) is .... m/s **[1987 - 2 Marks]**



8 Comprehension Based Questions

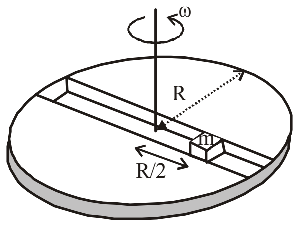
Passage

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{v}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega}.$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis



perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r}\left(R/2\right)\hat{i}$ at t = 0 and is constrained to move only along the slot.

12. The distance r of the block at time t is **[Adv. 2016]**

(a) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$ (b) $\frac{R}{2}\cos\omega t$

(c) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (d) $\frac{R}{2}\cos 2\omega t$

13. The net reaction of the disc on the block is **[Adv. 2016]**

(a) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

(b) $\frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

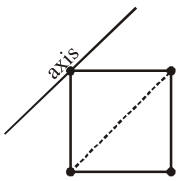
(c) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$

(d) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$

Topic-3 : Torque, Couple and Angular Momentum

1 MCQs with One Correct Answer

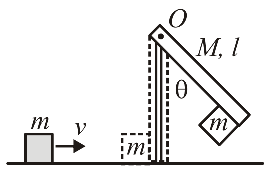
1. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is : **[Main Sep. 06, 2020 (I)]**



(a) $ml^2\omega$ (b) $4\,ml^2\omega$

(c) $3\,ml^2\omega$ (d) $2\,ml^2\omega$

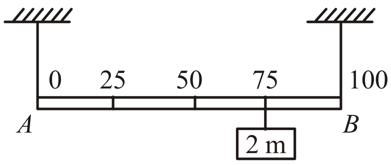
2. A block of mass $m = 1$ kg slides with velocity $v = 6$ m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2$ kg, and length $l = 1$ m, the value of θ is approximately: (take $g = 10$ m/s^2) **[Main Sep. 03, 2020 (I)]**



(a) 63° (b) 55°

(c) 69° (d) 49°

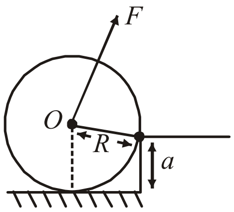
3.



Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is : **[Main Sep. 02, 2020 (I)]**

(a) 0.5 mg (b) 2 mg (c) 0.75 mg (d) 1 mg

4. A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is : **[Main Sep. 02, 2020 (I)]**



(a) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (b) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(c) $Mg\frac{a}{R}$ (d) $Mg\sqrt{1-\frac{a^2}{R^2}}$

5. Consider a uniform rod of mass $M = 4$m and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is: **[Main 8 Jan. 2020 I]**

(a) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (b) $\frac{3}{7}\frac{v}{l}$ (c) $\frac{3\sqrt{2}}{7}\frac{v}{l}$ (d) $\frac{4}{7}\frac{v}{l}$

6. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\cos\omega_2 t$

The torque, acting on the particle about the origin, at $t = 0$ is: **[Main 10 Apr. 2019 I]**

(a) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$ (b) $+my_0 a\omega_1^2\hat{k}$

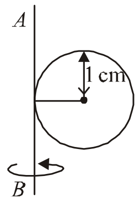
(c) zero (d) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

7. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is : **[Main 10 Apr. 2019 II]**

(a) $48(\hat{i}+\hat{j})$ (b) $36\hat{k}$

(c) $-34(\hat{k}-\hat{i})$ (d) $-48\hat{k}$

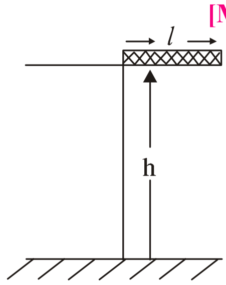
8. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to : **[Main 10 Apr. 2019 II]**



(a) 4.0×10^{-6} Nm (b) 1.6×10^{-5} Nm

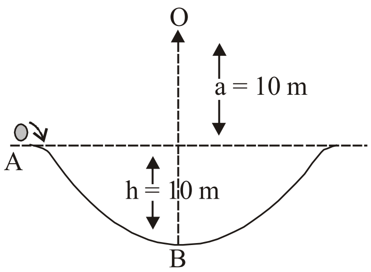
(c) 7.9×10^{-6} Nm (d) 2.0×10^{-5} Nm

9. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau =$ 0.01 s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to : **[Main 8 Apr. 2019 II]**



(a) 0.5 (b) 0.3 (c) 0.02 (d) 0.28

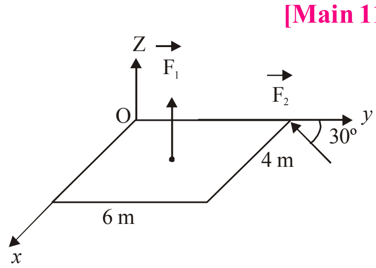
10. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g = 10 m/s^2) **[Main 12 Jan. 2019 II]**



(a) 2 kg-m^2/s (b) 8 kg-m^2/s

(c) 6 kg-m^2/s (d) 3 kg-m^2/s

11. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be : **[Main 11 Jan. 2019 I]**



(a) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$ (b) $\left(3\hat{i}-2\hat{j}-3\hat{k}\right)F$

(c) $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)F$ (d) $\left(3\hat{i}+2\hat{j}+3\hat{k}\right)F$

12. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians): **[Main 11 Jan. 2019 II]**

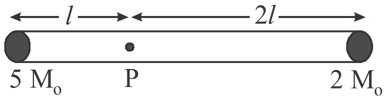
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

13. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is: **[Main 10 Jan. 2019 I]**

(a) μ FR/3 (b) μ FR/6

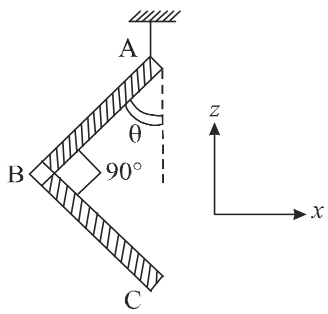
(c) μ FR/2 (d) $\frac{2}{3}\mu$ μFR

14. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be: **[Main 10 Jan. 2019 II]**



(a) $\frac{g}{13l}$ (b) $\frac{g}{3l}$ (c) $\frac{g}{2l}$ (d) $\frac{7g}{3l}$

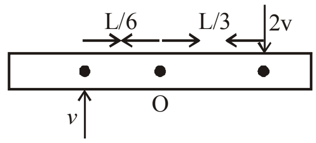
15. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: **[Main 9 Jan. 2019 I]**



(a) $\tan\theta = \frac{1}{2\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2}$

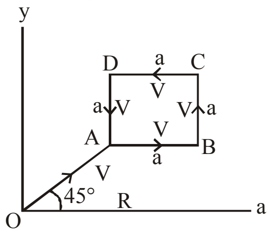
(c) $\tan\theta = \frac{2}{\sqrt{3}}$ (d) $\tan\theta = \frac{1}{3}$

16. A thin uniform bar of length L and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane from opposite sides of the bar with speeds 2v and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be: **[Main Online April 15, 2018]**



(a) $\frac{v}{6L}$ (b) $\frac{6v}{5L}$ (c) $\frac{3v}{5L}$ (d) $\frac{v}{5L}$

17. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure : **[Main 2016]**



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(a) $\vec{L} = mv\left[\frac{R}{\sqrt{2}}+a\right]\hat{k}$ when the particle is moving from B to C.

(b) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(c) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

(d) $\vec{L} = mv\left[\frac{R}{\sqrt{2}}-a\right]\hat{k}$ when the particle is moving from C to D.

18. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(< l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are $(g = 10\ m\,s^{-2})$ **[Adv. 2016]**

(a) $\frac{h}{l}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$ (b) $\frac{h}{l}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}N$

(c) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}N$ (d) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$

19. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is : **[Main Online April 11, 2015]**

(a) $14.4 \text{ kg m}^2\text{s}^{-1}$ (b) $8.64 \text{ kg m}^2\text{s}^{-1}$
(c) $20.16 \text{ kg m}^2\text{s}^{-1}$ (d) $11.52 \text{ kg m}^2\text{s}^{-1}$

20. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: **[Main 2014]**

(a) angular momentum is conserved.
(b) angular momentum changes in magnitude but not in direction.
(c) angular momentum changes in direction but not in magnitude.
(d) angular momentum changes both in direction and magnitude.

21. A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly ($g = 10 \text{ ms}^{-2}$) **[Main Online April 19, 2014]**

(a) $1.73 \text{ kg m}^2/\text{s}$ (b) $3.0 \text{ kg m}^2/\text{s}$
(c) $3.46 \text{ kg m}^2/\text{s}$ (d) $6.0 \text{ kg m}^2/\text{s}$

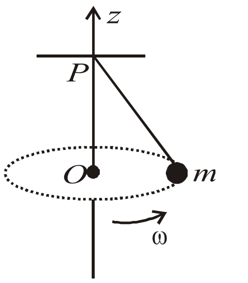
22. A particle of mass 2 kg is moving such that at time t, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2s$ about the origin in $\text{kg m}^{-2}\text{ s}^{-1}$ is : **[Main Online April 23, 2013]**

(a) $-80\hat{k}$ (b) $(10\hat{i} - 16\hat{j})$
(c) $-40\hat{k}$ (d) $40\hat{k}$

23. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. The angular speed of the door just after the bullet embeds into it will be : **[Main Online April 9, 2013]**

(a) 6.25 rad/sec (b) 0.625 rad/sec
(c) 3.35 rad/sec (d) 0.335 rad/sec

24. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then

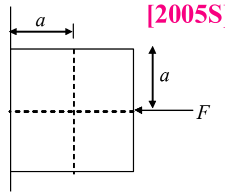


(a) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time
(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant
(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time
(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

25. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct? **[2005S]**

(a) $\vec{L}$ (angular momentum) is conserved about the centre
(b) only direction of angular momentum $\vec{L}$ is conserved
(c) It spirals towards the centre
(d) its acceleration is towards the centre.

26. A block of mass m is at rest under the action of force F against a wall as shown in figure. Which of the following statement is incorrect? **[2005S]**



(a) $f = mg$ [f friction force]
(b) $F = N$ [N normal force]
(c) F will not produce torque
(d) N will not produce torque

27. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_o. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be **[2004S]**

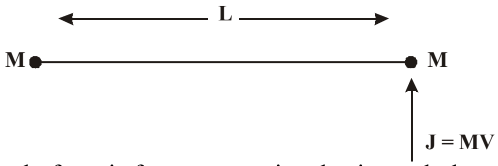
(a) $2K$ (b) $K/2$ (c) K (d) $K/4$

28. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved? **[2003S]**

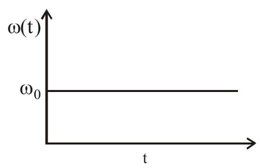
(a) centre of the circle
(b) on the circumference of the circle.
(c) inside the circle
(d) outside the circle.

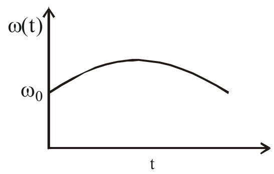
29. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J=MV$ is imparted to the body at one of its ends, what would be its angular velocity? **[2003S]**

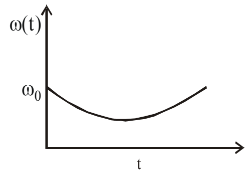
(a) V/L
(b) $2V/L$
(c) $V/3L$
(d) $V/4L$

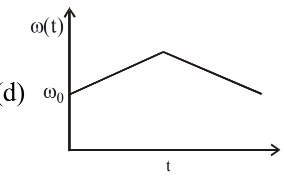


30. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as **[2002S]**

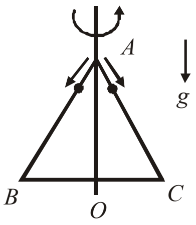
(a)


(b)


(c)


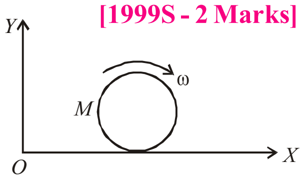
(d)


31. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are **[2000S]**



(a) angular velocity and total energy (kinetic and potential)
(b) Total angular momentum and total energy
(c) angular velocity and moment of inertia about the axis of rotation
(d) total angular momentum and moment of inertia about the axis of rotation

32. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Figure. The magnitude of angular momentum of the disc about the origin O is **[1999S - 2 Marks]**



(a) $(1/2)MR^2\omega$
(b) $MR^2\omega$
(c) $(3/2)MR^2\omega$
(d) $2MR^2\omega$

33. A mass M moving with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin **[1985 - 2 Marks]**

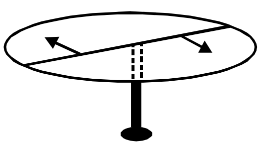
(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

34. A thin circular ring of mass 'M and radius r is rotating about its axis with a constant angular velocity ω, Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity **[1983 - 1 Mark]**

(a) $\frac{\omega M}{(M+m)}$ (b) $\frac{\omega(M-2m)}{(M+2m)}$
(c) $\frac{\omega M}{(M+2m)}$ (d) $\frac{\omega(M+2m)}{M}$

2 Integer Value Answer

35. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0 25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **[Adv. 2014]**



36. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[Adv. 2013]**

37. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is **[2010]**

3 Numeric Answer

38. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ______________. **[Main Sep. 05, 2020 (II)]**

39. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre __________.

[Main Sep. 03, 2020 (I)]

40. Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is ______. **[Adv. 2020]**

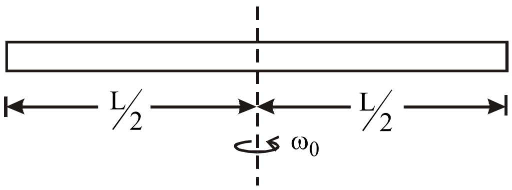
4 Fill in the Blanks

41. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$ where A is a constant, r is the instantaneous radius of the circle and n = ...

[1993 - 1 Mark]

42. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x–y plane) with its axis fixed along the y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A \cos(\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is .... **[1988 - 2 Marks]**

43. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpenducular to the rod and passing through the midpoint of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is .... **[1988 - 2 Marks]**

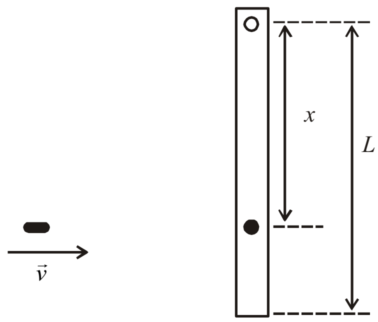


5 True / False

44. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. **[1986 - 3 Marks]**

6 MCQs with One or More than One Correct Answer

45. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for $x = x_M$. Then **[Adv. 2020]**



(a) $\omega = \dfrac{3vx}{L^2 + 3x^2}$ (b) $\omega = \dfrac{12vx}{L^2 + 12x^2}$

(c) $x_M = \dfrac{L}{\sqrt{3}}$ (d) $\omega_M = \dfrac{v}{2L}\sqrt{3}$

46. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ \text{Ns}^{-1}$ and $\beta = 1.0\ N$. The torque acting on the body about the origin at time $t = 1.0\ s$ is $\vec{\tau}$. Which of the following statements is (are) true? **[Adv. 2018]**

(a) $|\vec{\tau}| = \dfrac{1}{3} Nm$

(b) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(c) The velocity of the body at $t = 1\ s$ is $\vec{v} = \dfrac{1}{2}(\hat{i} + 2\hat{j}) ms^{-1}$

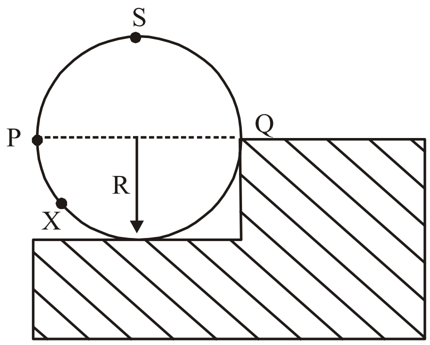
(d) The magnitude of displacement of the body at $t = 1 s$ is $\dfrac{1}{6} m$

47. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true? **[Adv. 2018]**

(a) $v = \sqrt{\dfrac{k}{2m}}R$ (b) $v = \sqrt{\dfrac{k}{m}}R$

(c) $L = \sqrt{mk}R^2$ (d) $L = \sqrt{\dfrac{mk}{2}}R^2$

48. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **[Adv. 2017]**



(a) If the force is applied at point P tangentially then decreases continuously as the wheel climbs

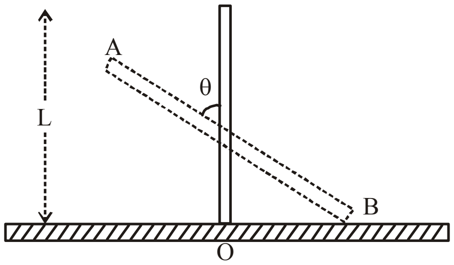
(b) If the force is applied normal to the circumference at point X then τ is constant

(c) If the force is applied normal to the circumference at point P then τ is zero

(d) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

49. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure).

At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **[Adv. 2017]**



(a) The midpoint of the bar will fall vertically downward
(b) The trajectory of the point A is a parabola
(c) Instantaneous torque about the point in contact with the floor is proportional to sinθ
(d) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cosθ)

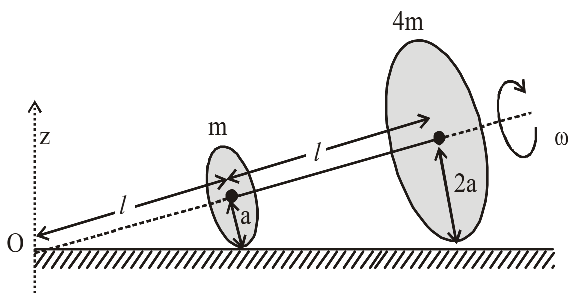
50. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j},$$

where $\alpha = 10/3\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[Adv. 2016]**

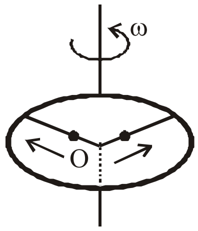
(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\ ms^{-1}$
(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -5/3)\ \hat{k}$ N m s
(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N
(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\ \hat{k}$ N m

51. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is (are) true? **[Adv. 2016]**



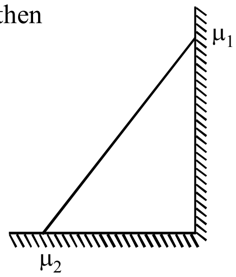
(a) The centre of mass of the assembly rotates about the z-axis with an angular speed of ω/5
(b) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$
(c) The magnitude of angular momentum of the assembly about its center of mass is 17 $ma^2\omega/2$.
(d) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$.

52. A ring of mass *M* and radius *R* is rotating with angular speed ω about a fixed vertical axis passing through its centre *O* with two point masses each of mass $\frac{M}{8}$ at rest at *O*. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from *O*. At this instant the distance of the other mass from *O* is **[Adv. 2015]**



(a) $\frac{2}{3}R$ (b) $\frac{1}{3}R$ (c) $\frac{3}{5}R$ (d) $\frac{4}{5}R$

53. In the figure, a ladder of mass *m* is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then **[Adv. 2014]**



(a) $\mu_1 = 0, \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$
(b) $\mu_1 \neq 0, \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$
(c) $\mu_1 \neq 0, \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$
(d) $\mu_1 = 0, \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

54. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[2009]**

(a) linear momentum of the system does not change in time
(b) kinetic energy of the system does not change in time
(c) angular momentum of the system does not change in time
(d) potential energy of the system does not change in time

55. The torque τ on a body about a given point is found to be equal to $\boldsymbol{A} \times \boldsymbol{L}$ where $\boldsymbol{A}$ is a constant vector, and $\boldsymbol{L}$ is the angular momentum of the body about that point. From this it follows that **[1998S - 2 Marks]**

(a) $\frac{d\boldsymbol{L}}{dt}$ is perpendicular to $\boldsymbol{L}$ at all instants of time.
(b) the component of $\boldsymbol{L}$ in the direction of $\boldsymbol{A}$ does not change with time.
(c) the magnitude of $\boldsymbol{L}$ does not change with time.
(d) $\boldsymbol{L}$ does not change with time

56. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[1990 - 2 Marks]**

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$
(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$

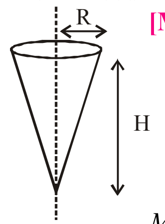
10 Subjective Problems

57. A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = v_0/g$ **[1984- 6 Marks]**

Topic-4 : Moment of Inertia and Rotational K.E.

1 MCQs with One Correct Answer

1. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is : **[Main Sep. 06, 2020 (I)]**



(a) $\frac{MR^2}{2}$ (b) $\frac{M(R^2+H^2)}{4}$
(c) $\frac{MH^2}{3}$ (d) $\frac{MR^2}{3}$

2. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is : **[Main Sep. 06, 2020 (II)]**

(a) $\frac{5}{12}ML^2$ (b) $\frac{7}{18}ML^2$ (c) $\frac{2}{5}ML^2$ (d) $\frac{3}{7}ML^2$

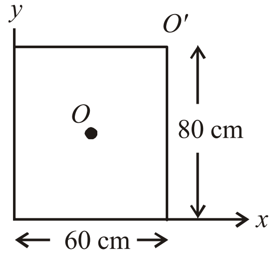
3. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is : **[Main Sep. 05, 2020 (I)]**

(a) $\frac{5}{6}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{3}{4}$

4. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is : **[Main Sep. 04, 2020 (II)]**

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) 4

5.



For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is : **[Main Sep. 04, 2020 (II)]**

(a) 2/3 (b) 1/4 (c) 1/8 (d) 1/2

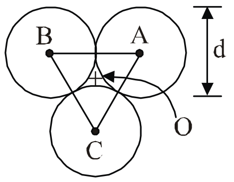
6. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is : **[Main Sep. 03, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{2}{3}}$

7. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is : **[Main Sep. 02, 2020 (II)]**

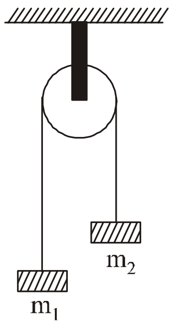
(a) $\frac{10}{3}$ J (b) $\frac{20}{3}$ J (c) $\frac{5}{3}$ J (d) $\frac{2}{3}$ J

8. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is: **[Main 9 Jan. 2020 I]**



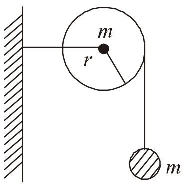
(a) $\frac{13}{23}$
(b) $\frac{15}{13}$
(c) $\frac{23}{13}$
(d) $\frac{13}{15}$

9. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is: **[Main 9 Jan. 2020 II]**



(a) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$
(b) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$
(c) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+1}\right]^{1/2} gh$
(d) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+1}\right]^{1/2} gh$

10. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be: **[Main 7 Jan. 2020 I]**



(a) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (b) $r\sqrt{\frac{3}{2gh}}$
(c) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (d) $r\sqrt{\frac{3}{4gh}}$

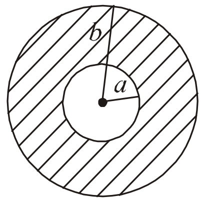
11. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is: **[Main 7 Jan. 2020 I]**

(a) $\frac{1}{4}l$ (b) $\frac{1}{8}l$ (c) $\sqrt{\frac{7}{48}}l$ (d) $\sqrt{\frac{3}{8}}l$

12. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is: **[Main 7 Jan. 2020 II]**

(a) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (b) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$
(c) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (d) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

13. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is : **[Main 12 Apr. 2019 I]**



(a) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (b) $\frac{a+b}{2}$
(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (d) $\frac{a+b}{3}$

14. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is : **[Main 10 Apr. 2019 I]**

(a) $-\frac{I_1\omega_1^2}{12}$ (b) $\frac{I_1\omega_1^2}{6}$ (c) $\frac{3}{8}I_1\omega_1^2$ (d) $-\frac{I_1\omega_1^2}{24}$

15. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : **[Main 10 Apr. 2019 I]**

(a) $\frac{MR^2}{3}$ (b) $\frac{2MR^2}{3}$
(c) $\frac{MR^2}{6}$ (d) $\frac{MR^2}{2}$

16. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second

part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : **[Main 10 Apr. 2019 II]**

(a) 185 (b) 140 (c) 285 (d) 65

17. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is: **[Main 9 April 2019 I]**

(a) $\frac{k}{4I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{2I}\theta$ (d) $\frac{2k}{I}\theta$

18. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be: **[Main 9 Apr. 2019 II]**

(a) $\frac{M\omega_0}{M+m}$ (b) $\frac{M\omega_0}{M+3m}$

(c) $\frac{M\omega_0}{M+6m}$ (d) $\frac{M\omega_0}{M+2m}$

19. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a\,MR^2$. The value of the coefficient a is: **[Main 8 April 2019 I]**

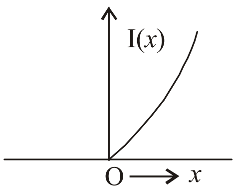
(a) $1/2$ (b) $3/5$ (c) $8/5$ (d) $3/2$

20. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be 1. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is: **[Main 12 Jan. 2019 I]**

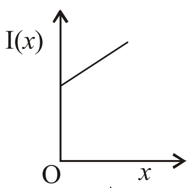
(a) 12 cm (b) 16 cm (c) 14 cm (d) 18 cm

21. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly? **[Main 12 Jan. 2019 II]**

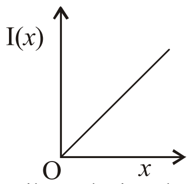
(a)



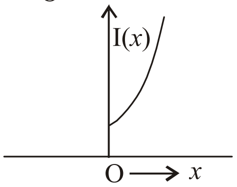
(b)



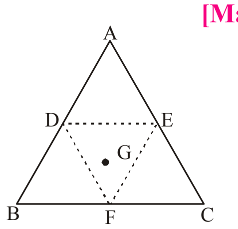
(c)



(d)



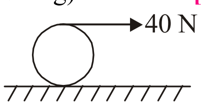
22. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then : **[Main 11 Jan. 2019 I]**



(a) $I = \frac{15}{16}I_0$ (b) $I = \frac{3}{4}I_0$

(c) $I = \frac{9}{16}I_0$ (d) $I = \frac{I_0}{4}$

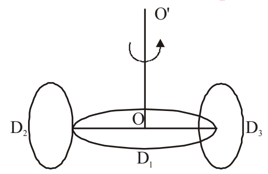
23. a string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) **[Main 11 Jan. 2019 II]**



(a) 20 rad/s^2 (b) 16 rad/s^2

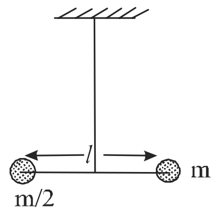
(c) 12 rad/s^2 (d) 10 rad/s^2

24. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be : **[Main 11 Jan. 2019 II]**



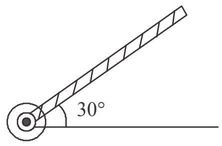
(a) MR^2 (b) $3MR^2$ (c) $\frac{4}{5}MR^2$ (d) $\frac{2}{3}MR^2$

25. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[Main 9 Jan. 2019 I]**



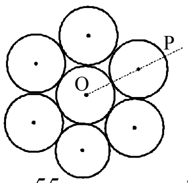
(a) $\frac{3k\theta_0^2}{l}$ (b) $\frac{2k\theta_0^2}{l}$ (c) $\frac{k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

26. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads^{-1}) will be (g = 10 ms^{-2}) **[Main 9 Jan. 2019 II]**



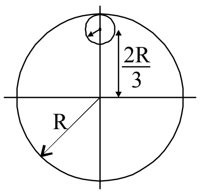
(a) $\sqrt{\frac{30}{7}}$ (b) $\sqrt{30}$ (c) $\sqrt{\frac{20}{3}}$ (d) $\sqrt{\frac{30}{2}}$

27. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is: **[Main 2018]**



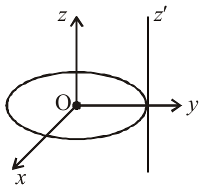
(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$ (c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

28. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is : **[Main 2018]**



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10MR^2$ (d) $\frac{37}{9}MR^2$

29. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z′ axes will be **[Main Online April 16, 2018]**

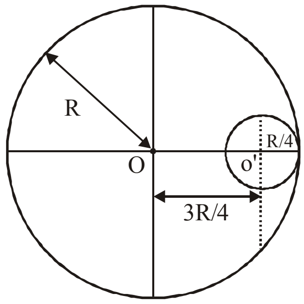


(a) 1 : 2 (b) 1 : 4 (c) 1 : 3 (d) 1 : 5

30. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ? **[Main 2017]**

(a) 1 (b) $\frac{3}{\sqrt{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

31. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is : **[Main Online April 9, 2017]**



(a) $\frac{219\,MR^2}{256}$

(b) $\frac{237\,MR^2}{512}$

(c) $\frac{19\,MR^2}{512}$

(d) $\frac{197\,MR^2}{256}$

32. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is : **[Main 2015]**

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$

(c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

33. Consider a thin uniform square sheet made of a rigid material. If its side is 'a' mass m and moment of inertia I about one of its diagonals, then : **[Main Online April 10, 2015]**

(a) $I > \frac{ma^2}{12}$ (b) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

(c) $I = \frac{ma^2}{24}$ (d) $I = \frac{ma^2}{12}$

34. A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be: **[Main Online April 25, 2013]**

(a) $\frac{m(M+2m)}{M}\omega^2R^2$ (b) $\frac{Mm}{(M+m)}\omega^2R^2$

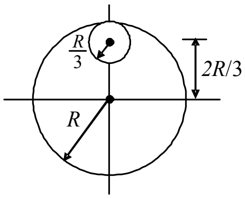
(c) $\frac{Mm}{(M+2m)}\omega^2R^2$ (d) $\frac{(M+m)M}{(M+2m)}\omega^2R^2$

35. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as **[2006 - 3M, –1]**

(a) $r = \sqrt{\frac{2}{15}}R$ (b) $r = \frac{2}{\sqrt{15}}R$

(c) $r = \frac{2}{15}R$ (d) $r = \frac{\sqrt{2}}{15}R$

36. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The m oment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is **[2005S]**



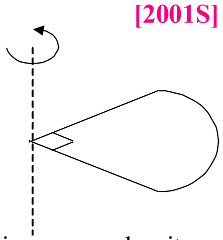
(a) $4MR^2$

(b) $\frac{40}{9}MR^2$

(c) $10\,MR^2$

(d) $\frac{37}{9}MR^2$

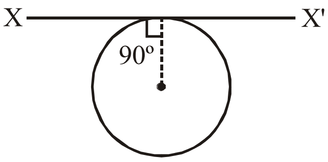
37. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is **[2001S]**



(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

(c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}\ MR^2$

38. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **[2000S]**



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

39. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to **[1998S - 2 Marks]**

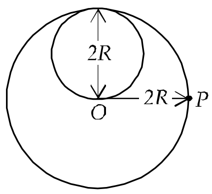
(a) I (b) $I\sin^2\theta$

(c) $I\cos^2\theta$ (d) $I\cos^2(\theta/2)$

2 Integer Value Answer

40. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is **[Adv. 2015]**

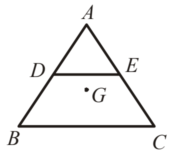
41. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is **[2012]**



42. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg– m², then N is. **[2011]**

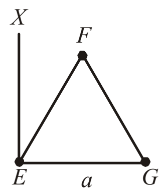
3 Numeric Answer

43. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _______. **[Main Sep. 04, 2020 (I)]**

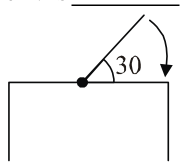


44. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is ___________. **[Main Sep. 04, 2020 (I)]**

45. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is _________. **[Main Sep. 03, 2020 (II)]**

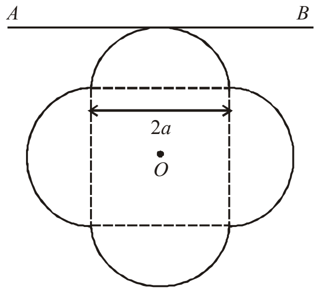


46. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}s^{-1}$, where n is an integer. The value of n is _________ . **[Main 9 Jan. 2020 I]**



4 Fill in the Blanks

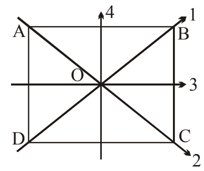
47. A symmetric lamina of mass M consists of a square shape with a semicircular section over of the edge of the square as shown in Fig. P-10. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6\ Ma^2$.



The moment of inertia of the lamina about the tangent AB in the plane of the lamina is.... **[1997 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

48. The moment of inertia of a thin square plate $ABCD$, Fig., of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is **[1992 - 2 Marks]**



(a) $I_1 + I_2$
(b) $I_3 + I_4$
(c) $I_1 + I_3$
(d) $I_1 + I_2 + I_3 + I_4$

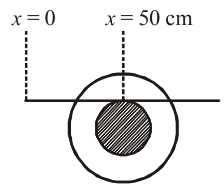
where I_1, I_2, I_3 and I_4 are respectively the moments of intertial about axis 1, 2, 3 and 4 which are in the plane of the plate.

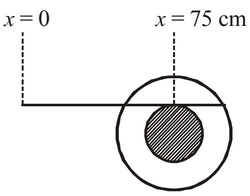
Topic-5 : Rolling Motion

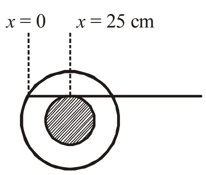
1 MCQs with One Correct Answer

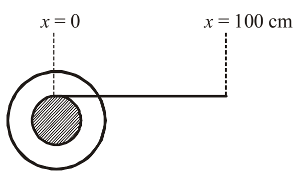
1. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is: **[Main 8 Jan. 2020 II]**
(a) 8.75×10^{-4} J
(b) 8.75×10^{-3} J
(c) 6.25×10^{-4} J
(d) 1.13×10^{-3} J

2. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale) **[Adv. 2020]**

(a)


(b)


(c)


(d)


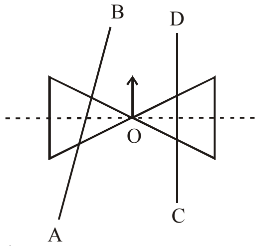
3. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by : **[Main 8 Apr. 2019 II]**

(a) $\frac{2}{\sqrt{5}}$ (b) 1 (c) $\frac{14}{15}$ (d) $\frac{4}{5}$

4. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane: (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is: **[Main 9 April 2019 I]**
(a) 4 : 3 : 2 (b) 10 : 15 : 7
(c) 14 : 15 : 20 (d) 2 : 3 : 4

5. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is: **[Main 10 Jan. 2019 I]**
(a) $\frac{3F}{2mR}$ (b) $\frac{F}{3mR}$ (c) $\frac{F}{2mR}$ (d) $\frac{2F}{3mR}$

6. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to: **[Main 2016]**

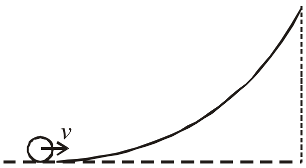


(a) go straight.
(b) turn left and right alternately.
(c) turn left.
(d) turn right.

7. A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ? **[Masin 2013]**

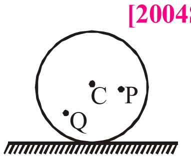
(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$ (c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

8. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[2007]**



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

9. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is **[2004S]**

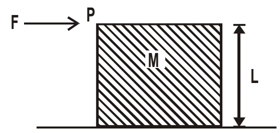


(a) $v_Q > v_C > v_P$
(b) $v_P > v_C > v_Q$
(c) $v_P = v_C, v_Q = v_C/2$
(d) $v_P < v_C > v_Q$

10. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are **[2002S]**

(a) up the incline while ascending and down the incline descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending.

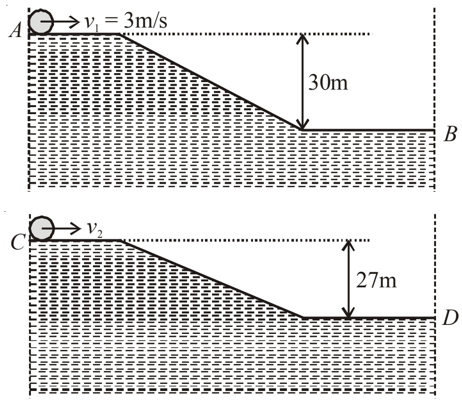
11. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is **[2000S]**



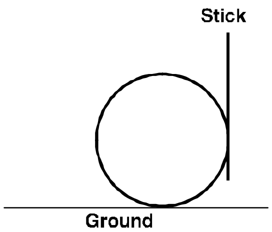
(a) infinitesimal
(b) mg/4
(c) mg/2
(d) mg(1 – μ)

2 Integer Value Answer

12. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s then v_2 in m/s is ($g = 10\,m/s^2$) **[Adv. 2015]**



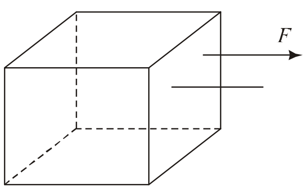
13. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2.



The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[2011]**

3 Numeric Answer

14.



Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is ________. **[Main 7 Jan. 2020 II]**

15. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take $g = 10\ \text{ms}^{-2}$.

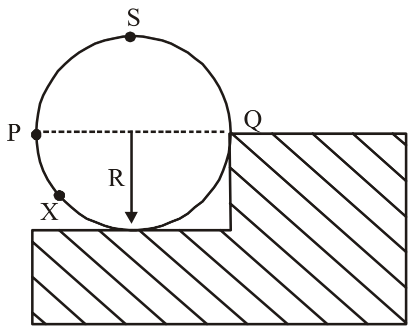
[Adv. 2018]

5 True / False

16. A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both cases is negligible. The cylinder will reach the wall first. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

17. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **[Adv. 2017]**



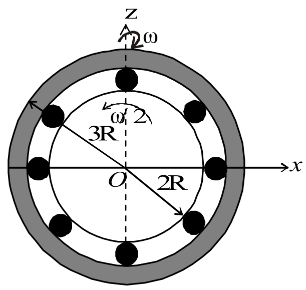
(a) If the force is applied at point P tangentially then decreases continuously as the wheel climbs

(b) If the force is applied normal to the circumference at point X then τ is constant

(c) If the force is applied normal to the circumference at point P then τ is zero

(d) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

18. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **[2012]**



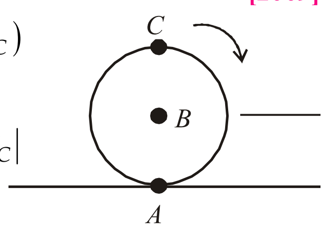
(a) the point O has linear velocity $3R\omega\,\hat{i}$

(b) the point P has linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$.

(c) the point P has linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(d) the point P has linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

19. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[2009]**



(a) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(b) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(c) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(d) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

20. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then **[2006 - 5M, –1]**

(a) The friction force is dissipative

(b) The friction force is necessarily changing

(c) The friction force will aid rotation but hinder translation

(d) The friction force is reduced if θ is reduced

9 Assertion and Reason Type Questions

21. **STATEMENT-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

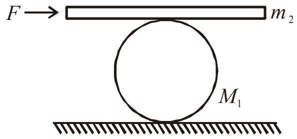
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement -1 is True, Statement-2 is False

(d) Statement -1 is False, Statement-2 is True

10 Subjective Problems

22. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in Figure. There in no slipping at any contact. The horizontal component of the force applied by the man is F. **[1999 - 10 Marks]**



Find

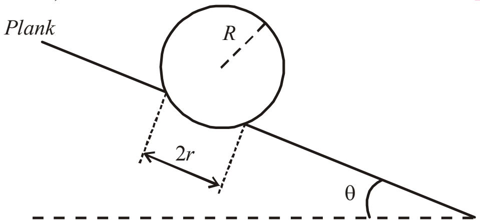
(a) the accelerations of the plank and the center of mass of the cylinder, and

(b) the magnitudes and directions of frictional forces at contact points.

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

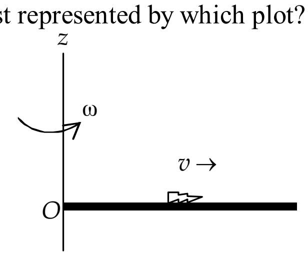
1. A football of radius R is kept on a hole of radius r $(r < R)$ made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale) **[Adv. 2020]**

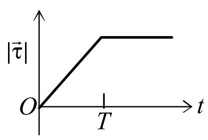
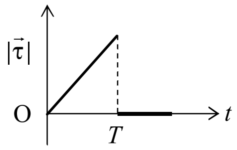


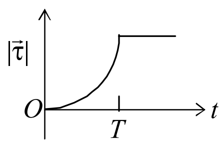
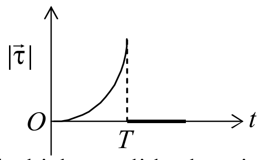
(a) $\sin\theta = \frac{r}{R}$ (b) $\tan\theta = \frac{r}{R}$

(c) $\sin\theta = \frac{r}{2R}$ (d) $\cos\theta = \frac{r}{2R}$

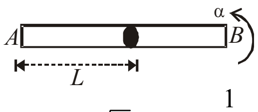
2. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ about O, as a function of time is best represented by which plot? **[2012]**



(a) 
(b) 

(c) 
(d) 

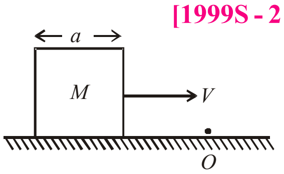
3. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is **[2000S]**



(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

4. A cubical block of side a is moving with velocity V on a horizontal smooth plane as shown in Figure. It hits a ridge at point O. The angular speed of the block after it hits O is **[1999S - 2 Marks]**

(a) $3V/(4a)$
(b) $3V/(2a)$
(c) $\sqrt{3V}/(\sqrt{2a})$
(d) zero



5. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity υ. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then **[1999S - 2 Marks]**

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

6. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of **[1995S]**

(a) 0.42 *m* from mass of 0.3 kg
(b) 0.70 *m* from mass of 0.7 kg
(c) 0.98 *m* from mass of 0.3 kg
(d) 0.98 *m* from mass of 0.7 kg

7. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A pendulum bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with track is **[1992 - 2 Mark]**

(a) zero (b) 30°
(c) 45° (d) 60°

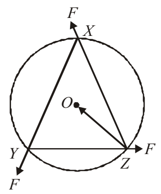
8. A tube of length L is filled completely with an incomressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is **[1992 - 2 Marks]**

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$

(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

2 Integer Value Answer

9. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure).



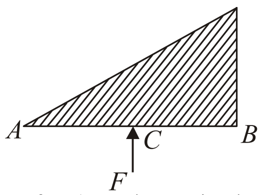
One second after applying the forces, the angular speed of the disc in rad s^{-1} is **[Adv. 2014]**

4 Fill in the Blanks

10. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is .... (Assume that the cube does not slide). **[1984 - 2 Marks]**

5 True / False

11. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (midpoint of AB) as shown in the figure. The angular acceleration in both the cases will be the same. **[1985 - 3 Marks]**



6 MCQs with One or More than One Correct Answer

12. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? **[Adv. 2019]**

[g is the acceleration due to gravity]

(a) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(b) The radical acceleration of the rod's center of mass will be $\frac{3g}{4}$

(c) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(d) The angular acceleration of the rod will be $\frac{2g}{L}$

13. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **[2012]**

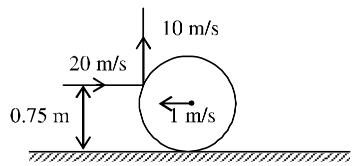
(a) Both cylinders P and Q reach the ground at the same time.

(b) Cylinders P has larger linear acceleration than cylinder Q.

(c) Both cylinders reach the ground with same translational kinetic energy.

(d) Cylinder Q reaches the ground with larger angular speed.

14. A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[2011]**



(a) the ring has pure rotation about its stationary CM.

(b) the ring comes to a complete stop.

(c) friction between the ring and the ground is to the left.

(d) there is no friction between the ring and the ground.

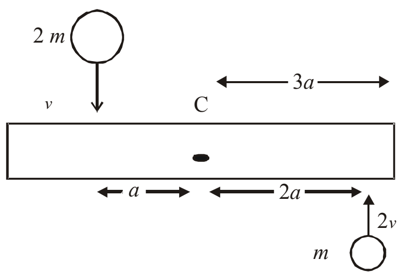
15. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v, respectively, strike the bar [as shown in the fig.] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision **[1991 - 2 Mark]**

(a) $v_c = 0$

(b) $\omega = \frac{3v}{5a}$

(c) $\omega = \frac{v}{5a}$

(d) $E = \frac{3mv^2}{5}$



7 Match the Following

16. In the Column-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$ In each case, the force acting on the particle is either zero or conservative. In Column-II, five physical quantities of the particle are mentioned $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path. **[Adv. 2018]**

	Column–I		Column–II
P.	$\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$	**1.**	$\vec{p}$
Q.	$\vec{r}(t) = \alpha \cos \omega t \ \hat{i} + \beta \sin \omega t \ \hat{j}$	**2.**	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos \omega t \ \hat{i} + \sin \omega t \ \hat{j})$	**3.**	K
S.	$\vec{r}(t) = \alpha t \ \hat{i} + \frac{\beta}{2} t^2 \ \hat{j}$	**4.**	U
		5.	E

(a) $P \to 1, 2, 3, 4, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 5$
(b) $P \to 1, 2, 3, 4, 5; Q \to 3, 5; R \to 2, 3, 4, 5; S \to 2, 5$
(c) $P \to 2, 3, 4; Q \to 5; R \to 1, 2, 4; S \to 2, 5$
(d) $P \to 1, 2, 3, 5; Q \to 2, 5; R \to 2, 3, 4, 5; S \to 2, 5$

17. **Column-II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column-I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**. **[2009]**

Column-I

(A) The force exerted by X on Y has a magnitude *Mg*.

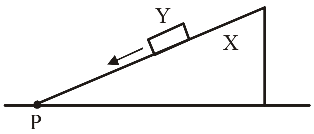
(B) The gravitational potential energy of X is continuously increasing.

(C) Mechanical energy of the system X + Y is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

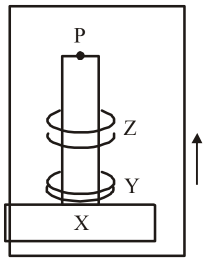
Column II

(p)



Block Y of mass *M* left on a fixed inclined plane X, slides on it with a constant velocity.

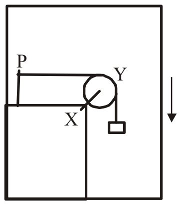
(q)



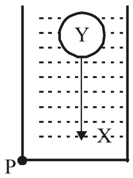
Two ring magnets Y and Z, each of mass *M*, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(r)

A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.



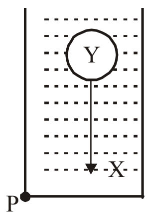
(s)



A sphere Y of mass *M* is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t)

A sphere Y of mass *M* is falling with its terminal velocity in a viscous liquid X kept in a container.



8 Comprehension Based Questions

Passage 1

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g. **[Adv. 2017]**

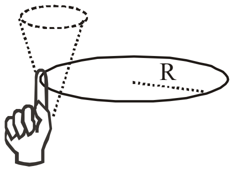


Figure 1

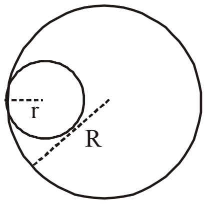


Figure 2

18. The total kinetic energy of the ring is

(a) $M\omega_0^2R^2$ (b) $\frac{1}{2}M\omega_0^2(R-r)^2$

(c) $M\omega_0^2(R-r)^2$ (d) $\frac{3}{2}M\omega_0^2(R-r)^2$

19. The minimum value of ω_0 below which the ring will drop down is

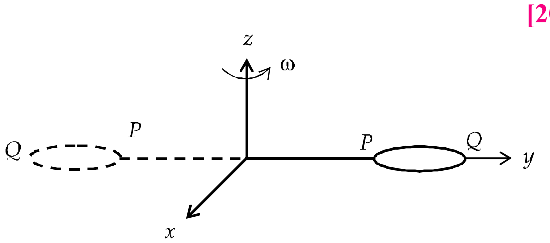
(a) $\sqrt{\frac{g}{\mu(R-r)}}$ (b) $\sqrt{\frac{2g}{\mu(R-r)}}$

(c) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\frac{g}{2\mu(R-r)}}$

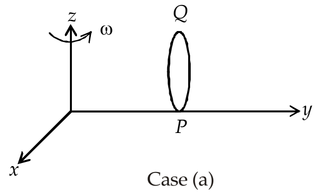
Passage 2

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass.

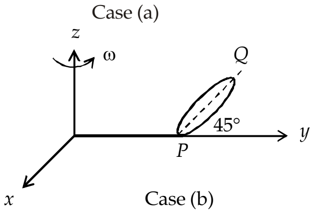
These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless, stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case **[2012]**



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

20. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(c) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(d) It is vertical for case (a); and is 45° to the x-z plane and is normal to the plane of the disc for case (b).

21. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\omega$ for both the cases

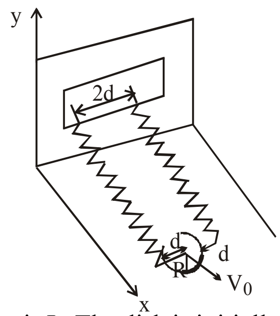
(b) It is ω for case (a); and $\omega/\sqrt{2}$ for case (b)

(c) It is ω for case (a); and $\sqrt{2\omega}$ for case (b)

(d) It is ω for both the cases.

Passage 3

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in horizontal plane.



The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. **[2008]**

22. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$ (c) $-2kx/3$ (d) $-4kx/3$

23. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to –

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$ (c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

24. The maximum value of V_0 for which the disk will roll without slipping is –

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$ (c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

Passage 4

Two discs *A* and *B* are mounted coaxially on a vertical axle. The discs have moments of inertia *I* and 2 *I* respectively about the common axis. Disc *A* is imparted an initial angular velocity 2 ω using the entire potential energy of a spring compressed by a distance x_1. Disc *B* is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

25. The loss of kinetic energy in the above process is **[2007]**

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$ (c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

26. When disc *B* is brought in contact with disc *A*, they acquire a common angular velocity in time *t*. The average frictional torque on one disc by the other during this period is **[2007]**

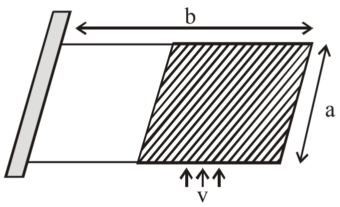
(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$ (c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

27. The ratio x_1/x_2 is **[2007]**

(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

10 Subjective Problems

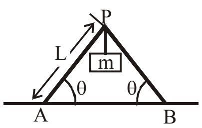
28. A rectangular plate of mass *M* and dimension $a \times b$ is held in horizontal position by striking n small balls (each of mass *m*) per unit area per second.



The balls are striking in the shaded half region of the plate. The collision of the balls with the plate is elastic. What is *v*? **[2006 - 6M]**

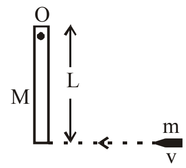
(Given $n = 100$, $M = 3$ kg, $m = 0.01$ kg; $b = 2$ *m*; $a = 1$ m; $g = 10\ m/s^2$).

29. Two identical ladders, each of mass *M* and length *L* are resting on the rough horizontal surface as shown in the figure. A block of mass *m* hangs from *P*.

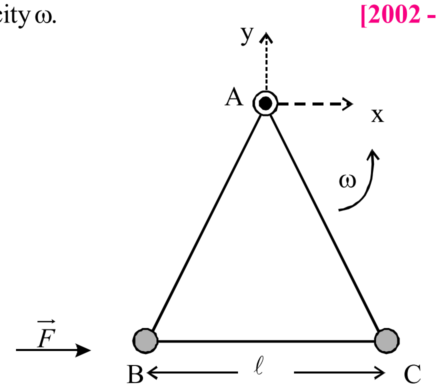


If the system is in equilibrium, find the magnitude and the direction of frictional force at *A* and *B*. **[2005 - 4 Marks]**

30. A wooden log of mass *M* and length *L* is hinged by a frictionless nail at *O*. A bullet of mass *m* strikes with velocity *v* and sticks to it. Find angular velocity of the system immediately after the collision about *O*. **[2005 - 2 Marks]**



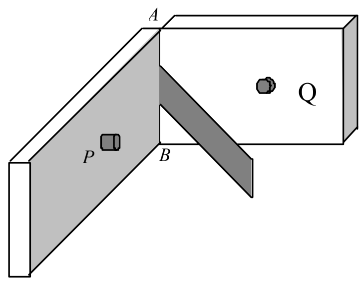
31. Three particles *A*, *B* and *C*, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictioness table (*x-y* plane) and is hinged to it at the point *A* so that it can move without friction about the vertical axis through *A* (see figure). The body is set into rotational motion on the table about *A* with a constant angular velocity ω. **[2002 - 5 Marks]**



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time *T*, when the side *BC* is parallel to the *x*-axis, a force *F* is applied on *B* along *BC* (as shown). Obtain the *x*-component and the *y*-component of the force exerted by the hinge on the body, immediately after time *T*.

32. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line *AB* joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to *AB* and passing through its center of mass is 1.2 kg-m^2. Two rubber obstacles *P* and *Q* are fixed, one on each metallic plate at a distance 0.5 m from the line *AB*. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. **[2001-10 Marks]**



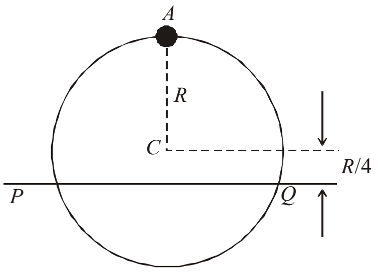
Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,

(a) Find the location of the center of mass of the laminar sheet from *AB*.

(b) At what angular velocity does the laminar sheet come back after the first impact?

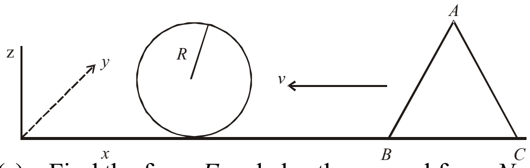
(c) After how many impacts, does the laminar sheet come to rest?

33. A uniform circular disc has radius R and mass m. A particle also of mass m, is fixed at a point A on the edge of the disc as shown in Figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ.



Initially, the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position. **[1998 - 8 Marks]**

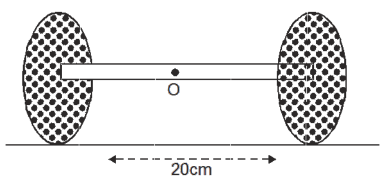
34. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $-v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in Figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time. Δt, during which the sphere exerts a constant force F on the wedge. **[1998 - 8 Marks]**



(a) Find the force F and also the normal force N exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of **F**. Find the magnitude of the torque due to the normal force **N** about the centre of the wedge, during the interval Δt.

35. Two thin circular disks of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm.



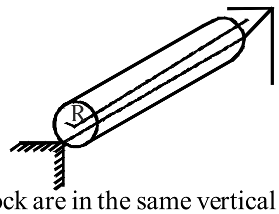
The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of the motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x axis as the direction of motion of the truck and z axis as the vertically upwards direction. If the truck has an acceleration of 9 m/s^2.

Calculate: **[1997 - 5 Marks]**

(i) The force of friction on each disk,

(ii) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in the x, y, and z directions.

36. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in the figure below. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine: **[1995 - 10 Marks]**

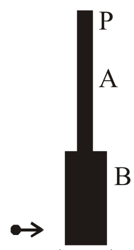


(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge,

(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and

(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

37. Two uniform thin rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in fig. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it.

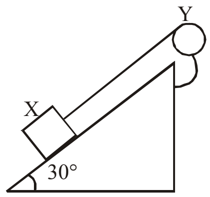


What should be the velocity of the object so that the system could just be raised to the horizontal position. **[1994 - 6 Marks]**

38. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in Figure. The drum is given an initial angular velocity such that the block X starts moving up the plane. **[1994 - 6 Marks]**

(i) Find the tension in the string during the motion.

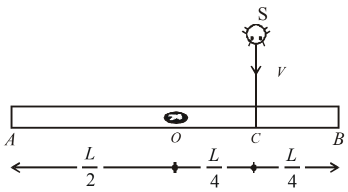
(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1} calculate the distance travelled by X from that instant of time until it comes to rest



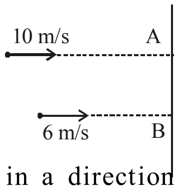
39. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (Fig). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. **[1992 - 8 Marks]**

(a) Determine the angular velocity ω in terms of V and L.

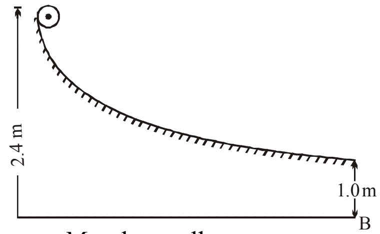
(b) If the insect reaches the end B when the rod has turned through an angle of 90°, determine V.



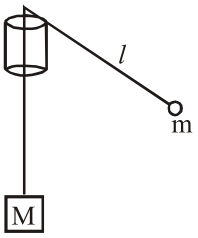
40. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length $\sqrt{3}$ meters. Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m/s, and other with 6 m/s as shown in fig. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of the kinetic energy of the system in the above collision process. **[1989 - 8 Marks]**



41. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part, The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in fig.) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass? Explain. **[1987 - 7 Marks]**



42. A large mass M and a small mass m hang at two ends of a string that passes over a smooth tube as shown in the figure. The mass m moves around a circular path which lies in a horizontal plane. The length of string from the mass m to the top of the tube is l and θ is the 'angle' this length makes with the vertical. What should be the frequency of rotation of mass m, so that the mass M remains stationary? **[1978]**



43. A 40 kg mass, hanging at the end of a rope of length l, oscillates in a vertical plane with an angular amplitude θ_0. What is the tension in the rope when it makes an engle θ with the vertical? If the breaking strength of the rope is 80 kg, what is the maximum amplitude with which the mass can oscillate without the rope breaking? **[1978]**

Answer Key

Topic-1 : Centre of Mass, Centre of Gravity & Principle of Moments

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (c)
11. (d)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** **(3)**	**18.** (23.00)	**19.** $\left(\frac{d-x}{d}\right)w, \frac{xw}{d}$	
20. False.	**21.** (b, c)	**22.** (a, c)	**23.** (d)						

Topic-2 : Angular Displacement, Velocity and Aceleration

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (c)
11. (0.6)	**12.** (a)	**13.** (b)							

Topic-3 : Torque, Couple and Angular Momentum

1. (c)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (c)
11. (a)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (c)
21. (c)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (d)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (b)
31. (b)	**32.** (c)	**33.** (b)	**34.** (c)	**35.** (4)	**36.** (8)	**37.** (6)	**38.** (20)	**39.** (9.00)	
40. (25.60 cm)		**41.** (–3)	**42.** $\frac{1}{3}$ MARω^2		**43.** $\frac{Mw_0}{M+6m}$		**44.** False	**45.** (a, c, d)	
46. (a, c)	**47.** (b, c)	**48.** (c)	**49.** (a, c, d)		**50.** (a, b)	**51.** (a, c)	**52.** (d)	**53.** (c,d)	**54.** (a)
55. (a,b,c)	**56.** (b, d)								

Topic-4 : Moment of Inertia and Rotational K.E.

1. (d)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (a)
11. (c)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (c)	**19.** (c)	**20.** (b)
21. (d)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (c)
31. (b)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (a)	**38.** (d)	**39.** (a)	**40.** (6)
41. (3)	**42.** (9)	**43.** (11)	**44.** (20)	**45.** (25)	**46.** (15)	**47.** (4.8 Ma^2)	**48.** (a, b, c)		

Topic-5 : Rolling Motion

1. (a)	**2.** (b)	**3.** (c)	**4.** (Bonus)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (c)
10. (d)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (7)	**15.** (4)	**16.** (50)	**17.** (0.75)	**18.** False.
19. (c)	**20.** (a, b)	**21.** (b, c)	**22.** (c, d)	**23.** (d)				

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** **(2)**	**10.** $\frac{2}{3}$mg
11. False	**12.** (a, b, c)	**13.** (d)	**14.** (c)	**15.** (a,c,d)	**16.** (a)				
17. A; B $\rightarrow$ (q, s, t) ; C $\rightarrow$ (p, r, t) D $\rightarrow$ (q, p)				**18.** (c)	**19.** (a)	**20.** (a)	**21.** (d)	**22.** (d)	**23.** (d)
24. (c)	**25.** (b)	**26.** (a)	**27.** (c)						

7 Gravitation

Topic-1 : Kepler's Laws of Planetary Motion

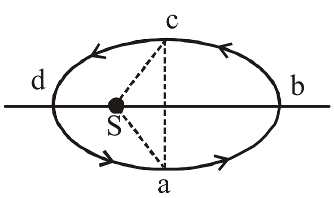
1 MCQs with One Correct Answer

1. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: **[Main 9 Jan. 2019 I]**

(a) $\frac{L}{m}$ (b) $\frac{4L}{m}$

(c) $\frac{L}{2m}$ (d) $\frac{2L}{m}$

2. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then: **[Main Online April 9, 2016]**



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$

(c) $t_1 = 3t_2$ (d) $t_1 = t_2$

3. A binary star system consists of two stars A and B which have time period T_A and T_B, radius R_A and R_B and mass M_A and M_B. Then **[2006 - 3M, –1]**

(a) if $T_A > T_B$ then $R_A > R_B$

(b) if $T_A > T_B$ then $M_A > M_B$

(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

(d) $T_A = T_B$

4. If the distance between the earth and the sun were half its present value, the number of days in a year would have been **[1996 - 2 Marks]**

(a) 64.5 (b) 129

(c) 182.5 (d) 730

6 MCQs with One or More than One Correct Answer

5. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$ **[1989 - 2 Mark]**

(a) T^2 is proportional to R^3

(b) T^2 is proportional to $R^{7/2}$

(c) T^2 is proportional to $R^{3/2}$

(d) T^2 is proportional to $R^{3/73}$

10 Subjective Problems

6. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1, find

(i) the speed of S_2 relative to S_1,

(ii) the angular speed of S_2 as actually observed by an astronaut in S_1. **[1986 - 6 Marks]**

Topic-2 : Newton's Universal Law of Gravitation

1 MCQs with One Correct Answer

1. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on point mass 'm' at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by: **[Main 12 Jan. 2019 I]**

(a) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(b) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

(c) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

2. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is: **[Main Online April 15, 2018]**

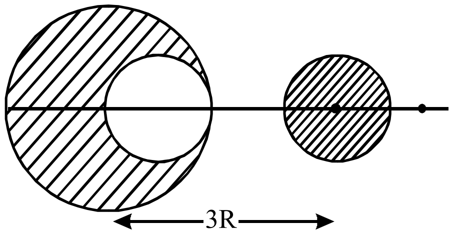
(a) 2 (b) 6 (c) 10^{-2} (d) 0.6

3. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: **[Main 2014]**

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

4. From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two sphere is: **[Main Online April 11, 2014]**



(a) $\frac{41\,GM^2}{3600\,R^2}$ (b) $\frac{41\,GM^2}{450\,R^2}$ (c) $\frac{59\,GM^2}{450\,R^2}$ (d) $\frac{GM^2}{225\,R^2}$

Topic-3 : Acceleration due to Gravity

1 MCQs with One Correct Answer

1. The value of acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 and a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals : **[Main 5 Sep. 2020 (I)]**

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{3}$ (d) $\frac{7}{9}$

2. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : (h<<R, where R is the radius of the earth) **[Main 5 Sep. 2020 (II)]**

(a) $\frac{R^2\omega^2}{2g}$ (b) $\frac{R^2\omega^2}{g}$

(c) $\frac{R^2\omega^2}{4g}$ (d) $\frac{R^2\omega^2}{8g}$

3. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) : **[Main 2 Sep. 2020 (II)]**

(a) $\frac{\sqrt{5}}{2}R - R$ (b) $\frac{R}{2}$

(c) $\frac{\sqrt{5}R - R}{2}$ (d) $\frac{\sqrt{3}R - R}{2}$

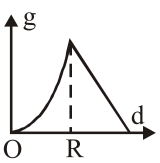
4. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ ms^{-2}$ at the north pole and the radius of the earth = 6400 km): **[Main 7 Jan. 2020 II]**
(a) 195.66 N (b) 194.32 N
(c) 194.66 N (d) 195.32 N

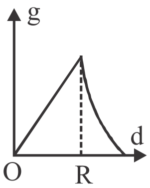
5. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9:4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density). **[Main 12 April 2019 II]**
(a) $\frac{R}{3}$ (b) $\frac{R}{4}$ (c) $\frac{R}{9}$ (d) $\frac{R}{2}$

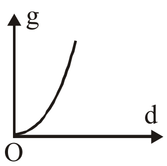
6. The value of acceleration due to gravity at Earth's surface is $9.8\ ms^{-2}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9\ ms^{-2}$, is close to : (Radius of earth = 6.4×10^6 m) **[Main 10 April 2019 I]**
(a) 2.6×10^6 m (b) 6.4×10^6 m
(c) 9.0×10^6 m (d) 1.6×10^6 m

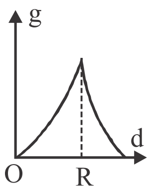
7. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence. **[Main Online April 16, 2018]**
(a) There will be no change in weight anywhere on the earth
(b) Weight of the object, everywhere on the earth, wild decrease
(c) Weight of the object, everywhere on the earth, will increase
(d) Except at poles, weight of the object on the earth will decrease

8. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): **[Main 2017]**

(a) 

(b) 

(c) 

(d) 

9. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight $\frac{3}{4}$W . Radius of the Earth is 6400 km and g = 10 m/s^2. **[Main Online April 8, 2017]**
(a) 1.1×10^{-3} rad/s (b) 0.83×10^{-3} rad/s
(c) 0.63×10^{-3} rad/s (d) 0.28×10^{-3} rad/s

10. A planet of radius $R = \frac{1}{10} \times (\text{radius of Earth})$ has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and a linear mass density $10^{-3}\ kg\ m^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is $10\ ms^{-2}$) **[Adv. 2014]**
(a) 96 N (b) 108 N (c) 120 N (d) 150 N

11. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is:
(mass of the moon = 7.36×10^{22} kg, radius of the moon's orbit = 3.8×10^8 m). **[Main Online April 22, 2013]**
(a) $6.73 \times 10^{-5}\ m/s^2$ (b) $6.73 \times 10^{-3}\ m/s^2$
(c) $6.73 \times 10^{-2}\ m/s^2$ (d) $6.73 \times 10^{-4}\ m/s^2$

12. If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would **[1981 - 2 Marks]**
(a) decrease (b) remain unchanged
(c) increase (d) be zero

4 Fill in the Blanks

13. The numerical value of the angular velocity of rotation of the earth should be ....rad/s in order to make the effective acceleration due to gravity equal to zero. **[1984 - 2 Marks]**

Topic-4 : Gravitational Field, Potential and Potential Energy

1 MCQs with One Correct Answer

1. Two planets have masses M and $16\ M$ and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach the surface of smaller planet, the minimum firing speed needed is : **[Main 6 Sep. 2020 (II)]**
(a) $2\sqrt{\frac{GM}{a}}$ (b) $4\sqrt{\frac{GM}{a}}$
(c) $\sqrt{\frac{GM^2}{ma}}$ (d) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

2. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is : **[Main 4 Sep. 2020 (I)]**

(a) $\frac{A}{(x^2+a^2)^{1/2}}$ (b) $\frac{A}{(x^2+a^2)^{3/2}}$

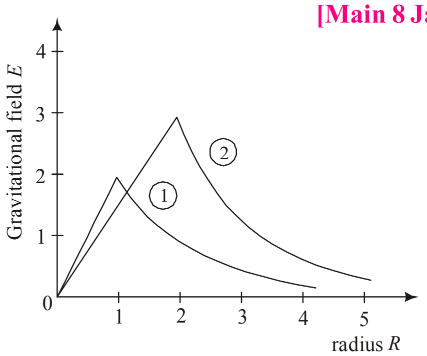
(c) $A(x^2+a^2)^{1/2}$ (d) $A(x^2+a^2)^{3/2}$

3. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at : **[Main 3 Sep. 2020 (II)]**

(a) $r=\sqrt{\frac{3}{4}}R$ (b) $r=R$

(c) $r=\frac{1}{\sqrt{3}}R$ (d) $r=\sqrt{\frac{5}{9}}R$

4. Consider two solid spheres of radii $R_1 = 1m$, $R_2=2m$ and masses M_1 and M_2, respectively. The gravitational field due to sphere 1 and 2 are shown. The value of $\frac{m_1}{m_2}$ is: **[Main 8 Jan. 2020 I]**

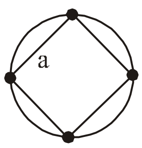


(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

5. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be: **[Main 9 April 2019 I]**

(a) $\frac{2GM}{9a^2}$ (b) $\frac{GM}{9a^2}$ (c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

6. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square ? **[Main 8 April 2019 I]**



(a) $1.35\sqrt{\frac{GM}{a}}$ (b) $1.16\sqrt{\frac{GM}{a}}$

(c) $1.21\sqrt{\frac{GM}{a}}$ (d) $1.41\sqrt{\frac{GM}{a}}$

7. A test particle is moving in circular orbit in the gravitational field produced by a mass density $r(r)=\frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T: **[Main 8 April 2019 II]**

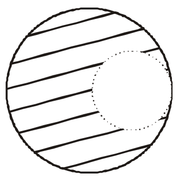
(a) T/R is a constant (b) T^2/R^3 is a constant

(c) T/R^2 is a constant (d) TR is a constant

8. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is: **[Main Online April 15, 2018]**

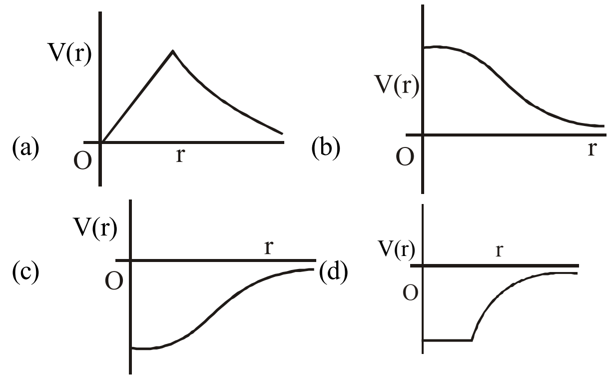
(a) $-\frac{GMm}{2R}$ (b) $+\frac{GMm}{6R}$ (c) $-\frac{GMm}{6R}$ (d) $\frac{GMm}{2R}$

9. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is :
(G = *gravitational constant*) **[Main 2015]**



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$ (c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

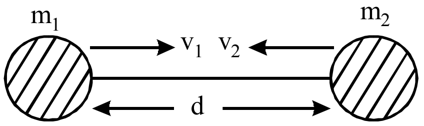
10. Which of the following most closely depicts the correct variation of the gravitational potential V(r) due to a large planet of radius R and uniform mass density ? (figures are not drawn to scale) **[Main Online April 11, 2015]**



11. The gravitational field in a region is given by $\vec{g} = 5N/kg\hat{i} + 12N/kg\hat{j}$. The change in the gravitational potential energy of a particle of mass 1 kg when it is taken from the origin to a point (7 m, – 3 m) is: **[Main Online April 19, 2014]**

(a) 71 J (b) $13\sqrt{58}$J (c) – 71 J (d) 1 J

12.



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd'? **[Main Online April 12, 2014]**

(Speed of m_1 is v_1 and that of m_2 is v_2)

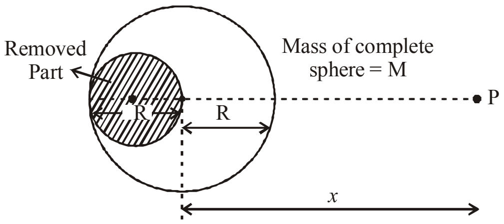
(a) $v_1 = v_2$

(b) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$ $v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(c) $v_1 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$ $v_2 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(d) $v_1 = m_2\sqrt{\dfrac{2G}{m_1}}$ $v_2 = m_2\sqrt{\dfrac{2G}{m_2}}$

13. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown, has been 'removed out'), at a very far off point, P, located as shown, would be (nearly) : **[Main Online April 9, 2013]**

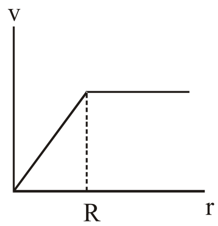


(a) $\dfrac{5}{6}\dfrac{GM}{x^2}$ (b) $\dfrac{8}{9}\dfrac{GM}{x^2}$ (c) $\dfrac{7}{8}\dfrac{GM}{x^2}$ (d) $\dfrac{6}{7}\dfrac{GM}{x^2}$

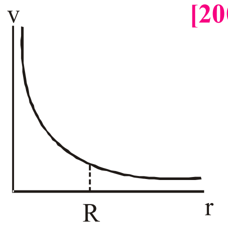
14. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by – **[2008]**

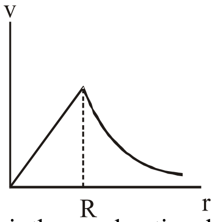
(a)



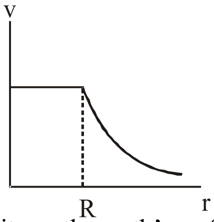
(b)



(c)



(d)



15. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is **[1983 - 1 Mark]**

(a) $\dfrac{1}{2}mgR$ (b) $2\,mgR$ (c) mgR (d) $\dfrac{1}{4}mgR$

3 Numeric Answer

16. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/ s)? Give your answer to the nearest integer in kilometer/s _____. **[Main 8 Jan. 2020 II]**

6 MCQs with One or More than One Correct Answer

17. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then: **[1994 - 2 Marks]**

(a) $\dfrac{F_1}{F_2} = \dfrac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

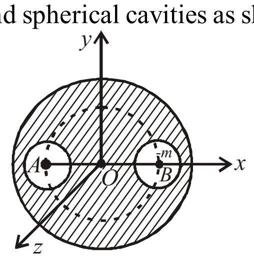
(b) $\dfrac{F_1}{F_2} = \dfrac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\dfrac{F_1}{F_2} = \dfrac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\dfrac{F_1}{F_2} = \dfrac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

18. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0 ,0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in fig

Then : **[1993-2 Marks]**



(a) The gravitational force due to this object at the origin is zero.

(b) the gravitational force at the point B (2, 0 ,0) is zero.

(c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

(d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

Topic-5 : Motion of Satellites, Escape Speed and Orbital Velocity

1. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is : **[Main 6 Sep. 2020 (I)]**
(a) 1 : 6 (b) 1 : 3 (c) 1 : 2 (d) 3 : 4

2. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is : **[Main 4 Sep. 2020 (II)]**
(a) $\frac{1}{\sqrt{2}}$ (b) 2
(c) 1 (d) $\sqrt{2}$

3. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is : **[Main 3 Sep. 2020 (I)]**
(a) $4R_e$ (b) $2.5R_e$ (c) $3R_e$ (d) $2R_e$

4. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as : **[Main 2 Sep. 2020 (I)]**
(a) $T^2 \propto R$ (b) $T^2 \propto R^3$ (c) $T^2 \propto \frac{1}{R^3}$ (d) $T \propto R$

5. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ or A. The collision is completely inelastic. Then, the combined body: **[Main 9 Jan. 2020 I]**
(a) continues to move in a circular orbit
(b) Escapes from the Planet's Gravitational field
(c) Falls vertically downwards towards the planet
(d) starts moving in an elliptical orbit around the planet

6. The energy required to take a satellite to a height 'h' above Earth surface (radius of Eareth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is: **[Main 9 Jan. 2019 II]**
(a) 1.6×10^3 km (b) 3.2×10^3 km
(c) 6.4×10^3 km (d) 28×10^4 km

7. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is : **[Main 9 Jan. 2020 II]**
(a) 4 (b) 1 (c) 2 (d) 3

8. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth): **[Main 7 Jan. 2020 I]**
(a) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$ (b) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$
(c) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$ (d) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

9. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ? [Given : Mass of Planet = 8×10^{22} kg, Radius of planet = 2×10^6 m, Gravitational constant G = 6.67×10^{-11} Nm^2/kg^2] **[Main 10 April 2019 II]**
(a) 9 (b) 17 (c) 13 (d) 11

10. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon. **[Main 8 April 2019 II]**
(a) $\frac{E}{64}$ (b) $\frac{E}{32}$
(c) $\frac{E}{4}$ (d) $\frac{E}{16}$

11. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth collides with the satellite completely in elastically. The speeds of the satellite and the meteorite are the same, Just before the collision. The subsequent motion of the combined body will be **[Main 12 Jan. 2019 I]**

(a) such that it escape to infinity
(b) In an elliptical orbit
(c) in the same circular orbit of radius R
(d) in a circular orbit of a different radius

12. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is : **[Main 12 Jan. 2019 II]**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\sqrt{\frac{1}{2}}$

13. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h << R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is: **[Main 11 Jan. 2019 I]**

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
(c) $\sqrt{\frac{gR}{2}}$ (d) $\sqrt{gR}\left(\sqrt{2}-1\right)$

14. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is: **[Main 10 Jan. 2019 I]**

(a) $2mv^2$ (b) mv^2
(c) $\frac{1}{2}mv^2$ (d) $\frac{3}{2}mv^2$

15. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpen-dicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 66 \times 10^{-11}$ $Nm^2\ kg^{-2}$) **[Main 10 Jan. 2019 II]**

(a) 2.4×10^4 m/s (b) 1.4×10^5 m/s
(c) 3.8×10^4 m/s (d) 2.8×10^5 m/s

16. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun–Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet) **[Adv. 2017]**

(a) $v_s = 22$ km s^{-1} (b) $v_s = 42$ km s^{-1}
(c) $v_s = 62$ km s^{-1} (d) $v_s = 72$ km s^{-1}

17. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[Main 2016]**

(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

18. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is : **[Main Online April 10, 2016]**

(a) Zero since astronaut feels weightless
(b) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$
(c) $F_G = \frac{GMm}{(R+h)^2}$
(d) $0 < F_G < \frac{GMm}{R^2}$

19. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R(R << L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then : **[Main Online April 10, 2015]**

(a) $T \propto r$ (b) $T \propto \sqrt{r}$ (c) $T \propto r^2$ (d) $T^2 \propto r^3$

20. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[Main 2013]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

21. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[2011]**

(a) $\frac{1}{2}mV^2$ (b) mV^2 (c) $\frac{3}{2}mV^2$ (d) $2mV^2$

2 Integer Value Answer

22. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}^{\text{th}}$ of its value of the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[Adv. 2015]**

23. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[2010]**

4 Fill in the Blanks

24. A particle is projected vertically upwards from the surface of earth (radius R_e) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is.... **[1997 - 2 Marks]**

25. The masses and radii of the Earth and the Moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is .... **[1988 - 2 Marks]**

26. A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is ....hours. **[1987 - 2 Marks]**

5 True / False

27. It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. **[1984 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

28. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then **[2012]**

(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$

(c) $V_R / V_P = 3$ (d) $V_P / V_Q = \frac{1}{2}$

29. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. **[1998S - 2 Marks]**

(a) The acceleration of S is always directed towards the centre of the earth.

(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.

(c) The total mechanical energy of S varies periodically with time.

(d) The linear momentum of S remains constant in magnitude.

7 Match the Following

30. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively, Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2, and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[Adv. 2018]**

	LIST–I		LIST–II
P.	v_1/v_2	**1.**	1/8
Q.	L_1/L_2	**2.**	1
R.	K_1/K_2	**3.**	2
S.	T_1/T_2	**4.**	8

(a) $P \to 4; Q \to 2; R \to 1; S \to 3$

(b) $P \to 3; Q \to 2; R \to 4; S \to 1$

(c) $P \to 2; Q \to 3; R \to 1; S \to 4$

(d) $P \to 2; Q \to 3; R \to 4; S \to 1$

9 Assertion and Reason Type Questions

31. **STATEMENT - 1 :** An astronaut in an orbiting space station above the earth experiences weightlessness.

because

STATEMENT - 2 : An object moving the earth under the influence of Earth's gravitational force is in a state of "free-fall". **[2008]**

(a) Statement- 1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement -1

(b) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement- 2 is False

(d) Statement -1 is False, Statement -2 is True

10 Subjective Problems

32. A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. **[2003 - 4 Marks]**

33. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$, respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a. **[1996 - 5 Marks]**

34. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. **[1990 - 8 Mark]**

(i) Determine the height of the satellite above the earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

35. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a. Find the intial velocity that should be given to each particle and also the time period of the circular motion. **[1988 - 5 Marks]**

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

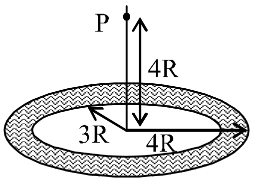
1. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radical distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \rho(r)/m$ is

[G is universal gravitational constant] **[Adv. 2019]**

(a) $\dfrac{3K}{\pi r^2 m^2 G}$ (b) $\dfrac{K}{2\pi r^2 m^2 G}$

(c) $\dfrac{K}{\pi r^2 m^2 G}$ (d) $\dfrac{K}{6\pi r^2 m^2 G}$

2. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is **[2010]**



(a) $\dfrac{2GM}{7R}(4\sqrt{2}-5)$

(b) $-\dfrac{2GM}{7R}(4\sqrt{2}-5)$

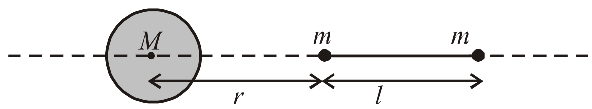
(c) $\dfrac{GM}{4R}$

(d) $\dfrac{2GM}{5R}(\sqrt{2}-1)$

2 Integer Value Answer

3. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\dfrac{M}{288}\right)$.

The value of k is **[Adv. 2015]**



6 MCQs with One or More than One Correct Answer

4. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) **[Adv. 2013]**

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\dfrac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\dfrac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\dfrac{2GM}{L}}$

(d) The energy of the mass m remains constant

Answer Key

Topic-1 : Kepler's Laws of Planetary Motion

1. (c)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)

Topic-2 : Newton's Universal Law of Gravitation

1. (d)	**2.** (a)	**3.** (d)	**4.** (a)

Topic-3 : Acceleration due to Gravity

1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (1.24×10^{-3} rad/s)							

Topic-4 : Gravitational Field, Potential and Potential Energy

1. (d)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (16.00)		**17.** (a,b)	**18.** (a,c,d)	

Topic-5 : Motion of Satellites, Escape Speed and Orbital Velocity

1. (a)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (d)
11. (b)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (a)
21. (b)	**22.** (2)	**23.** (3)	**24.** (R)	**25.** $\left(\sqrt{\frac{4G}{d}(M_1+M_2)}\right)$			**26.** (8.48 h)		**27.** False
28. (b, d)	**29.** (a, c)	**30.** (b)	**31.** (a)						

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. (b)	**2.** (a)	**3.** (7)	**4.** (b)

8 Mechanical Properties of Solids

Topic-1 : Hooke's Law & Young's Modulus

1 MCQs with One Correct Answer

1. If the potential energy between two molecules is given by $U=-\frac{A}{r^6}+\frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are: **[Main Sep. 06, 2020 (I)]**

(a) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$ (b) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$

(c) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{4B}$ (d) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

2. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to : **[Main 12 April 2019 II]**

(a) $9F/(\pi r^2 YT)$ (b) $6F/(\pi r^2 YT$

(c) $3F/(\pi r^2 YT)$ (d) $F/(3\pi r^2 YT)$

3. In an environment, brass and steel wires of length 1 m each with area of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,

[Given, the Young's modulus for steel and brass are, respectively, $120\times10^9 N/m^2$ and $60\times10^9 N/m^2$]

[Main 10 April 2019 II]

(a) $1.2\times10^6 N/m^2$ (b) $4.0\times10^6 N/m^2$

(c) $1.8\times10^6 N/m^2$ (d) $0.2\times10^6 N/m^2$

4. A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given that g = 3.1 À ms^{-2}, what will be the tensile stress that would be developed in the wire? **[Main 9 April 2019 I]**

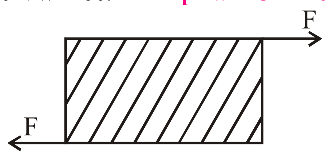
(a) $6.2\times10^6 Nm^{-2}$ (b) $5.2\times10^6 Nm^{-2}$

(c) $3.1\times10^6 Nm^{-2}$ (d) $4.8\times10^6 Nm^{-2}$

5. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to : **[Main 8 April 2019 II]**

(a) 1.5 mm (b) 1.9 mm (c) 1.7 mm (d) 1.3 mm

6. As shown in the figure, forces of 10^5N each are applied in opposite directions, on the upper and lower faces of a cube of side 10cm, shifting the upper face parallel to itself by 0.5cm. If the side of another cube of the same material is, 20cm, then under similar conditions as above, the displacement will be: **[Main Online April 15, 2018]**



(a) 1.00cm (b) 0.25cm (c) 0.37cm (d) 0.75cm

7. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal : **[Main Online April 9, 2016]**

(a) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (b) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

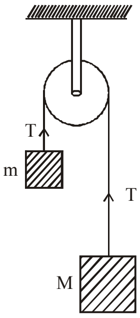
(c) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (d) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is:

(For steel Young's modulus is $2\times10^{11}\,\text{Nm}^{-2}$ and coefficient of thermal expansion is $1.1\times10^{-5}\,\text{K}^{-1}$) **[Main 2014]**

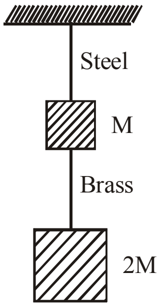
(a) 2.2×10^{8} Pa (b) 2.2×10^{9} Pa
(c) 2.2×10^{7} Pa (d) 2.2×10^{6} Pa

9. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If M = 2 m, then the stress produced in the wire is : **[Main Online April 25, 2013]**



(a) $\frac{2mg}{3A}$ (b) $\frac{4mg}{3A}$ (c) $\frac{mg}{A}$ (d) $\frac{3mg}{4A}$

10. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is : **[Main Online April 9, 2013]**

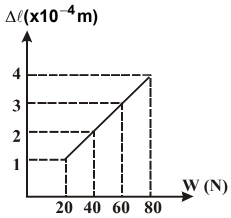


(a) $\frac{3c}{2ab^2}$ (b) $\frac{2a^2c}{b}$ (c) $\frac{3a}{2b^2c}$ (d) $\frac{2ac}{b^2}$

11. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is **[Adv. 2013]**

(a) 0.25 (b) 0.50 (c) 2.00 (d) 4.00

12. The adjacent graph shows the estension ($\Delta\ell$) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire. **[2003S]**



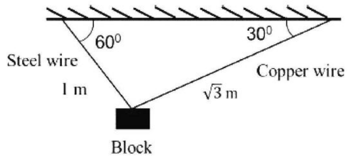
(a) 2×10^{11} N/m
(b) 2×10^{-11} N/m
(c) 3×10^{-12} N/m
(d) 2×10^{-13} N/m

13. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? **[1981- 2 Marks]**
(a) length = 50 cm , diameter = 0.5 mm
(b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm
(d) length = 300 cm, diameter = 3 mm.

2 Integer Value Correct Type

14. A block of weight 100N is suspended by copper and steel wires of same cross sectional area 0.5cm^2 and, length m and 1m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30º and 60º, respectively. If elongation in copper wire is and elongation in steel wire is (Δl_s), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is_____

[Young's modulus for copper and steel are $1\times10^{11}N/m^2$ and $2\times10^{11}N/m^2$, respectively.] **[Adv. 2019]**



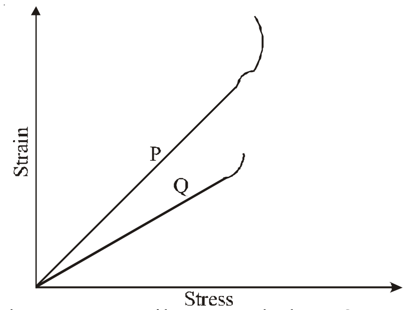
3 Numeric Answer

15. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = 4.8×10^{7} Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is _______ .

[Main 9 Jan 2020 I]

6 MCQs with One or More Than One Correct

16. In plotting stress versus strain curves for two materials P and Q,a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are) **[Adv. 2015]**



(a) P has more tensile strength than Q
(b) P is more ductile than Q
(c) P is more brittle than Q
(d) The Young's modulus of P is more than that of Q

Topic-2 : Bulk & Rigidity Modulus and Work Done in Stretching a Wire

1 MCQs with One Correct Answer

1. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is: **[Main 9 Jan 2020 II]**

(a) $\sqrt{2}:1$ (b) $1:2$ (c) $2:1$ (d) $1:\sqrt{2}$

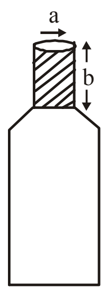
2. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to : **[Main 8 April 2019 I]**

(a) 10^6 N/m^{-2} (b) 10^4 N/m^{-2}
(c) 10^8 N/m^{-2} (d) 10^3 N/m^{-2}

3. A solid sphere of radius *r* made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass *m* is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is : **[Main 2018]**

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$ (c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

4. A bottle has an opening of radius a and length b. A cork of length b and radius $(a + \Delta a)$ where $(\Delta a << a)$ is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is : **[Main Online April 10, 2016]**



(a) $(\pi\mu B\,b)\,a$ (b) $(2\pi\mu Bb)\,\Delta a$
(c) $(\pi\mu B\,b)\,\Delta a$ (d) $(4\,\pi\mu B\,b)\,\Delta a$

5. Steel ruptures when a shear of 3.5×10^8 N m^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly: **[Main Online April 12, 2014]**

(a) 1.4×10^4 N (b) 2.7×10^4 N
(c) 3.3×10^4 N (d) 1.1×10^4 N

4 Fill in the Blanks

6. A wire of length *L* and cross sectional area *A* is made of a material of Young's modulus *Y*. If the wire is stretched by an amount *x*, the work done is .... **[1987 - 2 Marks]**

Answer Key

Topic-1 : Hooke's Law & Young's Modulus

1. (c)	**2.** (c)	**3.** (Bonus)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (c)	**12.** (a)	**13.** (a)	**14.** (2)	**15.** (4)	**16.** (a, b)				

Topic-2 : Bulk & Rigidity Modulus and Work Done in Stretching a Wire

1. (a)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** $\left(\frac{1}{2}\left(\frac{YA}{L}\right)x^2\right)$

9 Mechanical Properties of Fluids

Topic-1 : Pressure, Density, Pascal's Law and Archimedes' Principle

1 MCQs with One Correct Answer

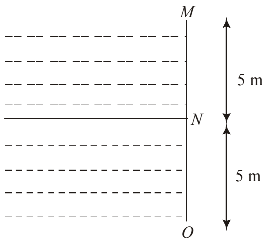
1. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is : **[Main 5 Sep. 2020 (I)]**
   (a) $\frac{8}{9}R$ (b) $\frac{4}{9}R$ (c) $\frac{2}{3}R$ (d) $\frac{1}{3}R$

2. An air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2) **[Main 4 Sep. 2020 (I)]**
   (a) 4.51 gm (b) 3.15 gm (c) 4.15 gm (d) 1.52 gm

3. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is : **[Main 4 Sep. 2020 (II)]**
   (a) $gdS(x_2^2 + x_1^2)$ (b) $gdS(x_2 + x_1)^2$
   (c) $\frac{3}{4}gdS(x_2 - x_1)^2$ (d) $\frac{1}{4}gdS(x_2 - x_1)^2$

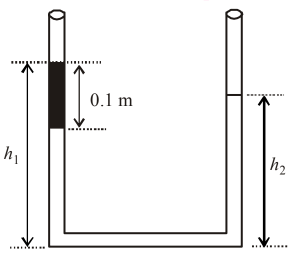
4. A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to: **[Main 8 Jan 2020 (I)]**
   (a) 1.26 (b) 1.04 (c) 1.01 (d) 1.03

5. Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is: **[Main 8 Jan 2020 (I)]**
   (a) $\frac{\rho_0}{3}$ (b) $\frac{\rho_0}{5}$ (c) $\frac{2\rho_0}{5}$ (d) $\frac{2\rho_0}{3}$

6. 

   Two liquids of densities ρ_1 and $\rho_2(\rho_2 = 2\rho_1)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing): **[Main 8 Jan 2020 (II)]**
   (a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4

7. An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kg m^{-3}). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is **[Adv. 2020]**
   (a) $\frac{15}{14}$ (b) $\frac{35}{33}$
   (c) $\frac{7}{6}$ (d) $\frac{5}{4}$

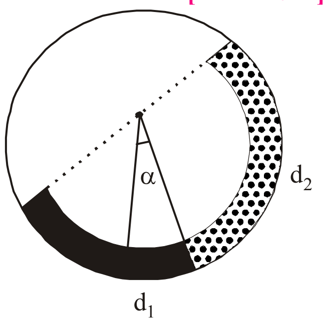


8. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m^3] **[Main 10 April 2019 (II)]**
   (a) 46.3 kg (b) 87.5 kg (c) 65.4 kg (d) 30.1 kg

9. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is: **[Main 9 April 2019 (II)]**

(a) 0.5 (b) 0.8 (c) 0.6 (c) 0.7

10. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90º angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[Main 2014]**



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$

(b) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$

(d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

11. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: **[Main 2013]**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

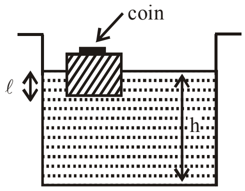
12. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **[Main 2012- II]**

(a) more than half-filled if ρ_c is less than 0.5.
(b) more than half-filled if ρ_c is more than 1.0.
(c) half-filled if ρ_c is more than 0.5.
(d) less than half-filled if ρ_c is less than 0.5.

13. When temperature of a gas is 20ºC and pressure is changed from $p_1 = 1.01 \times 10^5$ Pa to $p_2 = 1.165 \times 10^5$ Pa then the volume changed by 10%. The bulk modulus is **[2005S]**

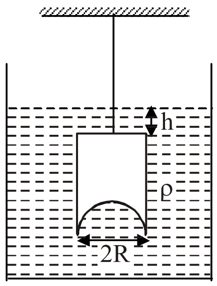
(a) 1.55×10^5 Pa (b) 0.115×10^5 Pa
(c) 1.4×10^5 Pa (d) 1.01×10^5 Pa

14. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance ℓ and h are shown here. After some time the coin falls into the water. Then **[2002S]**



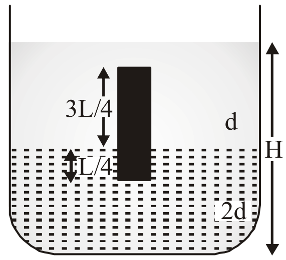
(a) ℓ decreases and h increases
(b) ℓ increases and h decreases
(c) both ℓ and h increase
(d) both ℓ and h decrease

15. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$ **[2001S]**
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

16. A homogeneous solid cylinder of length L ($L < H/2$), cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by **[1995S]**



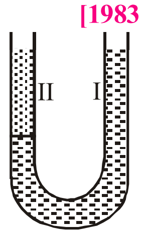
(a) $\frac{5}{4}d$

(b) $\frac{4}{5}d$

(c) $4d$

(d) $\frac{d}{5}$

17. A U-tube of uniform cross section (see Fig) is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be **[1983 - 1 Mark]**



(a) 1.12
(b) 1.1
(c) 1.05
(d) 1.0

3 Numeric Answer

18. A cubical solid aluminium (bulk modulus $= -V\frac{dP}{dV}$ = 70 GPa) block has an edge length of 1 m on the surface of the earth. It is kept on the floor of a 5 km deep ocean. Taking the average density of water and the acceleration due to gravity to be 10^3 kg m^{-3} and 10 ms^{-2}, respectively, the change in the edge length of the block in mm is _____. **[Adv. 2020]**

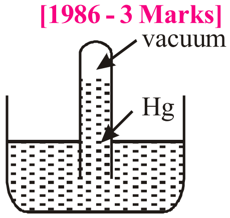
4 Fill in the Blanks

19. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless pistion of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere, $\delta R/R$, is .... **[1988 - 2 Mark]**

5 True / False

20. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises. **[1986 - 3 Marks]**

21. A barometer made of a very narrow tube (see Fig) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018 per C° and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C, but the temperature of the atmosphere does not change. Then the mercury height in the tube remains unchanged. **[1983 - 2 Marks]**



22. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, the level of the water in the pond decreases. **[1980]**

6 MCQs with One or More than One Correct Answer

23. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is (are) **[Adv. 2015]**

(a) $P(r=0)=0$

(b) $\dfrac{P(r=3R/4)}{P(r=2R/3)}=\dfrac{63}{80}$

(c) $\dfrac{P(r=3R/5)}{P(r=2R/5)}=\dfrac{16}{21}$

(d) $\dfrac{P(r=R/2)}{P(r=R/3)}=\dfrac{20}{27}$

24. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **[Adv. 2013]**

(a) The net elongation of the spring is $\dfrac{4\pi R^3 \rho g}{3k}$

(b) The net elongation of the spring is $\dfrac{8\pi R^3 \rho g}{3k}$

(c) The light sphere is partially submerged

(d) The light sphere is completely submerged

25. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is **[1988 - 2 Mark]**

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

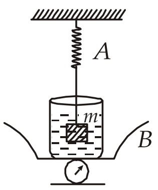
26. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation: **[1985 - 2 Marks]**

(a) the balance A will read more than 2 kg

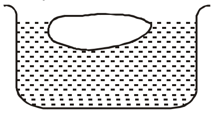
(b) the balance B will read more than 5 kg

(c) the balance A will read less than 2 kg and B will read more than 5 kg

(d) the balance A and B will read 2 kg and 5 kg respectively



27. A body floats in a liquid contained in a beaker. The whole system as shown in Figure falls freely under gravity. The upthrust on the body is/are **[1982 - 3 Marks]**



(a) zero

(b) equal to the weight of the liquid displaced

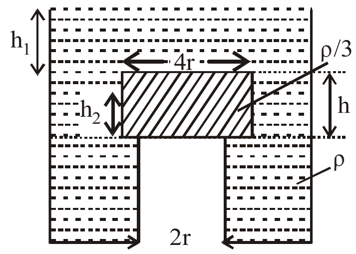
(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body

8 Comprehension/Passage Based Questions

Passage

A cylindrical tank has a hole of diameter $2r$ in its bottom. The hole is covered wooden cylindrical block of diameter $4r$, height h and density $\rho/3$.



Situation I : Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

Situation II : The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. The height h_2 is smaller than h (height of the block) and thus the block is exposed to the atmosphere.

28. Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up? **[2006 - 5M, –2]**

(a) $\dfrac{2h}{3}$ (b) $\dfrac{5h}{4}$

(c) $\dfrac{5h}{3}$ (d) $\dfrac{5h}{2}$

29. Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force **[2006 - 5M, –2]**

(a) $\dfrac{h}{3}$ (b) $\dfrac{4h}{9}$

(c) $\dfrac{2h}{3}$ (d) h

30. In situation 2, if h_2 is further decreased, then **[2006 - 5M, –2]**

(a) cylinder will not move up and remains at its original position

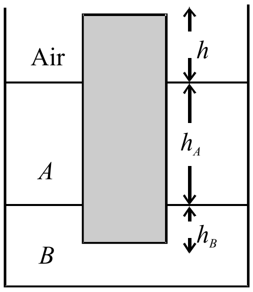
(b) for $h_2 = \dfrac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \dfrac{h}{4}$, cylinder again starts moving up

(d) for $h_2 = \dfrac{h}{5}$, cylinder again starts moving up

10 Subjective Problems

31. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. **[2002 - 5 Marks]**

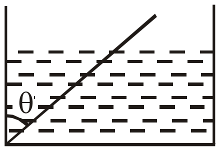


(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

32. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°) **[1984- 8 Marks]**



33. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? **[1979]**

34. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube ruses by 2cm. What is the size of the cube? **[1978]**

35. A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? **[1978]**

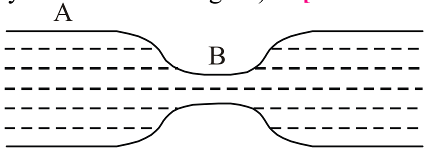
Topic-2 : Fluid Flow, Reynold's Number and Bernoulli's Principle

1 MCQs with One Correct Answer

1. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to : **[Main 6 Sep. 2020 (II)]**

(a) $\sqrt{\frac{P}{\rho}+v}$ (b) $\sqrt{\frac{2P}{\rho}+v^2}$ (c) $\sqrt{\frac{P}{2\rho}+v^2}$ (d) $\sqrt{\frac{P}{\rho}+v^2}$

2. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3}) **[Main 9 Jan. 2020 (I)]**



(a) 3020 cm^3/s (b) 2720 cm^3/s
(c) 2420 cm^3/s (d) 1810 cm^3/s

3. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is: **[Main 7 Jan. 2020 (II)]**

(a) $\frac{9}{16}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{4}$ (d) $\frac{81}{256}$

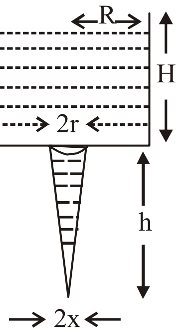
4. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be : [Take g = 10 ms^{-2}) **[Main 10 April 2019 (II)]**

(a) 2×10^{-5} m^2 (b) 5×10^{-5} m^2 (c) 5×10^{-4} m^2 (d) 1×10^{-5} m^2

5. The top of a water tank is open to air and its water lavel is maintained. It is giving out 0.74m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to: **[Main 9 Jan. 2019 (II)]**

(a) 6.0 m (b) 4.8 m (c) 9.6 m (d) 2.9 m

6. Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius r (r << R) at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross–section of water stream when it hits the ground is x. Then : **[Main Online April 9, 2016]**



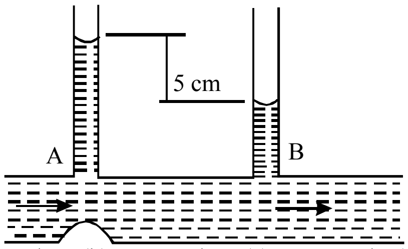
(a) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$ (b) $x = r\left(\frac{H}{H+h}\right)$

(c) $x = r\left(\frac{H}{H+h}\right)^{2}$ (d) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

7. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) **[Main 2014]**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

8. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is ($g = 10\ ms^{-2}$) **[Main Online April 19, 2014]**



(a) 7.5 cc/s (b) 8.0 cc/s (c) 10.0 cc/s (d) 12.5 cc/s

9. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $a/A = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) **[2005S]**

(a) 50 (b) 51 (c) 48 (d) 51.5

10. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to **[2000S]**

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\frac{L}{2\pi}$

2 Integer Value Answer

11. Water from a tap emerges vertically downwards with an initial spped of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is **[1998S - 2 Marks]**

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2

(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

12. A train with cross-sectional area S_t is moving with speed v_t inside a long tunnel of cross-sectional area S_0 ($S_0 = 4S_t$). Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then $p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of N is ________.

13. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.] **[2009]**

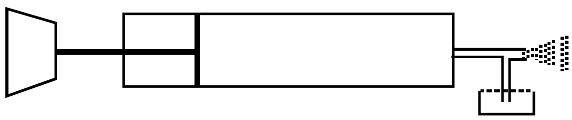
4 Fill in the Blanks

14. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross- sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is... Pa. (Density of water = 10^3 kg.m^{-3}) **[1994 - 2 Marks]**

8 Comprehension/Passage Based Questions

Passage

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of **[Adv. 2014]**

(a) 0.1 ms^{-1} (b) 1 ms^{-1} (c) 2 ms^{-1} (d) 8 ms^{-1}

16. If the density of air is ρ_a, and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to **[Adv. 2014]**

(a) $\sqrt{\frac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$ (c) $\sqrt{\frac{\rho_l}{\rho_a}}$ (d) ρ_l

9 Assertion and Reason Type Questions

17. **STATEMENT-1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

STATEMENT-2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

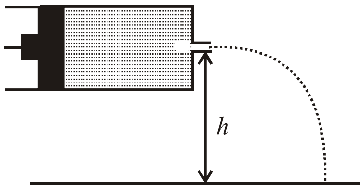
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

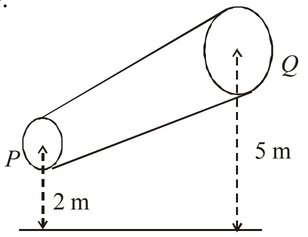
(d) Statement-1 is False, Statement-2 is True

10 Subjective Problems

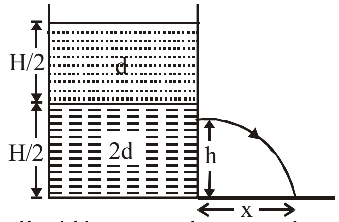
18. A tube has two area of cross-sections as shown in figure. The diameters of the tube are 8 mm and 2 mm. Find range of water falling on horizontal surface, if piston is moving with a constant velocity of 0.25 m/s, $h = 1.25$ m ($g = 10$ m/s^2) **[2004 - 2 Marks]**



19. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in Figure. The area of cross section of the tube two points P and Q at heights of 2 metres and 5 metres are respectively 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **[1997 - 5 Marks]**



20. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. **[1995 - 5 + 5 Marks]**



(a) A homogeneous solid cylinder of length $L (L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine:
  (i) the density D of the solid and
  (ii) the total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area $s (s << A)$ is punched on the vertical side of the container at a height $h (h < H/2)$. Determine :
  (i) the initial speed of efflux of the liquid at the hole,
  (ii) the horizontal distance x travelled by the liquid initially, and
  (iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.
  (Neglect the air resistance in these calculations.)

Topic-3 : Viscosity and Terminal Velocity

1 MCQs with One Correct Answer

1. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to : (ignore viscosity of air) **[Main 5 Sep. 2020 (II)]**

(a) r^4 (b) r (c) r^3 (d) r^2

2. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals: **[Main 12 April 2019 (II)]**

(a) 9 (b) 1/27 (c) 1/9 (d) 27

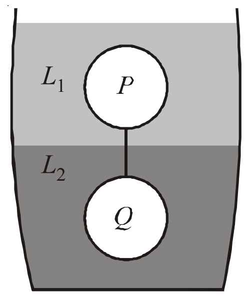
3. A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is 3m and that of kerosene 2m. When the hole is opened the velocity of fluid coming out from it is nearly: (take g = 10 ms^{-2} and density of water = 10^3 kg m^{-3}) **[Main Online April 11, 2014]**

(a) 10.7 ms^{-1} (b) 9.6 ms^{-1} (c) 8.5 ms^{-1} (d) 7.6 ms^{-1}

4. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is **[Adv. 2016]**

6 MCQs with One or More than One Correct Answer

5. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then **[Adv. 2015]**



(a) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$

(b) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$

(c) $\vec{V}_P \cdot \vec{V}_Q > 0$

(d) $\vec{V}_P \cdot \vec{V}_Q < 0$

Topic-4 : Surface Tension, Surface Energy and Capillarity

1 MCQs with One Correct Answer

1. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is : **[Main 3 Sep. 2020 (I)]**

   (a) 4 : 1 (b) 0.8 : 1 (c) 8 : 1 (d) 2 : 1

2. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to ($g = 10\ ms^{-2}$). **[Main 2 Sep. 2020 (II)]**

   (a) 0.049m (b) 0.087m (c) 0.137m (d) 0.172m

3. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet): **[Main 9 Jan. 2020 (II)]**

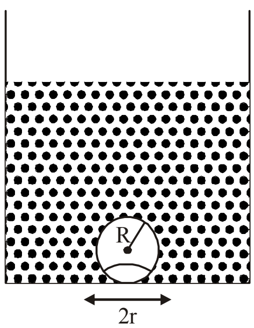
   (a) $r = \sqrt{\dfrac{2T}{3(d+\rho)g}}$ (b) $r = \sqrt{\dfrac{T}{(d-\rho)g}}$

   (c) $r = \sqrt{\dfrac{T}{(d+\rho)g}}$ (d) $r = \sqrt{\dfrac{3T}{(2d-\rho)g}}$

4. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to : **[Main 10 April 2019 (I)]**

   (a) 4/5 (b) 2/5 (c) 3/5 (d) 2/3

5. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r ≪ R and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) **[Main 2014]**



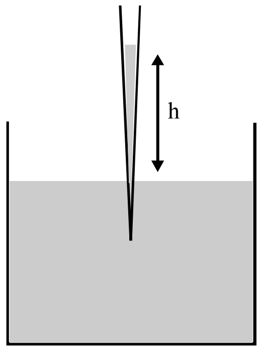
   (a) $R^2\sqrt{\dfrac{2\rho_w g}{3T}}$ (b) $R^2\sqrt{\dfrac{\rho_w g}{6T}}$

   (c) $R^2\sqrt{\dfrac{\rho_w g}{T}}$ (d) $R^2\sqrt{\dfrac{3\rho_w g}{T}}$

6. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio $\dfrac{x}{y}$ is: **[Main Online April 9, 2014]**

   (a) $\left(1-\dfrac{d}{R}\right)$ (b) $\left(1-\dfrac{2d}{R}\right)$ (c) $\left(\dfrac{R-d}{R+d}\right)$ (d) $\left(\dfrac{R+d}{R-d}\right)$

7. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **[Adv. 2014]**



   (a) $\dfrac{2S}{b\rho g}\cos(\theta-\alpha)$ (b) $\dfrac{2S}{b\rho g}\cos(\theta+\alpha)$

   (c) $\dfrac{2S}{b\rho g}\cos(\theta-\alpha/2)$ (d) $\dfrac{2S}{b\rho g}\cos(\theta+\alpha/2)$

2 Integer Value Answer

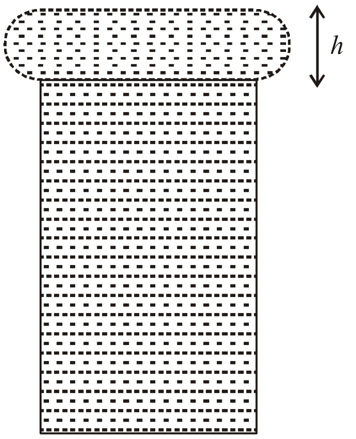
8. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \dfrac{0.1}{4\pi} Nm^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is **[Adv. 2017]**

9. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.] **[2009]**

3 Numeric Answer

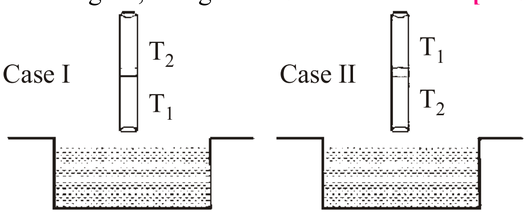
10. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is [$\rho_{(liquid)} = 900\ kgm^{-3}$, $g = 10\ ms^{-2}$] (Give answer in closest integer) __________. **[Main 3 Sep. 2020 (I)]**

11. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3}, 0.07 Nm^{-1} and 10 ms^{-2}, respectively, the value of h (in mm) is _________. **[Adv. 2020]**



6 MCQs with One or More than One Correct Answer

12. A cylindrical capillary tube of 0.2mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?
[surface tension of water = 0.075 N/m, density of water = 1000 kg/m³, take g = 10 m/s² **[Adv. 2019]**



(a) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
(b) For case II, if the capillary joint is 5cm above the water surface, the height of water column raised in the tube will be 3.75cm. (Neglect the weight of the water in the meniscus).
(c) For case I, if the joint is kept at 8cm above the water surface, the height of water column in the tubne will be 7.5cm. [Neglect the weight of the water in the meniscus]
(d) For case I, if the capillary joint is 5cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. [Neglect the weight of the water in the meniscus]

13. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true? **[Adv. 2018]**
(a) For a given material of the capillary tube, h decreases with increase in r
(b) For a given material of the capillary tube, h is independent of σ
(c) If this experiment is performed in a lift going up with a constant acceleration, then h decreases
(d) h is proportional to contact angle θ

8 Comprehension/Passage Based Questions

Passage

When liquid medicine of density ρ is to put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

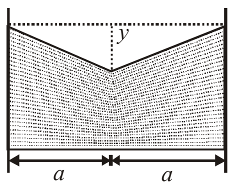
14. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is **[2010]**
(a) $2\pi rT$ (b) $2\pi RT$ (c) $\dfrac{2\pi r^2 T}{R}$ (d) $\dfrac{2\pi R^2 T}{r}$

15. If $r = 5\times10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately **[2010]**
(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m
(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

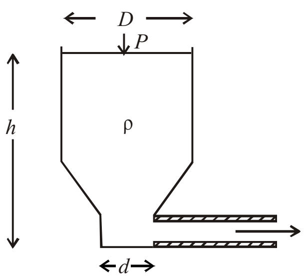
16. After the drop detaches, its surface energy is **[2010]**
(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J
(c) 5.4×10^{-6} J (d) 8.1×10^{-6} J

10 Subjective Problems

17. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by y ($y << a$) as shown in figure. Find surface tension of liquid. Neglect end effect. **[2004 - 2 Marks]**

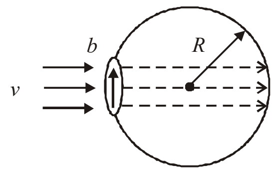


18. Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a. The volume flow rate in the capillary is Q.



If the capillary is removed the liquid comes out with a velocity of v_0. The density of the liquid is given as ρ. Calculate the coefficient of viscosity η. **[2003 - 4 Marks]**

19. A bubble having surface tension T and radius R is formed on a ring of radius b ($b << R$). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring. **[2003 - 4 Marks]**



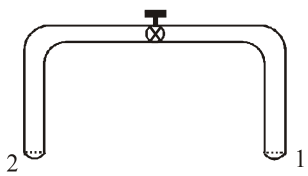
Topic-5 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to **[Adv. 2017]**

   (a) R (b) R^3 (c) $\frac{1}{R}$ (d) $R^{2/3}$

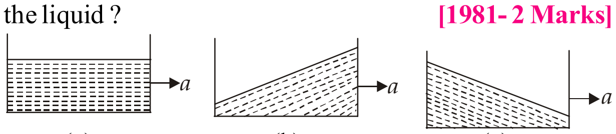
2. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position.



End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, **[2008]**

   (a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
   (b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
   (c) no changes occurs
   (d) air from end 2 flows towards end 1. volume of the soap bubble at end 1 increases

3. A vessel containing water is given a constant acceleration 'a' towards the right along a straight horizontal path. Which of the following diagrams in Fig. represents the surface of the liquid ? **[1981- 2 Marks]**



(a) (b) (c)

2 Integer Value Answer

4. A hot air balloon is carrying some passengers, and a few sandbags of mass 1 kg each so that its total mass is 480 kg. Its effective volume giving the balloon its buoyancy is V. The balloon is floating at an equilibrium height of 100 m. When N number of sandbags are thrown out, the balloon rises to a new equilibrium height close to 150 m with its volume V remaining unchanged. If the variation of the density of air with height h from the ground is $\rho(h) = \rho_0 e^{-\frac{h}{h_0}}$, where $\rho_0 = 1.25 \text{ kg m}^{-3}$ and $h_0 = 6000$ m, the value of N is __________.

4 Fill in the Blanks

5. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If the temperatures of both mercury and the metal are increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor .... **[1991 - 2 Mark]**

6 MCQs with One or More than One Correct Answer

6. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity μ_0. Which of the following statements is (are) true? **[Adv. 2018]**

   (a) The resistive force of liquid on the plate is inversely proportional to h
   (b) The resistive force of liquid on the plate is independent of the area of the plate
   (c) The tangential (shear) stress on the floor of the tank increases with μ_0
   (d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

7 Match the Following

7. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

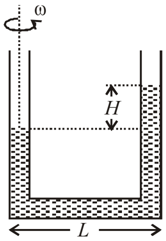
List - I		List - II	
P.	Lift is accelerating vertically up	**1.**	$d = 1.2$ m
Q.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration	**2.**	$d > 1.2$ m

R. Lift is moving vertically up with constant speed	**3.** $d < 1.2$ m
S. Lift is falling freely	**4.** No water leaks out of the jar

Code:
(a) P-2, Q-3, R-2, S-4 (b) P-2, Q-3, R-1, S-4
(c) P-1, Q-1, R-1, S-4 (d) P-2, Q-3, R-1, S-1

10 Subjective Problems

8. A **U** tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d << L$. **[2005 - 2 Marks]**



9. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of d_L **[1992 - 8 Marks]**
(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
(b) Is the motion of the ball simple harmonic?
(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

Answer Key

Topic-1 : Pressure, Density, Pascal's Law and Archimedes' Principle

1. (a)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (c)
11. (c)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (0.24)	**19.** $\frac{Mg}{3AK}$	
20. False	**21.** False	**22.** False	**23.** (b, c)	**24.** (a, d)	**25.** (c)	**26.** (b,c)	**27.** (a)	**28.** (c)	**29.** (b)
30. (a)									

Topic-2 : Fluid Flow, Reynold's Number and Bernoulli's Principle

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (a)
11. (c)	**12.** (9)	**13.** (6)	**14.** (500)	**15.** (c)	**16.** (a)	**17.** (a)			

Topic-3 : Viscosity and Terminal Velocity

1. (a)	**2.** (a)	**3.** (b)	**4.** (3)	**5.** (a, d)

Topic-4 : Surface Tension, Surface Energy and Capillarity

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (6)	**9.** (6)	**10.** (101)
11. (3.74)	**12.** (a, b, c)	**13.** (a, c)	**14.** (c)	**15.** (a)	**16.** (b)	**21. d/*l***			

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (a)	**2.** (b)	**3.** (c)	**4.** (4)	**5.**	**6.** (a, c, d)	**7.** (c)

10 Thermal Properties of Matter

Topic-1 : Thermometer & Thermal Expansion

1 MCQs with One Correct Answer

1. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is : **[Main Sep. 05, 2020 (II)]**

   (a) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ (b) $2\sqrt{\alpha_1\alpha_2}$

   (c) $\frac{\alpha_1 + \alpha_2}{2}$ (d) $4\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\frac{L_2 L_1}{(L_2 + L_1)^2}$

2. When the temperature of a metal wire is increased from 0°C to 10°C, its length increased by 0.02%. The percentage change in its mass density will be closest to : **[Main Sep. 02, 2020 (II)]**

   (a) 0.06 (b) 2.3 (c) 0.008 (d) 0.8

3. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to: **[Main 12 April 2019 (I)]**

   (Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m², respectively; $g = 10$ ms^{-2})

   (a) 9 kg (b) 0.5 kg (c) 1.5 kg (d) 0.9 kg

4. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is : **[Main 11 Jan. 2019 II]**

   (a) 230°C (b) 270°C (c) 200°C (d) 250°C

5. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is: **[Main 9 Jan. 2019 I]**

   (a) $\frac{F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$

   (c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{2F}{A\alpha\Delta T}$

6. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : **[Main 2017]**

   (a) $\frac{3\alpha}{PK}$ (b) $3PK\alpha$ (c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

7. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let l be the length of the rod, A its area of cross–section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to : **[Main Online April 8, 2017]**

   (a) $l^2 Y\alpha\Delta T$ (b) $lAY\alpha\Delta T$

   (c) $AY\alpha\Delta T$ (d) $\frac{AY}{\alpha\Delta T}$

8. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is **[Adv. 2016]**

   (a) 0.78 mm (b) 0.90 mm (c) 1.56 mm (d) 2.34 mm

9. On a linear temperature scale Y, water freezes at – 160° Y and boils at – 50° Y. On this Y scale, a temperature of 340 K would be read as : (water freezes at 273 K and boils at 373 K) **[Main Online April 9, 2013]**

(a) – 73.7° Y (b) – 233.7° Y
(c) – 86.3° Y (d) – 106.3° Y

10. Two rods, one of aluminum and the other made of steel, having initial length ℓ_1 and ℓ_2 are connected together to form a single rod of length $\ell_1 + \ell_2$. The coefficients of linear expansion for aluminum and steel are α_a and α_s and respectively. If the length of each rod increases by the same amount when their temperature are raised by t^0C, then find the ratio $\ell_1/(\ell_1 + \ell_2)$ **[2003S]**

(a) α_s/α_a (b) α_a/α_s
(c) $\alpha_s/(\alpha_a+\alpha_s)$ (d) $\alpha_a/(\alpha_a+\alpha_s)$

11. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60 °C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as **[2001S]**

(a) $\dfrac{1+60\gamma_{Fe}}{1+60\gamma_{hg}}$ (b) $\dfrac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$

(c) $\dfrac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\dfrac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

12. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C. The coefficient of expansion of cubical the metal is less than that of the alcohal. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that **[1980]**

(a) $W_1 > W_2$ (b) $W_1 = W_2$
(c) $W_1 < W_2$ (d) None of these

13. A constant volume gas thermometer works on **[1980]**

(a) The Principle of Archimedes
(b) Boyle's Law
(c) Pascal's Law
(d) Charle's Law

2 Integer Value Answer

14. Steel wire of lenght 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5} /° C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L>>diameter of the wire. Then the value of 'm' in kg is nearly **[2011]**

3 Numeric Answer

15. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ___________. **[Main Sep. 03, 2020 (I)]**

16. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5}/°C along the x-axis and 5×10^{-6}/°C along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}$/°C then the value of C is ______. **[Main 7 Jan. 2020 (I)]**

6 MCQs with One or More than One Correct Answer

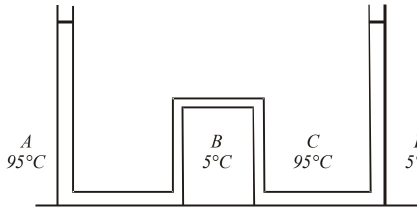
17. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_c and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is. **[1999S - 3 Marks]**

(a) proportional to ΔT
(b) inversely proportional to ΔT
(c) proportional to $|\alpha_B - \alpha_C|$
(d) inversely proportional to $|\alpha_B - \alpha_C|$

10 Subjective Problems

18. A cubical block of co-efficient of linear expansion α_s is submerged partially inside a liquid of co-efficient of volume expansion γ_ℓ. On increasing the temperature of the system by ΔT, the height of the cube inside the liquid remains unchanged. Find the relation between α_s and γ_ℓ. **[2004 - 4 Marks]**

19. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95° C while the columns B and D are maintained at 5°C. The height of the liquid in A and D measured from the base the are 52.8 cm and 51cm respectively. Determine the coefficient of thermal expansion of the liquid. **[1997 - 5 Marks]**

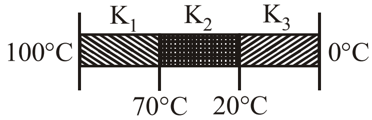


20. A sinker of weight w_0 has an apparent weight w_1 when weighed in a liquid at a temperature t_1 and w_2 when weight in the same liquid at temperature t_2. The coefficient of cubical expansion of the material of sinker is β. What is the coefficient of volume expansion of the liquid. **[1978]**

Topic-2 : Calorimetry and Heat Transfer

1 MCQs with One Correct Answer

1. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100ºC and the other at 0ºC (see figure). If the joints of the rod are at 70ºC and 20ºC in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is:
[Main Sep. 06, 2020 (II)]



(a) $K_1 : K_3 = 2 : 3, K_1 < K_3 = 2 : 5$
(b) $K_1 < K_2 < K_3$
(c) $K_1 : K_2 = 5 : 2, K_1 : K_3 = 3 : 5$
(d) $K_1 > K_2 > K_3$

2. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetics energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g – ºC) (1 cal = 4.2×10^7 ergs) close to :
[Main Sep. 05, 2020 (I)]

(a) 87.5ºC (b) 83.3ºC (c) 119.2ºC (d) 38.4ºC

3. The specific heat of water = 4200 J kg^{-1} K^{-1} and the latent heat of ice = 3.4×10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) : **[Main Sep. 04, 2020 (I)]**

(a) 61.7 (b) 63.8 (c) 69.3 (d) 64.6

4. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. '*m*' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of '*m*' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} °C^{-1}) **[Main Sep. 03, 2020 (II)]**

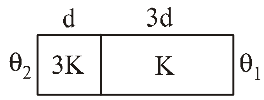
(a) 2 (b) 4 (c) 3.2 (d) 2.6

5. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/ g and heat of fusion of ice is 80 cal/g], the value of M is ________ **[Main 7 Jan. 2020 (II)]**

6. When M_1 gram of ice at –10 ºC (Specific heat = 0.5 cal g^{-1}ºC^{-1}) is added to M_2 gram of water at 50ºC, finally no ice is left and the water is at 0ºC. The value of latent heat of ice, in cal g^{-1} is: **[Main 12 April 2019 (I)]**

(a) $\frac{50M_2}{M_1} - 5$ (b) $\frac{5M_1}{M_2} - 50$
(c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

7. Two materials having coefficients of thermal conductivity '3*K*' and '*K*' and thickness '*d*' and '3*d*', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:
[Main 9 April 2019 (II)]



(a) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$ (b) $\frac{\theta_2 + \theta_1}{2}$
(c) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (d) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$

8. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:
[Main 12 Jan. 2019 (I)]

(a) $\frac{K_1 + K_2}{2}$ (b) $K_1 + K_2$
(c) $\frac{2K_1 + 3K_2}{5}$ (d) $\frac{K_1 + 3K_2}{4}$

9. Ice at –20°C is added to 50 g of water at 40°C, When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to **[Main 11 Jan. 2019 (I)]**
(Specific heat of water = 4.2J/g/°C
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334J/g)
(a) 50g (b) 100 g (c) 60 g (d) 40 g

10. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :
[Main 11 Jan. 2019 II]

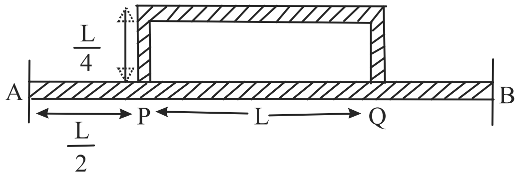
(a) 85ºC (b) 60ºC (c) 80ºC (d) 70ºC

11. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is: **[Main 10 Jan. 2019 I]**
(a) 90 Wm^{-2} (b) 120 Wm^{-2} (c) 65 Wm^{-2} (d) 200 Wm^{-2}

12. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stablizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1}) **[Main 10 Jan. 2019 II]**

(a) 458 J kg^{-1} K^{-1} (b) 1232 J kg^{-1} K^{-1}
(c) 916 J kg^{-1} K^{-1} (d) 654 J kg^{-1} K^{-1}

13. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to: **[Main 9 Jan. 2019 I]**



(a) 45°C (b) 75°C (c) 60°C (d) 35°C

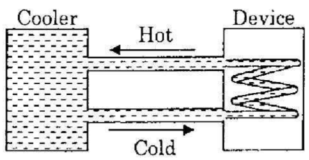
14. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is :
(take 4.2 Joule=1 calorie) **[Main Online April 8, 2017]**

(a) 378 J/kg – °C (b) 315 J/kg – °C
(c) 476 J/kg – °C (d) 434 J/kg – °C

15. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C **[Main 2017]**

(a) 1250°C (b) 825°C (c) 800°C (d) 885° C

16. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is **[Adv. 2016]**



(Specific heat of water is 4.2 kJ $kg^{-1}K^{-1}$ and the density of water is 1000 kg m^{-3})

(a) 1600 (b) 2067 (c) 2533 (d) 3933

17. An experiment takes 10 minutes to raise the temperature of water in a container from 0ºC to 100ºC and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal / g ºC, the heat of vapourization according to this experiment will come out to be : **[Main Online April 11, 2015]**

(a) 560 cal/ g (b) 550 cal/ g
(c) 540 cal/ g (d) 530 cal/ g

18. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross - section of each rod = 4 cm^2. End of copper rod is maintained at 100ºC where as ends of brass and steel are kept at 0ºC. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is: **[Main 2014]**

(a) 1.2 cal/s (b) 2.4 cal/s (c) 4.8 cal/s (d) 6.0 cal/s

19. A black coloured solid sphere of radius R and mass M is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature T_0. The initial temperature of the sphere is $3T_0$. If the specific heat of the material of the sphere varies as αT^3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature $2T_0$ will be (σ is Stefan Boltzmann constant) **[Main Online April 19, 2014]**

(a) $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$ (b) $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$

(c) $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$ (d) $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$

20. Parallel rays of light of intensity $I = 912$ Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ $Wm^{-2}K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to **[Adv. 2014]**

(a) 330 K (b) 660 K (c) 990 K (d) 1550 K

21. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[Main 2013]**

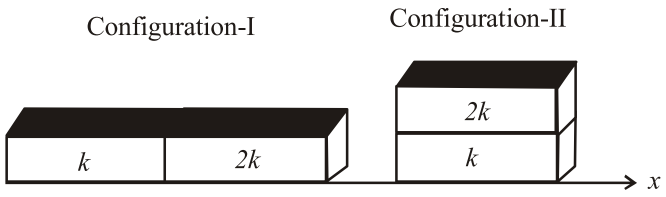
(a) $\rho L/T$ (b) $\sqrt{T/\rho L}$ (c) $T/\rho L$ (d) $2T/\rho L$

22. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30° C to 25° C. The specific heat of the liquid is :

(The water equivalent of the vessel is 30g.)
[Main Online April 25, 2013]

(a) 2.0 kcal/kg (b) 7 kcal/kg
(c) 3 kcal/kg (d) 0.5 kcal/kg

23. Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure. One of the blocks has thermal conductivity k and the other $2k$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration-I. The time to transport the same amount of heat in the configuration-II is **[Adv. 2013]**



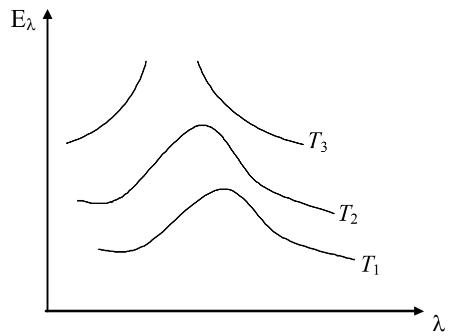
(a) 2.0 s (b) 4.5 s (c) 3.0 s (d) 6.0 s

24. Water of volume 2 litre in a container is heated with a coil of 1 kW at 27ºC. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27ºC to 77ºC [Given specific heat of water is 4.2 kJ/kg] **[2005 S]**
(a) 7 min (b) 6 min 2 s (c) 8 min 20 s (d) 14 min

25. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1ºC and it is defined under which of the following conditions? **[2005 S]**
(a) From 14.5 ºC to 15.5 ºC at 760 mm of Hg
(b) From 98.5 ºC to 99.5 ºC at 760 mm of Hg
(c) From 13.5 ºC to 14.5 ºC at 76 mm of Hg
(d) From 3.5 ºC to 4.5ºC at 76 mm of Hg

26. In which of the following process, convection does not take place primarily **[2005S]**
(a) sea and land breeze
(b) boiling of water
(c) heating air around a furnace
(d) warming of glass of bulb due to filament

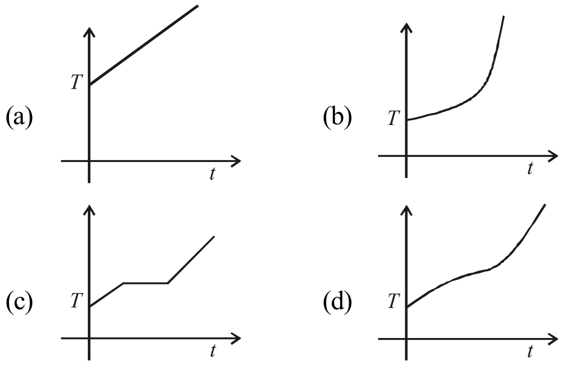
27. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following option is the correct match ? **[2005S]**



(a) Sun-T_3, tungsten filament - T_1, welding arc - T_2
(b) Sun-T_2, tungsten filament - T_1, welding arc - T_3
(c) Sun-T_3, tungsten filament - T_2, welding arc - T_1
(d) Sun-T_1, tungsten filament - T_2, welding arc - T_3

28. Two identical rods are connected between two containers one of them is at 100ºC and another is at 0ºC. If rods are connected in parallel then the rate of melting of ice is q_1 gm/sec. If they are connected in series then the rate is q_2. The ratio q_2/q_1 is **[2004S]**
(a) 2 (b) 4 (c) 1/2 (d) 1/4

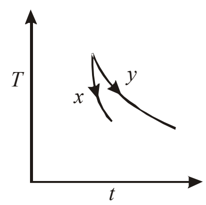
29. If liquefied oxygen at 1 atmospheric pressure is heated from 50 k to 300 k by supplying heat at constant rate. The graph of temperature vs time will be **[2004S]**



30. Three discs A, B and C having radii 2, 4, and 6 cm respectively are coated with carbon black. Wavelength for maximum intensity for the three discs are 300, 400 and 500 nm respectively. If Q_A, Q_B and Q_C are power emitted by A, B and D respectively, then **[2004S]**
(a) Q_A will be maximum (b) Q_B will be maximum
(c) Q_C will be maximum (d) $Q_A = Q_B = Q_C$

31. 2 kg of ice at –20^{0}C is mixed with 5kg of water at 20^{0}C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water & ice are 1kcal/kg/^{0}C & 0.5 kcal/kg/^{0}C while the latent heat of fusion of ice is 80 kcal/kg **[2003S]**
(a) 7 kg (b) 6 kg (c) 4 kg (d) 2 kg

32. The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies **[2003S]**

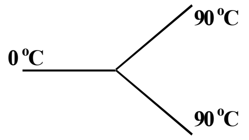


(a) $E_x > E_y$ & $a_x < a_y$
(b) $E_x < E_y$ & $a_x > a_y$
(c) $E_x > E_y$ & $a_x > a_y$
(d) $E_x < E_y$ & $a_x < a_y$

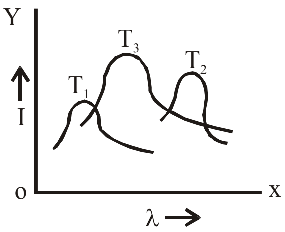
33. An ideal Black-body at room temperature is thrown into a furnace. It is observed that **[2002S]**
(a) initially it is the darkest body and at later times the brightest
(b) it is the darkest body at all times
(c) it cannot be distinguished at all times
(d) initially it is the darkest body and at later times it cannot be distinguished

34. Three rods made of same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be **[2001S]**

(a) 45°C
(b) 60°C
(c) 30°C
(d) 20°C

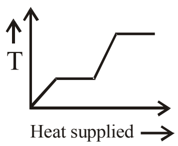


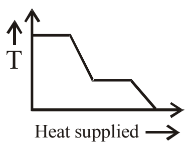
35. The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that **[2000S]**

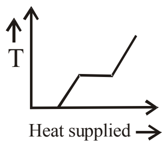


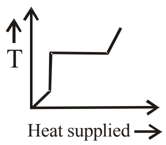
(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

36. A block of ice at –10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively ? **[2000S]**

(a) 

(b) 

(c) 

(d) 

37. A blackbody is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 *nm* and 500 nm is U_1, between 999 *nm* and 1000 *nm* is U_2 and between 1499 *nm* and 1500 *nm* is U_3. The Wien constant $b = 2.88 \times 10^6 nm\ K$. Then **[1998S - 2 Marks]**

(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

38. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. if the radius were halved and the temperature doubled, the power radiated in watt would be **[1997 - 1 Mark]**

(a) 225 (b) 450 (c) 900 (d) 1800

39. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is **[1995S]**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

40. Three rods of identical cross-sectional area and made from the same metal from the sides of an isosceles traingle *ABC*, right-angled at *B*. The points *A* and *B* are maintained at temperatures *T* and $(\sqrt{2})\,T$ respectively. In the steady state, the temperature of the point *C* is T_c. Assuming that only heat conduction takes place, T_c / T is **[1995S]**

(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$
(c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$

41. A cylinder of radius *R* made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius *R* and outer radius 2*R* made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is **[1988 - 2 Marks]**

(a) $K_1 + K_2$ (b) $K_1K_2 / (K_1 + K_2)$
(c) $(K_1 + 3K_2)/4$ (d) $(3K_1 + 3K_2)/4$

42. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilogram is **[1986 - 2 Marks]**

(a) 0.130 (b) 0.065 (c) 0.260 (d) 0.135

43. 70 calories of heat required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is : **[1985 - 2 Marks]**

(a) 30 (b) 50 (c) 70 (d) 90

44. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is **[1984- 2 Marks]**

(a) H_2 (b) F_2 (c) O_2 (d) Cl_2

45. A wall has two layers *A* and *B*, each made of different material. Both the layers have the same thickness. The thermal conductivity of the meterial of *A* is twice that of *B*. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer *A* is **[1980]**

(a) 6°C (b) 12°C (c) 18°C (d) 24°C

2 Integer Value Answer

46. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2$, (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C? **[Adv. 2016]**

47. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is **[Adv. 2015]**

48. Two spherical bodies A (radius 6 cm) and B(radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **[2010]**

49. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is **[2010]**

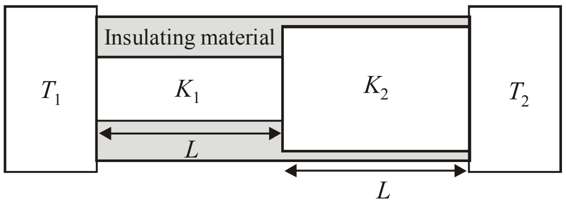
3 Numeric Answer

50. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5gm of the liquid completely evaporates. After pouring another 80gm of the liquid to its specific heat will be ___ °C.

[Neglect the heat exchange with surrounding]

[Adv. 2019]

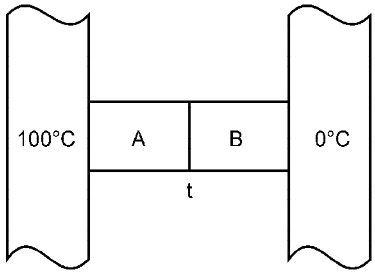
51. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = __________. **[Adv. 2018]**



4 Fill in the Blanks

52. Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 grams, the time taken to melt the ice will be... minutes. (Latent heat of fusion of ice = 3.3×10^5 J/kg.) **[1997 - 2 Marks]**

53. Two metal cubes A and B of same size are arranged as shown in Figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m°C, respectively. After steady state is reached the temperature t of the interface will be ..... **[1996 - 2 Marks]**



54. A substance of mass M kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The latent heat of fusion of the substance is .... **[1992 - 1 Mark]**

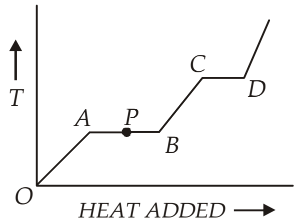
55. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than .... **[1991 - 1 Mark]**

56. A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0K. The time required for the temperature of the sphere to drop to 100K is .... **[1991 - 2 Marks]**

57. The earth receives at its surface radiation from the sun at the rate of 1400 W/m^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is...........K. **[1989 - 2 Marks]**

58. 300 grams of water at 25° C is added to 100 grams of ice at 0°C. The final temperature of the mixture is .....°C. **[1989 - 2 Marks]**

59. The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in solid state at the point O. The state of the material at the point P is .... **[1985 - 2 Marks]**

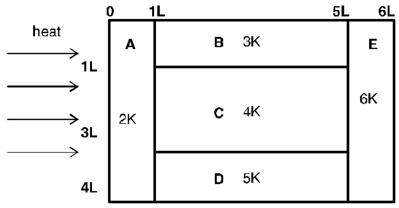


5 True / False

60. Two spheres of the same meterial have radii 1 m and 4 m and temperatures 4000K and 2000K respectively. The energy radiated per second by the first sphere is greater than that by the second. **[1988 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

61. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state **[2011]**



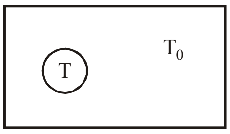
(a) heat flow through A and E slabs are same.

(b) heat flow through slab E is maximum.

(c) temperature difference across slab E is smallest.

(d) heat flow through C = heat flow through B + heat flow through D.

62. A black body of temperature T is inside chamber of T_0 temperature initially. Sun rays are allowed to fall from a hole in the top of chamber. If the temperature of black body (T) and chamber (T_0) remains constant, then **[2006 - 5M, –1]**



(a) Black body will absorb more radiation
(b) Black body will absorb less radiation
(c) Black body emit more energy
(d) Black body emit energy equal to energy absorbed by it

63. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelenth corresponding to maximum spectral radiancy in the radiation from A, by 1.00 μm. If the temperature of A is 5802 K : **[1994 - 2 Marks]**

(a) the temperature of B is 1934 K

(b) $\lambda_B = 1.5$ μm

(c) the temperature of B is 11604 K

(d) the temperature of B is 2901 K

10 Subjective Problems

64. 0.05 kg steam at 373 K and 0.45 kg of ice at 253K are mixed in an insulated vessel. Find the equilibrium temperature of the mixture. Given, $L_{fusion} = 80$ cal/g = 336 J/g, $L_{vaporization} = 540$ cal/g = 2268 J/g, $S_{ice} = 2100$ J/Kg K = 0.5 cal/gK and $S_{water} = 4200$ J/Kg K = 1 cal/gK **[2006 - 6M]**

65. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation $S = A + BT$, where $A = 100$ cal/kg-K and $B = 2 \times 10^{-2}$ cal/kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal/kg, Specific heat of water = 10^3 cal/kg-K). **[2001-5 Marks]**

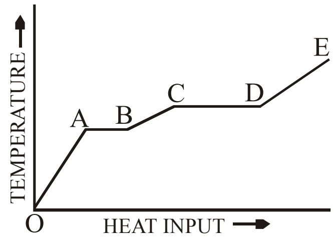
66. The temperature of 100g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose. **[1996 - 2 Marks]**

67. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius A are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster ? **[1982 - 2 Marks]**

68. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C.

(Melting point of lead = 327°C, specific heat of lead = 0.03 calories /gm/°C, latent heat of fusion of lead = 6 calories / gm, J = 4.2 joules /calorie). **[1981- 3 Marks]**

69. A Solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in the graph in figure. Study the graph carefully and answer the following questions : **[1980]**



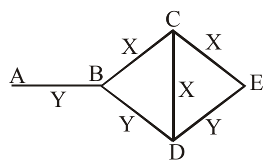
(i) What do the horizontal regions AB and CD represent?

(ii) If CD is equal to $2AB$, what do you infer?

(iii) What does the slope of DE represent?

(iv) The slope of OA > the slope of BC. What does this indicate?

70. Three rods of material X and three rods of material Y are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C. Calculate the temperature of the junctions B, C and D. The thermal conductivity of X is 0.92 cal/sec-cm-°C and that of Y is 0.46 cal/sec-cm-°C. **[1978]**



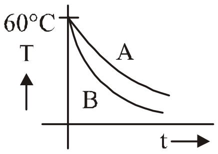
Topic-3 : Newton's Law of Cooling

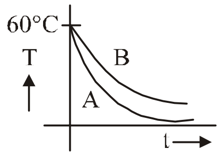
1 MCQs with One Correct Answer

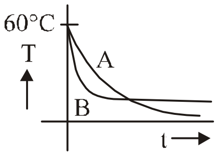
1. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to : **[Main Sep. 03, 2020 (II)]**

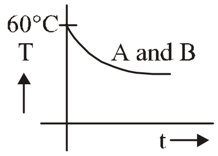
(a) 31°C (b) 33°C (c) 28°C (d) 35°C

2. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically ? (assume the emissivity of both the beakers to be the same) **[Main 8 April 2019 (I)]**

(a)


(b)


(c)


(d)


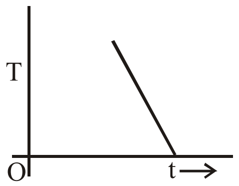
3. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately **[Main Online April 15, 2018]**

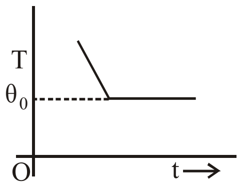
(a) 43°C (b) 47°C (c) 41°C (d) 45°C

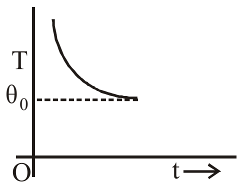
4. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is: **[Main Online April 12, 2014]**

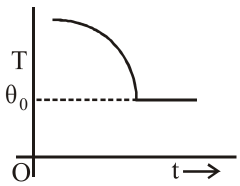
(a) 25°C (b) 10°C (c) 15°C (d) 20°C

5. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to **[Main 2013]**

(a)


(b)


(c)


(d)


Topic-4 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

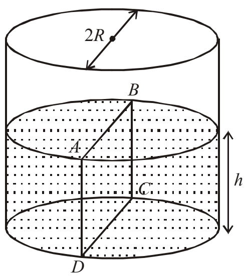
1. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) is the metal rod changes with time (t) as $T(t) = T_0(1+\beta t^{2/4})$

Where β is a constant with appropriate dimensions while T_0 is a constant with dimensions of temperature. The heat capacity of metal is : **[Adv. 2019]**

(a) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (b) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

(c) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (d) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

2. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **[2007]**



(a) $|2P_0Rh + \pi R^2 \rho gh - 2RT|$

(b) $|2P_0Rh + R\rho gh^2 - 2RT|$

(c) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$

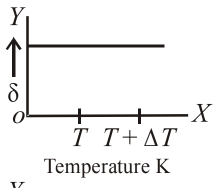
(d) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$

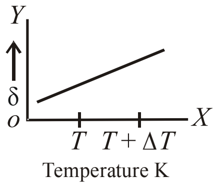
3. A spherical body of area A and emissivity $e = 0.6$ is kept inside a perfectly black body. Total heat radiated by the body at temperature T **[2005S]**

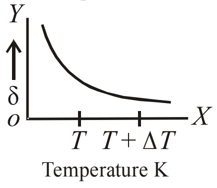
(a) $0.4\,AT^4$ (b) $0.8\,AT^4$

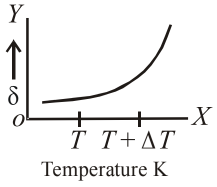
(c) $0.6\,AT^4$ (d) $1.0\,AT^4$

4. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \dfrac{\Delta V}{V\Delta T}$ varies with temperature as **[2000S]**

(a)


(b)


(c)


(d)


2 Integer Value Answer

5. A metal rod AB of length $10x$ has its one end A in ice at $0\,°C$, and the other end B in water at 100 °C. If a point P onthe rod is maintained at 400 °C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 *cal/g* and latent heat of melting of ice is 80 *cal/g*. If the point P is at a distance of λx from the ice end A, find the value of λ.

[Neglect any heat loss to the surrounding.] **[2009]**

3 Numeric Answer

6. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^{-2} and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be ____________. (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s^{-1}, Heat capacity of water = 4200 J kg^{-1} K^{-1}) **[Adv. 2020]**

6 MCQs with One or More than One Correct Answer

7. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of σT_0^4 = 460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct? **[Adv. 2017]**

(a) The amount of energy radiated by the body in 1 second is close to 60 joules

(b) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(c) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(d) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

Answer Key

Topic-1 : Thermometer & Thermal Expansion

1. (a)	2. (a)	3. (Bonus)	4. (a)	5. (a)	6. (c)	7. (c)	8. (a)	9. (c)	10. (c)
11. (a)	12. (c)	13. (d)	14. (3)	15. (20.00)	16. (60.00)	17. (b, d)			

Topic-2 : Calorimetry and Heat Transfer

1. (a)	2. (a)	3. (a)	4. (a)	5. (40)	6. (a)	7. (a)	8. (d)	9. (d)	10. (c)
11. (a)	12. (c)	13. (a)	14. (d)	15. (d)	16. (b)	17. (b)	18. (c)	19. (c)	20. (a)
21. (d)	22. (d)	23. (a)	24. (c)	25. (a)	26. (d)	27. (a)	28. (d)	29. (c)	30. (b)
31. (b)	32. (c)	33. (a)	34. (b)	35. (b)	36. (a)	37. (d)	38. (d)	39. (d)	40. (b)
41. (c)	42. (a)	43. (b)	44. (a)	45. (b)	46. (9)	47. (2)	48. (9)	49. (8)	50. (270°C)
51. (4.00)	52. (5.5)	53. (60°C)	54. $\left(\frac{P\times t}{m}\right)$	55. $\left(\frac{4\pi KTR^2}{P}\right)$	56. $\left(\frac{\rho rcdT}{3\sigma T^4}\right)$	57. (5803K)			
58. (0°C)	59. (liquid)	60. (False)	61. (a, c, d)	62. (a, c, d)	63. (a, b)				

Topic-3 : Newton's Law of Cooling

1. (b)	2. (b)	3. (a)	4. (b)	5. (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

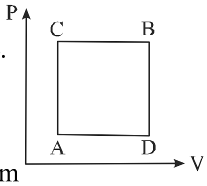
1. (d)	2. (b)	3. (d)	4. (c)	5. (9)	6. (8.33)	7. (c)

11 Thermodynamics

Topic-1 : First Law of Thermodynamics

1 MCQs with One Correct Answer

1. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is : **[Main 9 Jan. 2019 I]**



(a) 40 J (b) 80 J (c) 100 J (d) 20 J

2. 200g water is heated from 40°C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J/kgK): **[Main Online April 9, 2016]**
(a) 167.4 kJ (b) 8.4 kJ (c) 4.2 kJ (d) 16.7 kJ

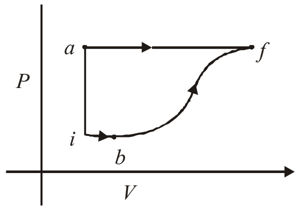
3. A gas is compressed from a volume of 2m^3 to a volume of 1m^3 at a constant pressure of 100 N/m^2. Then it is heated at constant volume by supplying 150 J of energy. As a result, the internal energy of the gas: **[Main Online April 19, 2014]**
(a) increases by 250 J (b) decreases by 250 J
(c) increases by 50 J (d) decreases by 50 J

4. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas **[2001S]**
(a) the temperature will decrease
(b) the volume will increase
(c) the pressure will remain constant
(d) the temperature will increase

2 Integer Value Answer

5. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J,

The ratio $\dfrac{Q_{bf}}{Q_{ib}}$ is **[Adv. 2014]**

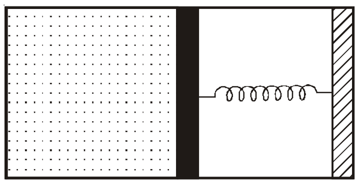


4 Fill in the Blanks

6. A container of volume 1m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300K and the other part is vacuum. The whole system is thermally isolated from the surroundings. When the partition is removed, the gas expands to occupy the whole volume. Its temperature will now be....... **[1993-1 Mark]**

6 MCQs with One or More than One Correct Answer

7. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is (are) **[Adv. 2015]**



(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4} P_1V_1$

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3} P_1V_1$

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6} P_1V_1$

8. During the melting of a slab of ice at 273 K at atmospheric pressure, **[1998S - 2 Marks]**

(a) positive work is done by the ice-water system on the atmosphere.

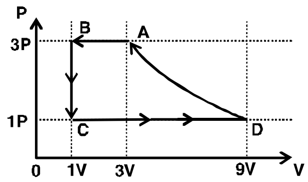
(b) positive work is done on the ice- water system by the atmosphere.

(c) the internal energy of the ice-water system increases.

(d) the internal energy of the ice-water system decreases.

7 Match the Following

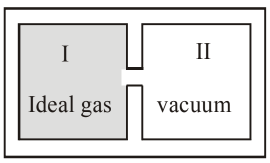
9. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I. **[2011]**

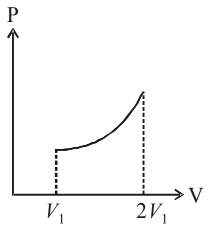


Column I	Column II
(A) Process A → B	(p) Internal energy decreases
(B) Process B → C	(q) Internal energy increases
(C) Process C → D	(r) Heat is lost
(D) Process D → A	(s) Heat is gained
	(t) Work is done on the gas

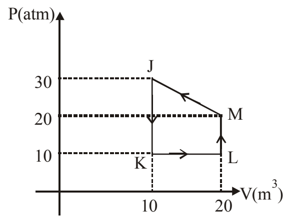
10. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[2008]**

Column I	Column II
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.	(p) The temperature of the gas decreases
(B) An ideal monoatomic gas expands to twice its remains original volume such that its pressure $P \propto 1/V^2$ where V is the volume of the gas	(q) The temperature of the gas increases or constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^{4/3}$	(r) The gas loses heat where V is its volume
(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph	(s) The gas gains heat





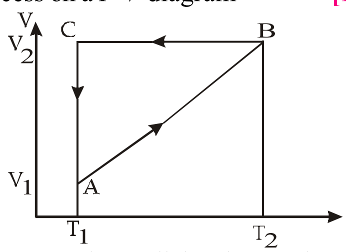
11. Heat given to process is positive, match the following option of Column I with the corresponding option of column II : **[2006 - 6M]**



Column I	Column II
(A) JK	(p) $\Delta W > 0$
(B) KL	(q) $\Delta Q < 0$
(C) LM	(r) $\Delta W < 0$
(D) MJ	(s) $\Delta Q > 0$

10 Subjective Problems

12. A cyclic process *ABCA* shown in the *V-T* diagram (fig) is performed with a constant mass of an ideal gas. Show the same process on a *P-V* diagram **[1981- 4 Marks]**

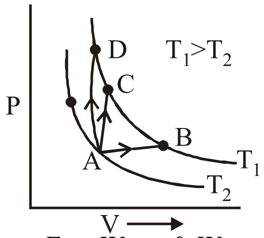


(In the figure, *CA* is parallel to the *V*-axis and *BC* is parallel to the *T*-axis)

Topic-2 : Specific Heat Capacity and Thermodynamical Processes

1 MCQs with One Correct Answer

1. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are lebelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are : **[Main 5 Sep. 2020 (I)]**



(a) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(c) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(d) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

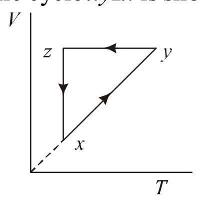
2. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is : **[Main 5 Sep. 2020 (II)]**

(a) 32 (b) 326 (c) 128 (d) $\frac{1}{32}$

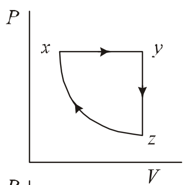
3. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as : **[3 Sep. 2020 (I)]**

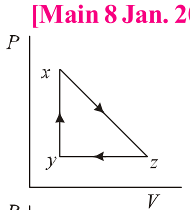
(a) irreversible isothermal (b) irreversible adiabatic
(c) reversible adiabatic (d) reversible isotherm7al

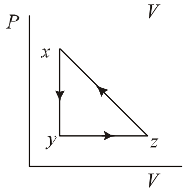
4. A thermodynamic cycle $xyzx$ is shown on a V-T diagram.

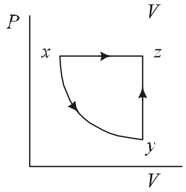


The P-V diagram that best describes this cycle is: (Diagrams are schematic and not to scale) **[Main 8 Jan. 2020 I]**

(a)


(b)


(c)


(d)


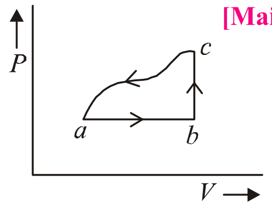
5. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is: $(3^{1.4} = 4.6555)$ [Take air to be an ideal gas] **[Main 7 Jan. 2020 I]**

(a) 60.7 J (b) 90.5 J (c) 100.8 J (d) 48 J

6. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by: **[Main 7 Jan. 2020 I]**

(a) 2 (b) $\frac{1}{2}$ (c) $\left(\frac{1}{2}\right)^{\gamma}$ (d) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

7. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is – 180 J, The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work down by the gas along the path abc is: **[Main 12 Apr. 2019 I]**



(a) 120 J (b) 130 J (c) 100 J (d) 140 J

8. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J $mol^{-1} K^{-1}$] **[Main 10 Apr. 2019 I]**

(a) 350 J (b) 374 J (c) 748 J (d) 700 J

9. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$.

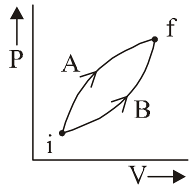
Here P_o and V_o are constants. Calculate the charge in the temperature of the gas if its volume changes from V_o to $2V_o$. **[Main 10 Apr. 2019 II]**

(a) $\frac{1}{2}\frac{P_oV_o}{R}$ (b) $\frac{5}{4}\frac{P_oV_o}{R}$

(c) $\frac{3}{4}\frac{P_oV_o}{R}$ (d) $\frac{1}{4}\frac{P_oV_o}{R}$

10. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:

[Main 9 April 2019 I]



(a) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$
(b) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$
(c) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$
(d) $\Delta Q_A = \Delta Q_B; \Delta U_A = \Delta U_B$

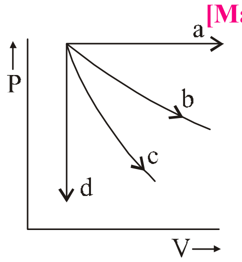
11. A thermally insulted vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closed to: (Latent heat of vaporization of water = 2.10×10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36×10^5 J kg^{-1})

[Main 8 April 2019 I]

(a) 150 g (b) 20 g (c) 130 g (d) 35 g

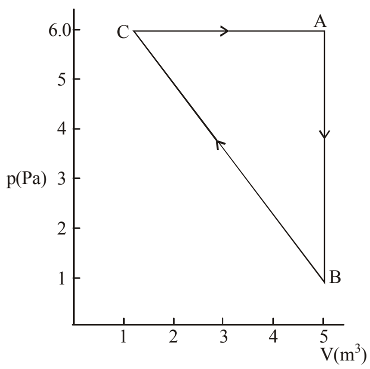
12. The given diagram shows four processes *i.e.*, isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :

[Main 8 Apr. 2019 II]



(a) a d b c (b) d a c b (c) a d c b (d) d a b c

13. For the given cyclic process CAB as shown for gas, the work done is: **[Main 12 Jan. 2019 I]**



(a) 30 J (b) 10 J (c) 1 J (d) 5 J

14. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is:

[Main 11 Jan. 2019 I]

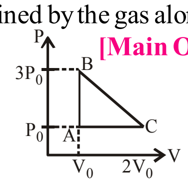
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$

15. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to: (Gas constant R = 8.31 J/mol-K)

[Main 10 Jan. 2019 II]

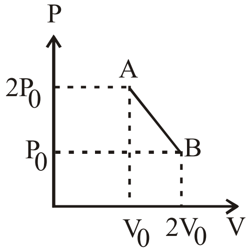
(a) 581 J (b) 291 J (c) 146 J (d) 73 J

16. One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by **[Main Online April 16, 2018]**



(a) $\frac{25}{8}\frac{P_0V_0}{R}$ (b) $\frac{25}{4}\frac{P_0V_0}{R}$
(c) $\frac{25}{16}\frac{P_0V_0}{R}$ (d) $\frac{5}{8}\frac{P_0V_0}{R}$

17. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be : **[Main 2016]**



(a) $\frac{9P_0V_0}{2nR}$ (b) $\frac{9P_0V_0}{nR}$ (c) $\frac{9P_0V_0}{4nR}$ (d) $\frac{3P_0V_0}{2nR}$

18. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is :

[Main Online April 9, 2016]

(a) $\frac{2}{5}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

19. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately **[Adv. 2016]**

(a) 112 J (b) 294 J (c) 588 J (d) 813 J

20. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$ **[Main 2015]**

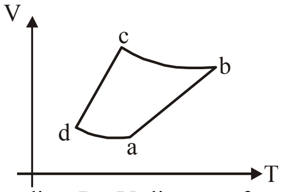
(a) $\frac{\gamma+1}{2}$ (b) $\frac{\gamma-1}{2}$ (c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

21. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is : **[Main 2015]**

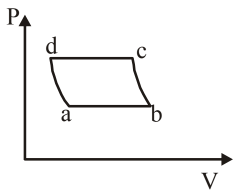
(a) $T \propto \frac{1}{R}$ (b) $T \propto \frac{1}{R^3}$

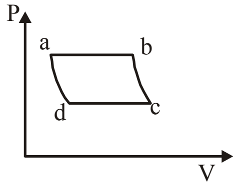
(c) $T \propto e^{-R}$ (d) $T \propto e^{-3R}$

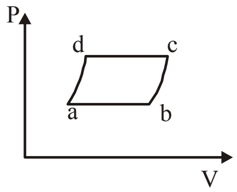
22. An ideal gas goes through a reversible cycle a→b→c→d has the V - T diagram shown below. Process d→a and b→c are adiabatic.

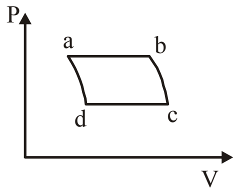


The corresponding P - V diagram for the process is (all figures are schematic and not drawn to scale) : **[Main Online April 10, 2015]**

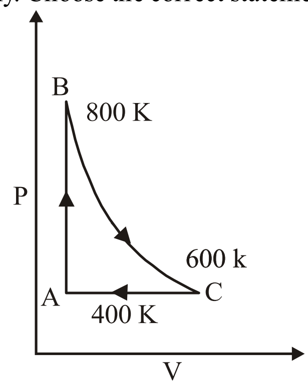
(a)


(b)


(c)


(d)


23. One mole of a diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement: **[Main 2014]**



(a) The change in internal energy in whole cyclic process is 250 R.
(b) The change in internal energy in the process CA is 700 R.
(c) The change in internal energy in the process AB is -350 R.
(d) The change in internal energy in the process BC is – 500 R.

24. During an adiabatic compression, 830 J of work is done on 2 moles of a diatomic ideal gas to reduce its volume by 50%. The change in its temperature is nearly: ($R = 8.3\ JK^{-1}\ mol^{-1}$) **[Main Online April 11, 2014]**

(a) 40 K (b) 33 K (c) 20 K (d) 14 K

25. An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of 'γ' of the gas is : **[Main Online April 22, 2013]**

(a) 1.5 (b) 1.4 (c) 1.3 (d) 1.6

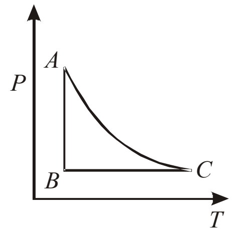
26. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is **[Main 2011]**

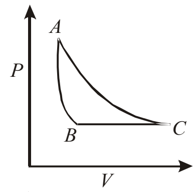
(a) $\frac{9}{8}RT_1$ (b) $\frac{3}{2}RT_1$ (c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

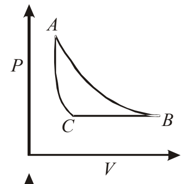
27. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process which of the following is true **[2004S]**

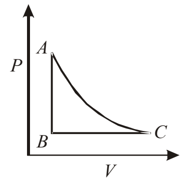
(a) $W > 0; P_3 > P_1$ (b) $W < 0; P_3 > P_1$
(c) $W > 0; P_3 < P_1$ (d) $W < 0; P_3 < P_1$

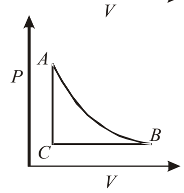
28. The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process, find the corresponding PV diagram. **[2003S]**



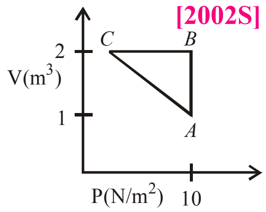
(a)


(b)


(c)


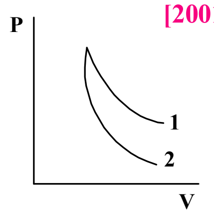
(d)


29. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process $C \to A$ is **[2002S]**



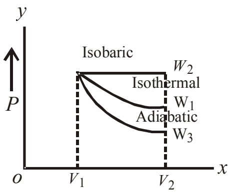
(a) –5 J
(b) –10 J
(c) –15 J
(d) –20 J

30. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to **[2001S]**



(a) He and O_2
(b) O_2 and He
(c) He and Ar
(d) O_2 and N_2

31. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then **[2000S]**



(a) $W_2 > W_1 > W_3$

(b) $W_2 > W_3 > W_1$

(c) $W_1 > W_2 > W_3$

(d) $W_1 > W_3 > W_2$

32. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the length of the gas column before and after expansion respectively, then $\frac{T_1}{T_2}$ is given by **[2000S]**

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$

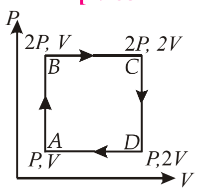
(c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

33. A given quantity of a ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is **[1998S - 2 Marks]**

(a) $\frac{2}{3}P$ (b) P

(c) $\frac{3}{2}P$ (d) $2P$

34. An ideal monatomic gas is taken round the cycle $ABCDA$ as shown in the $P-V$ diagram (see Fig.). The work done during the cycle is **[1983 - 1 Mark]**



(a) PV

(b) $2PV$

(c) $\frac{1}{2}PV$

(d) zero

2 Integer Value Answer

35. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is $a\,T_i$, the value of a is **[2010]**

3 Numeric Answer

36. An engine takes in 5 mole of air at 20°C and 1 atm, and compresses it adiabaticaly to 1/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ________. **[Main 2 Sep. 2020 (I)]**

37. Starting at temperature 300 K, one mole of an ideal diatomic gas (γ = 1.4) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ______. **[Main 9 Jan. 2020 II]**

38. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is ________. **[Adv. 2020]**

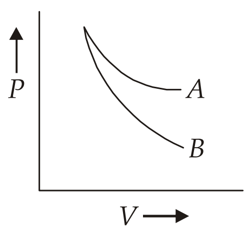
39. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R-a)$. For $a \ll R$ the magnitude of the work done in the process is given by, where X is a constant and $\gamma = C_p/C_v = 41/30$. The value of X is________. **[Adv. 2020]**

4 Fill in the Blanks

40. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is P_a. The ratio of the specific heats of the gas is 1.67. The ratio $\frac{P_a}{P_1}$ is ..... **[1994 - 2 Marks]**

5 True / False

41. The curves A and B in the figure shown P-V graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve A. **[1985 - 3 Marks]**



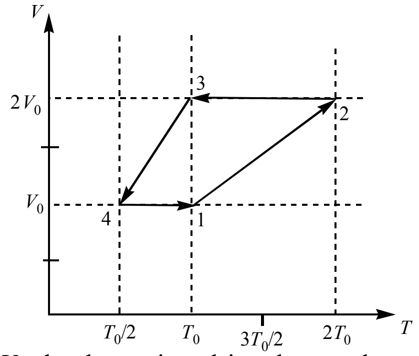
6 MCQs with One or More than One Correct Answer

42. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure

P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2}$ 9.2; R is gas constant) **[Adv. 2019]**

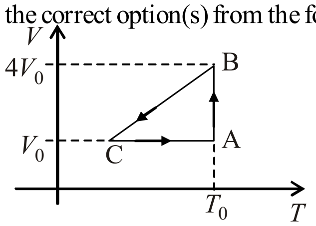
(a) The work |W| done during the process is $13RT_0$
(b) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$
(c) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$
(d) Adiabatic constant of the gas mixture is 1.6

43. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V–T) diagram. The correct statement(s) is/are: [R is the gas constant **[Adv. 2019]**



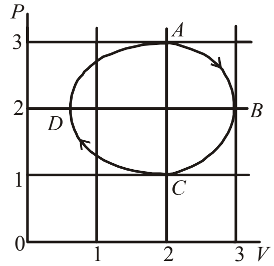
(a) Work done in this thermodynamic cycle $(1 \to 2 \to 3 \to 4 \to 1)$ is $|W| = \frac{1}{2}RT_0$
(b) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.
(c) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$
(d) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

44. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following **[2010]**



(a) Internal energies at A and B are the same
(b) Work done by the gas in process AB is $P_0V_0 \ln 4$
(c) Pressure at C is $\frac{P_0}{4}$
(d) Temperature at C is $\frac{T_0}{4}$

45. The figure shows the P-V plot of an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then, **[2009]**



(a) the process during the path $A \to B$ is isothermal
(b) heat flows out of the gas during the path $B \to C \to D$
(c) work done during the path $A \to B \to C$ is zero
(d) positive work is done by the gas in the cycle $ABCDA$

46. For an ideal gas : **[1989 - 2 Marks]**

(a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.
(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.
(c) the internal energy does not change in an isothermal process.
(d) no heat is added or removed in an adiabatic process.

7 Match the Following

47. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system. **[Main 4 Sep. 2020 (II)]**

	Process		Condition
(I)	Adiabatic	(A)	$\Delta W = 0$
(II)	Isothermal	(B)	$\Delta Q = 0$
(III)	Isochoric	(C)	$\Delta U \neq 0, \Delta W \neq 0, \Delta Q \neq 0$
(IV)	Isobaric	(D)	$\Delta U = 0$

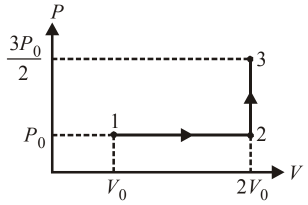
(a) (I)-(A), (II)-(B), (III)-(D), (IV)-(D)
(b) (I)-(B), (II)-(A), (III)-(D), (IV)-(C)
(c) (I)-(A), (II)-(A), (III)-(B), (IV)-(C)
(d) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

48. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ \ln\left(\frac{T}{T_A}\right) + R\ \ln\left(\frac{V}{V_A}\right)$. Here R is gas constant, V is volume of gas T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities. **[Adv. 2019]**

List-I		List-II	
(I)	Work done by the system in process 1→2→3	(P)	$\frac{1}{3}RT_0 / \ell n\, 2$
(II)	Change in internal energy in process 1→2→3	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process 1→2→3	(R)	RT_0
(IV)	Heat absorbed by the system in process 1→2	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3+\ell n\, 2)$
		(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is shown in the figure PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



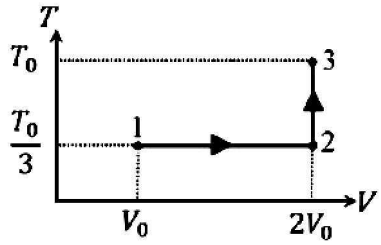
(a) I→Q, II→R, III→S, IV→U
(b) I→S, II→R, III→Q, IV→T
(c) I→Q, II→R, III→P, IV→U
(d) I→Q, II→S, III→R, IV→U

49. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ \ln\left(\frac{T}{T_A}\right) + R\ \ln\left(\frac{V}{V_A}\right)$. Here R is gas constant, V is volume of gas T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities. **[Adv. 2019]**

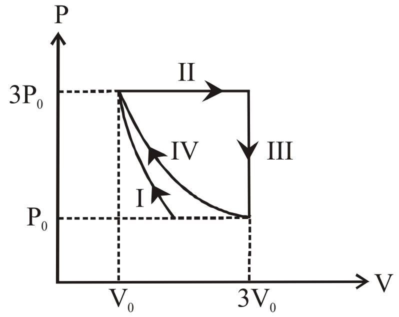
List-I		List-II	
(I)	Work done by the system in process 1→2→3	(P)	$\frac{1}{3}RT_0\ \ell n2$
(II)	Change in internal energy in process 1→2→3	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process 1→2→3	(R)	RT_0
(IV)	Heat absorbed by the system in process 1→2	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3+\ell n2)$
		(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(a) I→P, II→T, III→Q, IV→T
(b) I→S, II→T, III→Q, IV→U
(c) I→P, II→R, III→T, IV→P
(d) I→P, II→R, III→T, IV→S

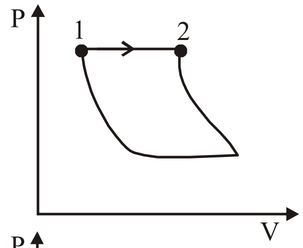
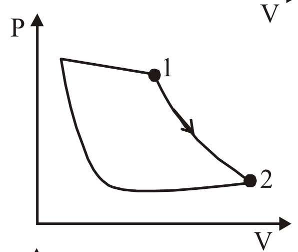
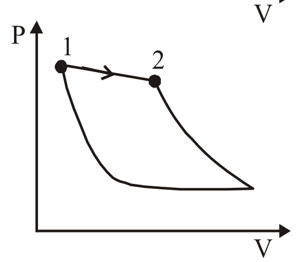
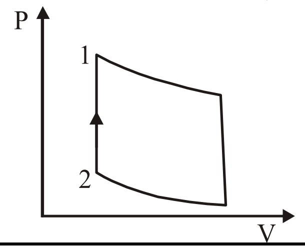
50. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II. **[Adv. 2018]**



LIST–I		LIST–II	
P.	In process I	**1.**	Work done by the gas is zero
Q.	In process II	**2.**	Temperature of the gas remains unchanged
R.	In process III	**3.**	No heat is exchanged between the gas and its surroundings
S.	In process IV	**4.**	Work done by the gas is $6P_0V0$

(a) $P \to 4; Q \to 3; R \to 1; S \to 2$
(b) $P \to 1; Q \to 3; R \to 2; S \to 4$
(c) $P \to 3; Q \to 4; R \to 1; S \to 2$
(d) $P \to 3; Q \to 4; R \to 2; S \to 1$

DIRECTIONS Q. No. 51, 52 and 53 : By appropriately matching the information given in the three columns of the following table. An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here Y is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n. **[Adv. 2017]**

Column 1	Column 2	Column 3
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

51. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(a) (I) (ii) (Q) (b) (IV) (ii) (R)
(c) (III) (iv) (R) (d) (I) (iv) (Q)

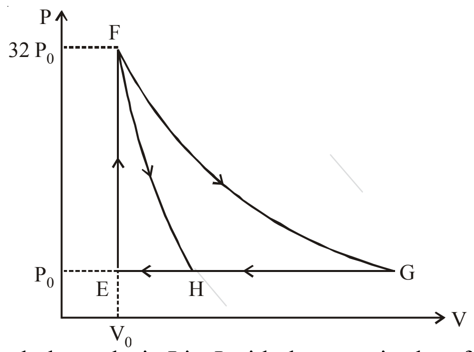
52. Which one of the following options is the correct combination?

(a) (IV) (ii) (S) (b) (III) (ii) (S)
(c) (II) (iv) (P) (d) (II) (iv) (R)

53. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(a) (II) (iv) (R) (b) (III) (iii) (P)
(c) (II) (iii) (S) (d) (II) (iii) (P)

54. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[Adv. 2013]**



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		**List II**
P.	$G \to E$	1.	$160\, P_0V_0 \ln 2$
Q.	$G \to H$	2.	$36\, P_0V_0$
R.	$F \to H$	3.	$24\, P_0V_0$
S.	$F \to G$	4.	$31\, P_0V_0$

Codes:

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

10 Subjective Problems

55. A cylinder of mass 1 kg is given heat of 20,000J at atmospheric pressure. If initially the temperature of cylinder is 20°C, find **[2005 - 6 Marks]**

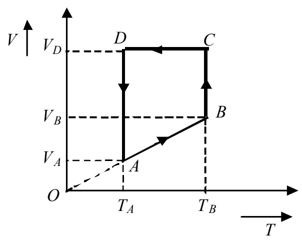
(a) final temperature of the cylinder.

(b) work done by the cylinder.

(c) change in internal energy of the cylinder

(Given that specific heat of cylinder = 400 J kg^{-1} $°C^{-1}$, coefficient of volume expansion = 9×10^{-5} $°C^{-1}$, Atmospheric pressure = 10^5 N/m^2 and Density of cylinder = 9000 kg/m^3)

56. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in figure. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C. **[2001-10 Marks]**



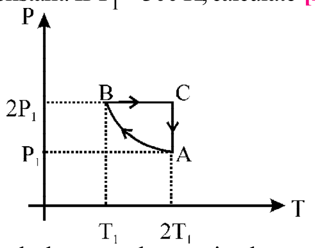
Calculate,

(a) the temperature of the gas at point B,

(b) heat absorbed or released by the gas in each process,

(c) the total work done by the gas during the complete cycle. Express your answer in terms of the gas constant R.

57. Two moles of an ideal monatomic gas is taken through a cycle $ABCA$ as shown in the P-T diagram. During the process AB, pressure and temperature of the gas vary such that PT = Constant. If T_1 = 300 K, calculate **[2000 - 10 Marks]**



(a) the work done on the gas in the process AB and

(b) the heat absorbed or released by the gas in each of the processes.

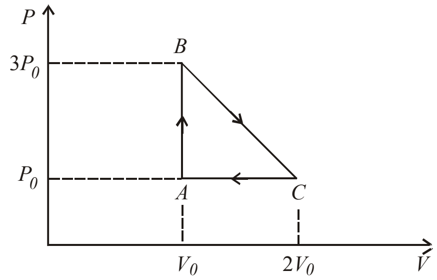
Give answer in terms of the gas constant R.

58. Two moles of an ideal monatomic gas, initially at pressure p_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2. **[1999 - 10 Marks]**

(a) Sketch the complete process on a $p - V$ diagram.

(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Give your answer in terms of p_1, V_1, V_2, Q and R]

59. One mole of an ideal monatomic gas is taken round the cyclic process $ABCA$ as shown in Figure. Calculate **[1998 - 8 Marks]**



(a) the work done by the gas.

(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB;

(c) the net heat absorbed by the gas in the path BC;

(d) the maximum temperature attained by the gas during the cycle.

60. One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \to B$ is an adiabatic compression, $B \to C$ is isobaric expansion, $C \to D$ is an adiabatic expansion, and $D \to A$ is isochoric. The volume ratios are $V_A / V_B = 16$ and $V_C / V_B = 2$ and the temperature at A is $T_A = 300$ K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **[1997 - 5 Marks]**

61. At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabaticallly to a volume $2V$. Calculate (i) the final temperature of the gas, (ii) change in its internal energy, and (iii) the work done by the gas during this process. **[1996 - 5 Marks]**

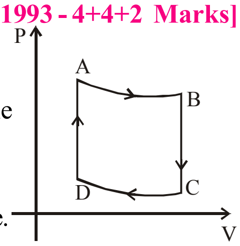
62. One mole of a mono atomic ideal gas is taken through the cycle shown in Fig: **[1993 - 4+4+2 Marks]**

$A \to B$: adiabatic expansion

$B \to C$: cooling at constant volume

$C \to D$: adiabatic compression

$D \to A$: heating at constant volume.



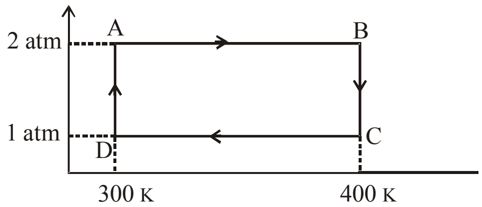
The pressure and temperature at A, B, etc. are denoted by P_A, T_A, P_B, T_B etc., respectively. Given that $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$, calculate the following quantities:

(i) The work done by the gas in the process $A \to B$

(ii) The heat lost by the gas in the process $B \to C$.

(iii) The temperature T_D. [Given : $(2/3)^{2/5} = 0.85$]

63. Two moles of helium gas undergo a cyclic process as shown in Fig. Assuming the gas to be ideal, calculate the following quantities in this process **[1992 - 8 Marks]**



(a) The net change in the heat energy
(b) The net work done
(c) The net change in internal energy

64. Three moles of an ideal gas $\left(C_p = \frac{7}{2}R\right)$ at pressure, P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure P_A. **[1991 - 4 + 4 Marks]**
(a) Sketch P - V and P - T diagrams for the complete process.
(b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.

65. An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $\frac{T}{2}$. **[1990 - 7 Mark]**
(i) How many degrees of freedom do the gas molecules have?
(ii) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.

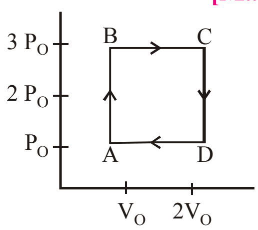
66. An ideal gas has a specific heat at constant pressure $C_P = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3, at a temperature of 300 K and a pressure of 1.6×10^6 N/m^2. An amount of 2.49×10^4 Joules of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. **[1987 - 7 Mark]**

67. Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 105 N/m^2 and 6 litres respectively. The final volume of the gas is 2litre. Molar specific heat of the gas at constant volume is 3R/2. **[1982 - 8 Marks]**

Topic-3 : Carnot Engine, Refrigerators and Second Law of Thermodynamics

1 MCQs with One Correct Answer

1. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to **[Main 6 Sep. 2020 (II)]**



(a) 19 (b) 16 (c) 8 (d) 4

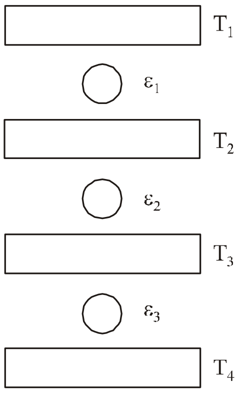
2. A heat engine is involved with exchange of heat of 1915 J, – 40 J, +125 J and – Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is: **[Main 2 Sep. 2020 (II)]**
(a) 640 J (b) 40 J (c) 980 J (d) 400 J

3. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is: **[Main 8 Jan. 2020 II]**
(a) 99 J (b) 100 J (c) 1 J (d) 90 J

4. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ? **[Main 7 Jan. 2020 II]**
(a) $T = \frac{2T_1T_2}{T_1 + T_2}$ (b) $T = \frac{T_1 + T_2}{2}$
(c) $T = \sqrt{T_1T_2}$ (d) $T = 0$

5. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively. **[Main 12 Apr. 2019 II]**
(a) 62°C, 124°C (b) 99°C, 37°C
(c) 124°C, 62°C (d) 37°C, 99°C

6. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T(4)$ The three engines are equally efficient if: **[Main 10 Jan. 2019 I]**



(a) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$

(b) $T_2 = (T_1^2 T_4)^{1/3}; T_3 = (T_1 T_4^2)^{1/3}$

(c) $T_2 = (T_1 T_4^2)^{1/3}; T_3 = (T_1^2 T_4)^{1/3}$

(d) $T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$

7. Two Carnot engines A and B are operated in series. The first one, A receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal: **[Main 9 Jan. 2019 II]**

(a) 600 K (b) 400 K (c) 300 K (d) 500 K

8. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature.The amount of work done in each cycle to operate the refrigerator is: **[Main Online April 15, 2018]**

(a) 420 J (b) 2100 J (c) 772 J (d) 2520 J

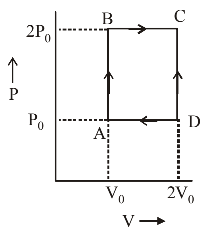
9. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency of the engine is : (Take C_v =1.5 R, where R is gas constant) **[Main Online April 8, 2017]**

(a) 0.24

(b) 0.15

(c) 0.32

(d) 0.08



10. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336 × 10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to : **[Main Online April 10, 2016]**

(a) 1.51×10^5 J (b) 1.68×10^6 J
(c) 1.71×10^7 J (d) 1.67×10^5 J

11. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways: **[Main 2015]**

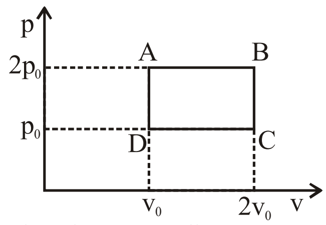
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.
In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(a) *ln2, 2ln2* (b) *2ln2, 8ln2*
(c) *ln2, 4ln2* (d) *ln2, ln2*

12. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be: **[Main Online April 12, 2014]**

(a) 20% and – 43°C (b) 40% and – 33°C
(c) 50% and – 20°C (d) 70% and – 10°C

13.



The above p-v diagramrepresents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is **[Main 2013]**

(a) $p_0 v_0$ (b) $\left(\frac{13}{2}\right) p_0 v_0$

(c) $\left(\frac{11}{2}\right) p_0 v_0$ (d) $4 p_0 v_0$

14. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A, and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then **[1998S - 2 Marks]**

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

3 Numeric Answer

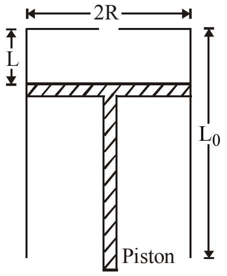
15. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer? **[Main 3 Sep. 2020 (II)]**

16. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is _______. **[Main 7 Jan. 2020 I]**

8 Comprehension/Passage Based Questions

Passage

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



17. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be **[2007]**

(a) P_0 (b) $\frac{P_0}{2}$

(c) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

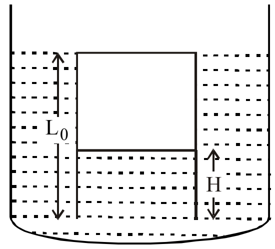
Therefore the pressure inside the cylinder is P_0 throughout the slow pulling process.

18. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is **[2007]**

(a) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (b) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(c) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (d) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

19. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies **[2007]**



(a) $\rho g(L_0-H)^2+P_0(L_0-H)+L_0P_0=0$
(b) $\rho g(L_0-H)^2-P_0(L_0-H)-L_0P_0=0$
(c) $\rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$
(d) $\rho g(L_0-H)^2-P_0(L_0-H)+L_0P_0=0$

Answer Key

Topic-1 : First Law of Thermodynamics

1. (a)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (2)	**6.** (Constant)	**7.** (a, b, c)	**8.** (b, c)
9. A-p,r,t; B-p,r; C-q,s; D-r,t							

Topic-2 : Specific Heat Capacity and Thermodynamical Processes

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)	**6** (Bonus)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (a)
21. (a)	**22.** (b)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (b)
31. (a)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (4)	**36.** (46)	**37.** (1818)	**38.** (1.78)	**39.** (2.05)	
40. (0.628)	**41.** (True)	**42.** (a, b, d)	**43.** (a, c)	**44.** (a, b)	**45.** (b, d)	**46.** (a, b, c, d)		**47.** (d)	**48.** (a)
49. (c)	**50.** (c)	**51.** (d)	**52.** (b)	**53.** (d)	**54.** (a)				

Topic-3 : Carnot Engine, Refrigerators and Second Law of Thermodynamics

1. (a)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (d)
11. (d)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (8791)	**16.** (600)	**17.** (a)	**18.** (d)	**19.** (c)	

12 Kinetic Theory

Topic-1 : Kinetic Theory of an Ideal Gas and Gas Laws

1 MCQs with One Correct Answer

1. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1, and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity) **[Main 12 Jan. 2019 II]**

(a) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$ (b) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1 l_2}\right]$

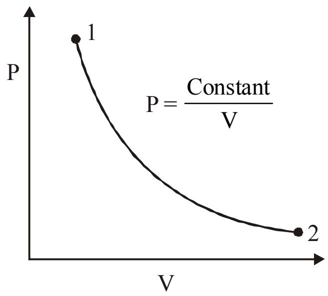
(c) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$ (d) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$

2. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : **[2017]**

(a) 2.5×10^{25} (b) -2.5×10^{25}

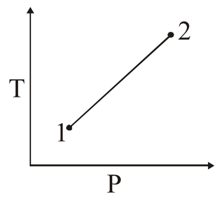
(c) -1.61×10^{23} (d) 1.38×10^{23}

3. For the P-V diagram given for an ideal gas,

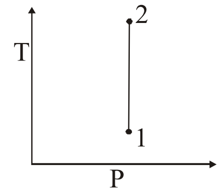


out of the following which one correctly represents the T-P diagram ? **[Main Online April 9, 2017]**

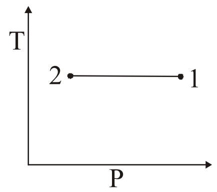
(a)



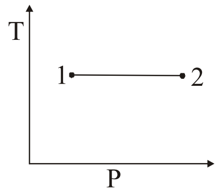
(b)



(c)



(d)



4. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is **[Adv. 2013]**

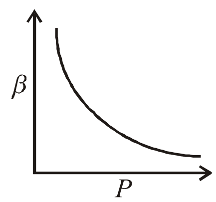
(a) 1 : 4 (b) 1 : 2 (c) 6 : 9 (d) 8 : 9

5. A real gas behaves like an ideal gas if its **[2010]**

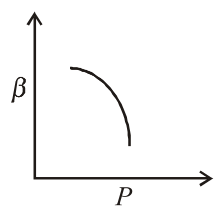
(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high

6. Which of the following graphs correctly represents the variation of $\beta = -\frac{dV/dP}{V}$ with P for an ideal gas at constant temperature? **[2002S]**

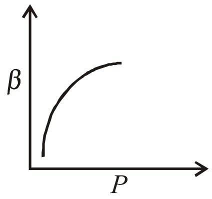
(a)



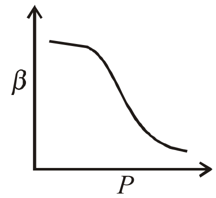
(b)



(c)



(d)

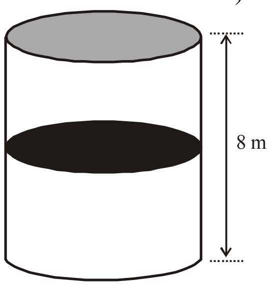


7. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A, and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and $1.5\ \Delta P$ respectively. Then **[1998S - 2 Marks]**

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

2 Integer Value Answer

8. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be _______ (take the acceleration due to gravity $= 10\ ms^{-2}$ and the universal gas constant $= 8.3\ J\ mol^{-1}K^{-1}$).



3 Numeric Answer

9. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ______. **[Main Sep. 06, 2020 (I)]**

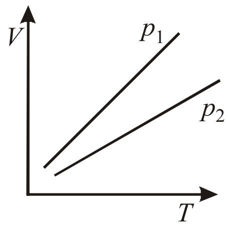
10. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$, then value of C in (K/atm.) is __________. **[Main Sep. 04, 2020 (II)]**

4 Fill in the Blanks

11. During an experiment, an ideal gas is found to obey an additional law VP^2 =constant, The gas is initially at a temperature T, and volume V. When it expands to a volume $2V$, the temperature becomes.......... **[1987 - 2 Marks]**

5 True / False

12. The volume V versus temperature T graphs for a certain amount of a perfect gas at two pressure p_1 and p_2 are as shown in Fig. It follows from the graphs that p_1 is greater than p_2. **[1982 - 2 Marks]**



Topic-2 : Speed of Gas, Pressure and Kinetic Energy

1 MCQs with One Correct Answer

1. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, $g = 980\ cm/s^2$, density of mercury $= 13.6\ g/cm^3$) **[Main Sep. 05, 2020 (I)]**

(a) 4.0×10^{18} (b) 4.0×10^{16}
(c) 5.8×10^{16} (d) 5.8×10^{18}

2. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be: **[Main 9 April 2019 I]**

(a) 100 m/s (b) $80\sqrt{5}$ m/s
(c) $100\sqrt{5}$ m/s (d) 80 m/s

3. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of : **[Main 8 April 2019 I]**

(a) $10^4\ N/m^2$ (b) $10^8\ N/m^2$
(c) $10^3\ N/m^2$ (d) $10^{16}\ N/m^2$

4. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to : **[Main 8 April 2019 II]**

[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K

Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg

Radius of Earth : 6.4×10^6 m

Gravitational acceleration on Earth $= 10\ ms^{-2}$]

(a) 800 K (b) 3×10^5 K
(c) 10^4 K (d) 650 K

5. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to : **[Main 9 Jan. 2019 I]**

(a) 3.16 (b) 0.32
(c) 0.45 (d) 2.24

6. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas? **[Main Online April 9, 2017]**

(a) $\frac{1}{2}$nRT (b) 0
(c) $\frac{3}{2}$nRT (d) $\frac{5}{2}$nRT

7. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is: **[Main Online April 10, 2015]**

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

8. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is: **[Main Online April 12, 2014]**

(a) H_2 (b) Cl_2 (c) O_2 (d) F_2

9. In the isothermal expansion of 10g of gas from volume V to 2V the work done by the gas is 575J. What is the root mean square speed of the molecules of the gas at that temperature? **[Main Online April 25, 2013]**

(a) 398m/s (b) 520m/s
(c) 499m/s (d) 532m/s

10. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is **[2012]**

(a) 0.32 (b) 0.45 (c) 2.24 (d) 3.16

11. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is **[1999S - 2 Marks]**

(a) $\sqrt{(2/7)}$ (b) $\sqrt{(1/7)}$
(c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

12. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is **[1999S - 2 Marks]**

(a) same everywhere (b) lower in the front side
(c) lower in the rear side (d) lower in the upper side

13. A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (relative molar mass 4) at a temperature 2T has a pressure of **[1997 - 1 Mark]**

(a) P/8 (b) P (c) 2P (d) 8P

14. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root-mean-square velocity of the gas molecules is v, at 480 K it becomes **[1996 - 2 Marks]**

(a) 4v (b) 2v (c) v/2 (d) v/4

15. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contain only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1 that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is **[1992 - 2 Marks]**

(a) $\frac{v_1+v_2}{2}$ (b) v_1 (c) $(v_1.v_2)^{\frac{1}{2}}$ (d) $\sqrt{\frac{3kT}{M}}$

where M is the mass of an oxygen molecule.

3 Numeric Answer

16. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is ____________. (Molar mass of N_2 gas 28 g); **[Main Sep. 05, 2020 (II)]**

5 True / False

17. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. **[1987 - 2 Marks]**

18. The ratio of the velocity of sound in Hydrogen gas ($\gamma = \frac{7}{5}$) to that in Helium gas ($\gamma = \frac{5}{3}$) at the same temperature is $\sqrt{\frac{21}{5}}$. **[1983 - 2 Marks]**

19. Two different gases at the same temperature have equal root mean square velocities. **[1982 - 2 Marks]**

20. The root-mean square speeds of the molecules of different ideal gases, maintained at the same temperature are the same. **[1981- 2 Marks]**

6 MCQs with One or More than One Correct Answer

21. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed. root mean square speed, and most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then **[1998S - 2 Marks]**

(a) no molecule can have a speed greater than $\sqrt{2}\,v_{rms}$
(b) no molecule can have a speed less than $v_p/\sqrt{2}$
(c) $v_p < \bar{v} < v_{rms}$
(d) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$.

Topic-3 : Degree of Freedom, Specific Heat Capacity and Mean Free Path

1 MCQs with One Correct Answer

1. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively, by: **[Main Sep. 06, 2020 (I)]**

(a) $U=\frac{5}{2}RT$ and $\gamma=\frac{6}{5}$ (b) $U=5RT$ and $\gamma=\frac{7}{5}$

(c) $U=\frac{5}{2}RT$ and $\gamma=\frac{7}{5}$ (d) $U=5RT$ and $\gamma=\frac{6}{5}$

2. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T is : **[Main Sep. 06, 2020 (II)]**

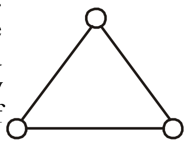
(a) T (b) $\frac{1}{\sqrt{T}}$ (c) $\frac{1}{T}$ (d) $\sqrt{T}$

3. Match the C_p/C_v ratio for ideal gases with different type of molecules : **[Main Sep. 04, 2020 (I)]**

Column-I Molecule Type	Column-II Cp/Cv
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(a) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(b) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(c) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

4. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :



[Main Sep. 03, 2020 (I)]

(a) $\frac{5}{2}RT$ (b) $\frac{3}{2}RT$ (c) $\frac{9}{2}RT$ (d) $3RT$

5. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? **[Main Sep. 03, 2020 (II)]**

(a) 5 (b) 6 (c) 3 (d) 7

6. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is : **[Main Sep. 02, 2020 (I)]**

(a) 15 (b) 13 (c) 20 (d) 11

7. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? **[Main Sep. 02, 2020 (II)]**

(1) The mean free path of the molecules decreases
(2) The mean collision time between the molecules decreases
(3) The mean free path remains unchanged
(4) The mean collision time remains unchanged

(a) (2) and (3) (b) (1) and (2)
(c) (3) and (4) (d) (1) and (4)

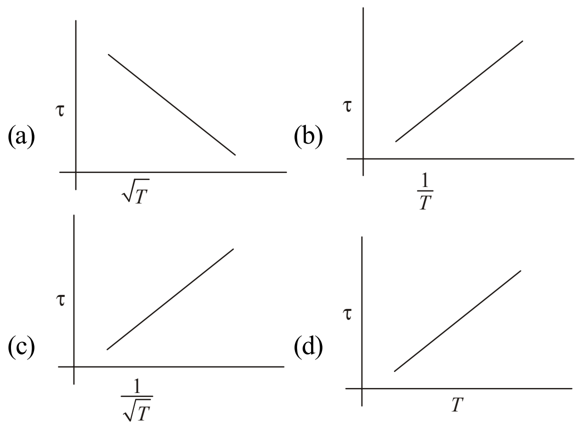
8. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats $(C_V^A$ and $C_V^B)$ of gas A and B, respectively is: **[Main 9 Jan 2020 I]**

(a) 7 : 9 (b) 5 : 9 (c) 3 : 5 (d) 5 : 7

9. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to: **[Main 9 Jan 2020 II]**

(a) 3.67 (b) 1.83 (c) 2.3 (d) 4.67

10. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale) **[Main 8 Jan. 2020 I]**



11. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be: **[Main 8 Jan. 2020 II]**

(a) 19/13 (b) 67/45
(c) 40/27 (d) 23/15

12. Two moles of an ideal gas with $\frac{C_p}{C_V}=\frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_V}=\frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is: [Main 7 Jan. 2020 I]
(a) 1.45 (b) 1.50 (c) 1.47 (d) 1.42

13. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K) [Main 12 April 2019 I]
(a) 19.7 J/mol L (b) 15.7 J/mol K
(c) 17.4 J/mol K (d) 21.6 J/mol K

14. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ? [Main 12 April 2019 II]
(a) 25 J (b) 35 J (c) 30 J (d) 40 J

15. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? [Main 10 April 2019 I]
(a) $\sim10^{12}$ (b) $\sim10^{11}$ (c) $\sim10^{10}$ (d) $\sim10^{13}$

16. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is : [Main 10 April 2019 II]
(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$ (c) $\frac{7}{5}Q$ (d) $\frac{3}{2}Q$

17. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of J mol^{-1} k^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then: [Main 9 April 2019 II]
(a) A is rigid but B has a vibrational mode.
(b) A has a vibrational mode but B has none.
(c) A has one vibrational mode and B has two.
(d) Both A and B have a vibrational mode each.

18. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas: [Main 12 Jan. 2019 I]
(a) 9×10^6 J (b) 6×10^4 J (c) 10^8 J (d) 3×10^2 J

19. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions wiil be close to: [Main 12 Jan. 2019 II]
(a) 2×10^{-7} s (b) 4×10^{-8} s
(c) 0.5×10^{-8} s (d) 3×10^{-6} s

20. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is : [Main 11 Jan. 2019 I]
(a) 15 RT (b) 12 RT (c) 4 RT (d) 20 RT

21. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion? [Main 10 Jan 2019 II]
(a) 10^3 J (b) 10^5 J (c) 10^4 J (d) 10^6 J

22. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole] [Main 9 Jan. 2019 II]
(a) 0.9 kJ (b) 6 kJ (c) 10 kJ (d) 14 kJ

23. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (1) the final temperature of the gas and (2) change in its internal energy. [Main 2018]
(a) (1) 189 K (2) 2.7 kJ
(b) (1) 195 K (2) –2.7 kJ
(c) (1) 189 K (2) –2.7 kJ
(d) (1) 195 K (2) 2.7 kJ

24. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V}=\frac{3}{2}$ for the mixture, then the value of n is [Main Online April 16, 2018]
(a) 3/2 (b) 2 (c) 1 (d) 3

25. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is : [Main 2017]
(a) a = 14 b (b) a = 28 b
(c) $a=\frac{1}{14}b$ (d) a = b

26. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is : [Main Online April 8, 2017]
(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

27. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) : [Main 2016]
(a) $n=\frac{C_P-C}{C-C_V}$ (b) $n=\frac{C-C_V}{C-C_P}$
(c) $n=\frac{C_P}{C_V}$ (d) $n=\frac{C-C_P}{C-C_V}$

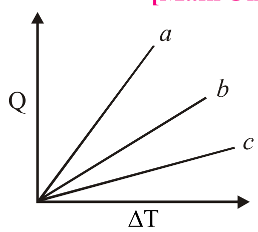
28. Using equipartition of energy, the specific heat (in J kg^{-1} K^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27) **[Main Online April 11, 2015]**

(a) 410 (b) 25 (c) 1850 (d) 925

29. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking $R = 8.0\ JK^{-1}$ $mole^{-1}$, 1 atm = 10^5 Pa and $N_{Avogadro}$ = 6×10^{23} $mole^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of: **[Main Online April 9, 2014]**

(a) 0.2 μm (b) 0.2 mm
(c) 0.2 cm (d) 0.2 nm

30. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines *a*, *b* and *c* respectively correspond to : **[Main Online April 9, 2013]**



(a) P, M and D (b) M, D and P
(c) P, D and M (d) D, M and P

31. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is **[1999S - 2 Marks]**

(a) 4 RT (b) 15 RT (c) 9 RT (d) 11 RT

32. The average translational kinetic energy of O_2 (relative molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (relative molar mass 28) molecules in eV at the same temperature is **[1997 - 1 Mark]**

(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768

33. From the following statements concerning ideal gas at any given temperature T, select the correct one(s) **[1995S]**

(a) The coefficient of volume expansion at constant pressure is the same for all ideal gases
(b) The average translational kinetic energy per molecule of oxygen gas is 3kT, k being Boltzmann constant
(c) The mean-free path of molecules increases with increases in the pressure
(d) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different

34. If one mole of a monatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for mixture is **[1988 - 1 Mark]**

(a) 1.40 (b) 1.50 (c) 1.53 (d) 3.07

35. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is **(1998S - 2 Marks)**

(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) depends on the moments of inertia of the two molecules

36. Two cylinders *A* and *B* fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of *A* is free to move, while that *B* is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in *A* is 30 K, then the rise in temperature of the gas in *B* is **[1998S - 2 Marks]**

(a) 30 K (b) 18 K (c) 50 K (d) 42 K

37. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is **[1990 - 2 Marks]**

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

3 Numeric Answer

38. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to ________. **[Main Sep. 04, 2020 (I)]**

4 Fill in the Blanks

39. One mole of a mono-atomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is .... **[1984- 2 Marks]**

5 True / False

40. At a given temperature, the specific heat of a gas at constant pressure is always greater than its specific heat at constant volume. **[1987 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

41. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature *T*. Assuming the gases are ideal, the correct statement(s) is (are) **[Adv. 2015]**

(a) The average energy per mole of the gas mixture is $2RT$
(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$
(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2
(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$

42. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then [2009]
(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas
(c) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas
(d) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

9 Assertion and Reason Type Questions

43. **Statement-1 :** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume. [2007]
because
Statement-2 : The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-2 is False, Statement-2 is True

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol.K) [2012]
(a) 62 J (b) 104 J (c) 124 J (d) 208 J

2. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is – [2008]
(a) 1/T (b) 2/T (c) 3/T (d) 4/T

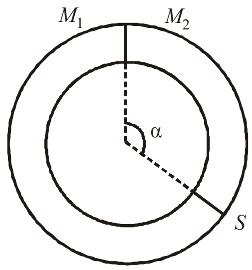
3. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is [1990 - 2 Marks]
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

3 Numeric Answer

4. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0\ J\,mol^{-1}\,K^{-1}$, the decrease in its internal energy, in *Joule*, is__________. [Adv. 2018]

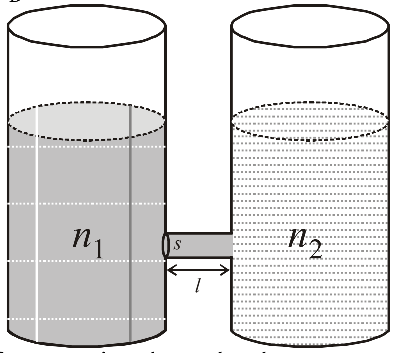
4 Fill in the Blanks

5. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figure is .... degrees. [1997 - 2 Marks]



6 MCQs with One or More than One Correct Answer

6. As shown schematically in the figure, two vessels contain water solutions (at temperature T) of potassium permanganate ($KMnO_4$) of different concentrations n_1 and n_2 ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) << n_1$. When they are connected by a tube of small length l and cross-sectional area S, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force $-\beta v$ on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant)



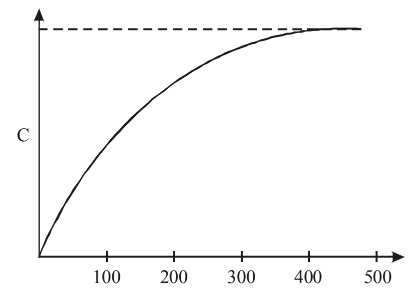
(a) the force causing the molecules to move across the tube is $\Delta n k_B TS$

(b) force balance implies $n_1 \beta v l = \Delta n k_B T$

(c) total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{l}\right)\left(\frac{k_B T}{\beta}\right) S$

(d) rate of molecules getting transferred through the tube does not change with time

7. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **[Adv. 2013]**



(a) The rate at which heat is absorbed in the range 0 -100 K varies linearly with temperature T.

(b) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

(c) There is no change in the rate of heat absorption in the range 400-500 K.

(d) The rate of heat absorption increases in the range 200-300 K.

8. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P-V diagram. Select the correct statement (s) from the following: **[1993-2 Marks]**

(a) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.

(b) In the T-V diagram, the path AB becomes a part of a parabola

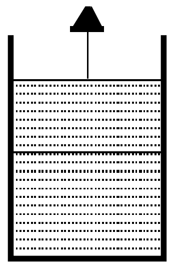
(c) In the P-T diagram, the path AB becomes a part of a hyperbola

(d) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

8 Comprehension/Passage Based Questions

Passage -1

In the figure, a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulated material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$.



9. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be **(Adv. 2014)**

(a) 550 K (b) 525 K

(c) 513 K (d) 490 K

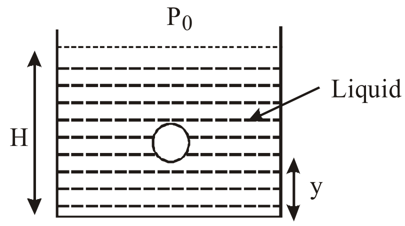
10. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. The total work done by the gases till the time they achieve equilibrium will be **[Adv. 2014]**

(a) 250 R (b) 200 R

(c) 100 R (d) – 100 R

Passage - 2

A small spherical monoatomic ideal gas bubble ($\gamma = 5/3$) is trapped inside a liquid of density ρ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension). **[2008]**



11. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(a) Only the force of gravity

(b) The force due to gravity and the force due to the pressure of the liquid

(c) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(d) The force due to gravity and the force due to viscosity of the liquid

12. When the gas bubble is at a height y from the bottom, its temperature is –

(a) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$ (b) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(c) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$ (d) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

13. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(a) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

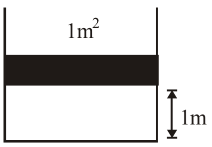
(b) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(c) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(d) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

10 Subjective Problems

14. A diatomic gas is enclosed in a vessel fitted with massless movable piston. Area of cross section of vessel is 1 m^2. Initial height of the piston is 1 m (see the figure). The initial temperature of the gas is 300 K. The temperature of the gas is increased to 400 K, keeping pressure constant, calculate the new height of the piston. The piston is brought to its initial position with no heat exchange. Calculate the final temperature of the gas. You can leave answer in fraction. **[2004 - 2 Marks]**



15. An insulated container containing monoatomic gas of molar mass *m* is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[2003 - 2 Marks]**

16. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take R = $\frac{25}{3}$ J/mol-K and k = 1.38×10^{-23} J/K **[2002 - 5 Marks]**

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

17. A gaseous mixture enclosed in a vessel of volume *V* consists of one mole of a gas *A* with $\gamma\,(=C_p/C_v) = 5/3$ and another gas *B* with $\gamma = 7/5$ at a certain temperature *T*. The relative molar masses of the gases *A* and *B* are 4 and 32, respectively. The gases *A* and *B* do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes. **[1995 - 10 Marks]**

(a) Find the number of moles of the gas *B* in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at *T* = 300 K.

(c) If *T* is raised by 1K from 300 K, find the % change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to 1/5 of its initial volume *V*. Find the change in its adiabatic compressibility in terms of the given quantities.

18. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of 1×10^5 Nm^{-2}. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to be ideal (Universal gas constant *R* = 8.314 J/mol – *K*). **[1994 - 6 Marks]**

19. An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross-section 8.0×10^{-3} m^2. Initially the gas is at 300 K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed (unstretched, uncompressed) state, fig. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure is 1.0×10^5 Nm^{-2}. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to be massless. **[1989 - 8 Mark]**

20. Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. **[1988 - 6 Marks]**

(i) Sketch the process on a *p-V* diagram.

(ii) What are the final volume and pressure of the gas ?

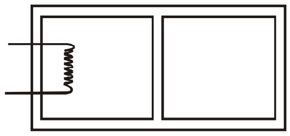
(iii) What is the work done by the gas ?

21. A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal end containing air at the same pressure P. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46cm and 44.5 cm respectively.

Calculate the pressure P in centimeters of mercury. (The temperature of the system is kept at 30°C). **[1986 - 6 Marks]**

22. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. **[1985 - 6 Marks]**

23. The rectangular box shown in Fig has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a mono-atomic ideal gas ($\gamma = 5/3$) at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $243\,P_0/32$. Determine (i) the final temperature of the gas in each chamber and (ii) the work done by the gas in the right chamber. **[1984- 8 Marks]**



24. One gram mole of oxygen at 27° and one atmospheric pressure is enclosed in a vessel. **[1983 - 8 Marks]**

(i) Assuming the molecules to be moving with V_{rms}, Find the number of collisions per second which the molecules make with one square metre area of the vessel wall.

(ii) The vessel is next thermally insulated and moved with a constant speed V_o. It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed V_o.

25. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperature of the jar is reduced by 1%. The saturated vapour pressures of water at the two temperatures are 30 and 25 mm of Hg.

Calculate the new pressure in the jar. **[1980]**

26. Given samples of 1 c.c. of hydrogen and 1c.c. of oxygen, both at N.T.P. which sample has a larger number of molecules? **[1979]**

Answer Key

Topic-1 : Kinetic Therory of an Ideal Gas and Gas Laws

1. (d) **2.** (b) **3.** (c) **4.** (d) **5.** (d) **6.** (a) **7.** (c)
8. (Bonus)/6 **9.** (5) **10.** (150) **11.** ($\sqrt{2T}$) **12.** (False)

Topic-2 : Speed of Gas, Pressure and Kinetic Energy

1. (a) **2.** (c) **3.** (Bouns) **4.** (c) **5.** (a) **6.** (a) **7.** (c) **8.** (a) **9.** (c) **10.** (d)
11. (c) **12.** (b) **13.** (c) **14.** (b) **15.** (b) **16.** (41) **17.** (False) **18.** (False) **19.** (False)
20. (False) **21.** (c, d)

Topic-3 : Degree of Freedom, Specific Heat Capacity and Mean Free Path

1. (c) **2.** (b) **3.** (c) **4.** (d) **5.** (b) **6.** (a) **7.** (a) **8.** (d) **9.** (Bonus)
10. (c) **11.** (a) **12.** (d) **13.** (c) **14.** (b) **15.** (c) **16.** (c) **17.** (b) **18.** (a) **19.** (b)
20. (a) **21.** (c) **22.** (c) **23.** (c) **24.** (b) **25.** (a) **26.** (d) **27.** (d) **28.** (d) **29.** (b)
30. (b) **31.** (d) **32.** (c) **33.** (a) **34.** (b) **35.** (a) **36.** (d) **37.** (d) **38.** (266.67)
39. (2R) **40.** (True) **41.** (a, b, c) **42.** (b, d) **43.** (b)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (d) **2.** (c) **3.** (d) **4.** (900J) **5.** (192°) **6.** (a, b, c) **7.** (b, c, d) **8.** (a, b, d) **9.** (d)
10. (d) **11.** (d) **12.** (b) **13.** (b)

13 Oscillations

Topic-1 : Displacement, Phase, Velocity and Acceleration in S.H.M.

1 MCQs with One Correct Answer

1. The position co-ordinates of a particle moving in a 3-D coordinate system is given by **[Main 9 Jan 2019, II]**
   $x = a \cos\omega t$
   $y = a \sin\omega t$
   and $z = a\omega t$
   The speed of the particle is:
   (a) $\sqrt{2}\, a\omega$ (b) $a\omega$ (c) $\sqrt{3}\, a\omega$ (d) $2a\omega$

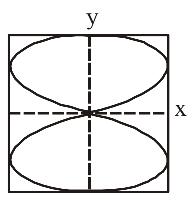
2. Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.
   $x(t) = A \sin (at + \delta)$
   $y(t) = B \sin (bt)$
   Identify the correct match below **[Main Online April 15, 2018]**
   (a) Parameters: $A = B$, $a = 2b$; $\delta = \frac{\pi}{2}$; Curve: Circle
   (b) Parameters: $A = B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Line
   (c) Parameters: $A \neq B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Ellipse
   (d) Parameters: $A \neq B$, $a = b$; $\delta = 0$; Curve: Parabola

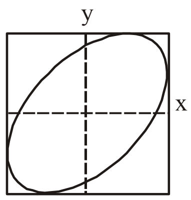
3. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $10\ s^{-1}$. At, $t = 0$ the displacement is 5 m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$ **[Main Online April 8, 2017]**
   (a) $500\ m/s^2$ (b) $500\sqrt{2}\ m/s^2$
   (c) $750\ m/s^2$ (d) $750\sqrt{2}\ m/s^2$

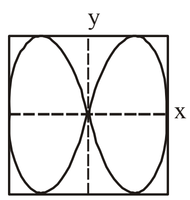
4. A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[Main 2016]**
   (a) $A\sqrt{3}$ (b) $\frac{7A}{3}$ (c) $\frac{A}{3}\sqrt{41}$ (d) 3A

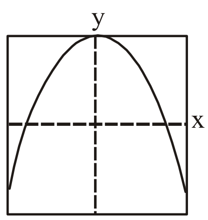
5. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time $t = 0$ one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is: **[Main Online April 9, 2016]**
   (a) $\frac{5T}{6}$ (b) $\frac{T}{3}$ (c) $\frac{T}{4}$ (d) $\frac{T}{6}$

6. x and y displacements of a particle are given as $x(t) = a \sin \omega t$ and $y(t) = a \sin 2\omega t$. Its trajectory will look like : **[Main Online April 10, 2015]**

   (a) 

   (b) 

   (c) 

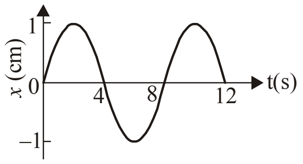
   (d) 

7. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants? **[Main Online April 12, 2014]**
   (a) $a + bx - cx^2$ (b) bx^2
   (c) $a - bx + cx^2$ (d) $-bx$

8. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **[2011]**

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$ (c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

9. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **[2009]**



(a) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (b) $\frac{-\pi^2}{32}$ cm/s^2

(c) $\frac{\pi^2}{32}$ cm/s^2 (d) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

4 Fill in the Blanks

10. Two simple harmonic motions are represented by the equations $y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

Their amplitudes are in the ratio of .... **(1986 - 2 Marks)**

6 MCQs with One or More than One Correct Answer

11. The function $x = A\sin^2 \omega t + B\cos^2 \omega t + C\sin \omega t \cos \omega t$ represent SHM for which of the option(s)

(a) for all value of A, B and C $(C \neq 0)$ **(2006 - 5M, –1)**

(b) $A = B, C = 2B$

(c) $A = -B, C = 2B$

(d) $A = B, C = 0$

12. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then. **(1999S - 3marks)**

(a) the resultant amplitude is $(1+\sqrt{2})a$

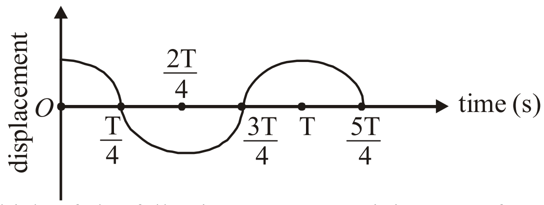
(b) the phase of the resultant motion relative to the first is 90°

(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion

(d) the resulting motion is not simple harmonic.

Topic-2 : Energy in Simple Harmonic Motion

1. The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion? **[Main Sep. 02, 2020 (II)]**

(1) The force is zero at $t = \frac{3T}{4}$

(2) The acceleration is maximum at $t = T$

(3) The speed is maximum at $t = \frac{T}{4}$

(4) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(a) (1), (2) and (4) (b) (2), (3) and (4)

(c) (1), (2) and (3) (d) (1) and (4)

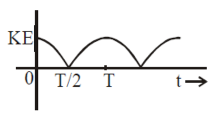
2. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210s will be : **[Main 11 Jan 2019 (I)]**

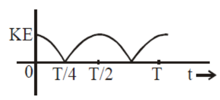
(a) $\frac{1}{9}$ (b) 1 (c) 2 (d) $\frac{1}{3}$

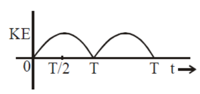
3. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be: **[Main 9 Jan 2019 (II)]**

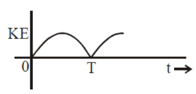
(a) $\frac{A}{2}$ (b) $\frac{A}{2\sqrt{2}}$ (c) $\frac{A}{\sqrt{2}}$ (d) A

4. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like: **[Main 2017]**

(a) 

(b) 

(c) 

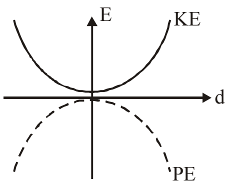
(d) 

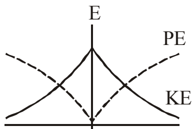
5. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm^{-1} and oscillates in a medium of constant 10^{-2} kg s^{-1}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to : **[Main Online April 9, 2017]**

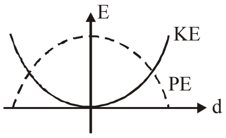
(a) 2 s (b) 3.5 s

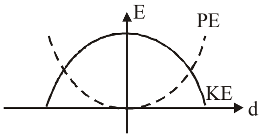
(c) 5 s (d) 7 s

6. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*) **[Main 2015]**

(a) 

(b) 

(c) 

(d) 

7. A particle free to move along the x-axis has potential energy given by $U(x) = k\,[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then **[1999S - 2marks]**
   (a) at points away from the origin, the particle is in unstable equilibrium
   (b) for any finite nonzero value of x, there is a force directed away from the origin
   (c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.
   (d) for small displacements from $x = 0$, the motion is simple harmonic

8. A particle executes simple harmonic motion with a frequency. f. The frequency with which its kinetic energy oscillates is **[1987 - 2marks]**
   (a) $f/2$ (b) f (c) $2f$ (d) $4f$

4 Fill in the Blanks

9. An object of mass 0.2 kg executes simple harmonic oscillation along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$, the object has Kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ....m. **[1994 - 2marks]**

6 MCQs with One or More than One Correct Answer

10. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its **[1989 - 2 Mark]**
    (a) maximum potential energy is 100 J
    (b) maximum kinetic energy is 100 J
    (c) maximum potential energy is 160 J
    (d) maximum potential energy is zero

Topic-3 : Time Period, Frequency, Simple Pendulum and Spring Pendulum

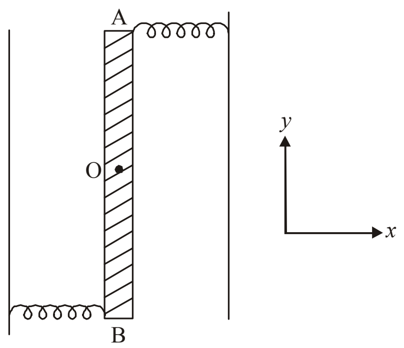
1. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is: **[Main Sep. 06, 2020 (I)]**
   (a) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (b) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$
   (c) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (d) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

2. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where '$y$' is measured from the lower end of unstretched spring. Then ω is : **[Main Sep. 06, 2020 (II)]**
   (a) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (b) $\sqrt{\frac{g}{y_0}}$
   (c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{2g}{y_0}}$

3. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is : **[Main Sep. 03, 2020 (II)]**
   (a) $\frac{1}{\sqrt{2}}$ (b) 1
   (c) $\frac{1}{2}$ (d) $\sqrt{2}$

4. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : **[Main 9 April 2019 I]**
   (a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$
   (c) $4T\sqrt{\frac{1}{15}}$ (d) $4T\sqrt{\frac{1}{14}}$

5. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is: **[Main 12 Jan 2019 (I)]**



(a) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

6. A simple harmonic motion is represented by :
$y = 5(\sin 3\pi t + \sqrt{3}\ \cos 3\pi t)$ cm
The amplitude and time period of the motion are :
[Main 12 Jan 2019 (II)]

(a) 10 cm, $\frac{2}{3}$ s (b) 10 cm, $\frac{3}{2}$ s

(c) 5 cm, $\frac{3}{2}$ s (d) 5 cm, $\frac{2}{3}$ s

7. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be: **[Main 11 Jan 2019 (II)]**

(a) $\frac{\sqrt{3}}{2}$ s (b) $\frac{2}{\sqrt{3}}$ s

(c) $\frac{3}{2}$ s (d) $2\sqrt{3}$ s

8. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water = 10^3 kg/m^3) **[Main 10 Jan 2019 (II)]**

(a) 3.75 rad s^{-1} (b) 1.25 rad s^{-1}

(c) 2.50 rad s^{-1} (d) 5.00 rad s^{-1}

9. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to : **[Main 9 Jan 2019 (II)]**

(a) 0.77 (b) 0.57

(c) 0.37 (d) 0.17

10. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[Main 2018]**

(a) 6.4 N/m (b) 7.1 N/m (c) 2.2 N/m (d) 5.5 N/m

11. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is : **[Main Online April 8, 2017]**

(a) $\frac{1}{4}$ Hz (b) $\frac{1}{2\sqrt{2}}$ Hz

(c) $\frac{1}{2}$ Hz (d) 2 Hz

12. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20° C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively : **[Main 2016]**

(a) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C (b) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C

(c) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C (d) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

13. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
(g = *gravitational acceleration*) **[Main 2015]**

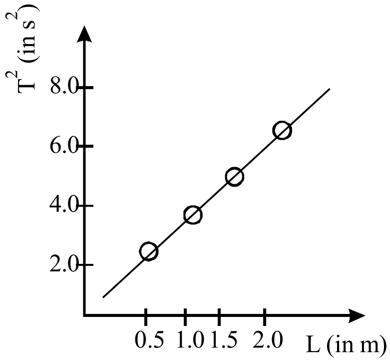
(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

14. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then: **[Main 2014]**

(a) amplitude of motion is 3a
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

15. In an experiment for determining the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure. **[Main Online April 19, 2014]**

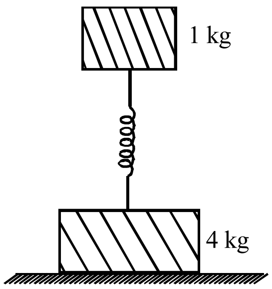


What is the value of g at the place?

(a) $9.81\ m/s^2$ (b) $9.87\ m/s^2$

(c) $9.91\ m/s^2$ (d) $10.0\ m/s^2$

16. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take g = $10\ ms^{-2}$) **[Main Online April 9, 2014]**



(a) 20 N (b) 10 N

(c) 60 N (d) 40 N

17. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[Main 2013]**

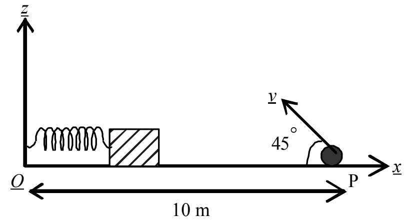
(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

18. Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to : **[Main Online April 9, 2013]**

(a) 2 (b) 7 (c) 5 (d) 3

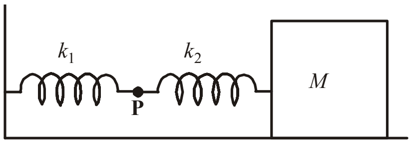
19. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v form point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10\ m/s^2$) **[2012]**



(a) $\sqrt{50}\ m/s$ (b) $\sqrt{51}\ m/s$

(c) $\sqrt{52}\ m/s$ (d) $\sqrt{53}\ m/s$

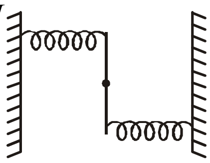
20. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[2009]**



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

21. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane.



The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is **[2009]**

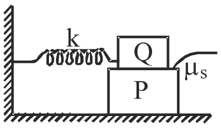
(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

22. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $(K = 1\ m/s^2)$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is ($g = 10\ m/s^2$) **[2005S]**

(a) 5/6 (b) 6/5 (c) 1 (d) 4/5

23. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks. **[2004S]**



(a) $kA/2$ (b) kA

(c) $\mu_s mg$ (d) zero

24. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by **[2000S]**

(a) $2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\dfrac{L}{g\sin\alpha}}$

(c) $2\pi\sqrt{\dfrac{L}{g}}$ (d) $2\pi\sqrt{\dfrac{L}{g\tan\alpha}}$

25. A particle of mass m is executing oscillations about the origin on the x axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is **[1998S - 2marks]**

(a) proportional to $1/\sqrt{a}$ (b) independent of a

(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

26. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to: **[1993-2marks]**

(a) $2\pi(m/K)^{1/2}$ (b) $2\pi\sqrt{\dfrac{m(YA+KL)}{YAK}}$

(c) $2\pi[(mYA/KL)]^{1/2}$ (d) $2\pi[(mL/YA)]^{1/2}$

27. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the sides faces of A. After the force is withdrawn, block A executes small oscillations the time period of which is given by **[1992 - 2mark]**

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\dfrac{M\eta}{L}}$

(c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$

28. A uniform cylinder of length L and mass M having cross sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half- submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is **[1990 - 2mark]**

(a) $\dfrac{1}{2\pi}\left(\dfrac{k-A\rho g}{M}\right)^{1/2}$ (b) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{M}\right)^{1/2}$

(c) $\dfrac{1}{2\pi}\left(\dfrac{k+\rho gL}{M}\right)^{1/2}$ (d) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{A\rho g}\right)^{1/2}$

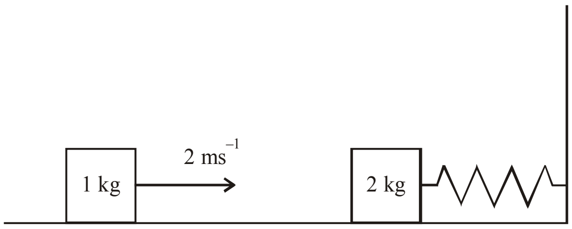
29. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is **[1988 - 1mark]**

(a) $\dfrac{k_1}{k_2}$ (b) $\sqrt{k_1/k_2}$

(c) $\dfrac{k_2}{k_1}$ (d) $\sqrt{k_2/k_1}$

3 Numeric Answer

30. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is __________. **[Adv. 2018]**



6 MCQs with One or More than One Correct Answer

31. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m(<M) is softly placed on the block after which they stick to each other. Which of the following

statement(s) is (are) true about the motion after the mass m is placed on the mass M? **[Adv. 2016]**

(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.

(b) The final time period of oscillation in both the cases is same.

(c) The total energy decreases in both the cases.

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

32. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision **[Adv. 2013]**

(a) The speed of the particle when it returns to its equilibrium position is u_0

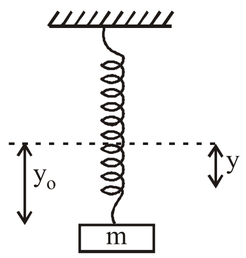
(b) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(c) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

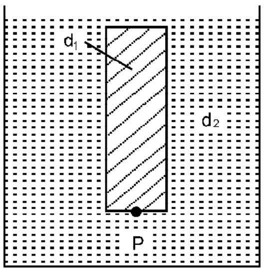
(d) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

10 Subjective Problems

33. A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency ω and amplitude a. If at a height y^* from the mean position, the body gets detached from the spring, calculate the value of y^* so that the height H attained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment. ($a\omega^2 > g$) **[2005 - 4 Marks]**



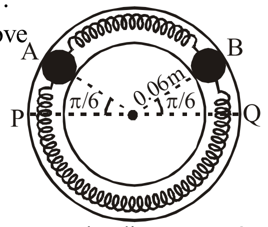
34. A thin rod of length L and area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid.



The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **[1996 - 5 Marks]**

35. Two identical balls A and B each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in Fig. The pipe is fixed in a horizontal plane.

The centres of the balls can move in a circle of radius 0.06 π meter. Each spring has a natural length zof 0.06π meter and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in Fig.) and released from rest.



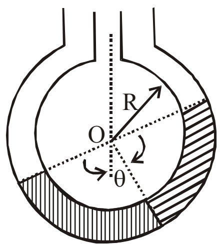
[1993 - 6 marks]

(i) Calculate the frequency of oscillation of ball B.

(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system

36. Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5ρ are poured into the two limbs of a circular tube of radius R and small cross section kept fixed in a vertical plane as shown in fig. Each liquid occupies one fourth the circumference of the tube.

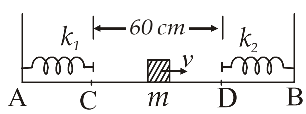


[1991 - 4 + 4 marks]

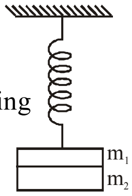
(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

37. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the springs is 60 cms. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block (k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 gm) **[1985 - 6 Marks]**



38. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.



[1981 - 3 marks]

39. A mass M attached to a spring, oscillates with a period of 2sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hook's Law is obeyed. **[1979]**

Topic-4 : Damped, Forced Oscillations and Resonance

1 MCQs with One Correct Answer

1. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to: **[Main 8 April 2019 (II)]**

(a) 50s (b) 100s (c) 20s (d) 10s

2. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}.\cos(10\pi t + \varphi)$. Here t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to : **[Main 9 Jan 2019 (II)]**

(a) 4s (b) 7s (c) 13s (d) 27s

3. An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is: $(M = 10, m = 5, u = 1, k = 1)$. **[Main Online April 16, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{3}{5}}$

4. The angular frequency of the damped oscillator is given by, $\omega = \sqrt{\left(\frac{k}{m} - \frac{r^2}{4m^2}\right)}$ where k is the spring constant, m is the mass of the oscillator and r is the damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period compared to the undamped oscillator is approximately as follows: **[Main Online April 11, 2014]**

(a) increases by 1% (b) increases by 8%
(c) decreases by 1% (d) decreases by 8%

5. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[Main 2013]**

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

6. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be : **[Main Online April 25, 2013]**

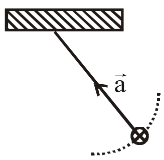
(a) Smaller than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

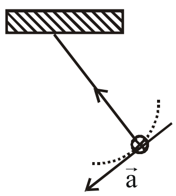
(b) $2\pi\sqrt{\frac{M}{k}}$

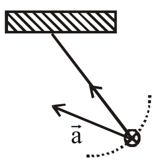
(c) Larger than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

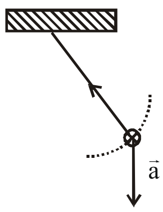
(d) $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

7. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in : **[2002S]**

(a)


(b)


(c)


(d)


8. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then **[2001S]**

(a) $T_1 < T_2$ (b) $T_1 > T_2$

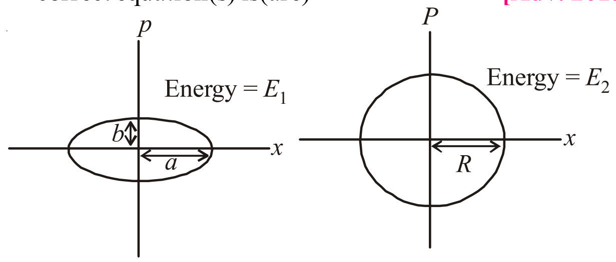
(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

2 Integer Value Answer

9. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its crosssectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is n × 10^9 Nm^{-2}, the value of n is **[2010]**

6 MCQs with One or More than One Correct Answer

10. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are) **[Adv. 2015]**

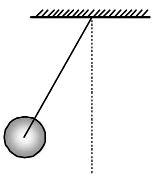


(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

11. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disc of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre.

The rod disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? **[2011]**



(a) Restoring torque in case A = Restoring torque in case B

(b) restoring torque in case A < Restoring torque in case B

(c) Angular frequency for case A > angular frequency for case B.

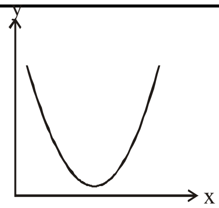
(d) Angular frequency for case A < Angular frequency for case B.

7 Match the Following

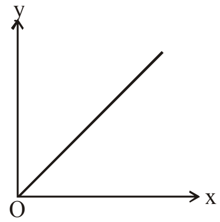
12. **[2008]**

Column I | **Column II**

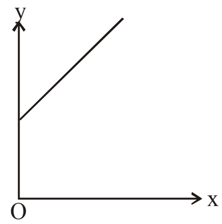
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

(p)


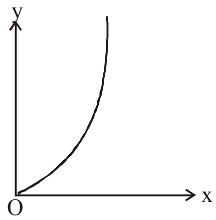
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.

(q)


(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.

(r)


(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

(s)


13. **Column I** **Column II** **[2007]**

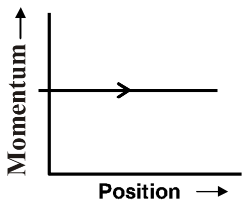
Column I	Column II
(A) The object moves on the x -axis under conservative force in such a way that its "speed " and position" satisfy $v = c_1\sqrt{c_2 - x^2}$ where c_1 and c_2 are positive constants.	(p) The object executes a a simple harmonic motion.
(B) The object moves on the x- axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.

(C)	The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r)	The kinetic energy of the object keeps on decreasing
(D)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where, M_e is the mass of the earth and R_e is the radius of the earth, Neglect forces from objects other than the earth.	(s)	The object can change its direction only once.

8 Comprehension/Passage Based Questions

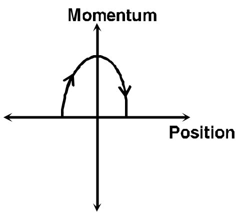
Passage- 1

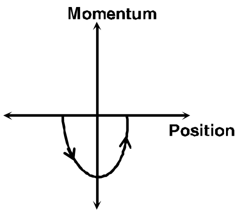
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momenum are changed. Here we consider some simple dynamical systems in one dimension.

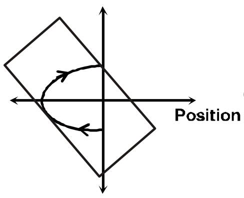


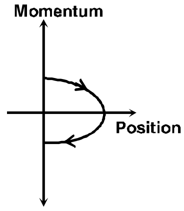
For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which positon or momentum upwards (or to right) is positive and downwards (or to left) is negative. **[2011]**

14. The phase space diagram for a ball thrown vertically up from ground is

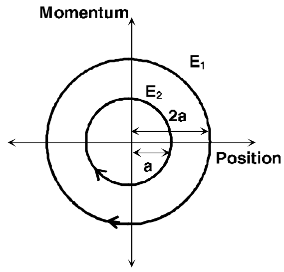
(a)


(b)


(c)


(d)


15. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



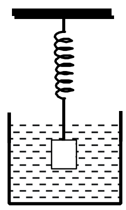
(a) $E_1 = \sqrt{2}E_2$

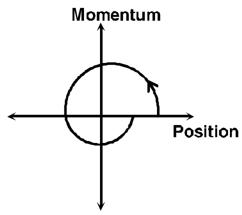
(b) $E_1 = 2E_2$

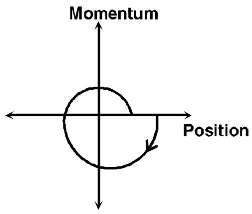
(c) $E_1 = 4E_2$

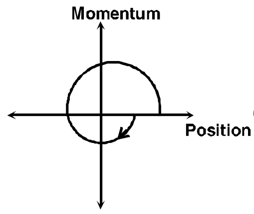
(d) $E_1 = 16E_2$

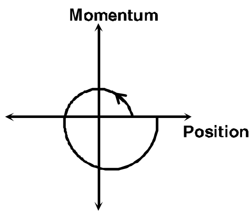
16. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(a)


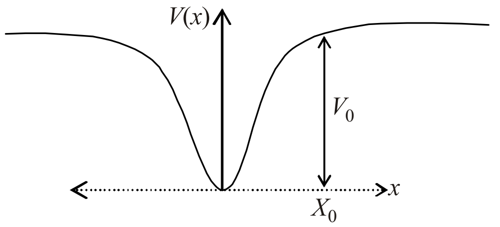
(b)


(c)


(d)


Passage - 2

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$,as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure). **[2010]**



17. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$
(b) $E > 0$
(c) $V_0 > E > 0$
(d) $E > V_0$

18. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$
(b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
(c) $A\sqrt{\frac{\alpha}{m}}$
(d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

19. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0
(b) proportional to $\frac{V_0}{mX_0}$
(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$
(d) zero

10 Subjective Problems

20. A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. **[1978]**

Answer Key

Topic-1 : Displacement, Phase, Velocity and Acceleration in S.H.M.

1. (a) **2.** (c) **3.** (b) **4.** (b) **5.** (d) **6.** (c) **7.** (d) **8.** (b) **9.** (d) **10.** (1)
11. (a, b, c) **12.** (a, c)

Topic-2 : Energy in Simple Harmonic Motion

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (b) **6.** (d) **7.** (d) **8.** (c) **9.** (0.06) **10.** (b, c)

Topic-3 : Time Period, Frequency, Simple Pendulum and Spring Pendulum

1. (b) **2.** (c) **3.** (a) **4.** (c) **5.** (c) **6.** (a) **7.** (d) **8.** (Bonus) **9.** (c) **10.** (b)
11. (c) **12.** (c) **13.** (c) **14.** (d) **15.** (b) **16.** (c) **17.** (c) **18.** (a) **19.** (a) **20.** (d)
21. (c) **22.** (b) **23.** (a) **24.** (a) **25.** (a) **26.** (b) **27.** (d) **28.** (b) **29.** (d) **30.** (2.09)
31. (a, b, d) **32.** (a, d)

Topic-4 : Damped, Forced Oscillations and Resonance

1. (c) **2.** (b) **3.** (b) **4.** (b) **5.** (c) **6.** (a) **7.** (c) **8.** (a) **9.** (4) **10.** (b, d)
11. (a, d) **12.** $(A \rightarrow p; B \rightarrow q; r, s; C \rightarrow s; D \rightarrow q)$ **13.** $(A \rightarrow p; B \rightarrow q, r; C \rightarrow p; D \rightarrow q, r)$ **14.** (d) **15.** (c)
16. (b) **17.** (c) **18.** (b) **19.** (d)

14 Waves

Topic-1 : Basic of Mechanical Waves, Progressive and Stationary Waves

1 MCQs with One Correct Answer

1. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$ Pa, $v \sim 300$ m/s, $\rho \sim 1$ kg/m^3 and $f \sim 1000$ Hz, then s will be of the order of (take the multiplicative constant to be 1) **[Main Sep. 05, 2020 (I)]**

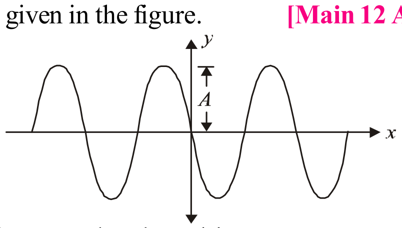
(a) $\frac{3}{100}$ mm (b) 10mm (c) $\frac{1}{10}$ mm (d) 1mm

2. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are: **[Main Sep. 04, 2020 (I)]**

(a) 1, 3, 5, ... (b) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, ...$

(c) 1, 2, 3, ... (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ...$

3. A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A\sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. **[Main 12 April 2019 I]**



For this wave, the phase ϕ is :

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

4. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? [Given reference intensity of sound as 10^{-12}W/m^2] **[Main 12 April 2019 II]**

(a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

5. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave? **[Main 12 Jan. 2019 I]**

(a) The wave is propagating along the negative x-axis with speed 25 ms^{-1}.

(b) The wave is propagating along the positive x-axis with speed 100 ms^{-1}.

(c) The wave is propagating along the positive x-axis with speed 25 ms^{-1}.

(d) The wave is propagating along the negative x-axis with speed 100 ms^{-1}.

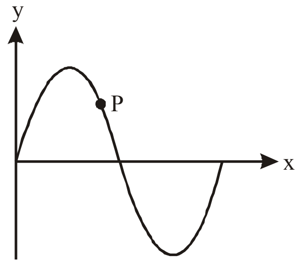
6. A transverse wave is represented by

$$y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

For what value of the wavelength the wave velocity is twice the maximum particle velocity? **[Main Online April 9, 2014]**

(a) 40 cm (b) 20 cm (c) 10 cm (d) 60 cm

7. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap–shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is – **[2008]**



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

(b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

(c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

(d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

8. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by **[2000S]**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

9. A travelling wave in a stretched string is described by the equation $y = A\sin(kx - \omega t)$ The maximum particle velocity is **[1997 - 1 Mark]**
(a) $A\omega$ (b) ω / k (c) $d\omega / dk$ (d) x / t

10. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if **[1984- 2 Marks]**
(a) $\lambda = \pi\frac{y_0}{4}$ (b) $\lambda = \pi\frac{y_0}{2}$
(c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

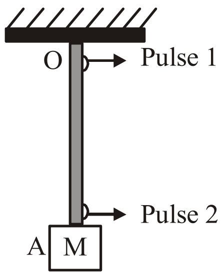
4 Fill in the Blanks

11. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ seconds, where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is .... m/s. **[1990 - 2 Marks]**

12. A travelling wave has the frequency n and the particle displacement amplitude A. For the wave the particle velocity amplitude is ---------- and the particle acceleration amplitude is ---------- . **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

13. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct? **[Adv. 2017]**



(a) The time $T_{AO} = T_{OA}$
(b) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(c) The wavelength of Pulse 1 becomes longer when it reaches point A
(d) The velocity of any pulse along the rope is independent of its frequency and wavelength

14. As a wave propagates, **[1999 - 3 Marks]**
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

15. In a wave motion $y = a\sin(kx - \omega t)$, y can represent **[1999 - 3 Marks]**
(a) electric field (b) magnetic field
(c) displacement (d) pressure

16. $y(x, t) = 0.8/[(4x+5t)^2+5]$ represents a moving pulse, where x and y are in meter and t in second. Then **[1999 - 3 Marks]**
(a) pulse is moving in $+x$ direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a sysmmetric pulse

17. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$m and $v = 10$ m s^{-1}, then λ and f are given by **[1998 - 2 Marks]**
(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m
(c) $f = 10^3\,Hz/(2\pi)$ (d) $f = 10^4\ Hz$

18. A wave is represented by the equation
$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$
where x is in meters and t is in seconds. The expression represents : **[1990 - 2 Marks]**
(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s.
(b) a wave traveling in the negative x-direction with a velocity 1.5 m/s.
(c) a wave travelling in the negative x-direction having a wavelength 0.2 m.
(d) a wave travelling in the positive x-direction having a wavelength 0.2 m.

19. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are : **[1987 - 2 Marks]**
(a) $\cos kx \sin \omega t$ (b) $k^2x^2 - \omega^2 t^2$
(c) $\cos^2(kx + \omega t)$ (d) $\cos(k^2x^2 - \omega^2 t^2)$

20. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4}\sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave **[1982 - 3 Marks]**
(a) travelling with a velocity of 30 m/s in the negative x direction
(b) of wavelength πm
(c) of frequency $30/\pi$ hertz
(d) of amplitude 10^{-4} m traveling along the negative x-direction

Topic-2 : Vibration of String and Organ Pipe

1 MCQs with One Correct Answer

1. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is : **[Main Sep. 05, 2020 (I)]**
(a) 2200 Hz (b) 550 Hz (c) 1100 Hz (d) 3300 Hz

2. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ? **[Main Sep. 03, 2020 (I)]**
(a) 3 (b) 6 (c) 12 (d) 9

3. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is: **[Main Sep. 02, 2020 (I)]**
(a) 2.25 (b) 0.44 (c) 1.25 (d) 1.5

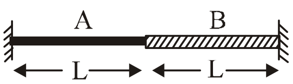
4. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to $v/2$. The value of T is close to: **[Main 8 Jan. 2020 (II)]**
(a) 2.50×10^4 N (b) 5.15×10^3 N
(c) 30.5×10^4 N (d) 10.2×10^2 N

5. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2} the extension of wire over its natural length is: **[Main 7 Jan. 2020 (I)]**
(a) 0.03 mm (b) 0.02 mm (c) 0.04 mm (d) 0.01 mm

6. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speeed of transverse waves in the string is 60 ms^{-1}. when the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to: **[Main 9 Jan. 2019 (I)]**
(a) $\frac{g}{30}$ (b) $\frac{g}{5}$ (c) $\frac{g}{10}$ (d) $\frac{g}{20}$

7. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then, v is equal to : **[Main 12 April 2019 (II)]**
(a) 332 ms^{-1} (b) 384 ms^{-1} (c) 338 ms^{-1} (d) 379 ms^{-1}

8. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio $p : q$ is : **[Main 8 April 2019 (I)]**

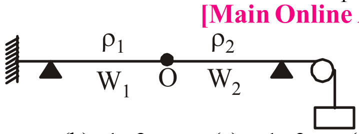


(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

9. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20,000 Hz) **[Main 10 Jan. 2019 (I)]**
(a) 6 (b) 4 (c) 7 (d) 5

10. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **[Main 2018]**
(a) 5 kHz (b) 2.5 kHz (c) 10 kHz (d) 7.5 kHz

11. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary waves is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is : **[Main Online April 8, 2017]**



(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 4 : 1

12. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is : **[Main 2016]**
(take g = 10 ms^{-2})
(a) $2\sqrt{2}$s (b) $\sqrt{2}$ s (c) $2\pi\sqrt{2}$ s (d) 2 s

13. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : **[Main 2016]**
(a) 2f (b) f (c) $\frac{f}{2}$ (d) $\frac{3f}{4}$

14. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[Main 2014]**
(a) 12 (b) 8 (c) 6 (d) 4

15. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? **[Main 2013]**
(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

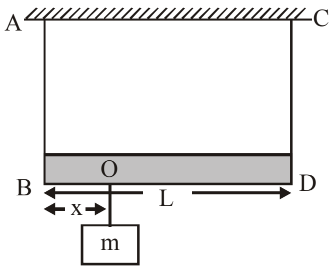
16. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ? **[Main Online April 23, 2013]**
(a) At 36 cm and 84 cm from one end
(b) At 24 cm and 72 cm from one end
(c) At 48 cm and 96 cm from one end
(d) At 72 cm and 96 cm from one end

17. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is **[2012]**
(a) 14.0 cm (b) 15.2 cm (c) 16.4 cm (d) 17.6 cm

18. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is **[2010]**
(a) 5 grams (b) 10 grams
(c) 20 grams (d) 40 grams

19. In the experiment to determine the speed of sound using a resonance column, **[2007]**
(a) prongs of the tuning fork are kept in a vertical plane
(b) prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

20. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is **[2006 - 3M, –1]**



(a) $\frac{L}{5}$
(b) $\frac{4L}{5}$
(c) $\frac{3L}{4}$
(d) $\frac{L}{4}$

21. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option **[2005S]**
(a) $n = 3,\ f_2 = \frac{3}{4} f_1$ (b) $n = 3,\ f_2 = \frac{5}{4} f_1$
(c) $n = 5,\ f_2 = \frac{3}{4} f_1$ (d) $n = 5,\ f_2 = \frac{5}{4} f_1$

22. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is **[2005S]**
(a) 51.2 cm/s (b) 102.4 cm/s
(c) 204.8 cm/s (d) 153.6 cm/s

23. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal **[2004S]**
(a) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$ (c) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

24. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **[2002S]**
(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

25. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental nodes, the one of length L with frequency v_1 and the other with frequency v_2. The raio v_1/v_2 is given by **[2000S]**
(a) 2 (b) 4 (c) 8 (d) 1

26. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **[1996 - 2 Marks]**
(a) 200 Hz (b) 300 Hz (c) 240 Hz (d) 480 Hz

27. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$ where x and y are in metre and t is in second **[1995S]**
(a) A node occurs at x = 0.15 m
(b) An antinode occurs at x = 0.3 m
(c) The speed wave is 5 ms^{-1}
(d) The wave length is 0.3 m

28. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **[1995S]**
(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$
(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

29. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is **[1988 - 1 Mark]**
(a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$
(c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$

30. A cylindrical tube open at both ends, has a fundamental frequency 'f' in air. The tube is dipped vertically in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now **[1981- 2 Marks]**
(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) f (d) $2f$

31. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is **[1988 - 2 Marks]**
(a) 8/3 (b) 3/8 (c) 1/6 (d) 1/3

32. A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz) **[1986 - 2 Marks]**
(a) 1024 (b) 512 (c) 256 (d) 128

33. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is : **[1985 - 2 Marks]**
(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

2 Integer Value Answer

34. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. **[2009]**

3 Numeric Answer

35. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is
(Young's modulus of wire $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer), ____________. **[Main Sep. 02, 2020 (II)]**

36. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is ________. **[Adv. 2020]**

4 Fill in the Blanks

37. A cylinder resonance tube open at both ends has fundamental frequency F in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is .... **[1992 - 1 Mark]**

38. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ....Hz. **[1987 - 2 Marks]**

39. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ..... metres. **[1984- 2 Marks]**

6 MCQs with One or More than One Correct Answer

40. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 *Hz* is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 *cm* and 83.9 *cm*. Which of the following statements is (are) true? **[Adv. 2017]**
(a) The speed of sound determined from this experiment is 332 *ms*$^{-1}$
(b) The end correction in this experiment is 0.9 *cm*
(c) The wavelength of the sound wave is 66.4 *cm*
(d) The resonance at 50.7 *cm* corresponds to the fundamental harmonic

41. One end of a taut string of length 3 m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) **[Adv. 2014]**
(a) $y(t) = A\sin\dfrac{\pi x}{6}\cos\dfrac{50\pi t}{3}$
(b) $y(t) = A\sin\dfrac{\pi x}{3}\cos\dfrac{100\pi t}{3}$
(c) $y(t) = A\sin\dfrac{5\pi x}{6}\cos\dfrac{250\pi t}{3}$
(d) $y(t) = A\sin\dfrac{5\pi x}{2}\cos 250\pi t$

42. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\text{ m})\sin[(62.8\text{ m}^{-1})x]\cos[(628\text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) **[Adv. 2013]**
(a) The number of nodes is 5
(b) The length of the string is 0.25 m
(c) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
(d) The fundamental frequency is 100 Hz

43. Standing waves can be produced **[1999 - 3 Marks]**
(a) on a string clamped at both the ends.
(b) on a string clamped at one end free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction

44. The (x, y) co-ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant) **[1998 - 2 Marks]**
(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$ (b) $a\sin(\pi x/L)\sin(\pi y/L)$
(c) $a\sin(\pi x/L)\sin(2\pi y/L)$ (d) $a\cos(2\pi x/L)\sin(\pi y/L)$

45. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency : **[1989 - 2 Marks]**
(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 Hz

7 Match the Following

46. A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List-I gives the above four strings while list-II lists the magnitude of some quantity. **[Adv. 2019]**

List-I		List-II	
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(a) $I \to Q, II \to P, III \to R, IV \to T$
(b) $I \to Q, II \to S, III \to R, IV \to P$
(c) $I \to P, II \to R, III \to S, IV \to Q$
(d) $I \to P, II \to Q, III \to T, IV \to S$

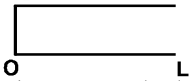
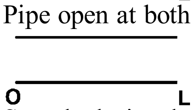
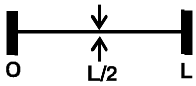
47. A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. **[Adv. 2019]**
List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I		List-II	
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}, \frac{5L_0}{4}$, and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3, and 4 are vibrated at their 1st, 3rd, 5th, and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

(a) $I \to T, II \to Q, III \to R, IV \to U$
(b) $I \to P, II \to Q, III \to T, IV \to U$
(c) $I \to P, II \to Q, III \to R, IV \to T$
(d) $I \to P, II \to R, III \to T, IV \to U$

48. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. **[2011]**

Column I	Column II
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends 	(q) Transverse waves
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

10 Subjective Problems

49. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[2005 - 4 Marks]**

50. A string tied between $x = 0$ and $x = \ell$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[2003 - 4 Marks]**

51. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. **[2003 - 2 Marks]**

52. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[2002 - 5 Marks]**

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

53. A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height H (≈ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurance of first two resonances. Speed of sound in air is 340 m/s and $g = 10$ m/s^2. **[2000 - 10 Marks]**

54. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 m s^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation. **[1998 - 8 Marks]**

(a) Find the length L of the air column.

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressures at the open end of the pipe?

(d) What are the maximum and minimum pressures at the closed end of the pipe?

55. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation—

$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ **[1985 - 6 Marks]**

Where x and y are in cm and t in seconds.

(i) What is the maximum displacement of a point at $x = 5$ cm?

(ii) Where are the nodes located along the string?

(iii) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ sec.?

(iv) Write down the equations of the component waves whose superposition gives the above wave

56. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/sec. Estimate the diameter of the tube. **[1980]**

One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.

Topic-3 : Beats, Interference and Superposition of Waves

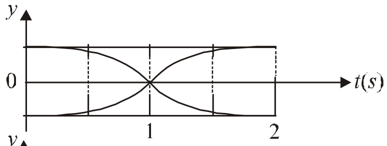
1 MCQs with One Correct Answer

1. There harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to: **[Main 9 Jan. 2020 I]**

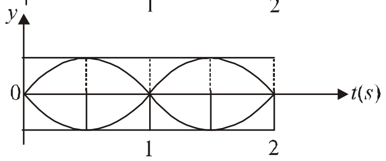
(a) $5.8\,I_0$ (b) $0.2\,I_0$ (c) $3\,I_0$ (d) I_0

2. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is : **[Main 10 April 2019 II]**

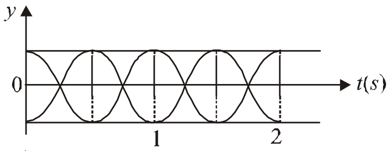
(a)



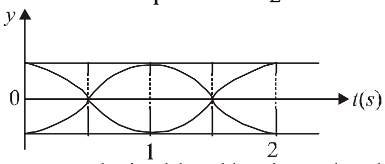
(b)



(c)



(d)



3. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: **[Main 12 Jan. 2019 II]**

(a) 322 ms^{-1} (b) 341 ms^{-1} (c) 335ms^{-1} (d) 328 ms^{-1}

4. 5 beats/ second are heard when a turning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95m or 1m . The frequency of the fork will be: **[Main Online April 15, 2018]**

(a) 195Hz (b) 251Hz (c) 150Hz (d) 300Hz

5. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **[2008]**

(a) 344 (b) 336 (c) 117.3 (d) 109.3

6. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is **[1998 - 2 Marks]**

(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0.40 s

7. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$

This expression may be considereed to be a result of the superposition of **[1992 - 2 Marks]**

(a) two (b) three (c) four (d) five

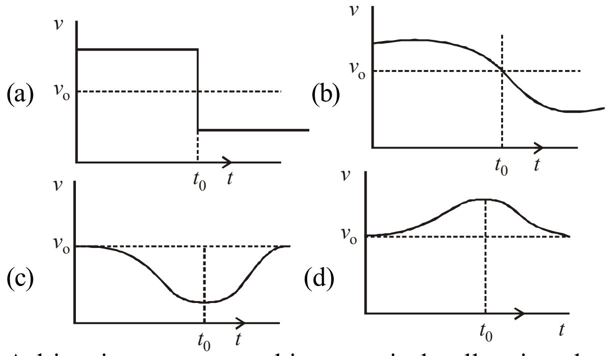
2 Integer Value Answer

8. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **[2010]**

Topic-4 : Musical Sound and Doppler's Effect

1 MCQs with One Correct Answer

1. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_o (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by: **[Main Sep. 06, 2020 (I)]**
(t_0 represents the instant when the distance between the source and observer is minimum)



2. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is : **[Main Sep. 05, 2020 (II)]**
(a) 54 km/hr (b) 36 km/hr (c) 18 km/hr (d) 24 km/hr

3. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}. **[Main Sep. 04, 2020 (II)]**
(a) 91 kmh^{-1} (b) 81 kmh^{-1} (c) 61 kmh^{-1} (d) 71 kmh^{-1}

4. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is v_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to: **[Main 7 Jan. 2020 I]**
(a) $\frac{1}{2}$ m/s (b) 1 m/s (c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}$ m/s

5. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then u equals: **[Main 12 April 2019 II]**
(a) 5.5 m/s (b) 15.0 m/s (c) 2.5 m/s (d) 10.0 m/s

6. A stationary source emits sounds waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 4801 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s) **[Main 10 April 2019 I]**
(a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

7. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = 340 ms^{-1}) **[Main 9 April 2019 II]**
(a) 2250 Hz (b) 2060 Hz (c) 2300 Hz (d) 2150 Hz

8. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 If the speed of the train is reduced to 17 m/s, the frequency registered is f_2 If speed of sound is 340 m/s, then the ratio f_1/f_2 is: **[Main 10 Jan. 2019 I]**
(a) 18/17 (b) 19/18 (c) 20/19 (d) 21/20

9. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to: **[Main 9 Jan. 2019 II]**
(a) 666 Hz (b) 753 Hz (c) 500 Hz (d) 333 Hz

10 Two engines pass each other moving in opposite directions with uniform speed of 30 m/s. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m/sec: **[Main Online April 9, 2016]**
(a) 450 Hz (b) 540 Hz (c) 270 Hz (d) 648 Hz

11. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to : **[Main 2015]**
(a) 18% (b) 24% (c) 6% (d) 12%

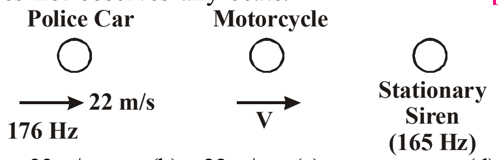
12. Two factories are sounding their sirens at 800 Hz. A man goes from one factory to other at a speed of 2m/s. The velocity of sound is 320 m/s. The number of beats heard by the person in one second will be: **[Main Online April 11, 2014]**
(a) 2 (b) 4 (c) 8 (d) 10

13. A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A, now, moves towards C with a speed 4 m/s. The number of beats heard at C is 6. When A moves away from C with speed 4 m/s, the number of beats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is : **[Main Online April 22, 2013]**

(a) 500 Hz (b) 506 Hz (c) 512 Hz (d) 494 Hz

14. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **[2011]**
(a) 8.50 kHz (b) 8.25 kHz
(c) 7.75 kHz (d) 7.50 kHZ

15. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats. **[2003S]**



(a) 33m/s (b) 22m/s (c) zero (d) 11m/s

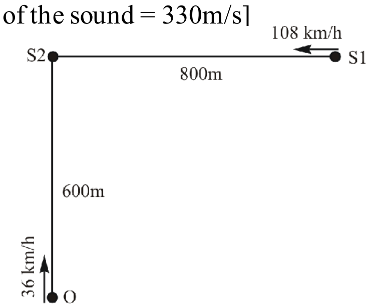
16. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train *A* records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train *B* he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train *B* to that train *A* is **[2002S]**

(a) 242/252 (b) 2 (c) 5/6 (d) 11/6

2 Integer Value Answer

17. A train S1, moving with a uniform velocity of 108 km/h approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36km/h towards S2 as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600m away from S2 and distance between S1 and S2 is 800m, the number of beats heard by O is _____

[Speed of the sound = 330m/s] **[Adv. 2019]**



18. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 *m* above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 *m* apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is __________. **[Adv. 2018]**

19. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received). **[Adv. 2017]**

20. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}, \frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of *n* is **[Adv. 2015]**

21. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. **[2010]**

3 Numeric Answer

22. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is ___________. **[Adv. 2020]**

4 Fill in the Blanks

23. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (Speed of sound in air = 342 ms^{-1}) **[1994 - 2 Marks]**

5 True / False

24. A source of sound with frequency 256 Hz is moving with a velocity *V* towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats **[1985 - 3 Marks]**

25. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. **[1980]**

7 Match the Following

26. Each of the properties of sound listed in the column *A* primarily depends on one of the quantities in column *B*. Write down the matching pairs from the two columns. **[1980]**

Column A	Column B
(A) pitch	(p) Waveform
(B) quality	(q) frequency
(C) loudness	(r) intensity

10 Subjective Problems

27. A boat is traveling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

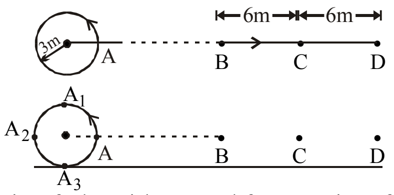
(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C; Density of river water = 10^3 kg/m^3;
Bulk modulus of the water = 2.088×10^9 Pa; Gas constant $R = 8.31$ J/mol-K;
Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4) **[2001 - 10 Marks]**

28. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist. **[1997 - 5 Marks]**

29. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. **[1996 - 3 Marks]**

30. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude $BC = CD = 6$ metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point A when the detector is at the point B. If the source emits a continous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. **[1990 - 7 Mark]**



31. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. **[1986 - 8 Marks]**

32. A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. **[1982 - 7 Marks]**

33. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? **[1981 - 4 Marks]**

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information: $\sqrt{167RT} = 640 J^{1/2} mole^{-1/2}$; $\sqrt{140RT} = 590 J^{1/2} mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.) **[Adv. 2014]**

(a) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(b) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(c) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(d) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

2. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. **[2003S]**

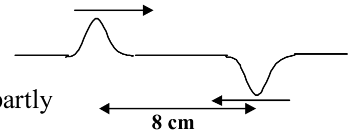
(a) 0.012m (b) 0.025m (c) 0.05m (d) 0.024m

3. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then **[2001S]**

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

4. Two pulses in a stretched string whose centers are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be **[2001S]**

(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential



5. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be **[1996 - 2 Marks]**

(a) 1.22v (b) 0.61v (c) 1.50v (d) 0.75v

5 True / False

6. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **[1984- 2 Marks]**

6 MCQs with One or More than One Correct Answer

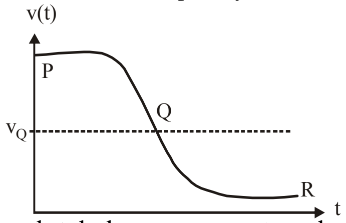
7. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses

Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person? **[Adv. 2016]**

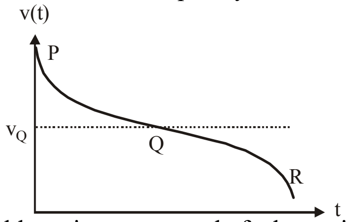
(a) $v_P + v_R = 2\,v_Q$

(b) The rate of change in beat frequency is maximum when the car passes through Q

(c) The plot below represents schematically the variation of beat frequency with time



(d) The plot below represents schematically the variation of beat frequency with time



8. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, **[2012]**

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

9. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then, **[2009]**

(a) the intensity of the sound heard at the first resonance was more than that at the second resonance

(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm

(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

8 Comprehension Based Questions

Passage - 1

Two trains A and B moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s. **[2007]**

10. The speed of sound of the whistle is

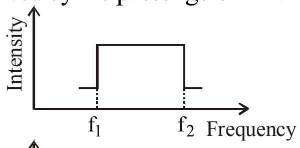
(a) 340 m/s for passengers in A and 310 m/s for passengers in B

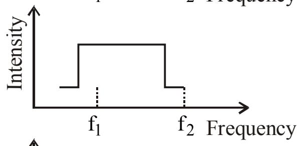
(b) 360 m/s for passengers in A and 310 m/s for passengers in B

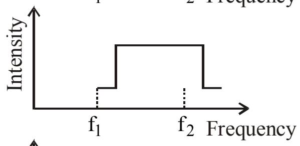
(c) 310 m/s for passengers in A and 360 m/s for passengers in B

(d) 340 m/s for passengers in both the trains

11. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(a)


(b)


(c)


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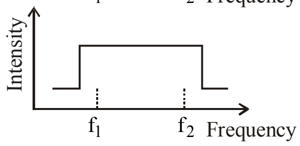


12. The spread of frequency as observed by the passengers in train B is

(a) 310 Hz (b) 330 Hz (c) 350 Hz (d) 290 Hz

Passage - 2

Waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling along x-axis. (Here x is in m and t is in second) **[2006 – 5M, –2]**

13. Find the number of times intensity is maximum in time interval of 1 sec.

(a) 4 (b) 6 (c) 8 (d) 10

14. The wave velocity of louder sound is

(a) 100 m/s (b) 192 m/s (c) 200 m/s (d) 96 m/s

15. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 sec is

(a) 100 (b) 46 (c) 192 (d) 96

10 Subjective Problems

16. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. **[1993 - 4 Marks]**

(i) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

17. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. **[1991 - 4 × 2 Marks]**
(a) What are the wavelength and frequency of incident wave?
(b) Write the equation for the reflected wave.
(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.
(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave ? What is the direction of propagation of travelling wave?

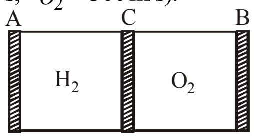
18. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train Find **[1988 - 5 Marks]**
(i) the Frequency of the whistle as heard by an observer on the hill,
(ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.
(Velocity of sound in air =1,200 km/hr)

19. The following equations represent transverse waves :
$z_1 = A\cos(kx - \omega t)$; **[1987 - 7 Marks]**
$z_2 = A\cos(kx + \omega t)$; $z_3 = A\cos(ky - \omega t)$
Identify the combination (s) of the waves which will produce (i) standing wave (s), (ii) a wave travelling in the directon making an angle of 45° degrees with the positive x and positive y axes. In each case, find the positions at which the resultant intensity is always zero.

20. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20° C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration.
Given for steel $Y = 2\times10^{11}$ N/m^2
$\alpha = 1.21\times10^{-5}$ per ^{o}C **[1984 - 6 Marks]**

21. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.
Given : Young modulus of copper $= 1.3\times10^{11}$ N/m^2.
Coefficient of linear expansion of copper
$= 1.7\times10^{-5}$ °C$^{-1.}$
Density of copper $= 9\times10^3$ kg/m^3. **[1979]**

22. AB is a cylinder of length 1m fitted with a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[1978]**



Answer Key

Topic-1 : Basic of Mechanical Waves, Progressive and Stationary Waves

1. (a)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (b)
11. 0.5 ms^{-1}	**12.** $4\pi^2\nu^2 A$	**13.** (a, d)	**14.** (a,c,d)	**15.** (a,b,c,d)	**16.** (b, c, d)	**17.** (a, c)	**18.** (b, c)	**19.** (a,c)	**20.** (a,b,c,d)

Topic-2 : Vibration of String and Organ Pipe

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (c)
31. (c)	**32.** (a)	**33.**	**34.** (5)	**35.** (35.00)	**36.** (0.62 to 0.63)		**37.** (f)	**38.** (240)	**39.** (0.5)
40. (a, b, c)	**41.** (a, c, d)	**42.** (b, c)	**43.** (a,b,c)	**44.** (b,c)	**45.** (a, b, d)	**46.** (c)	**47.** (b)		
48. A-p,t; B-p,s; C-q,s; D-q,r									

Topic-3 : Beats, Interference and Superpositon of Waves

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (5)

Topic-4 : Musical Sound and Doppler's Effect

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (d)
11. (d)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (8.13)	**18.** (5)	**19.** (6)	**20.** (3)
21. (7)	**22.** (0.62 to 0.63)		**23.** (6)	**24.** (False)	**25.** False	**26.** (A → q; B → p; C → r)			

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. **(d)**	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** True	**7.** (a,b,c)	**8.** (b,d)	**9.** (a,d)	**10.** (b)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)					

15 Electric Charges and Fields

Topic-1 : Electric Charges and Coulomb's Law

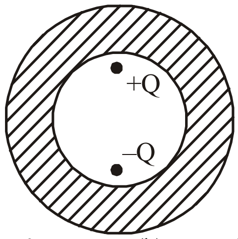
1 MCQs with One Correct Answer

1. Three charges + Q, q, + Q are placed respectively, at distance, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at $x = 0$, is zero, then value of q is: **[Main 9 Jan. 2019 (I)]**
(a) – Q/4 (b) + Q/2 (c) + Q/4 (d) – Q/2

2. Charge is distributed within a sphere of radius R with a volume charge density $p(r) = \frac{A}{r^2} e^{-2r/a}$ where A and a are constants. If Q is the total charge of this charge distribution, the radius R is: **[Main 9 Jan. 2019, (II)]**
(a) $a \log\left(1 - \frac{Q}{2\pi aA}\right)$ (b) $\frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$
(c) $a \log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$ (d) $\frac{a}{2}\log\left(1 - \frac{Q}{2\pi aA}\right)$

3. Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameter, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to **[Main Online April 16, 2018]**
(a) $\frac{3F}{4}$ (b) $\frac{F}{2}$ (c) F (d) $\frac{3F}{8}$

4. Shown in the figure are two point charges +Q and –Q inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then : **[Main Online April 10, 2015]**



(a) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$
(b) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 \neq 0, Q_2 = 0$
(c) $\sigma_1 = 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$
(d) $\sigma_1 \neq 0, Q_1 \neq 0$; $\sigma_2 \neq 0, Q_2 \neq 0$

5. Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to **[Main 2013]**
(a) y (b) –y (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

6. Two balls of same mass and carrying equal charge are hung from a fixed support of length l. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to : **[Main Online April 9, 2013]**
(a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

7. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **[2007]**
(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

8. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to : **[1987 - 2 Marks]**

(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

4 Fill in the Blanks

9. Five point charges, each of value $+q$ coul, are placed on five vertices of a regular hexagon of side L metres. The magnitude of the force on the point charge of value $-q$ coul. placed at the centre of the hexagen is .... newton. **[1992 - 1 Mark]**

10. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is .... and the tension in each string is .... newtons. **[1986 - 2 Marks]**

5 True / False

11. A ring of radius R carries a uniformly distributed charge $+Q$. *A* point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **[1988 - 2 Marks]**

12. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulombs and the other an equal negative charge. Their masses after charging are different. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

13. Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is α. The spheres are now immersed in a dielectric liquid of density 800 kg m^{-3} and dielectric constant 21. If the angle between the strings remains the same after the immersion, then **[Adv. 2020]**

(a) electric force between the spheres remains unchanged
(b) electric force between the spheres reduces
(c) mass density of the spheres is 840 kg m^{-3}
(d) the tension in the strings holding the spheres remains unchanged

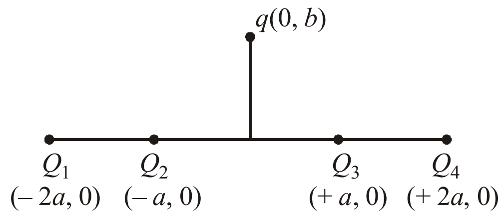
14. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y – axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x - axis. The charge Q will **[1984- 2 Marks]**

(a) execute simple harmonic motion about the origin
(b) move to the origin remain at rest
(c) move to infinity
(d) execute oscillatory but not simple harmonic motion

7 Match the Following

15. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

	List - I		List - II
P.	Q_1, Q_2, Q_3, Q_4 all positive	**1.**	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	**2.**	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	**3.**	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	**4.**	$-y$



Codes:

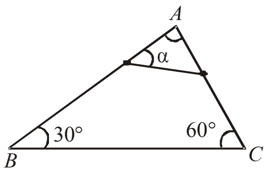
(a) P-3, Q-1, R-4, S-2 (b) P-4, Q-2, R-3, S-1 (c) P-3, Q-1, R-2, S-4 (d) P-4, Q-2, R-1, S-3

10 Subjective Problems

16. Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10$ m/s^2). **[1988 - 5 Marks]**

17. A thin fixed ring of radius 1 metre has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0. 9g and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 m from the centre of the ring. Show that the motion of the negatively charged particle is approaximately simple harmonic. Calculate the time period of oscillations. **[1982 - 5marks]**

18. A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine : **[1978]**



(i) the angle α,
(ii) the tension in the chord, and
(iii) the normal reactions on the beads.

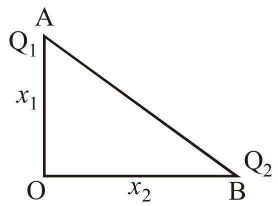
If the chord is now cut, what are the values of the charges for which the beads continue to remain stationary?

19. Three charges each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. **[1978]**
(i) If $Q = -q$, will the charges at the corners move towards centre or fly away from it.
(ii) For what value of Q will the charges remain stationary? In this situation how much work is done in removing the charges to infinity?

Topic-2 : Electric Field and Field Lines

1 MCQs with One Correct Answer

1. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to : **[Main Sep. 06, 2020 (I)]**



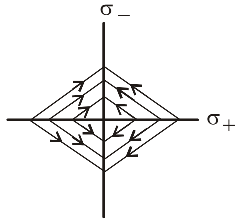
(a) $\frac{x_1^3}{x_2^3}$ (b) $\frac{x_2}{x_1}$ (c) $\frac{x_1}{x_2}$ (d) $\frac{x_2^2}{x_1^2}$

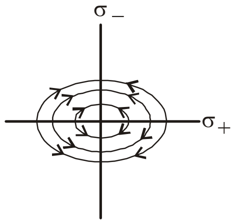
2. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell? **[Main Sep. 06, 2020 (II)]**

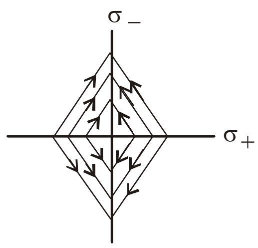
(a) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r < R$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} > F > 0$ for $r < R$

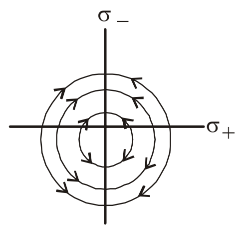
(c) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r > R$ (d) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for all r

3. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system ? **[Main Sep. 04, 2020 (I)]**

(a) 

(b) 

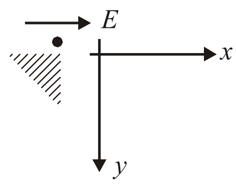
(c) 

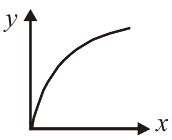
(d) 

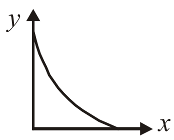
4. A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at $x = 0$. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is : **[Main Sep. 04, 2020 (II)]**

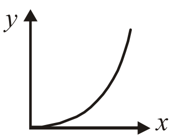
(a) a (b) $\sqrt{\frac{2}{a}}$ (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{1}{a}}$

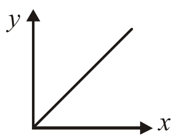
5. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale). **[Main Sep. 02, 2020 (II)]**



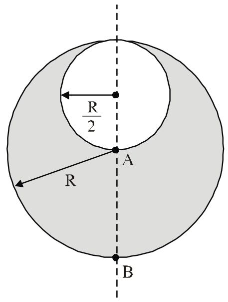
(a) 

(b) 

(c) 

(d) 

6. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is: **[Main 9 Jan. 2020, (I)]**



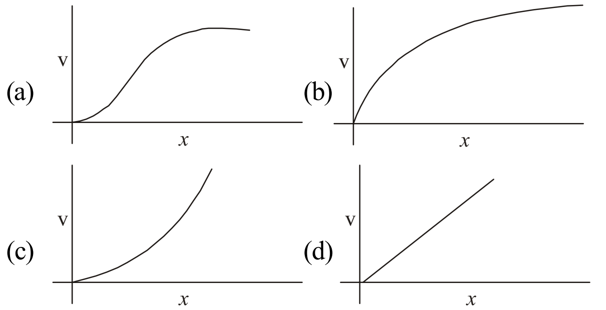
(a) $\frac{21}{34}$ (b) $\frac{18}{34}$ (c) $\frac{17}{54}$ (d) $\frac{18}{54}$

7. An electric dipole of moment $\vec{p} = (\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C.m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to: **[Main 9 Jan. 2020, (I)]**

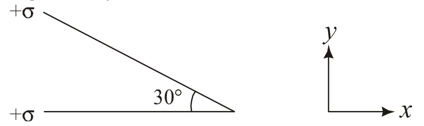
(a) $(+\hat{i} - 3\hat{j} - 2\hat{k})$ (b) $(-\hat{i} + 3\hat{j} - 2\hat{k})$

(c) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (d) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

8. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale) **[Main 8 Jan. 2020, (II)]**



9. Two infinite planes each with uniform surface charge density +σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by: **[Main 7 Jan. 2020, (I)]**



(a) $\frac{\sigma}{2\epsilon_0}\left[(1+\sqrt{3})\hat{y} - \frac{\hat{x}}{2}\right]$ (b) $\frac{\sigma}{\epsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y} + \frac{\hat{x}}{2}\right]$

(c) $\frac{\sigma}{2\epsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y} + \frac{\hat{x}}{2}\right]$ (d) $\frac{\sigma}{2\epsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$

10. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\vec{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time: **[Main 7 Jan 2020, (II)]**

(a) $\frac{2mv_0}{qE_0}$ (b) $\frac{3mv_0}{qE_0}$ (c) $\frac{\sqrt{3}mv_0}{qE_0}$ (d) $\frac{\sqrt{2}mv_0}{qE_0}$

11. Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x = D$, with D>> d, will behave as: **[Main 9 April 2019, (II)]**

(a) $E \propto \frac{1}{D^3}$ (b) $E \propto \frac{1}{D}$ (c) $E \propto \frac{1}{D^4}$ (d) $E \propto \frac{1}{D^2}$

12. The bob of a simple pendulum has mass 2 g and a charge of 5.0 ¼C. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : **[Main 8 April 2019, (I)]**
(take $g = 10$ m/s²)

(a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$
(c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

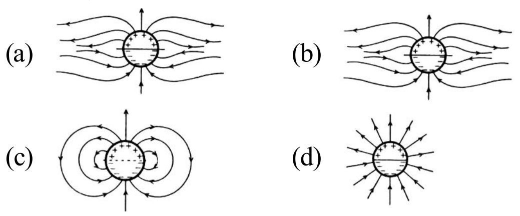
13. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is: **[Main 9 Jan. 2019, (I)]**

(a) $\frac{R}{\sqrt{5}}$ (b) $\frac{R}{\sqrt{2}}$ (c) R (d) $R\sqrt{2}$

14. A solid ball of radius R has a charge density ρ given by $\rho = \rho_0\left(1 - \frac{r}{R}\right)$ for $0 \le r \le R$. The electric field outside the ball is: **[Main Online April 15, 2018]**

(a) $\frac{\rho_0 R^3}{\varepsilon_0 r^2}$ (b) $\frac{4\rho_0 R^3}{3\varepsilon_0 r^2}$ (c) $\frac{3\rho_0 R^3}{4\varepsilon_0 r^2}$ (d) $\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$

15. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge - σ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) **[Main 2015]**



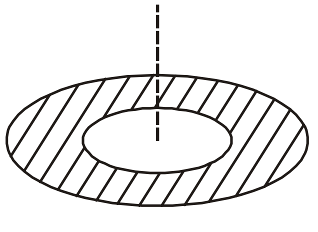
16. A thin disc of radius $b = 2a$ has a concentric hole of radius 'a' in it (see figure). It carries uniform surface charge 'σ' on it. If the electric field on its axis at height 'h' ($h << a$) from its centre is given as 'Ch' then value of 'C' is : **[Main Online April 10, 2015]**

(a) $\frac{\sigma}{4a\epsilon_0}$

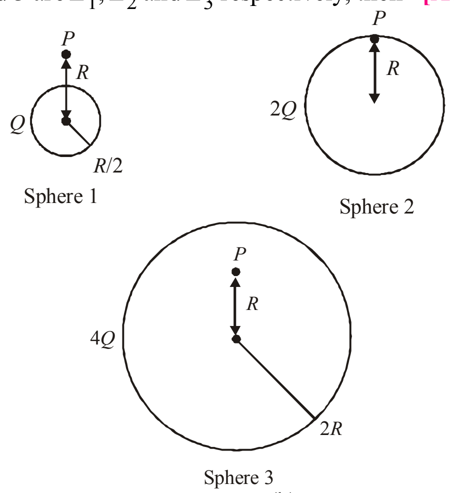
(b) $\frac{\sigma}{8a\epsilon_0}$

(c) $\frac{\sigma}{a\epsilon_0}$

(d) $\frac{\sigma}{2a\epsilon_0}$



17. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitude of the electric fields at point P at a distance R from the centre of sphere 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **[Adv. 2014]**



(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$
(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

18. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2\epsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc : **[Main Online April 25, 2013]**

(a) reduces by 70.7% (b) reduces by 29.3%
(c) reduces by 9.7% (d) reduces by 14.6%

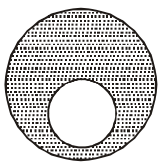
19. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ \text{ms}^{-1}$. Given $g = 9.8\ \text{m s}^{-2}$, viscosity of the air $= 1.8 \times 10^{-5}\ \text{Ns m}^{-2}$ and the density of oil $= 900\ \text{kg m}^{-3}$, the magnitude of q is **[2010]**

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

20. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is **[2009]**

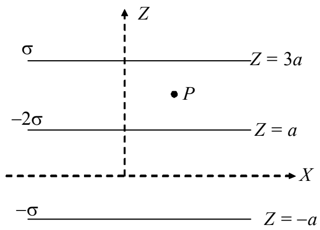
(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 1 : 4 : 9 (d) 1 : 8 : 18

21. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is **[2007]**



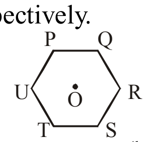
(a) zero everywhere
(b) non-zero and uniform
(c) non-uniform
(d) zero only at its center

22. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is **[2005S]**



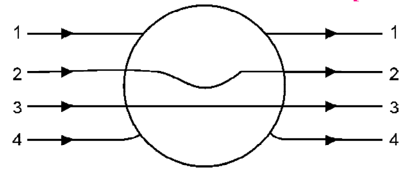
(a) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$ (c) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$ (d) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

23. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on $PQRSTU$ corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R. Which of the following arrangement of charge is possible for P, Q, R, S, T and U respectively. **[2004S]**



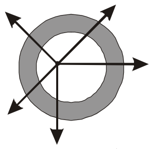
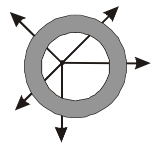
(a) +, +, +, –, –, – (b) –, +, +, +, –, –
(c) –, +, +, –, +, – (d) +, –, +, –, +, –

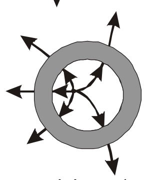
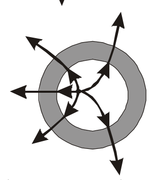
24. A metallic solid sphere is placed in a uniform electric fied. The lines of force follow the path(s) shown in Figure as **[1996 - 2 Marks]**



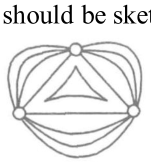
(a) 1 (b) 2 (c) 3 (d) 4

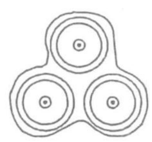
25. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces? **[2003S]**

(a)  (b) 

(c)  (d) 

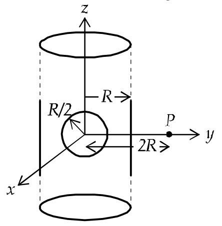
26. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[2001S]**

(a)  (b) 

(c)  (d) 

2 Integer Value Answer

27. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16K\varepsilon_0}$. The value of k is **[2012]**

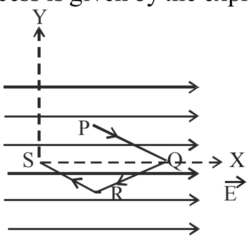


28. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre.

If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a. **[2009]**

4 Fill in the Blanks

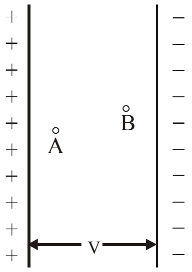
29. A point charge q moves from point P to point S along the path $PQRS$ (fig.) in a uniform electric field E pointing parallel to the positive direction of the X-axis. The cooridnates of the points P, Q, R and S are $(a, b, O), (2a, O, O)\ (a, -b, \underline{O})$ and (O, O, O) respectively. The work done by the field in the above process is given by the expression .... **[1989 - 2 Marks]**



5 True / False

30. An electric line of forces in the $x - y$ plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the $x - y$ plane, will move along the circular line of force. **[1988 - 2 Marks]**

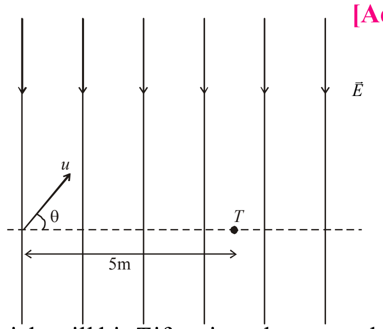
31. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. **[1986 - 3 Marks]**



32. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

33. A uniform electric field, $\vec{E} = -400\sqrt{3}\hat{y}\ \text{NC}^{-1}$ is applied in a region. A charged particle of mass m carrying positive charge q is projected in this region with an initial speed of $2\sqrt{10} \times 10^6\ \text{ms}^{-1}$. This particle is aimed to hit a target T, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10}\ \text{Ckg}^{-1}$. Then **[Adv. 2020]**



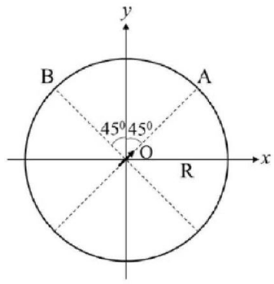
(a) the particle will hit T if projected at an angle 45° from the horizontal

(b) the particle will hit T if projected either at an angle 30° or 60° from the horizontal

(c) time taken by the particle to hit T could be $\sqrt{\frac{5}{6}}\ \mu\text{s}$ as well as $\sqrt{\frac{5}{2}}\ \mu\text{s}$

(d) time taken by the particle to hit T is $\sqrt{\frac{5}{3}}\ \mu\text{s}$

34. An electric dipole with moment $\frac{p_0}{\sqrt{2}}(\hat{i} + \hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are : (ε_0 is permittivity of free space. R >>dipole size) **[Adv. 2019]**



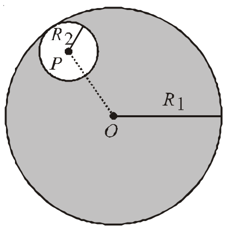
(a) The magnitude of total electric field on any two points of the circle will be same

(b) Total electric field at point B is $\vec{E}_B = 0$

(c) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i} + \hat{j})$

(d) $R = \left(\frac{p_0}{4\pi \epsilon_0 E_0}\right)^{1/3}$

35. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is (are) **[Adv. 2015]**



(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(d) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

36. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric field at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **[Adv. 2014]**

(a) $Q = 4\sigma\pi r_0^2$

(b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

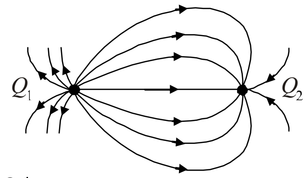
(c) $E_1(r_0/2) = 2E_2(r_0/2)$

(d) $E_2(r_0/2) = 4E_3(r_0/2)$

37. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\dfrac{\rho_1}{\rho_2}$ can be **[Adv. 2013]**

(a) -4
(b) $-\dfrac{32}{25}$
(c) $\dfrac{32}{25}$
(d) 4

38. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **[2010]**



(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

39. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre **[1998S - 2 Marks]**

(a) increases as r increases, for $r < R$.
(b) decreases as r increases, for $0 < r < \infty$.
(c) decreases as r increases, for $R < r < \infty$.
(d) is discontinuous at $r = R$.

7 Match the Following

40. The electric field E is measured at a point P (0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II. **[Adv. 2018]**

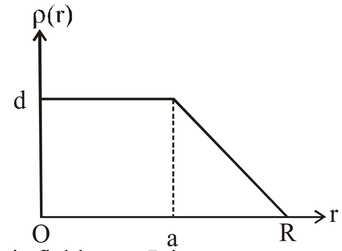
LIST–I		LIST–II	
P.	E is independent of d	**1.**	A point charge Q at the origin
Q.	$E \propto 1/d$	**2.**	A small dipole with point charges Q at $(0, 0, l)$ and – Q at $(0, 0, -l)$. Take $2l << d$
R.	$E \propto 1/d^2$	**3.**	An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S.	$E \propto 1/d^3$	**4.**	Two infinite wires carrying uniform linear charge density parallel to the x - axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$ Take $2l << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(a) $P \to 5$; $Q \to 3, 4$; $R \to 1$; $S \to 2$
(b) $P \to 5$; $Q \to 3$; $R \to 1, 4$; $S \to 2$
(c) $P \to 5$; $Q \to 3$; $R \to 1, 2$; $S \to 4$
(d) $P \to 4$; $Q \to 2, 3$; $R \to 1$; $S \to 5$

8 Comprehension Based Questions

Passage

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ (r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure The electric field is only along the radial direction. **[2008]**



41. The electric field at r = R is

(a) independent of a

(b) directly proportional to a

(c) directly proportional to a^2

(d) inversely proportional to a

42. For a = 0, the value of d (maximum value of ρ as shown in the figure) is –

(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$

(c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

43. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies.

(a) a = 0 (b) a = R/2

(c) a = R (d) a = 2R/3

10 Subjective Problems

44. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length $a(a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field **[2004]**

Topic-3 : Electric Dipole, Electric Flux and Gauss's Law

1 MCQs with One Correct Answer

1. Two identical electric point dipoles have dipole moments $\vec{P}_1 = P\hat{i}$ and $\vec{P}_2 = -P\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is : **[Main Sep. 06, 2020 (II)]**

(a) $\frac{P}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (b) $\frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$

(c) $\frac{P}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$ (d) $\frac{P}{a}\sqrt{\frac{2}{2\pi\varepsilon_0 ma}}$

2. In finding the electric field using Gauss law the formula $|\vec{E}| = \frac{q_{enc}}{\epsilon_0 |A|}$ is applicable. In the formula ϵ_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation? **[Main 8 Jan 2020, (I)]**

(a) Only when the Gaussian surface is an equipotential surface.

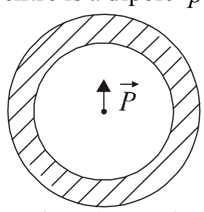
Only when the Gaussian surface is an

(b) equipotential surface and $|\vec{E}|$ is constant on the surface.

(c) Only when $|\vec{E}|$ = constant on the surface.

(d) For any choice of Gaussian surface.

3. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case : **[Main 12 April 2019, (I)]**



(a) surface change density on the inner surface is uniform and equal to $\frac{Q/2}{4\pi a^2}$

(b) electric field outside the shell is the same as that of a point charge at the centre of the shell.

(c) surface charge density on the outer surface depends on $|\vec{P}|$

(d) surface charge density on the inner surface of the shell is zero everywhere.-

4. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is : **[Main 8 April 2019, (II)]**

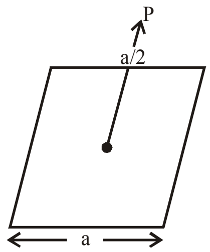
(a) $\sqrt{\frac{qE}{md}}$ (b) $\sqrt{\frac{2qE}{md}}$ (c) $2\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

5. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole? **[Main 11 Jan 2019, (II)]**

(a) -20×10^{-18} J (b) -7×10^{-27} J

(c) -10×10^{-29} J (d) -9×10^{-20} J

6. A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is: **[Main Online April 15, 2018]**

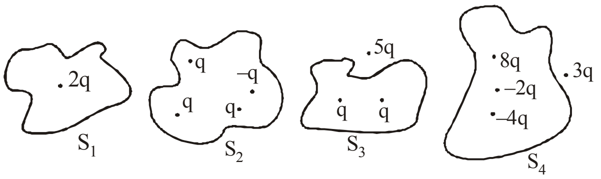


(a) $\frac{Q}{3\varepsilon_0}$
(b) $\frac{Q}{6\varepsilon_0}$
(c) $\frac{Q}{2\varepsilon_0}$
(d) $\frac{Q}{\varepsilon_0}$

7. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{i}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T_2} = -\vec{T_1}$. The angle θ is: **[Main 2017]**

(a) 60° (b) 90° (c) 30° (d) 45°

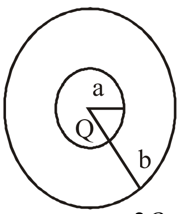
8. Four closed surfaces and corresponding charge distributions are shown below. **[Main Online April 9, 2017]**



Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3, and Φ_4. Then :

(a) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$ (b) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$
(c) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ (d) $\Phi_1 > \Phi_3 ; \Phi_2 < \Phi_4$

9. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $r = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[Main 2016]**

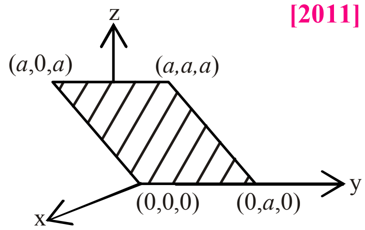


(a) $\frac{2Q}{\pi(a^2-b^2)}$ (b) $\frac{2Q}{\pi a^2}$
(c) $\frac{Q}{2\pi a^2}$ (d) $\frac{Q}{2\pi(b^2-a^2)}$

10. Two point dipoles of dipole moment $\vec{p}_1$ and $\vec{p}_2$ are at a distance x from each other and $\vec{p}_1 \parallel \vec{p}_2$. The force between the dipoles is : **[Main Online April 9, 2013]**

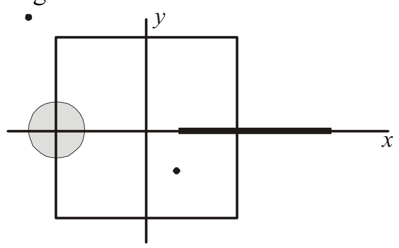
(a) $\frac{1}{4\pi\varepsilon_0}\frac{4p_1p_2}{x^4}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{x^3}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{x^4}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{8p_1p_2}{x^4}$

11. Consider an electric field $\vec{E} = E_0\hat{x}$ where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **[2011]**



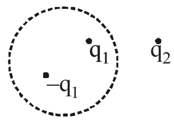
(a) $2E_0a_2$
(b) $\sqrt{2}E_0a^2$
(c) $E_0 a^2$
(d) $\frac{E_0a^2}{\sqrt{2}}$

12. A disc of radius $a/4$ having a uniformly distributed charge 6C is placed in the x - y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x - axis from $x = a/4$ to $x = 5a/4$. Two point charges – 7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is **[2009]**



(a) $\frac{-2C}{\varepsilon_0}$ (b) $\frac{2C}{\varepsilon_0}$ (c) $\frac{10C}{\varepsilon_0}$ (d) $\frac{12C}{\varepsilon_0}$

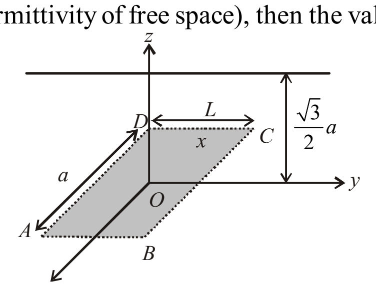
13. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be **[2004S]**



(a) due to q_1 and q_2 only
(b) due to q_2 only
(c) zero
(d) due to all

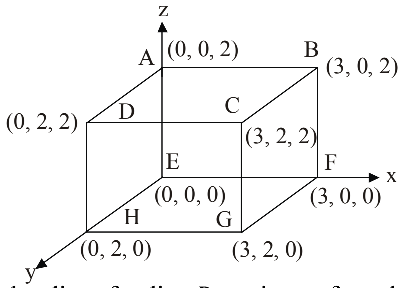
2 Integer Value Answer

14. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the $y-z$ plane at $z = \frac{\sqrt{3}}{2}$ a (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the $x-y$ plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is **[Adv. 2015]**



3 Numeric Answer

15. An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm^2/C) ________. **[Main 9 Jan 2020, (II)]**

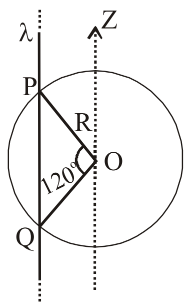


16. A circular disc of radius R carries surface charge density $\sigma(r) = \sigma_0\left(1 - \frac{r}{R}\right)$, where σ_0 is a constant and r is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is ϕ_0. Electric flux through another spherical surface of radius $\frac{R}{4}$ and concentric with the disc is ϕ. Then the ratio $\frac{\phi_0}{\phi}$ is_________. **[Adv. 2020]**

6 MCQs with One or More than One Correct Answer

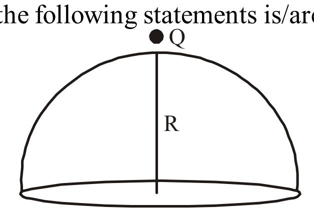
17. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ? **[Adv. 2019]**
[ε_0 is the permittivity of free space]
(a) If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\varepsilon_0$
(b) If $h > 2R$ and $r > R$ then $\phi = Q/\varepsilon_0$
(c) If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$
(d) If $h > 2R$ and $r > 4R/5$ then $\phi = Q/5\varepsilon_0$

18. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is $\in_0$. Which of the following statements is (are) true? **[Adv. 2018]**



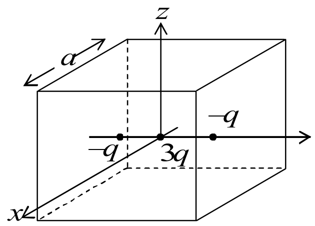
(a) The electric flux through the shell is $\sqrt{3}R\lambda/\in_0$
(b) The z-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}R\lambda/\in_0$
(d) The electric field is normal to the surface of the shell at all points

19. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct? **[Adv. 2017]**



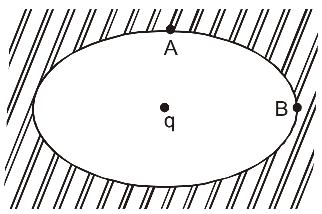
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$
(b) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$
(c) The component of the electric field normal to the flat surface is constant over the surface
(d) The circumference of the flat surface is an equipotential

20. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct options(s) **[2012]**



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.
(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

21. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **[1999S - 3 Marks]**



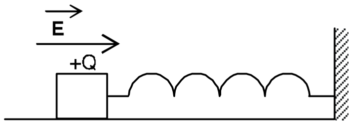
(a) electric field near A in the cavity = electric field near B in the cavity
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0

22. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor. **[1996 - 2 Marks]**
(a) is same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as $1/r$, where r is the distance from axis
(d) varies as $1/r^2$ where r is the distance from the axis.

Topic-4 : Miscellaneous (Mixed Concepts) Problems

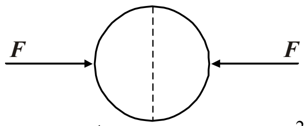
1 MCQs with One Correct Answer

1. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be **[2011]**



(a) of the same frequency and with shifted mean position.
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position.
(d) of changed frequency and with the same mean position.

2. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **[2010]**



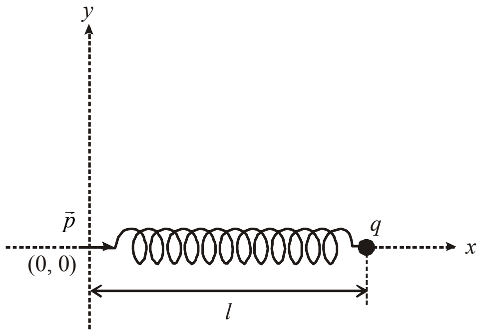
(a) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2 R$ (c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

2 Integer Value Answer

3. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is **[2011]**

3 Numeric Answer

4. One end of a spring of negligible unstretched length and spring constant k is fixed at the origin (0, 0). A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge q is fixed at the origin, the spring gets stretched to a length l and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta l << l$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of δ is ______.

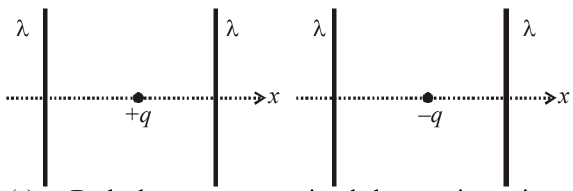


5 True / False

5. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **[1981- 2 Marks]**

6 MCQs with One or More than One Correct Answer

6. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are) **[Adv. 2015]**



(a) Both charges execute simple harmonic motion
(b) Both charges will continue moving in the direction of their displacement
(c) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement
(d) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement

7. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). **[2009]**
   (a) The angular momentum of the charge $-q$ is constant
   (b) The linear momentum of the charge $-q$ is constant
   (c) The angular velocity of the charge $-q$ is constant
   (d) The linear speed of the charge $-q$ is constant

8. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is **[1998S - 2 Marks]**
   (a) periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
   (b) simple harmonic, for all values of z_0 satisfying $0 < z_0 \leq R$
   (c) approximately simple harmonic, provided $z_0 << R$
   (d) such that P crosses O and continues to move along the negative z axis towards $z = -\infty$

Answer Key

Topic-1 : Electric Charges and Coulomb's Law

1. (a)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** $\left(\frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}\right)$
10. $\left(180°, \frac{KQ^2}{4L^2}\right)$	**11.** False	**12.** True	**13.** (a, c)	**14.** (d)	**15.** (a)			

Topic-2 : Electric Field and Field Lines

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (c)
11. (d)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (d)
21. (b)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (c)	**28.** (6)	**29.** (2)	**30.** (–qEA)
31. False	**32.** True	**33.** True	**34.** (b, c)	**35.** (b, d)	**36.** (d)	**37.** (c)	**38.** (b, d)	**39.** (a, d)	**40.** (a, c)
41. (b)	**42.** (a)	**43.** (b)	**44.** (c)						

Topic-3 : Electric Dipole, Electric Flux and Gauss's Law

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (c)
11. (c)	**12.** (a)	**13.** (d)	**14.** (6)	**15.** (–48)	**16.** (6.40)	**17.** (a,b,c)	**18.** (a, b)	**19.** (a, d)	**20.** (a,c,d)
21. (c, d)	**22.** (c)								

Topic-4 : Miscellaneous (Mixed Concepts)Problems

1. (a)	**2.** (a)	**3.** (3)	**4.** (3.14)	**5.** False	**6.** (c)	**7.** (a)	**8.** (a, c)

16 Electrostatic Potential and Capacitance

Topic-1 : Electrostatic Potential and Equipotential Surfaces

1 MCQs with One Correct Answer

1. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $(+q)$ each, while 2, 4, 6, 8, 10 have charge $(-q)$ each. The potential V and the electric field E at the centre of the circle are respectively :
(Take $V = 0$ at infinity) **[Main Sep. 05, 2020 (II)]**

(a) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = 0$

(b) $V = 0$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

(c) $V = 0$; $E = 0$

(d) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

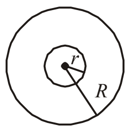
2. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively : **[Main Sep. 03, 2020 (I)]**

(a) 4.5 μC on both (b) +4.5 μC and –4.5 μC
(c) 3 μC and 6 μC (d) 6 μC and 3 μC

3. Concentric metallic hollow spheres of radii R and $4R$ hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference $V(R) - V(4R)$ is : **[Main Sep. 03, 2020 (II)]**

(a) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (b) $\frac{3Q_2}{4\pi\varepsilon_0 R}$ (c) $\frac{Q_2}{4\pi\varepsilon_0 R}$ (d) $\frac{3Q_1}{4\pi\varepsilon_0 R}$

4. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R $(R > r)$. If the surface charge densities on the two shells are equal, the electric potential at the common centre is : **[Main Sep. 02, 2020 (II)]**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

5. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at z = 4a. The minimum value of v such that it crosses the origin is : **[Main 10 April 2019 I]**

(a) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

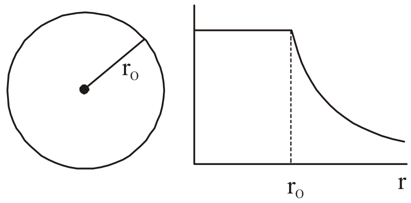
6. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of – 4 Q, the new potential difference between the same two surfaces is : **[Main 8 April 2019 I]**

(a) –2V (b) 2V (c) 4V (d) V

7. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is : **[Main 8 Jan. 2019 II]**

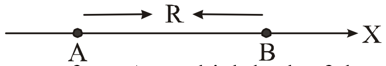
(a) 320 V (b) –48V (c) 180 V (d) –520 V

8. The given graph shows variation (with distance r from centre) of : **[Main 11 Jan. 2019 I]**



(a) Electric field of a uniformly charged sphere
(b) Potential of a uniformly charged spherical shell
(c) Potential of a uniformly charged sphere
(d) Electric field of a uniformly charged spherical shell

9. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4\,qa\,\hat{i}$ and $\vec{d}_B = -2\,qa\,\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is: **[Main 10 Jan. 2019 I]**

(a) $\frac{R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (c) $\frac{R}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

10. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ $(\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ? **[Adv. 2019[**

(a) The magnituide of electric field at a point, located on a line passing through the hole and shell's center, on a distance $2R$ from the centre of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$
(b) The ratio of the potential at the centre of the shell to that of the point at $\frac{1}{2}R$ from the centre towards the hole will be $\frac{1-\alpha}{1-2\alpha}$
(c) The magnitude of electric field at the centre of the shell is reduced by $\frac{\alpha V_0}{2R}$
(d) The potential at the centre of the shell is reduced by $2\alpha V_0$

11. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is: **[2018]**

(a) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$
(c) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

12. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between in the limits 589.0 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field ? **[Main Online April 8, 2017]**

(a) 589.5 V (b) 589.2 V (c) 589.4 V (d) 589.6 V

13. The potential (in volts) of a charge distribution is given by
$V(z) = 30 - 5z^2$ for $|z| \le 1$m
$V(z) = 35 - 10\,|z|$ for $|z| \ge 1$ m.
V(z) does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ε_0) which is spread over a certain region, then choose the correct statement. **[Main Online April 9, 2016]**

(a) $\rho_0 = 20\,\varepsilon_0$ in the entire region
(b) $\rho_0 = 10\,\varepsilon_0$ for $|z| \le 1$ m and $p_0 = 0$ elsewhere
(c) $\rho_0 = 20\,\varepsilon_0$ for $|z| \le 1$ m and $p_0 = 0$ elsewhere
(d) $\rho_0 = 40\,\varepsilon_0$ in the entire region

14. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[Main 2015]**

(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $2R = R_4$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \ne 0$ and $(R_2 - R_1) > (R_4 - R_3)$

15. An electric field $\vec{E} = (25\hat{i} + 30\hat{j})NC^{-1}$ exists in a region of space. If the potential at the origin is taken to be zero then the potential at $x = 2\ m$, $y = 2\ m$ is : **[Main Online April 11, 2015]**

(a) –110 J (b) –140 J (c) –120 J (d) –130 J

16. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is: **[Main 2014]**

(a) 120 J/C (b) –120 J/C (c) –80 J/C (d) 80 J/C

17. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating mass less strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of line joining them will be $\left(\frac{1}{4\pi\epsilon_0} = k\right)$. **[Main Online April 22, 2013]**

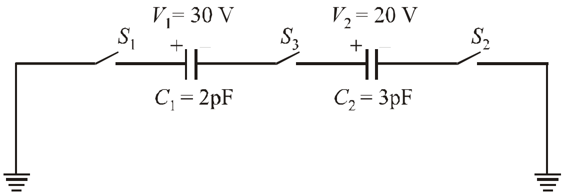
(a) $2\sqrt{k\,mg\tan\theta}$ (b) $\sqrt{k\,mg\tan\theta}$
(c) $4\sqrt{k\,mg/\tan\theta}$ (d) $\sqrt{k\,mg/\tan\theta}$

18. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. **[2007]**

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

19. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy: **[2001S]**
(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$

20. For the circuit shown in Figure, which of the following statements is true? **[1999 - 2 Marks]**



(a) With S_1 closed $V_1 = 15$ V, $V_2 = 20$ V
(b) With S_3 closed, $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

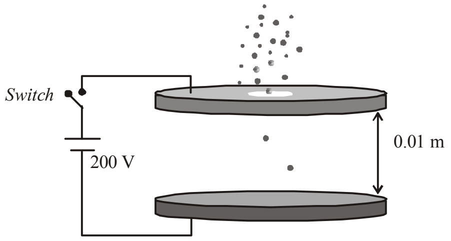
21. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$,..... $x = \infty$ on the x axis, and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$,.... $x = \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is **[1998S - 2 Marks]**

(a) 0 (b) $\dfrac{q}{8\pi\varepsilon_0\, x_0 \ln 2}$ (c) ∞ (d) $\dfrac{q \ln 2}{4\pi\varepsilon_0\, x_0}$

22. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is : **[1989 - 2 Marks]**
(a) V (b) $2V$ (c) $4V$ (d) $-2V$

23. A hollow metal sphere of radius 5 cms is charged such that the potential on its surface is 10 volts. The potential at the centre of the sphere is **[1983 - 1 Mark]**
(a) zero (b) 10 volts
(c) same as at a point 5 cms away from the surface
(d) same as at a point 25 cms away from the surface

24. Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg m^{-3} are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius 8×10^{-7} m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is ________. (neglect the buoyancy force, take acceleration due to gravity = 10 ms^{-2} and charge on an electron (e) = 1.6×10^{-19} C) **[Adv. 2020]**



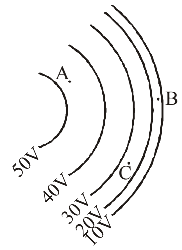
2 Integer Value Answer

25. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \hat{i}$, where $E_0 = 1.0$ NC^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is ____________. **[Adv. 2018]**

4 Fill in the Blanks

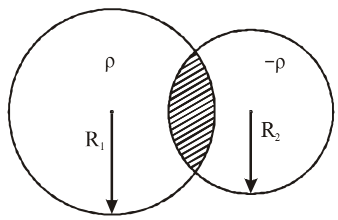
26. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. The electric field at the point (1m, 0, 2 m) is .... V/m. **[1992 - 1 Mark]**

27. Figure shows line of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point ... **[1984- 2 Marks]**



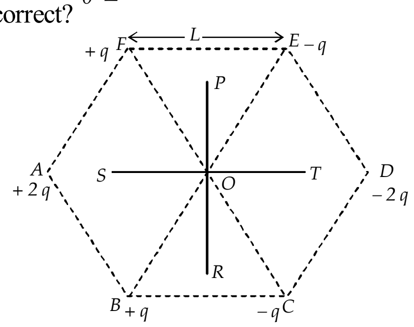
6 MCQs with One or More than One Correct Answer

28. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region **[Adv. 2013]**



(a) The electrostatic field is zero
(b) The electrostatic potential is constant
(c) The electrostatic field is constant in magnitude
(d) The electrostatic field has same direction

29. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{L^2}$, which of the following statement(s) is (are) correct? **[2012]**



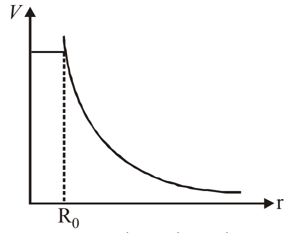
(a) The electric field at O is 6K along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

30. Which of the following statement(s) is/are correct? **[2011]**
(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(b) The Gauss law can be used to calculate the field distribution around an electric dipole.
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

31. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B(<R_A)$ are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then **[2011]**
(a) $E_A^{inside} = 0$ (b) $Q_A > Q_B$
(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (d) $E_A^{on\,surface} < E_B^{on\,surface}$

32. A spherical symmetric charge system is centered at origin. Given, Electric potential **[2006S - 5 Marks]**

$$V = \frac{Q}{4\pi\varepsilon_0 R_0} \ (r \le R_0), \ V = \frac{Q}{4\pi\varepsilon_0 r} \ (r > R_0)$$



(a) Within $r = 2R_0$ total enclosed net charge is Q
(b) Electric field is discontinued at $r = R_0$
(c) Charge is only present at $r = R_0$
(d) Electrostatic energy is zero for $r < R_0$

9 Assertion and Reason Type Questions

33. **STATEMENT-1 :** For practical purposes, the earth is used as a reference at zero potential in electrical circuits. and

STATEMENT-2 : The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$. **[2008]**
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement -1 is False, Statement-2 is True

10 Subjective Problems

34. A conducting liquid bubble of radius a and thickness t ($t << a$) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **[2005 - 2 Marks]**

35. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane. **[1991 - 4 + 2 + 2 Marks]**
(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.
(b) Give the expression $V(x)$ at a general point on the x - axis and sketch the function $V(x)$ on the whole x-axis.
(c) If a particle of charge $+q$ starts form rest at the centre of the circle, show by a short quantative argument that the particle eventually crosses the circle. Find its speed when it does so.

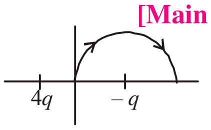
36. Three concentric spherical metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities σ, $-\sigma$ and σ respectively. **[1990 -7 Marks]**
(i) Find the potential of the three shells A, B and C.
(ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

37. A charge 'Q' is distributed over two concentric hollow spheres of radii 'r' and 'R' ($>r$) such that the surface densities are equal. Find the potential at the common centre. **[1981- 3 Marks]**

Topic-2 : Electric Potential Energy and Work Done in Carrying a Charge

1 MCQs with One Correct Answer

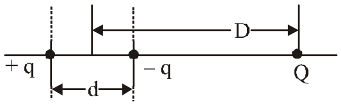
1. A two point charges $4q$ and $-q$ are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to $x = d$ along the semicircle as shown in the figure, the energy of the charge will : **[Main Sep. 04, 2020 (I)]**



(a) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$ (b) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$
(c) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$ (d) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

2. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to : **[Main Sep. 03, 2020 (II)]**
(a) 1 : 2 (b) 10 : 7 (c) 2 : 1 (d) 5 : 7

3. A system of three charges are placed as shown in the figure:

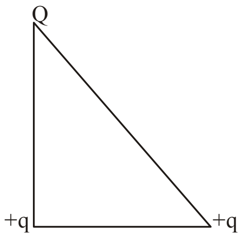


If D >> d, the potential energy of the system is best given by **[Main 9 April 2019 I]**

(a) $\frac{1}{4\pi\epsilon_0}\left[\frac{-q^2}{d}\frac{-qQd}{2D^2}\right]$ (b) $\frac{1}{4\pi\epsilon_0}\left[\frac{-q^2}{d}+\frac{2qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\epsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$ (d) $\frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

4. Three charges Q, + q and + q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is : **[Main 11 Jan. 2019 I]**



(a) + q (b) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (c) $\frac{-q}{1+\sqrt{2}}$ (d) –2q

5. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, – 2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be: **[Main 10 Jan. 2019 II]**

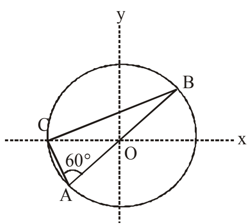
(a) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\,\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}$

6. **Statement 1 :** No work is required to be done to move a test charge between any two points on an equipotential surface.
Statement 2 : Electric lines of force at the equipotential surfaces are mutually perpendicular to each other. **[Main Online April 25, 2013]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is false, Statement 2 is true.

7. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ placed at points *A, B* and *C*, respectively, as shown in the figure. Take *O* to be the centre of the circle of radius R and angle $CAB = 60°$ **[2008]**



(a) The electric field at point *O* is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis
(b) The potential energy of the system is zero
(c) The magnitude of the force between the charges at *C* and *B* is $\frac{q^2}{54\pi\varepsilon_0 R^2}$
(d) The potential at point *O* is $\frac{q}{12\pi\varepsilon_0 R}$

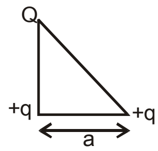
8. Positive and negative point charges of equal magnitude are kept at $\left(0,0,\frac{a}{2}\right)$ and $\left(0,0,\frac{-a}{2}\right)$ respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is **[2007]**

(a) positive (b) negative
(c) zero
(d) depends on the path connecting the initial and final positions

9. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the *x*-axis. Another point charge *Q* is placed at the origin. The change in the electrical potential energy of *Q*, when it is displaced by a small distance *x* along the *x*-axis, is approximately proportional to **[2002S]**

(a) x (b) x^2 (c) x^3 (d) $1/x$

10. Three charges *Q*, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if *Q* is equal to **[2000S]**



(a) $\frac{-q}{1+\sqrt{2}}$
(b) $\frac{-2q}{2+\sqrt{2}}$
(c) $-2q$
(d) $+q$

11. Two point charges $+q$ and $-q$ are held fixed at $(-d, o)$ and (d, o) respectively of a *x-y* coordinate system. Then **[1995S]**

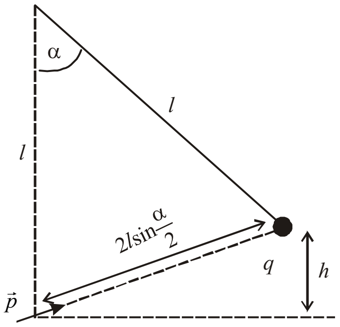
(a) The electric field *E* at all points on the *x*-axis has the same direction
(b) Electric field at all points on *y*-axis is along *x*-axis
(c) Work has to be done in bringing a test charge from ∞ to the origin
(d) The dipole moment is $2qd$ along the *x*-axis

12. Two identical thin rings, each of radius *R* metres, are coaxially placed a distance *R* metres apart. If Q_1 coulomb, and Q_2 coulomb, are respectively the charges uniformly spread on the two rings, the work done in moving a charge *q* from the centre of one ring to that of the other is **[1992 - 2 Marks]**

(a) zero (b) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

(c) $\frac{q\sqrt{2}(Q_1+Q_2)}{(4\pi\varepsilon_0 R)}$ (d) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

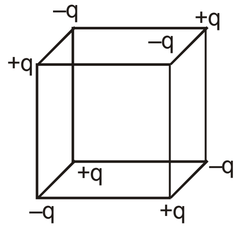
13. A point charge q of mass m is suspended vertically by a string of length l. A point dipole of dipole moment $\vec{p}$ is now brought towards q from infinity so that the charge moves away. The final equilibrium position of the system including the direction of the dipole, the angles and distances is shown in the figure below. If the work done in bringing the dipole to this position is $N \times (mgh)$, where g is the acceleration due to gravity, then the value of N is _________ . (Note that for three coplanar forces keeping a point mass in equilibrium, $\frac{F}{\sin\theta}$ is the same for all forces, where F is any one of the forces and θ is the angle between the other two forces) **[Adv. 2020]**



10 Subjective Problems

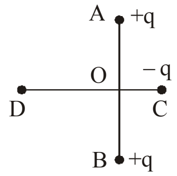
14. A charge $+Q$ is fixed at the origin of the co-ordinate system while a small electric dipole of dipole moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin. **[2003 - 4 Marks]**
(a) Calculate the K.E. of the dipole when it reaches to a point $(d, 0)$.
(b) Calculate the force on the charge $+Q$ at this moment.

15. Charges $+q$ and $-q$ are located at the corners of a cube of side as show in the figure. Find the work done to separate the charges to infinite distance. **[2003 - 2 Marks]**



16. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P. **[1993-4 Marks]**

17. Two fixed, equal, positive charges, each of magnitude 5×10^{-5} coul are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. **[1985 - 6 Marks]**



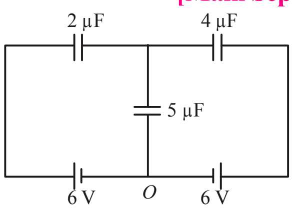
The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.

Topic-3 : Capacitors, Grouping of Capacitors and Energy Stored in a Capacitor

1 MCQs with One Correct Answer

1. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is : **[Main Sep. 05, 2020 (I)]**
(a) $\frac{25}{6}CV^2$ (b) $\frac{3}{2}CV^2$ (c) zero (d) $\frac{9}{2}CV^2$

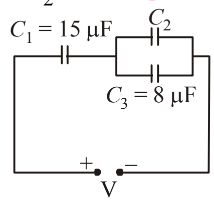
2. In the circuit shown, charge on the 5 μF capacitor is : **[Main Sep. 05, 2020 (II)]**



(a) 18.00 μC (b) 10.90 μC (c) 16.36 μC (d) 5.45 μC

3. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is : **[Main Sep. 04, 2020 (II)]**
(a) $\frac{1}{2}CV_0^2$ (b) $\frac{1}{3}CV_0^2$ (c) $\frac{1}{4}CV_0^2$ (d) $\frac{1}{6}CV_0^2$

4. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is : **[Main Sep. 03, 2020 (I)]**



(a) 450 μC (b) 590 μC (c) 160 μC (d) 650 μC

5. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is : **[Main Sep. 02, 2020 (II)]**

(a) 15 μF (b) 30 μF (c) 20 μF (d) 10 μF

6. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be: **[Main 8 Jan. 2020 I]**

(a) 4.2 μF (b) 3.2 μF (c) 1.6 μF (d) 8.4 μF

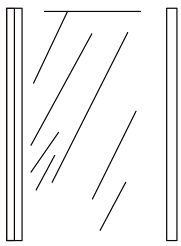
7. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression: **[Main 7 Jan. 2020 I]**

(a) $\dfrac{AK\in_0}{d}\left(1+\dfrac{\alpha d}{2}\right)$

(b) $\dfrac{A\in_0 K}{d}\left(1+\left(\dfrac{\alpha d}{2}\right)^2\right)$

(c) $\dfrac{A\in_0 K}{d}\left(1+\dfrac{\alpha^2 d^2}{2}\right)$

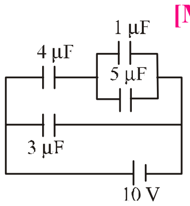
(d) $\dfrac{AK\in_0}{d}(1+\alpha d)$



8. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is: **[Main 9 April 2020 II]**

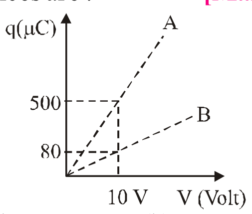
(a) $\dfrac{nV}{K+n}$ (b) V (c) $\dfrac{V}{K+n}$ (d) $\dfrac{(n+1)V}{(K+n)}$

9. In the given circuit, the charge on 4 μF capacitor will be : **[Main 12 April 2019 II]**



(a) 5.4 μC
(b) 9.6 μC
(c) 13.4 μC
(d) 24 μC

10. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are : **[Main 10 April 2019 I]**



(a) 40 μF and 10 μF (b) 60 μF and 40 μF
(c) 50 μF and 30 μF (d) 20 μF and 30 μF

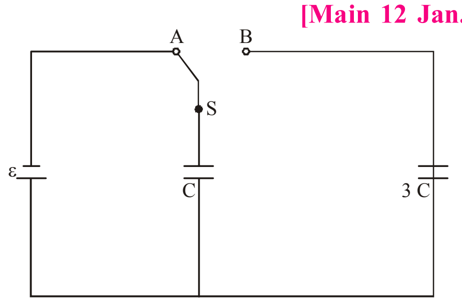
11. A capacitor with capacitance 5μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 ¼F, how much work is done? **[Main 9 April 2019 I]**

(a) 6.25×10^{-6} J (b) 3.75×10^{-6} J
(c) 2.16×10^{-6} J (d) 2.55×10^{-6} J

12. A parallel plate capacitor has 1μF capacitance. One of its two plates is given + 2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is : **[Main 8 April 2019 II]**

(a) 3V (b) 1V (c) 5V (d) 2V

13. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is: **[Main 12 Jan. 2019 I]**



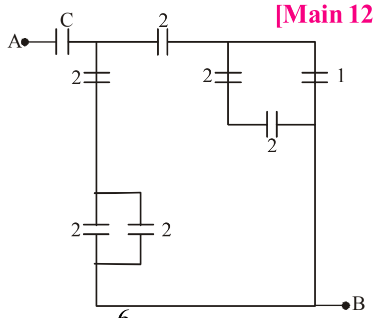
(a) $\dfrac{1}{8}\dfrac{Q^2}{C}$ (b) $\dfrac{3}{8}\dfrac{Q^2}{C}$ (c) $\dfrac{5}{8}\dfrac{Q^2}{C}$ (d) $\dfrac{3}{4}\dfrac{Q^2}{C}$

14. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is :

(Take $\in_0 = 8.85 \times 10^{-12} \dfrac{C^2}{N-M^2}$) **[Main 12 Jan. 2019 II]**

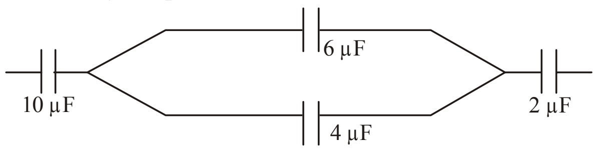
(a) 7.85×10^{-10} C (b) 6.85×10^{-10} C
(c) 8.85×10^{-10} C (d) 9.85×10^{-10} C

15. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF. **[Main 12 Jan. 2019 II]**



(a) $\dfrac{7}{11}$ μF (b) $\dfrac{6}{5}$ μF (c) 4 μF (d) $\dfrac{7}{10}$ μF

16. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30μC. The charge on the right plate of the 6μF capacitor is : **[Main 11 Jan. 2019 I]**

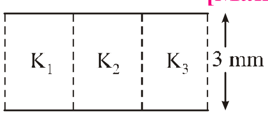


(a) –12 μC (b) +12 μC (c) –18 μC (d) +18 μC

17. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is: **[Main 10 Jan. 2019 II]**

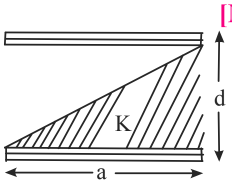
(a) 692 pJ (b) 508 pJ (c) 560 pJ (d) 600 pJ

18. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 1(4)$ The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be: **[Main 10 Jan. 2019 I]**



(a) 4 (b) 14 (c) 12 (d) 36

19. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is: **[Main 9 Jan. 2019 I]**



(a) $\dfrac{K \epsilon_0 a^2}{2d\,(K+1)}$ (b) $\dfrac{K \epsilon_0 a^2}{d\,(K-1)} \ln K$

(c) $\dfrac{K \epsilon_0 a^2}{d} \ln K$ (d) $\dfrac{1}{2}\dfrac{K \epsilon_0 a^2}{d}$

20. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $k = \dfrac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be: **[Main 2018]**

(a) 1.2 n C (b) 0.3 n C (c) 2.4 n C (d) 0.9 n C

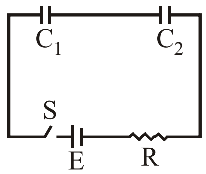
21. In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \dfrac{C_1C_2}{C_1 + C_2}\right)$. **[Main Online April 16, 2018]**

(a) $C_{eq}E[1 - \exp(-t/RC_{eq})]$

(b) $C_1E[1 - \exp(-tR/C_1)]$

(c) $C_2E[1 - \exp(-t/RC_2)]$

(d) $C_{eq}E \exp(-t/RC_{eq})$



22. A parallel plate capacitor with area 200cm^2 and separation between the plates 1.5cm, is connected across a battery of emf V. If the force of attraction between the plates is 25 × 10^{-6}N, the value of V is approximately: **[Main Online April 15, 2018]**

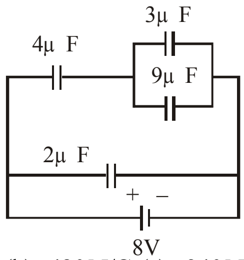
$\left(\varepsilon_0 = 8.85 \times 10^{-12} \dfrac{C^2}{N.m^2}\right)$

(a) 150V (b) 100V (c) 250V (d) 300V

23. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **[Main 2017]**

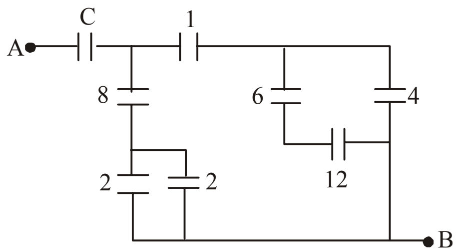
(a) 24 (b) 32 (c) 2 (d) 16

24. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distance 30 m from it, would equal : **[Main 2016]**



(a) 420 N/C (b) 480 N/C (c) 240 N/C (d) 360 N/C

25. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is : **[Main Online April 10, 2016]**

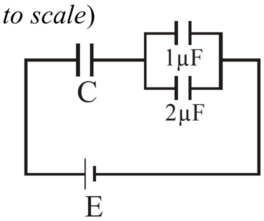


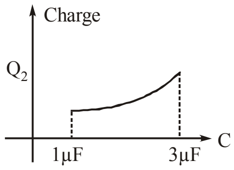
(a) $\dfrac{32}{23}$μF (b) $\dfrac{31}{23}$μF (c) $\dfrac{33}{23}$μF (d) $\dfrac{34}{23}$μF

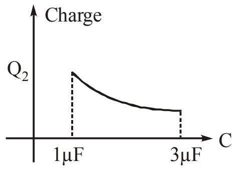
25. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6μF. This can be done by connecting them : **[Main Online April 9, 2016]**

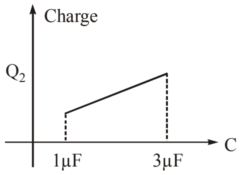
(a) all in series
(b) all in parallel
(c) two in parallel and one in series
(d) two in series and one in parallel

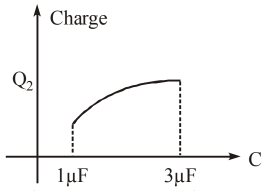
26. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by: (*figures are drawn schematically and are not to scale*) **[Main 2015]**



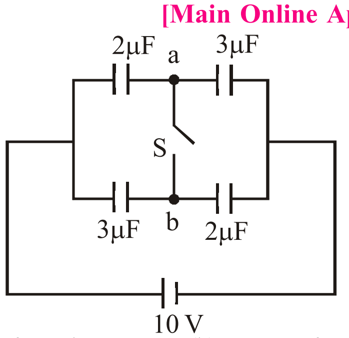
(a) 

(b) 

(c) 

(d) 

27. In figure a system of four capacitors connected across a 10 V battery is shown. Charge that will flow from switch S when it is closed is : **[Main Online April 11, 2015]**



(a) 5 μC from b to a (b) 20 μC from a to b
(c) zero (d) 5 μC from a to b

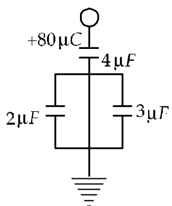
28. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[Main 2014]**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^{4} C/m^2 (d) 6×10^{4} C/m^2

29. A parallel plate capacitor having a separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one-third of the material is replaced by another material with dielectric constant 2K, so that effectively there are two capacitors one with area $\frac{1}{3}$ A, dielectric constant 2K and another with area $\frac{2}{3}$ A and dielectric constant K. If the capacitance of this new capacitor is C then $\frac{C}{C_0}$ is **[Main Online April 25, 2013]**

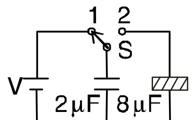
(a) 1 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

30. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **[2012]**



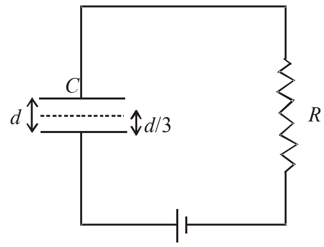
(a) +32 μC
(b) +40 μC
(c) +48 μC
(d) +80 μC

31. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[2011]**



(a) 0%
(b) 20%
(c) 75%
(d) 80%

32. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is d/3 initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is – **[2008]**



(a) $\frac{6\varepsilon_0 R}{5d+3vt}$

(b) $\frac{(15d+9vt)\,\varepsilon_0 R}{2d^2-3dvt-9v^2t^2}$

(c) $\frac{6\varepsilon_0 R}{5d-3vt}$

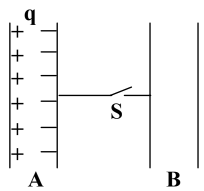
(d) $\frac{(15d-9vt)\,\varepsilon_0 R}{2d^2-3dvt-9v^2t^2}$

33. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is **[2002S]**

(a) $\frac{1}{4}C\left(V_1^2-V_2^2\right)$ (b) $\frac{1}{4}C\left(V_1^2+V_2^2\right)$

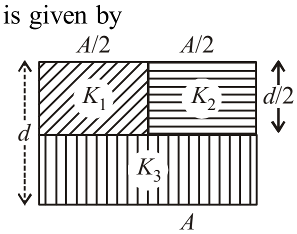
(c) $\frac{1}{4}C\left(V_1-V_2\right)^2$ (d) $\frac{1}{4}C\left(V_1+V_2\right)^2$

34. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **[2001S]**



(a) zero (b) q/2 (c) q (d) 2q

35. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2 and k_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by **[2000S]**



(a) $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{2K_3}$ (b) $\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$

(c) $K=\frac{K_1K_2}{K_1+K_2}+2K_3$ (d) $K=K_1+K_2+2K_3$

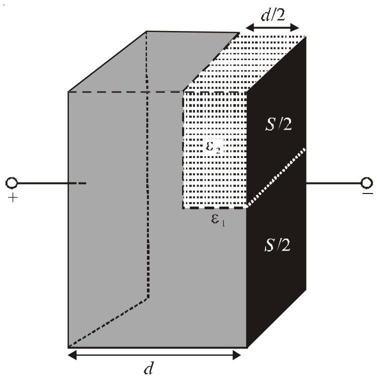
36. Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is **[1999 - 2 Marks]**

(a) $(Q_1+Q_2)/(2C)$ (b) $(Q_1+Q_2)/C$
(c) $(Q_1-Q_2)/C$ (d) $(Q_1-Q_2)/(2C)$

37. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is **[1995S]**

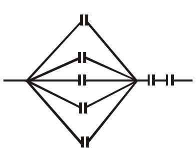
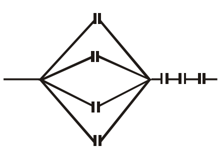
(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

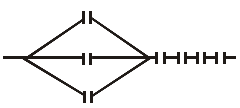
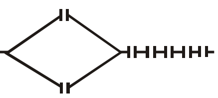
38. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative primitivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is **[Adv. 2015]**



(a) 6/5 (b) 5/3 (c) 7/5 (d) 7/3

39. Seven capacitors each of capacitance $2\mu F$ are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{10}{11}\right)\mu F$. Which of the combination (s) shown in figure will achieve the desired result ? **[1990 - 2 Marks]**

(a)  (b) 

(c)  (d) 

2 Integer Value Answer

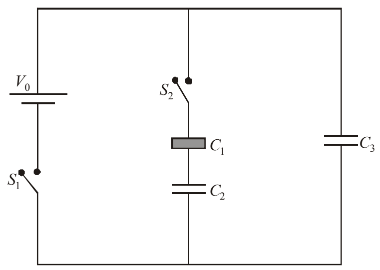
40. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the mth layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N($>10^3$), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d \ln 2}\right)$. The value of α will be ___

[$\in_0$ is the permittivity of free space] **[Adv. 2019]**

41. A 5 µF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 µF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is ________. **[Main Sep. 02, 2020 (I)]**

42. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) **[Main 7 Jan. 2020 II]**

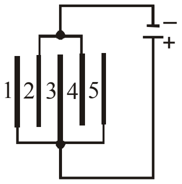
43. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 µF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 µC. The value of ε_r = ____________. **[Adv. 2018]**



4 Fill in the Blanks

44. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential differences across the capacitors now becomes............ **[1988 - 2 Marks]**

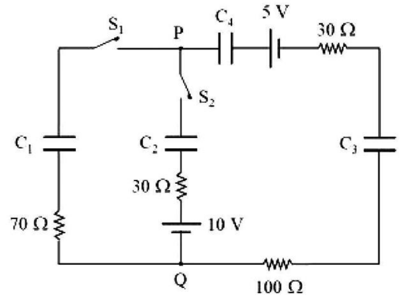
45. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance d apart, the plates are connected to a source of emf V as shown in the figure **[1984- 2 Marks]**



The charge on plate 1 is ..... and on plate 4 is .....

6 MCQs with One or More than One Correct Answer

46. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F$, $C_2 = 30\mu F$ and $C_3 = C_4 = 80\mu F$. **[Adv. 2019]**



Which of the statement(s) is/are correct?

(a) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10V.

(b) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30resistor (between points P and Q) will be 0.2A (round off to 1st decimal place)

(c) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25mA.

(d) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

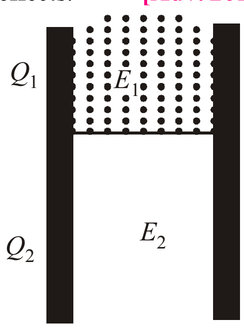
47. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **[Adv. 2014]**

(a) $\dfrac{E_1}{E_2} = 1$

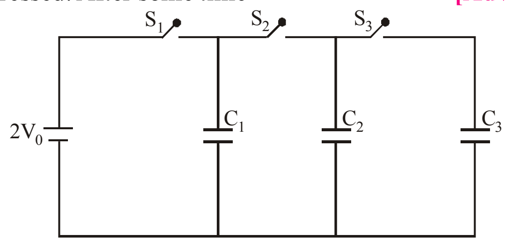
(b) $\dfrac{E_1}{E_2} = \dfrac{1}{K}$

(c) $\dfrac{Q_1}{Q_2} = \dfrac{3}{K}$

(d) $\dfrac{C}{C_1} = \dfrac{2+K}{K}$



48. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time **[Adv. 2013]**



(a) The charge on the upper plate of C_1 is $2CV_0$

(b) The charge on the upper plate of C_1 is CV_0

(c) The charge on the upper plate of C_2 is 0

(d) The charge on the upper plate of C_2 is $-CV_0$

49. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$, **[1998S - 2 Marks]**

(a) the magnitude of the electric field remains the same.

(b) the direction of the electric field remains the same.

(c) the electric potential increases continuously.

(d) the electric potential increases at first, then decreases and again increases.

50. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then **[1991 - 2 Marks]**

(a) $Q = \dfrac{\varepsilon_0 AV}{d}$ (b) $Q = \dfrac{\varepsilon_0 KAV}{d}$

(c) $E = \dfrac{V}{Kd}$ (d) $W = \dfrac{\varepsilon_0 AV^2}{2d}\left[1 - \dfrac{1}{K}\right]$

51. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles: **[1987 - 2 Marks]**

(a) the charge on the capacitor increases.

(b) the voltage across the plates increases.

(c) the capacitance increases.

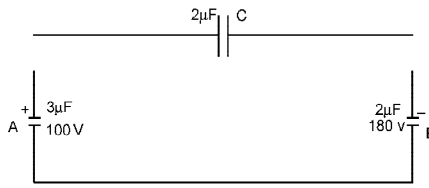
(d) the electrostatic energy stored in the capacitor increases

52. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as **[1985 - 2 Marks]**

(a) $Q > Q_0$ (b) $V > V_0$ (c) $E > E_0$ (d) $U > U_0$

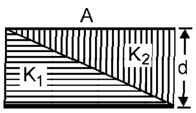
10 Subjective Problems

53. Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate **[1997 - 5 Marks]**



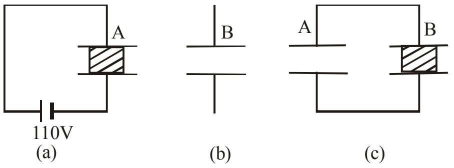
(i) the final charge on the three capacitors. and
(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.

54. The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2, respectively. Find the capacitance of the resulting capacitor. **[1996 - 2 Marks]**



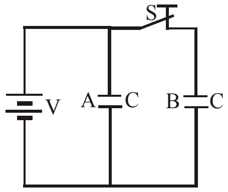
55. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-1}$) **[1994 - 6 Marks]**

56. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate area of A and B are 0.04 m^2 and 0.02m^2 respectively. A slab of dielectric constant (relative permittivity) K = 9 has dimensions such that it can exactly fill the space between the plates of capacitor B. **[1993 - 2 + 3 + 2 Marks]**



(i) The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110V. Calculate the capacitance of A and the energy stored in it.
(ii) The battery is disconnected and then the dielectric slab is moved from A. Find the work done by the external agency in removing the slab from A.
(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system.

57. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **[1983 - 6 Marks]**



Topic-4 : Miscellaneous (Mixed Concepts) Problems

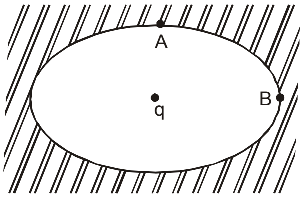
1 MCQs with One Correct Answer

1. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly **[2012]**

(a) 1×10^{-5} V
(b) 1×10^{-7} V
(c) 1×10^{-9} V
(d) 1×10^{-10} V

6 MCQs with One or More than One Correct Answer

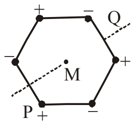
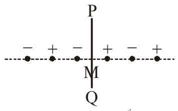
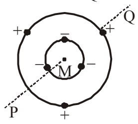
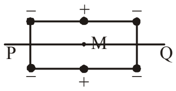
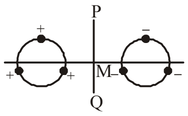
2. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **[1999S - 3 Marks]**



(a) electric field near A in the cavity = electric field near B in the cavity
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0

7 Match the Following

3. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column-II**. In each case, a point M and line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to the equivalent to a steady current. **[2009]**

	Column-I		Column-II	
A)	$E=0$	(p)		Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.
(B)	$V \neq 0$	(q)		Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.
(C)	$B=0$	(r)		Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
(D)	$\mu \neq 0$	(s)		Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.
		(t)		Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

8 Comprehension/Passage Based Questions

Passage-1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

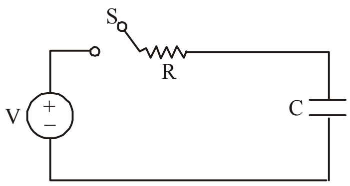


Figure 1

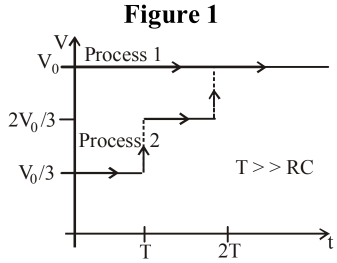


Figure 2

4. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by: **[Adv. 2017]**

(a) $E_C = E_D$ (b) $E_C = E_D \ln 2$

(c) $E_C = \frac{1}{2}E_D$ (d) $E_C = 2E_D$

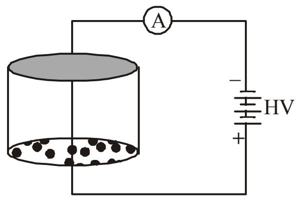
5. In Process 2, total energy dissipated across the resistance E_D is: **[Adv. 2017]**

(a) $E_D = \frac{1}{2}CV_0^2$ (b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(c) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (d) $E_D = 3CV_0^2$

Passage-2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r<<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



6. Which one of the following statements is correct? **[Adv. 2016]**

(a) The balls will stick to the top plate and remain there

(b) The balls will bounce back to the bottom plate carrying the same charge they went up with

(c) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(d) The balls will execute simple harmonic motion between the two plates

7. The average current in the steady state registered by the ammeter in the circuit will be **[Adv. 2016]**

(a) zero

(b) proportional to the potential V_0

(c) proportional to $V_0^{1/2}$

(d) proportional to V_0^2

10 Subjective Problems

8. Four point charges $+8mC$, $-1mC$, $-1mC$, and $+8mC$ are fixed at the points $-\sqrt{\frac{27}{2}}\text{m}, -\sqrt{\frac{3}{2}}\text{m}, +\sqrt{\frac{3}{2}}\text{m}$ and $+\sqrt{\frac{27}{2}}\text{m}$ respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\ \mu$C moves along the $-x$ direction. Its speed at $x = +\infty$ is V_0. Find the least value of V_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2/\text{C}^2$. **[2000 - 10 Marks]**

9. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $q/m = 4\in_0 g/\sigma$ **(1999 - 10 Marks)**

(a) Find the value of H if the particle just reaches the disc.

(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

10. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2, and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought into contact with S_2 and removed. This procedure is repeated n times. **[1998 - 8 Marks]**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.

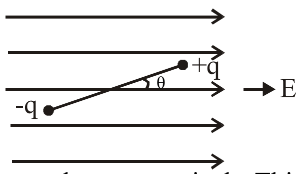
(b) What is the limiting value of this energy as $n \to \infty$?

11. (a) Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg. m. A charge of Q coulomb is uniformly distributed over a spherical volume of radius R metres. Obtain an expression for the energy of the system.

(b) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles?

(c) If the same charge of Q coulomb as in part (a) above is given to a spherical conductor of the same radius R, what will be energy of the system ? **[1992 - 10 Marks]**

12. A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degree) with the field direction, fig. Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free. **[1989 - 8mark]**



13. Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10\ \text{m/s}^2$). **[1988 - 5 Marks]**

Answer Key

Topic-1 : Electrostatic Potential and equipotential surfaces

1. (c)	2. (d)	3. (a)	4. (d)	5. (c)	6. (d)	7. (c)	8. (b)	9. (d)	10. (b)
11. (b)	12. (c)	13. (b)	14. (a)	15. (a)	16. (c)	17. (c)	18. (a)	19. (b)	20. (d)
21. (d)	22. (a)	23. (b)	24. (6)	25. (2)	26. (–2)	27. (B)	28. (c, d)	29. (a, b, c)	
30. (c, d)	31. (a, b, c, d)		32. (a, b, d)		33. (b)				

Topic-2 : Electric potential energy and work done in carrying a charge

1. (d)	2. (c)	3. (d)	4. (b)	5. (b)	6. (c)	7. (c)	8. (c)	9. (b)	10. (b)
11. (b)	12. (b)	13. (2)							

Topic-3 : Capacitors, grouping of capacitors and energy stored in a capacitor

1. (b)	2. (a)	3. (d)	4. (b)	5. (a)	6. (c)	7. (a)	8. (d)	9. (d)	10. (a)
11. (b)	12. (b)	13. (b)	14. (c)	15. (a)	16. (d)	17. (b)	18. (c)	19. (b)	20. (a)
21. (a)	22. (c)	23. (b)	24. (a)	25. (a)	26. (d)	27. (d)	28. (a)	29. (a)	30. (b)
31. (c)	32. (d)	33. (a)	34. (c)	35. (a)	36. (b)	37. (d)	38. (b)	39. (d)	40. (a)
41. (1)	42. (4)	43. (6)	44. (1.50)	45. $\left(\frac{3V}{K+2}\right)$	46. $\frac{E_0A}{d}V, \frac{-2E_0A}{d}V$		47. (c, d)		
48. (a, d)	49. (b, d)	50. (b, c)	51. (a, c, d)	52. (b, d)	53. (a, d)				

Topic-4 : Miscellaneous (Mixex Concepts) Problems

1. (c)	2. (c, d)	3. A-(p, r, s): B-(r, s), C-(p, q, t), D-(r, s)	4. (a)	5. (c)	6. (c)	7. (d)

17 Current Electricity

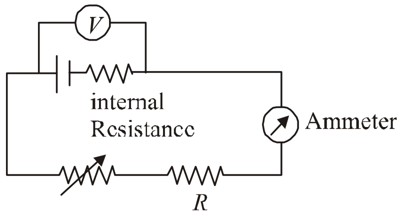
Topic-1 : Electric Current, Drift of Electrons, Ohm's Law, Resistance and Resistivity

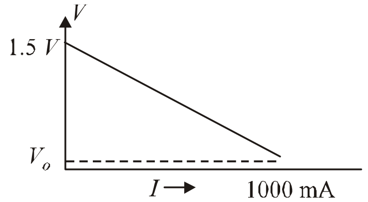
1 MCQs with One Correct Answer

1. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit : **[Main Sep. 06, 2020 (II)]**
(a) ammeter is always used in parallel and voltmeter is series
(b) Both ammeter and voltmeter must be connected in parallel
(c) ammeter is always connected in series and voltmeter in parallel
(d) Both, ammeter and voltmeter must be connected in series

2. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity ρ_C, ρ_T, ρ_M and ρ_A respectively. Then : **[Main Sep. 02, 2020 (I)]**
(a) $\rho_C > \rho_A > \rho_T$ (b) $\rho_M > \rho_A > \rho_C$
(c) $\rho_A > \rho_T > \rho_C$ (d) $\rho_A > \rho_M > \rho_C$

3. To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained : **[Main 12 Apr. 2019 (I)]**





If V_0 is almost zero, identify the correct statement:
(a) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω
(b) The value of the resistance R is 1.5 Ω
(c) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA
(d) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

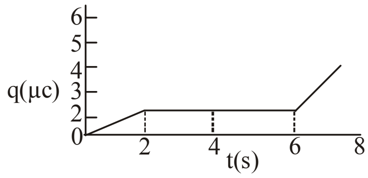
4. A current of 5 A passes through a copper conductor (resistivity) = $1.7\times10^{-8}\Omega$m) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s. **[Main 10 Apr. 2019 (I)]**
(a) 1.8m^2/Vs (b) 1.5 m^2/Vs
(c) 1.3 m^2/Vs (d) 1.0m^2/Vs

5. Space between two concentric conducting spheres of radii a and b ($b > a$) is filled with a medium of resistivity ρ. The resistance between the two spheres will be : **[Main 10 Apr. 2019 (II)]**
(a) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (b) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$
(c) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (d) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

6. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 *fs* (femto second), it's approximate resistivity is: (m_e = 9.1×10^{-31} kg) **[Main 9 Apr. 2019 (II)]**
(a) 10^{-6} Ω m (b) 10^{-7} Ωm
(c) 10^{-8} Ωm (d) 10^{-5} Ωm

7. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be : **[Main 8 April 2019 (I)]**
(a) 100 Ω (b) 400 Ω (c) 300 Ω (d) 500 Ω

8. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure: **[Main 12 Jan. 2019 (II)]**



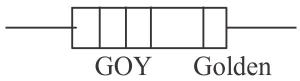
What is the value of current at t = 4 s ?

(a) Zero (b) 3 μA (c) 2 μA (d) 1.5 μA

9. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s close to (Take charge of electron to be = 1.6×10^{-19}C) **[Main 9 Jan. 2019, (I)]**

(a) 0.02 (b) 3 (c) 2 (d) 0.2

10. A carbon resistance has following colour code. What is the value of the resistance? **[Main 9 Jan. 2019, (II)]**



(a) 530 kΩ ± 5% (b) 5.3 kΩ ± 5%
(c) 6.4 MΩ ± 5% (d) 64 MΩ ± 10%

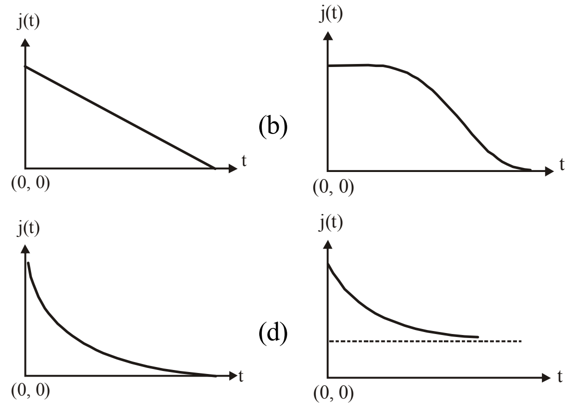
11. A heating element has a resistance of 100Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500°C more than room temperature. What is the temperature coefficient of resistance of the heating element? **[Main Online April 16, 2018]**

(a) 1×10^{-4}°C^{-1} (b) 5×10^{-4}°C^{-1}
(c) 2×10^{-4}°C^{-1} (d) 0.5×10^{-4}°C^{-1}

12. A uniform wire of length l and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be : **[Main Online April 9, 2017]**

(a) 1600 Ω (b) 400 Ω (c) 200 Ω (d) 100 Ω

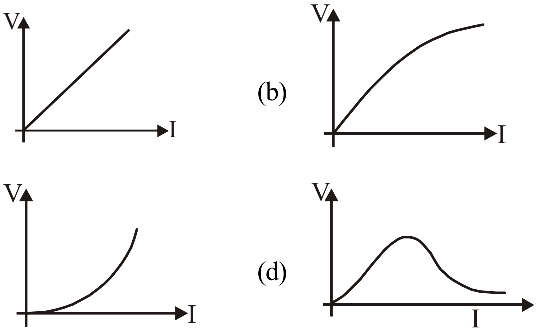
13. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material? **[Adv. 2016]**



14. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to : **[Main 2015]**

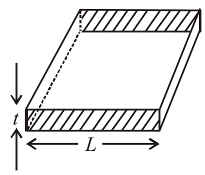
(a) 1.6×10^{-6} Ωm (b) 1.6×10^{-5} Ωm
(c) 1.6×10^{-8} Ωm (d) 1.6×10^{-7} Ωm

15. Suppose the drift velocity v_d in a material varied with the applied electric field E as $v_d \propto \sqrt{E}$. Then V – I graph for a wire made of such a material is best given by : **[Main Online April 10, 2015]**

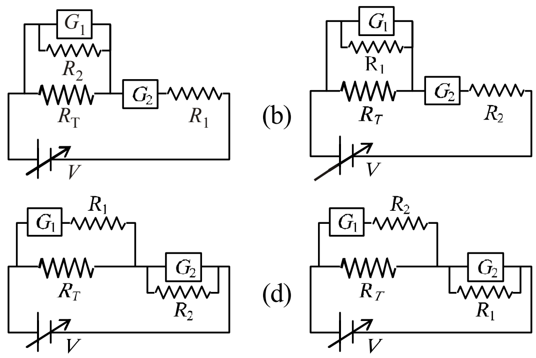


16. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **[2010]**

(a) directly proportional to L
(b) directly proportional to t
(c) independent of L
(d) independent of t

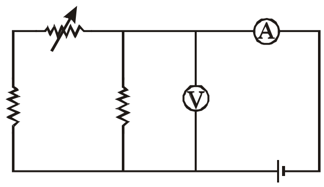


17. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is **[2010]**

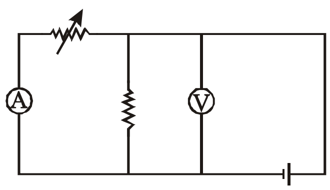


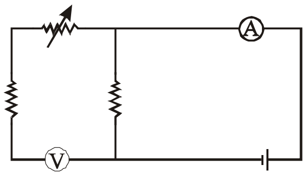
18. Express which of the following set ups can be used to verify Ohm's law? **[2003S]**

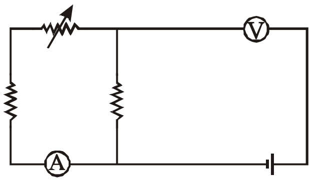
(a)



(b)



(c) 

(d) 

19. A piece of copper and another of germanium are cooled from room temperature to 80° K. The resistance of **[1988 - 1 Mark]**

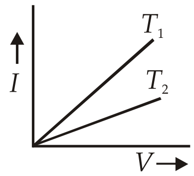
(a) each of them increases
(b) each of them decreases
(c) copper increases and germanium decreases
(d) copper decreases and germanium increases

20. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K, its resistance is 1 ohm. This resistance of the wire will be 2 ohm at. **[1980]**

(a) 1154 K (b) 1100 K
(c) 1400 K (d) 1127 K

5 True / False

21. The current –voltage graphs for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure. **[1985 - 3 Marks]**

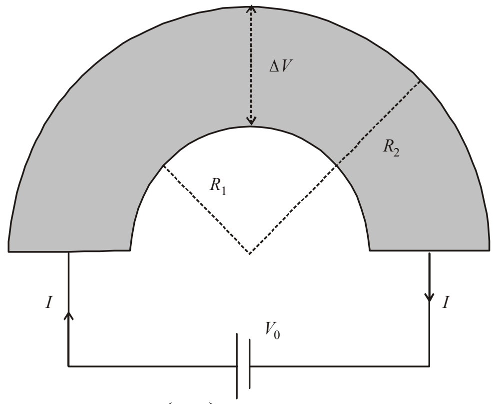


The temperature T_2 is greater than T_1.

22. Electrons in a conductor have no motion in the absence of a potential difference across it. **[1982 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

23. Shown in the figure is a semicircular metallic strip that has thickness t and resistivity ρ. Its inner radius is R_1 and outer radius is R_2. If a voltage V_0 is applied between its two ends, a current I flows in it. In addition, it is observed that a transverse voltage ΔV develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale) **[Adv. 2020]**



(a) $I = \dfrac{V_0 t}{\pi\rho} \ln\left(\dfrac{R_2}{R_1}\right)$
(b) the outer surface is at a higher voltage than the inner surface
(c) the outer surface is at a lower voltage than the inner surface
(d) $\Delta V \propto I^2$

9 Assertion and Reason Type Questions

24. Read the following statements carefully: **[1993-2 Marks]**

Y: The resistivity of a semiconductor decreases with increase of temperature.

Z: In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature

Select the correct statement(s) from the following;

(a) Y is true but Z is false
(b) Y is false but Z is true
(c) Both Y and Z are true
(d) Y is true and Z is the correct reason for Y

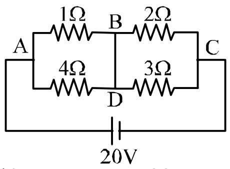
10 Subjective Problems

25. If a copper wire is stretched to make it 0.1% longer what is the percentage change in its resistance? **[1978]**

Topic-2 : Combination of Resistances

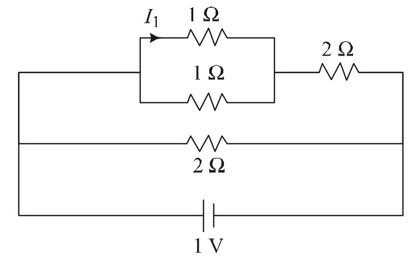
1 MCQs with One Correct Answer

1. In the given circuit diagram, a wire is joining points B and D. The current in this wire is: **[Main 9 Jan. 2020, (I)]**



(a) 0.4A (b) 2A (c) 4A (d) zero

2. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is: **[Main 7 Jan. 2020 (I)]**

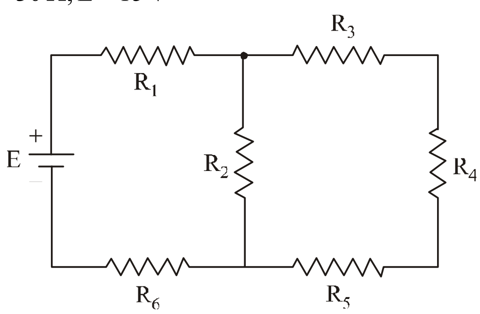


(a) 0.4 (b) 0.5 (c) 0.2 (d) 0.25

3. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on the circle make an angle 60° at the centre, the equivalent resistance between these two points will be: **[Main 9 Apr. 2019 (II)]**

(a) $\frac{12}{5}\Omega$ (b) $\frac{5}{2}\Omega$ (c) $\frac{5}{3}\Omega$ (d) $\frac{7}{2}\Omega$

4. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given : **[Main 8 Apr. 2019 (II)]**
$R_1 = 15\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 25\ \Omega$, $R_6 = 30\ \Omega$, E = 15 V

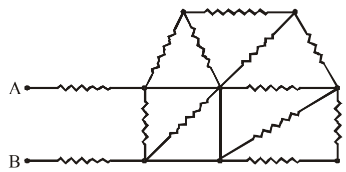


(a) 13/24 (b) 7/18 (c) 9/32 (d) 20/3

5. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is: **[Main 10 Jan. 2019 (I)]**

(a) 4 Ω (b) 8 Ω (c) 12 W (d) 2 W

6. In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is : **[Main Online April 15, 2018]**



(a) $2R$ (b) $\frac{5R}{2}$ (c) $\frac{5R}{3}$ (d) $3R$

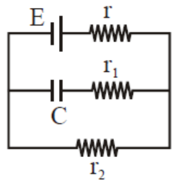
7. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[Main 2017]**

(a) $CE\frac{r_2}{(r+r_2)}$

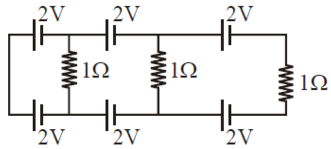
(b) $CE\frac{r_1}{(r_1+r)}$

(c) CE

(d) $CE\frac{r_1}{(r_2+r)}$



8.



In the above circuit the current in each resistance is **[Main 2017]**

(a) 0.5A (b) 0 A (c) 1 A (d) 0.25 A

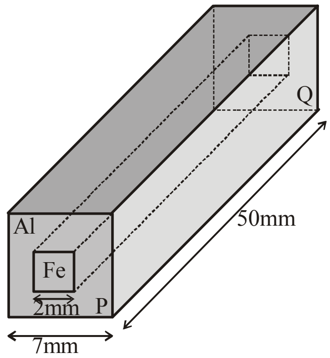
9. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\ \Omega$ m, respectively. The electrical resistance between the two faces P and Q of the composite bar is **[Adv. 2015]**

(a) $\frac{2475}{64}\mu\Omega$

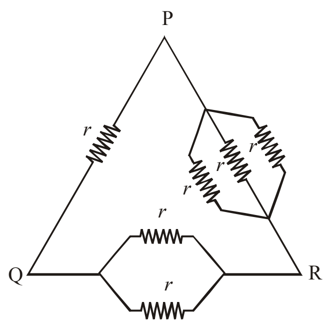
(b) $\frac{1875}{64}\mu\Omega$

(c) $\frac{1875}{49}\mu\Omega$

(d) $\frac{2475}{132}\mu\Omega$

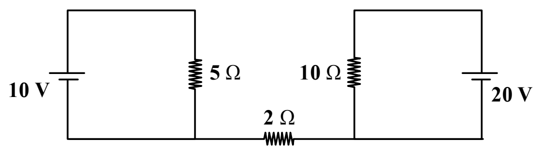


10. Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between : **[Main Online April 25, 2013]**



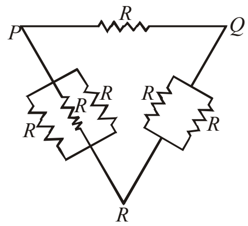
(a) P and R (b) P and Q
(c) Q and R (d) Any two points

11. Find out the value of current through 2Ω resistance for the given circuit. **[2005S]**



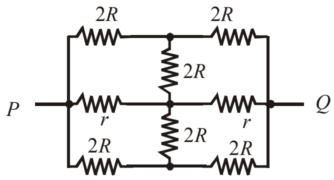
(a) zero (b) 2 A (c) 5 A (d) 4 A

12. Six identical resistors are connected as shown in the figure. The equivalent resistance will be **[2004S]**



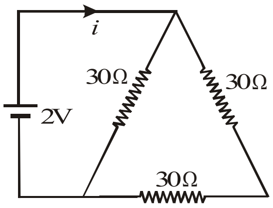
(a) Maximum between P and R
(b) Maximum between Q and R
(c) Maximum between P and Q
(d) All are equal

13. The effective resistance between points P and Q of the electrical circuit shown in the figure is **[2002S]**



(a) $\dfrac{2Rr}{R+r}$
(b) $\dfrac{8R(R+r)}{3R+r}$
(c) $2r+4R$
(d) $\dfrac{5R}{2}+2r$

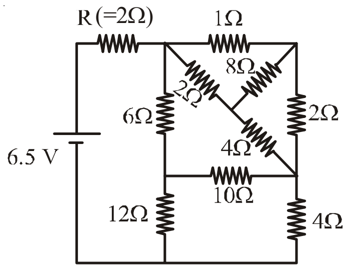
14. The current i in the circuit (see Fig) is **[1983 - 1 Mark]**



(a) $\frac{1}{45}$ A (b) $\frac{1}{15}$ A (c) $\frac{1}{10}$ A (d) $\frac{1}{5}$ A

2 Integer Value Answer

15. In the following circuit, the current through the resistor $R\,(=2\,\Omega)$ is I amperes. The value of I is **[Adv. 2015]**

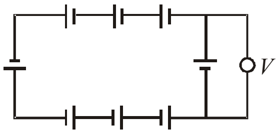


3 Numeric Answer

16. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20 Ω and 5 Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20 Ω is zero, the value of R (in Ω) is __________. **[Main 8 Jan. 2020, (II)]**

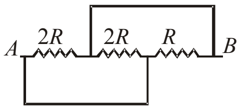
4 Fill in the Blanks

17. In the circuit shown below, each battery is 5V and has an internal resistance of 0.2 ohm.



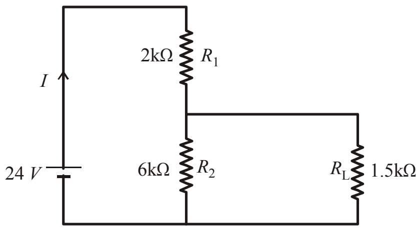
The reading in the ideal voltmeter V is ..... V. **[1997 - 2 Marks]**

18. The equivalent resistance between points A and B of the circuit given below is ......Ω . **[1997 - 2 Marks]**



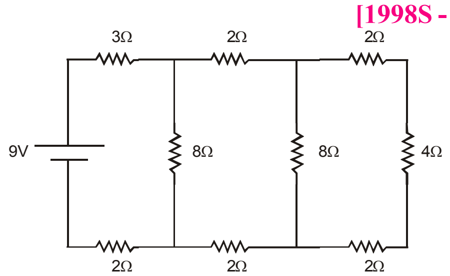
6 MCQs with One or More than One Correct Answer

19. For the circuit shown in the figure **[2009]**



(a) the current I through the battery is 7.5 mA
(b) the potential difference across R_L is 18 V
(c) ratio of powers dissipated in R_1 and R_2 is 3
(d) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

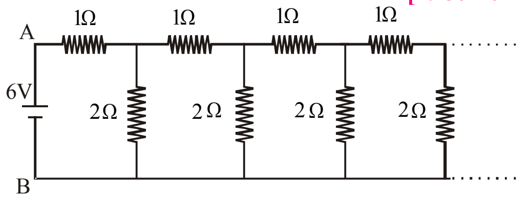
20. In the circuit shown in Figure the current through **[1998S - 2 Marks]**



(a) the 3 Ω resistor is 0.50 A.
(b) the 3 Ω resistor is 0.25 A.
(c) the 4 Ω resistor is 0.50 A
(d) the 4 Ω resistor is 0.25 A.

10 Subjective Problems

21. An infinite ladder network of resistances is constructed with a1 ohm and 2 ohm resistances, as shown in fig. **[1987 - 7 Marks]**

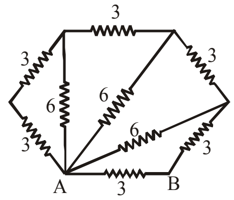


The 6 volt battery between A and B has negligible internal resistance :

(i) Show that the effective resistance between A and B is 2 ohms.

(ii) What is the current that passes through the 2 ohm resistance nearest to the battery ?

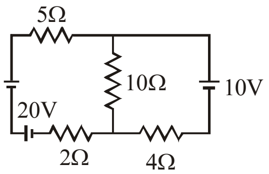
22. All resistances in the diagram below are in ohms. Find the effective resistance between the points A and B. **[1979]**



Topic-3 : Kirchhoff's Laws, Cells, Thermo e.m.f. & Electrolysis

1 MCQs with One Correct Answer

1.



In the figure shown, the current in the 10 V battery is close to : **[Main Sep. 06, 2020 (II)]**

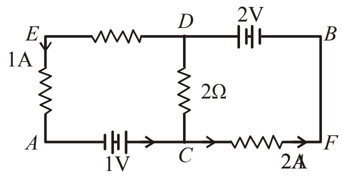
(a) 0.71 A from positive to negative terminal

(b) 0.42 A from positive to negative terminal

(c) 0.21 A from positive to negative terminal

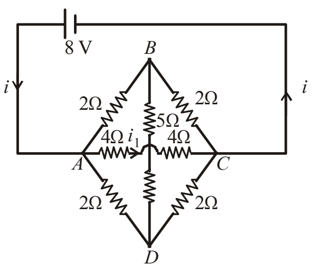
(d) 0.36 A from negative to positive terminal

2. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is : **[Main Sep. 05, 2020 (II)]**



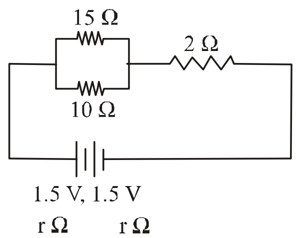
(a) +2 V (b) –2 V (c) –1 V (d) +1 V

3. The value of current i_1 flowing from A to C in the circuit diagram is : **[Main Sep. 04, 2020 (II)]**



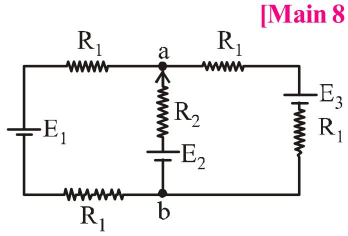
(a) 2 A (b) 4 A (c) 1 A (d) 5 A

4. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2V. The internal resistance r, of each cell is : **[Main 10 Apr. 2019 (I)]**



(a) 1 Ω (b) 0.5 Ω (c) 1.5 Ω (d) 0 Ω

5. For the circuit shown, with $R_1 = 1.0\,\Omega$, $R_2 = 2.0\,\Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points 'a' and 'b' is approximately (in V) : **[Main 8 April 2019 (I)]**



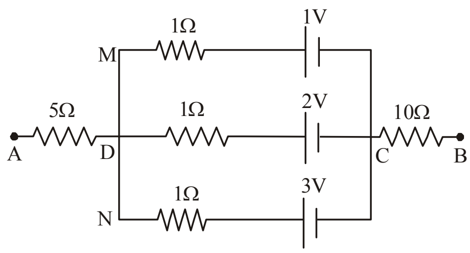
(a) 2.7 (b) 2.3 (c) 3.7 (d) 3.3

6. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when : **[Main 8 Apr. 2019 (II)]**

(a) $R = 0.001\,r$ (b) $R = 1000\,r$

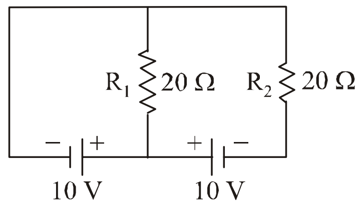
(c) $R = 2r$ (d) $R = r$

7. In the circuit shown, the potential difference between A and B is : **[Main 11 Jan. 2019 (II)]**



(a) 1 V (b) 2 V (c) 3 V (d) 6 V

8. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are: **[Main 10 Jan. 2019 (I)]**

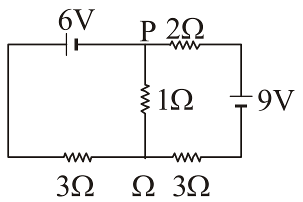


(a) 1, 2 (b) 2, 2 (c) 0.5, 0 (d) 0, 1

9. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between: **[Main 2018]**

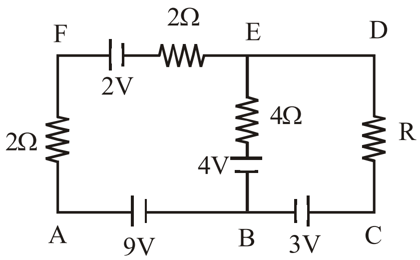
(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V
(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

10. In the circuit shown, the current in the 1Ω resistor is: **[Main 2015]**



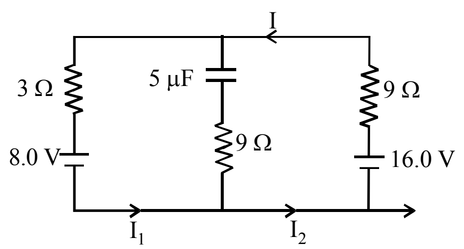
(a) 0.13 A, from Q to P (b) 0.13 A, from P to Q
(c) 1.3A from P to Q (d) 0A

11. In the electric network shown, when no current flows through the 4Ω resistor in the arm EB, the potential difference between the points A and D will be : **[Main Online April 11, 2015]**



(a) 6 V (b) 3 V (c) 5 V (d) 4 V

12. The circuit shown here has two batteries of 8.0 V and 16.0 V and three resistors 3 Ω, 9 Ω and 9 Ω and a capacitor of 5.0 μF. **[Main Online April 12, 2014]**

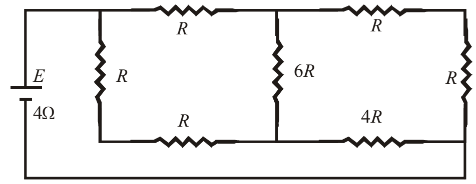


How much is the current I in the circuit in steady state?
(a) 1.6 A (b) 0.67 A (c) 2.5 A (d) 0.25 A

13. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. the value of N is **[2001S]**

(a) 4 (b) 6 (c) 8 (d) 9

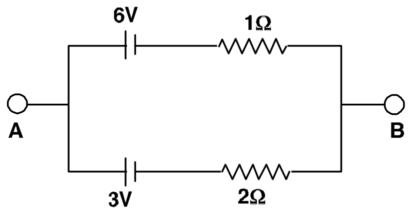
14. A battery of internal resistance 4Ω is connected to the network of resistances as shown. In order that the maximum power can be delivered to the network, the value of R in Ω should be **[1995S]**



(a) $\frac{4}{9}$ (b) 2 (c) $\frac{8}{3}$ (d) 18

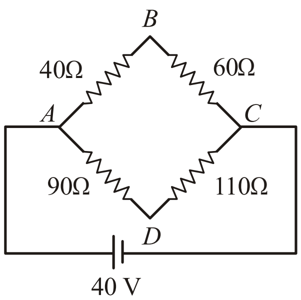
2 Integer Value Answer

15. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is **[2011]**



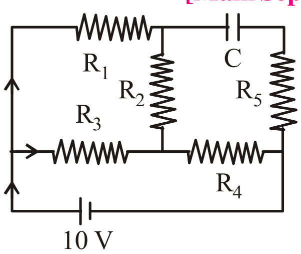
3 Numeric Answer

16.



Four resistances 40 Ω, 60 Ω, 90 Ω and 110 Ω make the arms of a quadrilateral $ABCD$. Across AC is a battery of emf 40 V and internal resistance negligible. The potential difference across BD in V is ____________. **[Main Sep. 04, 2020 (II)]**

17. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is ____________. **[Main Sep. 02, 2020 (II)]**

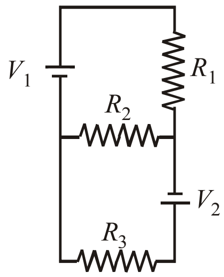


5 True / False

18. In an electrolytic solution the electric current is mainly due to the movement of free electrons. **[1980]**

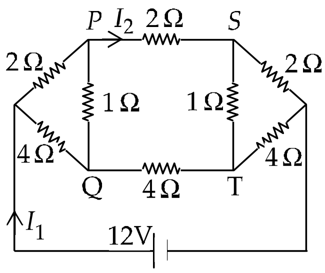
6 MCQs with One or More than One Correct Answer

19. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if **[Adv. 2014]**



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(b) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(c) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(d) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

20. For the resistance network shown in the figure, choose the correct option(s) **[2012- I]**



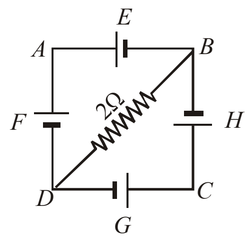
(a) The current through PQ is zero.
(b) $I_1 = 3A$
(c) The potential at S is less than that at Q.
(d) $I_2 = 2A$

10 Subjective Problems

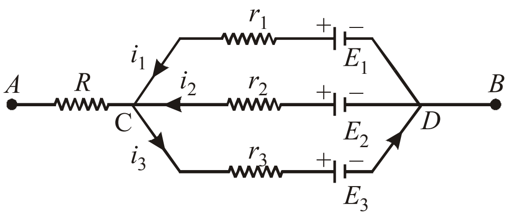
21. In the circuit shown in figure E, F, G, H are cells of emf 2, 1, 3 and 1 volt respectively, and their internal resistances are 2, 1, 3 and 1 ohm respectively. **[1984 - 6 Marks]**

Calculate :

(i) the potential difference between B and D and
(ii) the potential difference across the terminals of each cells G and H



22. In the circuit shown in fig E_1 =3 volts, E_2 = 2 volts, E_3 = 1 volt and R = r_1 = r_2 = r_3 = 1 ohm. **[1981 - 6 Marks]**



(i) Find the potential difference between the points A and B and the currents through each branch.
(ii) If r_2 is short circuited and the point A is connected to point B, find the currents through E_1, E_2 E_3 and the resistor R.

Topic-4 : Heating Effect of Current

1 MCQs with One Correct Answer

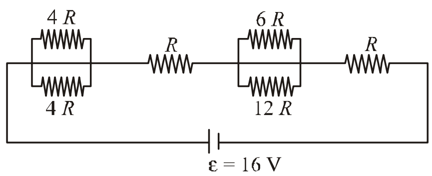
1. An electrical power line, having a total resistance of 2 Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately : **[Main Sep. 05, 2020 (I)]**

(a) 72% (b) 91% (c) 85% (d) 96%

2. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be: **[Main 7 Jan. 2020 (II)]**

(a) 10 A (b) 25 A (c) 15 A (d) 20 A

3. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is : **[Main 12 Apr. 2019 (I)]**



(a) 6Ω (b) 8Ω (c) 1Ω (d) 16Ω

4. One kg of water, at 20°C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to :

[Specific heat of water = 4200 J/(kg°C), Latent heat of water = 2260 kJ/kg] **[Main 12 Apr. 2019 (II)]**

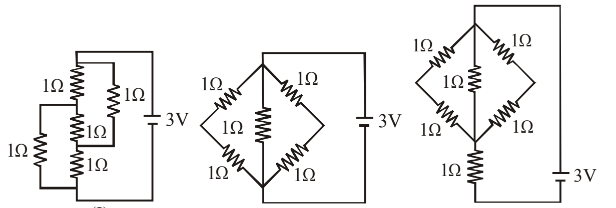
(a) 16 minutes (b) 22 minutes
(c) 3 minutes (d) 3 minutes

5. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then: **[Main 12 Jan. 2019 (I)]**
(a) P_1=16 W, P_2=4 W (b) P_1=16 W, P_2=9 W
(c) P_1=9 W, P_2=16 W (d) P_1=4 W, P_2=16 W

6. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistance are now connected in parallel combination to the same battery, the electric power consumed will be : **[Main 11 Jan. 2019 (I)]**
(a) 60 W (b) 240 W (c) 120 W (d) 30 W

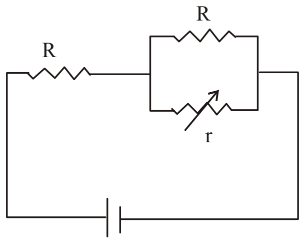
7. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is: **[Main 10 Jan. 2019 (II)]**
(a) 11×10^{-5} W (b) 11×10^{-3} W
(c) 11×10^{-4} W (d) 11×10^{5} W

8. The figure shows three circuits I, II and III which are connected to a 3V battery. If the powers dissipated by the configurations I, II and III are P_1, P_2 and P_3 respectively, then : **[Main Online April 9, 2017]**



(I) (II) (III)

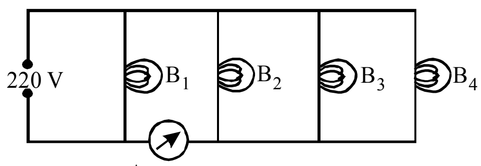
(a) $P_1 > P_2 > P_3$ (b) $P_1 > P_3 > P_2$
(c) $P_2 > P_1 > P_3$ (d) $P_3 > P_2 > P_1$

9.


In the circuit shown, the resistance r is a variable resistance. If for r = fR, the heat generation in r is maximum then the value of f is : **[Main Online April 9, 2016]**
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

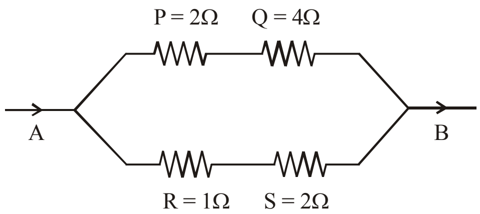
10. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of electric mains is 220 V. The minimum capacity of the main fuse of the building will be: **[Main 2014]**
(a) 8 A (b) 10 A (c) 12 A (d) 14 A

11. Four bulbs B_1, B_2, B_3 and B_4 of 100 W each are connected to 220 V main as shown in the figure. **[Main Online April 19, 2014]**



The reading in an ideal ammeter will be:
(a) 0.45 A (b) 0.90 A (c) 1.35 A (d) 1.80 A

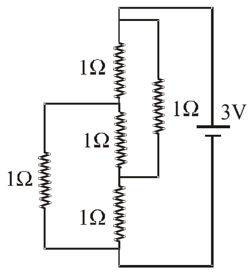
12. Which of the four resistances P, Q, R and S generate the greatest amount of heat when a current flows from A to B? **[Main Online April 23, 2013]**



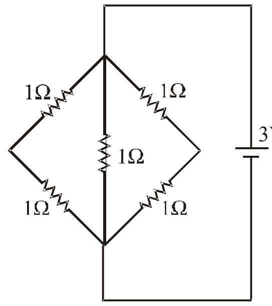
(a) Q (b) S (c) P (d) R

13. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is **[2010]**
(a) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (b) $R_{100} = R_{40} + R_{60}$
(c) $R_{100} > R_{60} > R_{40}$ (d) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

14. Figure shows three resistor configurations R_1, R_2 and R_3 connected to 3V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then – **[2008]**

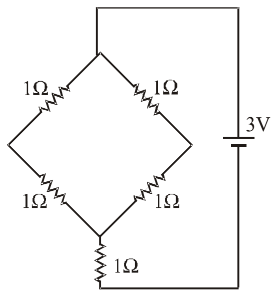


R_1



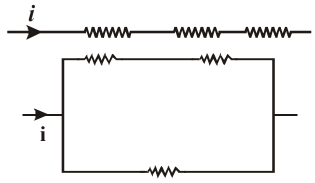
R_2

(a) $P_1 > P_2 > P_3$
(b) $P_1 > P_3 > P_2$
(c) $P_2 > P_1 > P_3$
(d) $P_3 > P_2 > P_1$

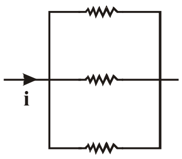


R_3

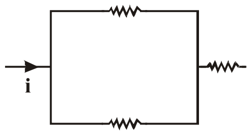
15. The three resistance of equal value are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation. **[2003S]**



(I) **(II)**



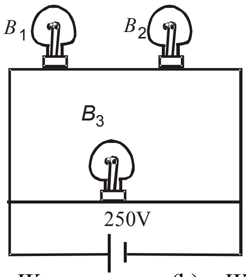
(III)



(IV)

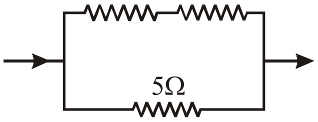
(a) III < II < IV < I (b) II < III < IV < I
(c) I < IV < III < II (d) I < III < II < IV

16. A 100 W bulb B_1, and two 60 W bulb B_2 and B_3, are connected to a 250 V source, as shown in figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3, respectively. Then **[2002S]**



(a) $W_1 > W_2 = W_3$ (b) $W_1 > W_2 > W_3$
(c) $W_1 < W_2 = W_3$ (d) $W_1 < W_2 < W_3$

17. In the circuit shown in fig the heat produced in the 5 ohm resistor due to the current flowing through it is 10 calories per second. **[1981- 2 Marks]**



The heat generated in the 4 ohms resistor is
(a) 1 calorie / sec (b) 2 calories /sec
(c) 3 calories /sec (d) 4 calories /sec

18. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if **[1980]**
(a) both the length and the radius of the wire are halved.
(b) both the length and the radius of the wire are doubled.
(c) the radius of the wire is doubled.
(d) the length of the wire is doubled.

2 Integer Value Answer

19. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\, J_2$ then the value of R in Ω is **[2010]**

4 Fill in the Blanks

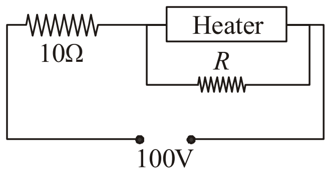
20. An electric bulb rated for 500 watts at 100 volts is used in a circuit having a 200 volts supply. The resistance R that must be put in series with the bulb, so that the bulb delivers 500 watt is ....ohm. **[1987 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

21. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K? **[Adv. 2014]**
(a) 4 if wires are in parallel (b) 2 if wires are in series
(c) 1 if wires are in series (d) 0.5 if wires are in parallel

10 Subjective Problems

22. A heater is designed to operate with a power of 1000 watts in a 100 volt line. It is connected in a combinations with a resistance of 10 ohms and a resistance R to a 100 volts mains as shown in the figure. What should be the value of R so that the heater operates with a power of 62.5 watts. **[1978]**



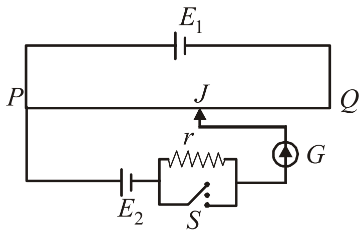
Topic-5 : Wheatstone Bridge and Different Measuring Instruments

1 MCQs with One Correct Answer

1. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400Ω resistor is close to: **[Main Sep. 03, 2020 (II)]**
(a) 2V (b) 1.8 V (c) 2.05 V (d) 1.95 V

2. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed? **[Main Sep. 03, 2020 (II)]**
(a) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.
(b) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.
(c) Multimeter shows a deflection, accompanied by a splash of light out of connected and NO deflection on reversing the probes if the chosen component is LED.
(d) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

3. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is : **[Main Sep. 02, 2020 (II)]**

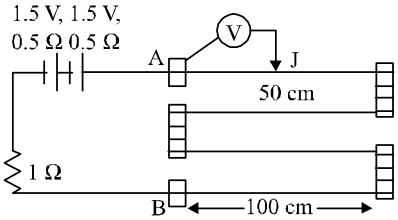


(a) 0.02 V/cm (b) 0.01 V/cm
(c) 0.03 V/cm (d) 0.04 V/cm

4. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of 20 Ω, the null point on it is found to be at 1000 cm. The resistance of whole wire is: **[Main 8 Jan. 2020 (I)]**

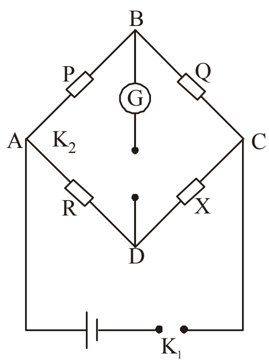
(a) 80 Ω (b) 120 Ω (c) 60 Ω (d) 100 Ω

5. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01$ Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be : **[Main 8 Apr. 2019 (II)]**



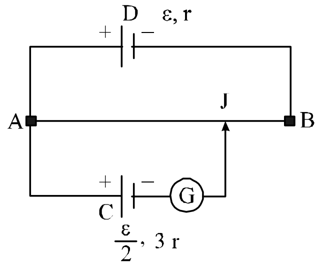
(a) 0.50 V (b) 0.75 V (c) 0.25 V (d) 0.20 V

6. In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405Ω. The value of X is close to : **[Main 11 Jan. 2019 (I)]**



(a) 401.5 ohm (b) 404.5 ohm
(c) 403.5 ohm (d) 402.5 ohm

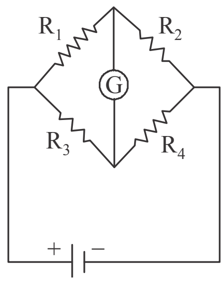
7. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is: **[Main 10 Jan. 2019 (I)]**



(a) $\frac{11}{12}$L (b) $\frac{11}{24}$L (c) $\frac{13}{24}$L (d) $\frac{5}{12}$L

8. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be: **[Main 10 Jan. 2019 (II)]**

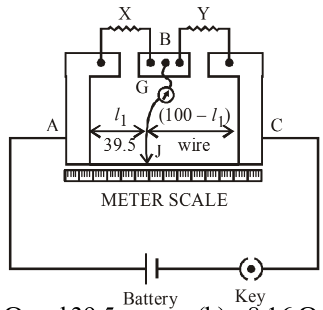


(a) Brown, Blue, Brown (b) Brown, Blue, Black
(c) Red, Green, Brown (d) Grey, Black, Brown

9. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[Main 2018]**

(a) 1 Ω (b) 1.5 Ω (c) 2 Ω (d) 2.5 Ω

10. In a meter bridge, as shown in the figure, it is given that resistance Y=12.5 Ω and that the balance is obtained at a distance 39.5 cm from end A (by jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2 ? **[Main Online April 15, 2018]**



(a) 19.15 Ω and 39.5 cm (b) 8.16 Ω and 60.5 cm
(c) 19.15 Ω and 60.5 cm (d) 8.16 Ω and 39.5 cm

11. Which of the following statements is false ? **[Main 2017]**

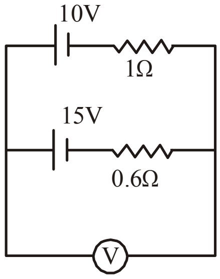
(a) A rheostat can be used as a potential divider

(b) Kirchhoff's second law represents energy conservation

(c) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

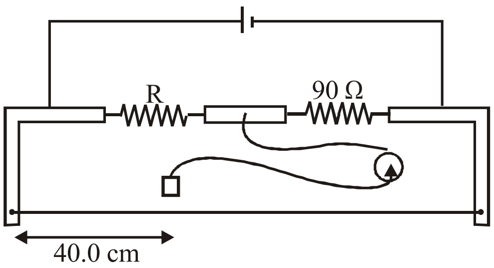
(d) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

12. A 10V battery with internal resistance 1Ω and a 15V battery with internal resistance $0.6\ \Omega$ are connected in parallel to a voltmeter (see figure). The reading in the voltmeter will be close to : **[Main Online April 10, 2015]**



(a) 12.5 V (b) 24.5 V (c) 13.1 V (d) 11.9 V

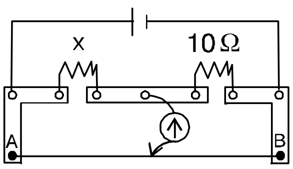
13. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of $90\ \Omega$, as shown in the figure. The least count of the scale used in the metre bridge is 1mm. The unknown resistance is **[Adv. 2014]**



(a) $60 \pm 0.15\ \Omega$ (b) $135 \pm 0.56\ \Omega$

(c) $60 \pm 0.25\ \Omega$ (d) $135 \pm 0.23\ \Omega$

14. A meter bridge is set up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is **[2011]**



(a) 10.2 ohm (b) 10.6 ohm

(c) 10.8 ohm (d) 11.1 ohm

15. A resistance of 2Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is **[2007]**

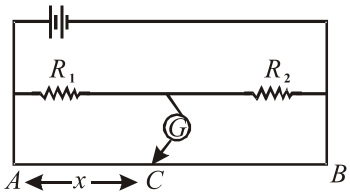
(a) 3Ω (b) 4Ω (c) 5Ω (d) 6Ω

16. A moving coil galvanometer of resistance $100\ \Omega$ is used as an ammeter using a resistance $0.1\ \Omega$. The maximum deflection current in the galvanometer is $100\ \mu A$. Find the minimum current in the circuit so that the ammeter shows maximum deflection **[2005S]**

(a) 100.1 mA (b) 1000.1 mA

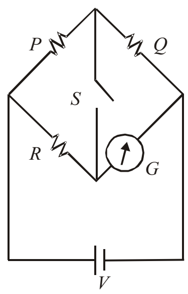
(c) 10.01 mA (d) 1.01 mA

17. In the shown arrangement of the experiment of the meter bridge if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled? **[2003S]**



(a) x (b) $x/4$ (c) $4x$ (d) $2x$

18. In the circuit $P \neq R$, the reading of the galvanometer is same with switch S open or closed. Then **[1999 - 2 Marks]**



(a) $I_R = I_G$

(b) $I_P = I_G$

(c) $I_Q = I_G$

(d) $I_Q = I_R$

3 Numeric Answer

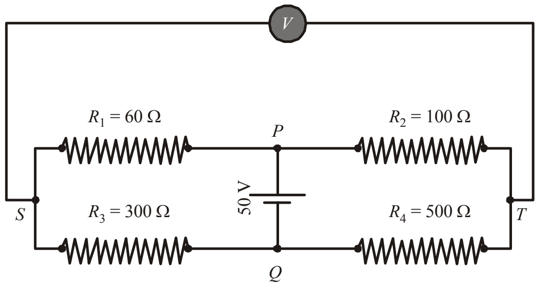
23. Four resistances of $15\ \Omega$, $12\ \Omega$, $4\ \Omega$ and $10\ \Omega$ respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of $10\ \Omega$ to balance the network is _____ Ω.

[Main 8 Jan. 2020 (I)]

24. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of $10\ \Omega$ is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\ \Omega$, where N is an integer then value of N is ________.

[Main 7 Jan. 2020 (II)]

25. In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient $0.0004°C^{-1}$. If the temperature of R_3 is increased by 100°C, the voltage developed between S and T will be ________ volt. **[Adv. 2020]**



9 Assertion and Reason Type Questions

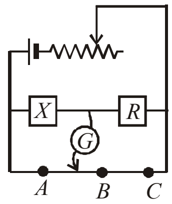
26. **STATEMENT-1 :** In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

STATEMENT-2 : Resistance of a metal increases with increase in temperature. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement -1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

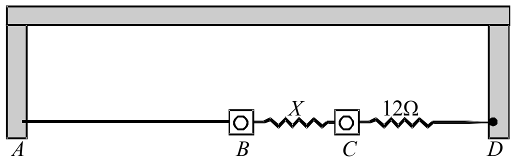
10 Subjective Problems

27. An unknown resistance X is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why? **[2005 - 2 Marks]**



$R = R_1$ or R_2 or R_3

28. A thin uniform wire AB of length 1m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions. **[2002 - 5 Marks]**

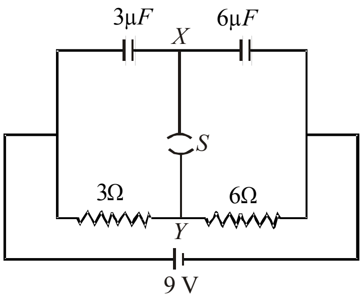


(a) Are there positive and negative terminals on the galvanometer?
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

Topic-6 : Miscellaneous (Mixed Concepts) Problems

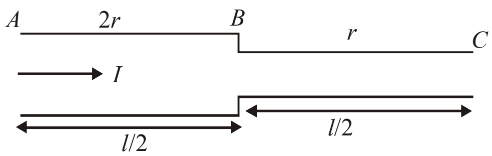
1 MCQs with One Correct Answer

1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **[2007]**



(a) 0 (b) 54 μC (c) 27μC (d) 81 μC

2. If a steady current I is flowing through a cylindrical element ABC. Choose the correct relationship **[2006]**



(a) $V_{AB} = 2V_{BC}$
(b) Power across BC is 4 times the power across AB
(c) Current densities in AB and BC are equal
(d) Electric field due to current inside AB and BC are equal

3. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 minutes supplies heat to the gas. The change in internal energy of the gas is **[2005S]**

(a) 10 kJ (b) 30 kJ (c) 20 kJ (d) 0 kJ

4. A 4 μF capacitor, a resistance of 2.5 $M\Omega$ is in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given $ln(2) = 0.693$] **[2005S]**

(a) 13.86s (b) 6.93s
(c) 7s (d) 14s

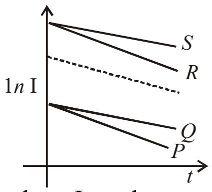
5. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln I with respect to time. If the resistance is changed to $2x$, the new graph will be **[2004S]**

(a) P

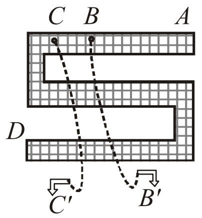
(b) Q

(c) R

(d) S



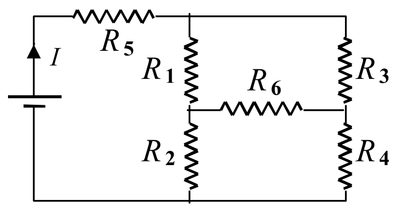
6. Shown in figure is a Post Office box. In order to calculate the value of external resistance, it should be connected between **[2004S]**



(a) B' and C' (b) A and D

(c) C and D (d) B and D

7. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy **[2001S]**



(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\dfrac{1}{R_5}+\dfrac{1}{R_6}=\dfrac{1}{R_1+R_2}+\dfrac{1}{R_3+R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4 = R_5R_6$

8. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. the value of N is **[2001S]**

(a) 4 (b) 6 (c) 8 (d) 9

9. The electrostatic field due to a point charge depends on the distance r as $\dfrac{1}{r^2}$. Indicate which of the following quantities shows same dependence on r. **[1980]**

(a) Intensity of light from a point source.

(b) Electrostatic potential due to a point charge.

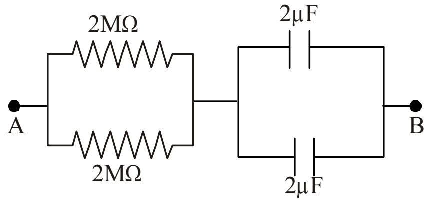
(c) Electrostatic potential at a distance r from the centre of a charged metallic sphere. Given $r <$ radius of the sphere.

(d) None of these

2 Integer Value Answer

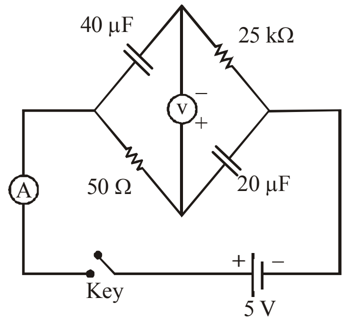
10. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\dfrac{2n}{249}\,\Omega$ resistance, it becomes an ammeter of range 0 – 1.5A. The value of n is **[Adv. 2014]**

11. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in sceonds) does the voltage across them become 4 V? [Take : ln5 =1.6, ln3 = 1.1] **[2010]**



6 MCQs with One or More than One Correct Answer

12. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true? **[Adv. 2016]**



(a) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(b) The voltmeter will display 0V at time t = ln 2 seconds

(c) The current in the ammeter becomes 1/e of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time.

13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black–body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non–uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? **[Adv. 2016]**

(a) The temperature distribution over the filament is uniform

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

14. When a potential difference is applied across, the current passing through **[1999S - 3 Marks]**
(a) an insulator at 0 K is zero
(b) a semiconductor at 0 K is zero
(c) a metal at 0 K is finite
(d) a *p-n* diode at 300K is finite, if it is reverse biased

15. Capacitor C_1 of capacitance 1 micro-farad and capacitor C_2 of capacitance 2 microfarad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$. **[1989 - 2 Marks]**
(a) The current in each of the two discharging circuits is zero at $t = 0$.
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.
(c) The currents in the two discharging circuits at $t = 0$ are unequal.
(d) Capacitor C_1, losses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

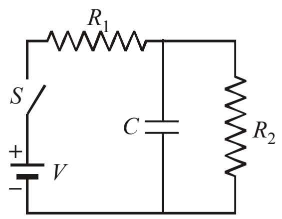
7 Match the Following

16. Column I gives some devices and Column II gives some processes on which the functioning of these devices depend. Match the devices in Column I with the processes in Column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS. **[2007]**

	Column I		Column II
(A)	Bimetallic strip	(p)	Radiation from a hot body
(B)	Steam engine	(q)	Energy conversion
(C)	Incandescent lamp	(r)	Melting
(D)	Electric fuse	(s)	Thermal expansion of solids

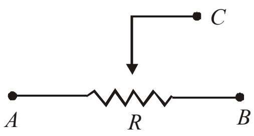
10 Subjective Problems

17. In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters as shown in the circuit. **[2005 - 4 Marks]**

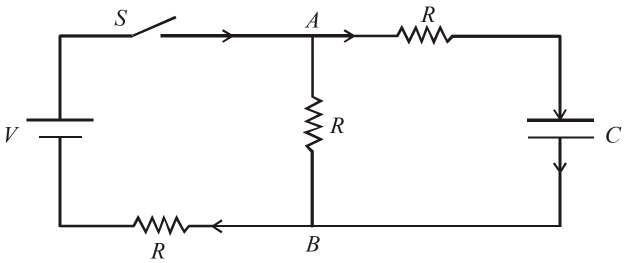


18. Draw the circuit diagram to verify Ohm's Law with the help of a main resistance of 100 Ω and two galvanometers of resistances 10^6 Ω and 10^{-3} Ω and a source of varying emf. Show the correct positions of voltmeter and ammeter. **[2004 - 4 Marks]**

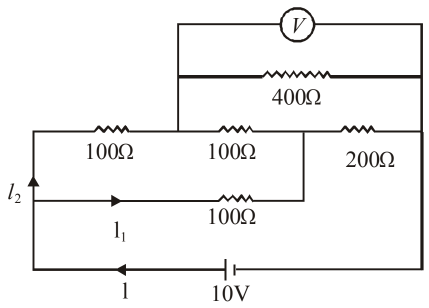
19. How a battery is to be connected so that the shown rheostat will behave like a potential divider? Also indicate the points about which output can be taken. **[2003 - 2 Marks]**



20. In the circuit shown in Figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$. **[1998 - 8 Marks]**
(a) Find the charge Q on the capacitor at time t.
(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$:



21. An electrical circuit is shown in Fig. Calculate the potential difference across the resistor of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchhoff's rules or otherwise. **[1996 - 5 Marks]**

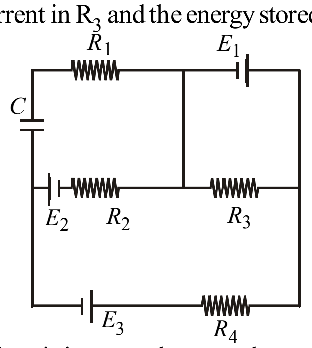


22. In the given circuit **[1988 - 5 Marks]**

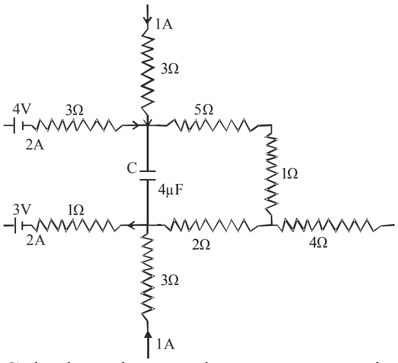
$E_1 = 3E_2 = 2E_3 = 6$ volts $\quad R_1 = 2R_4 = 6$ ohms

$R_3 = 2R_2 = 4$ ohms $\quad C = 5\mu f$.

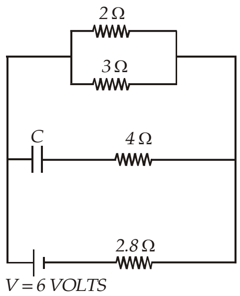
Find the current in R_3 and the energy stored in the capacitor.



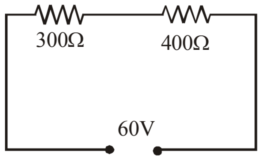
23. A part of ciucuit in a steady state along with the currents flowing in the branches, the values of resistances etc., is shown in the figure. Calculate the energy stored in the capacitor C (4μF) **[1986 - 4 Marks]**



24. Calculate the steady state current in the 2-ohm resistor shown in the circuit in the figure. The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 microfarad. **[1982 - 5 Marks]**

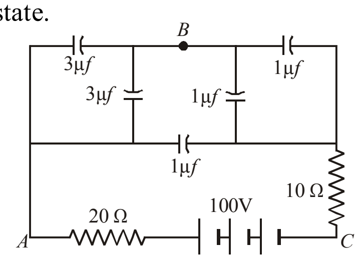


25. State ohm's law.
In the circuit shown in figure, a voltmeter reads 30 volts when it is connected across 400 ohm resistance. Calculate what the same voltmeter will read when it is connected across the 300 ohm resistance. **[1980]**



26. A battery of emf 2 volts and internal resistance 0.1 ohm is being charged with a current of 5 amps. **[1980]**
In what direction will the current flow inside the battery? What is the potential difference between the two terminal of the battery?

27. In the diagram shown find the potential difference between the points A and B and between the points B and C in the steady state. **[1979]**



Answer Key

Topic-1 : Electric Current, Drift of Electrons, Ohm's Law, Resistance and Resistivity

1. (c)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (a)
11. (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (d)
21. True,	**22.** False,	**23.** (a, c, d)	**24.** (c)						

Topic-2 : Combination of Resistances

1. (b)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (b)
11. (a)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (1)	**16.** (30.00)	**17.** (0 V)	**18.** $\left(\frac{R}{2}\right)$	**19.** (a, d)	**20.** (d)

Topic-3 : Kirchhoff's Laws, Cells, Thermo e.m.f. & Electrolysis

1. (c)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (5)	**16.** (2)	**17.** (08.00)	**18.** False	**19.** (a, b, d)	
20. (a, b, c, d)									

Topic-4 : Heating Effect of Current

1. (b)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (c)
11. (c)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (4)	**20.** (20 Ω)
21. (b, d)									

Topic-5 : Wheatstone Bridge and Defferent Measuring Instruments

1. (d)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (10)	**20.** (12)
21. (0.27)	**22.** (d)								

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (5)
11. (2)	**12.** (a, b, c, d)		**13.** (c, d)	**14.** (a,b,d)	**15.** (b, d)	**16.** A $\rightarrow$ s; B $\rightarrow$ q; C $\rightarrow$ p,q; D $\rightarrow$ q,r			

18 Moving Charges and Magnetism

Topic-1 : Motion of Charged Particle in Magnetic Field

1 MCQs with One Correct Answer

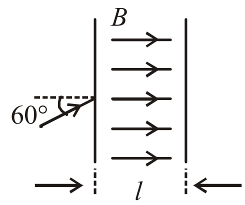
1. An electron is moving along $+x$ direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along $+y$ direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be : **[Main Sep. 06, 2020 (I)]**
   (a) 3×10^{-4} T, along $+z$ direction
   (b) 5×10^{-3} T, along $-z$ direction
   (c) 5×10^{-3} T, along $+z$ direction
   (d) 3×10^{-4} T, along $-z$ direction

2. A particle of charge q and mass m is moving with a velocity $-v\,\hat{i}(v \neq 0)$ towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is: **[Main Sep. 06, 2020 (I)]**
   (a) $\frac{qdB_0}{3m}$ (b) $\frac{2qdB_0}{m}$
   (c) $\frac{qdB_0}{m}$ (d) $\frac{qdB_0}{2m}$

3. A charged particle carrying charge 1 μC is moving with velocity $(2\hat{i}+3\hat{j}+4\hat{k})$ ms^{-1}. If an external magnetic field of $(5\hat{i}+3\hat{j}-6\hat{k})\times 10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times 10^{-9}$ N. The vector $\vec{F}$ is : **[Main Sep. 03, 2020 (I)]**
   (a) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$
   (b) $-30\hat{i}+32\hat{j}-9\hat{k}$
   (c) $-300\hat{i}+320\hat{j}-90\hat{k}$
   (d) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

4. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to : (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C) **[Main Sep. 02, 2020 (I)]**
   (a) 2 cm (b) 5 cm
   (c) 12 cm (d) 4 cm

5. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C) **[Main Sep. 02, 2020 (II)]**



   (a) 0.11 m
   (b) 0.88 m
   (c) 0.44 m
   (d) 0.22 m

7. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5T. If an electric field of 100V/m makes it to move in a straight path then the mass of the particle is (Given charge of electron = 1.6×10^{-19}C) **[Main 12 April 2019, I]**
   (a) 9.1×10^{-31} kg (b) 1.6×10^{-27} kg
   (c) 1.6×10^{-19} kg (d) 2.0×10^{-24} kg

8. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then, **[Main 10 April 2019, I]**
   (a) $r_e > r_p = r_{He}$ (b) $r_e < r_p = r_{He}$
   (c) $r_e < r_p < r_{He}$ (d) $r_e > r_p > r_{He}$

9. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths descrfibed by them will be: **[Main 12 Jan 2019, I]**

(a) $1:\sqrt{2}$ (b) $1:2$ (c) $1:3$ (d) $1:\sqrt{3}$

10. In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.
[Charge of the electron = 1.6×10^{-19} C
Mass of the electron = 9.1×10^{-31} kg] **[Main 11 Jan 2019, I]**

(a) 7.5×10^{-3} m (b) 7.5×10^{-2} m
(c) 7.5 m (d) 7.5×10^{-4} m

11. The region between $y = 0$ and $y = d$ contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. if $d = \dfrac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is : **[Main 11 Jan 2019, II]**

(a) $\dfrac{qvB}{m}\left(\dfrac{1}{2}\hat{i} - \dfrac{\sqrt{3}}{2}\hat{j}\right)$ (b) $\dfrac{qvB}{m}\left(\dfrac{\sqrt{3}}{2}\hat{i} + \dfrac{1}{2}\hat{j}\right)$

(c) $\dfrac{qvB}{m}\left(\dfrac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$ (d) $\dfrac{qvB}{m}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

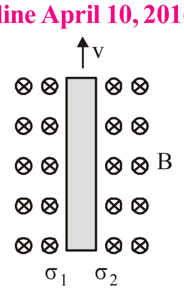
12. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is : **[Main 2018]**

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$
(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

13. A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is : **[Main Online April 9, 2017]**

(a) away from the wire
(b) towards the wire
(c) parallel to the wire along the current
(d) parallel to the wire opposite to the current

14. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed 'v' in a uniform magnetic field B going into the plane of the paper (See figure). If charge densities σ_1 and σ_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects): **[Main Online April 10, 2016]**



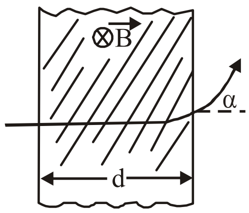
(a) $\sigma_1 = \dfrac{-\epsilon_0 vB}{2}, \sigma_2 = \dfrac{\epsilon_0 vB}{2}$

(b) $\sigma_1 = \epsilon_0 vB, \sigma_2 = -\epsilon_0 vB$

(c) $\sigma_1 = \dfrac{\epsilon_0 vB}{2}, \sigma_2 = \dfrac{-\epsilon_0 vB}{2}$

(d) $\sigma_1 = \sigma_2 = \epsilon_0 vB$

15. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width '*d*'. If α be the angle of deviation of proton from initial direction of motion (see figure), the value of *sin* α will be: **[Main Online April 10, 2015]**



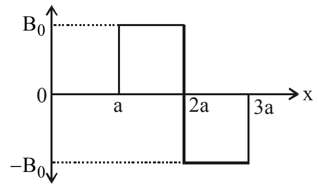
(a) $qV\sqrt{\dfrac{Bd}{2m}}$ (b) $\dfrac{B}{2}\sqrt{\dfrac{qd}{mV}}$

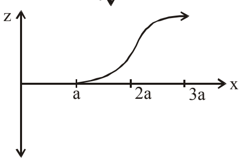
(c) $\dfrac{B}{d}\sqrt{\dfrac{q}{2mV}}$ (d) $Bd\sqrt{\dfrac{q}{2mV}}$

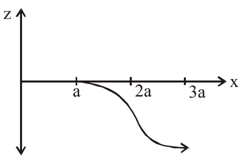
16. A particle of charge 16×10^{-16} C moving with velocity 10 ms^{-1} along x-axis enters a region where magnetic field of induction $\vec{B}$ is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along x-axis, the magnitude of $\vec{B}$ is : **[Main Online April 23, 2013]**

(a) 16×10^3 Wb m^{-2} (b) 2×10^3 Wb m^{-2}
(c) 1×10^3 Wb m^{-2} (d) 4×10^3 Wb m^{-2}

17. A magnetic field $\vec{B} = B_0\hat{j}$, exists in the region $a < x < 2a$, and $\vec{B} = -B_0\hat{j}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like **[2007]**



(a)


(b)


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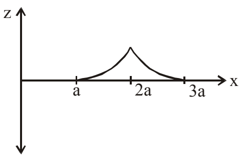

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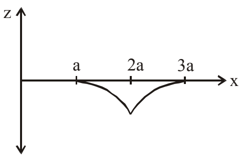
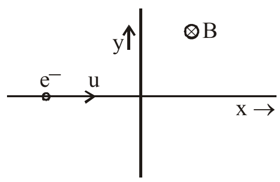
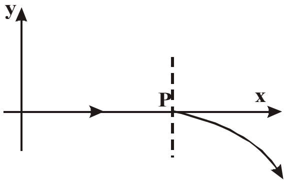


18. An electron travelling with a speed u along the positive x-axis enters into a region of magnetic field where $B = -B_0\hat{k}$ $(x > 0)$. It comes out of the region with speed v then **[2004S]**



(a) $v = u$ at $y > 0$
(b) $v = u$ at $y < 0$
(c) $v > u$ at $y > 0$
(d) $v > u$ at $y < 0$

19. For a positively charged particle moving in a x-y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x-y plane and is found to be non-circular. Which one of the following combinations is possible? **[2003S]**



(a) $\vec{E} = 0; \vec{B} = b\hat{i} + c\hat{k}$

(b) $\vec{E} = a\hat{i}; \vec{B} = c\hat{k} + a\hat{i}$

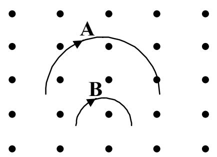
(c) $\vec{E} = 0; \vec{B} = c\hat{j} + b\hat{k}$

(d) $\vec{E} = a\hat{i}; \vec{B} = c\hat{k} + b\hat{j}$

20. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is **[2002S]**

(a) $\frac{qbB}{m}$ (b) $\frac{q(b-a)B}{m}$ (c) $\frac{qaB}{m}$ (d) $\frac{q(b+a)B}{2m}$

21. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in the figure. Then **[2001S]**



(a) $m_A v_A < m_B v_B$

(b) $m_A v_A > m_B v_B$

(c) $m_A < m_B$ and $v_A < v_B$

(d) $m_A = m_B$ and $v_A = v_B$

22. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the +x-direction and a magnetic field along the +z-direction, then **[2000S]**

(a) positive ions deflect towards +y-direction and negative ions towards -y direction

(b) all ions deflect towards +y-direction

(c) all ions deflect towards –y-direction

(d) positive ions deflect towards –y-direction and negative ions towards + y-direction.

23. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on **[2000S]**

(a) ω and q (b) ω, q and m

(c) q and m (d) ω and m

24. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a **[1999S - 2 Marks]**

(a) straight line (b) circle

(c) helix (d) cycloid

25. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is **[1995S]**

(a) proportional to 2 $(180° - \theta)$

(b) inversely proportional to r

(c) zero, only if $\theta = 180°$

(d) zero for all values of θ

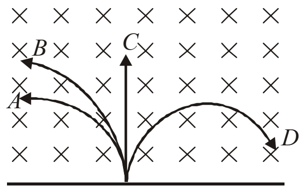
26. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is **[1988 - 2 Marks]**

(a) $(R_1 / R_2)^{1/2}$ (b) R_2 / R_1

(c) $(R_1 / R_2)^2$ (d) R_1 / R_2.

4 Fill in the Blanks

27. A neutron, a proton, and an electron and an alpha particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labelled in fig. The electron follows track ...... and the alpha particle follows track ..... **[1984- 2 Marks]**



5 True / False

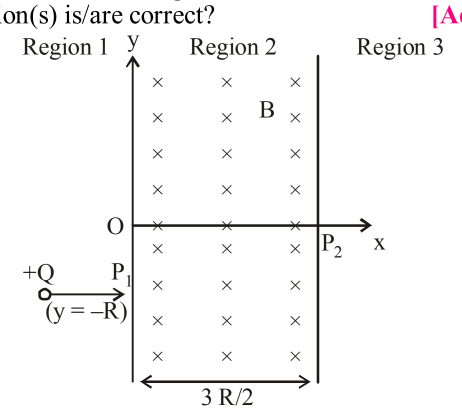
28. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular paths of the same radius. **[1985 - 3 Marks]**

29. A charged particle enters a region of uniform magnetic field at an angle of 85° to the magnetic line of force . The path of the particle is a circle. **[1983 - 2 Marks]**

30. There is no change in the energy of a charged particle moving in a magnetic field although a magnetic force is acting on it. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

31. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x–axis enters region 2 from region 1 at point P_1 ($y = -R$). Which of the following option(s) is/are correct? **[Adv. 2017]**



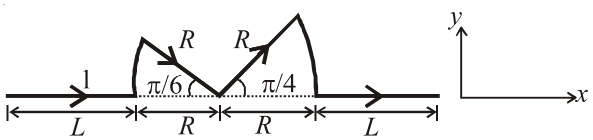
(a) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re–enter region 1

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x–axis

(c) When the particle re–enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y–axis is $p/\sqrt{2}$

(d) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re–entry into region 1 is inversely proportional to the mass of the particle

32. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are) **[Adv. 2015]**



(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(b) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

33. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ \mathrm{ms}^{-1}$, enters a region of uniform static magnetic field, normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})\ \mathrm{ms}^{-1}$. The correct statement(s) is (are) **[Adv. 2013]**

(a) The direction of the magnetic field is $-z$ direction

(b) The direction of the magnetic field is $+z$ direction

(c) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units

(d) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units

34. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time $t > 0$? **[2012]**

(a) If θ = 0°, the charge moves in a circular path in the x-z plane.

(b) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.

(c) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(d) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis.

35. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is / are true? **[2011]**

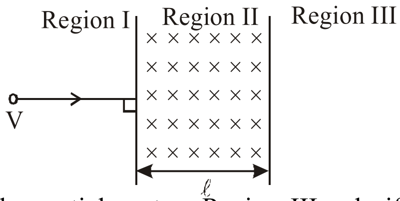
(a) They will never come out of the magnetic field region.

(b) They will come out travelling along parallel paths.

(c) They will come out at the same time.

(d) They will come out at different times.

36. A particle of mass m and charge q, moving with velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ Choose the correct choice(s). **[2008]**



(a) The particle enters Region III only if its velocity $v > \frac{q\ell B}{m}$

(b) The particle enters Region III only if its velocity $v < \frac{q\ell B}{m}$

(c) Path length of the particle in Region II is maximum when velocity $v = \frac{q\ell B}{m}$

(d) Time spent in Region II is same for any velocity v as long as the particle returns to Region I

37. H^+, He^+ and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4 amu and 16 amu respectively. Then

(a) H^+ will be deflected most **[1994 - 2 Marks]**

(b) O^{2+} will be deflected most

(c) He^+ and O^{2+} will be deflected equally

(d) all will be deflected equally

38. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively, this region of space may have : **[1985 - 2 Marks]**

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$

(c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

8 Comprehension/Passage Based Questions

Directions (Qs. 41 to 43) : By appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

	Column 1		Column 2		Column 3
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E} = E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

39. In which case will the particle move in a straight line with constant velocity?
(a) (III) (ii) (R) (b) (IV) (i) (S)
(c) (III) (iii) (P) (d) (II) (iii) (S)

40. In which case will the particle describe a helical path with axis along the positive z direction?
(A) (IV) (i) (S) (B) (II) (ii) (R)
(C) (III) (iii) (P) (D) (IV) (ii) (R)

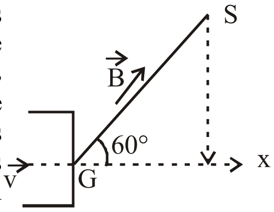
41. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$)?
(a) (II) (iii) (Q) (b) (III) (ii) (R)
(c) (IV) (ii) (S) (d) (III) (ii) (P)

10 Subjective Problems

42. A proton and an α-particle are accelerated with same potential difference and they enter in the region of constant magnetic field B perpendicular to the velocity of particles. Find the ratio of radius of curvature of proton to the radius of curvature of α - particle. **[2004 - 2 Marks]**

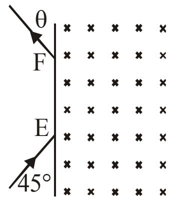
43. The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of the magnetic field. Neglect gravity throughout the question. **[1999 - 10 Marks]**
(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at angle 30° to its initial velocity.
(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto $2.1L$.

44. An electron gun G emits electrons of energy 2keV travelling in the positive x-direction. The electrons are required to hit the spot S where $GS = 0.1$m, and the line GS makes an angle of 60° with the x-axis as shown in the fig. A uniform magnetic field $\vec{B}$ parallel to GS exists. Find $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S. **[1993-7 Marks]**



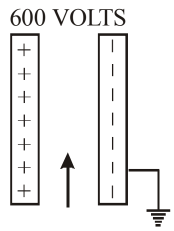
45. A beam of protons with a velocity 4×10^5 m/sec enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). **[1986 - 6 Marks]**

46. A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$ C enters a region of uniform magnetic field of strength 1 tesla along the direction shown in fig. The speed of the particle is 10^7 m/s. (i) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ. (ii) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.



(1984- 8 Marks)

47. A potential difference of 600 volts is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 m/sec moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects). (Charge of the electron $= -1.6 \times 10^{-19}$ coulomb) **[1981- 3 Marks]**



Topic-2 : Magnetic Field Lines, Biot-Savart's Law and Ampere's Circuital Law

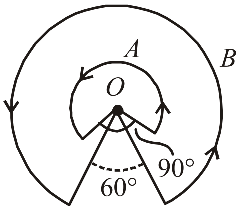
1 MCQs with One Correct Answer

1. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane wit speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle : **[Main Sep. 06, 2020 (II)]**

(a) $\frac{mv^2 \vec{B}}{2B^2}$ (b) $-\frac{mv^2 \vec{B}}{2\pi B^2}$

(c) $-\frac{mv^2 \vec{B}}{B^2}$ (d) $-\frac{mv^2 \vec{B}}{2B^2}$

2. A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is: **[Main Sep. 04, 2020 (I)]**

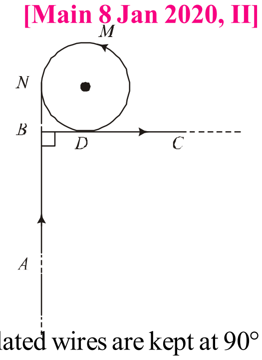


(a) 4 : 6 (b) 6 : 4 (c) 2 : 5 (d) 6 : 5

3. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is: **[Main Sep. 03, 2020 (I)]**

(a) $250\sqrt{3}$ (b) $50\sqrt{3}$ (c) $500\sqrt{3}$ (d) $5\sqrt{3}$

4. A very long wire $ABDMNDC$ is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn $DMND$ of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is: **[Main 8 Jan 2020, II]**



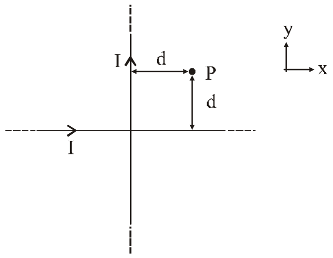
(a) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$

(b) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$

(c) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$

(d) $\frac{\mu_0 I}{2R}$

5. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure.

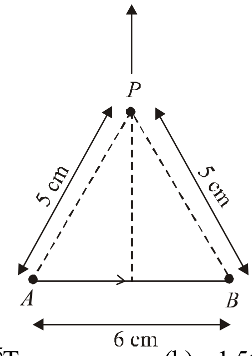


These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be : **[Main 12 April 2019, I]**

(a) Zero (b) $-\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$

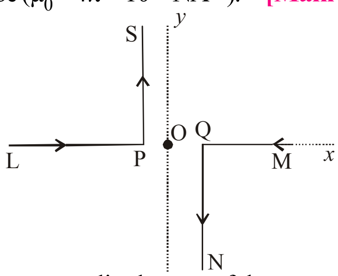
(c) $\frac{+\mu_0 I}{\pi d}(\hat{z})$ (d) $\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$

6. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure) ($\mu_o = 4\pi \times 10^{-7}$ N-A^{-2}) **[Main 12 April 2019, II]**



(a) 2.0×10^{-5}T (b) 1.5×10^{-5}T
(c) 3.0×10^{-5}T (d) 2.5×10^{-5}T

7. As shown in the figure, two infinitely long, identical wires are bent by 90º and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4 cm, and the magnitude of the magneticf field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}): **[Main 12 Jan 2019, I]**



(a) 20 A, perpendicular out of the page
(b) 40 A, perpendicular out of the page
(c) 20 A, perpendicular into the page
(d) 40 A, perpendicular into the page

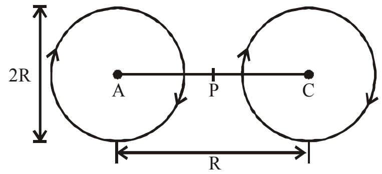
8. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_1) to that at the centre of the coil (B_C), *i.e.*, $\frac{B_L}{B_C}$ will be: **[Main 9 Jan 2019, II]**

(a) N (b) $\frac{1}{N}$ (c) N^2 (d) $\frac{1}{N^2}$

9. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is: **[Main 2018]**

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

10. A Helmholtz coil has pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by (Refer to figure): **[Main Online April 15, 2018]**



(a) $\frac{4N\mu_0 I}{5^{3/2}R}$ (b) $\frac{8N\mu_0 I}{5^{3/2}R}$ (c) $\frac{4N\mu_0 I}{5^{1/2}R}$ (d) $\frac{8N\mu_0 I}{5^{1/2}R}$

11. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is: **[Main 2016]**

(a) $\frac{\pi^2}{16}$ (b) $\frac{\pi^2}{8\sqrt{2}}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{16\sqrt{2}}$

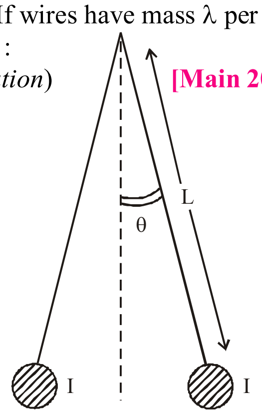
12. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :
(*g = gravitational acceleration*) **[Main 2015]**

(a) $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$

(b) $\sqrt{\frac{\pi\lambda g L}{\mu_0}\tan\theta}$

(c) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$

(d) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$



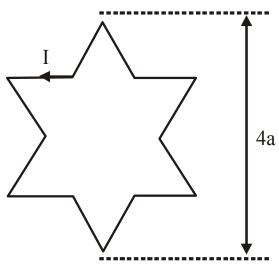
13. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current I passes through it. The ratio $B_1 : B_2$ is: **[Main Online April 12, 2014]**

(a) 1 : 1 (b) 1 : 3 (c) 1 : 9 (d) 9 : 1

14. An electric current is flowing through a circular coil of radius R. The ratio of the magnetic field at the centre of the coil and that at a distance $2\sqrt{2}R$ from the centre of the coil and on its axis is : **[Main Online April 9, 2013]**

(a) $2\sqrt{2}$ (b) 27 (c) 36 (d) 8

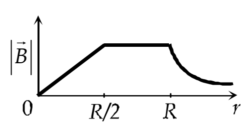
15. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is **[Adv. 2017]**

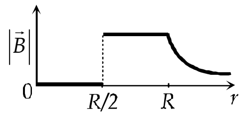


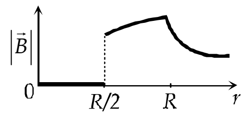
(a) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$ (b) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$

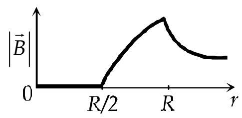
(c) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$ (d) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

16. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by **[2012]**

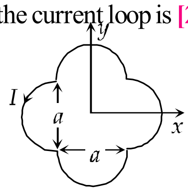
(a)


(b)


(c)


(d)


17. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **[2012]**



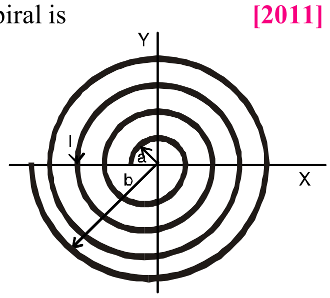
(a) $a^2 I\,\hat{k}$

(b) $\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$

(d) $(2\pi+1)a^2 I\,\hat{k}$

18. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the XY plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the centre of the spiral is **[2011]**



(a) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$

(b) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$

(c) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$

(d) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$

19. A long straight wire along the Z-axis carries a current I in the negative Z-direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $Z = 0$ plane is **[2002S]**

(a) $\frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$ (b) $\frac{\mu_0 I(x\hat{i}+y\hat{j})}{2\pi(x^2+y^2)}$

(c) $\frac{\mu_0 I(x\hat{j}-y\hat{i})}{2\pi(x^2+y^2)}$ (d) $\frac{\mu_0 I(x\hat{i}-y\hat{j})}{2\pi(x^2+y^2)}$

20. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the center is **[2001S]**

(a) $\frac{\mu_o NI}{b}$ (b) $\frac{2\mu_o NI}{a}$

(c) $\frac{2\mu_o NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{2\mu_o NI}{2(b-a)}\ln\frac{a}{b}$

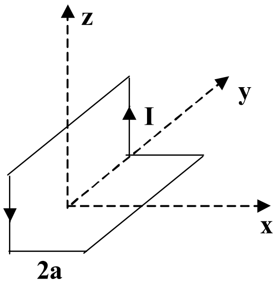
21. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ points in the direction **[2001S]**

(a) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

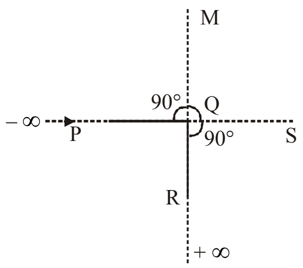
(b) $\frac{1}{\sqrt{3}}\left(-\hat{j}+\hat{k}+\hat{i}\right)$

(c) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(d) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

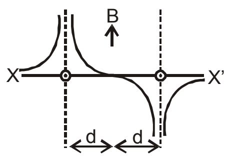


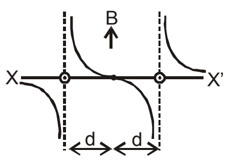
22. An infinitely long conductor PQR is bent to form a right angle as shown in Figure. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by **[2000S]**

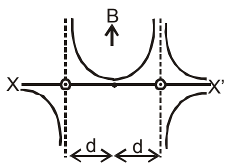


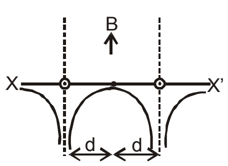
(a) 1/2 (b) 1 (c) 2/3 (d) 2

23. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the line XX' is given by **[2000S]**

(a)


(b)


(c)


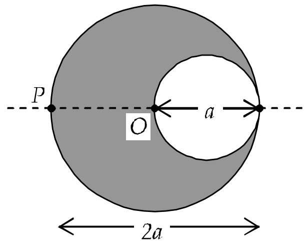
(d)


24. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is **[1995S]**

(a) proportional to 2 (180° – θ)
(b) inversely proportional to r
(c) zero, only if θ = 180°
(d) zero for all values of θ

2 Integer Value Answer

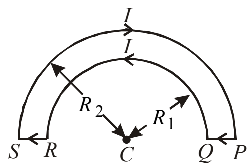
25. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinity long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is **[2012]**



26. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k. **[2009]**

4 Fill in the Blanks

27. The wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown. The magnitude of the magnetic induction at the centre C is .... **[1988 - 2 Marks]**



6 MCQs with One or More than One Correct Answer

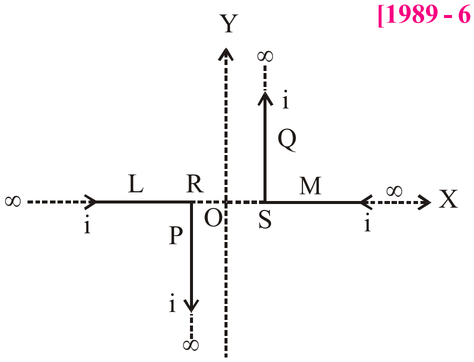
28. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) **[Adv. 2013]**

(a) In the region 0 < r < R, the magnetic field is non-zero
(b) In the region R < r < 2R, the magnetic field is along the common axis
(c) In the region R < r < 2R, the magnetic field is tangential to the circle of radius r, centered on the axis
(d) In the region r > 2R, the magnetic field is non-zero

29. A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then **[1993-2 Marks]**
(a) the magnetic field at all points inside the pipe is the same, but not zero.
(b) the magnetic field at any point inside the pipe is zero
(c) the magnetic field is zero only on the axis of the pipe
(d) the magnetic field is different at different points inside the pipe.

10 Subjective Problems

30. A pair of stationary and infinitely long bent wires are placed in the XY plane as shown in fig. The wires carry currents of i = 10 amperes each as shown. The segments L and M are along the X-axis. The segments P and Q are parallel to the Y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O. **[1989 - 6 Marks]**



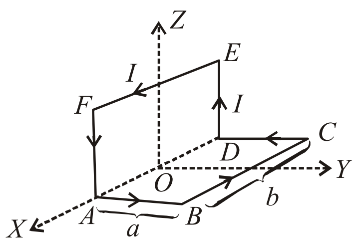
Topic-3 : Force and Torque on Current Carrying Conductor

1 MCQs with One Correct Answer

1. A square loop of side $2a$, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point $(0, b, 0)$, $(b >> a)$. The magnitude of the torque on the loop about z-axis is given by: **[Main Sep. 05, 2020 (I)]**

(a) $\dfrac{\mu_0 I^2 a^2}{2\pi b}$
(b) $\dfrac{\mu_0 I^2 a^3}{2\pi b^2}$
(c) $\dfrac{2\mu_0 I^2 a^2}{\pi b}$
(d) $\dfrac{2\mu_0 I^2 a^3}{\pi b^2}$

2. A wire carrying current I is bent in the shape $ABCDEFA$ as shown, where rectangle $ABCDA$ and $ADEFA$ are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop $ABCDEFA$ is : **[Main Sep. 02, 2020 (II)]**

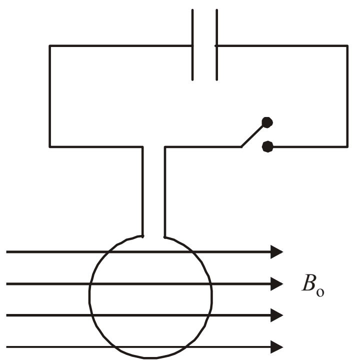


(a) abI, along $\left(\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}\right)$
(b) $\sqrt{2}abI$, along $\left(\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}\right)$
(c) $\sqrt{2}abI$, along $\left(\dfrac{\hat{j}}{\sqrt{5}} + \dfrac{2\hat{k}}{\sqrt{5}}\right)$
(d) abI, along $\left(\dfrac{\hat{j}}{\sqrt{5}} + \dfrac{2\hat{k}}{\sqrt{5}}\right)$

3. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then : **[Main 9 Jan 2020, II]**

(a) $T = \sqrt{\dfrac{2m}{IB}}$
(b) $T = \sqrt{\dfrac{\pi m}{2IB}}$
(c) $T = \sqrt{\dfrac{2\pi m}{IB}}$
(c) $T = \sqrt{\dfrac{\pi m}{IB}}$

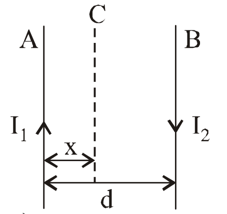
4. A circular coil of radius R and N turns has negligible resistance. As shown in the schematic figure, its two ends are connected to two wires and it is hanging by those wires with its plane being vertical. The wires are connected to a capacitor with charge Q through a switch. The coil is in a horizontal uniform magnetic field B_o parallel to the plane of the coil. When the switch is closed, the capacitor gets discharged through the coil in a very short time. By the time the capacitor is discharged fully, magnitude of the angular momentum gained by the coil will be (assume that the discharge time is so short that the coil has hardly rotated during this time)



(a) $\frac{\pi}{2}NQB_oR^2$ (b) πNQB_oR^2

(c) $2\pi NQB_oR^2$ (d) $4\pi NQB_oR^2$

5. Two wires A & B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are: **[Main 10 April 2019, I]**



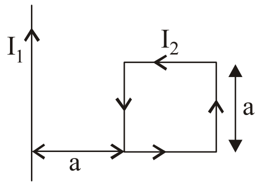
(a) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

(b) $x = \left(\frac{I_2}{(I_1 + I_2)}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

(c) $x = \left(\frac{I_1}{(I_1 + I_2)}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

(d) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

6. A rigid square of loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be: **[Main 9 April 2019 I]**



(a) Repulsive and equal to $\frac{\mu_o I_1 I_2}{2\pi}$

(b) Attractive and equal to $\frac{\mu_o I_1 I_2}{3\pi}$

(c) Repulsive and equal to $\frac{\mu_o I_1 I_2}{4\pi}$

(d) Zero

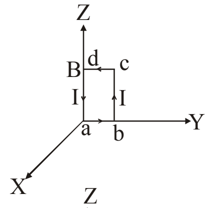
7. A circular coil having N turns and radius *r* carries a current *I*. It is held in the XZ plane in a magnetic field B. The torque on the coil due to the magnetic field is : **[Main 8 April 2019 I]**

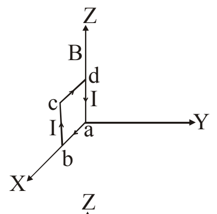
(a) $\frac{Br^2 I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\frac{B\pi r^2 I}{N}$ (d) Zero

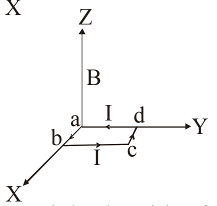
8. A charge q is spread uniformly over an insulated loop of radius r . If it is rotated with an angular velocity ω with respect to normal axis then the magnetic moment of the loop is **[Main Online April 16, 2018]**

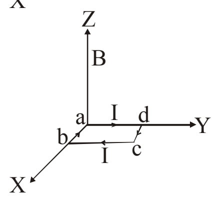
(a) $\frac{1}{2}q\omega r^2$ (b) $\frac{4}{3}q\omega r^2$ (c) $\frac{3}{2}q\omega r^2$ (d) $q\omega r^2$

9. A uniform magnetic field B of 0.3 T is along the positive Z-direction. A rectangular loop (abcd) of sides 10 cm × 5 cm carries a current 1 of 12 A. Out of the following different orientations which one corresponds to stable equilibrium ? **[Main Online April 9, 2017]**

(a)


(b)


(c)


(d)


10. Two coaxial solenoids of different radius carry current I in the same direction. $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : **[Main 2015]**

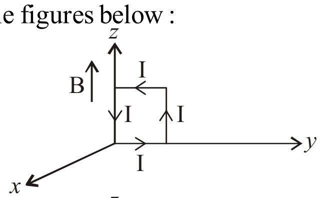
(a) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$

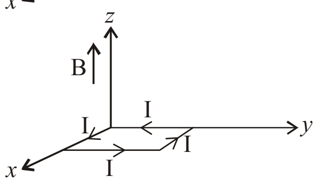
(b) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

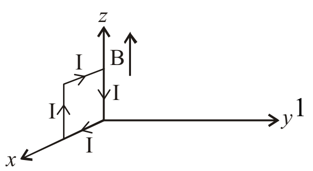
(c) $\vec{F_1} = \vec{F_2} = 0$

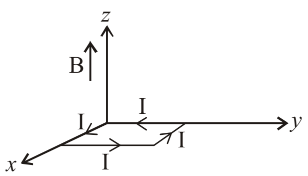
(d) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

11. A rectangular loop of sides 10 cm and 5 cm carrying a current 1 of 12 A is placed in different orientations as shown in the figures below :

(A)


(B)


(C)


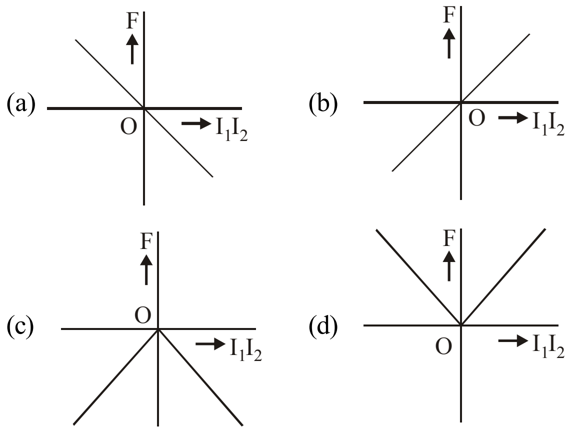
(D)


If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium? **[Main 2015]**

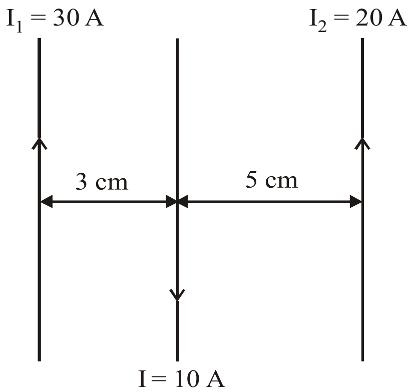
(a) (B) and (D), respectively (b) (B) and (C), respectively

(c) (A) and (B), respectively (d) (A) and (C), respectively

12. Two long straight parallel wires, carrying (adjustable) current I_1 and I_2, are kept at a distance d apart. If the force 'F' between the two wires is taken as 'positive' when the wires repel each other and 'negative' when the wires attract each other, the graph showing the dependence of 'F', on the product $I_1 I_2$, would be: **[Main Online April 11, 2015]**

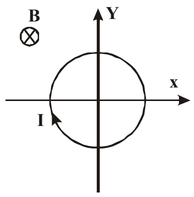


13. Three straight parallel current carrying conductors are shown in the figure. The force experienced by the middle conductor of length 25 cm is: **[Main Online April 11, 2014]**



(a) 3×10^{-4} N toward right (b) 6×10^{-4} N toward right
(c) 9×10^{-4} N toward right (d) Zero

14. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **[2003S]**

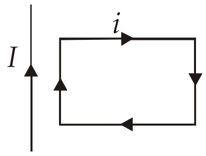


(a) contract
(b) expand
(c) move towards +ve x-axis
(d) move towards -ve x-axis.

15. Two thin long parallel wires seperated by a distance 'b' are carrying a current 'i' amp each. The magnitude of the force per unit lenght exerted by one wire on the other is **[1986 - 2 Marks]**

(a) $\frac{\mu_0 i^2}{b^2}$ (b) $\frac{\mu_0 i^2}{2\pi b}$ (c) $\frac{\mu_0 i}{2\pi b}$ (d) $\frac{\mu_0 i}{2\pi b^2}$

16. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I is established in the wire as shown in the figure, the loop will : **[1985 - 2 Marks]**



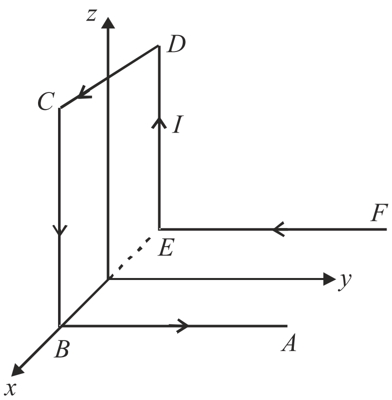
(a) rotate about an axis parallel to the wire
(b) move away from the wire
(c) move towards the wire
(d) remain stationary

2 Integer Value Answer

17. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is **[Adv. 2014]**

4 Fill in the Blanks

18. A wire $ABCDEF$ (with each side of length L) bent as shown in figure and carrying a current I is placed in a uniform magnetic induction B parallel to the positive y-direction. The force experienced by the wire is .... in the .... direction. **[1990 - 2 Marks]**



5 True / False

19. No net force acts on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field. **[1981- 2 Marks]**

10 Subjective Problems

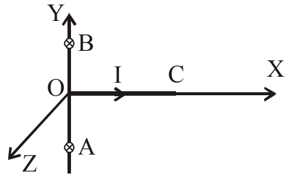
20. Three infinitely long thin wires, each carrying current i in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$. **[1997 - 5 Marks]**

(i) Find the locus of the points for which the magnetic field B is zero.

(ii) If the central wire is displaced along the Z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

21. A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of $20A$, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of $30A$, as shown in figure . Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations. **[1994 - 6 Marks]**

22. A straight segment OC (of length L meter) of a circuit carrying a current I amp is placed along the x-axis (Fig.). Two infinetely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed at $y = -a$ meter and $y = +a$ meter respectively, as shown in the figure.



If the wires A and B each carry a current I amp into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed ? **[1992 - 10 Marks]**

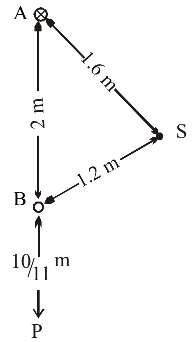
23. Two long straight parallel wires are 2 metres apart, perpendicular to the plane of the paper (see figure). The wire A carries a current of 9.6 amps, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $\frac{10}{11}$ metre from the wire B, is zero.

Find : **[1987 - 7 Marks]**

(i) The magnitude and direction of the current in B .

(ii) The magnitude of the magnetic field of induction at the point S.

(iii) The force per unit length on the wire B.



Topic-4 : Galvanometer and its Conversion into Ammeter and Voltmeter

1 MCQs with One Correct Answer

1. A galvanometer of resistance G is converted into a voltmeter of ragne 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be : **[Main Sep. 05, 2020 (I)]**

(a) G (b) R_1 (c) $R_1 - G$ (d) $R_1 + G$

2. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to : **[Main Sep. 05, 2020 (II)]**

(a) 333° A/div. (b) 6×10^{-3} A/div.
(c) 666° A/div. (d) 3×10^{-3} A/div.

3. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0 – 5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0 – 10 mA is : **[Main 10 April 2019, I]**

(a) 500 Ω (b) 100 Ω (c) 200 Ω (d) 10 Ω

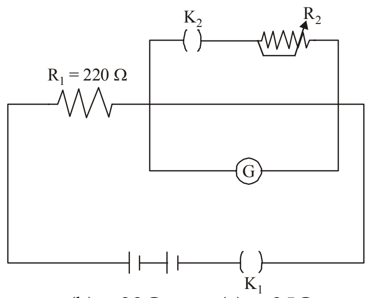
4. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 k Ω resistance. The maximum voltage, that can be measured using this voltmeter, will be close to: **[Main 9 April 2019 I]**

(a) 40 V (b) 15 V (c) 20 V (d) 10 V

5. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1mA. The value of B (in Tesla) is approximately: **[Main 9 April 2019, II]**

(a) 10^{-4} (b) 10^{-2} (c) 10^{-1} (c) 10^{-3}

6. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]: **[Main 12 Jan 2019, I]**



(a) 5Ω (b) 22Ω (c) 25Ω (d) 12Ω

7. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of : **[Main 12 Jan 2019, II]**

(a) 250 ohm (b) 200 ohm (c) 6200 ohm (d) 6250 ohm

8. A galvanometer having a resistance of 20 Ω and 30 division on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is: **[Main 11 Jan 2019, II]**

(a) 100 Ω (b) 120 Ω (c) 80 Ω (d) 125 Ω

9. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V? **[Main 8 Jan 2019, II]**

(a) 10 kΩ (b) 8.9 kΩ (c) 7.9 kΩ (d) 9.9 kΩ

10. In a circuit for finding the resistance of a galvanometer by half deflection method, a 6 V battery and a high resistance of 11kΩ are used. The figure of merit of the galvanometer 60μA/division. In the absence of shunt resistance, the galvanometer produces a deflection of $\theta = 9$ divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$, is closest to **[Main Online April 16, 2018]**

(a) 55Ω (b) 110Ω (c) 220Ω (d) 550Ω

11. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is **[Main 2017]**

(a) $2.535 \times 10^3\ \Omega$ (b) $4.005 \times 10^3\ \Omega$

(c) $1.985 \times 10^3\ \Omega$ (d) $2.045 \times 10^3\ \Omega$

12. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a currect of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is : **[Main 2016]**

(a) 0.1 Ω (b) 3Ω (c) 0.01Ω (d) 2Ω

13. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S related by the equation: **[Main Online April 9, 2016]**

(a) $S(R+G) = RG$ (b) $2S(R+G) = RG$

(c) $2G = S$ (d) $2S = G$

3 Numeric Answer

14. A moving coil galvanometer has 50 turns and each turn has an area $2 \times 10^{-4}\ m^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} *N m rad*$^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 –1.0 *A*. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in *ohms*, is __________. **[Adv. 2018]**

6 MCQs with One or More than One Correct Answer

15. Two identical moving coil galvanometers have 10 resistance and full scale deflection at 2μ*A* current. One of them is converted into a voltmeter of 100mV full scale reading and the other into an Ammeter of 1mA full scale current using appropriate resistors. There are then used to measure the voltage and current in the Ohm's law experiment with R – 1000Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct? **[Adv. 2019]**

(a) The resistance of the voltmeter will be 100*k*

(b) The measured value of *R* will be 978<*R*< 982

(c) If the ideal cell is replaced by a cell having internal resistance of 5then the measured value of *R* will be more than 1000

(d) The resistance of the Ammeter will be 0.02 (round off to 2nd decimal place)

16. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is (are) true? **[Adv. 2016]**

(a) The maximum voltage range is obtained when all the components are connected in series

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(c) The maximum current range is obtained when all the components are connected in parallel

(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

17. A microameter has a resistance of 100Ω and a full scale range of 50μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination (s) **[1991 - 2 Marks]**

(a) 50 V range with 10 kΩ resistance in series

(b) 10 V range with 200 kΩ resistance in series

(c) 5 mA range with 1Ω resistance in parallel

(d) 10 mA range with 1Ω resistance in parallel

9 Assertion and Reason Type Questions

18. **Statement-1 :** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

Statement-2 : Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized. **[2008]**

(a) **Statement**-1 is True, **Statement**-2 is True; **Statement**-2 is a correct explanation for **Statement**-1

(b) **Statement**-1 is True, **Statement**-2 is True; **Statement**-2 is NOT a correct explanation for **Statement**-1

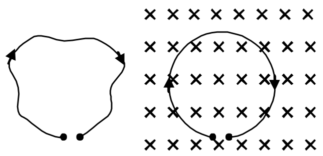
(c) **Statement** -1 is True, **Statement**-2 is False

(d) **Statement** -1 is False, **Statement**-2 is True

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is **[2010]**



(a) IBL (b) $\frac{IBL}{\pi}$ (c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

2. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is **[1999S - 2 Marks]**

(a) directly proportional to I

(b) directly proportional to R

(c) inversely proportional to R

(d) zero

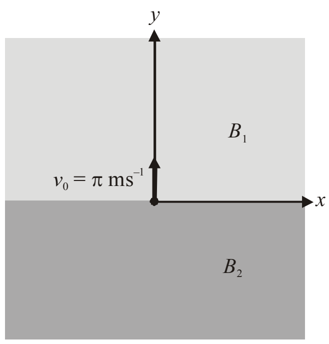
3. Two very long, straight, parallel wires carry steady currents I & -I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity **v** is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is **[1998S - 2 Marks]**

(a) $\frac{\mu_0 Iqv}{2\pi d}$ (b) $\frac{\mu_0 Iqv}{\pi d}$

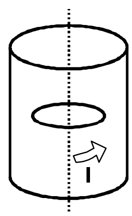
(c) $\frac{2\mu_0 Iqv}{\pi d}$ (d) 0

2 Integer Value Answer

4. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi$ ms^{-1} at $t = 0$, as shown in the figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is __________. **[Adv. 2018]**



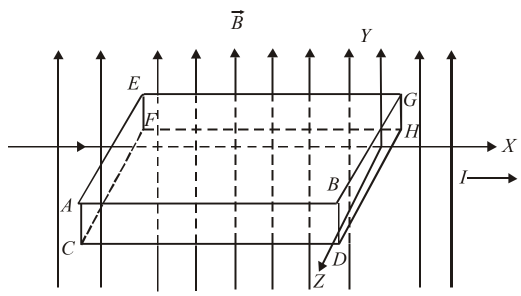
5. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300\,t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300\,t)$, then 'N' is



[2011]

4 Fill in the Blanks

6. A metallic block carrying current I is subjected to a uniform magnetic induction $\vec{B}$ as shown in Figure .



The moving charges experience a force $\vec{F}$ given by .... which results in the lowering of the potential of the face .... Assume the speed of the carriers to be v. **[1996 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

7. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true? **[Adv. 2018]**

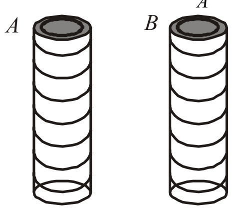
(a) If $I_1 = I_2$, then $\vec{B}$ **cannot** be equal to zero at the origin (0, 0, 0)

(b) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(c) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\dfrac{\mu_0 I}{2R}\right)$

8. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are) **[2009]**



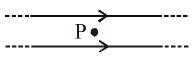
(a) $\rho_A > \rho_B$ and $m_A = m_B$ (b) $\rho_A < \rho_B$ and $m_A = m_B$
(c) $\rho_A > \rho_B$ and $m_A > m_B$ (d) $\rho_A < \rho_B$ and $m_A < m_B$

7 Match the Following

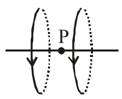
9. Two wires each carrying a steady current I are shown in four configurations in Column I. Some of the resulting effects are described in Column II. Match the statements in Column I with the statements in column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS. **[2007]**

Column I

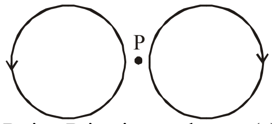
(A) Point P is situated midway between the wires.



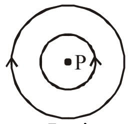
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.



(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.



(D) Point P is situated at the common center of the wires.



Column II

(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.

(q) The magnetic fields (B) at P due to the currents in the wires are in oppo site directions.

(r) There is no magnetic field at P.

(s) The wires repel each other.

10. Column I gives certain situations in which a straight metallic wire of resistance R is used and Column II gives some resulting effects. Match the statements in Column I with the statements in Column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS. **[2007]**

Column I	Column II
(A) A charged capacitor is connected to the ends of the wire	(p) A constant current flows through the wire
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q) Thermal energy is generated in the wire
(C) The wire is placed in a constant electric field that has a direction along the length of the wire	(r) A constant potential difference develops between the ends of the wire
(D) A battery of constant emf is connected to the ends of the wire.	(s) charges of constant magnitude appear at the ends of the wire

11. Match the following columns : **[2006, 6M]**

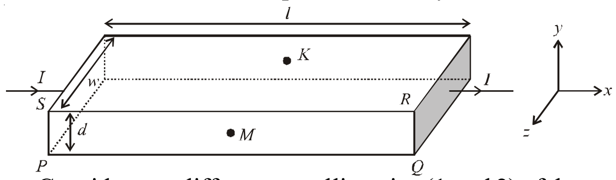
Column I	Column II
(A) Dielectric ring uniformly charged	(p) Constant electrostatic field out of system
(B) Dielectric ring uniformly charged rotating with angular velocity ω	(q) Magnetic field strength
(C) Constant current in ring i	(r) Electric field (induced)
(D) $i = i_0 \cos\omega t$	(s) Magnetic dipole moment

8 Comprehension/Passage Based Questions

Passage-1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are ℓ, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross-section of the strip and carried by electrons.



12. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2 respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are) **[Adv. 2015]**

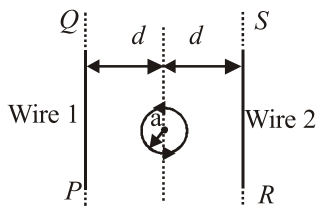
(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

13. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are) **[Adv. 2015]**

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Passage-2

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wire are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



14. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case **[Adv. 2014]**

(a) current in wire 1 and wire 2 in the direction PQ and RS, respectively and $h \approx a$
(b) current in wire 1 and wire 2 in the direction PQ and SR, respectively and $h \approx a$
(c) current in wire 1 and wire 2 in the direction PQ and SR, respectively and $h \approx 1.2a$
(d) current in wire 1 and wire 2 in the direction PQ and RS, respectively and $h \approx 1.2a$

15. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop). **[Adv. 2014]**

(a) $\dfrac{\mu_0 I^2 a^2}{d}$ (b) $\dfrac{\mu_0 I^2 a^2}{2d}$
(c) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (d) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

Passage-3

Advanced countries are making use of powerful electromagnets to move trains at very high speed. These trains are called maglev trains (abbreviated from magnetic levitation). These trains float on a guideway and do not run on steel rail tracks.
Instead of using a engine based on fossil fuels, they make use of magnetic field forces. The magnetized coils are arranged in the guide way which repels the strong magnets placed in the train's under carriage. This helps train move over the guideway, a technic called electro-dynamic suspension. When current passes in the coils of guideway, a typical magnetic field is set up between the undercarriage of train and guideway which pushes and pull the train along the guideway depending on the requirement.
The lack of friction and its aerodynamic style allows the train to more at very high speed.

16. The levitation of the train is due to **[2006 - 5M, –2]**

(a) Mechanical force (b) Electrostatic attraction
(c) Electrostatic repulsion (d) Magnetic repulsion

17. The disadvantage of maglev trains is that **[2006 - 5M, –2]**

(a) More friction (b) Less pollution
(c) Less wear & tear (d) High initial cost

18. The force which makes maglev move **[2006 - 5M, –2]**

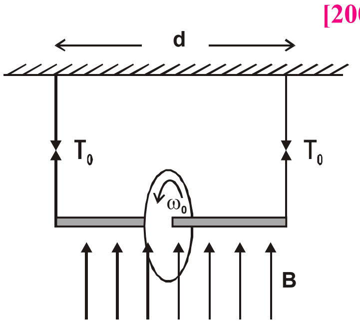
(a) Gravitational field (b) Magnetic field
(c) Nuclear forces (d) Air drag

10 Subjective Problems

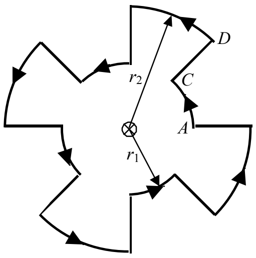
19. In a moving coil galvanometer, torque on the coil can be expressed as $\tau = ki$, where i is current through the wire and k is constant. The rectangular coil of the galvanometer having number of turns N, area A and moment of inertia I is placed in magnetic field B. Find **[2005 - 6 Marks]**

(a) k in terms of given parameters N, I, A and B
(b) the torsion constant of the spring, if a current i_0 produces a deflection of $\pi/2$ in the coil.
(c) the maximum angle through which the coil is deflected, if charge Q is passed through the coil almost instantaneously. (ignore the damping in mechanical oscillations).

20. A wheel of radius R having charge Q, uniformly distributed on the rim of the wheel is free to rotate about a light horizontal rod. The rod is suspended by light inextensible strings and a magnetic field B is applied as shown in the figure. The initial tensions in the strings are T_0. If the breaking tension of the strings are $\frac{3T_0}{2}$, find the maximum angular velocity ω_0 with which the wheel can be rotated. **[2003 - 4 Marks]**



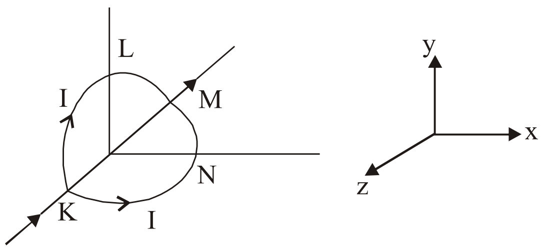
21. A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_1 = 0.12$ m. Each arc subtends the same angle at the center. **[2001-10 Marks]**



(a) Find the magnetic field produced by this circuit at the center.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the center of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the center due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the center?

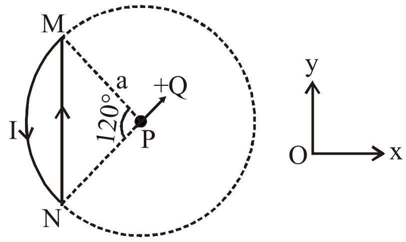
22. A circular loop of radius R is bent along a diameter and given a shape as shown in the figure. One of the semicircles (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at the origin. Current I is flowing through each of the semi circles as shown in figure. **[2000 - 10 Marks]**



(a) A particle of charge q is released at the origin with a velocity $\vec{v} = -v_0\hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_o\hat{j}$ is applied, determine the force $\vec{F_1}$ and $\vec{F_2}$ on the semicircles KLM and KNM due to the field and the net force $\vec{F}$ on the loop.

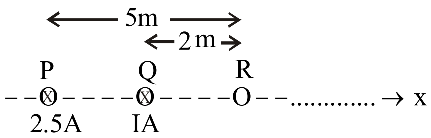
23. A wire loop carrying a current I is placed in the x-y plane as shown in fig. **[1991 - 4 + 4 Marks]**



(a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

24. Two long parallel wires carrying current 2.5 amperes and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 metres and 2 metres respectively from a collinear point R (see figure) **[1990 - 8 Marks]**



(i) An electron moving with a velocity of 4×10^5 m/s along the positive x - direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(ii) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 amperes may be placed so that the magnetic induction at R is zero.

Answer Key

Topic-1 : Motion of Charged Particle in Magnetic Field

1. (c)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (Bonus)
11. (b)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (c)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (Path D, Path B)	**27.** (False)	**28.** (False)		
29. (True)	**30.** (a, b)	**31.** (a,b,c)	**32.** (a, c)	**33.** (c, d)	**34.** (b,d)	**35.** (a, c, d)	**36.** (a, c)	**37.** (a,b,d)	
38. (d)	**39.** (a)	**40.** (b)							

Topic-2 : Magnetic Field Lines, Biot-Savart's Law and Ampere's Circuital Law

1. (d)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (b)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (5)	**26.** (7)	**27.** $B=\frac{\mu_0 I}{4}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$	**28.** (a, d)	**29.** (b)	

Topic-3 : Force and Torque on Current Carrying Conductor

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (3)	**18.** $ILB\hat{k}$, positive Z direction		
19. True									

Topic-4 : Galvanometer and its Conversion into Ammeter and Voltmeter

1. (d)	**2.** (d)	**3.** (Bonus)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (b)
11. (c)	**12.** (c)	**13.** (a)	**14.** (5.56)	**15.** (b, d)	**16.** (a, c)	**17.** (b, c)	**18.** (c)		

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (c)	**2.** (d)	**3.** (d)	**4.** (2)	**5.** (6)	**6.**	**7.** (a, b, d)	**8.** (b, d)

9. $A \to q, r$, $B \to p$, $C \to q, r$, $D \to q, s$

10. $A \to p$, $B \to q, s$ $C \to q, s$ $D \to q, r, s$

11. $A \to p$, $B \to q, s$ $C \to q, s$ $D \to q, r, s$

12. (a, d)	**13.** (a, c)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (d)

18. (b)

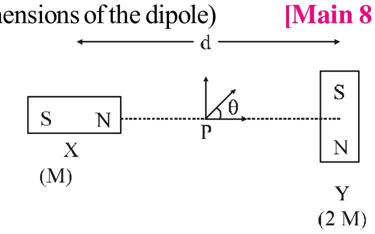
19 Magnetism and Matter

Topic-1 : Magnetism, Gaus's Law, Magnetic Moment, Properties of Magnet

1. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is : **[Main Sep. 04, 2020 (I)]**
(a) 6.4×10^{-2} J (b) 9.2×10^{-3} J
(c) 7.2×10^{-2} J (d) 11.7×10^{-3} J

2. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be : **[Main Sep. 04, 2020 (II)]**
(a) 10 rad s^{-1} (b) 10π rad s^{-1}
(c) 20π rad s^{-1} (d) 20 rad s^{-1}

3. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole) **[Main 8 April 2019 II]**



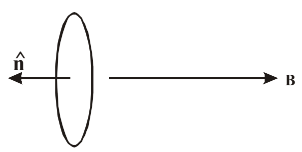
(a) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(d/2\right)^3} \times qv$ (b) 0
(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(d/2\right)^3} \times qv$ (d) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(d/2\right)^3} \times qv$

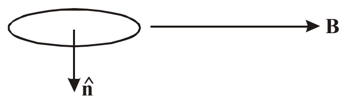
4. A magnet of total magnetic moment $10^{-2}\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\hat{i}$ (cosωt)where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is: **[Main 10 Jan. 2019 I]**
(a) 0.01 J (b) 0.007 J (c) 0.028 J (d) 0.014 J

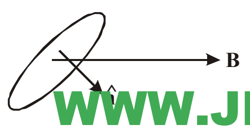
5. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of 30° with this field. The magntidue of the other field (in mT) is close to: **[Main Online April 9, 2016]**
(a) 1 (b) 11 (c) 36 (d) 1060

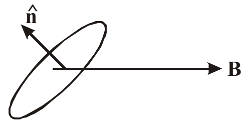
6. A 25 cm long solenoid has radius 2 cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetization $\vec{M}$ (magnetic moment/volume), then $|\vec{M}|$ is : **[Main Online April 10, 2015]**
(a) 30000π Am^{-1} (b) 3π Am^{-1}
(c) 30000 Am^{-1} (d) 300 Am^{-1}

7. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III & IV arrange them in the decreasing order of Potential Energy **[2003S]**

I. 

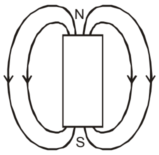
II. 

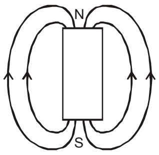
III 

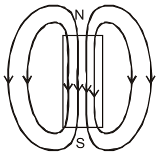
IV 

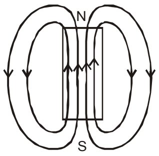
(a) I > III > II > IV (b) I > II > III > IV
(c) I > IV > II > III (d) III > IV > I > II

8. The magnetic field lines due to a bar magnet are correctly shown in **[2002S]**

(a) 

(b) 

(c) 

(d) 

9. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on **[2000S]**

(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

10. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a pependicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is **[1998S - 2 Marks]**

(a) $\frac{q}{2m}$ (b) $\frac{q}{m}$ (c) $\frac{2q}{m}$ (d) $\frac{q}{\pi m}$

11. A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\vec{B}_0$ such that $\vec{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is **[1983 - 1 Mark]**

(a) $ir B_0$ (b) $2\pi ir B_0$ (c) zero (d) $\pi ir B_0$

12. A magnetic needle is kept in a non uniform magnetic field. It experiences **[1982 - 3 Marks]**

(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) neither a force nor a torque

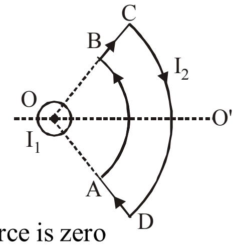
4 Fill in the Blanks

13. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolutions per second. The magnetic moment associated with the orbital motion of the electron is .... **[1988 - 2 Marks]**

14. A wire of length L metre, carrying a current i ampere is bent in the form of a circle. The magnitude of its magnetic moment is ....in MKS units. **[1987 - 2 Marks]**

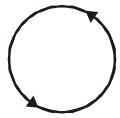
6 MCQs with One or More than One Correct Answer

15. A long current carrying wire, carrying current I_1 such that I_1 is flowing out from the plane of paper is placed at O. A steady state current I_2 is flowing in the loop $ABCD$ **[2006 - 5M, –1]**



(a) the net force is zero
(b) the net torque is zero
(c) as seen from O, the loop will rotate in clockwise along OO' axis
(d) as seen from O, the loop will rotate in anticlockwise direction along OO' axis

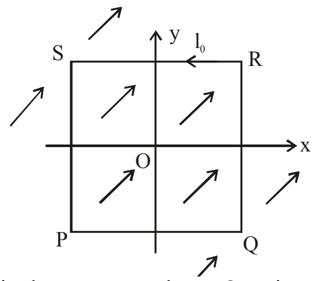
16. The following field line can never represent **[2006 - 5M, –1]**



(a) induced electric field
(b) magnetostatic field
(c) gravitational field of a mass at rest
(d) electrostatic field

10 Subjective Problems

17. A uniform, constant magnetic field **B** is directed at an angle of 45° to the x axis in the xy-plane. $PQRS$ is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position as shown in Figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.

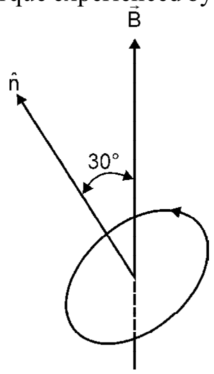


(a) What is the torque τ about O acting on the frame due to the magnetic field?
(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about this rotation occurs. (Δt is so short that any variation in the torque during this interval may be neglected.) Given : the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2$. **[1998 - 8 Marks]**

18. An electron in the ground state of hydrogen atom is revolving in anticlock-wise direction in a circular orbit of radius R. **[1996 - 5 Marks]**

(i) Obtain an expression for the orbital magnetic dipole moment of the electron.

(ii) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane-normal of the electron-orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.



19. A bar magnet is placed with its north pole pointing north and its south pole pointing south. Draw a figure to show the location of neutral points. **[1979]**

20. A bar magnet with poles 25 cm apart and of strength 14.4 amp-m rests with centre on a frictionless pivot. It is held in equilibrium at an angle of 60° with respect to a uniform magnetic field of induction 0.25 Wb/m^2, by applying a force F at right angles to its axis at a point 12 cm from pivot. Calculate F. What will happen if the force F is removed? **[1978]**

Topic-2 : The Earth Magnetism, Magnetic Materials and their Properties

1 MCQs with One Correct Answer

1. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be : **[Main Sep. 05, 2020 (II)]**

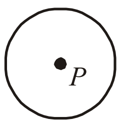
(a) 50×10^2 Am^2 (b) 5×10^2 Am^2
(c) 500×10^2 Am^2 (d) 0.5×10^2 Am^2

2. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be : **[Main Sep. 04, 2020 (II)]**

(a) 1 A/m (b) 4 A/m (c) 2.25 A/m (d) 0.75 A/m

3. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance.

The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is : **[Main Sep. 03, 2020 (II)]**

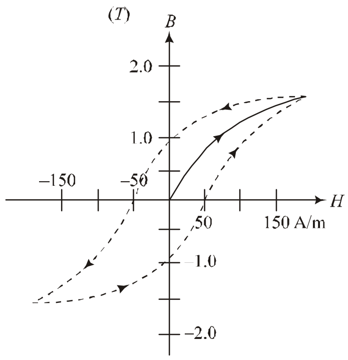


(a) $\vec{B}$
(b) zero
(c) much large than $|\vec{B}|$ and parallel to $\vec{B}$
(d) much large than $|\vec{B}|$ but opposite to $\vec{B}$

4. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required? **[Main Sep. 02, 2020 (I)]**

(a) T : Large retentivity, small coercivity
(b) P : Small retentivity, large coercivity
(c) T : Large retentivity, large coercivity
(d) P : Large retentivity, large coercivity

5.



The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are: **[Main 7 Jan. 2020 II]**

(a) 1.5 T, 50 A/m and 1.0 T
(b) 1.5 T, 50 A/m and 1.0 T
(c) 150 A/m, 1.0 T and 1.5 T (d) 1.0 T, 50 A/m and 1.5 T

6. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is: **[Main 12 Jan. 2019 II]**

(a) 3.267×10^{-4} (b) 3.672×10^{-4}
(c) 3.726×10^{-4} (d) 2.672×10^{-4}

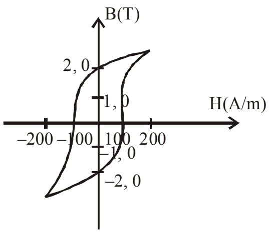
7. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is: **[Main 11 Jan. 2019 II]**

(a) 3.3×10^{-2} (b) 4.3×10^{-2}
(c) 2.3×10^{-2} (d) 3.3×10^{-4}

8. A bar magnet is demagnetized by inserthing it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is: **[Main 9 Jan. 2019 I]**

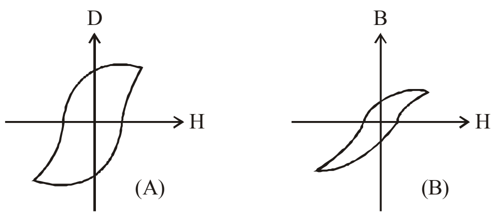
(a) 285 A/m (b) 2600 A/m
(c) 520 A/m (d) 1200 A/m

9. The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm.. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is: **[Main Online April 15, 2018]**



(a) 2 mA (b) 1 mA (c) 40 μA (d) 20 μA

10. Hysteresis loops for two magnetic materials A and B are given below :



These materials are used to make magnets for elecric generators, transformer core and electromagnet core. Then it is proper to use : **[Main 2016]**

(a) A for transformers and B for electric generators.
(b) B for electromagnets and transformers.
(c) A for electric generators and trasformers.
(d) A for electromagnets and B for electric generators.

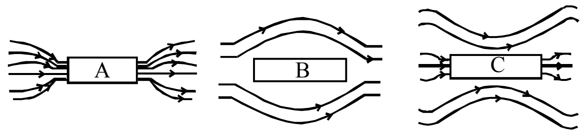
11. A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of 15m and height 5m is lying towards east over Delhi. Its speed is 240 ms^{-1}. The earth's magnetic field over Delhi is 5×10^{-5}T with the declination angle ~0° and dip of θ such that sin θ $= \frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to: **[Main Online April 10, 2016]**

(a) $V_B = 40$ mV; $V_W = 135$ mV with left side of pilot at higher voltage
(b) $V_B = 45$ mV; $V_W = 120$ mV with right side of pilot at higher voltage
(c) $V_B = 40$ mV; $V_W = 135$ mV with right side of pilot at higher voltage
(d) $V_B = 45$ mV; $V_W = 120$ mV with left side of pilot at higher voltage

12. The coercivity of a small magnet where the ferromagnet gets demagnetized is $3 \times 10^3\,Am^{-1}$. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is: **[Main 2014]**

(a) 30 mA (b) 60 mA
(c) 3 A (d) 6 A

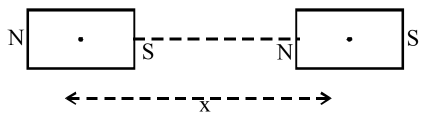
13. Three identical bars A, B and C are made of different magnetic materials. When kept in a uniform magnetic field, the field lines around them look as follows:



Make the correspondence of these bars with their material being diamagnetic (D), ferromagnetic (F) and paramagnetic (P): **[Main Online April 11, 2014]**

(a) $A \leftrightarrow D, B \leftrightarrow P, C \leftrightarrow F$
(b) $A \leftrightarrow F, B \leftrightarrow D, C \leftrightarrow P$
(c) $A \leftrightarrow P, B \leftrightarrow F, C \leftrightarrow D$
(d) $A \leftrightarrow F, B \leftrightarrow P, C \leftrightarrow D$

14. The mid points of two small magnetic dipoles of length d in end-on positions, are separated by a distance x, (x > > d). The force between them is proportional to x^{-n} where n is: **[Main Online April 9, 2014]**



(a) 1 (b) 2 (c) 3 (d) 4

15. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultand horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth.s magnetic induction is 3.6× 10.5Wb/m^2) **[Main 2013]**

(a) 3.6 × 10.5 Wb/m^2
(b) 2.56 × 10.4 Wb/m^2
(c) 3.50 × 10.4 Wb/m^2
(d) 5.80 × 10.4 Wb/m^2

16. The earth's magnetic field lines resemble that of a dipole at the centre of the earth. If the magnetic moment of this dipole is close to 8×10^{22} Am^2, the value of earth's magnetic field near the equator is close to (radius of the earth $= 6.4 \times 10^6$ m) **[Main Online April 25, 2013]**

(a) 0.6 Gauss
(b) 1.2 Gauss
(c) 1.8 Gauss
(d) 0.32 Gauss

Topic-3 : Magnetic Equipment

1 MCQs with One Correct Answer

1. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\frac{T_1}{T_2}$ will be: **[Main Sep. 05, 2020 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$

(c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

2. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth and the two places, then the ratio B_1/B_2 is best given by : **[Main 12 April 2019 I]**

(a) 1.8 (b) 0.7 (c) 3.6 (d) 2.2

3. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then: **[Main 10 Jan. 2019 II]**

(a) $T_h = T_c$ (b) $T_h = 2\, T_c$

(c) $T_h = 1.5\, T_c$ (d) $T_h = 0.5\, T_c$

4. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is : **[Main 2017]**

(a) 6.98 s (b) 8.76 s

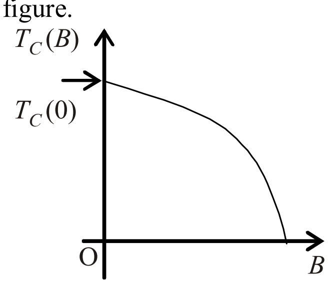
(c) 6.65 s (d) 8. 89 s

Topic-4 : Miscellaneous (Mixed Concepts) Problems

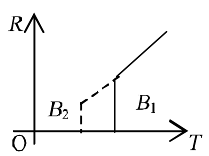
8 Comprehension/Passage Based Questions

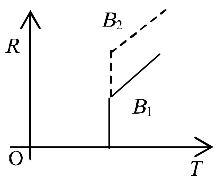
PASSAGE

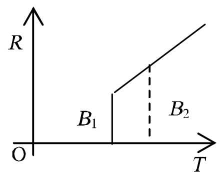
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure. **[2010]**

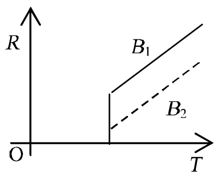


1. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1 which of the following graphs shows the correct variation of R with T in these fields?

(a)


(b)


(c)


(d)


2. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(a) $B = 5$ Tesla, T_C (B) = 80 K

(b) $B = 5$ Tesla, 75 K < T_C(B) < 100 K

(c) $B = 10$ Tesla, 75K < T_C(B) < 100 K

(d) $B = 10$ Tesla, T_C(B) = 70K

Answer Key

Topic-1 : Magnetism, Gaus's Law, Magnetic Moment, Properties of Magnet

1. (c) 2. (a) 3. (b) 4. (c) 5. (b) 6. (c) 7. (a) 8. (d) 9. (c) 10. (a)

11. (c) 12. (a) 13. ($1.25 \times 10^{-23} Am^2$) 14. ($\frac{iL^2}{4\pi}$) 15. (a, c) 16. (c, d)

Topic-2 : The Earth Magnetism, Magnetic Materials and their Properties

1. (b) 2. (d) 3. (b) 4. (d) 5. [illegible] 6. (a) 7. (d) 8. (b) 9. (b) 10 (b)

11. (d) 12. (c) 13. (b) 14. (d) 15. [illegible] 16. (a)

Topic-3 : Magnetic Equipment

1. (a) 2. (Bonus) 3. (a) 4. (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

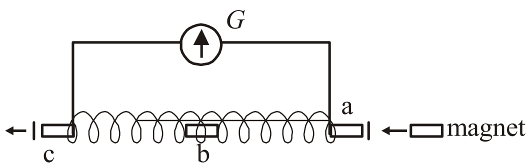
1. (a) 2. (b)

20 Electromagnetic Induction

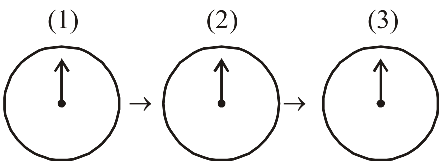
Topic-1 : Magnetic Flux, Faraday's and Lenz's Law

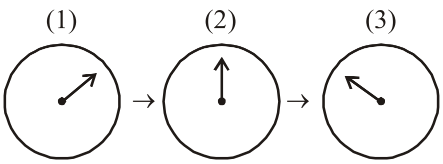
1 MCQs with One Correct Answer

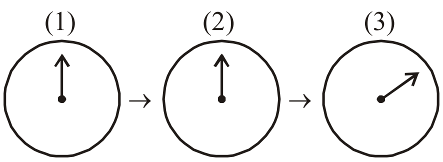
1. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil ? **[Main Sep. 04, 2020 (I)]**

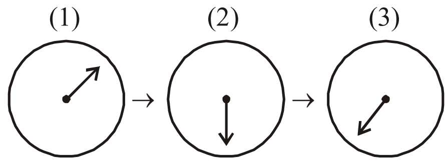


Three positions shown describe : (1) the magnet's entry (2) magnet is completely inside and (3) magnet's exit.

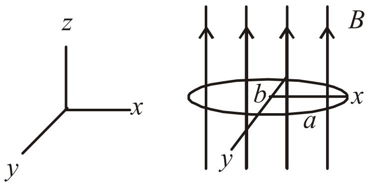
(a) 

(b) 

(c) 

(d) 

2. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is : **[Main Sep. 03, 2020 (I)]**

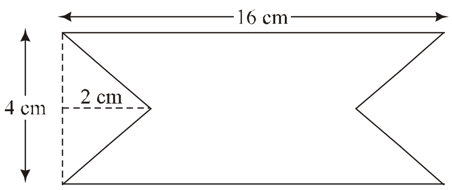


(a) $\dfrac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$ (b) zero

(c) $\dfrac{\pi abB\omega}{R}$ (d) $\dfrac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$

3. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.032\ \text{Ts}^{-1}$. The induced current in the loop is close to (Resistivity of the metal wire is $1.23 \times 10^{-8}\ \Omega\,\text{m}$) **[Main Sep. 03, 2020 (II)]**
(a) 0.43 A (b) 0.61 A (c) 0.34 A (d) 0.53 A

4. At time $t = 0$ magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is: **[Main 8 Jan. 2020 I]**



(a) $56\mu V$ (b) $28\mu V$ (c) $48\mu V$ (d) $36\mu V$

5. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct? **[Main 7 Jan. 2020 I]**

(a) $\phi_i = \phi_0$ (b) $\phi_i > \phi_0$ (c) $\phi_i < \phi_0$ (d) $\phi_i = -\phi_0$

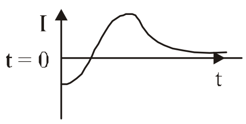
6. A long solenoid of radius R carries a time (t) - dependent current $I(t) = I_0 t(1-t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as: **[Main 7 Jan. 2020 I]**

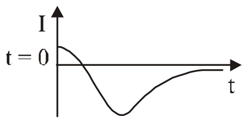
(a) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$
(b) At $t = 0.25$ direction of I_R reverses and V_R is maximum
(c) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$
(d) At $t = 0.5$ direction of I_R reverses and V_R is zero

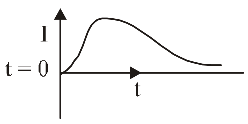
7. A planar loop of wire rotates in a uniform magnetic field. Initially, at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at: **[Main 7 Jan. 2020 II]**

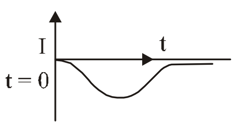
(a) 2.5 s and 7.5 s (b) 2.5 s and 5.0 s
(c) 5.0 s and 7.5 s (d) 5.0 s and 10.0 s

8. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ $(k > 0)$, as a function of time $(t \ge 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by: **[Main 9 Apr. 2019 II]**

(a)


(b)


(c)


(d)


9. The self induced emf of a coil is 25 volts. When the current in it is changed at uniiform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is: **[Main 9 Jan. 2019 II]**

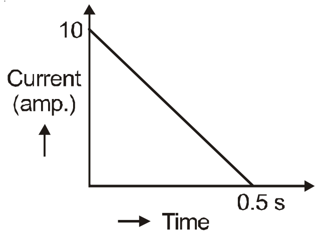
(a) 740 J (b) 437.5 J
(c) 540 J (d) 637.5 J

10. A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3} m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The the net charge flowing through the loop during t = 0 s and t = 10 ms is close to: **[Main 9 Jan. 2019 I]**

(a) 14 mC (b) 7 mC
(c) 21 mC (d) 6 mC

11. In a coil of resistance $100\,\Omega$, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **[Main 2017]**

(a) 250 Wb
(b) 275 Wb
(c) 200 Wb
(d) 225 Wb



12. A conducting metal circular–wire–loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-t/\tau}$, where B_0 and τ are constants, at time t = 0. If the resistance of the loop is R then the heat generated in the loop after a long time $(t \to \infty)$ is ; **[Main Online April 10, 2016]**

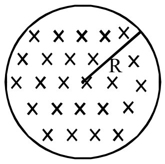
(a) $\dfrac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\dfrac{\pi^2 r^4 B_0^2}{2\tau R}$

(c) $\dfrac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\dfrac{\pi^2 r^4 B_0^2}{\tau R}$

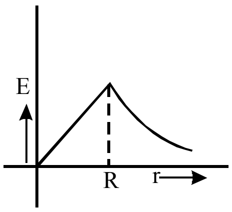
13. When current in a coil changes from 5 A to 2 A in 0.1 s, average voltage of 50 V is produced. The self - inductance of the coil is : **[Main Online April 10, 2015]**

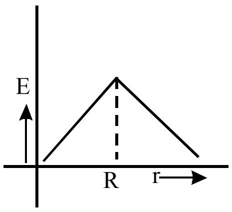
(a) 6 H (b) 0.67 H (c) 3 H (d) 1.67 H

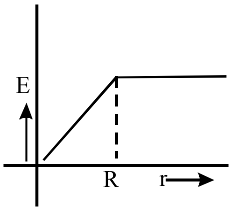
14. Figure shows a circular area of radius R where a uniform magnetic field $\vec{B}$ is going into the plane of paper and increasing in magnitude at a constant rate.

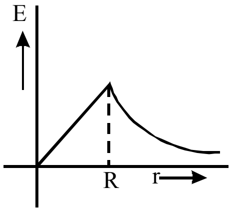


In that case, which of the following graphs, drawn schematically, correctly shows the variation of the induced electric field E(r)? **[Online April 19, 2014]**

(a) 

(b) 

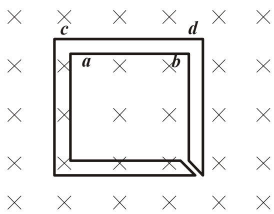
(c) 

(d) 

15. A circular loop of radius 0.3 cm lies parallel to amuch bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is **[Main 2013]**

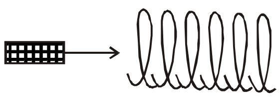
(a) 9.1×10^{-11} weber (b) 6×10^{-11} weber
(c) 3.3×10^{-11} weber (d) 6.6×10^{-9} weber

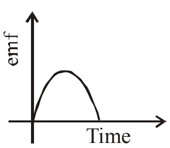
16. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then, **[2009]**

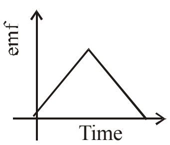


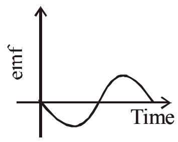
(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction *ba* and I_2 is in the direction *cd*
(d) I_1 is in the direction *ab* and I_2 is in the direction *dc*

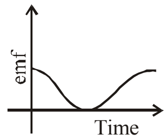
17. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced with time **[2004S]**



(a) 

(b) 

(c) 

(d) 

18. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is **[1995S]**

(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$ (c) ABR (d) $\frac{B^2A}{R^2}$

2 Integer Value Answer

19. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **[2011]**

3 Numeric Answer

20. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius current 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A Where t is in s. The emf induced in C_1 (in mV), at the instant $t = 1$ s is $\frac{4}{x}$. The value of x is ______. **[Main Sep. 05, 2020 (I)]**

21. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer _____. **[Main Sep. 02, 2020 (I)]**

22. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be ______. **[Main 9 Jan. 2020 I]**

23. A loop ABCDEFA of straight edges has six corner points $A(0, 0, 0)$, $B\{5, 0, 0)$, $C(5, 5, 0)$, $D(0, 5, 0)$, $E(0, 5, 5)$ and $F(0, 0, 5)$. The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})$T. The quantity of flux through the loop *ABCDEFA* (in Wb) is __________ . **[Main 7 Jan. 2020 I]**

4 Fill in the Blanks

24. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will .... **[1993 - 1 Marks]**

5 True / False

25. A coil of metal wire is kept stationary in a non-uniform magnetic field. An e.m.f. is induced in the coil. **[1986 - 3 Marks]**

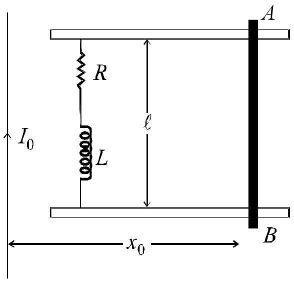
26. An e.m.f. can be induced between the two ends of a straight copper wire when it is moved through a uniform magnetic field. **[1980]**

6 MCQs with One or More than One Correct Answer

27. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is(are) **[2012]**
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

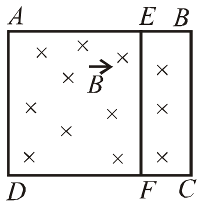
10 Subjective Problems

28. A metal bar AB can slide on two parallel thick metallic rails separated by a distance ℓ. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Answer the following questions. **[2002 - 5 Marks]**

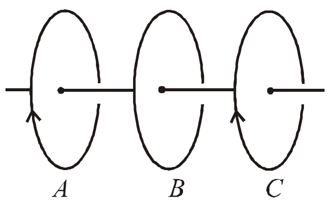


(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.
(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. Find the value of $\frac{L}{R}$ in terms of the other given quantities.

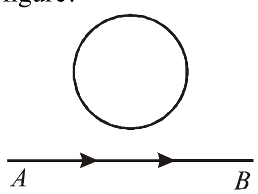
29. A rectangular frame $ABCD$, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in Fig. $AEFD$ is a square of side 1m, and $EB = FC = 0.5$m. The entire circuit is placed in steadily increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1T/s. The resistance per unit length of the wire is $1\Omega/\text{m}$. Find the magnitudes and directions of the currents in the segments AE, BE and EF. **[1993-5 Marks]**



30. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in Fig. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B ? If no, give reasons. If yes mark the direction of the induced current in the diagram. **[1982 - 2 Marks]**



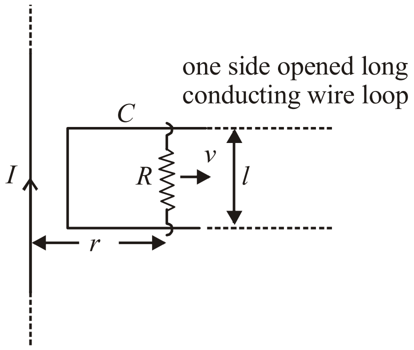
31. A current from A to B is increasing in magnitude. What is the direction of induced current, if any, in the loop as shown in the figure? **[1979]**



Topic-2 : Motional and Static EMI and Application of EMI

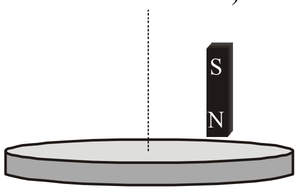
1 MCQs with One Correct Answer

1. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is : **[Main Sep. 05, 2020 (II)]**



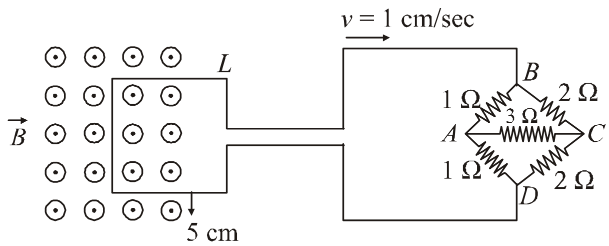
(a) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$ (b) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (c) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (d) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

2. A light disc made of aluminium (a nonmagnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (figure is schematic and not drawn to scale) **[Adv. 2020]**



(a) rotate in the direction opposite to the direction of magnet's motion
(b) rotate in the same direction as the direction of magnet's motion
(c) not rotate and its temperature will remain unchanged
(d) not rotate but its temperature will slowly rise

3. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 &!, the current in the loop at that instant will be close to : **[Main 12 Apr. 2019 I]**

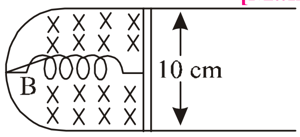


(a) 60μA (b) 170 μA (c) 150 μA (d) 115 μA

4. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to: **[Main 9 April 2019 I]**

(a) L (b) L^2 (c) $1/L^2$ (d) 1/L

5. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to: **[Main 8 April 2019 I]**



(a) 1000 (b) 50000 (c) 5000 (d) 10000

6. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is : **[Main 12 Jan. 2019 II]**

(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 2.5×10^{-3} V (d) 0.3×10^{-3} V

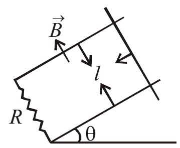
7. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil: **[Main 11 Jan. 2019 II]**

(a) decreases by a factor of 9
(b) increases by a factor of 27
(c) increases by a factor of 3
(d) decreases by a factor of $9\sqrt{3}$

8. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is: **[Main 10 Jan. 2019 I]**

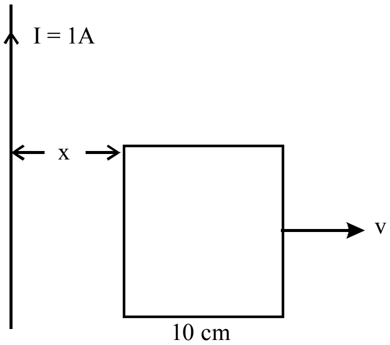
(a) $\pi n \rho l^3$ (b) $\frac{\pi}{3} n \rho l^3$ (c) $\frac{\pi}{4} n \rho l^3$ (d) $n \rho l^3$

9. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation 1 and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is: **[Main Online April 15, 2018]**



(a) $\frac{mgR\cos\theta}{B^2 l^2}$ (b) $\frac{mgR\sin\theta}{B^2 l^2}$
(c) $\frac{mgR\tan\theta}{B^2 l^2}$ (d) $\frac{mgR\cot\theta}{B^2 l^2}$

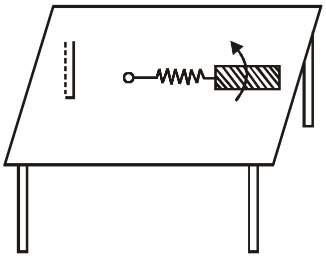
10. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plate of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 ms^{-1} (see figure). **[Adv. 2020]**



The e.m.f induced at the time the left arm of the frame is at x = 10 cm from the wire is: **[Online April 19, 2014]**

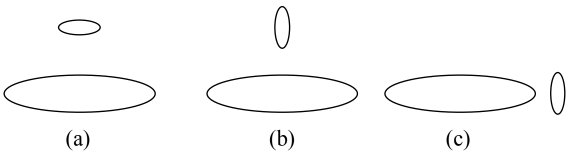
(a) 2 μV (b) 1 μV (c) 0.75 μV (d) 0.5 μV

11. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is **[2013]**



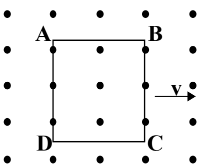
(a) $\frac{2B\omega\ell^2}{2}$ (b) $\frac{3B\omega\ell^2}{2}$ (c) $\frac{4B\omega\ell^2}{2}$ (d) $\frac{5B\omega\ell^2}{2}$

12. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be **[2001S]**



(a) maximum in situation (a) (b) maximum in situation (b)
(c) maximum in situation (c) (d) the same in all situations

13. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced **[2001S]**

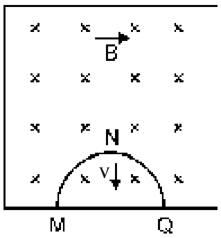


(a) in AD, but not in BC (b) in BC, but not in AD
(c) neither in AD nor in BC (d) in both AD and BC

14. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time **[1999S - 2 Marks]**

(a) 500 s (b) 25 s (c) 35 ms (d) 1 ms

15. A thin semi-circular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v, and the potential difference developed across the ring is



(a) zero **[1996 - 2 Marks]**
(b) $Bv\pi R^2/2$ and M is at higher potential
(c) πRBv and Q is at higher potential
(d) $2RBv$ and Q is at higher potential.

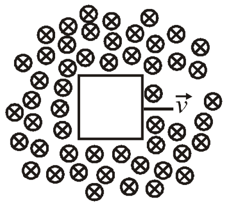
16. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement from the following **[1998S - 2 Marks]**

(a) The entire rod is at the same electric potential.
(b) There is an electric field in the rod.
(c) The electric potential is highest at the centre of the rod and decreases towards its ends.
(d) The electric potential is lowest at the centre of the rod, and increases towards its ends

17. A small square loop of wire of side l is placed inside a large square loop of wire of side L(L>>l). The loops are co-planar and their centres coincide. The mutual inductance of the system is proportional to **[1998S - 2 Marks]**

(a) l/L (b) l^2/L (c) L/l (d) L^2/l

18. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere.



The current induced in the loop is: **[1989 - 2 Marks]**

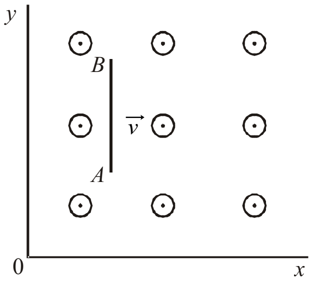
(a) BLv/R clockwise
(b) BLv/R anticlockwise
(c) $2BLv/R$ anticlockwise
(d) zero.

4 Fill in the Blanks

19. A uniformly wound solenoidal coil of self inductance 1.8×10^{-4} henry and resistance 6 ohm is broken up into two identical coils. These identical coils are then connected in parallel across a 15-volt battery of negligible resistance. The time constant for the current in the circuit is .... seconds and the steady state current through the battery is .... amperes. **[1989 - 2 Marks]**

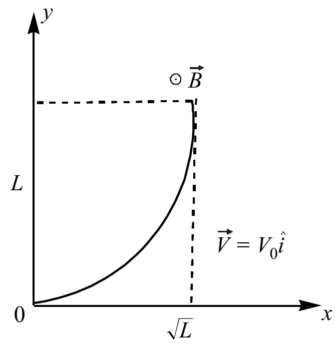
5 True / False

20. A conducting rod AB moves parallel to the x-axis (see Fig.) in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. **[1987 - 2 Marks]**



6 MCQs with One or More than One Correct Answer

21. A conducting wire of parabolic shape, initially $y = x^2$ is moving with velocity $\vec{V} = V_0\hat{i}$ in a non – uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V^0B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are: **[Adv. 2019]**



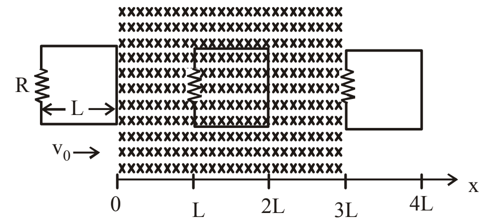
(a) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(b) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

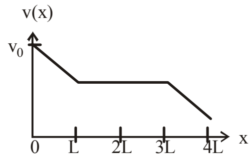
(c) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

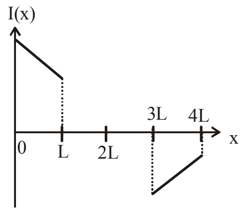
(d) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

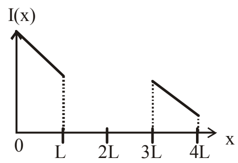
22. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **[Adv. 2016]**

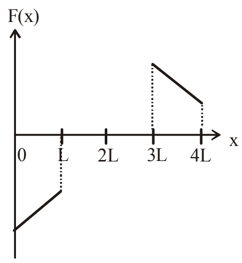


Which of the following schematic plot(s) is (are) correct? (Ignore gravity)

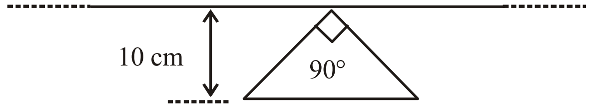
(a)


(b)


(c)


(d)


23. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? **[Adv. 2016]**



(a) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(b) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(c) The induced current in the wire is in opposite direction to the current along the hypotenuse

(d) There is a repulsive force between the wire and the loop

24. The *SI* unit of inductance, the henry, can be written as **[1998S - 2 Marks]**

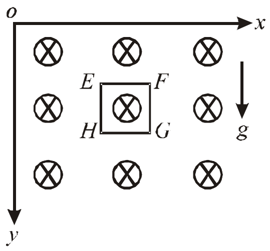
(a) weber/ampere (b) volt-second/ampere
(c) joule/(ampere)^2 (d) ohm-second

25. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then: **[1994 - 2 Marks]**

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = 4$

10 Subjective Problems

26. A magnetic field $B = B_0 (y/a)\hat{k}$ is into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R, in $x - y$ plane, starts falling under the influence of gravity see figure) Note the directions of x and y axes in figure. **[1999 - 10 Marks]**



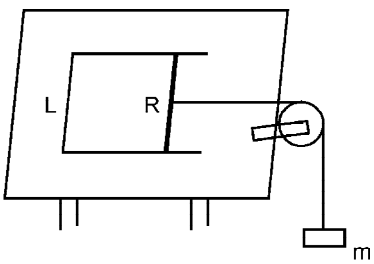
Find

(a) the induced current in the loop and indicate its direction.

(b) the total Lorentz force acting on the loop and indicate its direction, and

(c) an expression for the speed of the loop, $v(t)$ and its terminal value.

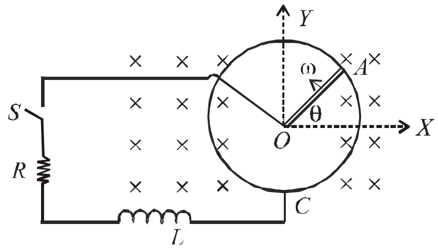
27. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate. **[1997 - 5 Marks]**



(i) the terminal velocity achieved by the rod, a nd

(ii) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

28. A metal rod OA of mass 'm' and length 'r' is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in the figure below. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. **[1995 - 10 Marks]**



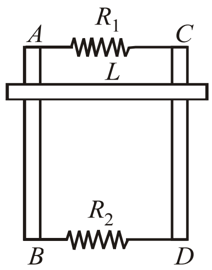
(a) What is the induced emf across the terminals of the switch?

(b) The switch S is closed at time $t = 0$.

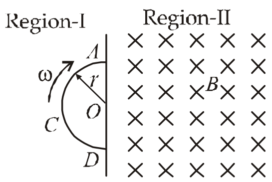
(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at $t = 0$.

29. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Figure. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 Tesla perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 Watt and 1.2 watt respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **[1994 - 6 Marks]**

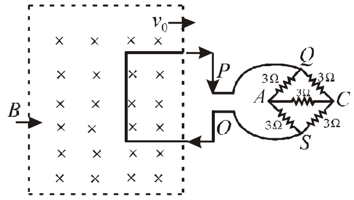


30. Space is divided by the line AD into two regions. Region I is field free and the Region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R. **[1985 - 6 Marks]**



(i) Obtain an expression for the magnitude of the induced current in the loop.
(ii) Show the direction of the current when the loop is entering into the Region II.
(iii) Plot a graph between the induced e.m.f and the time of rotation for two periods of rotation.

31. A square metal wire loop of side 10 cms and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2$ webers/m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohms. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? Give the direction of current in the loop. **[1983 - 6 Marks]**



32. The two rails of a railway track, insulated from each other and the ground, are connected to a milli voltmeter. What is the reading of the milli voltmeter when a train travels at a speed of 180 km/hour along the track, given that the vertical component of earth's magnetic field is 0.2×10^{-4} weber/m^2 & the rails are separated by 1 meter? **[1981- 4 Marks]**

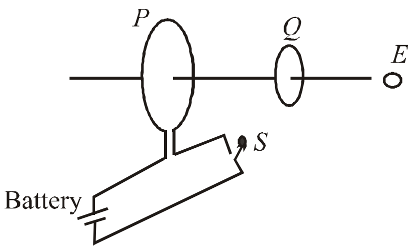
Topic-3 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be **[2005S]**
(a) zero
(b) anticlockwise of the +ve z axis
(c) clockwise of the +ve z axis
(d) along the magnetic field

2. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be **[2002S]**
(a) halved (b) the same
(c) doubled (d) quadrupled

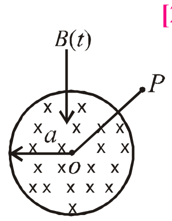
3. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the direction IQ_1 and IQ_2 (as seen by E) are **[2002S]**



(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

4. A coil of wire having inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $1_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time $(t > 0)$, the product $I_2(t)\,B(t)$ **[2000S]**
(a) increases with time (b) decreases with time
(c) does not vary with time (d) passes through a maximum

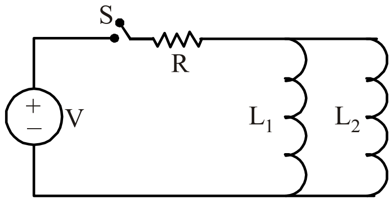
5. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region **[2000S]**
(a) is zero
(b) decreases as $1/r$
(c) increases as r
(d) decreases as $1/r^2$



6. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B
(a) remains stationary **[1999S - 2 Marks]**
(b) is attracted by the loop-A
(c) is repelled by the loop-A
(d) rotates about its CM, with CM fixed

6 MCQs with One or More than One Correct Answer

7. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct? **[Adv. 2017]**



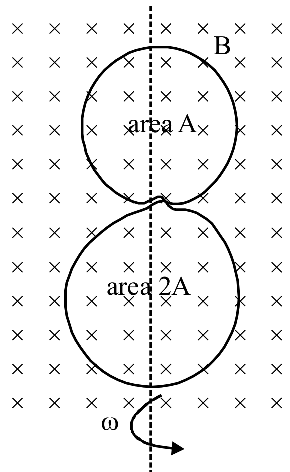
(a) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(b) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(c) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)

(d) At t = 0, the current through the resistance R is $\frac{V}{R}$

8. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **[Adv. 2017]**



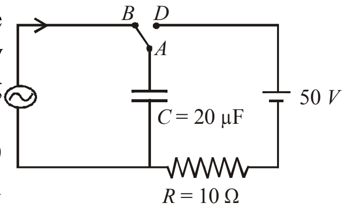
(a) The emf induced in the loop is proportional to the sum of the areas of the two loops

(b) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(c) The net emf induced due to both the loops is proportional to cos ωt

(d) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

9. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu F$, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **[Adv. 2014]**



(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(c) Immediately after A is connected to D, the current in R is 10 A

(d) $Q = 2 \times 10^{-3}$ C

10. L, C and R represent the physical quantities, inductance, capacitance and resistance respectively. The combination(s) which have the dimensions of frequency are **[1984- 2 Marks]**

(a) $1/RC$ (b) R/L (c) $1/\sqrt{LC}$ (d) C/L

8 Comprehension/Passage Based Questions

Passage

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ. **[Adv. 2013]**

11. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$ (c) BR (d) 2BR

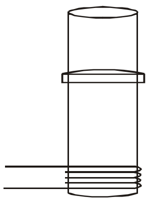
12. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(a) $-\gamma BQR^2$ (b) $-\gamma\frac{BQR^2}{2}$

(c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

9 Assertion and Reason Type Questions

13. **Statement-1 :** A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.

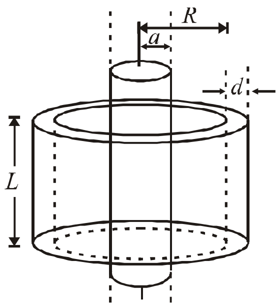


Statement-2 : In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. **[2007]**

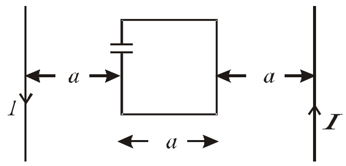
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

14. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R. thickness d ($d << R$) and length L. A variable current $i = i_0 \sin\omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **[2005 - 4 Marks]**

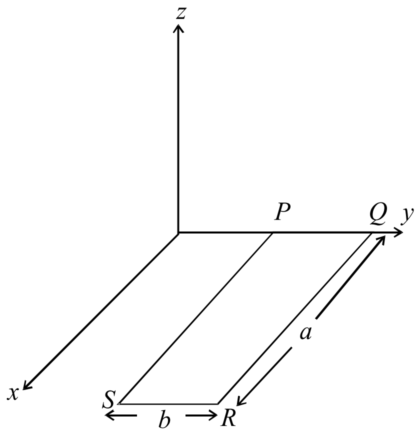


15. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as $I = I_0 \sin\omega t$. **[2003 - 4 Marks]**



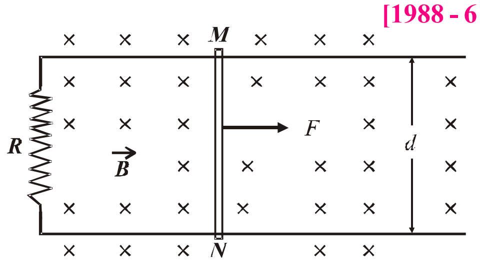
(a) Calculate maximum current in the square loop.
(b) Draw a graph between charges on the upper plate of the capacitor vs time.

16. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **[2002 - 5 Marks]**



(a) What is the direction of the current I in PQ?
(b) Find the magnetic force on the arm RS.
(c) Find the expression for I in terms of B_0, a, b and m.

17. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R. **[1988 - 6 Marks]**



(i) Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.
(ii) What fraction of the work done per second by F is converted into heat ?

Answer Key

Topic-1 : Magnetic Flux, Faraday's and Lenz's Law

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(d)	**6.**	(d)	**7.**	(b)	**8.**	(a)	**9.**	(b)	**10.**	(Bonus)
11.	(a)	**12.**	(b)	**13.**	(d)	**14.**	(a)	**15.**	(a)	**16.**	(d)	**17.**	(c)	**18.**	(b)	**19.**	(4)	**20.**	(5)
21.	(15)	**22.**	(10)	**23.**	(175.00)	**24.**	Left to right			**25.**	False	**26.**	True	**27.**	(a, c)	**28.**	(a)		
31.																			

Topic-2 : Motional and Static EMI and Application of EMI

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(c)	**9.**	(b)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(d)	**14.**	(d)	**15.**	(d)	**16.**	(b)	**17.**	(b)	**18.**	(d)	**19.**	0.3×10^{-4} s, 10 A		
20.	True	**21.**	(a, b, d)	**22.**	(a, b)	**23.**	(a, d)	**24.**	(a,b,c,d)	**25.**	(a,c)								

Topic-3 : Miscellaneous (Mixed Concepts) Problems

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(c)	**7.**	(a, b, c)	**8.**	(b, d)	**9.**	(c, d)	**10.**	(a,b,c)
11.	(b)	**12.**	(b)	**13.**	(a)														

21 Alternating Current

Topic-1 : Alternating Current, Voltage and Power

1 MCQs with One Correct Answer

1. An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is : [Main 8 April 2019 I]
(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

2. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_o \cos(\omega t)$. The emf induced in the smaller inner loop is nearly : [Main Online April 8, 2017]
(a) $\frac{\pi\mu_0 I_o}{2} \cdot \frac{a^2}{b} \omega \sin(\omega t)$ (b) $\frac{\pi\mu_0 I_o}{2} \cdot \frac{a^2}{b} \omega \cos(\omega t)$
(c) $\pi\mu_0 I_o \frac{a^2}{b} \omega \sin(\omega t)$ (d) $\frac{\pi\mu_0 I_o b^2}{a} \omega \cos(\omega t)$

3. A sinusoidal voltage $V(t) = 100 \sin(500t)$ is applied across a pure inductance of $L = 0.02$ H. The current through the coil is: [Main Online April 12, 2014]
(a) 10 cos (500 t) (b) – 10 cos (500t)
(c) 10 sin (500t) (d) – 10 sin (500t)

Topic-2 : AC Circuit, LCR Circuit, Quality and Power Factor

1 MCQs with One Correct Answer

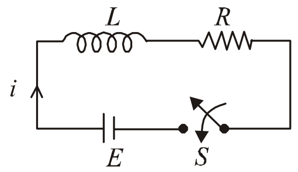
1. An AC circuit has $R = 100\ \Omega$, $C = 2\ \mu$F and $L = 80$ mH, connected in series. The quality factor of the circuit is : [Main Sep. 06, 2020 (I)]
(a) 2 (b) 0.5 (c) 20 (d) 400

2. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at $t = 0$, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is : [Main Sep. 04, 2020 (II)]
(a) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (b) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$
(c) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (d) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

3. A 750 Hz, 20 V (rms) source is connected to a resistance of $100\ \Omega$, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to : [Main Sep. 03, 2020 (I)]
(a) 418 s (b) 245 s (c) 365 s (d) 348 s

4. An inductance coil has a reactance of $100\ \Omega$. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is : [Main Sep. 02, 2020 (II)]
(a) 1.1×10^{-2} H (b) 1.1×10^{-1} H
(c) 5.5×10^{-5} H (d) 6.7×10^{-7} H

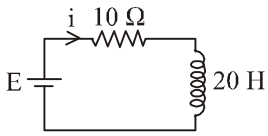
5. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is : [Main 12 April 2019 II]



(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$

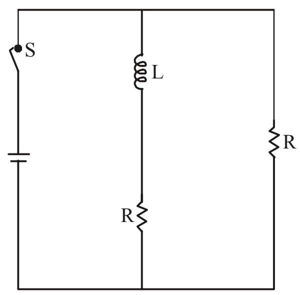
(c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

6. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is : **[Main 8 April 2019 I]**



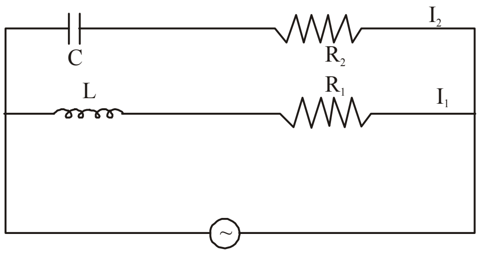
(a) $\frac{2}{ln\,2}$ (b) $\frac{1}{2}ln\,2$ (c) $2\,ln\,2$ (d) $ln\,2$

7. In the figure shown, a circuit contains two identical resistors with resistance R = 5 Ω and an inductance with L = 2 mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed? **[Main 12 Jan. 2019 I]**



(a) 5.5 A (b) 7.5 A (c) 3 A (d) 6 A

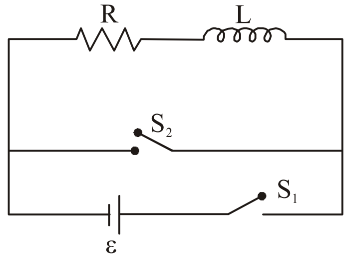
8.



In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by, $V = 200\sqrt{2}$ sin (100 t) volts. The phase difference between I_1 and I_2 is : **[Main 12 Jan. 2019 II]**

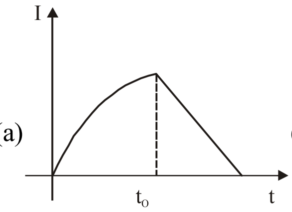
(a) 60° (b) 30° (c) 90° (d) 0

9. In the circuit shown,

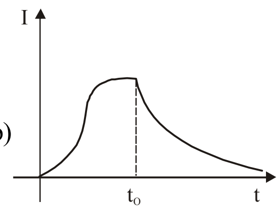


the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. the behaviour of the current I as a function of time 't' is given by: **[Main 11 Jan. 2019 II]**

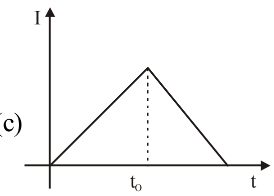
(a)



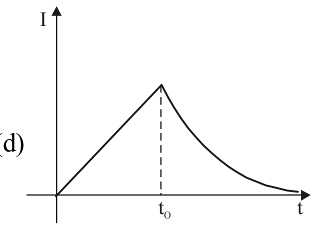
(b)



(c)



(d)

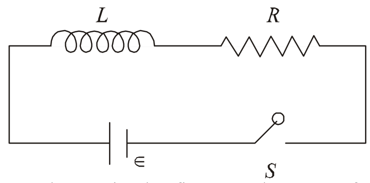


10. In LC circuit the inductance L = 40 mH and capacitance C = 100 .00F. If a voltage $V(t) = 10 \sin(314\,t)$ is applied to the circuit, the current in the circuit is given as: **[Main 9 Jan. 2019]**

(a) 0.52 cos 314 t (b) 10 cos 314 t

(c) 5.2 cos 314 t (d) 0.52 sin 314 t

11.



As shown in the figure, a battery of emf ∈ is connected to an inductor *L* and resistance *R* in series. The switch is closed at $t = 0$. The total charge that flows from the battery, between $t = 0$ and $t = t_c$ (t_c is the time constant of the circuit) is: **[Main 8 Jan. 2019 II]**

(a) $\frac{\in R}{eL^2}$ (b) $\frac{\in L}{R^2}\left(1-\frac{1}{e}\right)$

(c) $\frac{\in L}{R^2}$ (d) $\frac{\in R}{eL^2}$

12. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: **[Main 7 Jan. 2019 I]**

(a) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(b) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(c) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(d) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

13. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to:

(Take $e^2 = 7.389$) **[Main 7 Jan. 2019 II]**

(a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

14. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively: **[Main 2018]**

(a) 50W, 10A (b) $\frac{1000}{\sqrt{2}}$ W, 10A

(c) $\frac{50}{\sqrt{2}}$ W, 0 (d) 50W, 0

15. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by: **[Main 2018]**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{(\omega_0 C)}$ (d) $\frac{CR}{\omega_0}$

16. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that R = 5 Ω, L = 25 mH and C = 1000 μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be : **[Main Online April 9, 2017]**

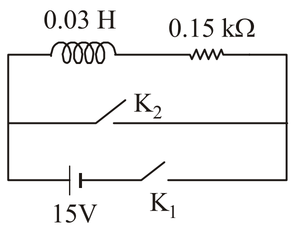
(a) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (b) 7 Ω and 45°

(c) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$ (d) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

17. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to : **[Main 2016]**

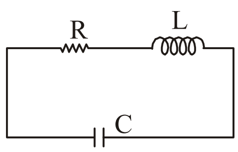
(a) 0.044 H (b) 0.065 H
(c) 80 H (d) 0.08 H

18. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : $(e^5 \cong 150)$ **[2015]**

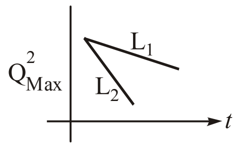


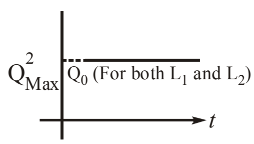
(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA

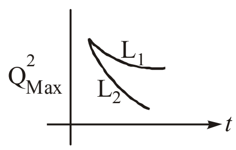
19. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

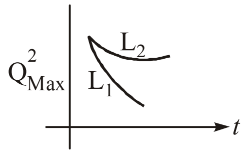


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time(t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly ? (*plots are schematic and not drawn to scale*) **[Main 2015]**

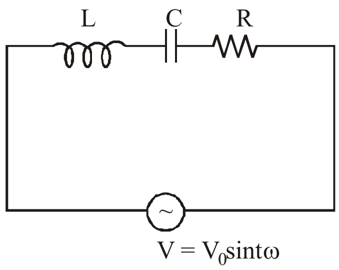
(a)


(b)


(c)


(d)


20. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in : **[Main Online April 11, 2015]**



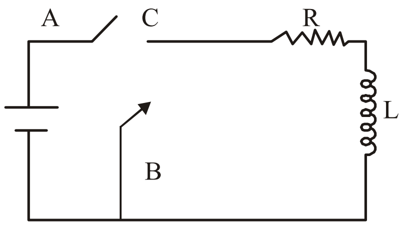
(a) series with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

(b) series with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(c) parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(d) parallel with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

21. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to: **[Main 2014]**

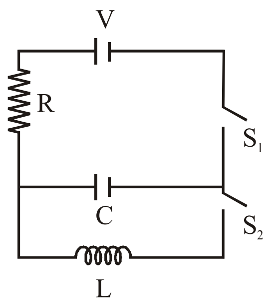


(a) $\frac{e}{1-e}$ (b) 1 (c) –1 (d) $\frac{1-e}{e}$

22. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC sources is 100 V, the V_R is close to: **[Main Online April 9, 2014]**

(a) 50 V (b) 70 V (c) 90 V (d) 100 V

23. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct ? **[Main 2013]**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At, $t = \tau$, $q = CV/2$

(c) At, $t = 2\tau$, $q = CV(1 - e^{-2})$

(d) At, $t = 2\tau$, $q = CV(1 - e^{-1})$

24. A series LR circuit is connected to an ac source of frequency ω and the inductive reactance is equal to 2R. A capacitance of capacitive reactance equal to R is added in series with L and R. The ratio of the new power factor to the old one is : **[Main Online April 25, 2013]**

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{5}{2}}$

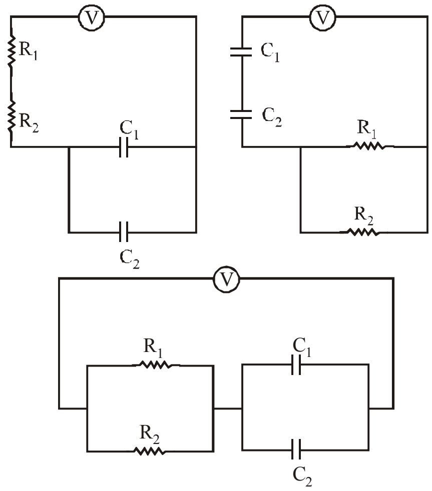
25. In a series L-C-R circuit, $C = 10^{-11}$ Farad, $L = 10^{-5}$ Henry and R = 100 Ohm, when a constant D.C. voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9} C. The D.C. source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant D.C. voltage E. At resonance the peak value of the charge acquired by the capacitor will be : **[Main Online April 22, 2013]**

(a) 10^{-15} C (b) 10^{-6} C (c) 10^{-10} C (d) 10^{-8} C

26. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased **[2010]**

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

27. Find the time constant (in μs) for the given RC circuits in the given order respectively **[2006 - 3M, –1]**



$R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 4\mu F$, $C_2 = 2\mu F$

(a) $18, 4, \frac{8}{9}$ (b) $18, \frac{8}{9}, 4$ (c) $4, 18, \frac{8}{9}$ (d) $4, \frac{8}{9}, 18$

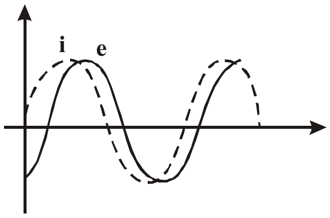
28. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements **[2003S]**

(a) $R = 1k\Omega\,, C = 10\,\mu F$

(b) $R = 1k\Omega\,, C = 1\,\mu F$

(c) $R = 1k\Omega\,, L = 10\,H$

(d) $R = 1k\Omega\,, L = 1\,H$

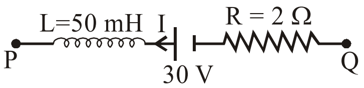


2 Integer Value Answer

29. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is **[Adv. 2016]**

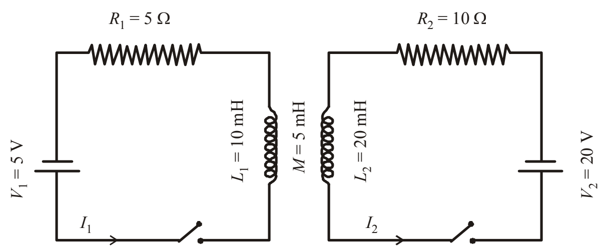
3 Numeric Answer

30. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is ______. **[Main Sep. 06, 2020 (I)]**



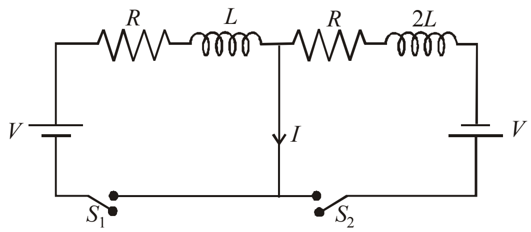
31. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is ______. **[Main Sep. 06, 2020 (II)]**

32. The inductors of two *LR* circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced *EMF* in the inductors by the time the currents reach their steady state values is________ mJ. **[Adv. 2020]**



6 MCQs with One or More than One Correct Answer

33. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true? **[Adv. 2018]**



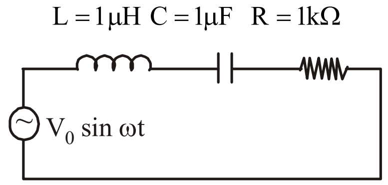
(a) $I_{max} = \dfrac{V}{2R}$

(b) $I_{max} = \dfrac{V}{4R}$

(c) $\tau = \dfrac{L}{R}\ln 2$

(d) $\tau = \dfrac{2L}{R}\ln 2$

34. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **[Adv. 2017]**



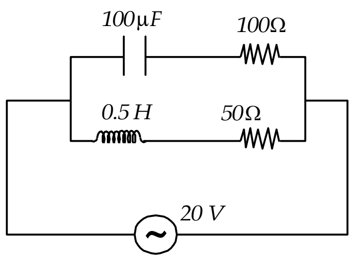
(a) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}

(b) The frequency at which the current will be in phase with the voltage is independent of R

(c) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(d) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

35. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are) **[2012]**



(a) The current through the circuit, I is 0.3 A.

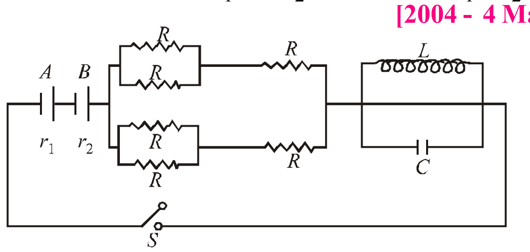
(b) The current through the circuit, I is $0.3\sqrt{2}A$

(c) The voltage across 100 Ω resistor = $10\sqrt{2}V$

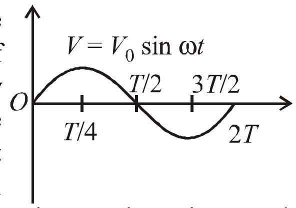
(d) The voltage across 50 Ω resistor = 10 V

10 Subjective Problems

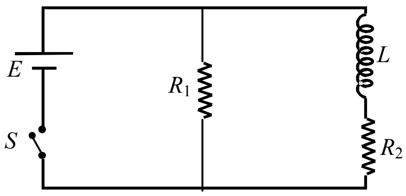
36. In the figure both cells A and B are of equal emf. Find R for which potential difference across battery A will be zero, long time after the switch is closed. Internal resistance of batteries A and B are r_1 and r_2 respectively ($r_1 > r_2$). **[2004 - 4 Marks]**



37. In a series L–R circuit (L = 35 mH and R = 11 Ω), a variable emf source ($V = V_0 \sin \omega t$) of V_{rms} = 220 V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph (π=22/7). **[2004 - 4 Marks]**

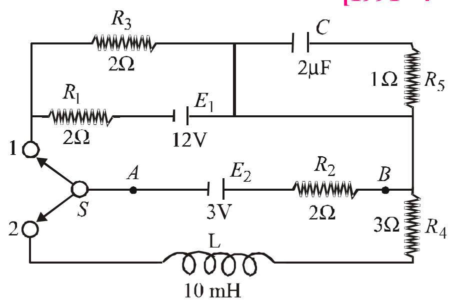


38. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2Ω and R_2 = 2Ω are connected to a battery of emf E = 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time t = 0. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? **[2001-5 Marks]**



39. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? **[1996 - 3 Marks]**

40. A circuit containing a two position switch S is shown in fig. **[1991 - 4 + 4 Marks]**



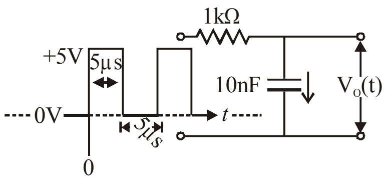
(a) The switch S is in position '1'. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

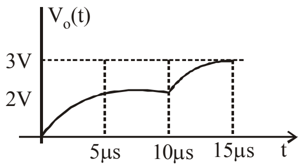
(b) If now the switch S is put in position 2 at t = 0 find
  (i) steady current in R_4 and
  (ii) the time when current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time

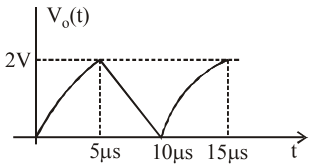
Topic-3 : Transformers and LC Oscillations

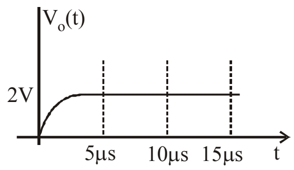
1 MCQs with One Correct Answer

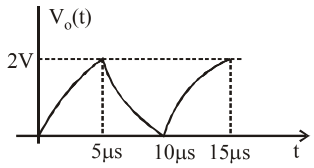
1. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_o(t)$, across the capacitor is correctly depicted by : **[Main Sep. 06, 2020 (I)]**



(a) 

(b) 

(c) 

(d) 

2. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are : **[Main 10 April 2019 I]**

(a) 220 V and 20 A (b) 440 V and 20 A
(c) 440 V and 5 A (d) 220 V and 10 A

3. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be: **[Main 9 Jan. 2019 II]**

(a) 50 A (b) 45 A (c) 35 A (d) 25 A

4. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5 A, and its efficiency is 90%, the output current would be: **[Main Online April 16, 2018]**

(a) 20 A (b) 40 A (c) 45 A (d) 25 A

Topic-4 : Miscellaneous (Mixed Concepts) Problems

6 MCQs with One or More than One Correct Answer

1. A series R– C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_c across the capacitor are compared in the two cases. Which of the following is/are true? **[2011]**

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$ (c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

7 Match the Following

2. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2, (indicated in circuits) are related as shown in **Column I.** Match the two **[2010]**

Column I

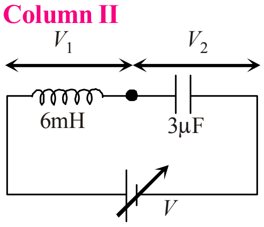
(A) $I \neq 0, V_1$ is proportional to I

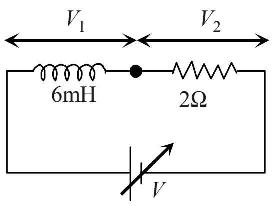
(B) $I \neq 0, V_2 > V_1$

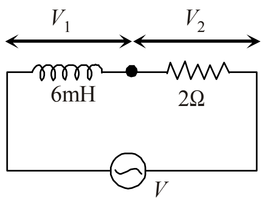
(C) $V_1 = 0, V_2 = V$

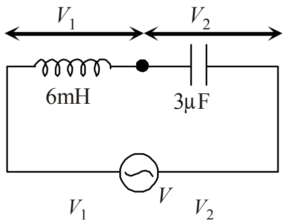
(D) $I \neq 0, V_2$ is proportional to I

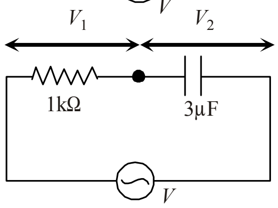
Column II

(p) 

(q) 

(r) 

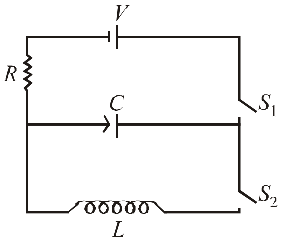
(s) 

(t) 

8 Comprehension/Passage Based Questions

Passage 1

In the given circuit the capacitor (C) may be charged through resistance R by a battery V by closing switch S_1. Also when S_1 is opened and S_2 is closed the capacitor is connected in series with inductor (L).



3. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is τ. Which of the following is correct **[2006 – 5M, –2]**

(a) after time interval τ, charge on the capacitor is $\frac{CV}{2}$

(b) after time interval 2τ, charge on the capacitor of $CV(1-e^{-2})$

(c) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(d) after time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$

4. When the capacitor gets charged completely, S_1 is opened and S_2 is closed. Then, **[2006 – 5M, –2]**

(a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy

(b) at any time $t > 0$, current in the circuit is in the same direction

(c) at $t > 0$, there is no exchange of energy between the inductor and capacitor

(d) at any time $t > 0$, instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$

5. Given that the total charge stored in the LC circuit is Q_0, for $t \geq 0$, the charge on the capacitor is **[2006 – 5M, –2]**

(a) $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$ (b) $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

(c) $Q = -LC\frac{d^2Q}{dt^2}$ (d) $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Passage 2

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method , a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with power factor unity. All the currents and voltages mentioned are rms values. **[Adv. 2013]**

6. If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation| (in %) during transmission is

(a) 20 (b) 30 (c) 40 (d) 50

7. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1 (c) 100 : 1 (d) 50 : 1

Answer Key

Topic-1 : Alternating Current, Voltage and Power

1. **(d)** **2.** **(a)** **3.** **(b)**

Topic-2 : AC Circuit, LCR Circuit, Quality and Power Factor

1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (Bonus)	**9.** (b)	**10.** (a)
11. (a)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (8)	**30.** (33)
31. (400)	**32.** (55)	**33.** (b, d)	**34.** (b, c)	**35.** (a,c)	**36.** $\frac{4}{3}(r_1 - r_2)$	**37.** $20A, \frac{\pi}{4}$	**38.** $12e^{-5t}$ V		
39. 3.465 s									

Topic-3 : Transformers and LC Oscillation

1. **(a)** **2.** **(c)** **3.** **(b)** **4.** **(c)**

Topic-4 : Miscellaneous (Mixed Concepts)Problems

1. (a,c) **2.** A-r,s,t; B-q,r,s,t; C-p,q; D-q,r,s,t **3.** (b) **4.** (d) **5.** (c) **6.** (a) **7.** (b)

22 Electromagnetic Waves

Topic-1 : Electromagnetic Waves, Conduction and Displacement Current

1 MCQs with One Correct Answer

1. For a plane electromagnetic wave, the magnetic field at a point x and time t is
$\vec{B}(x,t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]$ T
The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is:
(speed of light c = 3 × 10^8 ms^{-1}) **[Main Sep. 06, 2020 (II)]**
(a) $\vec{E}(x,t) = [-36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$
(b) $\vec{E}(x,t) = [36\sin(1 \times 10^3 x + 0.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$
(c) $\vec{E}(x,t) = [36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]\frac{V}{m}$
(d) $\vec{E}(x,t) = [36\sin(1 \times 10^3 x + 1.5 \times 10^{11} t)\hat{i}]\frac{V}{m}$

2. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\hat{j}$ $\sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$ V/m. The maximum magnetic force experienced by the electron will be :
(given $c = 3 \times 10^8$ ms^{-1} & electron charge = 1.6×10^{-19}C)
[Main Sep. 05, 2020 (I)]
(a) 3.2×10^{-18} N (b) 2.4×10^{-18} N
(c) 4.8×10^{-19} N (d) 1.6×10^{-19} N

3. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$
Its magnetic field will be given by: **[Main Sep. 04, 2020 (II)]**
(a) $\frac{E_0}{c}(-\hat{x} + \hat{y})\sin(kz - \omega t)$
(b) $\frac{E_0}{c}(\hat{x} + \hat{y})\sin(kz - \omega t)$
(c) $\frac{E_0}{c}(\hat{x} - \hat{y})\sin(kz - \omega t)$
(d) $\frac{E_0}{c}(\hat{x} - \hat{y})\cos(kz - \omega t)$

4. The magnetic field of a plane electromagnetic wave is
$\vec{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{i}T$
where $c = 3 \times 10^8$ ms^{-1} is the speed of light.
The corresponding electric field is :
[Main Sep. 03, 2020 (I)]
(a) $\vec{E} = 9\sin[200\pi(y + ct)]\hat{k}$ V/m
(b) $\vec{E} = -10^{-6}\sin[200\pi(y + ct)]\hat{k}$ V/m
(c) $\vec{E} = 3 \times 10^{-8}\sin[200\pi(y + ct)]\hat{k}$ V/m
(d) $\vec{E} = -9\sin[200\pi(y + ct)]\hat{k}$ V/m

5. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment $t = 0$ is : **[Main Sep. 03, 2020 (II)]**
(a) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{k}$ (b) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{j}$
(c) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{k}$ (d) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{j}$

6. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close to
($\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and speed of light = 3×10^8 ms^{-1}) :
[Main Sep. 02, 2020 (I)]
(a) 150 nT (b) 160 nT (c) 180 nT (d) 190 nT

7. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i}-2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave. [Main Sep. 02, 2020 (II)]

(a) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

(c) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ (d) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$

8. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant): [Main 9 Jan 2020, (II)]

(a) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(b) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t+k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(c) $B_0\hat{k}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(d) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

9. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B}=5\times10^{-8}\hat{j}\,T$. The corresponding electric field $\vec{E}$ is (speed of light $c=3\times10^8$ ms^{-1}) [Main 8 Jan 2020, (II)]

(a) $1.66\times10^{-16}\,\hat{i}$ V/m (b) $-1.66\times10^{-16}\,\hat{i}$ V/m

(c) $-15\,\hat{i}$ V/m (d) $15\,\hat{i}$ V/m

10. If the magnetic field in a plane electromagnetic wave is given by $\vec{B}=3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{j}$ T, then what will be expression for electric field? [Main 7 Jan 2020, (I)]

(a) $\vec{E}=(60\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

(b) $\vec{E}=(9\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

(c) $\vec{E}=(3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

(d) $\vec{E}=(3\times10^{-8}\sin(1.6\times10^3x+48\times10^{10}t)\,\hat{k}$ v/m)

11. The electric field of a plane electromagnetic wave is given by

$$\vec{E}=E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz+\omega t)$$

At $t=0$, a positively charged particle is at the point $(x,y,z)=\left(0,0,\frac{\pi}{k}\right)$. If its instantaneous velocity at $(t=0)$ is $v_0\hat{k}$, the force acting on it due to the wave is: [Main 7 Jan 2020, (II)]

(a) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (b) zero

(c) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (d) parallel to $\hat{k}$

12. An electromagnetic wave is represented by the electric field $\vec{E}=E_0\hat{n}\sin[\omega t+(6y-8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i},\hat{j},\hat{k}$, the direction of propogation $\hat{s}$ is : [Main 12 April 2019, (I)]

(a) $\hat{s}=\frac{3\hat{i}-4\hat{j}}{5}$ (b) $\hat{s}=\frac{-4\hat{k}+3\hat{j}}{5}$

(c) $\hat{s}=\left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$ (d) $\hat{s}=\frac{3\hat{j}-3\hat{k}}{5}$

13. A plane electromagnetic wave having a frequency $v=23.9$ GHz propagates along the positive z-direction in free space. The peak value of the Electric Field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave ? [Main 12 April 2019, (II)]

(a) $\vec{B}=2\times10^7\sin(0.5\times10^3z+1.5\times10^{11}t)\hat{i}$

(b) $\vec{B}=2\times10^{-7}\sin(0.5\times10^3z-1.5\times10^{11}t)\hat{i}$

(c) $\vec{B}=60\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{k}$

(d) $\vec{B}=2\times10^{-7}\sin(1.5\times10^2x+0.5\times10^{11}t)\hat{j}$

14. The electric field of a plane electromagnetic wave is given by $\vec{E}=E_0\hat{i}\cos(kz)\cos(\omega t)$
The corresponding magnetic field is then given by : [Main 10 April 2019, (I)]

(a) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\sin(\omega t)$

(b) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\cos(\omega t)$

(c) $\vec{B}=\frac{E_0}{C}\hat{j}\cos(kz)\sin(\omega t)$

(d) $\vec{B}=\frac{E_0}{C}\hat{k}\sin(kz)\cos(\omega t)$

15. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be: [Main 10 April 2019, II]

(a) 6.3×10^{-4} Ns (b) 1.4×10^{-6} Ns

(c) 5.0×10^{-3} Ns (d) 3.5×10^{-6} Ns

16. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E}=6.3\,\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be: [Main 9 April 2019 I]

(a) $18.9\times10^{-8}\,\hat{k}$T (b) $2.1\times10^{-8}\,\hat{k}$T

(c) $6.3\times10^{-8}\,\hat{k}$T (d) $18.9\times10^{8}\,\hat{k}$T

17. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1m^2 surface area will be close to (c = 3 × 10^8 m/s): **[Main 9 April 2019, II]**
(a) 15×10^{-8} N (b) 20×10^{-8} N
(c) 10×10^{-8} N (d) 35×10^{-8} N

18. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is E = 6 Vm^{-1} along y-direction. Its corresponding magnetic field component, B would be: **[Main 8 April 2019 I]**
(a) 2×10^{-8} T along z-direction
(b) 6×10^{-8} T along x-direction
(c) 6×10^{-8} T along z-direction
(d) 2×10^{-8} T along y-direction

19. The magnetic field of an electromagnetic wave is given by:
$$\vec{B} = 1.6 \times 10^{-6}\cos\left(2\times10^7 z + 6\times10^{15} t\right)\left(2\hat{i}+\hat{j}\right)\frac{Wb}{m^2}$$
The associated electric field will be : **[Main 8 April 2019, II]**
(a) $\vec{E} = 4.8 \times 10^2\cos(2 \times 10^7 z - 6 \times 10^{15} t)\left(2\hat{i}+\hat{j}\right)\frac{V}{m}$
(b) $\vec{E} = 4.8 \times 10^2\cos(2 \times 10^7 z - 6 \times 10^{15} t)(-2\hat{j}+\hat{i})\frac{V}{m}$
(c) $\vec{E} = 4.8 \times 10^2\cos(2 \times 10^7 z + 6 \times 10^{15} t)\left(-\hat{i}+2\hat{j}\right)\frac{V}{m}$
(d) $\vec{E} = 4.8 \times 10^2\cos(2 \times 10^7 z + 6 \times 10^{15} t)\left(\hat{i}-2\hat{j}\right)\frac{V}{m}$

20. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to : **[Main 12 Jan 2019, II]**
(a) 1 T (b) 10^2 T (c) 10^{-2} T (d) 10^{-4} T

21. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by : **[Main 11 Jan 2019, I]**
(a) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (b) $\left(\sqrt{n}, \sqrt{n}\right)$
(c) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

22. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by :
[Given permittivity of space $\epsilon_0 = 9 \times 10^{-12}$ SI units, Speed of light c = 3 × 10^8 m/s] **[Main 11 Jan 2019, II]**
(a) 2 kV/m (c) 0.7 kV/m
(b) 1 kV/m (d) 1.4 kV/m

23. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3 × 10^8 m/s)
$$B = 100 \times 10^{-6}\sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$$
then the maximum electric field associated with it is: **[Main 10 Jan. 2019 I]**
(a) 6×10^4 N/C (b) 3×10^4 N/C
(c) 4×10^4 N/C (d) 4.5 10^4 N/C

24. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression $\vec{E}(x, y) = 10\,\hat{j}\cos[(6x + 8z)]$
The magnetic field $\vec{B}(x, z, t)$ is given by: (c is the velocity of light) **[Main 10 Jan 2019, II]**
(a) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x-8z+10ct)]$
(b) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z-10ct)]$
(c) $\frac{1}{c}(6\hat{k}+8\hat{i})\cos[(6x+8z-10ct)]$
(d) $\frac{1}{c}(6\hat{k}-8\hat{i})\cos[(6x+8z+10ct)]$

25. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is nonmagnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct? **[Main 9 Jan 2019, I]**
(a) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$ (b) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
(c) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$ (d) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$

26. The energy associated with electric field is (U_E) and with magnetic fields is (U_B) for an electromagnetic wave in free space. Then : **[Main 9 Jan 2019, II]**
(a) $U_E = \frac{U_B}{2}$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = U_B$

27. A monochromatic beam of light has a frequency $v = \frac{3}{2\pi}\times10^{12}$ Hz and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is: **[Main Online April 15, 2018]**
(a) $k\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cos\left[10^4\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right).\vec{r}-(3\times10^{12})t\right]$
(b) $\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}-(3\times10^{12})t\right]$
(c) $\frac{E_0}{C}\hat{k}\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}+(3\times10^{12})t\right]$
(d) $\frac{E_0}{C}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r}+(3\times10^{12})t\right]$

28. The electric field component of a monochromatic radiation is given by

$\vec{E} = 2E_0\,\hat{i}\,\cos kz \cos \omega t$

Its magnetic field $\vec{B}$ is then given by : **[Main Online April 9, 2017]**

(a) $\frac{2E_0}{c}\hat{j}\sin kz\cos\omega t$ (b) $-\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$

(c) $\frac{2E_0}{c}\hat{j}\sin kz\sin\omega t$ (d) $\frac{2E_0}{c}\hat{j}\cos kz\cos\omega t$

29. Consider an electromagnetic wave propagating in vacuum. Choose the correct statement : **[Main Online April 10, 2016]**

(a) For an electromagnetic wave propagating in +y direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{z}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}}B_z(x,t)\hat{y}$

(b) For an electromagnetic wave propagating in +y direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}}E_{yz}(x,t)\hat{y}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}}B_{yz}(x,t)\hat{z}$

(c) For an electromagnetic wave propagating in +x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}}E_{yz}(y,z,t)(\hat{y}+\hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}}B_{yz}(y,z,t)(\hat{y}+\hat{z})$

(d) For an electromagnetic wave propagating in +x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}}E_{yz}(x,t)(\hat{y}-\hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}}B_{yz}(x,t)(\hat{y}+\hat{z})$

30. For plane electromagnetic waves propagating in the z-direction, which one of the following combination gives the correct possible direction for $\vec{E}$ and $\vec{B}$ field respectively? **[Main Online April 11, 2015]**

(a) $(2\hat{i}+3\hat{j})$ and $(\hat{i}+2\hat{j})$ (b) $(-2\hat{i}-3\hat{j})$ and $(3\hat{i}-2\hat{j})$

(c) $(3\hat{i}+4\hat{j})$ and $(4\hat{i}-3\hat{j})$ (d) $(\hat{i}+2\hat{j})$ and $(2\hat{i}-\hat{j})$

31. During the propagation of electromagnetic waves in a medium: **[Main 2014]**

(a) Electric energy density is double of the magnetic energy density.

(b) Electric energy density is half of the magnetic energy density.

(c) Electric energy density is equal to the magnetic energy density.

(d) Both electric and magnetic energy densities are zero.

32. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly: **[Main Online April 12, 2014]**

(a) 1.34 V/m (b) 2.68 V/m

(c) 4.02 V/m (d) 5.36 V/m

33. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is : **[Main 2013]**

(a) 3 V/m (b) 6 V/m (c) 9 V/m (d) 12 V/m

34. A plane electromagnetic wave in a non-magnetic dielectric medium is given by $\vec{E} = \vec{E}_0(4\times10^{-7}x - 50t)$ with distance being in meter and time in seconds. The dielectric constant of the medium is : **[Main Online April 22, 2013]**

(a) 2.4 (b) 5.8 (c) 8.2 (d) 4.8

Topic-2 : Electromagnetic Spectrum

1 MCQs with One Correct Answer

1. Chosse the **correct** option relating wavelengths of different parts of electromagnetic wave spectrum : **[Main Sep. 04, 2020 (I)]**

(a) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{X-\text{rays}}$

(b) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{x-\text{rays}}$

(c) $\lambda_{x-\text{rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

(d) $\lambda_{\text{visible}} > \lambda_{x-\text{rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

2. Arrange the following electromagnetic radiations per quantum in the order of increasing energy : **[Main 2016]**

A : Blue light B : Yellow light
C : X-ray D : Radiowave.

(a) C, A, B, D (b) B, A, D, C

(c) D, B, A, C (d) A, B, D, C

3. Microwave oven acts on the principle of : **[Main Online April 9, 2016]**

(a) giving rotational energy to water molecules

(b) giving translational energy to water molecules

(c) giving vibrational energy to water molecules

(d) transferring electrons from lower to higher energy levels in water molecule

4. If microwaves, X rays, infrared, gamma rays, ultra-violet, radio waves and visible parts of the electromagnetic spectrum are denoted by M, X, I, G, U, R and V then which of the following is the arrangement in ascending order of wavelength ? **[Main Online April 19, 2014]**

(a) R, M, I, V, U, X and G (b) M, R, V, X, U, G and I

(c) G, X, U, V, I, M and R (d) I, M, R, U, V, X and G

5. Photons of an electromagnetic radiation has an energy 11 keV each. To which region of electromagnetic spectrum does it belong ? **[Main Online April 9, 2013]**

(a) X-ray region (b) Ultra violet region

(c) Infrared region (d) Visible region

7 Match the Following

6. The correct match between the entries in column I and column II are : **[Main Sep. 05, 2020 (II)]**

I Radiation		II Wavelength	
(A)	Microwave	(i)	100 m
(B)	Gamma rays	(ii)	10^{-15} m
(C)	A.M. radio waves	(iii)	10^{-10} m
(D)	X-rays	(iv)	10^{-3} m

(a) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
(b) (A)-(i), (B)-(iii), (C)-(iv), (D)-(ii)
(c) (A)-(iii), (B)-(ii), (C)-(i), (D)-(iv)
(d) (A)-(iv), (B)-(ii), (C)-(i), (D)-(iii)

7. Given below in the left column are different modes of communication using the kinds of waves given in the right column. **[Main 10 April 2019, I]**

A.	Optical Fibre Communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.
(a) A – Q, B – S, C – R, D – P
(b) A – S, B – Q, C – R, D – P
(c) A – Q, B – S, C – P, D – R
(d) A – R, B – P, C – S, D – Q

8. Match List - I (Electromagnetic wave type) with List - II (Its association/application) and select the correct option from the choices given below the lists: **[Main 2014]**

	List 1		List 2
1.	Infrared waves	(i)	To treat muscular strain
2.	Radio waves	(ii)	For broadcasting
3.	X-rays	(iii)	To detect fracture of bones
4.	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

	1	2	3	4
(a)	(iv)	(iii)	(ii)	(i)
(b)	(i)	(ii)	(iv)	(iii)
(c)	(iii)	(ii)	(i)	(iv)
(d)	(i)	(ii)	(iii)	(iv)

9. Match the List-I (Phenomenon associated with electromagnetic radiation) with List-II (Part of electromagnetic spectrum) and select the correct code from the choices given below this lists: **[Main Online April 11, 2014]**

	List I		List II
I	Doublet of sodium	(A)	Visible radiation
II	Wavelength corresponding to temperature associated with the isotropic radiation filling all space	(B)	Microwave
III	Wavelength emitted by atomic hydrogen in interstellar space	(C)	Short radio wave
IV	Wavelength of radiation arising from two close energy levels in hydrogen	(D)	X-rays

(a) (I)-(A), (II)-(B), (III)-(B), (IV)-(C)
(b) (I)-(A), (II)-(B), (III)-(C), (IV)-(C)
(c) (I)-(D), (II)-(C), (III)-(A), (IV)-(B)
(d) (I)-(B), (II)-(A), (III)-(D), (IV)-(A)

10. Match List I (Wavelength range of electromagnetic spectrum) with List II (Method of production of these waves) and select the correct option from the options given below the lists. **[Main Online April 9, 2014]**

	List I		List II
(1)	700 nm to 1 mm	(i)	Vibration of atoms and molecules.
(2)	1 nm to 400 nm	(ii)	Inner shell electrons in atoms moving from one energy level to a lower level.
(3)	$< 10^{-3}$ nm	(iii)	Radioactive decay of the nucleus.
(4)	1 mm to 0.1 m	(iv)	Magnetron valve.

(a) (1)-(iv), (2)-(iii), (3)-(ii), (4)-(i)
(b) (1)-(iii), (2)-(iv), (3)-(i), (4)-(ii)
(c) (1)-(ii), (2)-(iii), (3)-(iv), (4)-(i)
(d) (1)-(i), (2)-(ii), (3)-(iii), (4)-(iv)

Answer Key

Topic-1 : Electromagnetic Waves, Conduction and Displacement Current

1. (a)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (b)
11. (c)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (d)
21. (c)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (b)
31. (c)	**32.** (b)	**33.** (b)	**34.** (b)						

Topic-2 : Electromagnetic Spectrum

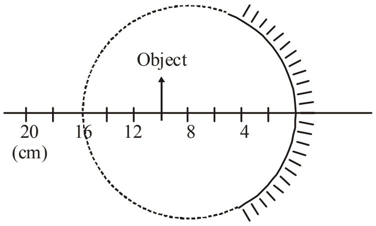
1. (b)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (d)

23 Ray Optics and Optical Instruments

Topic-1 : Plane Mirror, Spherical Mirror and Reflection of Light

1 MCQs with One Correct Answer

1.



A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale) **[Main Sep. 02, 2020 (I)]**

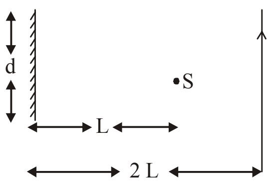
(a) Inverted, real and magnified
(b) Erect, virtual and magnified
(c) Erect, virtual and unmagnified
(d) Inverted, real and unmagnified

2. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is: **[Main 9 April 2019 I]**

(a) 0.24 m (b) 1.60 m (c) 0.32 m (d) 0.16 m

3. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2L as shown below. The distance over which the man can see the image of the light source in the mirror is: **[Main 12 Jan. 2019 I]**

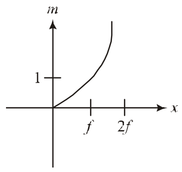
(a) d
(b) 2d
(c) 3d
(d) $\frac{d}{2}$

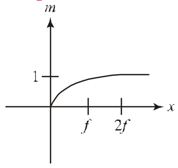


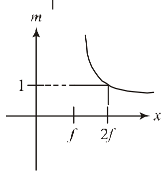
4. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be: **[Main 9 Jan. 2019 II]**

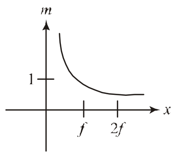
(a) 45° (b) 60° (c) 75° (d) 90°

5. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by
(Graphs are drawn schematically and are not to scale) **[Main 8 Jan. 2019 II]**

(a)


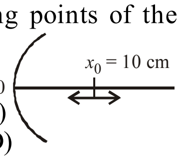
(b)


(c)


(d)


6. A particle is oscillating on the X-axis with an amplitude 2 cm about the point $x_0 = 10$ cm with a frequency ω. A concave mirror of focal length 5 cm is placed at the origin (see figure) Identify the correct statements: **[Main Online April 15, 2018]**

(A) The image executes periodic motion
(B) The image executes non-periodic motion
(C) The turning points of the image are asymmetric w.r.t the image of the point at $x = 10$ cm
(D) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$



(a) (B), (D) (b) (B), (C)
(c) (A), (C), (D) (d) (A), (D)

7. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be: **[Main Online April 10, 2015]**

(a) 60 cm (b) –24 cm (c) – 60 cm (d) 24 cm

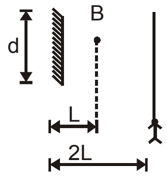
8. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. The angle of incidence is **[Adv. 2013]**

(a) 30° (c) 60° (b) 45° (d) 75°

9. In an experiment to determine the focal length (f) of a concave mirror by the u - v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **[2007]**

(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

10. A point source of light B is placed at a distance L in front of the centre of a mirror of width 'd' hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in fig. The greatest distance over which he can see the image of the light source in the mirror is **[2000S]**



(a) $d/2$ (b) d
(c) $2d$ (d) $3d$

11. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distanee u from the pole of the mirror. The size of the image is approximately equal to **[1988 - 2 Mark]**

(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$

(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

2 Integer Value Answer

12. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour? **[2010]**

3 Numeric Answer

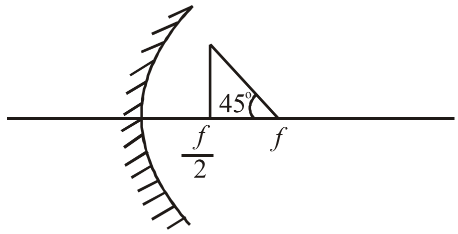
13. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is ________. **[Main Sep. 03, 2020 (II)]**

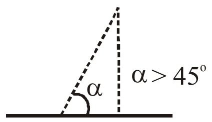
4 Fill in the Blanks

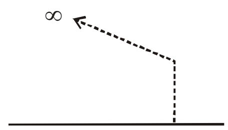
14. A thin rod of length $\frac{f}{3}$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is .... **[1991 - 1 Mark]**

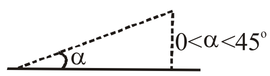
6 MCQs with One or More than One Correct Answer

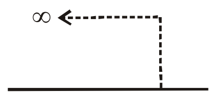
15. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.) **[Adv. 2018]**



(a)


(b)


(c)


(d)


16. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are :
(42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are) **[2009]**

(a) (42, 56) (b) (48, 48)
(c) (66, 33) (d) (78, 39)

9 Assertion and Reason Type Questions

17. **STATEMENT-1** The formula connecting u, v and f for a spherical mirror is valid for mirrors whose sizes are very small compared to their radii of curvature.

STATEMENT-2 Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces. **[2007]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

Topic-2 : Refraction of Light at Plane Surface and Total Internal Reflection

1 MCQs with One Correct Answer

1. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be: **[Main 9 Jan. 2020 I]**

(a) $\frac{h}{\sqrt{2}}$ (b) $\frac{h}{2(\sqrt{2}+1)}$

(c) $\frac{h}{3\sqrt{2}}$ (d) $\frac{3}{4}h\sqrt{2}$

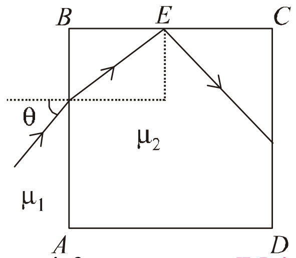
2. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly):
[Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$] **[Main 9 Jan. 2020 II]**

(a) 21% (b) 34% (c) 17% (d) 50%

3. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be: **[Main 8 Jan. 2020 I]**

(a) 15° (b) 30° (c) 45° (d) 60°

4. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.

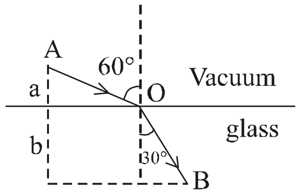


Then θ must satisfy : **[Main 12 Apr. 2019 II]**

(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

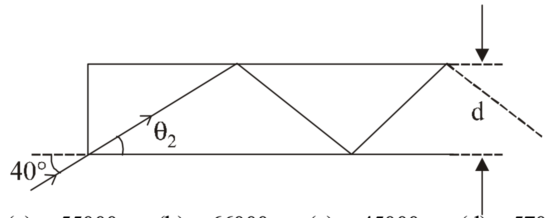
5. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is : **[Main 10 Apr. 2019 I]**



(a) $\frac{2\sqrt{3}}{a}+2b$ (b) $2a+\frac{2b}{3}$

(c) $2a+\frac{2b}{\sqrt{3}}$ (d) $2a+2b$

6. In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20$ μm. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to :
(refractive index of fiber is 1.31 and sin 40° = 0.64) **[Main 8 April 2019 I]**



(a) 55000 (b) 66000 (c) 45000 (d) 57000

7. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be: **[Main 12 Jan. 2019 I]**

(a) 30 V/m (b) 10 V/m (c) 24 V/m (d) 6 V/m

8. Let the refractive index of a denser medium with respect to a rarer medium be n_{12} and its critical angle be θ_C. At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A is given by : **[Main Online April 8, 2017]**

(a) $\frac{1}{\cos^{-1}(\sin\theta_C)}$ (b) $\frac{1}{\tan^{-1}(\sin\theta_C)}$

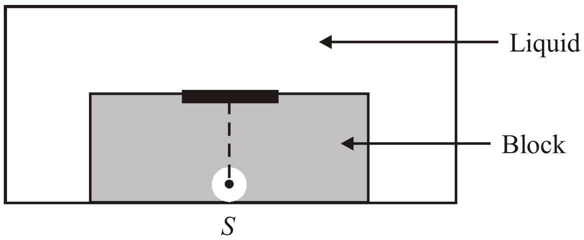
(c) $\cos^{-1}(\sin\theta_C)$ (d) $\tan^{-1}(\sin\theta_C)$

9. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$, and the diver's eyes are 15 cm below the surface of water. Then the radius of the circle is: **[Main Online April 9, 2014]**

(a) $15\times3\times\sqrt{5}$ cm (b) $15\times3\sqrt{7}$ cm

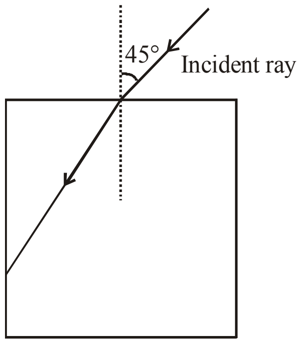
(c) $\frac{15\times\sqrt{7}}{3}$ cm (d) $\frac{15\times3}{\sqrt{7}}$ cm

10. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **[Adv. 2014]**



(a) 1.21 (b) 1.30 (c) 1.36 (d) 1.42

11. A light ray falls on a square glass slab as shown in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to : **[Main Online April 23, 2013]**



(a) $\frac{(\sqrt{2}+1)}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

12. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4.3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take g = 10 m/s^2.] **[2009]**

(a) 9 m/s (b) 12 m/s (c) 16 m/s (d) 21.33 m/s

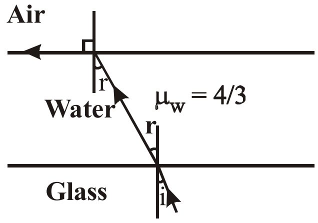
13. A ray of light traveling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be **[2007]**

(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflected ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

14. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is **[2004S]**

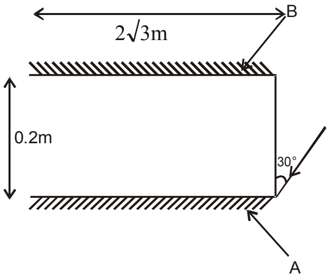
(a) 6 cm (b) 4 cm (c) 12 cm (d) 9 cm

15. A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be **[2003S]**



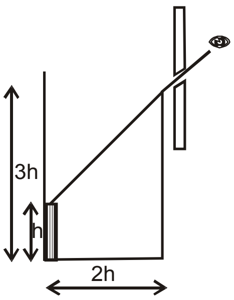
(a) (4/3)sin$i$ (b) 1/sin$i$ (c) 4/3 (d) 1

16. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **[2002S]**



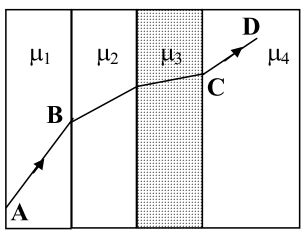
(a) 28 (b) 30 (c) 32 (d) 34

17. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is 3h and its radius h. When the beaker is filled with a liquid up to a height 2h, he can see the lower end of the rod. Then the refractive index of the liquid is **[2002S]**



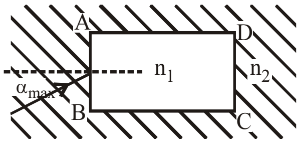
(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

18. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have **[2001S]**



(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$
(c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

19. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2(n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max} such that the ray comes out only from the other surface CD is given by **[2000S]**

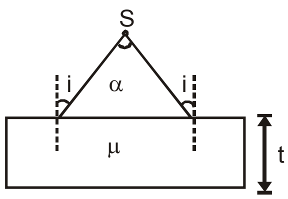


(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$

(b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$

(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$ (d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

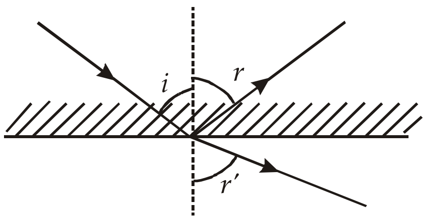
20. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is **[2000S]**



(a) zero (b) α

(c) $\sin^{-1}\left(\frac{1}{n}\right)$ (d) $2\sin^{-1}\left(\frac{1}{n}\right)$

21. A ray of light from a denser medium strike a rarer medium at an angle of incidence i (see Fig). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r' The critical angle is **[1983 - 1 Mark]**



(a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\tan i)$

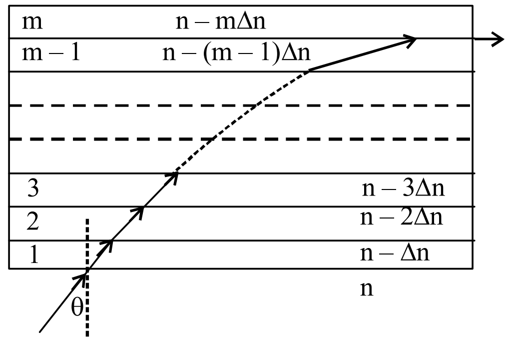
(c) $\sin^{-1}(\tan r')$ (d) $\tan^{-1}(\sin i)$

22. When a ray of light enters a glass slab from air, **[1980]**

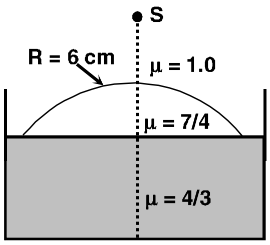
(a) its wavelength decreases.
(b) its wavelength increases.
(c) Its frequency decreases.
(d) neither its wavelength nor its frequency changes.

2 Integer Value Answer

23. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m? **[Adv. 2017]**



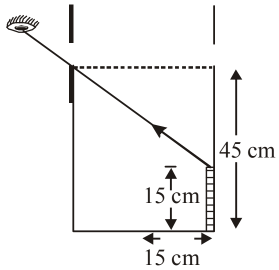
24. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is **[2011]**



25. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **[2010]**

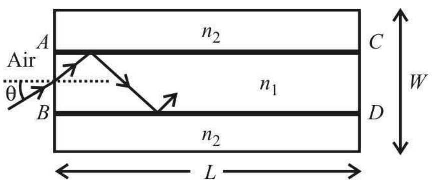
3 Numeric Answer

26. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, the value of N is ___________. **[Main Sep. 03, 2020 (I)]**



27. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is ____________. **[Main Sep. 02, 2020 (II)]**

28. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L >> W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6m$, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}s$, where t is _____ **[Adv. 2019]**



4 Fill in the Blanks

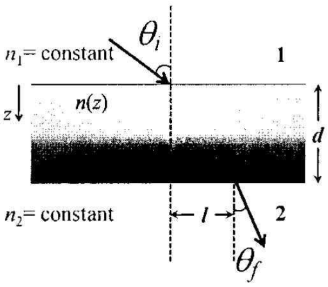
29. A light of wavelength 6000Å in air, enters a medium with refractive index 1.5 Inside the medium its frequency is .... Hz and its wavelength is .... Å. **[1997 - 2 Marks]**

30. A monochromatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ....., its frequency is ...... **[1985 - 2 Marks]**

31. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is .... and its wavelength is ....**[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

32. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i, from medium 1 and emerges in medium 2 with refraction angle θ_t with a lateral displacement l. **[Adv. 2016]**



Which of the following statement(s) is(are) true?

(a) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(b) $n_1 \sin\theta_i = (n_2 - n_1) \sin\theta_f$

(c) l is independent of n_2

(d) l is dependent on n(z)

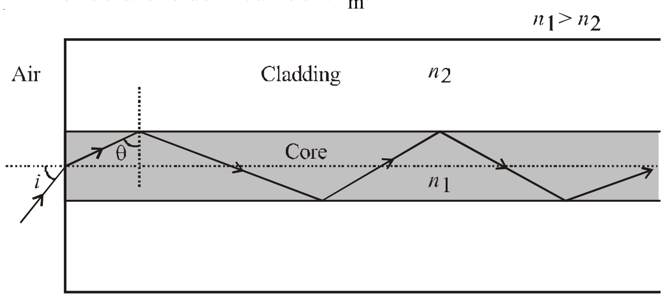
33. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following : **[1998 - 2 Marks]**

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

8 Comprehension/Passage Based Questions

Passage

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



34. For two structure namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are) **[Adv. 2015]**

(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

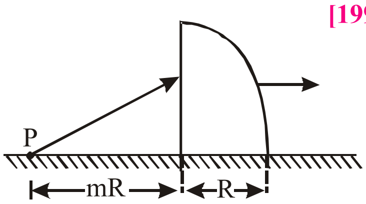
35. If two structure of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is **[Adv. 2015]**

(a) $\frac{NA_1\, NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

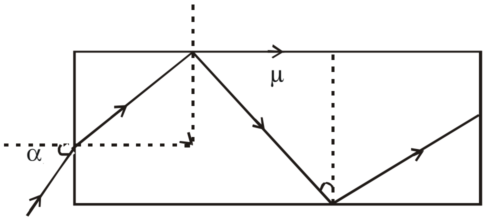
10 Subjective Problems

36. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in Figure. **[1999 - 5 Marks]**



37. The $x-y$ plane is the boundary between two transparent media. Medium –1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium –2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium –1 given by the vector $A = 6\sqrt{3}i + 8\sqrt{3}\,j - 10\,\mathbf{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium –2. **[1999 - 10 Marks]**

38. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index μ. Determine the least value of μ so that the light entering the rod does not emerge from the curved surface of rod irrespective of the value of α **[1992 - 8 Marks]**

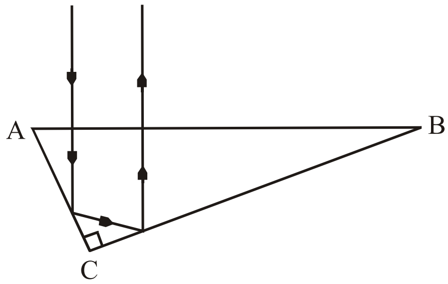


39. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial **[1988 - 6 Marks]**

(i) Find the position of the image due to refraction at the first surface and the position of the final image.

(ii) Draw a ray diagram showing the positions of both the images.

40. A right prism is to be made by selecting a proper material and the angles A and B $(B \leq A)$, as shown in Figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections.



(i) What should be the minimum refractive index n for this to be possible ?

(ii) For $n = \dfrac{5}{3}$ is it possible to achieve this with the angle B equal to 30 degrees ? **[1987 - 7 Marks]**

Topic-3 : Refraction at Curved Surface Lenses and Power of Lens

1 MCQs with One Correct Answer

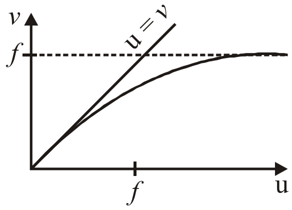
1. A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is : **[Main Sep. 06, 2020 (I)]**

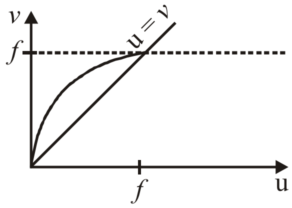
(a) 2.6 m from the mirror, real
(b) 1 m from the mirror, virtual
(c) 1 m from the mirror, real
(d) 2.6 m from the mirror, virtual

2. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is : **[Main Sep. 06, 2020 (II)]**

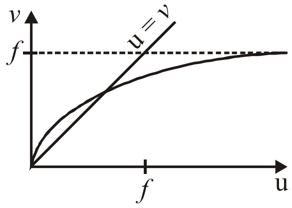
(a) $2R$ (b) $\dfrac{R}{2}$ (c) $\dfrac{3R}{2}$ (d) $\dfrac{R}{3}$

3. For a concave lens of focal length f, the relation between object and image distances u and v, respectively, from its pole can best be represented by ($u = v$ is the reference line) : **[Main Sep. 05, 2020 (I)]**

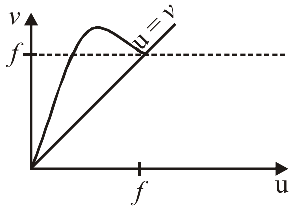
(a)


(b)


(c)



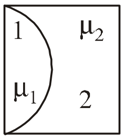
(d)



4. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_l, then the ratio f_l/f is closest to the integer: **[Main 7 Jan. 2020 II]**

(a) 1 (b) 9 (c) 5 (d) 17

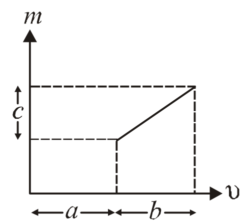
5. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is : **[Main 10 Apr. 2019 I]**



(a) $\frac{R}{\mu_1-\mu_2}$ (b) $\frac{2R}{\mu_1-\mu_2}$

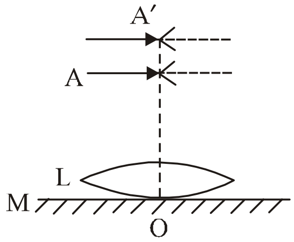
(c) $\frac{2R}{2(\mu_1-\mu_2)}$ (d) $\frac{R}{2-(\mu_1-\mu_2)}$

6. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used ? **[Main 10 Apr. 2019 II]**



(a) $\frac{b^2}{ac}$ (b) $\frac{b^2c}{a}$ (c) $\frac{a}{c}$ (d) $\frac{b}{c}$

7. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_i will be: **[Main 9 Apr. 2019 II]**



(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

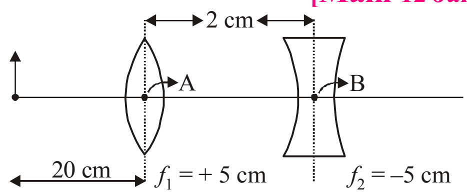
8. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be : **[Main 8 April 2019 I]**

(a) 20 cm from the convergent mirror, same size as the object
(b) 40 cm from the convergent mirror, same size as the object
(c) 40 cm from the convergent lens, twice the size of the object
(d) 20 cm from the convergent mirror, twice the size of the object

9. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be : **[Main 8 Apr. 2019 II]**

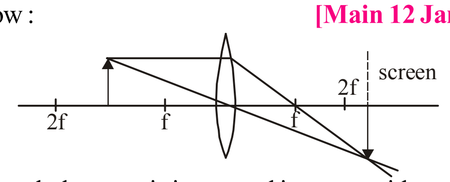
(a) 30 cm (b) 25 cm (c) 10 cm (d) 20 cm

10. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths) **[Main 12 Jan. 2019 I]**



(a) 70 cm from point B at left; virtual
(b) 40 cm from point B at right; real
(c) $\frac{20}{3}$ cm from point B at right, real
(d) 70 cm from point B at right; real

11. Formation of real image using a biconvex lens is shown below : **[Main 12 Jan. 2019 II]**



If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen ?

(a) Image disappears (b) Magnified image
(c) Erect real image (d) No change

12. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5m/s, the speed and direction of the image will be: **[Main 11 Jan. 2019 I]**
(a) 2.26×10^{-3} m/s away from the lens
(b) 0.92×10^{-3} m/s away from the lens
(c) 3.22×10^{-3} m/s towards the lens
(d) 1.16×10^{-3} m/s towards the lens

13. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2 If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as: **[Main 10 Jan. 2019 I]**
(a) $\mu_1 + \mu_2 = 3$ (b) $2\mu_1 - \mu_2 = 1$
(c) $3\mu_2 - 2\mu_1 = 1$ (d) $2\mu_2 - \mu_1 = 1$

14. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **[Main 10 Jan. 2019 II]**
(a) 1 cm (b) 2 cm (c) 4.0 cm (d) 3.1 cm

15. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screem is shifted by a distance d. Then d is: **[Main 9 Jan. 2019 I]**
(a) 1.1 cm away from the lens
(b) 0
(c) 0.55 cm towards the lens
(d) 0.55 cm away from the lens

16. A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A. If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig. B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens if: **[Main Online April 15, 2018]**

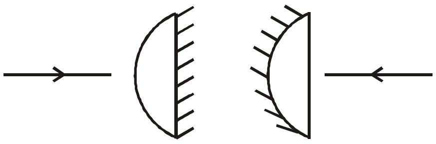
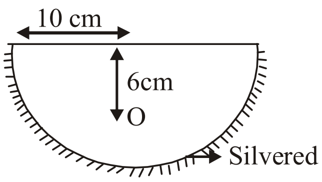


Fig. A Fig. B

(a) 1.50 (b) 1.55 (c) 1.75 (d) 1.51

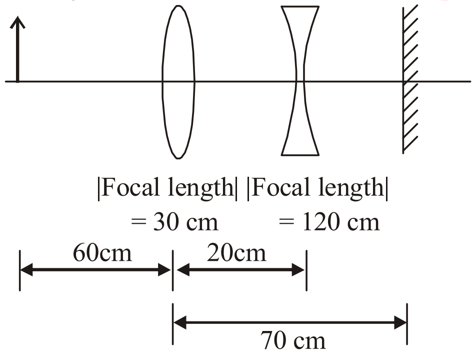
17. In an experiment a convex lens of focal length 15 cm is placed coaxially on an optical bench in front of a convex mirror at a distance of 5 cm from it. It is found that an object and its image coincide, if the object is placed at a distance of 20 cm from the lens. The focal length of the convex mirror is : **[Main Online April 9, 2017]**
(a) 27.5 cm (b) 20.0 cm (c) 25.0 cm (d) 30.5 cm

18. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen : **[Main Online April 10, 2016]**



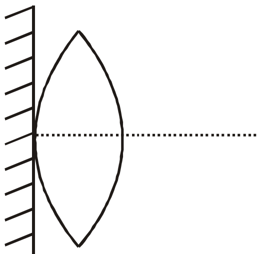
(a) 14 cm below flat surface
(b) 20 cm below flat surface
(c) 16 cm below flat surface
(d) 30 cm below flat surface

19. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and a plane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image, formed by the combination, is a real image, at a distance of : **[Main Online April 9, 2016]**



(a) 60 cm from the convex lens
(b) 60 cm from the concave lens
(c) 70 cm from the convex lens
(d) 70 cm from the concave lens

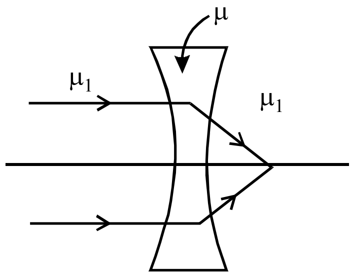
20. A thin convex lens of focal length 'f' is put on a plane mirror as shown in the figure. When an object is kept at a distance 'a' from the lens - mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of 'a' is: **[Main Online April 11, 2015]**

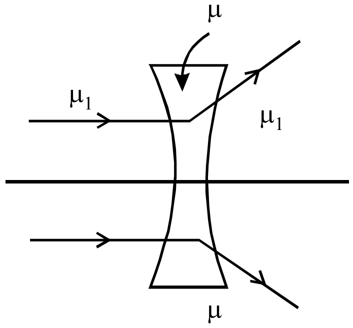


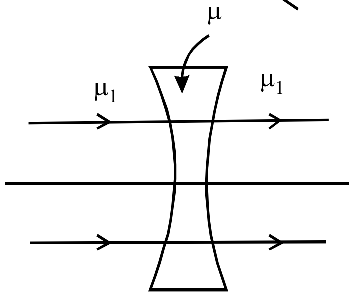
(a) 3f (b) $\frac{3}{2}$f (c) f (d) 2f

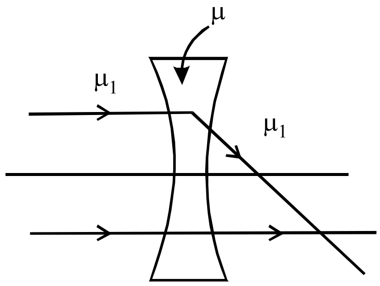
21. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is: **[Main 2014]**
(a) $f_1 = f_2 < f$
(b) $f_1 > f$ and f_2 becomes negative
(c) $f_2 > f$ and f_1 becomes negative
(d) f_1 and f_2 both become negative

22. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is: **[Main Online April 12, 2014]**

(a)


(b)


(c)


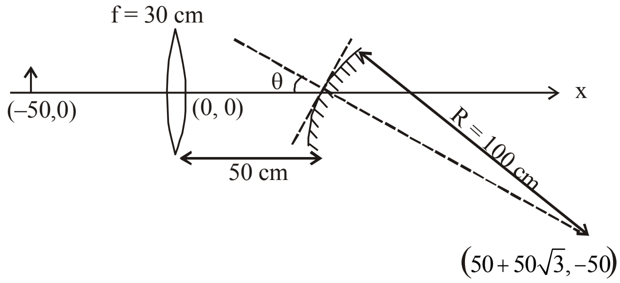
(d)


23. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[Main 2013]**
(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

24. The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is: **[Main Online April 22, 2013]**
(a) 36 cm (b) 27 cm (c) 60 cm (d) 30 cm

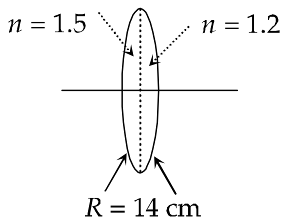
25. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is **[Adv. 2013]**
(a) 1m (b) 2m (c) 3m (d) 6m

26. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are **[Adv. 2016]**
(a) (0, 0) (b) $(50-25\sqrt{3}, 25)$
(c) $(25, 25\sqrt{3})$ (d) $(125/3, 25\sqrt{3})$

27. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be **[2012]**

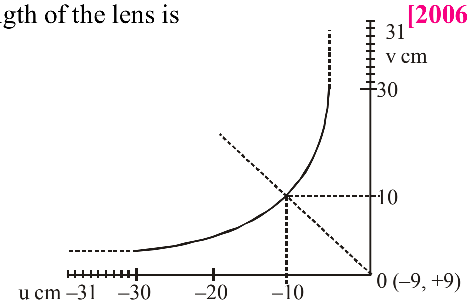


(a) -280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

28. Rays of light from Sun falls on a biconvex lens of focal length f and the circular image of Sun of radius r is formed on the focal plane of the lens. Then **[2007]**
(a) Area of image is πr^2 and area is directly proportional of f
(b) Area of image is πr^2 and area is directly proportional to f^2
(c) Intensity of image increases if f is increased
(d) If lower half of the lens is covered with black paper area will become half

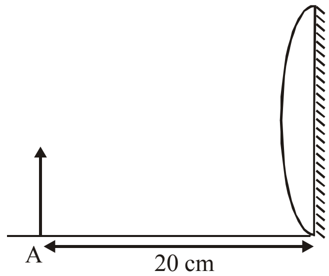
29. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **[2010]**
(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

30. The graph shows relationship between object distance and image distance for a equiconvex lens. Then, focal length of the lens is **[2006 - 3M, –]**



(a) 0.50 ± 0.05 cm (b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm (d) 5.00 ± 0.10 cm

31. Focal length of the plano-convex lens is 15 cm. A small object is placed at A as shown in the figure. The plane surface is silvered. The image will form at **[2006 - 3M, –1]**



(a) 60 cm to the left of lens
(b) 12 cm to the left of lens
(c) 60 cm to the right of lens
(d) 30 cm to the left of lens

32. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths? **[2005S]**
(a) –15, 10 (b) –10, 15 (c) 75, 50 (d) –75, 50

33. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **[2003S]**
(a) 1.25 cm (b) 2.5 cm (c) 1.05 cm (d) 2 cm

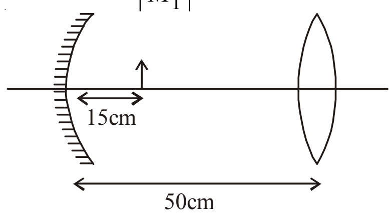
34. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with **[2000S]**
(a) air and placed in air (b) air and immersed in L_1
(c) L_1 and immersed in L_2 (d) L_2 and immersed in L_1

35. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a **[1999S - 2 Marks]**
(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

36. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing **[1995S]**
(a) a concave mirror of suitable focal length
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

37. Spherical aberration in a thin lens can be reduced by **[1994 - 1 Mark]**
(a) using a monochromatic light
(b) using a doublet combination
(c) using a circular annular mark over the lens
(d) increasing the size of the lens.

38. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



[Adv. 2015]

2 Integer Value Answer

39. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is **[2010]**

3 Numeric Answer

40. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)D$ where N is an integer, the value of N is ___________. **[Main Sep. 04, 2020 (II)]**

41. A point object in air is in front of the curved surface of a *plano-convex* lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is___________. **[Main 8 Jan. 2020 I]**

4 Fill in the Blanks

42. Two thin lenses, when in contact, produce a combination of power +10 diopters. When they are 0.25 m apart, the power reduces to +6 diopters. The focal length of the lenses are .... *m* and ... *m*. **[1997 - 2 Marks]**

43. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index $\frac{4}{3}$, its focal length will become ....cm. **[1987 - 2 Marks]**

44. A convex lens *A* of focal length 20 cm and a concave lens *B* of focal length 5 cm are kept along the same axis with a distance *d* between them. If a parallel beam of light falling on *A* leaves *B* as a parallel beam, then *d* is equal to ...... cm. **[1985 - 2 Marks]**

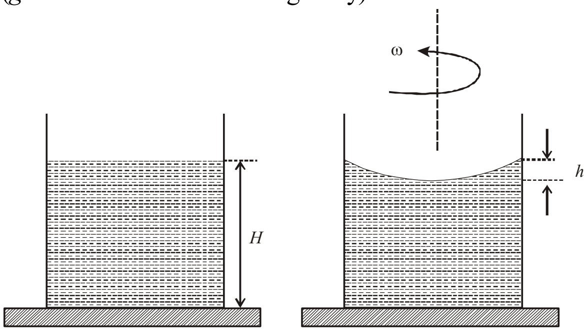
5 True / False

45. A parallel beam of white light fall on a combination of a concave and a convex lens, both of the same meterial. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet colour at the outer edge. **[1988 - 2 Marks]**

46. A convex lens of focal length 1 meter and a concave lens of focal length 0.25 meter are kept 0.75 meter apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

47. A beaker of radius *r* is filled with water (refractive index $\frac{4}{3}$) up to a height *H* as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is $h(h \ll H,\ h \ll r)$, as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature *R*. Which of the following is/are correct? (*g* is the acceleration due to gravity)



(a) $R = \dfrac{h^2 + r^2}{2h}$

(b) $R = \dfrac{3r^2}{2h}$

(c) Apparent depth of the bottom of the beaker is close to $\dfrac{3H}{2}\left(1+\dfrac{\omega^2 H}{2g}\right)^{-1}$

(d) Apparent depth of the bottom of the beaker is close to $\dfrac{3H}{4}\left(1+\dfrac{\omega^2 H}{4g}\right)^{-1}$

48. A plano–convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? **[Adv. 2016]**
(a) The refractive index of the lens is 2.5
(b) The radius of curvature of the convex surface is 45 cm
(c) The faint image is erect and real
(d) The focal length of the lens is 20 cm

49. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image *Q* in the glass. The line *PQ* cuts the surface at a point *O*, and *PO* = *OQ*. The distance *PO* is equal to **[1998 - 2 Marks]**
(a) 5*R* (b) 3*R* (c) 2*R* (d) 1.5*R*

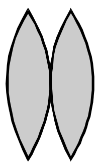
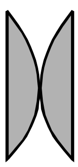
50. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration **[1998 - 2 Marks]**
(a) is absent.
(b) is smaller if the curved surface of the lens faces the object.
(c) is smaller if the plane surface of the lens faces the object.
(d) is the same whichever side of the lens faces the object

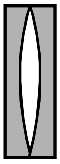
51. Which of the following form(s) a virtual and erect image for all positions of the object ? **[1996 - 2 Marks]**
(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror.

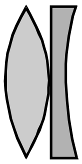
52. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen **[1986 - 2 Marks]**
(a) half the image will disappear.
(b) complete image will be formed.
(c) intensity of the image will increase.
(d) intensity of the image will decrease.

53. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm . The power of the combination is **[1982 - 3 Marks]**
(a) – 1.5 dioptres (b) – 6.5 dioptres
(c) + 6.5 dioptres (d) + 6.67 dioptres

54. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is *r* and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

List - I		List - II	
P.		**1.**	$2r$
Q.		**2.**	$\frac{r}{2}$

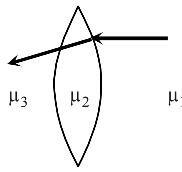
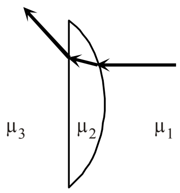
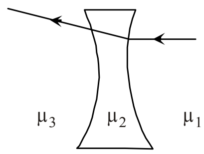
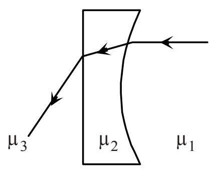
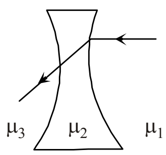
R.  3. $-r$

S.  4. r

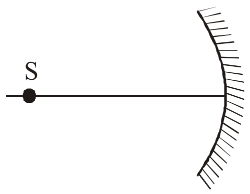
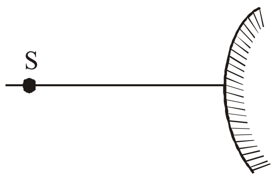
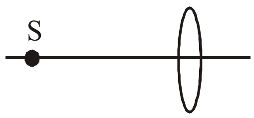
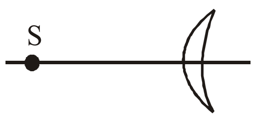
Codes:

(a) P-1, Q-2, R-3, S-4 (b) P-2, Q-4, R-3, S-1
(c) P-4, Q-1, R-2, S-3 (d) P-2, Q-1, R-3, S-4

55. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2, and μ_3 are given. Match them to the ray diagrams shown in **Column II.** **[2010]**

Column I	**Column II**
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 
(D) $\mu_2 > \mu_3$	(s) 
	(t) 

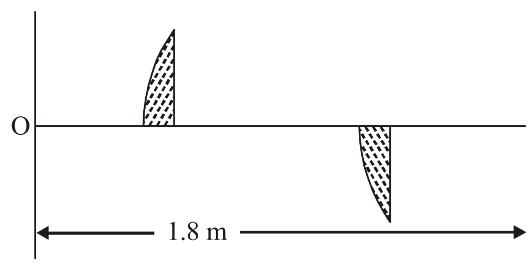
56. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[2008]**

Column I	**Column II**
(A) 	(p) real image
(B) 	(q) virtual image
(C) 	(r) magnified image
(D) 	(s) image at infinity

10 Subjective Problems

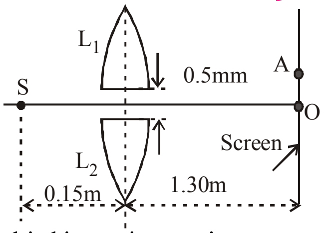
57. An object is moving with velocity 0.01 m/s towards a convex lens of focal length 0.3 m. Find the magnitude of rate of separation of image from the lens when the object is at a distance of 0.4 m from the lens. Also calculate the magnitude of the rate of change of the lateral magnification. **[2004 - 4 Marks]**

58. A thin plano-convex lens of focal length f is split into two halves: one of the halves is shifted along the optical axis . The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half-lenses is 2. Find the focal-length of the lens and separation between the two halves. Draw the ray diagram for image formation. **[1996 - 5 Marks]**



59. An image Y is formed of point object X by a lens whose optic axis is AB as shown in figure. Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (Having the same axis as AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. **[1994 - 6 Marks]**

60. In Fig., S is a monochromatic point source emitting light of wavelength $\lambda = 500\text{nm}$. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO. **[1993 - 5+1 Marks]**

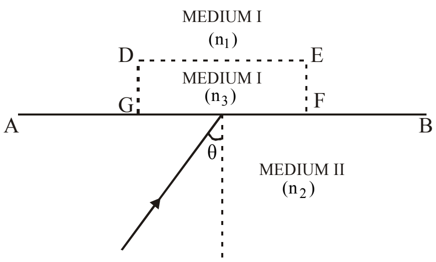


(i) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5mm, will the distance OA increase, decrease, or remain the same?

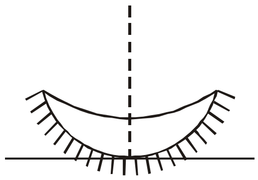
61. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 $(n_2 > n_1)$ at an angle of incidence θ as shown in fig. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II. Consider separately the cases **[1986 - 6 Marks]**

(i) $n_3 < n_1$ and (ii) $n_3 > n_1$.



62. A plano convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. **[1984- 6 Marks]**

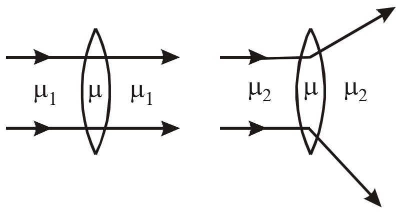
63. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. **[1981- 6 Marks]**



(i) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(ii) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.

64. What is the relation between the refractive indices μ_1 and μ_2, if the behaviour of light rays is as shown in the figure? **[1979]**

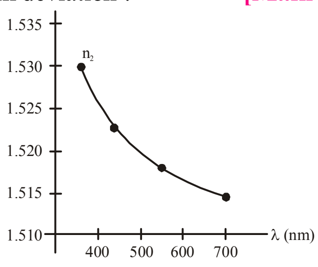


65. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made a material of refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature are of 22 cm. Determine the position of the final image. Is the image real as virtual? **[1978]**

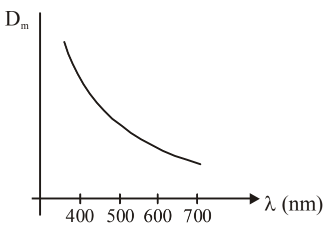
Topic-4 : Prism and Dispersion of Light

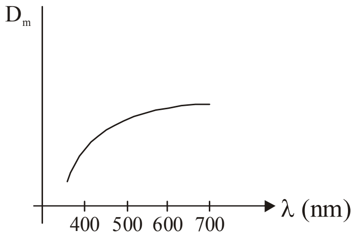
1 MCQs with One Correct Answer

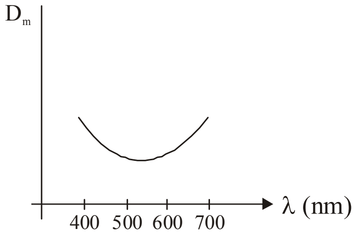
1. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation? **[Main 11 Jan. 2019, I]**

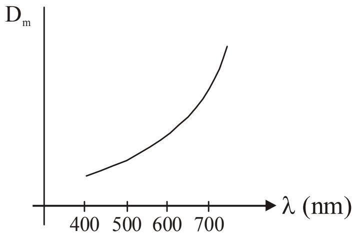


(a)



(b) 

(c) 

(d) 

2. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is : **[Main 11 Jan. 2019 II]**

(a) 90° (b) 30° (c) 60° (d) 45°

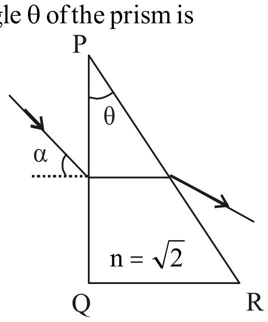
3. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be: **[Main Online April 16, 2018]**

(a) 30° (b) 90° (c) 0° (d) 45°

4. In an experiment for determination of refractive index of glass of a prism by $i-\delta$, plot it was found thata ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index? **[Main 2016]**

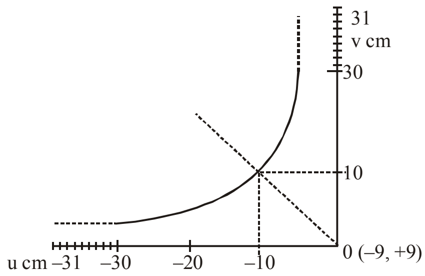
(a) 1.7 (b) 1.8 (c) 1.5 (d) 1.6

5. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is **[Adv. 2016]**



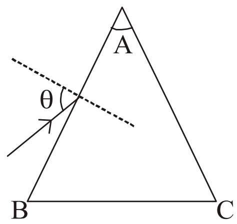
(a) 15° (b) 22.5° (c) 30° (d) 45°

6. The graph shows relationship between object distance and image distance for a equiconvex lens. Then, focal length of the lens is **(2006 - 3M, –1)**



(a) 0.50 ± 0.05 cm (b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm (d) 5.00 ± 0.10 cm

7. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided : **[Main 2015]**



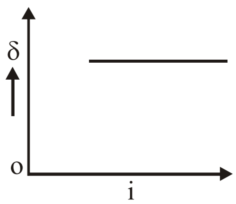
(a) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

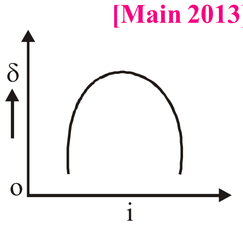
(b) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

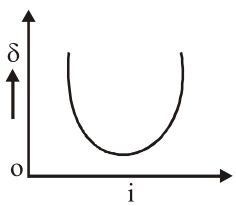
(c) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

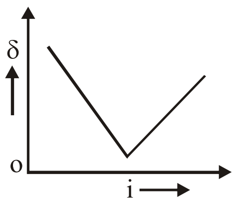
(d) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

8. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[Main 2013]**

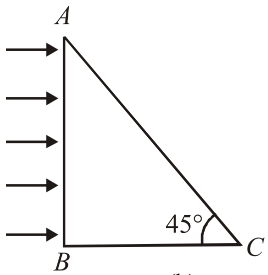
(a) 

(b) 

(c) 

(d) 

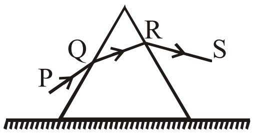
9. A beam of light consisting of red, green and blue colours is incident on a right-angled prism on face AB. The refractive indices of the material for the above red, green and blue colours are 1.39, 1.44 and 1.47 respectively. A person looking on surface AC of the prism will see **[Main Online May 26, 2012]**



(a) no light (b) green and blue colours
(c) red and green colours (d) red colour only

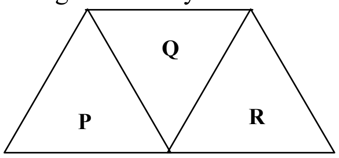
10. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **[2008]**
(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

11. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation **[2004S]**



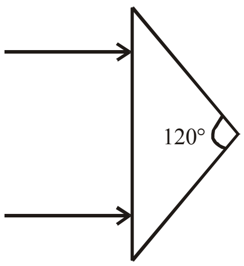
(a) PQ is horizontal (b) QR is horizontal
(c) RS is horizontal (d) Any one will be horizontal

12. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will now suffer **[2001S]**



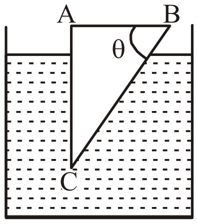
(a) greater deviation (b) no deviation
(c) same deviation as before (d) total internal reflection

13. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel monochromatic rays enter the prism parallel to each other in air as shown. The rays emerge from the opposite faces **[1995S]**



(a) are parallel to each other
(b) are diverging
(c) make an angle $2[\sin^{-1}(0.72) - 30°]$ with each other
(d) make an angle $2\sin^{-1}(0.72)$ with each other

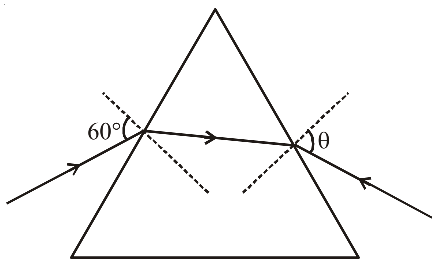
14. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam incident normally on the face AB is totally reflected to reach on the face BC if **[1981- 2 Marks]**



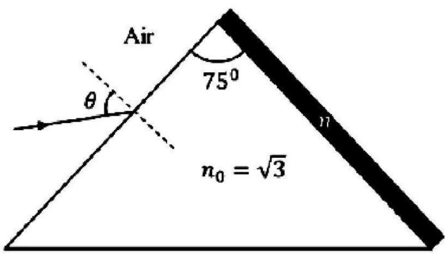
(a) $\sin\theta > \frac{8}{9}$
(b) $\frac{2}{3} < \sin\theta < \frac{8}{9}$
(c) $\sin\theta < \frac{2}{3}$
(d) None of these

2 Integer Value Answer

15. The monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **[JEE Adv. 2015]**



16. A monochromatic light is incident from air on a refracting surface of a prism of angle 75^0 and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60^0$. The value of n^2 is **[Adv. 2019]**



4 Fill in the Blanks

17. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is... degrees. **[1997 - 2 Marks]**

18. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is .... **[1992 - 1 Mark]**

5 True / False

19. A beam of white light passing through a hollow prism give no spectrum. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

20. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. **[Adv. 2017]**

Which of the following options is/are correct?

(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(b) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(c) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

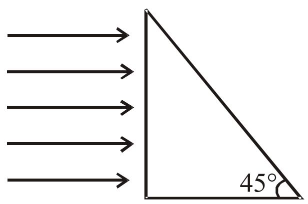
(d) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

21. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is **[1990 - 2 Marks]**

(a) 5.33° (b) 4° (c) 3° (d) 2.6°

22. A beam of light consisting of red, green and blue colours is incident on a right angled prism, fig. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will **[1989 - 2 Mark]**



(a) separate part of the red colour from the green and blue colours

(b) separate part of the blue colour from the red and green colours

(c) separate all the three colours from one another

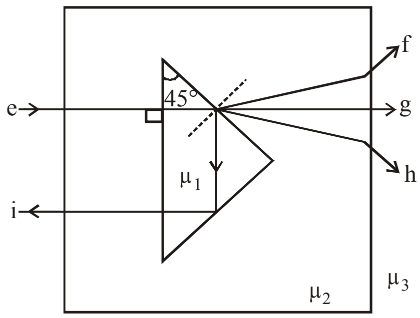
(d) not separate even partially any colour from the other two colours

7 Match the Following

23. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'.

Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists: **[Adv. 2013]**

	List I		**List II**
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

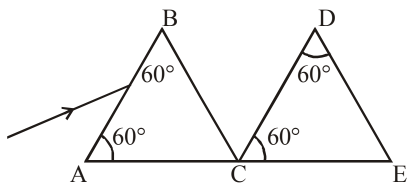


Codes:

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

10 Subjective Problems

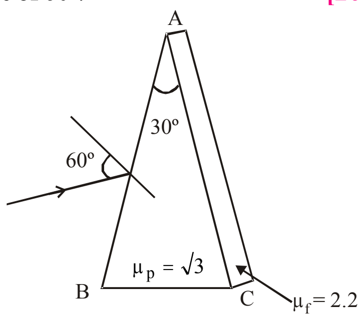
24. Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in the figure. A light ray strikes the first prism at face AB. Find, **[2005 - 4 Marks]**



(a) the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

(b) through what angle the prism DCE should be rotated about C so that the final emergent ray also has minimum deviation.

25. Shown in the figure is a prism of angle 30° and refractive index $\mu_p = \sqrt{3}$. Face AC of the prism is covered with a thin film of refractive index $\mu_f = 2.2$. A monochromatic light of wavelength $\lambda = 550$ nm fall on the face *AB* at an angle of incidence of 60°. **[2003 - 4 Marks]**

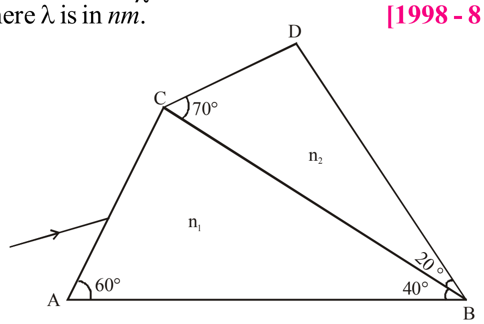


Calculate
(a) angle of emergence.
(b) minimum value of thickness *t* so that intensity of emergent ray is maximum.

26. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in Figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light, according to

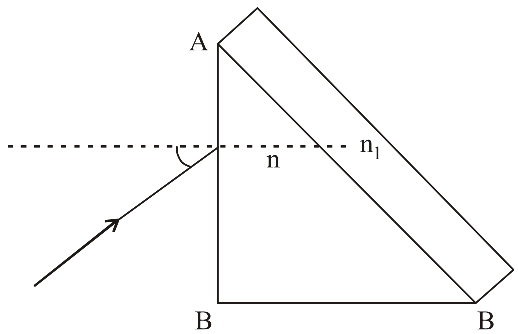
$$n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2} \quad \text{and} \quad n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$$

where λ is in *nm*. **[1998 - 8 Marks]**



(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface *BC* pass through without bending at that interface.
(b) For light of wavelength λ_0, find the angle of incidence i on the face *AC* such that the deviation produced by the combination of prisms is minimum.

27. A right angled prism (45° –90°–45°) of refractive index n has a plate of refractive index n_1 $(n_1 < n)$ cemented to its diagonal face. The assembly is in air. A ray is incident on *AB*.



(i) Calculate the angle of incidence at *AB* for which the ray strikes the diagonal face at the critical angle.
(ii) Assuming $n = 1.352$ calculate the angle of incidence at *AB* for which the refracted ray passes through the diagonal face undeviated. **[1996 - 3 Marks]**

28. A ray of light is incident at an angle of 60° on one face of prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism. **[1978]**

Topic-5 : Optical Instruments

1 MCQs with One Correct Answer

1. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eye piece? **[Main 8 Jan. 2020 I]**
(a) 20 cm (b) 40 cm (c) 30 cm (d) 10 cm

2. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to: **[Main 7 Jan. 2020 I]**
(a) 22mm (b) 12mm (c) 2mm (d) 33mm

3. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears : **[Main 2016]**
(a) 20 times taller (b) 20 times nearer
(c) 10 times taller (d) 10 times nearer

4. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are : **[Main Online April 10, 2016]**
(a) Two (b) Four (c) Three (d) Five

5. A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to : **[Main Online April 10, 2015]**
0 (a) 30° (b) 15° (c) 60° (d) 1°

6. In a compound microscope, the focal length of objective lens is 1.2 cm and focal length of eye piece is 3.0 cm. When object is kept at 1.25 cm in front of objective, final image is formed at infinity. Magnifying power of the compound microscope should be: **[Main Online April 11, 2014]**
(a) 200 (b) 100 (c) 400 (d) 150

7. In a compound microscope, the intermediate image is
(a) virtual, erect and magnified **[2000S]**
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

8. The focal lengths of the objective and the eye piece of a compound microscope are 2.0 cm and 3.0 cm, respectively. The distance between the objective and the eye piece is 15.0 cm. The final image formed by the eye piece is at infinity. The two lenses are thin. The distance in cm of the object and the image produced by the objective, measured from the objective lens, are respectively **[1995S]**

(a) 2.4 and 12.0 (b) 2.4 and 15.0
(c) 2.0 and 12.0 (d) 2.0 and 3.0

3 Numeric Answer

9. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm.
The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm.
The value of n is ______. **[Main Sep. 05, 2020 (I)]**

10. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is __________. **[Main Sep. 04, 2020 (I)]**

4 Fill in the Blanks

11. The resolving power of electron microscope is higher that that of an optical microscope because the wavelength of electrons is .... than the wavelength of visible light. **[1992 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

12. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm. **[1992 - 2 Marks]**

(a) The distance between the objective and the eyepiece is 16.02 m
(b) The angular magnification of the planet is – 800
(c) The image of the planet is inverted
(d) The objective is larger then the eyepiece

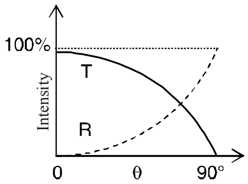
13. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and the focal length f_0 of the eyepiece are **[1989 - 2 Marks]**

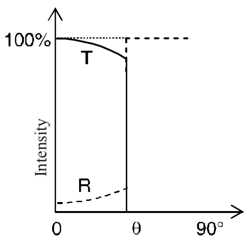
(a) $f_0 = 45$ cm and $f_e = -9$ cm
(b) $f_0 = 50$ cm and $f_e = 10$ cm
(c) $f_0 = 7.2$ cm and $f_e = 5$ cm
(d) $f_0 = 30$ cm and $f_e = 6$ cm.

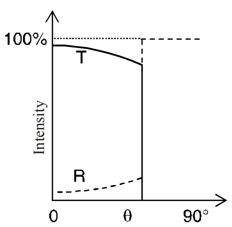
Topic-6 : Miscellaneous (Mixed Concepts) Problems

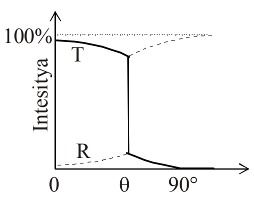
1 MCQs with One Correct Answer

1. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **[2011]**

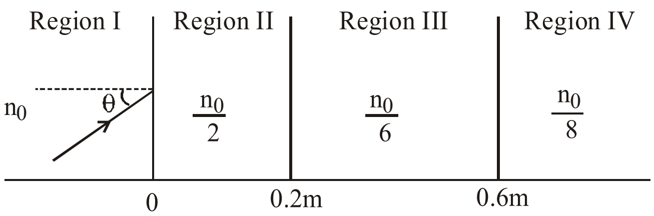
(a)


(b)


(c)


(d)


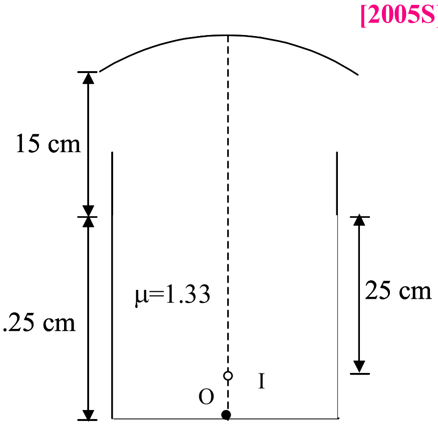
2. A light beam is travelling from Region I to IV (figure). The refractive index in regionals I, II, III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$ respectively. The angle of incidence θ for which the beam just misses entering region IV is – **[2008]**



(a) $\sin^{-1}(3/4)$ (b) $\sin^{-1}(1/8)$
(c) $\sin^{-1}(1/4)$ (d) $\sin^{-1}(1/3)$

3. A container is filled with water $(\mu = 1.33)$ upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is **[2005S]**

(a) 15 cm
(b) 20 cm
(c) –18.31 cm
(d) 10 cm



4. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is **[2004S]**

(a) 6 cm (b) 4 cm (c) 12 cm (d) 9 cm

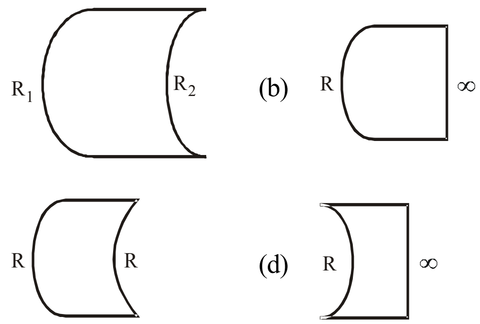
5. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m/s, velocity of sound in air = 300 m/s) **[2004S]**

(a) 3000 Hz (b) 120 Hz
(c) 600 Hz (d) 6000 Hz

6. A beam of white light is incident on glass air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are **[2004S]**

(a) Violet, Indigo, Blue
(b) All colors except green
(c) Yellow, Orange, Red
(d) White light

7. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams. **[2002S]**



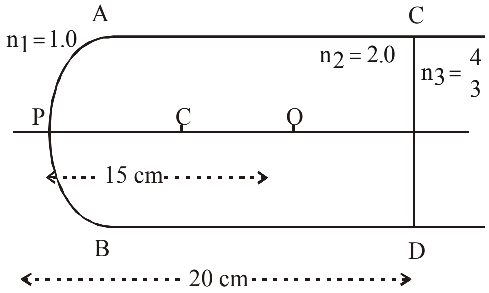
2 Integer Value Answer

8. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is ___________. **[Adv. 2018]**

4 Fill in the Blanks

9. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is .... **[1992 - 1 Mark]**

10. A slab of a material of refractive index 2 shown in fig. has a curved surface *APB* of radius of curvature 10 cm and a plane surface *CD*. On the left of *APB* is air and on the right of *CD* is water with refractive indices as given in the figure. An object *O* is placed at a distance of 15 cm from the pole *P* as shown. The distance of the final image of *O* from *P*, as viewed from the left is .... **[1991 - 2 Marks]**



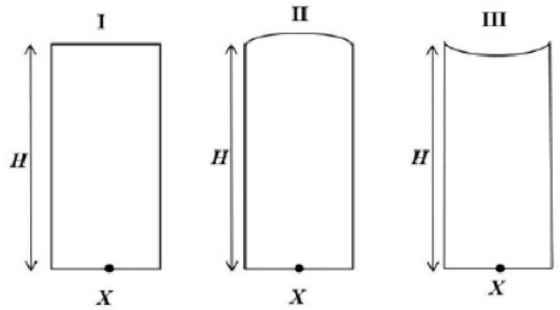
5 True / False

11. The intensity of light at a distance '*r*' from the axis of a long cylindrical source is inversely proportional to '*r*'. **[1981- 2 Marks]**

12. The setting sun appears higher in the sky than it really is. **[1980]**

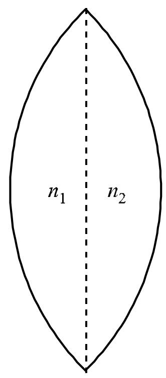
6 MCQs with One or More than One Correct Answer

13. Three glass cylinders of equal height H = 30 cm and same refractive index $n = 1.5$ are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3 m), If H_1, H_2, and H_3 are the apparent depths of a point *X* on the bottom of the three cylinders, respectively, the correct statement(s) is/are : **[Adv. 2019]**



(a) $0.8\text{ cm} < (H_2 = H_1) < 0.9\text{ cm}$
(b) $H_2 > H_1$
(c) $H_3 > H_1$
(d) $H_2 > H_3$

14. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surface are equal. *f* is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are. **[Adv. 2019]**



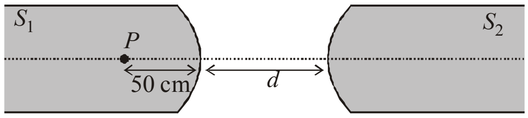
(a) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(b) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surface of the same radius of curvature.

(c) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

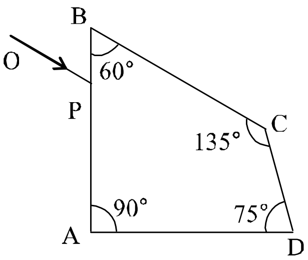
(d) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

15. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is **[Adv. 2015]**



(a) 60 cm (b) 70 cm (c) 80 cm (d) 90 cm

16. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **[2010]**

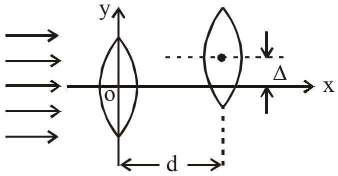


(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) The angle between the incident ray and the emergent ray is 120°

17. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be. **[1998 - 2 Marks]**

(a) real, and will remain at C.
(b) real, and located at a point between C and ∞ .
(c) virtual, and located at a point between C and O.
(d) real, and located at a point between C and O

18. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1$, $d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the fig. **[1993-2 Marks]**



Taking the origin of coordinates O, at the centre of the first lens the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by:

(a) $x = \frac{f_1 f_2}{f_1 + f_2}, y = \Delta$

(b) $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}, y = \frac{\Delta}{f_1 + f_2}$

(c) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(d) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = 0$

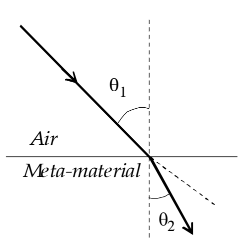
8 Comprehension/Passage Based Questions

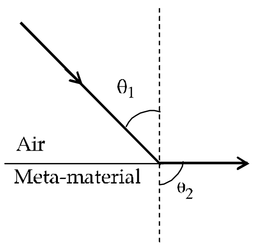
PASSAGE

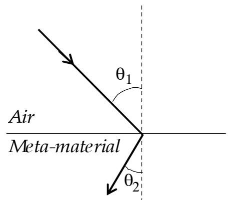
Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = c/v = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

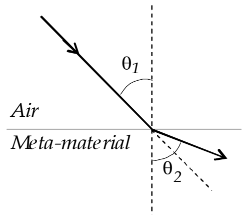
In normal materials, both ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[2012]**

19. For light incident from air on a meta-material, the appropriate ray diagram is

(a) 

(b) 

(c) 

(d) 

20. Choose the correct statement.

(a) The speed of light in the meta-material is $v = c|n|$

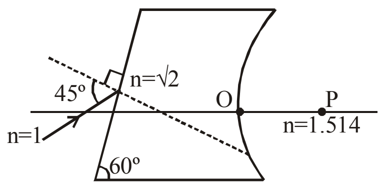
(b) The speed of light in the meta-material is $v = \dfrac{c}{|n|}$

(c) The speed of light in the meta-material is $v = c$.

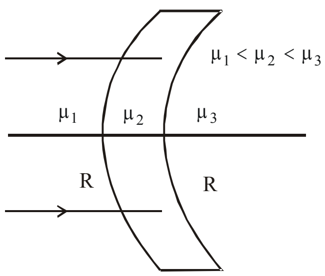
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}\,|n|$, where λ_{air} is wavelength of the light in air.

10 Subjective Problems

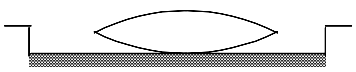
21. A ray is incident on a medium consisting of two boundaries, one plane and other curved as shown in the figure. The plane surface makes an angle 60º with horizontal and curved surface has radius of curvature 0.4 m. The refractive indices of the medium and its environment are shown in the figure. If after refraction at both the surfaces the ray meets principle axis at P, find OP. **[2004 - 2 Marks]**



22. Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R. **[2003-2 Marks]**

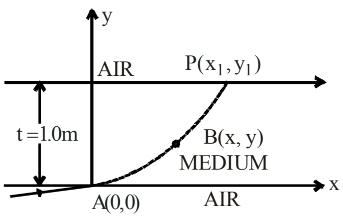


23. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. **[2001-5 Marks]**



24. The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively and those of flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6º is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. (a) Determine the angle of the flint glass prism. (b) Calculate the net dispersion of the combined system. **[2001 - 5 Marks]**

25. A ray of light travelling in air is incident at grazing angle (incident angle $\cong 90°$) on a long rectangular slab of a transparent medium of thickness t = 1.0 m (see figure below). The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction $n(y)$ given by



$$n(y) = [ky^{3/2} + 1]^{1/2},$$

where k = 1.0 (metre)$^{-3/2}$

The refractive index of air is 1.0. **[1995 - 10 Marks]**

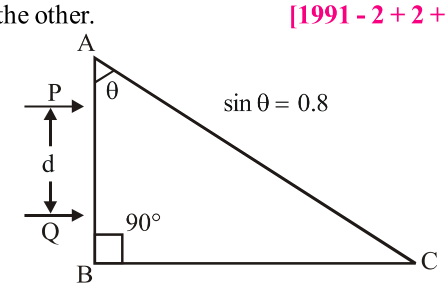
(a) Obtain a relation between the slope of the trajectory of the ray at a pint $B(x, y)$ in the medium and the incident angle at that point.

(b) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

26. Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in fig. The refractive index of the prism as a function of wavelength is given by the relation. $\mu(\lambda) = 1.20 + \dfrac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection of the face AC is just satisfied for one wave length and is not satisfied for the other. **[1991 - 2 + 2 + 4 Marks]**

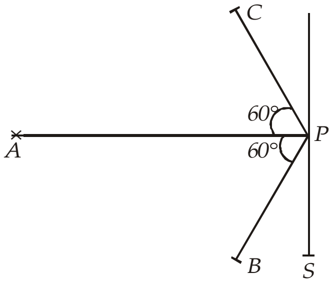


(a) Find the value of b.

(b) Find the deviation of the beams transmitted through the face AC

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of transmission form the face AC, are 41 and I respectively, find the resultant intensity at the focus.

27. Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen (see figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distance AP, BP and CP are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of sources A and B are 90 watts and 180 watts respectively. The beam from C is of intensity 20 watts/m2. Calculate the intensity at P on the screen. **[1982 - 5 Marks]**



28. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror.
Find the position of the final image formed. (You may take the distance of the near surface of the slab from the mirror to be 1 cm. **[1980]**

29. A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. **[1979]**

Answer Key

Topic-1 : Plane Mirror, Spherical Mirror and Reflection of Light

1. (d) **2.** (c) **3.** (c) **4.** (b) **5.** (c) **6.** (c) **7.** (c) **8.** (a) **9.** (b) **10.** (d)
11. (d) **12.** (3) **13.** (1) **14.** (1.5) **15.** (d) **16.** (c, d) **17.** (c)

Topic-2 : Refraction of Light at Plane Surface and Total Internal Reflection

1. (d) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (d) **7.** (c) **8.** (d) **9.** (d) **10.** (c)
11. (d) **12.** (c) **13.** (c) **14.** (a) **15.** (b) **16.** (b) **17.** (b) **18.** (d) **19.** (a) **20.** (b)
21. (a) **22.** (a) **23.** (8) **24.** (2) **25.** (6) **26.** (158) **27.** (90) **28.** (50) **29.** (4000 Å)
30. (4000 Å, 5×10^{14}Hz) **31.** (2×10^8m/s, 4×10^{-7}m) **32.** (a,c,d) **33.** (c, d) **34.** (a, c) **35.** (d)

Topic-3 : Refraction at Curved Surface Lenses and Power of Lens

1. (d) **2.** (d) **3.** (d) **4.** (b) **5.** (a) **6.** (d) **7.** (a) **8.** (Bouns) **9.** (c) **10.** (d)
11. (a) **12.** (d) **13.** (b) **14.** (d) **15.** (d) **16.** (b) **17.** (a) **18.** (b) **19.** (a) **20.** (d)
21. (b) **22.** (a) **23.** (c) **24.** (d) **25.** (c) **26.** (c) **27.** (b) **28.** (b) **29.** (b) **30.** (c)
31. (b) **32.** (a) **33.** (b) **34.** (d) **35.** (a) **36.** (c) **37.** (c) **38.** (7) **39.** (2) **40.** (476.19)
41. (60) **42.** (0.125m, 0.5m) **43.** (60cm) **44.** (15cm) **45.** True; **46.** False; **47.** (a,d) **48.** (a, d) **49.** (a)
50. (b) **51.** (b,c) **52.** (b,d) **53.** (a) **54.** (b) **55.** A-p, r; B-q,s,t; C-p,r,t, D-q,s
56. A-p, q, r, s; B-q; C-p, q, r, s; D-p, q, r, s

Topic-4 : Prism and Dispersion of Light

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (c) **7.** (c) **8.** (c) **9.** (d) **10.** (a)
11. (b) **12.** (c) **13.** (c) **14.** (a) **15.** (2) **16.** (1.5) **17.** (15°) **18.** (0°) **19.** True; **20.** (a, c, d)
21. (c) **22.** (a) **23.** (d)

Topic-5 : Optical Instruments

1. (d) **2.** (a) **3.** (b) **4.** (c) **5.** (c) **6.** (a) **7.** (c) **8.** (a) **9.** (50) **10.** (4.48)
11. (smaller) **12.** (a,b,c,d) **13** (d)

Topic-6 : Miscellaneous (Mixed Concepts) Problems

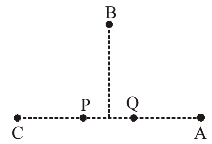
1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (130) **9.** $\left(\frac{\sqrt{\mu\varepsilon}}{\mu_0\varepsilon_0}\right)$
10. (–30cm) **11.** True; **12.** True; **13.** (b,d) **14.** (a, b, c) **15.** (b) **16.** (a, b, c) **17.** (d) **18.** (c) **19.** (c)
20. (b)

24 Wave Optics

Topic-1 : Wavefront, Interference of Light, Coherent and Incoherent Sources

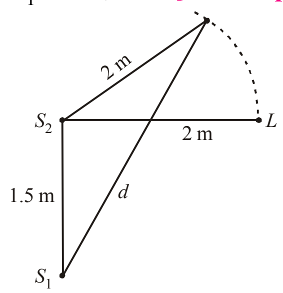
1 MCQs with One Correct Answer

1. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90º. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio : **[Main Sep. 06, 2020 (I)]**



(a) 0 : 1 : 4 (b) 2 : 1 : 0 (c) 0 : 1 : 2 (d) 4 : 1 : 0

2. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig.). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is : **[Main Sep. 05, 2020 (II)]**



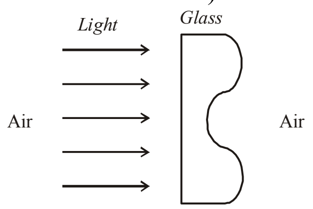
(a) 12m (b) 5m (c) 2m (d) 3m

3. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is : **[Main Sep. 03, 2020 (II)]**

(a) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1}-\frac{L_1}{n_2}\right)$ (b) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1}-\frac{L_2}{n_2}\right)$

(c) $\frac{2\pi}{\lambda}(n_1L_1-n_2L_2)$ (d) $\frac{2\pi}{\lambda}(n_2L_1-n_1L_2)$

4. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wavefront will be (figures are schematic and not drawn to scale) **[Adv 2020]**



(a)  (b)  (c)  (d) 

5. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2}=\frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be : **[Main 8 April 2019 I]**

(a) 2 (b) 18 (c) 4 (d) 9

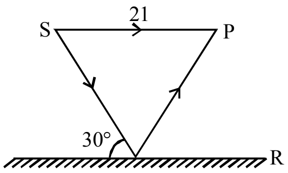
6. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: **[Main 9 Jan. 2019 I]**

(a) 16 : 9 (b) 25 : 9 (c) 4 : 1 (d) 5 : 3

7. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam : **[Main 2015]**

(a) bends downwards (b) bends upwards
(c) becomes narrower
(d) goes horizontally without any deflection

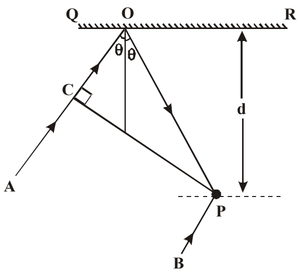
8. Interference pattern is observed at 'P' due to superimposition of two rays coming out from a source 'S' as shown in the figure. The value of 'l' for which maxima is obtained at 'P' is: **[Main Online April 12, 2014]**
(R is perfect reflecting surface)



(a) $l = \dfrac{2n\lambda}{\sqrt{3}-1}$ (b) $l = \dfrac{(2n-1)\lambda}{2(\sqrt{3}-1)}$

(c) $l = \dfrac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$ (d) $l = \dfrac{(2n-1)\lambda}{\sqrt{3}-1}$

9. In the adjacent diagram, *CP* represents a wavefront and *AO* & *BP*, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray *BP* and reflected ray *OP*. **[2003S]**



(a) $\cos\theta = 3\lambda/2d$

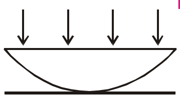
(b) $\cos\theta = \lambda/4d$

(c) $\sec\theta - \cos\theta = \lambda/d$

(d) $\sec\theta - \cos\theta = 4\lambda/d$

10. Two beams of light having intensities *I* and 4*I* interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point *A* and π at point *B*. Then the difference between the resultant intensities at *A* and *B* is **[2001S]**
(a) 2*I* (b) 4*I* (c) 5*I* (d) 7*I*

11. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown in Figure.
The observed interference fringes from this combination shall be **[1999S - 2 Marks]**

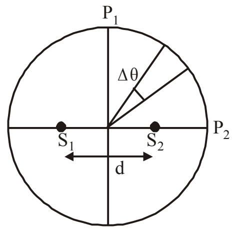


(a) straight
(b) circular
(c) equally spaced
(d) having fringe spacing which increases as we go outwards

12. Two coherent monochromatic light beams of intensities *I* and 4 *I* are superposed. The maximum and minimum possible intensities in the resulting beam are **[1988 - 1 Mark]**
(a) 5*I* and *I* (b) 5*I* and 3*I*
(c) 9*I* and *I* (d) 9*I* and 3*I*

6 MCQs with One or More than One Correct Answer

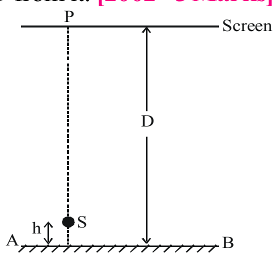
13. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct? **[Adv. 2017]**



(a) A dark spot will be formed at the point P_2
(b) At P_2 the order of the fringe will be maximum
(c) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000
(d) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

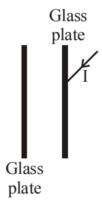
10 Subjective Problems

14. A point source *S* emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface *AB* (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance *D* from it. **[2002 - 5 Marks]**



(a) What is the shape of the interference fringes on the screen?
(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point *P* (shown in the figure).
(c) If the intensity at point *P* corresponds to a maximum, calculate the minimum distance through which the reflecting surface *AB* should be shifted so that the intensity at *P* again becomes maximum.

15. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and transmits the remaining. Find the ratio of the minimum and the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate. **[1990 - 7 Mark]**



Topic-2 : Young's Double Slit Experiment

1 MCQs with One Correct Answer

1. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :
[Main Sep. 03, 2020 (I)]
(a) 0.17° (b) 0.57° (c) 1.7° (d) 0.07°

2. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to : **[Main Sep. 02, 2020 (I)]**

(a) 1.27 μm (b) 2.87 nm (c) 2 nm (d) 2.05 μm

3. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be : **[Main Sep. 02, 2020 (II)]**

(a) 24 (b) 30 (c) 18 (d) 28

4. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$ th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is: **[Main 8 Jan 2020 II]**

(a) 0.853 (b) 0.672 (c) 0.568 (d) 0.760

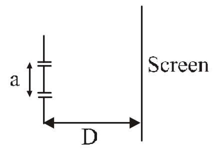
5. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is: **[Main 7 Jan 2020 II]**

(a) 6.9 mm (b) 3.9 mm (c) 5.9 mm (d) 4.9 mm

6. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is ($\approx$ is the wavelength of the light used) : **[Main 12 April 2020 I]**

(a) $\frac{2\lambda}{(\mu-1)}$ (b) $\frac{\lambda}{2(\mu-1)}$ (c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{(2\mu-1)}$

7. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index ¼ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is », t will be: **[Main 9 April 2019 I]**



(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{nD\lambda}{a(\mu-1)}$ (c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{2D\lambda}{a(\mu-1)}$

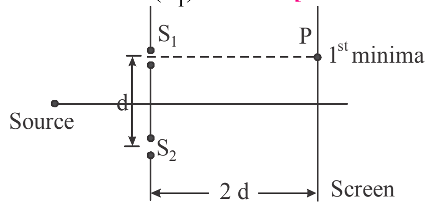
8. In a Young's double slit experiment, the path difference, at a certain point on the screen, betwen two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to: **[Main 11 Jan 2019 I]**

(a) 0.74 (b) 0.85 (c) 0.94 (d) 0.80

9. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1 When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are: **[Main 10 Jan. 2019 I]**

(a) 625 nm, 500 nm (b) 380 nm, 525 nm
(c) 380 nm, 500 nm (d) 400 nm, 500 nm

10. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? **[Main 10 Jan 2019 II]**

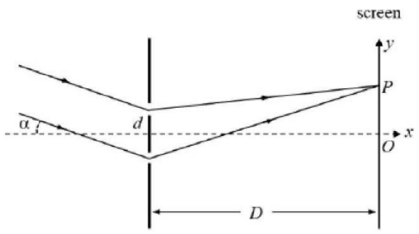


(a) $\frac{\lambda}{2(\sqrt{5}-2)}$ (b) $\frac{\lambda}{(\sqrt{5}-2)}$ (c) $\frac{\lambda}{2(5-\sqrt{2})}$ (d) $\frac{\lambda}{(5-\sqrt{2})}$

11. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is **[Main 9 Jan 2019 II]**

(a) 640 (b) 320 (c) 321 (d) 641

12. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ? **[Adv. 2019]**



(a) For $\alpha = 0$, there will be constructive interference at point P.
(b) Fringe spacing depends on α
(c) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.
(d) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

13. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is : **[Main 2017]**

(a) 9.75 mm (b) 15.6 mm (c) 1.56 mm (d) 7.8 mm

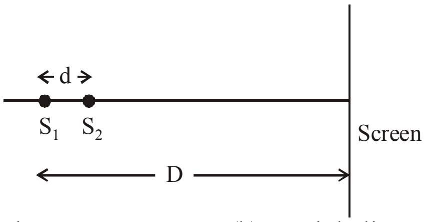
14. In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that D >> d >> λ. If the fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is: **[Main Online April 11, 2015]**

(a) $\frac{\beta}{6}$ (b) $\frac{\beta}{3}$ (c) $\frac{\beta}{4}$ (d) $\frac{\beta}{2}$

15. In a Young's double slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width. Then the number of intensity maxima observed within the central maximum of the single slit diffraction pattern is: **[Main Online April 19, 2014]**

(a) 3 (b) 6 (c) 12 (d) 24

16. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be **[Main 2013]**



(a) points (b) straight lines
(c) semi-circles (d) concentric circles

17. A thin glass plate of thickness is $\frac{2500}{3}\lambda$ (λ is wavelength of light used) and refractive index μ = 1.5 is inserted between one of the slits and the screen in Young's double slit experiment. At a point on the screen equidistant from the slits, the ratio of the intensities before and after the introduction of the glass plate is : **[Main Online April 25, 2013]**

(a) 2 : 1 (b) 1 : 4 (c) 4 : 1 (d) 4 : 3

18. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is **[Adv. 2013]**

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$ (c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

19. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are b_G, b_R and b_B, respectively. Then, **[2012]**

(a) $b_G > b_B > b_R$ (b) $b_B > b_G > b_R$
(c) $b_R > b_B > b_G$ (d) $b_R > b_G > b_B$

20. In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this point is **[2005S]**

(a) $\sin^{-1}(\lambda/d)$ (b) $\sin^{-1}(\lambda/2d)$
(c) $\sin^{-1}(\lambda/3d)$ (d) $\sin^{-1}(\lambda/4d)$

21. Monochromatic light of wavelength 400 nm and 560 nm are incident simultaneously and normally on double slits apparatus whose slits separation is 0.1 mm and screen distance is 1m. Distance between areas of total darkness will be **[2004S]**

(a) 4mm (b) 5.6mm (c) 14mm (d) 28mm

22. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wave-lenght λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is **[2002S]**

(a) 2λ (b) $2\lambda/3$ (c) $\lambda/3$ (d) λ

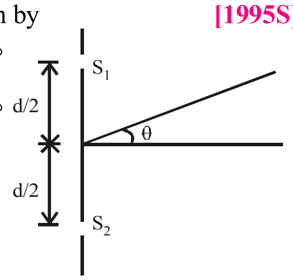
23. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **[2001S]**

(a) 12 (b) 18 (c) 24 (d) 30

24. In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern **[2000S]**

(a) the intensities of both the maxima and the minima increase
(b) the intensity of the maxima increases and the minima has zero intensity
(c) the intensity of the maxima decreases and that of the minima increases
(d) the intensity of the maxima decreases and the minima has zero intensity

25. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance d = 150.0 m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **[1995S]**



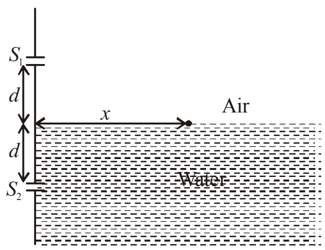
(a) $I(\theta) = I_0/2$ *for* $\theta = 30°$
(b) $I(\theta) = I_0/4$ *for* $\theta = 90°$
(c) $I(\theta) = I_0$ *for* $\theta = 0°$
(d) $I(\theta)$ is constant for all values of θ.

26. In Young's double-slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is **[1981- 2 Marks]**

(a) unchanged. (b) halved.
(c) doubled (d) quadrupled

2 Integer Value Answer

27. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = $\frac{4}{3}$) as shown in the figure. The positions of maximum on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is **[Adv. 2015]**



3 Numeric Answer

28. A young's double-slit experiment is performed using monochromatic light of wavelength λ. The inntensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is ______. **[Main Sep. 06, 2020 (II)]**

29. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) ______. **[Main 9 Jan 2020 II]**

4 Fill in the Blanks

30. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude 'A' and of wavelength 'λ'. In another experiment with the same set-up the two slits are sources of equal amplitude 'A' and wavelength 'λ', but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is .... **[1986 - 2 Marks]**

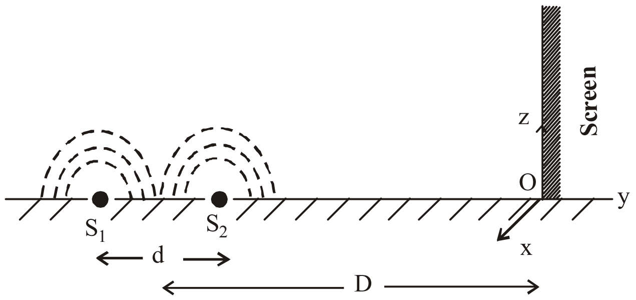
5 True / False

31. In a Young's double slit experiment performed with a source of white light, only black and white fringes are observed. **[1987 - 2 Marks]**

32. The two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. **[1984- 2 Marks]**

6 MCQs with One or More than One Correct Answer

33. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3 m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is(are) true of the intensity pattern on the screen? **[Adv. 2016]**



(a) Straight bright and dark bands parallel to the x-axis
(b) The region very close to the point O will be dark
(c) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction
(d) Semi circular bright and dark bands centered at point.

34. A light source, which emits two wavelength $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2 respectively, then **[Adv. 2014]**

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) Form the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) The angular separation of fringes for λ_1 is greater than λ_2.

35. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s). **[2008]**

(a) If $d = \lambda$, the screen will contain only one maximum
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

36. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d (> b) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are **[1984- 2 Marks]**

(a) $\lambda = \frac{b^2}{d}$ (b) $\lambda = \frac{2b^2}{d}$ (c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

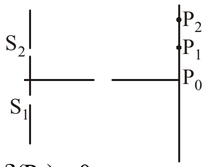
37. In the Young' s double slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that **[1982 - 3 Marks]**

(a) the intensities at the screen due to the two slits are 5 units and 4 units respectively
(b) the intensities at the screen due to the two slits are 4 units and 1 units respectively
(c) the amplitude ratio is 3
(d) the amplitude ratio is 2

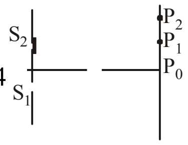
7 Match the Following

38. **Column-I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by δ (P) and the intensity by I(P). Match each situation given in **Column-I** with the statetment(s) in **Column-II** valid for that situation. **[2009]**

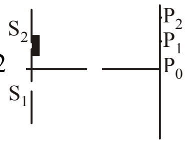
Column-I
Column-II

(A) 

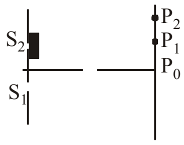
(p) $\delta(P_0) = 0$

(B) $(\mu - 1)\,t = \lambda/4$ 

(q) $\delta(P_1) = 0$

(C) $(\mu - 1)\,t = \lambda/2$ 

(r) $I(P_1) = 0$

(D) $(\mu - 1)\,t = 3\lambda/4$ 

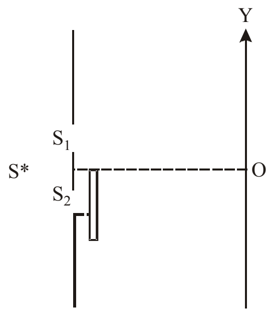
(s) $I(P_0) > I(P_1)$

(t) $I(P_2) > I(P_1)$

10 Subjective Problems

39. In *YDSE* a light containing two wavelengths 500 nm and 700 nm are used. Find the minimum distance where maxima of two wavelengths coincide. Given $D/d = 10^3$, where D is the distance between the slits and the screen and d is the distance between the slits. **[2004 - 4 Marks]**

40. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 µm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in Figure. **[1999 - 10 Marks]**



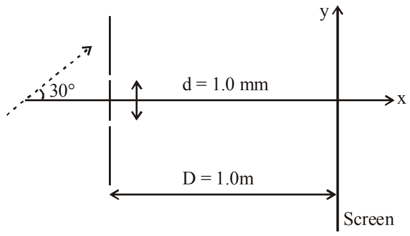
(a) Find the location of the central maximum (bright fringe with zero path difference) on the y – axis.

(b) Find the light intensity at point O relative to the maximum fringe intensity.

(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

[All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]

41. A coherent parallel beam of microwaves of wavelength λ = 0.5 mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in Fig.



(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x axis (as in the dotted arrow shown in Figure), find the y-coordinate of the first minima on either side of the central maximum. **[1998 - 8 Marks]**

42. In Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum (n = 0) fells before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the fifth maximum earlier, lies below the point P while the sixth minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected.) **[1997 - 5 Marks]**

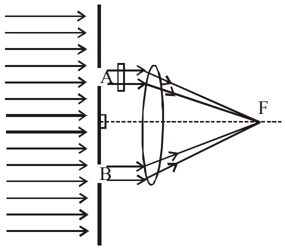
43. A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1mm, and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. **[1996 - 3 Marks]**

(i) Calculate the fringe-width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis.

44. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ W m^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A, see fig. Calculate the power (in watts) received at the focal spot F of the lens.

The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. **[1989 - 8 Mark]**



45. A beam of light consisting of two wavelengths, 6500Å and 5200Å, is used obtain interference fringes in a Young's double slit experiment : **[1985 - 6 Marks]**

(i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

Topic-3 : Diffraction, Polarisation of Light and Resolving Power

1 MCQs with One Correct Answer

1. A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to : **[Main Sep. 04, 2020 (I)]**

(a) 1.0×10^{-5} J (b) 1.0×10^{-4} J
(c) 1.5×10^{-4} J (d) 5.0×10^{-4} J

2. The aperture diameter of telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to: **[Main 9 Jan. 2020 I]**

(a) 60m (b) 20m (c) 200m (d) 600m

3. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is: **[Main 7 Jan. 2020 I]**

(a) 71.6° (b) 18.4° (c) 90° (d) 45°

4. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be : **[Main 12 April 2019 I]**

(a) 0.24 μm (b) 0.38 μm (c) 0.12 μm (d) 0.48 μm

5. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_o is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_o/I) equals (nearly) : **[Main 12 April 2019 II]**

(a) 5.33 (b) 16.00
(c) 10.67 (d) 1.80

6. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. Coming from a distant object, the limit of resolution of the telescope is close to: **[Main 9 April 2019 II]**

(a) 1.5×10^{-7} rad (b) 2.0×10^{-7} rad
(c) 3.0×10^{-7} rad (d) 4.5×10^{-7} rad

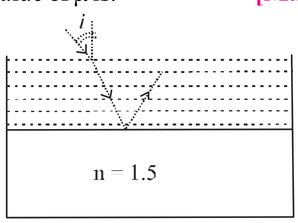
7. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star. **[Main 8 April 2019 II]**

(a) 305×10^{-9} radian (b) 610×10^{-9} radian
(c) 152.5×10^{-9} radian (d) 457.5×10^{-9} radian

8. In a double-slit experiment, green light (5303Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is : **[Main 11 Jan 2019 II]**

(a) 10 (b) 05 (c) 04 (d) 09

9. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is: **[Main 9 Jan. 2019 I]**



(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{5}{\sqrt{3}}$ (d) $\frac{4}{3}$

10. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.) **[Main 2018]**
(a) 25 μm (b) 50 μm (c) 75 μm (d) 100 μm

11. Unpolarized light of intensity I passes through an ideal polarizer A. Another indentical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is: **[Main 2018]**
(a) 0° (b) 30° (c) 45° (d) 60°

12. Light of wavelength 550 nm falls normally on a slit of width 22.0×10^{-5}cm. The angular position of the second minima from the central maximum will be (in radians)
[Main Online April 15, 2018]
(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

13. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is I/2. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to I/3. The angle between the polarizers A and C is θ. Then
[Main Online April 16, 2018]
(a) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$ (b) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$
(c) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$ (d) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$

14. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1}) **[Main 2017]**
(a) 17.3 GHz (b) 15.3GHz (c) 10.1 GHz (d) 12.1 GHz

15. A single slit of width *b* is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourth minima in the diffraction pattern at a distance 1 m from the slit are at 3 cm and 6 cm respectively from the central maximum, what is the width of the central maximum? (i.e. distance between first minimum on either side of the central maximum) **[Main Online April 8, 2017]**
(a) 1.5 cm (b) 3.0 cm (c) 4.5 cm (d) 6.0 cm

16. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when : **[Main 2016]**
(a) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$
(b) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$
(c) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$
(d) $a = \sqrt{\lambda l}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

17. Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm. The wavelength of light is 600 nm. To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year = 9.46×10^{15} m) of the order of :
[Main Online April 10, 2016]
(a) 10^8 km (b) 10^{10} km (c) 10^{11} km (d) 10^6 km

18. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is : **[Main 2015]**
(a) 100 μm (b) 300 μm (c) 1 μm (d) 30 μm

19. Unpolarized light of intensity I_0 is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true ?
[Main Online April 11, 2015]
(a) reflected light is completely polarized with intensity less than $\frac{I_0}{2}$
(b) transmitted light is completely polarized with intensity less than $\frac{I_0}{2}$
(c) transmitted light is partially polarized with intensity $\frac{I_0}{2}$
(d) reflected light is partially polarized with intensity $\frac{I_0}{2}$

20. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals: **[Main 2014]**
(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{3}$

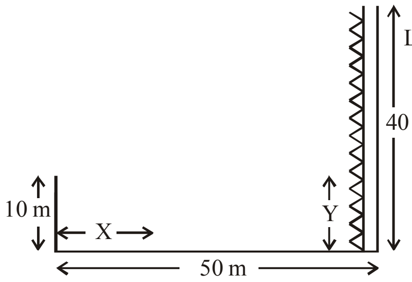
21. A ray of light is incident from a denser to a rarer medium. The critical angle for total internal reflection is θ_{iC} and Brewster's angle of incidence is θ_{iB}, such that $\sin\theta_{iC}/\sin\theta_{iB} = \eta = 1.28$. The relative refractive index of the two media is: **[Main Online April 19, 2014]**
(a) 0.2 (b) 0.4 (c) 0.8 (d) 0.9

22. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of some other wavelength. If wavelength of red light is 6600 Å, then wavelength of first maximum will be:
[Main Online April 12, 2014]
(a) 3300Å (b) 4400Å (c) 5500Å (d) 6600Å

23. Abeam of unpolarised light of intensity I_0 is passed through a polaroidAand then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **[Main 2013]**

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) $I_0/8$

24. A person lives in a high-rise building on the bank of a river 50 m wide. Across the river is a well lit tower of height 40 m. When the person, who is at a height of 10 m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming from distance X from his building is the least and this corresponds to the light coming from light bulbs at height 'Y' on the tower. The values of X and Y are respectively close to (refractive index of water $\simeq \frac{4}{3}$) **[Main Online April 9, 2013]**



(a) 25 m, 10 m (b) 13 m, 27 m
(c) 22 m, 13 m (d) 17 m, 20 m

25. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X– rays, then the observed pattern will reveal, **[1999S - 2 Marks]**

(a) that the central maximum is narrower
(b) more number of fringes
(c) less number of fringes
(d) no diffraction pattern

26. Consider Fraunhoffer diffraction pattern obtained with a single slit illuminated at normal incidence. At the angular position of the first diffraction minimum the phase difference (in radians) between the wavelets from the opposite edges of the slit is **[1995S]**

(a) $\pi/4$ (b) $\pi/2$ (c) 2π (d) π

27. A beam of light of wave length 600 nm from a distance source falls on a single slit 1 mm wide and a resulting diffraction pattern is observed on a screen 2m away. The distance between the first dark fringes on either side of central bright fringe is **[1994 - 1 Mark]**

(a) 1.2 cm (b) 1.2 mm (c) 2.4 cm (d) 2.4 mm

28. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is **[1998 - 2 Marks]**

(a) 0 (b) $\pi/2$ (c) π (d) 2π

3 Numeric Answer

29. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is ____________. **[Main Sep. 04, 2020 (II)]**

Topic-4 : Miscellaneous (Mixed Concepts) Problems

4 Fill in the Blanks

1. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meters and 25 meters respectively from the source. The ratio of amplitudes of the waves at P and Q is .... **[1989 - 2 Marks]**

7 Match the Following

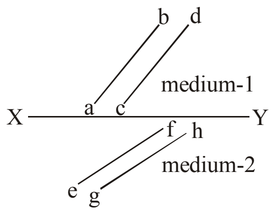
2. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_o, respectively. Then **[2006 - 6M]**

Column I	Column II
(A) Intensity of light received by lens	(p) Radius of aperture
(B) Angular magnification	(q) Dispersion of lens
(C) Length of telescope	(r) Focal length of objective lens and eyepiece lens
(D) Sharpness of image	(s) Spherical aberration

8 Comprehension/Passage Based Questions

Passage

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The line ab and cd represent wave-forms of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction. **[2007]**



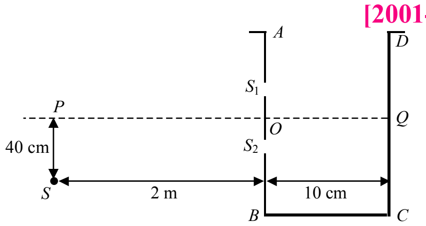
3. Light travels as a
(a) parallel beam in each medium
(b) convergent beam in each medium
(c) divergent beam in each medium
(d) divergent beam in one medium and convergent beam in the other medium.

4. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.
(a) ϕ_c cannot be equal to ϕ_d
(b) ϕ_d can be equal to ϕ_e
(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

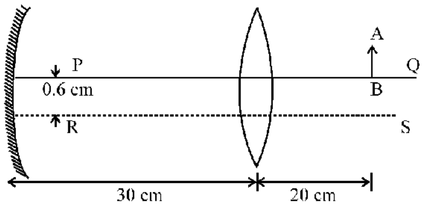
5. Speed of light is
(a) the same in medium-1 and medium-2
(b) larger in medium-1 than in medium-2
(c) larger in medium-2 than in medium-1
(d) different at b and d.

10 Subjective Problems

6. A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure below. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. **[2001-5 Marks]**



7. (a) A convex lens of focal length 15 cm and a cancave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also locate position of A' and B' with respect to the optic axis RS. **[2000 - 6 Marks]**



(b) A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surface of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the rays interfere constructively. **[2000 - 4 Marks]**

Answer Key

Topic-1 : Wavefront, Interference of Light, Coherent and Incoherent Sources

1. (b)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b, c)							

Topic-2 : Young's Double Slit Experiment

1. (b)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (Bonus)	**8.** (b)	**9.** (a)	**10.** (a)
11. (d)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (c)
21. (d)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (3)	**28.** (9)	**29.** (750)	**30.**
31. FALSE	**32.** TRUE	**33.** (b,d)	**34.** (a, b, c)		**35.** (a, b)	**36.** (a, c)	**37.** (b, d)		
38. (A-p, s; B-q; C-t; D-r, s, t)									

Topic-3 : Diffraction, Polarisation of Light and Resolving Power

1. (d)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (c)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (d)
21. (c)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (c)	**27.** (d)	**28.** (d)	**29.** (198)	

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. $\frac{25}{9}$ **2.** (A) $\rightarrow$ (p); (B) $\rightarrow$ (r); (C) $\rightarrow$ (r); (D) $\rightarrow$ (p), (q), (r); **3.** (a) **4.** (c) **5.** (b) **6.** (a)
7. (a)

25 Dual Nature of Radiation and Matter

Topic-1 : Matter Waves, Cathode and Positive Rays

1 MCQs with One Correct Answer

1. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, λ_{He++} and λ_p is : **[Main Sep. 06, 2020 (I)]**
   (a) $\lambda_e > \lambda_{He++} > \lambda_p$ (b) $\lambda_e < \lambda_{He++} = \lambda_p$
   (c) $\lambda_e > \lambda_p > \lambda_{He++}$ (d) $\lambda_e < \lambda_p < \lambda_{He++}$

2. Assuming the nitrogen molecule is moving with r.m.s.velocity at 400 K, the de-Broglie wavelength of nitrongen molecule is close to :
   (Given : nitrogen molecule weight : 4.64×10^{-26} kg, Boltzman constant : 1.38×10^{-23} J/K, Planck constant : 6.63×10^{-34} J.s) **[Main Sep. 06, 2020 (II)]**
   (a) 0.24 Å (b) 0.20 Å (c) 0.34 Å (d) 0.44 Å

3. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is : **[Main Sep. 04, 2020 (I)]**
   (a) $\Delta\lambda = \frac{3}{2}\lambda_0$ (b) $\Delta\lambda = \frac{5}{2}\lambda_0$
   (c) $\Delta\lambda = 2\lambda_0$ (d) $\Delta\lambda = 4\lambda_0$

4. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to: **[Main Sep. 02, 2020 (II)]**
   (a) 4.8×10^{-27} kg (b) 9.1×10^{-31} kg
   (c) 1.2×10^{-28} kg (d) 9.7×10^{-28} kg

5. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become $\frac{\lambda}{2}$. Value of ΔE, is: **[Main 9 Jan. 2020 (I)]**
   (a) E (b) 4E (c) 3E (d) 2E

6. An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is: **[Main 9 Jan. 2020 (II)]**
   (a) $-\frac{h}{|e|E\sqrt{t}}$ (b) $\frac{|e|Et}{h}$
   (c) $-\frac{h}{|e|Et}$ (d) $\frac{-h}{|e|Et^2}$

7. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wave length at time t is given by: **[Main 8 Jan. 2020 (II)]**
   (a) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$ (b) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$
   (c) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$ (d) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$

8. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'γ_x' and 'γ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is: **[Main 9 Apr. 2019 (II)]**
   (a) $\frac{\gamma_x\gamma_y}{\gamma_x+\gamma_y}$ (b) $\frac{\gamma_x\gamma_y}{|\gamma_x-\gamma_y|}$ (c) $\gamma_x-\gamma_y$ (d) $\gamma_x+\gamma_y$

9. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50v Another particle B of mass '4m' and charge'q' is accelerated by a potential differnce of 2500V. The ratio of de-Broglie wavelength $\frac{\lambda_A}{\lambda_B}$ is **[Main 12 Jan. 2019 (I)]**

(a) 10.00 (b) 0.07 (c) 14.14 (d) 4.47

10. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to :

(Speed of light = 3×10^8 m/s)

Planck's constant = 6.63×10^{-34}J.s

Mass of electron = 9.1×10^{-31} kg) **[Main 11 Jan. 2019 (I)]**

(a) 1.1×10^6 m/s(b) 1.7×10^6 m/s

(c) 1.8×10^6 m/s(d) 1.45×10^6 m/s

11. Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is: **[Main Online April 15, 2018]**

(a) $\lambda_{CM} = \lambda_1 = \lambda_2$ (b) $\frac{1}{\lambda_1} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

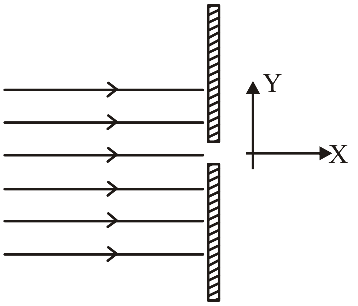
(c) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ (d) $\lambda_{CM} = \left(\frac{\lambda_1 + \lambda_2}{2}\right)$

12. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is **[Main 2017]**

(a) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = 2$

13. A parallel beam of electrons travelling in x-direction falls on a slit of width d (see figure). If after passing the slit, an electron acquires momentum p_y in the y-direction then for a majority of electrons passing through the slit (h is Planck's constant) : **[Main Online April 10, 2015]**



(a) $|P_y| d > h$ (b) $|P_y| d < h$

(c) $|P_y| d \simeq h$ (d) $|P_y| d >> h$

14. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship? **[Main Online April 9, 2014]**

(a) an electron (b) a proton

(c) an α-particle (d) a dust particle

15. Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference 4 V. The de-Broglie wavelengths are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by : (given m_e is mass of electron and m_p is mass of proton). **[Main Online April 23, 2013]**

(a) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ (b) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$

(c) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (d) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$

16. A proton has kinetic energy $E = 100$ keV which is equal to that of a photon. The wavelength of photon is λ_2 and that of proton is λ_1. The ration of λ_2/λ_1 is proportional to **[2004S]**

(a) E^2 (b) $E^{1/2}$ (c) E^{-1} (d) $E^{-1/2}$

17. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1/λ_2, is **[1999S - 2 Marks]**

(a) m_1/m_2 (b) m_2/m_1

(c) 1.0 (d) $\sqrt{m_2}/\sqrt{m_1}$

2 Integer Value Answer

18. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is **[2010]**

10 Subjective Problems

19. The potential energy of a particle of mass m is given by

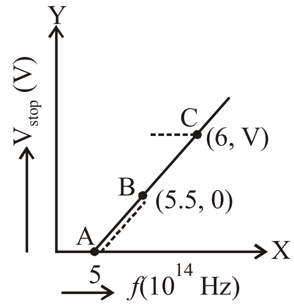
$$V(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find λ_1 / λ_2. **[2005 - 2 Marks]**

Topic-2 : Photon, Photoelectric Effect X-rays and Davisson-Germer Experiment

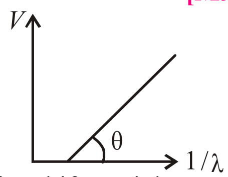
1 MCQs with One Correct Answer

1. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is : (Plancks constant $h = 6.62 \times 10^{-34}$ J.s) **[Main Sep. 04, 2020 (I)]**



(a) 2.27eV (b) 2.59 eV (c) 1.93 eV (d) 2.10 eV

2. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased : **[Main Sep. 04, 2020 (II)]**



(a) Straight line shifts to right
(b) Slope of the straight line get more steep
(c) Straight line shifts to left
(d) Graph does not change

3. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to : **[Main Sep. 03, 2020 (I)]**

(a) 0.81 eV (b) 1.02 eV (c) 0.52 eV (d) 0.61 eV

4. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is: **[Main Sep. 03, 2020 (II)]**

(a) $\frac{1}{500}$ (b) 250 (c) $\frac{1}{250}$ (d) 500

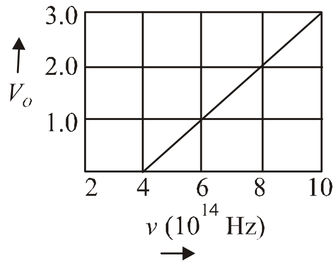
5. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to: **[Main 9 Jan. 2020 (I)]**

(a) 1.1 ev (b) 0.8 ev (c) 1.6 ev (d) 1.8 ev

6. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is: **[Main 8 Jan. 2020 I]**

(a) 4 eV (b) 2 eV (c) 1.5 eV (d) 3 eV

7. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be: (Given : Planck's constant $(h) = 6.63 \times 10^{-34}$ Js, electron charge $e = 1.6 \times 10^{-19}$ C) **[Main 12 Apr. 2019 I]**



(a) 1.82eV (b) 1.66 eV
(c) 1.95 eV (d) 2.12 eV

8. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is : [Given Planck's constant h= 6.6×10^{-34} Js, speed of light $c = 3.0 \times 10^8$ m/s] **[Main 10 Apr. 2019 (II)]**

(a) 5×10^{15} (b) 1.5×10^{16} (c) 2×10^{16} (d) 1×10^{16}

9. The electric field of light wave is given as

$$\vec{E} = 10^3 \cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi \times 6 \times 10^{14} t\right)\hat{x}\frac{N}{C}$$

This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is:

Given, E (in eV) $= \frac{12375}{\lambda(\text{in Å})}$ **[Main 9 April 2019 (I)]**

(a) 2.0 V (b) 0.72 V (c) 0.48 V (d) 2.48 V

10. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stoppoing potential is $-V_0$. The threshold frequency for photoelectric emission is : **[Main 12 Jan. 2019 II]**

(a) $\frac{5\nu}{3}$ (b) $\frac{4}{3}\nu$

(c) $2v$ (d) $\frac{3v}{2}$

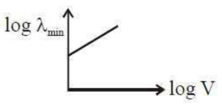
11. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to: **[Main 11 Jan. 2019 II]**
$\left(\frac{hc}{e} = 1240 \text{ nm-V}\right)$
(a) 0.5 V (b) 1.5 V (c) 1.0 V (d) 2.0 V

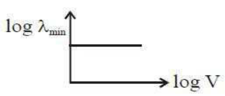
12. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be:
[1 eV = 1.6×10^{-19} J] **[Main 10 Jan. 2019 (II)]**
(a) 10^{14} and 10 eV (b) 10^{12} and 5 eV
(c) 10^{11} and 5 eV (d) 10^{10} and 5 eV

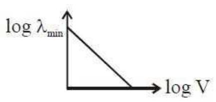
13. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of (2) The work function of the metal (in eV) is close to:
(Energy of photon = $\frac{1240}{\lambda\,(\text{in nm})}$ eV) **[Main 9 Jan. 2019 I]**
(a) 1.8 (b) 2.5 (c) 5.6 (d) 1.4

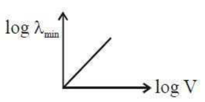
14. The magnetic field associated with a light wave is given at the origin by
$B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$.
If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons? **[Main 9 Jan. 2019 II]**
($c = 3 \times 10^8$ ms^{-1}, $h = 6.6 \times 10^{-34}$ J-s)
(a) 6.82 eV (b) 12.5 eV (c) 8.52 eV (d) 7.72 eV

15. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{min}$ with log V is correctly represented in : **[Main 2017]**

(a)


(b)


(c)


(d)


16. The maximum velocity of the photoelectrons emitted from the surface is v when light of frequency n falls on a metal surface. If the incident frequency is increased to 3n, the maximum velocity of the ejected photoelectrons will be : **[Main Online April 8, 2017]**
(a) less than $\sqrt{3}$v (b) v
(c) more than $\sqrt{3}$v (d) equal to $\sqrt{3}$v

17. A photoelectric material having work–function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de–Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ result in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to **[Adv. 2017]**
(a) λ_d/λ (b) λ_d^2/λ^2
(c) λ_d^3/λ (d) λ_d^3/λ^2

18. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be: **[Main 2016]**
(a) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$
(c) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (d) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

19. A photoelectric surface is illuminated successively by monochromatic light of wavelengths λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is : **[Main Online April 10, 2016]**
(a) $\frac{hc}{2\lambda}$ (b) $\frac{hc}{\lambda}$
(c) $\frac{hc}{3\lambda}$ (d) $\frac{3hc}{\lambda}$

20. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below :

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is **[Adv. 2016]**
(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

21. Match **List - I** (Fundamental Experiment) with **List - II** (its conclusion) and select the correct option from the choices given below the list: **[Main 2015]**

List-I	List-II
A. Franck-Hertz Experiment	(i) Particle nature of light
B. Photo-electric experiment	(ii) Discrete energy levels of atom
C. Davison-Germer experiment	(iii) Wave nature of electron
	(iv) Structure of atom

(a) (A)-(ii); (B)-(i); (C)-(iii)
(b) (A)-(iv); (B)-(iii); (C)-(ii)
(c) (A)-(i); (B)-(iv); (C)-(iii)
(d) (A)-(ii); (B)-(iv); (C)-(iii)

22. A beam of light has two wavelengths of 4972Å and 6216Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed among the two wavelengths. The beam falls normally on an area of 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photoelectrons liberated in 2s is approximately: **[Main Online April 12, 2014]**

(a) 6×10^{11} (b) 9×10^{11} (c) 11×10^{11} (d) 15×10^{11}

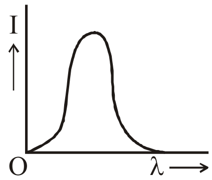
23. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly **[Adv. 2014]**

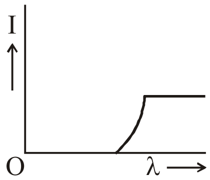
(a) 3.7 eV (b) 3.2 eV
(c) 2.8 eV (d) 2.5 eV

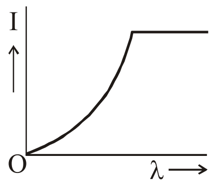
24. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to **[Adv. 2014]**

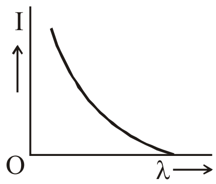
(a) 1.99 (b) 2.14
(c) 0.50 (d) 0.48

25. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[Main 2013]**

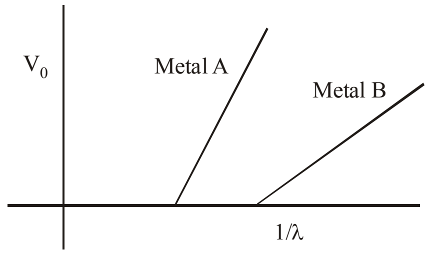
(a)


(b)


(c)


(d)


26. In an experiment on photoelectric effect, a student plots stopping potential V_0 against reciprocal of the wavelength λ of the incident light for two different metals A and B. These are shown in the figure. **[Main Online April 25, 2013]**



Looking at the graphs, you can most appropriately say that:

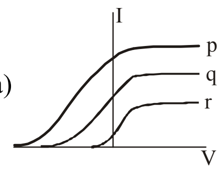
(a) Work function of metal B is greater than that of metal A
(b) For light of certain wavelength falling on both metal, maximum kinetic energy of electrons emitted from A will be greater than those emitted from B.
(c) Work function of metal A is greater than that of metal B
(d) Students data is not correct

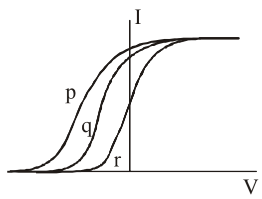
27. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is **[Adv. 2013]**

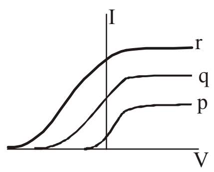
(a) 0.3×10^{-17} kg ms^{-1} (c) 3.0×10^{-17} kg ms^{-1}
(b) 1.0×10^{-17} kg ms^{-1} (d) 9.0×10^{-17} kg ms^{-1}

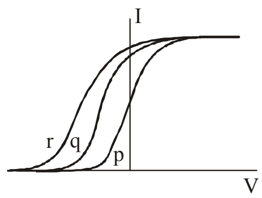
28. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is

[Take $hc = 1240$ eV nm] **[Main 2009]**

(a)


(b)


(c)


(d)


29. Which one of the following statements is WRONG in the context of X-rays generated from a X-ray tube? **[2008]**

(a) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.
(b) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(c) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(d) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

30. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **[2007]**

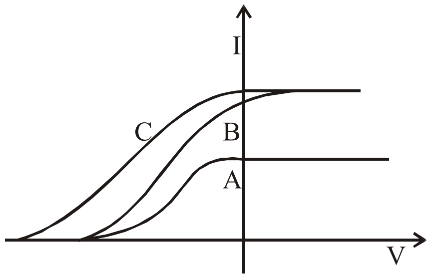
(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$

(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

31. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ. **[2005S]**

(a) $Z = 6$ (b) $Z = 4$ (c) $Z = 11$ (d) $Z = 44$

32. In a photoelectric experiment anode potential is plotted against plate current. **[2004S]**



(a) A and B will have different intensities while B and C will have different frequencies

(b) B and C will have different intensities while A and C will have different frequencies

(c) A and B will have different intensities while A and C will have equal frequencies

(d) B and C will have equal intensities while A and B will have same frequencies

33. The potential difference applied to an X-ray tube is 5kV and the current through it is $3.2mA$. Then the number of electrons striking the target per second is **[2002S]**

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

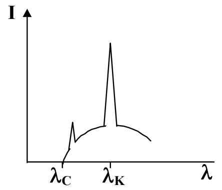
34. The intensity of X-rays from a Coolidge tube is plotted against wavelength λ as shown in the figure. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased **[2001S]**

(a) $\lambda_K - \lambda_C$ increases

(b) $\lambda_K - \lambda_C$ decreases

(c) λ_K increases

(d) λ_K decreases



35. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K-shell electrons of tungsten have 72.5 keV energy. X-rays emitted by the tube contain only **[2000S]**

(a) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of 0.155Å

(b) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths

(c) the characteristic X-ray spectrum of tungsten

(d) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of 0.155Å and the characteristic X-ray spectrum of tungsten.

36. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[1998S - 2 Marks]**

(a) 540nm (b) 400nm (c) 310nm (d) 220nm

37. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from **[1998S - 2 Marks]**

(a) 0 to ∞

(b) λ_{min} to ∞ where $\lambda_{min} > 0$

(c) 0 to λ_{max} where $\lambda_{max} < \infty$

(d) λ_{min} to λ_{max} where $0 < \lambda_{min} < \lambda_{max} < \infty$

38. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential, in volt, is **[1997 - 1 Mark]**

(a) 2 (b) 4 (c) 6 (d) 10

39. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy in (eV) required to remove both the electrons from a neutral helium atom is **[1995S]**

(a) 38.2 (b) 49.2 (c) 51.8 (d) 79.0

40. The X-ray beam coming from an X-ray tube will be **[1985 - 2 Marks]**

(a) monochromatic

(b) having all wavelengths smaller than a certain maximum wavelength

(c) having all wavelengths larger than a certain minimum wavelength

(d) having all wavelengths lying between a minimum and a maximum wavelength

41. The shortest wavelength of X-rays emitted from an X-ray tube depends on **[1982 - 3 Marks]**

(a) the current in the tube

(b) the voltage applied to the tube

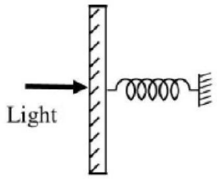
(c) the nature of the gas in tube

(d) the atomic number of the target material

2 Integer Value Answer

42. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency such that = $10^{24} m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is ______

[Consider the spring as massless] **[Adv. 2019]**



Mirror

43. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is **[Adv. 2013-I]**

44. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in freespace. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of 'z' is **[2011]**

3 Numeric Answer

45. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$. Then E (in eV) is ______.
(Plank constant $h = 6.64 \times 10^{-34}$ Js, 1 eV $= 1.6 \times 10^{-19}$ J, electron mass $m = 9.1 \times 10^{-31}$ kg) **[Main Sep. 05, 2020 (I)]**

46. The surface of a metal is illuminated alternately with photons of energies $E_1 = 4$ eV and $E_2 = 2.5$ eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is ______________. **[Main Sep. 05, 2020 (II)]**

47. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$. If the theshold wavelength for the metallic surface is $n\lambda$ then value of n will be _________. **[Main Sep. 02, 2020 (I)]**

48. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, $\lambda = 310$ nm. It falls normally on a metal (work function $\varphi = 2$eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. ($hc = 1240$ eVnm, 1 eV $= 1.6 \times 10^{-19}$ J), then x is ______. **[Main 7 Jan. 2020 I]**

49. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is __________. Mass of the electron $m_e = 9 \times 10^{-31}\,kg$ and $1.0\text{ eV} = 1.6 \times 10^{-19}\,J$. **[Adv. 2018]**

4 Fill in the Blanks

50. The wavelength of K_α X-rays produced by an X-ray tube is 0.76Å. The atomic number of the anode material of the tube is..... **[1996 - 2 Marks]**

51. A potential difference of 20 kV is applied across an X-ray tube. The minimum wavelength of X-rays generated is..........Å. **[1996 - 2 Marks]**

52. In an X-ray tube, electrons accelerated through a potential difference of 15, 000 volts strike a copper target. The speed of the emitted X-ray inside the tube is .... m/s **[1992 - 1 Mark]**

53. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen like element is 0.32 Å. The wavelength of the K_β line emitted by the same element will be .... **[1990 - 2 Marks]**

54. When the number of electrons striking the anode of an X-ray tube is increased, the .... of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased, the cut-off wavelength of the emitted X-rays......... **[1986 - 2 Marks]**

55. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the ...... of the incident radiation. **[1984- 2 Marks]**

56. To produce characteristic X-rays using a Tungsten target in an X-ray generator, the accelerating voltage should be greater than ___________ volts and the energy of the characteristic radiation is _____eV.
(The binding energy of the innermost electron in Tungsten is 40 keV). **[1983 - 2 Marks]**

5 True / False

57. In a photoelectric emission process the maximum energy of the photo-electrons increases with increasing intensity of the incident light. **[1986 - 3 Marks]**

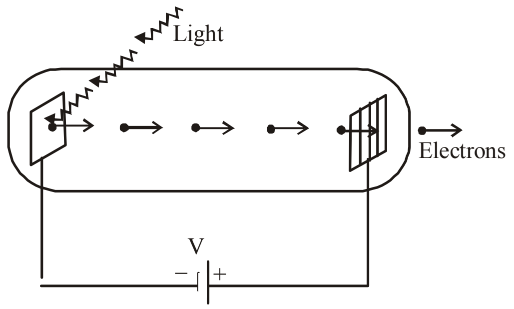
58. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. **[1981- 2 Marks]**

6 MCQs with One or More than One Correct Answer

59. In an X-ray tube, electrons emitted from a filament (cathode) carrying current I hit a target (anode) at a distance d from the cathode. The target is kept at a potential V higher than the cathode resulting in emission of continuous and characteristic X-rays. If the filament current I is decreased to $\frac{I}{2}$, the potential difference V is increased to $2V$, and the separation distance d is reduced to $\frac{d}{2}$, then

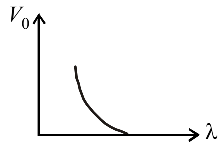
(a) the cut-off wavelength will reduce to half, and the wavelengths of the characteristic X-rays will remain the same
(b) the cut-off wavelength as well as the wavelengths of the characteristic X-rays will remain the same
(c) the cut-off wavelength will reduce to half, and the intensities of all the X-rays will decrease
(d) the cut-off wavelength will become two times larger, and the intensity of all the X-rays will decrease

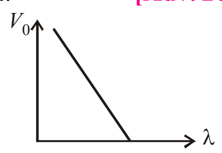
60. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is (are) true? **[Adv. 2016]**

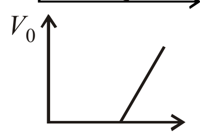


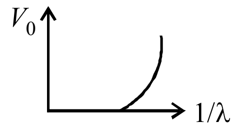
(a) λ_e decreases with increase in ϕ and λ_{ph}
(b) λ_e is approximately halved, if d is doubled
(c) For large potential difference (V >> ϕ/e), λ_e is approximately halved if V is made four times
(d) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

61. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$. **[Adv. 2015]**

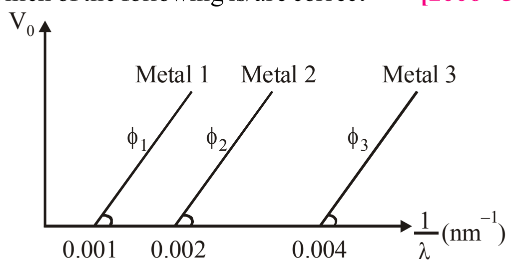
(a) 

(b) 

(c) 

(d) 

62. The graph between the stopping potential (V_0) and $\left(\frac{1}{\lambda}\right)$ is shown in the figure. ϕ_1, ϕ_2 and ϕ_3 are work functions, which of the following is/are correct **[2006 - 5M, –1]**



(a) $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(b) $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(c) $\tan\theta \propto \frac{hc}{e}$
(d) ultravioletlight can be used to emit photoelectrons from metal 2 and metal 3 only

63. When photons of energy 4.25 eV strike the surface of metal A, the ejected photoelectrons have maximum kinetic energy, T_A eV and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then **[1994 - 2 Marks]**
(a) The work function of A is 2.25 eV
(b) The work function of B is 4.20 eV
(c) $T_A = 2.00$ eV
(d) $T_B = 2.75$ eV

64. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then **[1992 - 2 Marks]**
(a) the stopping potential will be 0.2 volt
(b) the stopping potential will be 0.6 volt
(c) the saturation current will be 6.0 mA
(d) the saturation current will be 2.0 mA

65. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation **[1988 - 2 Marks]**
(a) the intensity increases
(b) the minimum wavelength increases
(c) the intensity remain unchanged
(d) the minimum wavelength decreases

66. Photoelectric effect supports quantum nature of light because **[1987 - 2 Marks]**
(a) there is a minimum frequency of light below which no photoelectrons are emitted
(b) the maximum kinetic energy of photo electrons depends only on the frequency of light and not on its intensity
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
(d) electric charge of the photoelectrons is quantized

67. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a **[1982 - 3 Marks]**
(a) 50 watt infrared lamp (b) 1-watt infra-red lamp
(c) 50 watt ultraviolet lamp (d) 1-watt ultraviolet lamp

9 Assertion and Reason Type Questions

68. **STATEMENT-1** **[2007]**
If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.
STATEMENT-2
When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

10 Subjective Problems

69. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4\mu A$ for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation 2×10^{-5} W/m^2. **[2003 - 2 Marks]**

70. Frequency of a photon emitted due to transition of electron of a certain element from L to K shell is found to be 4.2×10^{18} Hz. Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant $R = 1.1 \times 10^7$ m^{-1}. **[2003 - 2 Marks]**

71. Two metallic plates A and B, each of area 5×10^{-4} m^2, are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photons fall on it per square meter per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2eV. Determine **[2002 - 5 Marks]**

(a) the number of photoelectrons emitted up to $t = 10$ s,
(b) the magnitude of the electric field between the plates A and B at $t = 10$ s, and
(c) the kinetic energy of the most energetic photoelectron emitted at $t = 10$ s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B. Take $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\text{-}m^2$

72. Photoelectrons are emitted when 40 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α–particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. [Take $h = 4.14 \times 10^{-15}$ eV.s.] **[1999 - 5 Marks]**

73. Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form. **[1997 - 5 Marks]**

Topic-3 : Miscellaneous (Mixed Concepts) Problems

2 Integer Value Answer

1. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s/C; $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m **[2012-I]**

6 MCQs with One or More than One Correct Answer

2. The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then

(Take Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ Wm^{-2}K^{-4}, Wien's displacement constant $= 2.90 \times 10^{-3}$ m-K, Planck's constant $= 6.63 \times 10^{-34}$ Js, speed of light in vacuum $= 3.00 \times 10^8$ ms^{-1})

(a) power radiated by the filament is in the range 642 W to 645 W
(b) radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to 3.25×10^{-8} W
(c) the wavelength corresponding to the maximum intensity of light is 1160 nm
(d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

Answer Key

Topic-1 : Matter Waves, Cathode and Positive Rays

1. (c)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (c)	**18.** (3)		

Topic-2 : Photon, Photoelectric Effect X-rays and Davisson-Germer Experiment

1. (a)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (Bonus)
11. (c)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (b)
21. (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (d)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (a)
31. (a)	**32.** (d)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (c)
41. (b)	**42.** (1)	**43.** (1)	**44.** (7)	**45.** (50)	**46.** (2)	**47.** (9)	**48.** (11)	**49.** (24)	**50.** (41)

51. (0.62Å) **52.** (3×10^8 m/s) **53.** (0.27Å) **54.** (decreases) **55.** (Frequency)
56. (30×10^3 V) **57.** False **58.** False **59.** (a, c) **60.** (c) **61.** (a, c) **62.** (a, c) **63.** (a, b, c)
64. (b, d) **65.** (c, d) **66.** (a, b, c) **67.** (c, d) **68.** (b, d)

Topic-3 : Miscellaneous (Mixed Concepts) Problems

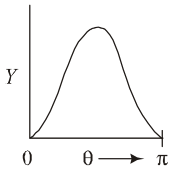
1. (7) **2.** (b, c, d)

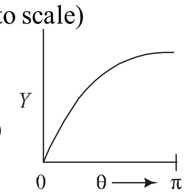
26 Atoms

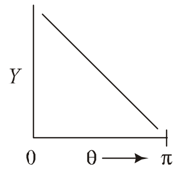
Topic-1 : Atomic Structure and Rutherford's Nuclear Model

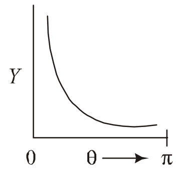
1 MCQs with One Correct Answer

1. The graph which depicts the results of Rutherford gold foil experiment with α-particles is: **[Main 8 Jan. 2020 I]**
θ: Scattering angle
Y: Number of scattered α-particles detected
(Plots are schematic and not to scale)

(a) 

(b) 

(c) 

(d) 

2. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of **[Main 1981- 2 Marks]**
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12}cm (d) 10^{-15} cm

Topic-2 : Bohr's Model and the Spectra of the Hydrogen Atom

1 MCQs with One Correct Answer

1. In a hydrogen atom the electron makes a transition from $(n + 1)^{th}$ level to the n^{th} level. If $n >> 1$, the frequency of radiation emitted is proportional to: **[Main Sep. 02, 2020 (II)]**
(a) $\frac{1}{n}$ (b) $\frac{1}{n^3}$ (c) $\frac{1}{n^2}$ (d) $\frac{1}{n^4}$

2. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state? **[Main 9 Jan. 2020 II]**
(a) 24.2 nm (b) 11.4 nm (c) 35.8 nm (d) 8.6 nm

3. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is: **[Main 7 Jan. 2020 I]**
(a) 1.6×10^{14} (b) 7.8×10^{14} (c) 6.2×10^{15} (d) 5.6×10^{12}

4. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is : **[Main 12 April 2019 II]**
(a) 20/7 (b) 27/5 (c) 7/5 (d) 9/7

5. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ? **[Main 10 April 2019 II]**
(Given : $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1})
(a) 11.4 nm (b) 9.4 nm (c) 12.3 nm (d) 10.8 nm

6. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be; **[Main 9 April 2019 I]**

(a) 889.2 nm (b) 488.9 nm
(c) 642.7 nm (d) 388.9 nm

7. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is : **[Main 8 April 2019 I]**

(a) $n = 2 \rightarrow n = 3$ (b) $n = 1 \rightarrow n = 4$
(c) $n = 2 \rightarrow n = 5$ (d) $n = 2 \rightarrow n = 4$

8. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be: **[Main 11 Jan 2019 I]**

(a) $25a_0$ (b) $9a_0$ (c) $16a_0$ (d) $4a_0$

9. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is l. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be: **[Main 11 Jan 2019 II]**

(a) $\frac{27}{20}\lambda$ (b) $\frac{16}{25}\lambda$
(c) $\frac{25}{16}\lambda$ (d) $\frac{20}{27}\lambda$

10. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants) **[Main 2018]**

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n$
(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

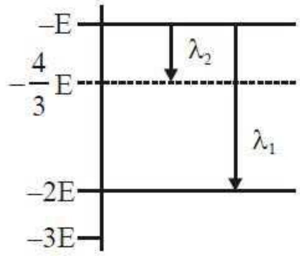
11. If the series limit frequency of the Lyman series is v_1, then the series limit frequency of the P-fund series is : **[Main 2018]**

(a) $25\,\nu_L$ (b) $16\,\nu_L$
(c) $\nu_L/16$ (d) $\nu_L/25$

12. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is: **[Main Online April 15, 2018]**

(a) 20 eV (b) 79 eV (c) 109 eV (d) 34 eV

13. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by **[Main 2017]**



(a) $r = \frac{3}{4}$ (b) $r = \frac{1}{3}$
(c) $r = \frac{4}{3}$ (d) $r = \frac{2}{3}$

14. According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the n^{th} orbit is proportional to : (n = principal quantum number) **[Main Online April 8, 2017]**

(a) n^{-4} (b) n^{-5} (c) n^{-3} (d) n^{-2}

15. A hydrogen atom makes a transition from n = 2 to n = 1 and emits a photon. This photon strikes a doubly ionized lithium atom (z = 3) in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is : **[Main Online April 9, 2016]**

(a) 2 (b) 4 (c) 5 (d) 3

16. As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom/ion : **[Main 2015]**

(a) kinetic energy decreases, potential energy increases but total energy remains same
(b) kinetic energy and total energy decrease but potential energy increases
(c) its kinetic energy increases but potential energy and total energy decrease
(d) kinetic energy, potential energy and total energy decrease

17. If one were to apply Bohr model to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a magnetic field 'B', the energy of the charged particle in the n^{th} level will be : **[Main Online April 10, 2015]**

(a) $n\left(\frac{hqB}{2\pi m}\right)$ (b) $n\left(\frac{hqB}{8\pi m}\right)$
(c) $n\left(\frac{hqB}{4\pi m}\right)$ (d) $n\left(\frac{hqB}{\pi m}\right)$

18. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to: **[Main 2014]**

(a) 1.8 eV (b) 1.1 eV
(c) 0.8 eV (d) 1.6 eV

19. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $\left({}_2He^4\right)^+$ and doubly ionised lithium $\left({}_3Li^6\right)^{++}$ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively then approximately which one of the following is correct? **[Main 2014]**

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

20. The binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is: **[Main Online April 9, 2014]**

(a) 122.4 eV (b) 30.6 eV
(c) 13.6 eV (d) 3.4 eV

21. Ina hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number (n – 1). If n>>1, the frequency of radiation emitted is proportional to : **[Main 2013]**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$

(c) $\frac{1}{n^{3/2}}$ (d) $\frac{1}{n^3}$

22. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as

$F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the n^{th} orbit in terms of the Bohr radius $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$ is : **[Main Online April 23, 2013]**

(a) $r_n = a_0 n - \beta$ (b) $r_n = a_0 n^2 + \beta$
(c) $r_n = a_0 n^2 - \beta$ (d) $r_n = a_0 n + \beta$

23. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 A°. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is **[2011]**

(a) 1215 A° (b) 1640 A° (c) 2430 A° (d) 4687 A°

24. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **[2007]**

(a) 802 nm (b) 823 nm (c) 1882 nm (d) 1648 nm

25. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector? **[2005S]**

(a) One photon of energy 10.2 eV and an electron of energy 1.4 eV
(b) 2 photon of energy of 1.4 eV
(c) 2 photon of energy 10.2 eV
(d) One photon of energy 10.2 eV and another photon of 1.4 eV

26. If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of $_{100}Fm^{257}$ is n times the Bohr radius, then find n. **[2003S]**

(a) 100 (b) 200 (c) 4 (d) 1/4

27. The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number? **[2003S]**

(a) $r_n \propto n$ (b) $r_n \propto 1/n$
(c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

28. A Hydrogen atom and a Li^{++} ion are both in the second excited state. If ℓ_H and ℓ_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then **[2002S]**

(a) $\ell_H > \ell_{Li}$ and $|E_H| > |E_{Li}|$

(b) $\ell_H = \ell_{Li}$ and $|E_H| < |E_{Li}|$

(c) $\ell_H = \ell_{Li}$ and $|E_H| > |E_{Li}|$

(d) $\ell_H < \ell_{Li}$ and $|E_H| < |E_{Li}|$

29. The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition **[2001S]**

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$

30. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true ? **[2000S]**

(a) Its kinetic energy increases and its potential and total energies decreases.
(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same.
(c) Its kinetic and total energies decrease and its potential energy increases.
(d) Its kinetic, potential and total energies decrease.

31. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to **[2000S]**

(a) $9/(5R)$ (b) $36/(5R)$ (c) $18/(5R)$ (d) $4/R$

32. In hydrogen spectrum the wavelength of H_α line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is, **[1999S - 2 Marks]**
(a) 2×10^8 m/s (b) 2×10^7 m/s
(c) 2×10^6 m/s (d) 2×10^5 m/s

33. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is **[1997 - 1 Mark]**
(a) 1.51 (b) 13.6 (c) 40.8 (d) 122.4

34. Consider the spectral line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength is produced by **[1983 - 1 Mark]**
(a) Hydrogen atom
(b) Deuterium atom
(c) Singly ionized Helium
(d) Doubly ionised Lithium

2 Integer Value Answer

35. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is __________. **[Adv. 2018]**

36. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is **[Adv. 2017]**

37. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking hc/e = 1.237×10^{-6} eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is **[Adv. 2016]**

38. An electron is an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **[Adv. 2015]**

39. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is ($hc = 1242$ eV nm) **[Adv. 2015]**

3 Numeric Answer

40. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is : **[Main Sep. 05, 2020 (I)]**
(Given the mass of the hydrogen atom to be 1 GeV/c^2)

41. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschan series in Å is : __________. **[Main Sep. 04, 2020 (I)]**

42. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is ______. **[Main 8 Jan. 2020 II]**

4 Fill in the Blanks

43. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is .... **[1992 - 1 Mark]**

44. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as a dopant in silicon (relative dielectric constant = 12) is .... Å **[1991 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

45. A particle of mass m moves in circular orbits with potential energy $V(r) = Fr$, where F is a positive constant and r is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by R and its speed and energy are denoted by v and E, respectively, then for the n^{th} orbit (here h is the Planck's constant) **[Adv. 2020]**
(a) $R \propto n^{1/3}$ and $v \propto n^{2/3}$
(b) $R \propto n^{2/3}$ and $v \propto n^{1/3}$
(c) $E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$
(d) $E = 2\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$

46. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a / \lambda_e = \frac{1}{5}$, which of the option(s) is/are correct?
[Use $hc = 1242$ eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively] **[Adv. 2019]**
(a) $\Delta p_a / \Delta p_e = \frac{1}{2}$
(b) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$
(c) $m = 2$
(d) $\lambda_e = 418$ nm

47. Highly excited states for hydrogen–like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n>>1. Which of the following statement(s) is(are) true? **[Adv. 2016]**

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as 1/n

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

48. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) **[Adv. 2013]**

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$

(c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

49. The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are **[1998S - 2 Marks]**

(a) $n_1 = 4,\ n_2 = 2$ (b) $n_1 = 8,\ n_2 = 2$

(c) $n_1 = 8,\ n_2 = 1$ (d) $n_1 = 6,\ n_2 = 3$

50. In Bohr's model of the hydrogen atom **[1984- 2 Marks]**

(a) the radius of the nth orbit is proportional to n^2

(b) the total energy of the electron in nth orbit is inversely proportional to n

(c) the angular momentum of the electron in an orbit is an integral multiple of $\frac{h}{2\pi}$

(d) the magnitude of potential energy of the electron in any orbit is greater than its K.E.

7 Match the Following

51. Some laws / processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[2007]**

Column I	**Column II**
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

8 Comprehension/Passage Based Questions

Passage-1

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **[2010]**

52. A diatomic molecule has moment of inertia *I*. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

53. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to

(Take $h = 2\pi \times 10^{-34}$ J s)

(a) $2.76 \times 10^{-46}\ \text{kg m}^2$ (b) $1.87 \times 10^{-46}\ \text{kg m}^2$

(c) $4.67 \times 10^{-47}\ \text{kg m}^2$ (d) $1.17 \times 10^{-47}\ \text{kg m}^2$

54. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. $= \frac{5}{3} \times 10^{-27}$ kg, is close to

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

Passage-2

When a particle is restricted to move along x - axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ...

(n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ $J\,s$ and $e = 1.6 \times 10^{-19}$ C.

55. The allowed energy for the particle for a particular value of n is proportional to **[2009]**

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

56. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to **[2009]**

(a) 0.8 meV (b) 8 meV
(c) 80 meV (d) 800 meV

57. The speed of the particle, that can take discrete values, is proportional to **[2009]**

(a) $n^{-3/2}$ (b) n^{-1} (c) $n^{1/2}$ (d) n

passage-3

In a mixture of H-He$^+$ gas (He+ is singly ionized He atom), H atoms and He+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He+ ions (by collisions). Assume that the Bohr model of atom is exactly valid. **[2008]**

58. The quantum number n of the state finally populated in He$^+$ ions is –

(a) 2 (b) 3 (c) 4 (d) 5

59. The wavelength of light emitted in the visible region by He+ ions after collisions with H atoms is –

(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m
(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

60. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He$^+$ ion is –

(a) 1/4 (b) 1/2 (c) 1 (d) 2

10 Subjective Problems

61. In hydrogen-like atom ($z = 11$), nth line of Lyman series has wavelength λ. The de-Broglie's wavelength of electron in the level from which it originated is also λ. Find the value of n? **[2006 - 6M]**

62. The photons from the Balmer series in Hydrogen spectrum having wavelength between 450 nm to 700 nm are incident on a metal surface of work function 2 eV. Find the maximum kinetic energy of ejected electron. (Given $hc = 1242$ eV nm) **[2004 - 4 Marks]**

63. A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85 eV and -0.544 eV (**includ ing** both these values). **[2002 - 5 Marks]**

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom = –13.6 eV)

64. A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. **[2000 - 6 Marks]**

65. An electron, in a hydrogen-like atom, is in an excited state. It has a total energy of –3.4 eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron. **[1996 - 3 Marks]**

66. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. **[1994 - 6 Marks]**

Determine the values of n and Z. (Ionization energy of H-atom = 13.6 eV)

67. A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system. **[1988 - 6 Marks]**

(i) Derive an expression for the radius of the nth Bohr orbit.

(ii) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) Find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit of the first orbit.

68. A double ionised Lithium atom is hydrogen-like with atomic number 3. **[1985 - 6 Marks]**

(i) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)

(ii) How many spectral lines are observed in the emission spectrum of the above excited system?

69. The ionization energy of a hydrogen like Bohr atom is 4 Rydbergs. (i) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state ? (ii) What is the radius of the first orbit for this atom? **[1984- 4 Marks]**

70. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975Å. How many different lines are possible in the resulting spectrum ? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.

Topic-3 : Miscellaneous (Mixed Concepts) Problems

10 Subjective Problems

1. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find **[1992 - 10 Marks]**
   (a) the energy of the photons causing the photoelectric emission,
   (b) the quantum numbers of the two levels involved in the emission of these photons,
   (c) the change in the angular momentum of the electron in the hydrogen atom in the above transition, and
   (d) the recoil speed of the emitting atom assuming it to be at rest before the transition.
   (Ionization potential of hydrogen is 13.6 eV)

2. Electrons in hydrogen like atom ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 volts. Calculate the work function of the metal and the stopping potential for the photoelectrons ejected by the longer wavelength.
   (Rydberg constant = $1.094 \times 10^7\ m^{-1}$) **[1990 - 7 Marks]**

3. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more and some have less than 2.7 eV. **[1989 - 8 Marks]**
   (i) Find the principal quantum number of the initially excited level B.
   (ii) Find the ionization energy for the gas atoms.
   (iii) Find the maximum and the minimum energies of the emitted photons.

4. A single electron orbits around a stationary nucleus of charge $+Ze$. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find **[1981- 10 Marks]**
   (i) The value of Z.
   (ii) The energy required to excite the electron from the third to the fourth Bohr orbit.
   (iii) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
   (iv) The kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.
   (v) The radius of the first Bohr orbit.
   (The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} metre, velocity of light = 3×10^8 m/sec. Planck's constant = 6.6×10^{-34} joules - sec).

Answer Key

Topic-1 : Atomic Structure and Rutherford's Nuclear Model

1. (c) **2.** (c)

Topic-2 : Bohr's Model and the Spectra of the Hydrogen Atom

1. (b)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (a)
11. (d)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (b)
21. (d)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (b)	**33.** (d)	**34.** (d)	**35.** (3)	**36.** (5)	**37.** (6)	**38.** (2)	**39.** (2)	**40.** (51)
41. (10553.14)		**42.** (486.00)	**43.** (1)	**44.** (3.81)	**45.** (b, c)	**46.** (b, c)	**47.** (a, b, d)	**48.** (a, c)	**49.** (a, d)
50. (a, c, d)	**51.** A $\rightarrow$ p, r; B $\rightarrow$ q, s; C $\rightarrow$ p; D $\rightarrow$ q				**52.** (d)	**53.** (b)	**54.** (c)	**55.** (a)	**56.** (b)
57. (d)	**58.** (c)	**59.** (c)	**60.** (a)						

27 Nuclei

Topic-1 : Composition and Size of the Nuclei

1 MCQs with One Correct Answer

1. The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of : $(M_{\text{prot.}} \cong M_{\text{neut.}} \simeq 1.67 \times 10^{-27}$ kg) **[Main Sep. 03, 2020 (II)]**

(a) 10^3 kg m^{-3} (b) 10^{10} kg m^{-3}
(c) 10^{24} kg m^{-3} (d) 10^{17} kg m^{-3}

2. The ratio of the mass densities of nuclei of ^{40}Ca and ^{16}O is close to : **[Main 8 April 2019 (II)]**

(a) 1 (b) 0.1 (c) 5 (d) 2

3. An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8:27. The ratio of the radii of the nuclei (assumed to be spherical) is: **[Main Online April 15, 2018]**

(a) 8 : 27 (b) 2 : 3 (c) 3 : 2 (d) 4 : 9

4. For uranium nucleus how does its mass vary with volume? **[2003S]**

(a) $m \propto V$ (b) $m \propto 1/V$ (c) $m \propto \sqrt{V}$ (d) $m \propto V^2$

5. Order of magnitude of density of uranium nucleus is, $[m_p = 1.67 \times 10^{-27}$kg] **[1999S - 2 Marks]**

(a) 10^{20}kg/m^3 (b) 10^{17}kg/m^3
(c) 10^{14}kg/m^3 (d) 10^{11}kg/m^3

5 True / False

6. The order of magnitude of the density of nuclear matter is 10^4 kg m^{-2}. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

7. The mass number of a nucleus is **[1986 - 2 Marks]**

(a) always less than its atomic number
(b) always more than its atomic number
(c) sometimes equal to its atomic number
(d) sometimes more than and sometimes equal to its atomic number

Topic-2 : Mass-Energy Equivalence and Nuclear Reactions

1 MCQs with One Correct Answer

1. You are given that mass of $^7_3$Li $= 7.0160$ u,
Mass of $^4_2$He $= 4.0026$ u
and Mass of $^1_1$H $= 1.0079$ u.
When 20 g of $^7_3$Li is converted into $^4_2$He by proton capture, the energy liberated, (in kWh), is :
[Mass of nucleon = 1 GeV/c^2] **[Main Sep. 06, 2020 (I)]**

(a) 4.5×10^5 (b) 8×10^6 (c) 6.82×10^5 (d) 1.33×10^6

2. Given the masses of various atomic particles $m_p = 1.0072$ u, $m_n = 1.0087$ u, $m_e = 0.000548$ u, $m_{\bar{\nu}} = 0$, $m_d = 2.0141$ u, where p ≡ proton, n ≡ neutron, e ≡ electron, $\bar{\nu}$ ≡ antineutrino and d ≡ deuteron. Which of the following process is allowed by momentum and energy conservation? **[Main Sep. 06, 2020 (II)]**

(a) $n + n \rightarrow$ deuterium atom (electron bound to the nucleus)
(b) $p \rightarrow n + e^+ + \bar{\nu}$
(c) $n + p \rightarrow d + \gamma$
(d) $e^+ + e^- \rightarrow \gamma$

3. Find the Binding energy per neucleon for $^{120}_{50}$Sn. Mass of proton $m_p = 1.00783$ U, mass of neutron $m_n = 1.00867$ U and mass of tin nucleus $m_{Sn} = 119.902199$ U. (take 1U = 931 MeV) **[Main Sep. 04, 2020 (II)]**

(a) 7.5 MeV (b) 9.0 MeV (c) 8.0 MeV (d) 8.5 MeV

4. In a reactor, 2 kg of ${}_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, N = 6.023 × 10^{26} per kilo mole and 1 eV = 1.6 × 10^{-19} J. The power output of the reactor is close to: **[Main Sep. 02, 2020 (I)]**
(a) 35 MW (b) 60 MW (c) 125 MW (d) 54 MW

5. Consider the nuclear fission
$Ne^{20} \rightarrow 2He^4 + C^{12}$
Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement: **[Main 10 Jan. 2019 II]**
(a) energy of 12.4 MeV will be supplied
(b) 8.3 MeV energy will be released
(c) energy of 3.6 MeV will be released
(d) energy of 11.9 MeV has to be supplied

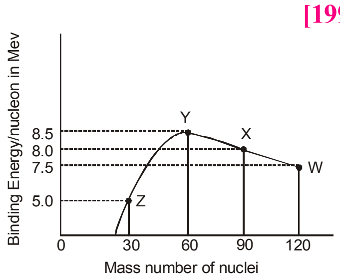
6. Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be : (velocity of light, c is 3 × 10^8 m/s) **[Main Online April 9, 2017]**
(a) 0.96 gm (b) 0.8 gm
(c) 4 × 10^{-2} gm (d) 6.6 × 10^{-5} gm

7. When Uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as :
$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3x + Q(\text{energy})$$
where three particles named x are produced and energy Q is released. What is the name of the particle x ? **[Main Online April 9, 2013]**
(a) electron (b) α-particle
(c) neutron (d) neutrino

8. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is **[Main 2007]**
(a) $E\left(\begin{matrix}236\\92\end{matrix}U\right) > E\left(\begin{matrix}137\\53\end{matrix}I\right) + E\left(\begin{matrix}97\\39\end{matrix}Y\right) + 2E(n)$
(b) $E\left(\begin{matrix}236\\92\end{matrix}U\right) < E\left(\begin{matrix}137\\53\end{matrix}I\right) + E\left(\begin{matrix}97\\39\end{matrix}Y\right) + 2E(n)$
(c) $E\left(\begin{matrix}236\\92\end{matrix}U\right) < E\left(\begin{matrix}140\\56\end{matrix}Ba\right) + E\left(\begin{matrix}94\\36\end{matrix}Kr\right) + 2E(n)$
(d) $E\left(\begin{matrix}236\\92\end{matrix}U\right) = E\left(\begin{matrix}140\\56\end{matrix}Ba\right) + E\left(\begin{matrix}94\\36\end{matrix}Kr\right) + 2E(n)$

9. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 amu and mass of Oxygen nucleus is 15.9994 amu] **[2005S]**
(a) 7.6 MeV (b) 56.12 MeV (c) 10.24 MeV (d) 23.9 MeV

10. Binding energy per nucleon vs mass number curve for nuclei is shown in the Figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **[1999S - 2 Marks]**



(a) $Y \rightarrow 2Z$ (b) $W \rightarrow X + Z$
(c) $W \rightarrow 2Y$ (d) $X \rightarrow Y + Z$

11. Fast neutrons can easily be slowed down by
(a) the use of lead shielding **[1994 - 1 Mark]**
(b) passing them through water
(c) elastic collisions with heavy nuclei
(d) applying a strong electric field.

4 Fill in the Blanks

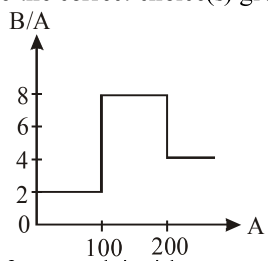
12. Consider the following reaction :
${}_1^2H + {}_1^2H = {}_2^4He + Q$
Mass of the deuterium atom = 2.0141 u
Mass of helium atom = 4.0024 u
This is a nuclear .... reaction in which the energy Q released is .... MeV. **[1996 - 2 Marks]**

13. In the nuclear process, ${}_6C^{11} \longrightarrow {}_5B^{11} + \beta^+ + X$, X stands for .... **[1992 - 1 Mark]**

14. The binding energies per nucleon for deuteron (${}_1H^2$) and helium (${}_2He^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus (${}_2He^4$) is .... **[1988 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

15. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below. **[2008]**



(a) Fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy.
(b) Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy
(c) Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments
(d) Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

16. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}Ne$ nucleus and M_2 the mass of a ${}_{20}^{40}Ca$ nucleus. Then **[1998S - 2 Marks]**
(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

17. Which of the following statement(s) is (are) correct? **[1994 - 2 Marks]**
(a) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons
(b) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons
(c) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu)
(d) In nuclear fission, energy is released by fragmentation of a very heavy nucleus

18. During a nuclear fusion reaction **[1987 - 2 Marks]**
(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products

19. From the following equations pick out the possible nuclear fusion reactions **[1984- 2 Marks]**
(a) ${}_6C^{13} + {}_1H^1 \to {}_6C^{14} + 4.3\text{MeV}$
(b) ${}_6C^{12} + {}_1H^1 \to {}_7N^{13} + 2\text{MeV}$
(c) ${}_7N^{14} + {}_1H^1 \to {}_8O^{15} + 7.3\text{MeV}$
(d) ${}_{92}U^{235} + {}_0n^1 \to {}_{54}Xe^{140} + {}_{38}Sr^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200\text{MeV}$

10 Subjective Problems

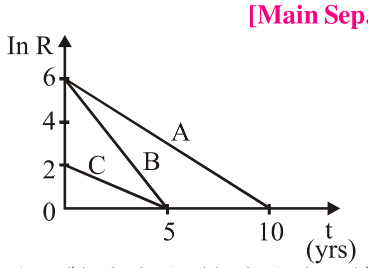
20. In a nuclear reaction ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required. **[2001 - 5 Marks]**

21. It is proposed to use the nuclear fusion reaction
$${}_1^2H + {}_1^2H \to {}_2^4He$$ **[1990 - 8 Marks]**
in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of ${}_1^2H$ and ${}_2^4He$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively)

Topic-3 : Radioactivity

1 MCQs with One Correct Answer

1. Acitvities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the ratio : **[Main Sep. 05, 2020 (I)]**



(a) 2 : 1 : 1 (b) 3 : 2 : 1 (c) 2 : 1 : 3 (d) 4 : 3 : 1

2. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to : **[Main Sep. 05, 2020 (II)]**
(a) 9 sec. (b) 6 sec. (c) 55 sec. (d) 12 sec.

3. In a radioactive material, fraction of active material remaining after time *t* is 9/16. The fraction that was remaining after *t*/2 is : **[Main Sep. 03, 2020 (I)]**
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{7}{8}$

4. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to: **[7 Jan. 2020, II]**
(a) 72 min (b) 62 min (c) 66 min (d) 52 min

5. Two radioactive substances A and B have decay constants 5λ and λ respectively. At *t* = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be : **[Main 10 April 2019, II]**
(a) 1/2λ (b) 1/4λ
(c) 1/λ (d) 2/λ

6. In a radioactive decay chain, the initial nucleus is ${}_{90}^{232}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is ${}_Z^AX$, A and Z are given by : **[Main 12 Jan. 2019, II]**
(a) A = 208 ; Z = 80 (b) A = 202 ; Z = 80
(c) A = 208 ; Z = 82 (d) A = 200; Z = 81

7. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to: **[Main 10 Jan. 2019 I]**
(a) 200 (b) 150
(c) 400 (d) 360

8. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is *l*n2, the half-life of B is: **[Main 9 Jan. 2019, II]**
(a) 4*l*n2 (b) $\frac{l\text{n}2}{2}$ (c) $\frac{l\text{n}2}{4}$ (d) 2*l*n2

9. A solution containing active cobalt ${}_{27}^{60}Co$ having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If 1 cm^3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body? (1Ci = 3.7 × 10^{10} decay per second and at t = 10 hrs $e^{-\lambda t}$ = 0.84) **[Main Online April 15, 2018]**
(a) 6 litres (b) 7 litres (c) 4 litres (d) 5 litres

10. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by **[Main 2017]**

(a) $t = T \log(1.3)$ (b) $t = \dfrac{T}{\log(1.3)}$

(c) $t = T\dfrac{\log 2}{\log 1.3}$ (d) $t = \dfrac{\log 1.3}{\log 2}$

11. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be : **[Main 2016]**
(a) 1 : 4 (b) 5 : 4 (c) 1 : 16 (d) 4 : 1

12. Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life = 15 hrs) in 7.5 hours, N_β is close to (Avogadro number = 6.023×10^{23}/g. mole): **[Main Online April 11, 2015]**
(a) 6.2×10^{21} (b) 7.5×10^{21}
(c) 1.25×10^{22} (d) 1.75×10^{22}

13. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of C^{14} is 5730 years, then age of the wooden piece placed in the museum is approximately: **[Main Online April 19, 2014]**
(a) 10439 years (b) 13094 years
(c) 19039 years (d) 39049 years

14. A radioactive nuclei with decay constant 0.5/s is being produced at a constant rate of 100 nuclei/s. If at t = 0 there were no nuclei, the time when there are 50 nuclei is: **[Main Online April 11, 2014]**

(a) 1s (b) $2 \ln\left(\dfrac{4}{3}\right)$ s

(c) $\ln 2$ s (d) $\ln\left(\dfrac{4}{3}\right)$ s

15. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{20}Ca$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be [Given: $\ln 10 = 2.3$] **[Adv. 2019]**
(a) 9.2 (b) 4.6 (c) 1.15 (d) 2.3

16. A radioactive sample S_1 having an activity 5µCi has twice the number of nuclei as another sample S_2 which has an activity of 10 µCi. The half lives of S_1 and S_2 can be **[2008]**
(a) 20 years and 5 years, respectively
(b) 20 years and 10 years, respectively
(c) 10 years each
(d) 5 years each

17. ${}^{221}_{87}Ra$ is a radioactive substance having half life of 4 days. Find the probability that a nucleus undergoes decay after two half lives **[2006 - 3M, –1]**

(a) 1 (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{1}{4}$

18. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later its activity becomes 3000 dps. What was its initial activity? **[2004S]**
(a) 20000 dps (b) 24000 dps
(d) 12000 dps (d) 6000 dps

19. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle **[2003S]**
(a) 4.4 MeV (b) 5.4 MeV (c) 5.6 MeV (d) 6.5 MeV

20. Which of the following processes represents a γ-decay? **[2002S]**

(a) ${}^{A}X_z + \gamma \longrightarrow {}^{A}X_{Z-1} + a + b$ dr

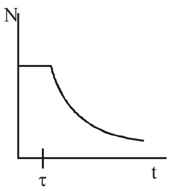
(b) ${}^{A}X_z + {}^{1}n_o \longrightarrow {}^{A-3}X_{Z-2} + c$

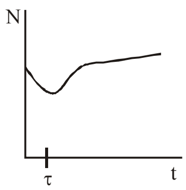
(c) ${}^{A}X_z \longrightarrow {}^{A}X_Z + f$

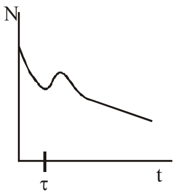
(d) ${}^{A}X_z + e_{-1} \longrightarrow {}^{A}X_{Z-1} + g$

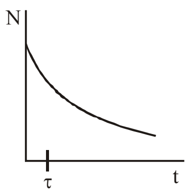
21. The half-life of ${}^{215}At$ is 100 μs. The time taken for the radioactivity of a sample of ${}^{215}At$ to decay to 1/16th of its initial value is **[2002S]**
(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

22. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represent the form of this plot? **[2001S]**

(a)


(b)


(c)


(d)


23. The electron emitted in beta radiation originates from **[2001S]**
(a) inner orbits of atoms
(b) free electrons existing in nuclei
(c) decay of a neutron in a nucleus
(d) photon escaping from the nucleus

24. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time **[2000S]**

(a) $\dfrac{1}{10\lambda}$ (b) $\dfrac{1}{11\lambda}$ (c) $\dfrac{11}{10\lambda}$ (d) $\dfrac{1}{9\lambda}$

25. Which of the following is a correct statement? **[1999S - 2 Marks]**
(a) Beta rays are same as cathode rays
(b) Gamma rays are high energy neutrons
(c) Alpha particles are singly ionised helium atoms
(d) Protons and neutrons have exactly the same mass

26. A radioactive material decays by simultaneous emission of two particles with respective half-lives 1620 and 810 years. The time, in years, after which one-fourth of the material remains is **[1995S]**
(a) 1080 (b) 2430 (c) 3240 (d) 4860

27. Consider α particles, β particles and γ - rays, each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are: **[1994 - 1 Mark]**
(a) α, β, γ (b) α, γ, β (c) β, γ, α (d) γ, β, α

28. The equation
$4\,{}_1^1H^+ \longrightarrow {}_2^4He^{2+} + 2e^- + 26\,\text{MeV}$ represents **[1983 - 1 Mark]**
(a) β-decay (b) γ-decay (c) fusion (d) fission

29. Beta rays emitted by a radioactive material are **[1983 - 1 Mark]**
(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by the nucleus
(d) neutral particles

30. The half life of radioactive Radon is 3.8 days. The time at the end of which $\frac{1}{20}$th of the radon sample will remain undecayed is (given $\log_{10} e = 0.4343$) **[1981- 2 Marks]**
(a) 3.8 days (b) 16.5 days (c) 33 days (d) 76 days.

2 Integer Value Answer

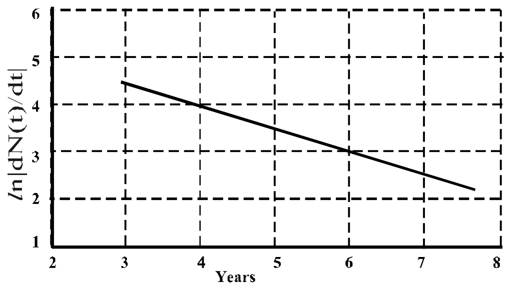
31. ^{131}I is an isotope of Iodine that B decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from person's body, and gives an activity of 115 Bq. The total volume of blood in the person¡¦s body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). **[Adv. 2017]**

32. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is **[Adv.2015]**

33. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is **[Adv. 2013-I]**

34. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **[2011]**

35. To determine the half life of a radioactive element, a student plots a graph of $ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\left|\frac{dN(t)}{dt}\right|$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[2010]**



4 Fill in the Blanks

36. When Boron nucleus $\left({}_5^{10}B\right)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element .... and has the mass number ...... **[1986 - 2 Marks]**

37. In the Uranium radioactive series the initial nucleus is ${}_{92}^{238}U$ and the final nucleus is ${}_{82}^{206}Pb$. When the Uranium nucleus decays to lead, the number of α-particles emitted is ......and the number of β-particles emitted is ..... **[1985 - 2 Marks]**

38. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration/second at a certain time. If the half life of the element is one second, the decay rate after one second is _______ and after three seconds is _______. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

39. In a radioactive decay chain, ${}_{90}^{232}Th$ nucleus decays to ${}_{82}^{212}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true? **[Adv. 2018]**
(a) $N_\alpha = 5$ (b) $N_\alpha = 6$ (c) $N_\beta = 2$ (d) $N_\beta = 4$

40. The half-life period of a radioactive element X is same as the mean-life time of another radioactive element Y. Initially both of them have the same number of atoms. Then **[1999S - 3 Marks]**
(a) X and Y have the same decay rate initially
(b) X and Y decay at the same rate always
(c) Y will decay at a faster rate than X
(d) X will decay at a faster rate than Y

41. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that **[1998S - 2 Marks]**
(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay at any time after $t = 0$

42. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are respectively given by **[1989 - 2 Marks]**

(a) $1/\lambda$ and $(ln\,2)/\lambda$ (b) $(ln\,2)/\lambda$ and $1/\lambda$

(c) $\lambda\,(ln\,2)$ and $1/\lambda$ (d) $\lambda/(ln\,2)$ and $1/\lambda$

43. A freshly prepared radioactive source of half life 2 hr emits radiation of intensity which is 64 times the permissibe safe level. The minimum time after which it would be possible to work safely with this source is **[1988 - 2 Marks]**

(a) 6 hr (b) 12 hr (c) 24 hr (d) 128 hr

44. During a negative beta decay **[1987 - 2 Marks]**

(a) an atomic electron is ejected

(b) an electron which is already present within the nucleus is ejected

(c) a neutron in the nucleus decays emitting an electron

(d) a part of the binding energy of the nucleus is converted into an electron

7 Match the Following

45. Match the nuclear processes given in column I with the appropriate option(s) in column II. **[Adv. 2015]**

Column I	Column II
(A) Nuclear fusion	(p) Absorption of thermal neutrons by $^{235}_{92}U$
(B) Fission in a nuclear reactor	(q) $^{60}_{27}Co$ nucleus
(C) β-decay	(r) Energy production in stars via hydrogen conversion to helium
(D) γ-ray emission	(s) Heavy water
	(t) Neutrino emission

46. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: **[Adv. 2013-II]**

List I	List II
P. Alpha decay	1. $^{15}_{8}O \rightarrow ^{15}_{7}O + ...$
Q. β+ decay	2. $^{138}_{92}U \rightarrow ^{234}_{90}Th + ...$
R. Fission	3. $^{185}_{83}Bi \rightarrow ^{184}_{82}Pb + ...$
S. Proton emission	4. $^{239}_{94}Pu \rightarrow ^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

47. Given below are certain matching type questions, where two columns (each having 4 items) are given. Immediately after the columns the matching grid is given, where each item of Column I has to be matched with the items of Column II, by encircling the correct match(es). Note that an item of column I can match with more than one item of column II. All the items of column II must be matched. Match the following : **[2006 - 6M]**

Column I	Column II
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number
(C) β-decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces

8 Comprehension/Passage Based Questions

Passage-1

The mass of a nucleus $^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

$^{1}_{1}H$	1.007825 u	$^{2}_{1}H$	2.014102 u
$^{3}_{1}H$	3.016050 u	$^{4}_{2}He$	4.002603 u
$^{6}_{3}Li$	6.015123 u	$^{7}_{3}Li$	7.016004 u
$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u
$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

$(1 u = 932\ MeV/c^2)$

[Adv. 2013]

48. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}Po$ at rest undergoes alpha decay, is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

49. The correct statement is

(a) The nucleus $^{6}_{3}Li$ can emit an alpha particle

(b) The nucleus $^{210}_{84}Po$ can emit a proton

(c) Deuteron and alpha particle can undergo complete fusion

(d) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion

10 Subjective Problems

50. A radioactive sample of ^{238}U decays to Pb through a process for which the half-life is 4.5×10^9 years. Find the ratio of number of nuclei of Pb to ^{238}U after a time of 1.5×10^9 years. Given $(2)^{1/3} = 1.26$. **[2004 - 2 Marks]**

51. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits 0.75 n β-particle, what is the mean life of the sample? **[2003 - 2 Marks]**

52. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1\ s^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = 1/30\ s^{-1}$. Initially, there are only X nuclei and their number is $N_o = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_Y(t) = (N_o\lambda_X/(\lambda_X - \lambda_Y))\{\exp(-\lambda_Y t)-\exp(-\lambda_x t)\}$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. **[2001-5 Marks]**

53. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element. **[1998 - 8 Marks]**
(a) Calculate the number N of nuclei of A at time t.
(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.

54. At a given instant there are 25% undecayed radio-active nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5%. Calculate (i) mean-life of the nuclei, and (ii) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number. **[1996 - 3 Marks]**

55. A small quantity of solution containing Na^{24} radio nuclide (half life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hour show an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that radioactive solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10} disintegrations per second) **[1994 - 6 Marks]**

56. A nucleus X, initially at rest, undergoes alpha decay according to the equation,

$${}^{A}_{92}X \to {}^{228}_{Z}Y + \alpha$$ **[1991 - 4 + 4 Marks]**

(a) Find the values of A and Z in the above process.
(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Tesla. Find the energy (In MeV) released during the process and the binding energy of the parent nucleus X.

Given that : $m(Y) = 228.03\,u$; $m\left({}^{1}_{0}n\right) = 1.009\,u$.

$m\left({}^{4}_{2}\mathrm{He}\right) = 4.003\,u$; $m\left({}^{1}_{1}\mathrm{H}\right) = 1.008\,u$

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? **[Adv. 2016]**
(a) 64 (b) 90 (c) 108 (d) 120

2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$ nuclei are same, 1 u = 931.5 Me V/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}N$ and ${}^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m) **[Adv. 2016]**
(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

3. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is **[1999S - 2 Marks]**
(a) nitrogen (b) carbon (c) boron (d) oxygen

4. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β–decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus $\left({}^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state. If ${}^{12}_{5}E$ decays to ${}^{12}_{6}C^*$, the maximum kinetic energy of the β–particle in units of MeV is (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum). **[Adv. 2016]**

5. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is **[Adv. 2015]**

4 Fill in the Blanks

6. Atoms having the same .... but different .... are called isotopes. **[1986 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

7. A fission reaction is given by ${}^{236}_{92}U \to {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are) **[Adv. 2015]**

(a) $x = n, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(b) $x = p, y = e^-, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(c) $x = p, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(d) $x = n, y = n, K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

8. A star initially has 10^{40} deuterons. It produces energy via the processes ${}_1H^2 + {}_1H^2 \to {}_1H^3 + p$, and ${}_1H^2 + {}_1H^3 \to {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of **[1993-2 Marks]**

(a) $10^6 s$. (b) $10^8 s$. (c) $10^{12} s$. (d) $10^{16} s$.

The masses of the nuclei are as follows :

$M(H^2) = 2.014$ amu;

$M(p) = 1.007$ amu; $M(n) = 1.008$ amu; $M(He^4) = 4.001$ amu.

7 Match the Following

9. Four physical quantities are listed in Column I. Their values are listed in Column II in a random order: **[1987 - 2 Marks]**

Column I	Column II
(a) Thermal energy of air molecules at room temp	(e) 0.02 eV
(b) Binding energy of heavy nuclei per nucleon	(f) 2 eV
(c) X-ray photon energy	(g) 1 keV
(d) Photon energy of visible light	(h) 7 MeV

The correct matching of Columns I and II is given by

(a) $a - e, b - h, c - g, d - f$ (b) $a - e, b - g, c - f, d - h$
(c) $a - f, b - e, c - g, d - h$ (d) $a - f, b - h, c - e, d - g$.

10. **Column-II** gives certain systems undergoing a process. **Column-I** suggests changes in some of the parameters related to the system. Match the statements in **Column-I** to the approapriate process(es) from **Column-II**. **[2009]**

Column-I	Column-II
(A) The energy of the system is increased	(p) *System :* A capacitor, initially uncharged *Process :* It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q) *System :* A gas in an adiabatic container fitted with an adiabatic piston *Process:* The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) *System:* A gas in a rigid container *Process:* The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(s) *System:* A heavy nucleus, initially at rest *Process:* The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) *System:* A resistive wire loop *Process:* The loop is placed in a time varying magnetic field perpendicular to its plane

11. In the following, column I lists some physical quantities and the column II gives approximate energy values associated with some of them. Choose the appropriate value of energy from column II for each of the physical quantities in column I and write the corresponding letter p, q, r, etc. against the number (A), (B), (C), (D) etc. of the physical quantity in the answer book. In your answer, the sequence of column I should be maintained. **[1997 - 4 Marks]**

Column I	Column II
(A) Energy of thermal neutrons	(p) 0.025 eV
(B) Energy of X-rays	(q) 0.5 eV
(C) Binding energy per nucleon	(r) 3 eV
(D) Photoelectric threshold of a metal	(s) 20 eV
	(t) 10 k eV
	(u) 8 M eV

8 Comprehension/Passage Based Questions

Passage-1

The β -decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

12. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron? **[2012]**

(a) $0 \le K \le 0.8 \times 10^6\, eV$

(b) $3.0\, eV \le K \le 0.8 \times 10^6\, eV$

(c) $3.0\, eV \le K < 0.8 \times 10^6\, eV$

(d) $0 \le K < 0.8 \times 10^6\, eV$

13. What is the maximum energy of the anti-neutrino? **[2012]**

(a) Zero

(b) Much less than 0.8×10^6 eV.

(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

Passage-2

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2H$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_1^2\text{H} + {}_1^2\text{H} \to {}_2^3\text{He} + n +$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away form the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following: Boltzmann constant

$k = 8.6 \times 10^{-5}$ eV/K; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm **[2009]**

14. In the core of nuclear fusion reactor, the gas becomes plasma because of

(a) strong nuclear force acting between the deuterons

(b) coulomb force acting between the deuterons

(c) coulomb force acting between deuteron-electron pairs

(d) the high temperature maintained inside the reactor core

15. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(a) $1.0 \times 10^9\,\text{K} < T < 2.0 \times 10^9\,\text{K}$

(b) $2.0 \times 10^9\,\text{K} < T < 3.0 \times 10^9\,\text{K}$

(c) $3.0 \times 10^9\,\text{K} < T < 4.0 \times 10^9\,\text{K}$

(d) $4.0 \times 10^9\,\text{K} < T < 5.0 \times 10^9\,\text{K}$

16. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(a) deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$ s

(b) deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s

(c) deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s

(d) deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

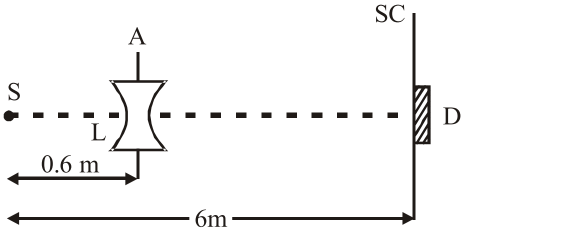
10 Subjective Problems

17. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find (a) atomic number of the nucleus. (b) the frequency of K_α line of the X-ray produced. ($R = 1.1 \times 10^7$ m^{-1} and $c = 3 \times 10^8$ m/s) **[2005 - 4 Marks]**

18. A nucleus at rest undergoes a decay emitting an α particle of de-Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α particles is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in amu. (1 amu = 931.470 MeV/c^2) **[2001-5 Marks]**

19. The element Curium $^{248}_{96}$Cm has a mean life of 10^{13} seconds. Its pirmary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α-decay are as follows: $^{248}_{96}$Cm = 248.072220u, $^{244}_{94}$Pu = 244.064100u and $^{4}_{2}$He=4.002603 u. Calculate the power output from a sample of 10^{20} Cm atoms. (1 u = 931 MeV/c^2.) **[1997 - 5 Marks]**

20. A monochromatic point source radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in fig. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9. **[1991 - 2 + 4 + 2 Marks]**



(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

(c) If the work function of the photoemissive surface is 1eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

21. How many electron, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24 ? **[1982 - 2 Marks]**

(i) number of electrons = (ii) number of protons =
(iii) number of neutrons =

Answer Key

Topic-1 : Composition and Size of the Nuclei

1. (d) **2.** (a) **3.** (c) **4.** (a) **5.** (b) **6.** (False) **7.** (c, d)

Topic-2 : Mass-Energy Equivalence and Nuclear Reactions

1. (d) **2.** (c) **3.** (d) **4.** (b) **5.** (d) **6.** (c) **7.** (c) **8.** (a) **9.** (c) **10.** (c)
11. (d) **12.** (Fusion, 24) **13.** (neutrino) **14.** (23.6) **15.** (b, d) **16.** (c, d) **17.** (a, d) **18.** (d)
19. (b, c)

Topic-3 : Radioactivity

1. (c) **2.** (a) **3.** (c) **4.** (b) **5.** (a) **6.** (c) **7.** (a) **8.** (c) **9.** (d) **10.** (d)
11. (b) **12.** (b) **13.** (c) **14.** (b) **15.** (a) **16.** (a) **17.** (c) **18.** (b) **19.** (b) **20.** (c)
21. (a) **22.** (d) **23.** (c) **24.** (d) **25.** (a) **26.** (a) **27.** (a) **28.** (c) **29.** (c) **30.** (b)
31. (5) **32.** (2) **33.** (4) **34.** (1) **35.** (8) **36.** (lithium, 7) **37.** (8, 6) **38.** (500 dps, 125 dps)
39. (a, c) **40.** (c) **41.** (d) **42.** (b) **43.** (b) **44.** (c) **45.** A→ r, t; B→ p, s; C→ p, q, r, t; D→ p, q, r, t
46. (c) **47.** A→ p, q; B→ p, r; C→ p, s; D→ p, q, r **48.** (a) **49.** (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (c) **2.** (c) **3.** (b) **4.** (9) **5.** (3) **6.** (atomic number, mass number) **7.** (a) **8.** (c)
9. (a) **10.** A→ p, q, t; B→ q; C→ s; D→ s **11.** A→ p; B→ t; C→ u; D→ r **12.** (d) **13.** (c) **14.** (d)
15. (a) **16.** (b)

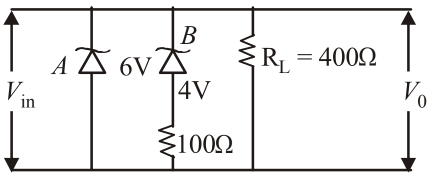
28 Semiconductor Electronics: Materials, Devices and Simple Circuits

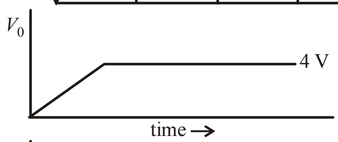
Topic-1 : Solid Semiconductors and P-N Junction Diode

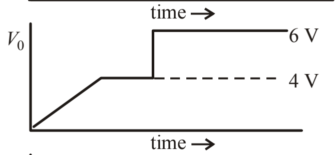
1 MCQs with One Correct Answer

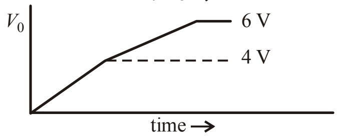
1. With increasing biasing voltage of a photodiode, the photocurrent magnitude : [Main Sep. 05, 2020 (I)]
   (a) remains constant
   (b) increases initially and after attaining certain value, it decreases
   (c) Increases linearly
   (d) increases initially and saturates finally

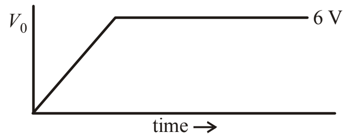
2. Two Zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit below. The output voltage V_0 variation with input voltage linearly increasing with time, is given by : ($V_{input} = 0$ V at $t = 0$) (figures are qualitative) [Main Sep. 05, 2020 (II)]



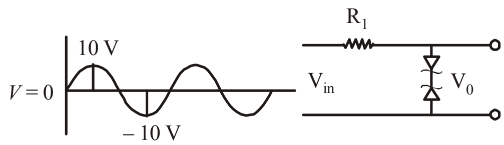
(a)


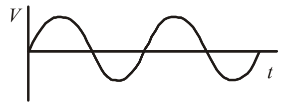
(b)


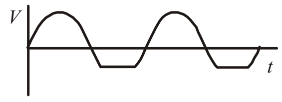
(c)


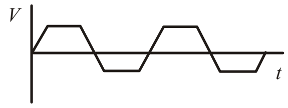
(d)


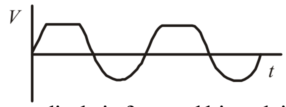
3. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale) [Main Sep. 04, 2020 (I)]



(a)


(b)


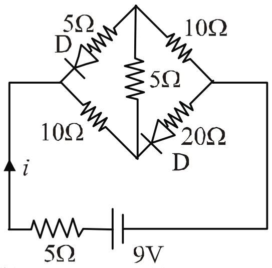
(c)


(d)


4. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is: [Main Sep. 03, 2020 (I)]
   (a) 300 Ω (b) 50 Ω
   (c) 100 Ω (d) 200 Ω

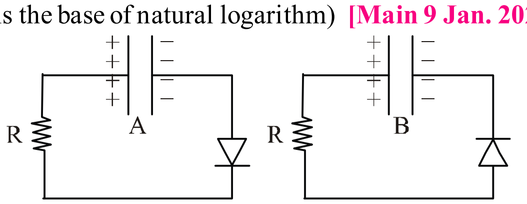
5. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is:
Planck's constant, $h = 6.63 \times 10^{-34}$ J.s.
Speed of light, $c = 3 \times 10^{8}$ m/s **[Main Sep. 03, 2020 (II)]**
(a) 1.1 eV (b) 2.0 eV
(c) 1.5 eV (d) 3.1 eV

6. The current i in the network is: **[Main 9 Jan. 2020 II]**



(a) 0.2 A (b) 0.6 A (c) 0.3 A (d) 0 A

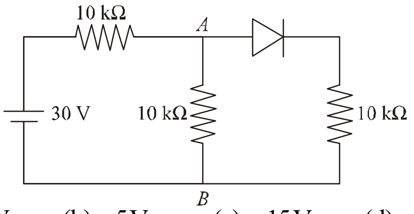
7. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time $t = 0$. If the charge on capacitors A and B at time $t = CR$ is Q_A and Q_B respectively, then (Here e is the base of natural logarithm) **[Main 9 Jan. 2020 II]**



(a) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$ (b) $Q_A = VC, Q_B = CV$

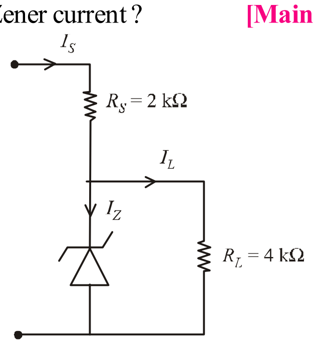
(c) $Q_A = VC, Q_B = \frac{VC}{e}$ (d) $Q_A = \frac{CV}{2}, Q_B = \frac{VC}{e}$

8. In the figure, potential difference between A and B is: **[Main 7 Jan. 2020 II]**



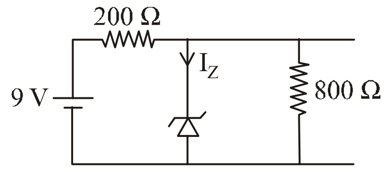
(a) 10 V (b) 5 V (c) 15 V (d) zero

9. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current? **[Main 12 Apr. 2019 II]**



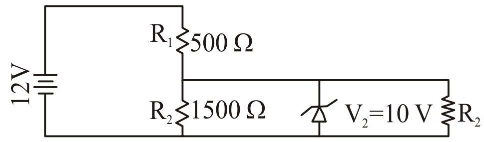
(a) 2.5 mA (b) 1.5 mA (c) 7.5 mA (d) 3.5 mA

10. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



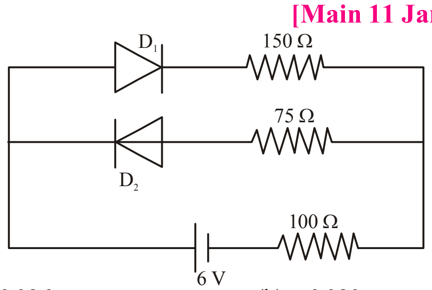
The current I_z through the Zener is : **[Main 8 April 2019 I]**
(a) 10 mA (b) 17 mA
(c) 15 mA (d) 7 mA

11. In the given circuit the current through Zener Diode is close to : **[Main 11 Jan. 2019 I]**



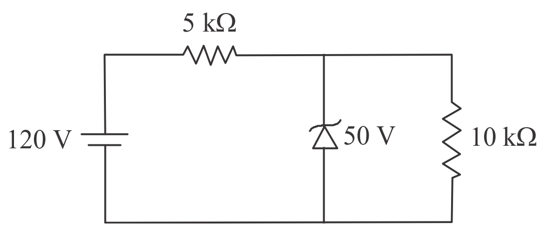
(a) 0.0 mA (b) 6.7 mA
(c) 4.0 mA (d) 6.0 mA

12. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6V, the current through the 100 Ω resistance (in Amperes) is : **[Main 11 Jan. 2019 II]**



(a) 0.036 (b) 0.020
(c) 0.027 (d) 0.030

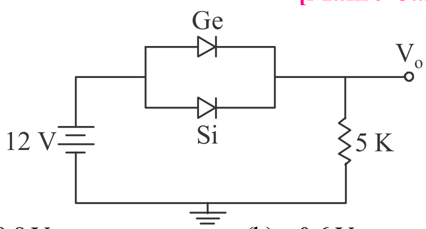
13. For the circuit shown below, the current through the Zener diode is: **[Main 10 Jan. 2019 II]**



(a) 9 mA (b) 5 mA
(c) Zero (d) 14 mA

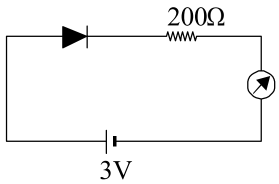
14. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 m^2/(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to: **[Main 9 Jan. 2019 I]**
(a) 2 Ωm (b) 4 Ωm (c) 0.4 Ωm (d) 0.2 Ωm

15. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by : (assume that the Ge diode has large breakdown voltage) **[Main 9 Jan. 2019 II]**



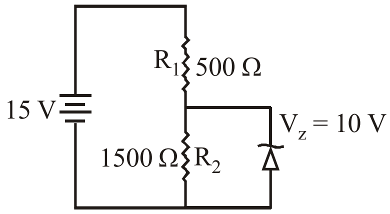
(a) 0.8 V (b) 0.6 V
(c) 0.2 V (d) 0.4 V

16. The reading of the ammeter for a silicon diode in the given circuit is : **[Main 2018]**



(a) 0 (b) 15 mA
(c) 11.5 mA (d) 13.5 mA

17. In the given circuit, the current through zener diode is: **[Main Online April 16, 2018]**



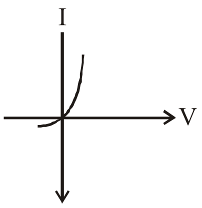
(a) 2.5mA (b) 3.3mA
(c) 5.5mA (d) 6.7mA

18. What is the conductivity of a semiconductor sample having electron concentration of $5 \times 10^{18}\ m^{-3}$, hole concentration of $5 \times 10^{19}\ m^{-3}$, electron mobility of $2.0\ m^2\ V^{-1}\ s^{-1}$ and hole mobility of $0.01\ m^2\ V^{-1}\ s^{-1}$? **[Main Online April 8, 2017]**

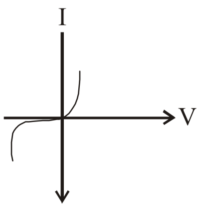
(Take charge of electron as 1.6×10^{-19} C)

(a) $1.68\ (\Omega - m)^{-1}$ (b) $1.83\ (\Omega - m)^{-1}$
(c) $0.59\ (\Omega - m)^{-1}$ (d) $1.20\ (\Omega - m)^{-1}$

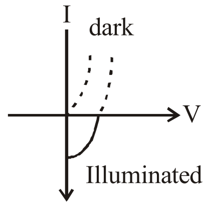
19. Identify the semiconductor devices whose characteristics are given below, in the order (i), (ii), (iii), (iv) : **[Main 2016]**



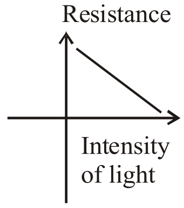
(i)



(ii)



(iii)



(iv)

(a) Solar cell, Light dependent resistance, Zener diode, simple diode
(b) Zener diode, Solar cell, simple diode, Light dependent resistance
(c) Simple diode, Zener diode, Solar cell, Light dependent resistance
(d) Zener diode, Simple diode, Light dependent resistance, Solar cell

20. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by : **[Main 2016]**

(a) Linear increase for Cu, exponential decrease of Si.
(b) Linear decrease for Cu, linear decrease for Si.
(c) Linear increase for Cu, linear increase for Si.
(d) Linear increase for Cu, exponential increase for Si.

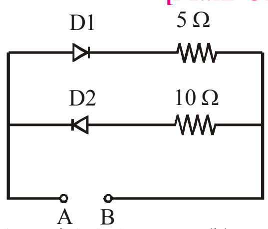
21. An experiment is performed to determine the 1–V characteristics of a Zener diode, which has a protective resistance of R = 100 Ω, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the DC source in the circuit is : **[Main Online April 9, 2016]**

(a) 0 – 5V (b) 0 – 24 V
(c) 0 – 12 V (d) 0 – 8V

22. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is : **[Main 2015]**

(a) 5.48 V/m (b) 7.75 V/m
(c) 1.73 V/m (d) 2.45 V/m

23. A 2V battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be: **[Main Online April 11, 2015]**



(a) 0.4 A and 0.2 A (b) 0.2 A and 0.4 A
(c) 0.1 A and 0.2 A (d) 0.2 A and 0.1 A

24. The forward biased diode connection is: **[Main 2014]**

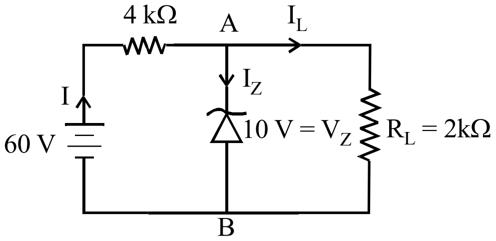
(a) 

(b) 

(c) 

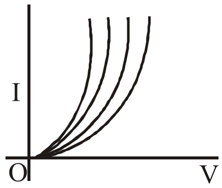
(d) 

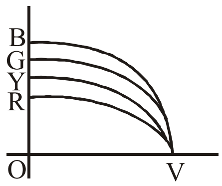
25. A Zener diode is connected to a battery and a load as show below: **[Main Online April 11, 2014]**

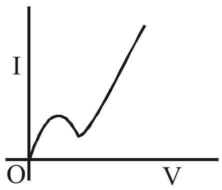


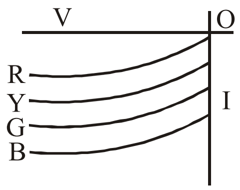
The currents, I, I_Z and I_L are respectively.
(a) 15 mA, 5 mA, 10 mA
(b) 15 mA, 7.5 mA, 7.5 mA
(c) 12.5 mA, 5 mA, 7.5 mA
(d) 12.5 mA, 7.5 mA, 5 mA

26. The I-V characteristic of an LED is **[Main 2013]**

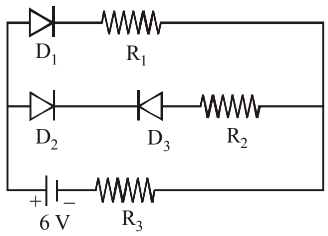
(a)


(b)


(c)


(d)


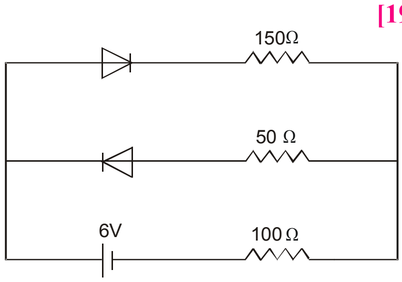
27. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance of 20 Ω and infinite backward resistance. Resistors $R_1 = R_2 = R_3 = 50\,\Omega$. Battery voltage is 6 V. The current through R_3 is : **[Main Online April 22, 2013]**



(a) 50 mA (b) 100 mA
(c) 60 mA (d) 25 mA

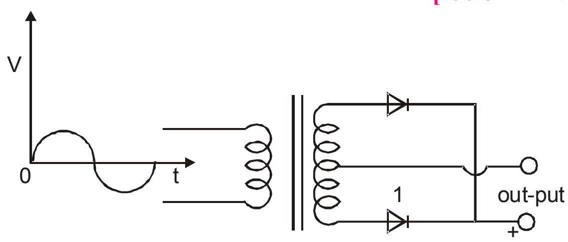
28. Which of the following statements is not true? **[1997 - 1 Mark]**
(a) The resistance of intrinsic semiconductors decrease with increase of temperature
(b) Doping pure Si with trivalent impurities give *p*-type semiconductors
(c) The majority carriers in *n*-type semiconductors are holes
(d) A *p-n* junction can act as a semiconductor diode

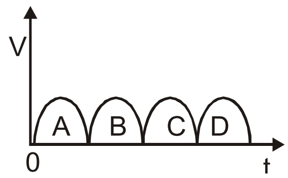
29. The circuit shown in the figure contains two diodes each with a forward resistance of 50 ohms and with infinite backward resistance. If the battery voltage is 6V, the current through the 100 ohm resistance (in amperes) is **[1997 - 1 Mark]**



(a) zero (b) 0.02 (c) 0.03 (d) 0.036.

30. A full-wave rectifier circuit along with the out-put is shown in Figure. The contribution (s) from the diode 1 is (are) **[1996 - 2 Marks]**



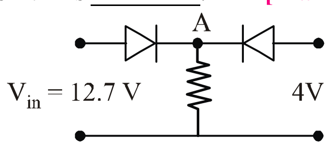


(a) C (b) A,C (c) B, D (d) A,B,C, D.

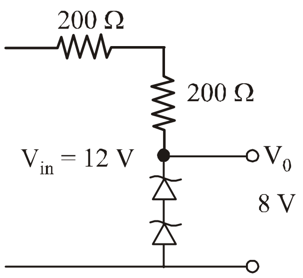
31. The probability of electrons to be found in the conduction band of an intrinsic semiconductor at a finite temperature **[1995S]**
(a) increases exponentially with increasing band gap
(b) decreases exponentially with increasing band gap
(c) decreases with increasing temperature
(d) is independent of the temperature and the band gap

3 Numeric Answer

32. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is ________. **[Main 9 Jan. 2020 I]**



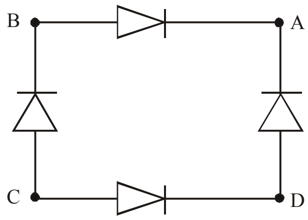
33. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both zener diodes are identical) _____. **[Main 9 Jan. 2020 II]**



4 Fill in the Blanks

34. In a ..... biased p-n junction, the net flow of holes is from the n region to the p region. **[1993 - 1 Mark]**

35. For the given circuit shown in fig. to act as full wave rectifier, the a.c. input should be connected across .... and .... and the d.c. output would appear across .... and .... **[1991 - 1 Mark]**

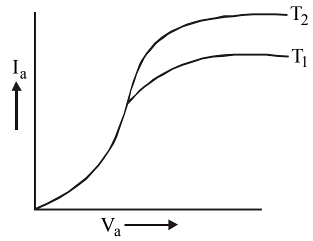


36. .... biasing of p-n junction offers high resistance to current flow across the junction. The biasing is obtained by connecting the p- side to the .... terminal of the battery. **[1990 - 2 Marks[**

37. In the forward bias arrangement of a p-n junction rectifier, the p end is connected to the ....terminal of the battery and the direction of the current is from ....to ....in the rectifier. **[1988 - 2 Marks]**

5 True / False

38. For a diode the variation of its anode current I_a with the anode voltage V_a at two different cathode temperatures T_1 and T_2 is shown in the figure. The temperature T_2 is greater than T_1. **[1986 - 3 Marks]**



6 MCQs with One or More than One Correct Answer

39. In a p-n junction diode not connected to any circuit, **[1998S - 2 Marks]**

(a) the potential is the same everywhere
(b) the p-type side is at a higher potential than the n-type side
(c) there is an electric field at the junction directed from the n-type side to the p-type side
(d) there is an electric field at the junction directed from the p-type side to the n-type side

40. Holes are charge carriers in **[1996 - 2 Marks]**

(a) intrinsic semiconductors (b) ionic solids
(c) p-type semiconductors (d) metals

41. Two identical p-n junctions may be connected in series with a battery in three ways, fig. The potential drops across the two $p-n$ junctions are equal in **[1989 - 2 Marks]**

42. The impurity atoms with which pure silicon should be doped to make a p-type semiconductor are those of **[1988 - 2 Marks]**

(a) phosphorus (b) boron
(c) antimony (d) aluminium

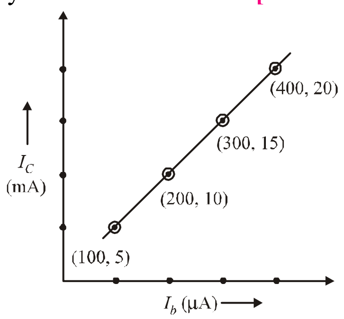
43. Select the correct statement from the following

(a) A diode can be used as a rectifier **[1984- 2 Marks]**
(b) A triode cannot be used as a rectifier
(c) The current in a diode is always proportional to the applied voltage
(d) The linear portion of the I–V characteristic of a triode is used for amplification without distortion

Topic-2 : Junction Transistor

1 MCQs with One Correct Answer

1. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 k Ω respectively, is shown in the figure. The Voltage and Power gain, are respectively : **[Main 12 Apr. 2019 I]**



(a) $2.5\times10^4, 2.5\times10^6$ (b) $5\times10^4, 5\times10^6$
(c) $5\times10^4, 5\times10^5$ (d) $5\times10^4, 2.5\times10^6$

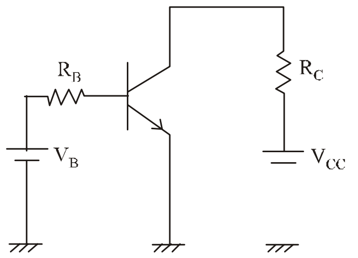
2. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is 10 kΩ. The common emitter current gain β is : **[Main 10 Apr. 2019 I]**

(a) 10^2 (b) 60 (c) 6×10^2 (d) 10^4

3. An NPN transistor is used in common emitterconfiguration as an amplifier with 1 k &! load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 ¼A change in the base current of the amplifier. The input resistance and voltage gain are: **[Main 9 April 2019 I]**

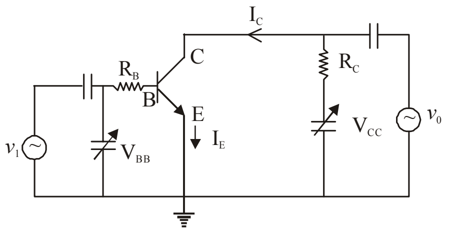
(a) 0.33 k Ω 1.5 (b) 0.67 k Ω 300
(c) 0.67 k Ω 200 (d) 0.33 k Ω 300

4. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, R_C = 1 k&! and V_{CC} = 10V. What is the minimum base current for V_{CE} to reach saturation ? **[Main 8 Apr. 2019 II]**



(a) 40 μA (b) 100 μA
(c) 7 μA (d) 10 μA

5.



In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, V_{CC} = 5 V, β_{dc} = 200, R_B = 100 kΩ, R_C = 1 KΩ and V_{BE} = 1.0 V. The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively : **[Main 12 Jan. 2019 II]**

(a) 25 μA and 3.5 V (b) 20 μA and 3.5 V
(c) 25 μA and 2.8 V (d) 20 μA and 2.8 V

6. In a common emitter configuration with suitable bias, it is given than R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by:
(β is current gain, I_B, I_C, I_E are respectively base, collector and emitter currents:) **[Main Online April 15, 2018]**

(a) $\beta\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_E}{\Delta I_B}, \beta^2\dfrac{R_L}{R_{BE}}$ (b) $\beta^2\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta\dfrac{R_L}{R_{BE}}$

(c) $\beta^2\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_E}, \beta^2\dfrac{R_L}{R_{BE}}$ (d) $\beta\dfrac{R_L}{R_{BE}}, \dfrac{\Delta I_C}{\Delta I_B}, \beta^2\dfrac{R_L}{R_{BE}}$

7. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be : **[Main Online April 2, 2017]**

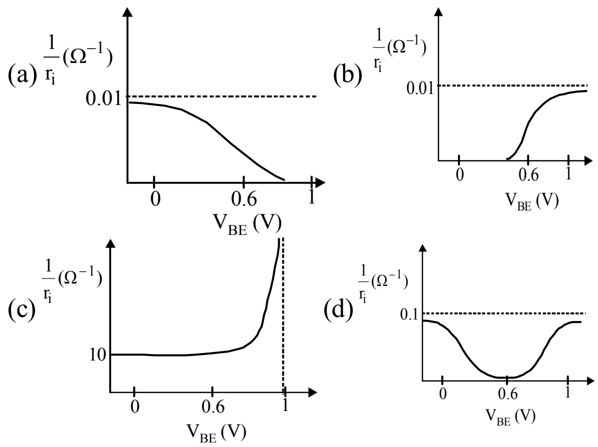
(a) 135° (b) 180° (c) 45° (d) 90°

8. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is : **[Main 2016]**

(a) $\alpha = \dfrac{\beta}{1+\beta}$ (b) $\alpha = \dfrac{\beta^2}{1+\beta^2}$

(c) $\dfrac{1}{\alpha} = \dfrac{1}{\beta} + 1$ (d) All of these

9. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configuration is : **[Main Online April 10, 2016]**



10. An n-p-n transistor has three leads A, B and C. Connecting B and C by moist fingers, A to the positive lead of an ammeter, and C to the negative lead of the ammeter, one finds large deflection. Then, A, B and C refer respectively to: **[Main Online April 9, 2014]**

(a) Emitter, base and collector
(b) Base, emitter and collector
(c) Base, collector and emitter
(d) Collector, emitter and base.

11. For a given plate voltage, the plate current in a triode valve is maximum when the potential of **[1985 - 2 Marks]**

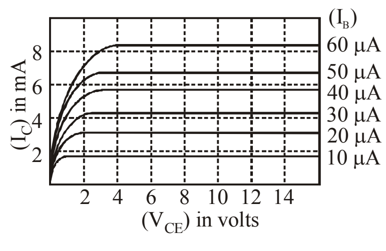
(a) the grid is positive and plate is negative
(b) the grid is zero and plate is positive
(c) the grid is negative and plate is positive
(d) the grid is positive and plate is positive

12. The plate resistance of a triode is 3×10^3 ohms and its mutual conductance is 1.5×10^{-3} amp/volt. The amplification factor of the triode is **[1981- 2 - 3 Marks]**

(a) 5×10^{-5} (b) 4.5 (c) 45 (d) 2×10^5

3 Numeric Answer

13. The output characteristics of a transistor is shown in the figure. When V_{CE} is 10 V and I_C = 4.0 mA, then value of β_{ac} is _______. **[Main Sep. 06, 2020 (II)]**



6 MCQs with One or More than One Correct Answer

14. A transistor is used in the common emitter mode as an amplifier. Then **[1998S - 2 Marks]**

(a) the base-emitter junction is forward-biased
(b) the base-emitter junction is reverse-biased
(c) the input signal is connected in series with the voltage applied to bias the base-emitter junction
(d) the input signal is connected in series with the voltage applied to bias the base-collector junction

15. In an *n-p-n* transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector,
(a) the emitter current will be 9 mA **[1992 - 2 Marks]**
(b) the base current will be 1 mA
(c) the emitter current will be 11 mA
(d) the base current will be – 1 mA

10 Subjective Problems

16. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of – 1 volt the anode current I (in milli amperes) is given in terms of plate voltage V (in volts) by the algebraic relation :

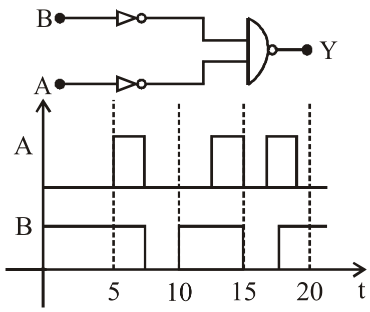
$$I = 0.125V - 7.5$$

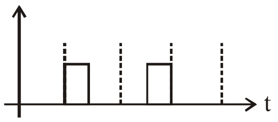
For grid voltage of – 3 volts, the current at anode voltage of 300 volts is 5 milliampere. Determine the plate resistance (r_p), transconductance (g_m) and the amplification factor (μ) for the triode. **[1987 - 7 Marks]**

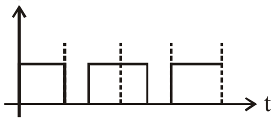
Topic-3 : Digital Electronics and Logic Gates

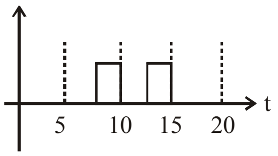
1 MCQs with One Correct Answer

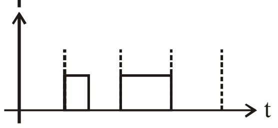
1. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B. **[Main Sep. 06, 2020 (I)]**



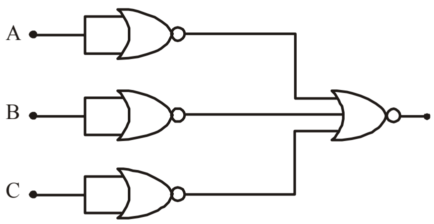
(a)


(b)


(c)


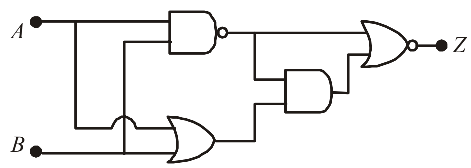
(d)


2. Identify the operation performed by the circuit given below: **[Main Sep. 04, 2020 (II)]**



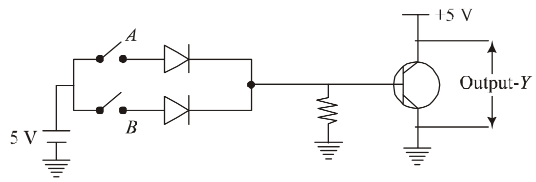
(a) NAND (b) OR (c) AND (d) NOT

3. In the following digitial circuit, what will be the output at 'Z', when the input (A, B) are (1, 0), (0, 0), (1, 1), (0, 1) : **[Main Sep. 02, 2020 (II)]**



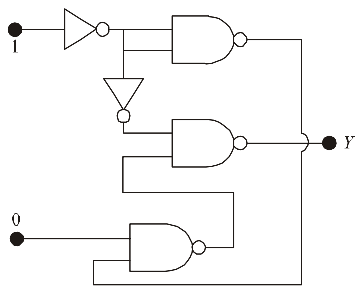
(a) 0, 0, 1, 0 (b) 1, 0, 1, 1 (c) 1, 1, 0, 1 (d) 0, 1, 0, 0

4. Boolean relation at the output stage-Y for the following circuit is: **[Main 8 Jan. 2020 I]**



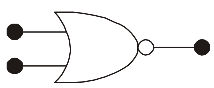
(a) $\overline{A}+\overline{B}$ (b) $A+B$ (c) $A.B$ (d) $\overline{A}.\overline{B}$

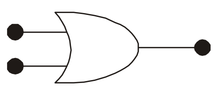
5. In the given circuit, value of Y is: **[Main 8 Jan. 2020 II]**

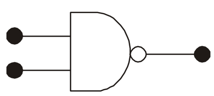


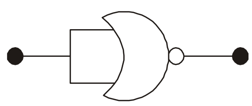
(a) 0 (b) toggles between 0 and 1
(c) will not execute (d) 1

6. Which of the following gives a reversible operation? **[Main 7 Jan. 2020 I]**

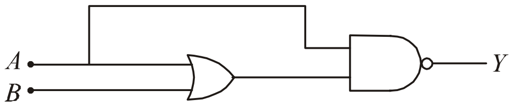
(a)


(b)


(c)


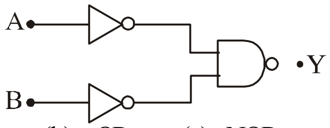
(d)


7. The truth table for the circuit given in the fig. is : **[Main 9 April 2019 I]**



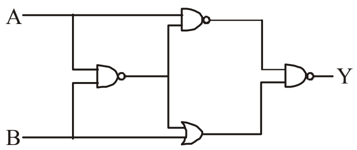
(a) $\begin{bmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} A & B & Y \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

8. The logic gate equivalent to the given logic circuit is: **[Main 9 Apr. 2019 II]**



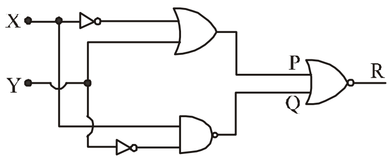
(a) NAND (b) OR (c) NOR (d) AND

9. The ouput of the given logic cirfcuit is: **[Main 12 Jan. 2019 I]**



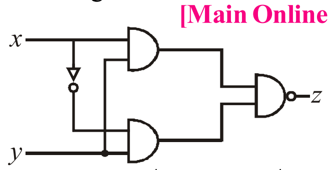
(a) $A\overline{B}+\overline{A}B$ (b) $AB+\overline{A}\,\overline{B}$
(c) $A\overline{B}$ (d) $\overline{A}B$

10. To get output '1' at R, for the given logic gate circuit the input values must be: **[Main 10 Jan. 2019 I]**



(a) X = 0, Y = 1 (b) X = 1, Y = 1
(c) X = 1, Y = 0 (d) X = 0, Y = 0

11. Truth table for the given circuit will be **[Main Online April 15, 2018]**



(a)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(b)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

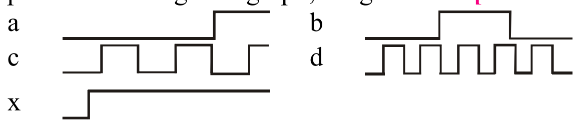
(c)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(d)

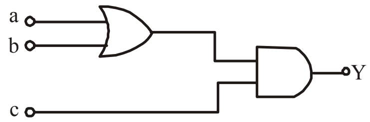
x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

12. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is : **[Main 2016]**



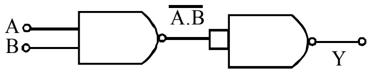
(a) OR (b) NAND
(c) NOT (d) AND

13. To get an output of 1 from the circuit shown in figure the input must be : **[Main Online April 10, 2016]**



(a) a = 0, b = 0, c = 1 (b) a = 1, b = 0, c = 0
(c) a = 1, b = 0, c = 1 (d) a = 0, b = 1, c = 0

14. Identify the gate and match A, B, Y in bracket to check. **[Main Online April 9, 2014]**



(a) AND (A = 1, B = 1, Y = 1)
(b) OR (A = 1, B = 1, Y = 0)
(c) NOT (A = 1, B = 1, Y = 1)
(d) XOR (A = 0, B = 0, Y = 0)

Answer Key

Topic-1 : Solid Semiconductors and P-N Junction Diode

1. (d) **2.** (c) **3.** (c) **4.** (c) **5.** (d) **6.** (c) **7.** (c) **8.** (a) **9.** (d) **10.** (a)
11. (a) **12.** (b) **13.** (a) **14.** (c) **15.** (d) **16.** (c) **17.** (b) **18.** (a) **19.** (c) **20.** (a)
21. (c) **22.** (d) **23.** (a) **24.** (a) **25.** (d) **26.** (a) **27.** (a) **28.** (c) **29.** (b) **30.** (b)
31. (b) **32.** (12) **33.** (40) **34.** (Reverse) **35.** (B & D; A & C) **36.** (Reverse, nagative)
37. (+(ve) *p*-side, *n*-side) **38.** True **39.** (c) **40.** (a,c)
41. (b) **42.** (b, d) **43.** (a)

Topic-2 : Junction Transistor

1. (Bonus) **2.** (a) **3.** (b) **4.** (a) **5.** (a) **6.** (d) **7.** (b) **8.** (b) **9.** (c) **10.** (c)
11. (d) **12.** (b) **13.** (150) **14.** (a,c) **15.** (b,c)

Topic-3 : Digital Electronics and Logic Gates

1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (a) **6.** (d) **7.** (c) **8.** (b) **9.** (c) **10.** (c)
11. (c) **12.** (a) **13.** (c) **14.** (a)

29 Communication Systems

Topic-1 : Communication Systems

1 MCQs with One Correct Answer

1. An amplitude modulated wave is represented by the expression $v_m = 5(1 + 0.6\cos 6280t)\sin(211 \times 10^4 t)$ volts
The minimum and maximum amplitudes of the amplitude modulated wave are, respectively : **[Main Sep. 02, 2020 (I)]**
(a) $\frac{3}{2}$ V, 5 V (b) $\frac{5}{2}$ V, 8 V
(c) 5 V, 8 V (d) 3 V, 5 V

2. In an amplitude modulator circuit, the carrier wave is given by, $C(t) = 4 \sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2 \sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are : **[Main 12 April 2019 (II)]**
(a) 0.5 and 10 kHz (b) 0.4 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

3. A message signal of frequency 100 MHz and peak voltage100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are : **[Main 10 April 2019 (II)]**
(a) 4 ; 1×10^8 Hz (b) 4 ; 2×10^8 Hz
(c) 0.25 ; 2×10^8 Hz (d) 0.25 ; 1×10^{-8}

4. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is: **[Main 9 April 2019 (I)]**
(a) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$
(b) $v_0 \sin[\omega_0(1 + 0.01\, A\sin\omega t)t]$
(c) $v_0 \sin\omega_0 t + A\cos\omega t$
(d) $(v_0 + A)\cos\omega t \sin\omega_0 t$

5. The physical sizes of the transmitter and receiver antenna in a communication system are: **[Main 9 April 2019 (II)]**
(a) independent of both carrier and modulation frequency
(b) inversely proportional to carrier frequency
(c) inversely proportional to modulation frequency
(d) proportional to carrier frequency

6. The wavelength of the carrier waves in a modern optical fiber communication network is close to : **[Main 8 April 2019 (I)]**
(a) 2400 nm (b) 1500 nm (c) 600 nm (d) 900 nm

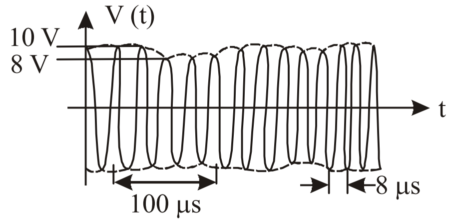
7. In a line of sight ratio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70m, then the minimum height of the transmitting antenna should be : **[Main 8 April 2019 (II)]**
(Radius of the Earth = 6.4×10^6 m).
(a) 20 m (b) 51 m
(c) 32 m (d) 40 m

8. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index? **[Main 12 Jan. 2019 (I)]**
(a) 0.3 (b) 0.5 (c) 0.6 (d) 0.4

9. To double the covering range of a TV transmittion tower, its height should be multiplied by: **[Main 12 Jan 2019 (II)]**
(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 4 (d) $\sqrt{2}$

10. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given $\pi = 22/7$] **[Main 11 Jan 2019 (II)]**
(a) 1785 and 1715 (b) 178.5 and 171.5
(c) 89.25 and 85.75 (d) 892.5 and 857.5

11. An amplitude modulated signal is plotted below :



Which one of the following best describes the above signal? **[Main 11 Jan. 2019 (II)]**

(a) $(9+\sin(2.5\pi\times10^5 t))\sin(2\pi\times10^4 t)$V

(b) $(1+9\sin(2\pi\times10^4 t))\sin(2.5\pi\times10^5 t)$ V

(c) $(9+\sin(2\pi\times10^4 t))\sin(2.5\pi\times10^5 t)$V

(d) $(9+\sin(4\pi\times10^4 t))\sin(5\pi\times10^5 t)$ V

12. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Given : radius of earth = 6.4×10^6 m). **[Main 10 Jan. 2019 (I)]**

(a) 65 km (b) 48 km (c) 80 km (d) 40 km

13. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot? **[Main 10 Jan. 2019 (I)]**

(a) 2750 kHz (b) 2900 kHz
(c) 2250 kHz (d) 2000 kHz

14. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3\times10^8$m/s, $h = 6.6\times10^{-34}$J-s) **[Main 9 Jan. 2019 (II)]**

(a) 3.75×10^6 (b) 3.86×10^6
(c) 6.25×10^5 (d) 4.87×10^5

15. A telephonic communication service is working at carrier frequency of 10 GHz. Only10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz? **[Main 2018]**

(a) 2×10^3 (b) 2×10^4 (c) 2×10^5 (d) 2×10^6

16. A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80% will be: **[Main 2018]**

(a) 11.2 V (b) 7 V (c) 22.4 V (d) 28 V

17. The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be: **[Main Online April 15, 2018]**

(a) 20 (b) 10 (c) 8 (d) 15

18. The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49μH and a capactiance of 2.5nF . It is modulated by an audio signal of 12kHz. The frequency range occupied by the side bands is: **[Main Online April 15, 2018]**

(a) 18kHz – 30kHz (b) 63kHz – 75kHz
(c) 442kHz – 466kHz (d) 13482kHz – 13494kHz

19. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m . The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m < \omega_c$. Which of the following frequencies is not contained in the modulated wave ? **[Main 2017]**

(a) $\omega_m + \omega_c$ (b) $\omega_c - \omega_m$
(c) ω_m (d) ω_c

20. A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length 1 of the antenna and effective power radiated P_{eff} will be given respectively as :
(K is a constant of proportionality) **[Main Online April 9, 2017]**

(a) $\lambda, P_{eff} = K\left(\frac{1}{\lambda}\right)^2$ (b) $\frac{\lambda}{8}, P_{eff} = K\left(\frac{1}{\lambda}\right)$

(c) $\frac{\lambda}{16}, P_{eff} = K\left(\frac{1}{\lambda}\right)^3$ (d) $\frac{\lambda}{5}, P_{eff} = K\left(\frac{1}{\lambda}\right)^{\frac{1}{2}}$

21. A signal of frequency 20 kHz and peak voltage of 5 Volt is used to modulate a carrier wave of frequency 1.2 MHz and peak voltage 25 Volts. Choose the correct statement. **[Main Online April 8, 2017]**

(a) Modulation index = 5, side frequency bands are at 1400 kHz and 1000 kHz

(b) Modulation index = 5, side frequency bands are at 21.2 kHz and 18.8 kHz

(c) Modulation index=0.8, side frequency bands are at 1180 kHz and 1220 kHz

(d) Modulation index=0.2, side frequency bands are at 1220 kHz and 1180 kHz

22. Choose the correct statement : **[Main 2016]**

(a) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(b) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(c) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(d) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

23. A modulated signal $C_m(t)$ has the form $C_m(t) = 30 \sin 300\pi t + 10 (\cos 200\pi t - \cos 400\pi t)$. The carrier frequency f_c, the modulating frequency (message frequency) f_ω and the modulation index μ are respectively given by : **[Main Online April 10, 2016]**

(a) $f_c = 200$ Hz; $f_w = 50$ Hz; $\mu = \frac{1}{2}$

(b) $f_c = 150$ Hz; $f_w = 50$ Hz; $\mu = \frac{2}{3}$

(c) $f_c = 150$ Hz; $f_w = 30$ Hz; $\mu = \frac{1}{3}$

(d) $f_c = 200$ Hz; $f_w = 30$ Hz; $\mu = \frac{1}{2}$

24. An audio signal consists of two distinct sounds: one a human speech signal in the frequency band of 200 Hz to 2700 Hz, while the other is a high frequency music signal in the frequency band of 10200 Hz to 15200 Hz. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth requried to send just the human speech is : **[Main Online April 9, 2016]**

(a) 2 (b) 5 (c) 6 (d) 3

25. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are : **[Main 2015]**

(a) 2005 kHz, 2000 kHz and 1995 kHz

(b) 2000 kHz and 1995 kHz

(c) 2 MHz only

(d) 2005 kHz and 1995 kHz

26. Long range radio transmission is possible when the radio waves are reflected from the ionosphere. For this to happen the frequency of the radio waves must be in the range: **[Main Online April 19, 2014]**

(a) 80 - 150 MHz (b) 8 - 25 MHz

(c) 1 - 3 MHz (d) 150 - 1500 kHz

27. For sky wave propagation, the radio waves must have a frequency range in between: **[Main Online April 12, 2014]**

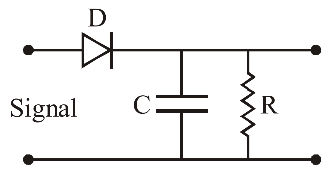
(a) 1 MHz to 2 MHz (b) 5 MHz to 25 MHz

(c) 35 MHz to 40 MHz (d) 45 MHz to 50 MHz

28. A transmitting antenna at the top of a tower has height 32 m and height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode? **[Main Online April 9, 2014]**

(a) 55.4 km (b) 45.5 km (c) 54.5 km (d) 455 km

29. A diode detector is used to detect an amplitudemodulated wave of 60% modulation by using a condenser of capacity 250 picofarad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it. **[Main 2013]**



(a) 10.62 MHz (b) 10.62 kHz

(c) 5.31 MHz (d) 5.31 kHz

30. Which of the following modulated signal has the best noise-tolerance ? **[Main Online April 25, 2013]**

(a) Long-wave (b) Short-wave

(c) Medium-wave (d) Amplitude-modulated

31. Which of the following statement is NOT correct? **[Main Online April 23, 2013]**

(a) Ground wave signals are more stable than the sky wave signals.

(b) The critical frequency of an ionospheric layer is the highest frequency that will be reflected back by the layer when it is vertically incident.

(c) Electromagnetic waves of frequencies higher than about 30 MHz cannot penetrate the ionosphere.

(d) Sky wave signals in the broadcast frequency range are stronger at night than in the day time.

32. This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-1: Short wave transmission is achieved due to the total internal reflection of the e-m wave from an appropriate height in the ionosphere.

Statement-2: Refractive index of a plasma is independent of the frequency of e-m waves. **[Main Online April 22, 2013]**

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true but Statement -2 is **not** the correct explanation of statement-1.

(d) Statement-1 is true, Statement-2 is true and Statement -2 is the correct explanation of Statement-1.

33. If a carrier wave $c(t) = A \sin \omega_c t$ is amplitude modulated by a modulator signal $m(t) = A \sin \omega_m t$ then the equation of modulated signal $[C_m(t)]$ and its modulation index are respectively **[Main Online April 9, 2013]**

(a) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$ and 2

(b) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_m t$ and 1

(c) $C_m(t) = A(1 + \sin \omega_m t) \sin \omega_c t$ and 1

(d) $C_m(t) = A(1 + \sin \omega_c t) \sin \omega_m t$ and 2

Answer Key

Topic-1 : Communication Systems

1. (b)	2. (d)	3. (c)	4. (a)	5. (b)	6. (b)	7. (c)	8. (c)	9. (c)	10. (c)
11. (c)	12. (a)	13. (d)	14. (c)	15. (c)	16. (a)	17. (b)	18. (c)	19. (c)	20. (a)
21. (d)	22. (c)	23. (b)	24. (c)	25. (a)	26. (b)	27. (b)	28. (b)	29. (b)	30. (b)
31. (c)	32. (a)	33. (c)							

Hints & Solutions

1 Physical World, Units and Measurements

Topic-1 : Unit of Physical Quantities

1. **(a)** Density of material in SI unit,

$= \dfrac{128\text{kg}}{\text{m}^3}$

Density of material in new system

$= \dfrac{128(50\,\text{g})(20)}{(25\,\text{cm})^3(4)^3} = \dfrac{128}{64}(20) = 40\,\text{units}$

2. **(b)** According to question

$E_y \propto J_x B_Z$

$\therefore$ Constant of proportionality

$K = \dfrac{E_y}{B_Z J_x} = \dfrac{C}{J_x} = \dfrac{m^3}{As}$

[As $\dfrac{E}{B} = C$ (speed of light) and $J = \dfrac{I}{\text{Area}}$]

3. **(a, b, c, d)**

(a) Inductance, $L = \dfrac{\text{Flux}(\phi)}{\text{Current}(I)} = \dfrac{\text{weber}}{\text{ampere}} \Rightarrow$ henry

(b) $\varepsilon = L\left(\dfrac{di}{dt}\right) \Rightarrow [L] = \dfrac{\varepsilon}{di/dt} = \dfrac{\text{volt}\times\text{sec}}{\text{amp}} \Rightarrow H = \dfrac{Vs}{A}$

(c) $U = \dfrac{1}{2}LI^2 \Rightarrow L = \dfrac{2U}{I^2} = \dfrac{J}{A^2} \Rightarrow H = \dfrac{J}{A^2}$

(d) $L = \dfrac{\varepsilon}{di/dt} = \left(\dfrac{\varepsilon}{di}\right)dt = \text{ohm}\times\text{sec}$

$\therefore$ henry = ohm – sec

4. **A → p, q**

Using $F = \dfrac{GM_eM_s}{r^2} \Rightarrow GM_eM_s = Fr^2 = \text{Nm}^2 = \text{kg}\dfrac{m}{s^2}\times m^2$

$= \text{kg m}^3\text{s}^{-2}$

Also (volt) (coulomb) (metre) = (joule) (metre)

($\because$ volt = Joule/coulomb)

$= (N - m)(m) = \text{Nm}^2 = \text{kg m}^3\text{s}^{-2}$

B → r, s

Using $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} \Rightarrow v_{rms}^2 = \dfrac{3RT}{M}$

Unit of $\dfrac{3RT}{M}$ is $m^2\ s^{-2}$

Also (farad) (volt)^2 (kg)$^{-1}$ = (joule) kg^{-1} ($\because u = \dfrac{1}{2}cv^2$)

$= \text{N-m kg}^{-1} = \text{kg ms}^{-2}\text{ m kg}^{-1} = \text{m}^2\text{s}^{-2}$

C → r, s

Using $F = qvB \Rightarrow v^2 = \dfrac{F^2}{q^2B^2}$

$\therefore$ Unit of v^2 is m^2s^{-2} which is further equal to FV^2 kg^{-1}.

D → r, s

Reason : Escape velocity $v_e = \sqrt{\dfrac{2GM}{R}} \Rightarrow v_e^2 = \dfrac{2GM}{R}$

$\therefore$ Unit of $\dfrac{GM}{R}$ is m^2 s^{-2}.

5. Capacitance $C = \dfrac{q}{v} = \dfrac{q}{\frac{\omega}{q}}$ coulomb-volt^{-1}, coulomb^2-joule^{-1}

Inductance $\dfrac{L}{R} = t$ and $R = \dfrac{v}{I}$ ohm-sec, volt-second (ampere)$^{-1}$

Magnetic Induction $F = IlB \Rightarrow B = \dfrac{F}{Il}$ newton (ampere-metre)$^{-1}$

6. (i) The M.K.S. unit of Young's modulus $\left(Y = \dfrac{F}{A}\Big/\dfrac{\Delta l}{l_0}\right)$ is Nm^{-2}.

(ii) The M.K.S. unit of magnetic induction $\left(B = \dfrac{\phi}{A}\right)$ is tesla or wb/m^2.

(iii) The M.K.S. unit of power of lens $P = \dfrac{1}{f\,(\text{in metre})}$ is dioptre or m^{-1}.

Topic-2 : Dimensions of Physical Quantities

1. **(a)** We know that

Speed of light, $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} = x$

Also, $c = \dfrac{E}{B} = y$

Time constant, $\tau = Rc = t$

$\therefore z = \dfrac{l}{Rc} = \dfrac{l}{t} = \text{Speed}$

Thus, x, y, z will have the same dimension of speed.

2. **(d)** From formula, $\dfrac{dQ}{dt} = kA\dfrac{dT}{dx}$

$$\Rightarrow k = \frac{\left(\frac{dQ}{dt}\right)}{A\left(\frac{dT}{dx}\right)},\ [k] = \frac{[ML^2T^{-3}]}{[L^2][KL^{-1}]} = [MLT^{-3}K^{-1}]$$

3. **(c)** Dimension of Force $F = M^1L^1T^{-2}$

Dimension of velocity $V = L^1T^{-1}$

Dimension of work $= M^1L^2T^{-2}$

Dimension of length = L

Moment of inertia = ML^2

$$\therefore x = \frac{IFv^2}{WL^4} = \frac{(M^1L^2)(M^1L^1T^{-2})(L^1T^{-2})^2}{(M^1L^2T^{-2})(L^4)}$$

$$= \frac{M^1L^{-2}T^{-2}}{L^3} = M^1L^{-1}T^{-2} = \text{Energy density}$$

4. **(b)** Solar constant $= \dfrac{\text{Energy}}{\text{Time Area}}$

Dimension of Energy, $E = ML^2T^{-2}$

Dimension of Time = T

Dimension of Area = L^2

$\therefore$ Dimension of Solar constant

$$= \frac{M^1L^2T^{-2}}{TL^2} = M^1L^0T^{-3}.$$

5. **(d)** Young's modulus, $Y = \dfrac{\text{stress}}{\text{strain}}$

$$\Rightarrow Y = \frac{F}{A}\Big/\frac{\Delta\ell}{\ell_0} = FA^{-1}V^0$$

6. **(c)** Energy, $E \propto A^a T^b P^c$
or, $E = kA^a T^b P^c$ ...(i)
where k is a dimensionless constant and a, b and c are the exponents.
Dimension of momentum, $P = M^1 L^1 T^{-1}$
Dimension of area, $A = L^2$
Dimension of time, $T = T^1$
Putting these value in equation (i), we get
$M^1 L^2 T^{-2} = M^c L^{2a+c} T^{b-c}$
by comparison
$c = 1$
$2a + c = 2$
$b - c = -2$
$c = 1, a = 1/2, b = -1 \quad \therefore \; E = A^{1/2} T^{-1} P^1$

7. **(a)** $\sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \sqrt{\dfrac{\mu_0^2}{\varepsilon_0 \mu_0}} = \mu_0 c \quad \left(\dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = c\right)$
$\mu_0 c \rightarrow MLT^{-2}A^{-2} \times LT^{-1}$
$ML^2T^{-3}A^{-2}$
Dimensions of resistance

8. **(a)** $X = 5YZ^2$
$\Rightarrow Y \propto \dfrac{X}{Z^2}$ ...(i)
$X = \text{Capacitance} = \dfrac{Q}{V} = \dfrac{Q^2}{W} = \dfrac{[A^2T^2]}{[ML^2T^{-2}]}$
$X = [M^{-1}L^{-2}T^4A^2]$
$Z = B = \dfrac{F}{IL} \quad [\because F = ILB]$
$Z = [MT^{-2}A^{-1}]$
$Y = \dfrac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$
$Y = [M^{-3}L^{-2}T^8A^4]$ (Using (i))

9. **(d)** $\left[\sqrt{\dfrac{\varepsilon_0}{\mu_0}}\right] = \sqrt{\dfrac{\varepsilon_0^2}{\mu_0 \varepsilon_0}} = \left[\dfrac{\varepsilon_0}{\sqrt{\mu_0 \varepsilon_0}}\right] = \varepsilon_0 C[LT^{-1}] \times [\varepsilon_0]$
$\left[\because \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = C\right]$
$\because F = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$
$\Rightarrow [\varepsilon_0] = \dfrac{[AT]^2}{[MLT^{-2}] \times [L^2]} = [A^2M^{-1}L^{-3}T^4]$
$\therefore \left[\sqrt{\dfrac{\varepsilon_0}{\mu_0}}\right] = [LT^{-1}] \times [A^2M^{-1}L^{-3}T^4] = [M^{-1}L^{-2}T^3A^2]$

10. **(b)** As we know,
$\left[\dfrac{\ell}{r}\right] = [T]$ and $[cv] = [AT]$
$\therefore \left[\dfrac{\ell}{rcv}\right] = \left[\dfrac{T}{AT}\right] = [A^{-1}]$

11. **(b)** Force of interaction between two atoms, $F = \alpha\beta e^{\left(\frac{-x^2}{\alpha kT}\right)}$
Since exponential terms are dimensionless
$\therefore \left[\dfrac{x^2}{\alpha kT}\right] = M^0L^0T^0$
$\Rightarrow \dfrac{L^2}{[\alpha]ML^2T^{-2}} = M^0L^0T^0 \quad \Rightarrow [\alpha] = M^{-1}T^2$
$[F] = [\alpha][\beta]$
$MLT^{-2} = M^{-1}T^2[\beta] \quad \Rightarrow [\beta] = M^2LT^{-4}$

12. **(d)** Let $[Y] = [V]^a [F]^b [A]^c$
$[ML^{-1}T^{-2}] = [LT^{-1}]^a [MLT^{-2}]^b [LT^{-2}]^c$
$[ML^{-1}T^{-2}] = [M^bL^{a+b+c} T^{-a-2b-2c}]$
Comparing power both side of similar terms we get,
$b = 1, a + b + c = -1, -a - 2b - 2c = -2$
solving above equations we get:
$a = -4, b = 1, c = 2$
so $[Y] = [V^{-4}FA^2] = [V^{-4}A^2F]$

13. **(b)** Dimension of $[h] = [ML^2T^{-1}]$
$[C] = [LT^{-1}]$
$[G] = [M^{-1}L^3T^{-2}]$
Hence dimension of
$\left[\sqrt{\dfrac{hC^5}{G}}\right] = \dfrac{[ML^2T^{-1}] \cdot [L^5T^{-5}]}{[M^{-1}L^3T^{-2}]}$
$= [ML^2T^{-2}] = \text{energy}$

14. **(c)** Let $t \propto G^x h^y C^z$
Dimensions of $G = [M^{-1}L^3T^{-2}]$,
$h = [ML^2T^{-1}]$ and $C = [LT^{-1}]$
$[T] = [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [LT^{-1}]^z$
$[M^0L^0T^1] = [M^{-x+y} L^{3x+2y+z} T^{-2x-y-z}]$
By comparing the powers of M, L, T both the sides
$-x + y = 0 \Rightarrow x = y$
$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ..... (i)
$-2x - y - z = 1 \quad \Rightarrow 3x + z = -1$ ..... (ii)
Solving eqns. (i) and (ii),
$x = y = \dfrac{1}{2}, z = -\dfrac{5}{2} \quad \therefore t \propto \sqrt{\dfrac{Gh}{C^5}}$

15. **(None)**
Stopping potential $(V_0) \propto h^x I^y G^Z C^r$
Here, h = Planck's constant $= [ML^2T^{-1}]$
I = current = $[A]$
G = Gravitational constant = $[M^{-1}L^3T^{-2}]$
and c = speed of light = $[LT^{-1}]$
V_0 = potential = $[ML^2T^{-3}A^{-1}]$
$\therefore [ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^x [A]^y [M^{-1}L^3T^{-2}]^z [LT^{-1}]^r$
$M^{x-z}; L^{2x+3z+r}; T^{-x-2z-r}; A^y$
Comparing dimension of M, L, T, A, we get
$y = -1, x = 0, z = -1, r = 5$
$\therefore V_0 \propto h^0 I^{-1} G^{-1} C^5$

16. **(d)** The quantity $\dfrac{B^2}{2\mu_0}$ is the energy density of magnetic field.
$\Rightarrow \left[\dfrac{B^2}{2\mu_0}\right] = \dfrac{\text{Energy}}{\textit{Volume}} = \dfrac{\text{Force} \times \text{displacement}}{(\text{displacement})^3}$
$= \left[\dfrac{ML^2T^{-2}}{L^3}\right] = ML^{-1}T^{-2}$

17. **(b)** Plank length is a unit of length, $l_p = 1.616229 \times 10^{-35}$ m
$l_p = \sqrt{\dfrac{hG}{c^3}}$

18. **(a)** Let mass, related as $M \propto T^xC^yh^z$
$M^1L^0T^0 = (T^1)^x (L^1T^{-1})^y (M^1L^2T^{-1})^z$
$M^1L^0T^0 = M^z \, L^{y+2z} + T^{x-y-z}$

z = 1
y + 2z = 0 $\quad$ x – y – z = 0
y = –2 $\quad$ x + 2 – 1 = 0
x = –1

M = $[T^{-1} C^{-2} h^1]$

19. **(b)** We know that resistivity

$$r = \frac{RA}{l}$$

$$\text{Conductivity} = \frac{1}{\text{resistivity}} = \frac{l}{RA}$$

$$= \frac{l\,I}{VA} \quad (\because V = RI)$$

$$= \frac{[L][I]}{\left[\frac{[ML^2T^{-2}]}{[I][T]}\right]\times[L^2]} \quad \because V = \frac{W}{q} = \frac{W}{it}$$

$$= [M^{-1}L^{-3}T^3][I^2] = [M^{-1}L^{-3}T^3I^2]$$

20. **(c)** Let μ_0 related with *e*, *m*, *c* and *h* as follows.

$\mu_0 = ke^am^bc^ch^d$

$[MLT^{-2}A^{-2}] = [AT]^a\ [M]^b\ [LT^{-1}]^c\ [ML^2T^{-1}]^d$

$= [M^{b+d}\ L^{c+2d}\ T^{a-c-d}\ A^a]$

On comparing both sides we get

$a = -2$ ...(i)
$b + d = 1$ ...(ii)
$c + 2d = 1$ ...(iii)
$a - c - d = -2$ ...(iv)

By equation (i), (ii), (iii) & (iv) we get,

$a = -2,\ b = 0,\ c = -1,\ d = 1$

$$\therefore [\mu_0] = \left[\frac{h}{ce^2}\right]$$

21. **(d)** Let unit '*u*' related with *e*, a_0, *h* and *c* as follows.

$[u] = [e]^a\ [a_0]^b\ [h]^c\ [C]^d$

Using dimensional method,

$[M^{-1}L^{-2}T^{+4}A^{+2}] = [A^1T^1]^a[L]^b[ML2T^{-1}]^c[LT^{-1}]^d$

$[M^{-1}L^{-2}T^{+4}A^{+2}] = [M^c\ L^{b+2c+d}\ T^{a-c-d}\ A^a]$

$a = 2,\ b = 1,\ c = -1,\ d = -1$

$$\therefore \quad u = \frac{e^2a_0}{hc}$$

22. **(b)** The dimensional formulae of

$$e = \left[M^0L^0T^1A^1\right]$$

$$\varepsilon_0 = \left[M^{-1}L^3T^4A^2\right]$$

$$G = \left[M^{-1}L^3T^{-2}\right] \text{ and } m_e = \left[M^1L^0T^0\right]$$

$$\text{Now, } \frac{e^2}{2\pi\varepsilon_0 Gm_e^2}$$

$$= \frac{\left[M^0L^0T^1A^1\right]^2}{2\pi\left[M^{-1}L^{-3}T^4A^2\right]\left[M^{-1}L^3T^{-2}\right]\left[M^1L^0T^0\right]^2}$$

$$= \frac{\left[T^2A^2\right]}{2\pi\left[M^{-1-1+2}\,L^{-3+3}\,T^{4-2}A^2\right]}$$

$$= \frac{\left[T^2A^2\right]}{2\pi\left[M^0L^0T^2A^2\right]} = \frac{1}{2\pi}$$

$\because \frac{1}{2\pi}$ is dimensionless thus the combination $\frac{e^2}{2\pi\varepsilon_0 Gm_e^2}$ would have the same value in different systems of units.

23. **(c)** Dimensions of $\mu = [MLT^{-2}A^{-2}]$

Dimensions of $\in = [M^{-1}L^{-3}T^4A^2]$

Dimensions of $R = [ML^2T^{-3}A^{-2}]$

$$\therefore \frac{\text{Dimensions of }\mu}{\text{Dimensions of }\in} = \frac{[MLT^{-2}A^{-2}]}{[M^{-1}L^{-3}T^4A^2]}$$

$= [M^2L^4T^{-6}A^{-4}] = [R^2]$

24. **(b)** As we know, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}$

$$\Rightarrow \varepsilon_0 = \frac{q_1q_2}{4\pi FR^2}$$

$$\text{Hence, } \varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$$

$= [M^{-1}\ L^{-3}\ T^4\ A^2]$

25. **(d)** Angular momentum = m × v × r = $ML^2\ T^{-1}$

$$\text{Latent heat } L = \frac{Q}{m} = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$$

$$\text{Capacitance } C = \frac{\text{Charge}}{\text{P.d.}} = M^{-1}L^{-2}T^4A^2$$

26. **(d)** Electric flux $\phi_E = \vec{E}.\vec{S}$

$\therefore$ Dimensionally $\phi_E \neq E$ (electric field)

Dipole moment, *p* = charge (*q*) × distance (*d*)

27. **(c)** The power of an exponent is a number or constant.

Therefore, dimensionally $\frac{\alpha z}{k\theta} = [M^\circ L^\circ T^\circ]$

$$\therefore \quad \alpha = \frac{k\theta}{z}$$

$$\therefore \quad \alpha = \frac{[ML^2T^{-2}\theta^{-1}][\theta]}{[L]} = \left[MLT^{-2}\right]$$

As $k = \frac{\text{joule}}{\text{kelvin}} = [ML^2T^{-2}\theta^{-1}]$ and $z = [L]$

And, dimensionally $P = \frac{\alpha}{\beta} \Rightarrow \beta = \frac{\alpha}{P}$

$$\therefore \quad [\beta] = \frac{MLT^{-2}}{ML^{-1}T^{-2}} = [M^0L^2T^0]$$

$$\left[\because P = \frac{F}{A} = [ML^{-1}T^{-2}]\right]$$

28. **(d)** Capacitance, $C = \frac{\Delta q}{\Delta t} = \frac{\varepsilon_0 A}{L}$

$$\therefore X = \varepsilon_0 L\frac{\Delta V}{\Delta t} = \frac{(\Delta q)L}{A(\Delta V)}L\frac{\Delta V}{\Delta t}$$

$$= \frac{(\Delta q)}{\Delta V}\frac{L^2}{L^2}\frac{\Delta V}{\Delta t} = \frac{\Delta q}{\Delta t} = I \quad \text{(current)}$$

29. **(c)** Here $\left(\frac{1}{2}\right)\varepsilon_0 E^2$ represents, energy density i.e., energy per unit volume.

$$\therefore \left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[\text{Energy}]}{[\text{Volume}]} = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$$

30. **(3)** Let $d \propto \rho^x S^y f^z = K\rho^x S^y f^z$ where K is a dimensionless constant.

$M^0L^1T^0 = M^xL^{-3x}M^yT^{-y}T^{-z}$

Equating dimensions both sides

$M^0L^1T^0 = M^{x+y}L^{-3x}T^{-y-z}$

$\therefore x+y=0,\ -3x=1$

$\therefore\ x=-\frac{1}{3}$ and $y=\frac{1}{3}$

$\therefore n=3$

31. Conductivity, $\sigma = \frac{J}{E} = \frac{i/A}{F/q} = \frac{i^2t}{FA}$

$=\left[M^{-1}L^{-3}T^3A^2\right]$

32. Here, $\left[\frac{a}{V^2}\right] = [P] \Rightarrow [a] = [PV^2] = \frac{MLT^{-2}}{L^2}L^6 = ML^5T^{-2}$

$\sigma = \frac{J}{E} = \frac{i/A}{F/q} = \frac{i^2t}{Fa} = [M^{-1}L^{-3}T^3A^2]$

33. $[X] = [C] = [M^{-1}L^{-2}T^2Q^2]$

$[Z] = [B] = [MT^{-1}Q^{-1}]$ $\left(\because Y = \frac{X}{3Z^2} \text{ given}\right)$

$\therefore\ [Y] = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$

34. Using $E = h\nu \Rightarrow$ Planck's constant,

$h = \frac{E}{\nu} = \frac{P.S.}{\frac{1}{T}} = \frac{\left[ML^2T^{-2}\right]}{\left[T^{-1}\right]} = \left[ML^2T^{-1}\right]$

35. **(a, b)** Given position, $L = [X^\alpha]$

Speed, $LT^{-1} = [X^\beta]$

Acceleration, $LT^{-2} = [X^p]$

Linear momentum, $MLT^{-1} = [X^q]$

Force, $MLT^{-2} = [X^r]$

$\frac{\text{Position}}{\text{Speed}} = \text{time, } T = -\frac{[X^\alpha]}{[X^\beta]} = X^{\alpha-\beta}$

Acceleration $= \frac{\text{Speed}}{\text{Time}} = \frac{X^\beta}{X^{\alpha-\beta}} = X^p$

$X^{\alpha-\beta+p} = X^\beta$

$\therefore \alpha + p = 2\beta$

Hence option (a) is correct.

Force $= \frac{\text{linear momentum}}{\text{time}}$

$[X^r] = \frac{[X^q]}{[X^{\alpha-\beta}]}$

$\Rightarrow r = q+\beta-\alpha \Rightarrow r = q+\beta-(2\beta-p)$

$\Rightarrow r = q-\beta+p \Rightarrow p+q-r=\beta$

Hence option (b) is correct.

36. **(a, b, c)** According to question, dimensions of angular momentum $[mvr] = M^0L^0T^0$ and of mass $[m] = M^0L^0T^0$

$ML^2T^{-1} = ML^0T^0 \Rightarrow T = L^2$

Momentum $p = mv = \frac{mvr}{r} = \frac{M^0L^0T^0}{L} = L^{-1}$

Energy $E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = L^{-2}$

Power $P = \frac{E}{t} = \frac{L^{-2}}{T} = \frac{L^{-2}}{L^2} = L^{-4}$

Force $F = \frac{E}{x} = \frac{L^{-2}}{L} = L^{-3}$

37. **(b, d)**

Dimensions of $[n] = [L^{-3}]$, $[q] = [AT]$

$\left[\frac{q^2}{\varepsilon}\right] = [Fr^2] = [U\times r] = [ML^3T^{-2}]$

$[k_BT] = [U] = [ML^2T^{-2}]$

Dimension of $l = [L]$

(a) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right)\times\frac{1}{(k_BT)}\times n}$

$= \sqrt{[U\times r]\times\frac{1}{[U]}\times n} = \sqrt{[n]\times[r]}$

$= \sqrt{[L^{-3}][L]} = [L^{-1}]$

(b) R.H.S. $= \sqrt{\frac{\varepsilon(k_BT)}{nq^2}} = \sqrt{\frac{(k_BT)}{n(q^2/\varepsilon)}} = \sqrt{\frac{[U]}{[n][U\times r]}}$

$= \sqrt{\frac{1}{[n][r]}} = \sqrt{\frac{1}{[L^{-3}][L]}} = [L]$

(c) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right)\frac{1}{(k_BT)}\times\frac{1}{n^{2/3}}}$

$= \sqrt{[U\times r]\frac{1}{[U]}\frac{1}{[L^{-2}]}} = [L^{3/2}]$

(d) R.H.S. $= \sqrt{\left(\frac{q^2}{\varepsilon}\right)\times\frac{1}{(k_BT)}\times\frac{1}{n^{1/3}}}$

$= \sqrt{[U\times r]\times\frac{1}{[U]}\times\frac{1}{[n^{1/3}]}} = \sqrt{[r]\times\frac{1}{[n^{1/3}]}}$

$= \sqrt{[L]\frac{1}{[L^{-1}]}} = [L]$

38. **(a, c)** Using, $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $R = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

(a) $\mu_0 I^2 = \varepsilon_0 V^2 \Rightarrow \frac{\mu_0}{\varepsilon_0} = \frac{V^2}{I^2} = R^2 = \sqrt{\left(\frac{\mu_0}{\varepsilon_0}\right)^2}$

(b) $\varepsilon_0 I = \mu_0 V \Rightarrow \frac{\mu_0}{\varepsilon_0} = \frac{I}{V} = \frac{1}{R}$ $(\because V = RI)$

Dimensionally incorrect.

(c) $I = \varepsilon_0 C V$

$\therefore \frac{1}{\varepsilon_0 C} = \frac{V}{I} = R \quad \therefore \frac{1}{\varepsilon_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} = R$

Dimensionally correct.

(d) $\mu_0 C I = \varepsilon_0 V$

$\therefore \frac{\mu_0}{\varepsilon_0} = \frac{V}{IC} = \frac{R}{C} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{1}{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}} = \mu_0$

Dimensionally incorrect.

39. **(a, c, d)**

As Planck's constant h, speed of light c and gravitational constant G are used as basic units for length L and Mass M so, $L \propto h^x c^y G^z$ .... (i)

and $M \propto h^p c^q G^r$ .... (ii)

Dimensions of $[h] = [M L^2 T^{-1}]$, $[c] = [L\ T^{-1}]$

$[G] = [M^{-1} L^3 T^{-2}]$

Using principle of homogeneity of dimensions

For eqn. (i)

$[M^0 L T^0] = [M^x\ L^{2x}\ T^{-x}][L^y\ T^{-y}][M^{-z}\ L^{3z}\ T^{-2z}]$

$M^0\ L\ T^0 = M^{(x-z)}\ L^{(2x+y+3z)}\ T^{(-x-y-2z)}$

On comparing powers from both sides, we get

$x - z = 0,\ 2x + y + 3z = 1,\ -x - y - 2z = 0$

On solving these eqns., we get

$x = \frac{1}{2},\ y = -\frac{3}{2},\ z = \frac{1}{2}$

$\therefore \quad L = K\sqrt{\frac{hG}{c^3}}$; K is a constant.

In the same way solving eqn. (ii) we get,

$M = K'\sqrt{\frac{hc}{G}}$; K' is a constant.

40. **(b, c)** Using $F = \frac{Q_1 Q_2}{(4\pi\varepsilon_0)r^2}$ and $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Hence, $[\varepsilon_0] = \frac{[Q]^2}{[F][r^2]} = \frac{I^2 T^2}{MLT^{-2}.L^2} = M^{-1}L^{-3}T^4 I^2$

$[\mu_0] = \frac{1}{[\varepsilon_0][C]^2} = \frac{1}{[M^{-1}L^{-3}T^4 I^2]}[LT^{-2}]^2$

$= [MLT^{-2}I^{-2}]$

41. **(a, b, c)**

Reynold's number

= Coefficient of friction = $[M^0 L^0 T^0]$ i.e., dimensionless.

Curie is the unit of radioactivity (number of atoms decaying per second) and frequency also has the unit per second *i.e.*, $[M^0 L^0 T^{-1}]$.

Latent heat = $\frac{Q}{m}$ and Gravitation potential = $\frac{W}{m}$.

Dimensionally same $[M^0 L^2 T^{-2}]$

42. **(a, d)** Torque, $\tau = F \times r \times \sin\theta$

and work done, $W = F \times d \times \cos\theta$

Dimensionally, $\tau = W = [ML^2 T^{-2}]$

Dimensionally, light year = wavelength = $[L]$

43. **(c)** Boltzmannn constant $[K_B] = \frac{u}{\theta} \quad \left(\because u = \frac{1}{2}K_B T\right)$

$\therefore [K_B] = ML^2T^{-2}K^{-1}$

Coefficient of viscosity $[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$

Planck constant, $h = \frac{E}{v} = \frac{ML^2T^{-2}}{T^{-1}} = ML^{-2}T^{-1}$

Thermal conductivity $K_{conductivity} = \frac{H\ell}{tA\Delta T}$

$= \frac{ML^2T^{-2} \times L}{T \times L^2 \times K} = MLT^{-3}K^{-1}$

44. Angular Momentum = $mvr = [ML^2T^{-1}]$

Latent heat $[L] = Q/m$ $[L] = [L^2T^{-2}]$

Torque $\tau = F \times r$ $[\tau] = [ML^2T^{-2}]$

Capacitance $C = \frac{1}{2} = \frac{q^2}{u}$; $[C] = [M^{-1}L^{-2}T^2Q^2]$

Inductance $L = \frac{2u}{i^2}; [L] = [ML^2Q^{-2}]$

Resistivity $\rho = \frac{RA}{l}$ and $R = \frac{L}{t}$ $\therefore [\rho] = [ML^3T^{-1}Q^{-2}]$

45. **(c)** Using, $C = \frac{E}{B}$ where C = speed of light

$\therefore$ $E = CB = LT^{-1}B$

46. **(d)** We know that

$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \therefore C^2 = \frac{1}{\mu_0 \varepsilon_0}$

$\therefore \mu_0 = \varepsilon_0^{-1} L^{-2} T^2 \quad [\because C = LT^{-1}]$

47. (i) Magnetic Flux $\phi = BA = \frac{F}{qv}A = [M^1L^2T^{-1}Q^{-1}]$

(ii) Modulus of Rigidity = $\frac{F}{A} = [ML^{-1}T^{-2}]$

48. As per question, $T \propto P^a d^b E^c$ or $[T] = [P]^a[d]^b[E]^c$

or $[M^0L^0T^1] = [ML^{-1}T^{-2}]^a\ [ML^{-3}]^b\ [ML^2T^{-2}]^c$

or $M^0L^0T^1 = M^{a+b+c}\ L^{-a-3b+2c}\ T^{-2a-2c}$

$\therefore$ $a + b + c = 0$ ..... (i)

$-a - 3b + 2c = 0$ ..... (ii)

$-2a - 2c = 1$ ..... (iii)

Solving, eqns. (i), (ii) and (iii) we get

$a = -5/6,\ b = 1/2,\ c = 1/3.$

Topic-3 : Errors in Measurements & Experimental Physics

1. **(b)** Given : No. of division on circular scale of screw gauge = 50

Pitch = 0.5 mm

Least count of screw gauge

$= \frac{\text{Pitch}}{\text{No. of division on circular scale}}$

$= \frac{0.5}{50}\text{mm} = 1 \times 10^5\ \text{m} = 10\ \mu\text{m}$

And nature of zero error is positive.

2. **(d)** Average diameter, $d_{av} = 5.5375$ mm

Deviation of data, $\Delta d = 0.07395$ mm

As the measured data are upto two digits after decimal, therefore answer should be in two digits after decimal.

$\therefore d = (5.54 \pm 0.07)$ mm

3. **(d)** Given : $Z = \frac{a^2 b^{2/3}}{\sqrt{c}d^3}$

Percentage error in Z,

$$= \frac{\Delta Z}{Z} = \frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{3\Delta d}{d}$$

$$= 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5 = 14.5\%.$$

4. **(a)** Thickness = M.S. Reading + Circular Scale Reading (L.C.)

Here LC $= \frac{\text{Pitch}}{\text{Circular scale division}} = \frac{0.1}{50} = 0.002$ cm per division

So, correct measurement is measurement of integral multiple of L.C.

5. **(c)** L.C. of vernier callipers = 1 MSD – 1 VSD

$$= \left(1 - \frac{9}{10}\right) \times 1 = 0.1 \text{ mm} = 0.01 \text{ cm}$$

Here 7^{th} division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale.

Zero error = 7 × 0.1 = 0.7 mm = 0.07 cm.

Length of the cylinder = measured value – zero error

= (3.1 + 4 × 0.01) – 0.07 = 3.07 cm.

6. **(d)** When screw on a screw-gauge is given six rotations, it moves by 3mm on the main scale

$$\therefore \quad \text{Pitch} = \frac{3}{6} = 0.5 \text{mm}$$

$$\therefore \quad \text{Least count L.C.} = \frac{\text{Pitch}}{CSD} = \frac{0.5 \text{mm}}{50}$$

$$= \frac{1}{100} \text{mm} = 0.01 mm = 0.001 \text{cm}$$

7. **(Bonus)**

$D_1 = A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2 = 280.6$

$D_2 = A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2 = 280.7$

$D_3 = A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183 = 280.7$

$D_4 = A_4 + B_4 + C_4 = 25 + 236.191 + 19.5 = 281$

None of the option matches.

8. **(c)** Given, Length of simple pendulum, $l = 25.0$ cm

Time of 40 oscillation, $T = 50s$

Time period of pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = \frac{4\pi^2 \ell}{g} \quad \Rightarrow g = \frac{4\pi^2 \ell}{T^2}$$

$$\Rightarrow \text{Fractional error in g} = \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \left(\frac{0.1}{25.0}\right) + 2\left(\frac{1}{50}\right) = 0.044$$

$$\therefore \text{Percentage error in } g = \frac{\Delta g}{g} \times 100 = 4.4\%$$

9. **(Bonus)** $\delta = \frac{M}{V} = \frac{M}{l^3} = Ml^{-3}$

$$\frac{\Delta\delta}{\delta} = \frac{\Delta M}{M} + 3\frac{\Delta l}{l} = \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31 \text{ kg/m}^3$$

10. **(d)** $A = 7 \times 5.29 = 37.03 \text{ cm}^2$

The result should have three significant figures, so $A = 37.0 \text{ cm}^2$

11. **(d)** We have

$$T = 2\pi\sqrt{\frac{\ell}{g}} \text{ or } g = 4\pi^2 \frac{\ell}{T^2}$$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta R}{Q} \times 100 + 2\frac{\Delta T}{T} \times 100$$

$$= \frac{0.1}{55} \times 100 + 2\left(\frac{1}{30}\right) \times 100$$

= 0.18 + 6.67 = 6.8%

12. **(b)** Least count of main scale of screw gauge = 1 mm

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of division on circular scale}}$$

$$5 \times 10^{-6} = \frac{10^{-3}}{\text{N}}$$

$\Rightarrow$ N = 200

13. **(c)** Least count of screw gauge,

$$\text{LC} = \frac{\text{Pitch}}{\text{No. of division}}$$

= $0.5 \times 10^{-3} = 0.5 \times 10^{-2}$ mm + ve error = $3 \times 0.5 \times 10^{-2}$ mm

= 1.5×10^{-2} mm = 0.015 mm

Reading = MSR + CSR – (+ve error)

= 5.5 mm + (48 × 0.5 × 10^{-2}) – 0.015

= 5.5 + 0.24 – 0.015 = 5.725 mm

14. **(c)** Maximum error = 1.5 % + 3 (1%) = 4.5%

15. **(d)** Maximum percentage error in A

$$= 3(\% \text{ error in P}) + 2(\% \text{ error in Q}) + \frac{1}{2}(\% \text{ error in R}) + 1(\% \text{ error in S})$$

$$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1 \times 1.5$$

= 1.5 + 2 + 1.5 + 1.5 = 6.5%

16. **(d)** Least count $= \frac{\text{Value of 1 part on main scale}}{\text{Number of parts on vernier scale}}$

$$= \frac{0.25}{5 \times 100} \text{cm} = 5 \times 10^{-4} \text{ cm}$$

Reading = 4 × 0.05 cm + 30 × 5 × 10^{-4} cm

= (0.2 + 0.0150) cm = 0.2150 cm (Thickness of wire)

17. **(d)** Surface tension, $T = \frac{rhg}{2} \times 10^3$

Relative error in surface tension,

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0 \quad (\because \text{ g, 2 \& } 10^3 \text{ are constant})$$

Percentage error

$$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times 0.01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times 0.01}{1.45 \times 10^{-2}}\right) 100$$

= (0.8 + 0.689)

= (1.489) = 1.489% ≅ 1.5%

18. **(c)** Given, $P = a^{1/2} b^2 c^2 d^{-4}$,
Maximum relative error,
$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$
$$= \frac{1}{2}\times 2 + 2\times 1 + 3\times 3 + 4\times 5 = 32\%$$

19. **(b)** Depth of the well = L = 20 m
$\therefore$ Time taken by stone to reach the bottom of well,
$$t_1 = \sqrt{\frac{2L}{g}} \quad \left(\text{using, } L = \frac{1}{2}gt^2\right)$$
Time taken by impact sound to reach the person,
$$t_2 = \frac{L}{v}$$
Total time taken in the process is given by,
$$T = t_1 + t_2 = \sqrt{\frac{2L}{g}} + \frac{L}{v}$$
$$\therefore \quad \delta T = \sqrt{\frac{2}{g}}\frac{1}{2}\frac{\delta L}{\sqrt{L}} + \frac{\delta L}{v}$$
On substituting the given values we get,
$$\delta T = \frac{16}{300}\delta L$$
$$\therefore \quad \text{Fractional error, } \frac{\delta L}{L}\times 100 = \frac{300}{16}\frac{\delta T}{L}\times 100$$
$$= \frac{300}{16}\times\frac{0.01}{20}\times 100 \simeq 1\%$$

20. **(d)** L.C. $= \frac{0.5}{50} = 0.01$ mm
Zero error = 5 × 0.01 = 0.05 mm (Negative)
Reading = (0.5 + 25 × 0.01) + 0.05 = 0.80 mm

21. **(c)** $\Delta T = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4|}{4}$
$$= \frac{2+1+3+0}{4} = 1.5$$
As the resolution of measuring clock is 1.5 therefore the mean time should be 92 ± 1.5

22. **(b)** For C_1 vernier calliper,
L.C. = 1MSD – 1VSD = 1mm – 0.9 mm = 0.1 mm = 0.01 cm
[$\because$ 10 VS D = 9 MSD = 9 mm]
Reading = MSR + L.C. × VSR = 2.8 + (0.01) × 7 = 2.87 cm
For C_2 vernier calliper,
L.C. = 1 mm – 1.1 mm [$\because$ 10 VSD 11 MSD = 11 mm]
L.C = – 0.1 mm = – 0.01 cm
Reading = 2.8 + (10 – 7) × 0.01 = 2.83 cm

23. **(d)** As, $g = 4\pi^2\frac{L}{T^2}$
$$\text{So, } \frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 + 2\frac{\Delta T}{T}\times 100$$
$$= \frac{0.1}{20}\times 100 + 2\times\frac{1}{90}\times 100 = 2.72 \simeq 3\%$$

24. **(b)** Least count $= \frac{0.1}{10} = 0.01$ cm
$d_1 = 0.5 + 8 \times 0.01 + 0.03 = 0.61$ cm
$d_2 = 0.5 + 4 \times 0.01 + 0.03 = 0.57$ cm
$d_3 = 0.5 + 6 \times 0.01 + 0.03 = 0.59$ cm
Mean diameter $= \frac{0.61 + 0.57 + 0.59}{3} = 0.59$ cm

25. **(a)** The current voltage relation of diode is
$I = (e^{1000\,V/T} - 1)$ mA (given)
When, $I = 5mA,\ e^{1000\,V/T} = 6mA$
Also, $dI = (e^{1000\,V/T})\times\frac{1000}{T}$
Error = ± 0.01 (By exponential function)
$= (6\,mA)\times\frac{1000}{300}\times(0.01) = 0.2$ mA

26. **(b)** Measured length of rod = 3.50 cm
For Vernier Scale with 1 Main Scale Division = 1 mm
9 Main Scale Division = 10 Vernier Scale Division,
Least count = 1 MSD –1 VSD = 0.1 mm

27. **(d)** In a voltmeter
$V \propto l$
$V = kl$
Now, it is given E = 1.1 volt for l_1 = 440 cm
and V = 0.5 volt for l_2 = 220 cm
Let the error in reading of voltmeter be ΔV then,
1.1 = 400 K and (0.5 – ΔV) = 220 K.
$$\Rightarrow \quad \frac{1.1}{440} = \frac{0.5 - \Delta V}{220}$$
$$\therefore \quad \Delta V = -0.05 \text{ volt}$$

28. **(b)** According to the question.
$t = (90 \pm 1)$ or, $\frac{\Delta t}{t} = \frac{1}{90}$
$l = (20 \pm 0.1)$ or, $\frac{\Delta l}{l} = \frac{0.1}{20}$
$\frac{\Delta g}{g}\% = ?$
As we know,
$$t = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{t^2}$$
$$\text{or, } \frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l} + 2\frac{\Delta t}{t}\right) = \left(\frac{0.1}{20} + 2\times\frac{1}{90}\right) = 0.027$$
$$\therefore \frac{\Delta g}{g}\% = 2.7\%$$

29. **(b)** In the measurement, the diameter of cylinder,
D = M.S.R + (V.S.R) × (L.C.) L.C. = (1MSD – 1VSD)
$$= (5.15 - 5.10) - \left(\frac{2.45}{50}\right) = .001\text{ cm}$$
$\therefore$ D = 5.10 + 24 × 0.001 = 5.124 cm

30. **(a)** The maximum possible error in Y due to l and d
$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + \frac{2\Delta d}{d}$$
Least count $= \frac{\text{Pitch}}{\text{No. of division on circular scale}}$
$$= \frac{0.5}{100}\text{mm} = 0.005 \text{ mm}$$
Here, $\Delta d = \Delta l = 0.005$ mm
Error contribution of $l = \frac{\Delta l}{l} = \frac{0.005\text{ mm}}{0.25\text{ mm}} = \frac{1}{50}$

Error contribution of $d = \frac{2\Delta d}{d} = \frac{2\times 0.005\text{ mm}}{0.5\text{ mm}} = \frac{1}{50}$

Hence contribution to the maximum possible error in the measurement of y due to l and d is the same.

31. (c) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{divisions on circular scale}} = \frac{0.5}{50} = 0.01\text{ mm} = \Delta r$$

Diameter, r = M.S.R. + (C.S.R) × (L.C.)

Diameter, $r = 2.5\text{ mm} + 20 \times \frac{0.5}{50} = 2.70\text{ mm}$

$$\frac{\Delta r}{r} = \frac{0.01}{2.70} \text{ or } \frac{\Delta r}{r}\times 100 = \frac{1}{2.7}$$

Now, density, $d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

$\therefore$ Percentage error in density, $\frac{\Delta d}{d}\times 100$

$$= \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\}\times 100$$

$$= \frac{\Delta m}{m}\times 100 + 3\times\left(\frac{\Delta r}{r}\right)\times 100$$

$$= 2\% + 3\times\frac{1}{2.7} = 3.11\%$$

32. (d) 20 divisions on the vernier scale

= 16 divisions of main scale

$\therefore$ 1 division on the vernier scale

$= \frac{16}{20}$ divisions of main scale $= \frac{16}{20}\times 1\text{mm} = 0.8$ mm

We know that least count, L.C. = 1MSD – 1VSD

= 1 mm – 0.8 mm = 0.2 mm

33. (b) The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T^2 = 4\pi^2\frac{\ell}{g} \Rightarrow g = 4\pi^2\frac{\ell}{T^2}$$

Percentage error in $g = \frac{\Delta g}{g}\times 100$

$$= \frac{\Delta \ell}{\ell}\times 100 + 2\frac{\Delta T}{T}\times 100$$

Putting the values of $\Delta\ell$, ℓ, ΔT and T we get $E_1 = 0.3125\%$ least among E_1, E_2 and E_3.

34. (b) $Y = \frac{F/A}{\ell/L} = \frac{4mgL}{\pi D^2\ell} = \frac{4\times 1\times 9.8\times 2}{\pi\left(0.4\times 10^{-3}\right)^2\times\left(0.8\times 10^{-3}\right)}$

$$= 2.0\times 10^{11}\text{ N/m}^2$$

Now $\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta \ell}{\ell}$

[$\because$ the value of m, g and L are exact]

$$= 2\times\frac{0.01}{0.4} + \frac{0.05}{0.8} = 2\times 0.025 + 0.0625$$

$$= 0.05 + 0.0625 = 0.1125$$

$\Rightarrow \Delta Y = Y\times 0.1125 = 2\times 10^{11}\times 0.1125 = 0.225\times 10^{11}$

$$= 0.2\times 10^{11}\text{N/m}^2$$

$\therefore$ Reading of Young's modulus $= (Y + \Delta Y)$

$$= (2\pm 0.2)\times 10^{11}\text{ N/m}^2$$

35. (d) Relative error in g, $\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

$\Delta\ell$ and ΔT are least and number of readings taken are maximum in option (d), therefore the measurement of g is most accurate.

36. (c) Least count $= \frac{\text{Pitch}}{\text{no. of divisions on circular scale}}$

$$= \frac{0.5}{50} = 0.01\text{mm}$$

Zero error = 5 × $L.C.$ = 5 × 0.01 mm = 0.05 mm

Diameter of ball = [Reading on main scale] + [Reading on circular scale × $L.C.$] – Zero error

= 0.5 × 2 + 25 × 0.01 – 0.05 = 1.20 mm

37. (a) Density, $\rho = \frac{m}{v} = \frac{m}{A\ell} = \frac{m}{\ell\pi r^2}$

Taking log and differentiate for errors

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta\ell}{\ell} \quad \text{.... (i)}$$

From question, putting the values of

$\Delta\ell = 0.06$ cm, $\ell = 6$ cm; $\Delta r = 0.005$ cm; $r = 0.5$ cm, $m = 0.3$ g; $\Delta m = 0.003$ g in eqn. (i) we get

$$\frac{\Delta\rho}{\rho} = \frac{4}{100}$$

$\therefore$ Percentage error in density,

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{4}{100}\times 100 = 4\%.$$

38. (a) Volume of cube, $V = \ell^3 = (1.2\times 10^{-2}\text{ m})^3$

$= 1.728\times 10^{-6}\text{ m}^3$

$\Rightarrow V = 1.7\times 10^{-6}\text{ m}^3$.

As length has two significant figures so volume has also two significant figures.

39. (4) $E_{(t)} = A^2 e^{-\alpha t} = A^2 e^{-0.2t}$

$\therefore$ $\log_e E = 2\log_e A - 0.2t$

On differentiating we get

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\frac{dt}{t}\times t$$

As errors always add up

$$\therefore \frac{dE}{E}\times 100 = 2\left(\frac{dA}{A}\times 100\right) + 0.2t\left(\frac{dt}{t}\times 100\right)$$

$$\Rightarrow \frac{dE}{E}\times 100 = 2\times 1.25\% + 0.2\times 5\times 1.5\% = 4\%$$

40. (4) Young's modulus $Y = \frac{FL}{A\times l}$

Here F, A and L are accurately known.

$\therefore$ Percentage error in Young's modulus

$$\frac{\Delta Y}{Y}\times 100 = \frac{\Delta L}{l}\times 100 = \frac{1.0\times 10^{-5}}{25\times 10^{-5}}\times 100 = 4\%$$

41. (1050)

Density, $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} \Rightarrow \rho = \frac{6}{\pi}MD^{-3}$

$$\therefore \%\left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta m}{m} + 3\left(\frac{\Delta D}{D}\right) = 6 + 3\times 1.5 = 10.5\%$$

$$\%\left(\frac{\Delta\rho}{\rho}\right)=\frac{1050}{100}\%=\left(\frac{x}{100}\right)\%$$

$\therefore x=1050.00$

42. **(1.30)**

Two capacitors, C_1 and C_2 are connected in series

$$\therefore \frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}\Rightarrow C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

$$C_{eq}=\frac{2000\times3000}{5000}=\frac{6000}{5}\text{pF}$$

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}\Rightarrow-\frac{\Delta C}{C_{eq}^2}=-\frac{\Delta C_1}{C_1^2}-\frac{\Delta C_2}{C_2^2}$$

$$\Rightarrow \Delta C=C_{eq}^2\left[\frac{\Delta C_1}{C_1^2}+\frac{\Delta C_2}{C_2^2}\right]$$

$$\Rightarrow \Delta C=\frac{6000}{5}\times\frac{6000}{5}\left[\frac{10}{2000\times2000}+\frac{15}{3000\times3000}\right]$$

$$=10\times\frac{9}{25}+15\times\frac{4}{25}=6\text{pF}$$

Energy stored in a capacitor,

$$U=\frac{1}{2}CV^2\Rightarrow\frac{\Delta U}{U}\times100=\frac{\Delta C}{C_{eq}}\times100+2\frac{\Delta V}{V}\times100$$

$$\Rightarrow\frac{\Delta U}{U}\times100=\frac{6\times5}{6000}\times100+2\times\frac{0.02}{5}\times100$$

$= 0.5 + 0.8 = 1.30\%$

Thus the percentage error in the calculation of the energy stored is 1.30%

43. **(1.39)**

$$u\pm\Delta u=(75-45)\pm\left(\frac{1}{4}+\frac{1}{4}\right)=(30\pm0.5)\text{ cm}$$

$$v\pm\Delta v=(135-75)\pm\left(\frac{1}{4}+\frac{1}{4}\right)=(60\pm0.5)\text{ cm}$$

We know that

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\therefore +\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}=\frac{\Delta f}{f^2} \quad \text{....(i)}$$

Now $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{60}-\frac{1}{-30}=\frac{1}{f}$

$$\Rightarrow\frac{1}{f}=\frac{1}{20}\quad\therefore f=20\text{cm}$$

Substituting the values in eqn. (i)

$$\frac{0.5}{(60)^2}+\frac{0.5}{(30)^2}=\frac{\Delta f}{(20)^2}$$

$\therefore \Delta f = 0.277$

Hence percentage error in the measurement of focal length

$$\frac{\Delta f}{f}\times100=\frac{0.277}{20}\times100=1.388\%=1.39\%$$

44. **(3.00)** We know that $\Delta l=\dfrac{Fl}{AY}$

$$=\frac{1.2\times10\times1}{\pi\left(\frac{5\times10^{-4}}{2}\right)^2\times2\times10^{11}}\simeq0.3\text{ mm}$$

L.C. of vernier calliper = 1 MSD – 1 VSD

$$=\left(1-\frac{9}{10}\right)=0.1\text{ mm}$$

As 9 MSD = 10 VSD

The third marking of vernier scale will coincide with the main scale because least count is 0.1 mm.

45. **(2.66 g cm^{-3})**

L.C. = 1 *MSD* – 1 *VSD*

$$=1MSD-\frac{9}{10}MSD\quad(\because\ 9\ MSD=10\ VSD)$$

$$=\left(1-\frac{9}{10}\right)MSD=\frac{1}{10}MSD=\left(\frac{1}{10}\times1\right)\text{mm}=0.1\text{ mm}$$

($\because$ 1 MSD = 1 mm)

Reading of side, l = MSR + VSR (LC) = 10 mm + 1 × 0.1 mm = 10.1 mm = 1.01 cm

Now, density $=\dfrac{\text{mass}}{\text{volume}}=\dfrac{m}{l^3}=\dfrac{2.736g}{(1.01)^3}=2.66\text{ g/cm}^3$

(Rounding off to 3 significant figures)

46. **(1.09 × 10^{10} N/m^2)**

Young's modulus, $Y=\dfrac{F}{A}\Big/\dfrac{X}{L}=\dfrac{F}{\frac{\pi D^2}{4}}\times\dfrac{L}{X}$

Maximum error in Y

$$\left(\frac{\Delta Y}{Y}\right)_{max}=2\left(\frac{\Delta D}{D}\right)+\frac{\Delta X}{X}+\frac{\Delta L}{L}$$

$$=2\left(\frac{0.001}{0.05}\right)+\left(\frac{0.001}{0.125}\right)+\left(\frac{0.1}{110}\right)=0.0489$$

Since, W = 50 N; D = 0.05 cm = 0.05 × 10^{-2}m;
X = 0.125 cm = 0.125 × 10^{-2}m;
L = 110 cm = 110 × 10^{-2}m

$$\therefore\ Y=\frac{50\times4\times110\times10^{-2}}{3.14(0.05\times10^{-2})\times(0.125\times10^{-2})}=2.24\times10^{11}\text{N/m}^2$$

$\therefore$ Maximum possible error in the measurement of Y

$\Delta Y = (0.0489)Y = 0.0489 \times 2.24 \times 10^{11}$
$= 1.09 \times 10^{10}$ N/m^2

47. **(2.6 cm^2)**

Least count, L.C. $=\dfrac{1\text{mm}}{100}=0.01\text{mm}$

Diameter = *MSR* + *CSR* × (L.C.)
= 1 mm + 47 × (0.01) mm = 1.47 mm

Curved surface area $=2\pi rl=2\pi\dfrac{D}{2}l=\pi Dl$

$$=\frac{22}{7}\times1.47\times56\text{ mm}^2=2.58724\text{ cm}^2$$

= 2.6 cm^2 (Rounding off to two significant figures)

48. **(a, b, d)**

$$T_{mean}=\frac{0.52+0.56+0.57+0.54+0.59}{6}=0.556\approx0.56\text{ S}$$

Error in reading $T_{mean}-T_1$; $T_{mean}-T_2$; $T_{mean}-T_3$; $T_{mean}-T_4$; $T_{mean}-T_5$

$$\text{Mean error}=\frac{0.04+0+0.01+0.02+0.03}{6}=0.016\approx0.02\text{S}$$

$\therefore$ % error in the measurement of 'T'

$$\frac{\Delta T}{T}\times 100 = \frac{0.02}{0.56}\times 100 = 3.57\%$$

% error in the measurement of g

$$\frac{\Delta g}{g}\times 100 = 2\frac{\Delta T}{T}\times 100 + \left(\frac{\Delta R + \Delta r}{R - r}\right)\times 100$$

$$= 2(3.57) + \left(\frac{1+1}{60-10}\right)\times 100 \approx 11\%$$

% error in the measurement of *r*

$$\frac{\Delta r}{r}\times 100 = \frac{1}{10}\times 100 = 10\%$$

49. **(b, c)**

Vernier callipers

$1\text{ MSD} = \frac{1\text{cm}}{8} = 0.125\text{cm}$

5 VSD = 4MSD

$\therefore 1\text{ VSD} = \frac{4}{5}\times 0.125 = 0.1\text{cm}$

L.C. = 1MSD – 1VSD

= 0.125cm – 0.1cm = 0.025cm

Screw gauge

If the pitch of screw gauge is twice the L.C of vernier callipers then pitch = 2 × L.C. of vernier calliper = 2 × 0.025 = 0.05cm.

L.C. of screw gauge

$$= \frac{\text{Pitch}}{\text{Total no. of divisions of circular scale}}$$

$$= \frac{0.05}{100}\text{cm} = 0.0005\text{cm} = 0.005\text{ mm.}$$

Now if the least count of the linear scale of the screw gauge is twice the least count of venier callipers then

L.C. of linear scale of screw gauge = 2 × 0.025 = 0.05cm.

Then pitch = 2 × 0.05 = 0.1cm.

$\therefore$ L.C. of screw gauge = $\frac{0.1}{100}$ cm = 0.001cm = 0.01mm.

50. **(d)** $\because 2d\sin\theta = \lambda \qquad \therefore \quad d = \frac{\lambda}{2}\text{cosec}\,\theta$ ... (i)

$$\therefore \frac{d(d)}{d\theta} = \frac{\lambda}{2}[-\text{cosec}\,\theta\cot\theta]$$

$\therefore \quad d(d) = -\frac{\lambda}{2}\text{cosec}\theta\cot\theta\, d\theta$ ... (ii)

Dividing (ii) by (i) we get

$$\left|\frac{d(d)}{d}\right| = \cot\theta\, d\theta$$

As θ increases from 0° to 90°, cot θ decreases and therefore $\left|\frac{d(d)}{d}\right|$ decreases

i.e., the fractional error in *d* decreases.

51. **(a, c)** The length of the string of simple pendulum is exactly l m (given), therefore the error in length $\Delta l = 0$.

Further the possibility of error in measuring time is 1s in 40s as the least count of stop watch is 1s.

$$\therefore \frac{\Delta t}{t} = \frac{\Delta T}{T} = \frac{1}{40}$$

Time period $T = \frac{\text{total time}}{\text{no. of oscillations}} = \frac{40}{20} = 2$ seconds

$$\therefore \frac{\Delta T}{T} = \frac{1}{40} \Rightarrow \frac{\Delta T}{2} = \frac{1}{40} \Rightarrow \Delta T = 0.05\text{ sec}$$

Now for measuring error in '*g*'

using, $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\frac{l}{g}$

$\therefore g = 4\pi^2\frac{l}{T^2}$

$\therefore$ Percentage error in '*g*'

$$\frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + 2\frac{\Delta T}{T}\times 100$$

$$\therefore \frac{\Delta g}{g}\times 100 = 0 + 2\left(\frac{1}{40}\right)\times 100 = 5\%$$

52. **(b)** $r = \left(\frac{1-a}{1+a}\right) \Rightarrow \frac{\Delta r}{r} = \frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1+a)}{(1+a)}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a} = \frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$$

$$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)}\frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$$

53. **(c)** Using, $N = N_0 e^{-\lambda t} \Rightarrow \ln N = \ln N_0 - \lambda t$

Differentiation with respect to λ

$$\Rightarrow \frac{1}{N}\frac{dN}{d\lambda} = 0 - t$$

$$\Delta\lambda = \frac{\Delta N}{Nt} = \frac{40}{2000\times 1} = 0.02$$

54. $(n+1)$ V.S.D = n M.S.D

$\therefore \quad 1\text{ V.S.D} = \frac{n}{n+1}$ M.S.D

Least count, (L.C.) = 1 M.S.D – 1V.S.D

$= \left(1 - \frac{n}{n+1}\right)$ M.S.D

$= \frac{1}{n+1}$ M.S.D. $= \frac{a}{n+1}$ units $\quad [\because 1\text{ MSD} = a\text{ units}]$

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** Charge, $e = It = [AT]$, angular frequency,

$\omega = \frac{1}{T} = [T^{-1}]$

No. of electrons per unit volume, $N = [L^{-3}]$, Permittivity of free space, $\in_o = [M^{-1}L^{-3}A^2T^4]$

Now substituting the dimensions of above quantities we get,

$$\sqrt{\frac{Ne^2}{m\in_o}} = \sqrt{\frac{L^{-3}A^2T^2}{M\,M^{-1}L^{-3}A^2T^4}} = \sqrt{T^{-2}} = T^{-1} = \omega_p$$

2. **(b)** $\omega_p = \sqrt{\frac{Ne^2}{m\in_o}} = 2\pi\nu = 2\pi\frac{c}{\lambda} \Rightarrow \lambda = 2\pi c\sqrt{\frac{m\in_o}{Ne^2}}$

$$= 2\times\frac{22}{7}\times 3\times 10^8\sqrt{\frac{10^{-30}\times 10^{-11}}{4\times 10^{27}\times(1.6\times 10^{-19})^2}} = 600\text{ nm}$$

2 Motion in a Straight Line

Topic-1 : Distance, Displacement & Uniform Motion

1. **(b)** Given, $v = b\sqrt{x}$

or $\frac{dx}{dt} = b\,x^{1/2}$ or $\int_0^x x^{-1/2}dx = \int_0^t bdt$

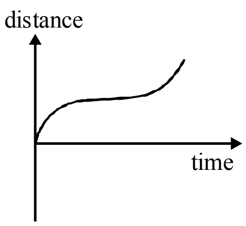
or $\frac{x^{1/2}}{1/2} = 6t$ or $x = \frac{b^2t^2}{4}$

Differentiating w. r. t. time, we get

$\frac{dx}{dt} = \frac{b^2 \times 2t}{4}$ $(t = \tau)$

or $v = \frac{b^2\tau}{2}$

2. **(b)** Graphs in option (c) position-time and option (a) velocity-position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.



3. **(b)** $|\text{Average velocity}| = \frac{|\text{displacement}|}{\text{time}}$

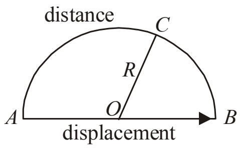
$= \frac{2r}{t} = 2\times\frac{1}{1} = 2\,\text{m/s}.$ $(\because\ r = 1\text{m}\,;\, t = 1\text{s})$

4. (2R, πR) Displacement = shortest distance

$= AB = AOB = 2R$

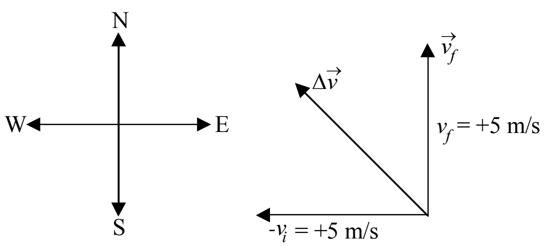
Distance = Path length

$= ACB = \frac{2\pi r}{2} = \pi R$



5. **(b)** Average acceleration

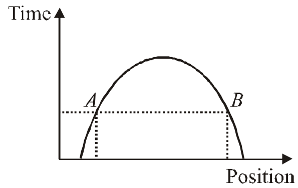
$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t} = \frac{\vec{v_f} + (-\vec{v_i})}{t} = \frac{\Delta\vec{v}}{t}$



$|\Delta\vec{v}| = \sqrt{v_f^2 + v_i^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2}\,\text{m/s}$

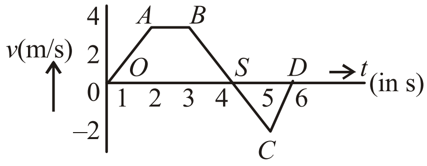
$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 in North-west direction.

6. No, At a given instant of time, the body is at two different positions *A* and *B* in the given position time graph, which is not possible.



Topic-2 : Non-uniform Motion

1. **(a)**



$OS = 4 + \frac{1}{3} = \frac{13}{3}$

$SD = 2 - \frac{1}{3} = \frac{5}{3}$

Distance covered by the body = area of *v*-*t* graph

= ar (*OABS*) + ar (*SCD*)

$= \frac{1}{2}\left(\frac{13}{3}+1\right)\times 4 + \frac{1}{2}\times\frac{5}{3}\times 2 = \frac{32}{3}+\frac{5}{3} = \frac{37}{3}$ m

2. **(b)** From the third equation of motion

$v^2 - u^2 = 2aS$

But, $a = \frac{F}{m}$ $\therefore v^2 = u^2 - 2\left(\frac{F}{m}\right)S$

$\Rightarrow v^2 = (1)^2 - (2)\left[\frac{2.5\times10^{-2}}{20\times10^{-3}}\right]\frac{20}{100}$

$\Rightarrow v^2 = 1 - \frac{1}{2} \Rightarrow v = \frac{1}{\sqrt{2}}\text{m/s} = 0.7\text{m/s}$

3. **(b)** $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(at + bt^2 + ct^3)$

$= a + 2bt - 3ct^2$

Acceleration, $\frac{dv}{dt} = \frac{d}{dt}(a + 2bt - 3ct^2)$

or $0 = 2b - 3c \times 2t$ $\therefore t = \left(\frac{b}{3c}\right)$

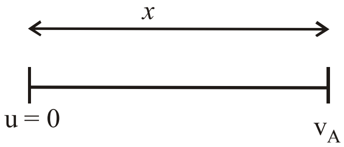
and $v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = \left(a + \frac{b^2}{3c}\right)$

4. **(d)** For constant acceleration, there is straight line parallel to t-axis on $\vec{a} - t$.

Inclined straight line on $\vec{v} - t$, and parabola on $\vec{x} - t$.

5. **(c)** Let time taken by A to reach finishing point is t_0

$\therefore$ Time taken by B to reach finishing point = $t_0 + t$



$v_A = a_1 t_0$

$v_B = a_2(t_0 + t)$

$v_A - v_B = v$

$\Rightarrow v = a_1 t_0 - a_2 (t_0 + t) = (a_1 - a_2)t_0 - a_2 t$ ...(i)

$$x_B = x_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2 (t_0 + t)^2$$

$$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t) \Rightarrow \left(\sqrt{a_1} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t$$

$$\Rightarrow t_o = \frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}}$$

Putting this value of t_0 in equation (i)

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2}t - a_2 t = \sqrt{a_1 a_2}t + a_2 t - a_2 t$$

or, $v = \sqrt{a_1 a_2}\,t$

6. **(c)** Using equation, $a = \frac{v-u}{t}$ and $S = ut + \frac{1}{2}at^2$

Distance travelled by car in 15 sec = $\frac{1}{2}\frac{(45)}{15}(15)^2$

$= \frac{675}{2}$ m

Distance travelled by scooter in 15 seconds = 30 × 15 = 450

($\because$ distance = speed × time)

Difference between distance travelled by car and scooter in 15 sec, 450 – 337.5 = 112.5 m

Let car catches scooter in time t;

$$\frac{675}{2} + 45(t - 15) = 30t$$

$337.5 + 45t - 675 = 30t \Rightarrow 15t = 337.5 \Rightarrow t = 22.5$ sec

7. **(a)** Let the car turn of the highway at a distance 'x' from the point M. So, RM = x

And if speed of car in field is v, then time taken by the car to cover the distance $QR = QM - x$ on the highway,

$$t_1 = \frac{QM - x}{2v} \quad .....(i)$$

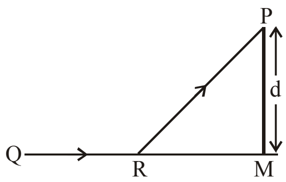
Time taken to travel the distance 'RP' in the field

$$t_2 = \frac{\sqrt{d^2 + x^2}}{v} \quad .....(ii)$$

Total time elapsed to move the car from Q to P

$$t = t_1 + t_2 = \frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}$$

For 't' to be minimum $\frac{dt}{dx} = 0$



$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$$

or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

8. **(c)** According to question, object is moving with constant negative acceleration i.e., a = – constant (C)

$$\frac{vdv}{dx} = -C$$

$vdv = -Cdx$

$$\frac{v^2}{2} = -Cx + k \qquad x = -\frac{v^2}{2C} + \frac{k}{C}$$

Hence, graph (3) represents correctly.

9. **(b)** Distance along a line i.e., displacement (s) $= t^3$ ($\because s \propto t^3$ given)

By double differentiation of displacement, we get acceleration.

$$V = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2 \text{ and } a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$$

$a = 6t$ or $a \propto t$

Hence graph (b) is correct.

10. **(a)** The equation for the given v-x graph is

$$v = -\frac{v_0}{x_0}x + v_0 \quad ...(i)$$

$$\frac{dv}{dx} = -\frac{v_0}{x_0}$$

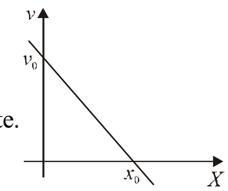
$$\therefore \quad a = v\frac{dv}{dx} = -\frac{v}{x_0} \times v = -\frac{v_0}{x_0}\left[-\frac{v_0}{x_0}x + v_0\right] \text{ from (i)}$$

$$\Rightarrow \quad a = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0} \quad ....(ii)$$

On comparing the equation (ii) with equation of a straight line

$y = mx + c$

we get $m = \frac{v_0^2}{x_0^2} = +$ve,



i.e. tan θ = + ve, i.e., θ is acute.

Also $c = -\frac{v_0^2}{x_0^2}$,

i.e., the y-intercept is negative

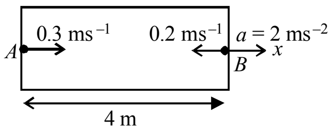
Hence graph (a) correctly depicts corresponding a-x graph.

11. **(b)** Change in velocity = area under the acceleration-time graph

$$= \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

Since, initial velocity is zero, final velocity i.e., maximum velocity is 55 m/s.

12. (8)



For ball A

$$u_1 = 0.3\ \text{ms}^{-1},\ a_1 = -2\text{ms}^{-2},\ s_1 = x,\ t_1 = t$$

$$\therefore \quad s_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$$

$$x = 0.3t - t^2 \qquad \text{...(i)}$$

For ball B

$$u_2 = 0.2\ \text{ms}^{-1},\ a_2 = 2\text{ms}^{-2},\ s_2 = 4 - x,\ t_2 = t$$

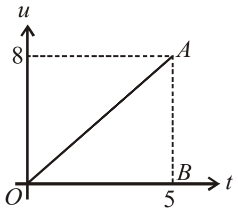
$$\therefore \quad s_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2$$

$$4 - x = 0.2\,t + t^2 \qquad \text{...(ii)}$$

Adding eq. (i) and (ii)

$$4 = 0.5t \quad \therefore \quad t = \frac{4}{0.5} = 8\text{s}.$$

13. (20)



Distance travelled = Area of speed-time graph

$$= \frac{1}{2} \times 5 \times 8 = 20\ \text{m}$$

14. (3) Distance X varies with time t as $x^2 = at^2 + 2bt + c$

$$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b \Rightarrow x\frac{dx}{dt} = at + b \Rightarrow \frac{dx}{dt} = \frac{(at+b)}{x}$$

$$\Rightarrow x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = a$$

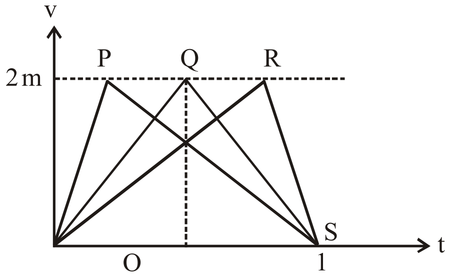
$$\Rightarrow \frac{d^2x}{dt^2} = \frac{a - \left(\frac{dx}{dt}\right)^2}{x} = \frac{a - \left(\frac{at+b}{x}\right)^2}{x}$$

$$= \frac{ax^2 - (at+b)^2}{x^3} = \frac{ac - b^2}{x^3}$$

$\Rightarrow \quad a \propto x^{-3}$ Hence, n = 3

15. (a, c, d) At $t = 0$ and $t = 1$ body is at rest
Initially α is positive so that the body acquires some velocity. Then α should be negative so that the body comes to rest. Hence α cannot remain positive for all time in the interval $0 \le t \le 1$
The journey is depicted in the following v-t graph.
Total time of journey = 1 sec
Total displacement = 1 m = Area under (v-t) graph

$$v_{max} = \frac{2s}{t} = \frac{2 \times 1}{1} = 2\ \text{m/s}$$



For path OQ, acceleration (α)

$$= \frac{\text{change in velocity}}{\text{time}} = \frac{2}{1/2} = 4\ \text{m/s}^2$$

For path QS is retardation = -4 m/s^2
For path OP, α (acceleration) > 4 m/s^2
For path PS (acceleration) < -4 m/s^2
For path OR, acceleration α < 4 m/s^2
For path RS, retardation α > 4 m/s^2
Hence $\alpha \ge 4$ at some point or points in its path.

16. (i) Let t_1 be the time taken by the car to attain the maximum velocity v_m while it is acceleration.
Using v = $u + at$

$$v_m = 0 + \alpha t_1 \text{ or } t_1 = \frac{v_m}{\alpha} \qquad \text{.... (i)}$$

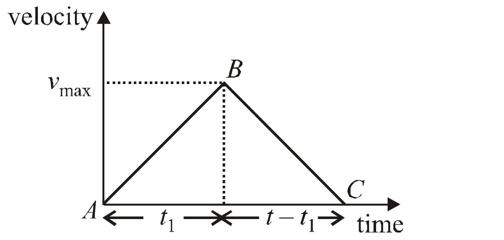
Since the total time elapsed is t, the car decelerators for time $t_2 = (t - t_1)$ to come by rest, $a = -\beta$ and $v = 0$
Usint $v = u + at_2$

$$0 = v_m - \beta(t - t_1) \text{ or } t_1 = t + \frac{v_m}{\beta} \qquad \text{.... (ii)}$$

Using (i) in (ii), we get

$$\frac{v_m}{\alpha} = t - \frac{v_m}{\beta} \text{ or } t = v_m\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$\text{or} \quad v_m = \frac{t\alpha\beta}{(\alpha+\beta)} \qquad \text{.... (iii)}$$



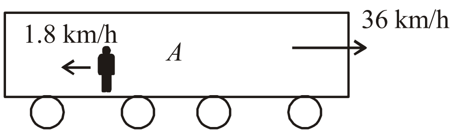
(ii) Total distance travelled = area of $\Delta\ ABC$

$$= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times t \times v_{max}$$

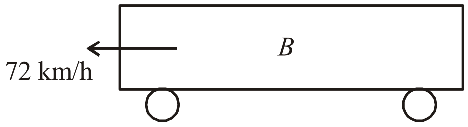
$$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha+\beta} t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right) t^2$$

Topic-3 : Relative Velocity in One Dimension

1. (a) According to question, train A and B are running on parallel tracks in the opposite direction.



$V_A = 36$ km/h $= 10$ m/s

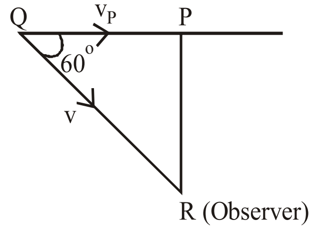


$V_B = -72$ km/h $= -20$ m/s

$V_{MA} = -1.8$ km/h $= -0.5$ m/s

$V_{man, B} = V_{man, A} + V_{A, B}$
$= V_{man, A} + V_A - V_B = -0.5 + 10 - (-20)$
$= -0.5 + 30 = 29.5$ m/s.

2. **(d)**



Distance, PQ $= v_p \times t$

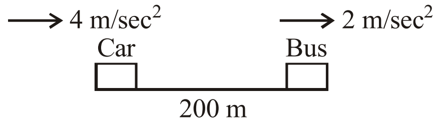
(Distance = speed × time)

Distance, QR = V.t

$$\cos 60° = \frac{PQ}{QR}$$

$$\frac{1}{2} = \frac{v_p \times t}{V.t} \Rightarrow v_p = \frac{v}{2}$$

3. **(c)**



Given, $u_C = u_B = 0$, $a_C = 4$ m/s^2, $a_B = 2$ m/s^2

hence relative acceleration, $a_{CB} = 2$ m/sec^2

Now, we know, $s = ut + \frac{1}{2}at^2$

$200 = \frac{1}{2} \times 2t^2 \quad \because u = 0$

Hence, the car will catch up with the bus after time

$t = 10\sqrt{2}$ second

4. **(c)** Person's speed walking only is $\frac{1}{60} \frac{\text{"escalator"}}{\text{second}}$

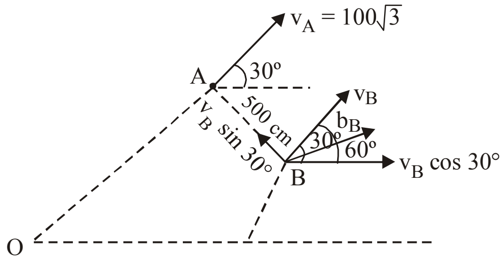
Standing the escalator without walking the speed is $\frac{1}{40} \frac{\text{"escalator"}}{\text{second}}$

Walking with the escalator going, the speed add.

So, the person's speed is $\frac{1}{60} + \frac{1}{40} = \frac{15}{120} \frac{\text{"escalator"}}{\text{second}}$

So, the time to go up the escalator $t = \frac{120}{5} = 24$ second.

5. **(5)**



The relative velocity of B with respect to A is perpendicular to the line of motion of A i.e., OA.

As the velocity component of A and B along the same line OA.

$\therefore \quad v_B \cos 30° = v_A = 100\sqrt{3}$

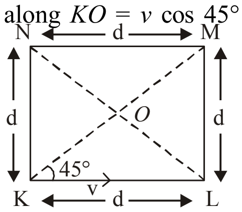
$$\Rightarrow \quad v_B = \frac{v_A}{\cos 30°} = \frac{100\sqrt{3}}{\sqrt{3}/2} = 200 \text{ ms}^{-1}$$

$$\therefore \quad t_0 = \frac{\text{displacement}}{\text{velocity}} = \frac{500}{v_B \sin 30°} = \frac{500}{200 \times \frac{1}{2}} = 5\text{s}$$

6. $\left(\frac{\mathbf{d}}{\mathbf{v}}\right)$ Each person K, L, M, N moves with a uniform speed v such that K always move directly towards L, L dierectly towards M, M directly towards N and N directly towards K.

Here on the basis of symmetry we can say that K, L, M and N will meet at the centre of the square 'O'.

At any instant velocity component along $KO = v \cos 45°$ $= \frac{v}{\sqrt{2}}$



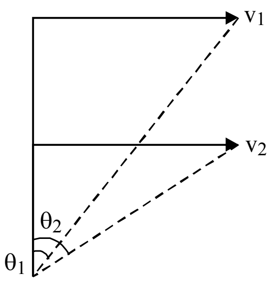
Distance $KO = d \cos 45° = \frac{d}{\sqrt{2}}$

$\therefore$ Time taken to meet at 'O' $= \frac{\text{distance}}{\text{velocity}}$

or, $t = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v}$

7. **(False)** As one train is moving in the direction of earth's rotation and other in the opposite direction. So, pressure exerted will be different.

8. **(b)** For a moving observer, the near by objects appear to move in the opposite direction at a large speed. This is because the angular speed of the near by object w.r.t observer is large. As the object moves away the angular velocity decreases and therefore its speed seems to be less. The distant object almost remains stationary.



$\because \theta_2 > \theta_1 \quad \therefore \omega_2 > \omega_1$

From the concept of relative velocity

$\vec{v}_{21} = \vec{v}_{2G} - \vec{v}_{1G}$

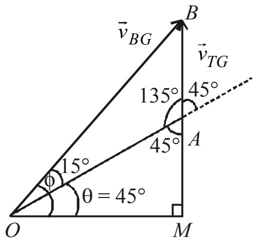
where G is the laboratory frame.

9. **(a)** (a) For the ball B to hit the trolley-A retative velocity of w.r.t. A, v_{BA} should be along OA. So, v_{BA} will make an angle 45° with +(ve) X-axis.

(b) Here $\theta = 45°$

$\therefore \quad \phi = \frac{4\theta}{3} = \frac{4 \times 45}{3} = 60°$

with +ve X-axis.



In ΔOMA,

$\theta = 45° \Rightarrow \angle OAM = 45°$

$\therefore \quad \angle OAB = 135°$

Also $\angle BOA = 60° - 45° = 15°$

Using sine law in ΔOBA

$$\frac{v_B}{\sin 135°} = \frac{v_T}{\sin 15°}$$

$$v_B = \frac{v_T \sin 135°}{\sin 15°} = \frac{(\sqrt{3}-1)\times 0.71}{0.2588} \simeq 2 \text{ m/s}$$

Hence speed of ball w.r.t ground, $v_B = 2$ m/s

Topic-4 : Motion Under Gravity

1. **(c)** For upward motion of helicopter,

$v^2 = u^2 + 2gh \Rightarrow v^2 = 0 + 2gh \Rightarrow v = \sqrt{2gh}$

Now, packet will start moving under gravity.

Let 't' be the time taken by the food packet to reach the ground.

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow -h = \sqrt{2gh}\,t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - \sqrt{2gh}\,t - h = 0$$

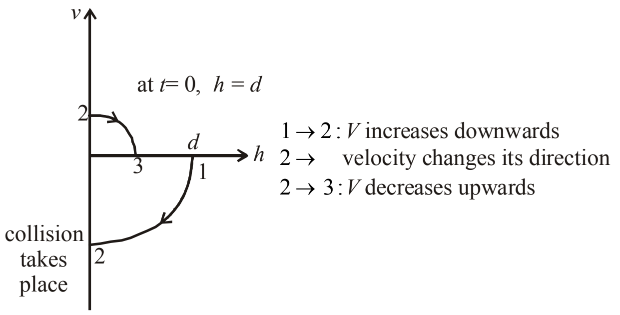
$$\text{or, } t = \frac{\sqrt{2gh} \pm \sqrt{2gh + 4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}}$$

$$\text{or, } t = \sqrt{\frac{2gh}{g}}(1+\sqrt{2}) \Rightarrow t = \sqrt{\frac{2h}{g}}(1+\sqrt{2}) \text{ or, } t = 3.4\sqrt{\frac{h}{g}}$$

2. **(c)** For uniformly accelerated/ deaccelerated motion :

$$v^2 = u^2 \pm 2gh$$

As equation is quadratic, so, v-h graph will be a parabola



Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve).

Note that time $t = 0$ corresponds to the point on the graph where $h = d$.

Next time collision takes place at 3.

3. **(a)** For a body thrown vertically upwards acceleration remains constant ($a = -g$) and velocity at anytime t is given by $V = u - gt$

During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.

Hence graph (a) correctly depicts velocity versus time.

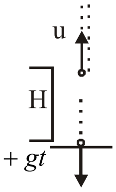
4. **(b)** $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

for $y_1 = -240$m, t = 8s $\quad \therefore \; y_2 - y_1 = 30t$ for $t \leq 8$s.

for $\quad t > 8$s,

$$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$$

5. **(c)** Speed on reaching ground



$v = \sqrt{u^2 + 2gh}$

Now, $v = u + at \quad \Rightarrow \quad \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g} \quad \text{(from question)}$$

$\Rightarrow \quad 2gH = n(n-2)u^2$

6. **(a)** For uniformly accelerated or deceleration motion $v^2 = u^2 \pm 2gh$ equation is quadratic hence $v - h$ graph will be a parabola.

Initial velocity downwards so negative after collision, it reverses its direction with smaller value hence velocity is upwards.

Further, $v'^2 = 2g \times \left(\frac{d}{2}\right) = gd$;

$$\therefore \quad \left(\frac{v}{v'}\right) = \sqrt{2} \text{ or } v = v'\sqrt{2} \Rightarrow v' = \frac{v}{\sqrt{2}}$$

As the direction is reversed and speed is decreased so graph (a) correctly depicts these conditions.

7. **(8.00)** Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2 \Rightarrow S = 0\times t + \frac{1}{2}gt^2$

$\Rightarrow \quad 200 = gt^2 \qquad [\because 2S = 100m]$

$$\Rightarrow t = \sqrt{\frac{200}{g}} \qquad \ldots\text{(i)}$$

In last $\frac{1}{2}s$, body travels a distance of 19 m, so in $\left(t - \frac{1}{2}\right)$

distance travelled = 81

Now, $\frac{1}{2}g\left(t-\frac{1}{2}\right)^2 = 81 \quad \therefore \; g\left(t-\frac{1}{2}\right)^2 = 81\times 2$

$$\Rightarrow \left(t - \frac{1}{2}\right) = \sqrt{\frac{81\times 2}{g}}$$

$$\therefore \; \frac{1}{2} = \frac{1}{\sqrt{g}}(\sqrt{200} - \sqrt{81\times 2}) \qquad \text{from eq. (i)}$$

$$\Rightarrow \sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2}) \quad \Rightarrow \sqrt{g} = 2\sqrt{2}$$

$\therefore$ g = 8 m/s^2

8. **(True)** As the ball falls back to point of projection, so $s = 0$

$\therefore \quad v^2 - u^2 = 2\times g\times s = 0$

$\therefore \quad v = u$

3 Motion in a Plane

Topic-1 : Vectors

1. **(d)** Using,

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2 \times 3 \times 5 \cos\theta$

or $\cos\theta = -0.3$

$\left(2\vec{A}_1 + 3\vec{A}_2\right) \cdot \left(3\vec{A}_1 - 2\vec{A}_2\right) = 2A_1 \times 3A_1$

$+ (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2 \times 2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2 \;\; 6A_2^2 + 5A^{-}_1A_2\cos\theta$

$= 6 \times 3^2 - 6 \times 5^2 + 5 \times 3 \times 5(-0.3) = -118.5$

2. **(c)** From figure,

$\vec{r}_G = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k} \quad \Rightarrow \quad \vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}\right) - \left(\frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}\right) = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$

3. **(a)** Let magnitude of two vectors $\vec{A}$ and $\vec{B}$ = a

$|\vec{A} + \vec{B}| = \sqrt{a^2 + a^2 + 2a^2\cos\theta}$ and

$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2\left[\cos(180° - \theta)\right]}$

$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$

and accroding to question,

$|\vec{A} + \vec{B}| = \quad n|\vec{A} - \vec{B}|$

or, $\frac{a^2 + a^2 + 2a^2\cos\theta}{a^2 + a^2 - 2a^2\cos\theta} = n^2$

$\Rightarrow \frac{\cancel{a^2}(1 + 1 + 2\cos\theta)}{\cancel{a^2}(1 + 1 - 2\cos\theta)} n^2 \Rightarrow \frac{(1 + \cos\theta)}{(1 - \cos\theta)} = n^2$

using componendo and dividendo theorem, we get

$\theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

4. **(a)** If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ..... (i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ..... (ii)

Solving equation (i) and (ii) we get

$a = \frac{1}{3}, b = \frac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

5. **(a)** Here $\vec{P} + b\vec{R} = \vec{S} \quad \Rightarrow \quad \vec{R} = \frac{\vec{S} - \vec{P}}{b}$

Also $\vec{R} = \vec{Q} - \vec{P}$

$\therefore \quad \frac{\vec{S} - \vec{P}}{b} = \vec{Q} - \vec{P} \quad \Rightarrow \quad \vec{S} - \vec{P} = b\vec{Q} - b\vec{P}$

$\therefore \quad \vec{S} = b\vec{Q} + (1 - b)\vec{P}$

6. **(195)**

Given : $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

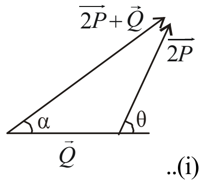
And, $\vec{r} = [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] = 3\hat{i} + \hat{j} - 2\hat{k}$

Torque, $\tau = \vec{r} \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$.

7. **(90)** Given,



$|\vec{R}| = |\vec{P}| \quad \Rightarrow \quad |\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ.\cos\theta = P^2$

$\Rightarrow \quad Q + 2P\cos\theta = 0$

$\Rightarrow \quad \cos\theta = -\frac{Q}{2P}$ ..(i)

$\tan\alpha = \frac{2P\sin\theta}{Q + 2P\cos\theta} = \infty \; (\because 2P\cos\theta + Q = 0)$

$\Rightarrow \quad \alpha = 90°$

8. **(2.00)** Given : $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$

$\therefore \quad |a\,\hat{i} + a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}| = \sqrt{3}|a\,\hat{i} - a\cos\omega t\,\hat{i} - a\sin\omega t\,\hat{j}|$

$\Rightarrow \quad |(1 + \cos\omega t)\hat{i} + \sin\omega t\,\hat{j}| = \sqrt{3}|(1 - \cos\omega t)\hat{i} - \sin\omega t\,\hat{j}|$

$\sqrt{2 + 2\cos\omega t} = \sqrt{3}\sqrt{2 - 2\cos\omega t}$

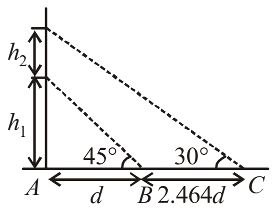
$\therefore \quad 1 + \cos\omega t = 3(1 - \cos\omega t)$

$\Rightarrow \quad 4\cos\omega t = 2 \quad \therefore \cos\omega t = \frac{1}{2} \quad$ or, $\omega t = \frac{\pi}{3}$

$\therefore \quad \frac{\pi}{6} \times \tau = \frac{\pi}{3} \quad \therefore \tau = 2.00$ seconds

Topic-2 : Motion in a Plane with Constant Acceleration

1. **(d)** From figure/ trigonometry,



$\frac{h_1}{d} = \tan 45° \quad \therefore h_1 = d$

And, $\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46d$

$\Rightarrow (h_1 + h_2) = \frac{3.46d}{\sqrt{3}} \Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$

$\therefore h_2 = d$

2. **(a)** Given : $\vec{u} = 5\hat{j}$ m/s

Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and

final coordinate $(20, y_0)$ in time t.

$S_x = u_x t + \frac{1}{2}a_x t^2 \qquad [\because u_x = 0]$

$$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2 \text{ s}$$

$$S_y = u_y \times t + \frac{1}{2} a_y t^2$$

$$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18 \text{ m}$$

3. **(c)** Using $S = ut + \frac{1}{2}at^2$

$$y = u_y t + \frac{1}{2} a_y t^2 \text{ (along } y \text{ Axis)}$$

$$\Rightarrow \quad 32 = 0 \times t + \frac{1}{2}(4)t^2 \Rightarrow \quad \frac{1}{2} \times 4 \times t^2 = 32 \Rightarrow t = 4\,s$$

$$S_x = u_x t + \frac{1}{2} a_x t^2 \qquad \text{(Along } x \text{ Axis)}$$

$$\Rightarrow \quad x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$$

4. **(d)** Given, Position vector,

$$\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin \omega t \hat{i} + \cos \omega t \hat{j})$

Acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 (\cos \omega t \hat{i} + \sin \omega t \hat{j})$$

$$\vec{a} = -\omega^2 \vec{r}$$

$\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0 \quad \therefore \vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

5. **(d)** $\vec{r} = 15t^2 \hat{i} + (4 - 20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$$\therefore a = \sqrt{30^2 + 40^2} = 50 \text{ m/s}^2$$

6. **(b)** As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4 = 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$$

$$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$$

[as $\vec{s}$ = change in position = $\vec{r}_f - \vec{r}_i$]

$$\vec{r}_r = 20\hat{i} + 20\hat{j}$$

$$|\vec{r}_r| = 20\sqrt{2}$$

7. **(c)** From given equation,

$$\vec{V} = K\left(y\hat{i} + x\hat{j}\right)$$

$$\frac{dx}{dt} = ky \text{ and } \frac{dy}{dt} = kx$$

Now $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx}, \Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

8. **(580)**

For pariticle 'A'	For particle 'B'
$X_A = -3t^2 + 8t + 10$	$Y_B = 5 - 8t^3$
$\vec{V}_A = (8 - 6t)\hat{i}$	$\vec{V}_B = -24t^2 \hat{j}$
$\vec{a}_A = -6\hat{i}$	$\vec{a}_B = -48t\hat{j}$

At t = 1 sec

$$\vec{V}_A = (8 - 6t)\hat{i} = 2\hat{i} \text{ and } \vec{v}_B = -24\hat{j}$$

$$\therefore \vec{V}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{i} - 24\hat{j}$$

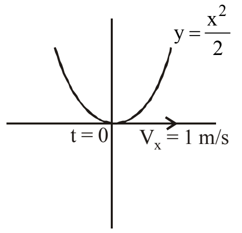
$\therefore$ Speed of B w.r.t. A, $\sqrt{v} = \sqrt{2^2 + 24^2}$

$= \sqrt{4 + 576} = \sqrt{580}$

$\therefore v = 580$ (m/s)

9. **(a, b, c, d)** According to question, equation

$$y = \frac{x^2}{2}$$



At t = 0, $\left.\begin{matrix} x = 0, y = 0 \\ u = 1 \end{matrix}\right\}$ given

$$y = \frac{x^2}{2}$$

$$\frac{dy}{dt} = \frac{1}{2} \cdot 2x \frac{dx}{dt} = x \frac{dx}{dt} \quad \Rightarrow v_y = xv_x$$

Now differenciate wrt time

$$\frac{d^2y}{dt^2} = x \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2$$

$$a_y = \frac{dx}{dt} \cdot V_x + xa_x$$

$$a_y = v_x^2 + xa_x$$

(a) If $a_x = 1$ and particle is at origin ($x = 0, y = 0$)

$a_y = v_x^2$

$a_y = 1^2 = 1$ m/s^2

(b) $a_x = 0$

$a_y = v_x^2 + xa_x \Rightarrow a_y = v_x^2$

If $a_x = 0$, v_x = constant = 1 $\Rightarrow a_y = I^2 = 1$

(c) At t = 0, x = 0 $\quad v_y = xv_x$

speed =1; $\quad v_y = 0 \Rightarrow v_x = 1$

(d) $a_x = 0$ implies that at t = 1s

$a_y = v_x^2 + xa_x \Rightarrow v_y = xv_x \Rightarrow a_y = v_x^2$

If $a_x = 0 \Rightarrow V_x$ = constant initially ($v_x = 1$) $\Rightarrow a_y = 1^2 = 1$

At t = 1 sec

$v_y = 0 + a_y \times t = 1 \times 1 = 1$

$$\tan\theta = \frac{v_y}{v_x} = x \quad (\theta \rightarrow \text{angle with x axis})$$

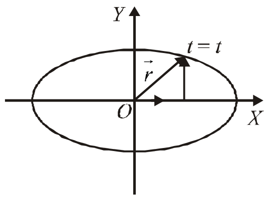
$$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{1}{1} = 1 \Rightarrow \theta = 45°$$

10. **(a, b, c)** $x = a\cos pt \Rightarrow \cos(pt) = \dfrac{x}{a}$ ... (i)

$y = b\sin pt \Rightarrow \sin(pt) = \dfrac{y}{b}$ ... (ii)

Squaring and adding eqn. (i) and (ii), we get,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



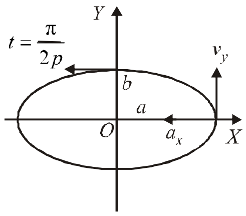
Hence path of the particle is an ellipse.

From the given equations we can find,

$$\frac{dx}{dt} = v_x = -ap\sin pt;\ \frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$$

$$\frac{dy}{dt} = v_y = pb\cos pt$$

and $\dfrac{d^2y}{dt^2} = a_y = -bp^2\sin pt$



At time $t = \dfrac{\pi}{2p}$ or $pt = \dfrac{\pi}{2}$

a_x and v_y become zero (because $\cos\dfrac{\pi}{2} = 0$). Only v_x and a_y are left, or we can say that velocity is along negative x-axis and acceleration along negative y-axis.

Hence, at $t = \dfrac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other.

At $t = t$, position of the particle

$$\vec{r}(t) = x\hat{i} + y\hat{j} = a\cos pt\,\hat{i} + b\sin pt\,\hat{j}$$

and acceleration of the particle

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} = -p^2[a\cos pt\,\hat{i} + b\sin pt\,\hat{j}]$$

$$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$$

Therefore, acceleration of the particle is always directed towards origin.

At $t = 0$, particle is at $(a, 0)$ and at $t = \dfrac{\pi}{2p}$, particle is at $(0, b)$. Therefore, the distance covered is one fourth of the elliptical path and not a.

Topic-3 : Projectile Motion

1. **(a)** Using principal of conservation of linear momentum for horizontal motion, we have

$2mv_x = mu + mu\cos 60°$

$$v_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Let R is the horizontal distance travelled by the body.

$$R = v_xT + \frac{1}{2}(0)(T)^2 \text{ (For horizontal motion)}$$

$$R = v_xT = \frac{3u}{4} \times \sqrt{\frac{2h}{g}}$$

$$\Rightarrow R = \frac{3\sqrt{3}u^2}{8g}$$

2. **(c)** Given, $y = 2x - 9x^2$

On comparing with,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have, $\tan\theta = 2$ or $\cos\theta = \dfrac{1}{\sqrt{5}}$

and $\dfrac{g}{2u^2\cos^2\theta} = 9$ or $\dfrac{10}{2u^2(1/\sqrt{5})^2} = 9$

$\therefore\ u = 5/3$ m/s

3. **(d)** R will be same for θ and $90° - \theta$.

Time of flights:

$t_1 = \dfrac{2u\sin\theta}{g}$ and

$$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right) = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$$

4. **(b)** For same range, the angle of projections are : θ and $90° - \theta$. So,

$h_1 = \dfrac{u^2\sin^2\theta}{2g}$ and

$$h_2 = \frac{u^2\sin^2(90° - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

Also, $R = \dfrac{u^2\sin 2\theta}{g}$

$$h_1h_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g}$$

$$= \frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2} = \frac{R^2}{16}$$

or $R^2 = 16\,h_1h_2$

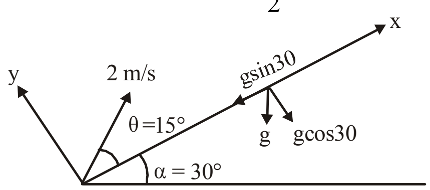
5. **(a)** On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

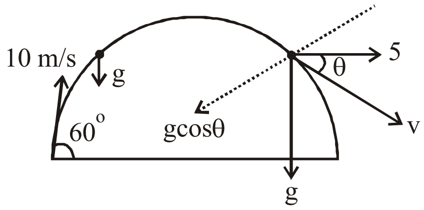
$$\text{Distance, S} = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{m} \simeq 20\text{cm}$$

6. **(b)**



$(10-5\sqrt{3})$

Horizontal component of velocity
$v_x = 10\cos 60° = 5$ m/s
vertical component of velocity

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s
After t = 1 sec.
Horizontal component of velocity $v_x = 5$ m/s
Vertical component of velocity

$v_y = |(5\sqrt{3}-10)|\text{ m/s} = 10-5\sqrt{3}$

Centripetal, acceleration $a_n = \dfrac{v^2}{R}$

$\Rightarrow R = \dfrac{v_x^2+v_y^2}{a_n} = \dfrac{25+100+75-100\sqrt{3}}{10\cos\theta}$ ...(i)

From figure (using (i))

$\tan\theta = \dfrac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$

$R = \dfrac{100(2-\sqrt{3})}{10\cos 15} = 2.8\text{m}$

7. **(a)** As we know, range $R = \dfrac{u^2\sin 2\theta}{g}$

and, area $A = \pi R^2$
$\therefore$ $A \propto R^2$ or, $A \propto u^4$

$\therefore \dfrac{A_1}{A_2} = \dfrac{u_1^4}{u_2^4} = \left[\dfrac{1}{2}\right]^4 = \dfrac{1}{16}$

8. **(c)** From question,
Horizontal velocity (initial),

$u_x = \dfrac{40}{2} = 20\text{m/s}$

Vertical velocity (initial), $50 = u_y t + \dfrac{1}{2}gt^2$

$\Rightarrow u_y \times 2 + \dfrac{1}{2}(-10)\times 4$

or, $50 = 2u_y - 20$ or, $u_y = \dfrac{70}{2} = 35\text{m/s}$

$\therefore \tan\theta = \dfrac{u_y}{u_x} = \dfrac{35}{20} = \dfrac{7}{4}$

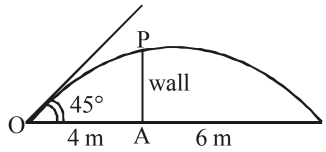
$\Rightarrow$ Angle $\theta = \tan^{-1}\dfrac{7}{4}$

9. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$
$\Rightarrow x = t$ … (i)

$y = 2t - \dfrac{1}{2}(10t^2)$ … (ii)

From (i) and (ii), $y = 2x - 5x^2$

10. **(b)**



As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $10 = 4H \Rightarrow H = \dfrac{10}{4} = 2.5\,\text{m}$

$(\because \text{Range} = 4\text{ m} + 6\text{ m} = 10\text{m})$

Maximum height, $H = \dfrac{u^2\sin^2\theta}{2g}$

$\therefore u^2 = \dfrac{H\times 2g}{\sin^2\theta} = \dfrac{2.5\times 2\times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$

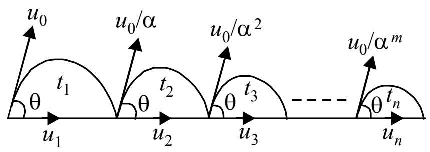
or, $u = \sqrt{100} = 10\text{ ms}^{-1}$

Height of wall PA

$= OA\tan\theta - \dfrac{1}{2}\dfrac{g(OA)^2}{u^2\cos^2\theta}$

$= 4 - \dfrac{1}{2}\times\dfrac{10\times 16}{10\times 10\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}} = 2.4\,\text{m}$

11. (4) Let u_1, u_2, u_3, ... be the horizontal velocity of the projectiles and t_1, t_2, t_3, ... be the time taken as shown in figure.



Average velocity $= \dfrac{\text{total displacement}}{\text{total time taken}}$

Time of flight $T = \dfrac{2u\sin\theta}{g}$

For given value of θ, value of T will change with the value of u.

$\therefore$ Total time taken $= t_1 + t_2 + t_3 + ...$

$= t_1 + \dfrac{t_1}{\alpha} + \dfrac{t_1}{\alpha^2} + ... = \dfrac{t_1}{1-\frac{1}{\alpha}} = \dfrac{t_1\alpha}{(\alpha-1)}$

Total displacement $= u_1t_1 + u_2t_2 + u_3t_3 + ...$

$= u_1t_1 + \dfrac{u_1}{\alpha}.\dfrac{t_1}{\alpha} + \dfrac{u_1}{\alpha^2}.\dfrac{t_1}{\alpha^2} + ... = \dfrac{u_1t_1}{1-\frac{1}{\alpha^2}} = \dfrac{u_1t_1\alpha^2}{(\alpha^2-1)}$

$\therefore$ Average velocity $= \dfrac{\text{total displacement}}{\text{total time}}$

$= \dfrac{u_1t_1\alpha^2}{(\alpha^2-1)} \times \dfrac{(\alpha-1)}{t_1\alpha} = \dfrac{u_1\alpha}{\alpha+1}$

According to question, average velocity = 0.8 V_1

or $\quad 0.8 V_1 = \dfrac{u_1\alpha}{\alpha+1}$ ..... (i)

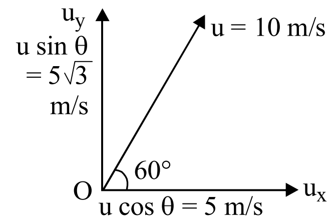
$V_1 = \dfrac{u_1 \times t_1}{t_1} = u_1$ ..... (ii)

From equation (i) and (ii)

$\alpha = 0.8\,\alpha + 0.8$ or $\alpha - 0.8\,\alpha = 0.8$

or $\quad 0.2\,\alpha = 0.8$ or $\alpha = \dfrac{0.8}{0.2} = 4$

12. **(5)** Here, $u = 10$ m/s, $\theta = 60°$



$\therefore \quad u_x = u\cos\theta = 10 \times \cos 60° = 5$ m/s

and $u_y = u\sin\theta = 10 \times \sin 60° = 5\sqrt{3}$ m/s

$\therefore \quad t = \dfrac{2u_y}{g} = \dfrac{2\times 5\sqrt{3}}{10} = \sqrt{3}$ s

Let a be the acceleration of train.

$$1.15 = u_x t - \frac{1}{2}at^2$$

$$\Rightarrow 1.15 = 5\times\sqrt{3} - \frac{1}{2}\times a \times(\sqrt{3})^2$$

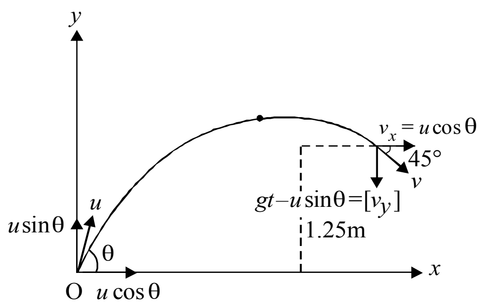
$$\Rightarrow \frac{3a}{2} = 5\sqrt{3} - 1.15 = 8.65 - 1.15 = 7.5$$

$$\therefore a = 7.5 \times \frac{2}{3} = 5 \text{ m/s}^2$$

13. **(True)** T.E. = P.E. + K.E. = Constant

At the top, K.E. is minimum and P.E. is maximum. Since K.E. is minimum so speed is also minimum.

14. Let 't' be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal.



According to question, we have following three conditions.

(i) Vertical displacement of stone is 1.25 m.

$\therefore \quad 1.25 = (u\sin\theta)\,t - \dfrac{1}{2}gt^2$

where g = 10 m/s^2

or $\quad (u\sin\theta)\,t = 1.25 + 5t^2$ ... (i)

(ii) Horizontal displacement of stone = 3 + displacement of object A.

$\therefore \quad (u\cos\theta)\,t = 3 + \dfrac{1}{2}at^2$

where $a = 1.5$ m/s^2

or $\quad (u\cos\theta)\,t = 3 + 0.75\,t^2$ ... (ii)

(iii) Horizontal component of velocity of stone = vertical component as velocity vector is inclined at 45° with horizontal.

$\therefore \quad (u\cos\theta) = gt - (u\sin\theta)$ ... (iii)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. $\therefore gt > u\sin\theta$. In upward motion $u\sin\theta > gt$).

Multiplying equation (iii) with t

$(u\cos\theta)\,t + (u\sin\theta)\,t = 10t^2$ ... (iv)

Now, (iv) – (ii) – (i) gives $4.25\,t^2 - 4.25 = 0$ or $t = 1$s

Substituting $t = 1$s in (i) and (ii), we get,

$u\sin\theta = 6.25$ m/s or $u_y = 6.25$ m/s

and $u\cos\theta = 3.75$ m/s.

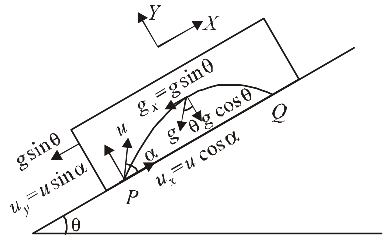
or $\quad u_x = 3.75$ m/s $\quad \therefore \quad \vec{u} = u_x\hat{i} + u_y\hat{j}$

or $\quad \vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

And the time after which the stone hits the object, $t = 1$ s

15. **(a)** The relative velocity of the particle with respect to the box is u. Now the relative velocity in x and y-direction u_x and u_y respectively.

Since, there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.



Taking relative terms w.r.t. box Y-direction motion

$u_y = + u\sin\alpha$

$a_y = - g\cos\theta$

$s_y = 0$ (activity is taken till the time the particle comes back to the box.)

$t_y = t$

$s_y = u_y t + \dfrac{1}{2}a_y t^2 \Rightarrow 0 = (u\sin\alpha)\,t - \dfrac{1}{2}g\cos\theta \times t^2$

$\Rightarrow \quad t = 0$ or $t = \dfrac{2u\sin\alpha}{g\cos\theta}$

Taking relative terms w.r.t. box X - direction motion

$u_x = + u\cos\alpha$; $a_x = 0$, $t_x = t$, $s_x = s_x$

$s_x = u_x t + \dfrac{1}{2}a_x t^2 \Rightarrow s_x = u\cos\alpha \times \dfrac{2u\sin\alpha}{g\cos\theta} = \dfrac{u^2\sin 2\alpha}{g\cos\theta}$

(b) The distance travelled by the box in time $\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle for the observer on ground to see horizontal displacement to be zero. Let the speed of the box at the time of projection of particle be U. Then for the motion of box with respect to ground.

$u_x = - U$; $a_x = - g\sin\theta$; $t_y = \dfrac{2u\sin\alpha}{g\cos\theta}$; $s_x = \dfrac{-u^2\sin 2\alpha}{g\cos\theta}$

$s_x = u_x t + \dfrac{1}{2}a_x t^2$

$$\frac{-u^2\sin 2\alpha}{g\cos\theta} = -U\left(\frac{2u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

Solving eqn. (i) we get $U = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

16. Till the bullets collide in air at point P they follow different path ACP and ABP.

Let t_1 = Time taken by first bullet in reaching P.

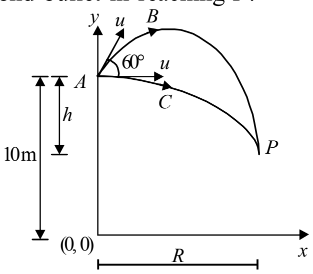
t_2 = Time taken by second bullet in reaching P.

For path ABP

Range, $R = (u\cos 60°)t_2$

Range, $R = (ut_1)$

For path ACP



$\therefore \quad \dfrac{ut_2}{2} = ut_1$

or $\quad t_2 = 2t_1$ .... (i)

Height for path ABP

$$h = -(u\sin 60°)t_2 + \frac{1}{2}gt_2^2$$

$$h = \frac{1}{2}gt_1^2$$

Height for path ACP $\therefore \quad \dfrac{-u\times\sqrt{3t_2}}{2} + \dfrac{1}{2}gt_2^2 = \dfrac{1}{2}gt_1^2$

or $\quad \dfrac{-5\sqrt{3}\times\sqrt{3t_2}}{2} + \dfrac{10t_1^2}{2} = \dfrac{10t_1^2}{2}$

or $\quad -15t_2 + 10t_2^2 = 10_1^2$ or $-3t_2 + 2t_2^2 = 2t_1^2$

or $\quad -(3\times 2t_1) + 2(2t_1)^2 = 2t_1^2$

or $\quad -6t_1 + (4\times 2)t_1^2 = 2t_1^2$ or $6t_1 = 6t_1^2$

or $\quad t_1 = 1$ sec .... (ii)

$\therefore \quad t_2 = 2t_1 = 2$ sec .... (iii)

Now $R = ut_1$ for path ACP

or $\quad R = 5\sqrt{3}\times 1 = 5\sqrt{3}$ m .... (iv)

Again $h = \dfrac{1}{2}gt_1^2$ for path ACP

$$= \frac{1}{2}\times 10\times(1)^2 = 5\text{ m}$$

$\therefore$ y-coordinate of $P = 10 - h = 10 - 5 = 5$ m .... (v)

(i) Time interval between the firings $= (t_2 - t_1)$
$= (2 - 1) = 1$ second

(ii) Coordinates of point, $P = (R, y) = (5\sqrt{3}\text{ m}, 5\text{ m})$

17. (i) Let t be the time taken for collision.

Till their point of collision objects thrown from A and C run along parabolic paths.

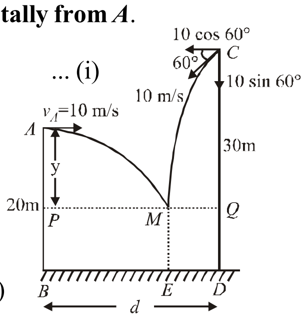
For mass m thrown horizontally from A.

For horizontal motion

PM $= V_A \times t = 10\,t$ ... (i)

For vertical motion

$u_y = 0$; $s_y = y$; $a_y = g$; $t_y = t$



$\therefore \quad y = \dfrac{1}{2}gt^2$ ... (ii)

Velocity in vertical direction

$v_y = u_y + a_y t = gt$ ... (iii)

For mass 2 m thrown from C

For horizontal motion $QM = [10\cos 60°]\,t$

$QM = 5\,t$ ... (iv)

For vertical motion $v_y = 10\sin 60° = 5\sqrt{3}$; $a_y = g$

$s_y = y + 10$; $t_y = t$

Now, $v_y = 5\sqrt{3} + gt$ ... (v)

and $(s_y) = u_y t + \dfrac{1}{2}a_y t^2$

$\Rightarrow \quad y + 10 = 5\sqrt{3}\,t + \dfrac{1}{2}gt^2$ ... (vi)

Now putting value of $y = \dfrac{1}{2}gt^2$ in eqn. (vi)

$\dfrac{1}{2}gt^2 + 10 = 5\sqrt{3}\,t + \dfrac{1}{2}gt^2 \Rightarrow t = \dfrac{2}{\sqrt{3}}$ sec

Distance between to towers

$$BD = PM + MQ = 10\,t + 5\,t = 15\,t = 15\times\frac{2}{\sqrt{3}} = 17.32\text{ m}$$

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

Let velocity of combined mass along horizontal $= v_x$

$m\times 10 - 2\,m\,10\cos 60° = 3\,m\times v_x$

$\Rightarrow \quad 10\,m - 10\,m = 3\,m\times v_x \Rightarrow v_x = 0$

Since, the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

$\therefore \quad \text{BE} = \text{PM} = 10\,t = 10\times\dfrac{2}{\sqrt{3}} = 11.547\text{ m}$

18. The range $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(49)^2\sin 2\times 45°}{9.8} = 245$ m.

Distance between the points of throw of particles is also 245 m. This means that the particles should collide at mid way between A and B at their maximum height $H =$

$$\frac{u^2\sin^2\theta}{2g} = \frac{(49)^2(\sin 45°)^2}{2\times 9.8} = 61.25\text{ m}$$

Each particle has the same speed of 49 m/s and horizontal velocity of each particle is same i.e.,

$u\cos 45° = 49/\sqrt{2}$ m/s

Applying conservation of linear momentum at the point of collision

$$m_1(u\cos 45°) - m_2(u\cos 45°) = m_2 v - m_1(u\cos 45°)$$

$$20\times 49\times\frac{1}{\sqrt{2}} - 40\times 49\times\frac{1}{\sqrt{2}} = -20\times 49\times\frac{1}{\sqrt{2}} + 40\,V$$

$\therefore \quad V = 0$

Particle Q is at rest after collision and will fall vertically downwards at the midway.

Time taken $= \sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2\times 61.25}{9.8}} = 3.53$ s

Topic-4 : Relative Velocity in Two Dimensions & Uniform Circular Motion

1. **(a)** Here, $R = 0.1$ m

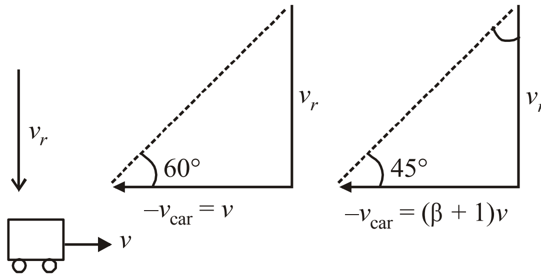
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105\text{ rad /s}$$

Acceleration of the tip of the clock second's hand,

$a = \omega^2 R = (0.105)^2(0.1) = 0.0011 = 1.1\times10^{-3}\ \text{m/s}^2$

Hence, average acceleration is of the order of 10^{-3}.

2. **(d)** The given situation is shown in the diagram. Here v_r be the velocity of rain drop.



When car is moving with speed v,

$\tan 60° = \dfrac{v_r}{v}$ ...(i)

When car is moving with speed $(1+\beta)v$,

$\tan 45° = \dfrac{v_r}{(\beta+1)v}$ ...(ii)

Dividing (i) by (ii) we get,

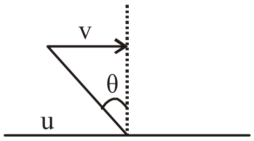
$\sqrt{3}v = (\beta+1)v \Rightarrow \beta = \sqrt{3}-1 = 0.732.$

3. **(c)** $\sin\theta = \dfrac{u}{v} = \dfrac{2}{4} = \dfrac{1}{2}$

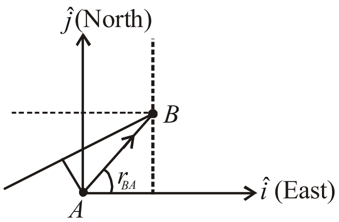
or $\theta = 30°$

with respect to flow,

$= 90° + 30° = 120°$



4. **(b)**



$\vec{v}_A = 30\hat{i} + 50\hat{j}$ km/hr

$\vec{v}_B = (-10\hat{i})$ km/hr

$r_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = 40\hat{i} - 50\hat{j}$

$t_{\text{minimum}} = \dfrac{|(\vec{r}_{BA})\cdot(\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2} = \dfrac{|(80\hat{i}+150\hat{j})(-40\hat{i}-50\hat{j})|}{(10\sqrt{41})^2}$

$\therefore\ t = \dfrac{10700}{10\sqrt{41}\times10\sqrt{41}} = \dfrac{107}{41} = 2.6$ hrs.

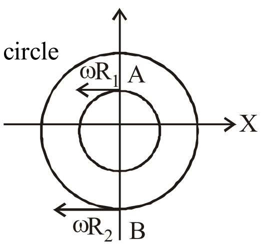
5. **(c)** From, $\theta = \omega t = \omega\dfrac{\pi}{2\omega} = \dfrac{\pi}{2}$

So, both have completed quater circle

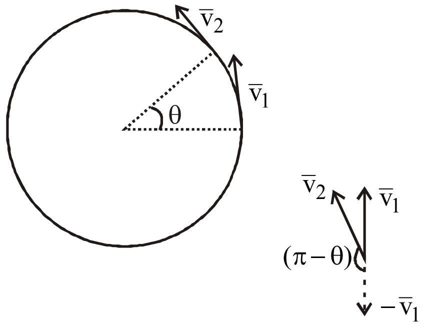
Relative velocity,

$v_A - v_B = \omega R_1(-\hat{i}) - \omega R_2(-i)$

$= \omega(R_2 - R_1)i$



6. **(d)**



Change in velocity,

$|\Delta\bar{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2\cos(\pi-\theta)}$

$= 2v\sin\dfrac{\theta}{2}$ $\qquad (\because |\vec{v}_1| = |\vec{v}_2|) = v$

$= (2\times10)\times\sin(30°) = 2\times10\times\dfrac{1}{2}$

$= 10$ m/s

7. **(c)** Speed, V = constant (from question)

Centripetal acceleration,

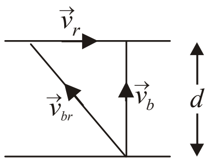
$a = \dfrac{V^2}{r}$

ra = constant

Hence graph (c) correctly describes relation between acceleration and radius.

8. **(b)** Shortest route corresponds to $\vec{v}_b$ perpendicular to river flow

$\therefore\ t = \dfrac{d}{v_b} = \dfrac{d}{\sqrt{v_{br}^2 - v_r^2}}$

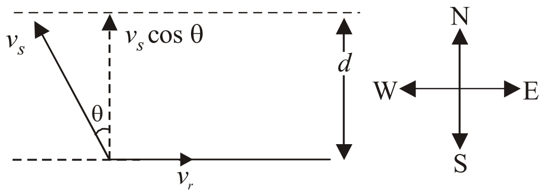


or $t = \dfrac{d}{v_b} = \dfrac{1\text{km}}{\dfrac{1}{\dfrac{1}{4}}}$

or $\dfrac{1}{4} = \dfrac{1}{\sqrt{25 - v_r^2}}$

$\Rightarrow v_r = 3$ km/h

9. **(a)** Time taken to cross the river $t = \dfrac{d}{v_s\cos\theta}$



For time to be minimum, $\cos\theta$ = maximum i.e., $\theta = 0°$

Hence swimmer should swim due north.

Laws of Motion

Topic-1 : Ist, IInd & IIIrd Laws of Motion

1. **(a)** Given

$\vec{F} = k(v_y\hat{i} + v_x\hat{j})$

$\therefore F_x = kv_y\hat{i},\ F_y = kv_x\hat{j}$

$\frac{mdv_x}{dt} = kv_y \Rightarrow \frac{dv_x}{dt} = \frac{k}{m}v_y$

Similarly, $\frac{dv_y}{dt} = \frac{k}{m}v_x$

$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$

$v_y^2 = v_x^2 + C$

$v_y^2 - v_x^2 =$ constant

$\vec{v} \times \vec{a} = (v_x\hat{i} + v_y\hat{j}) \times \frac{k}{m}(v_y\hat{i} + v_x\hat{j})$

$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m} = (v_x^2 - v_y^2)\frac{k}{m}\hat{k} =$ constant

2. **(b)** From the Newton's second law,

$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\left(\frac{dm}{dt}\right)$ ...(i)

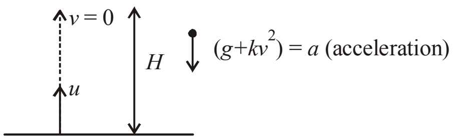
We have given, $\frac{dM(t)}{dt} = bv^2(t)$ ...(ii)

Thrust on the satellite,

$F = -v\left(\frac{dm}{dt}\right) = -v(bv^2) = -bv^3$ [Using (i) and (ii)]

$\Rightarrow F = M(t)a = -bv^3 \Rightarrow a = \frac{-bv^3}{M(t)}$

3. **(d)**



$\vec{F} = mkv^2 - mg$ ($\because mg$ and mkv^2 act opposite to each other)

$\vec{a} = \frac{\vec{F}}{m} = -[kv^2 + g]$

$\Rightarrow v\cdot\frac{dv}{dh} = -[kv^2 + g]$ $\left(\because a = v\frac{dv}{dh}\right)$

$\Rightarrow \int_u^0 \frac{v\cdot dv}{kv^2 + g} = \int_0^h dh$ $\Rightarrow \frac{1}{2k}\ln\left[kv^2 + g\right]_u^0 = -h$

$\Rightarrow \frac{1}{2k}\ln\left[\frac{ku^2 + g}{g}\right] = h$

4. **(a)** Net acceleration

$\frac{dv}{dt} = a = -(g + \gamma v^2)$

Let time t required to rise to its zenith (v = 0) so,

$\int_{v_0}^{0} \frac{-dv}{g + \gamma v^2} = \int_0^t dt$ [for H_{max}, v = 0]

$\therefore t = \frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\frac{\sqrt{\gamma}v_0}{\sqrt{g}}\right)$

5. **(d)** $v^2 = u^2 - 2gh$ or $v = \sqrt{u^2 - 2gh}$

Momentum, P = mv $= m\sqrt{u^2 - 2gh}$

At h = 0, P = mu and at $h = \frac{u^2}{f}, P = 0$

upward direction is positive and downward direction is negative.

6. **(b)** From Newton's second law

$\frac{dp}{dt} = F = kt$

Integrating both sides we get,

$\int_p^{3p} dp = \int_0^T kt\,dt \Rightarrow [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T \Rightarrow 2p = \frac{kT^2}{2} \Rightarrow T = 2\sqrt{\frac{p}{k}}$

7. **(a)** From $F = \frac{R}{t^2}v(t)$ Þ $m\frac{dv}{dt} = \frac{R}{t^2}v(t)$

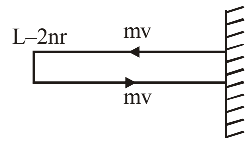
Integrating both sides $\int\frac{dv}{dt} = \int\frac{R\,dt}{mt^2}$

In $v = -\frac{R}{mt}$ $\therefore \ln v \propto \frac{1}{t}$

8. **(b)** Space between the supports for motion of beads is L–2nr

Average force experienced by each support,

$F = \frac{2mV}{\frac{2(L-2nr)}{V}} = \frac{mV^2}{L-2nr}$



9. **(a)** From question,

Mass of body, $m = 5$ kg

Velocity at $t = 0$, $u = (6\hat{i} - 2\hat{j})$ m/s

Velocity at $t = 10$s, $v = +6\hat{j}$ m/s

Force, $F = ?$

Acceleration, $a = \frac{v-u}{t} = \frac{6\hat{j} - (6\hat{i} - 2\hat{j})}{10} = \frac{-3\hat{i} + 4\hat{j}}{5}$ m/s²

Force, $F = ma$

$= 5 \times \frac{(-3\hat{i} + 4\hat{j})}{5} = (-3\hat{i} + 4\hat{j})N$

10. **(d)** Given : momentum $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$

And, force, $\vec{F} = \frac{d\vec{p}}{dt} = Ak\,[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$

Here, $\vec{F}.\vec{P} = 0$ But $\vec{F}.\vec{p} = Fp\cos\theta$

$\therefore$ $\cos\theta = 0 \Rightarrow \theta = 90°$.

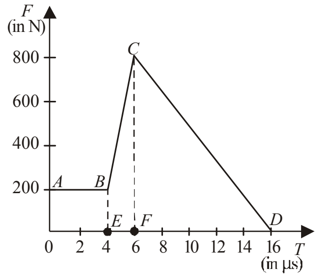
Hence, angle between the force momentum, θ = 90°

11. **(0.005)** Area under the $F - t$ graph gives the impulse imparted to the body.

The magnitude of total impulse of force on the body from $t = 4$ μs to $t = 16$ μs

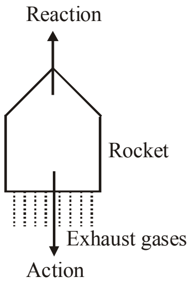
= area ($BCDFEB$)

= area of $BCFEB$ + area $CDFC$



$= \frac{1}{2}(200+800)\times 2\times 10^{-6} + \frac{1}{2}\times 10\times 800\times 10^{-6}$

$= 0.001 + 0.004 = 0.005$ Ns



12. **False :** The forward motion of rocket is due to the exhaust gases, thrown backward not due to surrounding air pushing backwards.
Here exhaust gases thrown backwards is action and rocket moving forward is reaction.
This phenomenon takes place in the absence of air as well.

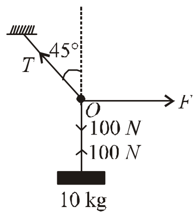
13. **(b, d)** Earth is an accelerated frame and hence, cannot be an inertial frame.
Earth is revolving round the sun and is rotating about its own axis.

14. **(b)** It is easier to pull a heavy object than to push it on a level ground. This is because the normal reaction in the case of pulling is less as compared by pushing. ($f = \mu N$). Therefore the frictional force is small in case of pulling.
The magnitude of frictional force depends on the nature of the two surfaces in contact. But is not the currect explanation of statement-1.

15. **(b)** Cloth can be pulled out without dislodging the dishes from the table because of inertia.
Law of inertia is the Newton's first law of motion.
For every action there is an equal and opposite reaction. This is Newton's third law of motion.

Topic-2 : Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. **(a)** From the free body diagram



$T\cos 45° = 100\ N$ ...(i)

$T\sin 45° = F$ ...(ii)

On dividing (i) by (ii) we get

$\frac{T\cos 45°}{T\sin 45°} = \frac{100}{F} \Rightarrow F = 100$ N

2. **(d)** Net force on the elevator = force on elevator + frictional force

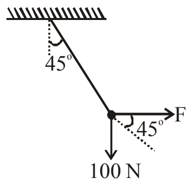
$\Rightarrow F = (10\,m + M)g + f$

where, m = mass of person, M = mass of elevator, f = frictional force

$\Rightarrow F = (10\times 68 + 920)\times 9.8 + 600 \Rightarrow F = 22000\ N$

$\Rightarrow P = FV = 22000\times 3 = 66000\ W$

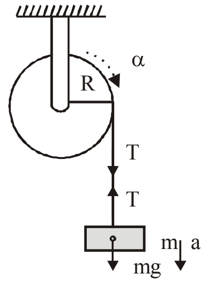
3. **(d)**
At equilibrium,



$\tan 45° = \frac{mg}{F} = \frac{100}{F}$

$\therefore$ F = 100 N

4. **(b)** From figure,



Acceleration $a = R\alpha$ ...(i)
and $mg - T = ma$ ...(ii)
From equation (i) and (ii)

$T\times R = mR^2\alpha = mR^2\left(\frac{a}{R}\right)$

or $T = ma$

$\Rightarrow mg - ma = ma$

$\Rightarrow a = \frac{g}{2}$

5. **(d)** Acceleration produced in upward direction

$a = \frac{F}{M_1 + M_2 + \text{Mass of metal rod}}$

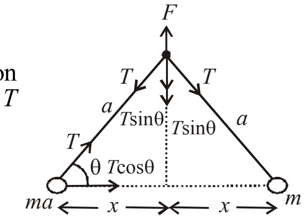
$= \frac{480}{20+12+8} = 12\ ms^{-2}$

Tension at the mid point

$T = \left(M_2 + \frac{\text{Mass of rod}}{2}\right)a$

$= (12+4)\times 12 = 192$ N

6. **(b)** From figure, acceleration of mass m is due to the force $T\cos\theta$



$\therefore\ T\cos\theta = ma$

$\Rightarrow a = \frac{T\cos\theta}{m}$ ... (i)

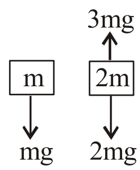
also, $F = 2T\sin\theta \Rightarrow T = \frac{F}{2\sin\theta}$

Putting this value of T in eqn. (i)

$a = \left(\frac{F}{2\sin\theta}\right)\frac{\cos\theta}{m}$

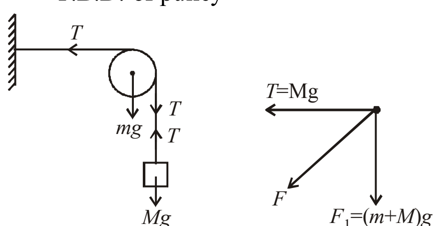
$= \frac{F}{2m\tan\theta} = \frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$ $\left[\because \tan\theta\frac{\sqrt{a^2-x^2}}{x}\right]$

7. **(c)** Before the string is cut the tension T has to hold both the masses $2m$ and m therefore,
$T = 3mg$
When the string is cut, the mass m is a freely falling body and its acceleration = acceleration due to gravity = g.
For mass $2m$, just after the string is cut, T remains $3mg$ because of the extension of string.



$\therefore\ 3mg - 2mg = 2m\times a \Rightarrow \frac{g}{2} = a$

8. **(d)** At equilibrium $T = Mg$
F.B.D. of pulley



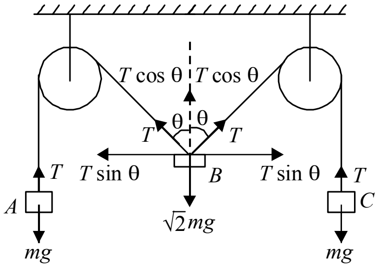
$F_1 = (m+M)\,g$

The resultant force on pulley

$F = \sqrt{F_1^2 + T^2} = [\sqrt{(m+M)^2 + M^2}\,]g$

9. **(c)** The tension in both strings will be same due to symmetry.

From figure,
T = mg



For equilibrium

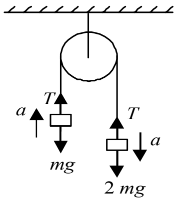
$\sqrt{2}\,mg = T\cos\theta + T\cos\theta = 2T\cos\theta$

$\therefore \quad \sqrt{2}\,mg = 2(mg)\cos\theta$

$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

10. **False :** From FBD, shown in case (a) for mass m

$T - mg = ma$ ... (i)



For mass $2m$

Case (a)

$2mg - T = 2ma$ ... (ii)

From (i) and (ii)

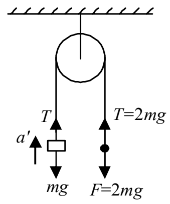
$a = g/3$ ... (iii)

Case (b) $T - mg = ma'$

$\Rightarrow \quad 2mg - mg = ma'$

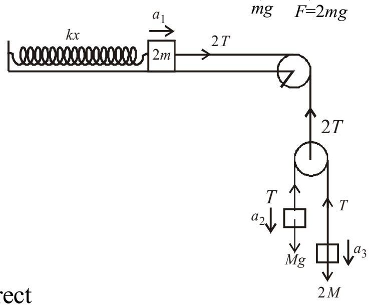
$[\because\ T = 2mg]$

$\therefore \quad a' = g$ ...(iv)



Hence, from eq (iii) & (iv) $a < a'$

11. **(d)** According to constraint relation from figure,



$a_1 = \frac{a_2 + a_3}{2}$

$\Rightarrow a_2 + a_3 = 2a_1$

$\Rightarrow \quad a_2 + a_3 = a_1 + a_1$

$\Rightarrow \quad a_1 - a_3 = a_2 - a_1$

$\Rightarrow$ Option (d) is correct

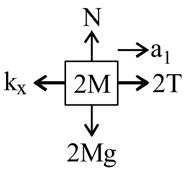
Let 'x' be the extension of the spring at a certain instant

$2T - k_x = 2Ma_1$

$2Mg - T = 2Ma_3$

$Mg - T = Ma_2$

On solving we get,



$a_1 = \frac{4g}{7} - \frac{2kx}{14M}$ ...(i)

$\therefore \quad \omega^2 = \frac{3k}{14M} \quad \therefore \quad \omega = \sqrt{\frac{3k}{14M}}$

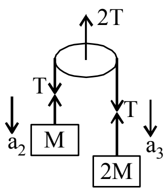
and $T = \frac{4Mg}{7} + \frac{2kx}{7}$ ...(ii)

For $a_1 = 0$ (Maximum extension of spring) we have from (i)

$\frac{4g}{7} - \frac{3kx}{14M} = 0$

$\therefore \quad 4g = \frac{3kx}{2M} \quad \therefore \quad x = \frac{8Mg}{3k}$

$\therefore \quad x_0 = 2x = \frac{16Mg}{3k}$



For $x = \frac{x_0}{4} = \frac{1}{4}\left(\frac{16Mg}{3k}\right) = \frac{4Mg}{3k}$

From eqn. (i) $a_1 = \frac{4g}{7} - \frac{3k}{14M} \times \frac{4Mg}{3x} = \frac{2g}{7}$

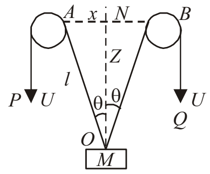
At $x = \frac{x_0}{2}$ particle is at mean position and its velocity = $A\omega$

$= \frac{x_0}{2}\sqrt{\frac{3k}{14M}} = \frac{8Mg}{3k}\sqrt{\frac{3k}{14M}}$

12. **(b)** Here from figure, $AN = x$ (= constant as pulley A and B are fixed), $NO = z$. Then velocity of mass $m = \frac{dz}{dt}$. Also,

let $OA = \ell$ then $\frac{d\ell}{dt} = U$

From ΔANO



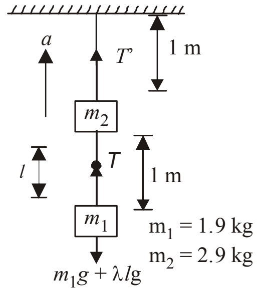
$x^2 + z^2 = \ell^2$ ...(i)

Differentiating equation (i) w.r.t to t

$0 + 2z\frac{dz}{dt} = 2\ell\frac{d\ell}{dt} \Rightarrow zv_M = \ell U$

$\Rightarrow \quad v_M = \frac{\ell}{z}U = \frac{U}{z/\ell} = \frac{U}{\cos\theta} \quad \left(\because \cos\theta = \frac{z}{\ell}\right)$

13. l = Mass of unit length of wire – 0.2 kg/m.



(i) Tension T at midpoint of lower wire :

l = Half-length = 0.5 m

$\therefore \quad T - (m_1 + \lambda l)g = (m_1 + \lambda l)a$

$T = (m_1 + \lambda l)(a + g)$

$= [1.9 + (0.2 \times 0.5)](0.2 + 9.8) = 2 \times 10 = 20$ N.

(ii) Tension T' at mid-point of upper wire :

$\therefore\ T' = [m_1 + (\tau \times 2l) + m_2]a + [m_2g + \lambda \times 2lg + m_1g]$

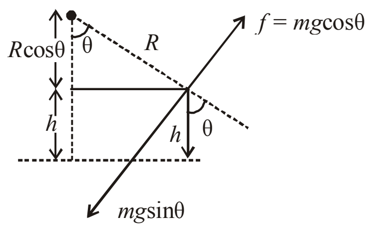
or $T' = [m_1 + (\lambda \times 2l) + m_2]\,(a + g)$

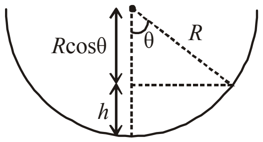
$= [1.9 + (0.2 \times 1) + 2.9][0.2 + 9.8] = 5 \times 10 = 50$N.

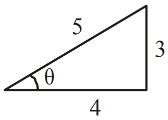
Topic-3 : Friction

1. **(a)** For balancing, $mg\sin\theta = f = \mu mg\cos\theta$

$\Rightarrow \tan\theta = \mu = \frac{3}{4} = 0.75$







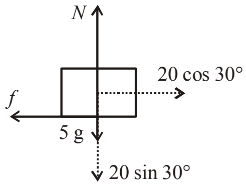
$$h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$$\therefore h = \frac{R}{5} = 0.2 \text{ m} \quad [\because \text{ radius, } R = 1\text{m}]$$

2. **(c)** A : $N = 5g + 20 \sin 30°$

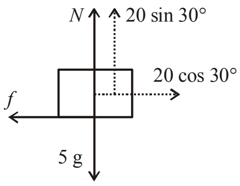
$= 50 + 20 \times \frac{1}{2} = 60$ N

Accelaration,



$$a_1 = \frac{F - f}{m} = \frac{20\cos 30° - \mu N}{5}$$

$$= \left[\frac{20\times\frac{\sqrt{3}}{2} - 0.2\times 60}{5}\right] = 1.06 \text{ m/s}^2$$



B : $N = 5g - 20 \sin 30°$

$= 50 - 20 \times \frac{1}{2} = 40$ N

$$a_2 = \frac{F - f}{m} = \left[\frac{20\cos 30° - 0.2\times 40}{5}\right] = 1.86 \text{ m/s}^2$$

Now $a_2 - a_1 = 1.86 - 1.06 = 0.8$ m/s²

3. **(b)** Taking (A + B) as system

$F - \mu(M + m)g = (M + m)a$

$$\Rightarrow a = \frac{F - \mu(M+m)g}{(M+m)}$$

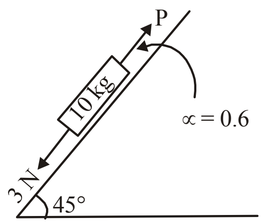
$$a = \frac{F - (0.2)4\times 10}{4} = \left(\frac{F-8}{4}\right) \quad ...(i)$$

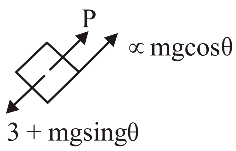
But, $a_{max} = \mu g = 0.2 \times 10 = 2$

$$\therefore \frac{F-8}{4} = 2$$

$\Rightarrow$ F = 16 N

4. **(a)**





Here, $mg \sin 45^\circ = \frac{100}{\sqrt{2}} = 50\sqrt{2}$

$[\because m = 10\text{kg}, g = 9.8 \text{ m s}^{-2}]$

$\mu mg \cos\theta = 0.6 \times mg \times \frac{1}{\sqrt{2}} = 0.6 \times 50\sqrt{2}$

$3 + mg \sin\theta = P + \mu mg \cos\theta$

$3 + 50\sqrt{2} = P + 30\sqrt{2}$

$\therefore$ P = 31.28 = 32 N

5. **(b)** Given : $m_1 = 5$kg; $m_2 = 10$kg; $\mu = 0.15$

FBD for m_1, $m_1g - T = m_1a$

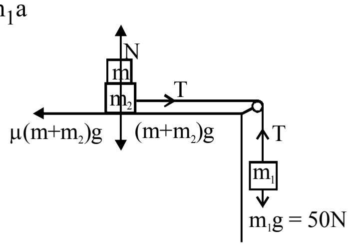
$\Rightarrow 50 - T = 5 \times a$

and, $T - 0.15\,(m + 10)g$

$= (10 + m)a$

For rest $a = 0$

or, $50 = 0.15\,(m + 10)\,10$



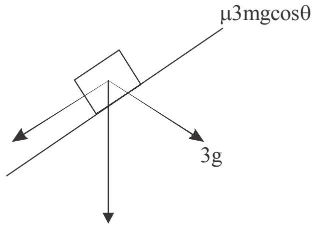
$$\Rightarrow \quad 5 = \frac{3}{20}(m + 10)$$

$\frac{100}{3} = m + 10 \;\therefore\; m = 23.3$kg; close to option (b)

6. **(b)** The coefficients of kinetic friction between the object and the incline

$$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) \Rightarrow \mu = 1 - \frac{1}{n^2} \qquad (\because \theta = 45°)$$

7. **(b)** Let μ be the minimum coefficient of friction

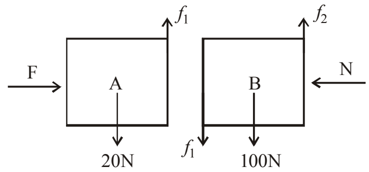


At equilibrium, mass does not move so,

$3mg \sin\theta = \mu 3mg \cos\theta$

$\therefore \mu_{min} = \tan\theta$

8. **(a)**



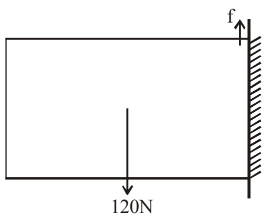
Assuming both the blocks are stationary

N = F

$f_1 = 20$ N

$f_2 = 100 + 20 = 120$N

Considering the two blocks as one system and due to equilibrium $f = 120$N



9. **(d)** $f = \mu(M + m)\,g$

$$a = \frac{f}{M+m} = \frac{\mu(M+m)g}{(M+m)} = \mu g$$

$= 0.05 \times 10 = 0.5$ ms^{-2}

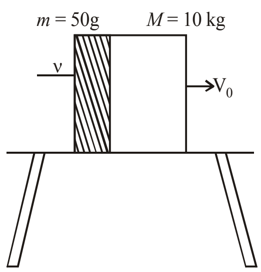
$$V_0 = \frac{\text{Initial momentum}}{(M+m)} = \frac{0.05V}{10.05}$$

$v^2 - u^2 = 2as$

$0 - u^2 = 2as$

$u^2 = 2as$

$$\left(\frac{0.05v}{10.05}\right)^2 = 2\times 0.5\times 2$$



Solving we get $v = 201\sqrt{2}$

Object falling from height H.

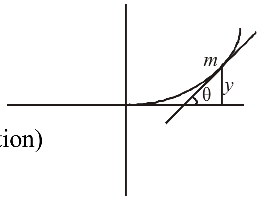
$$\frac{V}{10} = \sqrt{2gH}$$

$$\frac{201\sqrt{2}}{10} = \sqrt{2 \times 10 \times H}$$

$H = 40$ m $= 0.04$ km

10. **(a)** At limiting equilibrium,

$\mu = \tan\theta$



$$\tan\theta = \mu = \frac{dy}{dx} = \frac{x^2}{2} \quad \text{(from question)}$$

$\because$ Coefficient of friction $\mu = 0.5$

$$\therefore \quad 0.5 = \frac{x^2}{2} \quad \Rightarrow \quad x = \pm 1$$

$$\text{Now, } y = \frac{x^3}{6} = \frac{1}{6}m$$

11. **(d)** Force of friction at point P,

$$F_{friction} = \frac{1}{3}\text{Ma}\sin\theta$$

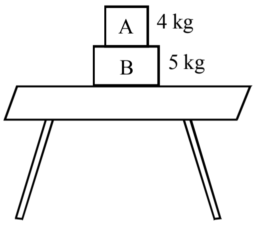
$$= \frac{1}{3}\text{Ma}\sin 90° \quad [\text{ here } \theta = 90°]$$

$$= \frac{\text{Ma}}{3}$$

12. **(c)** Minimum force on A

= frictional force between the surfaces

$= 12\,N$



Therefore maximum acceleration

$$a_{max} = \frac{12\text{N}}{4\text{kg}} = 3\text{m}/\text{s}^2$$

Hence maximum force,

F_{max} = total mass × a_{max}

$= 9 \times 3 = 27$ N

13. **(b)** Graph (b) correctly dipicts the acceleration-time graph of the block.

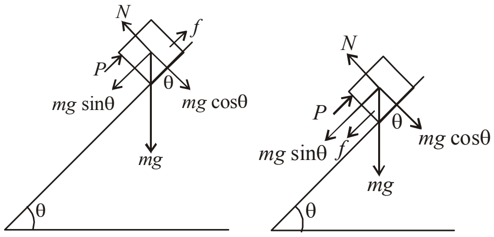
14. **(d)** When the body has maximum speed then

$\mu = 0.3x = \tan 45°$

$\therefore x = 3.33$ m

15. **(a)** According to question, $\tan\theta > \mu$, so block has a tendency to move down the incline. Force P is applied upwards along the incline to keep the block stationary. Here, at equilibrium $P + f = mg\sin\theta \;\Rightarrow f = mg\sin\theta - P$

Now as P increases, f decreases linearly with respect to P.



When $P = mg\sin\theta, f = 0$.

When force P is increased further, the block has a tendency to move upwards along the incline and hence frictional force acts downwards along the incline.

Here, at equilibrium $P = f + mg\sin\theta$

$\Rightarrow f = P - mg\sin\theta$

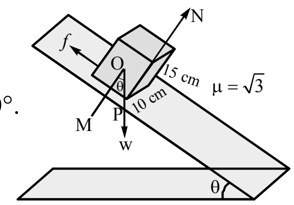
Now as P increases, f increases linearly w.r.t P.

Hence graph (a) correctly depicts the situation.

16. **(b)** Maximum angle not to slide the block, angle of inclination = angle of repose,

i.e., $\tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$.

For the block to topple, the condition of the block has been shown in the figure.



$$\text{In } \Delta POM, \; \tan\theta = \frac{PM}{OM} = \frac{10/2}{15/2} = \frac{5\,\text{cm}}{7.5\,\text{cm}} = \frac{2}{3}$$

So, $\theta < 60°$. From this we can conclude that the block will topple at lesser angle of inclination. Clearly the block will remain at rest on the plane up to a certain anlgle θ and then it will topple.

17. **(a)** Since the block is not moving forward for the maximum force F applied, therefore

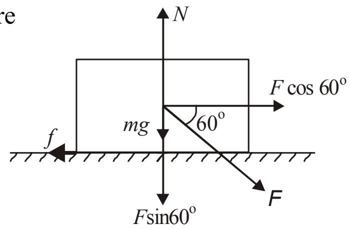
$F\cos 60° = f = \mu N$

(Horizontal direction)

For vertical equilibrium of the block,

$N = \text{mg} + F\sin 60°$

$\therefore \quad F\cos 60° = \mu N = \mu\,[F\sin 60° + mg]$



$$\Rightarrow \quad F = \frac{\mu mg}{\cos 60° - \mu\sin 60°} = \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20\,\text{N}$$

18. **(a)** The two forces acting on the insect are mg and N. Two components of mg are

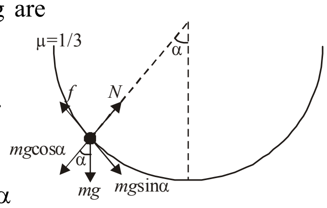
$mg\cos\alpha$ balances N.

$mg\sin\alpha$ is balanced by the frictional force.

$\therefore \quad N = mg\cos\alpha$

$f = mg\sin\alpha$

But $f = \mu N = \mu\, mg\cos\alpha$



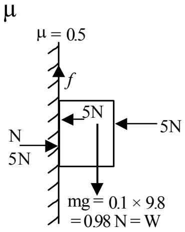
$$\therefore \quad \mu\, mg\cos\alpha = mg\sin\alpha \Rightarrow \cot\alpha = \frac{1}{\mu} \Rightarrow \cot\alpha = 3$$

19. **(b)** Limiting frictional force,

$f_l = \mu_s N = 0.5 \times 5 = 2.5\,N$.

For vertical equilibrium of the block,

Frictional force, $f = \text{mg} = 0.98\,N$.

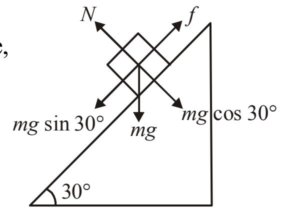


20. **(a)** The block is at rest.

For equilibrium, frictional force,

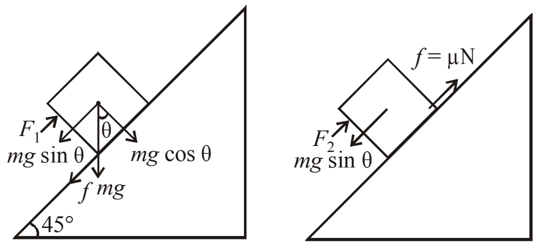
$f = mg \sin\theta = mg \sin 30°$

$= 2 \times 9.8 \times \frac{1}{2} = 9.8\text{N}$



21. **(5)**

Block moving upward | Block just remains stationary



For upward moving of block, pushing force $F_1 = mg \sin\theta + f$

$\therefore F_1 = mg \sin\theta + \mu\, mg \cos\theta = mg(\sin\theta + \mu\cos\theta)$

The force required to just prevent it from sliding down or block just remains stationary.

$F_2 = mg \sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$

Given , $F_1 = 3F_2$

$\therefore$ $\sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

$\therefore$ $1 + \mu = 3(1 - \mu)$ $[\because \sin\theta = \cos\theta]$

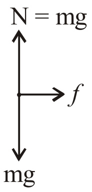
$\therefore$ $4\mu = 2$ $\Rightarrow \mu = 0.5$

$\therefore$ $N = 10\,\mu = 10 \times 0.5 = 5$

22. **(5N)** The frictional force is responsible to move the block of mass 1 kg with an acceleration of 5 m/s^2.

Therefore, frictional force,

$f = m \times a = 1 \times 5 = 5\,N.$



23. **(346)**

Acceleration of block while moving up an inclined plane,

$a_1 = g\sin\theta + \mu g\cos\theta$

$\Rightarrow a_1 = g\sin 30° + \mu g \cos 30°$

$$= \frac{g}{2} + \frac{\mu g\sqrt{3}}{2} \quad ..(i) \quad (\because \theta = 30°)$$

Using $v^2 - u^2 = 2a(s)$

$$\Rightarrow v_0^2 - 0^2 = 2a_1(s) \quad (\because u = 0)$$

$$\Rightarrow v_0^2 - 2a_1(s) = 0$$

$$\Rightarrow s = \frac{v_0^2}{a_1} \quad ...(ii)$$

Acceleration while moving down an inclined plane

$a_2 = g\sin\theta - \mu g\cos\theta \Rightarrow a_2 = g\sin 30° - \mu g\cos 30°$

$$\Rightarrow a_2 = \frac{g}{2} - \frac{\mu\sqrt{3}}{2}g \quad ...(iii)$$

Using again $v^2 - u^2 = 2as$ for downward motion

$$\Rightarrow \left(\frac{v_0}{2}\right)^2 = 2a_2(s) \Rightarrow s = \frac{v_0^2}{4a_2} \quad ...(iv)$$

Equating equation (ii) and (iv)

$$\frac{v_0^2}{a_1} = \frac{v_0^2}{4a_2} \Rightarrow a_1 = 4a_2 \Rightarrow \frac{g}{2} + \frac{\mu g\sqrt{3}}{2} = 4\left(\frac{g}{2} - \frac{\mu\sqrt{3}}{2}\right)$$

$\Rightarrow 5 + 5\sqrt{3}\mu = 4(5 - 5\sqrt{3}\mu)$ (Substituting, $g = 10$ m/s^2)

$\Rightarrow 5 + 5\sqrt{3}\mu = 20 - 20\sqrt{3}\mu \Rightarrow 25\sqrt{3}\mu = 15$

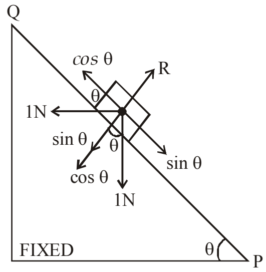
$$\Rightarrow \mu = \frac{\sqrt{3}}{5} = 0.346 = \frac{346}{1000}$$

So, $\frac{I}{1000} = \frac{346}{1000}$

24. **False :** Friction force opposes the relative motion of the surface of contact.

As the feet pushes the surface in backward direction, so frictional force exerted by the surface on the person is in the direction of his motion.

25. **(a, c)** The various forces acting on the block are as shown in the figure.



When θ = 45°, sinθ = cosθ

The block will remain stationary and the frictional force is zero.

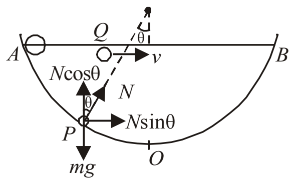
When θ > 45°, sinθ > cosθ

Therefore a frictional force acts towards Q.

When θ < 45°, cosθ > sinθ

Therefore a frictional force acts towards P.

26. **(a)** According to question, at A the horizontal speeds of both the masses is the same. As no force is acting in the horizontal direction the velocity of Q remains the same in horizontal.



In case of P as shown in figure at any intermediate position, the horizontal velocity first increases due to $N \sin\theta$, reaches a max value at O and then decreases. Hence it always remains greater than v. So, $t_P < t_Q$.

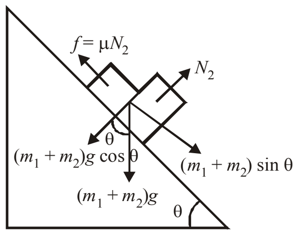
27. **(d)** Block will not be slip or will be at rest if

$(m_1 + m_2)g \sin\theta \le \mu m_2\, g\cos\theta$

$$\tan\theta \le \frac{\mu m_2 g}{(m_1 + m_2)g}$$

$$\Rightarrow \tan\theta \le \frac{\mu m_2}{m_1 + m_2}$$

$$\Rightarrow \tan\theta \le \frac{0.3 \times 2}{1 + 2} \le \frac{1}{5}$$



$\Rightarrow$ $\tan\theta \le 0.2$ i.e., $\theta \le 11.5°$

i.e., If the angle θ < 11.5° the frictional force is less than

$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6$ g

and is equal to $(m_1 + m_2)g \sin\theta$

Blocks will not slip on the inclined plane and friction is static.

At θ > 11.5° the bodies start moving on the inclined plane and friction is kinetic and equal to $\mu m_2 g \cos\theta$

28. Normal reaction, $N_1 = \mu ma \sin\theta$ and $N_2 = mg$

Applying pseudo force ma and resolving it.

$F_{net} = ma_r$

$ma\cos\theta - (f_1 + f_2) = ma_r$

$ma\cos\theta - \mu N_1 - \mu N_2 = ma_r$

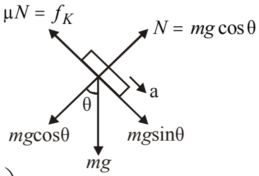
$$ma\cos\theta - \mu ma\sin\theta - \mu\, mg = ma_r$$

$$\Rightarrow a_r = a\cos\theta - \mu a\sin\theta - \mu g$$

$$= 25\times\frac{4}{5} - \frac{2}{5}\times 25\times\frac{3}{5} - \frac{2}{5}\times 10 = 10\,\text{m/s}^2$$

29. Acceleration of block down the plane

$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m}$$



$\therefore$ $a_A = g\sin\theta - \mu_{k,A}\, g\cos\theta$

$= g\sin 45° - \mu\cos 45°$

$$= 10\left(\frac{1}{\sqrt{2}}\right) - (0.2)(10)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}$$

And $a_B = g\sin\theta - \mu_{k,B}\, g\cos\theta$

$= g\sin 45°\ \mu_{kB}\ g\cos 45°$

$$= 10\left(\frac{1}{\sqrt{2}}\right) - (0.3)(10)\left(\frac{1}{\sqrt{2}}\right) = 3.5\sqrt{2}\,\text{m/s}^2$$

Let a_{AB} is relative acceleration of A w.r.t. B. Then

$a_{AB} = a_A - a_B$

The relative distance between A and B, L.

$$L = \frac{1}{2}a_{AB}t^2$$

or $$t^2 = \frac{2L}{a_{AB}} = \frac{2L}{a_A - a_B} = \frac{2(\sqrt{2})}{\left(4\sqrt{2}\right) - \left(3.5\sqrt{2}\right)}$$

$\Rightarrow t^2 = 4$ or $t = 2s$.

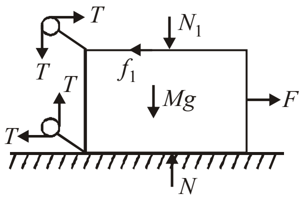
Distance moved by A during that time is given by

$$S_A = \frac{1}{2}a_4 t^2 = \frac{1}{2}\times 4.5\sqrt{2}\times 4 = 8\sqrt{2}\,\text{m}$$

Similarly for $B = 7\sqrt{2}$ m.

Hence both the blocks will come in line after A has travelled a distance $8\sqrt{2}$ m down the plane

30. **(a)** Free body diagram of mass M



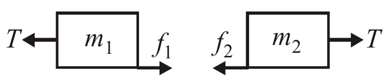
(b) The maximum value of force of friction between m_1 and M

$(f_1)_{max} = (0.3)(20)(10) = 60\,N$ ...(i)

The maximum value of force of friction between m_2 and M

$(f_2)_{max} = (0.3)(5)(10) = 15\,N$ ...(ii)

Forces on m_1 and m_2 in horizontal direction are as follows:



There are only two possibilities.

Case I Either both m_1 and m_2 will remain stationary (w.r.t. ground)

Case II both m_1 and m_2 will move (w.r.t. ground). First case is possible when

or $T \le (f_1)_{max}$ or $T \le 60\,N$ and $T \le (f_2)_{max}$ or $T \le 15\,N$

These conditions will be satisfied when $T \le 15\,N$ say $T = 14$ then $f_1 = f_2 = 14\,N$.

Therefore the condition $f_1 = 2f_2$ will not be satisfied.

Thus m_1 and m_2 both can't remain stationary.

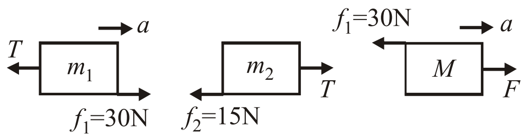
In the second case, when m_1 and m_2 both move

$f_2 = (f_2)\max = 15\,N$

$\therefore$ $f_1 = 2f_2 = 30\,N$

Since $f_1 < (f_1)_{max}$, there is no relative motion between m_1 and M, *i.e.*, all the masses move with same acceleration, say 'a'.

Free body diagrams and equations of motion are as follows:



For mass, m_1 : $30 - T = 20\,a$ ... (iii)

For mass, m_2 : $T - 15 = 5\,a$ ... (iv)

For mass, M : $F - 30 = 50\,a$ ... (v)

Adding eq. (iii) & (iv), we get.

acceleration $a = \frac{3}{5}\text{m/s}^2$.

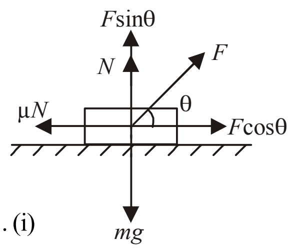
From eq. (iv) $T - 15 = \frac{5\times 3}{5}$ $\Rightarrow$ T = 18 N

From eq. (v) $F - 30 = 50\times\frac{3}{5}$ $\Rightarrow$ F = 60 N

31. Let force F be applied to move the body at an angle θ to the horizontal.

The body will move when

$F\cos\theta = \mu N$



from figure, normal reaction

$N = mg - F\sin\theta$

$F\cos\theta = \mu(mg - F\sin\theta)$

$$\Rightarrow F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \quad \text{... (i)}$$

Differentiating the above equation w.r.t. θ, we get

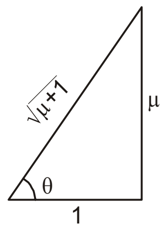
$$\frac{dF}{d\theta} = \frac{\mu mg}{(\cos\theta + \mu\sin\theta)^2}[-\sin\theta + \mu\cos\theta] = 0$$

$\therefore$ $\theta = \tan^{-1}\mu$

This is the angle for minimum force.

To find the minimum force substituting these values in equation (i)

$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}},\quad \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$



$$F = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu}{\sqrt{\mu^2+1}}\times\mu}$$

$$\Rightarrow F = \frac{\mu mg\left(\sqrt{\mu^2+1}\right)}{\mu^2+1} = \frac{\mu mg}{\sqrt{\mu^2+1}}$$

$\Rightarrow$ $F = mg\sin\theta$

32. According to question, mass M_1 moves downwards with a uniform velocity i.e., net acceleration of the system is zero. Or net pulling force on the system is zero.

For equilibrium,

(a) $M_1 g = M_2 g \sin 37° + \mu M_2 g \cos 37° + \mu M_3 g$

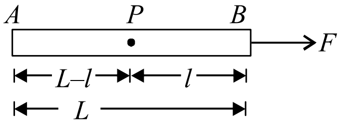
or $M_1 = M_2 \sin 37° + \mu M_2 \cos 37° + \mu M_3$

$$= (4)\left(\frac{3}{5}\right) + (0.25)(4)\left(\frac{4}{5}\right) + (0.25)(4) = 4.2\,\text{kg}$$

(b) Since, M_3 is moving with uniform velocity $T = \mu_1 m_2 g = (0.25) \times 4 \times 9.8 = 9.8\,\text{N}$

33. Let T be the tension in the rope at point P and a be the acceleration produced in the rope.

Mass per unit length of the rope is $\mu = \dfrac{F}{L}$



For the part AP;

$T = \mu(L - l)a$ ......(i)

For the part PB;

$F - T = \mu l a$ ......(ii)

$$F - T = \mu l\left[\frac{T}{\mu(L-l)}\right] \quad \text{[Using eq. (i)]}$$

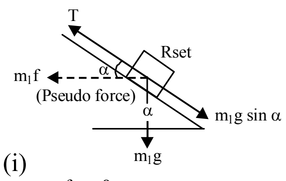
$$F - T = \frac{Tl}{L-l} \Rightarrow T\left[\frac{l}{L-l} + l\right] = F;$$

$$T\left[\frac{L}{L-l}\right] = F \Rightarrow T = F\left(\frac{L-l}{L}\right)$$

$$\text{or, } T = F\left[1 - \frac{l}{L}\right]$$

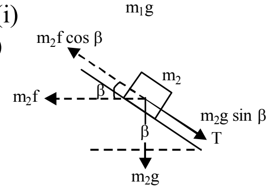
34. As cubes do not slide down the planes hence they have same acceleration.

Consider the FBD of the cubes along incline



$T + m_1 f \cos\alpha = m_1 g \sin\alpha \rightarrow$ (i)

$T + m_2 g \sin\beta = m_2 g \cos\beta \rightarrow$ (ii)



Eq (i) – Eq (ii)

$(m_1 \cos\alpha + m_2 \cos_\beta) f$
$= (m_1 \sin\alpha + m_2 \sin\beta) g$

$$\Rightarrow \quad f = \frac{(m_1 \sin\alpha + m_2 \sin\beta)}{(m_1 \cos\alpha + m_2 \cos\beta)} g$$

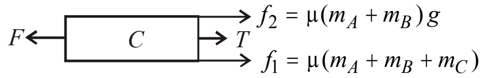
$$\text{from eq (i) } T = (m_1 g \sin\alpha) - (m_1 \cos\alpha)\left[\frac{m_1 \sin\alpha + m_2 \sin\beta}{m_1 \cos\alpha + m_2 \cos\beta}\right] g$$

$$\text{or, } T = g\left[\frac{m_1^2 \cos\alpha \sin\alpha + m_1 m_2 \cos\beta \sin\alpha - m_1^2 \cos\alpha \sin\alpha - m_1 m_2 \cos\alpha \sin\beta}{(m_1 \cos\alpha + m_2 \cos\beta)}\right]$$

$$\text{or, } T = \frac{m_1 m_2 [\cos\beta \sin\alpha - \sin\beta \cos\alpha] g}{(m_1 \cos\alpha + m_2 \cos\beta)}$$

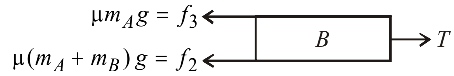
35. When force F is applied on block C will move towards left and the block 'B' will move towards right due to reaction of C on B, while block A always remains at rest.

The F.B.D. for mass C is



As C is moving with constant speed $F = f_1 + f_2 + T$ ... (i)

F.B.D. for mass B is



As B is moving with constant speed $f_2 + f_3 = T$ ... (ii)

Subtracting eq. (ii) from (i)

$F - (f_2 + f_3) = f_1 + f_2 + T - T = f_1 + f_2$

$\Rightarrow F = f_1 + 2f_2 + f_3 = \mu(m_A + m_B + m_C) g + 2\mu(m_A + m_B) g + \mu m_A g$

$F = \mu(4m_A + 3m_B + m_C) g$

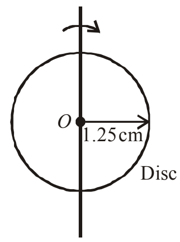
(Given: $m_A = 3\text{kg}$, $m_B = 4\text{kg}$, $M_e = 5$ kg and $\mu = 0.25$)

$= 0.25[4 \times 3 + 3 \times 4 + 5] \times 9.8 = 71.05\,N$

Hence, force necessary to drag, F = 71.05 N

Topic-4 : Circular Motion & Banking of Road

1. **(d)** Using, $\mu mg = \dfrac{mv^2}{r} = mr\omega^2$



$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi$ rad/sec

Radius, $r = 1.25\text{ cm} = 1.25 \times 10^{-2}\text{ m}$

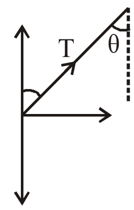
Coefficient of friction, μ = ?

$$\mu mg = \frac{m(r\omega)^2}{r} \quad (\because \; v = r\omega)$$

$$\mu mg = mr\omega^2$$

$$\Rightarrow \mu = \frac{r\omega^2}{g} = \frac{1.25 \times 10^{-2} \times \left(7 \times \frac{22}{7}\right)^2}{10} = \frac{1.25 \times 10^{-2} \times 22^2}{10} = 0.6$$

2. **(d)** Given, $\theta = 45°$, $r = 0.4$ m, $g = 10$ m/s²



$T\sin\theta = \dfrac{mv^2}{r}$ ...... (i)

$T\cos\theta = mg$ ...... (ii)

From equation (i) & (ii) we have,

$$\tan\theta = \frac{v^2}{rg}$$

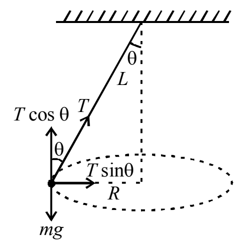
$v^2 = rg \quad \because \; \theta = 45°$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{0.4 \times 10} = 2\text{ m/s}$$

3. **(c)**

4. **(d)** The horizontal component of tension provides the necessary centripetal force.



$$\therefore T\sin\theta = \frac{mv^2}{R} = mR\omega^2$$

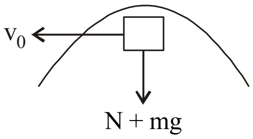
$$T \times \frac{R}{L} = mR\omega^2 \; [\because \sin\theta = \frac{R}{L}]$$

$$\therefore \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$= \frac{18}{0.5} = 36\text{ rad/s}$$

5. **(a)** According to question, the speed with which the block enters the track is the same in all the tracks and the block rises to the same height so from law of conservation of energy, speed of the block at highest point will be same in all four cases.

Let the velocity at the highest point be v



(N + mg) provides the centripetal force $\frac{mv^2}{R}$ to the body

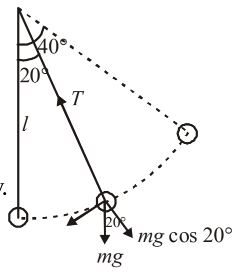
$$N + mg = \frac{mv^2}{R}$$

or $$N = \frac{mv^2}{R} - mg$$

R (the radius of curvature) in first case is minimum.
Hence, normal reaction N will be maximum in first case.

6. **False :** The angular amplitude of the pendulum is 40° given, when its angular displacement is 20° then

For equilibrium of the bob, $T - mg\cos 20° = \frac{mv^2}{l}$, where l is the length of the pendulum and v is the velocity of the bob.

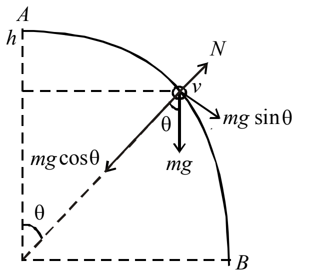


$\therefore \quad T = mg\cos 20° + \frac{mv^2}{l}$

$\frac{mv^2}{l}$ is always a positive quantity.

Hence, clearly $T > mg\cos 20°$.

7. **(d)** As the bead is moving A to B i.e., in the circular path



$\therefore \; mg\cos\theta - N = \frac{mv^2}{R}$

$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$ ...(i)

And by energy conservation,

$\frac{1}{2}mv^2 = mg\left[R - R\cos\theta\right]$

$\therefore \; \frac{v^2}{R} = 2g\left(1 - \cos\theta\right)$

Putting this value of $\frac{v^2}{R}$ in eqn. (i)

$N = mg\cos\theta - m\left[2g - 2g\cos\theta\right]$

$\Rightarrow \quad N = mg\cos\theta - 2mg + 2mg\cos\theta$

$\Rightarrow \quad N = 3mg\cos\theta - 2mg$

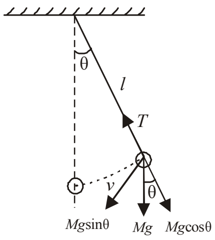
$\Rightarrow \quad N = mg\left(3\cos\theta - 2\right)$

Clearly when $\cos\theta > \frac{2}{3}$ N is positive acts radially outwards

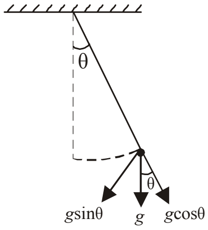
and if $\cos\theta < \frac{2}{3}$ N acts radially inwards

8. **(b, c)** A long radius net force = centripetal force $\left(\frac{Mv^2}{\ell}\right)$.

And along tangent net force $= ma_t$ as the motion of a pendulum is the part of circular motion.



$\therefore \quad T - Mg\cos\theta = \frac{Mv^2}{\ell}$



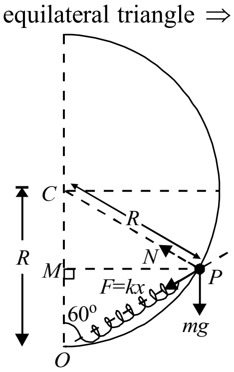
And, $ma_t = mg\sin\theta \Rightarrow a_t = g\sin\theta$

9. Radius of circle = R

In ΔOCP, $OC = CP = R$

$\therefore \quad \angle COP = \angle CPO = 60° \Rightarrow \angle OCP = 60°$

$\therefore \quad \Delta OCP$ is an equilateral triangle $\Rightarrow OP = R$



$\therefore \quad$ Extension of string $= R - \frac{3R}{4} = \frac{R}{4} = x$

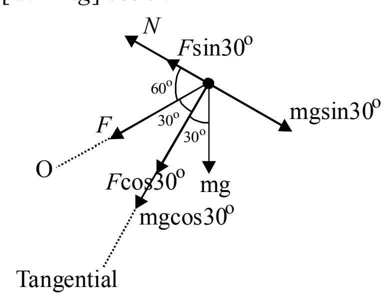
The forces acting are shown in the figure (i)

From FBD of the ring

Force in the tangential direction

$= F\cos 30° + mg\cos 30$

$= [kx + mg]\cos 30°$



$F_t \; = \frac{5mg}{8}\sqrt{3} \quad \therefore F_t = ma_t \quad \Rightarrow \quad a_t = \frac{5\sqrt{3}}{8}g$

Also, when the ring is just released

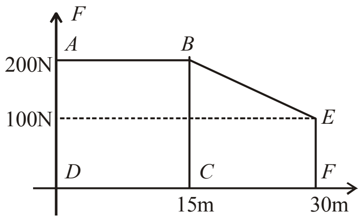
$N + F\sin 30° = mg\sin 30°$

$\Rightarrow \; N = (mg - F)\sin 30° = \left(mg - \frac{mg}{4}\right) \times \frac{1}{2} = \frac{3mg}{8}$

5 Work, Energy & Power

Topic-1 : Work Done by Constant & Variable Force

1. **(d)** The given situation can be drawn graphically as shown in figure.



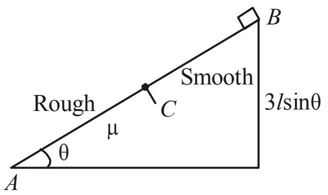
Work done = Area under F-x graph

= Area of rectangle $ABCD$ + Area of trapezium $BCFE$

$$W = (200 \times 15) + \frac{1}{2}(100+200) \times 15 = 3000 + 2250$$

$$\Rightarrow W = 5250 \text{ J}$$

2. **(c)** If $AC = l$ then according to question, $BC = 2l$ and $AB = 3l$.



Here, work done by all the forces is zero.

$$W_{\text{friction}} + W_{mg} = 0$$

$$mg(3l)\sin\theta - \mu mg\cos\theta(l) = 0$$

$$\Rightarrow \mu mg\cos\theta l = 3mgl\sin\theta \;\Rightarrow \mu = 3\tan\theta = k\tan\theta$$

$$\therefore k = 3$$

3. **(c)** Work done, $W = \int \vec{F}\cdot\vec{ds}$

$$= \left(-x\hat{i} + y\hat{j}\right)\cdot\left(d \times \hat{i} + dy\hat{j}\right)$$

$$\Rightarrow W = -\int_1^0 x dx + \int_0^1 y dy = \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$$

4. **(d)** Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

N = normal reaction

Now, work done by normal reaction 'N' on block in time t, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/_2 t^2\right)$

or, $W = \frac{3mg^2t^2}{8}$

5. **(a)** From question, $v = a\sqrt{s} = \frac{ds}{dt}$

or, $2\sqrt{s} = at \Rightarrow S = \frac{a^2t^2}{4}$

$$F = m \times \frac{a^2}{2}$$

Work done $= \frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}ma^4t^2$

6. **(c)** Work done in stretching the rubber-band by a distance dx is

$$dW = F\,dx = (ax + bx^2)dx$$

Integrating both sides,

$$W = \int_0^L ax dx + \int_0^L bx^2 dx = \frac{aL^2}{2} + \frac{bL^3}{3}$$

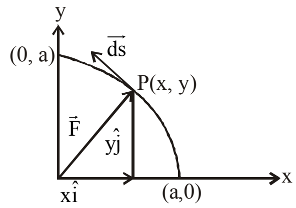
7. **(d)** Radius of circular path = a

The equation of circle is $x^2 + y^2 = a^2$

Given : force

$$\vec{F} = K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}} + \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\right]$$

$$\vec{F} = K\left[\frac{x\hat{i}}{(a^2)^{3/2}} + \frac{y\hat{j}}{(a^2)^{3/2}}\right]$$



$$\vec{F} = \frac{K}{a^3}\left[x\hat{i} + y\hat{j}\right]$$

The force acts radially outwards as shown in the figure and the displacement is tangential to the circular path.

Here the angle between the force which acts radially outwards and displacement which is tangential to the circular path is 90°

$\therefore$ Work done, $W = FS\cos\theta = 0$

8. **(b)** In a conservative field work done does not depend on the path *i.e.*, path independent. The gravitational field is a conservative field.

$\therefore$ $W_1 = W_2 = W_3$

9. **(c)** $dW = \vec{F}\,.\,d\vec{S}$ and $\vec{F} = -K(y\hat{i} + x\hat{j})$ given

$$d\vec{S} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dW = -K(y\,\hat{i} + x\,\hat{j})\,.\,(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

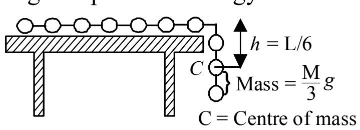
$$= -K(ydx + xdy) = -K\,[d(xy)]$$

$$\therefore \int_{0,0}^{a,a} dW = -K\int_{0,0}^{a,a} d\,(xy)$$

or $W = -K[xy]_{0,0}^{a,a}$ or $W = -Ka^2$.

Hence total work done by the force on the particle, $W = -Ka^2$

10. **(d)** The work done in bringing the mass up will be equal to the change in potential energy of the mass.



i.e., W = Change in potential energy

$$= mgh = \frac{M}{3} \times g \times \frac{L}{6} = \frac{MgL}{18}$$

11. **(8)** When the system is released,

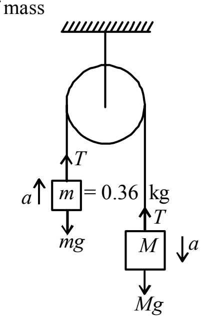
$T - mg = ma$ ...(i)

$Mg - T = Ma$ ...(ii)

From eq. (i) & (ii)

$$a = \frac{(M-m)g}{M+m} = g/3$$

and $T = 4\,mg/3$



For block m = 0.36 kg

$u = 0, a = g/3, \quad t = 1, s = ?$

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{g}{3} \times 1^2 = g/6 \quad (m = 0.72 \text{ kg})$

$\therefore$ Work done by the string on m

$T s \cos 0° = 4\frac{mg}{3} \times \frac{g}{6} \times 1 = \frac{4 \times 0.36 \times 10 \times 10}{3 \times 6} = 8 \text{ J}$

12. (0.75)

Given : Force, $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$

and $\alpha = -1 \text{ Nm}^{-1}$

We know that $dW = \vec{F} \cdot d\vec{r} = (\alpha y\,\hat{i} + 2\alpha x\,\hat{j}) \cdot (dx\,\hat{i} + dy\,\hat{j})$

$\therefore dW = \alpha y dx + 2\alpha x dy$

Work done

From A → B dy = 0, as y = 1

$\therefore W_1 = \int_0^1 \alpha y\, dx = \alpha \int_0^1 dx = \alpha$

From B → C dx = 0, as x = 1

$\therefore W_2 = \int_1^{0.5} 2\alpha n\, dy = \int_1^{0.5} 2\alpha dy = 2\alpha(-0.5) = -\alpha$

From C → D dy = 0, as y = 0.5

$\therefore W_3 = \int_1^{0.5} \alpha \times 0.5\, dx = -\frac{\alpha}{4}$

From D → E dx = 0, as x = 0.5

$\therefore W_4 = \int_{0.5}^{0} 2\alpha \times 0.5\, dy = -\frac{\alpha}{2}$

From E → F dy = 0, as y = 0

$\therefore W_5 = \int \alpha \times 0 \times dx = 0$

From F → A dx = 0 as x = 0

$\therefore W_6 = \int 2\alpha \times 0 dx = 0$

$\therefore$ Total work done $W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$= \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = 0.75\text{J}$

Topic-2 : Energy

1. **(c)** We know area under F-x graph gives the work done by the body

$\therefore W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5 \text{ J}$

Using work energy theorem,

Δ K.E = work done

$\therefore$ Δ K.E = 6.5 J

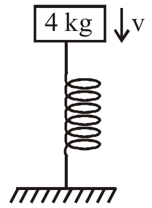
2. **(c)** $l_1 + l_2 = l$ and $l_1 = nl_2 \;\therefore\; l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

As $k \propto \frac{1}{l}$,

$\therefore \quad \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

3. **(b)** Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}$ m/s

Using principle of conservation of linear momentum just before and just after collision, we get



$1 \times \sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}$ m/s

Initial compression of spring

$1.25 \times 10^6 x_0 = 30 \Rightarrow x_0 \approx 0$

using work energy theorem,

$W_g + W_{sp} = \Delta KE$

$\Rightarrow 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 (0^2 - x^2) = 0 - \frac{1}{2} \times 4 \times v^2$

solving x ≈ 4 cm

4. **(a)** $W = u_f - u_i$

$= 0 - \left(-\frac{mg}{n} \times \frac{L}{2n}\right) = \frac{MgL}{2n^2}.$

5. **(c)** We know area under F-x graph gives the work done by the body

$\therefore W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$

$= 2.5 + 4 = 6.5 \text{ J}$

Using work energy theorem,

Δ K.E = work done

$\therefore$ Δ K.E = 6.5 J

6. **(d)** Maximum speed is at mean position or equilibrium At equilibrium Position

$F = kx \Rightarrow x = \frac{F}{k}$

From work-energy theorem,

$W_F + W_{sp} = \Delta KE$

$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$

$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2 \quad \Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2$

or, $v_{max} = \frac{F}{\sqrt{mk}}$

7. **(c)** $F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$

Since particle is moving in circular path

$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$

$\therefore$ K.E. $= \frac{1}{2}mv^2 = \frac{K}{2r^2}$

Total energy = P.E. + K.E.

$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero} \qquad (\because \text{P.E.} = -\frac{K}{2r^2} \text{ given})$

8. **(a)** Let V_f is the final speed of the body.

From questions,

$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \qquad \Rightarrow \qquad V_f = \frac{V_0}{2} = 5\,m/s$

$F = m\left(\frac{dV}{dt}\right) = -kV^2 \quad \therefore \quad (10^{-2})\frac{dV}{dt} = -kV^2$

$\int_{10}^{5} \frac{dV}{V^2} = -100K \int_0^{10} dt$

$$\frac{1}{5}-\frac{1}{10}=100K(10) \qquad \text{or,} \quad K=10^{-4}\,kgm^{-1}$$

9. **(a)** (K.E.)' = 50% of K.E. after hit i.e.,

$$\frac{1}{2}mv'^2=\frac{50}{100}\times\frac{1}{2}mv^2 \Rightarrow v'=\frac{v}{\sqrt{2}}$$

Coefficient of restitution $=\frac{1}{\sqrt{2}}$

Now, total distance travelled by object is

$$H=h\left(\frac{1+e^2}{1-e^2}\right)=h\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)=3h$$

10. **(c)** Using, $F = ma = m\frac{dV}{dt}$

$$6t=1.\frac{dV}{dt} \qquad [\because \quad m = 1 \text{ kg given}]$$

$$\int_0^v dV=\int 6t\,dt \quad V=6\left[\frac{t^2}{2}\right]_0^1=3\,ms^{-1} \quad [\because t = 1 \text{ sec given}]$$

From work-energy theorem,

$$W=\Delta KE=\frac{1}{2}m\left(V^2-u^2\right)=\frac{1}{2}\times1\times9=4.5\text{ J}$$

11. **(a)** Work done by friction at QR = μmgx

In triangle, $\sin 30°=\frac{1}{2}=\frac{2}{PQ} \Rightarrow PQ=4m$

Work done by friction at PQ = $\mu mg \times \cos 30° \times 4$

$=\mu mg\times\frac{\sqrt{3}}{2}\times4=2\sqrt{3}\,\mu mg$

Since work done by friction on parts *PQ* and *QR* are equal,

$\mu mg\,x=2\sqrt{3}\,umg$

$\Rightarrow x=2\sqrt{3}\cong 3.5m$

Using work energy theorem $mg\sin30°\times4=2\sqrt{3}\,\mu mg+\mu mgx$

$\Rightarrow 2=4\sqrt{3}\,\mu$

$\Rightarrow \mu = 0.29$

12. **(b)** $n=\frac{W}{\text{input}}=\frac{mgh\times1000}{\text{input}}=\frac{10\times9.8\times1\times1000}{\text{input}}$

$\text{Input}=\frac{98000}{0.2}=49\times10^4\text{J}$

$\text{Fat used}=\frac{49\times10^4}{3.8\times10^7}=12.89\times10^{-3}\text{kg.}$

13. **(b)** As we know, dU = F.dr

$$U=\int_0^r \alpha r^2dr=\frac{ar^3}{3} \qquad ...(i)$$

As, $\frac{mv^2}{r}=\alpha r^2$

$m^2v^2=m\alpha r^3$

$$\text{or, } 2m(KE)=\frac{1}{2}\alpha r^3 \qquad ...(ii)$$

Total energy = Potential energy + kinetic energy

Now, from eqn (i) and (ii)

Total energy = K.E. + P.E.

$$=\frac{\alpha r^3}{3}+\frac{\alpha r^3}{2}=\frac{5}{6}\alpha r^3$$

14. **(b)** Applying momentum conservation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.1u + m(0) = 0.1(0) + m(3)$

$0.1u = 3m$

$\frac{1}{2}0.1u^2=\frac{1}{2}m(3)^2$

Solving we get, $u = 3$

$$\frac{1}{2}kx^2=\frac{1}{2}K\left(\frac{x}{2}\right)^2+\frac{1}{2}(0.1)3^2$$

$$\Rightarrow \frac{3}{4}kx^2=0.9 \quad \Rightarrow \frac{3}{2}\times\frac{1}{2}kx^2=0.9$$

$\therefore \frac{1}{2}Kx^2 = 0.6$ J (total initial energy of the spring)

15. **(a)** Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases to v.

$$\therefore \quad u-v=\frac{4}{n} \text{ or, } v=u-\frac{4}{n}=\frac{u(n-1)}{n}$$

If F be the retarding force applied by each plank, then using work – energy theorem,

$$Fx=\frac{1}{2}mu^2-\frac{1}{2}mv^2=\frac{1}{2}mu^2-\frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$$

$$=\frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$$

$$Fx=\frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet.

Total distance travelled by the bullet before coming to rest = Px

Using work-energy theorem again,

$$F(Px)=\frac{1}{2}mu^2-0$$

$$\text{or, } P(Fx)=P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right]=\frac{1}{2}mu^2 \quad \therefore P=\frac{n^2}{2n-1}$$

16. **(a)** Given : $k_A=300\,N/m,\ k_B=400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B=(8.75-x)$ cm

$F=300\times x=400(8.75-x)$

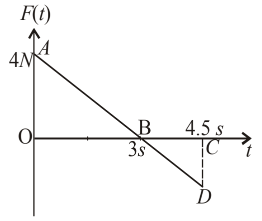
Solving we get, x = 5 cm

$x_B=8.75-5=3.75$ cm

$$\frac{E_A}{E_B}=\frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2}=\frac{300\times(5)^2}{400\times(3.75)^2}=\frac{4}{3}$$

17. **(c)** Area under $F-t$ graph gives the final linear momentum of the body.

Area of $\Delta AOB = \frac{1}{2}\times 3\times 4 = 6\text{ Ns}$



Also $\frac{OA}{OB} = \frac{CD}{CB}$

$\Rightarrow \frac{4}{3} = \frac{CD}{1.5} \quad \Rightarrow CD = 2$

$\therefore$ Area of $\Delta BCD = -\left[\frac{1}{2}\times 1.5\times 2\right] = -1.5\text{ Ns}$

$\therefore$ The final linear momentum = 6 – 1.5 = 4.5 Ns

$\therefore$ Kinetic energy of the block $= \frac{p^2}{2m} = \frac{(4.5)^2}{2\times 2} = 5.06\text{ J}$

18. **(c)** Heres when the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state as the other end of S_2 is free.

Therefore the energy stored in the system $= \frac{1}{2}k_1 x^2$.

When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. The P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.
It is because no friction anywhere.
So, energy is conserved

$\therefore \quad \frac{1}{2}k_1 x^2 = \frac{1}{2}k_2 y^2 \quad \Rightarrow \quad \frac{1}{2}\times kx^2 = \frac{1}{2}\times 4\, ky^2$

$\Rightarrow x^2 = 4y^2 \qquad \therefore \frac{y}{x} = \frac{1}{2}$

19. **(b)** For conservative forces $\Delta U = -W$

$\Delta U = -\int_0^x F dx$ or $\Delta U = -\int_0^x k\, x dx$

$\Rightarrow U_{(x)} - U_{(0)} = -\frac{kx^2}{2} \qquad (\because U_{(0)} = 0)$

$\therefore \quad U_{(x)} = -\frac{kx^2}{2} \quad \Rightarrow \quad x^2 = \frac{-2U_x}{k}$

It represents a parabola below x – axis symmetrical

20. **(b)** Let x be the maximum extension of the string. Here mechanical energy is conserved, so decrease in the gravitational potential energy of spring mass system (Mgx) = gain in spring elastic potential energy $\left(\frac{1}{2}kx^2\right)$

$$Mgx = \frac{1}{2}kx^2 \Rightarrow x = \frac{2Mg}{k}$$

21. **(d)** From, $F = -\frac{du}{dx}$

$dU_{(x)} = -Fdx$

$U_x = -\int_0^x (-kx + ax^3)dx$

$\therefore \quad U_x = -\int_0^x F dx = \frac{kx^2}{2} - \frac{ax^4}{4}$

$U_x = 0$ at $x = 0$ and $x = \sqrt{\frac{2k}{a}}$;

And U_x = negative for $x > \sqrt{\frac{2k}{a}}$

We have potential energy zero twice out of which one is at origin.
Also, when we put $x = 0$ in the given function, we get $F = 0$. But $F = -\frac{dU}{dx}$

$\Rightarrow$ At $x = 0$; $\frac{dU}{dx} = 0$ i.e. the slope of the graph should be zero.
These characteristics are represented by graph (d).

22. **(b)** When spring is cut into pieces then the length of longer piece of spring $= \frac{2L}{3}$

Here the original length of spring be L and spring constant is K (given)
For a spring, $K \times L$ = constant
Let K' be the spring cosntant of longer piece of spring

$\therefore \quad K \times L = \frac{2L}{3}\times K' \Rightarrow K' = \frac{3}{2}K$

23. **(c)** K.E. $= \frac{p^2}{2m}$; [K.E. = Kinetic energy; P = momentum]

$KE_1 = KE_2 \quad \therefore \frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$

$\therefore \quad \frac{p_1^2}{p_2^2} = \frac{m_1}{m_2} \Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

24. **(5)** Work done = Increase in P.E. + gain in K.E.
$F \times d = mgh$ + gain in K.E.
$18 \times 5 = 1 \times 10 \times 4$ + gain in K.E.
$\therefore$ Gain in K.E. = 50 J = 10n
$\therefore$ $n = 5$

25. **(4)**
Here, loss in K.E. of the block = gain in P.E. of the spring + work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mg.x \Rightarrow v = \frac{2\,\mu mgx + kx^2}{m}.$$

$$v = \sqrt{\frac{2\times 0.1\times 0.18\times 10\times 0.6 + 2\times 0.6\times 0.6}{0.18}}$$

$\therefore v = \frac{4}{10} = \frac{N}{10} \qquad \therefore N = 4$

26. **(150.00)** From work energy theorem,

$$W = F\cdot s = \Delta KE = \frac{1}{2}mv^2$$

Here $V^2 = 2gh$

$$\therefore F\cdot s = F\times \frac{2}{10} = \frac{1}{2}\times \frac{15}{100}\times 2\times 10\times 20$$

$$\therefore F = 150\text{ N}.$$

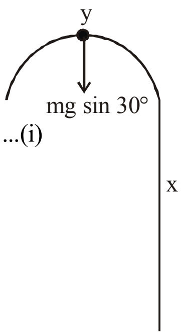
27. **(10.00)** Kinetic energy = change in potential energy of the particle,
KE = mgΔh
Given, $m = 1$ kg,
$\Delta h = h_2 - h_1 = 2 - 1 = 1m$
$\therefore KE = 1 \times 10 \times 1 = 10\,J$

28. **(a, d)**
At point Y the centripetal force provided by the component of weight mg

$$\therefore \text{mg sin } 30° = \frac{mv^2}{R}$$

$$\therefore v^2 = \frac{gR}{2} \qquad \text{...(ii)}$$

Now by the energy conservation between bottom point and point Y



$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$$\therefore v^2 = v_0^2 - 2gh \qquad \text{...(i)}$$

$\therefore$ From eq. (i)

$$\frac{gR}{2} = v_0^2 - 2gh$$

Hence option (a) is correct.
At point x and z of circula path, the points are at same height but less then h. So the velocity more than at point y.

So required centripetal force $= \dfrac{mv^2}{r}$ is maximum at points x and x.

29. **(a, b, d)** $G\dfrac{dk}{dt} = \gamma t$ and $k = \dfrac{1}{2}mV^2 \quad \therefore \dfrac{d}{dt}\left(\dfrac{1}{2}mV^2\right) = \gamma t$

$$\Rightarrow \frac{m}{2} \times 2V\frac{dV}{dt} = \gamma t \qquad \therefore mV\frac{dV}{dt} = \gamma t$$

$$\therefore m\int_0^V VdV = \gamma\int_0^t tdt \qquad \Rightarrow \frac{mV^2}{2} = \frac{\gamma t^2}{2}$$

$$\therefore V = \sqrt{\frac{\gamma}{m}} \times t. \text{ i.e., } V \propto t;\ V = \frac{ds}{dt} = \sqrt{\frac{\gamma}{m}}t \ \Rightarrow\ s = \sqrt{\frac{\gamma}{m}}\frac{t^2}{2}$$

So V is proportional to ‘t’ and distance cannot be proportional to 't'.

Now $F = ma = m\dfrac{dV}{dt} = m\dfrac{d}{dt}\left[\sqrt{\dfrac{\gamma}{m}} \times t\right] = m\sqrt{\dfrac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$

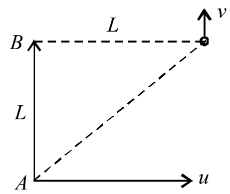
30. **(a, b)** From figure given in question,
Potential energy of the ball at point A = mgh$_A$
Potential energy of the ball at point B = 0
Potential energy of the ball at point C = mgh$_C$
Total energy at point A, $E_A = K_A + mgh_A$
Total energy at point B, $E_B = K_B$
Total energy at point C, $E_C = K_C + mgh_C$
According to the law of conservation of energy

$$E_A = E_B = E_C \qquad \text{... (i)}$$
$$E_A = E_B \Rightarrow E_C > K_C \qquad \text{...(ii)}$$
$$E_A = E_C$$
$$K_A + mgh_A = K_B + mgh_C$$

or, $$h_A - h_C = \frac{K_C - K_A}{mg} \qquad \text{...(iii)}$$

$\Rightarrow \quad h_A > h_C;\ K_C\,.\,K_A$ ...(iv)
From eq. (i),(ii) and (iv), we get $h_A > h_C$
$\therefore \quad K_B > K_C$

31. **(d)** From principle of conservation of energy
$(K.E.)_B + (P.E.)_B = (K.E.)_A + (P.E.)_A$,

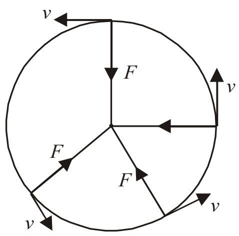


or, $\dfrac{1}{2}mv^2 + mgL = \dfrac{1}{2}mu^2 + 0$

$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$ ... (i)
Change in velocity $|\Delta \vec{v}|$
$= |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2}$
Putting the value of v from eq. (i)

$$|\Delta \vec{v}| = \sqrt{2(u^2 - g\ell)}$$

32. **(c, d)** As particle is acted upon by a force of constant magnitude and is perpendicular to the velocity of the particle so it is a case of uniform circulat motion.
The force is constant in magnitude, this show the speed is not changing and hence kinetic energy will remain constant.

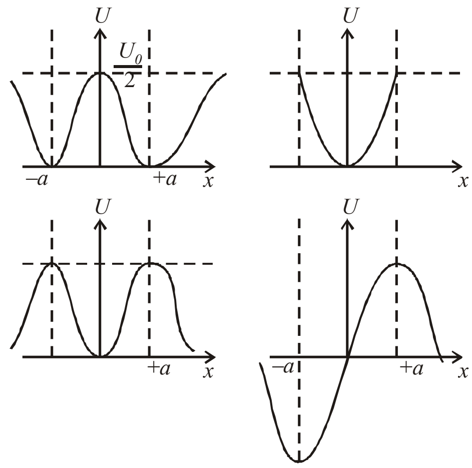


The velocity and acceleration changes continuously due to change in the direction.

33. **A → p, q, r, t; B → q, s; C → p, q, r, s; D → p, r, t**
For A

$$F_x = -\frac{dU}{dx} = \frac{-d}{dx}\left[\frac{U_0}{2}\left(1 - \left(\frac{x}{a}\right)^2\right)^2\right] = \frac{-2U_0}{a^3}(x-a)x(x+a)$$

F = 0 at $x = 0, x = a, x = -a$
and $u = 0$ at $x = -a, x = a$



For B $\quad F_x = \dfrac{-dU}{dx} = -U_0\left(\dfrac{x}{a}\right)$

For C $\quad F_x = \dfrac{-dU}{dx} = U_0\dfrac{e^{-x^2/a^2}}{a^3}x(x-a)(x+a)$

For D $\quad F_x = -\dfrac{dU}{dx} = -\dfrac{U_0}{2a^3}[(x-a)(x+a)]$

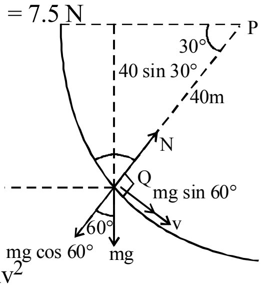
34. (b) $N - mg\cos\theta$ will provide required centripetal force

$$N - mg\cos\theta = \frac{mv^2}{R}$$

$$N = mg\cos\theta + \frac{mv^2}{R}$$

$$= 1 \times 10 \times \frac{1}{2} + \frac{1 \times (10)^2}{40} = 7.5\text{ N}$$

35. (b) Work done against friction = Initial mechanical energy – final mechanical energy.



$$150 = mgh - \frac{1}{2}\,\text{mv}^2$$

$\Rightarrow \quad 150 = mg\,R\sin 30° - \frac{1}{2}\,\text{mv}^2$

$(\because \quad h = R\sin 30°)$

$\Rightarrow \quad 150 = 1 \times 10 \times 40 \times \frac{1}{2} - \frac{1}{2} \times 1 \times v^2$

$\therefore \quad v = 10$ m/s

36. (c) In the first case the mechanical energy is completely converted into heat becuase of friction. *i.e.*, Decrease in mechanical energy = $\frac{1}{2}\,mv^2$.

While is second case, a part of mechanical energy is converted into heat due to fiction but another part of mechanical energy is retained in the form of potential energy of the block. *i.e.*, Decrease in mechanical energy $= \frac{1}{2}\,mv^2 - mgh$

Therefore statement 1 is correct.

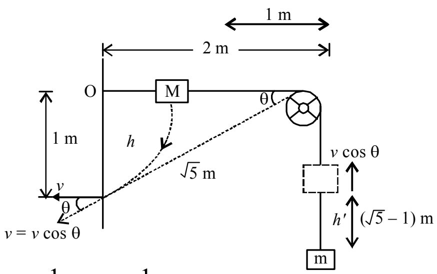
Statement 2 is wrong. The coefficient of friction between the block and the surface does not depend on the angle of inclination.

37. Let point mass hit the wall with the speed v.

Then, velocity of mass m at this instant $= v\cos\theta = \frac{2}{\sqrt{5}}\,v$.

Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5} - 1)$ m.

From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(\text{v}\cos\theta)^2$$

or, $\quad (2)(9.8)(1) = (0.5)(9.8)\,(\sqrt{5} - 1) + \frac{1}{2} \times 2 \times v^2$

$$+ \frac{1}{2} \times 0.5 \times \left(\frac{2v}{\sqrt{5}}\right)^2$$

Solving we get, $v = 3.29$ m/s.

38. K.E. of block = work against friction + P.E. of spring

$$\frac{1}{2}mv^2 = \mu_k mg\,(BD + x) + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2 = \mu_k mg\,(2.14 + x) + \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 0.5 \times 3^2 = 0.2 \times 0.5 \times 9.8\,(2.14 + x) + \frac{1}{2}\,2 \times x^2$$

$$2.14 + x + x^2 = 2.25$$

$\therefore \quad x^2 + x - 0.11 = 0$

On solving, we get $x = \frac{1}{10} = 0.1$

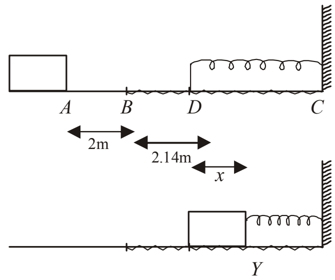
The spring gets compressed by 0.1 m

Restoring force at $Y = kx = 2 \times 0.1 = 0.2\,N$

Frictional force at $Y = \mu_s\,mg = 0.22 \times 0.5 \times 9.8 = 1.078\,N$

Since frictional force > restoring force, the body will stop here *i.e.*, after compressing the spring by x

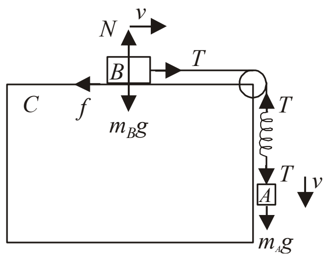
$\therefore$ The total distance travelled
$= AB + BD + x = 2 + 2.14 + 0.1 = 4.24$ m.



39. Here the net force acting on A and B is zero. Since the blocks A and B are moving with constant velocity.

Let the extension of the spring be x.

There will be no friction force between block A and C



$\because \quad f = \mu N$. Here there is no normal reaction on A because A is not pushing C so frictional force between block A and C,

$f = \mu\text{N} = 0$

Applying $F_{net} = ma$ on A,

$m_Ag - T = m_A \times 0$

$\therefore \quad T = m_Ag \quad$ ... (i)

Applying $F_{net} = ma$ on B,

$T - f = m_B \times 0$

$\therefore \quad T = f = \mu N$

$= \mu m_Bg \quad$ ... (ii)

From (i) and (ii)

$$\mu m_Bg = m_Ag \Rightarrow m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10\,\text{kg}$$

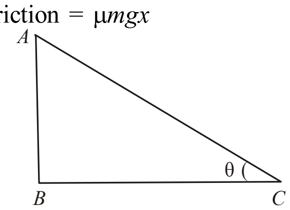
Here the force acting on the spring is the tension equal to restoring force

$\therefore \quad T = kx \quad \Rightarrow \quad x = \frac{T}{k} \qquad \therefore \quad x = \frac{m_ag}{k} = \frac{19.6}{k}$

Energy stored in spring

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times k \times \frac{19.6}{k} \times \frac{19.6}{k} = \frac{19.6 \times 19.6}{2 \times 1960} = 0.098\,\text{J}$$

40. Here, K.E. at C = Loss in P.E. – work done by friction
In both cases work done by friction = μmgx

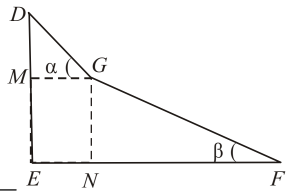


or, $\frac{1}{2}mv_c^2 = mg\,y - \mu mgx$

$= gy - \mu gx$

$\therefore \quad v_c = \sqrt{2g(y - \mu x)}$

Now, K.E. at F = loss in P.E. – work done by friction



$\frac{1}{2}mv_F^2 = mgy - \mu mgx$

$\Rightarrow \quad \frac{1}{2}v_F^2 = gy - \mu gx$

$\therefore \quad v_F = \sqrt{2g(y - \mu x)}$

i.e., $v_C = v_F = \sqrt{2gy - 2\mu gx}$

41. No. when a ball is thrown up the gravitational pull of earth, is acting on the ball which is responsible for the change in momentum.

42. For a spring, (spring constant) k × (length) l = Constant

If length is made one third, *i.e.*, $\frac{l}{3}$ the spring constant becomes three times *i.e.*, 3K.

43. Using Kinetic energy (K.E.) and momentum relation,

K.E. = $\frac{p^2}{2m}$ For equal value of p, K.E. $\propto \frac{1}{\text{mass}}$

Topic-3 : Power

1. **(b)** We know that
Power, $P = Fv$

But $F = mav = m\frac{dv}{dt}v$

$\therefore P = mv\frac{dv}{dt} \Rightarrow P\,dt = mv\,dv$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$P.\,t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$

Distance, $s = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt = \sqrt{\frac{2P}{m}} \cdot \frac{t^{3/2}}{3/2}$

$\Rightarrow s = \sqrt{\frac{8P}{9m}} \cdot t^{3/2} \Rightarrow s \propto t^{3/2}$

So, graph (b) is correct.

2. **(b)** Total force required to lift maximum load capacity against frictional force = 400 N

$F_{total} = Mg$ + friction
$= 2000 \times 10 + 4000$
$= 20{,}000 + 4000 = 24000\,N$

Using power, $P = F \times v$
$60 \times 746 = 24000 \times v$
$\Rightarrow v = 1.86$ m/s ≈ 1.9 *m/s*

Hence speed of the elevator at full load is close to 1.9 ms^{-1}

3. **(b)** Centripetel acceleration $a_c = n^2Rt^2$

$a_c = \frac{v^2}{R} = n^2Rt^2$

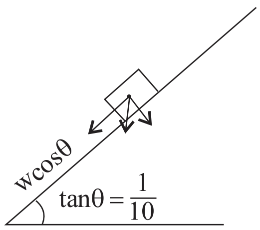
$v^2 = n^2R^2t^2$

$v = nRt$

$a_c = \frac{dv}{dt} = nR$

Power = ma_tv = m nR nRt = Mn^2R^2t.

4. **(c)** While moving downhill power



$P = \left(w\sin\theta + \frac{w}{20}\right)10$

$P = \left(\frac{w}{10} + \frac{w}{20}\right)10 = \frac{3w}{2}$

$\frac{P}{2} = \frac{3w}{4} = \left(\frac{w}{10} - \frac{w}{20}\right)V$

$\frac{3}{4} = \frac{v}{20} \Rightarrow v = 15$ m/s

$\therefore$ Speed of car while moving downhill v = 15 m/s.

5. **(d)** **6.** **(b)**

7. **(c)** $F = v\left(\frac{dm}{dt}\right) = v\frac{d}{dt}(\rho \times \text{volume}) = v\rho\frac{d}{dt}(\text{volume})$

$= v\rho \times (Av) = A\rho v^2$

Power = Force × velocity = $A\rho v^2 \times v = A\rho v^3$

Hence, power $P \propto v^3$

8. **(b)** The centripetal acceleration $a_c = k^2\,r\,t^2$

or, $\frac{v^2}{r} = k^2rt^2 \qquad \therefore \quad V = krt$

$a_t = \frac{dv}{dt} = kr$

Power, $P = F_tv = ma_tv$

$\therefore$ Power = mk^2r^2t

9. **(c)** Power, $P = \frac{w}{t} = \frac{E}{t}$ = constt $\quad \therefore \quad \frac{\frac{1}{2}mv^2}{t}$ = constt

From work-energy theorem, not work done = change in kinetic energy.

$\Rightarrow \quad \frac{v^2}{t}$ = constt (k) $\therefore \qquad v = kt^{1/2}$ and $\frac{ds}{dt} = kt^{1/2}$

or, $ds = kt^{1/2}\,dt$

By integrating, we get

$\Rightarrow \quad s = \frac{2kt^{3/2}}{3} + C \quad \Rightarrow \quad s \propto t^{3/2}$

i.e., Distance moved $S \propto t^{3/2}$

10. **(b)** If machine is lubricated with oil friction is reduced.

Mechanical efficiency = $\frac{\text{Output work}}{\text{Input energy}}$

Due to less friction output work will increase. Thus the mechanical efficiency increases.

Mechanical advantage, M.A. = $\frac{\text{load}}{\text{effort}}$

11. **(5)** Using, work – energy theorem, Δ K.E. = W = P × t

$\frac{1}{2}mv^2 = P \times t$

$\therefore v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5ms^{-1}$

12. **(18)** Given, Mass of the body, $m = 2$ kg
Power delivered by engine, $P = 1$ J/s
Time, $t = 9$ seconds
Power, $P = Fv$

$$\Rightarrow P = mav \qquad [\because F = ma]$$

$$\Rightarrow m\frac{dv}{dt}v = P \qquad \left(\because a = \frac{dv}{dt}\right)$$

$$\Rightarrow v\,dv = \frac{P}{m}dt$$

Integrating both sides we get

$$\Rightarrow \int_0^v v\,dv = \frac{P}{m}\int_0^t dt \quad \Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$\therefore \text{ Distance, } x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2}$$

$$\Rightarrow x = \sqrt{\frac{2\times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18.$$

Topic-4 : Collisions

1. **(d)** Before collision,
Velocity of particle A, $u_1 = (\sqrt{3}\hat{i} + \hat{j})$ m/s
Velocity of particle B, $u_2 = 0$
After collision,
Velocity of particle A, $v_1 = (\hat{i} + \sqrt{3}\hat{j})$
Velocity of particle B, $v_2 = 0$
Using principal of conservation of angular momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow 2m_2(\sqrt{3}\hat{i} + \hat{j}) + m_2 \times 0 = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2 \times \vec{v}_2$$

$$\Rightarrow 2\sqrt{3}\hat{i} + 2\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j} + \vec{v}_2$$

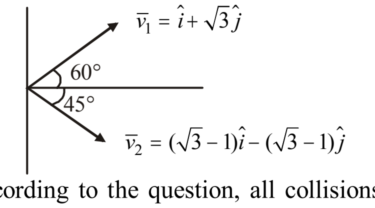
$$\Rightarrow \vec{v}_2 = (\sqrt{3}-1)\hat{i} - (\sqrt{3}-1)\hat{j} \qquad \Rightarrow \vec{v}_1 = \hat{i} + \sqrt{3}\hat{j}$$

For angle between $\vec{v}_1$ and $\vec{v}_2$,

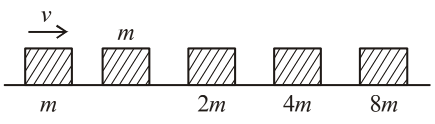
$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{\vec{v}_1\vec{v}_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2\times 2\sqrt{2}(\sqrt{3}-1)} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \theta = 105°$$

Angle between $\vec{v}_1$ and $\vec{v}_2$ is 105°



2. **(b)** According to the question, all collisions are perfectly inelastic, so after the final collision, all blocks are moving together.



Let the final velocity be v', using momentum conservation

$$mv = 16mv' \Rightarrow v' = \frac{v}{16}$$

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2}\times 16m \times \left(\frac{v}{16}\right)^2 = \frac{1}{2}\frac{mv^2}{16}$

Energy loss : $E_i - E_f = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$$

The total energy loss is P% of the original energy.

$$\therefore \%P = \frac{\text{Energy loss}}{\text{Original energy}} \times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

Hence, value of P is close to 94.

3. **(b)** Given,
Mass of block, $m_1 = 1.9$ kg
Mass of bullet, $m_2 = 0.1$ kg
Velocity of bullet, $v_2 = 20$ m/s
Let v be the velocity of the combined system. It is an inelastic collision.
Using conservation of linear momentum

$$m_1 \times 0 + m_2 \times v_2 = (m_1 + m_2)v$$
$$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times v$$
$$\Rightarrow v = 1 \text{ m/s}$$

Using work energy theorem
Work done = Change in Kinetic energy
Let K be the Kinetic energy of combined system.

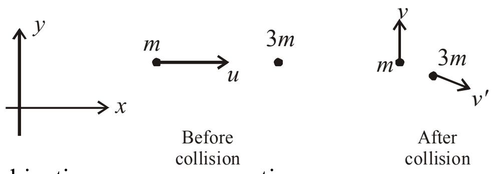
$$(m_1 + m_2)gh = K - \frac{1}{2}(m_1 + m_2)V^2$$

$$\Rightarrow 2 \times g \times 1 = K - \frac{1}{2} \times 2 \times 1^2 \Rightarrow K = 21 \text{ J}$$

4. **(c)** From conservation of linear momentum

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v'}$$

$$\vec{v'} = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$



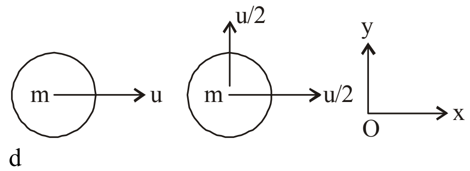
From kinetic energy conservation,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$$

$$\text{or, } mu^2 = mv^2 + \frac{mu^2}{3} + \frac{mv^2}{3}$$

$$\therefore v = \frac{u}{\sqrt{2}}$$

5. **(b)**



x-direction

$$mu + \frac{mu}{2} = 2mv_x' \Rightarrow V_x' = \frac{3u}{4}$$

y-direction $0 + \frac{mu}{2} = 2mv_y' \Rightarrow v_y' = \frac{u}{4}$

$$K.E._i = \frac{1}{2}m\,u^2 + \frac{1}{2}m\left[\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]$$

$$= \frac{1}{2}mu^2 + \frac{mu^2}{4} = \frac{3mu^2}{4}$$

$$K.E._f = \frac{1}{2}(2m)\left(v_x'\right)^2 + \frac{1}{2}(2m)\left(v_y'\right)^2$$

$$= \frac{1}{2}2m\left[\left(\frac{3u}{4}\right)^2 + \left(\frac{u}{4}\right)^2\right] = \frac{5}{8}mu^2$$

$\therefore$ Loss in $KE = KE_f - KE_i$

$$= mu^2\left(\frac{6}{8} - \frac{5}{8}\right) = \frac{mu^2}{8}$$

6. **(a)** Apply concervation of linear momentum in X and Y direction for the system then

M (10cos30°) + 2M (5cos45°) = 2M ($v_1$cos30°) + M($v_2$cos45°)

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}\,v_1 + \frac{v_2}{\sqrt{2}} \qquad ....(1)$$

Also

2M(5sin45°) – M(10sin30°) = 2M$v_1$sin30° – M$v_2$sin45°

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \qquad ....(2)$$

Solving equation (1) and (2)

$$\left(\sqrt{3}+1\right)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5 \Rightarrow v_1 = 6.5\text{m/s}$$

$v_2 = 6.3$ m/s

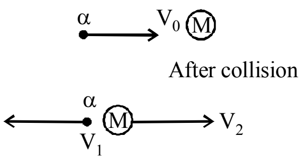
7. **(b)** $m_1v_1 + m_2v_2 = m_1v_2 + m_2v_1$

or $m_1v_1 + (0.5m_1)v_2 = m_10.5v_1 + (0.5m_1)v_4$

On solving, $v_1 = v_4 - v_2$

8. **(d)** Using conservation of momentum,

$mv_0 = mv_2 - mv_1$



$$\frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv_0^2$$

$\Rightarrow v_1 = 0.6v_0$

The collision is elastic. So,

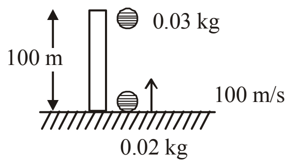
$$\frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mv_0^2 \,[\therefore \text{ M = mass of nucleus}]$$

$$\Rightarrow V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0 \quad mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$$

$$\Rightarrow 1.6m = 0.8\sqrt{mM} \qquad \Rightarrow 4m^2 = mM$$

$\therefore$ M = 4m

9. **(c)**



Time taken for the particles to collide,

$$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1\,\text{sec}$$

Speed of wood just before collision = gt = 10 m/s and speed of bullet just before collision = v – gt
= 100 – 10 = 90 m/s

$$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95\text{ m}$$

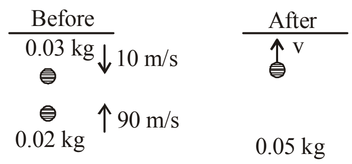
Now, using conservation of linear momentum just before and after the collision

$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v$ $\qquad \therefore\ v = 30$ m/s

Max. height reached by body

$$h = \frac{30 \times 30}{2 \times 10} = 45\,\text{m}$$



$\therefore$ Height above tower = 40 m

10. **(c)** Kinetic energy of block A

$$k_1 = \frac{1}{2}mv_0^2$$

$\therefore$ From principle of linear momentum conservation

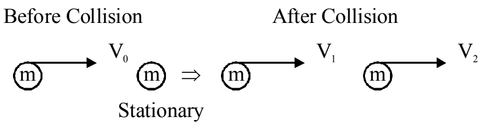
$$mv_0 = (2m + M)v_f \Rightarrow v_f = \frac{mv_0}{2m + M}$$

According to question, of $\frac{5}{6}$th the initial kinetic energy is lost in whole process.

$$\therefore \frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$$

$$\Rightarrow \frac{2m+M}{m} = 6 \therefore \frac{M}{m} = 4$$

11. **(b)**



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad ....(i)$$

From momentum conservation

$mv_0 = m(v_1 + v_2)$ $\qquad$ ....(ii)

Squarring both sides,

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

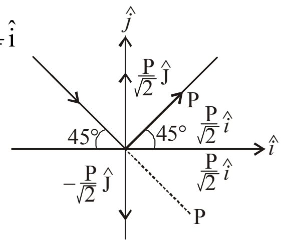
$2v_1v_2 = -\frac{v_0^2}{2}$

$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$

Solving we get relative velocity between the two particles

$v_1 - v_2 = \sqrt{2}v_0$

12. (a) Change in momentum



$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$

$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H$ molecule

$\Rightarrow I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$

Pressure, P

$= \frac{F}{A} = \frac{\sqrt{2}P}{A}n$ ($\because n =$ no. of particles)

$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}} = 2.35 \times 10^3 N/m^2$

13. (a) For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$v_1 = -\frac{11}{13}v$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

14. (a) For inelastic collision $v' = \frac{m_1}{(m_1 + m_2)}v$

$= \frac{1}{(1+1)}v = \frac{v}{2}$

$n \to v(H)$ Before

$(n)(H) \to \frac{v}{2}$ After

Loss in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$

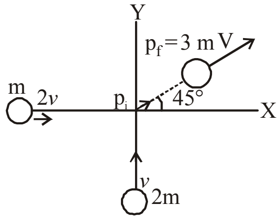
K.E. lost is used to jump from 1st orbit to 2nd orbit

ΔK.E. = 10.2ev

Minimum K.E. of neutron for inelastic collision

$\frac{1}{2}mv^2 = 2' \ 10.2 = 20.4$ eV

15. (a)



Initial momentum of the system

$p_i = \sqrt{[m(2V)^2 \times 2m(2V)^2]} = \sqrt{2}m \times 2V$

Final momentum of the system = 3mV

By the law of conservation of momentum

$2\sqrt{2}mv = 3mV \Rightarrow \frac{2\sqrt{2}v}{3} = V_{combined}$

Loss in energy

$\Delta E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 - \frac{1}{2}(m_1 + m_2)V_{combined}^2$

$\Delta E = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$

Percentage loss in energy during the collision $\simeq$ 56%

16. (d) From the law of conservation of momentum we know that,

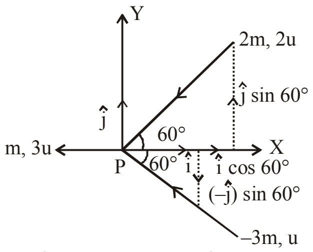
$m_1u_1 + m_2u_2 + ... = m_1v_1 + m_2v_2 + ...$

Given $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$

and $u_1 = 3u$, $u_2 = 2u$ and $u_3 = u$

Let the velocity when they stick $= \vec{v}$

Then, according to question,



$m \times 3u\ (\hat{i}) + 2m \times 2u\ (-\hat{i}\cos 60° - \hat{j}\sin 60°)$

$+ 3\ m \times u\ (-\hat{i}\cos 60° + \hat{j}\sin 60°) = (m + 2m + 3m)\ \vec{v}$

$\Rightarrow 3mu\hat{i} - 4mu\frac{\hat{i}}{2} - 4mu\left(\frac{\sqrt{3}}{2}\hat{j}\right) - 3mu\frac{\hat{i}}{2} + 3mu\left(\frac{\sqrt{3}}{2}\hat{j}\right) = 6m\vec{v}$

$\Rightarrow mu\hat{i} - \frac{3}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6m\vec{v}$

$\Rightarrow -\frac{1}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6m\vec{v}$

$\Rightarrow \vec{v} = \frac{u}{12}\left(-\hat{i} - \sqrt{3}\hat{j}\right)$

17. (b) As tennis ball is dropped, so initial velocity $u = 0$

$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m[u + at]^2 = \frac{1}{2}m[0 + gt]^2$

$\therefore K.E = \frac{1}{2}mgt^2$ $\therefore K.E \propto t^2$ ...(i) *i.e.*, The relation between k and t is parabolic.

First the kinetic energy will increase as per eq (i). As the balls touches the ground it starts deforming and loses its K.E., when the deformation is maximum K.E. = 0 As the ball moves up it loses K.E. and gain gravitational potential energy in the same time interval. These characteristics are best illustrated by kt graph shown in (b).

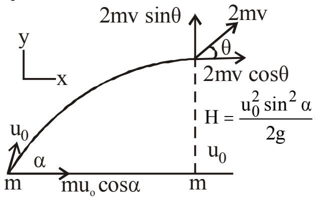
18. (a) Height, $h = \dfrac{u_0^2 \sin^2 \alpha}{2g}$

using $v^2 - u^2 = 2gh$

$$v_1^2 - u_0^2 = 2(-g)\left[\frac{u_0^2 \sin^2 \alpha}{2g}\right]$$

$$\Rightarrow \quad v_1^2 = u_0^2(1-\sin^2\alpha) = u_0^2 \cos^2 \alpha$$

$$\Rightarrow \quad v_1 = u_0 \cos \alpha$$



Applying conservation of linear momentum in Y-direction

$2mv \sin\theta = mv_1 = mu_0 \cos\alpha$ ...(i)

Applying conservation of linear momentum in X-direction

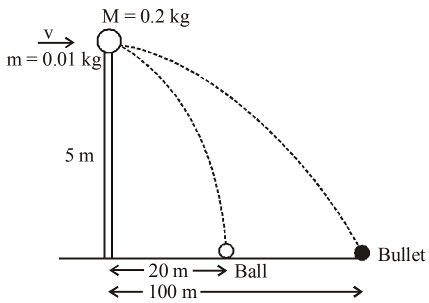
$2mv \cos\theta = mu_0 \cos\alpha$ ...(ii)

Dividing (i) and (ii) we get

$\tan\theta = 1 \quad \therefore \theta = 45° = \dfrac{\pi}{4}$

19. (d) *For vertical motion of bullet or ball*

$u = 0,\ h = 5\text{m},\ \ t = ?,\ \ g = 10\text{m/s}^2$



$$h = ut + \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2}\times 10 \times t^2$$

$$\Rightarrow t = 1 \text{ sec}$$

For horizontal motion of ball

$x_{ball} = V_{ball}\, t \Rightarrow 20 = V_{ball} \times 1 \Rightarrow V_{ball} = 20$ m/s

For horizontal motion of bullet

$x_{bullet} = V_{bullet} \times t \Rightarrow 100 = V_{bullet} \times 1 \Rightarrow V_{bullet} = 100$ m/s

Applying conservation of linear momentum during collision,

$$mV = mV_{bullet} + MV_{ball}$$

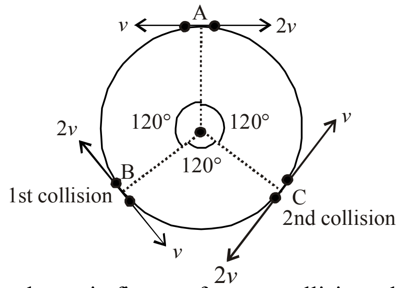
$$0.01\, V = 0.01 \times 100 + 0.2 \times 20$$

$$\therefore V = \frac{5}{0.01} = 500 \text{ m/s}$$

Hence initial velocity of the bullet, V = 500 m/s

20. (c) According to question, between collision, the particles move with constant speed.

At first collision one particle having speed $2v$ will rotate $2 \times 120° = 240°$ while other particle having speed v will rotate 120°. Hence, first collision takes place at B. At first collision, they will exchange their velocities as the collision is elastic and the particles have equal masses. Again second collision takes place at C.



Now, as shown in figure, after two collisions they will again reach at point A.

21. (d) $F_{ext} = (m_1 + m_2)g$ and $\Delta t = 2t_0$

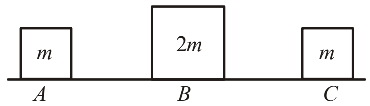
$$F_{ext} = \frac{\Delta p}{\Delta t}$$

$$\therefore \quad \Delta p = F_{ext}\ \Delta t = \left(m_1\vec{v}_1' + m_2\vec{v}_2'\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)$$

$$= (m_1 + m_2)\, g \times 2t_0$$

22. (4) Hence, Just after collision with A velocity of B

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B} = \frac{0 + 2m \times 9}{m + 2m} = 6\text{m/s}$$



The collision between B and C is completely inelastic

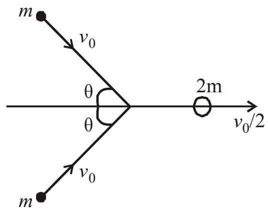
$$\therefore \quad m_B v_B = (m_B + m_c)\, v$$

$$\Rightarrow \quad v = \frac{6 \times 2m}{2m + m} = 4\text{m/s}.$$

23. (5) Due to elastic head on collision of equal mass m of bob velocity at the highest point of bob tied to string ℓ_1 is acquired by the bob tied to string ℓ_2.

$$\therefore \sqrt{g\ell_1} = \sqrt{5g\ell_2} \quad \Rightarrow \frac{\ell_1}{\ell_2} = 5$$

24. (120)



Momentum conservation along x direction,

$$2mv_0 \cos\theta = 2m\frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60°$$

Hence angle between the initial velocities of the two bodies

$$= \theta + \theta = 60° + 60° = 120°.$$

25. (10.00)

From momentum conservation in perpendicular direction of initial motion.

$mu_1 \sin\theta_1 = 10mv_1 \sin\theta_2$ ...(i)

It is given that energy of m reduced by half. If u_1 be velocity of m after collision, then

$\left(\frac{1}{2}mu^2\right)\frac{1}{2}=\frac{1}{2}mu_1^2 \qquad \Rightarrow u_1=\frac{u}{\sqrt{2}}$

If v_1 be the velocity of mass 10 m after collision, then

$\frac{1}{2}\times 10m\times v_1^2=\frac{1}{2^m}\frac{u^2}{2}\Rightarrow v_1=\frac{u}{\sqrt{20}}$

From equation (i), we have

$\sin\theta_1=\sqrt{10}\sin\theta_2$

26. **(1.00)** For elastic collision $KE_i = KE_f$

$\frac{1}{2}m\times 25+\frac{1}{2}\times m\times 9=\frac{1}{2}m\times 32+\frac{1}{2}mv_B^2$

$34=32+V_B^2\Rightarrow V_B=\sqrt{2}$

$KE_B=\frac{1}{2}mv_B^2=\frac{1}{2}\times 0.1\times 2=0.1J=\frac{1}{10}J$

$\therefore x=1$

27. **(30.00)** Maximum height,

$$H=\frac{u^2\sin^2\theta}{2g} \quad \Rightarrow 120=\frac{u^2\left(\frac{1}{2}\right)}{2g}$$

$\therefore \quad u^2=480\text{ g}$

Upon hitting the ground, it loses half of its kinetic energy

$\therefore \quad K.E_{initial}=\frac{1}{2}mu^2=240\text{ mg}$

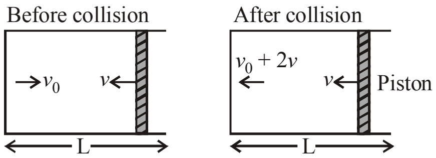
$K.E_{final}=\frac{1}{2}(240\text{ mg})=120\text{ mg}$

$\therefore \quad \frac{1}{2}mv^2=120\text{ mg} \qquad \therefore v^2=240\text{ g}$

After the bounce, the maximum height the ball reaches

$$\therefore \quad H'=\frac{v^2\sin^2\theta}{2g}=\frac{240g\times\left(\frac{1}{4}\right)}{2g}=30\text{ m}$$

28. **(a, d)**



When the small particle moving with velocity v_0 undergoes an elastic collision with the heavy movable piston moving with velocity v, it acquires a new velocity $v_0 + 2v$. So, the increase in velocity after every collision is $2v$.

Time period of collision when the piston is at a distance 'L' from the closed end

$T=\frac{\text{distance}}{\text{speed}}=\frac{2L}{v}$

Where v is the speed of the particle at that time.

$\therefore$ Frequency or rate at which the particle strikes the piston $=\frac{v}{2L}$

The rate of change of speed of the particle

$=\frac{dv}{dt}=(\text{frequency})\times 2v$

$\therefore \quad dv=\frac{v}{2L}2v\,dt$

$\therefore \quad \frac{dv}{v}=\frac{v\,dt}{L}=\frac{-dL}{L}$

Where dL is the distance travelled by the piston in time dt. The minus sign indicates decrease in 'L' with time.

$\therefore \quad \int_{V_0}^{V}\frac{dv}{v}=-\int_{L_0}^{x}\frac{dL}{L}$

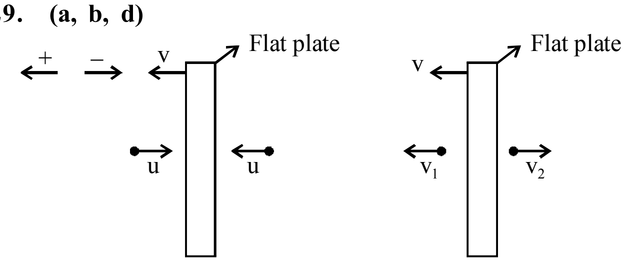
$\therefore \quad \ell n\frac{v}{v_0}=-\ell n\frac{L}{L_0} \qquad \text{or} \qquad |v|=\frac{v_0L_0}{L}$

when $L=\frac{L_0}{2}$ we have $|V|=\frac{V_0L_0}{L_0/2}=2V_0$

$\therefore \; K.E_{L_{0/2}}=\frac{1}{2}m(2V_0)^2$

$\therefore \; K.E_{L_0}=\frac{1}{2}mV_0^2 \qquad \therefore \frac{K.E_{L_{0/2}}}{K.E_{L_0}}=4$

29. **(a, b, d)**



Before collision | Just after collision

$1=\frac{v_1-v}{v+u}$ | $1=\frac{v+v_2}{u-v}$

$\therefore v_1=u+2v$ | $\therefore v_2=u-2v$

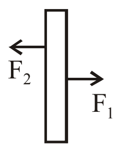
$\therefore \Delta v_1=2u+2v$ | and $\Delta v_2=2u-2v$

Now $F_1=\frac{dp_1}{dt}=\rho A(u+v)(2u+2v)$

and $F_2=\frac{dp_2}{dt}=\rho A(u-v)(2u-2v)$

$\therefore F_1=2\rho A(u+v)^2$ and $F_2=2\rho A(u-v)^2$

ΔF is the net force due to the air molecules on the plate.



$\Delta F=F_1-F_2=8\rho Auv \quad \therefore P=\frac{\Delta F}{A}=8\rho uv$

The net force $F_{net}=F-\Delta F=ma$

$\therefore F-8\rho Auv=ma$

30. **(a, c)**

According to law of conservation of linear momentum

$m_1u_1+m_2u_2=m_1v_1+m_2v_2$

$1\times u_1+5\times 0=1(-2)+5(v_2)$

$\Rightarrow u_1=-2+5v_2$ ...(i)

The coefficient of restituition

$e=\frac{v_2-v_1}{u_1-u_2} \Rightarrow 1=\frac{v_2-(-2)}{u_1-0}$

$\Rightarrow u_1=v_2+2$ ...(ii)

From eq (i) & (ii) $u_1=3$ m/s and $v_2=1$ m/s

Hence total momentum of the system = 3 kg m/s and K.E.cm = 0.75 J

31. (a, d)
From law of conservation of linear momentum

The initial linear momentum of the system, $p\hat{i} - p\hat{i} = 0$

$\therefore$ Final linear momentum should also be zero *i.e.*, $p_1' + p_2' = 0$

Option a :

$p_1' + p_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$ = Final momentum.

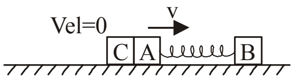
It is given that a_1, b_1, c_1, a_2, b_2 and c_2 have non-zero values. If a_1 = x and a_2 = –x. Also if b_1 = y and b_2 = –y then the $\hat{i}$ and $\hat{j}$ components become zero. But the third term having $\hat{k}$ component is non-zero. This gives a definite final momentum to the system which violates conservation of linear momentum, so this is a wrong option.

Option d:

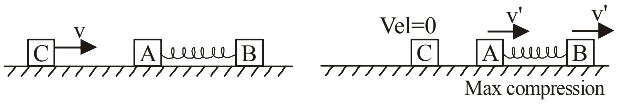
$p_1' + p_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0$ because $b_1 \neq 0$

Following the same reasoning as above the option d is also wrong.

32. (b, d) Just after the collision of *C* with *A*, *C* stops and *A* acquires a velocity *v* because of head-on elastic collision between identical masses.



When *A* starts moving towards right, the spring suffer a compression due to which *B* also starts moving towards right. The compression of the spring continues till there is relative velocity between *A* and *B*. When this relative velocity becomes zero, both *A* and *B* move with the same velocity *v*' and the spring is in a state of maximum compression say *x*.



From principle of conservation of linear momentum

$$mv = mv' + mv' \Rightarrow v' = \frac{v}{2}$$

$\therefore$ K.E. of *A* – *B* system at maximum compression,

$$\frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 = mv'^2 = \frac{mv^2}{4} \quad \left(\because v' = \frac{v}{2}\right)$$

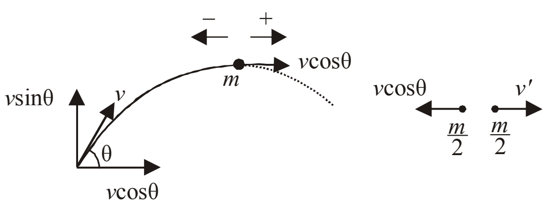
Applying energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{4}mv^2 \quad \therefore \quad x = v\sqrt{\frac{m}{2K}}$$

33. (a) Let *v*′ be the speed of other piece of shell after the collision.
As one piece retraces its path, the speed of this piece just after explosion should be *v* cos θ



Applying conservation of linear momentum at the highest point

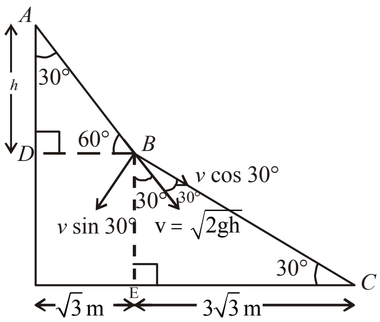
$$m\,(v\cos\theta) = \frac{m}{2} \times v' - \frac{m}{2} \times v\cos\theta$$

$\therefore \quad v' = 3v\cos\theta$

34. (c,d) In inelastic collision only momentum of the system may remain conserved.
(a) is incorrect because the momentum of ball changes in magnitude as well as direction.
(b) is incorrect because on collision, some mechanical energy is converted into heat, sound energy.
(c) is correct because for earth + ball system the impact force is an internal force.
(d) is correct. Total energy of the ball and the earth is conserved.

35. (b) As the inclined plane is frictionless,
The K. E . at B = P.E. at *A*

$$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh}$$



In Δ *ADB*, tan 60° = $\frac{h}{\sqrt{3}}$

$\therefore h = 3$ m

$\therefore v = \sqrt{6g} = \sqrt{60}$ m/s

This is the velocity of the block just before collision. This velocity makes an angle of 30° with the vertical. Also in right angled triangle *BEC*, ∠*EBC*= 60°. Therefore v makes an angle of 30° with the second inclined plane *BC*. The component of v along *BC* is *v* cos 30°.

It is given that the collision at *B* is perfectly inelastic therefore the impact forces act normal to the plane such that the vertical component of velocity becomes zero. The component of velocity along the incline *BC* remains unchanged and is equal to *v* cos 30°

$$= \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{180}{4}} = \sqrt{45}\ \text{m/s}$$

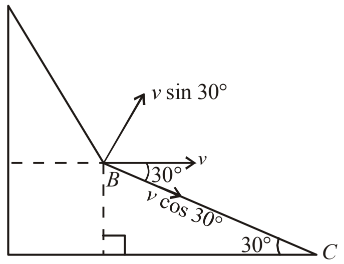
36. (b) In Δ *BCE*, tan30° $= \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{3\sqrt{3}} \Rightarrow BE = 3$m

From mechanical energy conservation principle,
Mechanical energy at *B* = mechanical energy at *C*

$$\frac{1}{2}M\left(\sqrt{45}\right)^2 + M \times 10 \times 3 = \frac{1}{2}Mv_c^2$$

$$45 + 60 = v_c^2 \qquad \therefore v_c = \sqrt{105}\ \text{m/s}$$

37. (c) The velocity of the block along *BC* just before collision is v cos 30°. The impact forces act perpendicular to the surface so the component of velocity along the incline remains unchanged.



Just after collision

Also since the collision is elastic, the vertical component of velocity (v sin 30°) before collision changes in direction, the magnitude remaining the same as shown in the figure. So the rectangular components of velocity after collision are as shown in the figure. This means that the final velocity of the block should be horizontal making an angle 30º with BC. Therefore the vertical component of the final velocity of the block is zero.

38. **(d)** Maximum energy loss $= \dfrac{P^2}{2m} - \dfrac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get\

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

39. **(d)** **Statement 1 :** For an elastic collision, the coefficient of restitution = 1

$$e = \frac{|v_2 - v_1|}{|u_1 - u_2|} \Rightarrow |v_2 - v_1| = |u_1 - u_2|$$

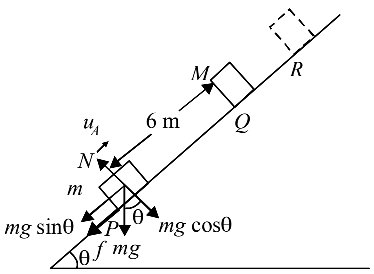
$\Rightarrow$ Relative velocity after collision is equal to relative velocity before collision. But in the statement relative speed is given.

Statement 2 : Linear momentum remains conserved in an elastic collision. This statement is true.

40. **From *P* to *Q*.**

Given : $u = 10$ m/s and $\mu = 0.25$; $s = 6$ m

$$a = -\left[\frac{mg\sin\theta + f}{m}\right] = -\left[\frac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$$



$= -[g\sin\theta + \mu g\cos\theta] = -g[\sin\theta + \mu\cos\theta]$

$= -10[0.05 + 0.25 \times 0.99] = -2.99$ m/s^2

Using, $v^2 - u^2 = 2as \Rightarrow v^2 = 100 + 2(-2.99) \times 6 \Rightarrow v = 8$ m/s

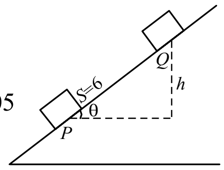
Hence velocity of mass m just before collision = 8 m/s. The velocity of mass M just before collision = 0 m/s (given). Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision. **Body of mass *M* moving from *Q* to *R* and coming to rest.** After collision,

$u = v_2$
$v = 0$
$a = -2.99$ m/s^2
$s = 0.5$

$v^2 - u^2 = 2as \Rightarrow (0)^2 - v_2^2 = 2(-2.99) \times 0.5$

$\Rightarrow v_2 = 1.73$ m/s

Body of mass *m* moving from *Q* to *P* after collision



$\sin\theta = \dfrac{h}{6}$

$h = 6\sin\theta = 6\sin 60° = 6 \times 0.05$

$u = v_1$

$v = +1$m/s

$(\text{K.E.} + \text{P.E.})_{\text{initial}} = (\text{K.E.} + \text{P.E.})_{\text{final}} + W_{\text{friction}}$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$$\frac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \frac{1}{2}(1)^2 + 0.25 \times 10 \times 6$$

$v_1 = -5$ m/s

$\therefore$ **Coefficient of restitution**

$$e = \left|\frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}\right|$$

$$= \left|\frac{-5 - 1.73}{8 - 0}\right| = 0.84$$

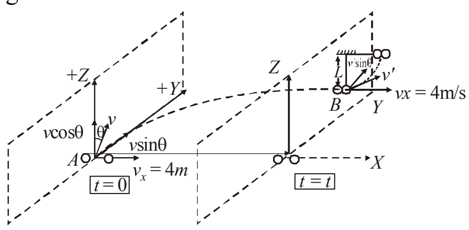
Applying conservation of linear momentum before and after collision,

$mv + M \times 0 = m \times v_1 + m \times v_2$

$2 \times 8 + M \times 0 = 2 \times (-5) + M(1.73)$

$\therefore M = \dfrac{26}{1.73} = 15.02$ kg

41. When the stone reaches the point B, the component of velocity in the $+Z$ direction ($v\cos\theta$) becomes zero due to the gravitational force in the $-Z$ direction.



The stone has two velocities at B

v_x in the $+X$ direction (4 m/s)

$v\sin\theta$ in the $+Y$ direction (6 sin 30° = 3 m/s)

Resultant velocity of the stone

$$v' = \sqrt{(v_x)^2 + (v\sin\theta)^2} = \sqrt{4^2 + 3^2} = 5\text{ m/s}$$

(i) Applying conservation of linear momentum at B for collision with a mass of equal magnitude.

Since, the collision is completely inelastic the two masses will stick together. v is the velocity of the two masses just after collision.]

$m \times 5 = 2m \times v$

$\therefore v = 2.5$ m/s

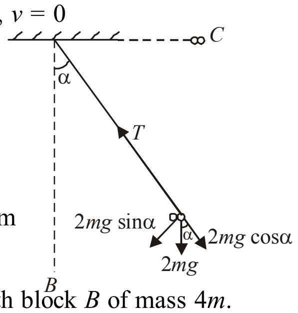
(ii) When the string is undergoing circular motion, at any arbitrary position, $T - 2mg\cos\alpha = \dfrac{2mv^2}{\ell}$

According to question, $T = 0$ when $\alpha = 90°$

$\therefore \quad 0 - 0 = \dfrac{2mv^2}{\ell} \Rightarrow v = 0$

i.e., in the horizontal position, $v = 0$

Applying energy conservation from B to C,



$$\frac{1}{2}2mv^2 = 2mg\ell$$

$$\Rightarrow \ell = \frac{v^2}{2g} = \frac{(2.5)^2}{2\times 9.8} = 0.318\text{ m}$$

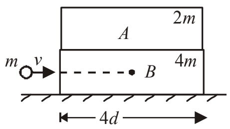
42. Object of mass m collides with block B of mass $4m$.
Since the collision is elastic in nature applying conservation of linear momentum

$$mv = (4\text{m})\,u + mv'$$

where u is the velocity of mass 4m after collision and v' is the velocity of mass m

$\Rightarrow \quad v' = v - 4u \qquad$ ... (i)

Applying conservation of kinetic energy



Also, $\dfrac{1}{2}mv^2 = \dfrac{1}{2}(4\text{m})u^2 + \dfrac{1}{2}mv'^2$

$\Rightarrow \quad v^2 = 4u^2 + v'^2 \qquad$ ... (ii)

From eq. (i) & (ii)

$$v^2 = 4u^2 + (v-4u)^2 \Rightarrow u = \frac{2v}{5}$$

block B starts moving but the block A remains at rest. As there is no friction between A and B

For block A to topple, block B should move a distance $2d$. Let the retardation produced in B due to friction force between B and the table be a

$$F = \mu N \Rightarrow (4\,m)a = \mu\,(6\,mg) \Rightarrow a = 1.5\,\mu g$$

For the motion of B,

$$u = \frac{2v}{5},\ v = 0,\ s = 2\text{d},\ a = -1.5\,\mu g$$

Now, $v^2 - u^2 = 2as \Rightarrow (0)^2 - \left(\dfrac{2v}{5}\right)^2 = 2(-1.5\mu g)\,2d$

$$\Rightarrow \quad \boldsymbol{v = \frac{5}{2}\sqrt{6\mu g d}}$$

For elastic collision between two bodies

$$v_1 = \frac{(m_1 - m_2)\,u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2}$$

Here $m_1 = m,\ m_2 = 4m,\ u_1 = 5\sqrt{6\mu g d},\ u_2 = 0$

$$\Rightarrow \quad v_1 = \frac{(m-4m)\,5\sqrt{6\mu g d}}{m+4m} + 0$$

$$= -3\times 5\frac{\sqrt{6\mu g d}}{5} = -3\sqrt{6\mu g d}$$

The negative sign shows that the mass m rebounds. It then follows a projectile motion and its path's parabolic.

$u_y = 0,\ s_y = d,\ a_y = g,\ t_y = ?$

For vertical motion,

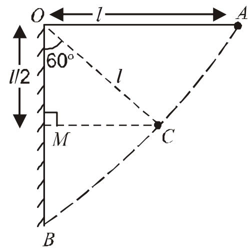
$$S = ut + \frac{1}{2}at^2 \Rightarrow d = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2d}{g}}$$

The horizontal distance travelled by mass m during this time t from P leftwards

$$x = 3\sqrt{6\mu g d}\times\sqrt{\frac{2d}{g}} = 6\sqrt{3\mu d^2} = \boldsymbol{6d\sqrt{3\mu}}$$

43. Initially when the bob of pendulum is at A, its P.E. = mgl
When the bob released from A and strikes to the wall at B, P.E. changes to K.E. and when it is at position 'C' the angular amplitude is 60°.

In ΔOCM



$$\cos 60° = \frac{OM}{\ell} \Rightarrow OM = \frac{\ell}{2}$$

The velocity of bob at B,

$$mg\ell = \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{2g\ell}$$

Let after n collisions, the angular amplitude is 60° when the bob again moves towards the wall from C, the velocity v'_B before collision

$$mg\frac{\ell}{2} = \frac{1}{2}mv'^2_B \Rightarrow v'_B = \sqrt{g\ell}$$

This means that the velocity of the bob should reduce from $\sqrt{2g\ell}$ to $\sqrt{g\ell}$ due to collisions with walls.

The final velocity after n collisions is $\sqrt{g\ell}$

$$\therefore \quad e^n(\sqrt{2g\ell}) = \sqrt{g\ell}$$

where e is coefficient of restitution.

$$\left(\frac{2}{\sqrt{5}}\right)^n \times\sqrt{2g\ell} = \sqrt{g\ell} \Rightarrow \left(\frac{2}{\sqrt{5}}\right)^n = \frac{1}{\sqrt{2}}$$

Taking log on both sides we get

$$n\log\left(\frac{2}{\sqrt{5}}\right) = \log\frac{1}{\sqrt{2}} \Rightarrow n = 3.1$$

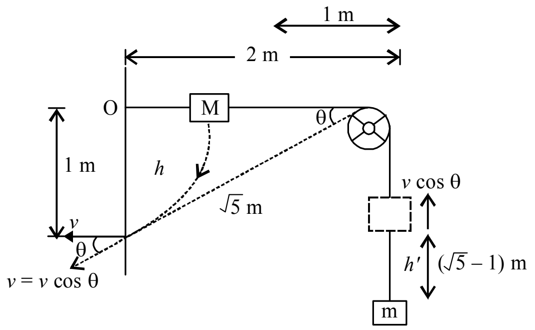
Hence, number of collisions = 4.

44. Let point mass hit the wall with the speed v.

Then, velocity of mass m at this instant = $v\cos\theta = \dfrac{2}{\sqrt{5}}v$.

Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)$ m.

From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(v\cos\theta)^2$$

or, $$(2)(9.8)(1) = (0.5)(9.8)(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$$

Solving we get, $v = 3.29$ m/s.

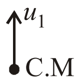
45. Initial position of C.M.

$$x_1 = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{100\times 0 + 100\times 98}{200} = 49 \text{ m}$$

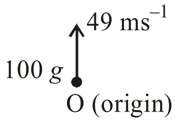
100 g vel = 0 (dropped)
Initial velocity of C.M

$$u_c = \frac{m_1u_1 + m_2u_2}{m_1+m_2} = \frac{100\times 49 + 100\times 0}{200} = 24.5 \text{ ms}^{-1}$$

Acceleration of C.M
$a_c = -9.8 \text{ ms}^{-2}$



Displacement of C.M is
$s_c = -49$ m
Applying



$$S = ut + \frac{1}{2}at^2$$

$$-49 = 24.5\,t - 4.9\,t^2$$

$$t^2 - 5t - 10 = 0$$

$$t = \frac{5\pm\sqrt{25+40}}{2} = \frac{5\pm\sqrt{65}}{2} = 6.53 \text{ s}$$

46. The collision between C and A is elastic and their masses are equal so they will exchange their velocities A was initially at rest, therefore the result of collision will be that C will come to rest and A will initially start moving with a velocity v_0. But since A is connected to B with a spring, the spring will get compressed.
Let v' be the common velocity of A and B.

At $t = t_0$, the velocities of A and B become same.
Applying energy conservation;

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}2mv'^2 + \frac{1}{2}kx_0^2$$

where x_0 is the compression in the spring at $t = t_0$

$$\therefore \quad v_0^2 = 3v'^2 + \frac{k}{m}x_0^2 \quad \text{... (i)}$$

Applying momentum conservation,

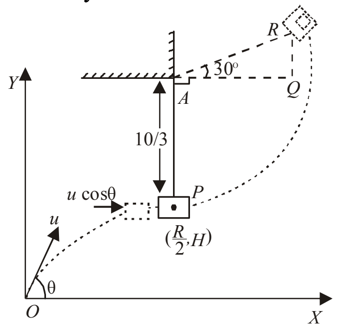
$$mv_0 = mv' + 2mv' \quad \therefore \quad v' = \frac{v_0}{3} \quad \text{... (ii)}$$

From eq. (i) and (ii)

$$v_0^2 - 3\times\frac{v_0^2}{9} = \frac{k}{m}x_0^2 \quad \Rightarrow k = \frac{2mv_0^2}{3x_0^2} \text{ (spring constant)}$$

47. At the highest point P, the velocity of the bullet = $u\cos\theta$
In ΔAQR
(i) From figure,

$$\sin 30° = \frac{QR}{10/3},\ QR = \frac{5}{3}$$



Now from, conservation of linear momentum at the highest point - P

$$M(u\cos\theta) + 3M\times 0 = (M + 3M)\,v$$

$$v = \frac{Mu\cos\theta}{4M} = \frac{u\cos\theta}{4}$$

From energy conservation principle $K.E.$ of the bullet-mass system at P = $P.E.$ of the bullet-mass system at R

$$\frac{1}{2}(4M)v^2 = (4M)gh$$

$$\frac{1}{2}(4M)\frac{u^2\cos^2\theta}{16} = 4Mg\times\left(\frac{10}{3}+\frac{5}{3}\right)$$

$$\cos^2\theta = \frac{9.8\times 5\times 2\times 16}{50\times 50} \Rightarrow \theta = 37°$$

($\because u = 50$ m/s given)

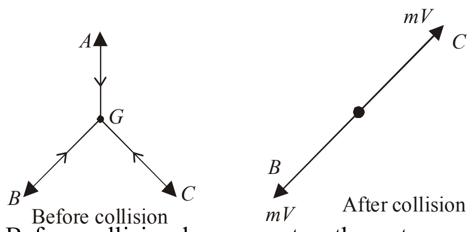
(ii) Vertical component,

$$\frac{R}{2} = \frac{u^2\sin 2\theta}{2g} = \frac{50\times 50\sin 2\times 37°}{2\times 9.8} = 122.6\,\text{m}$$

Horizontal component,

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{50\times 50\times(\sin 37°)^2}{2\times 9.8} = 46\,\text{m}$$

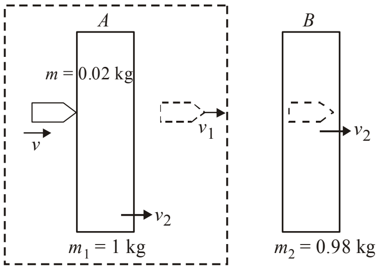
48.



Before collision by symmetry, the net momentum of the system is zero.
After collision, momentum is also zero as no esternal force is acting on the system. A comes to rest so B and C should have equal and opposite momenta, so velocity of C should be same in magnitude *i.e.*, V but in opposite direction of velocity of B.

49. Applying conservation of linear momentum for the system of bullet and plate A, $mv = mv_1 + m_1v_2$

or, $0.02v = 0.02\, v_1 + 1 \times v_2$ ...(i)



Again applying conservation of linear momentum for collision at B, $mv_1 = (m + m_2)v_2$

or, $0.02\, v_1 = (2.98 + 0.02)\, v_2 = 3v_2$

$$\Rightarrow \quad v_2 = \frac{0.02\, v_1}{3} \quad \text{... (ii)}$$

From eq. (i) and (ii)

$$0.02\, v = 0.02\, v_1 + \frac{0.02\, v_1}{3} \quad \Rightarrow \quad v = \frac{4}{3}v_1 \Rightarrow \frac{v}{v_1} = \frac{4}{3}$$

$$\frac{v_1}{v} = \frac{3}{4} \Rightarrow 1 - \frac{v_1}{v} = 1 - \frac{3}{4} = \frac{1}{4} = 0.25$$

$$\Rightarrow \frac{v - v_1}{v} = 0.25$$

$$\therefore \quad \text{% loss in velocity, } \frac{v - v_1}{v} \times 100 = \frac{1}{4} \times 100 = 25\%$$

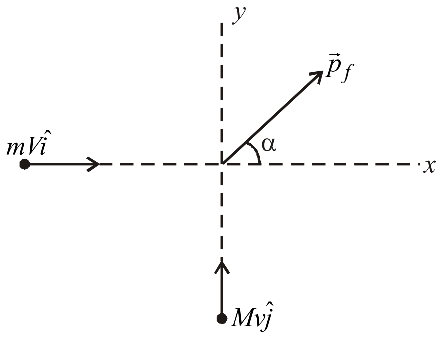
50. (i) According to consevation of linear momentum

$\vec{p}_f = \vec{p}_i = mV\hat{i} + Mv\hat{j}$ (Magnitude of the momentum of final body)

$$\therefore \quad p_f = \sqrt{m^2V^2 + M^2v^2}$$

$$\tan\alpha = \frac{Mv}{mV} \Rightarrow \alpha = \tan^{-1}\frac{Mv}{mV}$$

which gives the direction of the momentum.



$$\text{(ii)} \quad \frac{\text{K.E}_i - \text{K.E}_f}{\text{K.E}_i} = 1 - \frac{\text{K.E}_f}{\text{K.E}_i} = 1 - \frac{p_f^2(m+M)}{\frac{1}{2}mV^2 + \frac{1}{2}Mv^2}$$

$$= 1 - \frac{m^2V^2 + M^2v^2}{(m+M)(mV^2 + Mv^2)}$$

$$= \frac{m^2V^2 + mMv^2 + mMV^2 + M^2v^2 - m^2V^2 - M^2v^2}{(m+M)(mV^2 + Mv^2)}$$

$$\frac{\Delta K.E.}{K.E_i} = \frac{mM(v^2 + V^2)}{(m+M)(mV^2 + Mv^2)}$$

Topic-5 : Miscellaneous (Mixed Concept) Problems

1. **(6.30)** Given: mass $m = 0.4$ kg; impulse J = 1.0 Ns $V_{(t)} = v_0 e^{-t/\tau}$; $\tau = 4$s and $e^{-1} = 0.37$.

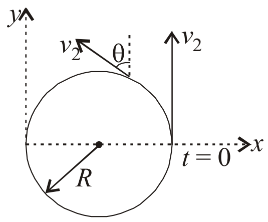
Impulse = Change in linear momentum

$$\text{or,} \quad J = mV_0 \Rightarrow \quad V_0 = \frac{J}{m} = \frac{1}{0.4} = 2.5\ \text{ms}^{-1}$$

$$\text{Also } (V_t) = v_0 e^{-t/\tau} \quad \therefore \frac{ds}{dt} = v_0 e^{-t/\tau} \quad \Rightarrow ds = v_0 e^{-t/\tau}\, dt$$

$$\therefore s = v_0 \int_0^\tau e^{-t/\tau} dt = v_0\tau(1 - e^{-1}) = 2.5 \times 4 \times 0.63 = 6.30\ \text{m}$$

2. Angular speed of particle about centre of the circular path



$$\omega = \frac{v_2}{R}, \theta = \omega t = \frac{v_2}{R}t$$

$$v_p = \left(-v_2 \sin\theta\hat{i} + v_2\cos\theta\hat{j}\right)$$

$$\text{or} \quad v_p = \left(-v_2 \sin\frac{v_2}{R}t\hat{i} + v_2\cos\frac{v_2}{R}t\hat{j}\right)$$

and $v_m = v_1\hat{j}$

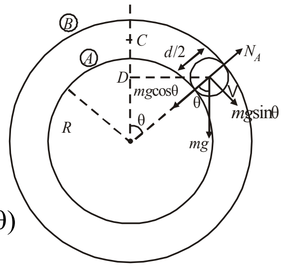
$\therefore$ Linear momentum of particle w.r.t. man as a function of time

$$L_{pm} = m(v_p - v_m)$$

$$= m\left[\left(-v_2\sin\frac{v_2}{R}t\right)\hat{i} + \left(v_2\cos\frac{v_2}{R}t - v_1\right)\hat{j}\right]$$

3. **(a)** For circular motion of the ball, the necessary centripetal force is provided by ($mg\cos\theta - N$).

$$mg\cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)} \quad \text{... (i)}$$



N = normal reaction at angle θ

According to energy conservation

$$\frac{1}{2}mv^2 = mg\left(R + \frac{d}{2}\right)(1 - \cos\theta)$$

$$\Rightarrow \quad v^2 = 2g\left(R + \frac{d}{2}\right)(1 - \cos\theta)$$

Putting this value of v^2 in eq. (i)

$N = mg\,(3\cos\theta - 2)$

(b) The ball will lose contact with the inner sphere when $N = 0$

or $\quad 3\cos\theta - 2 = 0 \;$ or $\; \theta = \cos^{-1}\left(\frac{2}{3}\right)$

After this it makes contact with outer sphere and normal reaction starts acting towards the centre.

So for $\quad \theta \le \cos^{-1}\left(\frac{2}{3}\right)$

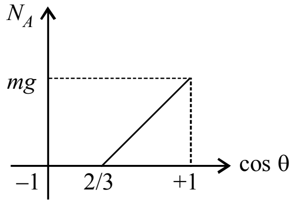
$N_B = 0$

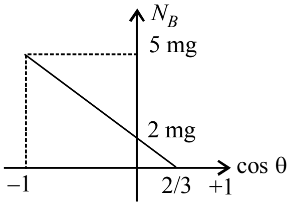
and $\quad N_A = mg\,(3\cos\theta - 2)$

and for $\quad \theta \ge \cos^{-1}\left(\frac{2}{3}\right)$

$N_A = 0 \quad$ and $\quad N_B = mg\,(2 - 3\cos\theta)$

The $N_A - \cos\theta$ and $N_B - \cos\theta$ graphs are as follows.



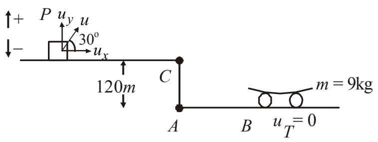


4. Consider the vertical motion of the cannon ball

$u_y = +\,100 \sin 30°$ (vertical component of velocity)

$s_y = -\,120$ m (vertical displacement)

$a_y = -\,10$ m/s^2



$t = t_0$ (Time of first impact on carriage)

$s_y = u_y t + \frac{1}{2} a_y t_0^2 \quad \therefore\; -120 = 50\,t_0 - 5\,t_0^2$

$\Rightarrow \quad t_0^2 - 10\,t_0 - 24 = 0$

$\therefore \quad t_0 = -\dfrac{(-10) \pm \sqrt{100 - 4(1)(-24)}}{2} = 12$ or -2 [Not valid]

$\therefore \quad t_0 = 12$ sec.

Horizontal component of velocity of the cannon ball remains the same

$\therefore \quad u_x = 100\cos 30° + 5\sqrt{3} = 55\sqrt{3}$

$\therefore$ Applying conservation of linear momentum to the cannon ball-trolley system in horizontal direction.

$mu_x + M \times 0 = (m + M)\,v_x$

$\therefore \quad v_x = \dfrac{mu_x}{m + M}$ (where m = mass of cannon ball, M = mass of trolley, v_x = velocity of the cannon ball-trolley system)

$\therefore \quad v_x = \dfrac{1 \times 55\sqrt{3}}{1 + 9} = 5.5\sqrt{3}\ \text{ms}^{-1}$

Horizontal distance covered by the car

$P = 12 \times 5\sqrt{3} = 60\sqrt{3}$ m ($\because$ Second ball was projected after 12 second.)

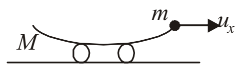
Since the second ball also struck the trolley,

$\therefore$ in time 12 seconds, the trolley covers a distance of $60\sqrt{3}$ m.

For trolley after 12 seconds;

$u = 5\sqrt{3}$ m/s, $\; v = ?, \quad t = 12$ s

$s = 60\sqrt{3}$ m,



$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 60\sqrt{3} = 5.5\sqrt{3} \times 12 + \frac{1}{2} \times a \times 144$

$\therefore \quad a = -\sqrt{3}/12\ \text{ms}^{-2} \quad \therefore\; v = u + at = 5\sqrt{3}$ m/s.

$v = u + at_0 = 5.5\sqrt{3} - \left(\frac{\sqrt{3}}{12}\right) \times 12$

$v = 4.5\sqrt{3}\ \text{ms}^{-1}$

To find the final velocity v_f of the carriage after the second impact.

Applying conservation of linear momentum in the horizontal direction

$mu_x + (M + m)\,v_x = (M + 2m)\,v_f$

$1 \times 55\sqrt{3} + (9 + 1)\; 4.5\sqrt{3} = (9 + 2)\,v_f$

$\therefore \quad v_f = 15.75$ m/s

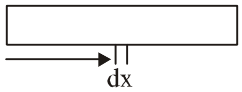
6 System of Particles and Rotational Motion

Topic-1 : Centre of Mass, Centre of Gravity & Principle of Moments

1. **(b)** Given,

Linear mass density, $\rho(x) = a + b\left(\frac{x}{L}\right)^2$

$$X_{CM} = \frac{\int x dm}{\int dm}$$



$$\int dm = \int_0^L \rho(x) dx$$

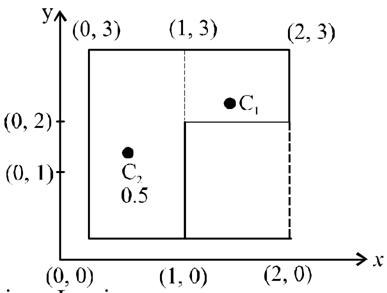
$$= \int_0^L \left[a + b\left(\frac{x}{L}\right)^2\right] dx = aL + \frac{bL}{3}$$

$$\int_0^L x dm = \int_0^L \left(ax + \frac{bx^3}{L^2}\right) dx = \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$$

$$\therefore \quad X_{CM} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)}{aL + \frac{bL}{3}}$$

$$\Rightarrow X_{CM} = \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$$

2. **(b)**



For given Lamina

$m_1 = 1, C_1 = (1.5, 2.5)$ (x y)
$m_2 = 3, C_2 = (0.5, 1.5)$

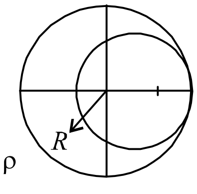
$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of centre of mass of flag shaped lamina (0.75, 1.75)

3. **(a)** Mass of sphere = volume of sphere x density of sphere

$$= \frac{4}{3}\pi R^3 \rho$$



Mass of cavity $M_{cavity} = \frac{4}{3}\pi(1)^3 \rho$

Mass of remaining

$$M_{(Remaining)} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi(1)^3 \rho$$

Centre of mass of remaining part,

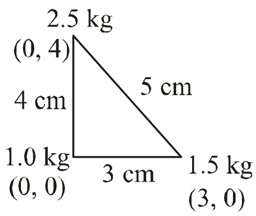
$$X_{COM} = \frac{M_1r_1 + M_2r_2}{M_1 + M_2}$$

$$\Rightarrow -(2-R) = \frac{\left[\frac{4}{3}\pi R^3 \rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R-1]}{\frac{4}{3}\pi R^3 \rho + \frac{4}{3}\pi(1)^3(-\rho)}$$

$$\Rightarrow \frac{(R-1)}{(R^3-1)} = 2 - R \quad \Rightarrow \frac{(R-1)}{(R-1)(R^2+R+1)} = 2 - R$$

$$\Rightarrow (R^2 + R + 1)(2 - R) = 1$$

4. **(d)**



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = \frac{1.5 \times 3}{5} = 0.9cm$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = \frac{2.5 \times 4}{5} = 2cm$$

Hence, centre of mass of system is at point (0.9, 2)

5. **(a)** Acceleration of centre of mass (a_{cm}) is given by

$$\therefore \vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$$

$$= \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

6. **(d)** With respect to point θ, the CM of the cut-off portion $\left(\frac{a}{4}, \frac{b}{4}\right)$.

Using, $x_{CM} = \frac{MX - mx}{M - m}$

$$= \frac{M \times 0 - \frac{M}{4} \times \frac{a}{4}}{M - \frac{M}{4}} = -\frac{a}{12}$$

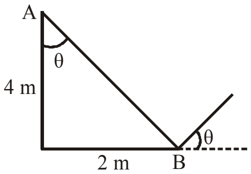
and $y_{CM} = -\frac{b}{12}$

So CM coordinates one

$$x_0 = \frac{a}{2} - \frac{a}{12} = \frac{5a}{12}$$

and $y_0 = \frac{b}{2} - \frac{b}{12} = \frac{5b}{12}$

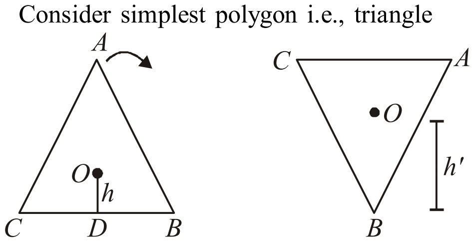
7. **(c)** To produce maximum moment of force line of action of force must be perpendicular to line AB.



$\therefore \tan\theta = \frac{2}{4} = \frac{1}{2}$

8. **(c)** According to principle of moments when a system is stable or balance, the anti-clockwise moment is equal to clockwise moment.
i.e., load × load arm = effort × effort arm
When 5 mg weight is placed, load arm shifts to left side, hence left arm becomes shorter than right arm.

9. **(b)** Consider simplest polygon i.e., triangle



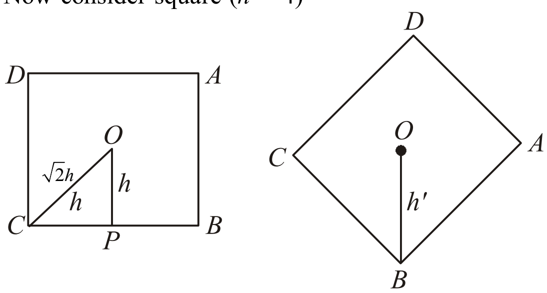
$(n = 3)$
$OD = h$ and also O is the centroid

$\therefore \frac{OA}{OD} = \frac{h'}{h} = \frac{2}{1}$ $\quad (\because OA = OB = h')$

$\Rightarrow h' = 2h$

$$\Delta = h' - h = 2h - h = h\left(\frac{1}{1/2} - 1\right) = h\left(\frac{1}{\cos\frac{\pi}{3}} - 1\right)$$

Now consider square $(n = 4)$

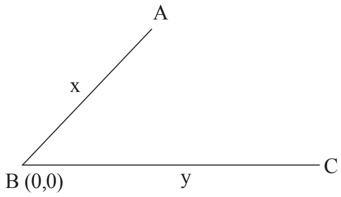


$$\Delta = h' - h = OB - OP = OC - OP$$

$$= \sqrt{2}h - h = h\left(\sqrt{2} - 1\right) = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$$

Hence for any value of $n, \Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

10. **(c)** Centre of mass $x_{cm} = \frac{x}{2} = \frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho y^{y/2}}{\rho(x+y)}$



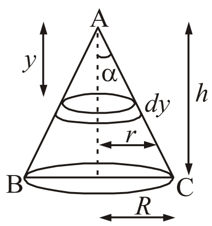
$$\Rightarrow \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$$

$$\therefore \frac{Bc}{AB}\frac{y}{x} = \frac{1+\sqrt{3}}{2} = 1.37$$

11. **(d)** Let density of cone = ρ.

Centre of mass, $ycm = \frac{\int ydm}{\int dm}$

$$= \frac{\int_0^h y\pi r^2 dy\rho}{\frac{1}{3}\pi R^2 h\rho} = \frac{\int_0^h r^2 y dy}{\frac{1}{3}R^2 h} \quad ...(i)$$



For a cone, we know that

$\frac{r}{R} = \frac{y}{n} \quad \therefore r = \frac{y}{n}R$

$$ycm = \frac{\int_0^h 3y^3 dy}{h^3} = \frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3} = \frac{3}{4}h$$

12. **(b)** Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right)dx}{\int_0^L \left(a + \frac{bx}{L}\right)dx}$$

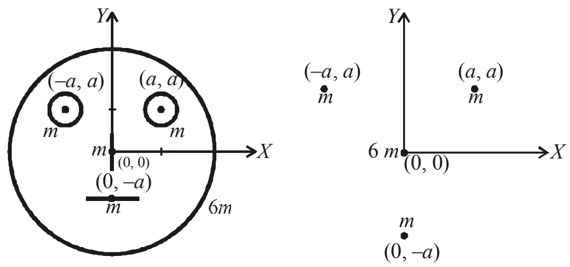
$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$

On solving we get, $b = 2a$

13. **(c)** 14. **(d)**

14. **(a)** The drawing given in the figure is made up of five bodies i.e., three circles and two straight line of uniform mass distribution or we can assume the system to be made up of five point masses where the mass of each is considered at its geometrical centre.



The y-coordinate of the centre of mass is

$$y_{cm}=\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5}{m_1+m_2+m_3+m_4+m_5}$$

$$\therefore\quad y_{cm}=\frac{6m\times 0+m\times 0+m\times a+m\times a+m(-a)}{6m+m+m+m+m}$$

$$=\frac{ma}{10m}=\frac{a}{10}$$

15. **(d)** Mutual force of attraction with which two particles A and B move towards each other is a internal force. There are no external forces acting on the system.

We know $F_{\text{ext}}=Ma_{\text{c.m.}}$

$a_{\text{c.m.}}=0 \Rightarrow v_{\text{c.m.}}=\text{constant.}$

Since, initially $v_{\text{c.m.}}=0$

$\therefore$ Final $V_{\text{cm}}=0$

16. **(3)** Centre of mass of solid hemisphere of radius R lies at a distance $\frac{3R}{8}$ above the centre of flat side of hemisphere.

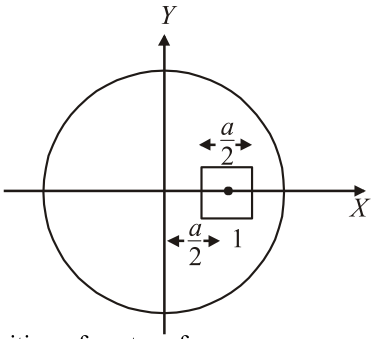
$$\therefore h_{\text{cm}}=\frac{3R}{8}=\frac{3\times 8}{8}=3\text{ cm}$$

17. **(23.00)** Let σ be the mass density of circular disc.

Original mass of the disc, $m_0=\pi a^2\sigma$

Removed mass, $m=\frac{a^2}{4}\sigma$

Remaining, mass, $m'=\left(\pi a^2-\frac{a^2}{4}\right)\sigma=a^2\left(\frac{4\pi-1}{4}\right)\sigma$



New position of centre of mass

$$X_{CM}=\frac{m_0x_0-mx}{m_0-m}=\frac{\pi a^2\times 0-\frac{a^2}{4}\times\frac{a}{2}}{\pi a^2-\frac{a^2}{4}}$$

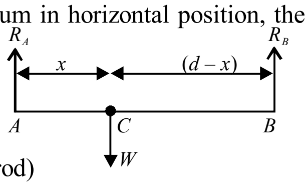
$$=\frac{-a^3/8}{\left(\pi-\frac{1}{4}\right)a^2}=\frac{-a}{2(4\pi-1)}=\frac{-a}{8\pi-2}=-\frac{a}{23}$$

$\therefore x=23$

18. Let R_A = Normal reaction at A

R_B = Normal reaction at B

For the rod to be in equilibrium in horizontal position, the moment of all the forces about A or about B should be zero



$\therefore$ Moment of forces about A,

$xW=R_Bd$ (W = weight of rod)

or $\quad R_B=\frac{xW}{d}$ ... (i)

Again, moment of forces about B,

$(d-x)W=R_Ad$ or $\quad R_A=\left(\frac{d-x}{d}\right)W$ ... (ii)

$\therefore$ Normal reaction on $A=\left(\frac{d-x}{d}\right)W$ and on $B=\frac{xW}{d}$.

19. **False.** There is no external force is acting on the two particle system

So, $a_{c.m}=0$ and hence $V_{c.m}$ = Constant.

20. **(b, c)** Let the block of mass M moves by distance x towards left.

$Mx=m(R-x)$

$\Rightarrow\quad x=\frac{mR}{M+m}$ towards left

$\therefore\quad x=-\frac{mR}{M+m}$

If v is the velocity of mass 'm' as it leaves the block and V is the velocity of block at that instant then according to conservation of linear momentum

mv = MV

By energy conservation

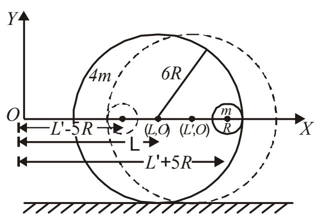
$$\text{mgR}=\frac{1}{2}mv^2+\frac{1}{2}MV^2$$

Solving we get, $v=\sqrt{\frac{2gR}{1+\frac{m}{M}}}$ and $V=\frac{m}{M}\sqrt{\frac{2gR}{1+\frac{m}{M}}}$

21. **(a, c)** When pedalling stop but the cycle is in motion the wheels move in the direction such that the centre of mass of the wheel move forward. Rolling friction will act in the opposite direction to the relative motion of the centre of mass of the body with respect to ground. Hence the rolling friction will act in backward direction in both the wheels. During pedalling sliding friction will act in the forward direction of rear wheel.

22. **(d)** There is no external torque on a body about its centre of mass, so no Ven is constant. For velocity of centre of mass to remain constant the net force acting on a body must be zero. The linear momentum of an isolated system remains constant.

23. Thin line of sphere represents initial state, dotted line of sphere represents final state.



When small sphere M changes its position to other extreme position, there is no external force in the horizontal direction. Therefore the x-coordinate of c.m. will not change.

$[x_{\text{c.m.}}]_{\text{initial}}=[x_{\text{c.m.}}]_{\text{final}}$

$$\Rightarrow\quad\frac{M_1x_1+M_2x_2}{M_1+M_2}=\frac{M_1x'_1+M_2x'_2}{M_1+M_2}$$

$$\Rightarrow\quad\frac{4m\times L+m\times(5R+L)}{4m+m}=\frac{4m\times L'+m\times(L'-5R)}{4m+m}$$

$\Rightarrow\quad 5L+5R=5L'-5R$

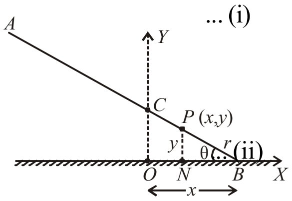
$\Rightarrow\quad 5L+10R=5L' \quad\Rightarrow L+2R=L'$

As the individual center of mass of the two spheres has a y co-ordinate zero in its initial state and its final state so, the y-coordinate of c.m. of the two sphere system will remain zero. Hence, the coordinate of centre of mass of bigger sphere $(L + 2R, 0)$.

24. **(i)** The rod stands vertical along they yr axis. While rotating, the force acting on the rod are its weight and normal reaction. These forces are vertical forces and cannot create a horizontal motion. Hence the center of mass moves vertically downwards. Thus the path of the center of mass of the rod is a straight line.

(ii) Consider an arbitrary point P on the rod located at (x, y) and at a distance r from the end B. The rod makes an angle θ with the horizontal at this position.

In $\Delta\, BNP$, $\sin\theta = \dfrac{y}{r}$ ... (i)



$$\cos\theta = \frac{x + BN}{L/2} = \frac{x + r\cos\theta}{L/2}$$

$$\Rightarrow \quad \cos\theta = \frac{x}{\frac{L}{2} - r}$$

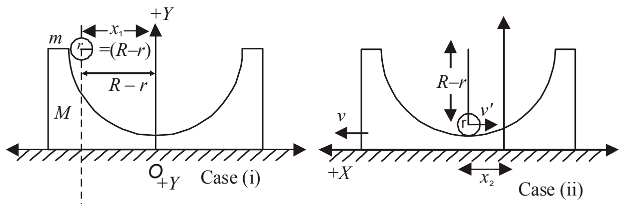
$$\because \ \sin^2\theta + \cos^2\theta = 1$$

$$\therefore \ \frac{y^2}{r^2} + \frac{x^2}{\left(\frac{L}{2} - r\right)^2} = 1$$

Hence shape of the path of this point is an ellipse.

25. Case (i) centre of mass of the system of two bodies in x-coordinate

$$x_C = \frac{M \times 0 + mx_1}{M + m} = \frac{mx_1}{M + m}$$



Case (ii) centre of mass of the system in x-coordinate

$$x'_C = \frac{M \times x_2 + m \times x_2}{M + m} = x_2$$

$x_C = x'_C$ ($\because$ Fexternal = 0 in x-direction)

$$\therefore \quad x_2 = \frac{mx_1}{M + m} = \frac{m(R - r)}{M + m}$$

i.e., the block more $x_2 = \dfrac{m(R - r)}{M + m}$ when

The cylinder reaches the bottom point β of the track

Applying conservation of linear momentum,

$P_i = P_f$

$0 = MV - mv$

$$\therefore \quad v = \frac{MV}{m}$$

Applying conservation of energy,

$$\Rightarrow \quad mg\,(R - r) = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2mg\,(R - r) = MV^2 + m\frac{M^2V^2}{m^2} \qquad \left[\because\ v = \frac{MV}{m}\right]$$

$$\Rightarrow \quad 2mg\,(R - r) = MV^2 + \frac{M^2V^2}{m}$$

$$2mg\,(R - r) = MV^2\left[1 + \frac{M}{m}\right] = MV^2\left[\frac{m + M}{m}\right]$$

$$\Rightarrow \quad \frac{2m^2g\,(R - r)}{M\,(m + M)} = V^2 \ \Rightarrow \quad V = m\sqrt{\frac{2g\,(R - r)}{M\,(m + M)}}$$

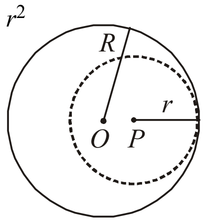
26. Let σ be the mass per unit area of uniform plate.

$\therefore$ Mass of the whole disc $= \sigma \times \pi\, R^2$

Mass of the portion removed $= \sigma \times \pi\, r^2$

$R = 28$ cm; $r = 21$ cm; $OP = 7$ cm

Position of centre of mass



$$x = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

$$= \frac{\sigma \times \pi R^2(0) - \sigma \times \pi r^2 \times 7}{\sigma\pi R^2 - \sigma\pi r^2}$$

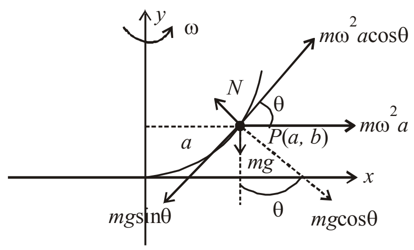
$$= \frac{-(21)^2 \times 7}{(28)^2 - (21)^2} = -\frac{21 \times 21 \times 7}{7 \times 49} = -9\text{ cm}$$

i.e., centre of mass lies at a distance of 9 cm from the origin towards left.

Topic-2 : Angular Displacement, Velocity and Aceleration

1. **(a)** $y = 4Cx^2 \Rightarrow \dfrac{dy}{dx} = \tan\theta = 8Cx$

At P, $\tan\theta = 8Ca$



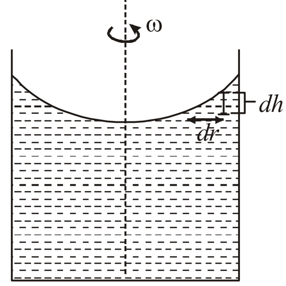
For steady circular motion

$$m\omega^2 a\cos\theta = mg\sin\theta$$

$$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega = \sqrt{\frac{g \times 8aC}{a}} = 2\sqrt{2gC}$$

2. **(c)** Here, $\rho dr\omega^2 r = \rho g dh$



$$\Rightarrow \omega^2\int_0^R r\,dr = g\int_0^h dh$$

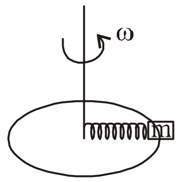
$$\Rightarrow \frac{\omega^2R^2}{2} = gh$$

(Given $R = 5$ cm)

$$\therefore h = \frac{\omega^2R^2}{2g} = \frac{25\omega^2}{2g}$$

3. (c) Free body diagram in the frame of disc

$$\xleftarrow{kx} \boxed{m} \longrightarrow m\omega^2(\ell_0 + x)$$



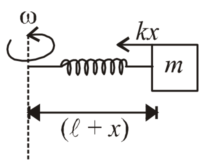
$\therefore\ m\omega^2(\ell_0 + x) = kx$

$\Rightarrow\ x = \dfrac{m\ell_0\omega^2}{k - m\omega^2}$

For $k >> m\omega^2$

$\Rightarrow\ \dfrac{x}{\ell_0} = \dfrac{m\omega^2}{k}$

4. (b) At elongated position (x),



$F_{\text{radial}} = \dfrac{mv^2}{r} = mr\omega^2$

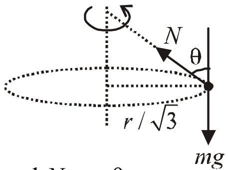
$\therefore\ kx = m(\ell + x)\omega^2$

($\because r = \ell + x$ here)

$kx = m\ell\omega^2 + mx\omega^2$

$\therefore\ x = \dfrac{m\ell\omega^2}{k - m\omega^2}$

5. (b) $N \sin\theta = m\omega^2 (r/2)$ ...(i)



$\sin\theta = \dfrac{r/2}{r} = \dfrac{1}{2} \Rightarrow \theta = 30°$

and $N\cos\theta = mg$ ...(ii)

or $\tan\theta = \dfrac{\omega^2 r}{2g}$

or $\tan 30° = \dfrac{\omega^2 r}{2g}$ or $\dfrac{1}{\sqrt{3}} = \dfrac{\omega^2 r}{2g}$

$\therefore\ \omega^2 = \dfrac{2g}{r\sqrt{3}}$.

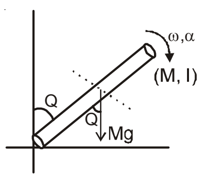
6. (c) $m\omega^2 R = \text{Force} \propto \dfrac{1}{R^n}$ $\left(\text{Force} = \dfrac{mv^2}{R}\right)$

$\Rightarrow\ \omega^2 \propto \dfrac{1}{R^{n+1}} \Rightarrow \omega \propto \dfrac{1}{R^{\frac{n+1}{2}}}$

Time period $T = \dfrac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

7. (c) Torque at angle θ



$\tau = Mg\sin\theta . \dfrac{l}{2}$

Also $\tau = l\alpha$

$\therefore\ l\alpha = Mg\sin\theta \dfrac{l}{2}$

$\dfrac{Ml^2}{3}.\alpha = Mg\sin\theta\dfrac{l}{2}$ $\left[\because I_{rod} = \dfrac{Ml^2}{3}\right]$

$\Rightarrow\ \dfrac{l\alpha}{3} = g\dfrac{\sin\theta}{2}$ $\therefore\ \alpha = \dfrac{3g\sin\theta}{2l}$

8. (a) For just complete rotation

$v = \sqrt{Rg}$ at top point

The rotational speed of the drum

Þ $\omega = \dfrac{v}{R} = \sqrt{\dfrac{g}{R}} = \sqrt{\dfrac{10}{1.25}}$

The maximum rotational speed of the drum in revolutions per minute

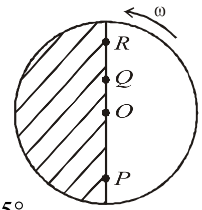
$\omega(\text{rpm}) = \dfrac{60}{2p}\sqrt{\dfrac{10}{1.25}} = 27$.

9. (a) At $t = 0$, $t = \dfrac{T}{2}$ and $t = T$ the relative velocity will be zero.

At $t = \dfrac{T}{4}$ and $t = \dfrac{3T}{4}$, the relative velocity will be maximum in magnitude

Hence graph (a) correctly depicts v_r versus t graph.

10. (c) For $\dfrac{1}{8}$ of rotation of disc,



$t = 0$

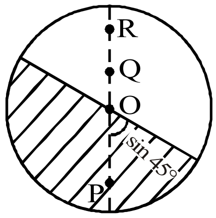
$t = \dfrac{1}{8} \times \dfrac{2\pi}{\omega} = \dfrac{\pi}{4\omega}$

x-coordinate of $P = \omega RT$

$= \dfrac{\pi R}{4} > R\sin 45°$

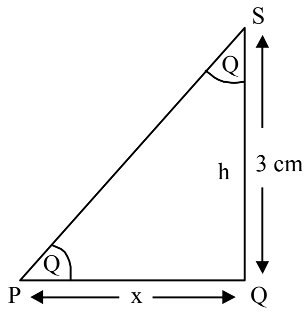
To reach the unshaded part, particle-P needs to travel horizontal range $> R\sin 45° \simeq 0.7\,R$

But its range is less than R/2 so it will land on shaded part. Q is near the origin, its velocity will be nearly along QR so it will land in unshaded part.



$t = \dfrac{\pi}{4\pi}$

11.



The velocity (v) of spot $= dx/dt$

and the angular speed (ω) of spot light $= \dfrac{d\phi}{dt}$

From $\Delta\,SOP$,

$\tan\phi = \dfrac{x}{h}$ $\Rightarrow\ x = h\tan\phi$

$\therefore\ \dfrac{dx}{dt} = h\sec^2\phi\,\dfrac{d\phi}{dt}$ or, $v = (h\sec^2\phi)\,\omega$

$\therefore\ v = 3\sec^2 45° \times 0.1$ $[\because \theta + \phi = 90°]$

$\therefore\ v = 3 \times 2 \times 0.1 = 0.6$ m/s

12. (a) Force on the block along slot $= m r\omega^2 = ma = m v\dfrac{dv}{dr}$

$\therefore\ \int_0^V V dv = \int_{R/2}^{r} \omega^2 r dr \Rightarrow V = \omega\sqrt{r^2 - \dfrac{R^2}{4}} = \dfrac{dr}{dt}$

$\therefore \int_{R/4}^{r} \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega\, dt$

On solving we get

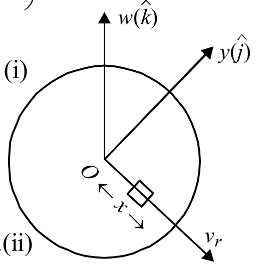
$r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2} e^{\omega t}$

or $r^2 - \frac{R^2}{4} = \frac{R^2}{4} e^{2\omega t} + r^2 - 2r\frac{R}{2} e^{\omega t}$

$\therefore \quad r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

13. (b) $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(V_{rot}\,\hat{i}\right)\times\omega\hat{k} + m\left(\omega\hat{k}\times r\hat{i}\right)\times\omega\hat{k}$

$\therefore m r\omega^2\,\hat{i} = \vec{F}_{in} + 2m\,V_{rot}\,\omega\left(-\hat{j}\right) + m\omega^2 r\,\hat{i}$



$\vec{F}_{in} = mrV_{rot}\,\omega\,\hat{j}$ .... (i)

But $r = \frac{R}{4}\left[e^{\omega t} + e^{-\omega t}\right]$

$\therefore \frac{dr}{dt} = V_r = \frac{R}{4}\left[\omega e^{\omega t} - \omega e^{-\omega t}\right]$ ....(ii)

From (i) and (ii)

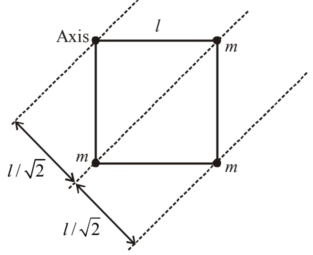
$\vec{F}_{in} = 2m\frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega\,\hat{j}$

$\therefore \quad \vec{F}_{in} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$

$\therefore \quad \vec{F}_{reaction} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\,\hat{K}$

Topic-3 : Torque, Couple and Angular Momentum

1. (c) Angular momentum, $L = I\omega$

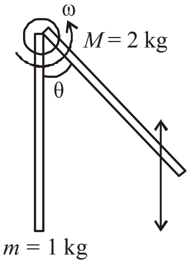


$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}l)^2$

$= \frac{2ml^2}{2} + 2ml^2 = 3ml^2$

Angular momentum $L = I\omega = 3ml^2\omega$

2. (a) Using conservation of angular momentum



$mvl = \left(ml^2 + \frac{2ml^2}{3}\right)\omega$

$\Rightarrow mvl = \frac{5}{3}ml^2\omega \Rightarrow \omega = \frac{3v}{5l}$

or, $\omega = \frac{3\times 6}{5\times 1} = \frac{18}{5}$ rad/s

Now using energy conservation, after collision

$\frac{1}{2}I\omega^2 = 2mg\frac{l}{2}(1-\cos\theta) + mgl(1-\cos\theta)$

$\Rightarrow \frac{1}{2}\left(\frac{5}{3}ml^2\right)\frac{9v^2}{25l^2} = 2mgl(1-\cos\theta)$

$\Rightarrow \frac{3}{5\times 2}mv^2 = 2mgl(1-\cos\theta)$

$\frac{3}{10}\times\frac{36}{2\times 10} = 1-\cos\theta \Rightarrow 1-\frac{27}{50} = \cos\theta$

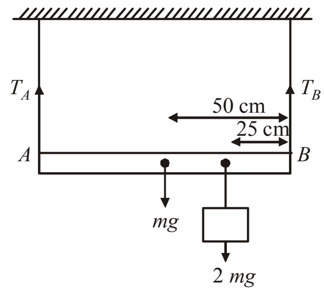
or, $\cos\theta = \frac{23}{50}$ $\quad \therefore \theta \simeq 63°$.

3. (d) Net torque, τ_{net} about B is zero at equilibrium

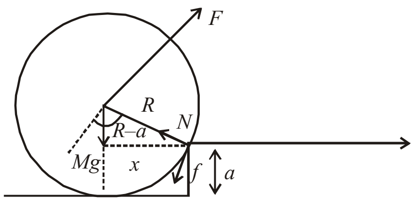
$\therefore T_A \times 100 - mg\times 50 - 2mg\times 25 = 0$

$\Rightarrow T_A \times 100 = 100mg$

$\Rightarrow T_A = 1mg$ (Tension in the string at A)



4. (a)

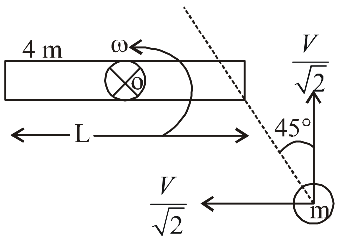


For step up, $F\times R \geq Mg\times x$

$x = \sqrt{R^2-(R-a)^2}$ from figure

$F_{min} = \frac{Mg}{R}\times\sqrt{R^2-(R-a)^2} = Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

5. (c)



About point O angular momentum

$L_{initial} = L_{final}$

$$\Rightarrow \frac{mV}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$

$$\therefore \omega = \frac{6V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$$

6. **(b)** Given that, $x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$$\frac{dx}{dt} = V_x$$

$$\Rightarrow v_x = -a\omega_1 \sin(\omega_1 t), \text{ and } \frac{dy}{dt} = v_y = b\omega_2 \cos(\omega_2 t)$$

$$\frac{dv_x}{dt} = a_x = -a\omega_1^2 \cos(\omega_1 t), \quad \frac{dv_y}{dt} = a_y = -b\omega_2^2 \sin(\omega_2 t)$$

At $t = 0$, $x = x_0 + a$, $y = y_0$

$a_x = -a\omega_1^2$, $a_y = 0$

Now, $\vec{\tau} = \vec{r} \times \vec{F} = m(\vec{r} \times \vec{a})$

$$= [(x_0 + a)\hat{i} + y_0\hat{j}] \times m(-a\omega_1^2 \hat{i}) = +my_0 a\omega_1^2 \hat{k}$$

7. **(d)** We have given $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$

$\vec{r}(\text{at } t = 2) = 4\hat{i} - 12\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$

$\vec{v}(\text{at } t = 2) = 2\hat{i} - 12\hat{j}$

$\vec{L} = mvr \sin\theta \hat{n} = m(\vec{r} \times \vec{v})$

$= 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$

8. **(d)** Angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{25 \times 2\pi - 0}{5} = 10\pi \text{ rad/s}^2$$

$\tau = I\alpha$

$$\Rightarrow \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$= 2.0 \times 10^{-5}$ Nm

9. **(a)**

10. **(c)** According to work-energy theorem

$$mgh = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$2gh = v_B^2 - v_A^2$$

$$2 \times 10 \times 10 = v_B^2 - 5^2$$

$\Rightarrow v_B = 15$ m/s

Angular momentum about O,

$L_O = mvr$

$= 20 \times 10^{-3} \times 20$

$L_O = 6$ kg.m^2/s

11. **(a)** Given, $\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$

$\vec{r}_1 = 0\hat{i} + 6\hat{j}$

Torque due to F_1 force

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 6\hat{j} \times \left(\frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})\right) = 3F(\hat{k})$$

Torque due to F_2 force

$\vec{\tau}_{F_2} = (2\hat{i} + 3\hat{j}) \times F\hat{k} = 3F\hat{i} + 2F(-\hat{j})$

$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} = 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$

$= (3\hat{i} - 2\hat{j} + 3\hat{k})F$

12. **(a)** Torque about the origin $= \vec{\tau} = \vec{r} \times \vec{F}$

$= r\ F \sin\theta \Rightarrow 2.5 = 1 \times 5 \sin\theta$

$$\sin\theta = 0.5 = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{6}$$

13. **(d)** Consider a strip of radius x and thickness dx,

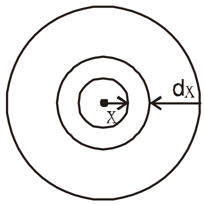
Torque due to friction on this strip

Net torque $= \sum$ Torque on ring

$$\int d\tau = \int_0^R \frac{\mu F.2\pi x dx}{\pi R^2}$$

$$\Rightarrow \tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$



14. **(a)** Applying torque equation about point P.

$\tau = I\alpha = [2M_0(2\ell)^2 + 5M_0\ell^2]\alpha$

$\Rightarrow 5M_0 g\ell - 4M_0 g\ell = [2M_0(2\ell)^2 + 5M_0\ell^2]\alpha$

$\Rightarrow M_0 g\ell = (13M_0 g\ell^2)\alpha$

$$\therefore \alpha = \frac{g}{13\ell}$$

15. **(d)** Given that, the rod is of uniform mass density and $AB = BC$

Let mass of one rod is m.

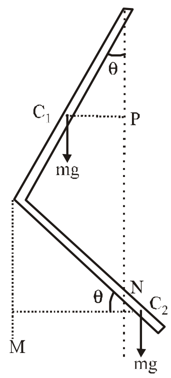
Balancing torque about hinge point.

$mg(C_1P) = mg(C_2N)$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

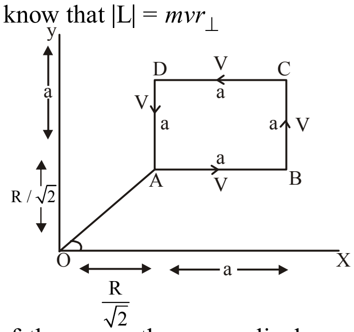
$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3} \text{ or, } \tan\theta = \frac{1}{3}$$



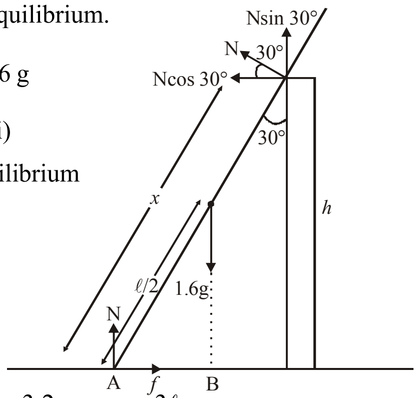
16. **(a)**

17. **(a)** We know that $|L| = mvr_\perp$



In none of the cases, the perpendicular distance $r_\perp$ is $\left(\frac{R}{\sqrt{2}} + a\right)$

18. **(d)** By vertical equilibrium.



$N + N\sin 30° = 1.6\,g$

$\Rightarrow N = \dfrac{3.2g}{3}$ … (i)

By horizontal equilibrium

$f = N\cos 30°$

$= \dfrac{\sqrt{3}}{2}N = \dfrac{16\sqrt{3}}{3}$

Torque about A

$1.6g \times AB = N \times x$

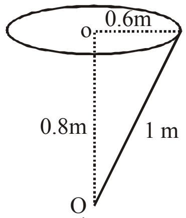
$1.6\,g \times \dfrac{\ell}{2}\cos 60° = \dfrac{3.2g}{3} \times x \quad \therefore \dfrac{3\ell}{8} = x$ … (i)

But $\cos 30° = \dfrac{h}{x} \qquad \therefore \; x = \dfrac{h}{\cos 30°}$ … (ii)

From eq. (i) and (ii)

$\dfrac{h}{\cos 30°} = \dfrac{3\ell}{8} \quad \therefore \; \dfrac{h}{\ell} = \dfrac{3\sqrt{3}}{16}$

19. **(a)** Angular momentum,



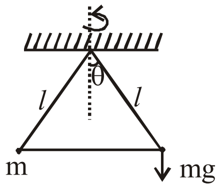
L_0 = mvr sin 90°

= 2 × 0.6 × 12 × 1 × 1

[As $V = r\omega$, Sin 90° = 1]

So, L_0 = 14.4 kgm^2/s

20. **(c)** Torque working on the bob of mass m is, $\tau = mg \times \ell \sin \theta$. (Direction parallel to plane of rotation of particle)



As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

21. **(c)** Given : m = 0.160 kg

θ = 60°

v = 10 m/s

Angular momentum $L = \vec{r} \times m\vec{v}$

= H mv cos θ

$= \dfrac{v^2 \sin^2\theta}{2g}\cos\theta \qquad \left[H = \dfrac{v^2\sin^2\theta}{2g}\right]$

$= \dfrac{10^2 \times \sin^2 60° \times \cos 60°}{2 \times 10}$

= 3.46 kg m^2/s

22. **(a)** Angular momentum L = m (v × r)

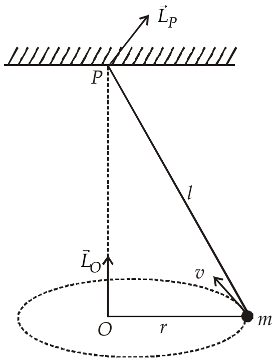
$= 2\,\text{kg}\left(\dfrac{dr}{dt} \times r\right) = 2\text{ kg}(4t\,j \times 5i - 2t^2\hat{j})$

$= 2\text{ kg}(-20\,t\,\hat{k}) = 2\text{ kg} \times -20 \times 2\text{ m}^{-2}\text{ s}^{-1}\,\hat{k}$

$= -80\,\hat{k}$

23. **(b)**

24. **(c)** For all locations of m the angular momentum of the mass m about O *i.e.*, L_o is $mr^2\omega$ and is directed toward $+z$ direction. The angular momentum of mass m about P i.e., L_p is mvl and is directed for the given location of m as shown in the figure. For different location of m, the direction of $\vec{L}_P$ remains changing.



25. **(b)** As particle is rotating in circular path and linear speed V is decreasing, L is not conserved in magnitude. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.

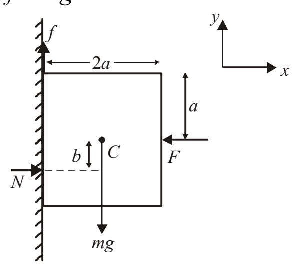
The direction of $\vec{L}$ remains same even when the speed decreases.

26. **(d)** The block is in equilibrium.

For translational equilibrium

$\Sigma F_x = 0 \Rightarrow F = N$

$\Sigma F_y = 0 \Rightarrow f = mg$



For rotational equilibrium $\quad \Sigma\tau_c = 0$

Torque created by frictional force (f) about $C = f \times a$ in clockwise direction.

There is another torque which should counter this torque. The normal reaction N on the block will create a torque $N \times b$ in the anticlockwise direction.

Such that $\quad f \times a = N \times b$

Hence N will produce torque.

27. **(b)** $(K.E.)_{\text{rotational}} = \dfrac{L^2}{2I}$.

Here , L = constant

$\therefore \quad (K.E.)_{\text{rotational}} \times I$ = constant.

Hence when I is doubled, $K.E._{\text{rotational}}$ becomes half.

28. **(a)** The net force acting on a particle undergoing uniform circular motion is centripetal force cowards centre of the circle. The torque due to this force about the centre is zero, therefore, angular momentum $\vec{L}$ about centre is conserved.

29. **(a)** Applying change in angular momentum of the system = angular impulse given to the system about the centre of mass (Angular momentum)$_f$ – (Angular momentum)$_i$

$= Mv \times \dfrac{L}{2} = I_C\,\omega$ ... (i)

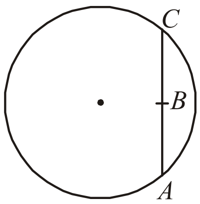
Here ω is the angular velocity of the rod.

Moment of Inertia of the system about its axis of rotation [centre of mass of the system]

$$I_C = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{2ML^2}{4} = \frac{ML^2}{2}$$

$$\therefore \quad \frac{MVL}{2} = \left(\frac{ML^2}{2}\right)\omega \Rightarrow \quad \omega = \frac{V}{L}$$

30. **(b)** There is no external torque, angular momentum remains conserved. As moment of inertia initially when the tortoise moves from A to B decreases and then increases when tortoise moves from B to C, so ω will increase initially and then decreases. So the variation of a ω is nonlinear.



31. **(b)** Due to increase in perpendicular distance of the bead with the axis of rotation *M.I.* about the axis of rotation is not constant. So I increases.

There is no external torque is acting

$$\therefore \tau_{ext} = \frac{dL}{dt} \Rightarrow L = \text{constant} \quad \Rightarrow I\omega = \text{constant}$$

But, I increases so ω decreases.

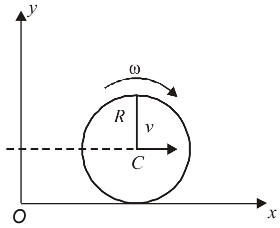
32. **(c)** When a disc rolls, it has two types of motion i.e., translational and rotational. Therefore there are two types of angular momentum and the total angular momentum is the vector sum of these two.

$\vec{L} = \vec{L}_T + \vec{L}_R$

L_T = angular momentum due to translational motion.

L_R = angular momentum due to rotational motion about *C.M.*

$L = MV \times R + I_{cm}\omega$



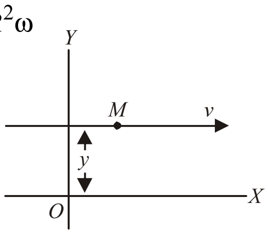
I_{cm} = *M.I.* about centre of mass C and $V = R\omega$

$$= M(R\omega)R + \frac{1}{2}MR^2\omega = \frac{3}{2}MR^2\omega$$

33. **(b)** Angular momentum

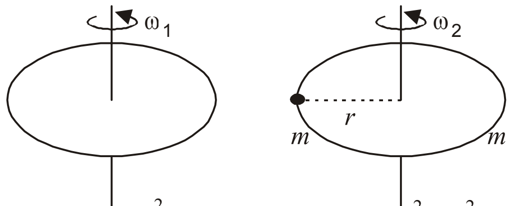
$$\vec{L} = r\perp \times \vec{p}$$

$$L = Mv \times y$$



According to question, M, v and y constant hence its angular momentum w.r.t. origin remains constant.

34. **(c)** Since the two objects are placed gently to the ring, therefore no external torque is acting on the system. Hence angular momentum remains constant.



$I = Mr^2$ $\qquad$ $I_2 = Mr^2 + mr^2$

$\therefore \quad I_1\omega_1 = I_2\,\omega_2$

$\Rightarrow \quad Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\,\omega_2 \qquad (\because \omega_1 = \omega)$

$$\therefore \quad \omega_2 = \frac{M\omega}{M + 2m}$$

35. **(4)** From conservation of angular momentum

$2\,(mvr) = I\omega$

$$2 \times 0.05 \times 9 \times 0.25 = \frac{1}{2} \times 0.45 \times (0.5)^2 \times \omega$$

$\therefore \ \omega = 4 \text{ rad s}^{-1}$

36. **(8)** From conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

$$\therefore \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\frac{1}{2}MR^2 \times \omega_1}{\left\{\frac{1}{2}MR^2 + 2[2mr^2]\right\}}$$

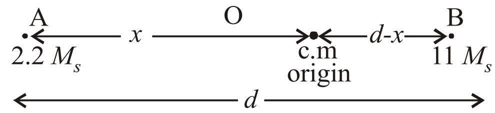
Given: $M = 50$ kg, $R = 0.4\ m$, $\omega_1 = 10$ rod/s

$m = 6.25$ kg and $r = 0.2\ m$

$$= \frac{\frac{1}{2} \times 50 \times 0.4 \times 0.4 \times 10}{\frac{1}{2} \times 50 \times 0.4 \times 0.4 + 2[2 \times 6.25 \times 0.2 \times 0.2]} = \frac{40}{4+1} = 8 \text{ rad / s}$$

37. **(6)**

Let the center of mass of the binary star system be at the origin. 'O'. Distance between $AO = x$ and $OB = (d - x)$



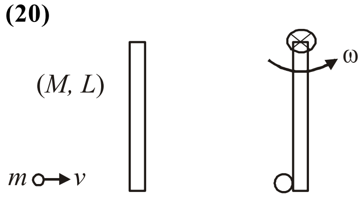
$$0 = \frac{2.2M_s(-x) + 11M_s(d - x)}{2.2M_s + 11M_s}$$

$$\Rightarrow 0 = 2.2\,M_s(-x) + 11M_s(d - x) \ \Rightarrow x = \frac{5d}{6}$$

For a binary star system, angular speed ω about the centre of mass. 'O' is same for both the stars.

$$\therefore \ \frac{L_{Total}}{L_B} = \frac{2.2M_s\left(\frac{5d}{6}\right)^2 \omega + 11M_s\left(\frac{d}{6}\right)^2 \times \omega}{11M_s\left(\frac{d}{6}\right)^2 \times \omega} = 6$$

38. **(20)**



Before collision $\qquad$ After collision

Using principal of conservation of angular momentum we have

$$\vec{L}_i = \vec{L}_f \Rightarrow mvL = I\omega$$

$$\Rightarrow mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega$$

$$\Rightarrow 0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$$

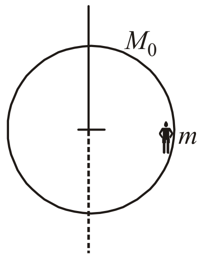
$$\Rightarrow 8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega \Rightarrow 8 = \frac{4}{10}\omega$$

$\Rightarrow \omega = 20$ rad/sec.

39. (9.00)

Here $M_0 = 200$ kg, $m = 80$ kg

Using conservation of angular momentum, $L_i = L_f$



$$I_1\omega_1 = I_2\omega_2$$

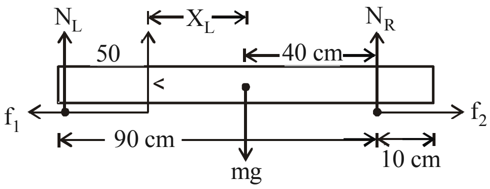
$$I_1 = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$$

$$I_2 = \frac{1}{2}M_0R^2 \text{ and } \omega_1 = 5 \text{ rpm}$$

$$\therefore \omega_2 = \left(\frac{M_0R^2}{2} + mR^2\right) \times \frac{5}{\frac{M_0R^2}{2}}$$

$$= \frac{5R^2}{R^2} \times \frac{(80+100)}{100} = 9 \text{ rpm.}$$

40. (25.60 cm)



Initially

$$N_L + N_R = Mg \qquad N_L = \frac{40}{90}Mg = \frac{4Mg}{9}$$

Torque about centre $\tau_{center} = 0$

$$\therefore N_1(50) = N_2(40) \qquad N_R = \frac{50}{90}Mg = \frac{5Mg}{9}$$

$$5N_L = 4N_R$$

$$f_{1_K} = \mu_K N_L \qquad f_{1_L} = \mu_S N_L$$

$$f_{1_K} = 0.32N_L \qquad f_{1_L} = 0.4N_L$$

$$f_{2_K} = 0.32N_{LR} \qquad f_{2_L} = 0.4N_R$$

If X_L = distance of left finger from centre when right finger starts moving

$(\tau_n = 0)_{\text{about centre}} \Rightarrow N_{LX_L} = N_R(40)$

$f_{K_1} = f_{L_2} \qquad \Rightarrow 0.32N_L = 0.40N_R$

$4N_L = 5N_R$

$$N_{LX_L} = \frac{4N_L}{5}(40) \Rightarrow X_L = 32$$

Now x_R = distance when right finger stops and left finger starts moving

$N_{LX_L} = N_R(x_R)$ [Torque about centre, $T_{centre} = 0$]

$f_{L_1} = f_{K_2} \qquad \Rightarrow 0.4N_L = 0.32N_R$

$4N_L = 4N_R$

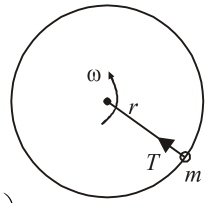
$$\frac{4N_2}{5}(32) = N_R X_R$$

$$\therefore x_R = \frac{128}{5} = 25.6 \text{ cm}$$

41. Let at any instant of time t, the radius of the horizontal surface = r.

$$T = \frac{mv^2}{r} = mr\omega^2 \qquad \text{... (i)}$$

Where m is the mass of stone and ω is the angular velocity at that instant of time t.



Also, $L = I\omega \Rightarrow w = \frac{L}{1}$

Putting this value of w is eq.(i)

$$T = \frac{mrL^2}{I^2} = \frac{mL^2}{(mr^2)^2} \times r \Rightarrow T = \left(\frac{L^2}{m}\right)r^{-3}$$

or, $T = Ar^{-3} \therefore n = -3 \qquad \left(\frac{L^2}{m} = A \text{ constant}\right)$

42. Considering the motion of the platform

$x = A\cos\omega t$ (given)

$$\Rightarrow \frac{dx}{dt} = -A\omega\sin\omega t \Rightarrow \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t$$

$\therefore$ Magnitude of the maximum acceleration of the platform

$\therefore$ | Max acceleration | = $A\omega^2$

When platform moves a torque acts on the cylinder and the cylinder rotates about its axis.

Acceleration of cylinder, $a_1 = \frac{f}{m}$

Torque $\tau = fR = I\alpha$

$$\Rightarrow \alpha = \frac{fR}{I} = \frac{fR}{MR^2/2}$$

or, $\alpha = \frac{2f}{MR}$ or $R\alpha = \frac{2f}{M} (= a_2)$

$\therefore$ Total or max linear acceleration,

$$a_{max} = a_1 + a_2 = \frac{f}{M} + \frac{2f}{M} = \frac{3f}{M}$$

or, $A\omega^2 = \frac{3f}{M} \Rightarrow f = \frac{MA\omega^2}{3}$

Thus, maximum torque,

$$\tau_{max} = f \times R = \frac{MA\omega^2R}{3} = \frac{1}{3}MAR\omega^2$$

43. There is no external force and hence no torque is applied, the angular momentum remains constant *i.e.*; $l_1 = l_2$

$\therefore \quad I_1\omega_1 = I_2\omega_2$

$$\therefore \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\frac{ML^2}{12} \times \omega_0}{\frac{ML^2}{12} + 2m \times \left(\frac{L}{2}\right)^2} = \frac{M\omega_0}{M + 6m}$$

44. False

$I_1\omega_1 = I_2\omega_2 \qquad (\because \vec{L} = \text{constant})$

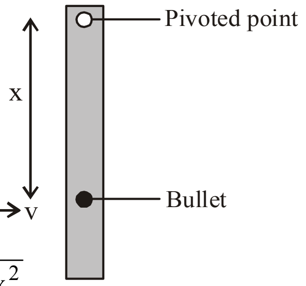
$$I_1 = \frac{1}{2}MR^2$$

$$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\frac{M}{4}R^2 = \left(\frac{4+1}{8}\right)MR^2 = \frac{5}{8}MR^2$$

$\omega_1 = \omega$; $\omega_2 = ?$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = + \frac{\frac{1}{2}MR^2 \times \omega}{\frac{5}{8}MR^2} = \frac{8}{2\times 5}\omega = \frac{4}{5}\omega$$

45. **(a, c, d)** From angular momentum conservation about the pivoted point.



$$mvx = \left(\frac{mL^2}{3} + mx^2\right)\omega$$

[As the combined system rotates with angular speed ω about the pivot]

$$\therefore \omega = \frac{mvx}{\frac{mL^2}{3} + mx^2} = \frac{3vx}{L^2 + 3x^2}$$

$$\omega = \frac{3vx}{L^2 + 3x^2}$$

Hence option (a) is correct.

For maximum angular velocity, $\frac{d\omega}{dx} = 0$

$$\frac{d}{dx}\left(\frac{L^2}{x} + 3x\right) = 0$$

$$\Rightarrow \frac{L^2}{x^2} + 3 = 0 \Rightarrow x = \frac{L}{\sqrt{3}}$$

$$\therefore X_m = \frac{L}{\sqrt{3}}$$

So option (c) is correct.

$$\omega_m = \frac{3vx}{L^2 + 3x^2} = \frac{3v\frac{L}{\sqrt{3}}}{L^2 + 3\left(\frac{L}{\sqrt{3}}\right)^2} = \frac{\sqrt{3}}{2L}V$$

Hence option (d) is correct.

46. **(a, c)** Given $\vec{F} = \alpha t\hat{i} + \beta\hat{j}$ or $\vec{F} = t\hat{i} + \hat{j}$

($\because \alpha = 1NS^{-1}$ and $\beta = 1N$)

$$\therefore \frac{m\,d\vec{v}}{dt} = t\,\hat{i} + \hat{j}$$

$$\therefore d\vec{v} = t\,dt\,\hat{i} + dt\,\hat{j} \quad [\because m = 1]$$

$$\therefore \int_0^v d\vec{v} = \int_0^t t\,dt\,\hat{i} + \int_0^t dt\,\hat{j}$$

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$

At $t = 1s$, $\vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j} \quad \therefore \quad d\vec{r} = \frac{t^2}{2}dt\,\hat{i} + t\,dt\,\hat{j}$

or, $\int_0^r d\vec{r} = \int_0^t \frac{t^2}{2}dt\,\hat{i} + \int_0^t t\,dt\,\hat{j} \quad \therefore \quad \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At $t = 1$, $\vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j} \quad \therefore |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j}) \quad (\text{at } t = 1\text{s})$$

or, $\vec{\tau} = -\frac{1}{3}\hat{k} \quad \therefore \quad |\vec{\tau}| = \frac{1}{3}$ Nm

47. **(b, c)** Given : potential energy, $V(r) = Kr^2/2$

Applying, $|F| = \frac{dv}{dr} = \frac{d}{dr}\left[\frac{kr^2}{2}\right] = kr$

For $r = R$, $F = kR$

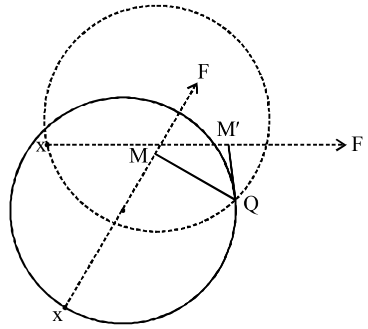
Also $F = \frac{mv^2}{R}$ ($\because$ Particle is moving in circular orbit)

or, $\frac{mv^2}{R} = kR \quad \Rightarrow v = \sqrt{\frac{k}{m}} \times R$

And angular momentum, $L = mvR = m\left(\sqrt{\frac{k}{m}}R\right)R = \sqrt{km}R^2$

48. **(c)** If the force (F) is applied at P tangential than the τ remains constant and τ = F × 2R.

If force is applied normal to X, then as the wheels climbs, then the perpendicular distance of force from Q will go on changing initially the perpendicular is QM, later it becomes QM'.



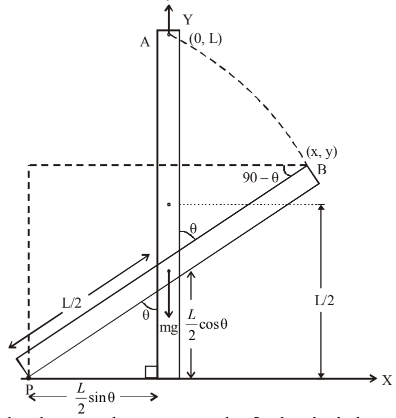
If the force (F) is applied normal to the circumference at point P then τ is zero.

If the force (F) is applied tangentially at point S then

$\tau = F \times R$ and the wheel climbs.

49. **(a, c, d)**

Force acting on com $F_x = 0$, $\therefore$ $a_x = 0$. Therefore the force acting in vertical direction will move the mid point or com of the bar fall vertically downwards.



When the bar makes an angle θ the height of its com

$= \frac{L}{2}\cos\theta$

$\therefore$ Displacement of its mid point from the initial position

$$\frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$$

Instantaneous torque about the point of contact P

$$\tau = mg \times \frac{L}{2}\sin\theta$$

Now $x = \frac{L}{2}\sin\theta$, $y = L\sin(90° - \theta) = L\cos\theta$

$\therefore \left(\frac{2x}{L}\right)^2 + \left(\frac{y}{L}\right)^2 = 1$ or $\frac{4x^2}{L^2} + \frac{y^2}{L^2} = 1$

Thus path of A is an ellipse not parabola.

50. (a, b)

Given: $\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$\vec{r} = \frac{10}{3}t^3\hat{i} + 5t^2\hat{j}m$

$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = 10t^2\hat{i} + 10t\hat{j}\ ms^{-1}$

and $\vec{a} = \frac{d\vec{v}}{dt} = 20t\hat{i} + 10\hat{j}ms^{-2}$

At t = 1s $\vec{r}_{t=1} = \frac{10}{3}\hat{i} + 5\hat{j}m$;

$\vec{v}_{t=1} = 10\hat{i} + 10\hat{j}\ ms^{-1}$ and $\vec{a}_{t=1} = 20\hat{i} + 10\hat{j}\ ms^{-2}$

$\vec{p}_{t=1} = \hat{i} + \hat{j}\ kgms^{-1}$

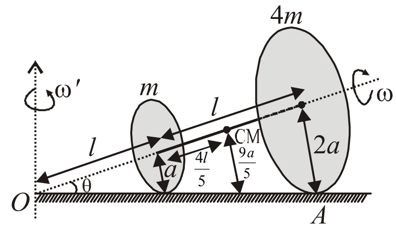
$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 5\right] = -\frac{5}{3}\hat{k}\ \text{kgms}^{-1}$$

$\vec{F} = m\vec{a} = (2\hat{i} + \hat{j})N$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 2 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 10\right] = \frac{-20}{3}\hat{k}\ \text{Nm}$$

51. (a, c) Position of CM on the axis of rod.

$$x_{CM} = \frac{m(0) + 4m(l)}{m + 4m} = \frac{4l}{5}$$



$$\cos\theta = \frac{1}{\sqrt{l^2 + a^2}} = \frac{\sqrt{24}a}{\sqrt{a^2 + 24a^2}} = \frac{\sqrt{24}}{5}$$

$\left[\because l = \sqrt{24}a \text{ given}\right]$

$$OA = \sqrt{(2l)^2 + (2a)^2} = \sqrt{96a^2 + 4a^2} = 10a$$

Let complete system rotates about z-axis with a constant angular velocity ω'

$$\therefore \quad \frac{\omega'}{\omega} = \frac{2\pi(2a)}{2\pi(10a)} \Rightarrow \omega' = \frac{\omega}{5}$$

Magnitude of angular momentum of the system about its center of mass

$$L_{CM} = I_{CM}\omega = \left[\frac{ma^2}{2} + \frac{4m(2a)^2}{2}\right]\omega = \frac{17}{2}ma^2\omega$$

Magnitude of angular momentum of CM of system about point O.

$$L' = 5m \times \frac{9l\omega}{5} \times \frac{9a}{5} = \frac{81m\omega la}{5} = \frac{81\sqrt{24}m\omega a^2}{5}$$

Magnitude of z-component of angular momentum of system about point 0.

$L_z = L'\cos\theta - L_{CM}\sin\theta$

$$= \frac{81\sqrt{24}m\omega a^2}{5} \times \frac{\sqrt{24}}{5} - \frac{17}{5}ma^2\omega \times \frac{1}{5}$$

$$= ma^2\omega\left(\frac{1944}{25} - \frac{17}{10}\right)$$

52. (d) Applying conservation of angular mumentum about the axis

$$MR^2 \times \omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8}r^2 \times \frac{8\omega}{9}$$

Solving we get

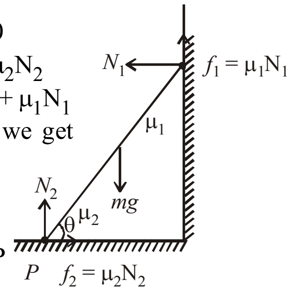
$\Rightarrow \quad r = \frac{4R}{5}$

53. (c, d) When $\mu_1 \neq 0$ and $\mu_2 \neq 0$

Horizontal equilibrium, $N_1 = \mu_2 N_2$

Vertical equilibrium, $mg = N_2 + \mu_1 N_1$

Solving the above equations we get



$$N_2 = \frac{mg}{1 + \mu_1\mu_2}$$

When $\mu_1 = 0$; Torque about P

$$mg \times \frac{l}{2}\cos\theta = N_1 \times l\sin\theta$$

$$\Rightarrow \quad N_1\tan\theta = \frac{mg}{2}$$

54. (a) Given $\sum \vec{F}_{ext} = 0$

$$\sum \vec{F}_{ext} = \frac{d\,\vec{p}_{\text{system}}}{dt} = 0 \Rightarrow \vec{p}_{\text{system}} = \text{constant}$$

i.e., Linear momentum of the system does not change in time.

Due to internal forces acting in the system, the kinetic and potential energy may change with time.

Also zero external force may create a torque. Thus the torque will change the angular momentum of the system in time.

55. (a, b, c)

Given $\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$ $\quad \left(\because \vec{\tau} = \frac{d\vec{L}}{dt}\right)$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is perpendicular to both $\vec{A}$ and $\vec{L}$.

From dot product rule, $\vec{L}.\vec{L} = L^2$

Differentiating with respect to time

$$\vec{L}\cdot\frac{\vec{dL}}{dt}+\vec{L}\cdot\frac{\vec{dL}}{dt}=2L\frac{dL}{dt} \Rightarrow 2\vec{L}\cdot\frac{\vec{dL}}{dt}=2L\frac{dL}{dt}$$

Since, $\vec{L}\perp\frac{dL}{dt}$

$$\therefore\ \vec{L}\cdot\frac{\vec{dL}}{dt}=0 \Rightarrow \frac{dL}{dt}=0$$

$\Rightarrow$ L = constant

Thus, the magnitude of L always remains constant.

As $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$,

Also, $\vec{L}$ is perpendicular to $\vec{A}$

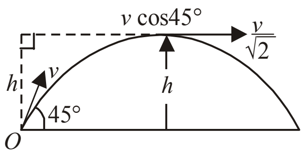
$\because\ \vec{L}\perp\vec{A}\quad \therefore\ \vec{L}.\vec{A}=0$

Hence, it can be concluded that component of $\vec{L}$ along $\vec{A}$ is zero i.e., constant.

56. **(b, d)** Angular momentum $L=r\perp\times P$

Angular momentum about point O

$$L=\frac{mv}{\sqrt{2}}\times h \qquad \text{... (i)}$$



Also, $h=\dfrac{v^2\sin^2\theta}{2g}=\dfrac{v^2}{4g}$ $\quad[\because \theta=45°]$ $\qquad$ ... (ii)

From eq. (i) and (ii)

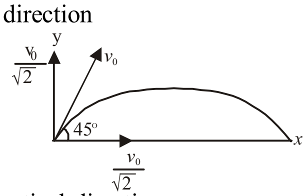
$$L=\frac{m}{\sqrt{2}}(2\sqrt{gh})\,h=m\sqrt{2gh^3}$$

$$\text{Also}\quad L=\frac{mv}{\sqrt{2}}\times\frac{v^2}{4g}=\frac{mv^3}{4\sqrt{2}\,g}$$

57. Let, After time $t=\dfrac{v_0}{g}$ (x, y) are the coordinates of the particle and v_x, v_y are the components of velocities at that time.

Here, velocity in horizontal direction

$$v_x=v_0\cos 45°=\frac{v_0}{\sqrt{2}}$$



Applying $v = u + at$ in the vertical direction

$$v_y=\left(v_0\sin 45°\right)-g\left(\frac{v_0}{g}\right)=\frac{v_0}{\sqrt{2}}-g\times\frac{v_0}{g}=\frac{v_0}{\sqrt{2}}-v_0$$

In horizontal direction $x=v_x\times t=\dfrac{v_0}{\sqrt{2}}\times\dfrac{v_0}{g}=\dfrac{v_0^2}{\sqrt{2}g}$

In vertical direction applying $S=ut+\dfrac{1}{2}at^2$

$$y=\frac{v_0}{\sqrt{2}}\times\frac{v_0}{g}-\frac{1}{2}g\frac{v_0^2}{g^2}=\frac{v_0^2}{\sqrt{2}g}-\frac{v_0^2}{2g}$$

Angular momentum, $L=m[V_y x-V_x y]$

$$\text{or,}\quad L=m\left[\frac{v_0^2}{\sqrt{2}g}\times\left(\frac{v_0}{\sqrt{2}}-v_0\right)-\left(\frac{v_0^2}{\sqrt{2}g}-\frac{v_0^2}{2g}\right)\frac{v_0}{\sqrt{2}}\right]$$

$$L=m\left[\frac{v_0^3}{2g}-\frac{v_0^3}{\sqrt{2}g}-\frac{v_0^3}{2g}+\frac{v_0^3}{2\sqrt{2}\,g}\right]$$

$$L=\frac{mv_0^3}{g}\left[\frac{1}{2\sqrt{2}}-\frac{1}{\sqrt{2}}\right]=\frac{-mv_0^3}{2\sqrt{2}\,g}$$

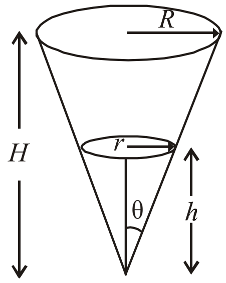
Again, $\vec{L}=\vec{r}\times\vec{p}$

Hence, the direction of L is perpendicular to the plane of motion and is directed away from the reader.

Topic-4 : Moment of Inertia and Rotational K.E.

1. **(d)** Hollow ice-cream cone can be assume as several parts of discs having different radius, so

$$I=\int dI=\int dm(r^2) \qquad \text{...(i)}$$



From diagram,

$$\frac{r}{h}=\tan\theta=\frac{R}{H}\ \text{or}\ r=\frac{R}{H}h \qquad \text{...(ii)}$$

Mass of element, $dm=\rho(\pi r^2)dh$ $\qquad$ ...(iii)

From eq. (i), (ii) and (iii),

Area of element, $dA=2\pi r dl=2\pi r\dfrac{dh}{\cos\theta}$

Mass of element, $dm=\dfrac{2Mh\tan dh}{R\sqrt{R^2+H^2}\cos\theta}$

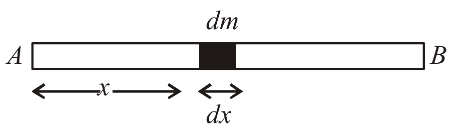
(here, $r=h\tan\theta$)

$$I=\int dI=\int_0^H dm(r^2)=\int_0^H \rho(\pi r^2)dh\left(\frac{R}{H}\cdot h\right)^2$$

$$=\int_0^H \rho\left(\pi\left(\frac{R}{H}\cdot h\right)^4\right)dh$$

Solving we get, $I=\dfrac{MR^2}{2}$

2. **(b)**



Mass of the small element of the rod

$dm = \lambda \cdot dx$

Moment of inertia of small element,

$$dI = dm \cdot x^2 = \lambda_0 \left(1+\frac{x}{L}\right) \cdot x^2 \ dx$$

Moment of inertia of the complete rod can be obtained by integration

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx$$

$$= \lambda_0 \left|\frac{x^3}{3} + \frac{x^4}{4L}\right|_0^L = \lambda_0 \left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$\Rightarrow I = \frac{7\lambda_0 L^3}{12} \qquad \text{...(i)}$$

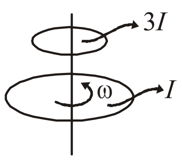
Mass of the thin rod,

$$M = \int_0^L \lambda \, dx = \int_0^L \lambda_0 \left(1+\frac{x}{L}\right) dx = \frac{3\lambda_0 L}{2}$$

$$\therefore \lambda_0 = \frac{2M}{3L}$$

$$\therefore I = \frac{7}{12}\left(\frac{2M}{3L}\right) L^3 \Rightarrow I = \frac{7}{18} ML^2$$

3. **(d)** By angular momentum conservation, $L_c = L_f$



$$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \frac{\omega}{4}$$

$$(KE)_i = \frac{1}{2} I\omega^2$$

$$(KE)_f = \frac{1}{2}(3I + I)\omega'^2$$

$$= \frac{1}{2} \times (4I) \times \left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$$

$$\Delta KE = \frac{1}{2} I\omega^2 - \frac{1}{8} I\omega^2 = \frac{3}{8} I\omega^2$$

$$\therefore \text{ Fractional loss in K.E. } = \frac{\Delta KE}{KE_{l_i}} = \frac{\frac{3}{8} I\omega^2}{\frac{1}{2} I\omega^2} = \frac{3}{4}.$$

4. **(c)** Let p be the density of the discs and t is the thickness of discs.

Moment of inertia of disc is given by

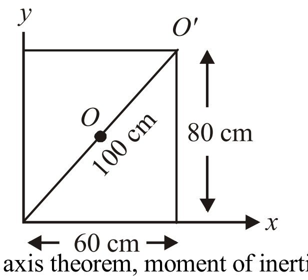
$$I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$$

$I \propto R^4$ (As ρ and t are same)

$$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4 \Rightarrow \frac{16}{1} = \alpha^4 \Rightarrow \alpha = 2$$

5. **(b)** Moment of inertia of rectangular sheet about an axis passing through O,

$$I_O = \frac{M}{12}(a^2 + b^2) = \frac{M}{12}[(80)^2 + (60)^2]$$

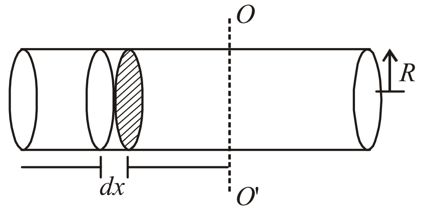


From the parallel axis theorem, moment of inertia about O',

$$I_{O'} = I_O + M(50)^2$$

$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(80^2 + 60^2)}{\frac{M}{12}(80^2 + 60^2) + M(50)^2} = \frac{1}{4}$$

6. **(c)** Let there be a cylinder of mass m length L and radius R. Now, take elementary disc of radius R and thickness dx at a distance of x from axis OO' then moment of inertia about OO' of this element.



$$dI = \frac{dmR^2}{4} + dmx^2$$

$$\Rightarrow I = \int dI = \int \frac{dmR^2}{4} + \int_{n=L/2}^{n=-L/2} \frac{M}{L} dx \times x^2$$

$$\text{Given : } I = \frac{MR^2}{4} + \frac{ML^2}{12}$$

$$\Rightarrow I = \frac{M}{4} \times \frac{V}{\pi L} + \frac{ML^2}{12} \Rightarrow I = \frac{MV}{4\pi L} + \frac{ML^2}{12}$$

$$\frac{dI}{dL} = -\frac{mV}{4\pi L^2} + \frac{M \times 2L}{12} = 0$$

$$\Rightarrow V = \frac{2}{3}\pi L^3 \Rightarrow \pi R^2 L = \frac{2}{3}\pi L^3$$

$$\therefore \frac{L}{R} = \sqrt{\frac{3}{2}}$$

7. **(b)** Initial angular momentum $= I_1\omega_1 + I_2\omega_2$

Let ω be angular speed of the combined system.

Final angular momentum $= I_1\omega + I_2\omega$

According to conservation of angular momentum

$$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$$

$$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{20}{3}$$

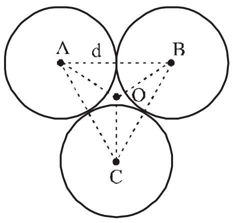
Final rotational kinetic energy

$$K_f = \frac{1}{2} I_1\omega^2 + \frac{1}{2} I_2\omega^2 = \frac{1}{2}(0.1 + 0.2) \times \left(\frac{20}{3}\right)^2$$

$$\Rightarrow K_f = \frac{20}{3} \text{ J}$$

8. **(a)** Moment of inertia,

$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$



and $AO = \frac{d}{\sqrt{3}}$

Moment of inertia about 'O'

$$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$$

$$\Rightarrow I_0 = \frac{13}{10}Md^2$$

And $I_A = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + Md^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2$

$$\Rightarrow I_A = \frac{23}{10}Md^2$$

$$\therefore \frac{I_O}{I_A} = \frac{\frac{13}{10}Md^2}{\frac{23}{10}Md^2} = \frac{13}{23}$$

9. **(a)** Using principal of conservation of energy

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2 \quad (\because v = \omega R)$$

$$\Rightarrow (m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$$

$$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

10. **(a)** When the bob covered a distance 'h'

Using $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\mathrm{m}(\omega \mathrm{r})^2 + \frac{1}{2} \times \frac{mr^2}{2} \times \omega^2 \quad (\because v = \omega r \text{ no slipping})$$

$$\Rightarrow mgh = \frac{3}{4}m\omega^2 r^2$$

$$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

11. **(c)** Moment inertia of the rod passing through a point $\frac{\ell}{4}$ away from the centre of the rod

$I = Ig + \mathrm{m}\ell^2$

$$\Rightarrow I = \frac{MI^2}{12} + M \times \left(\frac{I^2}{16}\right) = \frac{7MI^2}{48}$$

Using $I = MK^2 = \frac{7MI^2}{48}$ (K = radius of gyration)

$$\Rightarrow K = \sqrt{\frac{7}{48}}I$$

12. **(a)** Given,
mass per unit area of circular disc, $\sigma = A + Br$
Area of the ring = $2\pi rdr$
Mass of the ring, $dm = \sigma 2\pi rdr$
Moment of inertia,

$$I = \int dmr^2 = \int \sigma 2\pi rdr.r^2$$

$$\Rightarrow I = 2\pi\int_0^a (A + Br)r^3 dr = 2\pi\left[\frac{Aa^4}{4} + \frac{Ba^5}{5}\right]$$

$$\Rightarrow I = 2\pi a^4\left[\frac{A}{4} + \frac{Ba}{5}\right]$$

13. **(c)** $I = \int_a^b (dm)r^2$

$$= \int_a^b \left(\frac{\sigma_0}{r} \times 2\pi r\, dr\right)r^2 = \frac{2\pi\sigma_0}{3}\left| r^3 \right|_a^b$$

$$= \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

Mass of the disc,

$$m = \int_a^b \frac{\sigma_0}{r} \times 2\pi r\, dr = 2\pi\sigma_0(b - a)$$

Radius of gyration,

$$k = \sqrt{\frac{I}{m}}$$

$$= \sqrt{\frac{(2\pi\sigma_0/3)(b^3 - a^3)}{2\pi\sigma_0(b - a)}} = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

14. **(d)** As no external torque is acting so angular momentum should be conserved
$(\mathrm{I}_1 + \mathrm{I}_2)\,\omega = \mathrm{I}_1\omega_1 + \mathrm{I}_2\omega_2$ [ω_c = common angular velocity of the system, when discs are in contact]

$$\omega_c = \frac{\mathrm{I}_1\omega_1 + \frac{\mathrm{I}_1\omega_1}{4}}{\mathrm{I}_1 + \frac{\mathrm{I}_1}{2}}\left(\frac{5}{4} \times \frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$$\mathrm{E_f} - \mathrm{E_i} = \frac{1}{2}(\mathrm{I}_1 + \mathrm{I}_2)\omega_c^2 - \frac{1}{2}\mathrm{I}_1\omega_1^2 - \frac{1}{2}\mathrm{I}_2\omega_2^2$$

Put $\mathrm{I}_2 = \mathrm{I}_1/2$ and $\omega_c = \frac{5\omega_1}{6}$ $5\omega_1/6$

We get :

$$\mathrm{E_f} - \mathrm{E_i} = -\frac{\mathrm{I}_1\omega_1^2}{24}$$

15. **(b)** As from the question density (σ) = kr^2

Mass of disc $\mathrm{M} = \int_0^R (\mathrm{kr}^2)2\pi \mathrm{rdr} = 2\pi \mathrm{k}\frac{\mathrm{R}^4}{4} = \frac{\pi \mathrm{kR}^4}{2}$

$$\Rightarrow \mathrm{k} = \frac{2\mathrm{M}}{\pi \mathrm{R}^4} \quad \text{....(i)}$$

$\therefore$ Moment of inertia about the axis of the disc.

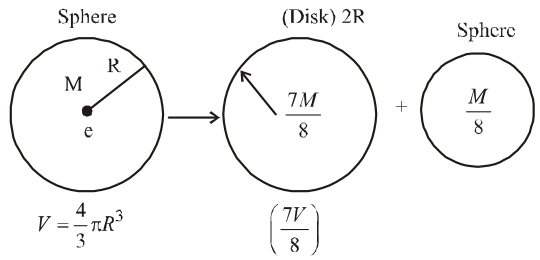
$$I = \int dI = \int (dm) r^2 = \int \sigma dA r^2$$

$$= \int (kr^2)(2\pi r dr) r^2$$

$$= \int_0^R 2\pi k\, r^5 dr = \frac{\pi k R^6}{3} = \frac{\pi \times \left(\frac{2M}{\pi R^4}\right) \times R^6}{3} = \frac{2}{3} MR^2$$

[putting value of k from eqn ....(i)]

16. (b)



$$I_1 = \left(\frac{7M}{8}\right)(2R)^2 \frac{1}{2} = \left(\frac{7}{16} \times 4\right) MR^2 = \frac{14}{8} mR^2$$

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right) r^2 \quad \Rightarrow I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$$

$$\left[\begin{array}{l} \frac{4}{3}\pi r^3 \rho = \frac{1}{8}\frac{4}{3}\pi R^3 \times \rho \\ \Rightarrow r = R/2 \end{array}\right]$$

$$\frac{I_1}{I_2} = \frac{14 \times 80}{8} = 140$$

17. (d) $\frac{1}{2} I\omega^2 = kQ^2$

or $\omega = \left(\sqrt{\frac{2k}{I}}\right) Q$

or $\alpha = \frac{d\omega}{\pi} = \sqrt{\frac{2K}{I}}\left(\frac{dQ}{dt}\right) = \left(\sqrt{\frac{2k}{I}}\right)\omega$

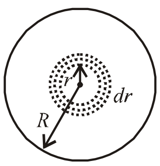
$$= \left(\sqrt{\frac{2k}{I}}\right)\left(\sqrt{\frac{2k}{I}}\right)\theta = \frac{2k\theta}{I}$$

18. (c) $I_i \omega_i = I_f \omega_f$

or $\left(\frac{ML^2}{12}\right)\omega_0 = \left(\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right)\omega_f$

$$\therefore \omega_f = \left(\frac{M\omega_0}{M + 6m}\right)$$

19. (c) Taking a circular ring of radius r and thickness dr as a mass element, so total mass,



$$M = \int_0^R \rho_0 r \times 2\pi r dr = \frac{2\pi\rho_0 R^3}{3}$$

$$I_C = \int_0^R \rho_0 r \times 2\pi r dr \times r^2 = \frac{2\pi\rho_0 R^5}{5}$$

Using parallel axis theorem

$$\therefore \; I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3} + \frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$$

$$= \frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right] R^2 = \frac{8}{5} MR^2$$

20. (b)

21. (d) According to parallel axes theorem

$$I = \frac{2}{5} mR^2 + mx^2$$

Hence graph (d) correctly depicts I vs x.

22. (a) Let mass of the larger triangle = M

Side of larger triangle = ℓ

Moment of inertia of larger triangle = ma^2

Mass of smaller triangle = $\frac{M}{4}$

Length of smaller triangle = $\frac{\ell}{2}$

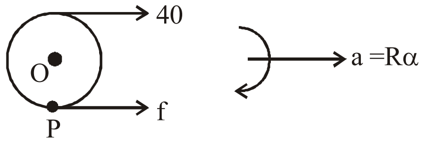
Moment of inertia of removed triangle $= \frac{M}{4}\left(\frac{a}{2}\right)^2$

$$\therefore \frac{I_{removed}}{I_{original}} = \frac{\frac{M}{4}}{M} \cdot \frac{\left(\frac{a}{2}\right)^2}{(a)^2}$$

$$I_{removed} = \frac{I_0}{16}$$

So, $I = I_0 - \frac{I_0}{16} = \frac{15 I_0}{16}$

23. (b)



From newton's second law

$40 + f = m(R\alpha)$ .....(i)

Taking torque about 0 we get

$40 \times R - f \times R = I\alpha$

$40 \times R - f \times R = mR^2 \alpha$

$40 - f = mR\,\alpha$ ...(ii)

Solving equation (i) and (ii)

$$\alpha = \frac{40}{mR} = 16\, rad / s^2$$

24. (b) Moment of inertia of disc D_1 about OO′ $= I_1 = \frac{MR^2}{2}$

M.O.I of D_2 about OO′

$$= I_2 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$$

M.O.I of D_3 about OO′

$$= I_3 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$$

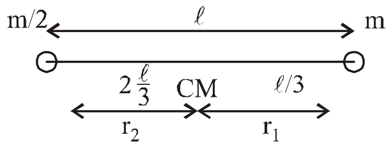
so, resultant M.O.I about OO′ is $I = I_1 + I_2 + I_3$

$$\Rightarrow I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3\,MR^2$$

25. **(c)** As we know, $\omega = \sqrt{\dfrac{k}{I}}$

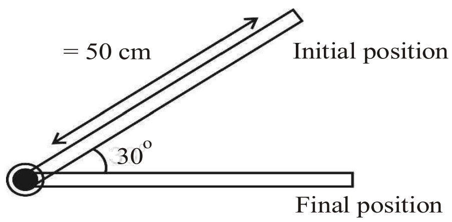
$$\omega = \sqrt{\frac{3k}{m\ell^2}} \quad \left[\because\ I_{rod} = \frac{1}{3}m\ell^2\right]$$



Tension when it passes through the mean position,

$$= m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

26. **(d)**



By the low of conservation of energy,
P.E. of rod = Rotational K.E.

$$mg\frac{\ell}{2}\text{Sin}\,\theta = \frac{1}{2}I\omega^2$$

$$\Rightarrow mg\frac{\ell}{2}\text{Sin}\,30° = \frac{1}{2}\frac{m\ell^2}{3}\omega^2 \quad \Rightarrow mg\frac{\ell}{2}\times\frac{1}{2} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$

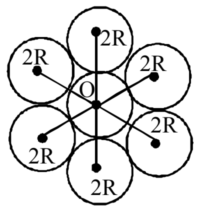
For complete length of rod,

$$\omega = \sqrt{3g/2(2\ell)} = \sqrt{\frac{30}{2}}\ \text{rod s}^{-1}$$

27. **(d)** Using parallel axes theorem, moment of inertia about 'O'

$$I_o = I_{cm} + md^2$$

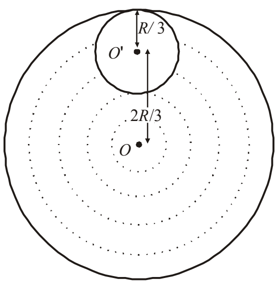
$$= \frac{7MR^2}{2} + 6(M\times(2R)^2) = \frac{55MR^2}{2}$$



Again, moment of inertia about point P, $I_p = I_o + md^2$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

28. **(a)** Let σ be the mass per unit area.



The total mass of the disc
$= \sigma\times\pi R^2 = 9M$
The mass of the circular disc cut

$$= \sigma\times\pi\left(\frac{R}{3}\right)^2 = \sigma\times\frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it.
Moment of inertia (I_1) of the complete disc $= \dfrac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.
M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2$$

$\therefore$ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2 + M\times\left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

$\therefore$ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc is

$$I = I_1 + I_2$$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2 + M\times\left(\frac{2R}{3}\right)^2\right]$$

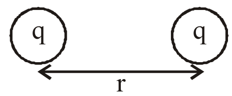
$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

29. **(c)** As we know, moment of inertia of a disc about an axis passing through C.G. and perpendicular to its plane,

$$I_z = \frac{mR^2}{2}$$

Moment of inertia of a disc about a tangential axis perpendicular to its own plane,

$$I_z' = \frac{3}{2}mR^2$$

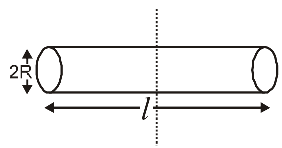


$$\therefore \quad I_z/I_z' = \frac{mR^2}{2}\bigg/\frac{3mR^2}{2} = 1/3$$

30. **(c)** As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3}\right]$$



$$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3}\right] \quad \Rightarrow \frac{dI}{dl} = \frac{m}{4}\left[\frac{-V}{\pi l^2} + \frac{2l}{3}\right] = 0$$

$$\frac{V}{\pi l^2} = \frac{2l}{3} \quad \Rightarrow V = \frac{2\pi l^3}{3}$$

$$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2} \text{ or, } \frac{l}{R} = \sqrt{\frac{3}{2}}$$

31. **(b)** Moment of Inertia of complete disc about 'O' point

$I_{total} = \frac{MR^2}{2}$

Radius of removed disc = R/4

$\therefore$ Mass of removed disc = M/16

[As $M \propto R^2$]

M.I of removed disc about its own axis (O')

$= \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{MR^2}{512}$

M.I of removed disc about O

$I_{removed\ disc} = I_{cm} + mx^2$

$= \frac{MR^2}{512} + \frac{M}{16}\left(\frac{3R}{4}\right)^2 = \frac{19\,MR^2}{512}$

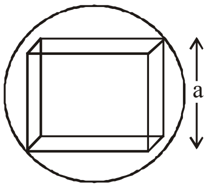
M.I of remaining disc

$I_{remaining} = \frac{MR^2}{2} - \frac{19}{512}MR^2 = \frac{237}{512}MR^2$

32. **(a)** Here $a = \frac{2}{\sqrt{3}}R$

Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.$



$M' = \frac{2M}{\sqrt{3}\pi}$

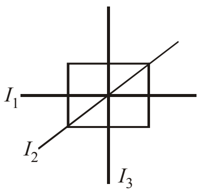
Moment of inertia of the cube about the given axis,

$I = \frac{M'a^2}{6}$

$= \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$

33. **(d)** For a thin uniform square sheet

$I_1 = I_2 = I_3 = \frac{ma^2}{12}$



34. **(c)** Kinetic energy $_{(rotational)}$ $K_R = \frac{1}{2}I\omega^2$

Kinetic energy $_{(translational)}$ $K_T = \frac{1}{2}Mv^2$ $(v = R\omega)$

M.I.$_{(initial)}$ $I_{ring} = MR^2$; $\omega_{initial} = \omega$

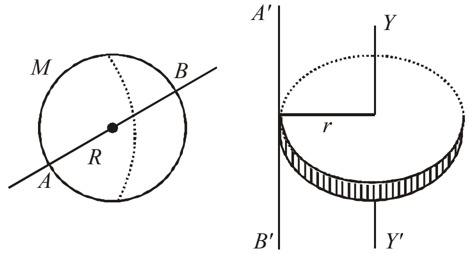
M.I.$_{(new)}$ $I'_{(system)} = MR^2 + 2mR^2$

$\omega'_{(system)} = \frac{M\omega}{M + 2m}$

Solving we get loss in K.E.

$= \frac{Mm}{(M + 2m)}\omega^2 R^2$

35. **(b)**



For solid sphere moment of inertia about diameter

$I_{AB} = \frac{2}{5}MR^2 = I$

For solid disc

$I_{A'B'} = I_{YY'} + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$

$I_{AB} = I_{A'B'}$ (given)

$\therefore \frac{2}{5}MR^2 = \frac{3}{2}Mr^2 \Rightarrow r = \frac{2}{\sqrt{15}}R$

36. **(a)** Let σ be the mass per unit area of the disc.

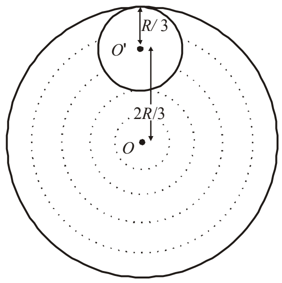
$\therefore$ Total mass of the disc

$= \sigma \times \pi R^2 = 9M$

Mass of the circular disc, cut

$= \sigma \times \pi\left(\frac{R}{3}\right)^2$

$= \sigma \times \frac{\pi R^2}{9} = M$



Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$

Using perpendicular axis theorem

$\therefore$ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$

$\therefore$ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc

$I = I_1 - I_2$

$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$

or, $I = \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$

37. **(a)** Mass of disc = 4M

I disc = $\frac{1}{2}MR^2$

$\therefore$ Moment of inertia of one quarter of disc

$= \frac{1}{4}\left[\frac{1}{2}(4M)R^2\right] = \frac{1}{2}MR^2$

38. **(d)** About the diameter of the circular loop (ring)

$$I=\frac{1}{2}MR^2$$

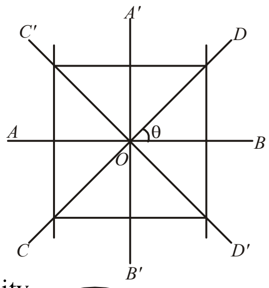
Using parallel axis theorem
Moment of inertia of the loop about XX' axis

$$I_{XX'}=\frac{MR^2}{2}+MR^2=\frac{3}{2}MR^2$$

Here mass $M=L\rho$ and radius $R=\frac{L}{2\pi}$;

$$\therefore\quad I_{XX'}=\frac{3}{2}(L\rho)\left(\frac{L}{2\pi}\right)^2=\frac{3L^3\rho}{8\pi^2}$$

39. **(a)** From symmetry $I_{AB}=I_{A'B'}$ and $I_{CD}=I_{C'D'}$
$A'B' \perp AB$ and $C'D' \perp CD$



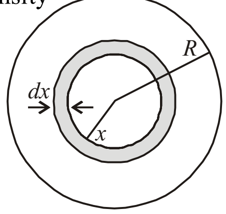
From theorem of perpendicular axes,

$$I_{zz}=I_{AB}+I_{A'B'}=I_{CD}+I_{C'D'}$$

$$\Rightarrow\quad 2I_{AB}=2I_{CD}$$

$$\therefore\quad I_{AB}=I_{CD}=I$$

40. **(6)** For solid sphere A, density



$$\rho_A(r)=k\left(\frac{r}{R}\right)$$

And $P_B(r)=K\left(\frac{r}{R}\right)^s$

Consider a spherical shell of radius x and thickness dx.
Mass of the shell, dm = density × volume

$$=\left(k\frac{x}{R}\right)(4\pi x^2\,dx)$$

So, moment of inertia of shell about its diameter,

$$dI=\frac{2}{3}(dm)x^2 \qquad =\frac{2}{3}\left(k\frac{x}{R}\right)(4\pi x^2dx)x^2$$

$$=\left(\frac{8\pi}{3}\frac{k}{R}\right)x^5dx$$

$\therefore$ Moment of inertia of the sphere A, $I_A=\int_0^R dI$

$$=\frac{8\pi k}{3R}\int_0^R x^5dx=\frac{8\pi k}{3R}\left[\frac{x^6}{6}\right]_0^R$$

$$\therefore\quad I_A=\left(\frac{8\pi k}{18}\right)R^5 \qquad \text{(i)}$$

Similarly, for sphere B

$$I_B=\frac{8\pi k}{3R^5}\int_0^R x^9dx=\left(\frac{8\pi k}{3R^5}\right)\left[\frac{x^{10}}{10}\right]_0^R$$

$$\therefore\quad I_B=\frac{8\pi k}{30}R^5 \qquad \text{(ii)}$$

From eqns. (i) and (ii)

$$\frac{I_B}{I_A}=\frac{18}{30}=\frac{6}{10}=\frac{n}{10}\quad \therefore n=6$$

41. **(3)** Let σ be the surface mass density.
Moment of inertia of the lamina about axes passing through 'O'

$$I_O=\frac{1}{2}\sigma[\pi(2R)^2]\times(2R)^2-\left[\frac{1}{2}(\sigma\pi R^2)^2+\sigma(\pi R^2)\times R^2\right]$$

$$=\frac{13}{2}\pi\sigma R^4$$

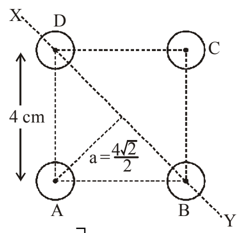
Moment of inertia of the lamina about axes passing through 'P'

$$I_P = 8\pi\sigma R^4+\sigma\pi(2R)^2\times(2R)^2$$
$$\left[\frac{1}{2}\sigma(\pi R^2)R^2+\sigma(\pi R^2)\left(\sqrt{(2R)^2+R^2}\right)^2\right]$$

$$= 24\pi\sigma R^4-5.5\sigma\pi R^4=18.5\pi\sigma R^4$$

$$\therefore\quad \frac{I_P}{I_O}=\frac{18.5\pi\sigma R^4}{\frac{13}{2}\pi\sigma R^4}=\frac{37}{13}\approx 3$$

42. **(9)** Let the four spheres be A, B, C, & D
$m_A=m_B=m_C=m_D=0.5$ kg
Moment of inertia of the system about the diagonal of the square



$$I_{XY}=I_1+I_B+I_C+I_D=2I_A+2I_B$$

$$=2\left[\frac{2}{5}MR^2+Ma^2\right]+2\left[\frac{2}{5}MR^2\right]$$

$$=4\times\frac{2}{5}MR^2+2Ma^2=M\left[\frac{8}{5}R^2+2(a)^2\right]$$

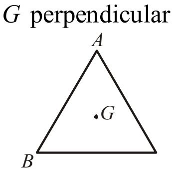
$$=0.5\left[\frac{8}{5}\times\left(\frac{\sqrt{5}}{2}\right)^2+2\times 8\right]\times 10^{-4}$$

$$=0.5[2+16]\times10^{-4}=9\times10^{-4}=N\times10^{-4}\text{ kg }m^2$$

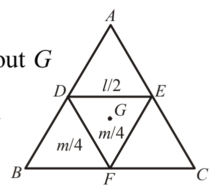
$\therefore\quad N=9$

43. **(11)** Let mass of triangular lamina = m, and length of side = l, then moment of inertia of lamina about an axis passing through centroid G perpendicular to the plane.

$$I_0\propto ml^2$$
$$I_0=kml^2$$



Let moment of inertia of $DEF=I_1$ about G



So, $I_1\propto\left(\frac{m}{4}\right)\left(\frac{l}{2}\right)^2\propto\frac{ml^2}{16}$ or $I_1=\frac{I_0}{16}$

Let $I_{ADE}=I_{BDF}=I_{EFC}=I_2$

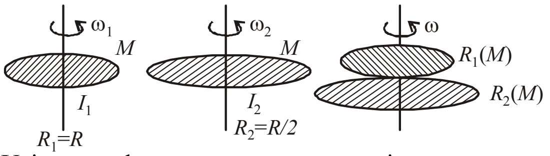
$$\therefore 3I_2+I_1=I_0\Rightarrow 3I_2+\frac{I_0}{16}=I_0\Rightarrow I_2=\frac{5I_0}{16}$$

Hence, moment of inertia of $DECB$ i.e., after removal part ADE

$$=2I_2+I_1=2\left(\frac{5I_0}{16}\right)+\left(\frac{I_0}{16}\right)=\frac{11I_0}{16}=\frac{NI_0}{16}$$

Therefore value of $N=11$.

44. (20) As we know moment of inertia disc, $I_{\text{disc}} = \frac{1}{2}MR^2$



Using angular momentum conservation

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\times\omega_f$$

$$\frac{MR^2}{2}\times\omega + 0 = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f \Rightarrow \omega_f = \frac{4}{5}\omega$$

Initial K.E., $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \frac{MR^2\omega^2}{4}$

Final K.E., $K_f = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\frac{16}{25}\omega^2 = \frac{MR^2\omega^2}{5}$

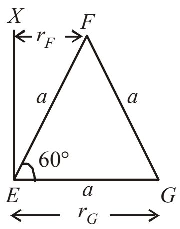
Percentage loss in kinetic energy % loss

$$= \frac{\frac{MR^2\omega^2}{4} - \frac{MR^2\omega^2}{5}}{\frac{MR^2\omega^2}{4}}\times 100 = 20\% = P\%$$

Hence, value of $P = 20$.

45. **(25)**

Moment of inertia of the system about axis XE.



$$I = I_E + I_F + I_G$$

$$\Rightarrow I = m(r_E)^2 + m(r_F)^2 + m(r_G)^2$$

$$\Rightarrow I = m\times 0^2 + m\left(\frac{a}{2}\right)^2 + ma^2 = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$$

$\therefore N = 25.$

46. **(15)** Here, length of bar, $l = 1$ m

angle, $\theta = 30°$

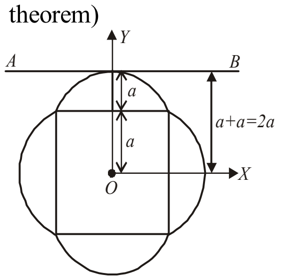
$$\Delta PE = \Delta KE \text{ or } mgh = \frac{1}{2}I\omega^2$$

$$\Rightarrow (mg)\frac{l}{2}\sin 30° = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 \Rightarrow mg\frac{l}{2}\times\frac{1}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{15} \text{ rad/s}$$

47. Assuming symmetric lamina to be in xy plane, $I_x = I_y$ By symmetry

$I_x + I_y = I_z$ (perpendicular-axis theorem)



$$I_x = I_y = \frac{I_z}{2} = 0.8\,Ma^2$$

$\because\ I_z = 1.6\,Ma^2$ (given)

Now, from to parallel-axes theorem,

$$I_{AB} = I_x + M(2a)^2$$
$$= 0.8\,Ma^2 + 4Ma^2$$
$$= 4.8\,Ma^2$$

48. **(a, b, c)** Since, $ABCD$ is a square lamina hence by symmetry $I_1 = I_2$ and $I_3 = I_4$

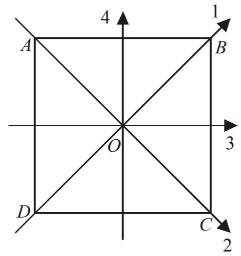
From perpendicular axes theorem,

Moment of inertia about an axis perpendicular to square plate and passing from centre, O

$I_o = I_1 + I_2 = I_3 + I_4$

or $I_o = 2I_2 = 2I_3 \quad \therefore I_2 = I_3$

$I_1 = I_2 = I_3 = I_4$



Therefore, I_o can be obtained by adding any two

i.e., $I_o = I_1 + I_2 = I_1 + I_3$
$= I_1 + I_4 = I_2 + I_3$
$= I_2 + I_4 = I_3 + I_4$

Topic-5 : Rolling Motion

1. **(a)** $K.E$ of the sphere = translational $K.E$ + rotational $K.E$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Where, I = moment of inertia,
ω = Angular, velocity of rotation
m = mass of the sphere
v = linear velocity of centre of mass of sphere

$\because$ Moment of inertia of sphere $I = \frac{2}{5}mR^2$

$$\therefore\ K.E = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\times\omega^2$$

$$\Rightarrow K.E = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\times\left(\frac{v}{R}\right)^2 \left(\because \omega = \frac{v}{R}\right)$$

$$\Rightarrow KE = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow KE = \frac{1}{2}mR^2\times\frac{7}{5}\times\frac{v^2}{R^2} = \frac{7}{10}\times\frac{1}{2}\times\frac{25}{10^4}$$

$$\Rightarrow KE = \frac{35}{4}\times 10^{-4} \text{ joule}$$

$\Rightarrow$ KE = 8.75×10^{-4} joule

2. **(b)** According to question, roller is moved 50 cm i.e., $V_{center}.\ t = 50$ cm

Radius of roller, R $= \frac{20}{2} = 10$ cm and of axle, r $= \frac{10}{2} = 5$ cm

For no slipping at the ground

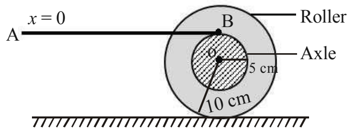
$V_{center} = \omega R$

$\therefore$ Velocity of scale = $(V_{center} + \omega r)$

$\therefore$ Distance moved by scale = $(V_{center} + \omega r)t$

$$= \left(V_{center} + \frac{V_{center}r}{R}\right)t \left(\because \omega = \frac{V}{R}\right)$$

$$= \frac{3V_{center}}{2}\cdot t = \frac{3}{2}\times 50 = 75 \text{ cm}$$



3. **(c)** For sphere,

$$\frac{1}{2}mv^2 + I\omega^2 = \frac{1}{2}mgh$$

$$\text{or}\quad \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = mgh \text{ or } h = \frac{7v^2}{10g}$$

For cylinder

$$\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{mR^2}{2}\right)=mgh' \quad \text{or} \quad h'=\frac{3v^2}{4g}$$

$$\therefore \quad \frac{h}{h'}=\frac{7v^2/10g}{3v^2/4g}=\frac{14}{15}$$

4. **(Bonus)** $mgh=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I_{cm}\omega^2$

$$=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2=\frac{1}{2}\left(m+\frac{I_{cm}}{R^2}\right)v_{cm}^2$$

For ring : mgh $=\frac{1}{2}\left(m+\frac{mR^2}{R^2}\right)v_{cm}^2$

$\therefore h=\frac{v_{cm}^2}{g}$.

For solid cylinder, $mgh=\frac{1}{2}\left(m+\frac{mR^2}{2R^2}\right)v_{cm}^2$

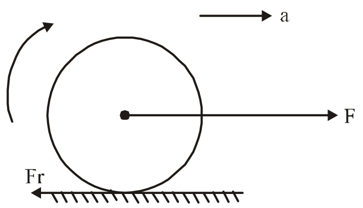
$\therefore h=\frac{3v_{cm}^2}{4g}$

For sphere, $mgh=\frac{1}{2}\left(m+\frac{2}{5}\frac{mR^2}{R^2}\right)v_{cm}^2$

$\therefore h=\frac{7v_{cm}^2}{10g}$

Ratio of heights $1:\frac{3}{4}:\frac{7}{10}\Rightarrow 20:15:14$

5. **(d)**



$F-f_r=ma$ ...(i)

$f_rR=I\alpha=\frac{mR^2}{2}\alpha$ ...(ii)

for pure rolling

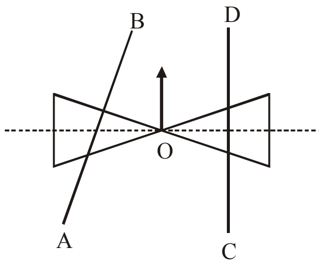
$a=\alpha R$ ...(iii)

from (1) (2) and (3)

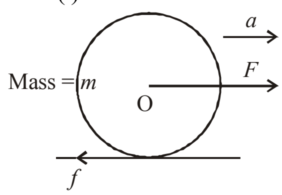
$$F-\frac{mR\alpha}{2}=m\alpha R, \quad F=\frac{3}{2}mR\alpha$$

$$\alpha=\frac{2F}{3mR}$$

6. **(c)** As shown in the diagram, the normal reaction of AB on roller will shift towards O.
This will lead to tending of the system of cones to turn left.



7. **(c)** From figure,
$ma=F-f$ ....(i)
And, torque $\tau=I\alpha$



$\frac{mR^2}{2}\alpha=fR$

$\frac{mR^2}{2}\frac{a}{R}=fR \quad \left[\because \alpha=\frac{a}{R}\right]$

$\frac{ma}{2}=f$ ...(ii)

Put this value in equation (i),

$ma=F-\frac{ma}{2}$ or $F=\frac{3ma}{2}$

8. **(d)** As we know,

Acceleration, $a=\frac{mg\sin\theta}{m+\frac{I}{r^2}}$

For cylinder, $a_c=\frac{M_c.g.\sin\theta_c}{M_c+\frac{1}{2}\frac{M_cR^2}{R^2}}=\frac{M_c.g.\sin\theta_c}{M_c+\frac{M_cR^2}{2R^2}}$

or, $a_c=\frac{2}{3}g\sin\theta_c$

For sphere,

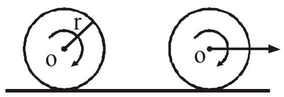
$a_s=\frac{M_sg\sin\theta_s}{M_s+\frac{I_s}{r^2}}=\frac{M_sg\sin\theta_s}{M_s+\frac{2}{5}\frac{MR^2}{R^2}}$

or, $a_s=\frac{5}{7}g\sin\theta_s$

given, $a_c=a_s$

i.e., $\frac{2}{3}g\sin\theta_c=\frac{5}{7}g\sin\theta_s \quad \therefore \quad \frac{\sin\theta_c}{\sin\theta_s}=\frac{\frac{5}{7}g}{\frac{2}{3}g}=\frac{15}{14}$

9. **(c)**



From conservation of angular momentum about any fix point on the surface,

$mr^2\omega_0=2mr^2\omega$

$\Rightarrow \omega=\omega_0/2 \Rightarrow v=\frac{\omega_0 r}{2} \quad [\because v=r\omega]$

10. **(d)** Applying energy conservation principle of

$$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=mg\left(\frac{3v^2}{4g}\right)$$

$$\therefore \quad \frac{1}{2}mv^2+\frac{1}{2}I\frac{v^2}{R^2}=\frac{3}{4}mv^2 \quad [\because v=R\omega]$$

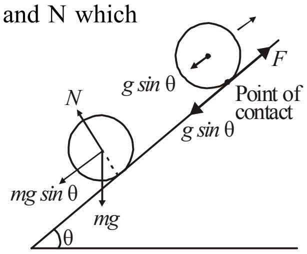
$$\therefore \quad \frac{1}{2}I\frac{v^2}{R^2}=\frac{3}{4}mv^2-\frac{1}{2}mv^2=\frac{1}{4}mv^2 \Rightarrow I=\frac{1}{2}mR^2$$

This is the moment of inertia of the disc hence the object is disc.

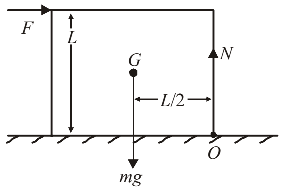
11. **(b)** In pure rolling, the point of contact is the instantaneous centre of rotation of all the particles of the disc. On applying $v=r\omega$
We find ω is same for all the particles then $v\propto r$.

$r_Q>r_C>r_P \quad \therefore \quad r_Q>r_C>r_P$

12. **(b)** Cylinder to be moving on a frictionless surface. In both the cases roll up roll down the acceleration of the centre of mass of the cylinder is g sin θ. Also no torque about the centre of cylinder is acting on the cylinder since we assumed the surface to be frictionless and the forces acting on the cylinder is mg and N which pass through the centre of cylinder. Therefore the net movement of the point of contact in both the cases is in the downward direction and hence the frictional force always acts in the upward direction.



13. **(c)** Here, due to applied force normal reaction shifts to one corner. At this situation, the cubical block starts topping about point O.



About point O torque

$$F \times L = mg \times \frac{L}{2} \Rightarrow F = \frac{mg}{2}$$

Hence minimum force required to topple the block,

$$F = \frac{mg}{2}$$

14. **(7)** $K.E_{Total}$ of a rolling disc $K.E_{trans} + K.E_{rot}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$K.E_{Total} = \frac{3}{4}mv^2$$

For surface AB

$k.E_i$ + loss in gravitational potential energy = $K.E_f$

$$\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}mV_B^2 \quad ...(i)$$

For surface CD

$$\frac{3}{4}m(v_2)^2 + mg(27) = \frac{3}{4}mV_D^2 \quad ...(ii)$$

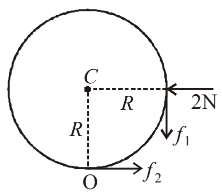
$\because V_B = V_D$. $\therefore$ from eq. (i) and (ii)

$$\frac{3}{4}m(3)^2 + mg \times 30 = \frac{3}{4}m(v_2)^2 + mg \times 27$$

$\therefore V_2 = 7\ m/s$

15. **(4)**

The stick applies (2N) force so, point of contact O of the ring with ground tends to slide. But the frictional force f_2 does not allow this and creates a torque about 'c' which starts rolling the ring. Between the ring & the stick a friction force f_1 also acts.



$$F - f_2 = ma$$

$$2 - f_2 = 2 \times 0.3 \quad \therefore f_2 = 1.4\text{ N}$$

Applying $\tau = I\alpha$ about C

$$(f_2 - f_1)R = I\alpha = I\frac{a}{R}$$

$[\because a = R\alpha]$

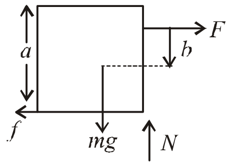
$$\Rightarrow [1.4 - \mu \times 2] \times 0.5 = 2 \times (0.5)^2 \times \frac{0.3}{0.5} \quad [\because I = MR^2]$$

$$\therefore \mu = 0.4 = \frac{4}{10} = \frac{P}{10} \quad \therefore P = 4$$

16. **(50)** For the box to be slide

$F = \mu mg = 0.4$ mg

For no toppling



$$F\left(\frac{a}{2}+b\right) \le mg\frac{a}{2}$$

$$\Rightarrow 0.4\,mg\left(\frac{a}{2}+b\right) \le mg\frac{a}{2} \Rightarrow 0.2\,a + 0.4\,b \le 0.5\,a$$

$$\Rightarrow \frac{b}{a} \le \frac{3}{4}$$

i.e. $b \le 0.75\,a$ but this is not possible.

As the maximum value of b can be equal to $0.5a$.

$$\Rightarrow \frac{100b}{a} = 50$$

17. **(0.75)**

The time taken to reach the ground or descend

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$$

For ring $t_1 = \frac{1}{\sin 60°}\sqrt{\frac{2h}{g}(1+1)} = \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} \quad \because \frac{K^2}{R^2} = 1$

For disc $t_2 = \frac{1}{\sin 60°}\sqrt{\frac{2h}{g}\left(1+\frac{1}{2}\right)} = \sqrt{\frac{4h}{g}} \quad \because \frac{K^2}{R^2} = \frac{1}{2}$

Given $t_1 - t_2 = \frac{2-\sqrt{3}}{\sqrt{10}}$

$$\therefore \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}} \Rightarrow 2\sqrt{h} - \sqrt{3h} = \sqrt{3} - \frac{3}{2}$$

$$\Rightarrow \sqrt{h}\,(2 - 1.732) = 1.732 - 1.5 \Rightarrow \sqrt{h} = \frac{0.232}{0.268}$$

$\therefore h \approx 0.75$ m (Height of the top of the in clined plane)

18. **False.** Total Kinetic energy of the ring

$$E_1 = (K.E.)_{\text{Rotation}} + (K.E.)_{\text{Translational}}$$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_c^2$$

$$= \frac{1}{2} \times mr^2\omega^2 + \frac{1}{2}m(r\omega)^2 \quad (\because I = mr^2,\ v_c = r\omega)$$

$$E_1 = mr^2\omega^2$$

Total kinetic energy of the cylinder

$$E_2 = (K.E.)_{\text{Rotation}} + (K.E.)_{\text{Translational}}$$

$$= \frac{1}{2}I'\omega'^2 + \frac{1}{2}Mv_c'^2$$

$$= \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega'^2 + \frac{1}{2}M(r\omega')^2$$

$$E_2 = \frac{3}{4}Mr^2\omega'^2$$

Equating $E_1 = E_2$

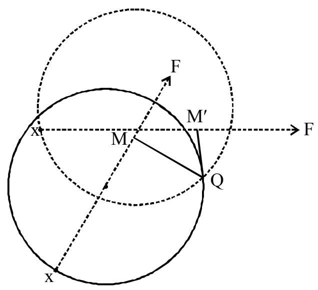
$$mr^2\omega^2 = \frac{3}{4}Mr^2\omega'^2$$

$$\Rightarrow \frac{\omega'^2}{\omega^2} = \frac{4m}{3M} = \frac{4}{3} \times \frac{0.3}{0.4} = 1 \Rightarrow \omega' = \omega$$

Hence both ring and cylinder will reach at the same time.

19. (c) If the force (F) is applied at P tangential than the τ remains constant and $\tau = F \times 2R$.

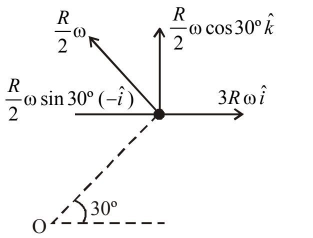
If force is applied normal to X, then as the wheels climbs, then the perpendicular distance of force from Q will go on changing initially the perpendicular is QM, later it becomes QM'.



If the force (F) is applied normal to the circumference at point P then τ is zero.

If the force (F) is applied tangentially at point S then $\tau = F \times R$ and the wheel climbs.

20. (a, b) Velocity at centre 'O' $\therefore\ \vec{v}_0 = 3R\omega\,\hat{i}$

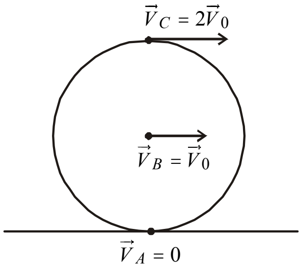


$$\vec{V}_P = 3R\omega\hat{i} - \frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 30°\hat{k}$$

$$\therefore\quad \vec{V}_P = \left[3R_\omega\,\hat{i} - \frac{R_\omega}{4}\hat{i}\right] + \frac{\sqrt{3}R_\omega}{4}\hat{k}$$

$$\text{or,}\quad \vec{V}_P = \frac{11}{4}R_\omega\,\hat{i} + \frac{\sqrt{3}}{4}R_\omega\,\hat{k}$$

21. (b, c) Here, $\vec{V}_A = 0$



$\vec{V}_B = \vec{V}_0$

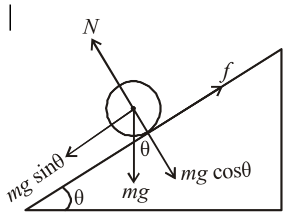
$\vec{V}_C = 2\vec{V}_0$

$\therefore \vec{V}_C - \vec{V}_B$

$= \vec{V}_B - 2\vec{V}_A$

and $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

22. (c, d) The sliding friction facts in the opposite direction of mg sin θ to oppose the relative motion. Because of frictional force the cylinder rolls.



Hence frictional force aids rotation but hinders translational motion.

Applying $F_{net} = ma$ along the direction of inclined plane,

$mg\sin\theta - f = ma_c$,

where a_c = acceleration of centre of mass of the cylinder

$\therefore\quad f = mg\sin\theta - ma_c$ ... (i)

$$\text{But } a_c = \frac{g\sin\theta}{1+\frac{I_c}{mR^2}} = \frac{g\sin\theta}{1+\frac{mR^2/2}{mR^2}} = \frac{2}{3}g\sin\theta$$

$$f = \frac{mg\sin\theta}{3}$$

Clearly, if θ is reduced, frictional force is reduced.

23. (d) In case of pure rolling on inclined plane acceleration of a body

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

For hollow cylinder $\frac{I}{MR^2} = \frac{MR^2}{MR^2} = 1$

For solid cylinder $\frac{I}{MR^2} = \frac{\frac{1}{2}MR^2}{MR^2} = \frac{1}{2}$

Hence acceleration of solid cylinder is more than hollow cylinder and therefore solid cylinder will reach the bottom of the inclined plane first.

In the case of rolling there will be no work done by friction. Therefore total mechanical energy remains conserved.

$\therefore\quad (KE)_{solid} = (KE)_{follow}$ = decrease in PE = mgh

24. The man applies a force. Its horizontal component F pushes the plank. Hence the point of contact of the plank with the cylinder will tend to move towards right the frictional force f_1 will act towards left on the plank.

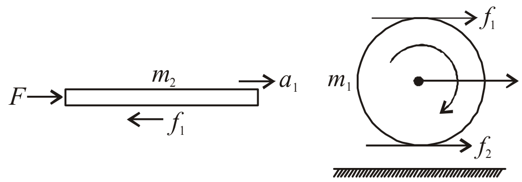
Let f_1 = friction between plank and cylinder

f_2 = friction between cylinder and ground

a_1 = acceleration of plank

a_2 = acceleration of centre of mass of cylinder

and α = angular acceleration of cylinder about its CM.



Since, there is no slipping anywhere

$\therefore\quad a_1 = 2a_2$ ... (i)

$$a_1 = \frac{F - f_1}{m_2} \quad \text{... (ii)}$$

$$a_2 = \frac{f_1 - f_2}{m_1} \quad \text{... (iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{\frac{1}{2}m_1R^2}$$

$$\alpha = \frac{2(f_1 - f_2)}{m_1R} \quad \text{... (iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad \text{... (v)}$$

(a) Solving eqs. (i) to (v), we get

Acceleration of plank, $a_1 = \frac{8F}{3m_1 + 3m_2}$

And Acceleration of centre of mass of cylinder, $a_2 = \frac{4F}{3m_1 + 8m_2}$

(b) Friction between plank and cylinder $f_1 = \frac{3m_1F}{3m_1 + 8m_2}$

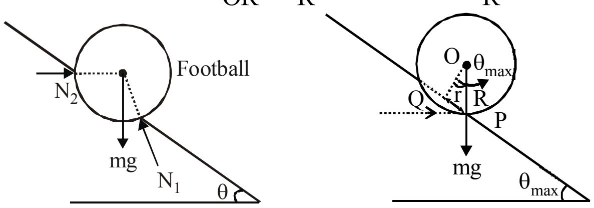
Friction between cylinder and ground $f_2 = \frac{m_1F}{3m_1 + 8m_2}$

Since, all quantities are positive, they are correctly shown in figures.

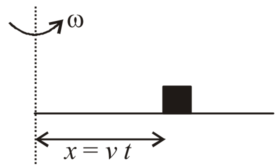
Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. **(a)** The maximum value of q i.e, q_{max}, the football is about to roll, then $N_2 = 0$ and all the forces (mg and N_1) must pass through contact point 'P'

$\therefore \cos(90° - \theta_{max}) = \frac{OQ}{OR} = \frac{r}{R}$ or, $\sin\theta_{max} = \frac{r}{R}$



2. **(b)** Angular momentum, $|\mathbf{L}|$ or $L = I\omega$ (about axis of rod)
Moment of inertia of the rod-insect system.
$I = I_{rod} + mx^2 = I_{rod} + mv^2t^2$
Here, m = mass of insect
$\therefore \quad L = (I_{rod} + mv^2t^2)\omega$



Now $|\tau| = \frac{dL}{dt} = (2mv^2t\omega)$ or $|\tau| \propto t$
i.e. the graph is straight line passing through origin.
After time T, L = constant

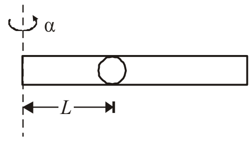
$\therefore \quad |\tau| \text{or} \frac{dL}{dt} = 0$

i.e., when the insect stops moving, L does not change and therefore T becomes constant.

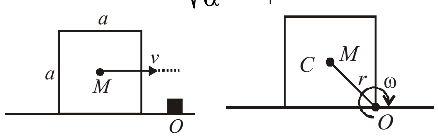
3. **(a)** When we are giving an angular acceleration (α) to the rod, the bead has instantaneous acceleration crinst $= L\alpha$. The bead has a tendency to move away from the centre. But due to the friction between the bead and the rod, this does not happen. If instantaneous angular velocity is ω then
Here, necessary frictional force is provided by frictional force
$mL\omega^2 = \mu(ma) \Rightarrow mL\omega^2 = \mu\, mL\alpha \Rightarrow \omega^2 = \mu\alpha$

Using $\omega = \omega_0 + \alpha t$,
$\Rightarrow \quad \omega = \alpha t$

$\therefore \quad \alpha^2t^2 = \mu\alpha \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$



4. **(a)**



$r^2 = \frac{a^2}{2}$ or $r = \sqrt{2}\frac{a}{2}$ By geometry

Net torque about point O is zero.
Hence, angular momentum (L) about O is conserved, or $L_i = L_f$

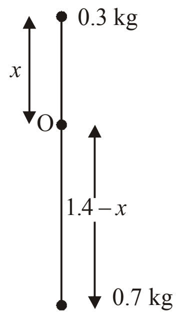
$$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{cm} + Mr^2)\omega$$

$$\omega = \left\{\frac{Ma^2}{6} + M\left(\frac{a^2}{2}\right)\right\}\omega = \frac{2}{3}Ma^2\omega = \frac{3v}{4a}$$

5. **(c)** As it is headon collision between two identical spheres, they will exchange their linear velocities. Since the spheres are smooth there will be no friction (no torque) and therefore there will be no transfer of angular momentum. Thus A, after collision will remain with its initial angular momentum. i.e., $\omega_A = \omega$ and $\omega_B = 0$

6. **(c)** For point mass $I = mr^2$
M.I. of the system about axis of rotation O
$I = I_1 + I_2 = 0.3x^2 + 0.7\,(1.4 - x)^2$
$= 0.3x^2 + 0.7\,(1.96 + x^2 - 2.8x)$
$= x^2 + 1.372 - 1.96x$

The work done in rotating the rod is converted into its rotational kinetic energy



$\therefore \quad W = \frac{1}{2}I\omega^2$

$= \frac{1}{2}[x^2 + 1.372 - 1.96x]\omega^2$

For work done to be minimum $\frac{dW}{dx} = 0$

$\therefore \quad \frac{d}{dx}\left[\left(x^2 + 1.372 - 1.96x\right)\right]\frac{w^2}{2}$

$\Rightarrow 2x - 1.96 = 0$

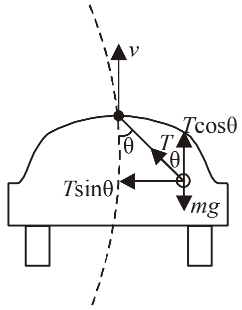
$\therefore \quad x = \frac{1.96}{2} = 0.98\,\text{m}$

7. **(c)** The bob is undergoing a circular motion as the car is moving in circular horizontal track with a constant speed. The bob is under the influence of two forces.

(i) Tension, T in the rod
(ii) Weight of the bob, mg

$T\cos\theta = mg$

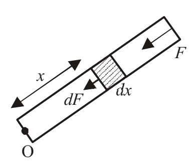
$T\sin\theta = \frac{mv^2}{r}$ is producing the necessary centripetal force for circular motion



$\therefore \quad \tan\theta = \frac{v^2}{rg} = \frac{10 \times 10}{10 \times 10} = 1$ or $\theta = 45°$

8. **(a)** The force acting on the mass of liquid dm of length dx at a distance x from the axis of rotation O.

$dF = (dm)\,x\,\omega^2 \qquad \left(\because F = \frac{mv^2}{r}\right)$



$\therefore \quad dF = \frac{M}{L}dx \times x\omega^2$

where $\frac{M}{L}$ is linear mass density of liquid.

$\therefore$ The force acting at the other end is for the whole liquid in tube.

$$F = \int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{M}{L}\omega^2\int_0^L x\,dx$$

$$= \frac{M}{L}\omega^2\left[\frac{x^2}{2}\right]_0^L = \frac{M}{L}\omega^2\left[\frac{L^2}{2} - 0\right] = \frac{ML\omega^2}{2}$$

9. **(2)** $3\left[F \times r \times \frac{1}{2}\right] = I\alpha$

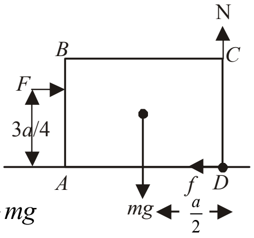
$$3 \times 0.5 \times 0.5 \times \frac{1}{2} = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5 \times \alpha$$

$\Rightarrow \alpha = 2$ rad s^{-1}

$\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + 2 \times 1 = 2$ rad s^{-1}

10. When the cube begins to tip about the edge the normal reaction N will pass through the edge D about which rotation takes place. The moments due to N and frictional force f will be zero.

Taking moment of force F and mg about D



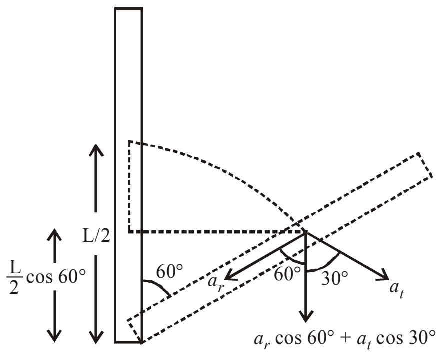
$$F \times \frac{3a}{4} = mg \times \frac{a}{2} \therefore F = \frac{2}{3}mg$$

Hence, minimum value of $F = \frac{2}{3}mg$

11. **False** torque, $\tau = I\alpha \quad \therefore \alpha = \frac{\tau}{I}$

$\tau = F \times r\perp$ Torque is same in both the cases. But since, I will be different due to different mass distribution about the axis, so α will be different.

12. **(a, b, c)** The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping.



Gain in kinetic energy = loss in potential energy

$$\frac{1}{2}I\omega^2 = \text{mg}\frac{l}{2}(1 - \cos 60°)$$

$$\therefore \quad \frac{ml^2}{3}\omega^2 = \text{mg}\frac{l}{2} \quad \Rightarrow \quad \omega = \sqrt{\frac{3\text{g}}{2l}}$$

Now, $\tau = I\alpha$

$$\therefore \quad \text{mg} \times \frac{l}{2}\sin 60° = \frac{1}{3}\text{m}l^2\alpha \quad \Rightarrow \alpha = \frac{3\sqrt{3}}{4}\frac{\text{g}}{l}$$

Further $a_t = \frac{l}{2}\alpha = \frac{3\sqrt{3}\text{g}}{8}$

Also $a_r = \omega^2\frac{l}{2} = \frac{3\text{g}}{2l} \times \frac{l}{2} = \frac{3\text{g}}{4}$

For vertical motion of centre of mass

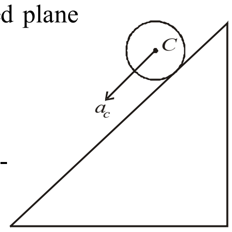
$\text{mg} - \text{N} = \text{m}(a_r \cos 60° + a_t \cos 30°)$

$$\therefore \quad \text{mg} - \text{N} = \text{m}\left[\frac{3\text{g}}{4} \times \frac{1}{2} + \frac{3\sqrt{3}\text{g}}{8} \times \frac{\sqrt{3}}{2}\right]$$

$$\therefore \quad \text{N} = \frac{\text{Mg}}{16}$$

13. **(d)** As we know, acceleration of the center of mass of cylinder rolling down an inclined plane

$$a_c = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

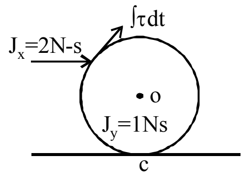


In case of P the mass is concentrated away from the axis,

So $I_P > I_Q$

$\therefore \quad a_P < a_Q \Rightarrow v_P < v_Q \Rightarrow \omega_P < \omega_Q$

14. **(c)** The angular impulse created by the frictional force between the ring and the ball tends to decrease the angular speed ω of the ring about O.

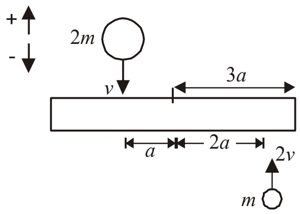


After the collision ω decreases but the ring remains rotating in the anticlockwise direction. Hence the friction between the ring and the ground at the point of contact is to the left.

15. **(a, c, d)**

Applying conservation of linear momentum i.e., $P_i = P_f$

$2m(-v) + m(2v) + 8m \times 0 = (2m + m + 8m)v_c \Rightarrow v_c = 0$



Now, applying conservation of angular momentum about centre of mass i.e., $L_i = L_f$

$2mv \times a + m(2v) \times 2a = I\omega$

Here, $I = \frac{1}{12}(8m) \times (6a)^2 + 2m \times a^2 + m \times 4a^2 = 30ma^2$

$$\therefore \quad 2mv(a) + m(2v) \times 2a = 30ma^2 \times \omega \Rightarrow \omega = \frac{v}{5a}$$

$\therefore$ After collision energy, $E = \frac{1}{2}I\omega^2$

$$= \frac{1}{2} \times 30ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}$$

16. **(a)** P $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j} \rightarrow \vec{v} = \frac{d\vec{r}}{dt}$

$= \alpha\hat{i} + \beta\hat{j}$ = constant

$\therefore \quad \vec{a} = 0$

Further $\vec{P} = m\vec{v}$ = constant

and K = $\frac{1}{2}$mv^2 = constant

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right) = 0 \qquad (\because \vec{a} \text{ is constant})$$

$\therefore$ U = constant

Also E = K + U = Constant

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0 \qquad \therefore \vec{L} = \text{constant}$$

Q $\vec{r}(t)=\alpha\cos\omega t\,\hat{i}+\beta\sin\omega t\,\hat{j} \rightarrow \vec{v}=\dfrac{d\vec{r}}{dt}$

$=-\alpha\omega(\sin\ \omega t)\hat{i}+\beta\omega(\cos\ \omega t)\hat{j}$

$\therefore\quad \vec{a}=\dfrac{d\vec{v}}{dt}=-\omega^2[\alpha\cos\omega t\ \hat{i}+\beta\sin\ \omega t\,\hat{j}]=-\omega^2\vec{r}$

Also $\vec{\tau}=\vec{r}\times\vec{F}=0$ $\therefore$ (L = constant)($\therefore$ $\vec{r}$ and $\vec{F}$ are 11)

$\Delta U=-\int\vec{F}.\vec{dr}=+\int_0^r m\omega^2 r dr=\dfrac{m\omega^2 r^2}{2}\qquad\therefore U\propto r^2$

Also r $=\sqrt{\alpha^2\cos^2\omega t+\beta^2\sin^2\omega t}\qquad\therefore r=f(t)$

Force is central $\therefore$ E =

R $\vec{r}(t)=\alpha t\hat{i}+\sin\omega t\hat{j}\rightarrow\vec{v}=\dfrac{d\vec{r}}{dt}$

$=\alpha[-\omega\sin\omega t\ \hat{i}+\omega\cos\omega t\ \hat{j}]$

$\therefore\quad$ v = $\alpha\omega$ = constant

$\vec{a}=\dfrac{d\vec{v}}{dt}=-\alpha\omega^2[\cos\ \omega t\,\hat{i}\ +\sin\ \omega t\,\hat{j}\,]$

$\therefore\quad \vec{a}=-\omega^2\vec{r}$

$\vec{\tau}=\vec{r}\times\vec{F}=0\quad\therefore\quad$ L = constant

Force is central in nature hence, U and K are also constant.

S $\vec{r}(t)=\alpha(\cos\omega t\hat{i}+\dfrac{\beta}{2}t^2\hat{j}\rightarrow\vec{v}=\dfrac{d\vec{r}}{dt}=\alpha t\ \hat{i}+\beta t\ \hat{j}\ \therefore$ V=$f(t)$

$\vec{a}=\beta\hat{j}$ = constant

$\vec{F}=m\vec{a}$ = constant

ΔU = $-\int\vec{F}\cdot d\vec{r}=-m\int_0^t\beta\hat{j}\cdot(\alpha\hat{i}+\beta t\hat{j})dt=\dfrac{-m\beta^2t^2}{2}$

K = $\dfrac{1}{2}mv^2=\dfrac{1}{2}m(\alpha^2+\beta^2t^2)$

Also, E = K + U = $\dfrac{1}{2}m\alpha^2$ = constant.

17. $\mathbf{A\rightarrow(p,t)\ ;\ B\rightarrow(q,s,t)\ ;\ C\rightarrow(p,r,t)\ \ D\rightarrow(q,p)}$

As the velocity is constant

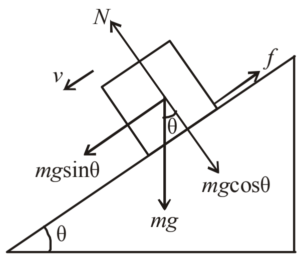
$\therefore\quad f=mg\sin\theta$

But $f=\mu N=\mu mg\cos\theta$

$\therefore\quad \mu mg\cos\theta=mg\sin\theta\ \Rightarrow\ \mu=\tan\theta$

The force by X on Y is the resultant of f and N

$$\sqrt{f^2+N^2}=\sqrt{\mu^2N^2+N^2}=\sqrt{\mu^2+1}N$$

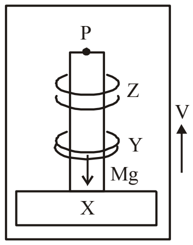


= $(\sqrt{\tan^2\theta+1})\ mg\cos\theta=\sec\theta mg\cos\theta=mg$

= weight of Y.

Again, due to the presence of frictional force between Y and X, the mechanical energy of the system $(X+Y)$ decreases continously as Y slides down.

(q) Lift moves up, X also moves up and therefore the gravitational energy of X is continuously increasing.

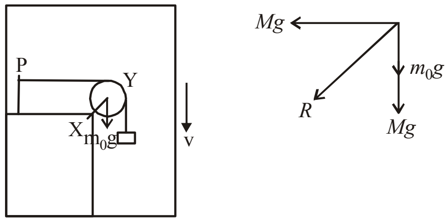


T of weight of Y about P as the perpendicular distance of the line of action of force from the point P is zero.

Force exerted by X on Y

$=Mg+Mg=2mg$ where Mg is wt. of Y and Mg is the force on Y due to Z.

(r)



In this case the force exerted by X on Y = force exerted by Y on X. The force on X due to Y is

$$R=\sqrt{(Mg)^2+(m_0+M)g]^2}\neq Mg$$

The mechanical energy of the system (X + Y) is continously decreasing as the system is coming down and its potential energy is decreasing, the kinetic energy remaining the same.

The torque of the weight of Y about $P\neq 0$

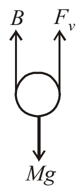
(s) Force on Y by X is = wt. of liquid displaced which cannot be equal to Mg as the density of Y > density of X (: Y is sinking)

The gravitational potential energy of X increases continously because as Y moves down, the centre of mass of X moves up.

(t) Sphere Y is moving with terminal velocity V_T.

$\therefore$ Net force on Y is zero i.e. $Mg=B+F_v$

$B+F_v$ are exerted by X on Y.



The gravitational potential energy of X is continously increasing because as Y moves down, the centre of mass of X moves up.

The mechanical energy of the system (X + Y) is continously decreasing to overcome the viscous forces.

18. **(c)** Here $\omega_0(R-r)=\omega R\quad\therefore\quad\omega=\omega_o\left(\dfrac{R-r}{R}\right)$

Total kinetic energy of the ring = (Kinetic rotational + kinetic energy translational)

$$K.E_{total}=\frac{1}{2}(2MR^2)\omega^2=M\omega_0{}^2(R-r)^2$$

19. **(a)** $\mu M\omega_{\min}^2(R-r)=Mg$

$\therefore\ \omega_{\min}=\sqrt{\dfrac{g}{\mu(R-r)}}$

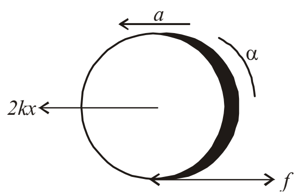
20. **(a)** Axis of rotation is parallel to z-axis. Hence for both the cases, in stantaneous axis passing through is vertical.

21. **(d)** For rigid body ω is same for any point of the body.

22. **(d)** $2kx-f=Ma$ and, $a=R\alpha$

$\therefore\quad \dfrac{2kx-f}{M}=R\left[\dfrac{fR}{\frac{1}{2}MR^2}\right]$

$=R\left[\dfrac{fR}{\frac{1}{2}MR^2}\right]$



Solving this equation, we get

$\therefore |F_{net}| = 2kx - f = 2kx - \dfrac{2kx}{3} = \dfrac{4kx}{3}$

This is opposite to displacement

$f = \dfrac{2kx}{3}$

$\therefore F_{net} = -\dfrac{4kx}{3}$ directed towards the equilibrium

23. **(d)** $F_{net} = 1 - \left(\dfrac{4kx}{3}\right)x$

$\therefore \quad a = \dfrac{F_{net}}{M} = -\left(\dfrac{4k}{3M}\right)x = -\omega^2 x \Rightarrow \omega = \sqrt{\dfrac{4k}{3M}}$

24. **(c)** Mechanical energy is conserved in case of pure rolling motion

$\therefore \quad \dfrac{1}{2}Mv_0^2 + \dfrac{1}{2}\left(\dfrac{1}{2}MR^2\right)\left(\dfrac{v_0}{R}\right)^2 = 2\left[\dfrac{1}{2}kx^2{}_{\max}\right]$

$\therefore \quad x_{\max} = \sqrt{\dfrac{3M}{4k}}v_0$

$F_{\max} = \mu Mg = \dfrac{2kx_{\max}}{3} = \dfrac{2k}{3}\sqrt{\dfrac{3M}{4k}}v_0$

$\therefore \quad v_0 = \mu g\sqrt{\dfrac{3M}{k}}$

25. **(b)** Loss in kinetic energy $= (K.E.)_{\text{initial}} - (K.E.)_{\text{final}}$

$= \left[\dfrac{1}{2}I(2\omega)^2 + \dfrac{1}{2}(2I)\omega^2\right] - \left[\dfrac{1}{2}(I+2I)\left(\dfrac{4}{3}\omega\right)^2\right]$

$= 3I\omega^2 - \dfrac{8}{3}I\omega^2 = \dfrac{I\omega^2}{3}$

26. **(a)** When disc B is brought in contact with disc A Let ω the common velocity. From conservation of angular momentum for the two disc system.

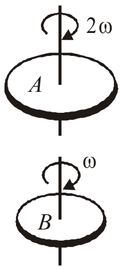
$I(2\omega) + 2I(\omega) = (I+2I)\omega' \Rightarrow \omega' = \dfrac{4}{3}\omega$

Torque on disc A

$\tau_A = \dfrac{\Delta L_A}{t} = \dfrac{L_f - L_i}{t} = \dfrac{I \times \frac{4}{3}\omega - I \times 2\omega}{t} = \dfrac{-2I\omega}{3t}$

Here negative sign indicates that the torque creates angular retardation.

27. **(c)** For disc A



$\dfrac{1}{2}kx_1^2 = \dfrac{1}{2}I(2\omega)^2 \Rightarrow kx_1^2 = 2I\omega^2$

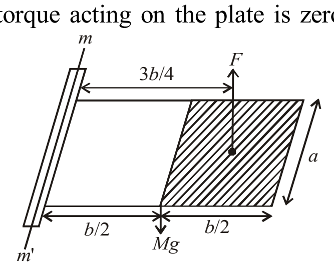
For disc B

$\dfrac{1}{2}kx_2^2 = \dfrac{1}{2} \times 2I\omega^2 \Rightarrow kx_2^2 = I\omega^2$

$\therefore \quad \dfrac{kx_1^2}{kx_2^2} = \dfrac{2I\omega^2}{I\omega^2} \Rightarrow \dfrac{x_1}{x_2} = \sqrt{2}$

28. Plate is held in horizontal position

$\therefore$ Net torque acting on the plate is zero.



$Mg \times \dfrac{b}{2} = F \times \dfrac{3b}{4}$ ... (i)

n number of balls each of mass m striking with velocity v to the shaded half portion of the plate

$F = n\dfrac{dp}{dt} \times A = n \times (2mv) \times a \times \dfrac{b}{2}$ $\quad (\because A = \text{area})$

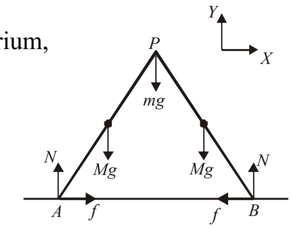
Putting this value of F in eqn. (i)

$Mg \times \dfrac{b}{2} = n \times (2mv) \times a \times \dfrac{b}{2} \times \dfrac{3b}{4}$

$\Rightarrow \quad 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \dfrac{3 \times 2}{4}$

$\therefore \quad v = 10$ m/s

29. Since the system is in equilibrium,



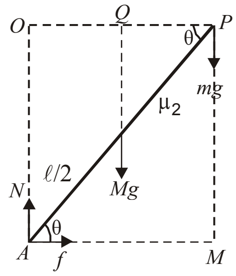
$\therefore \quad \Sigma F_y = 0$

$Mg + mg + Mg - N - N = 0$

$\Rightarrow \quad Mg + mg + Mg = N + N$

$\Rightarrow \quad N = \dfrac{(2M+m)g}{2}$ ... (i)

Let f be the frictional force at A and B. Calculating torques about P



$Mg \times PQ + f \times PM = N \times OP$

$\Rightarrow \quad Mg \times \dfrac{L}{2}\cos\theta + f \times L\sin\theta$

$= NL\cos\theta$

$\Rightarrow \quad f = \dfrac{NL\cos\theta - \dfrac{MgL}{2}\cos\theta}{L\sin\theta}$

$= N\cot\theta - \dfrac{Mg}{2}\cot\theta$

$\Rightarrow \quad f = \left[\left(\dfrac{2N+m}{2}\right)g - \dfrac{Mg}{2}\right]\cot\theta$

$\Rightarrow \quad f = \left[(M+m)\dfrac{g}{2}\right]\cot\theta$

30. A bullet of mass m strikes the wooden log of mass M and length L and sticks to it.

Torque $\vec{\tau} = \dfrac{d\vec{L}}{dt} \quad \Rightarrow \vec{\tau} \times dt = d\vec{L}$

When angular impulse $(\vec{\tau} \times d\vec{t})$ is zero, the angular momentum is constant. In this case for the wooden log-bullet system, the angular impulse about O is constant.

$\therefore$ [angular momentum of the system]$_{\text{initial}}$ = [angular momentum of the system]$_{\text{final}}$

$\Rightarrow \quad mv \times L = I_0 \times \omega$ ... (i)

where I_0 is the moment of inertia of the wooden log-bullet system after collision about O

$I_0 = I_{\text{wooden log}} + I_{\text{bullet}}$

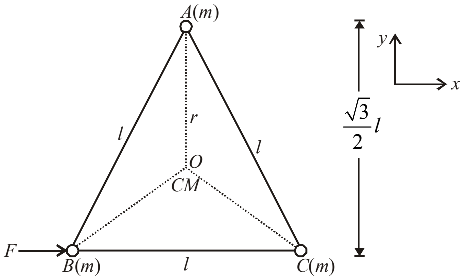
$= \dfrac{1}{3}ML^2 + ML^2$ ... (ii)

From eq. (i) and (ii)

$\omega = \dfrac{mv \times L}{\left[\dfrac{1}{3}ML^2 + mL^2\right]} = \dfrac{3mv}{(M+3m)L}$

31. (a) Let point 'O' denote the centre of mass (CM) of the system

$\therefore \quad OA = r = \dfrac{l}{\sqrt{3}}$ (By geometry)



F = centripetal force of $F = (3m)\, r\omega^2$

or $\quad F = (3m)\left(\dfrac{1}{\sqrt{3}}\right)\omega^2$

or $\quad F = \sqrt{3}ml\omega^2$

(b) Let α be the angular acceleration of system about point A

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\frac{\sqrt{3}}{2}l\right)}{2ml^2} = \frac{\sqrt{3}F}{4ml}$$

Now, acceleration of CM along x-axis is

$$a_x = r\alpha = \left(\frac{l}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4ml}\right) \quad \text{or} \quad a_x = \frac{F}{4m}$$

Let F_x be the force applied by the hinge along X-axis.

$\therefore \quad F_x + F = (3m)a_x$

or $\quad F_x + F = (3m)\left(\dfrac{F}{4m}\right)$ or $\quad F_x + F = \dfrac{3}{4}F$

or $\quad F_x = \dfrac{F}{4}$

Let F_y be the force applied by the hinge along Y-axis

$\therefore \quad F_y$ = centripetal force

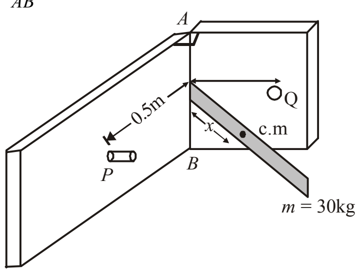
or $\quad F_y = \sqrt{3}ml\omega^2$

32. Let x be the distance of centre of mass from line AB.

$\therefore$ M.I. of laminar sheet about AB

$I_{AB} = I_{\text{c.m.}} + mx^2$ (From parallel axes theorem)

$I_{AB} = 1.2 + 30x^2$ ... (i)



Because of impulse the angular velocity of the laminar sheet will change after every impact.

Impulse = change in linear momentum

$6 = 30\,(V_f - V_i)$

$6 = 30 \times x\,(\omega_f - \omega_i)$ ... (ii)

Also, change in angular momentum = moment of impulse

$\therefore \quad I_{AB}\omega_f - I_{AB}\omega_i$ = Impulse × distance

$I_{AB}\,(\omega_f - \omega_i) = 6 \times 0.5 = 3$

$\therefore \quad \omega_f = \dfrac{3}{I_{AB}} + \omega_i = \dfrac{3}{1.2 + 30x^2} + (-1)$ ... (iii)

From eq. (ii) and (iii)

$6 = 30 \times x\left[\dfrac{3}{1.2+30x^2} - 1 + 1\right]$ ($\because \omega_i = 1$ rod/s)

$1 = 5x\left[\dfrac{3}{1.2+30x^2}\right]$

$\Rightarrow \quad 1.2 + 30x^2 = 5x\,[+3] = 15x$

$\therefore \quad 30x^2 - 15x - 1.2 = 0$

Solving, we get $x = 0.1$ or 0.4

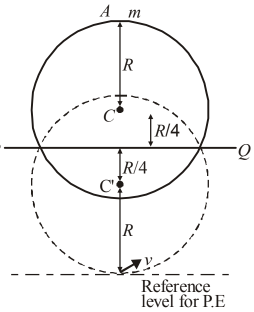
But at $x = 0.4$ m ω_f comes out to be negative (– 0.5 rod/s) which is not acceptable. Therefore,

(a) x = distance of CM from line AB = 0.1 m

(b) Substituting $x = 0.1$ m in eq. (ii) we get $\omega_f = 1$ rod/s the angular velocity with which sheet comes back after the first impact is 1 rad/s.

(c) As the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

33. Initially the disc is held vertical with the point A at its highest position. It starts rotating when allowed to fall. Hence its potential energy changes to rotational kinetic energy.



Applying energy conservation

Enitial energy = Final energy

$$mg\left(2R + \frac{2R}{4}\right) + mg\left(R + \frac{2R}{4}\right) = mgR + \frac{1}{2}I\omega^2$$

Where I = $M.I.$ of disc-mass system about PQ

$$mg \times \frac{10R}{4} + mg\frac{6R}{4} = mgR + \frac{1}{2}I\omega^2 \Rightarrow 3mgR = \frac{1}{2}I\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\dfrac{6mgR}{I}}$ ... (i)

$(I)_{PQ} = (I_{\text{disc}})_{PQ} + (I_{\text{mass}})_{PQ}$

$$= \left[\frac{mR^2}{4} + M\left(\frac{R}{4}\right)^2\right] + m\left(\frac{5R}{4}\right)^2$$

[$\because$ $M.I.$ of disc about diameter $= \dfrac{1}{4}MR^2$]

$$I = \frac{mR^2[4+1+25]}{16} = \frac{15mR^2}{8}$$

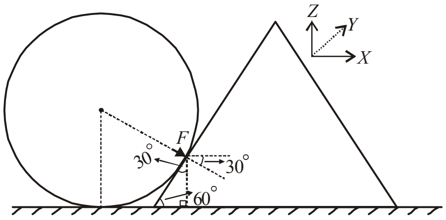
Putting this value of I in eq. (i)

$$\omega = \sqrt{\frac{6mgR \times 8}{15mR^2}} = \sqrt{\frac{16g}{5R}}$$

The linear speed of the particle as it reaches its lowest position

$$v = \omega\left(R + \frac{R}{4}\right) \quad \therefore \quad v = \sqrt{\frac{16g}{5R}} \times \frac{5R}{4} = \sqrt{5gR}$$

34. **(a)** The collision is elastic and the sphere is fixed, hence the wedge will return back with the same velocity (in magnitude).



The force responsible to change the velocity of the wedge in X-direction is F_x.

$F_x \times \Delta t = mv - (-mv)$

(Impulse) = (Change in momentum)

$\therefore \quad F_x = \dfrac{2mv}{\Delta t} \Rightarrow F\cos 30° = \dfrac{2mv}{\Delta t} \Rightarrow F = \dfrac{4mv}{\sqrt{3}\,\Delta t}$

$\because \quad F_x = F\cos 30°$ and $F_y = F\sin 30°$

In vector terms

$$\vec{F} = F_x\hat{i} + F_y(-\hat{k}) = F\cos 30°\hat{i} + F\sin 30°(-\hat{k})$$

$$= F \times \frac{\sqrt{3}}{2}\hat{i} + F \times \frac{1}{2}(-\hat{k})$$

$$\Rightarrow \quad \vec{F} = \frac{F}{2}(\sqrt{3}\,\hat{i} - \hat{k}) = \frac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\,\hat{i} - \hat{k})$$

Equilibrium of force in Z-direction (acting on wedge)

$F_y + mg = N$

$$\Rightarrow \quad N = \frac{F}{2} + mg = \frac{2mv}{\sqrt{3}\Delta t} + mg$$

$$N = \left(\frac{2mv}{\sqrt{3}\Delta t} + mg\right)\hat{k}$$

This is the normal force exerted by the table on the wedge during the time Δ t.

(b) Torques on wedge about the centre of mass of the wedge

$F \times h$ – Torque due to $N + mg \times 0 = 0$

Here h is the perpendicular distance between the centre of mass of the wedge and the line of action of F

$\therefore \quad$ Torque due to $N = F \times h = \dfrac{4mv}{\sqrt{3}\Delta t} \times h$

35. The disc-rod system can roll on the truck without slipping as the friction of the disc-rod system with the floor of the moving truck is large.

Given : Mass of each disc m = 2 kg

Radius of each disc = R = 10 cm = 0.1 m

Length of rod = 20 cm = 0.2 m

Acceleration of truck along x-axis = 9 m/s^2

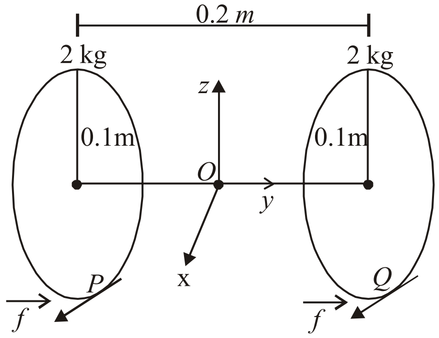
The axis of disc-rod object is horizontal and perpendicular to the direction of the motion of truck. z-axis is the vertically upward direction.

(i) Let a_0 = linear acceleration of centre of mass of disc
α = angular acceleration about its centre of mass

$\therefore \quad a_0 = \dfrac{f}{m} = \dfrac{f}{2}$ ... (i)

Also $\alpha = \dfrac{\tau}{I} = \dfrac{fR}{mR^2/2} = \dfrac{2f}{mR} = \dfrac{2f}{2 \times 0.1} = 10f$

$\therefore \quad \alpha = 10f$ ... (ii)



Since there is no slipping between disc and truck, acceleration of P = acceleration of Q

$$a_0 + R\alpha = a$$

or $\left(\dfrac{f}{2}\right) + (0.1)(10f) = a$ (from (i))

or $\dfrac{3f}{2} = a$ or $f = \dfrac{2a}{3}$ or $f = \dfrac{2 \times 9.0}{3} = 6N$

This force of friction f, is acting in positive x-direction. $\hat{i}$ is unit vector along x-axis.

$\therefore \quad$ In vector form, $\vec{f} = (6\hat{i})$ newton ... (iii)

(ii) Frictional torque, $\tau = r \times f$

At P, $\vec{r}_1 = -0.1\hat{j} - 0.1\hat{k}$

At Q, $\vec{r}_2 = 0.1\hat{j} - 0.1\hat{k}$

$\therefore \quad \vec{\tau}_1 = \vec{r}_1 \times \vec{f}$

or $\vec{\tau}_1 = \left(-0.1\hat{j} - 0.1\hat{k}\right) \times \left(6\hat{i}\right) N\text{–}m$

or $\vec{\tau}_1 = 0.6\hat{k} - 0.6\hat{j}$

or $\vec{\tau}_1 = 0.6(\hat{k} - \hat{j}) N\text{–}m$ ... (iv)

$|\vec{\tau}_1| = \sqrt{(0.6)^2 + (0.6)^2} = \sqrt{2} \times 0.6 = 1.414 \times 0.6$

$= 0.85$ N-m

Similarly, $\vec{\tau}_2 = \vec{r}_2 \times \vec{f}$ or $\tau_2 = 0.6\left(-\hat{j} - \hat{k}\right)$ N-m

$|\vec{\tau}_2| = \sqrt{(0.6)^2 + (0.6)^2} = \sqrt{2} \times 0.6 = 0.85$ N-m

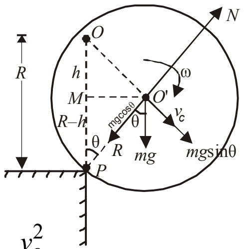
$|\vec{\tau}_1| = |\vec{\tau}_2| = 0.85$ N-m

36. **(a)** The cylinder of radius R mass 'm' rolls off the edge of the rectangular block without slipping. The rotation of the cylinder is about the point of contact so energy of the cylinder is conserved.

Let the original position of centre of mass of the cylinder be O. While rolling down off the edge, let the cylinder be at such a position that its centre of mass is at a position O'.

Let $\angle NPO$ be θ. As the cylinder is rolling, the c.m. rotates in a circular path. The centripetal force required for the circular motion.

$$mg\cos\theta - N = \frac{mv_c^2}{R}$$



The condition for the cylinder leaving the edge, $N = 0$

$$\therefore \quad mg\cos\theta = \frac{mv_c^2}{R} \Rightarrow \cos\theta = \frac{v_c^2}{Rg} \qquad \text{... (i)}$$

Applying energy conservation from O to O'.
Loss of potential energy of cylinder
= gain in translational $K.E.$ + gain in rotational $K.E.$

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2 \qquad \text{... (ii)}$$

Where I is the moment of inertia of the cylinder about O', $I = \frac{1}{2}mR^2$, ω is the angular speed, V_c is the velocity of center of mass.

Also for rolling, $v_c = \omega R \Rightarrow \omega = \frac{v_c}{R}$

Putting value of I and ω in eqn. (ii)

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}\times\frac{1}{2}mR^2\times\frac{v_c^2}{R^2}$$

$$\Rightarrow \; gh = \frac{1}{2}v_c^2 + \frac{1}{4}v_c^2 = \frac{3}{4}v_c^2 \;\Rightarrow\; v_c^2 = \frac{4gh}{3}$$

In $\Delta O'MP$, $\cos\theta = \frac{R-h}{R}$

$\Rightarrow h = R(1 - \cos\theta)$

$$\therefore \quad v_c^2 = \frac{4g}{3}R(1-\cos\theta) \qquad \text{... (iii)}$$

From eq. (i) and (iii), we get

$$\cos\theta = \frac{4gr}{3Rg}(1-\cos\theta)$$

$\Rightarrow 3\cos\theta = 4 - 4\cos\theta \quad \Rightarrow \cos\theta = \frac{4}{7}$

(b) Speed of C.M. of cylinder before leaving contact with edge.

$$v_c^2 = \frac{4gR}{3}\left(1-\frac{4}{7}\right) = \frac{4gR}{7} \Rightarrow v_c = \sqrt{\frac{4gR}{7}} \qquad \text{[from (iii)]}$$

(c) Before the centre of mass of the cylinder reaches the horizontal line of the edge, it leaves contact with the edge

$$\theta = \cos^{-1}\frac{4}{7} = 55.15^\circ$$

Hence the rotational $K.E.$, which the cylinder gains at the time of leaving contact with the edge remains the same in its further motion. Thereafter the cylinder gains translational $K.E.$

Again applying energy conservation from O to the point where centre of mass is in horizontal line with edge

$$mgR = \frac{1}{2}I\omega^2 + \frac{1}{2}m(v'_c)^2$$

$$mgR = \frac{1}{2}\times\frac{1}{2}mR^2\times\left(\sqrt{\frac{4g}{7R}}\right)^2 + \frac{1}{2}m(v'_c)^2$$

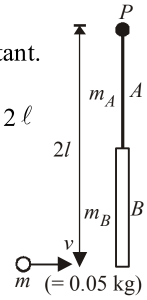
$$\because \quad \omega = \frac{v_c}{R} = \sqrt{\frac{4gR/7}{R}}$$

$$\Rightarrow \; mgR - \frac{mgR}{7} = \frac{6mgR}{7} = \text{translational } K.E.$$

Also, rotational $K.E. = \frac{1}{2}I\omega^2 = \frac{mgR}{7}$

$$\therefore \quad \frac{\text{Translational } K.E.}{\text{Rotational } K.E.} = 6$$

37. Since, $\tau = \frac{dL}{dt}$ and $\tau = 0 \quad \therefore \; L$ is constant.



Angular momentum before collision = $mv \times 2\ell$
Angular momentum after collision = $I\omega$

$I = I_m + I_A + I_B$

$$= \left[m(2\ell)^2 + \left\{m_A\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}\right)^2\right\} + \left\{m_B\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}+\ell\right)^2\right\}\right]$$

$$= \left[4m\ell^2 + m_A\left(\frac{\ell^2}{12}+\frac{\ell^2}{4}\right) + m_B\left(\frac{\ell^2}{12}+\frac{9\ell^2}{4}\right)\right]$$

$$= \left[4m\ell^2 + \frac{1}{3}m_A\ell^2 + \frac{7}{3}m_B\ell^2\right] = 0.09\,\text{kg}\,m^2$$

$I\omega = mv \times 2\ell$

$$\Rightarrow \quad \omega = \frac{mv\times 2\ell}{I} = \frac{0.05\times v\times 2\times 0.6}{0.09} = 0.67\,v$$

Applying conservation of mechanical energy after collision.
Loss of $K.E.$ = gain in $P.E.$

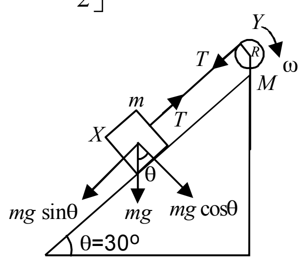
$$\frac{1}{2}I\omega^2 = mg(2\ell) + m_A\left(\frac{\ell}{2}\right)g + m_B g\left(\frac{3\ell}{2}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times 0.09\times(0.67v)^2$$

$$= \left[0.05\times 2 + 0.01\times\frac{1}{2} + 0.02\times\frac{3}{2}\right]\times 9.8\times 0.6$$

$\Rightarrow \quad v = 6.3$ m/s

38. **(i)** According to question, the drum is given an initial angular velocity such that the block X starts moving up the plane.



As the time passes and block X rises up the velocity of the block decreases. Let a be the linear retardation of the block X.

$$\therefore \quad mg\sin\theta - T = ma \qquad \text{... (i)}$$

The linear retardation of the block a and the angular acceleration of the drum (α) are related as

$$a = R\alpha \Rightarrow \alpha = \frac{a}{R}$$

where R is the radius of the drum.
The retarding torque of the drum is due to tension T in the string.

$$\tau = T \times R = I\alpha \ (\because \tau = I\alpha)$$

$$\therefore \quad T \times R = \frac{1}{2}MR^2\alpha \qquad \left[\because I = \frac{1}{2}MR^2\right]$$

$$\Rightarrow TR = \frac{1}{2}MR^2\frac{a}{R} \Rightarrow a = \frac{2T}{M}$$

Substituting this value of a in eq. (i)

$$mg\sin\theta - T = m \times \frac{2T}{M} \Rightarrow mg\sin\theta = \left(1 + \frac{2m}{M}\right)T$$

$$\therefore \quad T = \frac{(mg\sin\theta)\times M}{M+2m} = \frac{0.5\times 9.8\times\sin 30°\times 2}{2+2\times 0.5} = 1.63\text{ N}$$

(ii) Distance travelled by x

$$a = \frac{2T}{M} = \frac{2\times 1.63}{2} = 1.63\ m/s^2$$

using $V = R\omega \Rightarrow V = 0.2 \times 10 = 2$ m/s
($\because$ $R = 0.2$ m and $\omega = 10$ rod/s given)
$\therefore$ Distance travelled by block x

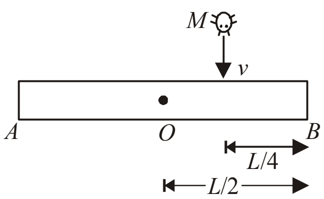
$$2as = v^2 - u^2 \Rightarrow s = \frac{v^2}{2a} = \frac{2\times 2}{2\times 1.63}$$

$\therefore$ Distance travelled by $x = 1.22$ m

39. **(a)** Angular momentum of the system (rod + insect) about the centre of mass 'O' before collision = angular momentum of the system after collision.

$$Mv \times \frac{L}{4} = I\omega$$

Where I is the moment of inertia of the system just after collision and ω is the angular velocity just after collision.



$$\Rightarrow \quad Mv\frac{L}{4} = \left[M\left(\frac{L}{4}\right)^2 + \frac{1}{12}ML^2\right]\omega$$

$$\Rightarrow \quad Mv \times \frac{L}{4} = \frac{ML^2}{4}\left[\frac{1}{4}+\frac{1}{3}\right]\omega = \frac{ML^2}{4}\left[\frac{3+4}{12}\right]$$

$$= \frac{ML^2}{4}\times\frac{7}{12}\times\omega \quad \Rightarrow \quad \omega = \frac{12}{7}\frac{v}{L}.$$

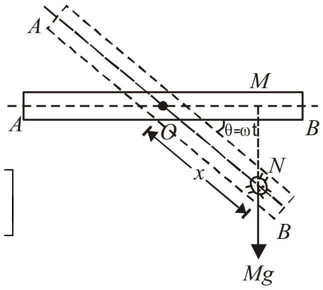
(b) After collision there is an extra mass M of the insect which creates a torque in the clockwise direction, which tends to create angular acceleration in the rod. But the same is compensated by the movement of insect towards B due to which moment of inertia I of the system increases. Let at any instant of time t the insect be at a distance x from the centre of the rod and the rod has turned through an angle θ (= ωt) w.r.t its original position

$$= \frac{dL}{dt} = \frac{d}{dt}(I\omega)$$

$$= \omega\frac{dI}{dt}$$

$$= \omega\frac{d}{dt}\left[\frac{1}{12}ML^2 + Mx^2\right]$$

$$= 2M\omega x\frac{dx}{dt}$$



This torque is balanced by the torque due to weight of insect.

$$\therefore \quad 2M\omega x\frac{dx}{dt} = Mg\,(x\cos\theta) \Rightarrow dx = \left(\frac{g}{2\omega}\right)\cos\omega t\,dt$$

On integration, taking limits

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega}\cos\omega t\,dt$$

when $\quad x = \frac{L}{4}, \omega t = 0$

$$[x]_{L/4}^{L/2} = \frac{g}{2\omega^2}[\sin\omega t]_0^{\pi/2\omega}$$

when $\quad x = \frac{L}{2}, \omega t = \frac{\pi}{2}$

$$\Rightarrow \left(\frac{L}{2} - \frac{L}{4}\right) = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$\Rightarrow \frac{L}{4} = \frac{g}{2\omega^2} \Rightarrow \omega = \sqrt{\frac{2g}{L}}$$

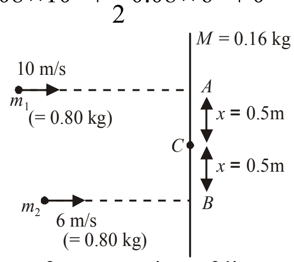
But $\omega = \frac{12}{7}\frac{v}{L} \Rightarrow \frac{12}{7}\frac{v}{L} = \sqrt{\frac{2g}{L}} \Rightarrow v = \frac{7}{12}\sqrt{2gL}$

$$\Rightarrow \quad v = \frac{7}{12}\sqrt{2\times 10\times 1.8} = 3.5\text{ ms}^{-1}$$

40. Before collision, Kinetic energy of the system

$$\text{K.E.}_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}0.08\times 10^2 + \frac{1}{2}0.08\times 6^2 + 0 = 5.44\text{ J} \qquad \text{... (i)}$$



Applying law of conservation of linear momentum during collision

$m_1 \times v_1 + m_2 \times v_2 = (M + m_1 + m_2)\,V_c$
where V_c is the velocity of centre of mass of the bar and particles sticked on it after collision
$0.08 \times 10 + 0.08 \times 6 = (0.16 + 0.08 + 0.08)\,V_c \Rightarrow V_c = 4$ m/s

$\therefore$ K.E. trans after collision

$$= \frac{1}{2}(M + m_1 + m_2)V_c^2 = 2.56\text{ J} \quad \text{... (ii)}$$

Applying conservation of angular momentum of the bar and two particle system about the centre of the bar.

$T_{External} = 0 \;\therefore\; L = \text{constant}$

$L_i = m_1 v_1 \times x - m_2 v_2 x$

$= 0.08 \times 10 \times 0.5 - 0.08 \times 6 \times 0.5$

$= 0.4 - 0.24 = 0.16\text{ kg m}^2\text{s}^{-1}$ (In clockwise direction)

$L_f = I\omega$

$$= \left[\frac{M\ell^2}{12} + m_1 x^2 + m_2 x^2\right]\omega$$

$$= \left[\frac{(0.16)(\sqrt{3})^2}{12} + 0.08 \times (0.5)^2 + (0.08)(0.5)^2\right]\omega$$

$= 0.08\,\omega$

$\therefore$ $0.08\,\omega = 0.16 \Rightarrow \omega = 2\text{ rad/s}$ ... (iii)

$$\text{K.E.}_{\text{Rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.08 \times 2^2 = 0.16\text{ J} \quad \text{... (iv)}$$

$K.E._f$ = Translational $K.E.$ + Rotational $K.E.$

$= 2.56 + 0.16 = 2.72\text{ J}$

$\Delta K.E. = K.E._i - \text{Final } K.E._f$

$= 5.44 - 2.72 = 2.72\text{ J}$

41. $P.E.$ at $D = P.E.$ at $Q + (K.E.)_{Trans} + (K.E.)_{Rot}$

From energy conservation principle

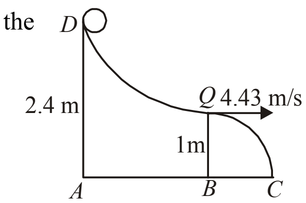
$$\Rightarrow \; mg(2.4) = mg(1) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg(2.4 - 1) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$$

$$\left(\because I = \frac{2}{5}mr^2 \;\&\; \omega = \frac{v}{r}\right)$$

or, $g \times 1.4 = \dfrac{7v^2}{10} \;\Rightarrow\; v = 4.43\text{ m/s}$

After point Q, the path of the body parabolic.



$h = \dfrac{1}{2}gt^2 \;\Rightarrow\; 1 = 4.9\,t_y^2$

$\therefore$ $t_y = \dfrac{1}{\sqrt{4.9}} = 0.45\text{ sec}$

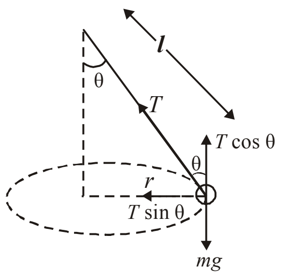
Distances $BC = V \times t_y = 4.43 \times 0.45 = 2\text{m}$

During its flight as a projectile, the sphere continues to rotate because of conservation of angular momentum.

42. Let small mass m moves around a circular path of radius r. Let the string makes an angle θ with the vertical and T be the tension

$T\cos\theta = mg$ ... (i)

$T\sin\theta = mr\omega^2$ ... (ii)



$\therefore$ $\tan\theta = \dfrac{r\omega^2}{g}$

From figure, $\sin\theta = \dfrac{r}{\ell}$

$\Rightarrow$ $r = \ell\sin\theta$

$\therefore$ $\tan\theta = \ell\sin\theta\dfrac{\omega^2}{g}$

$$\Rightarrow \omega^2 = \frac{\tan\theta . g}{\ell\sin\theta} \text{ or, } \omega = \sqrt{\frac{g}{\ell\cos\theta}}$$

$$\Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{g}{\ell\cos\theta}} \quad \because \omega = 2\pi\nu \quad \text{... (iii)}$$

For M to remain stationary, $T = Mg$

and $T\cos\theta = mg$

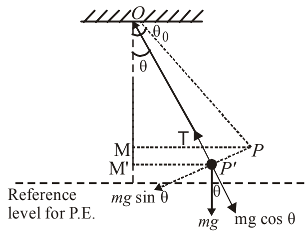
$\therefore$ $Mg\cos\theta = mg$

$\Rightarrow$ $\cos\theta = \dfrac{m}{M}$

Putting this value of cos θ is eq (iii), we get

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}\frac{M}{m}}$$

43.



In ΔOPM, $\cos\theta_0 = \dfrac{OM}{\ell}$

$\Rightarrow OM = \ell\cos\theta_0$

In $\Delta OP'M'$, $\cos\theta = \dfrac{OM'}{\ell}$

$\Rightarrow OM' = \ell\cos\theta$

$OM' - OM = \ell(\cos\theta - \cos\theta_0)$

Loss in potential energy = Gain in kinetic energy (Activity P to P')

$$\Rightarrow \; mg\ell(\cos\theta - \cos\theta_0) = \frac{1}{2}mv^2$$

$\Rightarrow$ $v^2 = 2g\ell(\cos\theta - \cos\theta_0)$ ... (i)

$T - mg\cos\theta = \dfrac{mv^2}{\ell}$

$\therefore$ $T = \dfrac{mv^2}{\ell} + mg\cos\theta$ ... (ii)

From eg. (i) and (ii)

$$T = \frac{m}{\ell} \times 2g\,\ell(\cos\theta - \cos\theta_0) + mg\cos\theta$$

$\therefore$ $T = 3mg\cos\theta - 2\,mg\cos\theta_0$

$\Rightarrow$ $T = mg\,(3\cos\theta - 2\cos\theta_0)$

From equation (ii) it is clear that the tension is maximum when $\cos\theta = 1$ i.e., $\theta = 0°$

Hence, $T_{\max} = \dfrac{mv^2}{\ell} + mg$ ... (iii)

From eqn. (i)

$v^2 = 2g\ell(1 - \cos\theta_0)$ ... (iv)

From (iii) and (iv)

$$T_{\max} = \frac{m}{\ell}[2g\ell(1 - \cos\theta_0)] + mg$$

$\therefore$ $T_{\max} = 3mg - 2mg\cos\theta_0$

$80 = 3 \times 40 - 2 \times 40\cos\theta_0$

$$\Rightarrow \; 80\cos\theta_0 = 40 \Rightarrow \cos\theta_0 = \frac{1}{2} \Rightarrow \theta_0 = 30°$$

7 Gravitation

Topic-1 : Kepler's Laws of Planetary Motion

1. **(c)** Areal velocity; $\frac{dA}{dt}$

$dA = \frac{1}{2}r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$

Also, $L = mvr = mr^2\omega$

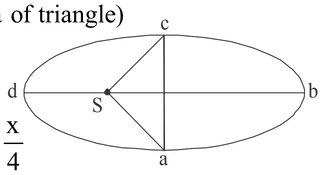
$\therefore \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

2. **(c)** Let area of ellipse abcd = x

Area of SabcS $= \frac{x}{2} + \frac{x}{4}$ (i.e., ar of abca + SacS)

(Area of half ellipse + Area of triangle)

$= \frac{3x}{4}$

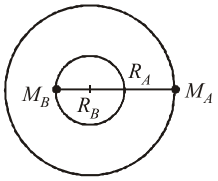


Area of SadcS $= x - \frac{3x}{4} = \frac{x}{4}$

$\frac{\text{Area of SabcS}}{\text{Area of SadcS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$

$\frac{t_1}{t_2} = 3$ or, $t_1 = 3t_2$

3. **(d)** Incase of binary star system, the gravitational force of attraction between the stars will provide the necessary centripetal forces. So angular velocity ω of both stars is the same. Therefore time period T $= \frac{2\pi}{\omega}$ remains the same.



4. **(b)** According to Kepler's law $T^2 \propto R^3$

$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

Here $T_1 = 365$ days ; $T_2 = ?$; $R_1 = R$ and $R_2 = \frac{R}{2}$

$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 365\left[\frac{R/2}{R}\right]^{3/2} = 129\text{ days}$

5. **(b)** The centripetal force is provided by Centripetal force,

$= \frac{mV^2}{R} = mR\omega^2$ the gravitational force of attraction

$mR\omega^2 = GMmR^{-5/2}$

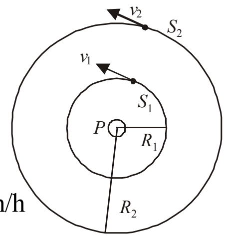
$\Rightarrow \frac{mR \times 4\pi^2}{T^2} = \frac{GMm}{R^{5/2}} \Rightarrow T^2 \propto R^{7/2}$

6. (i) According to Kepler's third law of planetary motion. $T^2 \propto R^3$

$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$

$\Rightarrow R_2^3 = \left(10^4\right)^3 \times \frac{8^2}{1^2} = 64 \times 10^{12}$

$\Rightarrow R_2 = 4 \times 10^4$ km.



Linear speed of satellite S_1

$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4$ km/h

Linear speed of satellite S_2,

$v_2 = \frac{2\pi R_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = \pi \times 10^4$ km/h

Hence the speed of satellite S_2 w.r.t. S_1

$= v_2 - v_1 = \pi \times 10^4 - 2\pi \times 10^4 = -\pi \times 10^4$ km/h

(ii) Angular speed of S_2 w.r.t. S_1

$= \frac{v_2 - v_1}{R_2 - R_1} = \frac{3.14 \times 10^4 \times 5/18}{3 \times 10^4 \times 10^3} = 3 \times 10^{-4}$ rad/s

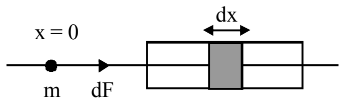
Topic-2 : Newton's Universal Law of Gravitation

1. **(d)** Given $\lambda = (A + Bx^2)$,

Taking small element dm of length dx at a distance x from x = 0

so, $dm = \lambda\, dx$

$dm = (A + Bx^2)dx$



$dF = \frac{Gmdm}{x^2}$

$\Rightarrow F = \int_a^{a+L}\frac{Gm}{x^2}(A + Bx^2)dx$

$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L} = Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

2. **(a)** As we know, Gravitational force of attraction,

$F = \frac{GMm}{R^2}$

$F_1 = \frac{GM_e m}{r_1^2}$ and $F_2 = \frac{GM_e M_s}{r_2^2}$

$\Delta F_1 = \frac{2GM_e m}{r_1^3}\Delta r_1$ and $\Delta F_2 = \frac{GM_e M_s}{r_2^3}\Delta r_2$

$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

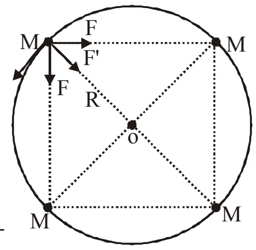
Using $\Delta r_1 = \Delta r_2 = 2R_{earth}$; $m = 8 \times 10^{22}$ kg;

$M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km and $r_2 = 150 \times 10^6$ km

$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8 \times 10^{22}}{2 \times 10^{30}}\right)\left(\frac{150 \times 10^6}{0.4 \times 10^6}\right)^3 \times 1 \cong 2$

3. **(d)** $2F\cos 45^\circ + F' = \frac{Mv^2}{R}$ (From figure)



Where $F = \frac{GM^2}{(\sqrt{2}R)^2}$ and

$F' = \frac{GM^2}{4R^2}$

$\Rightarrow \frac{2\times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$

$\Rightarrow \frac{GM^2}{R}\left[\frac{1}{4}+\frac{1}{\sqrt{2}}\right] = Mv^2$

$\therefore v = \sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

4. **(a)** Volume of removed sphere

$V_{\text{remo}} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi R^3\left(\frac{1}{8}\right)$

Volume of the sphere (remaining)

$V_{\text{remain}} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3\left(\frac{1}{8}\right) = \frac{4}{3}\pi R^3\left(\frac{7}{8}\right)$

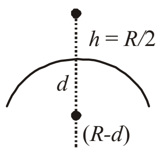
Therefore mass of sphere carved and remaining sphere are at respectively $\frac{1}{8}$M and $\frac{7}{8}$M.

Therefore, gravitational force between these two sphere,

$F = \frac{GMm}{r^2} = \frac{G\frac{7M}{8}\times\frac{1}{8}M}{(3R)^2} = \frac{7}{64\times 9}\frac{GM^2}{R^2} \simeq \frac{41}{3600}\frac{GM^2}{R^2}$

Topic-3 : Acceleration due to Gravity

1. **(b)** According to question, $g_h = g_d = g_1$



$g_h = \frac{GM}{\left(R+\frac{R}{2}\right)^2}$ and $g_d = \frac{GM(R-d)}{R^3}$

$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$

$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$

$\therefore \frac{d}{R} = \frac{5}{9}$

2. **(b)** Value of g at equator, $g_A = g\cdot - R\omega^2$

Value of g at height h above the pole,

$g_B = g\cdot\left(1-\frac{2h}{R}\right)$

As object is weighed equally at the equator and poles, it means g is same at these places.

$g_A = g_B$

$\Rightarrow g - R\omega^2 = g\left(1-\frac{2h}{R}\right) \Rightarrow R\omega^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2\omega^2}{2g}$

3. **(c)** The acceleration due to gravity at a height h is given by

$g = \frac{GM}{(R+h)^2}$

Here, G = gravitation constant

M = mass of earth

The acceleration due to gravity at depth h is

$g' = \frac{GM}{R^2}\left(1-\frac{h}{R}\right)$

Given, $g = g'$

$\therefore \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1-\frac{h}{R}\right)$

$\therefore R^3 = (R+h)^2(R-h) = (R^2+h^2+2hR)(R-h)$

$\Rightarrow R^3 = R^3 + h^2R + 2hR^2 - R^2h - h^3 - 2h^2R$

$\Rightarrow h^3 + h^2(2R-R) - R^2h = 0$

$\Rightarrow h^3 + h^2R - R^2h = 0 \quad \Rightarrow h^2 + hR - R^2 = 0$

$\Rightarrow h = \frac{-R\pm\sqrt{R^2+4(1)R^2}}{2}$

$= \frac{-R+\sqrt{5}R}{2} = \frac{(\sqrt{5}-1)}{2}R$

4. **(d)** Weight at pole, $w = mg = 196\ N$

$\Rightarrow \quad m = 19.6$ kg

Weight at equator, $w' = mg' = m(g - \omega^2 R)$

$= 19.6\left[10 - \left(\frac{2\pi}{24\times 3600}\right)^2 \times 6400\times 10^3\right]\text{N} \quad \left(\because \quad \omega = \frac{2\pi}{T}\right)$

$= 19.6\,[10 - 0.034] = 195.33\ N$

5. **(d)** $\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4}$ or $\frac{g_e}{g_p} = \frac{9}{4}$

or $\frac{GM/R^2}{G(M/9)/R_p^2} = \frac{9}{4}$

$\therefore \ \boldsymbol{R_p} = \frac{R}{2}$

6. **(a)** Given

Acceleration due to gravity at a height h from earth's surface is

$g_h = g\left(1+\frac{h}{R_e}\right)^{-2} \Rightarrow 4.9 = 9.8\left(1+\frac{h}{R_e}\right)^{-2}$

$\frac{1}{\sqrt{2}} = \left(1-\frac{h}{R_e}\right)$ [as h <<< R_e]

$h = R_e\left(\sqrt{2}-1\right)$

$h = 6400 \times 0.414\ \text{km} = 2.6\times 10^6\ \text{m}$

7. **(d)** With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - \omega^2 R\cos^2\phi$

Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90°$

At all other points as ω increases g' will decreases hence, weight, W = mg decreases.

8. **(b)** Variation of acceleration due to gravity, g with distance 'd' from centre of the earth

If $d < R, g = \frac{Gm}{R^2}.d$ *i.e.*, $g \propto d$ (straight line)

If $d = R, g_s = \frac{Gm}{R^2}$

If $d > R, g = \frac{Gm}{d^2}$ *i.e.*, $g \propto \frac{1}{d^2}$

9. **(c)** We know, $g' = g - \omega^2 R \cos^2 \theta$

$\frac{3g}{4} = g - \omega^2 R$

Given, $g' = \frac{3}{4}g$

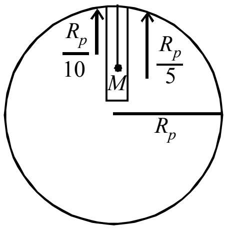
$\omega^2 R = \frac{g}{4}$

$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}}$

$= \frac{1}{2 \times 8 \times 100} = 0.6 \times 10^{-3}$ rad/s

10. **(b)** Given : $R_p = \frac{R_e}{10} = \frac{6 \times 10^6}{10} = 6 \times 10^5 m$

The mass of the wire
$= 10^{-3} \times 1.2 \times 10^5 = 120$ kg
Acceleration due to gravity at the surface of the planet and earth



$g_p = \frac{4}{3}\pi\rho G R_P$;

and $g_e = \frac{4}{3}\pi\rho G R_E$

$\therefore \frac{g_p}{g_e} = \frac{R_p}{R_E} = \frac{1}{10} \Rightarrow g_p = \frac{10}{10} = 1 \text{ ms}^{-2}$

Let g_{pM} be the acceleration due to gravity at point M which is the mid point of the wire and is at a depth of $\frac{R_p}{10}$.

$g_{pM} = g_p\left[1 - \frac{R_p/10}{R_p}\right] = 1[1 - 0.1] = 0.9 \text{ ms}^{-2}$

$\therefore$ Force = mass of wire $\times g_{pM} = 120 \times 0.9 = 108$ N

11. **(a)**

12. **(c)** From $mg = \frac{GMm}{R^2}$

$\Rightarrow g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(0.99R)^2}$

$\because$ Radius of the earth shrinks by 1%

$\therefore \frac{g'}{g} = \left(\frac{R}{0.99R}\right)^2 \Rightarrow g' > g$

13. **(1.24×10^{-3} rad/s)**

Using $g' = g - R\omega^2 \cos^2 \phi$

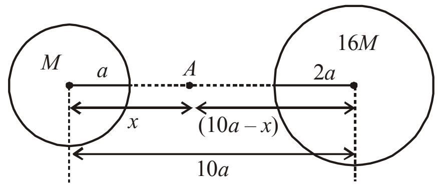
At equator, $\phi = 0$,

$\therefore g' = g - R\omega^2$

Here $g' = 0$ $\therefore \omega = \sqrt{\frac{g}{R}} = 1.24 \times 10^{-3}$ rad/s

Topic-4 : Gravitational Field, Potential and Potential Energy

1. **(d)**



Let A be the point where gravitation field of both planets cancel each other i.e. zero.

$\frac{GM}{x^2} = \frac{G(16M)}{(10a - x)^2}$

$\Rightarrow \frac{1}{x} = \frac{4}{(10a - x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a$ ... (i)

Using conservation of energy, we have

$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$

$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$

$\Rightarrow KE = GMm\left[\frac{1 + 64 - 4 - 16}{8a}\right]$

$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v = \sqrt{\frac{90GM}{8a}}$

$\Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

2. **(a)** Given : Gravitational field,

$E_G = \frac{Ax}{(x^2 + a^2)^{3/2}}, V_\infty = 0$

$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot \vec{d}_x$

$\Rightarrow V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$

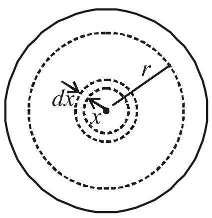
$\therefore V_x = \frac{A}{(x^2 + a^2)^{1/2}} - 0 = \frac{A}{(x^2 + a^2)^{1/2}}$

3. **(d)** Mass of small element of planet of radius x and thickness dx.

$dm = \rho \times 4\pi x^2 dx = \rho_0\left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$

Mass of the planet

$M = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx$



$\Rightarrow M = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$

Gravitational field,

$E = \frac{GM}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$

$$\Rightarrow E = 4\pi G\rho_0\left(\frac{r}{3}-\frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{dE}{dr} = 4\pi G\rho_0\left(\frac{1}{3}-\frac{3r^2}{5R^2}\right) = 0 \qquad \Rightarrow r = \frac{\sqrt{5}}{3}R$$

4. **(b)** Gravitation field at the surface

$$E = \frac{Gm}{r^2}$$

$$\therefore E_1 = \frac{Gm_1}{r_1^2} \text{ and } E_2 = \frac{Gm_2}{r_2^2}$$

From the diagram given in question,

$$\frac{E_1}{E_2} = \frac{2}{3} \quad (r_1 = 1\text{m}, R_2 = 2m \text{ given})$$

$$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2\left(\frac{m_1}{m_2}\right) \quad \Rightarrow \frac{2}{3} = \left(\frac{2}{1}\right)^2\left(\frac{m_1}{m_2}\right)$$

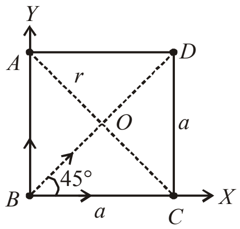
$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$$

5. **(c)** $E_g = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$

6. **(b)** $AC = a\sqrt{2} \quad \because r = \frac{AC}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Resultant force on the body

$$B = \frac{GM^2}{a^2}\hat{i} + \frac{GM^2}{a^2}\hat{j} + \frac{GM^2}{(a\sqrt{2})^2}(\cos 45°\hat{i} + \sin 45°\hat{j})$$



$$\Rightarrow |F| = \frac{GM^2}{a^2}(\sqrt{2}) + \frac{GM^2}{2a^2}$$

$\frac{Mv^2}{r}$ = Resultant force towards centre

$$\therefore \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = \sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

7. **(a)** $F = \frac{GMm}{r} = \int a\frac{\rho(dV)m}{r^2}$

$$= mG\int_0^R \frac{k}{r^2}\frac{4\pi r^2 dr}{r^2} = -4\pi kGm\left(\frac{1}{r}\right)_0^R = -\frac{4\pi kGm}{R}$$

Using Newton's second law, we have

$$\frac{mv_0^2}{R} = \frac{4\pi kGm}{R}$$

or v_0 = C (const.)

Time period, $T = \frac{2\pi R}{v_0} = \frac{2\pi R}{C}$ or $= \frac{T}{R}$ = constant.

8. **(c)** Initial gravitational potential energy, $E_i = -\frac{GMm}{2R}$

Final gravitational potential energy,

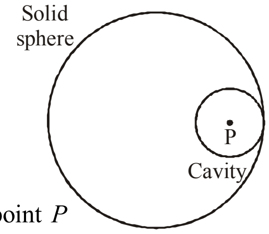
$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$$

$$= -\frac{4GMm}{6R} = -\frac{2GMm}{3R}$$

$\therefore$ Difference between initial and final energy,

$$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3}+\frac{1}{2}\right) = -\frac{GMm}{6R}$$

9. **(d)** Due to complete solid sphere, potential at point P



$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$

Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$$= V_{sphere} - V_{cavity}$$

$$= -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

10. **(c)** As, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

Graph (c) most closely depicts the correct variation of $v(r)$.

11. **(d)** Gravitational field, $I = (5\hat{i} + 12\hat{j})$ N/kg

$$I = -\frac{dv}{dr}$$

$$v = -\left[\int_0^x I_x dx + \int_0^y I_y dy\right] = -\left[I_x.x + I_y.y\right]$$

$$= -\left[5(7-0) + 12(-3-0)\right]$$

$$= -\left[35 + (-36)\right] = 1\ \text{J/kg}$$

i.e., change in gravitational potential 1 J/kg.
Hence change in gravitational potential energy 1 J

12. **(b)** We choose reference point, infinity, where total energy of the system is zero.
So, initial energy of the system = 0

$$\text{Final energy} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d}$$

From conservation of energy,
Initial energy = Final energy

$$\therefore 0 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 - \frac{Gm_1m_2}{d}$$

$$\text{or} \quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{d} \quad ...(1)$$

By conservation of linear momentum

$m_1v_1 + m_2v_2 = 0$ or $\frac{v_1}{v_2} = -\frac{m_2}{m_1} \Rightarrow v_2 = -\frac{m_1}{m_2}v_1$

Putting value of v_2 in equation (1), we get

$$m_1v_1^2 + m_2\left(-\frac{m_1v_1}{m_2}\right)^2 = \frac{2Gm_1m_2}{d}$$

$$\frac{m_1m_2v_1^2 + m_1^2v_1^2}{m_2} = \frac{2Gm_1m_2}{d}$$

$$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}} = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Similarly $v_2 = -m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

13. **(c)** Let mass of smaller sphere (which has to be removed) is m

Radius $= \frac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere

$$= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$$

14. **(c)** **For $r \geq R$**

$\frac{mv^2}{r} = m \times \left[\frac{GM}{r^2}\right]$ where M is the total mass of the spherical system.

$\therefore\ v \propto \frac{1}{\sqrt{r}}$ $\qquad \left(\because M = \left(\frac{4}{3}\pi R^3\right)\rho_0\right)$

For r < R

$\therefore\ \frac{mv^2}{r} = m\left[\frac{GM}{R^2}\right]$ and $M = \left(\frac{4}{3}\pi R^3\right)\rho_0 \quad \Rightarrow \quad v \propto r$

i.e., $v - r$ graph is a straight line passing through origin.

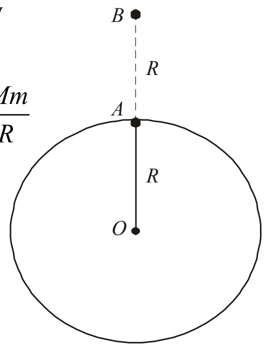
15. **(a)** Change in potential energy

$\therefore\ \Delta U = U_f - U_i$

$$= -GMm\left[\frac{1}{2R} - \frac{1}{R}\right] = \frac{GMm}{2R}$$

$\because\ g = \frac{GM}{R^2} \Rightarrow GM = gR^2$

$\therefore\ \Delta U = \frac{gR^2m}{2R} = \frac{mgR}{2}$



16. **(16.00)**

Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$$-\frac{GM_Em}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_Em}{R} + \frac{1}{2}mV^2$$

$$\left[\because \text{Gravitational potential energy} = -\frac{GMm}{r}\right]$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2} \Rightarrow V^2 = V_0^2 + \frac{9}{5}gR$$

$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16\ km/s$ $\quad [\because V_0 = 12$ km/s given]

17. **(a,b)** For $r > R$, the gravitational field, $F = \frac{GM}{r^2}$

$\therefore\ F_1 = \frac{GM}{r_1^2}$ and $F_2 = \frac{GM}{r_2^2}$ or, $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

For $r < R$, the gravitational field, $F = \frac{GM}{R^3} \times r$

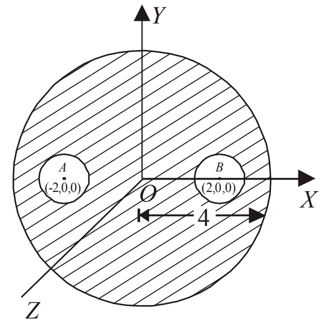
$\therefore\ F_1 = \frac{GM}{R^3} \times r_1$ and $F_2 = \frac{GM}{R^3} \times r_2$

$\therefore\ \frac{F_1}{F_2} = \frac{r_1}{r_2}$

18. **(a,c,d)** The gravitational field (E) intensity at the point O i.e., centre of a solid sphere is zero. Force acting on a test mass m_0 placed at O

$F = m_0E = m_0 \times 0 = 0$

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to x-axis.

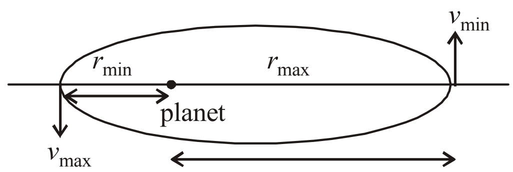


The gravitational field due to masses at A and B at 'O' is equal and opposite. As the plane of these circles is Y-Z ⊥ to X-axis so potential at any point on these two circles will be equal due to mass M and masses M and m.

Topic-5 : Motion of Satellites, Escape Speed and Orbital Velocity

1. **(a)** By angular momentum conservation

$mr_{min}v_{max} = mr_{max}v_{min}$



Given, $v_{min} = \frac{v_{max}}{6}$ $\quad \therefore\ \frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$

2. **(a)** Orbital speed of the body when it revolves very close to the surface of planet

$$V_0 = \sqrt{\frac{GM}{R}} \qquad \text{...(i)}$$

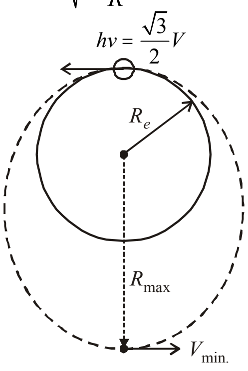
Here, G = gravitational constant
Escape speed from the surface of planet

$$V_e = \sqrt{\frac{2GM}{R}} \qquad \text{...(ii)}$$

Dividing (i) by (ii), we have

$$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

3. **(c)**



Orbital velocity, $V_0 = \sqrt{\frac{GM}{R_e}}$

From energy conversation,

$$-\frac{GMm}{R_e} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V\right)^2 = \frac{GMm}{R_{max}} + \frac{1}{2}mV_{min}^2 \qquad \text{...(1)}$$

From angular momentum conversation

$$\sqrt{\frac{3}{2}}VR_e = V_{min}R_{max} \qquad \text{...(2)}$$

Solving equation (1) and (2) we get,

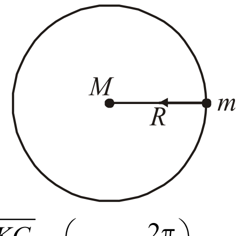
$R_{max} = 3R_e$

4. **(a)** According to question, mass density of a spherical galaxy varies as $\frac{k}{r}$.

Mass, $M = \int \rho dV$

$$\Rightarrow M = \int_0^{r=R_0} \frac{k}{r} 4\pi r^2\, dr \ \Rightarrow M = 4\pi k \int_0^{R_0} r\, dr$$

or, $M = \frac{4\pi k R_0^2}{2} = 2\pi k R^2$



$$F_G = \frac{GMm}{R_0^2} = m\omega_0^2 R \ (= F_C)$$

$$\Rightarrow \frac{G\frac{4\pi k R^2}{2}}{R^2} = \omega_0^2 R \Rightarrow \omega_0 = \sqrt{\frac{2\pi KG}{R}} \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}} \Rightarrow T^2 = \frac{2\pi R}{KG}$$

$\because 2\pi$, K and G are constants

$\therefore T^2 \propto R$.

5. **(d)** From law of conservation of momentum, $\vec{p}_i = \vec{p}_f$

$m_1u_1 + m_2u_2 = MV_f$

$$\Rightarrow v_f = \frac{\left(mv + \frac{mv}{4}\right)}{\frac{3m}{2}} = \frac{5v}{6}$$

Clearly, $v_f < v_i$ $\therefore$ Path will be elliptical

6. **(b)** K.E. of satellite is zero at earth surface and at height h from energy conservation

$U_{surface} + E_1 = U_h$

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re + h)}$$

$$\Rightarrow E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) \Rightarrow E_1 = \frac{GM_e m}{(R_e + h)} \times \frac{h}{R_e}$$

Gravitational attraction

$$F_G = ma_c = \frac{mv^2}{(R_e + h)} = \frac{GM_e m}{(R_e + h)^2}$$

$$mv^2 = \frac{GM_e m}{(R_e + h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e + h)}$$

$E_1 = E_2$

Clearly, $\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$

7. **(a)** Escape velocity of the planet A is $V_A = \sqrt{\frac{2GM_A}{R_A}}$

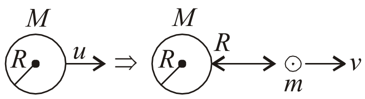
where M_A and R_A be the mass and radius of the planet A.
According to given problem

$$M_B = \frac{M_A}{2}, \ R_B = \frac{R_A}{2}$$

$$\therefore \ V_B = \sqrt{\frac{2G\frac{M_A}{2}}{\frac{R_A}{2}}} \quad \therefore \quad \frac{V_A}{V_B} = \sqrt{\frac{\frac{2GM_A}{R_A}}{\frac{2GM_A/2}{R_A/2}}} = \frac{n}{4} = 1$$

$\Rightarrow \quad n = 4$

8. **(b)**



$$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}m(v^2 - u^2) = \frac{-GMm}{2R}$$

$$\Rightarrow V = \sqrt{V = u^2 - \frac{GM}{R}} \qquad \text{...(i)}$$

$$v_0 = \sqrt{\frac{GM}{2R}} \quad \therefore \ v_{rad} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10\,v$$

Ejecting a rocket of mass $\frac{m}{10}$

$$\therefore \frac{9m}{10}\times\sqrt{\frac{GM}{2R}}=\frac{m}{10}\times v_\tau \Rightarrow V_\tau^2=81\frac{GM}{2R}$$

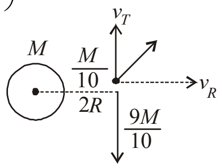
Kinetic energy of rocket,

$$KE_{rocket}=\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)$$

$$=\frac{1}{2}\times\frac{m}{10}\times\left((u^2-\frac{GM}{R})100+81\frac{GM}{R}\right)$$

$$=\frac{m}{20}\times 100\left(u^2-\frac{GM}{R}+\frac{81}{200}\frac{GM}{R}\right)$$

$$=5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$$



9. **(d)** Time period of revolution of satellite,

$$T=\frac{2\pi r}{v}$$

$$v=\sqrt{\frac{GM}{r}} \qquad \therefore T=2\pi r\sqrt{\frac{r}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

Substituting the values, we get

$$T=2\pi\sqrt{\frac{(202)^3\times 10^{12}}{6.67\times 10^{-11}\times 8\times 10^{22}}}\text{ sec}$$

T = 7812.2 s

$T \simeq 2.17$ hr $\Rightarrow$ 11 revolutions.

10. **(d)** Escape velocity,

$$v_c=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2G\rho V}{R}}=\sqrt{\frac{2GS\times 4\pi R^3}{R}}=\sqrt{\frac{8}{3}\pi\rho GR^2}$$

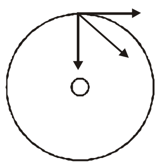
For moon, $v_c'=\sqrt{\frac{8}{3}\pi\rho GR_m^2}$

Given, $\frac{4}{3}\pi R^3=64\times\frac{4}{3}\pi R_m^3$ or $R_m=\frac{R}{4}$

$$\therefore \quad v_e'=\sqrt{\frac{8}{3}\pi\rho G\left(\frac{R}{4}\right)^2}=\frac{v_c}{4}$$

$$\frac{E}{E'}=\frac{\frac{1}{2}mv_e^2}{\frac{1}{2}mv_e'^2}=\frac{v_e^2}{v_c'^2}=\frac{v_e}{\left(\frac{v_e}{4}\right)}=16 \quad \text{or} \quad E'=\frac{E}{16}$$

11. **(b)** $m v\hat{i}+m v\hat{j}=2m\vec{v}$



$$\Rightarrow \vec{v}=\frac{v}{2}\hat{i}+\frac{v}{2}\hat{j}$$

$$\Rightarrow \vec{v}=\sqrt{\left(\frac{v}{2}\right)^2+\left(\frac{v}{2}\right)^2}=\frac{v}{\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\times\sqrt{\frac{GM}{R}}$$

12. **(b)** Orbital, velocity, $v=\sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$$T_A=\frac{1}{2}m_A V_A^2$$

Kinetic energy of satellite B,

$$T_B=\frac{1}{2}m_B V_B^2 \quad \Rightarrow \frac{T_A}{T_B}=\frac{m\times\frac{GM}{R}}{2m\times\frac{GM}{2R}}=1$$

13. **(d)** For a satellite orbiting close to the earth, orbital velocity is given by

$$v_0=\sqrt{g(R+h)}\approx\sqrt{gR}$$

Escape velocity (v_e) is

$$v_e=\sqrt{2g(R+h)}\approx\sqrt{2gR} \quad [\because h<<R]$$

$$\Delta v=v_e-v_0=(\sqrt{2}-1)\sqrt{gR}$$

14. **(b)** At height r from center of earth, orbital velocity

$$v=\sqrt{\frac{GM}{r}}$$

By principle of energy conservation

$$\text{KE of 'm'}+\left(-\frac{GMm}{r}\right)=0+0$$

($\because$ At infinity, PE = KE = 0)

$$\text{or} \quad \text{KE of 'm'}=\frac{GMm}{r}=\left(\sqrt{\frac{GM}{r}}\right)^2 m=mv^2$$

15. **(d)** Let M is mass of star m is mass of meteroite

By energy convervation between 0 and ∞.

$$-\frac{GMm}{r}+\frac{-GMm}{r}+\frac{1}{2}mV_{esc}^2=0+0$$

$$\therefore v=\sqrt{\frac{4GM}{r}}=\sqrt{\frac{4\times 6.67\times 10^{-11}\times 3\times 10^{31}}{10^{11}}}$$

$$\simeq 2.8\times 10^5 \text{ m/s}$$

16. **(b)** Applying energy conservation

$$\frac{1}{2}mV_s^2-\frac{GM_em}{R_e}-\frac{GM_em\times 3\times 10^5}{2.5\times 10^4 R_e}=0$$

$$\frac{V_s^2}{2}=\frac{GM_e}{R_e}\left[1+\frac{3\times 10^5}{2.5\times 10^4}\right]$$

$$V_s=\sqrt{13\left(\frac{2GM_e}{R_e}\right)}\left(\because V_e=\sqrt{\frac{2Gm_e}{R_e}}=11.2\text{ km/s}\right)$$

$$\therefore \quad V_s=\sqrt{13}\times 11.2\approx 42\text{ m/s}$$

17. **(b)** For h << R, the orbital velocity is $\sqrt{gR}$

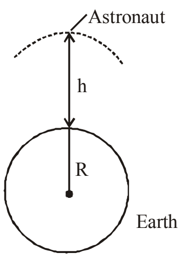
Escape velocity = $\sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$=\sqrt{2gR}-\sqrt{gR}=\sqrt{gR}\,(\sqrt{2}-1)$

18. **(c)** According to universal law of Gravitation,

Gravitational force $F=\frac{GMm}{(R+h)^2}$



19. **(a)** $F=\frac{2GM}{Lr}m$ or, $\frac{mv^2}{r}=\frac{2GM}{Lr}m$

$$mr\left(\frac{2\pi}{T}\right)^2=\frac{2GMm}{Lr}\left[\because v=r\omega \text{ and } \omega=\frac{2\pi}{T}\right]$$

$\Rightarrow \quad T\propto r$

20. **(a)** As we know,

Gravitational potential energy $= \frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

$$E_i = E_f$$

Therefore minimum required energy, $K = \frac{5GMm}{6R}$

21. **(b)** The relation between escape speed V_e and orbital velocity $V_e = \sqrt{2}V$. The kinetic energy at the time of ejection

$$KE = \frac{1}{2}mV_e^2 = \frac{1}{2}m(\sqrt{2}V)^2 = mV^2$$

22. **(2)** Let h be the height to which the bullet rises with the height acceleration due to gravity varies as

$$g^1 = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow \frac{g}{4} = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow h = R$$

We know escape speed, $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy principle

Loss of kinetic energy = gain in gravitaional potential energy

$$\therefore \quad \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\therefore \quad v = \sqrt{\frac{GM}{R}} \qquad ...(ii)$$

Comparing eq. (i) & (ii) we get $N = 2$

23. **(3)** We know that escape speed $v = \sqrt{2gR}$

$$\therefore \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g}\times\frac{R_p}{R}} \qquad ...(i)$$

Given $\frac{g_p}{g_e} = \frac{\sqrt{6}}{11}$ ...(ii)

Also $g = \frac{4}{3}\pi G\rho R \quad \therefore \frac{g_p}{g_e} = \frac{\rho_p}{\rho}\times\frac{R_p}{R}$

$$\therefore \frac{\sqrt{6}}{11} = \frac{2}{3}\times\frac{R_p}{R} \qquad \left[\because \frac{\rho_p}{\rho} = \frac{2}{3}\text{(given)}\right]$$

$$\therefore \frac{R_p}{R} = \frac{3\sqrt{6}}{22} \qquad ...(iii)$$

From (i), (ii) & (iii)

$$\frac{v_p}{v_e} = \sqrt{\frac{\sqrt{6}}{11}\times\frac{3\sqrt{6}}{22}} = \sqrt{\frac{3\times 6}{11\times 22}} = \frac{3}{11}$$

$$\therefore v_p = \frac{3}{11}\times v_e = \frac{3}{11}\times 11\text{km/s} = \mathbf{3}\text{ km/s}$$

24. **(R)** From conservation of mechanical energy

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{(R+h)}$$

KE provided $= \frac{1}{2}\times$ KE of escape

or $\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{GMm}{R}$

$$\therefore \quad \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow \quad -\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R+h = 2R \quad \text{or, } h = R$$

25. $\left(\sqrt{\frac{\mathbf{4G}}{\mathbf{d}}(\mathbf{M_1}+\mathbf{M_2})}\right)$ Total mechanical energy mass m at a midway point between the centres of earth and moon

$$= -\left\{\left[-\frac{GM_1M_2}{d} - \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}\right] - \left[-\frac{GM_1M_2}{d}\right]\right\}$$

$$= \frac{Gm}{d/2}(M_1+M_2) \text{ (= Binding energy)}$$

Kinetic energy required to escape the mass 'm' to infinity

$$\therefore \quad \frac{1}{2}mV_e^2 = \frac{Gm}{d/2}(M_1+M_2)$$

where V_e is the velocity with which mass m is projected.

$$\Rightarrow \quad V_e = \sqrt{\frac{4G}{d}(M_1+M_2)}$$

26. **(8.48 h)** According to Kepler's law $T^2 \propto R^3$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}.$$

Here $R_1 = R + 6R = 7R$ and $R_2 = 2.5R + R = 3.5\,R$

$T_1 = 24$ hours

$$\therefore \quad \frac{24\times 24}{T_2^2} = \frac{7\times 7\times 7\times R^3}{3.5\times 3.5\times 3.5\times R^3} \Rightarrow T_2 = 8.48\text{ h}$$

27. **False** New Delhi is not on the equatorial plane. Geostationary satellite is launched on the equatorial plane.

28. **(b, d)** Here Planets P and Q have the same uniform density 'ρ' and surface areas A and $4A$ respectively. Let the mass of P, M_P be m.

Then $m = \rho\times\frac{4}{3}\pi r^3 = \rho\times\frac{4}{3}\pi\left[\frac{A}{4\pi}\right]^{3/2}$

The mass of $M_Q = \rho\times\frac{4}{3}\pi\left[\frac{4A}{4\pi}\right]^{3/2} = 8$ m

$\therefore$ The mass of Planet R = 8 m + m = 9 m

If the radius of $P = r$

Then the radius of $Q = 2r$

$$\left[\because r_Q = \left(\frac{4A}{4\pi}\right)^{3/2} = 2\left(\frac{A}{4\pi}\right)^{3/2}\right]$$

and radius of $R = 9^{1/3}r$

$$\left[\begin{array}{l}\because M_R = M_P + M_Q \\ r_R^3 = r^3 + (2r)^3 = 9r^3\end{array}\right]$$

As we know, escape velocity from the planet

$$V_e = \sqrt{\frac{2GM}{R}} \quad \therefore\ v_P = \sqrt{\frac{2GM_P}{R_p}} = \sqrt{\frac{2Gm}{r}}$$

$$v_Q = \sqrt{\frac{2GM_Q}{R_Q}} = \sqrt{\frac{2G\,(8m)}{2r}} = 2v_P$$

$$v_R = \sqrt{\frac{2G\,(9\text{ m})}{9^{1/3}r}} = 9^{1/3}v_P$$

29. **(a, c)** Force on satellite is always towards earth which attracts the satellite with the gravitational force F, therefore, acceleration of satellite S is always directed towards centre of the earth.
The net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout.
As the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

30. **(b)** $\because$ Orbital velocity,

$$V = \sqrt{\frac{GM}{R}}, \text{ or, } V \propto \frac{1}{\sqrt{R}} \quad \therefore\ \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_1 v_2 R_2} = \frac{2\times2\times1}{1\times1\times4} = \frac{1}{1}$$

Kinetic energy, $K = \dfrac{GMm}{R}$.

$$\therefore\ \frac{k_1}{k_2} = \frac{m_1}{m_2}\times\frac{R_1}{R_2} = \frac{2\times4}{1\times1} = \frac{8}{1}$$

From Kepler's law of planetary motion.

$$T^2 \propto R^3 \quad \therefore\ \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \frac{1}{8}$$

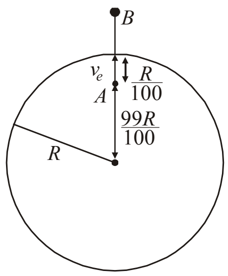
31. **(a)** An astronant in an orbiting space station above the earth experiences weight less as he is in a state of free fall. The force acting on astronant is utilised in providing necessary centripetal force.

32. Applying mechanical energy conservation,
Total energy at A = Total energy at B
$(\text{K.E.})_A + (\text{P.E.})_A = (\text{P.E.})_B$

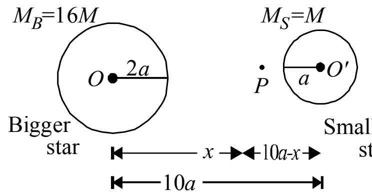
$$\Rightarrow\ \frac{1}{2}m\times\frac{2GM}{R} + \left[\frac{-GMm}{2R^3}\left\{3R^2 - \left(\frac{99R}{100}\right)^2\right\}\right] = -\frac{GMm}{R+h}$$

Solving above equation, we get $h = 99.5\,R$.

(Maximum height attained by the body from the surface of the earth)



33. Let the force of attraction is zero at a distance x from the bigger star.



Then force on mass m due to bigger star = force on mass m due to small star

$$\frac{GM_B m}{x^2} = \frac{GM_S m}{(10a-x)^2} \Rightarrow \frac{16M}{x^2} = \frac{M}{(10a-x)^2} \Rightarrow x = 8a$$

Let v denote velocity with which the body of mass m is fired so that it crosses P along OP. The energy is conserved.
The energy of the system (of these masses) initially = final energy when m is at P

$$-\frac{GM_B M_S}{10a} - \frac{GM_B m}{2a} - \frac{GM_S m}{8a} + \frac{1}{2}mv^2$$

$$= -\frac{GM_B M_S}{10a} - \frac{GM_B m}{8a} - \frac{GM_S m}{2a}$$

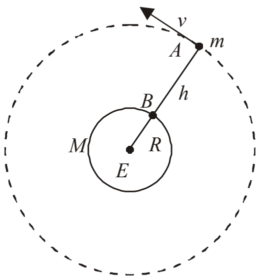
$$[\because M_B = 16M;\ \ M_S = M]$$

$$\therefore\ v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

34. (i) Here centripetal force is provided by the gravitational pull

$$\therefore\ \frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$\Rightarrow\ v^2 = \frac{GM}{R+h}$$



Given $v = \dfrac{1}{2}v_e = \dfrac{1}{2}\sqrt{\dfrac{2GM}{R}}$

$$\therefore\ \frac{1}{4}\left(\frac{2GM}{R}\right) = \frac{GM}{R+h} \Rightarrow 2R + 2h = 4R \therefore h = R = 6400\text{ km.}$$

(ii) Let V be the speed with which the satellite hits the surface of the earth. If the satellite is stopped, its kinetic energy is zero. When it falls freely on the Earth, its potential energy decreases and converts into kinetic energy.

$\therefore$ $(\text{P.E.})_A - (\text{P.E.})_B = \text{K.E.}$

$$\Rightarrow\ \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow\ v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{9.8\times6.4\times10^6}$$

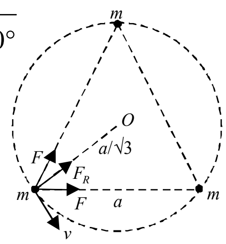
$= 7920\text{ m/s} = 7.92\text{ km/s}$

35. Let the initial velocity that should be given to each particle be 'V'. The centripetal force is provided by the resultant gravitational attraction of the two masses.

$$F_R = \sqrt{F^2 + F^2 + 2F^2\cos 60°}$$

$$= \sqrt{3}\,F = \sqrt{3}\,G\frac{m\times m}{a^2}$$

$$\therefore\ \sqrt{3}\,G\frac{m^2}{a^2} = \frac{mv^2}{r}$$



$$\left(\frac{mv^2}{r} = \text{centripetal force}\right)$$

Radius of the circular path $r = \frac{2}{3}\sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{\sqrt{3}}$

$$v^2 = \frac{\sqrt{3}Gmr}{a^2} = \frac{\sqrt{3}\,G\,ma}{a^2 \times \sqrt{3}} \Rightarrow v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

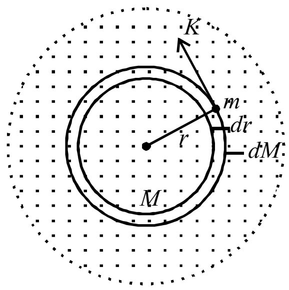
$$T = \frac{2\pi r}{v} = \frac{2\pi a/\sqrt{3}}{\sqrt{\frac{Gm}{a}}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$$

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. **(b)** Gravitational pull of the mass 'M' present in the sphere of radius 'r'. Provide the required centripetal force of particle of mass 'm' to revalue in a circular path.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2r} \Rightarrow K = \frac{GMm}{2r}$$

$\therefore \quad M = \frac{2Kr}{Gm}$



Differentiating the above equation w.r.t 'r' we get

$$\frac{dM}{dr} = \frac{2K}{Gm}$$

or $\quad dM = \frac{2K}{Gm}dr$

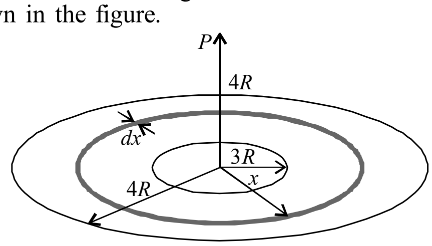
$\therefore \quad 4\pi r^2 dr\,\rho = \frac{2K}{Gm}dr \quad \Rightarrow \quad \rho = \frac{K}{2\pi r^2 mG}$

$\therefore \quad \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G} \quad$ or $\quad \frac{\rho(r)}{m} = \frac{K}{2\pi r^2 m^2 G}$

2. **(a)** Mass per unit area of the shaded part.

$$\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{\pi((4R)^2 - (3R)^2)} = \frac{M}{7\pi R^2}$$

Let us consider a ring of radius x and thickness dx as shown in the figure.



Mass of the ring, $dM = \sigma 2\pi x dx = \frac{2\pi Mxdx}{7\pi R^2}$

Potential at point P due to shaded part

$$V_P = \int_{3R}^{4R} -\frac{GdM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM2\pi}{7\pi R^2}\int_{3R}^{4R}\frac{xdx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

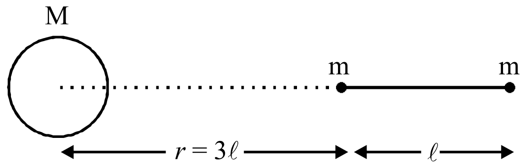
$$V_P = -\frac{GM2\pi}{7\pi R^2}\left[\sqrt{16R^2 + x^2}\right]_{3R}^{4R} = -\frac{2GM}{7R}(4\sqrt{2} - 5)$$

Workdone in moving a unit mass from P to $\infty = V_\infty - V_P$

or $\quad W_{P\infty} = 0 - \left(\frac{-2GM}{7R}(4\sqrt{2} - 5)\right) = \frac{2GM}{7R}(4\sqrt{2} - 5)$

3. **(7)** For point mass at distance $r = 4l$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma$$



For point mass at distance $r = 3l$

$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma$$

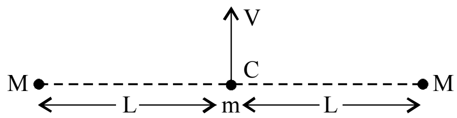
$$\therefore \quad \frac{GMm}{(4\ell)^2} + \frac{Gmm}{\ell^2} = \frac{GMm}{(3\ell)^2} - \frac{Gmm}{\ell^2}$$

$$\therefore \quad 2m = M\left[\frac{1}{9} - \frac{1}{16}\right] \Rightarrow m = \frac{7M}{288} \quad \therefore K = 7$$

4. **(b)** From conservation of mechanical energy,

$$\frac{-GMm}{L} - \frac{GMm}{L} + \frac{1}{2}mv^2 = 0 + 0$$

or, $\quad \frac{1}{2}mv^2 = \frac{2GMm}{L} \quad \therefore \quad V = \sqrt{\frac{4GM}{L}} = 2\sqrt{\frac{GM}{L}}$



8 Mechanical Properties of Solids

Topic-1 : Hook's Law & Young's Modulus

1. **(c)** Given : $U = \frac{-A}{r^6} + \frac{B}{r^{12}}$

For equilibrium,

$$F = \frac{dU}{dr} = -(A(-6r^{-7})) + B(-12r^{-13}) = 0$$

$$\Rightarrow 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}} \Rightarrow \frac{6A}{12B} = \frac{1}{r^6}$$

$\therefore$ Separation between molecules, $r = \left(\frac{2B}{A}\right)^{1/6}$

Potential energy,

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

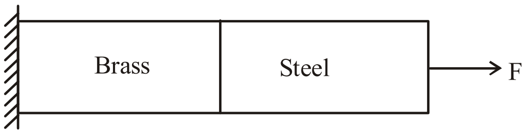
$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

2. **(c)** Here $L\alpha(\Delta T) = \frac{FL}{AY}$ $\therefore$ $\alpha = \frac{FL}{AYT} = \frac{F}{\pi r^2 YT}$

Coefficient of volume expression

$$\gamma = 3\alpha = \frac{3F}{\pi r^2 YT}.$$

3. **(Bonus)**



Young's modulus, $Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$

Let σ be the stress

Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] \quad [\because L_1 = L_2 = 1\text{m}]$$

$$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$$

$$= 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9 = 8 \times 10^6 \frac{N}{m^2}$$

4. **(c)** Given,
Radius of wire, $r = 2$ mm
Mass of the load m = 4 kg

$$\text{Stress} = \frac{F}{A} = \frac{mg}{\pi(r)^2}$$

$$= \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{ N/m}^2$$

5. **(c)** $\Delta l_1 = \Delta l_2$

or $\frac{Fl_1}{\pi r_1^2 y_1} = \frac{Fl_2}{\pi r_2^2 y_2}$ or $\frac{2}{R^2 \times 7} = \frac{1.5}{2^2 \times 4}$

$\therefore R = 1.75$ mm

6. **(b)** For same material the ratio of stress to strain is same
For first cube

$$\text{Stress}_1 = \frac{\text{force}_1}{\text{area}_1} = \frac{10^5}{(0.1^2)}$$

$$\text{Strain}_1 = \frac{\text{change in length}_1}{\text{original length}_1} = \frac{0.5 \times 10^{-2}}{0.1}$$

For second block,

$$\text{Stress}_2 = \frac{\text{force}_2}{\text{area}_2} = \frac{10^5}{(0.2^2)}$$

$$\text{Strain}_2 = \frac{\text{change in length}_2}{\text{original length}_2} = \frac{x}{0.2}$$

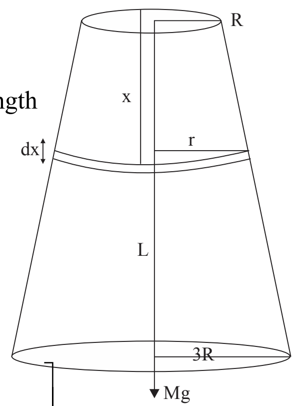
x is the displacement for second block.

For same material, $\frac{\text{stress}_1}{\text{strain}_1} = \frac{\text{stress}_2}{\text{strain}_2}$

or, $$\frac{\frac{10.5}{(0.1)^2}}{\frac{0.5 \times 10^{-2}}{0.1}} = \frac{\frac{10^5}{(0.2)^2}}{\frac{x}{0.2}}$$

Solving we get, $x = 0.25$ cm

7. **(c)** Consider a small element dx of radius r,



$$r = \frac{2R}{L}x + R$$

At equilibrium change in length of the wire

$$\int_0^1 dL = \int \frac{Mg\,dx}{\pi\left[\frac{2R}{L}x + R\right]^2 y}$$

Taking limit from 0 to L

$$\Delta L = \frac{Mg}{\pi y}\left[-\frac{1}{\left[\frac{2Rx}{L} + R\right]_0^L} \times \frac{L}{2R}\right] = \frac{MgL}{3\pi R^2 y}$$

The equilibrium extended length of wire = L + ΔL

$$= L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{1}{3}\frac{Mg}{\pi Y R^2}\right)$$

8. **(a)** Young's modulus $Y = \dfrac{\text{stress}}{\text{strain}}$

Stress = Y × strain
Stress in steel wire = Applied pressure
Pressure = stress = Y × strain

$$\text{Strain} = \frac{\Delta L}{L} = \alpha \Delta T$$

(As length is constant)
$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100 = 2.2 \times 10^{8}$ Pa

9. **(b)** Tension in the wire, $T = \left(\dfrac{2mM}{m+M}\right)g$

$$\text{Stress} = \frac{\text{Force / Tension}}{\text{Area}} = \frac{2mM}{A(m+M)}g$$

$$= \frac{2(m \times 2m)g}{A(m+2m)} \quad (M = 2\text{ m given})$$

$$= \frac{4m^2}{3mA}g = \frac{4mg}{3A}$$

10. **(c)** According to question,

$$\frac{\ell_s}{\ell_b} = a,\ \frac{r_s}{r_b} = b,\ \frac{y_s}{y_b} = c,\ \frac{\Delta\ell s}{\Delta\ell_b} = ?$$

As, $y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{Ay}$

$$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 . y_s} \quad [\because F_s = (M + 2M)g]$$

$$\Delta\ell_b = \frac{2Mg\ell_b}{\pi r_b^2 . y_b} \quad [\because F_b = 2Mg]$$

$$\therefore \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\dfrac{3Mg\ell_s}{\pi r_s^2 . y_s}}{\dfrac{2Mg.\ell_b}{\pi r_b^2 . y_b}} = \frac{3a}{2b^2C}$$

11. **(c)** Using, $Y = \dfrac{F/A}{\Delta\ell/\ell_0}$

$$Y = \frac{F/\pi(2R)^2}{\Delta\ell_1/2L} = \frac{F/\pi R^2}{\Delta\ell_2/L} \qquad \therefore \frac{\Delta\ell_2}{\Delta\ell_1} = 2$$

12. **(a)** Using, $Y = \dfrac{F}{A}\Big/\dfrac{\Delta\ell}{\ell} = \dfrac{F}{A}.\dfrac{\ell}{\Delta\ell} = \dfrac{20 \times 1}{10^{-6} \times 10^{-4}}$

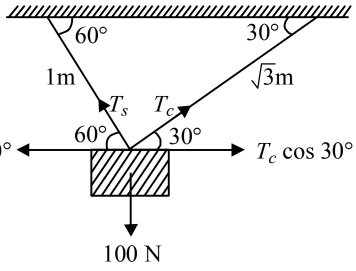
$$= 2 \times 10^{11}\,\text{N/m}^2.$$

13. **(a)** Using, $Y = \dfrac{T/A}{\Delta\ell/\ell} \Rightarrow \Delta\ell = \dfrac{T \times \ell}{A \times Y} = \dfrac{T}{Y} \times \dfrac{\ell}{A}$

$$\therefore \quad \Delta\ell \propto \frac{\ell}{A}. \qquad \left(\because \frac{T}{Y} \text{ is constant}\right)$$

$\dfrac{\ell}{A}$ is largest in (a) hence largest extension.

14. **(2)** Given : $l_c = \sqrt{3}$ m; $l_s = 1$ m; $Y_c = 1 \times 10^{11}$ N/m^2 and $Y_s = 2 \times 10^{11}$ N/m^2.



At equilibrium, $T_s \cos 60° = T_c \cos 30°$

$$\Rightarrow \frac{T_s}{2} = \frac{T_c\sqrt{3}}{2} \Rightarrow T_s = \sqrt{3}T_c \Rightarrow \frac{T_c}{T_s} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l_c}{l_s} = \frac{\sqrt{3}}{1} \text{ and } \frac{Y_c}{Y_s} = \frac{1 \times 10^{11}}{2 \times 10^{11}} = \frac{1}{2}$$

From, $Y = \dfrac{Fl}{A\Delta l} \Rightarrow \Delta l = \dfrac{Fl}{AY}$

Here, $A_s = A_c$

$$\therefore \frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right) \times \left(\frac{l_c}{l_s}\right) \times \left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{1}\right) \times \left(\frac{2}{1}\right) = 2$$

15. **(4)** Given : Wire length, $l = 0.3$ m

Mass of the body, m = 10 kg

Breaking stress, $\sigma = 4.8 \times 10^7$ Nm^{-2}

Area of cross-section, $a = 10^{-2}$ cm^2

Maximum angular speed ω = ?

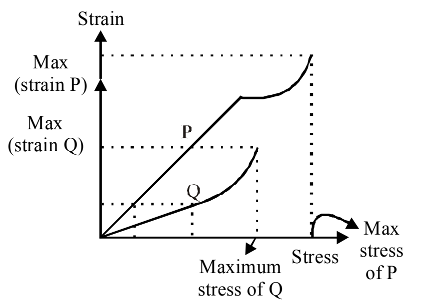
$T = Ml\omega^2$

$$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$$

$$\frac{ml\omega^2}{A} \le 48 \times 10^7 \Rightarrow \omega^2 \le \frac{(48 \times 10^7)A}{ml}$$

$$\Rightarrow \omega^2 \le \frac{(48 \times 10^7)(10^{-6})}{10 \times 3} = 16 \Rightarrow \omega_{max} = 4 \text{ rad/s}$$

16. **(a, b)** From graph, the maximum stress that P can withstand before breaking is greater than Q.



The strain of P is more than Q therefore P is more ductile.

$\because$ $Y = \frac{\text{stress}}{\text{strain}}$ So a given strain, stress is more for Q.

$\therefore$ $Y_Q > Y_P$.

Topic-2 : Bulk & Rigidity Modulus and Work Done in Stretching a Wire

1. **(a)** If force F acts along the length L of the wire of cross-section A, then energy stored in unit volume of wire is given by

Energy density $= \frac{1}{2}$ stress $\times$ strain

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{F}{AY} \left(\because \text{ stress} = \frac{F}{A} \text{ and strain} = \frac{X}{AY} \right)$$

$$= \frac{1}{2}\frac{F^2}{A^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{(\pi d^2)^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{\pi d^4 Y}$$

If u_1 and u_2 are the densities of two wires, then

$$\frac{u_1}{u_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{d_1}{d_2} = (4)^{1/4} \Rightarrow \frac{d_1}{d_2} = \sqrt{2} : 1$$

2. **(a)** When a catapult is stretched up to length l, then the stored energy in it = Δk. E $\Rightarrow$

$$\frac{1}{2} \cdot \left(\frac{YA}{L}\right)(\Delta l)^2 = \frac{1}{2} m v^2 \quad \Rightarrow y = \frac{m v^2 L}{\Delta (\Delta l)^2}$$

m = 0.02 kg

v = 20 ms^{-1}

L = 0.42 m

A = $(\pi d^2)/(4)$

d = 6×10^{-3} m

Δl = 0.2 m

$$y = \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04} = 2.3 \times 10^6 \text{ N/m}^2$$

So, order is 10^6.

3. **(c)** Bulk modulus, $K = \frac{\text{volumetric stress}}{\text{volumetric strain}}$

$$K = \frac{mg}{a\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \frac{dV}{V} = \frac{mg}{Ka} \quad \text{....(i)}$$

volume of sphere, $V = \frac{4}{3}\pi R^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\frac{3dr}{r} = \frac{mg}{Ka}$

$\therefore$ $\frac{dr}{r} = \frac{mg}{3Ka}$ (fractional decrement in radius)

4. **(d)** Stress $= \frac{\text{Normal force}}{\text{Area}} = \frac{N}{A} = \frac{N}{(2pa)b}$

Stress = B×strain

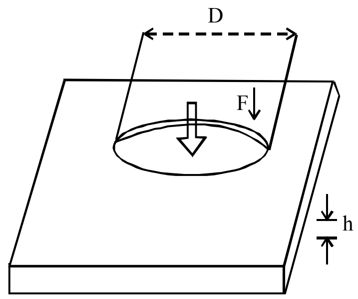
$$\frac{N}{(2pa)b} = B\frac{2paDa' b}{pa^2 b}$$

$$Þ \ N = B\frac{(2pa)^2 Dab^2}{pa^2 b}$$

Force needed to push the cork.

f = mN = $m4p$bDaB = $(4\pi\mu Bb)\Delta a$

5. **(c)** Shearing strain is created along the side surface of the punched disk. Note that the forces exerted on the disk are exerted along the circumference of the disk, and the total force exerted on its center only.



Let us assume that the shearing stress along the side surface of the disk is uniform, then

$$F = \int_{\text{surface}} dF_{max} = \int_{\text{surface}} \sigma_{max} dA = \sigma_{max} \int_{\text{surface}} dA$$

$$= \int \sigma_{max} . A = \sigma_{max} . 2\pi \left(\frac{D}{2}\right) h$$

$$= 3.5 \times 10^8 \times \left(\frac{1}{2} \times 10^{-2}\right) \times 0.3 \times 10^{-2} \times 2\pi$$

$$= 3.297 \times 10^4 \simeq 3.3 \times 10^4 \text{ N}$$

6. $\frac{1}{2}\left(\frac{YA}{L}\right) x^2$

Work done, $W = \frac{1}{2} K x^2$

where $K = \frac{YA}{L}$ and x = extension in wire

$$\therefore \quad W = \frac{1}{2}\left(\frac{YA}{L}\right) x^2$$

9 Mechanical Properties of Fluids

Topic-1 : Pressure, Density, Pascal's Law and Archimedes' Principle

1. **(a)** In equilibrium, $mg = F_e$

$F_B = V\rho_0 g$ and mass = volume × density

$$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$$

Given, relative density, $\dfrac{\rho_0}{\rho_w} = \dfrac{27}{8}$

$$\Rightarrow \left[1 - \left(\frac{r}{R}\right)^3\right]\frac{27}{8}\rho_w = \rho_w$$

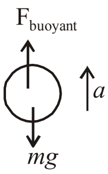
$$\Rightarrow 1 - \frac{r^3}{R^3} = \frac{9}{27} \Rightarrow 1 - \frac{1}{3} = \frac{r^3}{R^3} \Rightarrow \frac{2}{3} = \frac{r^3}{R^3}$$

$$\Rightarrow \frac{r}{R} = \left(\frac{2}{3}\right)^{1/3} \Rightarrow 1 - \frac{r^3}{R^3} = \frac{8}{27}$$

$$\Rightarrow \frac{r^3}{R^3} = 1 - \frac{8}{27} = \frac{19}{27} \qquad \therefore r = 0.89R = \frac{8}{9}R.$$

2. **(c)** Given :
Radius of air bubble = 1 cm,
Upward acceleration of bubble, $a = 9.8$ cm/s^2,

$\rho_{water} = 1$ g cm^{-3}



Volume $V = \dfrac{4\pi}{3}r^3 = \dfrac{4\pi}{3} \times (1)^3 = 4.19$ cm^3

$$F_{buoyant} - mg = ma \Rightarrow m = \frac{F_{buoyant}}{g + a}$$

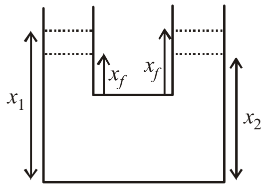
$$\therefore m = \frac{(V\rho_\omega g)}{g + a} = \frac{V\rho_\omega}{1 + \frac{a}{g}} = \frac{(4.19) \times 1}{1 + \frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15\,\text{g}$$

3. **(d)** Initial potential energy,

$$U_1 = (\rho S x_1)g \cdot \frac{x_1}{2} + (\rho S x_2)g \cdot \frac{x_2}{2}$$

Final potential energy,

$$U_f = (\rho S x_f)g \cdot \frac{x_f}{2} \times 2$$



By volume conservation,

$$Sx_1 + Sx_2 = S(2x_f)$$

$$x_f = \frac{x_1 + x_2}{2}$$

When valve is opened loss in potentail energy occur till water level become same.

$$\Delta U = U_i - U_f$$

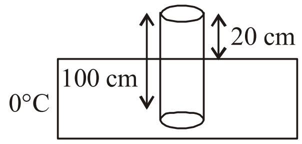
$$\Delta U = \rho S g\left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2}\right) - x_f^2\right]$$

$$= \rho S g\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2}\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2\right] = \frac{\rho S g}{4}(x_1 - x_2)^2$$

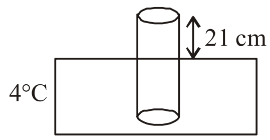
4. **(c)** When cylinder is floating in water at 0°C

Net thrust $= A(h_2 - h_1)\rho_{0°c}\,g$
$= A(100 - 80)\rho_{0°c}\,g$



When cylinder is floating in water at 4° C

Net thrust $= A(h_2 - h_1)\rho_{4°c}\,g$
$= A(100 - 21)\rho_{4°c}\,g$



$$\therefore \frac{\rho_{4°c}}{\rho_{0°c}} = \frac{80}{79} = 1.01$$

5. **(c)** For minimum density of liquid, solid sphere has to float (completely immersed) in the liquid.
$mg = F_B$ (also $V_{immersed} = V_{total}$)

$$\text{or} \int \rho\, dV = \frac{4}{3}\pi R^3 \rho_\ell$$

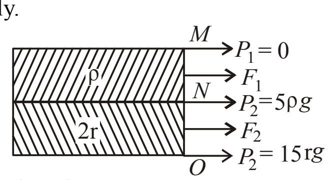
$$\left[\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right) 0 < r \le R \text{ given}\right]$$

$$\Rightarrow \int_0^R \rho_0 4\pi\left(1 - \frac{r^2}{R^2}\right).\, r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell$$

$$\Rightarrow 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]_0^R = \frac{4}{3}\pi R^3 \rho_\ell$$

$$\frac{4\pi\rho_0 R^3}{3} \times \frac{2}{5} = \frac{4}{3}\pi R^3 \rho_\ell \qquad \therefore \rho_\ell = \frac{2\rho_0}{5}$$

6. **(d)** Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.



Pressure is given by $P = \rho g h$

Now, $P_1 = 0$ ($\because$ $h = 0$)
$P_2 = \rho g(5)$
$P_3 = \rho g(15)$
$= 15\,\rho g$

Force on upper part, $F_1 = \dfrac{(P_1 + P_2)}{2}A$

Force on lower part, $F_2 = \dfrac{(P_2 + P_3)}{2}A$

$$\therefore \frac{F_1}{F_2} = \frac{5\rho g}{20\rho g} = \frac{5}{20} = \frac{1}{4}$$

7. **(b)** According to question, $h_1 + h_2 = 0.29 \times 2 + 0.1$

$\Rightarrow h_1 + h_2 = 0.68$ ...(1)

Pressure in two arms of u-tube is equal

$\therefore P_0 + \rho_k g(0.1) + \rho_w g(h_1 - 0.1) = P_0 + \rho_w g h_2$

[ρ_k = density of kerosene & ρ_w = density of water]

$\Rightarrow \rho_k g(0.1) + \rho_w g h_1 - \rho_w g \times (0.1) = \rho_w g h$

$\Rightarrow 800 \times 10 \times 0.1 + 1000 \times 10 \times h_1 - 1000 \times 10 \times 0.1 = 1000 \times 10 \times h_2$

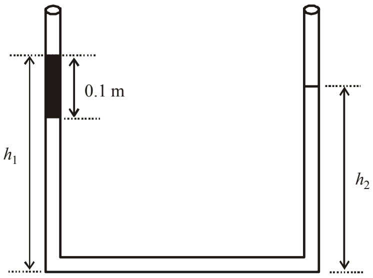
$\Rightarrow 10000 (h_1 - h_2) = 200$

$\therefore h_1 - h_2 = 0.02$ ...(2)

From eqns (1) & (2)

$h_1 = 0.35$ and $h_2 = 0.33$

$$\therefore \frac{h_1}{h_2} = \frac{0.35}{0.33} = \frac{35}{33}$$



8. **(b)** When a body floats then the weight of the body = upthrust

$$\therefore (50)^3 \times \frac{30}{100} \times (1) \times g = M_{cube} g \quad ...(i)$$

Let m mass should be placed, then

$(50)^3 \times (1) \times g = (M_{cube} + m)g$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$\Rightarrow mg = (50)^3 \times g (1 - 0.3) = 125 \times 0.7 \times 10^3$ g

$\Rightarrow m = 87.5$ kg

9. **(c)** $Mg = \left(\frac{4V}{5}\right)\rho\omega g$

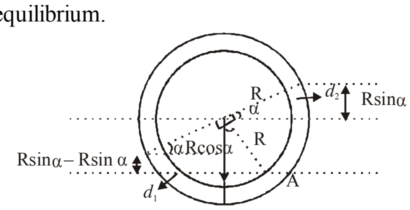
or $\left(\frac{M}{V}\right) = \frac{4\rho\omega}{5}$ or $\rho = \frac{4\rho\omega}{5}$

When block floats fully in water and oil, then

$Mg = F_{b_1} + F_{b_2}$

$(\rho V)g = \left(\frac{V}{2}\right)\rho_{oil}\, g + \frac{V}{2}\rho\omega g$ or $\rho_{oil} = \frac{3}{5}\rho\omega = 0.6\rho\omega$

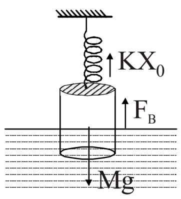
10. **(c)** Pressure at interface A must be same from both the sides to be in equilibrium.



$$\therefore (R\cos\alpha + R\sin\alpha)d_2 g = (R\cos\alpha - R\sin\alpha)d_1 g$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

11. **(c)** From figure, $kx_0 + F_B = Mg$



$kx_0 + \sigma\frac{L}{2}Ag = Mg$

[$\because$ mass = density × volume]

$\Rightarrow kx_0 = Mg - \sigma\frac{L}{2}Ag$

$$\Rightarrow x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, extension of the spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

12. **(a)** Let V_1 = total material volume of cylindrical shell

V_2 = total inside volume of cylindrical shell and

y = fraction of V_2 volume filled with water.

Total weight = Upthrust (Condition of floatation)

$$\therefore \quad V_1\rho_c g + (yV_2)(1)g = \left(\frac{V_1+V_2}{2}\right)(1)g$$

or $y = 0.5 + (0.5 - \rho_c)\frac{V_1}{V_2}$

So, if $\rho_c < 0.5$ then $y > 0.5$

13. **(a)** Bulk modulus,

$$B = \frac{\Delta p}{\Delta V/V} = \frac{(1.165\times10^5 - 1.01\times10^5)}{0.1} = 1.55 \times 10^5 \text{ Pa}$$

14. **(d)** When the coin falls from the top of block into water the block moves upwards because the weight of floating body becomes less and hence *l* decreases. Also, it will displace a volume of water, equal to its own volume, whereas when it is on the block it displaces more volume than to own volume because density of coin is greater than density of water. Hence *h* will also decrease.

15. **(d)** From Archimedes principle

Upthrust = wt. of fluid displaced

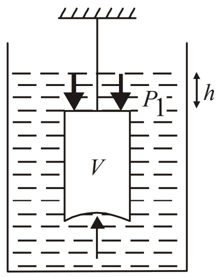
$F_{bottom} - F_{top} = V\rho g$

$\Rightarrow F_{bottom} = F_{top} + V\rho g$

$= P_1 \times A + V\rho g$

$= (h\rho g) \times (\pi R^2) + V\rho g$

$= \rho g\,[\pi R^2 h + V]$



16. **(a)** Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid.

$$D\left(\frac{A}{5} \times L \times g\right) = d\left(\frac{A}{5}\right)\left(\frac{3}{4}L\right)g + 2d\left(\frac{A}{5}\right)\left(\frac{L}{4}\right) \times g$$

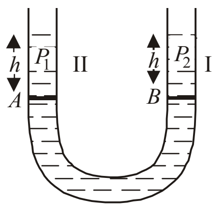
$$\therefore \quad D = \frac{5d}{4}$$

17. **(b)** Pressure in limb I at B = Pressure in limb II at A

$h\rho_1 g = h\rho_2 g$

$\Rightarrow \rho_1 = \rho_2$

Hence specific gravity of liquid II = *sp* gravity of liquid I = 1.1



18. **(0.24)**

As we know, bulk modules $B = \dfrac{P}{\left(-\dfrac{dv}{v}\right)}$

$$\Rightarrow \frac{dv}{v} = -\frac{P}{B} \Rightarrow 3\left(\frac{\Delta\ell}{\ell}\right) = -\frac{P}{B}$$

$$\Delta\ell = \left(\frac{-P}{B}\right)\frac{\ell}{3} = \left(\frac{\rho g h}{B}\right)\frac{\ell}{3}$$

$$= \frac{10^3 \times 10 \times 5 \times 10^3}{70 \times 10^9} \times \frac{1}{3} = 0.238 \times 10^{-3}$$

$\therefore \Delta\ell = 0.238$ mm $\simeq$ 0.24 mm

19. $K = \dfrac{-\Delta P}{\Delta V / V}$

where $\Delta P = \dfrac{Mg}{A}$ $\therefore$ $-\dfrac{\Delta V}{V} = \dfrac{Mg}{AK}$

$\Rightarrow -\dfrac{(V_f - V_i)}{V_i} = \dfrac{Mg}{AK}$ $\Rightarrow \dfrac{V_i - V_f}{V_i} = \dfrac{Mg}{AK}$

$$\Rightarrow \frac{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R - \delta R)^3}{\frac{4}{3}\pi R^3} = \frac{Mg}{AK}$$

$\Rightarrow \dfrac{R^3 - [R^3 - 3R^2\delta R]}{R^3} = \dfrac{Mg}{AK}$ $\Rightarrow \dfrac{\delta R}{R} = \dfrac{Mg}{3AK}$

20. **False,**

When the block of ice melts, the lead shot will ultimately sink in the water. When lead shot sinks, it will displace water equal to its own volume. But when lead shot was embedded in ice, it displaced more volume of water than its own volume because $d_{\text{lead}} > d_{\text{water}}$.

$\because$ volume $= \dfrac{\text{mass}}{\text{density}}$. Hence, level of water will fall.

21. **False,**

When temperature is increased, the density of mercury (ρ) decreases as $\rho_t = \rho_0 (1 - \gamma t)$ and hence, the level of mercury in barometer tube (h) increases

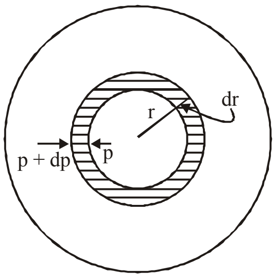
$\because$ $h\rho$ = constant

22. **False,**

The level of water will remain the same in the pond.

When the man drinks some water from the pond, his weight increases and therefore the boat will sink further. And this sinking of the boat will displace the same volume of water in pond as drunk by man.

23. **(b, c)** Let us consider an elemental mass dm shown in the shaded portion of the figure.



Here, $P\, 4\pi r^2 - (P + dP)\, 4\pi r^2$

$= \dfrac{GMr}{R^3} \rho\, (4\pi r^2)\, dr$

$\therefore -\displaystyle\int_0^P dp = \frac{GM\rho}{R^3} \int_R^r r\,dr$

$\therefore P = \dfrac{GM\rho}{2R^3}\left[R^2 - r^2\right]$

$$\therefore \frac{P\,(r = 3R/4)}{P\,(r = 2R/3)} = \frac{\left[R^2 - \frac{9R^2}{16}\right]}{\left[R^2 - \frac{4R^2}{9}\right]} = \frac{\frac{7R^2}{16}}{\frac{5R^2}{9}} = \frac{63}{80}$$

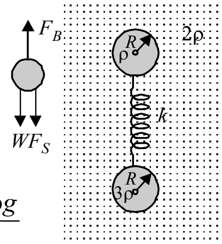
$$\text{and } \frac{P\,(r = 3R/5)}{P\,(r = 2R/5)} = \frac{\left[R^2 - \frac{9R^2}{25}\right]}{\left[R^2 - \frac{4R^2}{25}\right]} = \frac{16}{21}$$

24. **(a, d)** The complete system is as shown in figure.

Let x be the net elongation of the spring.

At equilibrium, for upper sphere

$W + F_S = F_B$



$\dfrac{4}{3}\pi R^3 rg + kx = \dfrac{4}{3}\pi R^3 (2\rho) g$

$\Rightarrow kx = \dfrac{4}{3}\pi R^3 2\rho g - \dfrac{4}{3}\pi R^3 \rho g$

$\Rightarrow kx = \dfrac{4\pi R^3 \rho g}{3}$ or $x = \dfrac{4\pi R^3 \rho g}{3k}$

25. **(c)** Let d be the density of the material of the sphere

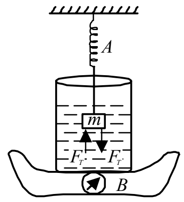
Weight of sphere

= Upthrust due to Hg + Upthrust due to oil

$Vdg = \dfrac{V}{2} d_{\text{Hg}}\, g + \dfrac{V}{2} d_{\text{oil}} \times g$

$\Rightarrow d = \dfrac{d_{\text{Hg}} + d_{\text{oil}}}{2} = \dfrac{13.6 + 0.8}{2} = 7.2\,\text{g/cm}^3$

26. **(b,c)** When the block of mass m is inside the liquid an upthrust F_T will act on the mass which will decrease the reading on A i.e., will read less than 2 kg. According to Newton's third law, to each and every action, there is equal and opposite reaction.



So by reaction an equal force will be exerted on the liquid of the beaker which will increase the reading in B i.e., will read more than 5 kg.

27. **(a)** The whole system falls freely under gravity, so $g = 0$

According to Archimedes principle

Upthrust = weight of fluid displaced

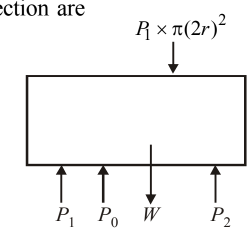
= (mass of fluid displaced) $\times$ g

$\therefore$ Upthrust = 0.

28. **(c)** Consider the equilibrium of wooden cylindrical block.

Forces acting in the downward direction are

(i) Weight of wooden cylinder



$W = \pi(2r)^2 \times h \times \dfrac{\rho}{3} \times g$

$= \pi \times 4r^2 \dfrac{h\rho}{3} g$

(ii) Force due to pressure (P_1) created by liquid of height h_1 above the wooden block

$F_1 = P_1 \times \pi\,(2r)^2 = [P_0 + h_1 \rho g] \times \pi\,(2r)^2$

(iii) Force acting on the upward direction due to pressure P_2 exerted from below the wooden block and atmospheric pressure

$$F_2 = P_2 \times \pi\left[(2r)^2 - r^2\right] + P_0 \times \pi(r)^2$$

$$= \left[P_0 + (h_1 + h)\rho g\right] \times \pi \times 3r^2 + P_0 \pi r^2$$

In situation 1, at the verge of rising the block, $F_2 = F_1 + W$

$$\left[P_0 + (h_1 + h)\rho g\right] \times \left(\pi \times 3r^2\right) + \pi r^2 P_0$$

$$= \left[P_0 + h_1 \rho g\right] \times 4\pi r^2 + \frac{\pi \times 4r^2 h\rho g}{3} \quad \text{or,} \quad h_1 = \frac{5h}{3}$$

29. **(b)** Again considering equilibrium of wooden block.

Total downward force = total force upwards

Wt. of block + force due to atmospheric pressure = force due to pressure of liquid + Force due to atmospheric pressure

$$\pi\,(16r^2)\frac{\rho}{3} \times g + P_0 \pi \times 16r^2 = [h_2 \rho g + P_0]\ \pi\,[(16 - 4)r^2] + P_0 \times 4r^2$$

when h_2 = height of the water level in situation – 2 for which the block remains in its original position.

$\therefore \quad h_2 = \frac{4}{9}h$

30. **(a)** In situation – 2 when the height h_2 of water level is further decreased, then the upward force acting on the wooden block decreases. The total force downward remains the same. This difference will be compensated by the normal reaction by the tank wall on the wooden block. Hence the block does not moves up and remains at its original position.

31. **(a)** As the hydrostatic force exerted by liquid A on the cylinder from all sides they cancels out and the net value is zero.

(b) In equilibrium, buoyant force = weight of the body

$\Rightarrow h_A\rho_A Ag + h_B\rho_B Ag = (h_A + h + h_B) A \rho_C g$

(where ρ_C = density of cylinder)

$$h = \left(\frac{h_A\rho_A + h_B\rho_B}{\rho_C}\right) - (h_A + h_B)$$

Substituting value of h_A, h_B ρ_A, ρ_B and ρ_e and solving we get h = 0.25 cm

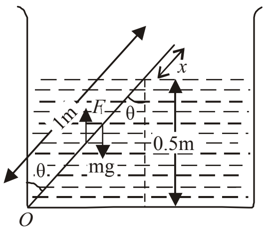
(c) Net acceleration, $a = \frac{F_{\text{Buoyant}} - Mg}{M}$

$$= \left[\frac{h_A\rho_A + \rho_B(h+h_B) - (h+h_A+h_B)\rho_C}{\rho_C(h+h_A+h_C)}\right]g$$

$= \frac{g}{6}$ upwards

32. Consider the equilibrium of the plank hinged at point 'O'

For equilibrium $F_{\text{net}} = 0$ and $\tau_{\text{net}} = 0$



Taking moment about point O

$$mg \times \frac{\ell}{2}\sin\theta = F_T\left(\frac{\ell - x}{2}\right)\sin\theta \quad \text{... (i)}$$

Also F_T = wt. of fluid displaced = $[(\ell - x)A] \times \rho_w g$ ... (ii)

And $m = (\ell A)0.5\rho_w$ ... (iii)

Where A is the area of cross section of the rod.

From eq. (i), (ii) and (iii)

$$(\ell A)0.5\rho_w g \times \frac{\ell}{2}\sin\theta = [(\ell - x)A]\rho_w g \times \left(\frac{\ell - x}{2}\right)\sin\theta$$

Here, $\ell = 1$ m

$\therefore \quad (1 - x)^2 = 0.5 \quad \Rightarrow \quad x = 0.293$ m

From the diagram

$$\cos\theta = \frac{0.5}{1-x} = \frac{0.5}{0.707} \quad \therefore \quad \theta = 45°$$

33. Let V_ℓ, V_s = volume of liquid and stone respectively and d_ℓ, d_s = density of liquid and stone respectively

When the stones were in the boat, the weight of stones were balanced by the buoyant force.

$V_s d_s = V_\ell d_\ell$

Since, $d_s > d_\ell \quad \therefore \quad V_s < V_\ell$

Hence when stones are put in water, the level of water falls

34. Let the size or edge of cube be ℓ. When mass m = 200 g is on the cube of wood

$$200g + \ell^3 d_{\text{wood}} g = \ell^3 d_{\text{water}} g$$

$$\Rightarrow \quad \ell^3 d_{\text{wood}} = \ell^3 d_{\text{water}} - 200 \quad \text{... (i)}$$

When the mass m = 200 g is removed

$$\ell^3 d_{\text{wood}} = (\ell - 2)\ell^2 d_{\text{water}} \quad \text{... (ii)}$$

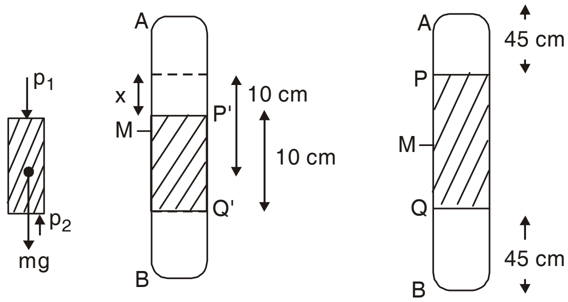
From eq. (i) and (ii)

$$\ell^3 d_{\text{water}} - 200 = (\ell - 2)\ell^2 d_{\text{water}}$$

$(\because \; d_{\text{water}} = 1)$

$$\therefore \quad \ell^3 - 200 = \ell^2(\ell - 2) \;\Rightarrow\; \ell = 10\,\text{cm}$$

35. Let by distance x the column of mercury be displaced.



Let A be the area of cross-section of the tube.

Let x distance the column of mercury be displaced if the tube is held vertically.

Applying $P_1V_1 = P_2V_2$

For air present in column AP

$p \times 45 \times A = p_1 \times (45 + x) \times A$

$$\Rightarrow \; p_1 = \frac{45}{45+x} \times 76 d_{\text{Hg}} \times g \quad \text{... (i)}$$

For air present in column QB

$p \times 45 \times A = p_2 \times (45 - x) \times A$

$$\Rightarrow \; p_2 = \frac{45}{45-x} \times 76 d_{\text{Hg}} \times g \quad \text{... (ii)}$$

M is the mid-point of tube AB.

At equilibrium

$p_1 \times A + mg = p_2 \times A$

$p_1 \times A + 10 \times A \times d_{\text{Hg}}g = p_2 \times A$

$\Rightarrow \; p_1 + 10 d_{\text{Hg}} \times g = p_2$ ... (iii)

From eq. (i), (ii) and (iii)

$$\frac{45 \times 76 \times d_{\text{Hg}} g}{45 + x} + 10 d_{\text{Hg}} \times g = \frac{45}{45 - x} \times 76 \times d_{\text{Hg}} \times g$$

$$\Rightarrow \; \frac{45 \times 76}{45 + x} + 10 = \frac{45 \times 76}{45 - x} \quad \therefore \; x = 2.95 \text{ cm.}$$

Topic-2 : Fluid Flow, Reynold's Number and Bernoulli's Principle

1. **(d)** Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g h_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P, P_2 = \dfrac{P}{2}$, we get

$$\Rightarrow P + \frac{1}{2}\rho\upsilon^2 = \frac{P}{2} + \frac{1}{2}\rho V^2 \Rightarrow \frac{P}{2} + \frac{1}{2}\rho\upsilon^2 = \frac{1}{2}\rho V^2$$

$$\Rightarrow V = \sqrt{\upsilon^2 + \frac{P}{\rho}}$$

2. **(b)** According to question, area of cross-section at A, $aA = 40$ cm^2 and at B, $aB = 20$ cm^2
Let velocity of liquid flow at A, $= V_A$ and at B, $= V_B$
Using equation of continuity $a_A V_A = a_B V_B$
$40V_A = 20V_B \Rightarrow 2V_A = V_B$
Now, using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2 \Rightarrow P_A - P_B = \frac{1}{2}\rho\left(V_B^2 - V_A^2\right)$$

$$\Rightarrow \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right) \Rightarrow \Delta P = 500 \times \frac{3V_B^2}{4}$$

$$\Rightarrow V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}} \text{m/s} = 1.37\times 10^2 \text{ cm/s}$$

Volume flow rate Q = $a_B \times v_B$
$= 20 \times 100 \times V_B = 2732$ cm^3/s ≈ 2720 cm^3/s

3. **(a)** From the equation of continuity
$A_1 v_1 = A_2 v_2$
Here, v_1 and v_2 are the velocities at two ends of pipe.
A_1 and A_2 are the area of pipe at two ends

$$\Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi(4.8)^2}{\pi(6.4)^2} = \frac{9}{16}$$

4. **(b)** Using Bernoullie's equation

$$P + \frac{1}{2}(v_1^2 - v_2^2) + \rho g h = P$$

$$\Rightarrow v_2^2 = v_1^2 + 2gh \Rightarrow v_2 = \sqrt{v_1^2 + 2gh}$$

Equation of continuity
$A_1 v_1 = A_2 v_2$

$$(1 \text{ cm}^2)\ (1 \text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2\times 10 \times \frac{15}{100}}\right)$$

$10^{-4} \times 1 = A_2 \times 2$

$$\therefore\ A_2 = \frac{10^{-4}}{2} = 5 \times 10^{-5} \text{ m}^2$$

5. **(b)** Here, volume tric flow rate

$$= \frac{0.74}{60} = \pi r^2 v = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74\times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740\times 740}{24\times 24\times 10}\left(\because \pi^2 = 10\right) \Rightarrow h = \frac{74\times 74}{2\times 24\times 24} \approx 4.8\text{m}$$

i.e., The depth of the centre of the opening from the level of water in the tank is close to 4.8 m

6. **(a)** According to Bernoulli's Principle,

$$\frac{1}{2} r v_1^2 + r\,gh = \frac{1}{2} r v_2^2$$

$$v_1^2 + 2gh = v_2^2$$

$$2gH + 2gh = v_2^2 \qquad \text{...(i)}$$

$a_1 v_1 = a_2 v_2$

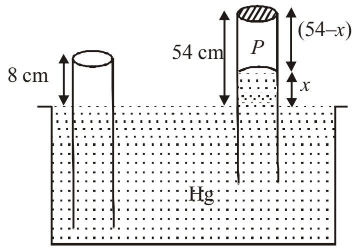
$$p r^2 \sqrt{2gh} = p x^2 v_2$$

$$\frac{r^2}{x^2}\sqrt{2gh} = v_2$$

Substituting the value of v_2 in equation (i)

$$2gH + 2gh = \frac{r^4}{x^4} 2gh \ \text{ or, } \ x = r\left[\frac{H}{H+h}\right]^{\frac{1}{4}}$$

7. **(a)**



Length of the air column above mercury in the tube is,
$P + x = P_0$
$\Rightarrow\ P = (76 - x) \ \Rightarrow 8 \times A \times 76 = (76 - x) \times A \times (54 - x)$
$\therefore\ x = 38$
Thus, length of air column
$= 54 - 38 = 16$ cm.

8. **(a)** According to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore\ v_2^2 - v_1^2 = 2gh \qquad \text{...(1)}$$

According to the equation of continuty

$$A_1 v_1 = A_2 v_2 \qquad \text{...(2)}$$

$$\frac{A_1}{A_2} = \frac{6 \text{ mm}^2}{10 \text{ mm}^2}$$

From equation (2), $\dfrac{A_1}{A_2} = \dfrac{v_2}{v_1} = \dfrac{6}{10}$

or, $v_2 = \dfrac{6}{10} v_1$

Putting this value of v_2 in equation (1)

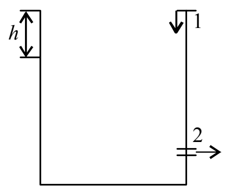
$$\left(\frac{6}{10}v_1\right)^2 - (v_1)^2 = 2\times 10^3 \times 5$$

$$\left[\begin{array}{l}\because g = 10\text{m/s}^2 = 10^3 \text{cm/s}^2 \\ \text{and h} = 5\text{ cm}\end{array}\right]$$

Solving we get $v_1 = \dfrac{10}{8}$

Therefore the rate at which water flows through the tube $= A_1 v_1 = A_2 v_2 = \dfrac{6\times 10}{8} = 7.5\text{cc/s}$

9. **(a)** From Bernoulli's theorem at point 1 and 2



$P_0 + \rho gh + \frac{1}{2}\rho V_1^2$

$= P_0 + 0 + \frac{1}{2}\rho V_2^2$ ...(i)

And equation of continuity $A_1V_1 = A_2V_2$

or $V_2 = \frac{A_1V_1}{A_2}$ ...(ii)

From eq. (i) & (ii)

$v^2 = \frac{2gh}{\sqrt{1-\left(\frac{a}{A}\right)^2}}$ or, $v^2 = \frac{2\times10\times2.475}{\sqrt{1-(0.1)^2}} = 50\ m^2/s^2$

$(\because h = 3 - 0.525 = 2.475\ m)$

10. **(a)** As we know, velocity of efflux $V = \sqrt{2gh}$

From equation of continuity $A_1V_1 = A_2V_2$

$\sqrt{(2gy)} \times L^2 = \sqrt{(2g\times4y)}\,\pi R^2$

$\Rightarrow L^2 = 2\pi R^2 \quad \therefore R = \frac{L}{\sqrt{2\pi}}$

11. **(c)** Given : $v_1 = 1.0\ ms^{-1}$

$A_1 = 10^{-4}\ m^2$

v_2 = velocity of water stream at 0.15 m below the tap

$A_2 = ?$

For calculating v_2 using, $v^2 - u^2 = 2as$

$u = 1$ m/s; $s = 1.5$ m, $a = g = 10\ m/s^2$ and $v = v_2 = ?$

$v^2 - 1 = 2 \times 10 \times 0.15 \Rightarrow v = v_2 = 2$ m/s

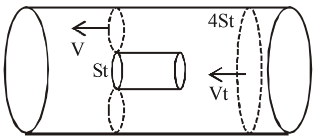
By equation of continuity $v_1A_1 = v_2A_2$

$\therefore A_2 = \frac{v_1A_1}{v_2} = \frac{1\times10^{-4}}{2} = 5\times10^{-5}m^2$

12. **(9)** Applying Bernoulli's equation

$P_0 + \frac{1}{2}\rho v_t^2 = P + \frac{1}{2}\rho v^2$

$P_0 - P = \frac{1}{2}\rho(v^2 - v_t^2)$ ...(i)



From equation of continuity

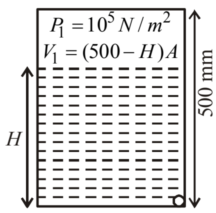
$4S_tv_t = v\times(4S_t - S_t) = v\times3S_t$

$\Rightarrow v = \frac{4}{3}v_t$ ...(ii)

From eqs. (i) and (ii)

$P_0 - P = \frac{1}{2}\rho\left(\frac{16}{9}v_t^2 - v_t^2\right) = \frac{1}{2}\rho\frac{7v_t^2}{9} = \frac{7}{2N}\rho v_t^2 \quad \therefore N = 9$

13. **(6)** Initially, pressure of air column above water $P_1 = 10^5\ Nm^{-2}$ and volume $V_1 = (500 - H)A$, where A is the area of cross-section of the vessel.



Initially

Finally, the volume of air column above water $V_2 = (500-200)A = 300$ A. If P_2 is the pressure of air then

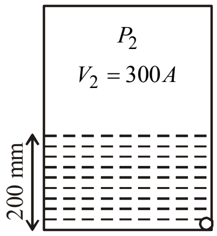
$P_2 + \rho gh = P_1$

$\therefore P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$

$\therefore P_2 = 9.8\times10^4\ N/m^2$

Assuming the temperature remains constant, according to Boyle's law

$P_1V_1 = P_2V_2$



Finally

$\therefore 10^5 \times (500 - H)A = (9.8\times10^4)\times300A \Rightarrow H = 206$ mm

$\therefore$ Fall in height of water level due to the opening of orifice = 206 – 200 = 6 mm

14. According to equation of continuity

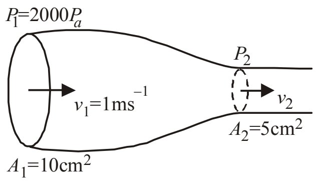
$A_1v_1 = A_2v_2 \Rightarrow 10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2$m/s

Now, using Bernoulli's theorem

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow 2000 + \frac{1}{2}\times1000\times1^2$

$= P_2 + \frac{1}{2}\times1000\times2^2$



$\therefore P_2 = 500\ Pa$

15. **(c)** Here, piston is pushed at a speed, $v_1 = 5$ m/s

Let air comes out of nozzle with a speed v_2

From principle of continuity,

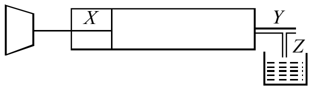
$a_1v_1 = a_2v_2$

$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$

$\Rightarrow (20)^2 \times 5 = (1)^2 \times v_2$

$\therefore v_2 = 2000\ mms^{-1} = 2\ ms^{-1}$

16. **(a)** $P_X - P_Y = \frac{1}{2}\rho_a v_a^2$



$P_Z - P_Y = \frac{1}{2}\rho_l v_l^2$

But $P_Z = P_X$

$\therefore \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}}\times v_a$

$\therefore$ Volume flow rate $\propto \sqrt{\frac{\rho_a}{\rho_l}}$

17. **(a)** The volume flow rate (Q) of an incompressible fluid in steady flow remains constant

From equation of continuity,

av = constant

$\therefore Q = a \times v$ = constant or, $a \propto \frac{1}{v}$

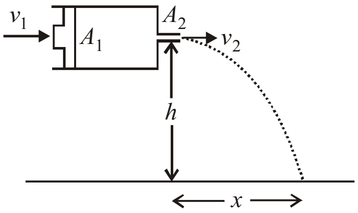
where a = area of cross-section and v = velocity

$\Rightarrow$ If v decreases a increases and vice - versa.

When stream of water moves up, its speed (v) decreases and therefore 'a' increases i.e. the water spreads out as a fountain. When stream of water from hose pipe moves down, its speed increases and therefore area of cross-section decreases.

18. Given $A_1 = \pi \times (4 \times 10^{-3}\ m)^2$, $A_2 = \pi \times (1\times 10^{-3}\ m)^2$

$v_1 = 0.25$ m/s



From equation of continuity $A_1v_1 = A_2v_2$

$$\therefore \quad v_2 = \frac{A_1}{A_2}v_1 = \frac{\pi \times (4\times10^{-3})^2 \times 0.25}{\pi \times (1\times10^{-3})^2} = 4 \text{ m/s}$$

From $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Hence horizontal range

$$x = v_2 \times t = v_2\sqrt{\frac{2h}{g}} = 4\times\sqrt{\frac{2\times1.25}{10}} = 2\text{m}$$

19. Given : $\rho = 1000$ kg/m^3, $h_1 = 2$m, $h_2 = 5$ m
$A_1 = 4\times10^{-3}$m^2, $A_2 = 8\times10^{-3}$ m^2, $v_1 = 1$ m/s
From equation of continuity

$$A_1v_1 = A_2v_2 \quad \therefore v_2 = \frac{A_1v_1}{A_2} = 0.5 \text{ m/s}$$

Applying Bernoulli's theorem, at P and Q

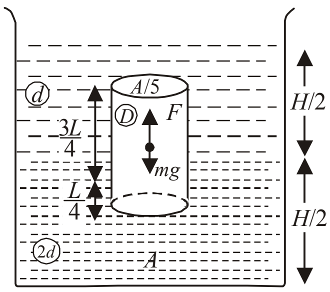
$$(p_1 - p_2) = \rho g\,(h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Now, work done/vol. by gravity forces
$= \rho g\,(h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3$ J/m^3.

$$\text{And } \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\times10^3\left[\frac{1}{4}-1\right] = -\frac{3}{8}\times10^3 \text{ J/m}^3$$
$$= -0.375 \times 10^3 \text{ J/m}^3$$

$\therefore$ Work done / vol. by pressure
$P_1 - P_2 = 29.4\times10^3 - 0.375\times10^3$ J/m^3 $= 29.025\times10^3$ J/m^3.

20. **(a)** (i)
Since the cylinder is in equilibrium in the liquid
$\therefore$ Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid



$$\frac{A}{5}\times L\times D\times g = \frac{A}{5}\times\frac{L}{4}\times2d\times g + \frac{A}{5}\times\frac{3L}{4}\times d\times g$$

$$\Rightarrow D = \frac{2d}{4}+\frac{3d}{4} = \frac{5d}{4}$$

(ii) Considering vertical equilibrium of two liquids and the cylinder.
Total pressure at the bottom of the cylinder = Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density $2d$ + Pressure due to cylinder [Weight/Area]

$$P = P_0 + \frac{H}{2}dg + \frac{H}{2}\times2d\times g + \frac{\frac{A}{5}\times L\times D\times g}{A}$$

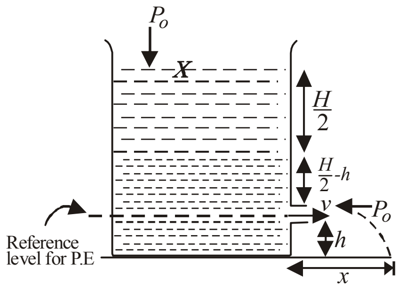
$$\Rightarrow P = P_0 + \left(\frac{3H}{2}+\frac{L}{4}\right)dg \quad \left[\because D = \frac{5d}{4}\right]$$

(b) (i) Applying Bernoulli's theorem

$$P_0 + \left[\frac{H}{2}\times d\times g + \left(\frac{H}{2}-h\right)2d\times g\right]$$
$$= P_0 + \frac{1}{2}(2d)v^2 \Rightarrow v = \sqrt{\frac{(3H-4h)}{2}g}$$

(ii) For vertical motion of liquid falling from hole time taken to reach the liquid to the bottom
$u_y = 0, S_y = h, a_y = g, t_y = t$

$$S = ut + \frac{1}{2}at^2 \quad \Rightarrow h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$



$$x = v\times t = \sqrt{(3H-4h)\frac{g}{2}}\times\sqrt{\frac{2h}{g}} = \sqrt{(3H-4h)h}$$

(iii) For finding the value of h for which x is maximum,

$$\frac{dx}{dh} = 0$$

$$\frac{dx}{dh} = \frac{1}{2}[3H-4h)h]^{-1/2}\{3H-8h\}$$

$$\frac{1}{2}[(3H-4h)]^{-1/2}[3H-8h] = 0 \Rightarrow h = \frac{3H}{8}$$

Hence, x will be maximum at $h = \frac{3}{4}$H

The maximum value of x

$$x_m = \sqrt{\left[3H - 4\left(\frac{3H}{8}\right)\right]\frac{3H}{8}}$$
$$= \sqrt{\frac{12H}{8}\times\frac{3H}{8}} = \frac{6H}{8} = \frac{3H}{4}$$

Topic-3 : Viscosity and Terminal Velocity

1. **(a)** Using, $v^2 - u^2 = 2gh$

$$\Rightarrow v^2 - 0^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

Terminal velocity,

$$V_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

After falling through h the velocity should be equal to terminal velocity

$$\therefore \sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\Rightarrow 2gh = \frac{4}{81}\frac{r^4 g^2(\rho-\sigma)^2}{\eta^2}$$

$$\Rightarrow h = \frac{2r^4 g(\rho-\sigma)^2}{81\eta^2} \Rightarrow h \,\alpha\, r^4$$

2. **(a)** $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$ or $r = \frac{R}{3}$.

Terminal velocity, $v \propto r^2$

$$\therefore \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$$

or $v_2 = \left(\frac{r_2}{r_1}\right)^2 v_1 = \left(\frac{R/3}{R}\right)^2 v_1 = \frac{1}{9}$ or $\frac{v_1}{v_2} = 9.$

3. **(b)** According to Toricelli's theorem,
Velocity of efflex,

$$V_{eff} = \sqrt{2gh} = \sqrt{2\times 9.8\times 5} \cong 9.8\,\text{ms}^{-1}$$

4. **(3)** As we know, terminal velocity

$$V_T = \frac{2r^2}{9\eta}(\rho-\sigma)g$$

$$\frac{V_P}{V_Q} = \frac{\dfrac{2r_1^2(\sigma-\rho_1)g}{9\eta_1}}{\dfrac{2r_2^2(\sigma-\rho_2)g}{9\eta_2}} = \frac{r_1^2(\sigma-\rho_1)}{r_2^2(\sigma-\rho_2)}\times\frac{\eta_2}{\eta_1}$$

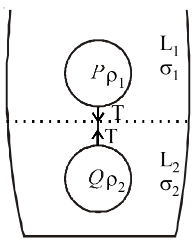
$$= \frac{1^2}{(0.5)^2}\frac{[8-0.8]}{[8-1.6]}\times\frac{2}{3} = 3$$

5. **(a, d)** Since string is taut,
$\rho_1 < \sigma_1$ and $\rho_2 < \sigma_2$
For floating, net weight of system = net upthrust
$(\rho_1+\rho_2)V_g = (\sigma_1+\sigma_2)V_g$
Upward terminal velocity

$$V_P = \frac{2r^2(\sigma_2-\rho_1)g}{9\eta_2}$$

Where r is radius of sphere.
Downward terminal velocity

$$V_Q = \frac{2r^2(\rho_2-\sigma_1)g}{9\eta_1}$$



$$\therefore \left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2}$$

$(\because \quad \rho_1 - \sigma_2 = \sigma_1 - \rho_2)$

Again $V_P \,.\, V_Q < 0$ *i.e.*, negative as V_P and V_Q are opposite to each other.

Topic-4 : Surface Tension, Surface Energy and Capillarity

1. **(c)** According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \qquad ...(i)$$

And $\Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2} \qquad ...(ii)$

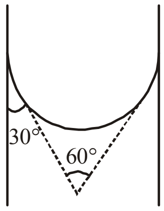
Dividing, equation (ii) by (i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Volume $V = \frac{4}{3}\pi R^3 \quad \therefore \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$

2. **(b)** Given,
Angle of contact $\theta = 30°$
Surface tension, $T = 0.05\ \text{Nm}^{-1}$
Radius of capillary tube, $r = 0.15$ mm $= 0.15 \times 10^{-3}$m
Density of methylene iodide, $\rho = 667$ kg m^{-3}

Capillary rise, $h = \frac{2T\cos\theta}{\rho g r}$



$$= \frac{2\times 0.05\times\frac{\sqrt{3}}{2}}{667\times 10\times 0.15\times 10^{-3}} = 0.087\ \text{m}$$

3. **(d)** For the drops to be in equilibrium upward force on drop = downward force on drop

$$T.2\pi R = \frac{4}{3}\pi R^3 dg - \frac{2}{3}\pi R^3 \rho g$$

$$\Rightarrow T(2\pi R) = \frac{2}{3}\pi R^3(2d-\rho)g$$

$$\Rightarrow T = \frac{R^2}{3}(2d-\rho)g \Rightarrow R = \sqrt{\frac{3T}{(2d-\rho)g}}$$

4. **(b)** As we know that

$$\frac{2T\cos\theta}{r\rho g} = R\ h$$

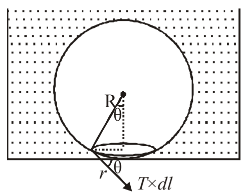
$$\frac{T_{Hg}}{T_{Water}} = 7.5$$

$$\frac{\rho_{Hg}}{\rho_W} = 13.6\ \& \frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135°}{\cos 0°} = \frac{1}{\sqrt{2}}$$

$$\frac{R_{Hg}}{R_{Water}} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$= 7.5\times\frac{1}{13.6}\times\frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

5. **(a)** When the bubble gets detached,
Buoyant force = force due to surface tension



Force due to excess pressure = upthrust

Access pressure in air bubble = $\frac{2T}{R}$

$$\frac{2T}{R}(\pi r^2) = \frac{4\pi R^3}{3T}\rho_w g$$

$$\Rightarrow r^2 = \frac{2R^4\rho_w g}{3T} \Rightarrow r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

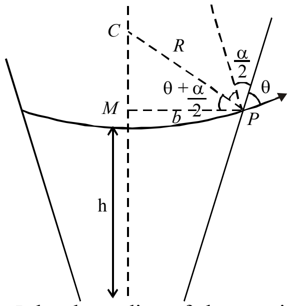
6. **(a)** Acceleration due to gravity changes with the depth,

$$g' = g\left(1 - \frac{d}{R}\right)$$

Pressure, $P = \rho g h$

Hence ratio, $\frac{x}{y}$ is $\left(1 - \frac{d}{R}\right)$

7. **(d)**



Here R be the radius of the meniscus formed with a constant angle θ and P_0 be the atmospheric pressure.

In ΔPCM $\cos\left(\theta + \frac{\alpha}{2}\right) = \frac{b}{R} \quad \Rightarrow R = \frac{b}{\left(\theta + \frac{\alpha}{2}\right)}$

Also $\left(P_0 - \frac{2S}{R}\right) + h\rho g = P_0 \quad \Rightarrow \quad h\rho g = \frac{2S}{R}$

$\therefore \quad h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos(\theta + \alpha/2)$

8. **(6)** $\frac{4}{3}\pi R^3 = k \times \frac{4}{3}\pi r^3 \quad \therefore \quad R = K^{1/3} r$

$\Delta U = S[k \times 4\pi r^2 - 4\pi R^2]$

$\therefore \Delta U = 4\pi s\left[k \times \frac{R^2}{k^{2/3}} - R^2\right] = 4\pi S R^2\left[k^{1/3} - 1\right]$

$\therefore \Delta U = 4\pi S R^2\left[10^{\alpha/3} - 1\right]$ $[\because \ K = 10^{\alpha}]$

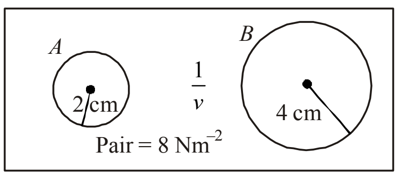
$\therefore 10^{-3} = 4\pi \times \frac{0.1}{4\pi} \times (10^{-2})^2\left[10^{\alpha/3} - 1\right]$

$\therefore 10^2 = 10^{\alpha/3} - 1$ Neglecting

$10^2 = 10^{\alpha/3} \quad \Rightarrow \quad \frac{\alpha}{3} = 2 \quad \therefore \alpha = 6$

9. **(6)** Pressure, $P_A = P_{air} + \frac{4T}{R_A}$

($\because$ Excess pressure due to surface tension (T) in soap bubble $= 4T/r$)



$P_A = 8 + \frac{4T}{R_A} = \frac{4 \times 0.04}{0.02} + 8 \Rightarrow P_A = 16 N/m^2$

Similarly, $P_B = 8 + \frac{4T}{R_B} = 8 + \frac{4 \times 0.04}{0.04} = 12$ N/m^2

According to ideal gas equation, $P_V = nRT$

$P_A V_A = n_A R T_B \quad \Rightarrow \quad 16 \times \frac{4}{3}\pi(0.02)^3 = n_A R T_A$ ...(i)

$P_B V_B = n_B R T_B \Rightarrow \quad 12 \times \frac{4}{3}\pi(0.04)^3 = n_B R T_B$ ... (ii)

Dividing eq. (ii) by (i)

$$\frac{12 \times \frac{4}{3}\pi(0.04)^3}{16 \times \frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \qquad [\because T_A = T_B]$$

$\therefore \quad \frac{n_B}{n_A} = 6$

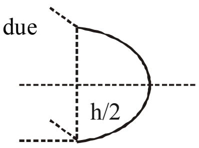
10. **(101)**
Given : Radius of capillary tube,
$r = 0.015$ cm $= 15 \times 10^{-5}$ mm
$h = 15$ cm $= 15 \times 10^{-2}$ mm

Using, $h = \frac{2T\cos\theta}{\rho g r}$ $\qquad [\cos\theta = \cos 0° = 1]$

Surface tension,

$T = \frac{rh\rho g}{2} = \frac{15 \times 10^{-5} \times 15 \times 10^{-2} \times 900 \times 10}{2} = 101$ milli newton m^{-1}

11. **(3.74)** According to question, the water above the rim as a disc of thickness 'h' having semicircular edges.
r = h/2
Pressure at the bottom of disc = pressure due to surface tension



$\rho g h = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$R_1 >>> R_2 \therefore \frac{1}{R_1} <<< \frac{1}{R_2}$ and $R_2 = h/2$

$\therefore \rho g h = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = T\left[0 + \frac{1}{h/2}\right] = \frac{2T}{h}$

$\Rightarrow h^2 = \frac{2T}{\rho g} \Rightarrow h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{20 \times 0.07}{10^3 \times 10}} = \sqrt{\frac{14 \times 100}{10^4 \times 100}}$

$\therefore h = \sqrt{14}$ mm = 3.741

12. **(a, b, c) For case I**

$$h_1 = \frac{2T\cos\theta_1}{r\rho g} = \frac{2 \times 0.75 \times \cos 0°}{2 \times 10^{-4} \times 1000 \times 10} = 7.5\text{cm}$$

For case II

$$h_2 = \frac{2T\cos\theta_2}{r\rho g} = \frac{2 \times 0.75 \times \cos 60°}{2 \times 10^{-4} \times 1000 \times 10} = 3.75\text{cm}$$

The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus will be different for both cases.

In case II, if the capillary joint is 5 cm above the water surface then water in capillary will not reach the interface. Water will reach only till 3.75 cm.

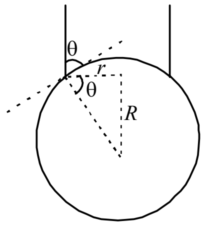
13. **(a, c)** As we know $h = \frac{2\sigma\cos\theta}{r\rho g_{eff}}$

As 'r' increases, h decreases $h \propto \frac{1}{r}$

[all other parameters remaining constant]

Also $h \propto \sigma$
Further if lift is going up with an acceleration 'a' then $g_{eff} = g + a$. As g_{eff} increases, 'h' decreses.
Also $h \propto \cos\theta$ not $h \propto \theta$

14. **(c)** Vertical force due to surface tension



$F_V = (T\cos\theta) \times 2\pi r$

$$= T\left(\frac{r}{R}\right) \times 2\pi r = \frac{2\pi r^2 T}{R}$$

$$\left[\because \cos\theta = \frac{r}{R}\right]$$

15. **(a)** When the drop is about to detach from the dropper
Weight = vertical force due to surface tension ($mg = F_V$)

$$\therefore \frac{4}{3}\pi R^3 \rho g = \frac{2\pi r^2 T}{R}$$

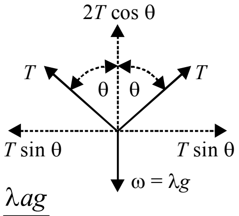
$$\Rightarrow R^4 = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right) = \frac{3}{2} \times \frac{(5\times10^{-4})^2 \times 0.11}{1000\times10} = 4.12 \times 10^{-12}$$

or, $R = 1.42 \times 10^{-3}$ m

16. **(b)** Surface energy = $T \times 4\pi R^2$

$$= 0.11 \times 4 \times \frac{22}{7} \times (1.42\times10^{-3})^2 = 2.7\times10^{-6}\,\text{J}$$

17. For equilibrium of wire in vertical direction, from free body diagram of the wire shown in the figure.



$\therefore\ 2Tl\cos\theta = \lambda l g$

or $\quad 2T\cos\theta = \lambda g$

$$\cos\theta = \frac{y}{a}$$

while $y << a$

$$\therefore \frac{2T\times y}{a} = \lambda g \quad \text{or} \quad T = \frac{\lambda a g}{2y}$$

18. When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow P + \rho g H = \frac{1}{2}\rho(v_0^2 - v_1^2)$$

From equation of continuity $A_1 v_1 = A_2 v_2$

$$v_1 = \frac{A_2 v_0}{A_1} \qquad [\text{here } v_2 = v_0]$$

$$\therefore P + \rho g H = \frac{1}{2}\rho\left[v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho g H = \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Here, $P + \rho g H = \Delta P$
According to Poisseuille's equation

$$Q = \frac{\pi(\Delta P)a^4}{8\eta l} \Rightarrow \eta = \frac{\pi(\Delta P)a^4}{8Q\ell}$$

$$\therefore \eta = \frac{\pi(P+\rho g H)a^4}{8Q\ell} = \frac{\pi}{8Q\ell} \times \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \times a^4$$

$$\eta = \frac{\pi}{8Ql} \times \frac{1}{2}\rho v_0^2\left[1 - \frac{d^4}{D^4}\right] \times a^4 \quad \left[\because \frac{A_2}{A_1} = \frac{d^2}{D^2}\right]$$

19. The bubble will separate from the ring, when the force due to air $= \rho A V^2$ in the bubble equals the surface tension force $4T/R$ inside the bubble.

$$\therefore \rho A v^2 = \frac{4T}{R} \times A \quad \Rightarrow R = \frac{4T}{\rho v^2}$$

Radius at which bubbles separate from the ring $R = \dfrac{4T}{\rho v^2}$

Topic-5 : Miscellaneous Problems

1. **(a)** Given : $\dfrac{1}{\rho}\dfrac{d\rho}{dt} = \text{constant}$

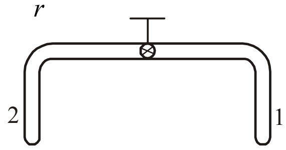
$$\therefore \frac{4\pi R^3}{3m}\frac{d}{dt}\left[\frac{m}{\frac{4}{3}\pi R^3}\right] = \text{constant}$$

$$\Rightarrow R^3\frac{d}{dt}(R^{-3}) = \text{constant}$$

$$\Rightarrow R^3(-3R^{-4})\frac{dR}{dt} = \text{constant} \quad \therefore \left|\frac{dR}{dt}\right| \propto R$$

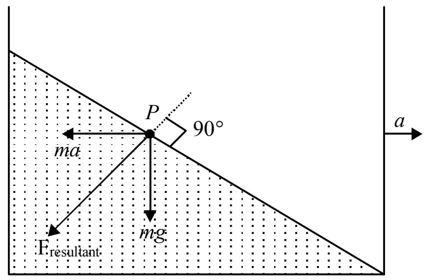
2. **(b)** We know that excess pressure in a soap bubble is inversely proportional to its radius. *i.e.*, $P = \dfrac{4T}{r}$

The soap bubble at end 1 has small radius as compared to the soap bubble at end 2 (given). Therefore excess pressure at 1 is more.



Hence, air flows from end 1 to end 2 and the volume of soap bubble at end 1 decreases.

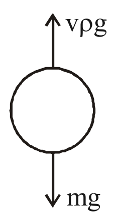
3. **(c)** It force acts on water towards right, when a vessel containing water is given a constant acceleration a towards the right. As an action the water exerts a force on the wall of vessel towards right. As a reaction the wall pushes the water towards left. According to Newton's 3rd law of motion. The slope of water is as shown in figure.



P is a particle of water.
The surface of water should set it self at 90° to $F_{resultant}$.

4. **(4)** According to question, variation of the density of air with height h from the ground.

$$\rho(h) = \rho_0 e^{-\frac{h}{h_0}}$$



At h = 100 m, mass m = 480 kg
$\therefore mg = vg\rho_{(100)}$
$\Rightarrow 480 \times g = v\rho_1 g$
A h = 150 m, mass m = (480 – N)kg
$\therefore mg = vg\rho_{(150)}$
$\Rightarrow (480 - N)g = v\rho_2 g$

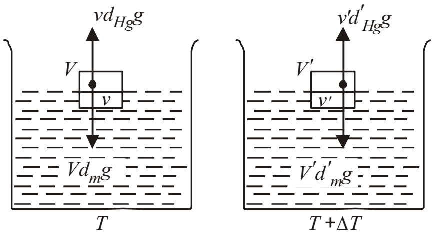
$$\frac{480-N}{480}=\frac{\rho_2}{\rho_1}\Rightarrow\left(1-\frac{N}{480}\right)=\frac{e^{-h_2/h_0}}{e^{-h_1/h_0}}=e^{\frac{h_1-h_2}{h_0}}=e^{-\frac{50}{6000}}$$

$$\Rightarrow 1-\frac{N}{480}=1-\frac{50}{6000}\therefore N=\frac{50\times 480}{6000}=4$$

5. The condition for floatation, $vd_{Hg}g = Vd_m g$

Fraction of volume of metal submerged in mercury

$$=\frac{v}{V}=\frac{d_m}{d_{Hg}}=x\,(\text{say})$$



In second case, when temperature is increased by ΔT.

$v'd'_{Hg}g = V'd'_m\, g$

$$\Rightarrow\frac{v'}{V'}=\frac{d'_m}{d'_{Hg}}=\text{Fraction of volume of metal submerged in}$$

mercury = x' (say)

$$\therefore\frac{x'}{x}=\frac{d'_m\times d_{Hg}}{d'_{Hg}\times d_m}=\frac{d'_m\times d'_{Hg}(1+\gamma_2\Delta T)}{d'_{Hg}\times d'_m(1+\gamma_1\Delta T)}=\frac{(1+\gamma_2\Delta T)}{(1+\gamma_1\Delta T)}$$

$= (1+\gamma_2\,\Delta T)\,(1+\gamma_1\,\Delta T)^{-1}$

$= (1+\gamma_2\,\Delta T)\,(1-\gamma_1\,\Delta T) = 1+(\gamma_2-\gamma_1)\,\Delta T-\gamma_1\gamma_2\,(\Delta T)^2$

$$\frac{x'}{x}-1=(\gamma_2-\gamma_1)\Delta T\quad\Rightarrow\quad\frac{x'-x}{x}=(\gamma_2-\gamma_1)\Delta T$$

6. **(a, c, d)** $F=-\eta A\left(\frac{dv}{dx}\right)$ or $|F|=\eta A\,\frac{u_0}{h}$

where $\frac{u_0}{h}$ = velocity gradient $\left(\frac{dv}{dx}\right)$

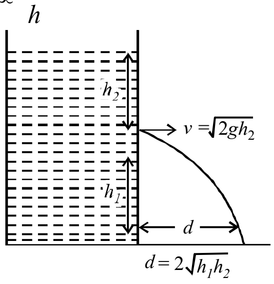
$F\propto\eta$; $F\propto A$; $F\propto\mu_0$ and $F\propto\frac{1}{h}$

7. **(c)** Horizontal distance,

$d = v\times t$

$$=\sqrt{2gh_2}\times\sqrt{\frac{2h_1}{g}}$$

$$=2\sqrt{h_1h_2}$$



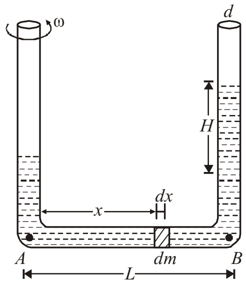
If $g_{eff}>g$
$g_{eff}=g$
$g_{eff}<g$

In all the three cases $d=2\sqrt{h_1h_2}=1.2$ m

If $g_{eff}=0$, then no water leaks out as there will be no pressure difference.

8. Let us consider a small elemental mass dm at a distance x from A as shown in the figure. Centripetal force required for the mass dm to rotate = $(dm)\,x\omega^2$

$\therefore$ Total centripetal force required for the mass of length L to rotate



$$=\int_0^L (dm)\,x\omega^2\quad\text{where } dm=\rho\times\frac{\pi d^2}{4}\times dx$$

$$\therefore\quad\text{Total centripetal force}=\int_0^L\left(\rho\times\frac{\pi d^2}{4}\times dx\right)\times\left(x\omega^2\right)$$

$$=\rho\times\frac{\pi d^2}{4}\times\omega^2\int_0^L x\,dx$$

$$=\rho\times\frac{\pi d^2}{4}\times\omega^2\times\frac{L^2}{2}$$

This centripetal force is provided by the weight of liquid of height H

Weight of liquid of height H

$$=\frac{\pi d^2}{4}\times H\times\rho\times g$$

$$\therefore\quad\frac{\pi d^2}{4}\times H\times\rho\times g=\rho\times\frac{\pi d^2}{4}\times\frac{\omega^2\times L^2}{2}\quad\text{or, } H=\frac{\omega^2L^2}{2g}$$

9. **(a)** Since density of ball (d) < density of liquid (d_L) upthrust > weight.

$\therefore$ The ball falls with retardation inside liquid.

$$\therefore\quad\text{Retardation}=\frac{\text{upthrust}-\text{weight}}{\text{mass}}$$

$$\text{or}\quad a=\frac{Vd_Lg-Vdg}{Vd}\quad\text{or}\quad a=\left(\frac{d_L-d}{d}\right)g\qquad\text{...(i)}$$

Let the ball come to rest inside liquid in time t.

$$\therefore\quad t=\frac{v}{a}=\frac{gt_1}{2a}$$

[$\because$ Velocity of the ball just before it collides with liquid, $V=gt_1/2$]

$$\text{or}\quad t=\frac{gt_1}{2\left(\frac{d_L-d}{d}\right)g}=\frac{dt_1}{2(d_L-d)}\qquad\text{...(ii)}$$

$\therefore$ The time of rise to liquid surface = t.

(a) $\because$ t_2 = Time taken by the ball to come back to the position from which it was released.

or $t_2=t_1+2t$ substitute t from (ii)

$$\text{or}\quad t_2=t_1+\frac{2\times dt_1}{2(d_L-d)}\quad\text{or}\quad t_2=t_1\left[1+\frac{d}{(d_L-d)}\right]$$

$$\text{or}\quad t_2=\frac{t_1d_L}{d_L-d}$$

(b) Since the retardation = $\left(\frac{d_L-d}{d}\right)g$ is not propor-tional to displacement, the motion of the ball is not simple harmonic.

(c) If $d=d_L$ then the retardation = retardation = 0. Since the ball strikes the water surface with some velocity, it will continue with the same velocity $v=gt_1/2$ in downward direction.

10 Thermal Properties of Matter

Topic-1 : Thermometer & Thermal Expansion

1. **(a)** Let L_1' and L_2' be the lengths of the wire when temperature is changed by $\Delta T°C$.
At $T°C$,
$$L_{eq} = L_1 + L_2$$
At $T + \Delta°C$
$$L'_{eq} = L'_1 + L'_2$$
$$\therefore \; L_{eq}(1+\alpha_{eq}\Delta T) = L_1(1+\alpha_1\Delta T) + L_2(1+\alpha_2\Delta T)$$
$$[\because L' = L(1+\alpha\,\Delta T)]$$
$$\Rightarrow (L_1+L_2)(1+\alpha_{eq}\Delta T) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$$
$$\Rightarrow \alpha_{eq} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$$

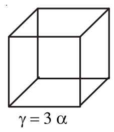
2. **(a)** Change in length of the metal wire (Δl) when its temperature is changed by ΔT is given by
$\Delta l = l\alpha\Delta T$
Here, α = Coefficient of linear expansion
Here, $\Delta l = 0.02\%$, $\Delta T = 10°C$
$$\therefore \alpha = \frac{\Delta l}{l\Delta T} = \frac{0.02}{100 \times 10} \Rightarrow \alpha = 2 \times 10^{-5}$$
Volume coefficient of expansion, $\gamma = 3\alpha = 6 \times 10^{-5}$
$$\because \rho = \frac{M}{V}$$
$$\frac{\Delta V}{V} \times 100 = \gamma\Delta T = (6 \times 10^{-5} \times 10 \times 100) = 6 \times 10^{-2}$$
Volume increase by 0.06% therefore density decrease by 0.06%.

3. **(Bonus)** $\Delta_{temp} = \Delta_{load}$ and $A = \pi r^2 = \pi(10^{-3})^2 = \pi \times 10^{-6}$
$$L\,\alpha\,\Delta T = \frac{FL}{AY}$$
or $\quad 0.2 \times 10^{-5} \times 20 = \dfrac{F \times 0.2}{(\pi \times 10^{-6}) \times 10^{11}}$
$$\therefore \quad F = 20\pi\,N \;\therefore\; m = \frac{f}{g} = 2\pi = 6.28\text{ kg}$$

4. **(a)** Change in length in both rods are same i.e.
$$\Delta\ell_1 = \Delta\ell_2$$
$$\ell\alpha_1\Delta\theta_1 = \ell\alpha_2\Delta\theta_2$$
$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$
$$\frac{4}{3} = \frac{\theta - 30}{180 - 30}$$
$$\boxed{\theta = 230°C}$$

5. **(a)** Young's modulus $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{A(\Delta\ell/\ell)}$
Using, coefficient of linear expansion,
$$\alpha = \frac{\Delta\ell}{\ell\Delta T} \Rightarrow \frac{\Delta\ell}{\ell} = \alpha\Delta T \quad \therefore Y = \frac{F}{A(\alpha\Delta T)}$$

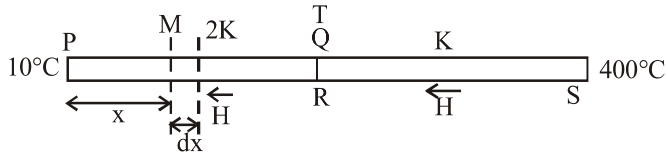
6. **(c)** As we know, Bulk modulus
$$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \quad \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$



$$V = V_0(1 + \gamma\Delta t)$$
$$\frac{\Delta V}{V_0} = \gamma\Delta t \quad \therefore \; \frac{P}{K} = \gamma\Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

7. **(c)** Due to thermal exp., change in length $(\Delta l) = l\,\alpha\,\Delta T$ ... (i)
Young's modulus (Y) $= \dfrac{\text{Normal stress}}{\text{Longitudinal strain}}$
$$Y = \frac{F/A}{\Delta l/l} \Rightarrow \frac{\Delta l}{l} = \frac{F}{AY}$$
$$\Delta l = \frac{Fl}{AY}$$
From eq^n (i), $\dfrac{Fl}{AY} = l\,\alpha\,\Delta T$
$F = AY\,\alpha\,\Delta T$

8. **(a)** At steady state, heat flow rate is same from P to Q and from Q to S



$$\therefore \; \frac{KA(400 - T)}{\ell} = \frac{2KA(T - 10)}{\ell} \Rightarrow T = 140°C$$
The temperature gradient
$$\frac{dT}{dx} = \frac{140 - 10}{1} \qquad \therefore dt = 130\,dx$$
Therefore change temperature at a cross-section M distant 'x' from P is
$\Delta T = 130\,x$
Extension in a small elemental length 'dx' is
$$dl = dx\alpha\,\Delta T = dx\,\alpha(130x)$$
$$\therefore \int dl = 130\alpha\int_0^1 x\,dx$$
$$\therefore \Delta l = 130 \times 1.2 \times 10^{-5} \times \frac{1}{2} = 78 \times 10^{-5}\text{ m}$$

9. **(c)** $\frac{\text{Reading on any scale} - \text{LFP}}{\text{UFP} - \text{LFP}}$

= constant for all scales

$$\frac{340-273}{373-273} = \frac{°Y-(-160)}{-50-(-160)} \Rightarrow \frac{67}{100} = \frac{y+160}{110}$$

$\therefore$ $Y = -86.3° Y$

10. **(c)** When the lengths of each rod increases by the same amount

$\therefore$ $\Delta\ell_a = \Delta\ell_s \Rightarrow \ell_1\alpha_a t = \ell_2\alpha_s t$

$$\Rightarrow \frac{\ell_2}{\ell_1} = \frac{\alpha_a}{\alpha_s} \Rightarrow \frac{\ell_2}{\ell_1}+1 = \frac{\alpha_a}{\alpha_s}+1$$

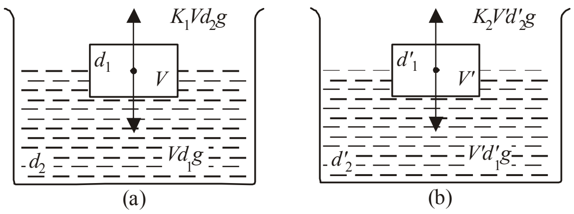
$$\Rightarrow \frac{\ell_2+\ell_1}{\ell_1} = \frac{\alpha_a+\alpha_s}{\alpha_s} \Rightarrow \frac{\ell_1}{\ell_1+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$$

11. **(a)** **For equilibrium when temperature 0°C fig (a)**

Upthrust = Wt. of body

$\therefore$ $K_1 V d_2 g = V d_1 g$

$$\Rightarrow K_1 = \frac{d_1}{d_2} \quad \text{... (i)}$$



For equilibrium when temperature increases to 60° fig. (b)

When the temperature is increased the density will decrease.

$\therefore$ $d_1' = d_1(1+\gamma_{Fe}\times 60)$

and $d_2' = d_2(1+\gamma_{Hg}\times 60)$

Again upthrust = Wt. of body

$\therefore$ $K_2 V' d_2' g = V' d_1' g$

$$\therefore \quad K_2\left[\frac{d_2}{1+\gamma_{Hg}\times 60}\right] = \frac{d_1}{1+\gamma_{Fe}\times 60}$$

$$\therefore \quad K_2\left[\frac{1+\gamma_{Fe}\times 60}{1+\gamma_{Hg}\times 60}\right] = \frac{d_1}{d_2} \Rightarrow \frac{K_1}{K_2} = \frac{1+\gamma_{Fe}\times 60}{1+\gamma_{Hg}\times 60}$$

12. **(c)** $W_1 = mg - Vd_a g$

$$W_2 = mg - V'd'_a g = mg - V(1+50\gamma_b)\frac{d_a g}{(1+50\gamma_a)}$$

$$= mg - Vd_a g\left[\frac{1+50\gamma_b}{1+50\gamma_a}\right]$$

Given $\gamma_b < \gamma_a$

$\therefore$ $1+50\gamma_b < 1+50\gamma_a$ or, $\frac{1+50\gamma_b}{1+50\gamma_a} < 1$

$\therefore$ $W_2 > W_1$ or $W_1 < W_2$

13. **(d)** As constant volume gas thermometer, works on Charle's law.

14. **(3)** We know that, $Y = \frac{F}{A}/\frac{\Delta l}{l}$

$$\therefore \quad Y = \frac{mg/A}{\Delta\ell/\ell} = \frac{mg\ell}{A\Delta\ell} \quad \text{....(i)}$$

Also $\Delta\ell = \ell\,\alpha\,\Delta T$ ...(ii)

From (i) and (ii)

$$Y = \frac{mg\ell}{A\ell\alpha\Delta T} = \frac{mg}{A\alpha\Delta T}$$

$$\therefore \quad m = \frac{YA\alpha\Delta T}{g} = \frac{10^{11}\times\pi(10^{-3})^2\times 10^{-5}\times 10}{10} = \pi \approx \mathbf{3}$$

15. **(20.00)**

Volume capacity of beaker, $V_0 = 500$ cc

$V_b = V_0 + V_0\gamma_{\text{beaker}}\Delta T$

When beaker is partially filled with V_m volume of mercury,

$V_b^1 = V_m + V_m\gamma_m\Delta T$

Unfilled volume $(V_0 - V_m) = (V_b - V_m^1)$

$$\Rightarrow V_0\gamma_{\text{beaker}} = V_m\gamma_M \quad \therefore V_m = \frac{V_0\gamma_{\text{beaker}}}{\gamma_M}$$

$$\text{or, } V_m = \frac{500\times 6\times 10^{-6}}{1.5\times 10^{-4}} = 20 \text{ cc.}$$

16. **(60.00)** Volume, $V = lbh$

$$\therefore \quad \gamma = \frac{\Delta V}{V} = \frac{\Delta\ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

(γ = coefficient of volume expansion)

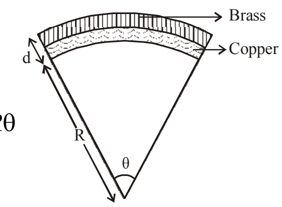
$\Rightarrow \gamma = 5\times 10^{-5} + 5\times 10^{-6} + 5\times 10^{-6}$

$= 60\times 10^{-6}/°C$

$\therefore$ Value of $C = 60.00$

17. **(b, d)** Let L_0 be the original length of the strip.

Co-efficient of linear expansion of brass is greater than that of copper i.e., $\alpha_B > \alpha_C$.



$L_B = L_0(1+\alpha_B\Delta T)(R+d)\theta$

$= L_0(1+\alpha_B\Delta T)$

Again, $L_C = L_0(1+\alpha_C\Delta T) = R\theta$

$$\therefore \quad \frac{(R+d)\theta}{R\theta} = \frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}$$

$$\text{or, } \frac{R+d}{R} = (1+\alpha_B\Delta T)(1-\alpha_C\Delta T),$$

[By binomial expansion]

$$\text{or, } 1+\frac{d}{R} = 1+(\alpha_B-\alpha_C)\Delta T - \alpha_B\alpha_C(\Delta T)^2$$

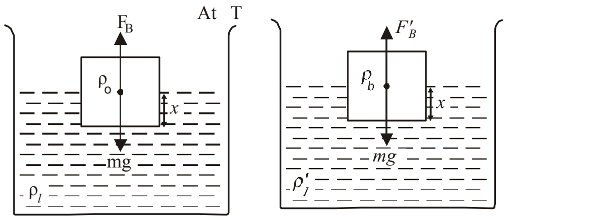
$$\text{or, } \frac{d}{R} = (\alpha_B-\alpha_C)\Delta T \text{ or } R = \frac{d}{(\alpha_B-\alpha_C)\Delta T}$$

$$\therefore \quad R \propto \frac{1}{\Delta T} \text{ and } R \propto \frac{1}{|\alpha_B-\alpha_C|}.$$

18. Initially, at temperature T bnoyant force

$F_B = \text{mg or, } Ax\rho_\ell g = AL\rho_b g$

$\therefore$ $x\rho_\ell = L\rho_b$ ... (i)



At temperature $T + \Delta T$ the volume of the cube increases but the density of liquid decreases so depth upto which the cube is immersed in the liquid remains same.

$\therefore F_B' = mg$

or, $A'x\rho'_\ell\, g = AL\rho_b g$

Now, $A' = A(1 + 2\alpha\Delta T)$

$\rho'_\ell = \rho_\ell(1-\gamma\Delta T)$

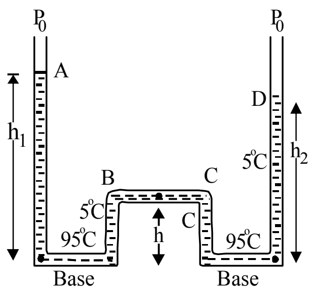
$\therefore \quad A(1+2\alpha\,\Delta T)\,x\,\rho_\ell(1-\gamma\Delta T)g = AL\rho_b g$

$\Rightarrow \quad x\rho_\ell(1+2\alpha\,\Delta T)(1-\gamma\Delta T) = L\rho_b$

$\Rightarrow \quad x\rho_\ell(1+2\alpha\,\Delta T)(1-\gamma\Delta T) = x\rho_\ell$ [from eq. (i)]

$\Rightarrow \quad 1 + 2\alpha\,\Delta T - \gamma\,\Delta T = 1 \quad \Rightarrow \gamma = 2\alpha$ or, $\gamma_i = 2 \propto s$

19. Density of a liquid decreases as temperature rises according to relatoin $d_0 = d_t(1+\gamma t)$ where γ = co-efficient of cubical expansion of liquid.



$h_1 = 52.8$cm, $h_2 = 51$ cm, $h = 49$ cm

Pressure at B = Pressure at C

$P_0 = d_{95°C}\, gh_1 - d_{5°C}\, gh = P_0 + d_{5°C}\, gh_2 - d_{95°C}\, gh$

or $\quad d_{95°C}(h_1 + h) = d_{5°C}(h_2 + h)$

or $\quad \dfrac{d_{95°C}}{d_{5°C}} = \dfrac{h_2+h}{h_1+h}$ or $\dfrac{d_0/(1+95\gamma)}{d_0/(1+5)} = \dfrac{h_2+h}{h_1+h}$

or $\quad \dfrac{1+5\gamma}{1+95\gamma} = \dfrac{51+49}{52.8+49} = \dfrac{100}{101.8} = \dfrac{1}{1.018}$

or $\quad 1 + 95\gamma = 1.018 + 5 \times 1.018\gamma$

or $\quad 95\gamma - 5.090\gamma = 0.018$ or $\gamma = \dfrac{0.018}{89.91} \times 2 \times 10^{-4}$

$\therefore \quad$ Coefficient of linear expansion $\alpha = \dfrac{\gamma}{3} = \dfrac{2\times10^{-4}}{3}$

$\therefore \quad \alpha = 6.67 \times 10^{-5}$ per °C.

20. Let γ_l be the coefficient of volume expansion of actual weight – apparent weight = up thrust

$W_0 - W_1 = V \times d_\ell \times g$ ... (i)

$W_0 - W_2 = V' \times d'_\ell \times g$ ... (ii)

Also, $V' = V(1 + \beta\,\Delta T)$ ... (iii)

and $d_\ell = d_\ell'(1+\gamma_\ell \Delta T)$ ... (iv)

From eq. (ii), (iii) and (iv)

$$W_0 - W_2 = \frac{V(1+\beta\Delta T)\times d_\ell}{1+\gamma_\ell\Delta T} \times g \quad \text{... (v)}$$

Dividing (i) by (v), we get

$$\frac{W_0 - W_1}{W_0 - W_2} = \frac{Vd_\ell g(1+\gamma_\ell\Delta T)}{V(1+\beta\Delta T)d_\ell g}$$

$$\Rightarrow \frac{W_0-W_1}{W_0-W_2} = \frac{1+\gamma_\ell\Delta T}{1+\beta\Delta T} \Rightarrow \frac{W_0-W_1}{W_0-W_2} = \frac{1+\gamma_\ell(t_2-t_1)}{1+\beta(t_2-t_1)}$$

$$\Rightarrow (W_0 - W_1)\,[1+\beta(t_2-t_1)] = (W_0 - W_2)\,[1+\gamma_\ell(t_2-t_1)]$$

$$\Rightarrow \gamma_\ell = \frac{W_2 - W_1}{(W_0-W_2)(t_2-t_1)} + \frac{\beta(W_0-W_1)}{(W_0-W_2)}$$

Topic-2 : Calorimetry and Heat Transfer

1. **(a)** As the rods are identical, so they have same length (l) and area of cross-section (A). They are connected in series. So, heat current will be same for all rods.

$$\text{Heat current} = \left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD}$$

$$\Rightarrow \frac{(100-70)K_1A}{l} = \frac{(70-20)K_2A}{l} = \frac{(20-0)K_3A}{l}$$

$\Rightarrow K_1(100-70) = K_2(70-20) = K_3(20-0)$

$\Rightarrow K_1(30) = K_2(50) = K_3(20)$

$\Rightarrow \dfrac{K_1}{10} = \dfrac{K_2}{6} = \dfrac{K_3}{15} \Rightarrow K_1 : K_2 : K_3 = 10:6:15$

$\Rightarrow K_1 : K_3 = 2:3.$

2. **(a)** According to question, one half of its kinetic energy is converted into heat in the wood.

$$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

$$\Rightarrow \Delta T = \frac{v^2}{4\times s} = \frac{210\times210}{4\times4.2\times0.3\times1000} = 87.5°\text{C}$$

3. **(a)** Here ice melts due to water.

Let the amount of ice melts = m_{ice}

$$m_w s_w \Delta\theta = m_{ice}L_{ice} \quad \therefore m_{ice} = \frac{m_w s_w \Delta\theta}{L_{ice}}$$

$$= \frac{0.2\times4200\times25}{3.4\times10^5} = 0.0617 \text{ kg} = 61.7 \text{ g}$$

4. **(a)** Heat given by water $= m_w C_w (T_{mix} - T_w)$

$= 200 \times 1 \times (31 - 25)$

Heat taken by steam $= m\,L_{stem} + m\,C_w\,(T_s - T_{mix})$

$= m \times 540 + m(1) \times (100-31) = m \times 540 + m(1) \times (69)$

From the principal of calorimeter,

Heat lost = Heat gained

$\therefore (200)(31-25) = m \times 540 + m(1)(69)$

$\Rightarrow 1200 = m(609) \Rightarrow m \approx 2.$

5. **(40)** Using the principal of calorimetry

$M_{ice} L_f + m_{ice}(40-0)\,C_w = m_{stream} L_v + m_{stream}(100-40)\,C_w$

$\Rightarrow M(540) + M \times 1 \times (100-40) = 200 \times 80 + 200 \times 1 \times 40$

$\Rightarrow 600\,M = 24000 \Rightarrow M = 40g$

6. **(a)** $M_1 C_{ice} \times (10) + M_1 L = M_2 C_\omega (50)$

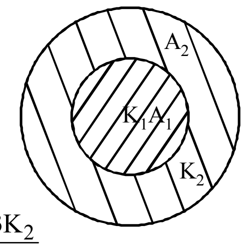
or $M_1 \times C_{ice} (=0.5) \times 10 + M_1 L = M_2 \times 1 \times 50$

$\Rightarrow L = \frac{50M_2}{M_1} - 5$

7. **(a)** $H_1 = H_2$ $\theta_2 \boxed{3k \,|\, k} \theta_1$ (widths d, $3d$; junction temperature θ)

or $(3k)A\left(\frac{\theta_2-\theta}{d}\right) = kA\left(\frac{\theta-\theta_1}{3d}\right)$ or $\theta = \left(\frac{\theta_1+9\theta_2}{10}\right)$

8. **(d)** Effective thermal conductivity of system



$K_{eq} = \frac{K_1A_1+K_2A_2}{A_1+A_2}$

$= \frac{K_1\pi R^2 + K_2[\pi(2R)^2 - \pi R^2]}{\pi(2R)^2}$

$= \frac{K_1(\pi R^2)+K_2(3\pi R^2)}{4\pi R^2} = \frac{K_1+3K_2}{4}$

9. **(d)** Let m gram of ice is added.

From principal of calorimeter

heat gained (by ice) = heat lost (by water)

$\therefore\ 20 \times 2.1 \times m + (m-20) \times 334 = 50 \times 4.2 \times 40$

$376\,m = 8400 + 6680$

$m = 40.1$

10. **(c)** Heat loss = Heat gain = $mS\Delta\theta$

So, $m_A S_A \Delta\theta_A = m_B S_B \Delta\theta_B$

$\Rightarrow 100 \times S_A \times (100-90) = 50 \times S_B \times (90-75)$

$2S_A = 1.5S_B \Rightarrow S_A = \frac{3}{4} S_B$

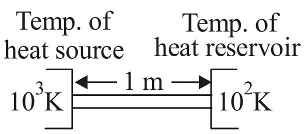
Now, $100 \times S_A \times (100-\theta) = 50 \times S_B \times (\theta - 50)$

$2 \times \left(\frac{3}{4}\right) \times (100-\theta) = (\theta-50)$

$300 - 3\theta = 2\theta - 100$

$400 = 5\theta \Rightarrow \theta = 80°C$

11. **(a)**



$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$

Energy flux, $\frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{k\Delta T}{\ell}$

$= \frac{(0.1)(900)}{1} = 90\ \text{W/m}^2$

12. **(c)** Let specific heat of unknown metal be 's' According to principle of calorimetry, Heat lost

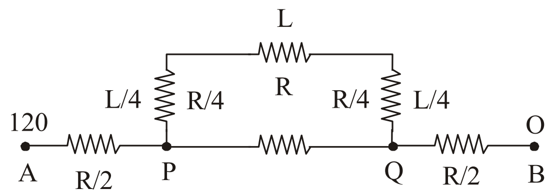
= Heat gain m × $s\Delta\theta = m_1 s_{brass}(\Delta\theta_1 + m_2\, s_{water} + \Delta\theta_2)$

$\Rightarrow 192 \times S \times (100-21.5) = 128 \times 394 \times (21.5-8.4)$

Solving we get,$+\ 240 \times 4200 \times (21.5-8.4)$

$S = 916\ \text{Jkg}^{-1}\text{k}^{-1}$

13. **(a)** $\frac{\Delta T_{AB}}{R_{AB}} = \frac{120}{\frac{8}{5}R} = \frac{120 \times 5}{8R}$



In steady state temperature difference between P and Q,

$\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5}R = \frac{360}{8} = 45°C$

14. **(d)** According to principle of calorimetry,

$Q_{given} = Q_{used}$

$0.2 \times S \times (150-40) = 150 \times 1 \times (40-27) + 25 \times (40-27)$

$0.2 \times S \times 110 = 150 \times 13 + 25 \times 13$

Specific heat of aluminium

$S = \frac{13 \times 25 \times 7}{0.2 \times 110} = 434\ \text{J/kg-°C}$

15. **(d)** According to principle of calorimetry,

Heat lost = Heat gain

$100 \times 0.1(T-75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$10\,T - 750 = 450 + 7650 = 8100 \Rightarrow T - 75 = 810$

$T = 885°C$

16. **(b)** $P_{heater} - P_{cooler} = \frac{mc\Delta T}{t} = \frac{V\rho c\Delta T}{t}$

$\therefore (3000 - P_{cooler}) = \frac{0.12 \times 1000 \times 4.2 \times 10^3 \times 20}{3 \times 60 \times 60}$

$\therefore P_{cooler} = 2067\text{W}$

17. **(b)** As $Pt = mC\Delta T$

So, $P \times 10 \times 60 = mC\ 100$ ...(i)

and $P \times 55 \times 60 = \text{mL}$ ...(ii)

Dividing equation (i) by (ii) we get

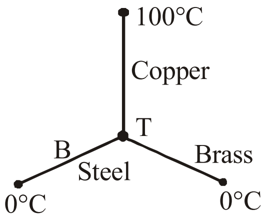
$\frac{10}{55} = \frac{C \times 100}{L}$

$\therefore\ L = 550$ cal./g.

18. **(c)** Rate of heat flow is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)}{l}$$

Where, K = coefficient of thermal conductivity
l = length of rod and A = area of cross-section of rod



If the junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$$\frac{0.92 \times 4(100 - T)}{46}$$

$$= \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times (T - 0)}{12}$$

$\Rightarrow \quad 200 - 2T = 2T + T \quad \Rightarrow \quad T = 40°C$

$$\therefore \quad Q_{Copper} = \frac{0.92 \times 4 \times 60}{46} = 4.8 \text{ cal/s}$$

19. **(c)** In the given problem, fall in temperature of sphere,

$dT = (3T_0 - 2T_0) = T_0$

Temperature of surrounding, $T_{surr} = T_0$
Initial temperature of sphere, $T_{initial} = 3T_0$
Specific heat of the material of the sphere varies as,

$c = \alpha T^3$ per unit mass (α = a constant)
Applying formula,

$$\frac{dT}{dt} = \frac{\sigma A}{McJ}\left(T^4 - T_{surr}^4\right)$$

$$\Rightarrow \frac{T_0}{dt} = \frac{\sigma 4\pi R^2}{M\alpha(3T_0)^3 J}\left[(3T_0)^4 - (T_0)^4\right]$$

$$\Rightarrow dt = \frac{M\alpha 27 T_0^4 J}{\sigma 4\pi R^2 \times 80 T_0^4}$$

Solving we get,
Time taken for the sphere to cool down temperature $2T_0$,

$$t = \frac{M\alpha}{16\pi R^2 \sigma} \ln\left(\frac{16}{3}\right)$$

20. **(a)** Let T be the final steady state temperature of the black body.
In steady state,
Energy lost = Energy gained
$\sigma(T^4 - T_0^4) \times 4\pi R^2 = I(\pi R^2)$

$$\therefore \; 5.7 \times 10^{-8}\left[T^4 - (300)^4\right] \times 4 = 912$$

$\therefore \quad T = 330$ K

21. **(d)** When radius is decrease by ΔR,

$$4\pi R^2 \Delta R \rho L = 4\pi T[R^2 - (R - \Delta R)^2]$$

$$\Rightarrow \rho R^2 \Delta R L = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$$

$$\Rightarrow \rho R^2 \Delta R L = T 2R \Delta R \qquad [\,\Delta R \text{ is very small}]$$

$$\Rightarrow R = \frac{2T}{\rho L}$$

22. **(d)** As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30°C to 25°C) is same 2 minutes, therefore

$$\left(\frac{dQ}{dt}\right)_{water} = \left(\frac{dQ}{dt}\right)_{liquid}$$

$$\text{or, } \frac{(m_W C_W + W)\Delta T}{t} = \frac{(m_\ell C_\ell + W)\Delta T}{t}$$

(W = water equivalent of the vessel)
or, $m_W C_W = m_\ell C_\ell$

$$\therefore \text{ Specific heat of liquid, } C_\ell = \frac{m_W C_W}{m_\ell}$$

$$= \frac{50 \times 1}{100} = 0.5 \text{ kcal/kg}$$

23. **(a)** Equivalent thermal resistance in configuration-I

$$R_3 = R_1 + R_2 = \frac{L}{KA} + \frac{L}{2KA} = \frac{3}{2}\frac{L}{KA}$$

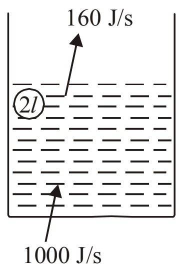
Equivalent thermal resistance in configuration-II

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l} \quad \text{or,} \quad R_P = \frac{l}{3KA} = \frac{R_s}{4.5}$$

i.e., Thermal resistance in configuration–II, R_P is 4.5 times less than thermal resistance in configuration – I, R_s

$$\therefore \quad 4.5 t_P = t_s \;\Rightarrow\; t_P = \frac{t_s}{4.5} = \frac{9}{4.5} s = 2s$$

24. **(c)** As shown in the figure, the net heat gained by the water to raise its temperature
$= (1000 - 160) = 840$ J/s
Now, the heat required to raise the temperature of water from 27°C to 77°C
$Q = mc\,\Delta t = 2 \times 4200 \times 50$ J
Hence time required to gain Q amount of heat



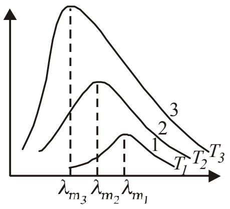
$$t = \frac{Q}{840} = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

25. **(a)** 1 Calorie is the amount of heat required to raise temperature of 1 gram of water from 14.5°C to 15.5°C at 760 mm of Hg.

26. **(d)** Warming of glass of bulb due to filament is primarily due to radiation. A medium is required for convection process. As a bulb is almost evacuated, heat from the filament is transmitted through radiation.

27. **(a)** According to Wein's displacement law

$\lambda_m \times T = \text{constant}$



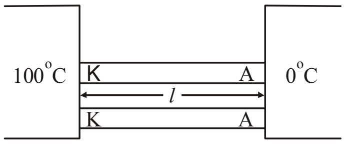
From the graph, $\lambda_{m_3} < \lambda_{m_2} < \lambda_{m_1}$

$\therefore \quad T_3 > T_2 > T_1$

The temperature of Sun – T_3 is higher than that of welding arc T_2 which in turn is greater than tungsten filament -T_1.

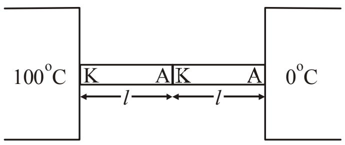
28. **(d)** In parallel combination of rods

$K_P = K_1 + K_2 = K + K = 2K$



In series combination

$$K_S = \frac{K_1 - K_2}{K_1 + K_2} = \frac{KK}{2K} = \frac{K}{2}$$



$$q_1 = \frac{2K\,A(100)}{\ell} \text{ and } q_2 = \frac{KA(100)}{2}$$

$$\therefore \quad \frac{q_2}{q_1} = \frac{KA(100)}{2\ell} \times \frac{\ell}{2K\,A(100)} = \frac{1}{4}$$

29. **(c)** From 50 K to boiling temperature, T increases linearly as $Q = mc\,\Delta T$. Hence $T - t$ graph will be a straight line inclined to time axis.

During boiling, $\quad Q = mL$

Temperature remains constant till boiling is complete and graph will be a straight line paralls to time axis.

After that, temperature increases linearly T-t graph will be a straight line inclined to time axis.

30. **(b)** From wein's displacement law

$\lambda_m T$ = Constant

$$\therefore \quad T = \frac{\text{constant}}{\lambda m}$$

$$\left\{\therefore T_A = \frac{C}{3 \times 10^{-7}}, T_B = \frac{C}{4 \times 10^{-7}}, T_C = \frac{C}{5 \times 10^{-7}}\right\}$$

Again from stefan's law

$$Q = \sigma A T^4 \;\therefore\; Q_A = \sigma.\pi\,(2 \times 10^{-2})^2 \times \frac{C^4}{81 \times 10^{-28}}$$

$$Q_B = \sigma.\pi\,(4 \times 10^{-2})^2 \times \frac{C^4}{256 \times 10^{-28}}$$

$$\text{and } Q_C = \sigma.\pi\,(6 \times 10^{-2})^2 \times \frac{C^4}{625 \times 10^{-28}}$$

Hence $Q_B > Q_C > Q_A$

31. **(b)** Heat released when 5kg of water at 20°C falls to 0°C temperature

$= mC_\omega\,\Delta T = 5 \times 1 \times 20 = 100$ kcal

Heat required by 2 kg ice at – 20°C to convert into 2 kg of ice at 0°C

$= mC_{ice}\,\Delta T = 2 \times 0.5 \times 20 = 20$ k cal.

Hence remaining $100 - 20 = 8$ kcal of heat will melt m kg ice at 0°C to water at 0°C

$Q = mL \Rightarrow 80 = m \times 80 \Rightarrow m = 1$ kg

Therefore the amount of water at 0°C

$= 5$ kg $+ 1$kg $= 6$ kg

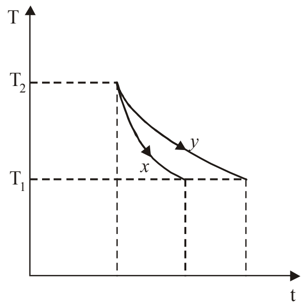
And amount of ice at 0°C = 2 – 1 = 1 kg.

32. **(c)** From the graph, $\left(\frac{-dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

Rate of cooling, $\left(\frac{-dT}{dt}\right) \propto$ emissivity (e)

$\therefore \quad E_x > E_y$

Also $a_x > a_y$ as good absorbers are good emitters.



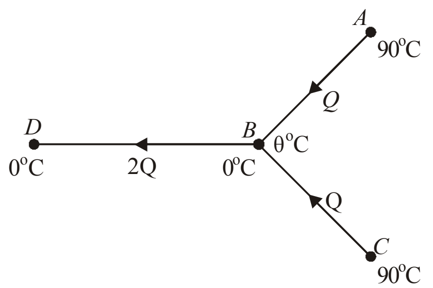
33. **(a)** According to Kirchoff's law, good absorbers are good emitters and bad reflectors.

At high temperature (in the furnace), since it absorbs more energy, it emits more radiations as well and hence is the brightest and it is the darkest body initially.

34. **(b)** Let θ°C be the temperature of junction at B. Let Q is the heat flowing per second from A at 90°C to B at θ°C on account of temperature difference.

$$\therefore \quad Q = \frac{KA(90 - \theta)}{\ell} \qquad \text{... (i)}$$

And same for C to B. $\quad Q = \dfrac{KA(90 - \theta)}{l}$



$\therefore$ The heat flowing per second from B to D

$$2Q = \frac{KA(\theta - 0)}{\ell} \quad \text{... (ii)}$$

Dividing eq. (ii) by (i)

$$2 = \frac{\theta}{90 - \theta} \Rightarrow \theta = 60°$$

Hence temperature of the junction $\theta = 60°C$

35. **(b)** According to Wien's displacement law, λT = constant

From graph $\lambda_1 < \lambda_3 < \lambda_2 \quad \therefore \; T_1 > T_3 > T_2$.

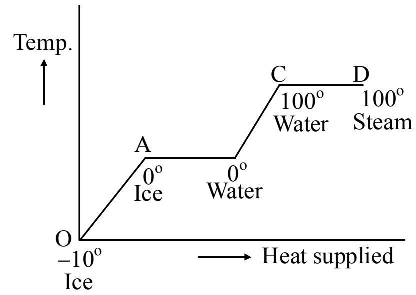
36. **(a)**

O $\rightarrow$ A, the temperature of ice changes from –10°C to 0°C.

A $\rightarrow$ B, ice at 0°C melts into water at 0°C.

B $\rightarrow$ C, water at 0°C changes into water at 100°C.

C $\rightarrow$ D, water at 100°C changes into steam at 100°C.

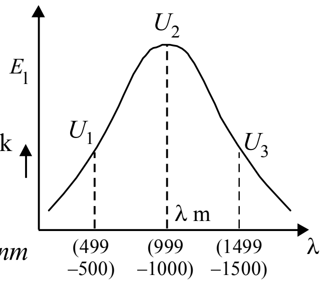


37. **(d)** According to Wien's displacement law,

$\lambda_m T = 2.88 \times 10^6\, nmK$

The wavelength at the peak of the spectrum

$$\lambda_m = \frac{2.88 \times 10^6\, nmK}{2880\, K} = 10^3\, nm$$



It follows that the energy radiated between 499 nm to 500 nm will be less than that emitted between 999 nm to 1000 nm, i.e., $U_1 < U_2$ or $U_2 > U_1$.

38. **(d)** From Stefan's law energy radiated per second by a black body

$$\frac{E}{t} = \sigma T^4 \times A = \sigma T^4 \times 4\pi r^2$$

$$\frac{E}{t} = 450 \text{ when } T = 500\,K, r = 0.12 \text{ m}$$

$$\therefore \quad 450 = 4\pi\sigma (500)^4 (0.12)^2 \quad \text{... (i)}$$

when $T = 1000\,K$, $r = 0.06$ m, $\frac{E}{t} = ?$

$$\therefore \quad \frac{E}{t} = 4\pi\sigma (1000)^4 (0.06)^2 \quad \text{... (ii)}$$

Dividing eq. (ii) by (i), we get

$$\frac{E/t}{450} = \frac{(1000)^4 (0.06)^2}{(500)^4 (0.12)^2} = \frac{2^4}{2^2} = 4$$

$$\text{or, } \frac{E}{t} = 450 \times 4 = 1800 \text{ W}$$

39. **(d)** According to Stefan's law, $\Delta Q = e\sigma A T^4 \Delta t$

Also, $\Delta Q = mc\,\Delta T$

or, $mc\,\Delta T = e\sigma A T^4\,\Delta t$

$$\text{or, } \frac{\Delta T}{\Delta t} = \frac{e\sigma A T^4}{mc} = \frac{e\sigma T^4}{mc}\left[\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = k\left(\frac{1}{m}\right)^{1/3}$$

Therefore for the given two bodies.

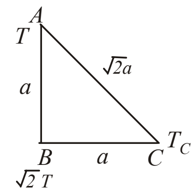
$$\frac{\Delta T_1 / \Delta t_1}{\Delta T_2 / \Delta t_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

40. **(b)** According to question, temperature at $B = \sqrt{2}T >$ temperatue at $A = T$ so heat flow from B to A, A to C and C to B

For steady state condition, $\Delta Q/\Delta t$ is same.

Applying heat conduction formula $\frac{\Delta Q}{\Delta t} = \frac{k\,A\,\Delta T}{\ell}$

For sides AC and CB $\left(\frac{\Delta T}{\sqrt{2}a}\right)_{AC} = \left(\frac{\Delta T}{a}\right)_{CB}$

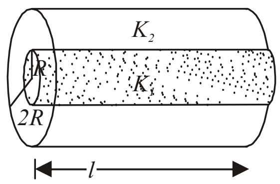


$$\Rightarrow \frac{T - T_c}{\sqrt{2}a} = \frac{T_c - \sqrt{2}\,T}{a} \Rightarrow T - T_c = \sqrt{2}\,T_c - 2T$$

$$\Rightarrow 3T = T_c(\sqrt{2} + 1) \Rightarrow \frac{T_c}{T} = \frac{3}{\sqrt{2} + 1}$$

41. **(c)** Let K = thermal conductivity of the system

Total transfer of heat per second through the combined system = Heat transfer per second from material with thermal conductivity K_1 + Heat transfer per second from material with thermal conductivity K_2.



$$\frac{KA\Delta T}{\ell} = \frac{K_1 A_1 \Delta T}{\ell} + \frac{K_2 A_2 \Delta T}{\ell}$$

or, $K\pi (2R)^2 = K_1 \pi R^2 + K_2 \pi [(2R)^2 - R^2]$

or, $K = \dfrac{K_1 + 3K_2}{4}$

42. **(a)** From principle of calorimetry,

Heat lost by steam = Heat gained by (water + calorimeter)

$mL + m \times c \times (100 - 80) = 1.12 \times c \times (80 - 15)$

$m\,[540 + 1 \times 20] = 1.12 \times 1 \times 65$

$m = 0.13$ kg

43. **(b)** $\dfrac{Q_2}{Q_1} = \dfrac{nC_v \Delta T}{nC_p \Delta T} = \dfrac{1}{\gamma} \Rightarrow Q_2 = \dfrac{Q_1}{\gamma}$

$$\Rightarrow \quad Q_2 = \frac{70}{1.4} = 50 \text{ cal}$$

44. **(a)** From, $V_{rms} = \sqrt{\frac{3RT}{m}}$

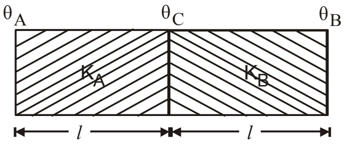
$$M = \frac{3RT}{V_{rms}^2} = \frac{3 \times 8.314 \times 298}{1930 \times 1930} \times 1000 = 2\,g$$

Hence, the gas is H_2

45. **(b)** Under thermal equilibrium, $\theta_A - \theta_B = 36°C$ (Given)

$K_A = 2K_B$ (Given)

$$\theta_C = \frac{\frac{K_A}{\ell}\theta_A + \frac{K_B}{\ell}\theta_B}{\frac{K_A}{\ell} + \frac{K_B}{\ell}}$$



$$\therefore \quad \theta_C = \frac{2\theta_A + \theta_B}{3} = \frac{2\theta_A + \theta_A - 36}{3} = \frac{3(\theta_A - 12)}{3}$$

$$\therefore \quad \theta_A - \theta_C = 12°C$$

Hence, the temperature difference across the layer $A = 12°C$

46. **(9)** According to stefan's law, $P \propto T^4$ or $P = P_0 T^4$

$$\therefore \log_2 P = \log_2 P_0 + \log_2 T^4 \quad \therefore \log_2 \frac{P}{P_0} = 4\log_2 T$$

At $T = 487°C = 760$ K, $\log_2 \frac{P}{P_0} = 4\log_2 760 = 1$ ... (i)

At $T = 2767°C = 3040K$,

$$\log_e \frac{\rho}{\rho_0} = 4\log_2 3040 = 4\log_2(760 \times 4)$$

$$= 4\left[\log_2 760 + \log_2 2^2\right] = 4\log_2 760 + 8 = 1 + 8 = 9$$

47. **(2)** From (i) Stefan-Boltzmann law, $P = \sigma A T^4$ and (ii) Wein's displacement law $= \lambda_m \times T =$ constant

$$\frac{P_A}{P_B} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = 2$$

48. **(9)** According to stefan-Boltzmann law,

$$\frac{\text{Rate of total energy radiated by A}}{\text{Rate of total energy radiated by B}}$$

$$= \frac{\sigma T_1^4 (4\pi r_1^2)}{\sigma T_2^4 (4\pi r_2^2)} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4 \left(\frac{r_1}{r_2}\right)^2 \quad \left[\because \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \text{by Wein's law}\right]$$

$$= \left(\frac{1500}{500}\right)^4 \left(\frac{6}{18}\right)^2 = \mathbf{9}$$

49. **(8)** As there is no other heat exchange in this process, So Heat supplied = Heat used in converting m grams of ice from –5°C to 0°C + Heat used in converting 1 gram of ice at 0°C to water at 0°C

or, $420 = mc\Delta\theta + ml$

$$\Rightarrow 420 = m \times \frac{2100}{1000} \times 5 + \frac{1 \times 3.36 \times 10^5}{1000}$$

$$\Rightarrow 420 = m \times 10.5 + 336 \qquad \therefore m = \frac{84}{10.5} = \mathbf{8}\,g$$

50. **(270°C)** Let C be the specific heat capacity of liquid and L be the latent heat of vapourisation.

From principle of calorimetry,

Heat lost = heat gain

$m_C S_C \Delta T = mC\,\Delta T + mL$

or $m_C S_C (110 - 80) = 5C\,(80 - 30) + 5L$ ...(i)

Where, m_C = mass of calorimeter

S_C = sp. heat of calorimeter

Again, when 80g liquid is poured and equilibrium temperature is 50°C

$m_C S_C (80 - 50) = 80C\,(50 - 30)$ ...(ii)

From eq. (i) & (ii)

$1600\,C = 250C + 5L$

$$\therefore \frac{L}{C} = \frac{1350}{5} = 270°C$$

51. **(4.00)**

Rate of heat flow will be same,

$$\text{Rate of heat flow } \frac{dQ}{dt} = \frac{\text{temp. difference}}{\text{thermal resistance}} = \frac{1}{R}(T_2 - T_1)$$

where $R = \frac{L}{KA}$

$$\frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \text{ or } R_1 = R_2$$

$$\frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \therefore \frac{K_1}{K_2} = \frac{A_2}{A_1} = \frac{\pi(2r)^2}{\pi r^2} = 4\left[\because L_1 = L_2 = L\right]$$

52. Solar power received by 0.2 m^2 area

$= (1400\ W/m^2)(0.2\ m^2) = 280$ W

Mass of ice = 280 g = 0.280 kg

Heat required to melt ice = mL

$= (0.280)(3.3 \times 10^5) = 9.24 \times 10^4$ J

If t is the time taken for the ice to melt,

$$\therefore \quad (280)t = 9.24 \times 10^4 \text{ J} \qquad \left[\because P = \frac{E}{t}\right]$$

$$\text{or,} \quad t = \frac{9.24 \times 10^4}{280}\text{s} = 330\text{s} \equiv 5.5\,\text{min}$$

53. Let t be the temperature of the interface after the steady state is reached.

The heat transferred per second through A

$Q_1 = K_1 A\,(100 - t)$

The heat transferred per second through B

$Q_2 = K_2 A\,(t - 0)$

At steady state $K_1 A\,(100 - t) = K_2 A\,(t - 0)$

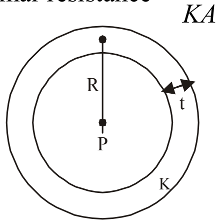
$\Rightarrow 300\,(100 - t) = 200\,(t - 0) \Rightarrow 300 - 3t = 2t \Rightarrow t = 60°$ C

54. It takes t seconds for the substance to solidify (given). Therefore total heat released in t seconds $= P \times t = mL_{\text{fusion}}$

$$\therefore \quad L_{\text{fusion}} = \frac{P \times t}{m}$$

55. Rate of heat transfer, $P = \dfrac{\text{temperature differ}}{\text{thermal resistance}}$

or $\quad P = \dfrac{T}{t/4\pi R^2 K} \quad \left[\because \text{ Thermal resistance} = \dfrac{l(-t)}{KA}\right]$



$$\Rightarrow \quad t = \frac{4\pi KTR^2}{P}$$

i.e., thickness t should not exceed, $\dfrac{4\pi KTR^2}{P}$

56. Energy radiated per second

$$\sigma T^4 A = mc\frac{dT}{dt}$$

$$\Rightarrow \quad dt = \frac{mcdT}{\sigma T^4 A} = \frac{\rho \times \frac{4}{3}\pi r^3 cdT}{\sigma T^4 \times 4\pi r^2} \quad \left[\because m = \rho \times \frac{4}{3}\pi r^3 \text{and } A = 4\pi r^2\right]$$

$$\Rightarrow \quad dt = \frac{\rho rc}{3\sigma}\frac{dT}{T^4}$$

Integrating both sides

$$\int_0^t dt = \frac{\rho rc}{3\sigma}\int_{200}^{100}\frac{dT}{T^4} = \frac{\rho rc}{3\sigma}\left[-\frac{1}{3T^3}\right]_{200}^{100}$$

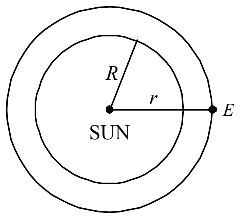
$$t = -\frac{\rho rc}{9\sigma}\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

$$t = \frac{7\rho rc}{(72\times 10^6)\sigma} \approx \frac{7\rho rc}{72\times 10^6 (5.67\times 10^{-8})} = 1.71\rho rc$$

Hence time required for the temperature of the sphere to drop to 100 k = 1.71 ρrc

57. According to Stefan's law,

Energy radiated per unit time per unit area $= \sigma T^4$



$$\therefore \quad \frac{\text{Energy}}{\text{time}} = \left(\sigma T^4\right)\times\left(4\pi R^2\right)$$

$\therefore$ Energy received on earth per unit time $= 1400 \times 4\pi r^2$

$$\therefore \quad \sigma T^4 \times 4\pi R^2 = 1400 \times 4\pi r^2 \quad \text{or} \quad T^4 = \frac{1400 \times r^2}{\sigma R^2}$$

$$\text{or} \quad T^4 = \frac{1400 \times (1.5\times 10^{11})^2}{(5.67\times 10^{-8})(7\times 10^8)^2}$$

$$\text{or} \quad T^4 = \frac{14 \times 2.25 \times 10^{24}}{5.67 \times 49 \times 10^8} \quad \text{or} \quad T = 5803 \text{ K.}$$

58. The heat required for 100 g of ice at 0° C to change into water at 0°C $= mL = 100 \times 80 \times 4.2 = 33{,}600$ J

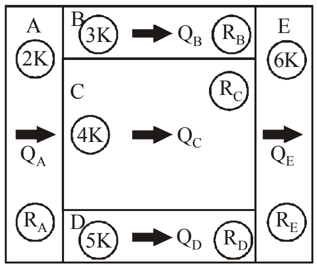
The heat released by 300g of water at 25°C to change its temperature to 0°C $= mc\Delta T = 300 \times 4.2 \times 25 = 31{,}500$ J

Hence complete ice will not melt, so the final temperature of the mixture will be 0°C.

59. O $\to$ A material is in solid state only temperature is increasing. From A $\to$ B material is partly solid and partly liquid.
Since P is a point between A and B, therefore the material is partly solid and partly liquid.

60. **(False)** Energy radiated per second by the first sphere
$E_1 = \varepsilon\sigma T^4 A = \varepsilon\sigma(4000)^4 \times 4\pi \times 11 \times 11$
$= 1024 \times \pi \times 10^{12} \times \varepsilon\sigma$
Energy radiated per second by the second sphere
$E_2 = \varepsilon\sigma \times (2000)^4 \times 4\pi \times 4 \times 4$
$= 1024\,\pi \times 10^{12} \times \varepsilon\sigma$
$\therefore\ E_1 = E_2$

61. **(a, c, d)**
According to question, heat Q flows only from left to right through the blocks. Hence heat flow through slab A and E are the same.



$Q_A = Q_B + Q_C + Q_D = Q_E$

We know that thermal resistance R $= \dfrac{\ell}{KA}$

Let the width of slabs be W. Then

$$R_A = \frac{L}{2K(4L)W} = \frac{1}{8KW},\ R_B = \frac{4L}{3K(LW)} = \frac{4}{3KW}$$

$$R_C = \frac{4L}{4K(2LW)} = \frac{1}{2KW},\ R_D = \frac{4L}{5K(LW)} = \frac{4}{5KW}$$

$$R_E = \frac{L}{6K(4LW)} = \frac{1}{24KW}$$

Now, $\Delta T = QR$
Since the resistance to heat flow is least for slab E, the temperature difference across E is smallest.
Also

$$Q_C = \frac{\Delta T_C}{R_C} = \frac{\Delta T_C}{1/2\,KW} = 2KW(\Delta T_C)$$

$$Q_B = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{4/3KW} = \frac{3KW(\Delta T_C)}{4} \quad [\because \Delta T_B = \Delta T_C]$$

$$Q_D = \frac{\Delta T_D}{R_D} = \frac{\Delta T_C}{4/5KW} = \frac{5KW(\Delta T_c)}{4} \quad [\because \Delta T_D = \Delta T_C]$$

$$Q_B + Q_D = \frac{3KW(\Delta T_C)}{4} + \frac{5KW(\Delta T_C)}{4}$$

$$= \frac{8KW(\Delta T_C)}{4} = 2KW(\Delta T_C) = Q_C$$

i.e., $\quad Q_C = Q_B + Q_D$

62. **(a, c, d)**
Since the temperature of black body and chamber remains constant so energy emitted = energy absorbed by black body.

63. **(a, b)**

Energy emitted per second by body A and B are same.

$\therefore = \varepsilon_A \sigma T_A^4 A = \varepsilon_B \sigma T_B^4 A$

$\therefore T_B = \left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{1/4} \times T_A = 1934\,\text{K}$

According to Wein's displacement law $\lambda_m \propto \frac{1}{T}$

$(\lambda_m)_A T_A = (\lambda_m)_B T_B$

$\Rightarrow \frac{(\lambda_m)_A}{(\lambda_m)_B} = \frac{T_B}{T_A} = \frac{5802}{1934}$ ... (i)

Since temperature of A is more therefore $(\lambda_m)_A$ is less

$\therefore (\lambda_m)_B - (\lambda_m)_A = 1 \times 10^{-6}\,\text{m}$ (given) ... (ii)

Solving eq. (i) and (ii), we get

$\lambda_B = 1.5 \times 10^{-6}\,\text{m}.$

64. Heat lost by steam at 100°C to change to 100°C water

$Q_1 = mL_{vap} = 0.05 \times 2268 \times 1000 = 1{,}13{,}400\,\text{J}$

Heat lost by 100°C water to change to 0°C water

$Q_2 = MC\Delta T = 0.05 \times 4200 \times 100 = 21{,}000\,\text{J}$

Heat required by 0.45 kg of ice to change its temperature from 253 K to 273 K

$Q_3 = m \times C_{ice} \times \Delta T = 0.45 \times 2100 \times 20 = 18{,}900\,\text{J}$

Heat required by 0.45 kg ice at 273 K to convert into 0.45 kg water at 273 K

$Q_4 = mL_{fusion} = 0.45 \times 336 \times 1000 = 151{,}200\,\text{J}$

From the above data it is clear that $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ so whole ice will not melt.

Therefore the final temperature will be 273 K or 0°C.

65. Let m be the mass of the container

According to principle of calorimetry, heat lost = heat gain

Heat lost by container = $msdT$ or $dQ = m(A + BT)dT$

or $\int_0^Q dQ = m\int_{500}^{300} (A + BT)\,dT$

or $Q = m\left[AT + \frac{BT^2}{2}\right]_{500}^{300}$

or $Q = m\left[100T + \frac{2\times 10^{-2}}{2}T^2\right]_{500}^{300}$

or $Q = m\left[100(300-500) + \frac{(300)^2 \times (500)^2}{100}\right]$

or $Q = m\,[-2000 - 1600]$ calorie

or $Q = -21600\,m$ calorie ...(i)

Heat gained by ice in melting = mL

$\therefore Q_1 = 0.1 \times 80000 = 8000$ cal

Heat gained by water of above ice = $ms\Delta T$

$\therefore Q_2 = 0.1 \times 1000 \times 27 = 2700$ cal

$\therefore$ Total heat gained = 8000 + 2700

$Q_1 + Q_2 = 10700$ cal ...(ii)

$\therefore$ Heat lost = Heat gained or 21600 m = 10700

or $m = \frac{10700}{21600} = 0.495\,\text{kg}$ $\therefore$ Mass of container = 0.495 kg.

66. Heat lost by steam = Heat gained by water

$m_s L_{fus} = m_w c\,\Delta T$

$\Rightarrow m_s = \frac{m_w c\Delta T}{L_{fus}} = \frac{0.1 \times 4200 \times 66}{540 \times 10^3 \times 4.2} = 0.0122\,\text{kg}$

67. The energy emitted per second by both spheres will be same as the temperature and surface area are same.

We know that $Q = mc\Delta T$

or, $\frac{dQ}{dt} = \frac{mcdT}{dt}$

Since Q is same and c is also same (both of copper).

$\therefore \frac{dQ}{dt} \propto \frac{1}{m}$

Mass of hollow sphere is less so hollow sphere will cool faster.

68. Given 25% of the heat is absorbed by the obstacle. Therefore 75% heat is used in melting of lead. Initial temp. = 27°C

Metting point of lead = 327°C $\therefore$ ΔT = 327 – 27 = 300°C

$(0.75) \times \frac{1}{2}Mv^2 = Mc\,\Delta T + ML$

$(0.75) \times \frac{1}{2}v^2 = (0.03 \times 300 + 6) \times 4.2$

$v = 12.96$ m/s

69. (i) In region - AB, heat is absorbed by the material at a constant temperature and is used in phase changing from solid to liquid.

In region CD heat is absorbed by the material at a constant temperature and is used in phase changing from liquid to gas.

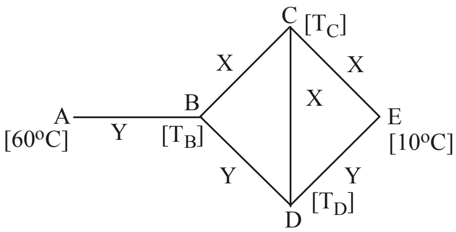
(ii) Latent heat of vaporisation = 2 (latent heat of fusion)

(iii) The slope DE indicates that the temperature of the solid begins to rise.

(iv) The slope of OA > slope of BC. This indicates that there is a rise in specific heat.

70. Given: Thermal conductivity K_X = 0.92 cal/sec-cm-°C

K_Y = 0.46 cal/sec-cm-°C



From figure,

$\frac{K_Y A(60 - T_B)}{\ell} = \frac{K_X A(T_B - 10)}{2\ell} + \frac{K_Y A(T_B - 10)}{2\ell}$

Solving the above equation, we get T_B = 30°C

No heat will flow through CD

As C is a point at the middle of BE therefore temperature at

$C = \frac{T_B + T_E}{2} = \frac{30°C + 10°C}{2} = 20°C$

Similarly temperature at D is also 20°C.

Topic-3 : Newton's Law of Cooling

1. **(b)** From Newton's Law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Here, $T_1 = 50°C$, $T_2 = 40°C$
an d $T_o = 20°C$, t = 600S = 5 minutes

$$\Rightarrow \frac{50-40}{5\,\text{Min}} = K\left(\frac{50+40}{2} - 20\right) \quad ...(i)$$

Let T be the temperature of sphere after next 5 minutes. Then

$$\frac{40-T}{5} = K\left(\frac{40+T}{2} - 20\right) \quad ...(ii)$$

Dividing eqn. (ii) by (i), we get

$$\frac{40-T}{10} = \frac{40+T-40}{50+40-40} = \frac{T}{50}$$

$$\Rightarrow 40 - T = \frac{T}{5} \Rightarrow 200 - 5T = T$$

$$\therefore T = \frac{200}{6} = 33.3°C$$

2. **(b)** Rate of Heat loss = $mS\left(\frac{dT}{dt}\right) = e\sigma A T^4$

$$-\frac{dT}{dt} = \frac{e\sigma \times A \times T^4}{\rho \times Vol. \times S} \Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho S}$$

$$\frac{\left(-\frac{dT}{dt}\right)_A}{\left(-\frac{dT}{dt}\right)_B} = \frac{\rho_B}{\rho_A} \times \frac{S_B}{S_A} = \frac{10^3}{8 \times 10^2} \times \frac{4000}{2000}$$

$$\Rightarrow \left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

3. **(a)** According to Newton's law of cooling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60-50}{10}\right) = K\left(\frac{60+50}{2} - 25\right) \quad ..... (i)$$

$$\text{and, } \left(\frac{50-\theta}{10}\right) = K\left(\frac{50+\theta}{2} - 25\right) \quad ..... (ii)$$

Dividing eq. (i) by (ii),

$$\frac{10}{(50-\theta)} = \frac{60}{\theta} \Rightarrow \theta = 42.85°C \cong 43°C$$

4. **(b)** By Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = -K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

where θ_0 is the temperature of surrounding.

Now, hot water cools from 60°C to 50°C in 10 minutes,

$$\frac{60-50}{10} = -K\left[\frac{60+50}{2} - \theta_0\right] \quad ...(i)$$

Again, it cools from 50°C to 42°C in next 10 minutes.

$$\frac{50-42}{10} = -K\left[\frac{50+42}{2} - \theta_0\right] \quad ...(ii)$$

Dividing equations (i) by (ii) we get

$$\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$$

$$\frac{10}{8} = \frac{55-\theta_0}{46-\theta_0}$$

$$460 - 10\theta_0 = 440 - 8\theta_0$$

$$2\theta_0 = 20$$

$$\theta_0 = 10°C$$

5. **(c)** According to Newton's law of colling the temperature goes on decreasing with time non-linearly.

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. **(d)** Power,

$$P = \frac{dQ}{dt} = \frac{d}{dt}(mc)T = (H)\frac{dT}{dt}$$

$$= (H)\frac{d}{dt}\left[T_0(1+\beta t^{1/4})\right]$$

$$P = (H)T_0 = \frac{\beta t^{-3/4}}{4} \quad \text{where } H = \text{heat capacity}$$

$$\therefore \quad (H) = \frac{4Pt^{3/4}}{T_0\beta} \quad ..... (i)$$

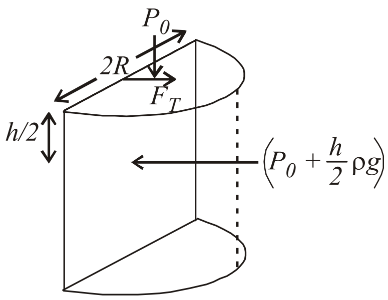
$$\text{But } t^{1/4} = \frac{T(t) - T_0}{\beta T_0}$$

$$\therefore \quad t^{3/4} = \frac{[T(t)-T_0]^3}{\beta^3 T_0^3} \quad ..... (ii)$$

From eq. (i) & (ii),

$$(H) = \frac{4P}{T_0\beta}\frac{[T(t)-T_0]^3}{\beta^3 T_0^3} = \frac{4P[T(t)-T_0]^3}{\beta^4 T_0^4}$$

2. **(b)** In the first part the force is created due to pressure and in the second part the force is due to surface tension T.



The force is $\left[\left(P_0 + \frac{h\rho g}{2}\right) \times (2R \times h)\right] - 2RT$

$\therefore$ Force = $2P_0Rh + R\rho gh^2 - 2RT$

3. **(d)** When a spherical body is kept inside a perfectly black body then the total heat radiated by the body is equal to that of the black body.

In this question the given options are wrong as all the four options contain e in place of σ.

4. **(c)** For a given mass of gas at constant pressure,

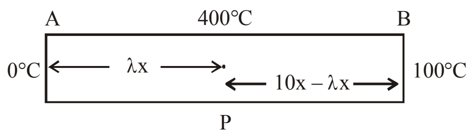
$$\frac{V}{T} = \text{Constant} \qquad \therefore \frac{V+\Delta V}{T+\Delta T} = \frac{V}{T}$$

or $VT + V\Delta T = VT + T\Delta V$ or $V\Delta T = T\Delta V$

or $\frac{1}{T} = \frac{\Delta V}{V\Delta T}$ or $\frac{1}{T} = \delta$ or $\delta T = 1$

The equation represents a rectangular hyperbola of the form $xy = c^2$ depicted by graph (c).

5. **(9)**



For heat flow from P to 0

$$\frac{dQ_1}{dt} = L_f \frac{dm_1}{dt} = \frac{KA\,400}{\lambda x} \qquad \text{..... (i)}$$

For heat flow from P to B

$$\frac{dQ_2}{dt} = L_{vap}\frac{dm_2}{dt} = \frac{KA\,300}{10x - \lambda x} \text{.....(ii)} \left[\text{Given } \frac{dm_1}{dt} = \frac{dm_2}{dt}\right]$$

Dividing eq. (i) by (ii) and solving we get λ = **9**

6. **(8.33)**

Rate of loss of heat,

$$\frac{dQ}{dt} = \sigma eA(T^4 - T_0^4) \qquad \text{...(i)}$$

$$\Rightarrow \frac{dQ}{Adt} = e\sigma(T_0 + \Delta T)^4 - T_0^4) = \sigma T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= e\sigma T_0^4\left[\left(1 + 4\frac{\Delta T}{T_0}\right) - 1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \qquad \text{...(ii)}$$

Now from eq. (i)

$$ms\frac{dT}{dt} = \sigma eA(T^4 - T_0^4) \qquad [\because Q = ms\Delta T]$$

$$\Rightarrow \frac{dT}{dt} = \frac{\sigma eA}{ms}[(T_0 + \Delta T)^4 - T_0^4]$$

$$= \frac{\sigma eA}{ms}T_0^4 \times \left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}T_0^4 \cdot 4\Delta T$$

$$\frac{dT}{dt} = K\Delta T;$$

$$\left(K = \frac{4\sigma eAT_0^3}{ms} \text{ Constant for Newton's law of cooling}\right)$$

$$\Rightarrow 4\sigma eAT_0^3 = \frac{K}{A}(ms)$$

From eq. (i)

$$\frac{dQ}{Adt} = e\sigma T_0^3 \cdot 4\Delta T$$

Since, rate of loss of heat = heat received per second

$700 = (K/A)\,(ms)\,\Delta T \qquad [K \times ms = 4200 \times 10^{-3}]$

$$\Rightarrow \Delta T = \frac{700 \times A}{K \times ms} = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200} = \frac{50}{6} = \frac{25}{3}$$

$\therefore \Delta T = 8.33$

7. **(c)** Energy radiated by the body = $\sigma A(T^4 - T_0{}^4)t$

[For a black body e = 1]

$$= \sigma A[(T_0 + 10)^4 - T_0{}^4]t$$

$$= \sigma AT_0{}^4\left[\left(1 + \frac{10}{T_0}\right)^4 - 1\right]t$$

$$= \sigma AT_0{}^4\left[\frac{40}{T_0}\right] \times t = 460 \times 1 \times \frac{40}{300} \times 1 = 61.33J$$

$$P = \frac{\text{Energy radiated}}{\text{time}} = \sigma AT^4 - \sigma AT_0{}^4$$

$$\therefore \left|\frac{dp}{dT_0}\right| = \sigma A(4T_0{}^3) \quad \therefore |dp| = \sigma A(4T_0{}^3)dT_0$$

$$\therefore |\Delta P| = 4\sigma AT_0{}^3$$

Here as human body is not a black body. So option (a) and (b) are incorrect.

Energy radiated $\propto$ A where A is the surface area of the body. Hence option (c) is correct.

11 Thermodynamics

Topic-1 : First Law of Thermodynamics

1. **(a)** ΔU remains same for both paths ACB and ADB

$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

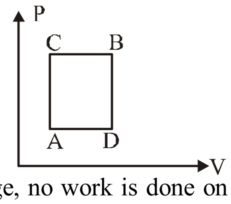
$\Rightarrow 60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

$\Rightarrow U_{ACB} = 30\text{ J}$

$\therefore \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}$

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$



2. **(d)** Volume of water does not change, no work is done on or by the system (W = 0)

According to first law of thermodynamics

$Q = DU + W$

For Isochoric process $Q = DU$

$\Delta U = \mu c dT = 2 \times 4184 \times 20 = 16.7\text{ kJ}.$

3. **(a)** As we know,

$\Delta Q = \Delta u + \Delta w$ (Ist law of thermodynamics)

$\Rightarrow \Delta Q = \Delta u + P\Delta v$

or $150 = \Delta u + 100(1-2)$

$= \Delta u - 100$

$\therefore \quad \Delta u = 150 + 100 = 250\text{J}$

Thus the internal energy of the gas increases by 250 J

4. **(a)** From the first law of thermodynamics

$dQ = dU + dW$

Here $dW = 0$ (given)

$\therefore \quad dQ = dU$

Now since $dQ < 0$ (given)

i.e., dQ is negative so dU decreases.

Internal energy 'U' decrease when temperature T decreases.

5. **(2)** Applying first law of thermodynamics to path iaf

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

$500 = \Delta U_{iaf} + 200 \qquad \therefore \Delta U_{iaf} = 300\text{ J}$

Now,

$Q_{ibf} = \Delta U_{ibf} + W_{ib} + W_{bf} = 300 + 50 + 100$

$Q_{ib} + Q_{bf} = 450\text{ J}$ ...(i)

Also $Q_{ib} = \Delta U_{ib} + W_{ib}$

$\therefore Q_{ib} = 100 + 50 = 150\text{ J}$ ...(ii)

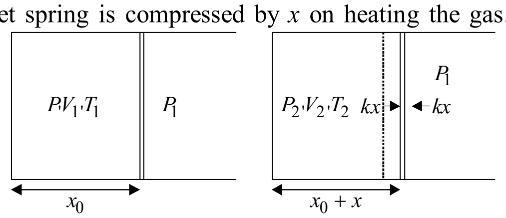
From eq. (i) & (ii) $\dfrac{Q_{bf}}{Q_{ib}} = \dfrac{Q_{ib}\gamma - Q_{ib}}{Q_{ib}} = \dfrac{300}{150} = 2$

6. **(Constant)** In this expansion, no work is done because the gas expands constant in vacuum. $\therefore \ \Delta W = 0$

Also, $Q = 0$ as the process is adiabatic. Hence from first law of thermodynamics, $\Delta U = 0$ *i.e.,* temperature remains constant.

7. **(a, b, c)**

Let spring is compressed by x on heating the gas.



(a) As gas is ideal monoatomic,

$\therefore \ \dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$ ...(i)

Force on spring by gas = kx

$\therefore P_2 = P_1 + \dfrac{kx}{A}$ (A = area of cross – section of piston) ...(ii)

When $V_2 = 2V_1$, $T_2 = 3T_1$

$\therefore \ \dfrac{P_1V_1}{T_1} = \dfrac{P_2 \times (2V_1)}{3T_1} \Rightarrow P_2 = \dfrac{3}{2}P_1$

Putting this value of P_2 in eqn. (ii) we get

$\dfrac{3}{2}P_1 = P_1 + \dfrac{kx}{A} \Rightarrow kx = \dfrac{P_1A}{2}$

$x = \dfrac{V_2 - V_1}{A} = \dfrac{2V_1 - V_1}{A} = \dfrac{V_1}{A}$

Energy stored in the spring

$= \dfrac{1}{2}kx^2 = \dfrac{1}{2}(kx)(x) = \dfrac{P_1V_1}{4}$

(b) Change in internal energy,

$\Delta U = \dfrac{f}{2}(P_2V_2 - P_1V_1) = \dfrac{3}{2}\left(\dfrac{3}{2}P_1 \times 2V_1 - P_1V_1\right) = 3P_1V_1$

(c) Again, when $V_2 = 3V_1$ and $T_2 = 4T_1$ then

From eqn. (i),

$\dfrac{P_1V_1}{T_1} = \dfrac{P_2(3V_1)}{4T_1} \Rightarrow P_2 = \dfrac{4}{3}P_1$

$x = \dfrac{V_2 - V_1}{A} = \dfrac{2V_1}{A}$

From eqn. (ii),

$\dfrac{4}{3}P_1 = P_1 + \dfrac{kx}{A} \Rightarrow kx = \dfrac{P_1A}{3}$

Work done by gas = Work done by gas on atmosphere + Energy stored in spring.

$W_g = P_1Ax + \dfrac{1}{2}kx^2 = P_1(2V_1) + \dfrac{1}{2}\left(\dfrac{P_1A}{3}\right)\left(\dfrac{2V_1}{A}\right)$

$= 2P_1V_1 + \dfrac{1}{3}P_1V_1 = \dfrac{7}{3}P_1V_1$

(d) $\Delta Q = W_g + \Delta U$

$= \dfrac{7}{3}P_1V_1 + \dfrac{3}{2}(P_2V_2 - P_1V_1)$

$= \dfrac{7}{3}P_1V_1 + \dfrac{3}{2}\left(\dfrac{4}{3}P_1 \times 3V_1 - P_1V_1\right)$

8. **(b, c)**

There is a decrease in volume during melting of an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, option (b) is correct.

Secondly heat is absorbed during melting (i.e. dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative.) Therefore, from first law of thermodynamics $dU = dQ - dW$

change in internal energy of ice-water system, dU will be positive or internal energy will increase.

9. **A-p,r,t; B-p,r; C-q,s; D-r,t**
(A) Process A $\rightarrow$ B
This is an isobaric process, P = constant and volume (V) of the gas decreases. Therefore work is done on the gas.
$W = P(3V - V) = 2PV$
Also V decreases so temerature at B decreases
$\therefore$ Internal energy U decreases.
From, $Q = U + W$ as U and W decreases so Q decreases that means heat is lost.
(B) Process B $\rightarrow$ C
This is an isochoric process V = constant pressure decreases $P \propto T$ so temperature also decreases.
$W = 0$; ΔU = negative so ΔQ negative
Hence heat is lost.
(C) Process C $\rightarrow$ D
This is isobaric, Pressure P = constant V increases and $V \propto T$ so T increases. Hence ΔW, ΔU and ΔQ + ve so heat gained by the gas.
(D) Process D $\rightarrow$ A
Applying PV = nRT

for D $P(9V) = 1\,RT_D \therefore T_D = \dfrac{9PV}{R}$

for A $3P(3V) = 1RT_A \therefore T_A = \dfrac{9PV}{R}$

i.e., the process is isothermal $\quad\therefore \Delta U = 0$
Now, $\Delta Q = \Delta U + W \quad \therefore \Delta Q = W$.
As volume decreaes in this process so W negative i.e., wom done on the gas aand ΔQ negative hence heat is lost.

10. **(A)-(q) :** As the ideal gas expands in vacuum, and the container is insulated therefore $W = 0$ & $Q = 0$ and according to first law of thermodynamics
$\Delta U = Q + W \quad \Rightarrow \quad \Delta U = 0$
Hence there is no change in the temparature of the gas of T is constant.

(B)-(p, r) : Given $P \propto \dfrac{1}{V^2}$ or, PV^2 = constant

or, $nRTV$ = constant $\quad \therefore V \times T$ = constant
As the gas expands its volume increases so temperature decreases
We know that $Q = n\,C\,\Delta T$ ...(i)
For a polytropic process

$C = C_v + \dfrac{R}{1 - n}$ and PV^n = constant

Here PV^2 = Constant. $\quad \therefore n = 2$

$\therefore C = C_v + \dfrac{R}{1-2} = C_v - R = \dfrac{3}{2}R - R = \dfrac{R}{2}$

$Q = nC\,\Delta T = n \times \dfrac{R}{2} \times \Delta T.$

i.e. ΔT is negative, Q is negative so heat is lost.
As volume increases, so temperature decreases given

$P \propto \dfrac{1}{V^{4/3}} \Rightarrow PV^{4/3}$ = constant.

$\therefore n = \dfrac{4}{3}$.

$\therefore C = C_v + \dfrac{R}{1 - \dfrac{4}{3}} = \dfrac{3}{2}R + \dfrac{3R}{-1} = \dfrac{3}{2}R - 3R = \dfrac{-3R}{2}$

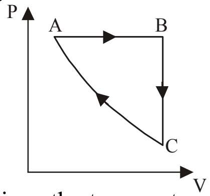
$\therefore Q = nC_V\,\Delta T = n\left(\dfrac{-3R}{2}\right)\Delta t$

As ΔT is negative, Q is positive. So gas gains heat.

(D)-(q, s) : From $PV = nRT \Rightarrow T = \dfrac{PV}{nR}$ PV increase So T increases volume increases so W increases.
From $Q = \Delta U + W$, Q increases.
Hence the gas gains heat.

11. From the given P–V graph, In process $J \rightarrow K$ volume, V = constant p is decreasing and $P \propto T$ Therefore, T should also decrease.
$\therefore W = Pdv = 0, \Delta V = \Delta Q < 0$ (negative)
From $\Delta Q = \Delta U + \Delta W$
In process $K \rightarrow L$ p = constant = $nC_v\Delta T$ V is increasing So temperature should also increase.
$\therefore \Delta W = Pdv > 0, \Delta U = nC_v\Delta T > 0$ and $Q = mC\Delta T > 0$
In process $L \rightarrow M$ V = constant P in creases to T increases
$\therefore W = 0, \Delta U > 0$ and $Q > 0$
In process $M \rightarrow J$
V is decreasing $\therefore \Delta W < 0$
T is also decreasing $\therefore \Delta U < 0$ and $\Delta Q < 0$

12. The corresponding P-V vgraph of the given V-T graph is shown in figure.



Process **A $\rightarrow$ B** A straight line in V-T graph indicates $V \propto T$ $\therefore$ Pressure is constant. Both V and T are increasing.
Process **B $\rightarrow$ C** Volume is constant. Since the temperature is decreasing, So pressure will also decrease.
Process **C $\rightarrow$ A** The temperature is constant but volume decreases. Hence pressure will increase as $P \propto \dfrac{I}{V}$.

Topic-2 : Specific Heat Capacity and Thermodynamical Processes

1. **(b)** Temperature change ΔT is same for all three processes
$A \rightarrow B$; $A \rightarrow C$ and $A \rightarrow D$
$\Delta U = nC_v\Delta T$ = same
$E_{AB} = E_{AC} = E_{AD}$
Work done, $W = P \times \Delta V$
$AB \rightarrow$ volume is increasing $\Rightarrow W_{AB} > 0$
$AD \rightarrow$ volume is decreasing $\Rightarrow W_{AD} < 0$
$AC \rightarrow$ volume is constant $\Rightarrow W_{AC} = 0$

2. **(c)** In adiabatic process
PV^γ = constant

$\therefore P\left(\dfrac{m}{\rho}\right)^\gamma$ = constant $\qquad \left(\because V = \dfrac{m}{\rho}\right)$

As mass is constant
$\therefore P \propto \rho^\gamma$
If P_i and P_f be the initial and final pressure of the gas and ρ_i and ρ_f be the initial and final density of the gas. Then

$\dfrac{P_f}{P_i} = \left(\dfrac{\rho_f}{\rho_i}\right)^\gamma = (32)^{7/5}$

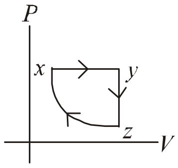
$\Rightarrow \dfrac{nP_i}{P_i} = (2^5)^{7/5} = 2^7$

$\Rightarrow n = 2^7 = 128.$

3. **(b)** Bursting of helium balloon is irreversible and in this process $\Delta Q = 0$, so adiabatic.

4. **(a)** From the corresponding V-T graph given in question,
Process $xy \rightarrow$ Isobaric expansion,
Process $yz \rightarrow$ Isochoric (Pressure decreases)

Process $zx \rightarrow$ Isothermal compression
Therefore, corresponding PV graph is as shown in figure



5. **(b)** Given, $V_1 = 1$ litre, $P_1 = 1$ atm
$V_2 = 3$ litre, $\gamma = 1.40$,
Using, PV^r = constant $\Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555}\ atm$$

$\therefore$ Work done, $W = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1}$

$$= \frac{\left(1\times1 - \frac{1}{4.6555}\times 3\right)1.01325\times10^5\times10^{-3}}{0.4} = 90.1\ J$$

Closest value of $W = 90.5\ J$

6. **(Bonus)** We know that Relaxation time,

$T \propto \dfrac{V}{\sqrt{T}}$ ...(i)

Equation of adiabatic process is
$TV^{\gamma-1}$ = constant

$$\Rightarrow \quad T \propto \frac{1}{V^{\gamma-1}}$$

$$\Rightarrow \quad T \propto V^{1+\frac{\gamma-1}{2}} \quad \text{using (i)}$$

$$\Rightarrow \quad T \propto V^{\frac{1+\gamma}{2}}$$

$$\Rightarrow \quad \frac{T_f}{T_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}} = (2)^{\frac{1+\gamma}{2}}$$

7. **(b)** $\Delta U_{ac} = -(\Delta U_{ca}) = -(-180) = 180$ J
$Q = 250 + 60 = 310$ J
Now $Q = \Delta U + W$
or $310 = 180 + W$
or $W = 130$ J

8. **(c)** As the process is isochoric so,

$$Q = nc_V\Delta T = \frac{67.2}{22.4}\times\frac{3R}{2}\times 20 = 90R = 90\times 8.31 \simeq 748\text{j}.$$

9. **(b)** We have given,

$$P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$$

When $V_1 = V_0 \quad \Rightarrow P_1 = P_0\left[1 - \frac{1}{2}\right] = \frac{P_0}{2}$

When $V_2 = 2V_0$

$$\Rightarrow P_2 = P_0\left[1 - \frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$$

$$\Delta T = T_2 - T_1 = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right| \left[\because T = \frac{PV}{nR}\right]$$

$$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right| = \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$$

$$= \frac{5P_0V_0}{4nR} = \frac{5P_0V_0}{4R} \qquad (\because n = 1)$$

10. **(c)** Internal energy depends only on initial and final state
So, $\Delta U_A = \Delta U_B$
Also $\Delta Q = \Delta U + W$
As $W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$

11. **(b)** Suppose amount of water evaporated be M gram.
Then (150 – M) gram water converted into ice.
so, heat consumed in evoporation = Heat released in fusion
$M \times L_v = (150 - M) \times L_s$
$M \times 2.1 \times 10^6 = (150 - M) \times 3.36 \times 10^5$
$\Rightarrow M = 20$ g

12. **(d)** a $\rightarrow$ Isobasic, b $\rightarrow$ Isothermal, c $\rightarrow$ Adiabatic, d $\rightarrow$ Isochoric

13. **(b)** Total work done by the gas during the cycle is equal to area of triangle ABC.

$$\therefore \ \Delta W = \frac{1}{2}\times 4\times 5 = 10\text{ J}$$

14. **(b)** Equation of adiabatic change is
$TV^{\gamma-1}$ = constant

Put $\gamma = \dfrac{7}{5}$, we get: $\gamma - 1 = \dfrac{7}{5} - 1$

$$\therefore x = \frac{2}{5}$$

15. **(b)** Work done,

$$W = P\Delta V = nR\Delta T = \frac{1}{2}\times 8.31\times 70 \simeq 291\text{J}$$

16. **(a)** Equation of the BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

using PV = nRT

$$\text{Temperature,}\ \ T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1\times R}$$

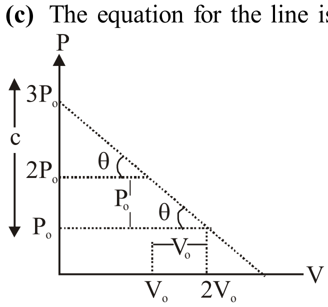
($\because$ n = 1 mole given)

$$T = \frac{P_0}{F}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV} = 0 \Rightarrow 5 - \frac{4V}{V_0} = 0 \Rightarrow V = \frac{5}{4}V_0$$

$$T = \frac{P_0}{R}\left[5\times\frac{5V_0}{4} - \frac{2}{V_0}\times\frac{25}{16}V_0^2\right] = \frac{25}{8}\frac{P_0V_0}{R}$$

17. **(c)** The equation for the line is



$$P = \frac{-P_0}{V_0}V + 3P \qquad \left[\text{slope} = \frac{-P_0}{V_0},\ c = 3P_0\right]$$

$PV_0 + P_0V = 3P_0V_0$ ...(i)
But pV = nRT

$$\therefore P = \frac{nRT}{V} \qquad \text{...(ii)}$$

From (i) & (ii) $\frac{nRT}{V}V_0 + P_0V = 3P_0V_0$

$\therefore$ nRT $V_0 + P_0V^2 = 3P_0V_0V$ ...(iii)

For temperature to be maximum $\frac{dT}{dV} = 0$

Differentiating e.q. (iii) by 'V' we get

$nRV_0\frac{dT}{dV} + P_0(2V) = 3P_0V_0$

$\therefore nRV_0\frac{dT}{dV} = 3P_0V_0 - 2P_0V$

$\frac{dT}{dV} = \frac{3P_0V_0 - 2P_0V}{nRV_0} = 0$

$V = \frac{3V_0}{2} \quad \therefore \quad P = \frac{3P_0}{2}$ [From (i)]

$\therefore T_{max} = \frac{9P_0V_0}{4nR}$ [From (iii)]

18. **(a)** Efficiency of heat engine is given by

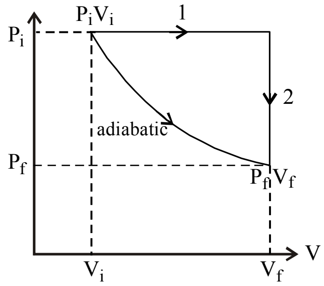
$h = \frac{w}{Q} = 1 - \frac{C_V}{C_P} = \frac{R}{C_p} = \frac{R}{\frac{5R}{2}} = \frac{2}{5}$

$(\because C_p - C_v = R)$

For monoatomic gas $C_P = \frac{5}{2}R$.

19. **(c)** $\because PV^r$ = constant

P^3V^5 = constant $\Rightarrow PV^{5/3}$ = constant $\therefore \gamma = \frac{5}{3}$



For the two step process

$W = p_i(V_f - V_i) = 10^5(7\times10^{-3}) = 7\times10^2$ J

$\Delta U = \frac{f}{2}(P_fV_f - P_iV_i) = \frac{1}{\gamma-1}\left(\frac{1}{4}\times10^2 - 10^2\right)$

$\Delta U = \frac{3}{2}.\frac{3}{4}\times10^2 = -\frac{9}{8}\times10^2$ J

From first law of thermody namics

$Q - W = \Delta U$

$\therefore \quad Q = 7\times10^2 - \frac{9}{8}\times10^2 = \frac{47}{8}\times10^2$ J $= 588$J

20. **(a)** $\tau = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}}$

$\tau \propto \frac{V}{\sqrt{T}}$

As, $TV^{\gamma-1} = K$

So, $\tau \propto V^{\gamma + 1/2}$

Therefore, $q = \frac{\gamma+1}{2}$

21. **(a)** As, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

But $\frac{U}{V} = KT^4$

So, $P = \frac{1}{3}KT^4$

or $\frac{uRT}{V} = \frac{1}{3}KT^4$ [As PV = u RT]

$\frac{4}{3}\pi R^3T^3$ = constant

Therefore, $T \propto \frac{1}{R}$

22. **(b)** In *VT* graph

ab-process : Isobaric, temperature increases.

bc process : Adiabatic, pressure decreases.

cd process : Isobaric, volume decreases.

da process : Adiabatic, pressure increases.

The above processes correctly represented in *P-V* diagram (b).

23. **(d)** In cyclic process, change in total internal energy is zero.

$\Delta U_{cyclic} = 0$

$\Delta U_{BC} = nC_v\Delta T = 1\times\frac{5R}{2}\Delta T$

Where, C_v = molar specific heat at constant volume.

For BC, $\Delta T = -200$ K

$\therefore \quad \Delta U_{BC} = -500R$

24. **(c)** **Given :** work done, $W = 830$ J

No. of moles of gas, $\mu = 2$

For diatomic gas $\gamma = 1.4$

Work done during an adiabatic change

$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$

$\Rightarrow \quad 830 = \frac{2\times8.3(\Delta T)}{1.4-1} = \frac{2\times8.3(\Delta T)}{0.4}$

$\Rightarrow \quad \Delta T = \frac{830\times0.4}{2\times8.3} = 20$ K

25. **(b)** Volume of the gas

$v = \frac{m}{d}$ and

Using PV^γ = constant

$\frac{P'}{P} = \frac{V}{V'} = \left(\frac{d'}{d}\right)^\gamma$

or $128 = (32)^\gamma$

$\therefore \gamma = \frac{7}{5} = 1.4$

26. **(a)** Initially, volume $V_1 = 5.6\ell$, temperature $= T_1$ and

$\gamma = \frac{5}{3}$ (for monoatomic gas)

The number of moles of gas, $n = \frac{5.6\ell}{22.4\ell} = \frac{1}{4}$

Finally (after adiabatic compression) $V_2 = 0.7\ell$

For adiabatic process $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

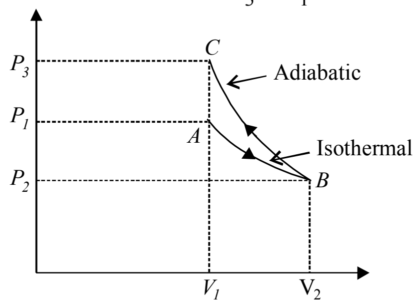
$\therefore T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} = T_1(8)^{2/3} = 4T_1$

Work done in adiabatic process

$$W = \frac{nR\Delta T}{\gamma - 1} = \frac{\frac{1}{4}R[T_1 - 4T_1]}{\left[\frac{5}{3} - 1\right]} = \frac{9}{8}RT_1$$

27. **(b)** In the first process W_{AB} is + ve as ΔV is positive, in the second process W_{BC} is – ve as ΔV is – ve and area under the curve of second process is more

$\therefore$ Net Work < 0 and also $P_3 > P_1$.



28. **(b)** From the $P - T$ graph we find AB to be a isothermal process, AC is adiabatic process given. Also for an expansion process, the slope of adiabatic curve is more (or we can say that the area under the $P - V$ graph for isothermal process is more than adiabatic process for same increase in volume). Hence graph (b) correctly depicts corresponding P – V graph.

29. **(a)** For cyclic process;

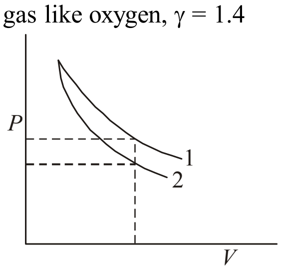
$Q_{cyclic} = W_{AB} + W_{BC} + W_{CA} = 10\text{ J} + 0 + W_{CA} = 5\text{ J}$

$\therefore \quad W_{CA} = -5\text{ J}$

30. **(b)** For adiabatic process PV^{γ} = constant

Also for monoatomic gas like helium, $\gamma = 1.67$

for diatomic gas like oxygen, $\gamma = 1.4$



Since, $\gamma_{diatomic} < \gamma_{monoatomic}$ $\therefore P_{diatomic} > P_{monoatomic}$

Hence graph 1 is for diatomic *i.e.*, oxygen and graph 2 is for monoatomic *i.e.*, for helium.

31. **(a)** Work done is equal to area under the curve on PV diagram. Hence from the diagram $w_2 > w_1 > w_3$.

32. **(d)** From $TV^{\gamma-1}$ = constant

For monatomic gas $\gamma = \frac{5}{3}$, hence $TV^{2/3}$ = constant

$$\therefore \quad T_1L_1^{2/3} = T_2L_2^{2/3} \qquad (\because V \propto L)$$

$$\text{Hence, } \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

33. **(b)** Bulk modulus, $K = \frac{-dPV}{dV}$

For an isothermal process; PV = constant

$\therefore \; PdV + VdP = 0$

$$\therefore \; P = \frac{dP}{dV/V} = K \qquad \text{(Bulk modulus)}$$

34. **(a)** The work done during the cycle = area enclosed in the $P - V$ curve

35. **(4)** For an adiabatic process, applying $TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\gamma = 1.4 \text{ (for diatomic gas) } V_2 = \frac{V_1}{32}, T_1 = T_i, T_2 = aT_i$$

$$\therefore T_i = aT_i\left[\frac{1}{32}\right]^{1.4-1} = aT_i\left[\frac{1}{2^5}\right]^{0.4} = \frac{aT_i}{4} \qquad \therefore a = \mathbf{4}$$

36. **(46)**

For adiabatic process, $TV^{\gamma-1}$ = constant

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = 20°\text{C} + 273 = 293\text{ K}$, $V_2 = \frac{V_1}{10}$ and $\gamma = \frac{7}{5}$

$$T_1(V_1)^{\gamma-1} = T_2\left(\frac{V_1}{10}\right)^{\gamma-1}$$

$$\Rightarrow 293 = T_2\left(\frac{1}{10}\right)^{2/5} \Rightarrow T_2 = 293(10)^{2/5} \simeq 736\text{ K}$$

$\Delta T = 736 - 293 = 443\text{ K}$

During the process, change in internal energy

$$\Delta U = NC_V\Delta T = 5 \times \frac{5}{2} \times 8.3 \times 443 \simeq 46 \times 10^3 \text{ J} = X \text{ kJ}$$

$\therefore X = 46$.

37. **(1818)** For an adiabatic process,

$TV^{\gamma-1}$ = constant

$\therefore \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow \quad T_2 = (300) \times \left(\frac{V_1}{\frac{V_1}{16}}\right)^{1.4-1} \quad \Rightarrow \quad T_2 = 300 \times (16)^{0.4}$$

Ideal gas equation, $PV = nRT$

$$\therefore \quad V = \frac{nRT}{P}$$

$\Rightarrow \quad V = kT$ (since pressure is constant for isobaric process)

So, during isobaric process

$V_2 = kT_2$ ...(i)

$2V_2 = kT_f$ ...(ii)

Dividing (i) by (ii)

$$\frac{1}{2} = \frac{T_2}{T_f}$$

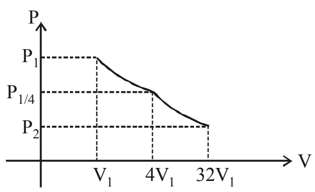
$T_f = 2T_2 = 300 \times 2 \times (16)^{0.4} = 1818\ K$

38. **(1.77 to 1.78)**

For monatomic gas, $\gamma = \frac{5}{3}$

In adiabatic process

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$



$$\Rightarrow \frac{P_1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$$

$\Rightarrow P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$

And $W_{adi} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{2} - 1}$

$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8}P_1V_1$

In isothermal process,

$W_{iso} = 2.303\ \mu RT \log_{10}\left(\frac{V_2}{V_1}\right)$

$\Rightarrow W_{iso} = P_1V_1 \ \ln\left(\frac{4V_1}{V_1}\right) = 2P_1V_1 \ \ln 2$

$\therefore \frac{W_{iso}}{W_{adia}} = \frac{2P_1V_1 \ \ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9} \ \ln 2 = f \ln 2$

So, $f = \frac{16}{9} = 1.7778 \approx 1.78$

39. (2.05)

40. For adiabatic expansion

$PV^\gamma = P_a \times (2V)^\gamma \Rightarrow P_a = \frac{P}{2^\gamma} = \frac{P}{2^{1.67}}$

For isothermal expansion

$P \times V = P_i \times 2V \Rightarrow P_i = \frac{P}{2}$

$\therefore \frac{P_a}{P_i} = \frac{P}{2^{1.67}} \times \frac{2}{P} = \frac{2}{2^{1.67}} = 0.628$

41. (True) The slope of *P-V* curve is more for adiabatic process than for isothermal process. From the graph it is clear that slope for *B* is greater than the slope for *A*. So the isothermal process is represented by the curve *A*.

42. (a, b, d)

Adiabatic constant of the gas mixture,

$\gamma_m = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{v1} + n_2C_{v2}} = \frac{5 \times \frac{5R}{2} + 1 \times \frac{7R}{2}}{5 \times \frac{3R}{2} + 1 \times \frac{5R}{2}} = 1.6$

For an adiabatic process, $P_v \gamma$ = Constant

$\therefore P = P_0\left(\frac{V_0}{V}\right)^{1.6} = P_0(4)^{1.6}$

$= P_0(2^2)^{1.6} = P_0 2^{3.2} = 9.2 P_0$

Work done during the process,

$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{9.2P_0 \times (V_0/4) - P_0V_0}{1-1.6} = \frac{-13P_0V_0}{6}$

But $P_0V_0 = 6RT_0$ (as $n = 5 + 1 = 6$)

$\therefore W = \frac{-13(6RT_0)}{6} = -13RT_0 \therefore |W| = 13 RT_0$

The average K.E. of the gas mixture,

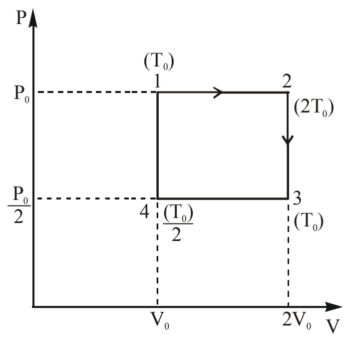
$\overline{K.E} = nCV_{mi} \times T_2$

From, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or, $T_2 = T_1(2)^{6/5} = 23 T_0$

$\therefore \overline{K.E} = nCV_{mi} \times T_2 = 23RT_0$

43. (a, c) The *P–V* graph of the given *V–T* graph is given below.



(a) Work done during cyclic process (1→2→3→4–1) *W*

= area enclosed in the loop $= \frac{P_0}{2}V_0$

$\because \ P_0V_0 = nRT_0 \quad \therefore \frac{P_0V_0}{2} = \frac{nRT_0}{2}$

$\therefore$ Work done $W = \frac{nRT_0}{1} = \frac{RT_0}{2} \quad [\text{as } n = 1]$

(b) Process $\underline{1 \to 2}$ is isobaric

Process $\underline{2 \to 3}$ is isochoric

Process $\underline{3 \to 4}$ is isobaric

Process $\underline{4 \to 1}$ is isochoric

Hence no adiabatic process is imvolved.

(c) $|\Delta Q_{1\to2}| = |nC_p\Delta T| = |nC_p(2T_0 - T_0)| = |nC_pT_0|$

$|\Delta Q_{2\to3}| = |\Delta U| = |nC_v\Delta T| = |nC_vT_0|$

$\therefore \left|\frac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right| = \frac{C_p}{C_v} = \frac{5}{3}$

(d) $|\Delta Q_{3\to4}| = nC_p\frac{T_0}{2}$

$\therefore \left|\frac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right| = \frac{nC_pT_0}{nC_p\frac{T_0}{2}} = \frac{2}{1}$

44. (a, b)

Process A to B

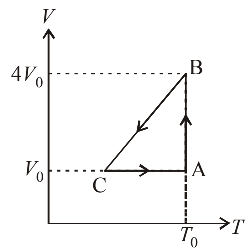
Here, $T_A = T_B \therefore U_A = U_B$

Also $P_0 V_0 = P_B \times 4 V_0$

$\Rightarrow P_B = \frac{P_0}{4}$

Work done

$W_{AB} = nRT_0 \log_e \frac{4V_0}{V_0}$



$= P_0V_0 \log_e 4 \qquad [\because P_0V_0 = nRT_0]$

The process BC is not clear. Therefore no judgement can be made at *C* regarding *P* and *T*.

45. (b, d)

(a) Process *A – B* is not isothermal. In case of an isothermal process we get a rectangular hyperbola in a P-V diagram.

(b) In process B → C → D, ΔU is negative. PV decreases and volume also decreases, therefore W is negative. From first law of thermodynamic, Q is negative i.e., there is a heat loss.

(c) $W_{AB} > W_{BC}$. Therefore work done during path $A \to B \to C$ is positive.
(d) Work done is clockwise cycle in a PV diagram is positive.

46. (a, b, c, d)
(a) For all thermal processes, change is internal energy in a constant pressure process
$\Delta U = nC_v \Delta T$ where $\Delta T = (T_2 - T_1)$
(b) According to first law of thermodynamics.
$\Delta Q = \Delta U + \Delta W$
In an adiabatic process $\Delta Q = 0$.
or, $0 = \Delta U + \Delta W$
or, $|\Delta U| = |\Delta W|$
(c) In the isothermal process, $\Delta T = 0 \therefore \Delta U = 0$
$\because \Delta U = nC_v \Delta T$
(d) In the adiabatic process, $\Delta Q = 0$.

47. (d)
(I) **Adiabatic process :** No exchange of heat takes place with surroundings.
$\Rightarrow \Delta Q = 0$
(II) **Isothermal process :** Temperature remains constant
$\therefore \Delta T = 0 \Rightarrow \Delta U = \frac{f}{2} nR\Delta T \Rightarrow \Delta U = 0$
No change in internal energy [$\Delta U = 0$].
(III) Isochoric process volume remains constant
$\Delta V = 0 \Rightarrow W = \int P \cdot dV = 0$
Hence work done is zero.
(IV) In isobaric process pressure remains constant.
$W = P \cdot \Delta V \neq 0$
$\Delta U = \frac{f}{2} nR\Delta T = \frac{f}{2}[P\Delta V] \neq 0$
$\therefore \Delta Q = nC_p \Delta T \neq 0$

48. (a) I. $W_{1\to2\to3} = W_{1\to2} + W_{2\to3}$ [$W_{2\to3} = 0, \Delta V = 0$]
$= P_0 \times V_0 + O$
$= P_0 V_0 = \frac{RT_0}{3}$ $\quad [\because P_0 V_0 = \frac{1}{3} RT_0 \text{ given}]$
$\therefore$ I $\to$ Q
II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$
$= nC_v \Delta T_{1\to2} + nC_v \Delta T_{2\to3}$
$= 1 \times \frac{3R}{2}(T_f - T_i)_{1\to2} + 1 \times \frac{3R}{2}[T_f - T_i]_{2\to3}$
$= \frac{3}{2}[2P_0V_0 - P_0V_0] + \frac{3}{2}\left[\frac{3P_0}{2} \times 2V_0 - P_0 \times 2V_0\right]$
$= 3P_0V_0 = 3 \times \frac{1}{3} RT_0 = RT_0$
II $\to$ R
III. $Q_{1\to2\to3} = Q_{1\to2} + Q_{2\to3}$
$= nC_p \Delta T_{1\to2} + nC_v \Delta T_{2\to3}$
$= \frac{5}{2} P_0 V_0 + 1 \times \frac{3}{2}\left[\frac{3P_0}{2} \times 2V_0 - P_2(2V_0)\right]$
$= \frac{8}{2} P_0 V_0 = \frac{8}{2} \times \frac{RT_0}{3} = \frac{4}{3} RT_0$
III – S
IV. $Q_{1\to2} = nC_p \Delta T_{1\to2} = nCp(T_f - T_i)$
$= 1 \times \frac{5}{2} R\left[\frac{P_0(2V_0)}{R} - \frac{P_0V_0}{R}\right] [\because PV = nRT]$
$= \frac{5}{2} P_0 V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right) = \frac{5RT_0}{6}$
$\therefore$ IV – U

49. (c) I. $W_{1\to2\to3} = W_{1\to2} + W_{2-3}$
$= nRT \ln \frac{V_f}{V_i} + Pdv$
$= 1R \frac{T_0}{3} \ell n \frac{2V_0}{V_0} + \text{zero}$ $\left[\begin{array}{l}\because n = 1, \text{ and } W_{2\to3} \text{ isochroic} \\ \text{no change in volume, } \Delta V = 0\end{array}\right]$
$= \frac{RT_0}{3} \ln 2$
$\therefore$ I – P
II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$
$= O + nC_v \Delta T = n\frac{f}{2} R\Delta T$ [Process $1 \to 2$ isothermic $\Delta T = 0$]
$= 1 \times \frac{3}{2} R\left(T_0 - \frac{T_0}{3}\right) = RT_0$ [For monoatomic gas f = 3]
$\therefore$ II – R
III. $Q_{1\to2\to3} = W_{1\to2\to3} + \Delta U_{1\to2\to3}$
From first law of thermodynamics
$= \frac{RT_0}{3} \ell n2 + RT_0$
$= \frac{RT_0}{3}[\ell n2 + 3]$
$\therefore$ III – T
IV. $Q_{1\to2} = W_{1\to2} + \Delta U_{1\to2}$
$= \frac{RT_0}{3} \ell n2 + 0 = \frac{RT_0}{3} \ell n2 \, [\because \Delta U_{1\to2} = 0]$
IV – P

50. (c) Process I is adiabatic therefore $\Delta Q = 0$
Process II is isobaric P = constant therefore $W = P(V_2 - V_1)$
$= 3P_0(3V_0 - V_0) = 6P_0V_0$
Process III is isochoric V = constant therefore
$W = P(V_2 - V_1) = 0$
Process IV is isothermal, temperature T = constant, $\therefore \Delta u = 0$

51. (d) Laplace's correction of the speed of sound in ideal gas is related to adiabatic process.
$P - V$ curve in adiabatic process is steeper than isothermal

52. (b) Work done in isochoric process is zero for which we get a vertical line in P-V graph, graph (s).

53. (d) $\Delta U = \Delta Q - P\Delta V$
$W = -P\Delta V = -P(V_2 - V_1) = -PV_2 + PV_1$
which is the formula for isobaric process, so graph (P)

54. (a) $W_{GE} = P_0(V_0 - 32V_0) = -31 P_0V_0$
$W_{GH} = P_0(8V_0 - 32V_0) = -24 P_0V_0$
$(W_{FH})_{adiabatic} = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{P_0(8V_0) - 32P_0(V_0)}{1 - \frac{5}{3}} = 36P_0V_0$
$(W_{FG})_{isothermal} = nRT \ln\left(\frac{V_f}{V_i}\right) = P_0V_0 \ln\left(\frac{V_f}{V_i}\right)$

$= 1(32\, P_0V_0) \log_e \dfrac{32V_0}{V_0}$

$= 32\, P_0V_0 \log_e 2^5 = 160\, P_0V_0 \log_e 2$

(a) is the correct option

55. (a) From $\Delta Q = ms\Delta T$

$\Delta T = \dfrac{\Delta Q}{ms} = \dfrac{20000}{1 \times 400} = 50°C$

(b) $\Delta T = V\gamma\Delta T = \left(\dfrac{1}{9000}\right)\left(9 \times 10^{-5}\right)(50)$

$= 5 \times 10^{-7}\, m^3$

$\therefore W = p\,\Delta V = (10)^5 (5 \times 10^{-7}) = 0.05 J$

(c) $\Delta U = \Delta Q - W = (2000 - 0.05)J = 19999.95J$

56. (a) In the given V-T graph, AB is a straight line

$\therefore \dfrac{V}{T} =$ constant (Isobaric process)

$\therefore \dfrac{V_A}{T_A} = \dfrac{V_B}{T_B}$

$\Rightarrow T_B = \dfrac{V_B}{V_A} \times T_A = 2 \times 300 = 600\, K \left[\because \dfrac{V_B}{V_A} = 2\right]$

(b) Heat absorbed or released

In process, A to B is a (isobaric process)

$\therefore Q = nC_p\Delta T = 2 \times \dfrac{5}{2} R \times 300 = 1500\, R$

$\left[\because C_p = \dfrac{5}{2} R \text{ for monoatomic gas}\right]$

Heat is absorbed,

In process, B to C is an (isothermal process).

$\therefore$ Internal energy change $dU = 0 \because \Delta T = 0$

$\therefore$ From first law of thermodynamics $dQ = dW$

$\therefore Q = 2.303 \times nRT \log_{10} \dfrac{V_f}{V_i}$

$= 2.30. \times 2 \times R \times 600 \times \log_{10} 2$

$= 2763.6 \times \log_{10} 2 \times R = 831.8\, R$ (absorbed)

In process, C to D is a (isochoric process)

$dW = 0 \because dV = 0$

$\therefore Q = nC_v \Delta T = n\left(\dfrac{3}{2}R\right)(T_A - T_B)$

$= 2 \times \dfrac{3}{2} R \times (-300) = -900\, R$ (released).

In process, D to A (isothermal process)

$\therefore Q = 2.303 \times nRT \log_{10} \dfrac{V_f}{V_i}$

$= 2.303 \times 2\, R \times 300 \times \log\left(\dfrac{V_f}{V_i}\right) = -831.8\, R$

Heat is released

(c) Total work done

$W = Q_{A \to B} + Q_{B \to C} + Q_{C \to D} + Q_{D \to A}$

$= (1500\, R + 831.8\, R) - (900\, R + 831.8\, R) = 600\, R$

57. (a) Number of moles, $n = 2$, $T_1 = 300$ K

During the process A $\to$ B

$pT =$ constant or $p^2V =$ constant $= K$ (say)

$\therefore p = \dfrac{\sqrt{K}}{\sqrt{V}}$

$\therefore W_{A \to B} = \int_{V_A}^{V_B} p.dV = \int_{V_A}^{V_B} \dfrac{\sqrt{K}}{\sqrt{V}} dV$

$= 2\sqrt{K}\,[\sqrt{V_B} - \sqrt{V_A}]$

$= 2[\sqrt{KV_B} - \sqrt{KV_A}]$

$= 2[\sqrt{(P_B^2V_B)V_B} - \sqrt{(P_A^2V_A)V_A}]$

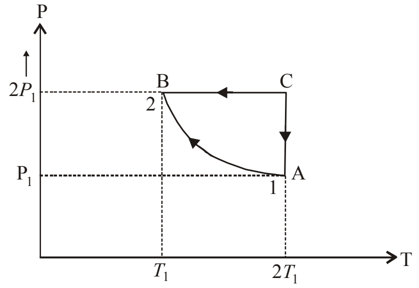
$= 2\,[P_BV_B - P_AV_A]$

$= 2\,[nRT_B - nRT_A] = 2\,nR\,[T_1 - 2T_1]$

$= (2)(2)(R)[300 - 600] = -1200R$

$\therefore$ Work done on the gas in the process AB is $1200R$.

Alternate method



$pV = nRT$

$\therefore pdV + Vdp = nRdT$

or $pdV + \dfrac{(nRT)}{P}.dp = nRdT$ ...(i)

From the given condition

$pT =$ constant

$pdT + Tdp = 0$ ...(ii)

From Eqs. (i) and (ii), we get

$pdV = 2nRdT$

$\therefore W_{A \to B} = \int pdV = 2nR\int_{T_A}^{T_B} dT = 2nR(T_B - T_A)$

$= 2nR\,(T_1 - 2T_1) = (2)(2)(R)(300 - 600)$

or $W_{A \to B} = -1200R$

(b) For PVx = Constt., Molar heat capacity

$C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1 - x} = \dfrac{R}{\frac{5}{3} - 1} + \dfrac{R}{1 - \frac{1}{2}}$

Here $P^2V =$ constant or $PV^{1/2} =$ constant

$\therefore x = \dfrac{1}{2}$

$\Rightarrow C = 3.5R$

$Q_{A \to B} = nC\,\Delta T = 2(3.5\, R)(300 - 600) = -2100\, R$

Process B – C: Process is isobaric

$Q_{B \to C} = nC_p\, \Delta T = (2)\left(\dfrac{5}{2}R\right)(T_C - T_B)$

$= 2\left(\dfrac{5}{2}R\right)(2T_1 - T_1) = (5R)(600 - 300) = 1500R$

Heat is absorbed

Process C – A: Process is isothermal

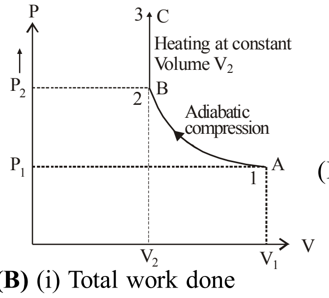
$\Delta T = 0 \therefore \Delta u = 0$ So, $Q_{C \to A} = W_{C \to A} = nRT_C \ln\left(\dfrac{P_C}{P_A}\right)$

$= nR(2T_1) \ln\left(\dfrac{2P_1}{P_1}\right) = (2)(R)(600)\ln(2) = 1200R \times 0.6932$

$Q_{C \to A} = 831.6\, R$ (absorbed)

58. n = no. of moles = 2,

(A) The complete process on P-V diagram is shown below.



(Isochoric process)

(B) (i) Total work done

$$W = W_{AB} + W_{BC} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} + 0$$

$$[\because W_{BC} = P\Delta V = P \times 0 = 0]$$

According to Poisson's law, $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

$$\therefore \quad W = \frac{1}{\gamma - 1}\left[P_1V_1 - P_1\left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right]$$

$$= \frac{1}{\gamma - 1}\left[P_1V_1 - P_1 V_2 . \frac{V_1}{V_2} . \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right]\left(\because \gamma = \frac{5}{3}\right)$$

$$W = \frac{3}{2}\left[P_1V_1 - P_1V_1\left(\frac{V_1}{V_2}\right)^{2/3}\right] = \frac{3}{2}P_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

(ii) Total change in internal energy. $\Delta U = \Delta U_{AB} + \Delta U_{BC} = Q - W$

$$= Q - \frac{3}{2}P_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

[according to first law of thermodynamics]

$$\begin{bmatrix} B \to C & Q = \Delta U_{BC} + 0 \\ A \to B & Q = \Delta U_{AB} + W \end{bmatrix}$$

(iii) For process $BC : \Delta U_{BC} = nC_v \Delta T = Q \quad [\because W_{BC} = 0\,]$

For monoatomic gas $C_v = \dfrac{R}{\gamma - 1} = \dfrac{3}{2}R,$

$$\therefore \quad \Delta U_{BC} = Q = 2 \times \frac{3R}{2} . \Delta T \Rightarrow \Delta T = \frac{Q}{3R}.$$

According to Poission's Law :

For the process AB, $T_A V_B^{\gamma - 1} = T_B V_B^{\gamma - 1}$

$$\text{or} \quad T_B = T_A\left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \frac{P_1V_1}{nR}\left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\therefore \quad T_B = \frac{P_1}{2R} . V_1^{\gamma} . V_2^{1-\gamma} = \frac{P_1V_1^{5/3}V_2^{-2/3}}{2R}$$

Hence, $T_C = T_B + \Delta T = \dfrac{P_1V_1^{5/3}V_2^{-2/3}}{2R} + \dfrac{Q}{3R}$

59. Number of mole, $n = 1$, For monoatomic gas :

$$C_p = \frac{5R}{2}, C_v = \frac{3R}{2}$$

(a) Work done by the gas, w = Area of ΔABC

$$W = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}\ V_0 \times 2P_0 = P_0V_0$$

(b) Heat rejected by the gas in the path CA i.e., isobaric process

$$\Delta Q_{CA} = nC_p\,\Delta T = 1 \times (5R/2)\,(T_A - T_C)$$

$$T_C = \frac{2P_0V_0}{I \times R}, T_A = \frac{P_0V_0}{I \times R}$$

$$\Delta Q_{CA} = \frac{5R}{2}\left[\frac{P_0V_0}{R} - \frac{2P_0V_0}{R}\right] = -\frac{5}{2}P_0V_0$$

Heat absorbed by the gas in the path AB *i.e.*, isochoric process

$$\Delta Q_{AB} = nC_v\Delta T = 1 \times (3R/2)\,(T_B - T_A)$$

$$= \frac{3R}{2}\left[\frac{3P_0V_0}{1 \times R} - \frac{P_0V_0}{1 \times R}\right] = 3P_0V_0$$

(c) As $\Delta U = 0$ in cyclic process, hence, from first law of thermodynamics $\Delta Q = \Delta U + \Delta W$

$\therefore\ \Delta Q = \Delta W$

$$\Delta Q_{AB} + \Delta Q_{CA} + \Delta Q_{BC} = \Delta W$$

$$\Delta Q_{BC} = P_0V_0 - \frac{P_0V_0}{2} = \frac{P_0V_0}{2}$$

(d) Equation for line BC $P = -\left[\dfrac{2P_0}{V_0}\right] V + 5P_0,$

$$(\because y = mx + c \text{ or}, P = -mV + C)$$

$P = \dfrac{RT}{V}$ [for one mole]

$$\therefore \quad RT = -\frac{2P_0}{V_0}V^2 + 5P_0V \qquad \text{... (i)}$$

For maximum; $\dfrac{dT}{dV} = 0,\ -\dfrac{2P_0}{V_0} \times 2V + 5P_0 = 0;$

$$\therefore \quad V = \frac{5V_0}{4} \qquad \text{... (ii)}$$

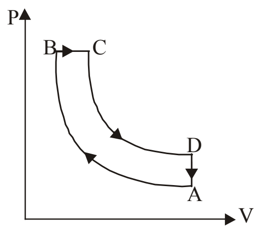
Hence from equation (i) and (ii)

$$RT_{\max} = \frac{-2P_0}{V_0} \times \left(\frac{5V_0}{4}\right)^2 + 5P_0\left(\frac{5V_0}{4}\right)$$

$$= -\,2P_0V_0 \times \frac{25}{16} + \frac{25P_0V_0}{4} = \frac{25}{8}P_0V_0$$

$$\therefore \quad T_{\max} = \frac{25}{8}\frac{P_0V_0}{R}$$

60. Given $\dfrac{V_A}{V_B} = 16, \dfrac{V_C}{V_B} = 2$



$T_A = 300\,K, T_B = ?, T_D = ?, \eta = ?$

$n = 1$, for diatomic gas, $\gamma = 1.4$

For adiabatic process $A \to B$

$$T_A V_A^{\gamma - 1} = T_B V_B^{\gamma - 1}$$

$$\text{or } T_B = \left(\frac{V_A}{V_B}\right)^{\gamma - 1} T_A = (16)^{2/5} \times 300 = 909\,K$$

$\therefore$ For isobaric process $B \to C$ Pressure = constant

$$\frac{V_B}{T_B} = \frac{V_C}{T_C} \quad \text{or} \quad T_C = T_B\left(\frac{V_C}{V_B}\right) = 909\,[2] = 1818\,K$$

For adiabatic process $C \to D$:

$\because \frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$; $\therefore \frac{V_A}{V_C} = 8$

$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$

$\therefore T_D = T_C\left[\frac{V_C}{V_D}\right]^{\gamma-1} = 1818\left[\frac{1}{8}\right]^{2/5} = \frac{1818}{(64)^{1/5}} = 791K$

For $B \to C$ process : Heat absorbed $Q_1 = nC_p (T_C - T_B)$

$= n\frac{\gamma R}{\gamma-1}(T_C - T_B) = 1\frac{(7/5)R}{(2/5)}(1818-909)$

$= \frac{7R}{2} \times 909 \cong 3182\, R$

For $D \to A$ process : Heat released

$Q_2 = nC_v (T_D - T_A) = n\frac{R}{\gamma-1}(T_D - T_A)$

$= 1.\frac{R}{(2/5)}(791-300) = \frac{5R}{2} \times 491$

($\because$ No heat is exchanged in adiabatic processes).

Now, $W_{AB} = -\frac{nR}{\gamma-1}(T_B - T_A)$

$= -\frac{R}{(2/5)}(900-300) = -\frac{5R}{2} \times 609$

$W_{BC} = -nR(T_C - T_B) = 1 \times R(1818-909) = 909\, R$

$W_{CD} = -\frac{nR}{\gamma-1}(T_C - T_D) = +\frac{R}{(2/5)}(1818-791)$

$= \frac{5R}{2} \times 1027$

$W_{net} = 909\, R + \frac{5R}{2}(1027-609) = 909\, R + \frac{5R}{2} \times 418$

$= 909\, R + 1045\, R = 1954\, R$

$\therefore$ Efficiency $= 100 \times (W_{net}/Q_1) = 100 \times \frac{1954\, R}{3182\, R} = 61.4\%$

61. (i) $T_i = 27 + 273 = 300\, K$; $\gamma = \frac{5}{3}$ (for monoatomic gas)

Since the gas expands adiabatically, so using

$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

$\Rightarrow T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\left[\frac{V}{2V}\right]^{5/3-1} = 189\, K$

(ii) Work done is adiabatic process

$W = \frac{-nR(T_2 - T_1)}{\gamma-1} = \frac{-2 \times 8.31(189-300)}{5/3-1}$

$= \frac{+8.31 \times 111 \times 3}{2} = +2767\text{J}$

According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$ But $\Delta Q = 0$

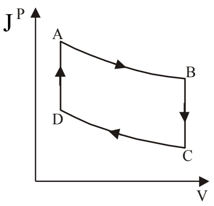
(as the process is adiabatic)

$\therefore \Delta U = -\Delta W = -2767$ J

(iii) Work done by the gas, $W = 2767$ J

62. Given $T_A = 1000$ K

$P_B = \frac{2}{3}P_A$; $P_C = \frac{1}{3}P_A$



No. of moles, $n = 1$, $R = 8.31$J mol^{-1} k^{-1}, $T_A = 1000$ K

$\frac{C_P}{C_V} = \gamma = \frac{5}{3}$ (For mono atomic gas)

(i) W_{AB} (adiabatic process)

$W_{AB} = \frac{nR[T_A - T_B]}{\gamma-1}$

From $T^\gamma P^{1-\gamma}$ = constant

$T_A^\gamma P_A^{1-\gamma} = T_B^\gamma P_B^{1-\gamma} \Rightarrow \left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^\gamma$ ... (i)

$\Rightarrow T_B = T_A\left[\frac{P_A}{P_B}\right]^{\frac{1-\gamma}{\gamma}} = 1000\left[\frac{3}{2}\right]^{\frac{1-5/3}{5/3}} = 850$ K

$\therefore W_{AB} = \frac{1 \times 8.31[1000-850]}{5/3-1} = 1870$ J

(ii) Heat lost in the process B $\to$ C isochoric process.

$Q_{BC} = nC_v \Delta T = nC_v (T_B - T_C)$

To find T_C, we use $\frac{P_B}{T_B} = \frac{P_C}{T_C}$ (volume V = constant)

$\Rightarrow T_C = \frac{P_C}{P_B} \times T_B = \frac{1}{2} \times 850 = 425\, K\left[\because \frac{P_C}{P_A} = \frac{\frac{1}{3}P_A}{\frac{2}{3}P_A} = \frac{1}{2}\right]$

$\therefore Q = 1 \times \frac{3}{2} \times 8.31\,[425-850] = -5298$ J

(iii) C $\to$ D and A $\to$ B are adiabatic processes.

$\therefore P_C^{1-\gamma} T_C^\gamma = p_D^{1-\gamma} T_D^\gamma \Rightarrow \frac{P_C}{P_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}}$ ...(i)

$P_A^{1-\gamma} T_A^\gamma = p_B^{1-\gamma} T_B^\gamma$

$\Rightarrow \frac{P_A}{P_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}}$ ...(ii)

Multiplying Eqs. (i) and (ii), we get

$\frac{P_C P_A}{P_D P_B} = \left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}}$

Processes B $\to$ C and D $\to$ A are isochoric.

(V = constant)

Therefore, $\frac{P_C}{P_B} = \frac{T_C}{T_B}$ and $\frac{P_A}{P_D} = \frac{T_A}{T_D}$

Multiplying these two equations, we get

$\frac{P_C\, P_A}{P_B\, P_B} = \frac{T_C\, T_A}{T_B\, P_B}$ ...(iv)

From Eqs. (iii) and (iv), we have

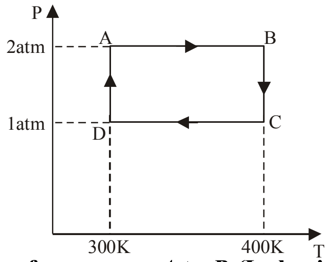
$\left(\frac{T_D\, T_B}{T_C\, T_A}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_C\, T_A}{T_B\, T_D}\right)$ or $\left(\frac{T_C\, T_A}{T_D\, T_B}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_C\, T_A}{T_B\, T_D}\right)$

$\Rightarrow \frac{T_C\, T_A}{T_D\, T_B} = 1$

or $T_D = \dfrac{T_C\ T_A}{T_B} = \dfrac{(425)(1000)}{850}$

or $T_D = 500$ K

63. For the cyclic process given



Work done for process *A* to *B* (Isobaric process)

$W_{AB} = nR\,(T_B - T_A)$

$= nR \times 100 = 2 \times 200 \times 8.32 = 1664$ J

Work done for process *C* to *D* (Isobaric process)

$W_{CD} = -W_{AB} = -1664J$

$= -100nR = -200 \times 8.32 = -1664$ J

Work done for process *B* to *C* (Isothermal process)

$$W_{BC} = 2.303 nRT \log_{10} \frac{P_B}{P_C}$$

$$= 2.303 nR \times 400 \log_{10} \frac{2}{1} = 277.2\ nR$$

$= 554.4 \times 8.32 = 4612.6$

Work done for process *D* to *A* (Isothermal process)

$$W_{DA} = 2.303 nRT \log_{10} \frac{P_D}{P_A} = 2.303 nR \times 300 \log_{10} \frac{1}{2}$$

$= -207.9 nR$

$= -415.8 \times 8.32 = -3459.5$ J

The total work done, $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$= 1153$ J

(a) From first law of thermodynamics $\Delta U = Q - W$

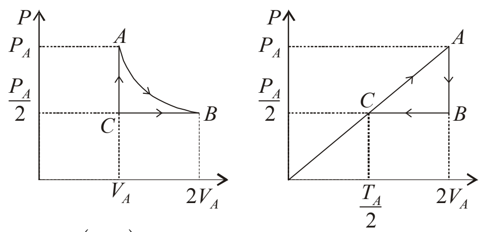
For complete cycle $\Delta U = 0$

$\therefore\ Q = W = 1153$ J

(b) Net work done $W = 1153$ J

(c) $\Delta U = 0$. Since, the process is cyclic i.e., $\Delta T = 0$

64. **(a)** The *p-V* and *p-T* diagrams are shown below



$$(pV)_C = \frac{(pV)_A}{2} \Rightarrow \therefore T_C = \frac{T_A}{2}$$

(b) Process *A-B* $\quad T$ = constnat

$\therefore P \propto \dfrac{1}{V}$, V is doubled so, p will become half.

Also, $V_A = \dfrac{nRT_A}{P_A} = \dfrac{3RT_A}{P_A}$

$\Delta U_{AB} = 0$

$$\therefore Q_{AB} = W_{AB} = nRT_A \text{ In}\left(\frac{2V_A}{V_A}\right)$$

$= 3RT_A$ in $(2) = 2.80RT_A$

Process B-C

$Q_{BC} = nC_P\,(T_C - T_B)$

$$= (3)\left(\frac{7}{2}R\right)\left(\frac{T_A}{2} - T_A\right) = -\frac{21}{4}RT_A$$

$= -5.25RT_A$

Process C-A V = constant

$\therefore\ W_{CA} = 0$

or $Q_{CA} = \Delta U_{CA} = nC_V\,(T_A - T_C)$

$$= (3)\left(\frac{5}{2}R\right)\left(T_A - \frac{T_A}{2}\right) = 3.75RT_A$$

In a cyclic process,

$\Delta U = 0$

$\therefore\ Q_{net} = Q_{AB} + Q_{BC} + Q_{CA} = 0.58\ RT_A$

65. (i) For adiabatic process, $T.V.^{\gamma\,-\,1}$ = constant

$$TV^{\gamma-1} = \frac{T}{2}(5.66V)^{\gamma-1} \Rightarrow 2 = (5.66)^{\gamma-1}$$

Taking log on both sides, log 2 = $(\gamma - 1)$ log 5.66

$\therefore\ \gamma = 1.4$

$$\text{But } \gamma = 1 + \frac{2}{f} \Rightarrow 1.4 = 1 + \frac{2}{f}$$

$$\Rightarrow f = \frac{2}{0.4} = 5 \text{ (Degree of freedom)}$$

(ii) For adiabatic process now using PV^r = constant

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow P_2 = \frac{P}{(5.66)^{1.4}} = \frac{P}{11.32}$$

Work done for adiabatic process

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{PV - \left(\dfrac{P}{11.32}\right)(5.66V)}{1.4 - 1} = 1.25\ PV$$

66. Applying ideal gas equation $PV = nRT$

$$\therefore\ n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3} = 5.33 \text{ moles}$$

Given: $\quad C_p = \dfrac{5R}{2} \quad \therefore\ C_v = \dfrac{3R}{2}$

2.49×10^4 J of heat energy is supplied at constant volume hence from

$Q = nC_v\,\Delta T$

$$\therefore\ \Delta T = \frac{Q}{nC_v} = \frac{2.49 \times 10^4}{5.33 \times \frac{3}{2} \times 8.3} = 375 \text{ K}$$

$\therefore\ $ final temperature $= 300 + 375 = 675$ K

Now from Gay Lussac's law,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = \frac{P_1T_2}{T_1}$$

$$= \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6\ \text{Nm}^{-2}$$

67. In adiabatic process , work done

$$W = \frac{1}{1 - \gamma}[P_2V_2 - P_1V_1]$$

Here, $P_1 = 10^5$ N/m^2, $V_1 = 6\ \ell = 6 \times 10^{-3}$ m^3

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$, $V_2 = 2\,\ell = 2\times 10^{-3}$

Given $C_v = \frac{3}{2}R$ $\therefore C_p = \frac{5}{2}R$ [$\because C_p - C_v = R$]

$\therefore \quad \gamma = \frac{C_p}{C_v} = 1.67$

$\therefore \quad P_2 = 10^5\left[\frac{6}{2}\right]^{1.67} = 10^5\times(3)^{1.67} = 6.26\times 10^5$ N/m^2

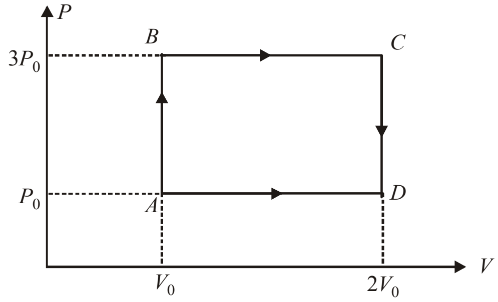
$\therefore \quad W = \frac{1}{1-1.67}[6.26\times 10^5\times 2\times 10^{-3} - 10^5\times 6\times 10^{-3}]$

$W = \frac{1}{-0.67}[1252-600] = -\frac{652}{0.67} = -973.1$ J

Here negative sign indicates that the gas is compressed.

Topic-3 : Carnot Engine, Refrigerators and Second Law of Thermodynamics

1. **(1)**



From the figure,

Work, $W = 2P_0V_0$

Heat given, $Q_{in} = W_{AB} + W_{BC} = n\cdot C_V\Delta T_{AB} + nC_P\Delta T_{BC}$

$= n\frac{3R}{2}(T_B - T_A) + \frac{n5R}{2}(T_C - T_B)$

$\left(\because C_v = \frac{3R}{2} \text{ and } C_P = \frac{5R}{2}\right)$

$= \frac{3}{2}(P_BV_B - P_AV_A) + \frac{5}{2}(P_CV_C - P_BV_B)$

$= \frac{3}{2}\times[3P_0V_0 - P_0V_0] + \frac{5}{2}[6P_0V_0 - 3P_0V_0]$

$= 3P_0V_0 + \frac{15}{2}P_0V_0 = \frac{21}{2}P_0V_0$

Efficiency, $\eta = \frac{W}{Q_{in}} = \frac{2P_0V_0}{\frac{21}{2}P_0V_0} = \frac{4}{21}$

$\eta\% = \frac{400}{21} \approx 19.$

2. **(c)** Efficiency, $\eta = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{W}{\Sigma Q}$

$= \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$

Here, $Q_1 = 1915$ J, $Q_2 = -40$ J and $Q_3 = 125$ J

$\therefore \frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$

$\Rightarrow 1915 - 40 + 125 + Q_4 = 1020$

$\Rightarrow Q_4 = 1020 - 2000$

$\Rightarrow Q_4 = -Q = -980$ J

$\Rightarrow Q = 980$ J

3. **(d)** For carnot refrigerator

Efficiency $= \frac{Q_1 - Q_2}{Q_1}$

Where,

Q_1 = heat lost from sorrounding

Q_2 = heat absorbed from reservoir at low temperature.

Also, $\frac{Q_1 - Q_2}{Q_1} = \frac{w}{Q_1}$

$\Rightarrow \frac{1}{10} = \frac{w}{Q_1}$

$\Rightarrow Q_1 = w\times 10 = 100\,J$

So, $Q_1 - Q_2 = w$

$\Rightarrow Q_2 = Q_1 - w$

$\Rightarrow 100 - 10 = Q_2 = 90\,J$

4. **(b)** Let Q_H = Heat taken by first engine

Q_L = Heat rejected by first engine

Q_2 = Heat rejected by second engine

Work done by 1st engine = work done by 2nd engine

$W = Q_H - Q_L = Q_L - Q_2 \quad \Rightarrow 2Q_L = Q_H + Q_2$

$2 = \frac{\theta_H}{\theta_L} + \frac{\theta_2}{\theta_L}$

Let T be the temperature of cold reservoir of first engine. Then in carnot engine.

$\frac{Q_H}{Q_L} = \frac{T_1}{T}$ and $\frac{Q_L}{Q_2} = \frac{T}{T_2}$

$\Rightarrow \quad 2 = \frac{T_1}{T} + \frac{T_2}{T}$ using (i)

$\Rightarrow \quad 2T = T_1 + T_2 \quad \Rightarrow \quad T = \frac{T_1 + T_2}{2}$

5. **(b)** Using, $n = 1 - \frac{T_2}{T_1}$

$n = \frac{1}{6} = 1 - \frac{T_2}{T_1}$

and $\frac{T}{3} = 1 - \frac{T_2 - 62}{T_1}$

On solving, we get

$T_1 = 99°C$ and $T_2 = 37°C$

6. **(b)** According to question, $\eta_1 = \eta_2 = \eta_3$

$\therefore 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$

[$\because$ Three engines are equally efficient]

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1T_3}$ ...(i)

$T_3 = \sqrt{T_2T_4}$ ...(ii)

From (i) and (ii)

$T_2 = (T_1^2\,T_4)^{1/3} \qquad T_3 = (T_1\,T_4^2)^{1/3}$

7. **(d)** $\eta_A = \frac{T_1 - T_2}{T_1} = \frac{w_A}{Q_1}$

and, $\eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$

According to question,

$W_A = W_B$

$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$

$\therefore T_2 = \frac{T_1 + T_3}{2}$

$= \frac{600 + 400}{2} = 500K$

8. **(a)** Given: Temperature of cold body, T_2 = 250 K temperature of hot body; T_1 = 300 K

Heat received, Q_2 = 500 cal work done, W = ?

$$\text{Efficiency} = 1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W} \Rightarrow 1 - \frac{250}{300} = \frac{W}{Q_2 + W}$$

$$W = \frac{Q_2}{5} = \frac{500 \times 4.2}{5} J = 420 \text{ J}$$

9. **(b)** Work-done (W) = P_0V_0

According to principle of calorimetry

Heat given = $Q_{AB} = Q_{BC}$

$= nC_VdT_{AB} + nC_PdT_{BC}$

$= \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$

$= \frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V)$

$= \frac{13}{2}P_0V_0$

Thermal efficiency of engine $(\eta) = \frac{W}{Q_{given}} = \frac{2}{13} = 0.15$

10. **(d)** $\Delta H = mL = 5 \times 336 \times 10^3 = Q_{sink}$

$\frac{Q_{sink}}{Q_{source}} = \frac{T_{sink}}{T_{source}}$

$\therefore Q_{source} = \frac{T_{source}}{T_{sink}} \times Q_{sink}$

Energy consumed by freezer

$\backslash \; w_{output} = Q_{source} - Q_{sink} = Q_{sink}\left(\frac{T_{source}}{T_{sink}} - 1\right)$

Given: $T_{source} = 27°C + 273 = 300K$,

$T_{sink} = 0°C + 273 = 273\,k$

$W_{output} = 5 \times 336 \times 10^3 \left(\frac{300}{273} - 1\right) = 1.67 \times 10^5 J$

11. **(d)** The entropy change of the body in the two cases is same as entropy is a state function.

12. **(b)** Given : $Q_1 = 1000$ J

$Q_2 = 600$ J

$T_1 = 127°C = 400$ K

$T_2 = ?$

$\eta = ?$

Efficiency of carnot engine,

$\eta = \frac{W}{Q_1} \times 100\%$

or, $\eta = \frac{Q_2 - Q_1}{Q_1} \times 100\%$

or, $\eta = \frac{1000 - 600}{1000} \times 100\%$

$\eta = 40\%$

Now, for carnot cycle $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\frac{600}{1000} = \frac{T_2}{400}$

$T_2 = \frac{600 \times 400}{1000}$

$= 240$ K

$= 240 - 273$

$\therefore T_2 = -33°C$

13. **(b)** Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right)$$

$$\Rightarrow \frac{3}{2}P_0V_0 + \frac{5}{2}2P_0V_0 = \left(\frac{13}{2}\right)P_0V_0$$

14. **(c)**

Container A	Container B
Mass of gas = m_A	Mass of gas = m_B
Change in pressure = ΔP	Change in pressure = $1.5\Delta P$
$P_AV = \frac{m_A}{M}RT$	$P_BV = \frac{m_B}{M}RT$
$P'_A(2V) = \frac{m_A}{M}RT$	$P'_B(2V) = \frac{m_B}{M}RT$

$\Rightarrow P_A - P'_A = \frac{m_ART}{MV} - \frac{m_ART}{M(2V)}$

$\Rightarrow \Delta P = \frac{m_ART}{2MV}$ ... (i)

and $P_B - P'_B = \frac{m_BRT}{MV} - \frac{m_BRT}{M(2V)}$

$1.5\,\Delta P = \frac{m_BRT}{2MV}$ ... (ii)

Dividing eq. (ii) by (i)

$$\frac{1.5\Delta P}{\Delta P}=\frac{m_B}{M_A} \Rightarrow \frac{3}{2}=\frac{m_B}{m_A} \Rightarrow 3m_A=2m_B$$

15. **(8791)**
Given,
Heat absorbed, $Q_2 = mL = 80 \times 100 = 8000$ Cal
Temperature of ice, $T_2 = 273$ K
Temperature of surrounding,
$T_1 = 273 + 27 = 300$ K

$$\text{Efficiency} = \frac{w}{Q_2}=\frac{Q_1-Q_2}{Q_2}=\frac{T_1-T_2}{T_2}=\frac{300-273}{273}$$

$$\Rightarrow \frac{Q_1-8000}{8000}=\frac{27}{273} \Rightarrow Q_1 = 8791 \text{ Cal}$$

16. **(600.00)** Given; $T_1 = 900\,K$, $T_2 = 300K$, $W = 1200\,J$

Using, $1-\dfrac{T_2}{T_1}=\dfrac{W}{Q_1}$

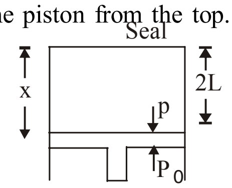
$$\Rightarrow 1-\frac{300}{900}=\frac{1200}{Q_1}$$

$$\Rightarrow \frac{2}{3}=\frac{1200}{Q_1} \Rightarrow Q_1 = 1800$$

Therefore heat energy delivered by the engine to the low temperature reservoir, $Q_2 = Q_1 - W = 1800 - 1200 = 600.00\,J$

17. **(a)** As the cylinder is open at its bottom and has a smell hole at its tope, hence the pressure inside the cylinder is atmosphenic pressure, P_0 throughout the slow pulling process.

18. **(d)** Let x be the distance of the piston from the top.



At equilibrium
$Mg = (P_0 - p)\,\pi R^2$

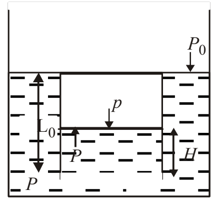
$$\Rightarrow p = \frac{-Mg}{\pi R^2}+P_0$$

Since the cylinder isothermally conducting
$\therefore$ temperature, T = constant
Applying $P_1V_1 = P_2V_2$

$$P_0 \times (2L \times \pi R^2) = px \times \pi R^2 \Rightarrow x = \frac{P_0}{p}\times 2L$$

$$=\frac{P_0}{\left[P_0-\dfrac{Mg}{\pi R^2}\right]}\times 2L=\left[\frac{P_0\times \pi R^2}{P_0\,\pi R^2-Mg}\right]\times 2L$$

19. **(c)** At equilibrium, $p = P$
$\Rightarrow\ p = P_0 + (L_0 - H)\,\rho g$ ... (i)
Also $P_0 \times (\pi R^2 L_0) = p\,[\pi R^2 (L_0 - H)]$ $\quad (\because P_1V_1 = P_2V_2)$



$$\Rightarrow p = \frac{L_0P_0}{L_0-H}$$

Substituting this value of E from (i) and (ii)

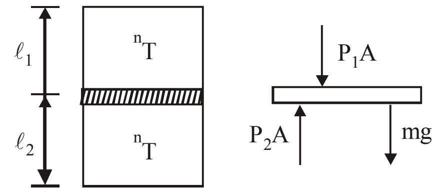
$$\frac{L_0P_0}{L_0-H}=P_0+(L_0-H)\rho g$$

$\Rightarrow\ L_0P_0 = P_0\,(L_0 - H) + (L_0 - H)^2\,\rho g$
$\Rightarrow\ \rho g\,(L_0 - H)^2 + P_0\,(L_0 - H) - L_0P_0 = 0$

12 Kinetic Theory

Topic-1 : Kinetic Theory of an Ideal Gas and Gas Laws

1. **(d)** Clearly from figure,
$P_2A = P_1A + mg$
or, $\dfrac{nRT \cdot A}{A\ell_2} = \dfrac{nRT \cdot A}{A\ell_1} + mg$



$\Rightarrow \quad nRT\left(\dfrac{1}{\ell_2} - \dfrac{1}{\ell_1}\right) = mg$

$\therefore \quad m = \dfrac{nRT}{g}\left(\dfrac{\ell_1 - \ell_2}{\ell_1 \cdot \ell_2}\right)$

2. **(b)** Given: Temperature $T_i = 17 + 273 = 290\ K$
Temperature $T_f = 27 + 273 = 300\ K$
Atmospheric pressure, $P_0 = 1 \times 10^5\ Pa$
Volume of room, $V_0 = 30\ m^3$
Difference in number of molecules, $n_f - n_i = ?$
Using ideal gas equation, $PV = nRT(N_0)$,
N_0 = Avogadro's number

$\Rightarrow \quad n = \dfrac{PV}{RT}(N_0)$

$\therefore \quad n_f - n_i = \dfrac{P_0V_0}{R}\left(\dfrac{1}{T_f} - \dfrac{1}{T_i}\right) N_0$

$= \dfrac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23}\left(\dfrac{1}{300} - \dfrac{1}{290}\right)$

$= -2.5 \times 10^{25}$

3. **(c)** From P-V graph,
$P \propto \dfrac{1}{V}$, T = constant and Pressure is increasing from 2 to 1 so option (c) represents correct T-P graph.

4. **(d)** $P_1M_1 = \rho_1RT$ and $P_2M_2 = \rho_2RT$
$\therefore \dfrac{P_1}{P_2} \times \dfrac{M_1}{M_2} = \dfrac{\rho_1}{\rho_2}$

$\dfrac{4}{3} \times \dfrac{2}{3} = \dfrac{\rho_1}{\rho_2} \qquad \therefore \dfrac{\rho_1}{\rho_2} = \dfrac{8}{9}$

5. **(d)** A real gas behaves as an ideal gas at low pressure and high temperature.

6. **(a)** According to Boyle's law, PV = constant.
$\therefore \quad PdV + VdP = 0$

$\Rightarrow \quad \dfrac{PdV}{dP} = -V; \beta = -\left(\dfrac{1}{V}\right)\left(\dfrac{dV}{dP}\right) = \left(\dfrac{1}{P}\right) \Rightarrow \beta \times P = 1$

Hence graph between β and P will be a rectangular hyperbola.

7. **(c)**

Container A	**Container B**
Mass of gas = m_A	Mass of gas = m_B
Change in pressure = ΔP	Change in pressure = $1.5\Delta P$
$P_AV = \dfrac{m_A}{M}RT$	$P_BV = \dfrac{m_B}{M}RT$
$P'_A(2V) = \dfrac{m_A}{M}RT$	$P'_B(2V) = \dfrac{m_B}{M}RT$

$\Rightarrow \quad P_A - P'_A = \dfrac{m_ART}{MV} - \dfrac{m_ART}{M(2V)}$

$\Rightarrow \quad \Delta P = \dfrac{m_ART}{2MV}$ ... (i)

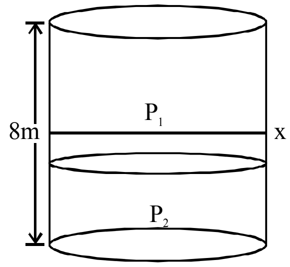
and $P_B - P'_B = \dfrac{m_BRT}{MV} - \dfrac{m_BRT}{M(2V)}$

$1.5\ \Delta P = \dfrac{m_BRT}{2MV}$ ... (ii)

Dividing eq. (ii) by (i)

$\dfrac{1.5\Delta P}{\Delta P} = \dfrac{m_B}{M_A} \Rightarrow \dfrac{3}{2} = \dfrac{m_B}{m_A} \Rightarrow 3m_A = 2m_B$

8. **(Bonus)/6**
Initially partition is held at a distance of 4m from the top.
Temperature, T = 300 K
PV = PAh = nRT
$P_04A = 0.1R \times 300$ (For both)
Let piston shifts by × meter and final temerature is T, then
$P_2A(4 - x) = 0.1RT$



$\therefore P_2A = \dfrac{0.1RT}{4-x} = \dfrac{RT}{10(4-x)}$ and $P_1A = \dfrac{RT}{10(4+x)}$

Finally $P_2A - P_1A = mg = 83 = \dfrac{RT}{10}\left(\dfrac{1}{4-x} - \dfrac{1}{4+x}\right)$

$\Rightarrow \dfrac{RT}{10}\left(\dfrac{2x}{16-x^2}\right) = 83$

$0.2C_v \times 300 + mg\ .\ x = 0.2C_vT$

$\Rightarrow \dfrac{3}{2}R \times \dfrac{2}{10} \times 300 + 83x = \dfrac{2}{10} \times \dfrac{3}{2}RT$

$\Rightarrow \left(\dfrac{900R}{10} + 83x\right) = 3\left(\dfrac{RT}{10}\right)$

$\Rightarrow \left(\dfrac{300R}{10} + \dfrac{83}{3}x\right) = \dfrac{RT}{10} \quad \Rightarrow \left(30R + \dfrac{83x}{3}\right)\left(\dfrac{2x}{16-x^2}\right) = 83$

$\Rightarrow 83\left(3 + \dfrac{x}{3}\right)\left(\dfrac{2x}{16-x^2}\right) = 83$

$\Rightarrow \dfrac{(9+x)}{3}\dfrac{(2x)}{(16-x^2)} = 1$

$\Rightarrow 18x + 2x^2 = 48 - 3x^2 \Rightarrow 5x^2 + 18x - 48 = 0$

$\therefore x = 1.78 \approx 2$

Hence distance from top when reached equilibrium = 4 + 2 = 6 m

9. **(5)** Using ideal gas equation, $PV = nRT$

$\Rightarrow P_1V_1 = nR \times 250 \quad [\because T_1 = 250\text{ K}] \quad ...(i)$

$P_2(2V_1) = \frac{5n}{4}R \times 2000 \quad [\because T_2 = 2000\text{ K}] \quad ...(ii)$

Dividing eq. (i) by (ii),

$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000} \Rightarrow \frac{P_1}{P_2} = \frac{1}{5} \quad \therefore \frac{P_2}{P_1} = 5.$

10. **(150)** In first case,

From ideal gas equation

$PV = nRT$

$P\Delta V + V\Delta P = 0$ (As temperature is constant)

$\Delta V = -\frac{\Delta P}{P}V \quad ...(i)$

In second case, using ideal gas equation again

$P\Delta V = -nR\Delta T$

$\Delta V = -\frac{nR\Delta T}{P} \quad ...(ii)$

Equating (i) and (ii), we get

$\frac{nR\Delta T}{P} = -\frac{\Delta P}{P}V \Rightarrow \Delta T = \Delta P\frac{V}{nR}$

Comparing the above equation with $|\Delta T| = C|\Delta P|$, we have

$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300\text{ K}}{2\text{ atm}} = 150\text{ K/atm}$

11. **($\sqrt{2}T$)** $PV = nRT$ (Ideal gas equation)

$\Rightarrow P = \frac{nRT}{V} \quad ...(i)$

Given VP^2 = const ... (ii)

From (i) and (ii)

$\therefore \frac{T^2}{V} = \text{constant} \quad \text{or,} \quad T \propto \sqrt{v}$

$\because V_1 = 2V$ and $T_1 = T$

$\therefore \frac{T_1^2}{V_1} = \frac{T_2^2}{V_2} \Rightarrow T_2 = T_1\sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2}T$

12. **(False)** For a particular temperature T, $V \propto \frac{1}{P}$

Volume is greater for pressure $P_1 \quad \therefore \quad p_1 < p_2$

Topic-2 : Speed of Gas, Pressure and Kinetic Energy

1. **(a)** Given : $K.E._{mean} = \frac{3}{2}kT = 4 \times 10^{-14}$

$P = 2$ cm of Hg, $V = 4\text{ cm}^3$

$N = \frac{PV}{KT} = \frac{P\rho gV}{KT} = \frac{2 \times 13.6 \times 980 \times 4}{\frac{8}{3} \times 10^{-14}} \simeq 4 \times 10^{18}$

2. **(c)** $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \frac{(273+127)}{(273+237)} = \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\therefore v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2} \times 200 = 100\sqrt{5}\text{ m/s.}$

3. **(Bouns)** Rate of change of momentum during collision

$= \frac{mv-(-mv)}{\Delta t} = \frac{2mv}{\Delta t}N$

so pressure $P = \frac{N \times (2mv)}{\Delta t \times A}$

$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1} = 2\, N/m^2$

4. **(c)** $v_{rms} = v_e$

$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3 \quad \text{or} \quad \sqrt{\frac{3kT}{m}} = 11.2 \times 10^3$

or $\sqrt{\frac{3 \times 1.38 \times 10^{-23}T}{2 \times 10^{-3}}} = 11.2 \times 10^3 \quad \therefore T = 10^4\text{ K}$

5. **(a)** Using $\frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{M_2}{M_1}}$

$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$

6. **(a)** Energy associated with N moles of diatomic gas,

$U_i = N\frac{5}{2}RT$

Energy associated with n moles of monoatomic gas

$= n\frac{3}{2}RT$

Total energy when n moles of diatomic gas converted into monoatomic $(U_f) = 2n\frac{3}{2}RT + (N-n)\frac{5}{2}RT$

$= \frac{1}{2}nRT + \frac{5}{2}NRT$

Now, change in total kinetic energy of the gas

$\Delta U = Q = \frac{1}{2}nRT$

7. **(c)** Pressure, $P = \frac{1}{3}\frac{mN}{V}V_{rms}^2$

or, $P = \frac{(mN)T}{V}$

If the gas mass and temperature are constant then

$P \propto (V_{rms})^2 \propto T$

So, force $\propto (V_{rms})^2 \propto T$

i.e., Value of $q = 1$

8. **(a)** $\because \quad C = \sqrt{\frac{3RT}{M}}$

$(1930)^2 = \frac{3 \times 8.314 \times 300}{M}$

$M = \frac{3 \times 8.314 \times 300}{1930 \times 1930} \approx 2 \times 10^{-3}\text{ kg}$

The gas is H_2.

9. **(c)** $v_{rms} = \sqrt{\frac{3\rho v}{\text{mass of the gas}}}$

10. **(d)** Using $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{rms} \propto \frac{1}{\sqrt{M}}$

$\frac{v_{rms\,(helium)}}{v_{rms\,(argon)}} = \sqrt{\frac{M_{argon}}{M_{helium}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$

11. **(c)** Velocity of sound in a gas $v=\sqrt{\frac{\gamma p}{p}}=\sqrt{\frac{\gamma RT}{M}}$

$$\frac{V_{N_2}}{V_{He}}=\sqrt{\frac{\gamma_{N_2}M_{He}}{\gamma_{He}M_{N_2}}}=\sqrt{\frac{7/5\times 4}{5/3\times 28}}=\frac{\sqrt{3}}{5}$$

12. **(b)** When a enclosed gas is accelerated in the positive x-direction then the pressure of the gas decreases along the positive x-axis and follows the equation

$$\Delta P=-\rho\, a\, dx$$

So more pressure on the rear side and less pressure on the front side.

13. **(c)** From, PV = nRT $\Rightarrow$ P = $\frac{nRT}{V}$ or P $\propto$ T

($\because$ V and n are same.)

Therefore, if T is doubled, pressure also becomes two times, i.e., 2P.

14. **(b)** $\frac{(V_{rms})_1}{(V_{rms})_2}=\sqrt{\frac{T_1}{T_2}}\Rightarrow\frac{V}{(V_{rms})_2}=\sqrt{\frac{120}{480}}$

$$\Rightarrow\frac{V}{(V_{rms})_2}=\frac{1}{2}\Rightarrow(V_{rms})_2=2V$$

15. **(b)** According to Maxwell's distribution of speed, average speed of molecules of an ideal gas v $\propto\sqrt{T}$.

$\therefore$ The velocity of oxygen molecules will be same in A as well as C as the temperature of A and C are same

16. **(41)** Room mean square speed is given by

$$v_{rms}=\sqrt{\frac{3RT}{M}}$$

Here, M = Molar mass of gas molecule

T = temperature of the gas molecule

We have given $v_{N_2}=v_{H_2}$

$$\therefore\sqrt{\frac{3RT_{N_2}}{M_{N_2}}}=\sqrt{\frac{3RT_{H_2}}{M_{H_2}}}\Rightarrow\frac{T_{H_2}}{2}=\frac{573}{28}\Rightarrow T_{H_2}=41\text{ K}$$

17. **(False)** We know that, rms speed

$$v=\sqrt{\frac{3RT}{M}}\text{ then }v'=\sqrt{\frac{3R(2T)}{M/2}}\quad\therefore v'=2v$$

18. **(False)** $\frac{(C_{H_2})_1}{(C_{H_e})_2}=\frac{\sqrt{\frac{\gamma_1 RT}{M_1}}}{\sqrt{\frac{\gamma_2 RT}{M_2}}}=\sqrt{\frac{\gamma_1}{\gamma_2}\times\frac{M_2}{M_1}}$

$$=\sqrt{\frac{7/5}{5/3}\times\frac{4}{2}}=\sqrt{\frac{7}{5}\times\frac{3}{5}\times 2}=\sqrt{\frac{42}{25}}$$

19. **(False)** For a particular termperature root mean square speed,

$$V_{rms}\propto\frac{1}{\sqrt{M}}$$

i.e., V_{rms} will have different values for different gases.

20. **(False)** RMS speed, $V_{rms}=\sqrt{\frac{3RT}{M}}$

i.e., V_{rms} depends on temperature and molar mass and hence rms speed will be different for different ideal gases.

21. **(c, d)**

We know that

$$\bar{v}=\sqrt{\frac{8RT}{\pi M}};v_{rms}=\sqrt{\frac{3RT}{M}}\text{ and }v_p=\sqrt{\frac{2RT}{M}}$$

From these expressions, we can conclude that

$v_p<\bar{v}<v_{rms}$

Also, $V_{rms}=\sqrt{\frac{3}{2}}VP$

And, average kinetic energy of gaseous molecules

$$\bar{E}=\frac{1}{2}mv_{rms}^2=\frac{1}{2}m\left(\frac{3}{2}v_p^2\right)=\frac{3}{4}mv_p^2$$

Topic-3 : Degree of Freedom, Specific Heat Capacity, and Mean Free Path

1. **(c)** Total degree of freedom f = 3 + 2 = 5

Total energy, $U=\frac{nfRT}{2}=\frac{5RT}{2}$

And $\gamma=\frac{C_p}{C_v}=1+\frac{2}{f}=1+\frac{2}{5}=\frac{7}{5}$

2. **(b)** Mean free path, $\lambda=\frac{1}{\sqrt{2}\pi nd^2}$

where, d = diameter of the molecule

n = number of molecules per unit volume

But, mean time of collision, $\tau=\frac{\lambda}{v_{rms}}$

But $v_{rms}=\sqrt{\frac{3kT}{R}}\quad\therefore\tau=\frac{\lambda}{\sqrt{\frac{3kT}{m}}}\Rightarrow t\propto\frac{1}{\sqrt{T}}$

3. **(c)** As we know,

$\gamma=\frac{C_p}{C_v}=1+\frac{2}{f}$, where f = degree of freedom

(A) Monatomic, f = 3

$\therefore\gamma=1+\frac{2}{3}=\frac{5}{3}$

(B) Diatomic rigid molecules, f = 5

$\therefore\gamma=1+\frac{2}{5}=\frac{7}{5}$

(C) Diatomic non-rigid molecules, f = 7

$\therefore\gamma=1+\frac{2}{7}=\frac{9}{7}$

(D) Triatomic rigid molecules, f = 6

$\therefore\gamma=1+\frac{2}{6}=\frac{4}{3}$

4. **(d)** Here degree of freedom, f = 3 + 3 = 6 for triatomic non-linear molecule.

Internal energy of a mole of the gas at temperature T,

$$U=\frac{f}{2}nRT=\frac{6}{2}RT=3RT$$

5. **(b)** Let C_p and C_v be the specific heat capacity of the gas at constant pressure and volume.

At constant pressure, heat required

$\Delta Q_1=nC_p\Delta T$

$\Rightarrow 160=nC_p\cdot 50$ ...(i)

At constant volume, heat required

$\Delta Q_2=nC_v\Delta T$

$\Rightarrow 240=nC_v\cdot 100$ ...(ii)

Dividing (i) by (ii), we get

$$\frac{160}{240}=\frac{C_p}{C_v}\cdot\frac{50}{100}\Rightarrow\frac{C_p}{C_v}=\frac{4}{3}$$

$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f}$ (Here, f = degree of freedom)

$\Rightarrow f = 6.$

6. **(a)** Total energy of the gas mixture,

$$E_{mix} = \frac{f_1 n_1 RT_1}{2} + \frac{f_2 n_2 RT_2}{2}$$

$$= 3 \times \frac{5}{2} RT + \frac{5}{2} \times 3RT = 15RT$$

7. **(a)** As we know mean free path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

Here, N = no. of molecule
V = volume of container
d = diameter of molecule

But $PV = nRT = nNKT$

$$\Rightarrow \frac{N}{V} = \frac{P}{KT} = n \quad \Rightarrow \lambda = \frac{1}{\sqrt{2}} \frac{KT}{\pi d^2 P}$$

For constant volume and hence constant number density n of gas molecules $\frac{P}{T}$ is constant.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases.

8. **(d)** Specific heat of gas at constant volume

$C_v = \frac{1}{2} fR; f =$ degree of freedom

For gas A (diatomic)

f = 5 (3 translational + 2 rotational)

$$\therefore C_v^A = \frac{5}{2} R$$

For gas B (diatomic) in addition to (3 translational + 2 rotational) 2 vibrational degree of freedom.

$$\therefore \quad C_v^B = \frac{7}{2} R \text{ Hence } \frac{C_v^A}{C_v^B} = \frac{\frac{5}{2}R}{\frac{7}{2}R} = \frac{5}{7}$$

9. **(Bonus)** Mean free path of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Here, n = number of collisions per unit volume
d = diameter of the molecule
If average speed of molecule is v then

Mean free time, $\tau = \frac{\lambda}{v}$

$$\Rightarrow \tau = \frac{1}{\sqrt{2}\pi n d^2 v} = \frac{1}{\sqrt{2}\pi n d^2}\sqrt{\frac{M}{3RT}} \qquad \left(\because v = \sqrt{\frac{3RT}{M}}\right)$$

$$\therefore \quad \tau \propto \frac{\sqrt{M}}{d^2} \quad \therefore \quad \frac{\tau_1}{\tau_2} = \frac{\sqrt{M_1}}{d_1^2} \times \frac{d_2^2}{\sqrt{M_2}}$$

$$= \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = 1.09$$

10. **(c)** Relaxation time $(\tau) \propto \frac{\text{mean free path}}{\text{speed}} \Rightarrow \tau \propto \frac{1}{v}$

and, $v \propto \sqrt{T} \quad \therefore \tau \propto \frac{1}{\sqrt{T}}$

Hence graph between τ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

11. **(a)** Helium is a monoatomic gas and Oxygen is a diatomic gas.

For helium, $C_{V_1} = \frac{3}{2}R$ and $C_{P_1} = \frac{5}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$ and $C_{P_2} = \frac{7}{2}R$

$$\gamma = \frac{N_1 C_{P_1} + N_2 C_{P_2}}{N_1 C_{V_1} + N_2 C_{V_2}} \Rightarrow \gamma = \frac{n.\frac{5}{2}R + 2n.\frac{7}{2}R}{n.\frac{3}{2}R + 2n.\frac{5}{2}R} = \frac{19nR \times 2}{2(13nR)}$$

$$\therefore \left(\frac{C_P}{C_V}\right)_{mixture} = \frac{19}{13}$$

12. **(d)** Using, $\gamma_{mixture} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

$$\Rightarrow \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1} \Rightarrow \frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$\Rightarrow \frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1} \Rightarrow \gamma_m - 1 = \frac{5}{12} \Rightarrow \gamma_m = \frac{17}{12} = 1.42$$

13. **(c)** $[C_v]_{min} = \frac{n_1[C_{v_1}] + n_2[C_{v_2}]}{n_1 + n_2}$

$$= \left[\frac{2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}}{2 + 3}\right]$$

= 2.1 R = 2.1 × 8.3 = 17.4 J/mol–k

14. **(b)** $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

or $\frac{W}{Q} = 1 - \frac{5}{7} = \frac{2}{7}$ or $Q = \frac{7}{2}W = \frac{7 \times 10}{2} = 35$ J

15. **(c)** $V = 25 \times 10^{-3}$ m^3, N = 1 mole of O_2

T = 300 K

$V_{rms} = 200$ m/s

$$\therefore \lambda = \frac{1}{\sqrt{2} N \pi r^2}$$

Average time $\frac{1}{\tau} = \frac{<V>}{\lambda} = 200.N\pi r^2.\sqrt{2}$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} . \pi \times 10^{-18} \times 0.09$$

The closest value in the given option is = 10^{10}

16. **(c)** Amount of heat required (Q) to raise the temperature at constant volume

$Q = nC_v\Delta T$ ...(i)

Amount of heat required (Q_1) at constant pressure

$Q_1 = nC_p\Delta T$ ...(ii)

Dividing equation (ii) by (i), we get $\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v}$

$$\Rightarrow Q_1 = (Q)\left(\frac{7}{5}\right) \left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)$$

17. **(b)** $\gamma_A = \frac{C_P}{C_v} = \frac{29}{22} = 1.32 < 1.4$ (diatomic)

and $\gamma_B = \frac{30}{21} = \frac{10}{7} = 1.43 > 1.4$

Gas A has more than 5-degrees of freedom.

18. **(a)** Energy of the gas, E

$= \frac{f}{2}nRT = \frac{f}{2}PV = \frac{f}{2}(3\times10^6)(2) = f\times3\times10^6$

Considering gas is monoatomic i.e., f = 3

Energy, E = 9×10^6 J

19. **(b)** Using, $\tau = \frac{1}{2n\pi d^2 V_{avg}}$

$\therefore t \propto \frac{\sqrt{T}}{P}\left[\because n = \frac{\text{no.of molecules}}{\text{Volume}}\right]$

or, $\frac{t_1}{6\times10^{-8}} = \frac{\sqrt{500}}{2P}\times\frac{P}{\sqrt{300}} \approx 4\times10^{-8}$

20. **(a)** $U = \frac{f_1}{2}n_1RT + \frac{f_2}{2}n_2RT$

Considering translational and rotational modes, degrees of freedom $f_1 = 5$ and $f_2 = 3$

$\therefore u = \frac{5}{2}(3RT) + \frac{3}{2}\times5RT$

U = 15RT

21. **(c)** Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right) = \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT) = \frac{3}{2}PV$

$= \frac{3}{2}P\left(\frac{m}{\rho}\right) = \frac{3}{2}P\left(\frac{2}{8}\right) = \frac{3}{2}\times4\times10^4\times\frac{2}{8} = 1.5\times10^4$ J

therefore, order = 10^4 J

22. **(c)** Heat transferred,

$Q = nC_v\Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times i.e., T' = 4T as $v_{rms} = \sqrt{3RT/M}$

$\therefore Q = \frac{15}{28}\times\frac{5\times R}{2}\times(4T - T)$

$\left[\therefore \frac{CP}{CV} = \gamma_{diatomic} = \frac{7}{5} \& C_p - C_v = R\right]$

or, Q = 10000 J = 10 kJ

23. **(c)** In an adiabatic process

$TV^{\gamma-1}$ = Constant or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$(300)V^{2/3} = T_2(2V)^{2/3} \Rightarrow T_2 = \frac{300}{(2)^{2/3}}$

T_2 = 189 K (final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\ \Delta T$

$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7$ kJ

24. **(b)** Using formula,

$$\gamma_{mixture} = \left(\frac{C_p}{C_v}\right)_{mix} = \frac{\frac{n_1\gamma_1}{\gamma_1-1} + \frac{n_2\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}}$$

Putting the value of $n_1 = 2$, $n_2 = n$, $\left(\frac{C_p}{C_v}\right)_{mix} = \frac{3}{2}$

$\gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$ and solving we get, n = 2

25. **(a)** As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \frac{R}{M}$

For hydrogen (M = 2) $C_p - C_v = a = \frac{R}{2}$

For nitrogen (M = 28) $C_p - C_v = b = \frac{R}{28}$

$\therefore \frac{a}{b} = 14$ or, $a = 14b$

26. **(d)** The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$\frac{C_p}{C_v} = \gamma = \left(1 + \frac{2}{f}\right)$

where f is degree of freedom

$\frac{C_p}{C_v} = \left(1 + \frac{2}{5}\right) = \frac{7}{5}$

27. **(d)** For a polytropic process

$C = C_v + \frac{R}{1-n} \quad \therefore C - C_v = \frac{R}{1-n}$

$\therefore 1 - n = \frac{R}{C - C_v} \quad \therefore 1 - \frac{R}{C - C_v} = n$

$\therefore n = \frac{C - C_v - R}{C - C_v} = \frac{C - C_v - C_p + C_v}{C - C_v}$

$= \frac{C - C_p}{C - C_v} (\because C_p - C_{v=R})$

28. **(d)** Using equipartition of energy, we have

$\frac{6}{2}KT = mCT$

$C = \frac{3\times1.38\times10^{-23}\times6.02\times10^{23}}{27\times10^{-3}}$

$\therefore$ C = 925 J/kgK

29. **(b)**

30. **(b)** On giving same amount of heat at constant pressure, there is no change in temperature for mono, dia and polyatomic.

$(\Delta Q)_P = \mu C_p\Delta T\left(\mu = \frac{\text{No. of molecules}}{\text{Avogedro's no.}}\right)$

or $\Delta T \propto \frac{1}{\text{no. of molecules}}$

31. **(d)** The internal energy of n moles of an ideal gas

$$U = \frac{1}{2} nfRT$$

where f = number of degrees of freedom.

$f_{O_2} = 5$ and $f_{Ar} = 3$

$$\therefore \quad U = U_{O_2} + U_{Ar} = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right)$$

$\therefore$ Total internal energy $U = 11$ RT

32. **(c)** Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2}$kT which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

33. **(a)** For an ideal gas PV = nRT

Coefficient of volume expansion

$$\left(\frac{\Delta V}{\Delta T}\right)_p = \frac{nR}{P} = \text{Constant}$$

Average translation K.E. for $O_2 = \frac{3}{2}$kT

(Three degrees of freedom for translational motion).

Now decrease in pressure increases the volume.

It increases mean free path of the molecules. Also average K.E. does not depend on the gas, so molecules of each component of mixture of gases have same average translational energy.

34. **(b)** $\gamma_1 = \frac{5}{3}$ means gas is monoatomic or $C_{r_1} = \frac{3}{2}R$

$\gamma_2 = \frac{7}{5}$ means gas is diatomic or $C_{v_2} = \frac{5}{2}R$

C_v (of the mixture)

$$= \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

C_p (of the mixture) = $C_v + R = 3R$

$$\therefore \quad \gamma_{\text{mixture}} = \frac{C_p}{C_v} = \frac{3R}{2R} = 1.5$$

35. **(a)** According to the law of equipartition of energy the average kinetic energy associated with each degree of freedom per molecule is $\frac{1}{2}$k.T. In this case, O_2 and N_2 both have two degrees of rotational kinetic energy and since the temperature is also same, the ratio of the average rotational kinetic energy $\dfrac{\frac{2 \times KT}{2}}{\frac{2 \times KT}{2}}$ *i.e.*, 1 : 1.

36. **(d)** Piston A is free to move, therefore heat will be supplied at constant pressure

$\therefore \quad \Delta Q_A = nC_p \Delta T_A$ ... (i)

Piston B is held fixed, therefore heat will be supplied at constant volume.

$\therefore \quad \Delta Q_B = nC_v \Delta T_B$ ... (ii)

But $\Delta Q_A = \Delta Q_B$ (given)

$$\therefore \quad nC_p \Delta T_A = nC_v \Delta T_B \quad \therefore \; \Delta T_B = \left(\frac{C_p}{C_v}\right) \Delta T_A$$

$\Delta T_B = \gamma\,(\Delta T_A)$ [γ = 1.4 (diatomic)]

= (1.4) (30 K)

$\therefore \quad \Delta T_B = 42$ K

37. **(d)** $\dfrac{\Delta U}{Q_p} = \dfrac{nC_v \Delta T}{nC_p \Delta T} = \dfrac{C_v}{C_p} = \dfrac{1}{\gamma} = \dfrac{1}{7/5} = \dfrac{5}{7}$

38. **(266.67)** Here work done on gas and heat supplied to the gas are zero.

Let T be the final equilibrium temperature of the gas in the vessel.

Total internal energy of gases remain same.

i.e., $u_1 + u_2 = u'_1 + u'_2$

or, $n_1 C_v \Delta T_1 + n_2 C_v \Delta T_2 = (n_1 + n_2) C_v T$

$$\Rightarrow (0.1)C_v(200) + (0.05)C_v(400) = (0.15)C_v T$$

$$\therefore T = \frac{800}{3} = 266.67 \text{ K}$$

39. Molar specific heat of the mixture at constant volume

$$\bar{C}_v = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \frac{1 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{1+1} = 2R$$

40. **(True)** Clearly, $C_p > C_v$ $C_p - C_v = R \Rightarrow C_P = C_V + R$

41. **(a, b, c)**

According to question no. of mole of hydrogen = no. of mole of helium = 1

Total internal energy, u

$$= \frac{f_1}{2}nRT + \frac{f_2}{d}nRT \Rightarrow u = \frac{3}{2}RT + \frac{5}{2}RT = 4RT$$

$\therefore$ Average internal energy per mole = $\dfrac{u}{2n} = \dfrac{4RT}{2} = 2RT$

We know that $V_{\text{sound}} = \sqrt{\dfrac{\gamma RT}{M}}$

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \Rightarrow \frac{2}{\gamma_{mix} - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

$$\frac{2}{\gamma_{mix} - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_{mix} - 1 = \frac{1}{2} \;\therefore\; \gamma_{mix} = \frac{3}{2}$$

$$\frac{(V_s)_{mix}}{(V_s)_{He}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma_{He}}} = \sqrt{\frac{\frac{3}{2} \times 4}{3 \times \frac{5}{3}}} = \sqrt{\frac{6}{5}}$$

$$\left[\because M_{mix} = \frac{1 \times 2 + 1 \times 4}{2} = 3\right]$$

We know that $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$\therefore \frac{(V_{rms})_{He}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{H_E}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

42. **(b, d)**

We known for all gases $C_p - C_v = R$

For monoatomic gas : $C_v = \frac{3}{2}R$; $C_p = \frac{5}{2}R$; $\gamma = \frac{5}{3}$

$\therefore C_p.C_v = \dfrac{15}{4}$ and $C_p + C_v = 4$

For diatomic gas : $C_v = \frac{5}{2}R$; $C_p = \frac{7}{2}R$; $\gamma = \frac{7}{5}$

$\therefore C_p.C_v = \dfrac{35}{4}$ and $C_p + C_v = 6$

43. **(b)** The total translational kinetic energy of n moles of gas = $\frac{3}{2}$nRT 1.5 PV $(\because P_V = \text{nRT})$

Yes, the molecules of a gas collide with each other and the velocities of the molecules change due to collision.

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. **(d)** The heat is supplied at constant pressure. *i.e.*, the process is isobaric

$\therefore Q = n\, C_p\, \Delta t$

$$= 2\left[\frac{5}{2}R\right] \times \Delta t = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208 \text{ J}$$

$$\left(\because C_p = \frac{5}{2}R \text{ for mono-atomic gas}\right)$$

2. **(c)** pT^2 = constant (given)

$$\therefore \left(\frac{nRT}{V}\right)T_2 = \text{constant or } T^3V^{-1} = \text{constant}$$

$(\therefore PV = nRT)$

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0 \text{ or } 3dT = \frac{T}{V}.dV \quad ...(i)$$

From the equation

$dV = V\gamma\, dT$

$$\gamma = \text{coefficient of volume expansion of gas} = \frac{dV}{V.\text{dT}}$$

From eq. (i) $\gamma = \dfrac{dV}{V.\text{dT}} = \dfrac{3}{T}$

3. **(d)** $\dfrac{\Delta U}{Q_p} = \dfrac{nC_v\Delta T}{nC_p\Delta T} = \dfrac{C_v}{C_p} = \dfrac{1}{\gamma} = \dfrac{1}{7/5} = \dfrac{5}{7}$

4. **(900*J*)** Given: $T_i = 100\text{k}$, $V_f = 8V_i$

For and adiabatic process, $TV^{\gamma-1}$ = constant

or, $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$$\Rightarrow \frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1} \Rightarrow \frac{T_i}{T_f} = \left(\frac{8V_i}{V_i}\right)^{\gamma-1}$$

For monoatomic gas $\gamma = \dfrac{5}{3}$

$$\therefore T_f = \frac{T_i}{\left(8^{5/3} - 1\right)} = \frac{T_i}{4}$$

Change in internal energy $\Delta u = nC_V\Delta T$

$$= 1 \times \frac{3}{2}R\left(\frac{T_i}{4} - T_i\right) = \frac{3}{2} \times 8\left(\frac{-3}{4}\right) \times 100 = -900J$$

5. The movable stopper will adjust to a position with equal pressure on either sides. Therefore, $P_1 = P_2$

$$P_1 = \frac{n_1RT}{V_1} = \frac{m}{M_1}RT,\ P_2 = \frac{n_2RT}{V_2} = \frac{m}{M_2}RT$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7} \quad \therefore \alpha = \frac{360°}{(8+7)} \times 8 = 192°$$

6. **(a, b, c)** Force = Pressure × Area

$$p_1 = \frac{n_1RT}{N_A} \text{ and } p_2 = \frac{n_2RT}{N_A}$$

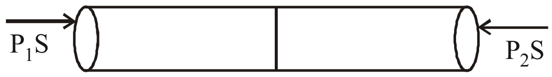
$$F = \Delta p.A = \left(\frac{n_1RT}{N_A} - \frac{n_2RT}{N_A}\right)S$$

$F = (n_1 - n_2)k_BTS = \Delta nk_BTS$

$$\left(\because \frac{R}{N_A} = K_B(\text{Boltzmann constant})\right)$$

Hence, option (a) is correct.

$$V = \frac{\Delta n\, k_B\, TS}{\beta}$$



Force balance = pressure × area = total number of molecules × βv

$\Delta n\, k_BTS = \ell n_1S\beta v$ [βv = viscous force (given)]

$\Rightarrow n_1\beta v\ell = \Delta nk_BT$

So option (b) is correct.

Total number of molecules/sec, $\dfrac{\Delta N}{\Delta t} = \dfrac{(n_1vdt)S}{\Delta t}$

$$= n_1vS = \frac{\Delta nk_BTvS}{\beta v\ell} = \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_BT}{\beta}\right)S$$

Option (c) is correct.

As Δn will decrease with time so rate of molecules getting transferred through the tube decreases with time.

Hence option (d) is incorrect.

7. **(b, c, d)**

(a) As we know, $Q = mc\,\Delta T$

$$\Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dT} \text{ or, } \frac{dQ}{dt} \propto C \text{ i.e., rate of heat absorption} \propto \text{C.}$$

In the range 0 to 100K from the graph, C increases with temperature but not linearly therefore the rate at which heat is absorbed varies with temperature. But not linearly.

(b) As the value of C is greater in the temperature range 400-500K, the heat absorbed in increasing the temperature from 0 - 100K is less than the heat required for increasing the temperature from 400 - 500K.

(c) From the graph the value of C does not change in the temperature range 400-500K, therefore there is no change in the rate of heat absorption in this range.

(d) As the value of C increases from 200-300K, the rate of heat absorption increases in the range 200-300K.

8. **(a, b, d)**

(a)

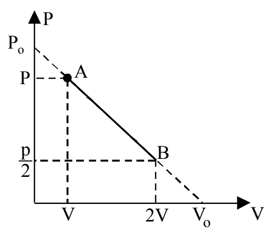


Fig (i)

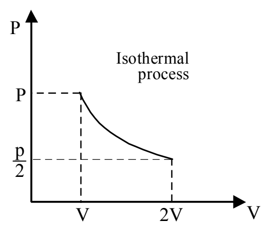


Fig (ii)

Workdone = Area under $P - V$ curve and V-axis

$\therefore W_1 > W_2$

(b) To study V-T diagram.

In the given process, AB is a straight line. It has a negative slope and a positive intercept.

The equation of line is $P = -\alpha V + \beta$ ...(i)

Where α and β are positive constants.

$\because PV = nRT$ ...(ii)

$\therefore P = \frac{nRT}{V} \quad \therefore \frac{nRT}{V} = -\alpha V + \beta$

or $T = \frac{-\alpha V^2}{nR} + \frac{\beta V}{nR}$ ...(iii)

This represents a parabola in terms of T and V.

$\therefore$ The path AB becomes a part of parabola.

(c) For P-T diagram, eliminate V in (i) and (ii)

$\therefore P = -\alpha \frac{nRT}{P} + \beta$ or $P^2 - P\beta = -\alpha nRT$

or $T = \frac{-P^2}{\alpha nR} + \frac{P\beta}{\alpha nR}$ ...(iv)

This represents a parabola in terms of T and P.

$\therefore$ The path AB becomes a part of parabola.

(d) Variation of T along AB

From $eq(iii), T = \frac{-\alpha V^2}{nR} + \frac{\beta V}{nR}$

$\therefore \frac{dT}{dV} = \frac{-2\alpha V}{nR} + \frac{\beta}{nR}$ ...(v)

When $\frac{dT}{dV} = 0, V = \frac{\beta}{2\alpha}$ ...(vi)

$\frac{d^2T}{dV^2} = \frac{-2\alpha}{nR} + 0 = \frac{-2\alpha}{nR}$

$\frac{d^2T}{dV^2}$ is negative. It means T has some maximum value.

$V = \frac{\beta}{2\alpha}$ is the value of maxima of temperature.

Also $P_AV_A = P_BV_B, RT_A = RT_B$ or $T_A = T_B$

In going from A to B, the temperature of the gas first increases of maximum at $V = \beta/2\alpha$

Then the temperature decreases and restored to original value.

9. **(d)** Let T be the final temperature of the gases when equilibrium is achieved.

Heat lost by monatomic gas at constant volume

= Heat gained by diatomic gas at constant pressure

$\therefore nC_{v_1}(700 - T) = nC_{p_2}(T - 400)$

$\frac{3}{2}R(700 - T) = \frac{7}{2}R(T - 400)$

$\Rightarrow 2100 - 3T = 7T - 2800 \Rightarrow 10T = 4900 \therefore T = 490$ K

10. **(d)** As the pressure of gases in both compartments is the same

$\therefore nC_{p_1}(700 - T) = nC_{p_2}(T - 400)$

$\frac{5}{2}R(700 - T) = \frac{7}{2}R(T - 400)$

$3500 - 5T = 7T - 2800 \Rightarrow 12T = 6300 \quad \therefore T = 525$ K

Applying first law of thermodynamics

$$\Delta W_1 + \Delta U_1 = \Delta Q_1$$

and $\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

or, $-(\Delta W_1 + \Delta W_2) = \Delta U_1 + \Delta U_2$

$= nC_{v_1}(525 - 700) + n_2C_{v_2}(525 - 400)$

$= -2 \times \frac{3R}{2} \times 175 + 2 \times \frac{5R}{2} \times 125 = -525R + 625R = -100R$

Therefore, total work done = –100 R

11. **(d)** The forces acting besides buoyancy force which is the force due to pressure of the liquid are

(i) Force of gravity (vertically downwards)

(ii) Viscous force (vertically downwards)

12. **(b)** As the bubble does not exchange any heat with the liquid. So, the process is adiabatic.

Applying relation, $T^\gamma P^{1-\gamma}$ = constant

$T_2 = T_1 \left[\frac{P_1}{P_2}\right]^{\frac{1-\gamma}{\gamma}}$

Here $T_1 = T_0, P_1 = P_0 + H\rho_\ell g$, $T_2 = T, P_2 = P$

$P = P_0 + (H - y)\rho_\ell g$, $\gamma = \frac{5}{3}$

$\therefore T = T_0 \left[\frac{P_0 + H\rho_\ell g}{P_0 + (H - y)\rho_\ell g}\right]^{\left(1-\frac{5}{3}\right)/5/3}$

$= T_0 \left[\frac{P_0 + H\rho_\ell g}{P_0 + (H - y)\rho_\ell g}\right]^{\frac{-2}{3} \times \frac{3}{5}}$

$T = T_0 \left[\frac{P_0 + (H - y)\rho_\ell g}{P_0 + H\rho_\ell g}\right]^{\frac{2}{5}}$

13. **(b)** $\because PV = nRT$

$\Rightarrow V = \frac{nRT}{P} = \frac{nRT}{P_0 + (H - y)\rho_\ell g}$

Where P is pressure of the bubble at an arbitrary location distant y from the bottom.

Substituting the value of P and T from above we get

$V = \frac{nR}{[P_0 + (H - y)\rho_\ell g]} \times \frac{T_0[P_0 + (H - y)\rho_\ell g]^{\frac{2}{5}}}{[P_0 + H\rho_\ell g]^{\frac{2}{5}}}$

$= \frac{nRT_0}{[P_0 + (H - y)\rho_\ell g]^{\frac{3}{5}} [P_0 + H\rho_\ell g]^{\frac{2}{5}}}$

$\therefore$ Buoyancy force

$= V\rho_\ell g = \frac{nRT_0\rho_\ell g}{[P_0 + (H - y)\rho_\ell g]^{\frac{3}{5}} [P_0 + H\rho_\ell g]^{\frac{2}{5}}}$

14. At constant pressure, $V \propto T$ or, $\frac{T_1}{V_1} = \frac{T_2}{V_2}$

$\therefore \frac{T_1}{Ah_1} = \frac{T_2}{Ah_2}$ ($\therefore V = A \times h$)

$\Rightarrow h_2 = \frac{T_2h_1}{T_1} = \frac{400}{300} \times 1 = \frac{4}{3}$ m

Gas is compressed without heat exchange, So process is adiabatic

From $T_1V_1r^{-1} = T_2V_2^{\gamma 1}$

$\therefore T'_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma - 1} = 400\left(\frac{4}{3}\right)^{2/5} = 448.8$ K

15. When container is stopped, velocity decreases by v_0.

Hence kinetic energy decreases by $\frac{1}{2}MV_0^2$

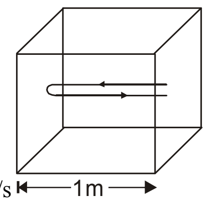
Decrease in kinetic energy = Increase in internal energy of the gas

$$\frac{1}{2}mv_0^2 = nC_V\Delta T$$

$$\frac{1}{2}mv_0^2 = \frac{m}{M}\left(\frac{3}{2}R\right)\Delta T \therefore \Delta T = \frac{mv_0^2}{3R}$$

16. Let T be temperature of the gas

The distance travelled by an atom of helium in $\frac{1}{500}$ s i.e., time between two successive collisions $= 2l = 2m$ ($\because l = 1$ m)



$$V_{rms} = \frac{\text{distance}}{\text{time}} = \frac{2}{1/500} = 1000 \text{ m/s}$$

(a) $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow 1000 = \sqrt{\frac{3 \times 25/3 \times T}{4 \times 10^{-3}}} \therefore T = 160\text{ K}$

(b) Average kinetic energy of an atom of a monoatomic helium gas $K.E = \frac{3}{2}kT$

$$\therefore \quad K.E = \frac{3}{2}kT = \frac{3}{2} \times \left(1.38 \times 10^{-23}\right) \times 160 = 3.312 \times 10^{-21}$$

(c) From ideal gas equation $PV = nRT = \frac{m}{M}RT$

$$\therefore m = \frac{PVM}{RT} = \frac{100 \times 1 \times 4}{25/3 \times 160} \Rightarrow m = 0.3012 \text{ gm}$$

Hence mass of helium gas in the box, m = 0.3012 g

17. (a) Let the number of moles of gas $B = n$

The number of moles of gas A = 1

$$\because U = \frac{nRT}{\gamma - 1} \quad m = \frac{19}{13}$$

$$\therefore U_m = U_A + U_B$$

$$\therefore \frac{(n_A + n_B)RT}{\gamma_m - 1} = \frac{n_A RT}{\gamma_A - 1} + \frac{n_B RT}{\gamma_B - 1}$$

or $\frac{1+n}{\left(\frac{19}{13} - 1\right)} = \frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{n}{\left(\frac{7}{5} - 1\right)}$

or $\frac{13(1+n)}{6} = \frac{3}{2} + \frac{5n}{2}$ or $13 + 13n = 9 + 15n$

or $4 = 2n$ or $n = 2$

(b) Speed of sound $v = \sqrt{\frac{\gamma RT}{M}}$

For mixture, $M = \frac{n_A M_A + n_B M_B}{n_A + n_B}$

$$M = \frac{(1 \times 4) + (2 \times 32)}{1 + 2} = \frac{68}{3} g \text{ mol}^{-1}$$

$$\therefore v = \sqrt{\frac{19}{13} \times \frac{8.31 \times 300 \times 3}{68 \times 10^{-3}}} \text{ or } v = 401 \text{m/s}$$

(c) Velocity of sound $v \propto \sqrt{T}$ $\therefore v$ = (constant k) $T^{1/2}$

or $\frac{dv}{dT} = \frac{1}{2}k\,T^{-1/2}$

or $dv = \frac{kdT}{2\sqrt{T}}$ where $dT = 1K, T = 300\,K$.

or $\frac{dv}{v} = \frac{kdT}{2\sqrt{T}} \times \frac{1}{k\sqrt{T}} = \frac{1}{2}\left(\frac{dT}{T}\right)$

$$\therefore \frac{dv}{v} \times 100 = \frac{1}{2}\frac{dT}{T} \times 100$$

or % change in $v = \frac{1}{2} \times \left(\frac{1}{300}\right) \times 100 = \frac{1}{6} = 0.167\%$

(d) For adiabatic change. PV = constant

$$\therefore V(dP) + P\gamma V^{\gamma - 1}(dV) = 0$$

or $\frac{dP}{dV} = -\frac{\gamma P}{V}$ or $-\frac{dPV}{dV} = \gamma P$

or Bulk modulus = E = γP

$$\because \text{Compressibility}(C) = \frac{1}{\text{Bulk modulus }(E)}$$

$$\therefore C = \frac{1}{\gamma P} \therefore \Delta C = C_2 - C_1$$

or $\Delta C = \frac{1}{\gamma P_2} - \frac{1}{\gamma P_1} = \frac{1}{\gamma}\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$ ...(i)

Again, $P_1 = \frac{(n_A + n_B)RT}{V}$

or $P_1 = \frac{(1+2) \times 8.31 \times 300}{V}$ or $P_1 = \frac{7479}{V}$ ...(ii)

Again, $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{5V_1}{V_1}\right)^\gamma = 5^\gamma$

or $P_2 = P_1 \times 5^\gamma \left(\text{where } \gamma = \frac{19}{13} \text{given}\right)$ ...(iii)

Putting value of P_1, P_2 in eq (i)

$$\therefore \Delta C = \frac{1}{\gamma}\left(\frac{1}{P_1 \times 5^\gamma} - \frac{1}{P_1}\right) = \frac{1}{\gamma P_1}\left[\frac{1}{5^\gamma} - 1\right]$$

or $\Delta C = \frac{13V}{19 \times 7479}\left[\frac{1}{5^{19/13}} - 1\right]$

$$= \frac{13V}{19 \times 7479}[0.1 - 1] = -\frac{(13 \times 0.90)V}{19 \times 7479} = -\frac{(13 \times 1)V}{19 \times 8310}$$

$= -0.0000827V = -8.27 \times 10^{-5}\,V$

18. The total pressure exerted by the mixture $P = 10^5$ Nm^{-2}

Temperature $T = 300$ K ; Volume = 0.02 m^3

Let there be x gram of Ne. $\therefore$ mass of Ar = $(28 - x)g$

Number of gram moles of Neon, $n_1 = \frac{x}{20}$;

Number of gram moles of Argon, $n_2 = \frac{28 - x}{40}$

But according to Dalton's law of partial pressure

$$P = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V}$$

or, $10^5 = \frac{xRT}{20V} + \frac{(28 - x)RT}{40V}$

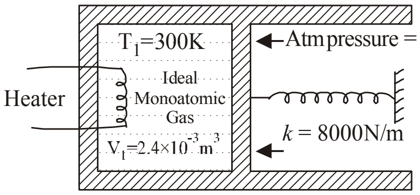
$$\Rightarrow \quad \frac{10^5 \times 40 \times 0.02}{8.314 \times 300} = x + 28 \quad \Rightarrow \quad x = 4.074\ g$$

$\therefore$ Mass of Neon = 4.074 g and
mass of Argon = 28 – 4.074 = 23.926 g

19. Final pressure

$$P_f = P_{atm} + \frac{k\,x}{A}$$

$$\Rightarrow \quad P_f = 10^5 + \frac{8000 \times 0.1}{8 \times 10^{-3}} = 2 \times 10^5\ \text{N/m}^2$$



Final volume,

$V_f = V_1 + xA$
$= 2.4 \times 10^{-3} + 0.1 \times 8 \times 10^{-3} = 3.2 \times 10^{-3}\ \text{m}^3$

Applying $\dfrac{P_i V_i}{T_i} = \dfrac{P_f V_f}{T_f} \Rightarrow T_f = \dfrac{P_f V_f T_i}{P_i V_i}$

or, $T_f = \dfrac{2 \times 10^5 \times 3.2 \times 10^{-3} \times 300}{10^5 \times 2.4 \times 10^{-3}} = 800\,\text{K}.$

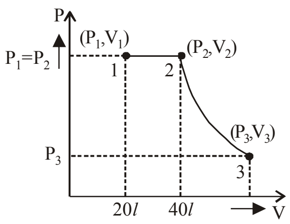
Here heat supplied by the heater is used for expansion of the gas, increasing its temperature and storing potential energy in the spring.

$$Q = P\Delta V + nC_v\Delta T + \frac{1}{2}k\,x^2$$

$$= 10^5\,[0.8 \times 10^{-3}] + \frac{P_1 V_1}{RT_1} C_v \Delta T + \frac{1}{2}kx^2$$

$$= 80 + \frac{10^5 \times 2.4 \times 10^{-3}}{2 \times 300} \times \frac{3}{2} \times 2 \times 500 + \frac{1}{2} \times 8000 \times 0.1 = 720\ \text{J}$$

20. (i) The process on $P - V$ diagram is as shown below.



(ii) To find the final volume and pressure of the gas.

Applying $P_1 V_1 = nRT_1$

$\therefore$ $P_1 \times 20 \times 10^{-3} = 2 \times 8.3 \times 300$
$P_1 = 2.49 \times 10^5\ \text{Nm}^{-2}$

For process 1 $\rightarrow$ 2 (isobaric)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ or, } \frac{20}{300} = \frac{40}{T_2} \quad \Rightarrow \quad T_2 = 600\ \text{K}$$

For process, 2 $\rightarrow$ 3 (adiabatic)

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$$

$$\therefore \quad V_3 = V_2 \left[\frac{T_2}{T_3}\right]^{\frac{1}{\gamma-1}} = 40\left[\frac{600}{300}\right]^{\frac{1}{\frac{5}{3}-1}} = 113\ell$$

$\left[\because \gamma = \dfrac{5}{3} \text{ for mono atomic gas}\right]$

Now, $P_3 V_3 = nRT_3$

$$\therefore \quad P_3 = \frac{nRT_3}{V_3} = \frac{2 \times 8.3 \times 300}{113 \times 10^{-3}} = 0.44 \times 10^5\ \text{N/m}^2$$

$\because$ $T_3 = T_1$ given

(iii) Work done by the gas $W = W_{12} + W_{23}$

$$= P_1(V_2 - V_1) + \frac{nR}{\gamma - 1}(T_2 - T_3)$$

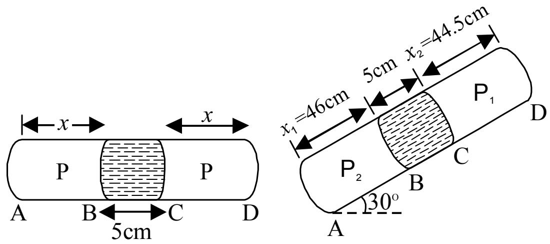
$$= 2.49 \times 10^5\,(40 - 20)\,10^{-3} + \frac{2 \times 8.3}{\frac{5}{3} - 1}(600 - 300)$$

$= 4980 + 7470 = 12450$ J

21. Let A be the area of cross-section of the tube.
Since temperature is the same, applying Boyle's law i.e., PV = constant in two sides of mercur column

$P \times (x \times A) = P_2 \times (x_2 \times A)$
and $P \times (x \times A) = P_1 \times (x_1 \times A)$
$\therefore$ $P_1 \times (x_1 \times A) = P_2 \times (x_2 \times A)$
or, $P_1 x_1 = P_2 x_2$ ...(i)

where $P_2 = P_1$ + Pressure due to mercury column



Pressure due to mercury column

$$P = \frac{F}{A} = \frac{mg\sin 30°}{A} = \frac{Vdg\sin 30°}{A}$$

$$= \frac{(A \times 5) \times dg\sin 30°}{A} = 5 \sin 30° \text{ cm of Hg}$$

$P_2 = P_1 + 5 \sin 30° = P_1 + 2.5$

Substituting this value in (iii)

$P_1 \times x_1 = [P_1 + 2.5] \times x_2$
or, $P_1 \times 46 = [P_1 + 2.5] \times 44.5$

$$\therefore \quad P_1 = \frac{44.5 \times 2.5}{1.5}$$

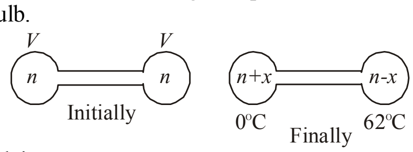
Putting this value of P_1 in $px = P_1 x_1$

$$P \times x = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\Rightarrow \quad P \times \left[\frac{46 + 44.5}{2}\right] = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\left[\because x = \frac{x_1 + x_2}{2}\right] \Rightarrow P = 75.4 \text{ cm}$$

22. Let x moles transfer from high temperature side to low temperature side bulb.



$\because$ Applying $PV = nRT$

For left bulb

76 × V = nR × 273	Initially
P' × V = (n + x) R × 273	Finally

Dividing, we get

$$\frac{P'}{76} = \frac{n + x}{n} \qquad \text{... (i)}$$

For right bulb
$76 \times V = nR \times 273$ Initially
$P' \times V = (n - x) R \times 335$ Finally
Again dividing, we get
$$\frac{P'}{76} = \frac{n-x}{x} \times \frac{335}{273} \quad \text{... (ii)}$$
From eq. (i) and (ii)
$$\frac{n+x}{n} = \frac{n-x}{n} \times \frac{335}{273}$$
$$\Rightarrow \quad n = \frac{608}{62} x. \quad \text{... (iii)}$$
Substituting this value of n in eq (i)
$$\frac{P'}{76} = 1 + \frac{62}{608} \qquad \therefore P' = \frac{670}{608} \times 76 = 83.75 \text{ cm Hg}$$

23. (i) For the left chamber
$$\frac{P_0 V_0}{T_0} = \frac{P_0 \times 243}{32 \times T_1} \times V_1$$
$$\Rightarrow \quad T_1 = \frac{243}{32} \times \frac{V_1 T_0}{V_0} \quad \text{... (i)}$$
For the right chamber, adiabatic compression occurs
$$\therefore P_0 V_0^{\gamma} = P_0 \times \frac{243}{32} \times V_2^{\gamma}$$
$$\Rightarrow \quad \frac{V_2}{V_0} = \left(\frac{32}{243}\right)^{3/5} \quad \Rightarrow \quad V_2 = \frac{8}{27} V_0$$
But $V_1 + V_2 = 2V_0$
$$\therefore \quad V_1 = 2V_0 - V_2 = 2V_0 - \frac{8}{27} V_0 = \frac{46}{27} V_0 \quad \text{... (ii)}$$
From eq (i) and (ii) $T_1 = \frac{243}{32} \times \frac{46 \times V_0}{V_0 \times 27} \times T_0$
$$\text{or,} \quad T_1 = \frac{207}{16} T_0 = 12.9 T_0$$
Again using P–T equation for right chamber i.e., $P_{1-\gamma} T_{\gamma}$ = constant
$$\left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$$
$$\Rightarrow \quad \left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243 P_0}{32 P_0}\right)^{1-5/3} \qquad \text{or,} \quad T_2 = 2.25\ T_0$$
(ii) Work done by the gas in right chamber (adiabatic process)
$$W = \frac{1}{1-\gamma}(P_2 V_2 - P_0 V_0)$$
$$= -\frac{3}{2}\left[\frac{243}{32} P_0 \times \frac{8}{27} V_0 - P_0 V_0\right]$$
$$= -\frac{3}{2}\left(\frac{9}{4} - 1\right) P_0 V_0 = -\frac{15}{8} \times RT_0 = -1.875\ RT_0$$

24. **(i)** Force $F = P \times A = 10^5 \times 1 = 10^5$ N
$$F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F \times \Delta t = 10^5 \times 1 = 10^5 \quad \text{.. (i)}$$
Now, momentum change per second
$(\Delta p) = n \times 2mv$ ...(ii)
Where n = number of collisions per second per square metre area
From (i) and (ii)
$$n \times 2mv = 10^5 \qquad \therefore \ n = \frac{10^5}{2mv}$$
Root mean square velocity
$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32/1000}} = 483.4 \text{ m/s}$$
$$\therefore \quad n = \frac{10^5 \times 6.023 \times 10^{23}}{2 \times 32 \times 483.4} = 1.97 \times 10^{27}$$
(ii) Kinetic energy of motion of oxygen molecules will be converted into heat energy.
K.E. of 1 gm mole of oxygen $\frac{1}{2} M v_0^2$
$= nC_v \Delta T = 1 \times C_v \times 1 = C_v$
$$\therefore \ \frac{1}{2} M v_0^2 = C_v \quad \text{But,} \ C_v = \frac{R}{\gamma - 1}$$
$$\therefore \ \frac{1}{2} M v_0^2 = \frac{R}{\gamma - 1}$$
$$\text{or, } v_0 = \sqrt{\frac{2R}{M(\gamma-1)}} = \sqrt{\frac{2 \times 8.314}{\frac{32}{100} \times (1.41 - 1)}} = 35.6 \text{ ms}^{-1}$$

25. $P_1 = 830 - 30 = 800$ mm Hg ; P_2 ?
$V_1 = V$; $V_2 = V$; $T_1 = T$; $T_2 = T - 0.01\ T = 0.99\ T$
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \ \therefore \ P_2 = \frac{P_1 T_2}{T_1} = \frac{800 \times 0.099 T}{T} = 792 \text{ mmHg}$$
$\therefore$ Total pressure in the jar = 792 + 25 = 817 mm Hg

26. From, PV = nRT
When P, T are same $n \propto V$
As volumes are same, i.e., 1c.c of each hydrogen and oxygen, So both samples will have equal number of molecules

Oscillations

Topic-1 : Displacement, Phase, Velocity and Acceleration in S.H.M.

1. **(a)** Here, $v_x = -a\,\omega \sin \omega t$, $v_y = a\,\omega \cos \omega t$ and $v_z = a\omega$

$\Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$\Rightarrow v = \sqrt{(-a\omega \sin \omega t)^2 + (a\omega \cos \omega t)^2 + (a\omega)^2}$

$v = \sqrt{2}a\omega$

2. **(c)** From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A \sin (at + \delta)$

$y = B \sin (bt + r)$

Clearly $A \neq B$ hence ellipse.

3. **(b)** Maximum velocity in SHM, $v_{max} = a\omega$

Maximum acceleration in SHM, $A_{max} = a\omega^2$

where a and ω are maximum amplitude and angular frequency.

Given that, $\frac{A_{max}}{v_{max}} = 10$

i.e., $\omega = 10\ s^{-1}$

Displacement is given by

$x = a \sin (\omega t + \pi/4)$

at $t = 0$, $x = 5$

$5 = a \sin \pi/4$

$5 = a \sin 45° \Rightarrow a = 5\sqrt{2}$

Maximum acceleration $A_{max} = a\omega^2 = 500\sqrt{2}$ m/s²

4. **(b)** We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

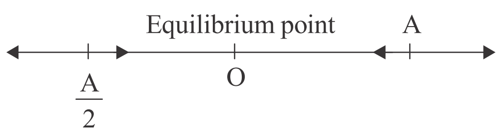
Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

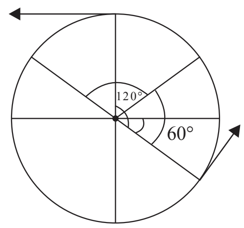
$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9} \quad \therefore A' = \frac{7A}{3}$$

5. **(d)**



(at time t = 0)



Angle covered to meet $\theta = 60° = \frac{p}{3}$ rad.

If they cross each other at time t then

$$t = \frac{q}{2p} = \frac{p}{3' \ 2p}T = \frac{T}{6}$$

6. **(c)** At $t = 0$, $x(t) = 0$; $y(t) = 0$

$x(t)$ is a sinusoidal function

At $t = \frac{\pi}{2\omega}$; $x(t) = a$ and $y(t) = 0$

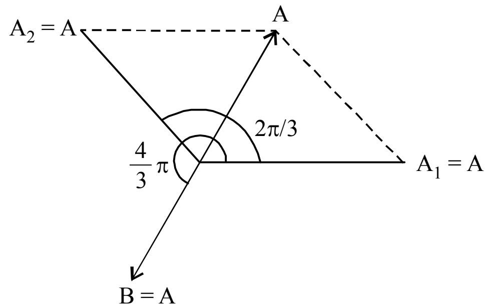
Hence trajectory of particle will look like as (c).

7. **(d)** In linear S.H.M., the restoring force acting on particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

i.e., $F \propto x$

or $F = -bx$ where b is a positive constant.

8. **(b)** Two sinusoidal displacements $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2}{3}\pi\right)$ have amplitude A each, with a phase difference of $2\frac{\pi}{3}$. It is given that sinusoidal displacement $x_3(t) = B(\sin \omega t + \phi)$ brings the mass to a complete rest. This is possible when the amplitude of third $B = A$ and is having a phase difference of $\phi = 4\frac{\pi}{3}$ with respect to $x_1(t)$ as shown in the figure.



9. **(d)** The equation for the S.H.M. $x = a \sin \omega t$

$$\Rightarrow \quad x = a\sin\left(\frac{2\pi}{T}\right) \times t = 1\sin\left(\frac{2\pi}{8}\right)t = \sin\frac{\pi}{4}t$$

($\because$ $a = 1$ cm, $T = 8s$ from graph)

or, velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left[\sin\left(\frac{\pi}{4}\right)t\right] = \frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)t$

$\therefore$ Acceleration $a = \frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{\pi}{4}\right)t$

At $t = \frac{4}{3}s$ acceleration

$$a = \frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\frac{\pi}{4}\times\frac{4}{3} = \frac{-\pi^2}{16}\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{3}\pi^2}{32}\text{cm}/s^2$$

10. In SHM $Y = A\sin(\omega t + \varphi)$

$y_1 = 10\sin(3\pi t + \pi/4)$ ... (i)

$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$ ... (ii)

$\therefore\ y_2 = 5\times 2\left[\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right] = 10\sin(3\pi t + \pi/3)$

The ratio of amplitudes $= \frac{A_1}{A_2} = 10 : 10 = 1 : 1$

11. **(a, b, c)** The given equation

$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

Rearranging the equation for SHM the sine and cosine functions should have linear power.

$$\therefore\ x = \frac{A}{2}(2\sin^2\omega t) + \frac{B}{2}(2\cos^2\omega t) + \frac{C}{2}(2\sin\omega t\cos\omega t)$$

$$= \frac{A}{2}[1-\cos 2\omega t] + \frac{B}{2}[1+\cos 2\omega t] + \frac{C}{2}[\sin 2\omega t]$$

(a) For $A = 0$ and $B = 0$, $x = \frac{C}{2}\sin(2\omega t)$

The above equation represents SHM.

(b) If $A = B$ and $C = 2B$ then $x = B + B\sin 2\omega t$

This is an equation of SHM.

(c) $A = -B$, $C = 2B$;

$\therefore$ $x = B\cos 2\omega t + B\sin 2\omega t$

Two SHMs are superposed to give another SHM equation.

(d) $A = B$, $C = 0$ $\therefore$ $x = A$.

This equation does not represents SHM.

12. **(a, c)** Applying superposition principle

$y = y_1 + y_2 + y_3$

$= a\sin\omega t + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$

$= a[\sin\omega t + \sin(\omega t + 90°)] + a\sin(\omega t + 45°)$

$= 2a\sin(\omega t + 45°)\cos 45° + a\sin(\omega t + 45°)$

$= (\sqrt{2}+1)\,a\sin(\omega t + 45°)$

$= A\sin(\omega t + 45°)$

Clearly, resultant motion is SHM of amplitude $A = (\sqrt{2}+1)\,a$ and differ in phase by 45° not by 90° relative to the first.

Energy E in SHM $\propto$ (amplitude)^2 $\left[\because E = \frac{1}{2}mA^2\omega^2\right]$

$$\frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$\therefore$ $E_{\text{resultant}} = (3+2\sqrt{2})E_{\text{single}}$

Topic-2 : Energy in Simple Harmonic Motion

1. **(c)** From graph equation of SHM

$X = A\cos\omega t$

(1) At $\frac{3T}{4}$ particle is at mean position.

$\therefore$ Acceleration = 0, Force = 0

(2) At T particle again at extreme position so acceleration is maximum.

(3) At $t = \frac{T}{4}$, particle is at mean position so velocity is maximum.

Acceleration = 0

(4) When KE = PE

$\Rightarrow \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

Here, A = amplitude of SHM

x = displacement from mean position

$\Rightarrow A^2 = 2x^2 \Rightarrow x = \frac{+A}{\sqrt{2}}$

$\Rightarrow \frac{A}{\sqrt{2}} = A\cos\omega t \quad \Rightarrow t = \frac{T}{2}$

$\therefore$ $x = -A$ which is not possible

$\therefore$ 1, 2 and 3 are correct.

2. **(d)** Kinetic energy, $k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

Potential energy, $U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$$

3. **(c)** Potential energy (U) $= \frac{1}{2}kx^2$

Kinetic energy (K) $= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\Rightarrow x^2 = A^2$ or, $x = \pm\frac{A}{\sqrt{2}}$

4. **(b)** For a particle executing SHM

At mean position; $t = 0$, $\omega t = 0$, $y = 0$, $V = V_{max} = a\omega$

$\therefore$ $K.E. = KE_{max} = \frac{1}{2}m\omega^2a^2$

At extreme position : $t = \frac{T}{4}$, $\omega t = \frac{\pi}{2}$, $y = A$, $V = V_{min} = 0$

$\therefore$ $K.E. = KE_{min} = 0$

Kinetic energy in SHM, $KE = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2}m\omega^2a^2\cos^2\omega t$

Hence graph (b) correctly depicts kinetic energy time graph.

5. **(b)** Since system dissipates its energy gradually, and hence amplitude will also decreases with time according to

$a = a_0 e^{-bt/m}$ .... (i)

$\because$ Energy of vibration drop to half of its initial value (E_0), as

$E \propto a^2 \Rightarrow a \propto \sqrt{E}$

$$a = \frac{a_0}{\sqrt{2}} \Rightarrow \frac{bt}{m} = \frac{10^{-2}t}{0.1} = \frac{t}{10}$$

From eq^n (i),

$$\frac{a_0}{\sqrt{2}} = a_0e^{-t/10}$$

$\frac{1}{\sqrt{2}} = e^{-t/10}$ or $\sqrt{2} = e^{\frac{t}{10}}$

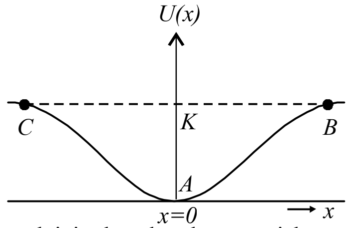
$\ln\sqrt{2} = \frac{t}{10}$ $\quad\therefore\quad$ t = 3.5 seconds

6. **(d)** K.E = $\frac{1}{2}k(A^2 - d^2)$ and P.E. = $\frac{1}{2}kd^2$

At mean position $d = 0$. At extreme positions d = A

7. **(d)** Given: $U(x) = K\left[1 - e^{-x^2}\right]$, $\therefore F = -\frac{dU}{dx} = 2kxe^{-x^2}$

The corresponding $u - x$ graph of the given $u(x) = K\,[1 - e^{-x^2}]$ which is an exponentially increasing graph of u with x^2 is as shown in the figure.



From the graph it is clear that the potential energy is minimum at $x = 0$. Hence, $x = 0$ is the state of stable equilibrium. Now if we displace the particle from $x = 0$ then for displacements the particle tends to regain the position $x = 0$ with a force $F = \frac{2kx}{e^{x^2}}$. Hence, for small values of x we have $F \propto x$.

Hence for small displacement from $x = 0$ the motion is simple harmonic.

8. **(c)** During one complete oscillation, the kinetic energy of particle executing SHM will become maximum twice.

Therefore the frequency with which its kinetic energy oscillates will be $2f$.

9. Given: $m = 0.2$ kg, $\omega = \frac{25}{\pi}$, K.E. = 0.5 J, P.E. = 0.4 J

and a = ?

T.E. = K.E. + P.E. = (0.5 + 0.4) J = 0.9 J

Also, T.E. = $\frac{1}{2}m\omega^2a^2 = \frac{1}{2}m \times 4\pi^2\nu^2a^2$

$\Rightarrow\ 0.9 = \frac{1}{2} \times 0.2 \times 4\pi^2 \times \frac{25}{\pi} \times \frac{25}{\pi} \times a^2 \quad \therefore\ a = 0.06\text{m}$

10. **(b, c)** Energy of the oscillator

$= \frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100\,\text{J}$

Total mechanical energy E = 160 J

Extreme position	Mean position
$x = A$	$x = 0$
K.E. = 0	K.E. = 100J
E = 160 J	E = 160 J
P. E. = 160J	P. E. = 60J

Topic-3 : Time Period, Frequency, Simple Pendulum and Spring Pendulum

1. **(b)** An elastic wire can be treated as a spring and its spring constant.

$k = \frac{YA}{L}$ $\qquad \left[\because Y = \frac{F}{A}\Big/\frac{\Delta l}{l_0}\right]$

Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

2. **(c)** $y = y_0 \sin^2\omega t$

$\Rightarrow y = \frac{y_0}{2}(1 - \cos 2\omega t)$ $\quad \left(\because \sin^2\omega t = \frac{1 - \cos 2\omega t}{2}\right)$

$\Rightarrow y - \frac{y_0}{2} = \frac{-y_0}{2}\cos 2\omega t$

$\Rightarrow y = A\cos 2\omega t$

$\therefore$ Amplitude $= \frac{y_0}{2}$

Angular velocity = 2ω

For equilibrium of mass, $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$

3. **(a)** Potential energy of spring $= \frac{1}{2}kx^2$

Here, x = distance of block from mean position,

k = spring constant

At mean position, potential energy $= \frac{1}{2}kA^2$

At equilibrium position, half of the mass of block breaks off, so its potential energy becomes half.

Remaining energy $= \frac{1}{2}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}kA'^2$

Here, A' = New distance of block from mean position

$\Rightarrow A' = \frac{A}{\sqrt{2}}$

4. **(c)** $T = 2\pi\sqrt{\frac{l}{g}}$

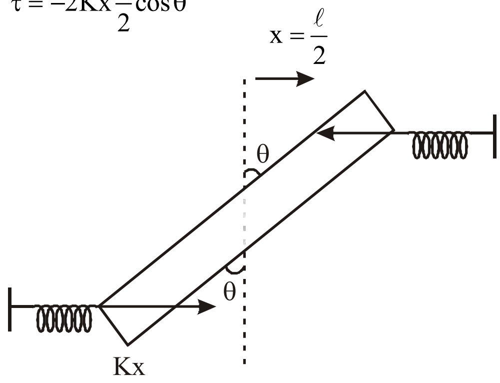
When immersed non viscous liquid

$a_{mt} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$

Now $T' = 2\pi\sqrt{\frac{l}{0_{\text{net}}}} = 2\pi\sqrt{\frac{l}{\sqrt{\frac{15g}{16}}}} = \frac{4}{\sqrt{15}}T$

5. **(c)** Net torque due to spring force:

$\tau = -2\text{Kx}\frac{\ell}{2}\cos\theta$



$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta \quad \left[\text{let } C = \frac{K\ell^2}{2}\right]$

$\Rightarrow$ So, frequency of resulting oscillations

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

6. **(a)** Given : $y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$

$\Rightarrow y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$\therefore$ Amplitude = 10 cm

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3}$s

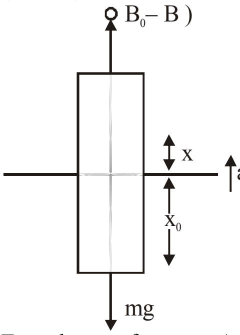
7. **(d)** Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$

$\Rightarrow T_p = 2\sqrt{3}$ s

8. **(Bonus)**



at equilibriu

$B_0 = mg$

Extra boyant force = ρAxg

$B_0 + B = mg + ma$

$\therefore B = ma = \rho Axg = (\pi r^2 \rho g)x$

$$a = \frac{(\pi r^2 \rho g)x}{m}$$

using, $a = \omega^2 x$

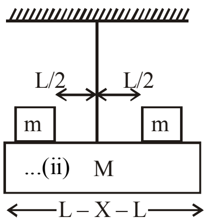
$$\Rightarrow \omega = \sqrt{\frac{\pi r^2 \rho g}{m}}$$

$W \simeq 7.95$ rads^{-1}

9. **(c)** $f_1 = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$ ...(i)

$$= \frac{1}{2}\sqrt{\frac{3C}{ML^2}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{C}{L^2\left(\frac{M}{3} + \frac{M}{2}\right)}} \quad \text{...(ii)}$$



As frequency reduces by 80%

$\therefore f_2 = 0.8 f_1 \Rightarrow \frac{f_2}{f_1} = 0.8$ ...(iii)

Solving equations (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

10. **(b)** As we know, frequency in SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

where m = mass of one atom

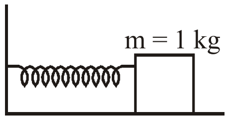
Mass of one atom of silver, $= \frac{108}{(6.02 \times 10^{23})} \times 10^{-3}$ kg

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

Solving we get, spring constant,
K = 7.1N/m

11. **(c)** Frequency of spring (f) $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1$ Hz

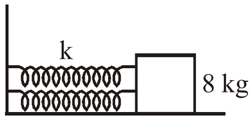
$\Rightarrow 4\pi^2 = \frac{k}{m}$



If block of mass m = 1 kg is attached then,
$k = 4\pi^2$

Now, identical springs are attached in parallel with mass m = 8 kg. Hence,
$k_{eq} = 2k$

$$F = \frac{1}{2\pi}\sqrt{\frac{k \times 2}{g}} = \frac{1}{2}\text{Hz}$$



12. **(c)** Time lost/gained per day $= \frac{1}{2} \propto \Delta\theta \times 86400$ second

$12 = \frac{1}{2}\alpha(40 - \theta) \times 86400$ .... (i)

$4 = \frac{1}{2}\alpha(\theta - 20) \times 86400$ ....(ii)

On dividing we get, $3 = \frac{40 - \theta}{\theta - 20}$

$3\theta - 60 = 40 - \theta$

$4\theta = 100 \Rightarrow \theta = 25°C$

13. **(c)** As we know, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell + \Delta\ell}{\ell}}$$

$$\Rightarrow \left(\frac{T_M}{T}\right)^2 = \frac{\ell + \Delta\ell}{\ell}$$

or, $\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY} \quad \left[\because \Delta\ell = \frac{Mg\ell}{AY}\right]$

$$\therefore \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

14. **(d)** In simple harmonic motion, starting from rest,
At $t = 0$, $x = A$
$X = A\cos\omega t$ ..... (i)
When $t = \tau$, $x = A - a$
When $t = 2\tau$, $x = A - 3a$
From equation (i)
$A - a = A\cos\omega\tau$ ....(ii)
$A - 3a = A\cos 2\omega\tau$ ....(iii)
As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$...(iv)
From equation (ii), (iii) and (iv)

$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$\Rightarrow 2a^2 = aA \Rightarrow A = 2a \Rightarrow \frac{a}{A} = \frac{1}{2}$$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{A-a}{A} \Rightarrow \cos\omega\tau = \frac{1}{2} \quad \text{or} \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$\Rightarrow T = 6\tau$$

15. **(b)** From graph it is clear that when
$L = 1\text{m}$, $T^2 = 4\text{s}^2$
As we know,

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$$

$$= 4\times\left(\frac{22}{7}\right)^2 \times \frac{1}{4} = \left(\frac{22}{7}\right)^2$$

$$\therefore \quad g = \frac{484}{49} = 9.87\text{m/s}^2$$

16. **(c)** Mass of bigger body M = 4 kg
Mass of smaller body m = 1 kg
Smaller mass (m = 1 kg) executes S.H.M of angular frequency ω = 25 rad/s
Amplitude x = 1.6 cm = 1.6×10^{-2}
As we know,

$$T = 2\pi\sqrt{\frac{m}{K}} \text{ or, } \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$\text{or, } \frac{1}{25} = \sqrt{\frac{1}{K}} \left[\because m = 1\text{kg}; \omega = 25 \text{ rad/s}\right]$$

or, $K = 625 \text{ Nm}^{-1}$.
The maximum force exerted by the system on the floor
= Mg + Kx + mg
= $4 \times 10 + 625 \times 1.6 \times 10^{-2} + 1 \times 10 = 40 + 10 + 10 = 60$ N

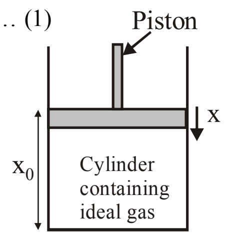
17. **(c)** $\frac{Mg}{A} = P_0$

$$P_0 V_0^{\gamma} = PV^{\gamma}$$

$Mg = P_0 A$ … (1)
Let piston is displaced by distance x

$$P_0 A x_0^{\gamma} = PA(x_0 - x)^{\gamma}$$

$$P = \frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}$$



$$Mg - \left(\frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) A = F_{\text{restoring}}$$

$$P_0 A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) = F_{\text{restoring}} \quad [x_0 - x \approx x_0]$$

$$F = -\frac{\gamma P_0 A x}{x_0}$$

$\therefore$ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

18. **(a)** Let T_1 and T_2 be the time period of the two pendulums

$$T_1 = 2\pi\sqrt{\frac{1}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{4}{g}}$$

As $\ell_1 < \ell_2$ therefore $T_1 < T_2$
Let at t = 0 they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillations differ by an integer, the two pendulums will again begin to swing together
Let longer length pendulum complete n oscillation and shorter length pendulum complete (n + 1) oscillation. For unison swinging

$$(n+1)T_1 = nT_2$$

$$(n+1)\times 2\pi\sqrt{\frac{l}{g}} = (n)\times 2\pi\sqrt{\frac{4}{g}}$$

$\Rightarrow n = 1$
$\therefore n + 1 = 1 + 1 = 2$

19. **(a)** Time of flight of projectile,

$$T = \frac{2V\sin\theta}{g}$$

$$\therefore \quad 1 = \frac{2V\sin 45^\circ}{g} \quad \therefore \ V = \sqrt{50} \text{ ms}^{-1}$$

Hence pebble is projected with a speed $V = \sqrt{50}$ ms^{-1}

20. **(d)** If the spring of spring constant k_1 is compressed by x_1 and that of spring constant k_2 is compressed by x_2 then

$$x_1 + x_2 = A \quad \text{… (i)}$$

$$\text{and} \quad k_1 x_1 = k_2 x_2 \Rightarrow x_2 = \frac{k_1 x_1}{k_2} \quad \text{…(ii)}$$

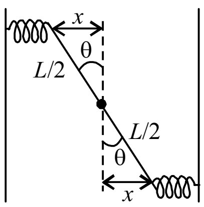
Solving eqs. (i) & (ii) we get

$$x_1 = \frac{k_2 A}{k_2 + k_1}$$

21. **(c)** From figure, compression of the spring, $x = \frac{L}{2}\theta$.

$$\text{Torque} = \left\{(k)\cdot\frac{L}{2}\theta\right\}L$$

$$\text{Also torque} = I\alpha = -\frac{kL^2}{2}$$



$$\frac{ML^2}{12}\cdot\alpha = -\left(\frac{kL^2}{2}\right)\theta$$

$$\left(\because I = \frac{1}{2}ML^2\right)$$

$\therefore \quad \alpha = -\left(\frac{6k}{M}\right)\theta = -\omega^2\theta$

$\therefore \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

22. **(b)** Given : $y = kt^2$

$\therefore \quad \frac{dy}{dt} = v = 2kt \quad$ and, $\quad \frac{d^2y}{dt^2} = a = 2k$

or $\quad a_y = 2\,\text{m/s}^2 \quad (\because k = 1\ \text{m/s}^2$ given)

We know that $T = 2\pi\sqrt{\frac{\ell}{g}}$ or, $T_1 = 2\pi\sqrt{\frac{l}{g}}$

$T_2 = 2\pi\sqrt{\frac{l}{g+a_y}}$

$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$

23. **(a)** Here acceleration is same for both P and Q as block Q oscillates but does not slip. The $(P-Q)$ system oscillates with angular frequency ω. The spring is stretched by A.
Angular frequency of the system,

$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$

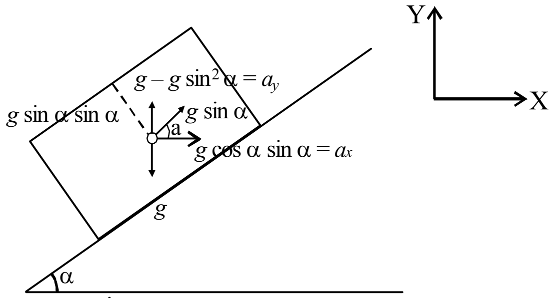
Maximum acceleration of the system in SHM

$a_{max} = A\omega^2 = A\sqrt{\left(\frac{k}{2m}\right)^2} = \frac{KA}{2m}$

This acceleration to the lower block is provided by friction
$\therefore$ Maximum force of friction

$f_{max} = ma_{max} = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$

24. **(a)** As shown in the figure, acceleration down the plane $a = g\sin\alpha$ is the pseudo acceleration applied by the observer in the accelerated frame



$a_x = g\sin\alpha\cos\alpha = a\cos\alpha$
$a_y = g - g\sin^2\alpha = g(1-\sin^2\alpha) = g\cos^2\alpha$
The effective acceleration due to gravity acting on the bob

$g_{eff} = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{g^2\sin^2\alpha\cos^2\alpha + g^2\cos^4\alpha}$

$= g\cos\alpha\sqrt{\sin^2\alpha + \cos^2\alpha} = g\cos\alpha$

$\therefore \quad T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$

25. **(a)** Given : $U(x) = k|x|^3$

$\therefore \quad [k] = \frac{[U]}{[x^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

Now time period may depend on $T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$

$\therefore \quad [M^0L^0T] = [M]^x [L]^y [ML^{-1}T^{-2}]^z$
$= [M^{x+z} L^{y-z} T^{-2z}]$

Equating the powers, we get
$-2z = 1$ or $z = -1/2$
$y - z = 0$ or $y = z = -1/2$
Hence $T \propto (\text{amplitude})^{-1/2} \propto a^{-1/2}$

Therefore, $T \propto \frac{1}{\sqrt{a}}$

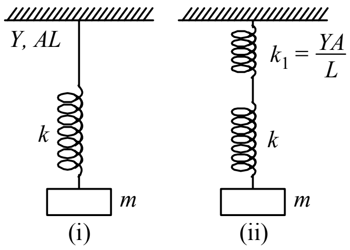
26. **(b)** Let us consider the wire also as a spring of spring constant k'. Then the case becomes that of two spring attached in series. The equivalent spring constant

$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k'} \Rightarrow k_{eq} = \frac{kk'}{k+k'}$

Force constant of spring $= k$

$\therefore \quad \frac{\text{force}}{\text{length}} = k \quad$ ...(i)

For wire,



$Y = \frac{F}{A}\frac{L}{l}$

or $\quad \frac{F}{l} = \frac{YA}{L}$

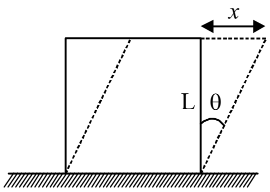
$\therefore \quad \frac{\text{force}}{\text{length}} = \frac{YA}{L}$

or $\quad k_1 = \frac{YA}{L} \quad$ ...(ii)

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(k+k_1)}{kk_1}}$

$= 2\pi\sqrt{\frac{m\left(k+\frac{YA}{L}\right)}{k\times\frac{YA}{L}}} = 2\pi\sqrt{\frac{m(YA+kL)}{YAk}}$

27. **(d)** Here, $A = L^2$ and $\theta = \frac{x}{L}$



Modulus of rigidity, $\eta = F/A\,\theta$
$\therefore$ Restoring force
$F = -\eta A\theta = -\eta Lx$
or, acceleration,

$a = \frac{F}{M} = -\frac{\eta L}{M}x$

Since, $a \propto -x$ So, oscillations are SHM. Therefore, time period of which is given by

$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|} = 2\pi\sqrt{\left|\frac{x}{a}\right|}$

or, $\quad T = 2\pi\sqrt{\frac{M}{\eta L}}$

28. **(b)** If x is the displacement then,
Net restoring force = extra upthrust + extra spring force

$\therefore \quad M\omega^2 x = [\rho A g + k]x$

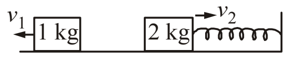
$\Rightarrow \omega = \left[\dfrac{\rho A g + k}{M}\right]^{1/2} \quad \Rightarrow v = \dfrac{1}{2\pi}\left[\dfrac{\rho A g + k}{M}\right]^{1/2}$

29. **(d)** Given maximum velocity, $v_1 = v_2$ or, $a_1\omega_1 = a_2\omega_2$
$\because \; V_{maximum} = a\omega$

$\Rightarrow a_1 \times \dfrac{2\pi}{T_1} = a_2 \times \dfrac{2\pi}{T_2}$

$\therefore \quad \dfrac{a_1}{a_2} = \dfrac{T_1}{T_2} = \dfrac{2\pi\sqrt{\dfrac{m}{k_1}}}{2\pi\sqrt{\dfrac{m}{k_2}}} = \sqrt{\dfrac{k_2}{k_1}}$

30. **(2.09)**
Let velocities of 1 kg and 2 kg blocks just after collision be v_1 and v_2 respectively.
Just after collision



From momentum conservation principle,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
$0 + 1 \times 2 = -1v_1 + 2v_2$ .... (i)

Collision is elastic. Hence $e = 1$

$e = 1 = \dfrac{2-0}{v_1 + v_2}$

$\Rightarrow \quad v_2 + v_1 = 2$ .... (ii)

From eqs. (i) and (ii),

$v_2 = \dfrac{4}{3}$ m/s, $v_1 = \dfrac{2}{3}$ m/s

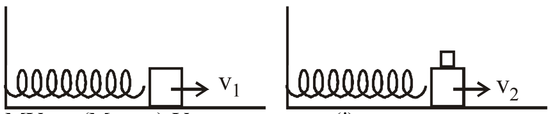
2 kg block will perform SHM after collision, so spring returns to its unstretched position for the first time after.

$t = \dfrac{T}{2} = \pi\sqrt{\dfrac{m}{k}} = \pi\sqrt{\dfrac{2}{2}} = \pi = 3.14$ s

Distance or required separation between the blocks

$= |v_1|t = \dfrac{2}{3} \times 3.14 = 2.093 = 2.09$ m

31. **(a, b, d)**
Case (i) : Applying principle of conservation of linear momentum.



$MV_1 = (M + m)\,V_2$ ... (i)
$M(A_1 \times \omega_1) = (M+m)(A_2 \times \omega_2)$

$\therefore \quad MA_1 \times \sqrt{\dfrac{K}{M}} = (M+m)\,A_2 \times \sqrt{\dfrac{K}{M+m}}$

$\therefore \quad A_2 = \sqrt{\dfrac{M}{M+m}}\,A_1 \quad \Rightarrow \quad \dfrac{A_2}{A_1} = \sqrt{\dfrac{M}{M+m}}$

Also $E_1 = \dfrac{1}{2}MV_1^2$

and $E_2 = \dfrac{1}{2}(M+m)V_2^2 = \dfrac{1}{2}(M+m)$

$\left(\because V_2 = \left(\dfrac{M}{M+m}\right)V_1 \text{ from eq (i)}\right)$

$\times \dfrac{M^2V_1^2}{(M+m)^2} = \dfrac{1}{2}\left(\dfrac{M}{M+m}\right)^2 V_1^2$

Clearly $E_1 > E_2$

The new time Period $T_2 = 2\sqrt{\dfrac{m+M}{K}}$

Instantaneous speed at X_0 of the combined masses

$V_2 = \dfrac{MV_1}{M+m} < V_1$

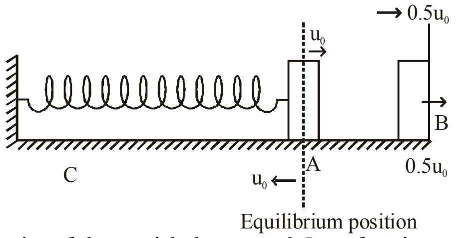
Case (ii) : The new time Period $T_2 = 2\sqrt{\dfrac{m+M}{K}}$

Also $A_2 = A_1$ and $E_2 = E_1$
In this case also, instantaneous speed at X_o of the combined masses decreases.

32. **(a, d)** The particle collides elastically with rigid wall.

$\therefore \quad e = \dfrac{V}{0.5u_0} = 1 \Rightarrow V = 0.5u_0$

i.e., the particle rebounds with the same speed. Therefore the particle will return to its equilibrium position with speed u_0.



The velocity of the particle becomes $0.5u_0$ after time t. Using, equation $V = V_{max}\cos\omega t$

$0.5u_0 = u_0\cos\omega t$

$\therefore \quad \dfrac{\pi}{3} = \dfrac{2\pi}{t} \times T \quad \Rightarrow \quad t = \dfrac{T}{6}$

The time period $T = 2\pi\sqrt{\dfrac{m}{k}} \quad \therefore t = \dfrac{\pi}{3}\sqrt{\dfrac{m}{k}}$

The time taken by the particle to pass through the equilibrium for the first time $= 2t = \dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$.

The time taken for the maximum compression
$= t_{AB} + t_{BA} + t_{AC}$

$= \dfrac{\pi}{3}\sqrt{\dfrac{m}{k}} + \dfrac{\pi}{3}\sqrt{\dfrac{m}{k}} + \dfrac{\pi}{3}\sqrt{\dfrac{m}{k}} = \pi\sqrt{\dfrac{m}{k}}\left[\dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{2}\right]$

$= \dfrac{7\pi}{6}\sqrt{\dfrac{m}{k}}$.

The time taken for particle to pass through the equilibrium position second time

$= 2\left[\dfrac{\pi}{3}\sqrt{\dfrac{m}{k}}\right] + \pi\sqrt{\dfrac{m}{k}} = \pi\sqrt{\dfrac{m}{k}}\left(\dfrac{2}{3} + 1\right) = \dfrac{5}{3}\pi\sqrt{\dfrac{m}{k}}$

33. Under SHM, at a distance y above the mean position velocity of block, $v = \omega\sqrt{a^2 - y^2}$

After detaching from spring, net downward acceleration of the block = g.

$\therefore \quad$ Height attained by the block = h

$$\therefore \quad h = y + \frac{v^2}{2g} \quad \text{or} \quad h = y + \frac{\omega^2\left(a^2 - y^2\right)}{2g}$$

For h to be maximum, $\frac{dh}{dy} = 0, y = y^*$.

$$\therefore \quad \frac{dh}{dy} = 1 + \frac{\omega^2}{2g}(-2y^*) \quad \text{or} \quad 0 = 1 - \frac{2\omega^2 y^*}{2g}$$

or $\quad \frac{\omega^2 y^*}{g} = 1$ or $y^* = \frac{g}{\omega^2}$

Since $a\omega^2 > g$ (given)

$\therefore\ a > \frac{g}{\omega^2}\ \therefore a > y^*.y^*$ from mean position $< a$.

Hence $y^* = \frac{g}{\omega^2}$.

34. Volume of the rod = LS

Weight of the rod = $LS\, d_1 g$

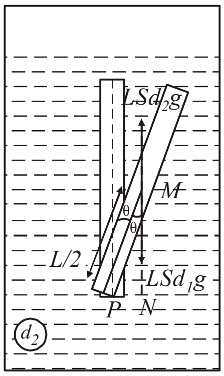
Upthrust acting on rod = $LS\, d_2 g$

Since, $d_2 > d_1$ (given).

$\therefore$ Net force acting at the centre of mass of the rod at tilted position $(LS\, d_2 g - LS d_1 g)$

Torque about this force about P

$\tau = F \times r_1 = (LSd_2 g - LSd_1 g) \times PN$



$\therefore \quad \tau = LSg\,(d_2 - d_1) \times \frac{L}{2}\sin\theta$

when θ is small, $\sin\theta \approx \theta$

$$\therefore \quad \tau = \frac{L^2 Sg}{2}(d_2 - d_1)\theta. \quad \text{...(i)}$$

Since, $\tau \propto \theta$, hence motion is simple harmonic.

On comparing it with $\tau = C\,\theta$, we get

$$C = \frac{L^2 Sg}{2}(d_2 - d_1)$$

$$\Rightarrow \quad I\omega^2 = \frac{L^2 Sg}{2}(d_2 - d_1) \quad \text{...(ii)}$$

The moment of inertia I of the rod about P,

$$I = \frac{1}{3}ML^2 = \frac{1}{3}LSd_1 L^2$$

Putting this value of I in eq. (ii)

$$\omega^2 \times \frac{L^3}{3}Sd_1 = \frac{L^2 Sg}{2}(d_2 - d_1)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Sg\,(d_2 - d_1)}{2LSd_1}} \Rightarrow \omega = \sqrt{\frac{3(d_2 - d_1)g}{2d_1 L}}$$

Hence angular frequency of oscillation,

$$\omega = \sqrt{\frac{3g\,(d_2 - d_1)}{2d_1 L}}$$

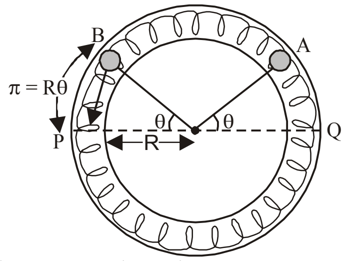
35. (i) Given : Natural length of spring, $l_0 = 0.06\,\pi = \pi R$

Spring constant, $k = 0.1$ N/m

Mass of each block A and B, $m = 0.1$ kg

Radius of circle, R= 0.06 m

As both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. It results into compression of spring in upper segment and an equal elongation of spring in lower segment. Let it be x. PB and QA denote x in the figure.



Compression = $R\theta$ = elongation = x

$\therefore$ Force exerted by each spring on each ball = 2 kx

$\therefore$ Total force on each ball due to two springs = 4 kx

$\therefore$ Restoring torque about origin $O = -(4\,kx)R$

$\therefore$ $\tau = -4k(R\theta)R$, where θ = Angular displacement

or $\tau = -4kR^2\theta$

Since torque (τ) is proportional to θ, each ball executes angular SHM about the centre O.

Again, $\tau = -4kR^2\theta$

or $\quad I\alpha = -4kR^2\theta$ where α = angular acceleration

or $\quad (mR^2)\,\alpha = -4kR^2\theta$ or $\alpha = -\left(\frac{4k}{m}\right)\theta$

$\therefore$ Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{\alpha}{\theta}}$

$\therefore$ Frequency of each ball $= \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$

$$= \frac{1}{2\pi}\sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}\text{ sec}^{-1} \quad \text{..(ii)}$$

(ii) v_{max} be the velocity at the mean position

$\therefore$ Loss in elastic potential energy = gain in kinetic energy

$$2\left[\frac{1}{2}K\left(2R\frac{\pi}{6}\right)^2\right] = 2 \times \left[\frac{1}{2}mv_{max}^2\right]$$

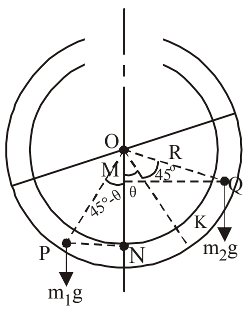
$$\therefore \quad v_{max} = \sqrt{\frac{K}{m}} \times \frac{R\pi}{3} = 0.02\,\pi \text{ m/s} = 0.628 \text{ m/s}$$

(iii) Total energy $= 2\left[\frac{1}{2}mv_{max}^2\right]$ (= K.E. at mean position)

$$= 2\left[\frac{1}{2} \times 0.1(0.02\pi)^2\right]$$

$$= 3.95 \times 10^{-4} \text{ J}$$

36.



At equilibrium,

$$\begin{pmatrix}\text{Torque at q due to}\\ \text{liquid of density}\,\rho\end{pmatrix} = \begin{pmatrix}\text{Torque at p due to}\\ \text{liquid of density}\,1.5\,\rho\end{pmatrix}$$

$m_2 g \times QM = m_1 g \times PN$

$\therefore \quad m_2 g\, R \sin(45° + \theta) = m_1 gR \sin(45° - \theta)$

$V\rho gR \sin(45° + \theta) = 1.5 V\rho gR \sin(45° - \theta)$ ... (i)

$$\Rightarrow \frac{\sin(45+\theta)}{\sin(45-\theta)} = 1.5$$

$$\Rightarrow \frac{\sin 45° \cos\theta + \cos 45° \sin\theta}{\sin 45° \cos\theta - \cos 45° \sin\theta} = \frac{3}{2}$$

Solving we get,

$$\tan\theta = \frac{1}{5}$$

(b) Let us now displace the liquids in anticlockwise direction along the circumference of tube through an angle α.

$\therefore$ Net torque

$\tau = m_2 gR \sin(45° + \theta + \alpha) - m_1 gR \sin(45° - \theta - \alpha)$

$= V\rho gR \sin(45° + \theta + \alpha) - 1.5 V\rho gR (45° - \theta - \alpha)$

$= V\rho gR \sin(\theta + 45°) \cos\alpha + V\rho gR \cos(45° + \theta) \sin\alpha - 1.5\, V\rho gR \sin(45° - \theta) \cos\alpha + 1.5\, V\rho gR \cos(45° - \theta) \sin\alpha$

Using eq. (i) we get

$$\tau = V\rho gR\left[\cos(45° + \theta)\sin\alpha + 1.5\cos(45° - \theta)\sin\alpha\right]$$

$\tau = V\rho gR\,[\cos(45° + \theta) + 1.5 \cos(45° - \theta)] \sin\alpha$

when α is small (given) $\therefore \sin\alpha \approx \alpha$

$\therefore \quad \tau = V\rho gR\,[\cos(45° + \theta) + 1.5 \cos(45° - \theta)]\alpha$

Since, τ and α are proportional, motion is simple harmonic.

Moment of inertia about O

$I = m_1 R^2 + m_2 R^2 = V\rho R^2 + 1.5\, V\rho R^2 = 2.5\, V\rho R^2$

Now from $T = 2\pi\sqrt{\frac{I}{C}}$

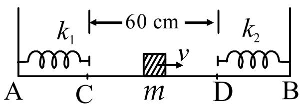
$$= 2\pi \frac{\sqrt{(V\rho \times 2.5R^2)}}{\sqrt{[\cos(45+\theta) + 1.5\cos(45-\theta)]V\rho gR}}$$

or, $T = 2\pi \frac{\sqrt{1.803R}}{\sqrt{g}} \qquad \left(\because \tan\theta = \frac{1}{5}\right)$

37. Since AB is a smooth table. So no acceleration or retardation in this region so motion from C to D to C is uniform with speed 120 m/s.

The mass will strike the right spring, compress it. The K.E. of the mass will convert into P.E. of the spring. Again the spring will return to its normal size thereby converting its P.E. to K.E. of the block.

For this process, time taken $= \frac{T}{2}$, where $T = 2\pi\sqrt{\frac{m}{k}}$.



$$\therefore \quad t_1 = \frac{T}{2} = \pi\sqrt{\frac{m}{k_2}} = \pi\sqrt{\frac{0.2}{3.2}} = 0.785 \text{ sec}$$

Now, time taken to travel from D to C

$$t_2 = \frac{\text{distance}}{\text{velocity}} = \frac{60}{120} = 0.5$$

Now the block will compress the left spring and then the spring again attains its normal length. The time taken C to A

$$t_3 = \pi\sqrt{\frac{m}{k_1}} = \pi\sqrt{\frac{0.2}{1.8}} = 1.05 \text{ sec.}$$

Again time taken to travel from C to D

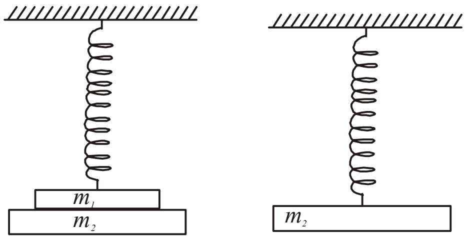
$$t_4 = \frac{60}{120} = 0.5$$

$\therefore$ Total time of oscillation

$T = t_1 + t_2 + t_3 + t_4$

$= 0.785 + 0.5 + 1.05 + 0.5 = 2.83$ (Approximately)

38.



Let x_1 be the extension in equilibrium when both m_1 and m_2 are suspended.

$\therefore \quad (m_1 + m_2)g = kx_1 \Rightarrow x_1 = \frac{(m_1 + m_2)g}{k}$ ...(i)

Let x_2 be the extension when only m_2 is left.

$\therefore \quad kx_2 = m_2 g$ or $x_1 = \frac{m_2 g}{k}$ ...(ii)

From eqs. (i) and (ii), amplitude of oscillation

$$A = x_1 - x_2 = \frac{m_1 g}{k}$$

Angular frequency when m_1 is removed only m_2 is left,

$$\omega = \sqrt{\frac{k}{m_2}}$$

39. According to question, if the mass is increased by 2 kg the period increases by 1 s. Time period the spring

$$T = 2\pi\sqrt{\frac{M}{k}}$$

or, $2 = 2\pi\sqrt{\frac{M}{k}}$ .... (i)

When mass is increased by 2 kg

$3 = 2\pi\sqrt{\frac{M+2}{k}}$ ... (ii)

Dividing eq. (i) by (ii)

$$\frac{2}{3} = \sqrt{\frac{M}{M+2}} \Rightarrow \frac{4}{9} = \frac{M}{M+2} \Rightarrow M = 1.6 \text{ kg.}$$

Topic-4 : Damped, Forced Oscillations and Resonance

1. **(c)** Time of half the amplitude is $= 2s$

Using, $A = A_0 e^{-kt}$

$\frac{A_0}{2} = A_e\, e^{-k \times 2}$ ...(i)

and $\frac{A_0}{1000} = A_e\, e^{-kt}$ ...(ii)

Dividing (i) by (ii) and solving, we get

$t = 20$ s

2. **(b)** Amplitude of vibration at time t = 0 is given by

$A = A_0 e^{-0.1 \times 0} = 1 \times A_0 = A_0$

also at t = t, if $A = \frac{A_0}{2}$

$\Rightarrow \frac{1}{2} = e^{-0.1t}$

t = 10 ln 2 ≃7s

3. **(b)** In first collision mu momentum will be imparted to system, in second collision when momentum of (M + m) is in opposite direction mu momentum of particle will make its momentum zero.

On 13th collision, $\boxed{m} \rightarrow \boxed{M+12}$; $\boxed{M+13m} \rightarrow V$

$$mu = (M+13m)v \Rightarrow v = \frac{mu}{M+13m} = \frac{u}{15}$$

$$v = \omega A \Rightarrow \frac{u}{15} = \sqrt{\frac{K}{M-13m}} \times A$$

Putting value of M, m, u and K we get amplitude

$$A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

4. **(b)** The change in time period compared to the undamped oscillator increases by 8%.

5. **(c)** $\because A = A_0 e^{-\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)

According to the questions, after 5 second,

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \ldots (i)$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots (ii)$$

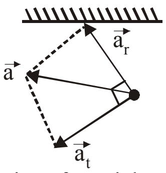
From eqns (i) and (ii)

$A = 0.729\, A_0$

$\therefore\ \alpha = 0.729$

6. **(a)**

7. **(c)** The resultant of transverse $\vec{a}_t$ and radial $\vec{a}_r$ component of the acceleration is represented by $\vec{a}$



8. **(a)** In SHM, velocity of particles goes on decreasing from maximum value to zero as the particles travel from mean position to extreme position.

Therefore if the time taken for the body to go from O to $A/2$ is T_1 and to go from $A/2$ to A is T_2 then obviously $T_1 < T_2$

9. **(4)** As we know, $Y = \frac{FL}{Al} \Rightarrow F = \left(\frac{YA}{L}\right) l$

Also, $F = Kl$

or, $Kl = \left(\frac{YA}{L}\right) l \Rightarrow K = \frac{YA}{L}$

Angular frequency $\omega = \sqrt{\frac{K}{m}}$ or, $\omega = \sqrt{\frac{YA}{ml}}$

or, $140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}}$ ($\because\ Y = 9 \times 10^9$ given)

$\therefore\ n = 4$

10. **(b, d)** For first harmonic oscillator,

Mass = m

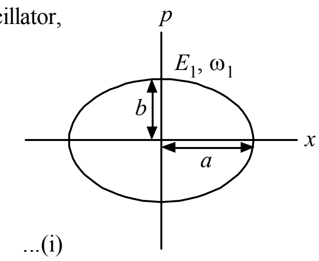
Angular frequency = ω_1

Amplitude = a

Total energy = E_1

Maximum momentum,

$p_{max} = b$



$$E_1 = \frac{1}{2} m\omega_1^2 a^2 \quad \ldots (i)$$

$p_{max} = mv_{max} = m\,a\,\omega_1 \Rightarrow b = m\,a\,\omega_1$

$$\frac{a}{b} = \frac{1}{m\omega_1} \quad \ldots (ii)$$

For second harmonic oscillator,

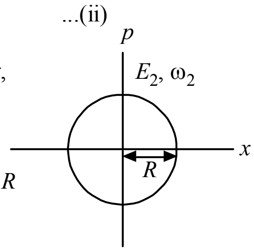
Mass = m

Angular frequency = ω_2

Amplitude = R

Maximum momentum, $p_{max} = R$

Total energy = E_2



$$E_2 = \frac{1}{2} m\omega_2^2 R^2 \quad \ldots (iii)$$

$p_{max} = mv_{max} = m\omega_2 R$

$R = m\omega_2 R \Rightarrow m\omega_2 = 1 \quad \ldots (iv)$

From eqns. (ii) and (iv).

$$\frac{a}{b} = \frac{\omega_2}{\omega_1} \quad \ldots (v)$$

From eqns. (i) and (iii),

$$\frac{E_1}{E_2} = \frac{\omega_1^2 a^2}{\omega_2^2 R^2}$$

If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then from eqn. (v)

$$\frac{\omega_2}{\omega_1} = n^2$$

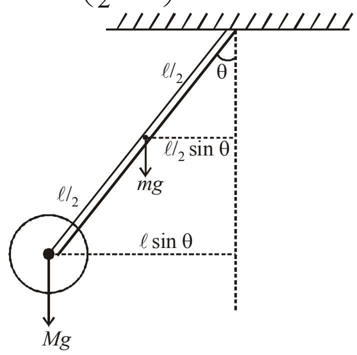
and from eqn. (vi)

$$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} \times n^2 = \frac{\omega_1}{\omega_2} \quad \therefore \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

11. **(a, d)**

Applying, torque $\tau = I\alpha$

For case A : $mg\left(\frac{\ell}{2}\sin\theta\right) + Mg(\ell \sin\theta) = I_A \alpha_A \quad \ldots (i)$



For case B : $mg\left(\frac{\ell}{2}\sin\theta\right) + Mg(\ell\sin\theta) = I_B\alpha_B$ ...(ii)

From eq. (i) & (ii)

$I_A\,\alpha_A = I_B\,\alpha_B$

Since $I_A > I_B \quad \therefore \alpha_A < \alpha_B$

Hence, $\omega_A < \omega_B$

12. **A → p; B → q; r, s; C → s; D → q**

(A) : Potential energy k minimum at mean position and maximum at extreme position. In case of a S.H.M. we get a parabola for potential energy versus displacement graph.

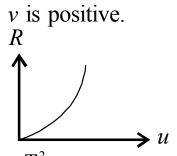
(B) : S = ut for a = 0. Therefore we get a straight line passing through the origin.

$S = ut + \frac{1}{2}at^2$ for constant positive acceleration. In this case we get a part of parabola as a graph line between s versus t as shown by graph (s).

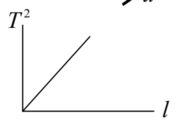
(p) is ruled out because if a is –ve and v is positive.

$S = S_0 + vt$ graph (r)

(C) : $R = \dfrac{u^2\sin 2\theta}{g} \quad \Rightarrow \quad R \propto u^2$



(D) : $T = 2\pi\sqrt{\dfrac{\ell}{g}} \quad \Rightarrow \quad T^2 \propto \ell$



13. **A → p; B → q, r; C → p; D → q, r**

(A) : For a simple harmonic motion $v = \omega\sqrt{a^2 - x^2}$. On comparing it with $v = c_1\sqrt{c_2 - x^2}$ this equation is SHM with w $= c_1$ and $a^2 = c_2$

(B) : $v = -k\,x$

when x is positive; v is – ve, and as x decreases, v decreases. Therefore kinetic energy will decrease. When $x = 0$, $v = 0$. Therefore the object does not change its direction.

When x is negative, v is positive. But as x decreases in magnitude, v also decreases. Therefore kinetic energy decreases. When $x = 0$, $v = 0$. Therefore the object does not change its direction.

(C) : When $a = 0$, let the spring have an extension x. Then $k\,x = mg$.

When the elevator starts going upwards with a constant acceleration, as seen by the observer in the elevator, the object is at rest.

$\therefore \quad ma + mg = k\,x'$

$\Rightarrow \quad ma = k\,(x' - x)$ (Since a is constant)

(D) : The object is projected with a speed is $\sqrt{2}$ times the escape speed $V_e = \sqrt{\dfrac{2GM_e}{R_e}}$. Therefore the object will leave the earth. It will therefore not change the direction keeps on moving with decreasing speed.

14. **(d)** When the ball is thrown upwards, at the point of throw (O) the linear momentum is in upwards direction (and has a maximum value) and the position is zero. As the time passes, the ball moves upwards and its momentum goes on decreasing and the position becomes positive. The momentum becomes zero at the topmost point.

As the time increases, the ball starts moving down with an increasing linear momentum in the downward direction (negative) and reaches back to its original position.

15. **(c)** In SHM mechanical energy, $E \propto$ (amplitude)^2

$\therefore \quad E_1 \propto (2a)^2 \quad \& \quad E_2 \propto a^2$

$\therefore \quad \dfrac{E_1}{E_2} = 4$

16. **(b)** When the position of the mass is at one extreme end in the positive side (the topmost point), the momentum is zero. As the mass moves towards the mean position the momentum increases in the negative direction.

As the mass is osicillating in water its amplitude will go on decreasing and the amplitude will decrease with time.

17. **(c)** The particle will not perform oscillations if energy ≤ 0. Therefore E > 0. If $E = V_0$, the potential energy will become constant as depicted in the graph given. In this case also the particle will not oscillate.

$\therefore \quad E > V_0$

$\therefore \quad V_0 > E > 0$

18. **(b)** Potential energy, $V = \propto x^4$ given

$$\therefore \quad \alpha = \frac{\text{Potential energy}}{x^4} = \frac{ML^2T^{-2}}{L^4} = [ML^{-2}T^{-2}]$$

$$\text{Now} \quad \frac{1}{A}\sqrt{\frac{m}{\alpha}} = \frac{1}{L}\sqrt{\frac{M}{ML^{-2}T^{-2}}} = T$$

19. **(d)** $F = \dfrac{-dV(x)}{dx}$

$\because \quad$ As $V(x)$ = constant for $x > X_0$

$\therefore \quad F = 0$ and hence $a = 0$ for $x > X_0$

20. A point mass m is suspended at the end of a massless wire of length L fig. (a).

From fig. (b), due to equilibrium

$T = mg \quad$ ... (i)

From $\quad Y = \dfrac{T/A}{\ell/L}$

$\Rightarrow T = \dfrac{YA\ell}{L} \quad$ ... (ii)

From eq. (i) and (ii)

$mg = \dfrac{YA\ell}{L} \quad$ ... (iii)

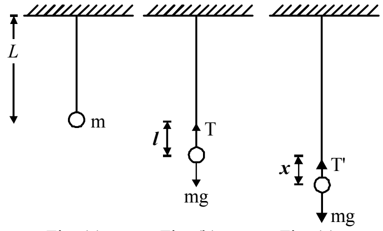


Fig. (a) Fig. (b) Fig. (c)

From fig. (c)

Restoring force

$$= -[T' - mg] = -\left[\frac{YA(\ell + x)}{L} - \frac{YA\,\ell}{L}\right] \quad \text{[from (iii)]}$$

$$= \frac{-YAx}{L}$$

On comparing this equation with $F = -m\omega^2 x$

$$m\omega^2 = \frac{YA}{L} \Rightarrow \omega = \sqrt{\frac{YA}{mL}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{YA}{mL}}$$

$$\text{or, frequency } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

14 Waves

Topic-1: Basic of Mechanical Waves Progressive and Stationary Waves

1. **(a)** As we know,

Pressure amplitude, $\Delta P_0 = aKB = S_0 KB = S_0 \times \frac{\omega}{V} \times \rho V^2$

$$\left[\because K = \frac{\omega}{V}, V = \sqrt{\frac{B}{\rho}}\right]$$

$$\Rightarrow S_0 = \frac{\Delta P_0}{\rho V \omega} \approx \frac{10}{1 \times 300 \times 1000} \text{ m} = \frac{1}{30} \text{ mm} \approx \frac{3}{100} \text{ mm}$$

2. **(b)** Given : Distance between one crest and one trough = 1.5 m

$= (2n_1 + 1)\frac{\lambda}{2}$

Distance between two crests = 5 m $= n_2\lambda$

$\frac{1.5}{5} = \frac{(2n_1+1)}{2n_2} \Rightarrow 3n_2 = 10n_1 + 5$

Here n_1 and n_2 are integer.

If $n_1 = 1, n_2 = 5$ $\quad \therefore \lambda = 1$

$n_1 = 4, n_2 = 15$ $\quad \therefore \lambda = 1/3$

$n_1 = 7, n_2 = 25$ $\quad \therefore \lambda = 1/5$

Hence possible wavelengths $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$ metre.

3. **(b)** At $t = 0, x = 0, y = 0$

$\phi = \pi$ rad

4. **(a)** Using, $\beta = 10$

or $120 = 10 \log 10\left(\frac{I}{10^{-12}}\right)$ ...(i)

Also $I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$ ...(ii)

On solving above equations, we get

$r = 40$ cm.

5. **(a)** Comparing the given equation

$y = 10^{-3}\sin(50t + 2x)$ with standard equation,

$y = a\sin(\omega t - kx)$

$\Rightarrow$ wave is moving along –ve x-axis with speed

$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25$ m/sec.

6. **(a)** Given, amplitude a = 10 cm

wave velocity = 2 × maximum particle velocity

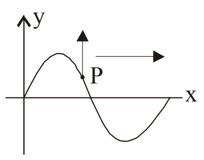
i.e, $\frac{\omega\lambda}{2\pi} = 2\frac{a\omega}{\pi}$

or, $\lambda = 4a = 4 \times 10 = 40$ cm

7. **(a)** Particle velocity v_p is related to the displacement of the particle from the mean position as

$v_p = 2\pi v\sqrt{A^2 - y^2}$

$v_p = 2\pi\left(\frac{v}{\lambda}\right)\sqrt{A^2 - y^2}$



$= \frac{2\pi}{0.5} \times 0.1\sqrt{(0.1)^2 - (0.05)^2} = \frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

Since the wave is sinusoidal moving in positive x-axis the point will move parallel to y-axis therefore options (c) and (d) are ruled out. As the wave moves forward in positive X-direction, the point should move upwards i.e. in the positive Y-direction.

8. **(b)** Speed of sound in a gas,

$C_{rms} = \sqrt{\left(\frac{\gamma RT}{M}\right)}$ Here $C_{rms} \propto \sqrt{\frac{1}{m}}$;

$\left[\gamma_{monoatomic} = \frac{5}{3}, R \,\&\, T \text{ are constant}\right]$

$\therefore \quad \frac{C_{rms_1}}{C_{rms_2}} = \sqrt{\left(\frac{m_2}{m_1}\right)}$

9. **(a)** $v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t) \quad \therefore \quad v_{max} = A\omega$

Maximum particle velocity of the particle in SHM, $v_{max} = A\omega$

10. **(b)** $y = y_0 \sin 2\pi\left[ft - \frac{x}{\lambda}\right]$

$\therefore \quad \frac{dy}{dt} = \left[y_0 \cos 2\pi\left(ft - \frac{x}{\lambda}\right)\right] \times 2\pi f$

or, $\left[\frac{dy}{dt}\right]_{max} = y_0 \times 2\pi f$

Given: Maximum particle velocity = 4c = 4 ($f \times T$)

$y_0 \times 2\pi f = 4(f \times \lambda) \quad \therefore \quad \lambda = \frac{\pi y_0}{2}$

11. Given : $y = \frac{1}{(1+x)^2}$

At $t = 0$ we get $y = 1$ when $x = 0$

Again, $y = \frac{1}{1 + (x-1)^2}$

So, at $t = 2$ $y = 1$ when $x = 1$

The wave pulse has travelled a distance of 1m in 2 sec.

$\therefore$ Velocity of wave pulse, $v = \frac{1}{2} = 0.5 \text{ ms}^{-1}$

12. Particle velocity amplitude = $V_{max} = A\omega = A2\pi v$

Particle acceleration amplitude = $a_{max} = A\omega^2 = 4\pi^2 v^2 A$

13. **(a, d)** Wavelength of pulse, $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$ or, $T \propto \sqrt{T}$

Where T = tension of string.

Here $T_1 > T_2 \quad \therefore \lambda_1 > \lambda_2$

The velocities of the two pulses cannot be same at mid-point as velocity being vector quantity has direction.

$V = \sqrt{\frac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same i.e., $T_{AO} = T_{OA}$

14. **(a, c, d)** For a plane wave, intensity i.e., energy crossing per unit area per unit time is constant at all points.

But for a spherical wave, intensity at a distance r from a point source

$I \propto \frac{1}{r^2}$

But the **total intensity** of the spherical wave over the spherical surface centered at the source remains constant at all times.

For line source $I \propto \frac{1}{r}$ spherical wave is not produced by the line source.

15. **(a, b, c, d)** In the wave motion $y = a(kx - \omega t)$, y can represent, electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

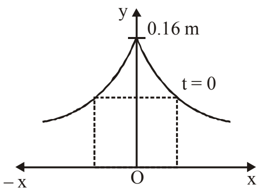
16. **(b, c, d)** Comparing the given equation, $y(x, t)$

$$= \frac{0.8}{(4x+5t)^2+5} = \frac{0.8}{16\left[x+\frac{5}{4}t\right]^2+5}$$

With the equation of moving pulse
$y = f(x + vt)$

$$v = \frac{5}{4}\text{ms}^{-1} = \frac{2.5}{2}\text{ms}^{-1}$$

So, the wave will travel a distance of 2.5 m in 2 sec.



At $x = 0, t = 0$, $y = \frac{0.8}{(4x+5t)^2+5}$

$$= \frac{0.8}{5} = 0.16 \text{ m}$$

$\therefore$ Maximum displacement is 0.16 m
The shape of the pulse at $x = 0$ and $t = 0$ is as shown in figure and it is symmetric.

17. **(a, c)** For a transverse sinusodial wave travelling on a string, the maximum velocity $v_{max} = a\omega$.

Given maximum velocity $= \frac{v}{10} = \frac{10}{10} = 1\text{m/s}$

$\therefore \quad a\omega = 1 \quad \Rightarrow \quad 10^{-3} \times 2\pi v = 1 \; [\because \omega = 2\pi v]$

$$\Rightarrow \quad v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\text{Hz}$$

And, $\lambda = \frac{v}{v} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\text{ m}$

18. **(b, c)** Given equation, $y = A \sin(10\pi x + 15\pi t + \pi/3)$
Comparing this equation with standard equation of a wave travelling in $-X$ direction

$$y = A \sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right] \Rightarrow y = A \sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$$

$$\frac{2\pi v}{\lambda} = 15\pi \text{ and } \frac{2\pi}{\lambda} = 10\pi$$

$$\Rightarrow \quad \lambda = \frac{1}{5} = 0.2 \text{ m and } v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5 \text{ m/s}$$

19. **(a,c)** For wave motion, the differential equation is $\frac{\partial^2 y}{\partial t^2} = \left(\text{constant } \frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2} \quad \left[\because v = \frac{\omega}{k}\right]$

or $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ....(i)

The wave motion is characterized by the two conditions

$f(x, t) = f(x, t + T)$ ....(ii)

$f(x, t) = f(x + \lambda, t)$ ....(iii)

20. **(a,b,c,d)** Given: $y = 10^{-4} \sin(60t + 2x)$
Comparing the given equation with the standard wave equation
$y = a \sin(\omega t + kx)$
Amplitude $a = 10^{-4}$m; $k = 2\text{ m}^{-1}$

And, $\omega = 60$ rad/s $\Rightarrow 2\pi f = 60 \quad \therefore \quad f = \frac{30}{\pi}$Hz

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi \text{ m}$$

Speed of wave $v = \frac{\omega}{k} = \frac{60}{2} = 30$ m/s

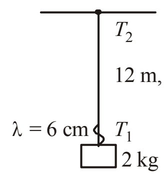
Topic-2 : Vibration of String and Organ Pipe

1. **(a)** Here, $l_1 = 17$ cm and $l_2 = 24.5$ cm, $V = 330$ m/s, $f = ?$
$\lambda = 2(l_2 - l_1) = 2 \times (24.5 - 17) = 15$ cm
Now, from $v = f\lambda \Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$$\therefore \lambda = \frac{330}{15} \times 100 = \frac{1100 \times 100}{5} = 2200 \text{ Hz}$$

2. **(c)** Using, $V = f\lambda$

$$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2} \Rightarrow \lambda_2 = \frac{V_2}{V_1}\lambda_1$$



Again using,

$$n = \frac{V}{\lambda} = \sqrt{\frac{T}{M}}\lambda_2 = \sqrt{\frac{T_2}{T_1}}\lambda_1$$

$T_2 = 8\text{g (Top)}$

$$= \sqrt{\frac{8g}{2g}}\lambda_1 = 2\lambda_1 = 12 \text{ cm} \quad T_1 = 2\text{g (Bottom)}$$

3. **(a)** Using $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$,

where, T = tension and $\mu = \frac{\text{mass}}{\text{length}}$

$$f_x = \frac{1}{2\ell}\sqrt{\frac{T_x}{\mu}} \text{ and } f_z = \frac{1}{2\ell}\sqrt{\frac{T_z}{\mu}}$$

$$\frac{f_x}{f_z} = \frac{450}{300} = \sqrt{\frac{T_x}{T_z}} \qquad \therefore \frac{T_x}{T_z} = \frac{9}{4} = 2.25.$$

4. **(b)** The velocity of a transverse wave in a stretched wire is given by

$$v = \sqrt{\frac{T}{\mu}}$$

Where,
T = Tension in the wire
μ = linear density of wire $\quad (\because V \propto T)$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{v}{v} \times 2 = \sqrt{\frac{2.06 \times 10^4}{T_2}}$$

$$\Rightarrow T_2 = \frac{2.06 \times 10^4}{4} = 0.515 \times 10^4 N$$

$\Rightarrow T_2 = 5.15 \times 10^3 N$

5. **(a)** Given, $l = 60$ cm, $m = 6$ g, $A = 1\text{ mm}^2$, $v = 90$ m/s and $Y = 16 \times 10^{11}\text{ Nm}^{-2}$

Using, $v = \sqrt{\frac{T}{m} \times l} \Rightarrow T = \frac{mv^2}{l}$

Again from, $Y = \frac{T}{A}\Delta L / L_0$

$$\Delta L = \frac{Tl}{YA} = \frac{mv^2 \times l}{l(\text{YA})} = \frac{6 \times 10^{-3} \times 90^2}{16 \times 10^{11} \times 10^{-6}} = 3 \times 10^{-4} m$$
$= 0.03$ mm

6. **(b)** Wave speed $V = \sqrt{\frac{T}{\mu}}$

when car is at rest a = 0

$$\therefore 60 = \sqrt{\frac{Mg}{\mu}}$$

Similarly when the car is moving with acceleration a,

$$60.5=\sqrt{\frac{M\left(g^2+a^2\right)^{1/2}}{\mu}} \Rightarrow \frac{60.5}{60}=\sqrt{\sqrt{\frac{g^2+a^2}{g^2}}}$$

$$\left(1+\frac{0.5}{60}\right)^4=\frac{g^2+a^2}{g^2}=1+\frac{2}{60} \Rightarrow g^2+a^2=g^2+g^2\times\frac{2}{60}$$

$a=g\sqrt{\frac{2}{60}}=\frac{g}{\sqrt{30}}$ [which is closest to g/5]

7. **(b)** $\frac{3\lambda}{2}=2$ or $\lambda=\frac{4}{3}m$

Velocity, $v=f\lambda=240\times\frac{4}{3}=320\,\text{m/sec}$

Also $f_1=\frac{240}{3}=80\,\text{Hz}$

8. **(c)** As there must be node at both ends and at the joint of the wire *A* and *B* so

$$\frac{V_A}{V_B}=\sqrt{\frac{u_B}{u_A}}=\frac{r_B}{r_A}=2=\frac{\lambda_A}{\lambda_B} \Rightarrow \lambda_A=2\lambda_B \Rightarrow \frac{P}{q}=\frac{1}{2}$$

9. **(a)** If a closed pipe vibration in N^{th} mode then frequency of vibration $n=\frac{(2N-1)v}{4l}=(2N-1)n_1$

(where n_1 = fundamental frequency of vibration)

Hence $20{,}000=(2N-1)\times 1500$

$\Rightarrow N=7.1\approx 7$

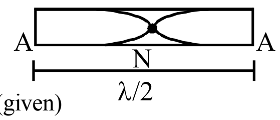
$\therefore$ Number of over tones = (No. of mode of vibration) – 1 = 7 – 1 = 6

10. **(a)** In solids, Velocity of wave

$$V=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}$$

$v=5.85\times10^3$ m/sec

Since rod is clamped at middle fundamental wave shape is as follow



$\frac{\lambda}{2}=L \Rightarrow \lambda=2L$

$\lambda=1.2$m ($\because$ L = 60 cm = 0.6m (given)

Using $v=f\lambda$

$$\Rightarrow f=\frac{v}{\lambda}=\frac{5.85\times10^3}{1.2}$$

$=4.88\times10^3$ Hz $\simeq$ 5 KHz

11. **(b)** $n_1=n_2$

T $\to$ Same

r $\to$ Same

$l\to$ Same

Frequency of vibration

$$n=\frac{p}{2l}\sqrt{\frac{T}{\pi r^2\rho}}$$

As T, r, and l are same for both the wires

$n_1=n_2$

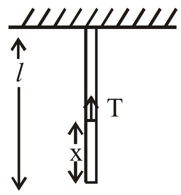
$$\frac{p_1}{\sqrt{\rho_1}}=\frac{p_2}{\sqrt{\rho_2}} \Rightarrow \frac{p_1}{p_2}=\frac{1}{2} \qquad \because \rho_2=4\rho_1$$

12. **(a)** We know that velocity in string is given by

$$v=\sqrt{\frac{T}{\mu}} \qquad ...(i)$$

where $\mu=\frac{m}{l}=\frac{\text{mass of string}}{\text{length of string}}$

The tension $T=\frac{m}{\ell}\times x\times g$ ..(ii)



From (1) and (2)

$\frac{dx}{dt}=\sqrt{gx}$

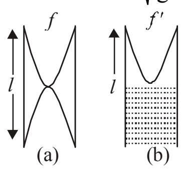
$x^{-1/2}dx=\sqrt{g}\,dt$

$\therefore \int_0^{\ell}x^{-1/2}dx-\sqrt{g}\int_0^{\ell}dt$

$\Rightarrow 2\sqrt{l}$

$=\sqrt{g}\times t \quad \therefore t=2\sqrt{\frac{\ell}{g}}=2\sqrt{\frac{20}{10}}=2\sqrt{2}$

13. **(b)**



The fundamental frequency in case (a) is $f=\frac{v}{2\ell}$

The fundamental frequency in case (b) is

$$f'=\frac{v}{4(\ell/2)}=\frac{v}{2\ell}=f$$

14. **(c)** Length of pipe = 85 cm = 0.85m

Frequency of oscillations of air column in closed organ pipe is given by,

$$f=\frac{(2n-1)\upsilon}{4L} \Rightarrow f=\frac{(2n-1)\upsilon}{4L}\le 1250$$

$$\Rightarrow \frac{(2n-1)\times340}{0.85\times4}\le 1250 \Rightarrow 2n-1\le 12.5\approx 6$$

15. **(b)** Fundamental frequency,

$$f=\frac{v}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{A\rho}} \quad \left[\because v=\sqrt{\frac{T}{\mu}} \text{ and } \mu=\frac{m}{\ell}\right]$$

Also, $Y=\frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A}=\frac{Y\Delta\ell}{\ell}$

$$\Rightarrow f=\frac{1}{2\ell}\sqrt{\frac{y\Delta\ell}{\ell\rho}} \qquad ....(i)$$

$\ell=1.5$ m, $\frac{\Delta\ell}{\ell}=0.01$,

$\rho=7.7\times10^3$ kg/m^3 (given)

$y=2.2\times10^{11}$ N/m^2 (given)

Putting the value of ℓ, $\frac{\Delta\ell}{\ell}$, ρ and y in eq^n. (i) we get,

$$f=\sqrt{\frac{2}{7}}\times\frac{10^3}{3} \quad \text{or} \quad f\approx 178.2\text{ Hz}$$

16. **(d)** Total length of the wire, L = 114 cm

$n_1:n_2:n_3=1:3:4$

Let L_1, L_2 and L_3 be the lengths of the three parts

As $n\propto\frac{1}{L}$

$$\therefore L_1:L_2:L_3=\frac{1}{1}:\frac{1}{3}:\frac{1}{4}=12:4:3$$

$$\therefore L_1=\left(\frac{12}{12+4+3}\times114\right)=72\text{ cm}$$

$$L_2 = \left(\frac{4}{19}\times 114\right) = 24 \text{ cm}$$

and $L_3 = \left(\frac{3}{19}\times 114\right) = 18$ cm

Hence the bridges should be placed at 72 cm and 72 + 24 = 96 cm from one end.

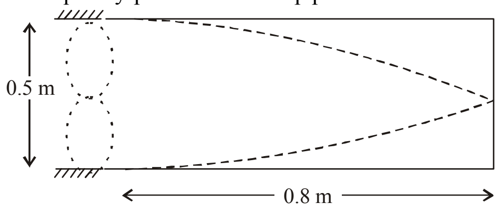
17. **(b)** Considering the end correction [$e = 0.3\ D$ where D = diameter]

$f = n\left[\frac{v}{4(l+e)}\right]$ For first resonance, $n = 1$

$$\therefore f = \frac{v}{4(l+0.3D)} \Rightarrow l = \frac{v}{4f} - 0.3D$$

$$\text{or, } l = \left(\frac{336\times 100}{4\times 512}\right) - 0.3\times 4 = 15.2\text{cm}$$

18. (b) Frequency of 2nd harmonic of string = fundamental frequency produced in the pipe



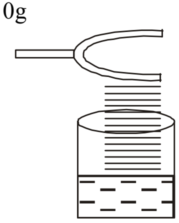
$$2\left(\frac{v_1}{2l_1}\right) = \frac{v_2}{4l_2}$$

$$\therefore\ 2\times\left[\frac{1}{2l_1}\sqrt{\frac{T}{\mu}}\right] = \frac{v}{4l_2}$$

$$\therefore\ \frac{1}{0.5}\sqrt{\frac{50}{\mu}} = \frac{320}{4\times 0.8} \Rightarrow \mu = 0.02 \text{ kg m}^{-1}$$

Hence, the mass of the string, $m_1 = \mu\ l_1$
$= 0.02 \times 0.5$ kg = 10g

19. **(a)** To determine the speed of sound using a resonance column, prongs of the tuning fork are kept in a verticle plane. As shown in the figure, the fringes of the tuning fork are kept in a vertical plane.



20. **(a)** Frequency of first harmonic in AB = $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

Frequency of second harmonic in CD = $f_2\ \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$f_1 = f_2$ (given)

$$\therefore\ \frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{1}{l}\sqrt{\frac{T_2}{\mu}}$$

or $T_1 = 4T_2$ ...(i)

Equating torques due to T_1 and T_2 about O for rotational equilibrium.

$\therefore\ T_1 x = T_2\ (L - x)$

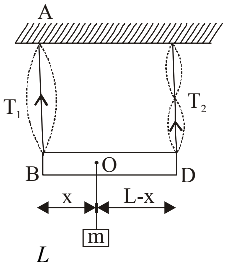
For translational equilibrium,

$T_1 + T_2 = \text{mg}$

From eg. (i) and (iii),

$T_1 = \frac{4mg}{5}$ and $T_2 = \frac{mg}{5}$

Now from eq. (ii).



$$\frac{4mg}{5}\times x = \frac{mg}{5}(L-x) \Rightarrow 4x = L - x \therefore x = \frac{L}{5}.$$

21. **(d)** Frequency of 2nd harmonic of open pipe,

$$f_1 = \frac{v}{\lambda} = \frac{v}{\ell} \quad \text{... (i)}$$

Frequency of n^{th} harmonic of closed pipe,

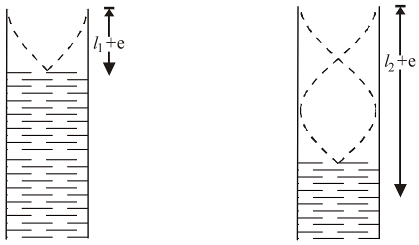
$$f_2 = \frac{v}{\lambda} = \frac{nv}{4\ell} \quad \text{... (ii)}$$

Here n is a odd number. From eq. (i) and (ii)

$f_2 = \frac{n}{4} f_1$, For first resonance, $n = 5$

$$\therefore f_2 = \frac{5}{4} f_1$$

22. **(c)** In a resonance tube,



For first resonance **For second resonance**

$\ell_1 + e = \frac{\lambda}{4}$ $\qquad \ell_2 + e = \frac{3\lambda}{4}$

But $v = \nu\lambda$

$\therefore\quad v = \nu\frac{4}{3}(\ell_2 + e) \qquad \Rightarrow \ell_2 + e = \frac{3v}{4\nu}$ ...(i)

$\therefore\quad v = \nu\ 4(\ell_1 + e) \qquad \Rightarrow \ell_1 + e = \frac{v}{4\nu}$ ... (ii)

Subtracting (i) and (ii),

$v = 2\nu(\ell_2 - \ell_1) \quad \therefore\ \Delta v = 2\nu(\Delta\ell_2 + \Delta\ell_1)$

$= 2 \times 512 \times (0.1 + 0.1)$ cm/s = 204.8 cm/s

Hence, maximum possible error in speed, $\Delta v = 204.8$ cm/s

23. **(b)** Frequency of first overtone in closed organ pipe,

$$\nu = \frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}}$$

Frequency of first overtone in open organ pipe,

$$\nu' = \frac{1}{\ell_2}\sqrt{\frac{P}{\rho_2}}$$

Here, $v = v'$

$$\frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}} = \frac{1}{l_2}\sqrt{\frac{P}{\rho_2}}$$

$$\therefore\quad \ell_2 = \frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$$

24. **(a)** Fundamental frequency, $f_0 = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$$= \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}} \Rightarrow M = 25 \text{ kg}$$

As frequency is corresponds to 5^{th} and 3^{rd} harmonic $P = 5$ and $P = 3$ respectively.

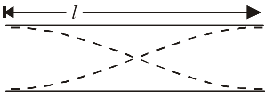
25. **(d)** $n_1 = \frac{1}{2\ell}\sqrt{\left(\frac{T}{4\pi r^2\rho}\right)}$ and $n_2 = \frac{1}{4\ell}\sqrt{\left(\frac{T}{\pi r^2\rho}\right)}$

$n = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$ [where $\frac{\lambda}{2}$ = length of string]

$$\therefore \frac{n_1}{n_2} = 2\times\frac{1}{2} = 1 \left[\because m = \frac{\text{mass}}{\text{length}} = \frac{\rho\times A\times \text{length}}{\text{length}} = \rho A\right]$$

26. **(a)** **For both end open organ pipe**

Fundamental frequency



$\nu_1 = \frac{c}{2\ell}$ ...(i)

For one end closed organ pipe

For third harmonic frequency

$\nu_2 = 3\left(\frac{c}{4l}\right)$ ... (ii)

Given $\nu_2 - \nu_1 = 100$ ... (iii)

From eq. (i) and (ii)

$\frac{\nu_2}{\nu_1} = \frac{3/4}{1/2} = \frac{3}{2} \Rightarrow \nu_1 = \frac{2}{3}\nu_2$ ...(iv)

Again, solving eq. (iii) & (iv) we get, $\nu_1 = 200$ Hz.

27. **(c)** Comparing the given equation, $y\,(x,\,t) = 0.02 \cos\left(50\pi t + \frac{\pi}{2}\right) \cos(10\pi x)$

$y\,(x, t) = A \cos(\omega t + \pi/2) \cos kx$

If $kx = \pi/2$, a node occurs;

$\therefore$ $10\pi x = \pi/2 \Rightarrow x = 0.05$ m

If $kx = \pi$, an antinode occurs

$\Rightarrow$ $10\pi x = \pi \Rightarrow x = 0.1$ m

Also speed of wave

$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5\,\text{m/s}$ and $\lambda = 2\pi/k = 2\pi/10\pi = 0.2$ m

28. **(a)** *In air* : $T = mg = \rho Vg$

$\therefore$ $f = \sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{\rho Vg}{m}}$ ... (i)

when the object is half immersed in water.

$T = mg$ – upthrust

$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$

$\therefore$ $f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}}$

$\frac{f'}{f} = \sqrt{\frac{2\rho - \rho_\omega}{2\rho}}$ $f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2}$

$= 300\left[\frac{2\rho - 1}{2\rho}\right]^{1/2}$ Hz

29. **(c)** Stationary wave is produced when two waves travel in opposite direction.

Now, $y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$

$\therefore$ $y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.

$y = -a\cos(kx - \omega t)$ cannot be as their directions are not opposite.

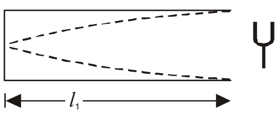
30. **(c)** When tube was open at both ends, then $\frac{\lambda}{2} = \ell$

$\therefore$ $f = \frac{v}{\lambda} = \frac{v}{2\ell}$

when tube is half dipped in water, then $\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$

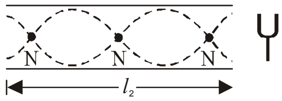
$\therefore f' = \frac{v}{\lambda'} = \frac{v}{2\ell} = f$

31. **(c)** For closed organ pipe,



$\nu_1 = \frac{v}{4\ell_1}$ for first harmonic

For open organ pipe,



$\nu_2 = \frac{3v}{2\ell_2}$ for third harmonic

$\because$ $\nu_1 = \nu_2$ $\therefore$ $\frac{v}{4\ell_1} = \frac{3v}{2\ell_2}$ $\Rightarrow$ $\frac{\ell_1}{\ell_2} = \frac{1}{6}$

32. **(a)** For tube closed at are end

$\frac{\lambda}{4} = \ell$ (Fundamental mode) $\therefore$ $\lambda = 4\ell$

For open tube

$\frac{\lambda'}{2} = \ell$ (Fundamental mode) $\therefore$ $\lambda' = 2\ell$

$\therefore$ $\nu = \frac{c}{\lambda} = \frac{c}{4\ell} = 512$ Hz (given)

and $\nu' = \frac{c}{\lambda'} = \frac{c}{2\ell} = 2\left(\frac{c}{4\ell}\right) = 2 \times 512 = 1024$ Hz.

33. For closed organ pipe,

$f = n\left(\frac{v}{41}\right)$ where, $n = 1, 3, 5$......

$\therefore$ $l = \frac{nv}{4f}$

For $n = 1$, $l_1 = \frac{(1)(330)}{4 \times 264} \times 100\text{ cm} = 31.25$ cm

For $n = 3$, $l_3 = 3l_1 = 93.75$ cm

For $n = 5$, $l_5 = 5l_1 = 156.25$ cm

34. **(5)** The distance between two successive nodes

$D = \frac{\lambda}{2} = \frac{v}{2f} = \frac{\sqrt{T/\mu}}{2f} = \frac{\sqrt{\frac{0.5 \times 0.2}{10^{-3}}}}{2 \times 100} = \frac{10}{2} = 5\text{cm}$

35. **(35.00)** Given,

Denisty of wire, $\sigma = 9 \times 10^{-3}$ kg cm^{-3}

Young's modulus of wire, $Y = 9 \times 10^{10}$ Nm^{-2}

Strain $= 4.9 \times 10^{-4}$

$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\text{Strain}}$

$\therefore \frac{T}{A} = Y \times \text{Strain} = 9 \times 10^9 \times 4.9 \times 10^{-4}$

Also, mass of wire, $m = Al\sigma$

Mass per unit length, $\mu = \frac{m}{J} = A\sigma$

Fundamental frequency in the string

$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{\sigma A}}$

$= \frac{1}{2 \times 1}\sqrt{\frac{9 \times 10^9 \times 4.9 \times 10^{-4}}{9 \times 10^3}}$

$= \frac{1}{2}\sqrt{49 \times 10^{9-4-3}} = \frac{1}{2} \times 70 = 35$ Hz

36. **(0.62 to 0.63)**

Let ℓ_1 = initial length of pipe

ℓ_2 = new length of pipe

V_T = Speed of tuning fork

In closed organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f = \left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2}$

$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V-V_T}\right) = \frac{V}{4\ell_2} \quad \Rightarrow \frac{V - V_T}{V} = \frac{\ell_2}{\ell_1}$

$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V - V_T}{V} - 1 \Rightarrow \frac{\ell_2 - \ell_1}{\ell_1} = \frac{-V_T}{V}$

Pecentage change required in the length of the pipe

$\frac{\ell_2 - \ell_1}{\ell_1} \times 100 = \frac{-2}{320} \times 100 = -0.625\%$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

37. In figure (a) both ends of the tube in air,

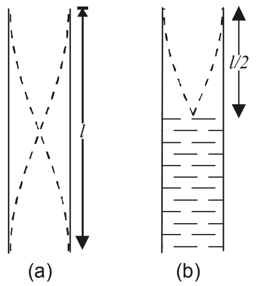
$\frac{\lambda}{2} = \ell \quad \Rightarrow \lambda = 2\ell$

$\therefore \quad f = \frac{c}{\lambda} = \frac{c}{2\ell}$

In figure (b) when half of the tube vertically dipped in water,

$\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$

$\therefore \quad f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f$



38. Fundamental frequency $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ or $f \propto \sqrt{T}$

$f \propto \sqrt{\text{weight of mass } (w)}$

$f' \propto \sqrt{\text{w} - \text{upthrust } (F)}$

$\therefore \quad \frac{f'}{f} = \sqrt{\frac{w-F}{w}}$ or, $f' = f\sqrt{\frac{w-F}{w}}$

Substituting the values, we get

$f' = 260\sqrt{\frac{(50.7)g - (0.0075)(10^3)g}{(50.7)g}} = 240\,Hz$

39. Shortest distance from the wall at which air particles have maximum amplitude of vibration $= \lambda/4 = \frac{0.5}{4} = 0.125$m

$\because \; c = v\lambda \quad \therefore \; \lambda = \frac{c}{v} = \frac{330}{660} = 0.5\,\text{m}$

40. **(a, b, c)** According to question, the length of the air column is varied by changing the level of water in the resonance tube,

so, $(2n+1)\frac{\lambda}{4} = 50.7 + e$ ...(i)

and $(2n+3)\frac{\lambda}{4} = 83.9 + e$ ...(ii)

Dividing eq (i) by (ii)

If $n = 1$, $\frac{3\lambda/4}{5\lambda/4} = \frac{50.7+e}{83.9+e}$ $\quad \therefore 3 \times 83.9 + 3e = 5 \times 50.7 + 5e$

$\Rightarrow \quad 2e = 1.8 \qquad \therefore e = 0.9$ cm

$\therefore \quad \frac{3\lambda}{4} = 50.7 + 0.9 = 51.6 \quad \Rightarrow \lambda = 66.4$ cm $= 0.664$ m

Also speed of sound, $V = v\lambda = 500 \times 0.664\ \text{ms}^{-1} = 332.0\ \text{ms}^{-1}$

41. **(a, c, d)**

There should be a displacement node at $x = 0$ and a displacement antinode at $x = 3$ m. Therefore, $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m.

Speed of wave, $v = \frac{\omega}{k} = 100\ \text{ms}^{-1}$.

Hence options (a), (c) & (d) satisfy the above conditions.

42. **(b, c)**

$y = [0.01 \sin(62.8x)] \cos(628\,t)$. [Given]

From the given equation, $k = \frac{2\pi}{\lambda} = 62.8 \;\therefore\; \lambda = \frac{2\pi}{62.8} = 0.1$m

Length of string, $l = 5 \times \frac{\lambda}{2} = 5 \times \frac{1}{20} = 0.25$ m

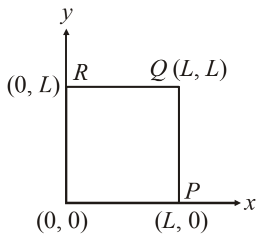
The midpoint M is an antinode and has the maximum displacement = 0.01 m

The fundamental frequency, $v = \frac{v}{2l} = \frac{\omega / k}{2l}$

$= \frac{628}{2 \times 0.25 \times 62.8} = 20\text{Hz}$

43. **(a,b,c)** Standing waves are produced by two identical waves superposing while travelling in opposite direction.

44. **(b,c)** The edges of the plate are clamped, so its displacements along the x and y axes will individually be zero at the edges.



Option (a) :

$u(x, y) = 0$ at $x = L, y = L$

$u(x, y) \neq 0$ at $x = 0, y = 0$

Option (b) :

$u(x, y) = 0$ at $x = 0, y = 0\,[\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L\,[\because \sin \pi = 0]$

Option (c) :

$u(x, y) = 0$ at $x = 0, y = 0\,[\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L\,[\because \sin \pi = 0, \sin 2\pi = 0]$

Option (d) :

$u(x, y) = 0$ at $y = 0, y = L\,[\because \sin 0 = 0, \sin \pi = 0]$

$u(x, y) \neq 0$ at $x = 0, x = L\,[\because \cos 0 = 1, \cos 2\pi = 1]$

45. **(a, b, d)** For closed organ pipe, $v = n\left(\frac{v}{4l}\right)$ where n = 1, 3, 5, 7....

$\therefore \quad \nu = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}, ... = 80, 240, 400....$

46. **(c)** Frequency, $\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ for first mode of vibration

For 'ν' to be maximum, 'ℓ' should be minimum.

String-1 $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

String-2 $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

String-3 $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

String-4 $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

47. **(b)** As $\nu = \frac{p}{2\ell}\sqrt{\frac{T}{m}} \;\therefore\; T = \frac{\nu^2\ell^2 m}{p^2}$

String-1 $T_0 = \frac{f_0^2 4L_0^2\mu}{\ell^2}$

String-2 $T_2 = \dfrac{f_0^2 4\left(\frac{3}{2}\right)^2 L_0^2(2\mu)}{(3)^2} = \dfrac{T_0}{2}$

String-3 $T_3 = \dfrac{f_0^2 4\left(\frac{5}{2}\right)^2 L_0^2(3\mu)}{5^2} = \dfrac{3}{16}T_0$

String-4 $T_4 = \dfrac{f_0^2 4\left(\frac{7}{4}\right)^2 L_0^2(4\mu)}{(14)^2} = \dfrac{T_0}{16}$

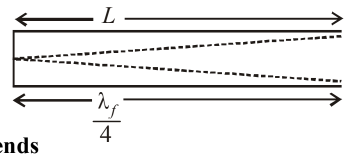
48. **A-p,t; B-p,s; C-q,s; D-q,r**

(A) **Pipe closed at one end**

Waves produced are longitudinal

$$\frac{\lambda_f}{4} = L$$

$\therefore \lambda_f = 4L$

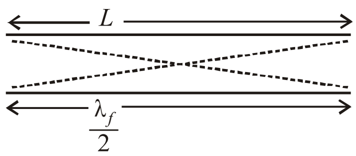


(B) **Pipe open at both ends**

waves produced are longitudinal

$$\frac{\lambda_f}{2} = L$$

$\therefore \lambda_f = 2L$

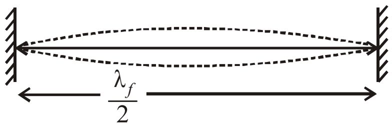


(c) **Stretched wire clamped at both ends**

Waves produced are transverse in nature.

$$\frac{\lambda_f}{2} = L$$

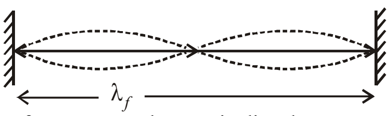
$\therefore \lambda_f = 2L$



(D) **Stretched wave clamped at both ends and at mid point**

Waves produced are transverse in nature

$\lambda_f = L$



49. The standard waveform of a transverse harmonic disturbance

$y = a \sin(\omega t \pm kx \pm \phi)$

Given $v_{max} = a\omega = 3$ m/s ... (i)

$A_{max} = a\omega^2 = 90$ m/s^2 ... (ii)

Velocity of wave $v = 20$ m/s ... (iii)

Dividing eq. (ii) by (i)

$$\frac{a\omega^2}{a\omega} = \frac{90}{3} \Rightarrow \omega = 30 \text{ rad/s} \quad \text{... (iv)}$$

Substituting the value of ω in eq. (i), we get

$$a = \frac{3}{30} = 0.1\text{m} \quad \text{... (v)}$$

Now, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{v/\nu} = \dfrac{\omega}{v} = \dfrac{30}{20} = \dfrac{3}{2}$ ... (vi)

Now putting the value of a, ω and k we get waveform

$$y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$$

50. The string vibrates in fundamental mode therefore $\ell = \dfrac{\lambda}{2}$

or $\lambda = 2\ell$ Since, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2\ell} = \dfrac{\pi}{\ell}$

Amplitude of vibration at a distance x from $x = 0$ $A = a \sin kx$

Mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a\sin kx)^2(2\pi\nu)^2$$
$$= 2\pi^2\mu\nu^2a^2\sin^2 kx\, dx$$

$\because v = \nu\lambda$

$\therefore \nu = \dfrac{v}{\lambda} \Rightarrow \nu^2 = \dfrac{v^2}{\lambda^2} = \dfrac{T/\mu}{4\ell^2}$ $\quad \left[\because v = \sqrt{T/\mu}\right]$

$\therefore dE = 2\pi^2\mu\dfrac{T/\mu}{4\ell^2}a^2\sin^2\left\{\left(\dfrac{\pi}{\ell}\right)x\right\}dx$

$\therefore$ Total energy of the string

$$E = \int dE = \int_0^{\ell} 2\pi^2\mu\frac{T/\mu}{4\ell^2}a^2\sin^2\left(\frac{\pi x}{\ell}\right)dx$$

or, $E = \dfrac{\pi^2 Ta^2}{4\ell}$

51. In the fundamental mode

$$(\ell + 0.6r) = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow v = 4f(\ell + 0.6r)$$

Here, $0.6\,r$ = end-correction in tube

r (= radius of tube) = 2.5 cm

l (= length of tube) = 16 cm

f (= frequency of tuning fork) = 480 Hz

$\therefore$ Velocity of sound in air, $v = 336$ m/s.

52. (a) Frequency of second harmonic in pipe A open at both ends = frequency of third harmonic in pipe B closed at one end

$$\therefore 2\left(\frac{v_A}{2l_A}\right) = 3\left(\frac{v_B}{4l_B}\right)$$

or $\dfrac{v_A}{v_B} = \dfrac{3}{4}$ or $\dfrac{\sqrt{\dfrac{\gamma_A RT_A}{4l_B}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \dfrac{3}{4}$ $\quad (\because l_A = l_B)$

or $\sqrt{\dfrac{\gamma_A}{\gamma_B}}\sqrt{\dfrac{M_B}{M_A}} = \dfrac{3}{4}(\text{as} T_A = T_B) \Rightarrow \dfrac{M_A}{M_B} = \dfrac{\gamma_A}{\gamma_B}\left(\dfrac{16}{9}\right)$

$= \left(\dfrac{5/3}{7/5}\right)\left(\dfrac{16}{9}\right)$ $\quad \left(\gamma_A = \dfrac{5}{3} \text{and } \gamma_B = \dfrac{7}{5}\right)$

$\therefore \dfrac{M_A}{M_B} = \left(\dfrac{25}{21}\right)\left(\dfrac{16}{9}\right) = \dfrac{400}{189}$

(b) Ratio of fundemental frequency in pipe A and in pipe B, which is now closed at both ends

$\dfrac{f_A}{f_B} = \dfrac{v_A/2l_A}{v_B/2l_B} = \dfrac{v_A}{v_B}$ $\quad (\text{as } l_A = l_B)$

$= \dfrac{\sqrt{\dfrac{\gamma_A RT_A}{M_A}}}{\dfrac{\gamma_B RT_B}{M_B}} = \sqrt{\dfrac{\gamma_A}{\gamma_B}\cdot\dfrac{M_B}{M_A}}$ $\quad (\because T_A = T_B)$

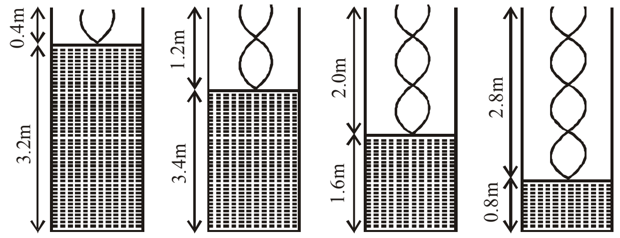
Substituting $\dfrac{M_B}{M_A} = \dfrac{189}{400} : \gamma_A = \dfrac{5}{3}$ and $\gamma_B = \dfrac{7}{3}$

$\therefore \dfrac{f_A}{f_B} = \sqrt{\dfrac{25}{21}\times\dfrac{189}{400}} = \dfrac{3}{4}$

53. Speed of sound, $v = 340$ m/s.

Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0} = 212.5 \Rightarrow \ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\text{m}.$$



In closed pipe only odd harmonics are obtained. Now, let $\ell_1, \ell_2, \ell_3, \ell_4$, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2 \text{ m};$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0 \text{ m}$$

$$7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8 \text{ m};$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6 \text{ m}$$

or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.

Hence heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$$A = \pi(2 \times 10^{-2})^2 = 1.26 \times 10^{-3} \text{ m}^{-2}$$

and $a = \pi(10^{-3})^2 = 3.14 \times 10^{-6} \text{ m}^2$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives,

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

So, rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14 \times 10^{-6}}{1.26 \times 10^{-3}}\sqrt{2 \times 10 \times H}$$

$$\Rightarrow \frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \frac{dH}{\sqrt{H}} = -1.11 \times 10^{-2} dt$$

$$\Rightarrow \int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\int_0^t dt$$

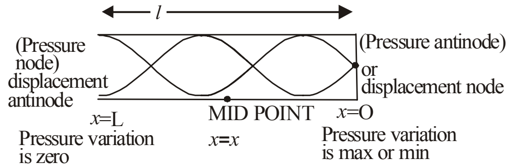
$$\Rightarrow 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.1 \times 10^{-2}).t$$

or, $t \approx 43\,s$

54. (a) For second overtone of the closed pipe

Frequency, $f = P\left(\frac{v}{4L}\right)$ or, $440 = 5\left(\frac{330}{4L}\right)$

$$\Rightarrow L = \frac{5 \times 330}{4 \times 440} \Rightarrow L = \frac{15}{16} \text{ m.}$$



(b) At any position x, the pressure

$\Delta P = \Delta P_0 \cos kx \cos \omega t$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos \frac{2\pi}{\lambda}x$

For $x = \frac{L}{2} = \frac{15}{2 \times 16} = \frac{15}{32}$ m (mid point)

$$\text{Amplitude} = \Delta P_0 \cos\left[\frac{2\pi}{(330/440)} \times \frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure $\because \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end of pipe

Maximum Pressure, $P_{max} = P_0 + \Delta P_0$

Minimum Pressure, $P_{min} = P_0 - \Delta P_0$

55. (i) $y = 4\sin\frac{\pi x}{15}\cos(96\pi t)$

Here amplitude, $A = 4\sin\left(\frac{\pi x}{15}\right)$

At $x = 5$ cm

Amplitude or maximum displacement,

$$A = 4\sin\left(\frac{\pi \times 5}{15}\right) = 4 \times 0.866 = 3.46 \text{ cm}$$

(ii) Nodes are the position where $A = 0$

$$\therefore \sin\left(\frac{\pi x}{15}\right) = 0 = \sin n\pi \quad \therefore \quad x = 15\text{ n}$$

where $n = 0, 1, 2$ $x = 15$ cm, 30 cm, 60 cm, ...

(iii) At $x = 7.5$ cm, $t = 0.25$ cm velocity of particle

$$v = \frac{dy}{dt} = 4\sin\left(\frac{\pi x}{15}\right)[-96\pi \sin(96\pi t)]$$

$$v = 4\sin\left(\frac{\pi \times 7.5}{15}\right)[-96\pi \sin(96\pi \times 0.25)]$$

$$= 4\sin\left(\frac{\pi}{2}\right)[-96\pi \sin(24\pi)] = 0$$

(iv) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos[96\pi t]$

$$= 2\left[2\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$$

$$= 2\left[\sin\left(96\pi t + \frac{\pi x}{15}\right) - \sin\left(96\pi t - \frac{\pi x}{15}\right)\right]$$

$$= 2\sin\left(96\pi t + \frac{\pi x}{15}\right) - 2\sin\left(96\pi t - \frac{\pi x}{15}\right)$$

$= y_1 + y_2$

Hence components waves

$$y_1 = 2\sin\left(96\pi t + \frac{\pi x}{15}\right)$$

and $y_2 = -2\sin\left(96\pi t - \frac{\pi x}{15}\right)$

56. Tube open at both ends

Length of tube l = 48 cm = 0.48m fundamental frequency, f = 320Hz velocity of sound in air v = 320 m/s

$$v = \frac{v}{2(\ell + 0.6D)} \quad \therefore \ 320 = \frac{320}{2(0.48 + 0.6 \times D)}$$

$0.48 + 0.6\,D = 0.5 \quad \Rightarrow 0.6\,D = 0.02$

$\Rightarrow \quad D = \dfrac{0.02}{60} \times 100 \text{ cm} = 3.33 \text{ cm}$ (Diameter of tube)

Tube closed at one end

Frequency, $v = \dfrac{v}{4(\ell + 0.3D)} = \dfrac{320}{4(0.48 + 0.3 \times 0.033)} \approx 163$ Hz

Topic-3: Beats, Interference and Superposition of Waves

1. **(a)**

2. **(c)** Beat frequency
= difference in frequencies of two waves
= 11 – 9 = 2 Hz

3. **(d)**

4. **(a)** Probable frequencies of tuning fork be $n \pm 5$

Frequency of sonometer wire, $n \propto \dfrac{1}{l}$

$\therefore \quad \dfrac{n+5}{n-5} = \dfrac{100}{95} \Rightarrow 95(n+5) = 100(n-5)$

or, $95\,n + 475 = 100\,n - 500$ or, $5\,n = 975$

or, $n = \dfrac{975}{5} = 195$ Hz

5. **(a)** With increase in tension, the frequency produced by string increases. As the beats/sec decreases therefore frequency of tuning fork

$$f = 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4 = 344\text{Hz}$$

6. **(b)** For a string frequency $f = \dfrac{1}{2\ell}\sqrt{\dfrac{F}{\mu}}$

$\therefore$ Time period, $T = 2\ell\sqrt{\dfrac{\mu}{F}}$

$F = 1.6$ N, $\mu = \dfrac{\text{mass}}{\text{length}} = \dfrac{10^{-2}}{0.4} = 2.5 \times 10^{-2}$

$\therefore\ T = 2 \times 0.4\sqrt{\dfrac{2.5 \times 10^{-2}}{1.6}} = 0.1$ sec.

The time required for constructive interference equal to the time period of a wave pulse, hence $\Delta t = 0.1$s

7. **(b)** Given equation, $y = 4\cos^2\left(\dfrac{t}{2}\right)\sin(1000\,t)$

$= 2\left(2\cos^2\dfrac{t}{2}\sin 1000t\right)$

or, $y = 2\,[\cos t + 1] \sin 1000t$ ...(1)

or, $y = 2\cos t \sin 1000t + 2\sin 1000t$ ...(2)

or, $y = \sin 1001t + \sin 999t + 2\sin 1000t$ ...(3)

Hence, for the given periodic motion, three independent harmonic motions are superposed.

8. **(5)** Here the phase difference between the two waves,

$\phi = \pi/2$

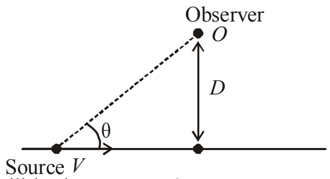
Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\dfrac{\pi}{2}} = \sqrt{16 + 9 + 0} = \mathbf{5}$

Topic-4: Musical Sound and Doppler's Effect

1. **(b)** Frequency heard by the observer

$$v_{\text{observed}} = \left(\frac{v_{\text{sound}}}{v_{\text{sound}} - v\cos\theta}\right)v_0$$



Initially θ will be less so $\cos\theta$ more.

$\therefore\ v_{\text{observed}}$ more, then it will decrease.

2. **(a)** Let f_1 be the frequency heard by wall, $f_1 = \left(\dfrac{v}{v - v_c}\right)f_0$

Here, v = Velocity of sound,
v_c = Velocity of Car,
f_0 = actual frequency of car horn

Let f_2 be the frequency heard by driver after reflection from wall.

$$f_2 = \left(\frac{v + v_c}{v}\right)f_1 = \left(\frac{v + v_c}{v - v_c}\right)f_0$$

$$\Rightarrow 480 = \left[\frac{345 + v_c}{345 - v_c}\right]440 \Rightarrow \frac{12}{11} = \frac{345 + v_c}{345 - v_c}$$

$\Rightarrow v_c = 54$ km/hr

3. **(a)** From the Doppler's effect of sound, frequency appeared at wall

$f_w = \dfrac{330}{330 - v} \cdot f$ ...(i)

Here, v = speed of bus,
f = actual frequency of source

Frequency heard after reflection from wall (f') is

$$f' = \frac{330 + v}{330} \cdot f_w = \frac{330 + v}{330 - v} \cdot f$$

$$\Rightarrow 490 = \frac{330 + v}{330 - v} \cdot 420$$

$\Rightarrow v = \dfrac{330 \times 7}{91} \approx 25.38$ m/s = 91 km/s

4. **(c)** From Doppler's effect, frequency of sound heard (f_1) when source is approaching

$f_1 = f_0\dfrac{c}{c - v}$

Here, c = velocity of sound
v = velocity of source

Frequency of sound heard (f_2) when source is receding

$f_2 = f_0\dfrac{c}{c + v}$

Beat frequency $= f_1 - f_2$

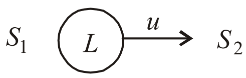
$\Rightarrow \quad 2 = f_1 - f_2 = f_0 c\left[\dfrac{1}{c - v} - \dfrac{1}{c + v}\right]$

$= f_0 c \dfrac{2v}{c^2\left[1 - \dfrac{v^2}{c^2}\right]}$

For $c >> v$

$\Rightarrow \quad v = \dfrac{2c}{2f_0} = \dfrac{c}{f_0} = \dfrac{350}{1400} = \dfrac{1}{4}$ m/s

5. **(c)** $f_1 = f\dfrac{v - v_0}{v}$ and $f_2 = f\dfrac{v + v_0}{v}$



But frequency,

$f_2 - f_1 = f \times \dfrac{2v_0}{v}$

or $10 = 660 \times \frac{2u}{330}$

$\therefore$ $u = 2.5$ m/s.

6. **(b)** Frequency of sound source $(f_0) = 500$ Hz

When observer is moving away from the source

Apparent frequency $f_1 = 480 = f_0\left(\frac{v - v'_0}{v}\right)$ ....(i)

And when observer is moving towards the source

$f_2 = 530 = f_0\left(\frac{v - v''_0}{v}\right)$ ....(ii)

From equation (i)

$480 = 500\left(\frac{300 - v'_0}{300}\right)$

$v'_0 = 12$ m/s

From equation (ii)

$530 = 500\left(1 + \frac{v''_0}{v}\right)$ $\therefore V''_0 = 18$ m/s

7. **(a)** $f' = f\frac{v - v_0}{v + v_s}$

or $2000 = f\frac{340 - 20}{340 + 20}$ $\therefore f = 2250$ Hz.

8. **(b)** According to Doppler's effect, when source is moving but observer at rest

$f_{app} = f_0\left[\frac{V}{V - V_s}\right] \Rightarrow f_1 = f_0\left[\frac{340}{340 - 34}\right]$

and, $f_2 = f_0\left[\frac{340}{340 - 17}\right]$

$\therefore \frac{f_1}{f_2} = \frac{340 - 17}{340 - 34} = \frac{323}{306}$ or, $\frac{f_1}{f_2} = \frac{19}{18}$

9. **(a)** Frequency of the sound produced by open flute.

$f = 2\left(\frac{v}{2\ell}\right) = \frac{2 \times 330}{2 \times 0.5} = 660 Hz$

Velocity of observer, $v_0 = 10 \times \frac{5}{18} = \frac{25}{9} m/s$

As the source is moving towards the observer therefore, according to Doppler's effect.

$\therefore$ Frequency detected by observer,

$f' = \left[\frac{v + v_0}{v}\right] f = \left[\frac{\frac{25}{9} + 330}{330}\right] 660$

$= \frac{2995}{9 \times 330} \times 660$ or, $f' = 665.55 \simeq 666$ Hz

10. **(d)** We know that the apperent frequency

$f' = \left(\frac{v - v_0}{v - v_s}\right) f$ from Doppler's effect

where $v_0 = v_s = 30$ m/s, velocity of observer and source

Speed of sound v = 330 m/s

$\therefore f' = \frac{330 + 30}{330 - 30} \times 540 = 648$ Hz.

$\because$ Frequency of whistle (f) = 540 Hz.

11. **(d)** $f_1 = f\left[\frac{v}{v - v_s}\right] = f \times \frac{320}{300} Hz$

$f_2 = f\left[\frac{v}{v + v_s}\right] = f \times \frac{320}{340} Hz$

$\left(\frac{f_2}{f_1} - 1\right) \times 100 = \left(\frac{300}{340} - 1\right) \times 100 \simeq 12\%$

12. **(d)** **Given:** Frequency of sound produced by siren, $f = 800$ Hz

Speed of observer, $u = 2$ m/s

Velocity of sound, $v = 320$ m/s

No. of beats heard per second = ?

No. of extra waves received by the observer per second = $\pm 4\lambda$

$\therefore$ No. of beats/ sec

$= \frac{2}{\lambda} - \left(-\frac{2}{\lambda}\right) = \frac{4}{\lambda} = \frac{2 \times 2}{\frac{320}{800}}$ $\left(\because \lambda = \frac{V}{f}\right)$

$= \frac{2 \times 2 \times 800}{320} = 10$

13. **(c)** f = 500 Hz

$A \xrightarrow{} 4$ m/s C B

Listener

Case 1 : When source is moving towards stationary listener

apparent frequency $\eta' = \eta\left(\frac{v}{v - v_s}\right) = 500\left(\frac{340}{336}\right) = 506$ Hz

Case 2 : When source is moving away from the stationary listener

$\eta'' = \eta\left(\frac{v}{v + v_s}\right) = 500\left(\frac{340}{344}\right) = 494$ Hz

In case 1 number of beats heard is 6 and in case 2 number of beats heard is 18 therefore frequency of the source at B = 512 Hz

14. **(a)** Apparent frequency of the siren heard by the car driver, f'

$= f\left[\frac{v + v_o}{v - v_s}\right]$

Here v = 320 m/s (given)

$v_0 = v_s = 36 \times \frac{5}{18} = 10$ m/s

or , $f' = 8\left[\frac{320 + 10}{320 - 10}\right] = 8 \times \frac{33}{31} \approx 8.5\,\text{kHz}$

15. **(b)** f_1= frequency of the police car heard by motorcyclist, f_2 = frequency of the siren heard by motorcyclist.

$f_1 = \frac{330 - v}{330 - 22} \times 176;\ f_2 = \frac{330 + v}{330} \times 165;$

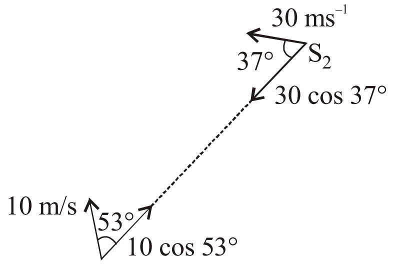
$\because$ $f_1 - f_2 = 0 \therefore v = 22$ m/s

16. **(b)** Using the formula $f' = f\left(\frac{v_A + v}{v}\right)$

$\frac{v_A + v}{v} = \frac{5.5}{5}$ and $\frac{V_B + V}{V} = \frac{6}{5}$ $\therefore \frac{v_B}{v_A} = 2$

17. **(8.13)** Apparent frequency heard by observer due to source S_1

$v_1 = v\left[\frac{v + v_0}{v}\right] = 120\left[\frac{330 + 10}{330}\right] = 120 \times \frac{34}{33} = 123.636$ Hz



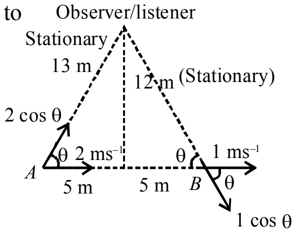
Apparent frequency heard by observer due to source S_2

$$v_2 = v\left[\frac{v+v_0}{v-v_s}\right] = 120\left[\frac{330+10\cos 53^\circ}{330-30\cos 37^\circ}\right]$$

$$\therefore v_2 = 120\left[\frac{330+10\times 0.6}{330-30\times 0.8}\right] = 120\left[\frac{336}{306}\right] = 131.764 \text{ Hz}$$

$\therefore$ Beat frequency, v_b = v2 – v1 131.764 – 123.636 = 8.125 Hz $\approx$ 8.13 Hz

18. **(5.00)**
Apparent frequency at O due to source at – A



$$\nu_A = \nu\left[\frac{v}{v-2\cos\theta}\right]$$

Apparent frequency at 'O' due to source at – B

$$\nu_B = \nu\left[\frac{v}{v+1\cos\theta}\right]$$

$\therefore$ Beat frequency, $v_b = \nu\left[\frac{v}{v-2\cos\theta}\right] - \nu\left[\frac{v}{v+\cos\theta}\right]$

$$= \nu\, v\left[\frac{1}{v-2\cos\theta} - \frac{1}{v+\cos\theta}\right]$$

$$= 1430\times 330\left[\frac{1}{330-2\times\frac{5}{13}} - \frac{1}{330+\frac{5}{13}}\right]$$

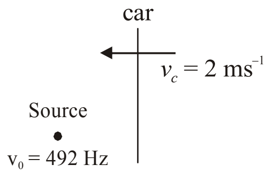
$$= 1430\times 330\times 13\left[\frac{1}{330\times 13-10} - \frac{1}{330\times 13+5}\right]$$

$$= 1430\times 330\times 13\left[\frac{1}{4280} - \frac{1}{4295}\right] \approx 5 \text{ Hz}$$

19. **(6)** Frequency observed at car

$$\upsilon_1 = \upsilon_0\left(\frac{v+v_c}{v}\right)$$

Frequency of reflected sound as observed at the source



$$\upsilon_2 = \upsilon_1\left(\frac{v}{v-v_c}\right) = \upsilon_0\left(\frac{v+v_c}{v-v_c}\right)$$

$\therefore$ Beat frequency $= v_2 - v_0$

$$= \upsilon_0\left[\frac{v+v_c-v+v_c}{v-v_c}\right] = \upsilon_0\left[\frac{v+v_c-v+v_c}{v-v_c}\right]$$

$$= \upsilon_0\left[\frac{2v_c}{v-v_c}\right] = 492\left[\frac{2\times 2}{330-2}\right] = \frac{492\times 4}{328} = 6 \text{ Hz}$$

20. **(3)** Resultant of amplitude, Ar

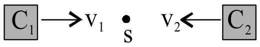
$$= \sqrt{I_0}\left[\sin O + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$$

$$= \sqrt{I_0}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \sqrt{3}\sqrt{I_0}$$

$\therefore \quad I_R = A_R^2 = 3I_0 \qquad \therefore \qquad n = 3$

21. **(7)** Firstly, car will be treated as an observer which is approaching the source. Then it will be treated as a source, which is moving in the direction of sound.
Frequency of sound reflected by the car C_1

$$f_1 = f_0\left(\frac{v+v_1}{v-v_1}\right)$$



And frequency of sound reffected by the car C_2

$$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$$

$$\therefore\ f_1 - f_2 = \left(\frac{1.2}{100}\right)f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$$

or $\left(\frac{1.2}{100}\right)f_0 = \frac{2v(v_1-v_2)}{(v-v_1)(v-v_2)}f_0$

$(v - v_1) = (v - v_2) \approx v$ as v_1 and v_2 are very very less than v.

$$\therefore \left(\frac{1.2}{100}\right)f0 = \frac{2(v_1-v_2)}{v}f_0$$

or $(v_1 - v_2) = \frac{v-1.2}{200} = \frac{330\times 1.2}{200} = 1.98\,\text{ms}^{-1} = 7\text{kmh}^{-1}$

22. **(0.62 to 0.63)**
Let ℓ_1 = initial length of pipe
ℓ_2 = new length of pipe
V_T = Speed of tuning fork

In closed organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f\left(\frac{V}{V-V_T}\right) = \frac{V}{4\ell_2}$

$$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V-V_T}\right) = \frac{V}{4\ell_2} \Rightarrow \frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$$

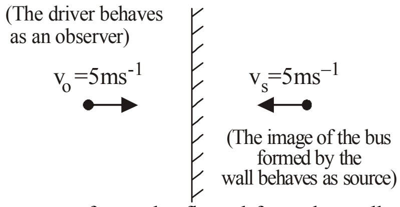
$$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1 \Rightarrow \frac{\ell_2-\ell_1}{\ell_1} = \frac{-V_T}{V}$$

Pecentage change required in the length of the pipe

$$\frac{\ell_2-\ell_1}{\ell_1}\times 100 = \frac{-2}{320}\times 100 = -0.625\%$$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

23. The observer and source are moving towards each other. The image of the source serves as source of reflected sound.

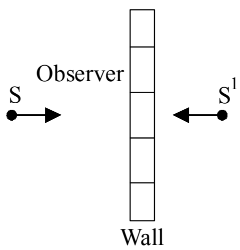


The frequency of sound reflected from the wall

$$\nu' = \nu\left[\frac{v+v_0}{v-v_s}\right] \Rightarrow \nu' = 200\left[\frac{342+5}{342-5}\right] \simeq 206 \text{ Hz.}$$

$\therefore$ Frequency of beats $= \nu' - \nu = 206 - 200 = 6$ Hz.

24. False, if the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound. Thus in both the cases, one sound coming directly from the source and the other coming after reflection will have the same apparent frequency. Hence, no beats will be heard.



25. False, intensity or loudness of sound, $I = \frac{1}{2}\rho V \omega^2 A^2$
Also intensity varies as distance from the point source as $I \propto \frac{1}{r^2}$
As none of the parameters are changing in case of a clear night or a clear day, so the intensity will remain the same.

26. **A → q; B → p; C → r**

(A) Pitch q.	frequency
(B) quality	p. waveform
(C) loudness	r. intensity

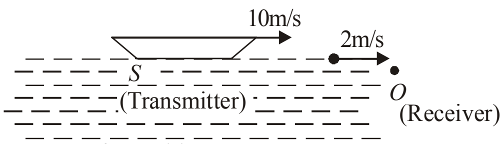
27. By Doppler's formula

$$v' = v\left[\frac{v + v_m \pm v_0}{v + v_m \pm v_s}\right]$$

Sign convention for V_m is as follows :
If medium is moving from s to O then + ve and vice versa.
Similarly v_0 and v_s are positive if these are directed from S to O and vice versa.

(a) Velocity of sound in water $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}}$

$= 1445$ m/s



Frequency of sound in water

$$v = \frac{v_{water}}{T_{water}} = \frac{1445}{14.45 \times 10^{-3}} = 10^5 \text{ Hz}$$

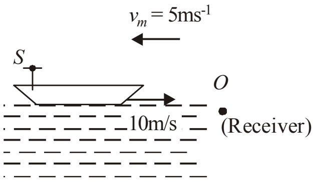
Frequency is independent of medium $\therefore$ $v_{water} = v_{air}$
$v_m = +2$ m/s; $v_0 = 0$; $v_s = 10$ m/s

Applying Doppler's formula, $v' = v\left[\frac{v + v_m - v_0}{v + v_m - v_s}\right]$

$\therefore \quad v' = (1 \times 10^5)\left[\frac{1445 + 2 - 0}{1445 + 2 - 10}\right]$ $\therefore$ $v' = 1.007 \times 10^5$ Hz

(b) In air velocity of sound

$$= \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4) \times (8.31) \times 293}{28.8 \times 10^{-3}}} = 344 \text{ m/s}$$



Applying Doppler's formula $v' = v\left(\frac{v - v_m - v_0}{v - v_m - v_s}\right)$

$\therefore \quad v' = (1 \times 10^5)\left[\frac{344 - 5 - 0}{344 - 5 - 10}\right] = 1.03 \times 10^5$ Hz

28. The motorist hears the beat frequency as he receives two different frequencies one directly from the sound source or band f and other reflected from the wall.
Apparent frequency when direct sound of band is heard by the motorist

$$f' = \left(\frac{v + v_m}{v + v_b}\right)f$$

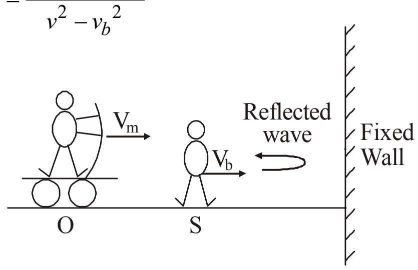
Again apparent frequency when heard both direct and reflected sound by the motorist

$$f'' = \left(\frac{v + v_m}{v - v_b}\right)f$$

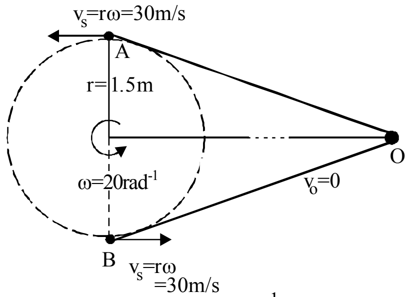
Hence beat frequency heard by motorist,

$$f_b = f'' - f' = \left(\frac{v + v_m}{v + v_b}\right)f - \left(\frac{v + v_m}{v - v_b}\right)f$$

or, $f_b = \frac{2v_b(v + v_m)}{v^2 - v_b^2}$



29. The whistle which is emitting sound is being rotated in a circle. The observer is at large distance from the whistle i.e., O



Given: $r = 1.5$ m and $\omega = 20$ rads^{-1}
Speed of source, $v_s = r\omega = 1.5 \times 20 = 30$ ms^{-1}
When the source is at the position A, then the frequency heard by the observer will be maximum

$$v' = v\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330 - 30}\right] = 484 \text{ Hz}$$

When the source is at the position B, then the frequency heard by the observer will be minimum

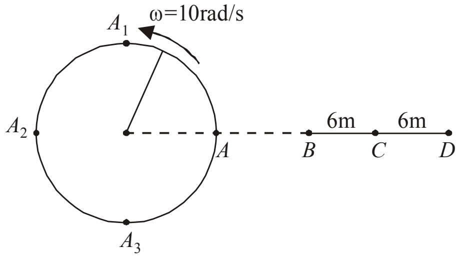
$$v'' = v\left[\frac{v}{v + v_s}\right] = 440\left[\frac{330}{330 + 30}\right] = 403.3 \text{ Hz}$$

Hence the range of frequencies heard by the observer = 403.3 Hz to 484 Hz.

30. The angular frequency of the detector, $\omega = 2\pi v = 2\pi \times \frac{5}{\pi}$

$= 10$ rad / s

The angular frequency of the detector and the source of sound are equal.



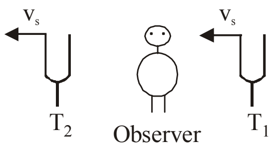
⇒ When the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

$$v_{min} = v\left[\frac{v - v_0}{v + v_s}\right] = 340 \times \frac{(340-60)}{(340+30)} = 257.3\,\text{Hz}$$

($\because v_0 = A\omega = 6 \times 10 = 60$ m/s
$v_s = R\omega = 3 \times 10 = 30$ m/s)

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$$v_{max} = v\left[\frac{v + v_0}{v - v_s}\right] = 340 \times \frac{(340+60)}{(340-30)} = 438.7\,\text{Hz}$$

31. 

For fork T_1 moving towards observer,

$$f' = \frac{v}{(v - v_s)} f \quad \text{where } v_s = \text{velocity of fork.}$$

For fork T_2 moving away from observer,

$$f'' = \frac{v}{v + v_s} f \quad \text{where } v = \text{velocity of sound.}$$

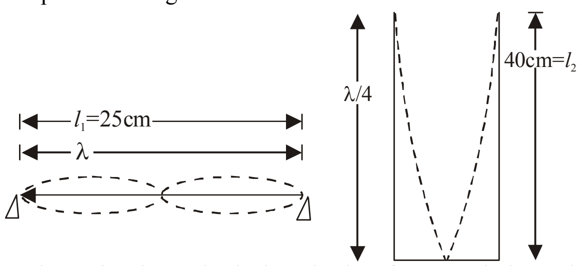
$\therefore$ Beat frequency $= f' - f'' = 3$

$$\text{or } 3 = \frac{vf}{v - v_s} - \frac{vf}{v + v_s} \quad \text{or } 3 = vf\left[\frac{(v+v_s)-(v-v_s)}{(v-v_s)(v+v_s)}\right]$$

$$\text{or } 3 = \frac{vf \times 2v_s}{v^2 - v_s^2} \quad \text{or } 3 = \frac{vf \times 2v_s}{v^2} \text{ as } v_s << v$$

$$\text{or } v_s = \frac{3v^2}{2vf} = \frac{3v}{2f} \text{ or } v_s = \frac{3 \times 340}{2 \times 340} = 1.5\text{m/s}$$

$\therefore$ Speed of tuning fork = 1.5 m/s

32. 

By decreasing the tension in the string beat frequency is decreasing.
$\therefore$ First overtone frequency of string – fundamental frequency of closed pipe = 8

$$\therefore \ 2\left(\frac{v_1}{2l_1}\right) - \left(\frac{v_2}{4l_2}\right) = 8 \text{ or } v_1 = l_1\left[8 + \frac{v_2}{4l_2}\right]$$

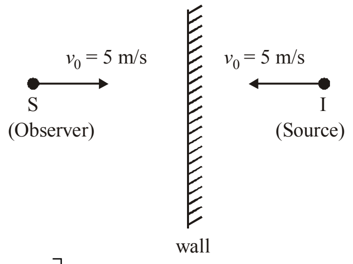
Substituting the value, we have

$$v_1 = 0.25\left[8 + \frac{320}{4 \times 0.4}\right] = 52 m/s$$

$$\text{Now, } v_1 = \sqrt{\frac{T}{\mu}} \therefore T = \mu v_1^2$$

$$\text{or, } T = \left(\frac{m}{l}\right)v_1^2 = \left(\frac{2.5 \times 10^{-3}}{0.25}\right)(52)2 = 27.04\,\text{N}$$

33. If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source (I) in the reflecting surface will become the source of the reflected sound.



$$v' = v\left[\frac{c - v_0}{c - v_s}\right]$$

v_0, v_s are + ve if they are directed from source to the observer and – ve if they are directed from observer to source.

$$v' = 256\left[\frac{330-(-5)}{330-5}\right] = 264\text{ Hz}$$

$\therefore$ Beat frequency $= v' - v = 264 - 256 = 8$

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. **(d)** Here, $v = \frac{v}{4l} = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} \times \frac{1}{4l} \Rightarrow v = v\lambda = v \times 4l$

$\Rightarrow$ v (244) × 4 × l = 336.7 m/s to 346.5 m/s
[$\therefore l = 0.350 \pm 0.005$]

For monatomic gas γ =1.67

$$\therefore \ v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}$$

$$= \sqrt{167RT} \times \sqrt{\frac{10}{M}} = 640\sqrt{\frac{10}{M}}$$

For Neon M = 20 $\therefore$ $v = 640 \times \frac{7}{10} = 448\ \text{ms}^{-1}$

For Argon M = 36, $\therefore$ $v = 640 \times \frac{17}{32} = 340\ \text{ms}^{-1}$

For diatomic gas γ = 1.4

$$v = \sqrt{140RT}\sqrt{\frac{10}{M}} = 590 \times \sqrt{\frac{10}{M}}$$

For Oxygen $M = 32$ $\therefore$ $v = 590 \times \frac{9}{16} = 331.87\ \text{ms}^{-1}$

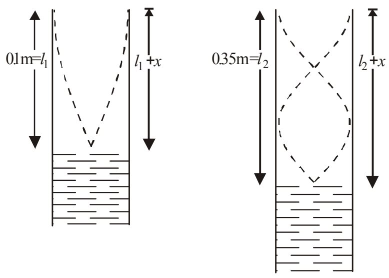
For Nitrogen $M = 28$ $\therefore$ $v = 590 \times \frac{3}{5} = 354\ \text{ms}^{-1}$

2. **(b)** Let x be the end correction.

$$\ell_1 + x = \frac{\lambda}{4} \quad \text{or, } \lambda = 4(\ell_1 + x)$$

$$(\ell_2 + x) = \frac{3\lambda}{4} \quad \text{or } \lambda = \frac{4}{3}(\ell_2 + x)$$

$$\therefore \ v_1 = \frac{v}{\lambda_1} = \frac{v}{4(\ell_1 + x)} \quad \therefore \ v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(\ell_2 + x)}$$



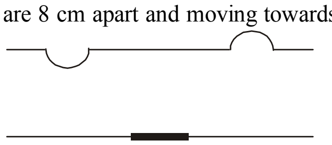
Given $\nu_1 = \nu_2$

$$\therefore \quad \frac{v}{4(\ell_1 + x)} = \frac{3v}{4(\ell_2 + x)} \text{ or, } \quad x = 0.025 \text{ m}$$

3. **(c)** Energy, $E \propto A^2\nu^2$ where A = amplitude and ν = frequency.
Also $\omega = 2\pi\nu \Rightarrow \omega \propto \nu$
Experiment = 1 : Amplitude = A and $\nu_1 = \nu$
Experiment = 2 : Amplitude = A and $\nu_2 = 2\nu$

$$\frac{E_2}{E_1} = \frac{A^2\nu_2^2}{A^2\nu_1^2} = 4 \quad \therefore E_2 = 4E_1$$

4. **(b)** The speed of each pulse is 2cm/s and initially two pulses are 8 cm apart and moving towards each other.



After two seconds pulses will overlap each other.
According to superposition principle the string will not have any distortion and will be straight.
Hence there will be no P.E. The total energy will be only kinetic.

5. **(a)** Velocity of sound by a stretched string $v = \sqrt{\frac{T}{\mu}}$
From Hooke's law, tension in string $T \propto$ extension x

$$\therefore \frac{v}{v'} = \sqrt{\frac{T}{T'}} \qquad v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22 \text{ v}$$

6. Speed of sound waves in water is greater than in air, so water behave as rarer medium and hence sound wave will be refract into water away from the normal.

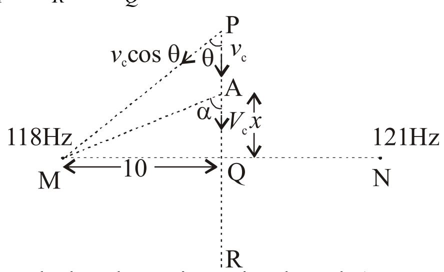
7. **(a, b, c)**

$$\nu_P = (V_n - V_m)\left[\frac{v + v_c \cos\theta}{v}\right] = 121 - 118\left[\frac{v + v_c\cos\theta}{v}\right]$$

$$\nu_Q = (v_N - v_M) = 121 - 118 = 3$$

$$\nu_R = (v_N - v_M)\left[\frac{v + v_c\cos\theta}{v}\right] = (121 - 118)\left[\frac{v - v_c\cos\theta}{v}\right]$$

$$\therefore \nu_P + \nu_R = 2\nu_Q$$



In general when the car is passing through A

$$\nu = 3\left[\frac{v + v_c\cos\alpha}{v}\right] \qquad \ldots(i)$$

$$\therefore \frac{d\nu}{d\alpha} = -3\left[\frac{v_c \sin\alpha}{v}\right] \quad \left|\frac{d\nu}{d\alpha}\right| \text{ is max when } \sin\alpha = 1$$

i.e., α =90° (at Q)

From eq. (i) $\frac{d\nu}{dt} = \frac{3v_c}{v}(-\sin\alpha)\frac{d\alpha}{dt}$ … (ii)

Also, $\tan\alpha = \frac{10}{x} \quad \therefore \quad \sec^2\alpha\frac{d\alpha}{dt} = -\frac{10}{x^2}\frac{dx}{dt}$

$$\therefore \quad \frac{d\alpha}{dt} = \frac{-10v}{x^2\sec^2\alpha} \qquad \ldots(iii)$$

From eq. (ii) & (iii)

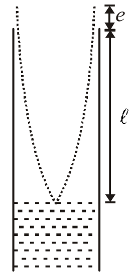
$$\frac{d\nu}{dt} = -\frac{3v_c}{v}\sin\alpha \times \left(\frac{-10v}{x^2\sec^2\alpha}\right) = \frac{30V_c\sin\alpha}{x^2\sec^2\alpha}$$

$$\therefore \frac{d\nu}{dt} = \frac{30v_c\sin\alpha}{(10\cot\alpha)^2\sec^2\alpha} = 0.3v_c\sin^3\alpha \text{. At } \alpha = 90°$$

$$\frac{d\nu}{dt} = max$$

8. **(b, d)**
When a sound pulse is reflected from open end of a pipe, phase changes by 180°. A high pressure pulse i.e., compression is reflected as a low pressure pulse i.e., rarefaction.
When sound pulse is reflected through a rigid boundary (closed end of a pipe), no phase change occurs so a high pressure pulse is reflected as a high pressure pulse.

9. **(a, d)**



The length of the air column at the first resonance is somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air due to end correction (e).

$$1 + e = \frac{\lambda}{4} \qquad \Rightarrow \qquad 1 = \frac{\lambda}{4} - e$$

Hence at second resonance the length of air column is more as compared to first resonance. Now, longer the length of air column, more is the absorption of energy and lesser is the intensity of sound heard.

10. **(b)** The speed of sound depends on the frame of reference of the observer.
V_{SA} = 340 + 20 = 360 m/s
and V_{SB} = 340 – 30 = 310 m/s

11. **(a)** There is no relative motion between source and observer for the passangers in train A. Since all the passengers in train A are moving with a velocity of 20 m/s therefore the distribution of sound intensity of the whistle by the passengers in train A is uniform.

12. **(a)** For the passangers in train B, source is approaching with velocity 20 m/s and observer is receding with velocity 30 m/s

$$\nu' = \nu_1\left[\frac{v - v_0}{v - v_s}\right] = 800\left[\frac{340 - 30}{340 - 20}\right] = 800 \times \frac{31}{32}$$

$$\nu'' = \nu_2\left[\frac{v - v_0}{v - v_s}\right] = 1120 \times \frac{31}{32}$$

$$\therefore \quad \nu'' - \nu' = (1120 - 800) \times \frac{31}{32} = 320 \times \frac{31}{32} = 310 \text{ Hz.}$$

13. **(a)** 14. **(c)**

15. **(d)** The given equations $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ represents two progressive wave travelling in the same direction along x-axis with slight difference in the frequency.
Comparing it with the standard equation
$y = A\cos(kx - \omega t)$, we get
$\omega_1 = 100\pi \Rightarrow 2\pi f_1 = 100\pi \Rightarrow f_1 = 50$ Hz and

$$K_1 = 0.5\pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4 \text{ m}$$

Wave velocity = $\lambda_1 f_1$ = 200 m/s [Alternatively use $v = \frac{\omega}{K}$]

$\omega_2 = 92\pi \Rightarrow 2\pi f_2 = 92\pi \Rightarrow f_2 = 46$ Hz

Therefore beat frequency = $f_1 - f_2$ = 4 Hz and

$$K_2 = 0.46\pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$$

Wave velocity = $\frac{200}{46} \times 46$ = 200 m/s

At $x = 0$,

$y_1 + y_2 = (A\cos 10\pi t) + (A\cos 92\pi t) = 0$

$\Rightarrow \cos 100\pi t = -\cos 92\pi t = \cos(-92\pi t)$

$= \cos[(2n+1)\pi - 92\pi t \quad \Rightarrow \quad t = \frac{2n+1}{192}$

when $t = 0$, $n = -\frac{1}{2}$ and when $t = 1$, $n = \frac{191}{2} = 95.2$

$\therefore$ In 1 second, $y_1 + y_2 = 0$ at $x = 0$ for 96 times

16. Let the two radio waves be represented by the equations

$y_1 = A\sin 2\pi\nu_1 t$

$y_2 = A\sin 2\pi\nu_2 t$

Since, $A_1 = A_2 = A$ and detector is at $x = 0$

The equation of resultant wave according to superposition principle

$y = y_1 + y_2 = A\sin 2\pi\nu_1 t + A\sin 2\pi\nu_2 t$

$= A[\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t]$

$$= A \times 2\sin\frac{(2\pi\nu_1 + 2\pi\nu_2)t}{2}\cos\frac{(2\pi\nu_1 + 2\pi\nu_2)t}{2}$$

$= 2A\sin\pi(\nu_1 + \nu_2)t\cos\pi(\nu_1 - \nu_2)t$

where the amplitude $A' = 2A\cos\pi(\nu_1 - \nu_2)t$

Now, intensity $\propto$ (Amplitude)^2

$\Rightarrow I \propto A'^2$

$I \propto 4A^2\cos^2\pi(\nu_1 - \nu_2)t$

Intensity will be maximum when

$\cos^2\pi(\nu_1 - \nu_2)t = 1$

or, $\cos\pi(\nu_1 - \nu_2)t = 1$

or, $\pi(\nu_1 - \nu_2)t = n\pi$

$$\Rightarrow \frac{(\omega_1 - \omega_2)}{2}t = n\pi \quad \text{or,} \quad t = \frac{2n\pi}{\omega_1 - \omega_2}$$

$\therefore$ Time interval between two successive maxima

$$\text{or,} \quad \frac{2n\pi}{\omega_1 - \omega_2} - \frac{2(n-1)\pi}{\omega_1 - \omega_2} \quad \text{or,} \quad \frac{2\pi}{\omega_1 - \omega_2} = \frac{2\pi}{10^3}\text{s}$$

Time interval between two successive maximas is $2\pi \times 10^{-3}$ sec

(ii) The detector can detect if resultant intensity $\geq 2A^2$

$\therefore$ Resultant amplitude $\geq \sqrt{2}A$

or, $2A\cos\pi(\nu_1 - \nu_2)t \geq \sqrt{2}A$

or, $\cos\pi(\nu_1 - \nu_2)t \geq \frac{1}{\sqrt{2}}$ or,

$$\cos\left[\frac{(\omega_1 - \omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$$

The detector lies idle when the values of $\cos\left[\frac{(\omega_1 - \omega_2)t}{2}\right]$ is between 0 and $\frac{1}{\sqrt{2}}$

$\therefore \frac{(\omega_1 - \omega_2)t}{2}$ is between $\frac{\pi}{2}$ and $\frac{\pi}{4}$

$\therefore \quad t_1 = \frac{\pi}{\omega_1 - \omega_2}$ and $t_2 = \frac{\pi}{2(\omega_1 - \omega_2)}$

$\therefore$ The time gap $= t_1 - t_2$

$$= \frac{\pi}{\omega_1 - \omega_2} - \frac{\pi}{2(\omega_1 - \omega_2)} = \frac{\pi}{2(\omega_1 - \omega_2)} = \frac{\pi}{2} \times 10^{-3}\text{ s}$$

17. **(a)** Compare the given equation $y_1 = A\cos(ax + bt)$ with the standard equation of a plane progressive wave.

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\nu t\right) \Rightarrow \frac{2\pi}{\lambda} = a \quad \Rightarrow \lambda = \frac{2\pi}{a}$$

Also, $2\pi\nu = b$

$\therefore$ Frequency of incident wave, $\nu = \frac{b}{2\pi}$

(b) The wave is reflected by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave $I' = 0.64\, I$

$\Rightarrow I' \propto A'^2 \Rightarrow 0.64\, I \propto A'^2 \Rightarrow 0.64\, A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of reflected wave

$y_r = 0.8A\cos(ax - bt + \pi) = -0.8A\cos(ax - bt)$

(c) The resultant wave equation

$y = y_i + y_r$

$= A\cos(ax + bt) + [-0.8A\cos(ax - bt)]$

Particle velocity

$v = \frac{dy}{dt} = -Ab\sin(ax + bt) - 0.8Ab\sin(ax - bt)$

$= -Ab[\sin(ax + bt) + 0.8\sin(ax - bt)]$

$= -Ab[\sin ax\cos bt + \cos ax\sin bt$

$+ 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$

$v = -Ab[1.8\sin ax\cos bt + 0.2\cos ax\sin bt]$

The maximum velocity will occur when $\sin ax = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore \quad |v_{max}| = 1.8\,Ab$

Also, $|v_{min}| = 0$

(d) $y = [A\cos(ax + bt)] - [0.8A\cos(ax - bt)]$

$= [0.8A\cos(ax + bt) + 0.2A\cos(ax + bt)]$

$- [0.8A\cos(ax - bt)]$

$= [0.8A\cos(ax + bt) - 0.8A\cos(ax - bt)]$

$+ [0.2A\cos(ax + bt)]$

$$= 0.8A\left[-2\sin\left\{\frac{(ax+bt)+(ax-bt)}{2}\right\}\sin\left\{\frac{(ax+bt)-(ax-bt)}{2}\right\}\right] 0.2A\cos(ax+bt)]$$

$\Rightarrow y = -1.6A\sin ax\sin bt + 0.2A\cos(ax + bt)$

where $(-1.6A\sin ax\sin bt)$ is the equation of a standing wave and $0.2A\cos(ax + bt)$ is the equation of travelling wave.

The wave is travelling in $-x$ direction.

Antinodes are the points where the amplitude is maximum,

i.e., $\sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$

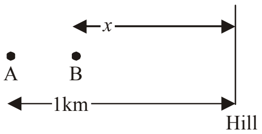
$$\text{or, } ax = \left[n\pi + (-1)^n\frac{\pi}{2}\right] \quad \text{or, } x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$$

18. (i) The frequency of the whistle heard by observer on the hill

$$n' = n\left[\frac{v + v_w}{v + v_w - v_s}\right] = 580\left[\frac{1200 + 40}{1200 + 40 - 40}\right] = 599\text{ Hz}$$

(ii) Let echo from the hill is heard by the driver at B which is at a distance x from the hill sound produced by train at a distance 1 km from the hill.

The time taken by the driver to reach from A to B



$t_1 = \dfrac{1-x}{40}$ ... (i)

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$t_2 = \dfrac{1}{(1200+40)} + \dfrac{x}{(1200-40)}$ ... (ii)

where t_{AH} = time taken by sound from A to hill with velocity (1200 + 40)

t_{HB} = time taken by sound from hill to B with velocity 1200 – 40

From eq. (i) and (ii)

$t_1 = t_2 \Rightarrow \dfrac{1-x}{40} = \dfrac{1}{1200+40} + \dfrac{x}{1200-40}$

$\Rightarrow x = 0.935$ km

The frequency of echo as heard by the driver

$n'' = n\left[\dfrac{(v-v_w)+v_s}{(v-v_w)-v_0}\right]$

$= 580\left[\dfrac{(1200-40)+40}{(1200-40)-40}\right] = 621$ Hz

19. (i) When two identical waves travelling in opposite direction superimpose, we get standing waves.

Hence following two equations will produce standing wave

$z_1 = A\cos(kx - \omega t)$

$z_2 = A\cos(kx + \omega t)$

The resultant wave is given by $z = z_1 + z_2$

$\Rightarrow z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$

$= 2A\cos kx \cos \omega t$

The resultant intensity will be zero when

$2A\cos kx = 0$

$\Rightarrow \cos kx = \cos\dfrac{(2n+1)}{2}\pi$

$\Rightarrow kx = \dfrac{2n+1}{2}\pi \Rightarrow x = \dfrac{(2n+1)\pi}{2k}$ where $n = 0, 1, 2, ...$

(ii) The transverse wave z_1 travelling in $+x$-axis and z_3 travelling in $+y$-axis

$z_1 = A\cos(kx - \omega t)$

$z_3 = A\cos(ky - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive x and positive y axes.

The resultant wave $z = z_1 + z_3$

$z = A\cos(kx - \omega t) + A\cos(ky - \omega t)$

$\Rightarrow z = 2A\cos\dfrac{(x-y)}{2}\cos\left[\dfrac{k(x+y)-2\omega t}{2}\right]$

The resultant intensity will be zero when

$2A\cos\dfrac{k(x-y)}{2} = 0 \Rightarrow \cos\dfrac{k(x-y)}{2} = 0$

$\Rightarrow \dfrac{k(x-y)}{2} = \dfrac{2n+1}{2}\pi$

or, $(x-y) = \dfrac{(2n+1)}{k}\pi$ where n = 0, ± 1, ± 2 etc.

20. Using, $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{T/A}{\Delta\ell/\ell} = \dfrac{T}{\alpha A \Delta\theta}$ $\therefore T = YA\alpha\Delta\theta$

$\left(\because \Delta\ell = \ell\alpha\Delta\theta \Rightarrow \dfrac{\Delta l}{l} = \bar{\alpha}\Delta\theta\right)$

Frequency of fundamental mode of vibration

$\nu = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}} = \dfrac{1}{2\ell}\sqrt{\dfrac{YA\alpha\Delta\theta}{m}}$

$= \dfrac{1}{2\times 1}\sqrt{\dfrac{2\times 10^{11}\times 10^{-6}\times 1.21\times 10^{-5}\times 20}{0.1}} = 11$ Hz

21. **(a)** Using $y = \dfrac{F/A}{\Delta l / l}$ and $\Delta l = l\alpha\Delta\theta$

$\therefore F = YA\alpha\Delta\theta$

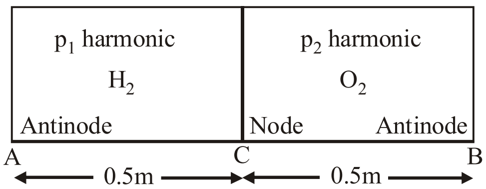
Speed of transverse wave

$v = \sqrt{\dfrac{T}{M}}$ $\left[\text{where } \mu = \text{mass per unit length} = \dfrac{Al\rho}{\ell} = A\rho\right]$

$= \sqrt{\dfrac{YA\alpha\Delta\theta}{A\rho}} = \sqrt{\dfrac{Y\alpha\Delta\theta}{\rho}}$

$\therefore v = \sqrt{\dfrac{1.3\times 10^{11}\times 1.7\times 10^{-5}\times 20}{9\times 10^3}} = 70$ m/s

22. According to question, the diaphragms A and B are set into vibrations of same frequency. Diaphragm C is a node. So, at A and B antinodes are formed.



$\dfrac{v_1}{4\ell}\times p_1 = \dfrac{v_2}{4\ell}\times p_2 \Rightarrow \dfrac{p_1}{p_2} = \dfrac{v_1}{v_2} = \dfrac{3}{11}$ or, $11p_1 = 3p_2$

i.e., The third harmonic in AC is equal to 11th harmonic in CB.

Now, the fundamental frequency in AC

$= \dfrac{v_1}{4\ell} = \dfrac{1100}{4\times 0.5} = 550\,\text{Hz}$

and the fundamental frequency in CB

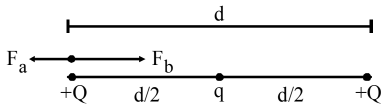
$= \dfrac{v_2}{4\ell} = \dfrac{300}{4\times 0.5} = 550\,\text{Hz}$

$\therefore$ Frequency in $AC = 3\times 550 = 1650$ Hz

and frequency in $CB = 11\times 150 = 1650$ Hz

Topic-1 : Electric Charges and Coulomb's Law

1. **(a)**



Force due to charge + Q, $F_a = \frac{KQQ}{d^2}$

Force due to charge q, $F_b = \frac{KQq}{\left(\frac{d}{2}\right)^2}$

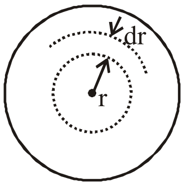
For equilibrium,

$\vec{F}_a + \vec{F}_b = 0 \Rightarrow \frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \quad \therefore q = -\frac{Q}{4}$

2. **(b)** $Q = \int \rho dv = \int_0^R \frac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right)$

$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R = 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$

$Q = 2\pi aA(1 - e^{-2R/a})$



$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$

3. **(d)** Spheres A and B carry equal charge say 'q'

$\therefore$ Force between them, $F = \frac{kqq}{r^2}$

When A and C are touched, charge on both $q_A = q_C = \frac{q}{2}$

Then when B and C are touched, charge on B

$q_B = \frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$

Now, the force between charge q_A and q_B

$F' = \frac{kq_Aq_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$

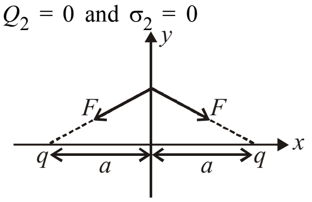
4. **(c)** Inside the cavity net charge is zero.

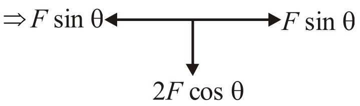
$\therefore$ $Q_1 = 0$ and $\sigma_1 = 0$

There is no effect of point charges $+Q$, $-Q$ and induced charge on inner surface on the outer surface.

$\therefore$ $Q_2 = 0$ and $\sigma_2 = 0$

5. **(a)**





$\Rightarrow F_{net} = 2F\cos\theta$

$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2 + a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$

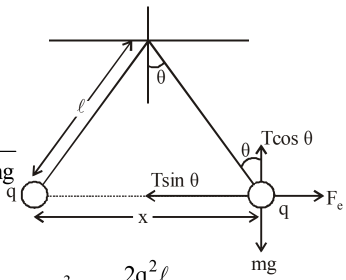
$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2 + a^2)^{3/2}}$ $\quad (\because y \ll a)$

$\Rightarrow \frac{kq^2 y}{a^3}$ So, $F \propto y$

6. **(d)**

In equilibrium, $F_e = T\sin\theta$

$mg = T\cos\theta$



$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$

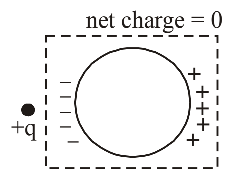
also $\tan\theta \approx \sin = \frac{x/2}{\ell}$

Hence, $\frac{x}{2\ell} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg} \Rightarrow x^3 = \frac{2q^2\ell}{4\pi\epsilon_0 mg}$

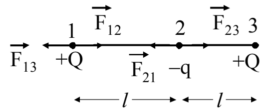
$\therefore x = \left(\frac{q^2\ell}{2\pi\epsilon_0 mg}\right)^{1/3}$

Therefore $x \propto \ell^{1/3}$

7. **(d)** When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface as shown in the figure. But net charge on the sphere is zero as no charge has left or entered the sphere.



8. **(b)** Charge q has to be negative for equilibrium.



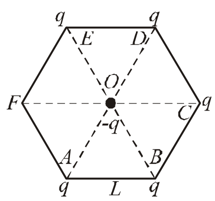
Considering equilibrium of charge $q + 1$

$F_{13} = F_{12}$

$\frac{KQ \times Q}{(2\ell)^2} = \frac{KQ(-q)}{\ell^2} \quad \therefore q = -\frac{Q}{4}$

9. $\left(\frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}\right)$

Force on (–q) at 0 due to charge at D will get cancelled out by force on (–q) due to change on A. Similarly force on –q due to charge at E will get cancelled out due to charge on B.

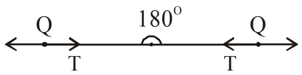


So the net force will be because of charge q on C only at '0'

$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}$ newton and attractive in nature.

10. **(**$180°, \frac{KQ^2}{4L^2}$**)** Here only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°.
The tension in each string will be equal to the electrostatic force of repulsion

Here, $Q_1 = Q_2 = Q$ and $R = 2L$



$$T = \frac{1}{4\pi\varepsilon_0} \times \frac{Q\times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{4L^2}$$

11. **False**
Electric field along axis of ring.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2+x^2\right)^{3/2}} = \frac{1Q\times 2R}{4\pi\varepsilon_0\left[R^2+(2R)^2\right]^{3/2}} = \frac{2QR}{4\pi\varepsilon_0 5\sqrt{5}R^3}$$

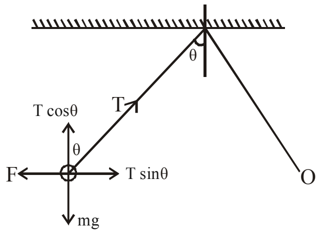
$\therefore$ Force $= -qE$

$\frac{-2QR}{4\pi\varepsilon_0 5\sqrt{5}R^2}$ *i.e.*, The force is not proportional to displacement.

Hence charge particle will not execute simple harmonic motion.

12. **True**
The metallic sphere which gets negatively charged gains electrons and hence its mass increases.
The metallic sphere which gets positively charged loses electrons and hence its mass decreases.

13. **(a, c)** The net electric force on any sphere is lesser but by Coulomb law the force due to one sphere to another remain the same.



In equilibrium
$T\cos\theta = mg$
and $T\sin\theta = F$

$$\therefore \tan\theta = \frac{F}{mg} \quad ...(i)$$

As force between two charge bodies according to coulomb's law does not depend upon the medium, hence force between them remain same.
Hence after immersed in dielectric medium
As given no change in angle θ.

$$T'\sin\theta = \frac{F}{\varepsilon_r} \text{ and } T'\cos\theta = mg\left(1-\frac{\rho_l}{\rho_s}\right)$$

$$\therefore \tan\theta = \frac{F}{mg(1-\frac{\rho_l}{\rho_s})\varepsilon_r} \quad ...(ii)$$

Now equating eqn. (i) & (ii)

$$1-\frac{\rho_l}{\rho_s} = \frac{1}{\varepsilon_r} \Rightarrow 1-\frac{800}{\rho_s} = \frac{1}{21}$$

$\therefore \rho_s = 840$ kg/m^3
Hence option (c) is correct

And tension, $T' = \frac{T}{\varepsilon_r} = \frac{T}{21}$

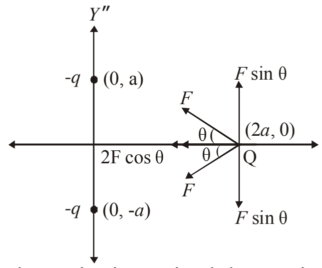
Hence option (d) is incorrect.

14. **(d)** Let us consider the positive charge Q at a distance x from the origin. Let force between Q and $-q$ be F. On resolving this force $F\sin\theta$ cancels out. The resultant force

$$F_R = 2F\cos\theta$$

$$= 2\times\frac{kQq}{(x^2+a^2)}\times\frac{x}{\sqrt{x^2+a^2}}$$

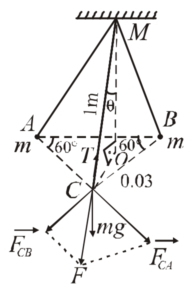
$$= \frac{2kQqx}{(x^2+a^2)^{3/2}}$$

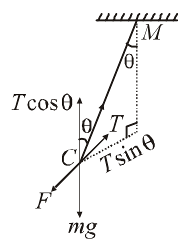


Since F_R is not proportional to x, the motion is not simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin the net force is zero but due to momentum it will cross the origin and move towards left. As it comes on negative x-axis, the force is again towards the origin.
Hence charge Q will execute oscillatory but not simple harmonic motion.

15. **(a)** If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along + y-direction as components of forces along x-axis cancell out each other.
If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along + x-direction as components of forees along y-axis cancell out each other.
If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along – y direction.
If Q_1, Q_3 positive and Q_2, Q_4 negative components of forces along y-axis can cel out each other. So net force on charge q along x-axis.

16. Each particle will be in equilibrium under the act of three force tension of string T, weighting, resultant electrostatic force F of the two other charges. Force F and mg are perpendicular.





Resolving T in the direction of mg and F and applying the condition of equilibrium,

$$T\cos\theta = mg; \qquad T\sin\theta = F$$

$$\therefore \tan\theta = \frac{F}{mg} \quad ... (i)$$

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA}F_{CB}\cos\alpha}$$

$$\therefore F = \sqrt{F_{CA}^2 + F_{CA}^2 + 2F_{CA}^2 \times \frac{1}{2}}$$

$$F = \sqrt{3}F_{CA} = \sqrt{3}\times\frac{kq^2}{(CA)^2} \quad ... (ii)$$

$[\,|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°\,]$

Let T make an angle θ with the vertical

$$OC = \frac{2}{3}\sqrt{(0.03)^2-(0.015)^2} = 0.0173 \text{ m}$$

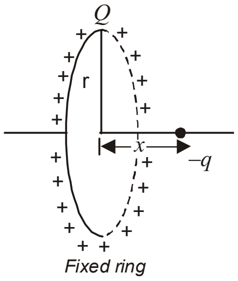
$\therefore$ $OM = 0.9997$

$$\therefore \tan\theta = \frac{OC}{OM} = \frac{0.0173}{0.9997} \quad .... (iii)$$

From eq. (i), (ii) and (iii)

$$\frac{0.0173}{0.9997} = \frac{\sqrt{3}\times 9\times 10^9\times q^2}{(0.03)^2\times 10^{-3}\times 9.8} \quad \therefore q = 3.16\times 10^{-9}\text{ C.}$$

17. Given : Mass of particle $m = 0.9 \times 10^{-3}$ kg
Charge on particle $q = -10^{-6}$ C
$r = 1$ m;
$Q = 10^{-5}$ C
The electric field due a uniformly charged ring of radius r at a point distant x from its center on its axis



$$E = k\frac{Qx}{(r^2+x^2)^{3/2}}$$

$\therefore$ Force on the negative charge
q $F = qE$

$$\therefore \quad F = \frac{-kQq}{(r^2+x^2)^{3/2}} \times x \quad \text{or,} \quad mA = \frac{-kQq}{(r^2+x^2)^{3/2}} \times x$$

$$\text{or,} \quad A = -k\frac{Qq}{m(r^2+x^2)^{3/2}} \times x$$

For $x << r$ $\quad A = -\dfrac{kQq}{r^3} \times x$

Hence motion is simple harmonic in nature.
Comparing the above equation with $A = -\omega^2 x$

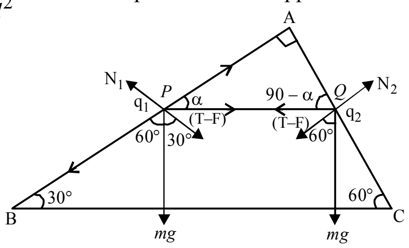
$$\therefore \quad \omega^2 = \frac{kQq}{mr^3} \quad \text{or} \quad \omega = \sqrt{\frac{kQq}{mr^3}}$$

$$\therefore \quad \frac{2\pi}{T} = \sqrt{\frac{kQq}{mr^3}} \Rightarrow T = 2\pi\sqrt{\frac{mr^3}{kQq}}$$

$$T = 2 \times 3.14\left[\frac{0.9 \times 10^{-3} \times 1^3}{9 \times 10^9 \times 10^{-5} \times 10^{-6}}\right]^{1/2}$$

$= 6.28\,[0.01]^{1/2} = 6.28\,[0.1] = 0.628$s

18. If the charges q_1 and q_2 are similar in nature, then electric force $F = \dfrac{q_1 q_2}{l^2}$ will be repulsive and is opposite to tension force T'



For equilibrium of bead at P
$mg\cos 60° = (T - F)\cos\alpha$ ...(i)
$N_1 = mg\cos 30° + (T - F)\sin\alpha$ ...(ii)
Similarly, for the equilibrium of bead at Q
$mg\sin 60° = (T - F)\sin\alpha$ ...(iii)
$N_2 = mg\cos 60° + (T - F)\cos\alpha$ ...(iv)
(i) Dividing equation (iii) by (i), we get : $\tan 60° = \tan\alpha$
i.e. $\alpha = 60°$ ...(v)
Now (ii) from equation (iii) $T - F = mg$
or $T = F + mg = (q_1q_2/l^2) + mg$ ...(vi)
(iii) From equation (ii) $N_1 = mg\cos 30° + (T - F)\sin 60°$
$N_1 = mg\cos 30° + mg\sin 60°$
$\therefore N_1 = \sqrt{3}mg$ ...(vii)
From equation (iv) $N_2 = mg\cos 60° + mg\cos 60° = mg$...(viii)
Again if the chord is cut, then T becomes zero. Hence from equation (vi)
$q_1q_2 = -mgl^2$ ... (ix)
This shows that q_1 and q_2 must have unlike charges for beads to remain stationary.

19. **(i)** Here three charges each of value q are placed at the corners of an equilateral triangle and a fourth charge Q is placed at the centre of the triangle as shown in the figure.

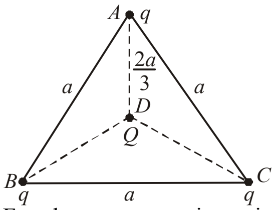
The force on charge q kept at A due to charges kept at B and C
$F_1 = 2F\cos 30°$

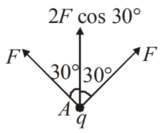
$$F_1 = \sqrt{3} \times \left(9 \times 10^9\frac{q^2}{a^2}\right)$$

The force on q due to charge $(-q)$ kept at D

$$F_2 = 9 \times 10^9\frac{q^2}{(2a/3)^2} = \frac{9}{4} \times \left(9 \times 10^9 \times \frac{q^2}{a^2}\right)$$

Clearly the two forces are not equal. Also as $F_2 > F_1$ the charges will move towards the centre.





(ii) For charges to remain stationary

$$2 \times K\frac{q^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{9}{4} \times K \times \frac{q^2 Q}{a^2} \Rightarrow \frac{4\sqrt{3}\,q}{9} = Q$$

The charge Q should be negative.
Potential energy of the system

$$= 3\left[K\frac{q^2}{a^2} + K\frac{q^2}{a^2}\right] + 3\left[K \times \frac{4\sqrt{3}}{9}\frac{q \times q}{(2a/3)^2}\right]$$

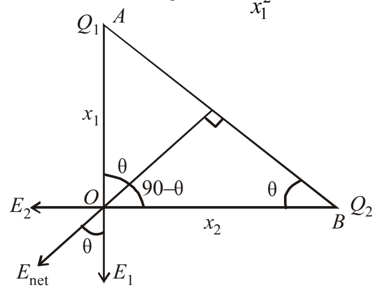
$$= 6K \times \frac{q^2}{a^2} + 3\sqrt{3}K\frac{q^2}{a^2} = 3(2+\sqrt{3})K\frac{q^2}{a^2}$$

This is the amount of work needed to move the charges to infinity.

Topic-2 : Electric Field and Field Lines

1. **(c)** Electric field due charge Q_2, $E_2 = \dfrac{kQ_2}{x_2^2}$

Electric field due charge Q_1, $E_1 = \dfrac{kQ_1}{x_1^2}$



From figure,

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2} \Rightarrow \frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

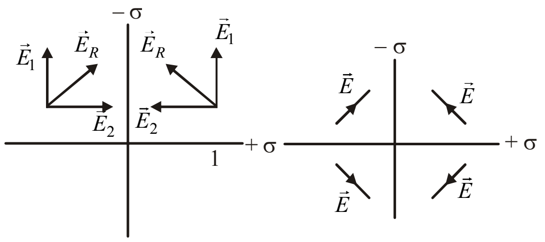
$$\Rightarrow \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1} \text{ or, } \frac{Q_1}{Q_2} = \frac{x_1}{x_2}.$$

2. **(c)** For spherical shell

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ (if $r \geq R$)
$= 0$ (if $r < R$)

Force on charge in electried field, $F = qE$
$\therefore F = 0$ (For $r < R$)
$F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$ (For $r > R$)

3. **(c)** The electric field produced due to uniformly charged infinite plane is uniform. So option (b) and (d) are wrong.
And +ve charge density σ_+ is bigger in magnitude so its field along Y direction will be bigger than field of –ve charge density σ_- in X direction. Hence option (c) is correct.



4. **(c)** Given,

Electric field, $E = E_0(1-x^2)$

$\therefore$ Force, $F = qE = qE_0(1-x^2)$

Also, $F = ma = mv\dfrac{dv}{dx}$ $\quad\left(\because a = v\dfrac{dv}{dx}\right)$

$\therefore mv\dfrac{dv}{dx} = qE_0(1-x^2)$

$\Rightarrow v\,dv = \dfrac{qE_0(1-x^2)dx}{m}$

Integrating both sides we get,

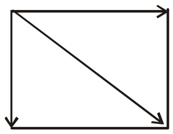
$\Rightarrow \int_0^v v\,dv = \int_0^x \dfrac{qE_0(1-x^2)dx}{m} \Rightarrow \dfrac{v^2}{2} = \dfrac{qE_0}{m}\left(x - \dfrac{9x^3}{3}\right) = 0$

$\Rightarrow x = \sqrt{\dfrac{3}{a}}$

5. **(d)** Net force acting on the particle,

$\vec{F} = qE\hat{i} + mg\hat{j}$

Net acceleration of particle is constant, initial velocity is zero therefore path is straight line.

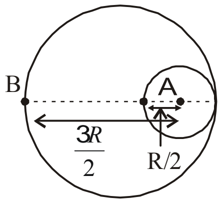


$a_x = \dfrac{2E}{m}$

$a = \sqrt{\left(\dfrac{2E}{m}\right)^2 + g^2}$

$ay = g$

6. **(b)** Electric field at A $\left(R' = \dfrac{R}{2}\right)$



$E_A.ds = \dfrac{q}{\varepsilon_0}$

$\Rightarrow \vec{E}_A = \dfrac{\rho\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0\cdot 4\pi\left(\frac{R}{2}\right)^2} \quad \Rightarrow \vec{E}_A = \dfrac{\sigma(R/2)}{3\varepsilon_0} = \left(\dfrac{\sigma R}{6\varepsilon_0}\right)$

Electric fields at 'B'

$\vec{E}_B = \dfrac{k\times\rho\times\frac{4}{3}\pi R^3}{R^2} - \dfrac{k\times\rho\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$

$\Rightarrow \vec{E}_B = \dfrac{\sigma R}{3\varepsilon_0} - \left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\dfrac{4\pi}{3}\left(\dfrac{R}{2}\right)^3$

$\Rightarrow \vec{E}_B = \dfrac{\sigma R}{3\varepsilon_0} - \dfrac{\sigma R}{54\varepsilon_0} \quad \Rightarrow E_B = \dfrac{17}{54}\left(\dfrac{\sigma R}{\varepsilon_0}\right)$

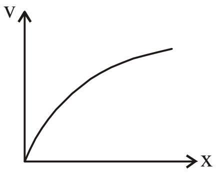
$\left|\dfrac{E_A}{E_B}\right| = \dfrac{1\times 54}{6\times 17} = \left(\dfrac{9}{17}\right) = \dfrac{9}{17}\times\dfrac{2}{2} = \dfrac{18}{34}$

7. **(c)** Since $\vec{r}\cdot\vec{p} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$

$\therefore\ \hat{E}$ is parallel to $\left(\hat{i} + 3\hat{j} - 2\hat{k}\right)$

8. **(b)**



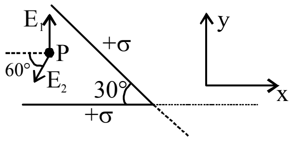
Using
$v^2 - u^2 = 2aS$ ...(i)
Here, $u = 0$, $s = x$
Also, $F_{electric} = ma$

$\Rightarrow qE = ma \ \Rightarrow a = \dfrac{qE}{m} \Rightarrow a = \dfrac{qE}{m}$

Substituting the values in (i) we get

$v^2 = \dfrac{2qE}{m}.x$

9. **(d)**



From figure,

$\vec{E}_1 = \dfrac{\sigma}{2\varepsilon_0}\hat{y}$ and $\vec{E}_2 = \dfrac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x} - \sin 60°\hat{y})$

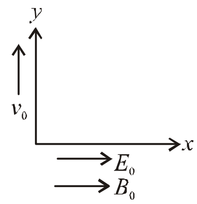
$= \dfrac{\sigma}{2\varepsilon_0}\left(-\dfrac{1}{2}\hat{x} - \dfrac{\sqrt{3}}{2}\hat{y}\right)$

Electric field in the region shown in figure (P)

$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \dfrac{\sigma}{2\varepsilon_0}\left[-\dfrac{1}{2}\hat{x} + \left(1 - \dfrac{\sqrt{3}}{2}\right)\hat{y}\right]$

or, $\vec{E}_P = \dfrac{\sigma}{2\varepsilon_0}\left[\left(1 - \dfrac{\sqrt{3}}{2}\right)\hat{y} - \dfrac{\hat{x}}{2}\right]$

10. **(c)** In the x direction



$F_x = qE$
$\Rightarrow \quad ma_x = qE$
$\Rightarrow \quad a_x = \dfrac{E_0 q}{m}$

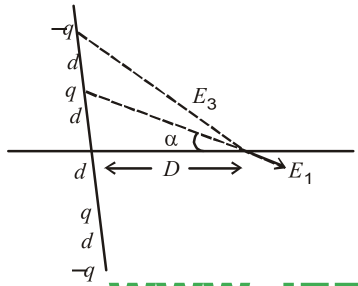
For speed to be double,

$v_0^2 + v_x^2 = (2v_0)^2$

$\Rightarrow \quad v_x = \sqrt{3}\ v_0 = a_x t \quad \Rightarrow \sqrt{3}v_0 = 0 + \dfrac{qE_0 t}{m} \quad \Rightarrow \quad t = \dfrac{\sqrt{3}v_0 m}{E_0 q}$

11. **(d)** $\vec{E} = (\vec{E}_1 + \vec{E}_2) + (\vec{E}_3 + \vec{E}_4)$

or $E = 2E\cos\alpha - 2E\cos\beta$



$$= \frac{2kq}{(D^2+d^2)} \times \frac{D}{\sqrt{D^2+d^2}} - \frac{2kq}{(D^2+(2d)^2} \times \frac{D}{\sqrt{D^2+(2d)^2}}$$

$$= \frac{2kqD}{(D^2+d^2)^{3/2}} - \frac{2kqD}{[D^2+(2d)^2]^{3/2}}$$

For d < < D

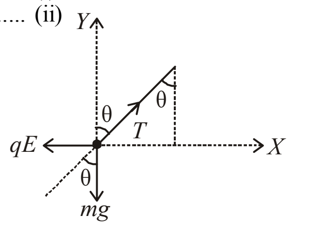
$$E \propto \frac{D}{D^3} \propto \frac{1}{D^2}$$

12. **(d)** At equilibrium resultant force on bob must be zero, so

$T\cos\theta = mg$ ..... (i)

$T\sin\theta = qE$ ..... (ii)

Solving (i) and (ii) we get



$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

[Here, $q = 5 \times 10^{-6}$ C, E = 2000 v/m, m = 2×10^{-3} kg]

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

13. **(b)** Electric field on the axis of a ring of radius R at a distance h from the centre,

$$E = \frac{kQh}{\left(h^2+R^2\right)^{3/2}}$$

Condition: for maximum electric field $\frac{dE}{dh} = 0$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2+h^2\right)^{3/2}}\right] = 0$$

By using the concept of maxima and minima we get, $h = \frac{R}{\sqrt{2}}$

14. **(d)** Charge density, $\rho = \rho_0\left(1-\frac{r}{R}\right)$

$dq = \rho dv$

$q_{in} = \int dq = \rho dv$

$$= \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr \quad (\because dv = 4\pi r^2 dr)$$

$$= 4\pi\rho_0\int_0^R\left(1-\frac{r}{R}\right)r^2 dr = 4\pi\rho_0\int_0^R r^2 dr - \frac{r^2}{R}dr$$

$$= 4\pi\rho_0\left[\left[\frac{r^3}{3}\right]_0^R - \left[\frac{r^4}{4R}\right]_0^R\right] = 4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^4}{4R}\right]$$

$$= 4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^3}{4}\right] = 4\pi\rho_0\left[\frac{R^3}{12}\right]$$

$$q = \frac{\pi\rho_0 R^3}{3}$$

$$E.4\pi r^2 = \left(\frac{\pi\rho_0 R^3}{3\epsilon_0}\right)$$

$\therefore$ Electric field outside the ball, $E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$

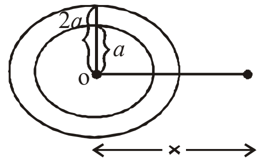
15. **(c)** Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

16. **(a)** Electric field due to complete disc ($R = 2a$) at a distance x and on its axis

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right] \quad E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{\sqrt{4a^2+h^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right] \quad \left[\begin{matrix}\text{here } x = h \\ \text{and, } R = 2a\end{matrix}\right]$$

Similarly, electric field due to disc ($R = a$)



$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{a}\right)$$

Electric field due to given disc

$E = E_1 - E_2$

$$\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right] - \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{a}\right] = \frac{\sigma h}{4\varepsilon_0 a}$$

Hence, $c = \frac{\sigma}{4a\varepsilon_0}$

17. **(c)** $E_1 = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{R^2}$;

$$E_2 = \frac{1}{4\pi\epsilon_0}\cdot\frac{2Q}{R^2};\ E_3 = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q/2}{R^2}$$

Clearly $E_2 > E_1 > E_3$

18. **(a)** Electric field intensity at the centre of the disc.

$$E = \frac{\sigma}{2\epsilon_0} \text{ (given)}$$

Electric field along the axis at any distance x from the centre of the disc

$$E' = \frac{\sigma}{2\epsilon_0}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

From question, x = R (radius of disc)

$$\therefore E' = \frac{\sigma}{2\epsilon_0}\left(1-\frac{R}{\sqrt{R^2+R^2}}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(\frac{\sqrt{2}R-R}{\sqrt{2}R}\right) = \frac{4}{14}E$$

$\therefore$ % reduction in the value of electric field

$$= \frac{\left(E-\frac{4}{14}E\right)\times100}{E} = \frac{1000}{14}\% \simeq 70.7\%$$

19. **(d)** **When the electric field is on**

In equilibrium, force due to electric field = weight

$qE = mg$

$$\Rightarrow qE = \frac{4}{3}\pi R^3\rho g \quad \therefore q = \frac{4\pi R^3\rho g}{3E} \quad ...(i)$$

When the electric field is switched off

Weight of the drop = viscous & force on the drop

$mg = 6\pi\eta R v_t$

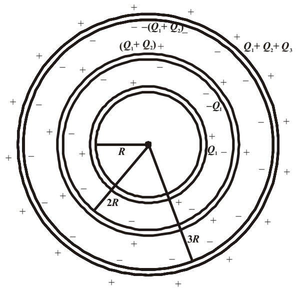
$$\frac{4}{3}\pi R^3\rho g = 6\pi\eta R v_t \quad \therefore R = \sqrt{\frac{9\eta v_t}{2\rho g}} \qquad ...(ii)$$

From eq. (i) & (ii) $q = \frac{4}{3}\pi\left[\frac{9\eta v_t}{2\rho g}\right]^{\frac{3}{2}} \times \frac{\rho g}{E}$

$$= \frac{4}{3}\times\pi\left[\frac{9\times 1.8\times 10^{-5}\times 2\times 10^{-3}}{2\times 900\times 9.8}\right]^{\frac{3}{2}} \times \frac{900\times 9.8\times 7}{81\pi\times 10^5}$$

or, $q = 7.8 \times 10^{-19}$ C

20. **(b)** The charges on the surfaces of the metallic spheres are shown in the figure charge density $\sigma = \frac{\text{charge}}{\text{area}}$ and as per question



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1+Q_2}{4\pi(2R)^2}$$

$$= \frac{Q_1+Q_2+Q_3}{4\pi(3R)^2}$$

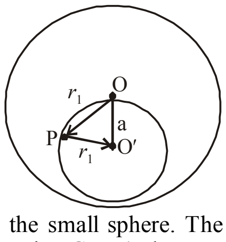
$$\therefore Q_1 = \frac{Q_1+Q_2}{4} \Rightarrow Q_2 = 3Q_1$$

$$\frac{Q_1+Q_2+Q_3}{9} = Q_1$$

$\Rightarrow 8Q_1 - Q_2 = Q_3 \quad \Rightarrow 8Q_1 - 3Q_1 = 5Q_1 = Q_3$

$\therefore Q_1 : Q_2 : Q_3 = 1:3:5$

21. **(b)** Let the charge per unit volume be σ and O be the centre of a uniforms charged solid sphere. Let us consider a uniformly charged sphere of negative charged density σ having its centre at O'. Also let OO' be equal to a.



Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere using Gauss's theorem

$$\vec{E}_1 = \frac{\sigma}{3\varepsilon_0}\vec{r}_1$$

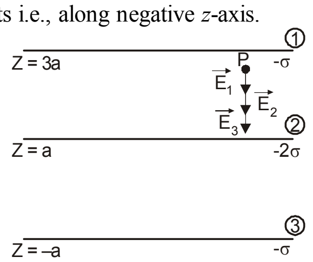
Also the electric field due to small sphere

$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\vec{r}_2{}'$ $\quad\therefore$ The total electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}a$$

Hence electric field will have a finite value which will be uniform.

22. **(c)** The direction of electric fields are according to the charge on the sheets i.e., along negative z-axis.



The total electric field at point P

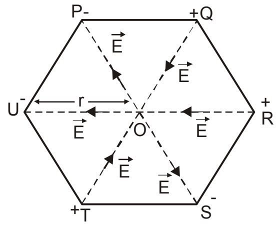
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

23. **(c)** Electric field, $|\vec{E}| = \frac{Kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O.



Similarly Electric field due to Q on O is cancelled by electric field due to T and O.

The electric field due to R on O is in the same direction as that of U on O. Hence the net electric field

$E + \vec{E} = 2\vec{E}$.

24. **(d)** The electric lines of force cannot enter the metallic sphere as E = 0 inside the solid metallic sphere. Also, the electric lines of force fall on the metallic surface normally.

25. **(c)** Electric field everywhere inside the metallic portion i.e., conductor of shell is zero.

Electric field lines are always normal to a surface.

26. **(c)** Electric lines of force do not form closed loops.

27. **(6)** The magnitude of the electric field at the point P which is at a distance $2R$ from the axis of the cylinder

$E = E_{total} - E_{cavity}$

$$= \frac{\gamma}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

$$Q_{sphere} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{\pi R^3\rho}{6}$$

$$\lambda_{cylinder} = \pi R^2\rho$$

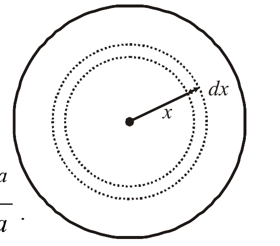
$$\therefore E = \frac{\pi R^2\rho}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}.\frac{\pi R^3\rho/6}{4R^2} = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{16\times 6\times\varepsilon_0}$$

$\therefore k = 6$

28. **(2)** Let there be a spherical shell of radius x and thickness dx. Volume of this shell, V = $4\pi x^2(dx)$. Charge enclosed in this shell

$$dq = (4\pi x^2)dx \times kx^a$$

or, $\quad dq = 4\pi k x^{2+a}\,dx$.



For $r = R$:

Total charge enclosed in the sphere of radius R

$$Q = \int_0^R 4\pi k\, x^{2+a}\,dx = 4\pi k\frac{R^{3+a}}{3+a}.$$

$\therefore$ Electric field at $r = R$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

For $r = R/2$:

Total charge enclosed in the sphere of radius $R/2$.

$$Q' = \int_0^{R/2} 4\pi k\, x^{2+a}dx = \frac{4\pi k(R/2)^{3+a}}{3+a}$$

$\therefore$ The electric field at $r = R/2$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\left(\frac{R}{2}\right)^{1+a}$$

$\because\ E_2 = \frac{1}{8}E_1$

$\therefore\ \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3}\times\frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$

$\Rightarrow\ 1 + a = 3 \therefore a = 2$

29. **(–qEA)** Since electrostatic field is a conservative field

$\therefore\ (W_{PQ} + W_{QR} + W_{RS}) + W_{SP} = 0$

or $W_{PS} + W_{SP} = 0$ or $W_{PS} = -W_{SP}$

$\therefore$ Work done = $q\vec{E}.\overrightarrow{SP}(\therefore F = qE)$

$W_{SP} = qE\hat{i}\left(\vec{r}_P - \vec{r}_S\right)(\because E\ 11\times\text{–axis})$

$W_{SP} - qE\hat{i}\left(a\hat{i} + b\hat{j}\right)$

$W_{SP} = qEa$

$\therefore\ W_{PS} = -W_{SP} = -qEa$

$\therefore$ Work done by field along process $PQRS = -qEa$

30. **False**

For a particle to move in circular motion, we need a centripetal force which is not available.

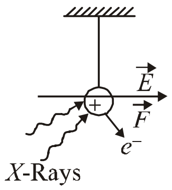
31. **True**

The electric field produced between the parallel plate capacitor is uniform. The force acting on charged particle placed in an electric field is given by $F = qE$.

Hence, the forces on the two protons are identical being equal charge and uniform electric field.

32. **True**

When high energy X-ray beam falls, it will knock out electrons from the small metal ball due to photoelectric effect, making it positively charged and an electrostatic force in the direction of electric field acts. Therefore the ball will be deflected in the direction of electric field.



33. **(b, c)** Given: electric field, $\vec{E}_y = -400\sqrt{3}\ \text{NC}^{-1}$

Initial speed of charged particle, $u = 2\sqrt{10}\times10^6$ m/s

Range, R = 5 m

F = ma and F = qE $\therefore a_y = \frac{q}{m}E_y = -400\sqrt{3}\times10^{10}$

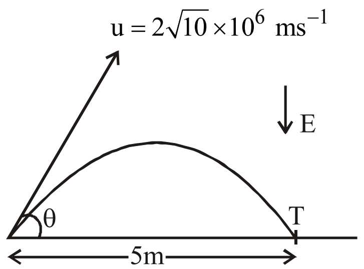
$\left[\because \frac{q}{m} = 10^{10}\,\text{Ckg}^{-1}\ \text{(given)}\right]$

$\text{Range}, R = \frac{u^2\sin2\theta}{a_y} \Rightarrow 5 = \frac{40\times10^{12}\sin2\theta}{400\sqrt{3}\times10^{10}}$

$\Rightarrow \sin2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow 2\theta = 60^\circ$ or, $120 \Rightarrow \theta = 30^\circ$ or, 60°

Particle hits the target, if $\theta = 30^\circ$ or $\theta = 60^\circ$



Time of flight $T_1 = \frac{2u\sin\theta}{a_y} = \frac{2\times2\sqrt{10}\times10^6\times\frac{1}{2}}{400\sqrt{3}\times10^{10}}$

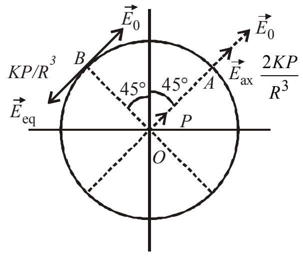
$= \sqrt{\frac{5}{6}}\mu\text{s (for }\theta = 30^\circ)$

Time of flight $T_2 = \frac{2u\sin\theta}{a_y} = \frac{2\times2\sqrt{10}\times10^6\times\frac{\sqrt{3}}{2}}{400\sqrt{3}\times10^{10}}$

$= \sqrt{\frac{5}{6}}\mu\text{s (for }\theta = 60^\circ)$

34. **(b, d)**

Given dipole moment of electric dipole $\vec{P} = \frac{P_0}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ and circle is equipotential. Also electric field is normal to such a line that is the direction of electric field is either radial or the magnitude of electric field should be zero at points on the circle.



Now considering point A, the electric field due to dipole $\frac{2\text{Kp}}{R^3}$ (directed from O to A) as point A lies on the axial line of electric dipole. The external electric field E_0 should also be in the direction of O to A.

Now considering point B which is a point on the equitorial line of the electric dipole. The electric field here due to dipole is $\frac{\text{Kp}}{R^3}$ in a direction opposite to the dipole. The external electric field should cancel out this field. i.e., $E_B = 0$

Further $\vec{E}_0 = \frac{-K\vec{p}}{R^3}$ ...(i)

The electric field at A

$\vec{E}_A = \frac{2K\vec{p}}{R^3} + \vec{E}_0 = -2\vec{E}_0 + \vec{E}_0 = -\vec{E}_0$

Again from eq. (i) $E_0 = \frac{1}{4\pi\epsilon_0}\frac{p_0}{\sqrt{2}(R^3)}\times\sqrt{2}$

$\therefore\ R^3 = \frac{p_0}{4\pi\epsilon_0 E_0}\quad \therefore\ R = \left[\frac{p_0}{4\pi\epsilon_0 E_0}\right]^{1/3}$

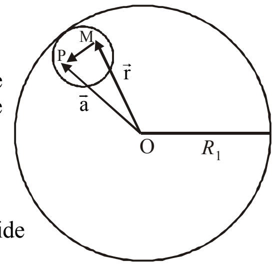
35. **(d)** Let us consider a point M inside the cavity where electric field has to be calculated. Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at M due to uniformly distributed charge of the whole sphere of radius R_1

$\vec{E}_1 = \frac{\rho}{3\varepsilon}\vec{r}$

Electric field at M due to the negative charge distribution in the cavity

$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$

$\therefore$ The total electric field at M inside the cavity,



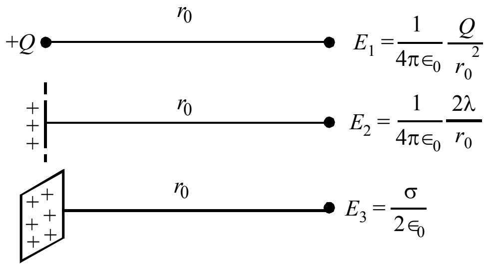
$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$

$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}(\vec{a} - \vec{r})\left[\because \vec{r} + \overrightarrow{MP} = \vec{a}\right]$

$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$

Hence inside the cavity $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$.

36. **(c)**



$E_1(r_0) = E_2(r_0) = E_3(r_0)$ (Given)

(a) $E_1(r_0) = E_3(r_0)$

$\therefore \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\epsilon_0} \Rightarrow Q = 2\pi\sigma r_0^2$

(b) $E_2(r_0) = E_3(r_0)$

$\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0} = \frac{\sigma}{2\epsilon_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$

(c) $E_1(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{4Q}{r_0^2}$

$= \frac{1}{4\pi\epsilon_0} \times \frac{4\times 2\lambda r_0}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

$\because \quad E_1(r_0) = E_2(r_0) \quad \therefore \; Q = 2\lambda\, r_0$

And $2E_2(r_0/2) = 2\left[\frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0}\right] = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$

(d) $E_2(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0/2} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0} = \frac{\lambda}{\pi\epsilon_0 r_0}$

$4E_3(r_0/2) = \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0} = \frac{2}{\epsilon_0}\times\frac{\lambda}{\pi r_0}$

37. **(b, d)**

Electric field E_1 due to smaller sphere 'A' at Q

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}$$

Electric field E_2 due to bigger sphere 'B' at Q

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2}{(5R)^2}$$

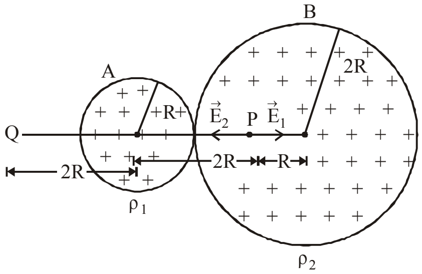
$E_1 + E_2 = 0$

$$\frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2} + \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho 2}{(5R)^2} = 0$$

$\therefore \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$

Electric field E_1 due to smaller sphere 'A' at P

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho_1 \times \frac{4}{3}\pi R^3}{(2R)^2} = \frac{1}{4\pi\epsilon_0}\times\frac{\rho_1\pi R}{3} = \frac{\rho_1 R}{4\epsilon_0\times 3}$$



Electric field E_2 due to bigger sphere 'B' at P

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2 R}{(2R)^3} = \frac{\rho_2 R}{3\varepsilon_0}$$

$E_1 = E_2 \qquad \therefore \frac{\rho_1 R}{4\epsilon_0\times 3} = \frac{\rho_2 R}{3\epsilon_0} \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$

38. **(a, d)**

Clearly from figure electric field lines are orginating from Q_1 and terminating on Q_2.

$\therefore \quad Q_1$ is positive and Q_2 is negative.

Number of electric field lines originating from Q_1 is more than terminating at Q_2.

$\therefore |Q_1| > |Q_2|$.

Since at a finite distance to the right of Q_2, the electric field is zero. The electric field created by Q_2 at a particular point will cancel out the electric field created by Q_1. But at a finite distance to the left of Q_1 the electric field is non-zero.

39. **(a, c)**

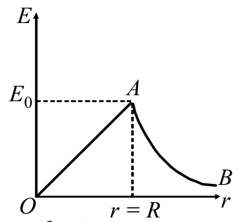
The electric field inside the sphere

$(r < R)\; E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}r$

or, $E \propto r$ Outside the sphere

$(R < r < \infty)\; E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$

or, $E \propto \frac{1}{r^2}$



Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

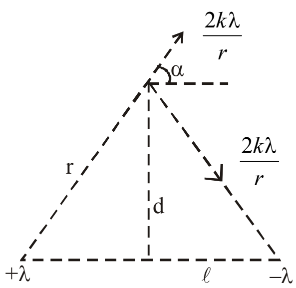
40. **(b)** For a point charge E = $\frac{kQ}{d^2}$ i.e.,

$E \propto \frac{1}{d^2}$ and for a dipole

$E = \frac{kp}{d^3}$ i.e., $E \propto \frac{1}{d^3}$

For an infinite long line charge

$E = \frac{2k\lambda}{d}$ i.e., $E \propto \frac{1}{d}$



For two infinite wires carrying uniform linear charge density.

$$E = \frac{2k\lambda}{r}\cos\alpha = \frac{2k\lambda}{\sqrt{d^2+\ell^2}}\times\frac{\ell}{\sqrt{d^2+\ell^2}} = \frac{2k\lambda\ell}{d^2+\ell^2}$$

or, $E \propto \frac{1}{d^2} \because 2l << d$

For infinite plane charge $E = \frac{\sigma}{2\epsilon_0}$ i.e., E is in dependent of d

41. **(a)** At $r = R$, area $A = 4\pi R^2$ and total enclosed charge, q_{in} = ze

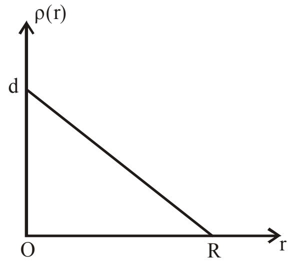
From Gauss's theorem $\phi = \frac{q_{in}}{\varepsilon_0}$

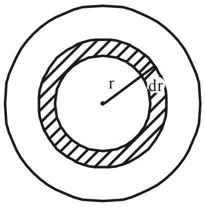
or, $E.A = \frac{q_{in}}{\varepsilon_0}$ or, $E\,(4\pi R^2) = \frac{ze}{\varepsilon_0}$

$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{ze}{R^2}$

Clearly electric field E is independent of A.

42. **(b)** For a = 0, the equation for the graph line $r = d - \frac{r}{R}dr$





Charge in the shaded element

$dq = r \times 4pr^2dr$

$\therefore dq = \left(d - \frac{d}{R}r\right)4pr^2dr$ or, $Ze = \int_o^R 4\pi dr^2 dr - \int_o^R \frac{4\pi d}{R}r^3 dr$

$\Rightarrow Ze = 4\pi d\frac{R^3}{3} - \frac{4\pi d}{R}\frac{R^4}{4}$

$\therefore \frac{Ze.}{4\pi dR^3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \Rightarrow d = \frac{3Ze}{\pi R^3}$

43. **(c)** If the volume charge density is constant then $E \propto r$.

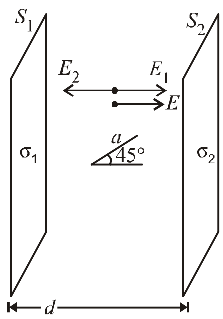
44. Electric field due to sheet S_1, $E_1 = \frac{\sigma_1}{\varepsilon_0}$

Electric field due to sheet

S_2, $E_2 = \frac{\sigma_2}{\varepsilon_0}$

$\therefore$ Net electric field, $E = E_1 - E_2$

$= \frac{\sigma_1 - \sigma_2}{\varepsilon_0}$ $(\because \sigma_1 > \sigma_2)$



Work done by electric field

$W = qv = (q_0 E)\,a\cos 45°$

$= q_0 E \times \frac{a}{\sqrt{2}}$

$\therefore W = \frac{q_0(\sigma_1 - \sigma_2)a}{\sqrt{2}\,\varepsilon_0}$

Topic-3 : Electric Dipole, Electric Flux and Gauss's Law

1. **(b)** Let v be the speed of dipole.

Using energy conservation

$K_i + U_i = K_f + U_f$

$\Rightarrow 0 - \frac{2k \cdot p_1}{r^3}p_2\cos(180°) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$

$\left(\because \text{Potential energy of interaction between dipole} = \frac{-2p_1p_2\cos\theta}{4\pi\epsilon_0 r^3}\right)$

$\Rightarrow mv^2 = \frac{2kp_1p_2}{r^3} \Rightarrow v = \sqrt{\frac{2kp_1p_2}{mr^3}}$

When $p_1 = p_2 = p$ and $r = a$

$v = \frac{p}{a}\sqrt{\frac{1}{2\pi\epsilon_0 ma}}$

2. **(a)**

3. **(b)** Surface charge density depends only due to Q. Also

$\oint \vec{E}.d\vec{A} = \frac{q_1\lambda}{\varepsilon_0}$

or $E \times 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, r \geq R$

4. **(b)** $\tau = -PE\sin\theta$

or $I\alpha = -PE\,(\theta)$

$\alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

5. **(b)** Potential energy of a dipole is given by

$U = -\vec{P}.\vec{E}$

$= -PE\cos\theta$

[Whereθ = angle between dipole and perpendicular to the field]

$= -(10^{-29})(10^3)\cos 45°$

$= -0.707 \times 10^{-26}$ J $= -7 \times 10^{-27}$J

6. **(b)** When cube is of side a and point charge Q is at the center of the cube then the total electric flux due to this charge will pass evenly through the six faces of the cube. So, the electric flux through one face will be equal to 1/6 of the total electric flux due to this charge.

Flux through 6 faces $= \frac{Q}{\epsilon_0}$

$\therefore$ Flux through 1 face, $= \frac{Q}{6\epsilon_0}$

7. **(a)** $T = PE\sin\theta$ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$

$\vec{E}_1 = E\vec{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta\,(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}E_1\hat{j}$

$\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}E_1\hat{j}$

$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

From eqns. (*i*) and (*ii*)

$pE\sin\theta = \sqrt{3}\,pE\cos\theta$

$\tan\theta = \sqrt{3}$ $\therefore$ $\theta = 60°$

8. **(c)** The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1, $\phi_1 = \frac{1}{\varepsilon_0}(2q)$

For surface S_2, $\phi_2 = \frac{1}{\varepsilon_0}(q+q+q-q) = \frac{1}{\varepsilon_0}2q$

For surface S_3, $\phi_3 = \frac{1}{\varepsilon_0}(q+q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\phi_4 = \frac{1}{\varepsilon_0}(8q-2q-4q) = \frac{1}{\varepsilon_0}(2q)$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4$ i.e. net electric flux is same for all surfaces.

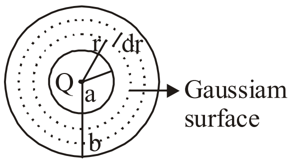
Keep in mind, the electric field due to a charge outside (S_3 and S_4), the Gaussian surface contributes zero net flux through the surface, because as many lines due to that charge enter the surface as leave it.

9. **(c)** Applying Gauss's law

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\epsilon_0} \quad \therefore E \times 4\pi r^2 = \frac{Q + 2\pi A r^2 - 2\pi A a^2}{\epsilon_0}$$

$\rho = \frac{dr}{dV}$

$Q = \rho 4\pi r^2$



$$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{Q - 2\pi A a^2}{r^2} + 2\pi A\right]$$

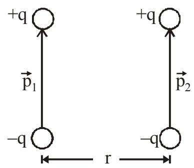
For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$\therefore A = \frac{Q}{2\pi a^2}$

10. **(c)** Force of interaction

$$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{6p_1p_2}{r^4}$$

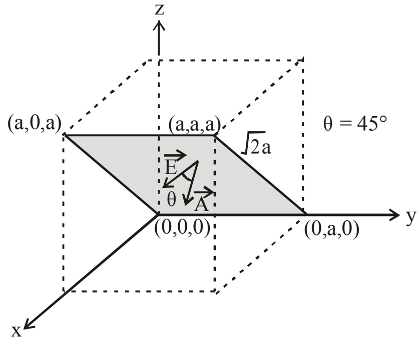


11. **(c)** Given $\vec{E} = E_0\hat{x}$

i.e, electric field $\vec{E}$ acts along + x direction and is a constant. Therefore the electric flux through the shaded portion whose area

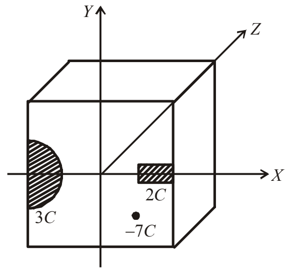
$\vec{A} = a \times \sqrt{2}a = \sqrt{2}a^2$

$\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}a^2)\cos 45° = E_0(\sqrt{2}a^2) \times \frac{1}{\sqrt{2}}$
$= E_0a^2$ ($\because$ Angle between E and A, Q = 45°)



12. **(a)** From the figure total charge enclosed in the cubical surface $q_{in} = 3C + 2C - 7C = -2C$. According to Gauss's theorem the electric flux through the cube

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$$



13. **(d)** The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface is the vector sum of electric fields due to all the charges *i.e.*, $q_1 - q_1$ and q_2.

14. **(6)** From figure $\tan\theta = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$

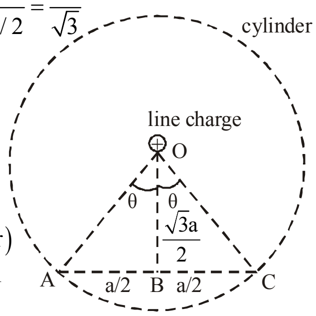
$\therefore$ θ = 30°

Electric flux through the complete cylinder by Gauss's theorem

ϕ cylinder $= \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

(Where L = length of cylinder)

$\therefore$ Electric flux passing through cylindrical surface i.e., for 60° angle $= \frac{\lambda L}{6\varepsilon_0}$ Hence, n = 6



15. **(–48)**

Flux of electric field $\vec{E}$ through any area $\vec{A}$ is defined as

$\phi = \int E.A\cos\theta$

Here, θ = angle between electric field and area vector of a surface

For surface $ABCD$ Angle, θ = 90°

$\therefore \quad \phi_1 = \int E.A\cos 90° = 0$

For surface $BCGF$ $\phi_n = \int \vec{E}.\vec{dA}$

$\therefore \quad \phi_{11} = \left[4\times\hat{i} - (y^2+1)\hat{j}\right].4\hat{i} = 16x$

$\phi_{11} = 48\frac{Nm^2}{C}$

$\phi_1 - \phi_{11} = -48$

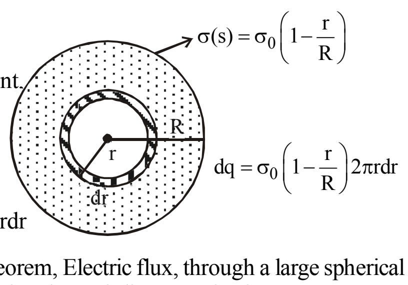
16. **(6.40)**

Let us consider a ring element of radius r and thickness dr

Surface charge density of disc of radius R,

$\sigma(r) = \sigma_0\left(1 - \frac{r}{R}\right)$

Charge of disc element



$dq = \sigma_0\left(1 - \frac{r}{R}\right)2\pi rdr$

Now from Gauss's theorem, Electric flux, through a large spherical surface that encloses the charged disc completely.

$$\phi_0 = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^R \sigma_0\left(1 - \frac{r}{R}\right)2\pi rdr}{\varepsilon_0}$$

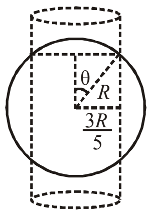
Electric flux through another spherical surface of radius R/4

$$\phi = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^{R/4} \sigma_0\left(1-\frac{r}{R}\right)2\pi r dr}{\varepsilon_0}$$

$$\therefore \frac{\phi_0}{\phi} = \frac{\sigma_0 2\pi \int_0^R \left(r-\frac{r^2}{R}\right)dr}{\sigma_0 2\pi \int_0^{R/4}\left(r-\frac{r^2}{R}\right)dr} = \frac{\frac{R^2}{2}-\frac{R^2}{3}}{\frac{R^2}{32}-\frac{R^2}{3\times 64}} = \frac{32}{5} = 6.40$$

17. **(a, b, c)**

(a) For $h > 2R$ and $r = \frac{3R}{5}$



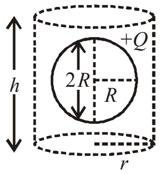
$$\sin\theta = \frac{3R/5}{R} = \frac{3}{5} = 37°$$

$$q_{in} = Q[1-\cos 37°] = Q\left[1-\frac{4}{5}\right] = \frac{Q}{5}$$

From Gauss's theorem $Q = \frac{q_{in}}{\varepsilon_0}$

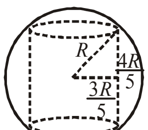
$$\therefore \phi = \frac{Q}{5\epsilon_0}$$

(b) For $h > 2R$ and $r > R$



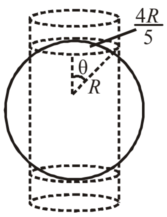
$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

(c) For $h < \frac{8}{5}R$ and $r = \frac{3}{5}R$



$$\phi = \frac{q_{in}}{\epsilon_0} = 0$$

(d) For $h > 2R$ and $r > \frac{4}{5}R$



$$\sin\theta = \frac{4R/5}{R} = \frac{4}{5} = 0.8 \Rightarrow \theta = 53°$$

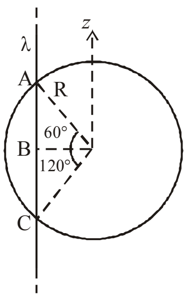
$$q_{in} = Q[1-\cos\theta] = Q\left[1-\frac{3}{5}\right] = \frac{2Q}{5}$$

$$\therefore \phi = \frac{2Q}{5\epsilon_0}$$

18. **(a, b)** According to Gauss's Law, Electric flux, ϕ

$$= \frac{1}{\epsilon_0} q_{in} = \frac{1}{\epsilon_0}[\lambda \times 2R\sin 60°]$$

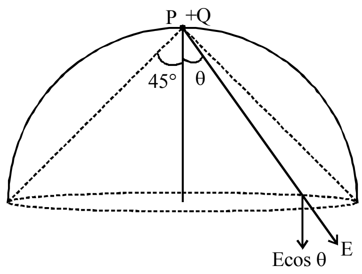
$$= \frac{\sqrt{3}\lambda R}{\epsilon_0}$$



AB = R sin 60° or AC = 2R sin 60°

Also, electric field is perpendicular to the wire therefore its z-com ponent will be zero.

19. **(a, d)** The circumference of the flat surface is an equipotential because the circumference is equidistant from + Q



The component of electric field perpendicular to the flat surface is $E\cos\theta$.

Here E as well as θ changes for different point on the flat surface.

The total flux through the curved and flat surface should be less than $\frac{Q}{\varepsilon_0}$.

The solid angle subtended by the flat surface at P

$$= 2\pi(1-\cos\theta) = 2\pi(1-\cos 45°) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

$\therefore$ Flux passing through curved surface

$$= -\frac{Q'}{\varepsilon_0}\frac{2\pi\left(1-\frac{1}{\sqrt{2}}\right)}{4\pi} = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right).$$

20. **(a, c, d)** Due to symmetry, the net electric flux passing through $x = +\frac{a}{2}$,

$x = -\frac{a}{2}$, $z = +\frac{a}{2}$ is same

According to Gauss's theorem, net electric flux net electric flux crossing through any closed surface $\phi = \frac{q_{in}}{\varepsilon_0}$

$$= \frac{-q+3q-q}{\varepsilon_0} = \frac{q}{\varepsilon_0}.$$

21. **(c, d)**

The potential of all points lie on the conductor is same. Thus potential at A = Potential at B

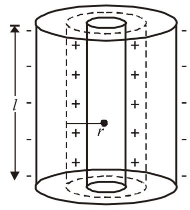
Total electric flux through cavity $= \frac{q}{\epsilon_0}$ this according to Gauss's theorem.

Option (a) and (b) are dependent on the curvature which is different at points A and B because cavity is elliptical.

22. **(c)** Electric field at a distance r from the axis of cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

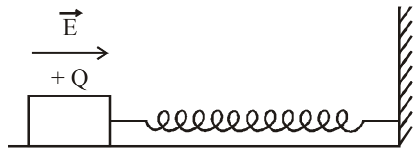
$\therefore E \propto \frac{1}{r}$ (λ= charge per unit length)



Topic-4 : Miscellaneous Problems

1. **(a)** Here frequency of SHM performed by wooden block

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



This value of frequency will remain unchange because when electric field is switched on, the value of k and m is not affected the mean position of SHM shifts towards right by $l = \frac{QE}{k}$ due to force acting $F = qE$.

2. **(a)** The electrostatic force per unit area electrostatic pressure at a point on the surface of a uniformly charged sphere $= \frac{1}{2}\varepsilon_0 E^2 = \frac{\sigma^2}{2\in_0}$

$\therefore$ The force on a hemispherical shell $F = \frac{\sigma^2}{2\in_0} \times \pi R^2$

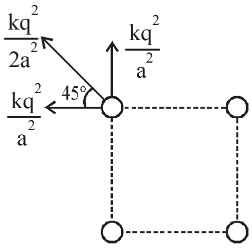
or, $F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$

3. **(3)** Net electrostatic force on one charge due to remaining three charges,

$$F_{electro} = \frac{kq^2}{2a^2} + 2\left[\frac{kq^2}{a^2} \times \frac{1}{\sqrt{2}}\right] = \frac{q^2}{a^2} \times \text{constant}$$

Surface tension force $F_{st} = \gamma a$

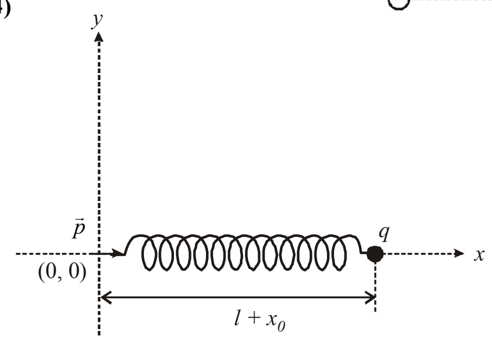
In equilibrium,



$$\frac{q^2}{a^2} \times \text{constant} = \gamma a$$

$\therefore \quad a = k\left(\frac{q^2}{\gamma}\right)^{1/3} \qquad \therefore N = 3$

4. **(3.14)**



Original frequency, $f = \frac{2}{\delta}\sqrt{\frac{K}{m}}$

If dipole appears at equilibrium,

$$\frac{2KP}{(\ell + x_0)^3} \cdot q = Kx_0 \qquad \text{...(i)}$$

When displaced towards right by length x_0

$$f_{net} = \frac{2KP}{(\ell + x_0 + x)^3} \cdot q - K(x_0 + x)$$

$$ma = \frac{2KPq}{(\ell + x_0)^3}\left[1 + \frac{x}{\ell - x_0}\right]^3 - K(x_0 + x)$$

$$= \frac{2KPq}{(\ell + x_0)^3}\left[1 - \frac{3x}{\ell + x_0}\right]^3 - K(x_0 + x)$$

$$= \frac{6KPqx}{(\ell + x_0)^4} Kx = \frac{3x}{\ell + x_0} Kx_0 - Kx = -Kx\left[\frac{3x_0}{\ell + x_0} + 1\right]$$

As 'ℓ' is negative, $ma = -Kx4 \Rightarrow a = \frac{-4K}{m}x$

New frequency, $f' = \frac{1}{2\pi}\sqrt{\frac{4K}{m}} = 2f = \frac{1}{\pi}\sqrt{\frac{K}{m}}$

$\therefore \delta = \pi = 3.14$

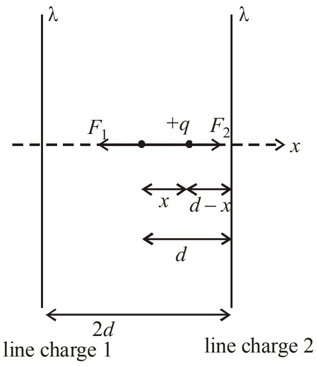
5. **False**

Electrostatic force is conservative in nature, Hence work done is path independent.

6 **(c)** Net force on charge q when it is given a small displacement x

$F_{net} = F_1 - F_2$

$$= \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d - x} - \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d + x}$$



$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{d + x - d + x}{d^2 - x^2}\right]$$

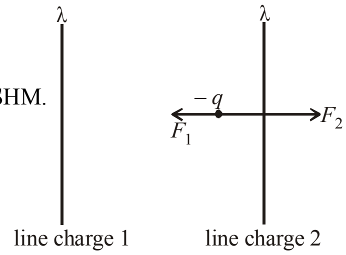
$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\frac{2x}{d^2 - x^2}$$

When $x << d$ then

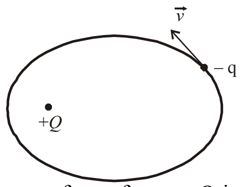
$F_{net} = \frac{\lambda}{\pi\varepsilon_0}x$ i.e., $f \propto x$

displacement therefore the charge $+q$ will execute SHM.

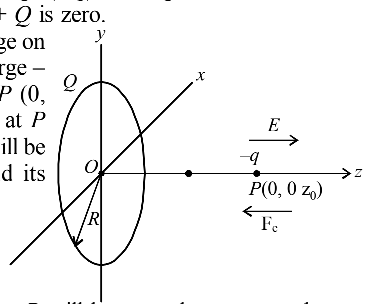
In case of charge $(-q)$ $F_2 > F_1$ *i.e.*, charge $-q$ is unstable, therefore the charge $-q$ continues to move in the direction of its displacement.



7. **(a)** According to `question, under the influence of coulomb field of charge +Q a charge $-q$ is moving around it in an elliptical orbit and this situation is shown in the figure which is similar to a planet revolving around sun. The distance of $-q$ from $+Q$ is changing, therefore, force between the charges will change. The speed of the charge $-q$ will be greater when the charge is nearer to $+Q$ as compared to when it is far. Hence, the angular velocity of charge $-q$ is not constant. The direction of the velocity changes continuously, therefore, linear momentum is not constant. The angular momentum of charge $(-q)$ about Q is constant because the torque about charge $+Q$ is zero.



8. **(a, c)** Let Q be the charge on the ring, the negative charge $-q$ is released from point P (0, 0, Z_0). The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qz_0}{(R^2 + z_0^2)^{3/2}}$$

Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(R^2 + Z_0^2)^{3/2}} \cdot Z_0 \quad \text{... (1)}$$

Similarly, when it crosses the origin, the force is again towards centre O.

Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly if $Z_0 << R$, $(R^2 + Z_0^2)^{3/2} \approx R^3$

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}\cdot Z_0 \qquad \text{[From equation 1]}$$

i.e. the restoring force $F_e \propto -Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position).

16 Electrostatic Potential and Capacitance

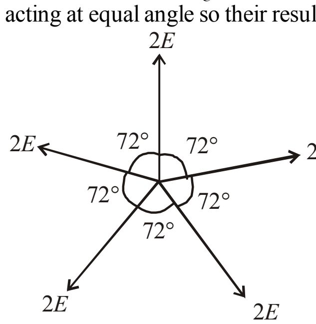
Topic-1 : Electrostatic Potential and Equipotential Surface

1. **(c)** Potential at the centre, $V_C = \dfrac{KQ_{net}}{R}$

$\because Q_{net} = 0$

$\therefore V_C = 0$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C = 0$, since equal electric field vectors are acting at equal angle so their resultant is equal to zero.



2. **(d)** Total charge $Q_1 + Q_2 = Q'_1 + Q'_2$

$$= 12\mu C - 3\mu C = 9\mu C$$

Two isolated conducting sphres S_1 and S_2 are now connected by a conducting wire.

$$\therefore V_1 = V_2 = \frac{KQ'_1}{\frac{2}{3}R} = \frac{KQ'_2}{\frac{R}{3}} = 12 - 3 = 9\ \mu C$$

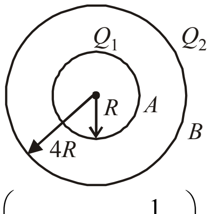
$$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$$

$$\therefore Q'_1 = 6\mu C \text{ and } Q'_2 = 3\mu C$$

3. **(a)** We have given two metallic hollow spheres of radii R and $4R$ having charges Q_1 and Q_2 respectively.

Potential on the surface of inner sphere (at A)

$$V_A = \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$



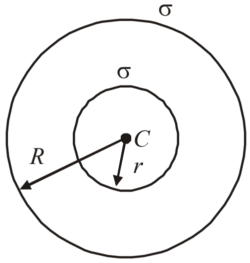
Potential on the surface of outer sphere (at B)

$$V_B = \frac{kQ_1}{4R} + \frac{kQ_2}{4R} \qquad \left(\text{Here, } k = \frac{1}{4\pi\varepsilon_0}\right)$$

Potential difference,

$$\Delta V = V_A - V_B = \frac{3}{4} \cdot \frac{kQ_1}{R} = \frac{3}{16\pi \epsilon_0} \cdot \frac{Q_1}{R}$$

4. **(d)** Let σ be the surface charge density of the shells.



Charge on the inner shell, $Q_1 = \sigma 4\pi r^2$

Charge on the outer shell, $Q_2 = \sigma 4\pi R^2$

$\therefore$ Total charge, $Q = \sigma 4\pi (r^2 + R^2)$

$$\Rightarrow \sigma = \frac{Q}{4\pi(r^2 + R^2)}$$

Potential at the common centre,

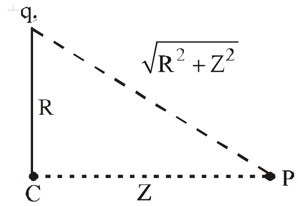
$$V_C = \frac{KQ_1}{r} + \frac{KQ_2}{R} \qquad \left(\text{where } K = \frac{1}{4\pi\varepsilon_0}\right)$$

$$= \frac{K\sigma 4\pi r^2}{r} + \frac{K\sigma 4\pi R^2}{R} = K\sigma 4\pi (r + R)$$

$$= \frac{KQ 4\pi (r+R)}{4\pi(r^2 + R^2)} = \frac{1}{4\pi\varepsilon_0} \frac{(r+R)Q}{(r^2 + R^2)}$$

5. **(c)** Potential at any point of the charged ring

$$V_p = \frac{Kq}{\sqrt{R^2 + Z^2}}$$



R = 3a

Z = 4a

$$\ell = \sqrt{R^2 + Z^2} = 5a$$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

$$\frac{1}{2} m v_0^2 = q\,[V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

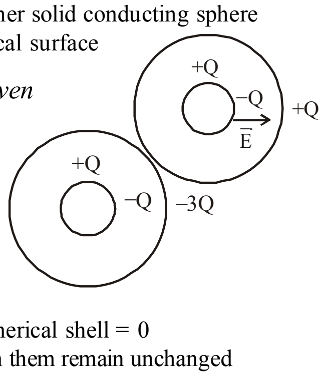
$$v_0^2 = \frac{4Kq^2}{15ma} = \frac{4}{15} \frac{1}{4\pi\varepsilon_0} \frac{q^2}{ma}$$

$$\Rightarrow v_0 = \sqrt{\frac{2}{m}} \left(\frac{2q^2}{15 \times 4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

6. **(d)** When charge Q is on inner solid conducting sphere

Electric field between spherical surface

$$\vec{E} = \frac{KQ}{r^2} \quad So \int \vec{E}.d\vec{r} = V \ \ given$$



Now when a charge – 4Q is given to hollow shell Electric field between surface remain unchanged.

$$\vec{E} = \frac{KQ}{r^2}$$

as, field inside the hollow spherical shell = 0

$\therefore$ Potential difference between them remain unchanged

i.e. $\int \vec{E}.d\vec{r} = V$

7. **(c)** Given, $\vec{E} = (Ax + B)\hat{i}$

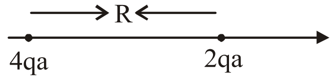
or $\quad E = 20x + 10$

Using $V = \int Edx$, we have

$$V_2 - V_1 = \int_{-5}^{1} (20x + 10)dx = -180 \text{ V or } V_1 - V_2 = 180 \text{ V}$$

8. **(b)** Electric potential is constant inside a charged spherical shell.

9. **(d)** Let at a distance 'x' from point B, both the dipoles produce same potential



$\therefore \dfrac{4qa}{(R+x)} = \dfrac{2qa}{(x^2)}$

$\Rightarrow \sqrt{2}x = R + x \Rightarrow x = \dfrac{R}{\sqrt{2}-1}$

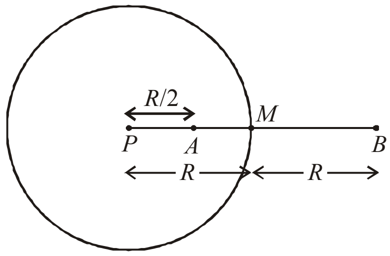
Therefore distance from A at which both of them produce the same potential

$= \dfrac{R}{\sqrt{2}-1} + R = \dfrac{\sqrt{2}R}{\sqrt{2}-1}$

10. **(b)** Let Q be the total charge on the surface of the spherical shell. Then surface charge density is $\dfrac{Q}{4\pi R^2}$. A hole is now cut of area $\alpha(4\pi R^2)$. The charge on this hole say q.

$\because \quad \dfrac{q}{\alpha(4\pi R^2)} = \dfrac{Q}{4\pi R^2} \quad \therefore q = \alpha Q$

The electric potential due to a uniformly charged thin spherical shell (at $r \geq R$) $V_0 = \dfrac{KQ}{R}$



Now net potential

At centre P, $V_P = \dfrac{KQ}{R} - \dfrac{K\alpha Q}{R} = \dfrac{KQ}{R}(1-\alpha)$

$= V_0(1-\alpha) = V_0 - \alpha V_0$

At point A $V_A = \dfrac{KQ}{R} - \dfrac{K\alpha Q}{R/2} = \dfrac{KQ}{R}(1-2\alpha)$

$\therefore \quad \dfrac{V_P}{V_A} = \dfrac{1-\alpha}{1-2\alpha}$

Electric field

$(E_B)_{\text{initial}} = \dfrac{KQ_0}{(2R)^2}$

$(E_B)_{\text{final}} = \dfrac{KQ}{(2R)^2} - \dfrac{K(\alpha Q)}{R^2} = \dfrac{KQ}{(2R)^2} - \alpha V_0$

$(E_P)_{\text{initial}} = 0$ and $(E_P)_{\text{final}} = \dfrac{K(\alpha Q)}{R^2} = \dfrac{\alpha V_0}{R}$

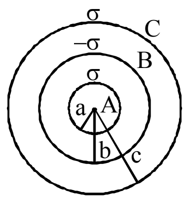
11. **(b)** Potential outside the shell, $V_{\text{outside}} = \dfrac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{\text{inside}} = \dfrac{KQ}{R}$

where 'R" is radius of the shell

$V_B = \dfrac{Kq_A}{r_b} + \dfrac{Kq_B}{r_b} + \dfrac{Kq_C}{r_c}$



$V_B = \dfrac{1}{4\pi \epsilon_0}\left[\dfrac{\sigma 4\pi a^2}{b} - \dfrac{\sigma 4\pi b^2}{b} + \dfrac{\sigma 4\pi c^2}{c}\right]$

$V_B = \dfrac{\sigma}{\epsilon_0}\left[\dfrac{a^2-b^2}{b} + c\right]$

12. **(c)** Potential gradient is given by,

$\Delta V = E.d$

$0.8 = Ed$ (max)

$\Delta V = Ed \cos\theta = 0.8 \times \cos 60 = 0.4$

Hence, maximum potential at a point on the sphere $= 589.4$ V

13. **(b)** $\Sigma_1 = \dfrac{-dv}{dr} = 10\,|z|$

$\Sigma_2 = \dfrac{-dv}{dr} = 10$ (constant : E)

$\therefore$ The source is an infinity large non conducting thick plate of thickness 2 m.

$\therefore 10Z \cdot 10A = \dfrac{\rho \cdot A \propto Z}{\varepsilon_0}$

$r_0 = 10e_0$ for $|z| \leq 1\text{m}$.

14. **(a)** As we know, $E = -\dfrac{dv}{dx}$

Potential at the point x = 2m, y = 2m is given by :

$\int_0^V dV = -\int_0^{2,2} (25dx + 30dy)$

on solving we get, V = – 110 volt.

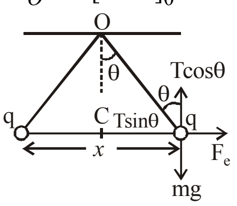
15. **(c)** Potential difference between any two points in an electric field is given by,

$dV = -\vec{E} \cdot \vec{dx}$

$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$

$V_A - V_O = -[10x^3]_0^2 = -80\ \text{J/C}$

16. **(c)**



In equilibrium, $F_e = T\sin\theta$

$mg = T\cos\theta$

$\tan\theta = \dfrac{F_e}{mg} = \dfrac{q^2}{4\pi \epsilon_0 x^2 \times mg} \quad \therefore x = \sqrt{\dfrac{q^2}{4\pi \epsilon_0 \tan\theta\, mg}}$

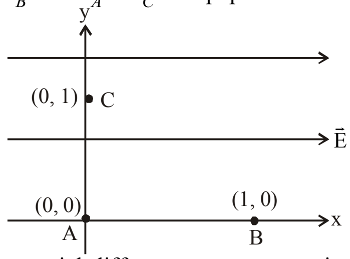
Electric potential at the centre of the line

$V = \dfrac{kq}{x/2} + \dfrac{kq}{x/2} = 4\sqrt{kmg/\tan\theta}$

17. **(a)** According to Gauss's theorem, electric field between two cylinders $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. This electric field will produce a potential differences, $dV = -\vec{E}.\vec{dr}$. When a charge density is given to the inner cylinder, the potential developed at its surface is different

from that on the outer cylinder. This is because the potential decreases with distance for a charged conducting cylinder when the point of consideration is outside the cylinder.
But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Hence no potential difference will be produced between the cylinders.

18. **(b)** As we move along the direction of electric field the potential decreases.
$\therefore \quad V_A > V_B$ and $V_A = V_C$ on equipotential surface.



19. **(d)** The potential difference across capacitor plates remains unchanged when switch S_3 is closed. With the closing of switch S_3 and S_1 the negative charge on C_2 will attract the positive charge on C_2 thereby maintaining the negative charge on C_1. The negative charge on C_1 will attract the positive charge on C_1. No transfer of charge will take place. Therefore p.d across C_1 and C_2 will be 30 V and 20 V respectively.

20. **(d)** Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + ...\right]$$

$$V = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

21. **(a)** The potential inside the shell is equal to the potential on its surface. When we add $-3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore the potential difference remains the same (v).

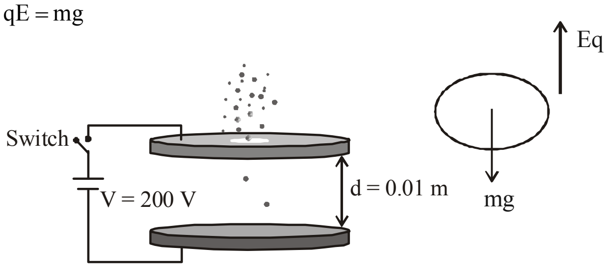
22. **(b)** The electric potential at the surface of a hollow or conducting sphere is same as the potential at the centre of the sphere and any point inside the sphere.
Hence electric protential at surface of the sphere = potential at centre of the sphere = 10V

23. **(6)** Let n no. of electrons present in the oil drop

Electric field, $E = \frac{V}{d} = \frac{200}{0.01} = 2 \times 10^4$ V/m

When terminal velocity is achieved

qE = mg



As q = ne and $m = \frac{4}{3}\pi R^3 \rho$

$\therefore$ n × 1.6 × 10^{-19} × 2 × 10^4

$= \frac{4\pi}{3}(8 \times 10^{-7})^3 \times 900 \times 10$

$$\Rightarrow n = \frac{4}{3}\pi \times \frac{(8\times10^{-7})\times 900\times 10}{2\times10^4\times1.6\times10^{-19}} \quad \therefore n = 6$$

24. **(2)** $\because$ F = ma $\therefore$ $qE = m\frac{dv}{dt}$

$$\Rightarrow dv = \frac{qEdt}{m} = \frac{q\sin 1000t\,\hat{i}}{m}dt$$

($\because$ E = sin 10^3 t $\hat{i}$ given)

$$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega}\sin 1000t\,dt\left[\text{max. speed is at } \frac{T}{2} = \frac{2\pi}{\omega\times 2}\right]$$

$$\therefore \quad V = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega} = -\frac{1}{10^{-3}}\times\frac{[\cos 1000t]_0^{\pi/\omega}}{1000}$$

$$\left(\begin{array}{l}\because m = 10^{-3}kg, q = 1C, E_0 = 1NC^{-1} \\ and\ w = 10^3 \text{ rads}^{-1} \text{ given}\end{array}\right)$$

$$\therefore \quad V = -\left[\cos 1000\times\frac{\pi}{1000} - \cos 0\right] = -[-1-1] = 2\ \text{ms}^{-1}$$

Hence maximum speed attained by the particle.

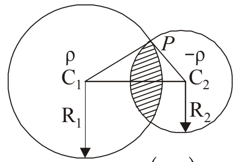
25. Given electric potential $V = 4x^2$ volts
Therefore, electric potential changes only along x-axis,

We know $E_x = \frac{-dV}{dx} \quad \Rightarrow E_x = -\frac{d(4x^2)}{dx} = -8x$

The electric field at point (1, 0, 2) (here x = 1)
$E_x = -8$ V/m

26. The magnitude of electric field is greatest at point B because at B the equipotential surfaces are closest.

27. **(c, d)** Electrostatic field at P is



$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overrightarrow{C_2P}$$

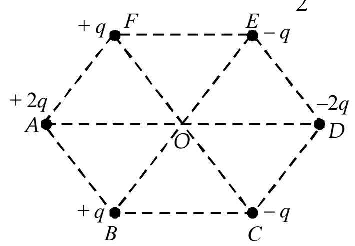
$$= \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} - \overrightarrow{C_2P}\right) = \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1P} + \overrightarrow{PC_2}\right) = \frac{\rho}{3\varepsilon_0}\left(\overrightarrow{C_1C_2}\right)$$

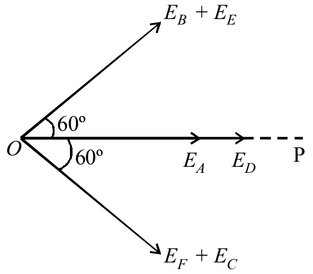
28. **(a, b, c)**

Here $\frac{|\overrightarrow{E_A}|}{2} = |\overrightarrow{E_B}| = |\overrightarrow{E_C}| = \frac{|\overrightarrow{E_D}|}{2} = |\overrightarrow{E_E}| = |\overrightarrow{E_F}| = K$

$\therefore \quad E_O = E_A + E_D + (E_F + E_C)\cos 60° + (E_B + E_C)\cos 60°$

$$= 2K + 2K + (K + K) \times \frac{1}{2} + (K + K) \times \frac{1}{2} = 6K$$





Electric potential at O

$$V_O = \frac{1}{4\pi \varepsilon_0 L}[2q + q + q - q - q - 2q] = 0$$

Potential at all points on the line PR is same not on line ST. PR is perpendicular bisector (the equatorial line) for the electric dipoles AB, FE and BC. Therefore the electric potential will be zero at any point on PR.

29. **(c, d)**
(a) Gauss's law is valid only when $E \propto r^{-2}$
(b) Gauss's law cannot be used to calculate the field distribution around an electric dipole.
(c) is correct as between two point charges we will get a point where the electric field due to the two point charges cancel out each other. If two point charges are of opposite signe then the two fields are along the lane direction hence they cannot be zero.
(d) $W_{AB} = q\,(\Delta V) = 1\,(V_B - V_A)$ or, $W_{AB} = V_B - V_A$

30. **(a, b, c, d)**
(a) Electric field inside a spherical metallic shell with charge on the surface is always zero. *i.e.*, E_A inside = 0
(b) When the shells are connected with a thin metal wire then electric potentials will be equal, *i.e.*, $V_A = V_B = V$.

$$\therefore \frac{1}{4\pi \epsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi \epsilon_0}\frac{Q_B}{R_B} = V$$

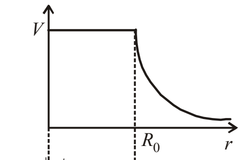
$\because R_A > R_B \therefore Q_A > A_B$.

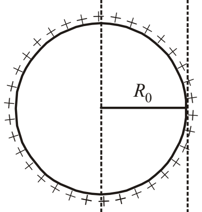
(c) As $\dfrac{\sigma_A}{\sigma_B} = \dfrac{\dfrac{Q_A}{4\pi R_A^2}}{\dfrac{Q_B}{4\pi R_B^2}} = \dfrac{R_B^2}{R_A^2}\times\dfrac{Q_A}{Q_B} = \dfrac{R_B^2}{R_A^2}\times\dfrac{4\pi\epsilon_0 R_A V}{4\pi\epsilon_0 R_B V} = \dfrac{R_B}{R_A}$

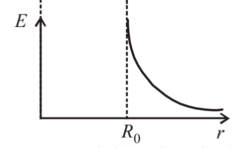
(d) $E_A = \dfrac{\sigma_A}{\epsilon_0}$ and $E_B = \dfrac{\sigma_B}{\epsilon_0}$

$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} < 1 \quad \therefore E_A < E_B$$

31. **(a, b, d)**







The given V-r graph is of charged conducting sphere of radius R_0
(a) The whole charge Q will be enclosed in a sphere of diameter $2R_0$.
(b) Electric field E = zero inside the sphere. Hence electric field is discontinued at $r = R_0$.
(c) Changes in V and E are continuously present for $r > R_0$. Option (c) is incorrect.
(d) For $r < R_0$, the potential V is constant and the electric intensity is zero. Obviously, the electrostatic energy is zero for $r < R_0$.

32. **(b)** The earth is used as a refrance at zero potential in electrical circuits for practical purposes.
For spherical capacitor, capacitance $C = 4\pi\varepsilon_0 R$ and electrical potential $V = \dfrac{Q}{C} = \dfrac{Q}{4\pi\varepsilon_0 R}$

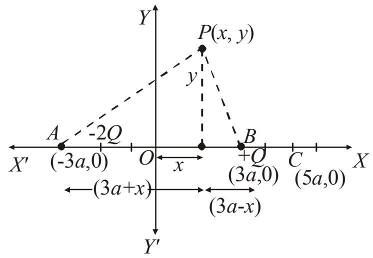
33. Potential of the liquid bubble $V = \dfrac{kq}{a} \Rightarrow q = \dfrac{Va}{K}$
Where q = charge on the liquid bubble.
Let after collapsing the radius of droplet becomes R but here volume remains constant

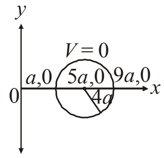
$$\therefore \frac{4}{3}\pi R^3 = 4\pi a^2 t \Rightarrow R = \left(3a^2 t\right)^{1/3}$$

Potential of the droplet

$$V^1 = \frac{Kq}{R} = \frac{K\left(\dfrac{Va}{k}\right)}{\left(3a^2 t\right)^{1/3}} = V\left(\frac{a}{3t}\right)^{1/3}$$

34. **(a)** Let P be a point in the X-Y plane with coordinates (x, y) at which the potential due to charges $-2Q$ (located at $-3a$, 0) and $+Q$ (located at + 3a, 0) be zero.





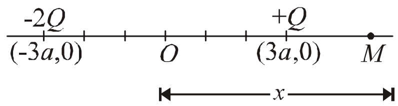
$$\therefore \quad \frac{K(2Q)}{\sqrt{(3a+x)^2 + y^2}} = \frac{K(+Q)}{\sqrt{(3a-x)^2 + y^2}}$$

$$\Rightarrow \quad 2\sqrt{(3a-x)^2 + y^2} = \sqrt{(3a+x)^2 + y^2}$$

$$\Rightarrow \quad (x - 5a)^2 + (y)^2 = (4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$.

(b) For $x > 3a$
Let us consider a point (arbitrary) M at a distance x from the origin on x-axis.



Potential at M

$$V(x) = \frac{K(-2Q)}{x+3a} + \frac{K(+Q)}{(x-3a)} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore \quad V(x) = KQ\left[\frac{1}{x-3a} - \frac{2}{x+3a}\right] \text{ for } |x| > 3a$$

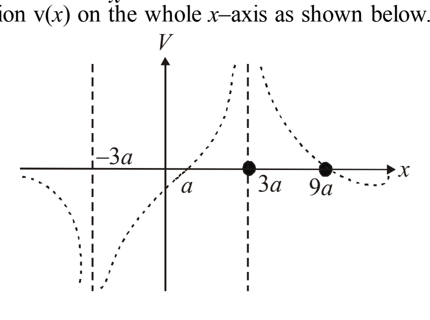
Similarly, for $0 < |x| < 3a$

$$V(x) = KQ\left[\frac{1}{3a-x} - \frac{2}{3a+x}\right]$$

Since circle of zero potential cuts the x-axis at $(a,0)$ and $(9a, 0)$
Hence, $V(x) = 0$ at $x = a$, at $x = 9a$

- From the above expressions
  $V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ at $x \to -3a$
- $V(x) \to 0$ as $x \to \pm\infty$
- $V(x)$ varies $\dfrac{1}{x}$ in general.

Function v(x) on the whole x–axis as shown below.



(c) From energy conservation principle

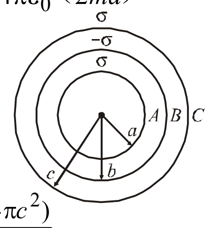
Let v be the speed of particle

$(\text{K.E.} + \text{P.E.})_{\text{centre}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$

$$0 + K\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + K\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{KQq}{4a}, \quad \therefore v = \sqrt{\frac{KQq}{2ma}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

35. Potential at any shell is due charges at shell A, B and C.



Potential of shell A

$$V_A = \frac{kq_A}{a} + \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$= \frac{k\sigma(4\pi a^2)}{a} + \frac{k(-\sigma)(4\pi b^2)}{b} + \frac{k\sigma(4\pi c^2)}{c}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \sigma \times \frac{4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\sigma\frac{(4\pi b^2)}{b} + \frac{1}{4\pi\varepsilon_0} \times \sigma\frac{(4\pi c^2)}{c}$$

$$= \frac{\sigma}{\varepsilon_0}[a - b + c]$$

Similarly, $V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$ or, $V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$

And $V_C = \frac{kq_A}{c} + \frac{kq_B}{c} + \frac{kq_C}{c}$

or, $V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$

(b) $\because \quad V_A = V_C$ (given)

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

or $\quad ac - bc + c^2 = a^2 - b^2 + c^2 \quad$ or $c = a + b$

36. Given surface charge densities are equal.

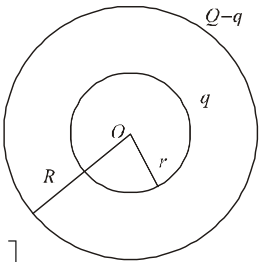
$$\therefore \quad \frac{q}{4\pi r^2} = \frac{Q - q}{4\pi R^2}$$

$$\left(\text{Surface charge density}, \sigma = \frac{q}{A}\right)$$

or, $qR^2 = (Q - q)\,r^2 \quad$ or, $\quad qR^2 = Qr^2 - qr^2$

$$\therefore \quad q = \frac{Qr^2}{R^2 + r^2} \qquad \begin{pmatrix} q = \text{charge on inner sphere} \\ Q - q = \text{charge on outer sphere} \end{pmatrix}$$

Potential at O due to inner sphere



$$V_i = K\frac{q}{r} = \frac{K}{r}\left(\frac{Qr^2}{R^2 + r^2}\right)$$

$$V_i = K\frac{Qr}{R^2 + r^2}$$

Potential at O due to outer sphere

$$V_0 = K\frac{(Q - q)}{R} = \frac{K}{R}\left[Q - \frac{Qr^2}{R^2 + r^2}\right]$$

$$= \frac{K}{R}\frac{\left[QR^2 + Qr^2 - Qr^2\right]}{(R^2 + r^2)} = \frac{K(QR)^2}{R\left(R^2 + r^2\right)} = \frac{KQR}{\left(R^2 + r^2\right)}$$

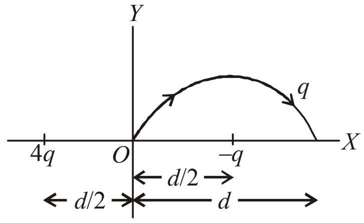
$\therefore$ Total potential at the common centre 'O'

$$V = V_i + V_0 = \frac{KQr}{R^2 + r^2} + \frac{KQR}{R^2 + r^2} = \frac{KQ(R + r)}{R^2 + r^2}$$

Topic-2 : Electric Potential Energy and Work Done in Carrying a Charge

1. **(d)** Change in potential energy, $\Delta u = q(V_f - V_i)$

Potential of $-q$ is same as initial and final point of the path.



$$\Delta u = q\left(\frac{k4q}{3d/2} - \frac{k4q}{d/2}\right) = -\frac{4q^2}{3\pi\varepsilon_0 d}$$

–ve sign shows the energy of the charge is decreasing.

2. **(c)** According to work energy theorem, gain in kinetic energy is equal to work done in displacement of charge.

$$\therefore \quad \frac{1}{2}mv^2 = q\Delta V$$

Here, ΔV = potential difference between two positions of charge q.

For same q and ΔV.

$$v \propto \frac{1}{\sqrt{m}}$$

Mass of hydrogen ion $m_H = 1$

Mass of helium ion $m_{He} = 4$

$$\therefore \quad \frac{v_H}{v_{He}} = \sqrt{\frac{4}{1}} = 2:1.$$

3. **(d)** $U = \frac{1}{4\pi\epsilon_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D + \frac{d}{2}\right)} + \frac{(-q)Q}{\left(D - \frac{d}{2}\right)}\right]$

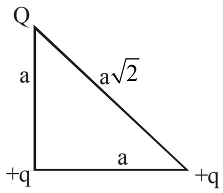
$$= \frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \text{ Ignoring } \frac{d^2}{4}$$

4. **(b)** Net electrostatic energy for the system

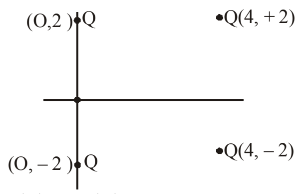
$$U = K\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right] = 0$$

$$\Rightarrow q = -Q\left[1 + \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow Q = \frac{-q\sqrt{2}}{\sqrt{2} + 1}$$



5. **(b)**



Potential at origin

$$v = \frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$$

and potential at $\infty = 0 = KQ\left(1 + \frac{1}{\sqrt{5}}\right)$

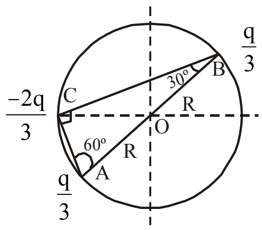
$\therefore$ Work required to put a fifth charge Q at origin W = VQ

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right)$$

6. **(c)** The work done in moving a charge along an equipotential surface is always zero.
The direction of electric field is perpendicular to the equipotential surface or lines.

7. **(c)** (a) The electric field due to charge $\frac{q}{3}$ at A and charge $\frac{q}{3}$ at B at O will get cancelled. The electric field at O due to charge $\left(\frac{-2q}{3}\right)$

$$E = \frac{1}{4\pi \in_0}\frac{2q/3}{R^2} = \frac{q}{6\pi \in_0 R^2}.$$



In Δ ABC $\frac{AC}{AB} = \sin 30º$

$\Rightarrow AC = \frac{AB}{2} = R$

Also $\frac{BC}{AB} = \sin 60º \Rightarrow BC = \frac{\sqrt{3}AB}{2} = \sqrt{3}\,R$

(b) Potential energy of the system K

$$\left[\frac{(q/3)(2/3)}{2R}\right] + K\left[\frac{(q/3)(-2q/3)}{R}\right] + K\left[\frac{(q/3)(-2q/3)}{\sqrt{3}R}\right]$$

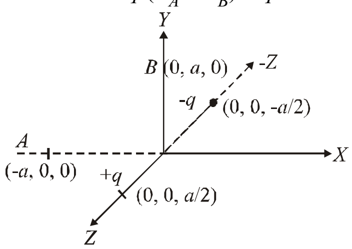
$$= \frac{kq^2}{9R}\left[\frac{1}{2} - 2 - \frac{2}{\sqrt{3}}\right] \neq 0$$

(c) Magnitude of force between B and C

$$F = \frac{1}{4\pi \in_0}\frac{(2q/3)(q/3)}{(\sqrt{3}R)^2} = \frac{q^2}{54\pi \in_0 R^2}$$

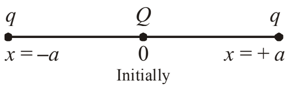
(d) Potential $V_O = K\left[\frac{+q/3}{R}\right] + K\left[\frac{+q/3}{R}\right] + K\left[\frac{-2q/3}{R}\right] = 0$

8. **(c)** Two charges make an electric dipole. A and B points lie on the equatorial plane of the dipole.
$\therefore$ Potential at A, V_A = potential at B, $V_B = 0$
Hence, work done $W = q(V_A - V_B) = q \times 0 = 0$

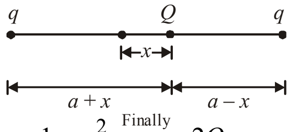


9. **(b)** Initial energy

$$U_i = \frac{2Qq}{4\pi\varepsilon_0 a} + \frac{1}{4\pi\varepsilon_0}\frac{a^2}{2a}$$



Final energy



$$U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a+x} + \frac{1}{a-x}\right] + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a} = \frac{2Qqa}{4\pi\varepsilon_0(a^2 - x^2)}$$

$$\therefore\ U_i - U_f = \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{a}{(a^2 - x^2)}\right]$$

$$= \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{a^2 - x^2 - a^2}{a(a^2 - x^2)}\right] = \frac{-2Qqx^2}{4\pi\varepsilon_0 a^3}$$

when $x << a$ then x^2 is neglected in denominator.

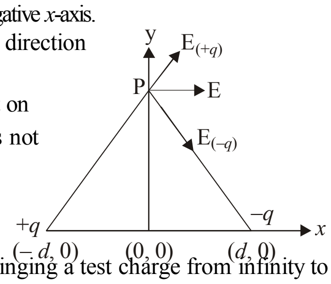
$$U_i - U_f = \left(\frac{-Qq}{2\pi\varepsilon_0 a^3}\right)x^2 \quad \therefore\ U_i - U_f \propto x^2$$

10. **(b)** Since net electrostatic energy = 0

$$\therefore\ \frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$$

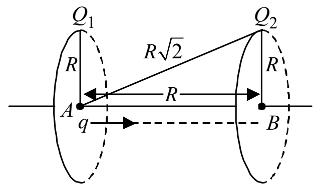
or $Q\sqrt{2} + q\sqrt{2} + Q = 0 \quad \therefore\ Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$

11. **(b)** (a) For $-d \le x \le d$, the electric field is along $+x$-axis.
For all other points, E is along negative x-axis.
Hence, E will not have same direction along entire, x-axis.
(b) Electric field at P, a point on y-axis is parallel to x-axis. It is not along x-axis itself.
(c) Electric potential at origin = zero



No work has to be done in bringing a test charge from infinity to the origin.
(d) The dipole moment is directed from $-q$ charge to $+q$ charge, along negative direction, of x-axis.
$\therefore$ Dipole moment = $-2\,qd$ along x-axis.

12. **(b)** The work done in moving a charge q from centre of one ring to other = change in potential energy



$W_{AB} = \mu$

$$\mu_A = \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right)\times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$$\mu_B = \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

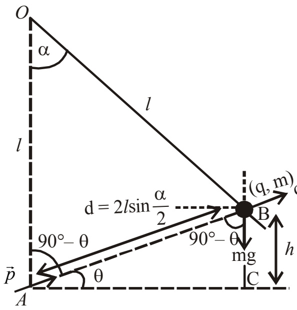
$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore \quad W_{AB} = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

13. **(2)** Initial potential energy, $U_i = 0$

Final potential energy



$$U_f = qV + mgh = \frac{qKp}{d^2} + mgh$$

$$\text{or, } U_f = \frac{kqP}{\left(2\ell\sin\frac{\alpha}{2}\right)^2} + mgh \qquad \text{...(i)}$$

From ΔOAB

$\alpha + 90° - \theta + 90° - \theta = 180°$

$\therefore \alpha = 2\theta$

From ΔABC, $h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\theta$

$$\therefore h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right) = 2\ell\sin^2\left(\frac{\alpha}{2}\right)$$

Now charge is in equilibrium at point B, therefore from sine rule

$$\frac{mg}{\sin\left[90° + \frac{\alpha}{2}\right]} = \frac{qE}{\sin[180° - 2\theta]} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin 2\theta}$$

$$\Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{\sin\alpha} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}} = \frac{qE}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$\Rightarrow qE = mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{q2kp}{\left[2\ell\sin\frac{\alpha}{2}\right]^3} mg2\sin\left(\frac{\alpha}{2}\right) \Rightarrow \frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2}$$

$$= mg\sin\left(\frac{\alpha}{2}\right)\left(2\ell\sin\frac{\alpha}{2}\right)$$

$$\Rightarrow \frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2} = mgh$$ Substituting this value in equation (i)

we get,

$U_f = 2mgh$

Work done in bringing the dipole

$W = U_f - U_i = 2mgh - 0 = 2mgh$

$W = \Delta U = 2mgh = Nmgh \therefore N = 2$

14. **(a)** $U_i = 0$ (since the charge is far away)

$$U_f = -Q \times \frac{1}{4\pi\varepsilon_0}\frac{p}{d^2}$$

$$\therefore \quad K.E. = |U_f - U_i| = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}$$

(b) Electric field at origin due to dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i}$$

Hence, force on charge Q

$$\vec{F} = Q\vec{E} = \frac{2pQ}{4\pi\varepsilon_0 d^3}\hat{i}$$

15. Here total charge-pairs $^8C_2 = \frac{8\times 7}{1\times 2} = 28$

12 pairs of dissimilar charges at a separation of a.

4 pairs of dissimilar charges at a separation of $\sqrt{3}a$.

12 pairs of similar charges at a separation of $\sqrt{2}a$.

Potential energy between two charges $= \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$

For 12 pairs of dissimilar charges, $P.E. = \frac{12(q)(-q)}{4\pi\varepsilon_0 a} = -\frac{12q^2}{4\pi\varepsilon_0 a}$

For 12 pairs of similar charges,

$$P.E. = \frac{12(q)(q)}{4\pi\varepsilon_0 a\sqrt{2}a} = -\frac{12q^2}{4\pi\varepsilon_0 a}\times 0.707$$

For 4 pairs of dissimilar charges

$$P.E. = \frac{4(q)(1-q)}{4\pi\varepsilon_0\sqrt{3}a} = -\frac{4q^2}{4\pi\varepsilon_0 a}\times 0.577$$

Total Potential energy of system

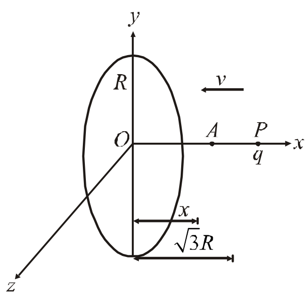
$$U = \frac{-12q^2}{4\pi\varepsilon_0 a} + \frac{12q^2}{4\pi\varepsilon_0 a}\times 0.707 + \left(\frac{-4q^2}{4\pi\varepsilon_0 a}\times 0.577\right)$$

$$= -5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$$

$\therefore$ Binding energy of system $= 5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

$\therefore$ Work done to separate the charges, $W = 5.824\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)$

16. We know, potential due to a ring at a distance of x from its centre on its axis.



$$V(x) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{R^2 + x^2}} \qquad \text{(Here } Q = (2\pi R)\lambda)$$

Potential at '0'

$$V_O = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi R\lambda}{R} \quad \text{or} \quad V_O = \frac{\lambda}{2\varepsilon_0}$$

Potential at 'P'

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{2\pi R\lambda}{\sqrt{R^2 + \left(\sqrt{3}R\right)^2}} = \frac{\lambda}{4\varepsilon_0}$$

Potential difference between points O and P

$$V = V_O - V_P = \frac{\lambda}{2\varepsilon_0} - \frac{\lambda}{4\varepsilon_0} = \frac{\lambda}{4\varepsilon_0}$$

Kinetic energy of the charged particle is converted into its potential energy at O.

$\therefore$ Potential energy of charge $(q) = qV$

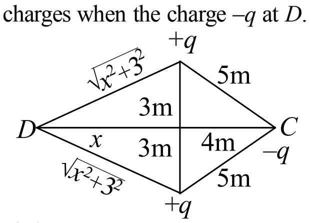
Kinetic energy of charged particle $= \frac{1}{2}mv^2$

For minimum speed of particle so that it does not return to P,

$$\frac{1}{2}mv^2 = qV \Rightarrow v^2 = \frac{2qV}{m} = \frac{2q \times \lambda}{m \times 4\varepsilon_0}$$

or $v = \sqrt{\dfrac{q\lambda}{2\varepsilon_0 m}}$

17. According to the principle of conservation of energy, total energy of system of charges when the charge $-q$ at 'c' = total energy of system of charges when the charge $-q$ at D.



P.E. + K.E. of charge $-q$ at c

$$= \left[\frac{Kq \times q}{6} + \frac{K(q)(-q)}{5} + \frac{Kq(-q)}{5}\right] + 4$$

= P.E. + K.E., when $-q$ is at D and momentarily at rest.

$$= \left[\frac{Kq \times q}{6} + \frac{Kq(-q)}{\sqrt{x^2+3^2}} + \frac{Kq(-q)}{\sqrt{x^2+3^2}}\right] + 0$$

$$= \frac{Kq \times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}}$$

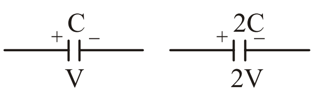
$$\frac{kq \times q}{6} + \frac{2kq(-q)}{5} + 4 = \frac{kq \times q}{6} + \frac{2kq(-q)}{\sqrt{x^2+3^2}}$$

$$\Rightarrow \quad 2 = kq^2\left[\frac{1}{5} - \frac{1}{\sqrt{x^2+3^2}}\right]$$

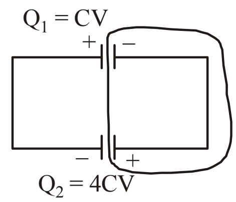
or, $x^2 + 9 = 81 \quad \therefore x = 8.48$ m

Topic-3 : Capacitors, Grouping of Capacitors and Energy Stored in a Capacitor

1. **(b)** When capacitors C and $2C$ capacitance are charged to V and 2V respectively.



$Q_1 = CV \quad Q_2 = 2C \times 2V = 4CV$



When connected in parallel

By conservation of charge

$$4CV - CV = (C + 2C)V_{\text{common}}$$

$$V_{\text{common}} = \frac{3CV}{3C} = V$$

Therefore final energy of this configuration,

$$U_f = \left(\frac{1}{2}CV^2 + \frac{1}{2} \times 2CV^2\right) = \frac{3}{2}CV^2$$

2. **(a)** Let q_1 and q_2 be the charge on the capacitors of 2μF and 4μF. Then charge on capacitor of 5μF

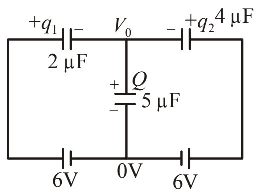
$Q = q_1 + q_2$

$\Rightarrow 5V_0 = 2(6 - V_0) + 4(6 - V_0)$

$\Rightarrow 5V_0 = 12 - 2V_0 + 24 - 4V_0$

$\Rightarrow 11V_0 = 36 \Rightarrow V_0 = \dfrac{36}{11}V$

$\Rightarrow Q = 5V_0 = \dfrac{180}{11}\mu C$



3. **(d)** When two capacitors with capacitance C_1 and C_2 at potential V_1 and V_2 connected to each other by wire, charge begins to flow from higher to lower potential till they acquire common potential. Here, some loss of energy takes place which is given by.

Heat loss, $H = \dfrac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

In the equation, put $V_2 = 0$, $V_1 = V_0$

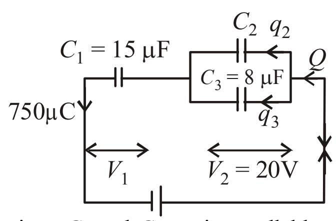
$$C_1 = C,\ C_2 = \frac{C}{2}$$

$$\text{Loss of heat} = \frac{C \times \frac{C}{2}}{2\left(C + \frac{C}{2}\right)}(V_0 - 0)^2 = \frac{C}{6}V_0^2$$

$$H = \frac{1}{6}CV_0^2$$

4. **(b)** According to question,

$Q = 750\mu C = q_2 + q_3$



Capacitors C_2 and C_3 are in parallel hence,

Voltage across C_2 = voltage across C_3 = 20 V

Change on capacitor C_3,

$q_3 = C_3 \times V_3 = 8 \times 20 = 160\mu C$

$\therefore q_2 = 750\mu C - 160\mu C = 590\mu C$

5. **(a)** Given,

Capacitance of capacitor, C_1 = 10 μF

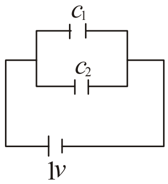
Potential difference before removing the source voltage, V_1 = 50 V

If C_2 be the capacitance of uncharged capacitor, then common potential is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \Rightarrow 20 = \frac{10 \times 50 + 0}{20 + C} \Rightarrow C = 15\,\mu F$$

6. **(c)** In parallel combination, $C_{eq} = C_1 + C_2 = 10\ \mu F$

When connected across 1 V battery, then



$$\frac{U_1}{U_2} = \frac{\left(\frac{1}{2}C_1V^2\right)}{\left(\frac{1}{2}C_2V^2\right)} = \frac{1}{4} \Rightarrow \frac{C_1}{C_2} = \frac{1}{4}$$

$\therefore C_2 = 8\ \mu F$ and $C_1 = 2\ \mu F$

Now C_1 and C_2 are connected in series combination,

$$\therefore C_{\text{equivalent}} = \frac{C_1C_2}{C_1 + C_2} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = 1.6\mu F$$

7. **(a)** Given, $K(x) = K(1 + \alpha x)$

Capacitance of element, $C_{el} = \dfrac{K\varepsilon_0 A}{dx}$

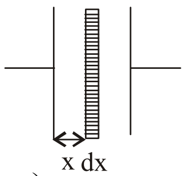
$$\Rightarrow C_{el} = \frac{\varepsilon_0 K(1+\alpha x)A}{dx}$$

$$\therefore \int d\left(\frac{1}{C}\right) = \frac{1}{C_{el}} = \int_0^d \left(\frac{dx}{\varepsilon_0 KA(1+\alpha x)}\right)$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha}[ln(1+\alpha x)]_0^d$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha} ln(1+\alpha d)[\alpha\, d << 1]$$

$$= \frac{1}{\varepsilon_0 KA\alpha}\left[\alpha d - \frac{\alpha^2 d^2}{2}\right]$$



$$= \frac{1}{\varepsilon_0 KA}\left[1 - \frac{\alpha d}{2}\right]$$

$$\therefore C = \frac{\varepsilon_0 KA}{d\left(1 - \frac{\alpha d}{2}\right)} \Rightarrow C = \frac{\varepsilon_0 KA}{d}\left(1 + \frac{\alpha d}{2}\right)$$

8. **(d)** $V' = \dfrac{CV + (nC)V}{kC + nC} \Rightarrow \dfrac{(n+1)V}{k+n}$

9. **(d)** $V_1 + V_2 = 10$ and $4V_1 = 6V_2$

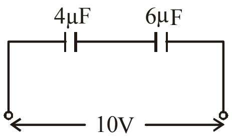
On solving above equations,

we get

$V_1 = 6$ V

Charge on 4 μf,

$q = CV_1 = 4 \times 6 = 24\ \mu C$.



10. **(a)** Equivalent capacitance in series combination (C') is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1C_2}{C_1 + C_2}$$

For parallel combination equivalent capacitance

$C'' = C_1 + C_2$

For parallel combination

$q = 10(C_1 + C_2)$

$q_1 = 500\ \mu C$

$500 = 10(C_1 + C_2)$

$C_1 + C_2 = 50\mu F$ ....(i)

For Series Combination–

$$q_2 = 10\frac{C_1C_2}{(C_1 + C_2)}$$

$80 = 10\dfrac{C_1C_2}{50}$ From equation ....(ii)

$C_1C_2 = 400$ ....(iii)

From equation (i) and (ii)

$C_1 = 10\mu F$ $C_2 = 40\mu F$

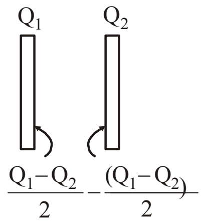
11. **(b)** $\omega = \omega_f - v_i = \dfrac{q}{2}\left(\dfrac{1}{C_f} - \dfrac{1}{C_i}\right)$

$$= \frac{(5 \times 10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right) \times 10^6 = 3.75 \times 10^{-6} J$$

12. **(b)** $V = \dfrac{Q}{C}$

$$= \left(\frac{Q_1 - Q_2}{2C}\right)$$

$$= \left(\frac{4 - 2}{2 \times 1}\right) = 1\ V$$



13. **(b)** Energy stored in the system initially

$$U_i = \frac{1}{2}CE^2$$

$$U_f = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{(CE)^2}{2 \times 4C} = \frac{1}{2}\frac{CE^2}{4}$$

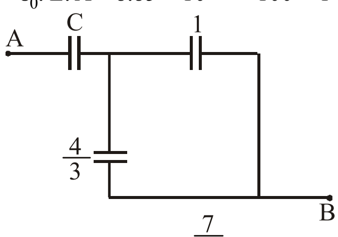
[As $Q = CE$, and $C_{eq} = 4C$]

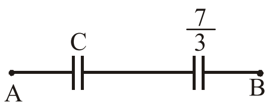
$$\Delta U = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2 = \frac{3}{8}\frac{Q^2}{C}$$

14. **(c)** $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$

$\therefore Q = \varepsilon_0 . E. A = 8.85 \times 10^{-12} \times 100 \times 1 = 8.85 \times 10^{-10} C$

15. **(a)**



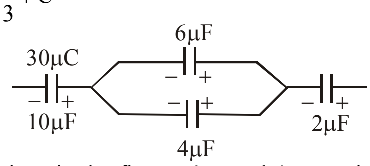


For series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{\frac{7C}{3}}{\frac{7}{3} + C} = \frac{1}{2} \Rightarrow 14\,C = 7 + 3\,C \Rightarrow C = \frac{7}{11}\mu F$$

16. **(d)**



As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation

Charge on 6μF capacitor $= \dfrac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

17. **(b)** $W = -\Delta u$

$$= (-1)\left|\frac{(c\varepsilon)^2}{2kc} - \frac{(c\varepsilon)^2}{2c}\right| = \frac{\varepsilon^2 c}{2}\frac{k-1}{k} = 508 \text{ J}$$

18. **(c)** Let dielectric constant of material used be K.

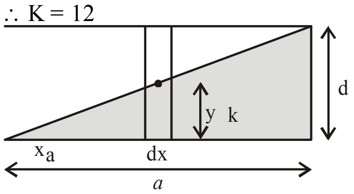
$$\frac{k_1 \in_0 A_1}{d} + \frac{k_2 \in_0 A_2}{d} + \frac{k_3 \in_0 A_3}{d} = \frac{k \in_0 A}{d}$$

or $\dfrac{10 \in_0 A/3}{d} + \dfrac{12 \in_0 A/3}{d} + \dfrac{14 \in_0 A/3}{d} = \dfrac{K \in_0 A}{d}$

$$\frac{\in_0 A}{d}\left(\frac{10}{3} + \frac{12}{3} + \frac{14}{3}\right) = \frac{K \in_0 A}{d}$$

$\therefore K = 12$

19. **(b)**



From figure, $\dfrac{y}{x} = \dfrac{d}{a} \Rightarrow y = \dfrac{d}{a}x$

$$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dc} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$$

$$\frac{1}{dc} = \frac{y}{\varepsilon_0 abx}\left(\frac{y}{k} + d - y\right)$$

$$\int dc = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y} \quad \text{or,} \quad c = \varepsilon_0 a.\frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$$

$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$$

20. **(a)** Charge on Capacitor, $Q_i = CV$

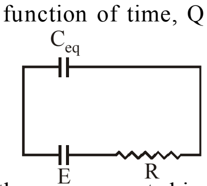
After inserting dielectric of dielectric constant $= K$ $Q_f = (kC)V$

Induced charges on dielectric

$Q_{ind} = Q_f - Q_i = KCV - CV$

$$(K-1)CV = \left(\frac{5}{3} - 1\right) \times 90 \text{ pF} \times 2\text{V} = 1.2\text{nc}$$

21. **(a)** During charging charge on the capacitor increases with time. Charge on the capacitor C_1 as a function of time, $Q = Q_0(1 - e^{-t/RC})$



$$Q = C_{eq}E\left[1 - e^{-t/RC_{eq}}\right]$$

$$(\because Q_0 = C_{eq}E)$$

Both capacitor will have charge as they are connected in series

22. **(c)** Given area of Parallel plate capacitor, $A = 200 \text{ cm}^2$

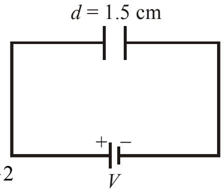
Separation between the plates, $d = 1.5$ cm

Force of attraction between the plates, $F = 25 \times 10^{-6}$N

$F = QE$

$$F = \frac{Q^2}{2A \in_0} \quad \left(\text{E due to parallel plate} = \frac{\sigma}{2\in_0} = \frac{Q}{A2\in_0}\right)$$

But $Q = CV = \dfrac{\in_0 A(V)}{d}$



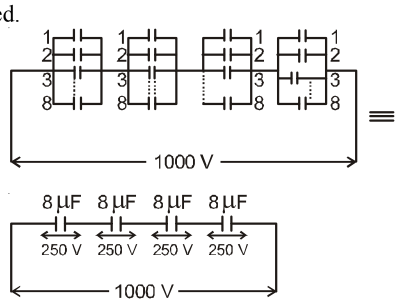
$$\therefore \quad F = \frac{(\in_0 AV)^2}{d^2 \times 2A\in_0}$$

$$= \frac{(\in_0 A)^2 \times V^2}{d^2 \times 2 \times (A\in_0)} = \frac{(\in_0 A) \times V^2}{d^2 \times 2}$$

or, $\quad 25 \times 10^{-6} = \dfrac{(8.85 \times 10^{-12}) \times (200 \times 10^{-4}) \times V^2}{2.25 \times 10^{-4} \times 2}$

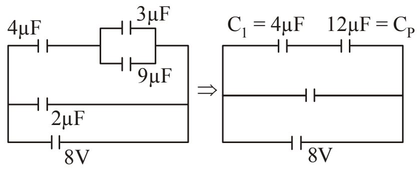
$$\Rightarrow \quad V = \sqrt{\frac{25 \times 10^{-6} \times 2.25 \times 10^{-4} \times 2}{8.85 \times 10^{-12} \times 200 \times 10^{-4}}} \approx 250 \text{ V}$$

23. **(b)** To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i.e., 32 such capacitors are required.



$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \qquad \therefore C_{eq} = 2 \ \mu F$$

24. **(a)**



Charge on C_1 is $q_1 = \left[\left(\dfrac{12}{4+12}\right) \times 8\right] \times 4 = 24\mu C$

The voltage across C_P is $V_P = \dfrac{4}{4+12} \times 8 = 2V$

∴ Voltage across 9μF is also 2V

∴ Charge on 9μF capacitor = 9 × 2 = 18μC

∴ Total charge on 4 μF and 9μF = 42μC

$$\therefore E = \frac{KQ}{r^2} = 9 \times 10^9 \times \frac{42 \times 10^{-6}}{30 \times 30} = 420 \text{ NC}^{-1}$$

25. **(a)** Capacitors 2μF and 2μF are parallel, their equivalent = 4 μF

6μF and 12 μF are in series, their equivalent = 4 μF

Now 4μF (2 and 2 μF) and 8μF in series = $\dfrac{3}{8}$μF

And 4μF (12 & 6 μF) and 4μF in parallel = 4 + 4 = 8μF

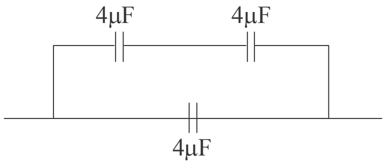
8μF in series with 1μF = $\dfrac{1}{8} + 1 \Rightarrow \dfrac{8}{9}$μF

Now $C_{eq} = \dfrac{8}{9} + \dfrac{8}{3} = \dfrac{32}{9}$

C_{eq} of circuit $= \dfrac{32}{9}$

With C $- \dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{9}{32} = 1 \Rightarrow C = \dfrac{32}{23}$

26. **(d)** To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of 4 μF capacitor in parallel with them.



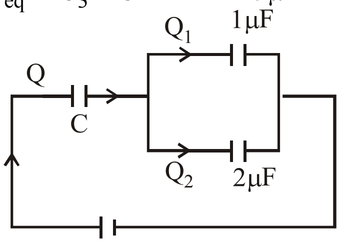
Two capacitances in series

$\therefore \dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

1 capacitor in parallel

$\therefore C_{eq} = C_3 + C = 4 + 2 = 6\,\mu F$

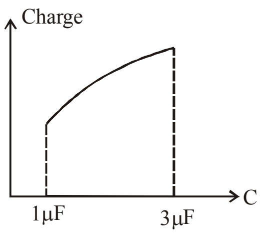
27. **(d)**



From figure, $Q_2 = \dfrac{2}{2+1}Q = \dfrac{2}{3}Q$

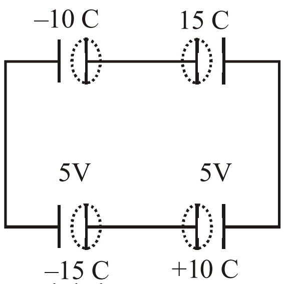
$Q = E\left(\dfrac{C \times 3}{C+3}\right)$

$\therefore Q_2 = \dfrac{2}{3}\left(\dfrac{3CE}{C+3}\right) = \dfrac{2CE}{C+3}$

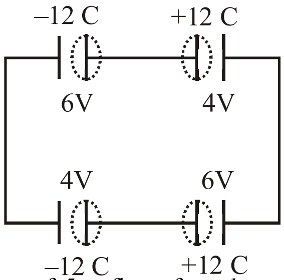


Therefore graph d correctly dipicts.

28. **(a)** when switch is closed



When switch is open



Charge of 5μc flows from b to a

29. **(a)** Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

$E = \dfrac{\sigma}{K\varepsilon_0}$

Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

$\approx 6 \times 10^{-7}\ C/m^2$

30. **(b)** $C_0 = \dfrac{k \in_0 A}{d}$

$C = \dfrac{k \in_0 2}{3d} + \dfrac{2k \in_0 A}{3d} = \dfrac{4}{3}\dfrac{k \in_0 A}{d}$

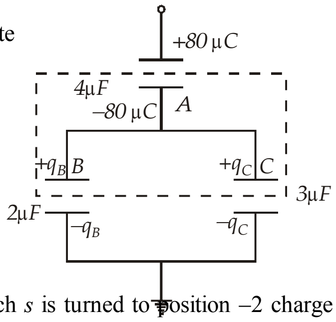
$\therefore \dfrac{C}{C_0} = \dfrac{\dfrac{4}{3}\dfrac{k \in_0 A}{d}}{\dfrac{k \in_0 A}{d}} = \dfrac{4}{3}$

31. **(c)** The total charge on plate A will be 80 μC.

2μF and 3μF capacitors are in parallel. Therefore,

$C_{eq} = 2 + 3 = 5$ HF

Charge on capacitor of 3μF capacitance



$q = \dfrac{3}{5} \times 80 = 48\mu C$

32. **(d)** Initially and when switch s is turned to position –2 charge (q) will remain constant.

Initially energy stored

$U_i = \dfrac{1}{2}\dfrac{q^2}{C_i} = \dfrac{1}{2} \times \dfrac{q^2}{2} = \dfrac{q^2}{4}$

When switch sis turned to position-Z then energy stored.

$U_f = \dfrac{1}{2}\dfrac{q^2}{C_f} = \dfrac{1}{2} \times \dfrac{q^2}{(2+8)} = \dfrac{q^2}{20}$

$\therefore$ Energy dissipated, $\Delta U = U_i - U_f = \dfrac{q^2}{4} - \dfrac{q^2}{20} = \dfrac{q^2}{5}$

$\therefore$ % of stored energy dissipated

$= \dfrac{\Delta U}{U_i} \times 100 = \dfrac{q2}{5} \times \dfrac{4}{q^2} \times 100 = 80\%$

33. **(a)** Time constant as a function of time $T_C = CR$

Capacitance $C = \dfrac{\varepsilon_0 A}{d - T + \dfrac{T}{k}}$

and, T (= thickness) of liquid after time t, $= \dfrac{d}{3} - vt$

$\therefore T_C = CR = \dfrac{\varepsilon_0 AR}{\left(d - \dfrac{d}{3} + vt\right) + \dfrac{\dfrac{d}{3} - vt}{k}}$

Given: $A = 1$ and K = 2

$\therefore T_C = \dfrac{\varepsilon_0 \times 1 \times R}{\left(d - \dfrac{d}{3} + vt\right) + \dfrac{\dfrac{d}{3} - vt}{2}}$

or, $T_C = \dfrac{6\varepsilon_0 R}{5d + 3vt}$

34. **(c)** Energy of a capacitor $U = \frac{1}{2}CV^2$

Initially, $U_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 = \frac{1}{2}C\left(V_1^2 + V_2^2\right)$

After contact, common potential = V

Finally $U_f = \frac{1}{2}(2C)\left(\frac{V_1+V_2}{2}\right)^2$

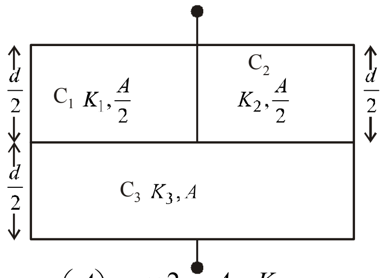
$\therefore \Delta U = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 - \frac{C}{4}(V_1+V_2)^2$

$= \frac{C}{4}\left[2V_1^2 + 2V_2^2 - V_1^2 - V_2^2 - 2V_1V_2\right]$

$= \frac{C}{4}\left[V_1^2 + V_2^2 - 2V_1V_2\right] = \frac{C}{4}(V_1 - V_2)^2$.

35. **(a)** When switch S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$ as unlike charges attract each other. Neither there will be any shifting of charge from B to A. Hence charge on capacitor $B = 0$

36. **(b)** Let C_1, C_2 and C_3 are the Capacity of capacitor with dielectric constant, K_1, K_2 and K_3 respectively



$\therefore \quad C_1 = K_1\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_1}{d}$

$\therefore \quad C_2 = K_2\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_2}{d}$

$\therefore \quad C_3 = K_3(A)\frac{\varepsilon_0 \times 2}{d} = \frac{2A\varepsilon_0 K_3}{d}$

C_1 and C_2 are in parallel

$\therefore \quad C_{eq} = \frac{A\varepsilon_0}{d}(K_1 + K_2)$

Again this C_{eq} and C_3 are in series

$\therefore \quad \frac{1}{C} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$

But $C = \frac{KA\varepsilon_0}{d}$ for single equivalent capacitor

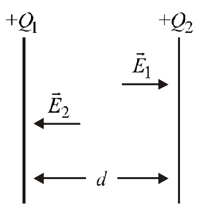
$\therefore \quad \frac{d}{KA\varepsilon_0} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$

or $\quad \frac{1}{K} = \frac{1}{K_1+K_2} + \frac{1}{2K_3}$.

37. **(d)** Within the capacitor plates, electric field,

$E_1 = \frac{Q_1}{2\varepsilon_0 A}; E_2 = \frac{Q_2}{2\varepsilon_0 A};$

$E = E_1 - E_2 \quad = \frac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$



Hence, potential differences $V = Ed$

Lor, $V = \frac{1}{2}\frac{d}{\varepsilon_0 A}(Q_1 - Q_2) \quad = \frac{Q_1 - Q_2}{2C}$

38. **(b)** When capacitors of capacitances C and $2C$ are connected in parallel to each other.

Resultant capacity $C_R = (2C + C) = 3C$

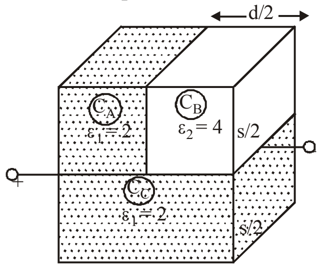
Net potential $V_R = 2V - V = V$

$\therefore$ Final energy, $E = \frac{1}{2}C_R(V_R)^2 = \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

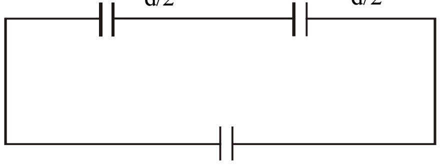
39. **(d)** As we know, the capacitance of a parallel, platecapacitor $C = \frac{\varepsilon_0 A}{d}$

Initialy, capacitance, $C_1 = \frac{\varepsilon_0 s}{d}$

When two dielectrics of permittivity $\varepsilon_1 = 2$ and $\varepsilon_2 = 4$ are introduced between the plates, then



$C_A = \frac{2\,\varepsilon_0\, s/2}{d/2} \qquad C_B = \frac{4\,\varepsilon_0\, s/2}{d/2}$



$C_C = \frac{2\,\varepsilon_0\, s/2}{d} = \frac{\varepsilon_0 s}{d}$

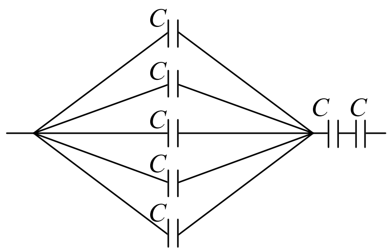
$C_2 = \frac{C_A \times C_B}{C_A + C_B} + C_C = \frac{\frac{2\varepsilon_0 s}{d} \times \frac{4\,\varepsilon_0 s}{d}}{\frac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$

$= \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$

$\therefore \quad C_2 = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 \Rightarrow \frac{C_2}{C_1} = \frac{7}{3} \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$

40. **(a)** The equivalent capacitance

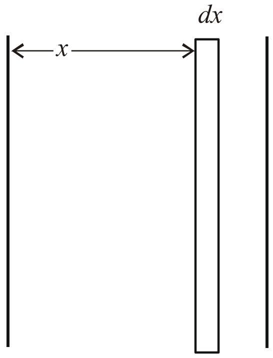
$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2 \times 5} = \frac{11}{10} \Rightarrow C_{eq} = \frac{10}{11}\mu F$



41. **(1)** Let the region between the plates is filled with N dielectric layers.

m = number of dielectric layers within x

d = distance between the plates.



Here, $\frac{x}{m}=\frac{d}{N}$

$$dc=\frac{k\left(1+\frac{m}{N}\epsilon_0\ A\right)}{dx}$$

$$\therefore\ \frac{1}{dc}=\frac{dx}{k\left(1+\frac{m}{N}\right)\epsilon_0\ A}$$

$$\frac{1}{C}=\int dc=\int_0^d\frac{dx}{k\left(1+\frac{m}{N}\right)\epsilon_0\ A}$$

$$=\int_0^d\frac{dx}{k\left(1+\frac{x}{d}\right)\epsilon_0\ A}$$

$$=\frac{d}{k\ \epsilon_0\ A}\int_0^d\frac{dx}{d+x}$$

or, $\frac{1}{C}=\frac{d}{k\ \epsilon_0\ A}ln_2\ \Rightarrow C=\frac{k\ \epsilon_0\ A}{d\,ln^2}$

Comparing this equivalent capacitance

$C=\frac{k\epsilon_0\ A}{d ln_2}$ with $\propto\left(\frac{k\epsilon_0\ A}{d ln_2}\right)$ we get, $\propto\ =1.00$

42. **(4)**

Given, $C_1 = 5\ \mu F$ and $V_1 = 220$ Volt

When capacitor C_1 fully charged it is disconnected from the supply and connected to uncharged capacitor C_2.

$C_2 = 2.5\ \mu F$, $V_2 = 0$

Energy change during the charge redistribution,

$$\Delta U=U_i-U_f=\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

$$=\frac{1}{2}\times\frac{5\times2.5}{(5+2.5)}(220-0)^2\ \mu J$$

$$=\frac{5}{2\times3}\times22\times22\times100\times10^{-6}\ J$$

$$=\frac{5\times11\times22}{3}\times10^{-4}\ J=\frac{55\times22}{3}\times10^{-4}\ J$$

$$=\frac{1210}{3}\times10^{-4}\ J=\frac{1210}{3}\times10^{-3}\ J\simeq4\times10^{-2}\ J$$

According to questions, $\frac{x}{100}=4\times10^{-2}$ $\therefore x=4$

43. **(6)** In the first condition, electrostatic energy is

$$U_i=\frac{1}{2}CV_0^2=\frac{1}{2}\times60\times10^{-12}\times400=12\times10^{-9}J$$

In the second condition $U_F=\frac{1}{2}C'V'^2$

$$U_f=\frac{1}{2}2C.\left(\frac{V_0}{2}\right)^2\qquad\left(\because\ C'=2C,V'=\frac{V_0}{2}\right)$$

$$=\frac{1}{4}\times60\times10^{-12}\times(20)^2\quad=6\times10^{-9}J$$

Energy lost = $U_i - U_f$ = $12\times10^{-9}J - 6\times10^{-9}J = 6\ nJ$

44. **(1.50)**

When switch S_1 is closed and S_2 is opened, capacitor C_3 becomes fully charged.

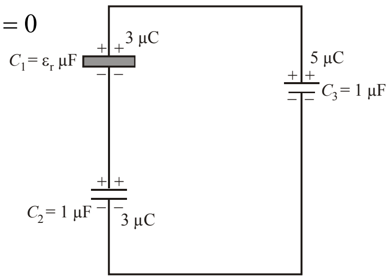
$\therefore$ Charge on capacitor C_3, $q_3 = C_3V = 1\times8\ \mu C = 8\ \mu C$

When switch S_2 is closed and S_1 is opened,

When all the capacitors reach equilibrium charge on C_3 is found to be 5 μC therefore charges on C_1 and C_2 are 3 μC each

Applying Kirchhoff loop rule

$$\frac{CV_0-q}{C}-\frac{q}{\varepsilon_rC}-\frac{q}{C}=0$$

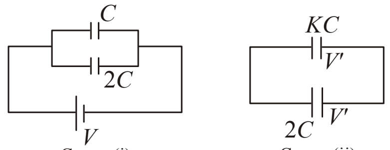


$$\Rightarrow\frac{5}{1}-\frac{3}{\varepsilon_r}-\frac{3}{1}=0$$

$$\therefore 5=3\left[1+\frac{1}{\varepsilon_r}\right]$$

$$\therefore\ \frac{1}{\varepsilon_r}=\frac{5}{3}-1=\frac{2}{3}\qquad\therefore\ \varepsilon_r=1.5$$

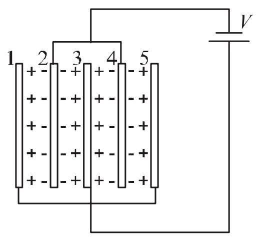
45. Total charge will remain conserved in two cases (i) & (ii)



Case - (i) Case - (ii)

$$\therefore\quad CV+2CV=KCV'+2CV'\ \text{ or, } V'=\frac{3V}{K+2}$$

46. Plate 1 is connected to +ve terminal so +ve charge on it

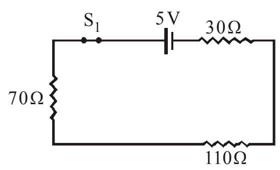


$$\therefore\quad q=CV\ =\frac{\varepsilon_0A}{d}\times V$$

Plate 4 is connected to –ve terminal so charge,

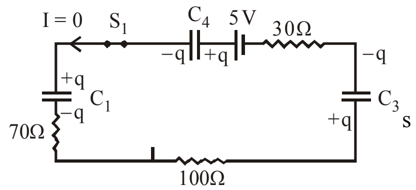
$$-2q=\frac{-2\varepsilon_0A}{d}\times V$$

47. **(c, d)** +S_1 closed and S_2 open then at $t = 0$, charge on each capacitor is zero.



$$\therefore\ I=\frac{V}{R}=\frac{5}{70+100+30}=0.025\ A=25\ mA.$$

When switch S_1 is closed for a long time all the capacitors are fully charged. As the capacitors are in series these carry equal charge q. Current in the circuit is now zero as circuit is in steady state.



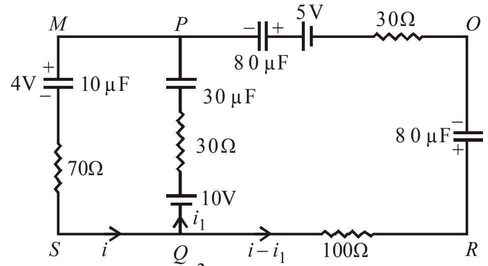
Applying Krichhoff's law
KVL

$5 - \frac{q}{80} - \frac{q}{10} - \frac{q}{80} = 0 \qquad \therefore\ q = 40\ \mu c$

Potential difference across C_1

$V = \frac{q}{C_1} = \frac{40 \times 10^{-6}}{10 \times 10^{-6}} = 4\,V$

Now just after closing the switch S_2 charge on each capacitor remains the same.



$V_P - 4 - 70 \times 25 \times 10^{-3} = V_Q$

$\therefore\ V_P - V_Q = 4 + 1.75 = 5.75$ V

In loop MPQS

$+10 - 30\,i_1 - 4 - 70\,i = 0$

$70\,i + 30\,i_1 = 6 \qquad .....\ (i)$

In loop QROPQ,

$+10 - 30\,i_1 + \frac{40}{80} - 5 + (i - i_1) \times 130 + \frac{40}{80} = 0$

$130\,i - 160\,i_1 = -6 \qquad .....\ (ii)$

Solving (i) & (ii), we get $i = 0.05$ A

$\therefore\ i_1 = 0.077$ A

48. **(a, d)**

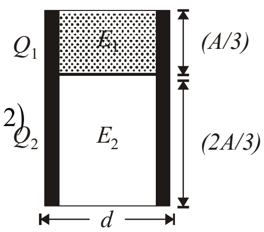
The given capacitor is equivalent to two capacitors in parallel with capacitances

$$C_1 = \frac{K\varepsilon_0 (A/3)}{d} = \frac{K\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 (2A/3)}{d} = \frac{2\varepsilon_0 A}{3d}$$

A = area of each plate and

d = distance between the plates



$\therefore C = C_1 + C_2$

$= \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{3d}(K+2)$

$\therefore \frac{C}{C_1} = \frac{\frac{\varepsilon_0 A(K+2)}{3d}}{\frac{\varepsilon_0 AK}{3d}} = \frac{K+2}{K}$

Let V be potential difference between the plates

$E_1 = \frac{V}{d}$ and $E_2 = \frac{V}{d}$

$\therefore\ \frac{E_1}{E_2} = 1$

$Q_1 = C_1 V = \frac{K\varepsilon_0 A}{3d} V$ and $Q_2 = C_2 V = \frac{2\varepsilon_0 A}{3d} V$

$\therefore \frac{Q_1}{Q_2} = \frac{K}{2}$

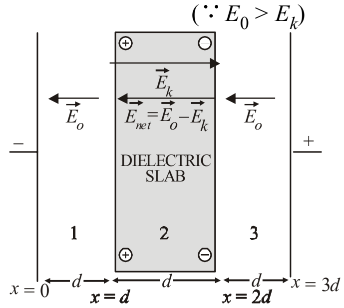
49. **(b, d)**

When switch S_1 is pressed, the capacitor C_1 gets charged such that its upper plate acquires a positive charge $+ 2\,CV_0$ and lower plate $- 2\,CV_0$.

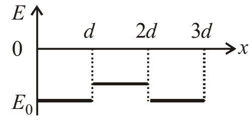
When switch S_2 is pressed and S_1 is release. As $C_1 = C_2$ the charge gets distributed equal. The upper plates of C_1 and C_2 now take charge $+ CV_0$ each and lower plate $- CV_0$ each.

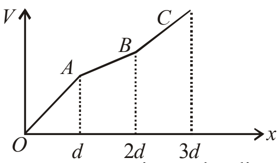
When S_2 is released and S_3 is pressed, the charge on the upper plate of C_1 is CV_0 and charge on upper plate of C_2 is $- CV_0$.

50. **(b, c)** (a) In region 1 and 3, there will be electric field $\vec{E}_0$ directed from positive to negative. In region 2, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $\vec{E}_0 - \vec{E}_k$ in the direction of $\vec{E}_0$ as shown in the diagram. $(\because E_0 > E_k)$



(b) The direction of electric field remains the same. In region-2 electric field is less an compared to region 1 and 3.



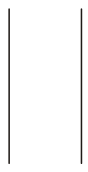


(c) When one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the potential increase. Since in region 2, the electric field is less as compared to 1 and 3 therefore the increase in potential will be less but there has to be increase in potential in all the regions from $x = 0$ to $x = 3d$.

Hence option (c) is correct and (d) is incorrect.

51. **(a, c, d)** Initially,

As $C = \frac{\varepsilon_0}{d} A$

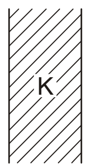


$Q = CV$

$= \frac{\varepsilon_0 A}{d} \times V$

When dielectric slab is inserted

$C' = \frac{K\varepsilon_0 A}{d}$



$V' = \frac{V}{K}$

$Q = \frac{\varepsilon_0 AV}{d} = C'V'$

Q will remain same as no charge is leaving or entering the plates during the process of slab insertion because battery is removed.

Now, $Q = C'V' = C'E'd$

$$\Rightarrow E' = \frac{Q}{C'd} = \frac{\frac{\varepsilon_0 AV}{d}}{\frac{K\varepsilon_0 A}{d}} \times \frac{1}{d} = \frac{V}{Kd}$$

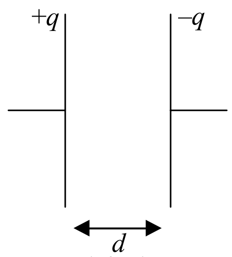
Work done is the change in energy stored

$$\therefore W = \frac{1}{2}CV^2 - \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d} - \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \left(\frac{V}{K}\right)^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$\text{or, } W = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[1 - \frac{1}{K}\right]$$

52. **(b, d)** Charge on plate is q

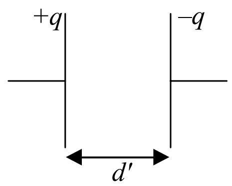


Capacitances, $C = \dfrac{\varepsilon_0 A}{d}$

$q = CV \Rightarrow V = \dfrac{q}{C}$,

$U = \dfrac{1}{2}q \times V$

If the plates of the capacitor are moved farther a part so $d^1 > d$



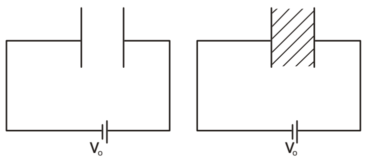
$C' = \dfrac{\varepsilon_0 A}{d'}$ $\therefore C' < C$,

$V' = \dfrac{q}{C'}$ $\therefore V' > V$

$U' = \dfrac{1}{2}qV' \Rightarrow U' = U$

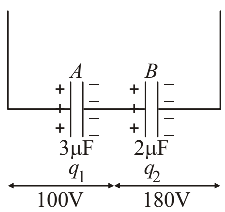
53. **(a, d)**

Capacitor without dielectric	Capacitor with dielectric



Capacitor without dielectric	Capacitor with dielectric
(i) Potential difference $= V_0$ Capacitance $= C$	Potential difference $= V_0$ Capacitance $= KC$
(ii) Charge $Q_0 = CV_0$	[K is the dielectric constant of slab $K > 1$] New charge $= KCV_0$
(iii) Potential Energy $= \dfrac{1}{2}CV_0^2$	New potential energy $= \dfrac{1}{2}KCV_0^2$
(iv) Electric field $E = \dfrac{V_0}{d}$	$E = \dfrac{V_0}{d}$

54. **(i)** Before uncharged capacitor falls



Charge on capacitor A

$q_A = 3 \times 10^{-6} \times 100 = 3 \times 10^{-4}$C

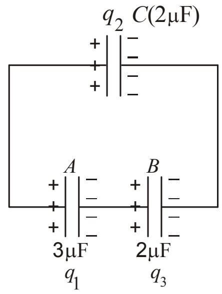
Charge on capacitor B

$q_B = 2 \times 10^{-6} \times 180 = 3.6 \times 10^{-4}$C

When uncharged capacitor falls,

Let the charge on capacitor A, C and B be q_1, q_2 and q_3 respectively.

By charge conservation.



The sum of charge on +ve plate of capacitor A and $C = q_A$

$\therefore \quad q_1 + q_2 = 3 \times 10^{-4}$C ... (i)

Similarly the sum of charge on –ve plates of capacitor C and $B = q_B$

$\therefore \quad -q_2 - q_3 = -3.6 \times 10^{-4}$C

or, $q_2 + q_3 = 3.6 \times 10^{-4}$C ... (ii)

Applying Kirchhoff's law in the closed loop,

$$\frac{q_1}{3\times10^{-6}} - \frac{q_2}{2\times10^{-6}} + \frac{q_3}{2\times10^{-6}} = 0$$

or, $2q_1 - 3q_2 + 3q_3 = 0$ ....(iii)

Solving (i), (ii) and (iii), we get

$q_1 = 90 \times 10^{-6}$ C, $q_2 = 210 \times 10^{-6}$ C, and $q_3 = 150 \times 10^{-6}$ C,

(ii) Amount of electrostatic energy in the system before uncharged capacitor falls.

$$U_i = U_A + U_B = \frac{1}{2}C_A(V_A)^2 + \frac{1}{2}C_B(V_B)^2$$

$$= \frac{1}{2} \times 3 \times 10^{-6}(100)^2 + \frac{1}{2} \times 2 \times 10^{-6}(180)^2$$

$= 4.74 \times 10^{-2}$J

Amount of electrostatic energy stored uncharged capacitor falls

$$U_f = \frac{1}{2}\frac{q_1^2}{C_A} + \frac{1}{2}\frac{q_2^2}{C_B} + \frac{1}{2}\frac{q_3^2}{C_C}$$

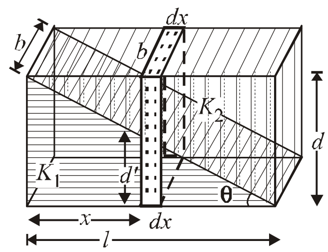
$$= \frac{1}{2}\frac{(90\times10^{-6})^2}{3\times10^{-6}} + \frac{1}{2}\frac{(210\times10^{-6})^2}{2\times10^{-6}} + \frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}}$$

$$+ \frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}} = 1.8 \times 10^{-2}\text{ J}$$

55. Let ℓ, b the length and breadth of the capacitor plate.

Therefore $\ell \times b = A$. d be the distance between the plates.

Let us consider a small dotted element of thickness dx as shown in the figure.



The small capacitance of the dotted portion $\dfrac{1}{dC} = \dfrac{1}{dC_1} + \dfrac{1}{dC_2}$

where dC_1 = capacitance of capacitor with dielectric K_1

dC_2 = capacitance of capacitor with dielectric K_2.

$$dC_1 = \frac{K_1(bdx)\varepsilon_0}{d'}$$

$$d' = d - x\frac{d}{\ell} = d\left[1 - \frac{x}{\ell}\right]$$

$$\therefore \quad dC_1 = \frac{K_1 b(dx)\,\varepsilon_0}{d\left[1 - \frac{x}{\ell}\right]} - \frac{K_1 b\ell(dx)\,\varepsilon_0}{d(\ell - x)} = \frac{K_1 A\,\varepsilon_0(dx)}{d(\ell - x)}$$

Similarly, $dC_2 = \dfrac{K_2\,\varepsilon_0(bdx)}{d-d'} = \dfrac{K_2\,\varepsilon_0 bdx}{d-d+\dfrac{xd}{\ell}}$

$\dfrac{K_2\varepsilon_0 b.\ell.dx}{xd} = \dfrac{K_2\,\varepsilon_0 A\,dx}{xd}$

$\therefore\quad \dfrac{1}{dC} = \dfrac{d(\ell-x)}{K_1 A\varepsilon_0(dx)} + \dfrac{xd}{K_2 A\varepsilon_0(dx)}$

$\Rightarrow\quad \dfrac{K_1K_2A\varepsilon_0 dx}{K_2\ell d + d(K_1-K_2)x} = dC$

Hence capacitance of the whole capacitor,

$C = \displaystyle\int_0^{\ell} \frac{K_1K_2A\varepsilon_0 dx}{K_2\ell d + d(K_1-K_2)x}$

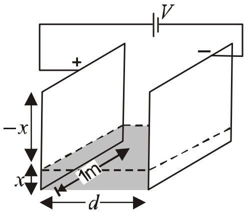
$= K_1K_2A\varepsilon_0 \displaystyle\int_0^{\ell} \frac{dx}{K_2\ell d + d(K_1-K_2)x}$

$= K_1K_2A\varepsilon_0 \left[\dfrac{\log[K_2\ell d + d(K_1-K_2)x]}{d(K_1-K_2)}\right]_0^{\ell}$

or, $C = \dfrac{K_1K_2A\varepsilon_0}{d(K_1-K_2)}\log\dfrac{K_1}{K_2}$

56. Here the system as shown in the figures, is equivalent to two capacitors in parallel one with air and other with oil with dielectric.

$C = C_1 + C_2$



$= \dfrac{k\varepsilon_0(x\times 1)}{d} + \dfrac{\varepsilon_0[(1-x)\times 1)}{d}$

$C = \dfrac{\varepsilon_0}{d}[kx + 1 - x]$ ... (i)

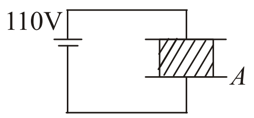
x = side of the dielectric = 1m and d = 0.01m

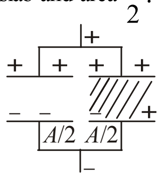
$\therefore q = CV = \dfrac{\varepsilon_0 V}{0.01}\left[1 + x(K-1)\right]$

$\therefore$ Current, $I = \dfrac{dq}{dt} = \dfrac{\varepsilon_0 V}{0.01}[K-1]\dfrac{dx}{dt}$

So, $I = \dfrac{8.85\times10^{-12}\times 500}{0.01}[11-1]\times 0.001 = 4.425\times10^{-9}\,A$

57. (i) Here capacitor A with dielectric slab can be considered as two capacitors in parallel, one having dielectric slab and area $\dfrac{A}{2}$ and another one is without dielectric slab and area $\dfrac{A}{2}$.





Now, capacitance $C_A = C_1 + C_2$

$= \dfrac{(A/2)\varepsilon_0}{d} + \dfrac{(A/2)\varepsilon_0\,\varepsilon_r}{d} = \dfrac{A}{2}\dfrac{\varepsilon_0}{d}[1+\varepsilon_r]$

$= \dfrac{0.4\times 8.85\times10^{-12}}{2\times 8.85\times10^{-4}}[1+9] = 2\times10^{-9}\,F$

Energy stored in this capacitor when connected to V = 110 volt battery.

$U_A = \dfrac{1}{2}C_A V^2 = \dfrac{1}{2}\times 2\times10^{-9}\times(110)^2$

$= 1.21\times10^{-5}$ J

(ii) While taking out the dielectric, the charge on the capacitor plate remains the same but capacitance of A will get changed, hence energy will change. This change in energy is equal to work done.

$W = \dfrac{q^2}{2C_A'} - \dfrac{q^2}{2C_A}$

$C_A' = \dfrac{A\varepsilon_0}{d} = \dfrac{0.04\times 8.85\times10^{-14}}{8.85\times10^{-4}} = 0.4\times10^{-9}\,F$

$q = C_A V = 2\times10^{-9}\times 110 = 2.2\times10^{-7}\,C$

$\therefore\quad W = \dfrac{(2.2\times10^{-7})^2}{2}\left[\dfrac{1}{0.4\times10^{-9}} - \dfrac{1}{2\times10^{-9}}\right]$

or, $W = 4.84\times10^{-5}$ J

(iii) Capacitance of $B = \dfrac{\varepsilon_0\,\varepsilon_r A_B}{d}$

$= \dfrac{8.85\times10^{-12}\times 9\times 0.02}{8.85\times10^{-4}} = 1.8\times10^{-9}\,F$

Charge on A, $q_A = 2.2\times10^{-7}\,C$ gets distributed into two parts.

$\therefore\quad q_1 + q_2 = 2.2\times10^{-7}\,C$

Also the potential difference across A = p.d. across B

$\dfrac{q_1}{C'_A} = \dfrac{q_2}{C_B}$

$\Rightarrow\; q_1 = \dfrac{C_A}{C_B}\,q_2 = \dfrac{0.4\times10^{-9}}{1.8\times10^{-9}}\,q_2 = 0.22\,q_2$

$\therefore\quad 0.22\,q_2 + q_2 = 2.2\times10^{-7}$

$\Rightarrow\; q_2 = \dfrac{2.2}{1.22}\times10^{-7} = 1.8\times10^{-7}\,C$

$\Rightarrow\; q_1 = 0.4\times10^{-7}\,C$

Total energy stored $= \dfrac{q_1^2}{2C'_A} + \dfrac{q_2^2}{2C_B}$

$= \dfrac{0.4\times0.4\times10^{-14}}{2\times0.4\times10^{-9}} + \dfrac{1.8\times1.8\times10^{-14}}{2\times1.8\times10^{-8}}$

$= 0.2\times10^{-5} + 0.9\times10^{-5} = 1.1\times10^{-5}$ J

58. Potential difference across each capacitor is V as both capacitors are in parallel.

Total energy = energy in A + energy in B

$= \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$ ...(i)

When the switch opened and a dielectric is inserted between the plates of capacitors, the new capacitance is 3C but potential difference across it is V

Energy in $A = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$

Energy in $B = \frac{1}{2}\frac{q^2}{KC} = \frac{1}{2}\times\frac{(CV)^2}{3C} = \frac{CV^2}{6}$

(charge on capacitor B remains same when switch is opened)

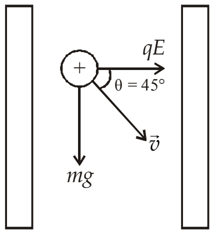
Total energy = energy in A + energy in B

$\therefore$ Total energy $= \frac{3}{2}CV^2 + \frac{1}{6}CV^2 = \frac{5}{3}CV^2$ ... (ii)

$$\frac{\text{Total energy initially}}{\text{Total energy finally}} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** According to the question, a proton is released at rest mid-way between the two plates and is found to move at 45°, so net force is at 45° from vertical and two forces acting on the proton just after the release are as shown in the figure.



$qE = mg$

$\therefore\ q\left(\frac{V}{d}\right) = mg$

$\therefore\ V = \frac{mgd}{q} = \frac{1.67\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}} = 10^{-9}$ V

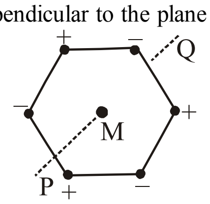
Hence X is nearly 1×10^{-9} V

2. **(c, d)** The potential of all points lie on the conductor is same. Thus potential at A = Potential at B

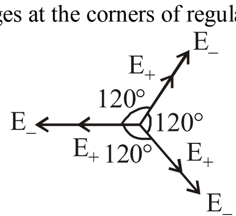
Total electric flux through cavity $= \frac{q}{\epsilon_0}$ this according to Gauss's theorem.

Option (a) and (b) are dependent on the curvature which is different at points A and B because cavity is elliptical.

3. (P) Here PQ is perpendicular to the plane of the hexagon.



Clearly electric field at the centre M of the hexagon due to the charges at the corners of regular hexagon is zero.



Here $|E_+| = |E_-|$.

i.e., $E = 0$ at M. (By symmetry)

The electric potential due to all the charges at M is zero.

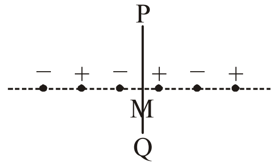
i.e., V = 0 at M

When the system of charges is rotated about line PM, the net current will be zero.

Hence magnetic field $B = 0$ at M.

When $B = 0$ then $\mu = 0$

(q) M is the mid-point between the two innermost charges.



Electric field due to the inner most positive and negative charges at M $E_1 = 2\left[k\frac{q}{r^2}\right]$ towards left.

Electic field due to the next positive and negative charges at M

$E_2 = 2\left[k\frac{q}{(2r)^2}\right]$ towards right. Electric field due to the outermost positive and negative charges at M

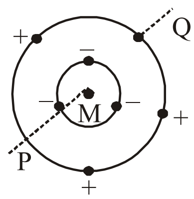
$E_3 = 2\left[k\frac{q}{(3r)^2}\right]$ towards left. i.e., $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{d}0$

The electric potential due to the charges at M

$$V = k\left[\frac{+q}{r}\frac{-q}{r} + \frac{q}{2r} - \frac{q}{2r} + \frac{q}{3r} - \frac{q}{3r}\right] = 0$$

Net current due to the innermost positive and negative charges is zero. Similarly the net current due to other charges in pairs is zero. Therefore, $B = 0$ $\therefore \mu = 0$

(r)



The net electric field due to negative charges $E_{inner} = 0$. Similarly the net electric field due to positive charges $E_{outer} = 0$.

$\therefore$ E = 0

The electric potential due to negative charges at M is different from the electric potential due to positive charges at M.

$\therefore$ V ≠ 0 at M.

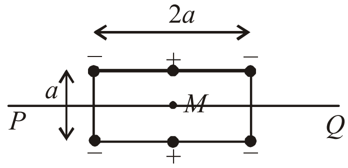
When the system of charges rotate, we get a current I_1 due to negative charges and another current I due to positive charges. The magnitude of B at M due to the currents is different.

$\therefore B \neq 0$ and $\mu \neq 0$.

Here electric field due to different charges cancel out in pairs.

$\therefore$ E = 0

(s)



The electric field at M due to all the charges is zero because

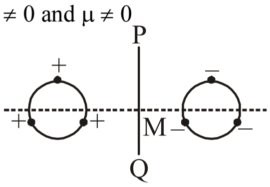
The potential at M due to the charges

$$V = k\left[\frac{+q}{a/2} + \frac{q}{a/2} - 4\left(\frac{q}{\frac{\sqrt{5}a}{4}}\right)\right] \neq 0$$

When the whole system is set into rotation with a constant angular velocity about the line *PQ* we get three loops in which current is flowing.

The magnetic field due to these currents produce a resultant magneic field at *M* which is not equal to zero. Therefore a net magnetic dipole moment will be produced.

$\therefore B \neq 0$ and $\mu \neq 0$

(t) 

Here $E \neq 0$ as there will be a net electric field due to the arrangement of charges at *M* towards the right side.

Electic potential $V = 0$ at *M* due to symmetrical arrangement of positive and negative charges.

Net current is zero when the system of charges rotates about *PQ*, due to symmetrical arrangement of charges. $\therefore B = 0$ and $\mu = 0$

4. **(a)** In process 1, work done by battery, $w = q \times V = CV_o \times V_o = CV_o^2$

Energy stored in the battery $E_C = \frac{1}{2}CV_0^2$

Heat dissipated across resistance,

$$E_D = W - E_C = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$$

$\therefore \quad E_C = E_D$

5. **(c)** Let V_i and V_f be the initial and final voltage in each step of process 2. Then

Energy dissipated = $W_{battery} - \Delta U$

$$= C(V_f - V_i)V_f - \frac{1}{2}C(V_f - V_i)^2$$

$$= \frac{1}{2}C(V_f - V_i)^2$$

$\therefore$ Total energy dissipated across the resistance,

$$E_D = \frac{1}{2}C\left[\left(\frac{V_o}{3} - 0\right)^2 + \left(\frac{2V_o}{3} - \frac{V_o}{3}\right)^2 + \left(V_o - \frac{2V_o}{3}\right)^2\right]$$

or, $E_d = \frac{1}{6}CV_o^2$

6. **(c)** After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate. But ball do not execute simple harmonic motion as force on it $\propto x$

7. **(d)** Average current, $I_{av} \propto \frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

From $S = ut + \frac{1}{2}at^2$

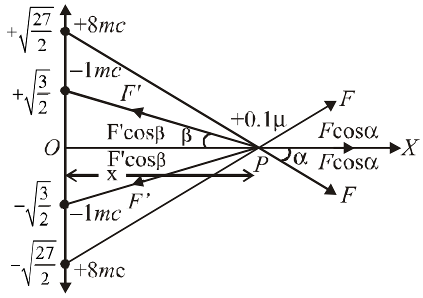
$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q \times 2V_0}{mh}\right) \times t^2 \left(\because a = \frac{F}{m} = \frac{qE}{m}\right)$$

$\therefore \quad t \propto \frac{1}{V_0}$ – (iii) $[\because Q \propto V_0]$

From eq. (i), (ii) and (iii)

$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^2$$

8. Let *P* be any point at a distance *x* from the origin. As shown in the figure, there are two forces of repulsion acting due to two charges of + 8 mC. The net force is $2F \cos\alpha$ towards right. Similarly there are two forces of attraction due to two charges of – 1 mC. The net force due to these force is $2F' \cos\beta$ towards left.



For net force to become zero.

$2F \cos\alpha = 2F' \cos\beta$

$$\frac{K \times 8 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{27}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{27}{2}}}$$

$$= \frac{K \times 1 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{3}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{3}{2}}} \Rightarrow x = \pm\sqrt{\frac{5}{2}}$$

At $x > \sqrt{\frac{5}{2}}$ *m* force is repulsive towards + *ve* *x*-axis and for $x < \sqrt{\frac{5}{2}}$ *m* force is attractive towards negative *x*-axis.

Electric potential of the four charges at $x = \sqrt{\frac{5}{2}}$

$$V = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{27}{2}}} - \frac{2 \times 9 \times 10^9 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{3}{2}}}$$

$$= 2 \times 9 \times 10^9 \times 10^{-6}\left[\frac{8}{4} - \frac{1}{2}\right] = 2.7 \times 10^4 \text{ V}$$

Kinetic energy is required to overcome the force of repulsion from ∞ to $x = \sqrt{\frac{5}{2}}$.

The work done in this process $W = q\,(V)$

$\therefore \quad W = 0.1 \times 10^{-6} \times 2.7 \times 10^4 = 2.7 \times 10^{-3}$ J

By energy conservation $\frac{1}{2}mV_0^2 = 2.7 \times 10^{-3}$

$\Rightarrow \frac{1}{2}\times 6\times 10^{-4} V_0^2 = 2.7\times 10^{-3} \Rightarrow V_0 = 3$ m/s

K.E. at the origin

Potential at origin

$$V_{x=0} = \frac{2\times 9\times 10^9 \times 8\times 10^{-6}}{\sqrt{\frac{27}{2}}} - \frac{2\times 9\times 10^9 \times 10^{-6}}{\sqrt{\frac{3}{2}}}$$

$= 2.4 \times 10^4$

Again by energy conservation

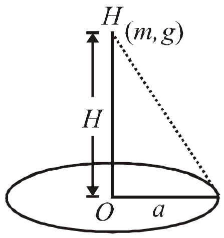
$$\text{K.E.} = q\left[V_{x=\frac{\sqrt{5}}{2}} - V_{x=0}\right]$$

$\therefore$ K.E. $= 0.1 \times 10^{-6}\,[2.7 \times 10^4 - 2.4 \times 10^4]$

$= 0.1 \times 10^{-6} \times 0.3 \times 10^4 = 3 \times 10^{-4}$ J

9. **(a)** Given a = radius of disc, σ = surface charge density, $q/m = 4\varepsilon_0 g/\sigma$

Potential due to a charged disc at any axial point situated at a distance x from O



$$V(x) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + x^2} - x]$$

At $x = H$

$$V(H) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H]$$

And $V(O) = \frac{\sigma a}{2\varepsilon_0}$

When the particle is dropped along the axis of dise, energy is conserved. Loss of gravitational potential energy = gain in electric potential energy

$mgH = q\Delta V = q\,[V(0) - V(H)]$

$$mgH = q\,\frac{\sigma}{2\varepsilon_0}\,[a - \{\sqrt{(a^2 + H^2)} - H\}] \quad \text{... (i)}$$

From the given relation $\frac{2}{m} = \frac{4\varepsilon_0 g}{\sigma} \Rightarrow \frac{\sigma q}{2\varepsilon_0} = 2mg$

Putting this value in equation (i),

$mgH = 2mg\;[a - \{\sqrt{(a^2 + H^2)} - H\}]$

or $H = \frac{4a}{3}$

(b) Total potential energy of the particle at height H

$U(x) = mgx + qV(x)$

$= mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2 + x^2} - x)$

$= mgx + 2mg\;[\sqrt{(a^2 + x^2)} - x]$ ....(ii)

$U_{(A)} = mgH + 2mg\left[\sqrt{a^2 + H^2} - H\right]$

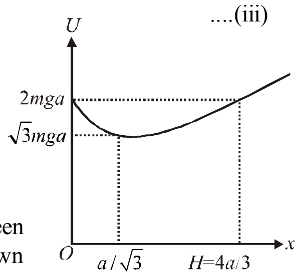
$= mg\left[2\sqrt{a^2 + H^2} - H^2\right]$ ....(iii)

For equilibrium : $\frac{dU}{dH} = 0$

at $H = \frac{a}{\sqrt{3}}$

$\therefore U_{\min} = \sqrt{3}$ mga

From equation (ii), graph between $U(x)$ and x a parabola, is as shown below.



Hence $H = \frac{a}{\sqrt{3}}$ is stable equilibrium position.

10. (a) Let S_1 contain charge Q before contact with S_2.

On contact, let a charge q_1 shift from S_1 to S_2.

$$\frac{k(Q - q_1)}{r} = \frac{kq_1}{R} \text{ where } k = \frac{1}{4\pi\varepsilon_0} \text{ or } q_1 = Q\left(\frac{R}{R+r}\right)$$

On second contact, similarly,

$$\frac{k\left[Q - (q_2 - q_1)\right]}{r} = \frac{kq_2}{R}$$

or $rq_2 = RQ - Rq_2 + Rq_1$

or $rq_2 = RQ - Rq_2 + RQ\left(\frac{R}{R+r}\right)$

or $q_2 = Q\left[\left(\frac{R}{R+r}\right) + \left(\frac{R}{R+r}\right)^2\right]$

On third contact,

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On n^th cotact,

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots\ldots + \left(\frac{R}{R+r}\right)^n\right]$$

In G. P.,

$$S_n = \frac{a\left(1 - r_1^n\right)}{\left(1 - r_1\right)} \text{ where } r_1 \text{ is common ratio.}$$

or $\quad q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$

Electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^2}{2C} = \frac{q_n^2}{2\left(4\pi\varepsilon_0 R\right)}$$

or $\quad U_n = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\times\left[\frac{QR}{r}\left\{1 - \left(\frac{R}{R+r}\right)^n\right\}\right]^2$

(b) Limiting value of energy as $n \to \infty$.

Let us calculate q_n when n tends to ∞.

For G.P., $S_\infty = \frac{a}{1 - r_1}$ where r_1 = common ratio

$$\therefore \quad q_\infty = \frac{QR}{R+r}\left[\frac{1}{1 - \frac{R}{R+r}}\right] \text{ or } q_\infty = \frac{QR}{r}$$

$$\therefore \quad U_\infty = \frac{q_\infty^2}{2C} = \left(\frac{QR}{r}\right)^2 \times \frac{1}{2\times\left(4\pi\varepsilon_0\right)\times(R)}$$

or $\quad U_\infty = \frac{Q^2R^2}{r^2 \times 2\times 4\pi\varepsilon_0 R}$ or $U_\infty = \frac{Q^2 R}{2\left(4\pi\varepsilon_0\right)r^2}$

11. **(a)** A charge is uniformly distributed over a non-conducting spherical volume of radius R. Let us consider a shell of the thickness dx at a distance x from the centre of a sphere

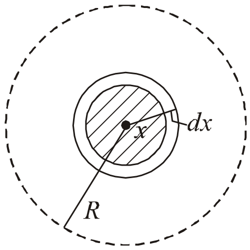
$$\text{Volume of the shell} = \frac{4}{3}\pi\left[(x + dx)^3 - \frac{4}{3}\pi x^3\right]$$

$= \frac{4}{3}\pi\left[(x+dx)^3 - x^3\right]$

$= \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$

$= \frac{4}{3}\pi x^3\left[1+\frac{3dx}{x} - 1\right]$

$= \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$



Let ρ be the charge per unit volume of the sphere

$\therefore$ Charge of the shell = $dq = 4\pi x^2 \rho dx$ ... (i)

Potential at the surface of the sphere of radius x

$= \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x}$ $\left[\because V = k\frac{q}{r}\right]$

$\therefore$ Potential at the surface of the sphere of radius $x = \frac{\rho x^2}{3\varepsilon_0}$

Work done in bringing the charge dq on the sphere of radius x dW

$= \frac{\rho x^2}{3\varepsilon_0} \times dq \Rightarrow dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$

$\therefore$ Work done in accumulating the charge Q over a spherical volume of radius R

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{4/3\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This work done is equal to the energy stored in the system.

(b) The corresponding energy needed to completely disassemble the planet earth against the gravitational pull.

$$E = \frac{3Q^2}{5\times(4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R} \text{ where } K = \frac{1}{4\pi\varepsilon_0}$$

Replacing $\frac{1}{4\pi\varepsilon_0}$ or K by G and Q^2 by M^2.

$\therefore$ $E = \frac{3GM^2}{5R}$

$g = \frac{GM}{R^2} \Rightarrow G = \frac{gR^2}{M}$

$\therefore$ $E = \frac{3}{5}\frac{gR^2}{M}\frac{M^2}{R} = \frac{3}{5}gMR = \frac{3}{5}\times 10 \times 2.5 \times 10^{31}$

$= 1.5 \times 10^{32}$ J

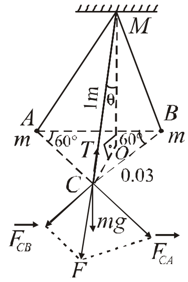
(c) If the same charge of Q conlomb is given to a spherical conductor of the same radius, then $C = 4\pi\varepsilon_0 R$ and energy

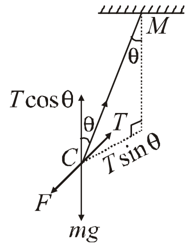
$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q2}{4\pi\varepsilon_0 R}$$

12. Minimum time for the dipole to align along the direction of electric field

$$t = \frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{ML}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}.$$

13. Each particle will be in equilibrium under the act of three force tension of string T, weighting, resultant electrostatic force F of the two other charges. Force F and mg are perpendicular.





Resolving T in the direction of mg and F and applying the condition of equilibrium,

$T\cos\theta = mg$; $T\sin\theta = F$

$\therefore$ $\tan\theta = \frac{F}{mg}$ ... (i)

$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA}F_{CB}\cos\alpha}$

$\therefore$ $F = \sqrt{F_{CA}^2 + F_{CA}^2 + 2F_{CA}^2 \times \frac{1}{2}}$

$F = \sqrt{3}F_{CA} = \sqrt{3} \times \frac{kq^2}{(CA)^2}$ ... (ii)

$|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°$]

Let T make an angle θ with the vertical

$OC = \frac{2}{3}\sqrt{(0.03)^2 - (0.015)^2} = 0.0173$ m

$\therefore$ $OM = 0.9997$

$\therefore \tan\theta = \frac{OC}{OM} = \frac{0.0173}{0.9997}$ .... (iii)

From eq. (i), (ii) and (iii)

$$\frac{0.0173}{0.9997} = \frac{\sqrt{3}\times 9\times 10^9 \times q^2}{(0.03)^2 \times 10^{-3} \times 9.8}$$

$\therefore q = 3.16 \times 10^{-9}$ C.

17 Current Electricity

Topic-1 : Electric Current, Drift of Electrons, Ohm's Law, Resistance and Resistivity

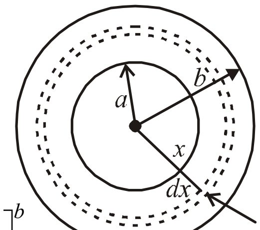
1. **(c)** **Ammeter :** In series connection, the same current flows through all the components. It aims at measuring the current flowing through the circuit and hence, it is connected in series.
Voltmeter : A voltmeter measures voltage change between two points in a circuit. So we have to place the voltmeter in parallel with the cicuit component.

2. **(b)** $\rho_M = 98 \times 10^{-8}$
$\rho_A = 2.65 \times 10^{-8}$
$\rho_C = 1.724 \times 10^{-8}$
$\rho_T = 5.65 \times 10^{-8}$
$\therefore \rho_M > \rho_T > \rho_A > \rho_C$

3. **(a)** When $i = 0$, $V = \varepsilon = 1.5$ volt

4. **(d)** Charge mobility
$(\mu) = \frac{V_d}{E}$ [Where V_d = drift velocity]
and resistivity $(\rho) = \frac{E}{j} = \frac{EA}{I} \Rightarrow E = \frac{I(\rho)}{A}$
$$\Rightarrow \mu = \frac{V_d}{E} = \frac{V_d A}{I_\rho} = \frac{1.1\times10^{-3}\times\pi\times\left(5\times10^{-3}\right)^2}{5\times1.7\times10^{-8}}$$
$$\mu = 1.0\frac{m^2}{V_s}$$

5. **(a)** $dR = \frac{(\rho)(dx)}{4\pi x^2}$



$R = \int dR$
$$\int dR = \rho\int_a^b \frac{dx}{4\pi x^2} \Rightarrow R = \frac{\rho}{4\pi}\left[\frac{-1}{x}\right]_a^b$$
$$R = \left(\frac{\rho}{4\pi}\right)\cdot\left(\frac{1}{a} - \frac{1}{b}\right)$$

6. **(c)** $\rho = \frac{m}{ne^2\tau}$
$$= \frac{9.1\times10^{-31}}{8.5\times10^{28}\times(1.6\times10^{-19})^2\times25\times10^{-15}}$$
$= 10^{-8}$ Ω-m

7. **(d)** Number 2 is associated with the red colour. This colour is replaced by green.
∵ Colour code figure for green is 5
∴ New resistance = 500 Ω

8. **(a)** Clearly, from graph
Current, $I = \frac{dq}{dt} = 0$ at $t = 4s$ [Since q is constant]

9. **(a)** Using, $I = neAv_d$
$\therefore$ Drift speed $v_d = \frac{1}{neA}$
$$\frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}} = 0.02 \text{ mms}^{-1}$$

10. **(a)** Colour code for carbon resistor
Bl, Br, R, O, Y, G, Blue, V, Gr, W
0 1 2 3 4 5 6 7 8 9
Resistance, R = AB × C ± D
Bands A and B are the first two significant figures of resistance
B and C indicates the decimal multiplier or the number of zeros that follow A and B
B and D is tolerance: Gold = ± 5%,
Silver = ± 10 % No colour = ± 20%
$R = 53\times10^4 \pm 5\% = 530\,k\Omega \pm 5\%$

11. **(c)** Resistance after temperature increases by 500°C i.e.,
$$R_t = \frac{V}{I} = \frac{220}{2} = 110\Omega$$
$R_0 = 100$ (given) temperature coefficient of resistance, α = ?
using $R_t = R_0(1 + \alpha t)$
$110 = 100(1 + \alpha 500)$
$$\alpha = \frac{10}{100\times500} \quad \text{or,} \quad \alpha = 2\times10^{-4}{}^\circ C^{-1}$$

12. **(a)** Given, $R_1 = 100\ \Omega$, $r' = r/2$, R_2 = ?
Resistivity of wire, $R = \frac{\rho l}{A}$ ∵ Area × length = volume
Hence, $R = \frac{\rho V}{A^2}$
Since, ρ → constant, V → constant
$R \propto \frac{1}{A^2}$ or $R \propto \frac{1}{r^4}$ ∵ $A = \pi r^2$
$\frac{R_2}{R_1} = 16 \Rightarrow R_2 = 16\times100 = 1600\ \Omega$, Resistance of new wire.

13. **(c)** Electric field at a distance r from line charge
$E = \frac{\lambda}{2\pi\varepsilon r} = \frac{dV}{dr}$ (λ = linear charge density of wire)
$$dV = -\frac{\lambda}{2\pi\varepsilon r}dr$$
Current through the elemental shell
$$I = \frac{|dV|}{dr} = \frac{\frac{\lambda}{2\pi\varepsilon r}dr}{\frac{1}{\sigma}\times\frac{dr}{2\pi r l}} = \frac{\lambda\sigma l}{\varepsilon}$$
$$\left(\because R = \rho\frac{l}{A} \therefore dR = \rho\frac{dr}{2\pi r l} = \frac{1}{\sigma}\frac{dr}{2\pi r l}\right)$$
This current is radially outwards,
$$\therefore \frac{d}{dt}(\lambda l) = \frac{-\lambda\sigma l}{\varepsilon} \Rightarrow \int_{\lambda_0}^{\lambda}\frac{d\lambda}{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right)\int_0^t dt$$
$$\Rightarrow \lambda = \lambda_0 e^{-(\sigma/\varepsilon)t}$$
$$\therefore J = \frac{I}{2\pi r l} = \frac{\lambda\sigma l}{2\pi\varepsilon r l} = \frac{\lambda\sigma}{2\pi\varepsilon r}$$

or, $J=\left(\frac{\lambda_0\sigma}{2\pi\varepsilon r}\right)e^{-(\sigma/\varepsilon)t} \Rightarrow J=J_0e^{-(\sigma/\varepsilon)t}$

14. **(b)** $V = IR = (neAv_d)\rho\frac{\ell}{A}$

$\therefore \quad \rho = \frac{V}{V_d \, lne}$

Here V = potential difference

l = length of wire

n = no. of electrons per unit volume of conductor.

e = no. of electrons

Placing the value of above parameters we get resistivity

$$\rho = \frac{5}{8\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-4}\times0.1}$$

$= 1.6 \times 10^{-5}\ \Omega m$

15. **(c)** $i = neAV_d$ and $V_d \propto \sqrt{E}$ (Given)

or, $i \propto \sqrt{E}$

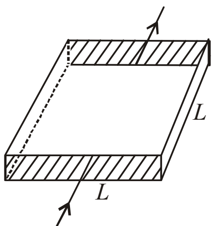
$i^2 \propto E$

$i^2 \propto V$

Hence graph (c) correctly dipicts the *V-I* graph for a wire made of such type of material.

16. **(c)** We know resistance, $R = \rho\frac{l}{a}$

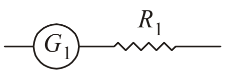
Here $l = L$ and $a = L \times t$



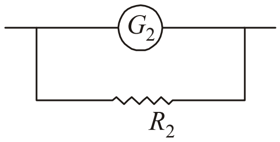
$\therefore \quad R = \frac{\rho L}{L \times t} = \frac{\rho}{t}$

Hence *R* is independent of *L* and inversely proportional to *t*.

17. **(c)** A voltmeter is made by connecting a high resistance R_1 in series with the galvanometer G_1.



An ammeter is made by connecting a low resistance R_2 in parallel with the galvanometer G_2.



Voltmeter is connected in parallel with the test resistor R_T in circuit.

Ammeter is connected in series with the test resistor R_T in circuit.

A variable voltage source *V* is connected in series with the test resistor R_T in circuit.

18. **(a)** In electric circuit ammeter is connected in series with resistance and voltmeter parallel with the net resistance.

In ohm's law, we check $V = IR$ by varying net resistance of the circuit.

19. **(d)** Copper is a metal whereas Germanium is Semi-conductor. Resistance of metal decreases and semi-conductor increases with decrease in temperature.

20. **(d)** Resistance increases with temperature as

$R_t + R_0(1 + \propto t)$

$R_1 = R_0(1 + \alpha t_1) \Rightarrow 1 = R_0[1 + 0.00125 \times 27]$

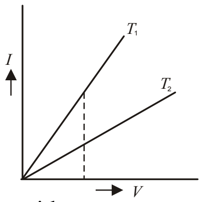
$R_2 = R_0(1 + \alpha t_2) \Rightarrow 2 = R_0[1 + 0.00125 \times t_2]$

Solving we get

$T_2 = 854°C = 854°C + 273°C = 1127$ K

21. **True,** From the given voltage, current graph

Slope of graph = $\frac{I}{V} = \frac{1}{R}$



$\Rightarrow$ Resistance, $R = \frac{1}{\text{slope of graph}}$

$(\text{Slope})_{T_2} < (\text{Slope})_{T_1} \therefore (\text{Resistance})_{T_2} > (\text{Resistance})_{T_1}$

For a metallic wire, resistance increases with temperature, $\therefore T_2 > T_1$

22. **False,** Due to thermal energy, electrons in a conductor are free and have thermal velocities. The electrons have motion in random directions even in the absence of potential difference.

23. **(a, c, d)** Resistance of elementary strips

$$\therefore \int\frac{1}{d_R} = \int_{R_1}^{R_2}\frac{tdx}{\rho\pi x} \Rightarrow \frac{1}{R} = \frac{t}{\pi\rho}ln\left(\frac{R_2}{R_1}\right)$$

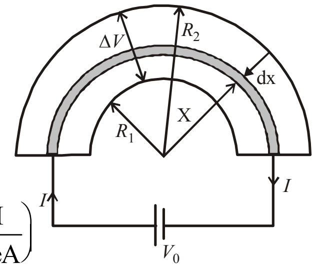
$$\text{Resistance, R} = \frac{\pi\rho}{t\,ln\left(\frac{R_2}{R_1}\right)} \quad \Rightarrow I = \frac{V_0}{R} = \frac{V_0 t\, ln\left(\frac{R_2}{R_1}\right)}{\pi\rho}$$

Hence option (a) is correct.

And, for circular motion of electron, ΔV develops

Inner surface at higher potential so that electric field develops radially outward.

So option (b) is correct.



$$\frac{mV_d^2}{r} = q\vec{E} \Rightarrow \vec{E} = \frac{mV_d^2}{qr}$$

$$\vec{E} = \frac{mI^2}{n^2e^2A^2qr}$$

$$\left(\because \text{Drift velocity, } V_d = \frac{I}{neA}\right)$$

$\Delta V = \int \vec{E}.\vec{dr}$ or, $\Delta V \propto V_d^2 \quad \therefore \Delta V \propto I^2$

Hence option (d) is correct.

24. **(c)** The conductivity (σ) of a semiconductor increases with increase in temperature i.e. the resistivity (*f*) decreases with increase in temperature as $P = \frac{1}{\sigma}$.

In a conducting solid, the collisions become more frequent with increase of temperature.

25. As we know, resistance, $R = \rho\frac{l}{A}$

And $V = Al \Rightarrow A = \frac{V}{l}(V = \text{volume of wire})$

$\therefore R = \frac{\rho l}{V/l} = \frac{\rho l^2}{V}$

Here ρ and V are constant $\quad \therefore R \propto l^2$

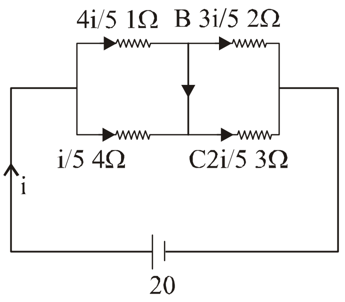
$\therefore$ Percentage change in resistance $\frac{\Delta R}{R}\times100 = 2$ (%change in l) = 2(0.1%) = 0.2%

Topic-2 : Combination of Resistances

1. **(b)** From circuit diagram,

$$\frac{1}{R_1} = \frac{1}{1} + \frac{1}{4} \Rightarrow R_1 = \frac{4}{5}$$

$$\frac{1}{R_2} = \frac{1}{2} + \frac{1}{3} \Rightarrow R_2 = \frac{6}{5}$$



$$R_{\text{eff}} = R_1 + R_2 = \frac{4}{5} + \frac{6}{5} = 2\Omega$$

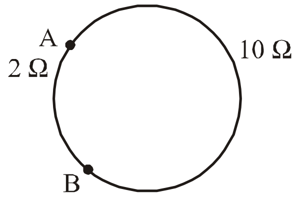
$$i = \frac{v}{R_{\text{eff}}} = \frac{20}{2} = 10A \quad \therefore I_{BC} = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = 2A$$

2. **(c)**

3. **(c)** When length becomes double its resistance becomes $(R\ \mu\ l^2)$

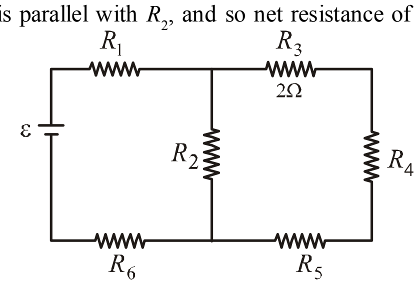
$R = 4 \times 3 = 12\ \Omega$

$$R_{eq} = \frac{2 \times 10}{12} = \frac{5}{3}\Omega$$



4. **(c)** R_3, R_4 and R_5 are in series so their equivalent $R = 20 + 5 + 25 = 50\ \Omega$

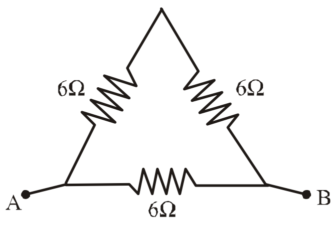
This is parallel with R_2, and so net resistance of the circuit



$$R_{eq} = \left(\frac{10 \times 50}{10 + 50}\right) + 15 + 30 = \frac{160}{3}\Omega$$

So, $$i = \frac{\varepsilon}{R_{eq}} = \frac{15}{(100/3)} = \frac{9}{32}A$$

5. **(a)**



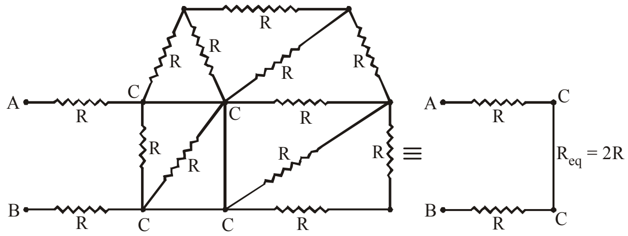
Resistance, R ∝ l so resistance of each side of the equilateral triangle = 6 Ω

Resistance R_{eq} between any two vertices

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq.} = 4\Omega$$

6. **(a)** $R_{series} = R_1 + R_2 + ... + R_n$

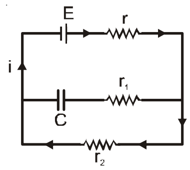
$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$



7. **(a)** In steady state, flow fo current through capacitor will be zero.

Current through the circuit,

$$i = \frac{E}{r + r_2}$$



Potential difference through capacitor

$$V_c = \frac{Q}{C} = E - ir = E - \left(\frac{E}{r + r_2}\right)r \quad \therefore Q = CE\frac{r_2}{r + r_2}$$

8. **(b)** The potential difference in each loop is zero.

∴ No current will flow or current in each resistance is Zero.

9. **(b)** As resistance of wire, $R = \int \frac{l}{A}$

$$R_{Fe} = \frac{\rho_{Fe} \times l_{Fe}}{A_{Fe}} = \frac{10^{-7} \times 50 \times 10^{-3}}{4 \times 10^{-6}} = \frac{25}{2} \times 10^{-4}$$

$$R_{Al} = \frac{\rho_{Al} \times l_{Al}}{A_{Al}} = \frac{2.7 \times 10^{-8} \times 50 \times 10^{-3}}{(49 - 4) \times 10^{-6}} = \frac{2.7 \times 50}{45} \times 10^{-5}$$

$= 0.3 \times 10^{-4}$

As potential difference across both resistors is same, so they are in parallel combination.

$$\therefore R_{PQ} = \frac{R_{Fe} \times R_{Al}}{R_{Fe} + R_{Al}} = \frac{12.5 \times 10^{-4} \times 0.3 \times 10^{-4}}{12.8 \times 10^{-4}} = \frac{1875}{64}\mu\Omega$$

10. **(b)** Resistance between P and Q

$$r_{PQ} = r \,||\left(\frac{r}{3} + \frac{r}{2}\right) = \frac{r \times \frac{5}{6}r}{r + \frac{5}{6}r} = \frac{5}{11}r$$

Resistance between Q and R

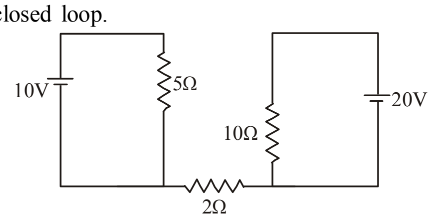
$$r_{QR} = \frac{r}{2} \,||\, (r + \frac{r}{3}) = \frac{\frac{r}{2} \times \frac{4}{3}r}{\frac{r}{2} + \frac{4}{3}r} = \frac{4}{11}r$$

Resistance between P and R

$$r_{PR} = \frac{r}{3} \,||\left(\frac{r}{2} + r\right) = \frac{\frac{r}{3} \times \frac{3}{2}r}{\frac{r}{3} + \frac{3}{2}r} = \frac{3}{11}r$$

Hence, it is clear that r_{PQ} is maximum

11. **(a)** The current in 2Ω resistor = zero because it is not a part of any closed loop.

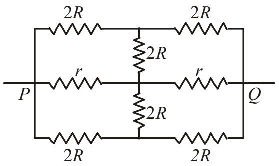


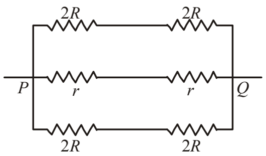
12. **(c)** Given resistance of each resistor 'R'

$$R_{PQ}=\frac{5}{11}R, R_{QR}=\frac{4}{11}R \text{ and } R_{PR}=\frac{3}{11}R$$

$\therefore$ R_{PQ} is maximum.

13. **(a)** The circuit above and below the axis POQ symmetrical and represents balanced wheatstone bridge. Hence the central resistance 2R is ineffective.





Therefore the equivalent circuit is redrawn as follows.

$$\therefore \quad \frac{1}{R_{PQ}}=\frac{1}{4R}+\frac{1}{4R}+\frac{1}{2r}=\frac{r+r+2R}{4Rr}$$

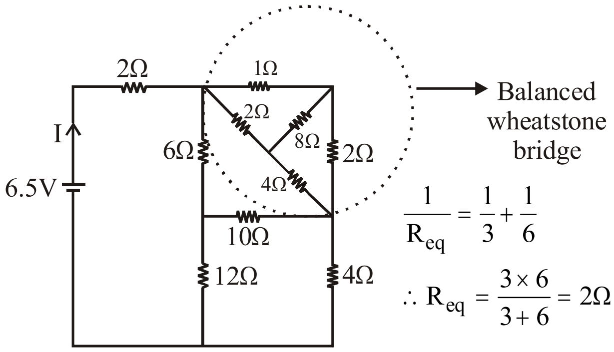
for, $R_{PQ}=\dfrac{2Rr}{R+r}$

14. **(c)** In the given circuit,

$$\frac{1}{R_{eq}}=\frac{1}{30}+\frac{1}{60}=\frac{90}{30\times 60}\Rightarrow R_{eq}=20\ \Omega$$

Current in the circuit, $I=\dfrac{V}{R}=\dfrac{2}{20}=0.1$ Amp.

15. **(1)** In the figure (a) no current flows through 8Ω resistance



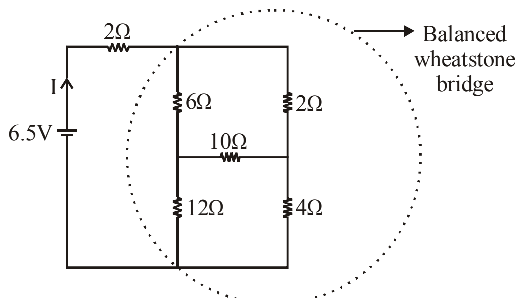
(a)

$$\frac{1}{R_{eq}}=\frac{1}{3}+\frac{1}{6}$$

$$\therefore R_{eq}=\frac{3\times 6}{3+6}=2\Omega$$

Again the equivalent resistance of balanced wheat stone bridge fig (b) no current through 10Ω resistance.

$$\therefore R_{eq}=\frac{6\times 18}{24}=\frac{9}{2}\Omega$$

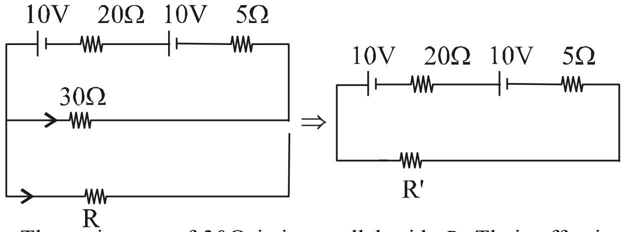


(b)

Therefore the current through the resistor $R = 2\Omega$

$$I=\frac{V}{R}=\frac{6.5}{2+\frac{9}{2}}=\frac{6.5}{6.5}=1A$$

16. **(30.00)**



The resistance of 30Ω is in parallel with R. Their effective resistance

$$\frac{1}{R'}=\frac{1}{30}+\frac{1}{R}$$

$$R'=\frac{30R}{30+R} \qquad \text{...(i)}$$

Also, V = IR $\Rightarrow 10=\dfrac{20\times 20}{R'+25}$

$\Rightarrow R'+25=40 \Rightarrow R'=15$

$$R'=15=\frac{30R}{30+R} \qquad \text{Using (i)}$$

$\Rightarrow 30+R=2R \quad \Rightarrow R=30\ \Omega$

17. **(0 V)** Here in the circuit 8 batteries each of 5V therefore, net emf of the circuit = 8 (5V) = 40 V

Internal resistance of each cell, $r = 0.2\Omega$.

Net resistance in the circuit = 8 (0.2 Ω) = 1.6 Ω

$\therefore$ Current flowing through the circuit,

$$I=\frac{40\text{ V}}{1.6\Omega}=25\text{ A}$$

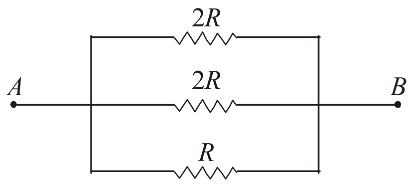
Hence voltmeter reading

$V = E - IR$ = (5V) – (25A) (0.2 Ω)

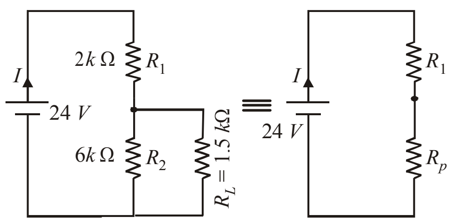
= 5V – 5V = 0

18. $\left(\dfrac{\boldsymbol{R}}{\boldsymbol{2}}\right)$ All the three resistances $2R$, $2R$ and R are in parallel as shown in figure.

Hence, $\dfrac{1}{R_{AB}}=\dfrac{1}{2R}+\dfrac{1}{2R}+\dfrac{1}{R}=\dfrac{2}{R}$ or, $R_{AB}=\dfrac{R}{2}$



19. **(a, d)**



$$R_p=\frac{R_2\times R_L}{R_2+R_L}=\frac{6\times 1.5}{6+1.5}=\frac{9}{7.5}k\Omega$$

$$\therefore R_{total}=R_1+R_p=2+\frac{9}{7.5}=3.2\ k\Omega$$

(a) $I=\dfrac{V}{R}=\dfrac{24}{3.2}\text{mA}=7.5\,\text{mA}$

$I = I_{R_1}$

$$I_{R_2} = \left(\frac{R_L}{R_L + R_2}\right) I = \left(\frac{1.5}{1.5+6}\right) \times 7.5 = 1.5\ mA$$

$\therefore I_{R_L} = I_{R_1} - I_{R_2} = 7.5 - 1.5 = 6\text{mA}$

(b) Potential difference across load

$$V_{RL} = (I_{RL}) R_L = 6 \times 1.5 = 9V$$

(c) Ratio of powers dissipated in R_1 and R_2

$$\frac{PR_1}{PR_2} = \frac{(I_{R_1})^2 R_1}{(I_{R_2})^2 R_2} = \frac{(7.5)^2 \times 2}{(1.5)^2 \times 6} = \frac{25}{3}$$

(d) When R_1 and R_2 are interchanged, then

$$R_P^1 = \frac{R_2 R_L}{R_2 + R_L} = \frac{2 \times 1.5}{3.5} = \frac{6}{7} k\Omega$$

$R'_{\text{total}} = 6 + R'_P = 6 + \dfrac{6}{7} k\Omega$

Now P.d. across R_L

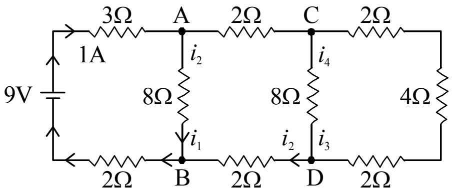
$$V'_{RL} = 24\left(\frac{6/7}{6+67}\right) = 3V$$

i.e., Now potential becomes $\dfrac{3}{9} = \dfrac{1}{3}$rd. Therefore power dissipated $P = \dfrac{V^2}{R}$ or $P \propto V^2$ will decrease by a factor of 9.

20. **(d)** Here net resistance of the circuit = 9Ω.

$\therefore$ Current drawn from the battery,

$i = \dfrac{V}{R} = \dfrac{9}{9} = 1A$ = current through 3Ω resistor



Potential difference between A and B

$V_A - V_B = 9 - 1(3+2) = 4V = 8i_1$

$\therefore i_1 = 0.5$ A $\quad \therefore i_2 = 1 - i_1 = 0.5$ A

Similarly, potential difference between C and D

$V_C - V_D = (V_A - V_B) - i_2(2+2)$

$= 4 - 4i_2 = 4 - 4\ (0.5) = 2V = 8i_3$

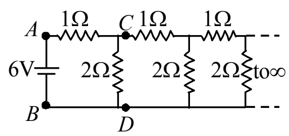
$\therefore i_3 = 0.25\text{A} \quad \therefore i_4 = i_2 - i_3 = 0.5 - 0.25 = 0.25\text{A}$

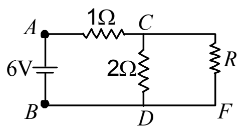
21. (i) In case of infinite ladder of resistances, the effective resistance, remains same when one identical item is added or removed from it.

Effective resistance between points C and D be R then the circuit can be redrawn as shown

Effective resistance between A and B

$$R_{eq} = 1 + \frac{2 \times R}{R+2}.$$





$\therefore \quad 1 + \dfrac{2 \times R}{R+2} = R$

$\Rightarrow \quad R + 2 + 2R = R^2 + 2R \Rightarrow R^2 - R - 2 = 0$

$\Rightarrow \quad R^2 - 2R + R - 2 = 0 \Rightarrow \quad R(R-2) + 1(R-2) = 0$

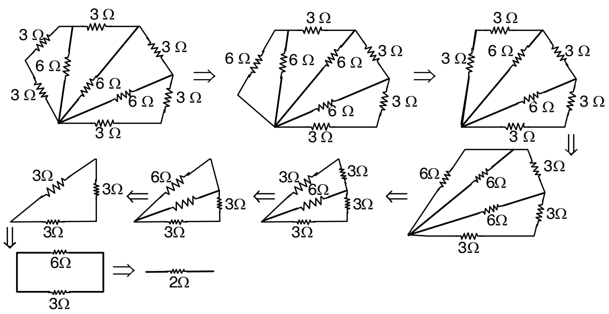
$\Rightarrow \quad [R+1][R-2] = 0 \Rightarrow \quad R = 2\Omega.$

(ii) $R_{AB} = 1\Omega + 1\Omega = 2\Omega \quad \therefore I_{AB} = \dfrac{6}{2} = 3\text{A}$

$i_{CD} = i_{EF}$ as resistances $R_{CD} = R_{EF}$

$\therefore \quad i_{CD} = i_{EF} = \dfrac{3}{2} = 1.5$ A

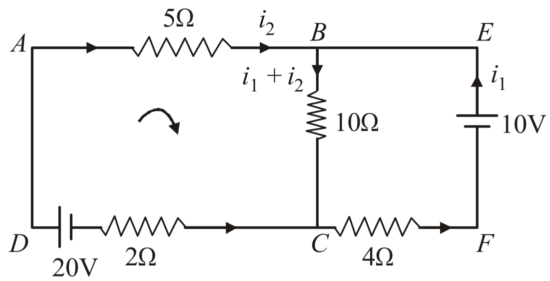
22.



Hence resistance between A and B, $R_{AB} = 2\Omega$

Topic-3 : Kirchhoff's Laws, Cells, Thermo e.m.f. & Electrolysis

1. **(c)**



Using Kirchoff's loop law in loop $ABCD$

$$-5i_2 - 10(i_1 + i_2) - 2i_2 + 20 = 0$$

$$\Rightarrow -10i_1 - 17i_2 + 20 = 0 \quad \text{...(i)}$$

Using Kirchoff's loop law in loop $BEFC$

$$\Rightarrow -10 + 4i_1 + 10(i_1 + i_2) = 0$$

$$\Rightarrow 14i_1 + 10i_2 + 10 = 0 \quad \text{...(ii)}$$

Multiplying equation (i) by 10, we have

$$(10i_1 + 17i_2 = 20) \times 10$$

$$\Rightarrow 100i_1 - 170i_2 = 200 \quad \text{...(iii)}$$

Multiplying equation (ii) by 17, we have

$$(14i_1 + 10i_2 = 10) \times 17$$

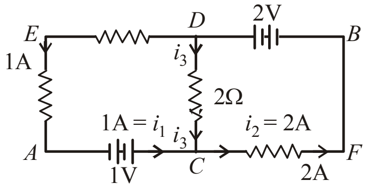
$$\Rightarrow 238i_1 - 170i_2 = 170 \quad \text{...(iv)}$$

On solving equations (iii) and (iv), we get

$$-138i_1 = 30 \Rightarrow i_1 = -\frac{30}{138} = -0.217$$

i_1 is negative it means current flows from positive to negative terminal.

2. **(d)**



Let us assume the potential at $A = V_A = 0$

Using Kirchoff's junction rule at C, we get

$i_1 + i_3 = i_2$

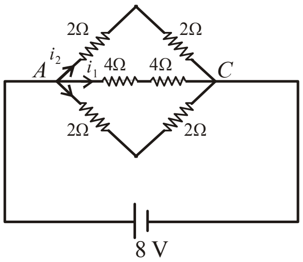
$1\text{A} + i_3 = 2\text{ A} \Rightarrow i_3 = 2A$

Now using Kirchoff's loop law along $ACDB$

$V_A + 1 + i_3(2) - 2 = V_B$

$\Rightarrow V_A + 1 + i_3(1) - 2 = V_B \;\Rightarrow V_B - V_A = 3 - 2 = 1\text{ volt}$

3. **(c)** The equivalent circuit can be drawn as

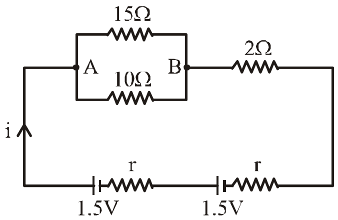


Voltage across $AC = 8$ V

Resistance $R_{AC} = 4 + 4 = 8\ \Omega$

$$i_1 = \frac{V}{R_{AC}} = \frac{8}{4+4} = 1\text{ Amp}$$

4. **(b)** For the given circuit



$$\text{i} = \frac{3}{8+2\text{r}}$$

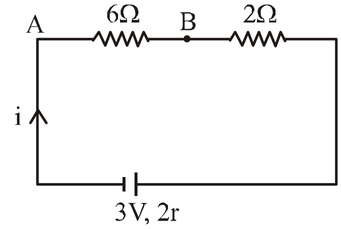
Now voltage across AB

$$\text{i} \times 6 = \frac{3}{8+2\text{r}} \times 6 = 2$$

$\Rightarrow 9 = 8 + 2\text{r}$

$\Rightarrow \text{r} = \frac{1}{2}\Omega$

$\equiv$

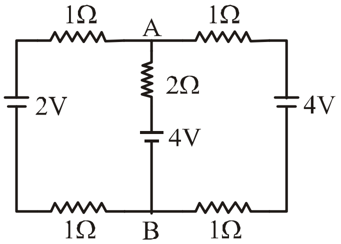


5. **(d)** Applying parallel combination of batteries

$$\frac{\frac{E_1}{1+1}+\frac{E_2}{2}+\frac{E_3}{1+1}}{\frac{1}{1+1}+\frac{1}{2}+\frac{1}{1+1}}$$

$$\frac{\frac{2}{2}+\frac{4}{2}+\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}} = \frac{5\times 2}{3}$$

$$= \frac{10}{3} = 3.3\text{ Volt}$$



6. **(d)** $i = \left(\frac{\varepsilon}{R+r}\right)$

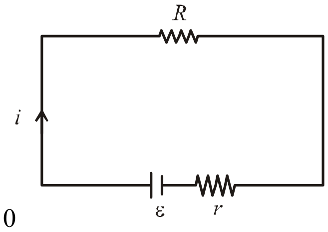
Power delivered to R.

$$P = i^2R = \left(\frac{\varepsilon}{R+r}\right)^2 R$$

P to be maximum, $\frac{dP}{dR} = 0$

or $\frac{d}{dR}\left[\left(\frac{\varepsilon}{R+r}\right)^2 R\right] = 0$

or $R = r$



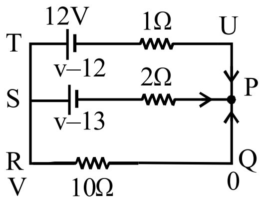
7. **(b)** Given, $E_1 = 1V$, $E_2 = 2V$, $E_3 = 3V$, $r_1 = 1\Omega$, $r_2 = 1\Omega$ and $r_3 = 1\Omega$

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}} = \frac{6}{3} = 2V$$

8. **(c)** Current passing through resistance R_1,

$$i_1 = \frac{v}{R_1} = \frac{10}{20} = 0.5A \text{ and, } i_2 = 0$$

9. **(b)**



Using Kirchhoff's law at P we get

$$\frac{V-12}{1} + \frac{V-13}{2} + \frac{V-0}{10} = 0$$

[Let potential at P, Q, U = 0 and at R = V

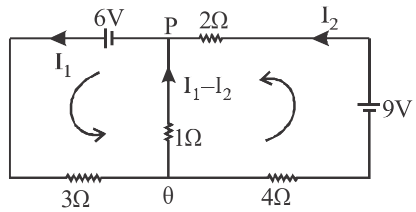
$$\Rightarrow \frac{V}{1} + \frac{V}{2} + \frac{V}{10} = \frac{12}{1} + \frac{13}{2} + \frac{0}{10}$$

$$\Rightarrow \frac{10+5+1}{10}V = \frac{24+13}{2} \;\Rightarrow V\left(\frac{16}{10}\right) = \frac{37}{2}$$

$$\Rightarrow V = \frac{37\times 10}{16\times 2} = \frac{370}{32} = 11.56\text{ volt}$$

10. **(a)** From KVL

$-6 + 3I_1 + 1(I_1 - I_2) = 0$



$6 = 3I_1 + I_1 - I_2$; $4I_1 - I_2 = 6$ ...(1)

$-9 + 2I_2 - (I_1 - I_2) + 3I_2 = 0$

$-I_1 + 6I_2 = 9$ ...(2)

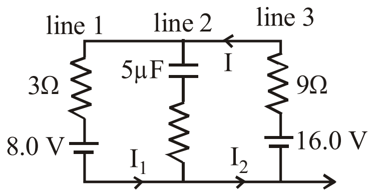
On solving (1) and (2)

$I_1 = 0.13A$

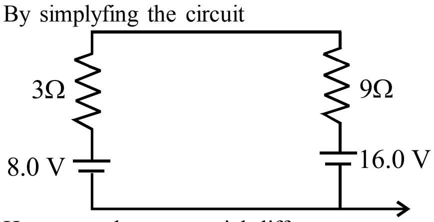
Direction Q to P, since $I_1 > I_2$.

11. **(c)** As no current flows through arm EB then
$V_D = 0V$
$V_E = 0V$
$V_B = -4V$
$V_A = 5V$
So, potential difference between the points A and D
$V_A - V_D = 5V$

12. **(b)**



In steady state capacitor is fully charged hence no current will flow through line 2.
By simplyfing the circuit



Hence resultant potential difference across resistances will be 8.0 V.

Thus current $I = \frac{V}{R} = \frac{8.0}{3+9} = \frac{8}{12}$ or, $I = \frac{2}{3} = 0.67$ A

13. **(b)** Let R be the resistance of wire.

Energy released in t second, $H = \frac{(3V)^2}{R} \times t \left(\because H = \frac{V^2}{R} t \right)$

Also, $H = mc\Delta T = \frac{9V^2}{R} \times t$ ($\because$ But $Q = mc\Delta T$) .... (i)

Let R' be the resistance of the wire of double length '$2l$'
$\therefore$ Now energy released in t-seconds H′

$= \frac{(NV)^2}{2R} \times t \;(\because R \propto l)$

Also $H' = m'c\Delta T = (2m)\, C\Delta T$

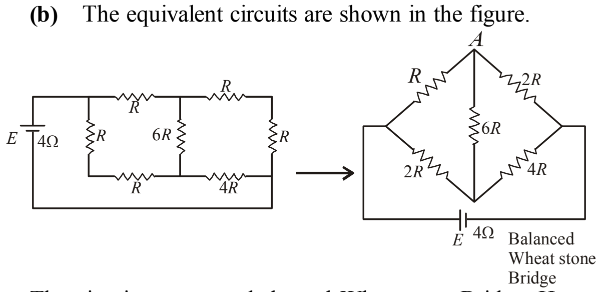
$2\,mc\Delta T = \frac{N^2V^2}{2R} \times t$ .... (ii)

Dividing eq. (i) by (ii)

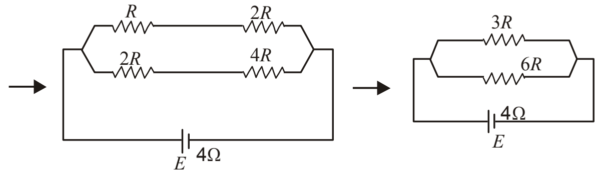
$\frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times t / R}{N^2V^2 t / 2R}$ or, $\frac{1}{2} = \frac{9 \times 2}{N^2}$

or, $N^2 = 18 \times 2 \quad \therefore \quad N = 6$

14. **(b)** The equivalent circuits are shown in the figure.



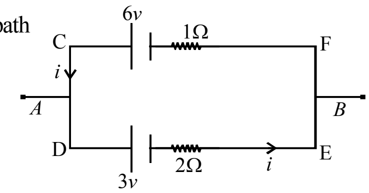
The circuit represents balanced Wheatstone Bridge. Hence no current will flow across. 6R Ω resistance



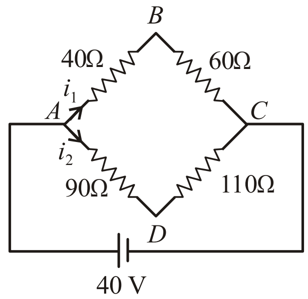
$$\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{6R}, \Rightarrow R_{eq} = \frac{(3R)(6R)}{(3R)+(6R)} = 2R$$

For maximum power, $R_{external} = R_{internal}$
$2R = 4\ \Omega \therefore R = 2\ \Omega$

15. **(5)** Let i be the current flowing in the circuit. Applying Kirchhoff's law KVL in the loop CDEFC
$-3 - 2i - i + 6 = 0 \qquad \Rightarrow 3i = 3$
$\therefore i = 1$ A
Now for upper or lower path
$V_A - 6 + 1 \times 1 = V_B$
$\therefore V_A - V_B = 5$ V
or, $V_A - 3 - 2 \times 1 = V_B$
$\therefore \quad V_A - V_B = 5V$



16. **(2)**



Current through AB, $i_1 = \frac{40}{40+60} = 0.4$

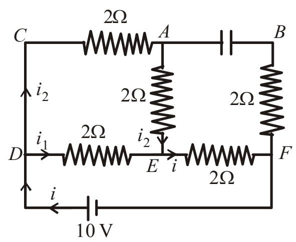
Current through AD, $i_2 = \frac{40}{90+110} = \frac{1}{5}$

Using KVL in BAD loop

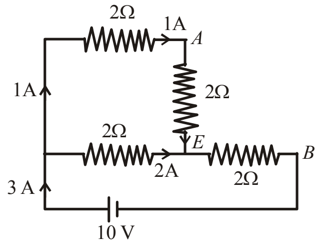
$V_B + i_1(40) - i_2(90) = V_D$

$\Rightarrow V_B - V_D = \frac{1}{5}(90) - \frac{4}{10}(40) \Rightarrow V_B - V_D = 18 - 16 = 2$ V

17. **(08.00)**



As capacitor is fully charged no current will flow through it.



We have the current distribution as shown in the figure.

Equivalent resistance, $R_{eq} = \left(\frac{4 \times 2}{4+2}\right) + 2$

Net current, $i = \frac{10}{\frac{4}{3}+2} = \frac{10 \times 3}{10} = 3$ Amp

$i_1 = 2$ A and $i_2 = 1$ A

$V_{AEB} = 1 \times 2 + 3 \times 2 = 8$ V

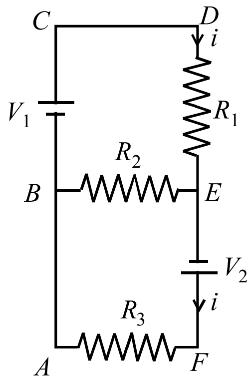
18. **False,** In an electrolytic solution, the electric current is mainly due to the movement of ions not electrons.

19. **(a, b, d)** No current is flowing through resistance R_2.

Applying KVL in loop

$$V_1 - iR_1 + V_2 - iR_3 = 0$$

$$\therefore\ i = \frac{V_1 + V_2}{R_1 + R_3} \quad \text{... (i)}$$



Applying KVL in loop BCDEB

$$v_1 - iR_1 = 0\ \therefore\ i = \frac{V}{R_1} \quad \text{...(ii)}$$

From eq. (i) & (ii) $\frac{V_1}{R_1} = \frac{V_1 + V_2}{R_1 + R_3}$

$\therefore\quad V_1R_1 + V_1R_3 = V_1R_1 + V_2R_1$

$\Rightarrow\quad V_1R_3 = V_2R_1$

If $V_1 = V_2$ then $R_1 = R_3 = R_2$

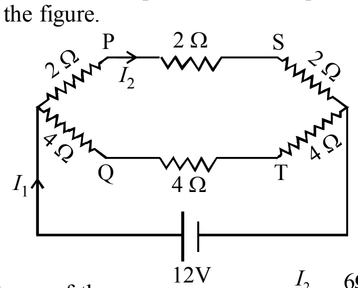
If $V_1 = V_2$ then $R_1 = R_3 = 2R_2$

If $V_1 = 2V_2$ then $2R_3 = R_1$

If $2V_1 = V_2$ then $R_3 = 2R_1 = R_2$

20. **(a, b, c, d)**

Resistance of arm PQ and ST becomes ineffective as P & Q and S & T are at the same potential. The equivalent circuit is as shown in the figure.

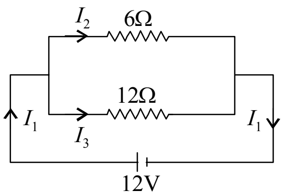


The resistance of the upper arm

$R_1 = 2\Omega + 2\Omega + 2\Omega = 6\Omega$

The resistance of the lower arm

$R_2 = 4\Omega + 4\Omega + 4\Omega = 12\Omega$



Equivalent resistance of the circuit,

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{(6\Omega)(12\Omega)}{6\Omega + 12\Omega} = 4\Omega$$

$$\therefore I_1 = \frac{12V}{4\Omega} = 3A$$

$$I_2 = \left(\frac{12}{6+12}\right) \times 3 = 2A$$

$$I_3 = I_1 - I_2 = 1A$$

Potential difference across A and P,

$$V_A - V_P = I_2 \times 2\Omega = (2A)(2\Omega)$$

$$12V - V_P = 4V \text{ or } V_P = 8V$$

Potential difference across A and Q,

$$V_A - V_Q = I_3 \times 2\Omega = (1A)(4\Omega)$$

$$12V - V_Q = 4V$$

$$V_Q = 12V - 4V = 8V$$

Potential difference across P and S,

$$V_P - V_S = (2A)(2W) = 4V$$

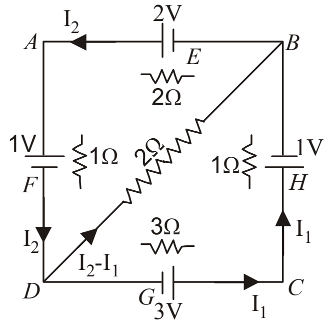
$$8V - V_S = 4V \Rightarrow V_S = 4V$$

$\therefore V_s < V_Q$

21. Applying Kirchoff's law KVL in loop $BDAB$

$+ 2(I_2 - I_1) + 1 + 1 \times I_2 - 2 + 2I_2 = 0$

$\Rightarrow\quad 2I_1 - 5I_2 = -1$ ... (i)



Applying Kirchoff's law KVL in loop $BCDB$, we get

$-2(I_2 - I_1) + 1 + I_1 - 3 + 3I_1 = 0$

$\Rightarrow\quad 3I_1 - I_2 = 1$ ... (ii)

Solving eq. (i) and (ii), we get

$I_1 = 6/13$ A and $I_2 = \frac{5}{13}$ A

(i) Potential difference between B and D,

$$V_B + \left[\frac{5}{13} - \frac{6}{13}\right] \times 2 = V_D \quad \therefore V_B - B_D = \frac{2}{13}\text{ V}$$

(ii) p.d. across $G = 3 - \frac{6}{13} \times 3 = \frac{39-18}{13} = \frac{21}{13}$ V

[$\because$ the cell is in discharging mode]

p.d. across $H = 1 + 1 \times \frac{6}{13} = \frac{19}{13}$ V

[$\because$ cell is in charging mode]

22. (i) Since no current is taken from the battery

$$\therefore V_{AB} = E_{eq} = \frac{\Sigma E/r}{\Sigma \frac{1}{r}} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$= \frac{\left(3/1 + 2/1 + 1/1\right)}{\left(1/1\right) + \left(1/1\right) + \left(1/1\right)} = 2V$$

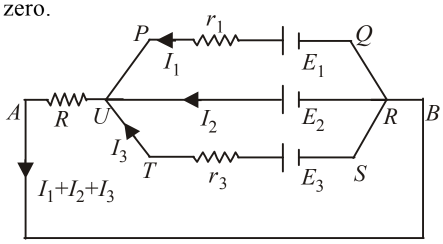
Internal resistance of equivalent battery $= \frac{1}{3}\Omega$

$$i_1 = \frac{V_B - V_A + E_1}{r_1} = \frac{-2+3}{1} = 1A$$

$$i_2 = \frac{V_B - V_A + E_2}{r_2} = \frac{-2+2}{1} = 0$$

and $$i_3 = \frac{V_B - V_A + E_3}{r_3} = \frac{-2+1}{1} = -1A$$

(ii) If r_2 is short-circuited then resistance of this branch in zero.



Applying Kirchoff's law in *PQRUP* starting from *P* moving clockwise

$I_1r_1 - E_1 + E_2 = 0$ or $I_1 - 3 + 2 = 0$ $\therefore I_1 = 1A$

Applying Kirchoff's law in *URSTU* starting from *U* moving clockwise

$-E_2 + E_3 - I_3r_3 = 0$ or $-2 + 1 - I_3 = 0$ $\therefore I_3 = -1A$

Here – ve sign of I_3 indicates that the direction of current in branch *UTSR* is opposite to that assumed.

Applying Kirchoff's law in *AURBA* starting from *A* moving clockwise.

$(I_1 + I_2 + I_3) R - E_2 = 0$ or $(1 + I_2 - 1) R = 2$ $\therefore I_2 = 2A$

Hence, current through R, $= I_1 + I_2 + I_3 = 2A$

Topic-4 : Heating Effect of Current

1. **(b)** Given : Power, $P = 1$ kW = 1000 W

$R = 2\Omega$, $V = 220\,V$

Current, $I = \frac{P}{V} = \frac{1000}{220}$

$$P_{loss} = I^2R = \left(\frac{1000}{220}\right)^2 \times 2$$

$$\therefore \text{ Efficiency } = \frac{1000}{1000 + P_{loss}} \times 100 = 96\%.$$

2. **(d)** Net Power, P

$= 15 \times 45 + 15 \times 100 + 15 \times 10 + 2 \times 1000$

$= 15 \times 155 + 2000$ W

Power, $P = VI \Rightarrow I = \frac{P}{V}$

$$\therefore I_{main} = \frac{15 \times 155 + 2000}{220} = 19.66\ A \approx 20A$$

3. **(b)** Equivalent resistance,

$$R_{eq} = \frac{4R \times 4R}{4R + 4R} + R + \frac{6R \times 12R}{6R + 12R} + R = 2R + R + 4R + R = 8R.$$

Using, $P = \frac{V^2}{R_{eq}} \Rightarrow 4 = \frac{16^2}{8R}$

$$\therefore R = \frac{16^2}{4 \times 8} = 8\,\Omega$$

4. **(b)**

5. **(a)** As $R = \frac{V^2}{P}$, so $R_1 = \frac{220^2}{25}$ and $R_2 = \frac{220^2}{100}$

Current flown $i = \frac{220}{R_1 + R_2}$

$$P_1 = i^2R_1 = \frac{220^2}{\left(\frac{220^2}{25} + \frac{220^2}{100}\right)} \times \frac{220^2}{25} = 16\text{ W}$$

Similarly, $P_2 = i^2R_2 = 4$ W

6. **(b)** When two resistances are connected in series, $R_{eq} = 2R$

Power consumed, $P = \frac{\varepsilon^2}{R_{eq}} = \frac{\varepsilon^2}{2R}$

In parallel condition, $R_{eq} = R/2$.

New power, $P' = \frac{\varepsilon^2}{(R/2)}$

or $P' = 4P = 240$ W ($\because P = 60$ W)

7. **(a)** Power, $P = I^2R$

$4.4 = 4 \times 10^{-6} \times R \Rightarrow R = 1.1 \times 10^6\Omega$

When supply of 11 v is connected

Power, $P' = \frac{v^2}{R} = \frac{11^2}{1.1} \times \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5}$ W

8. **(c)** From the given circuit, net resistances

$R_I = 1\,\Omega$, $R_{II} = 1/2\,\Omega$, $R_{III} = 3/2\,\Omega$

It is clear that $R_3 > R_1 > R_2$

Hence, $P_3 < P_1 < P_2$

As Power (P) $= \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R}$

9. **(c)** Heat energy will be maximum when resistance will be minimum.

10. **(c)** Total power consumed by electrical appliances in the building, $P_{total} = 2500$W

Watt = Volt × ampere

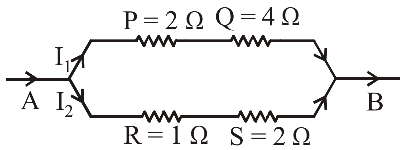
$$\Rightarrow 2500 = V \times I \Rightarrow 2500 = 220\,I \Rightarrow I = \frac{2500}{220} = 11.36 \approx 12A$$

(Minimum capacity of main fuse)

11. **(c)** Current in each bulb $= \frac{\text{Power}}{\text{Voltage}} = \frac{100}{220} = 0.45A$

Current through ammeter $= 0.45 \times 3 = 1.35$ A

12. **(b)**



$R_1 = P + Q = 2\,\Omega + 4\,\Omega = 6\,\Omega$

$R_2 = R + S = 1\Omega + 2\,\Omega = 3\,\Omega$

$I_1R_1 = I_2R_2$

$$I_1 = \frac{R_2}{R_1}I_2 = \frac{3}{6}I_2 = \frac{I_2}{2}$$

or $I_2 = 2I_1$

Heat flow $H = I^2 Rt$

For Q, $H_Q = I_1^2\,Qt = \frac{I_2^2}{4} \times 4t = I_2^2 t$

For S, $H_S = I_2^2\,St = I_2^2 \cdot 2t = 2I_2^2 t$

$\therefore$ Greatest amount of heat generated by S.

13. **(d)** We know power $P = \frac{V^2}{R}$

$\therefore$ For a constant V at a particular temperature $P \propto \frac{1}{R}$

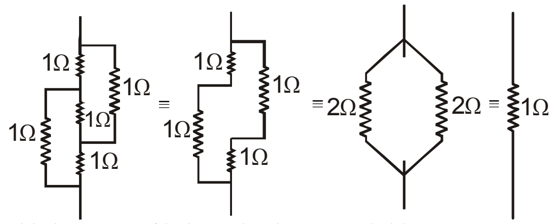
It is given that the powers of the bulbs are in the order 100W > 60 W > 40W

$\therefore \quad \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

14. **(c)** We know power, $P = \frac{V^2}{R}$

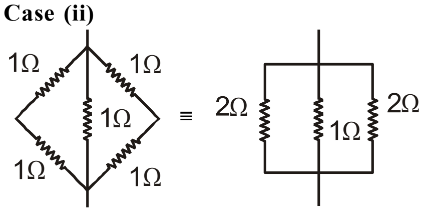
And V is constant in all three cases $\therefore P \propto \frac{1}{R}$

Case (i)



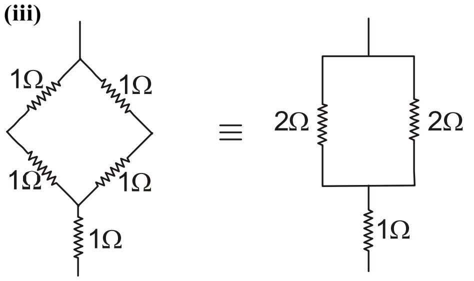
This is a case of balanced Wheatstone bridge $R_1 = 1\Omega$

Case (ii)



$\frac{1}{R^2} = \frac{1}{2} + \frac{1}{1} + \frac{1}{2} \Rightarrow R_2 = \frac{2}{4} = \frac{1}{2}52$

Case (iii)

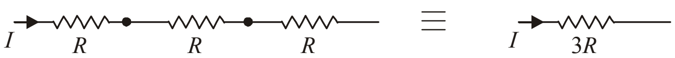


Thus $R_3 = 2\Omega$

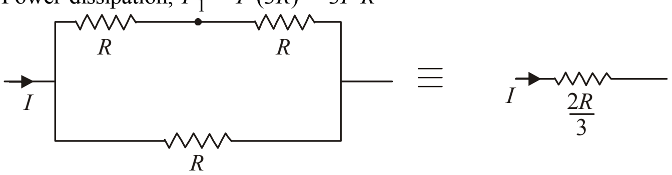
Since, $R_2 < R_1 < R_3$ $\therefore P_2 > P_1 > P_3$

15. **(a)** As we know, $P = I^2R$

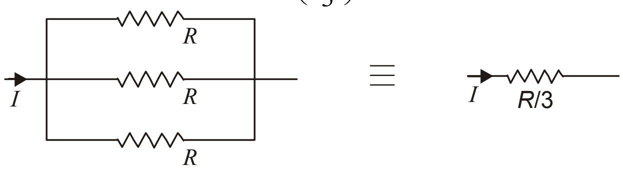
Here I constant $\therefore P \propto R$



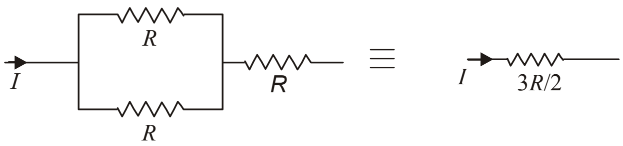
Power dissipation, $P_1 = I^2(3R) = 3I^2R$



Power dissipation, $P_2 = I^2\left(\frac{2R}{3}\right) = 0.67I^2R$



Power dissipation, $P_3 = I^2(R/3) = 0.33I^2R$

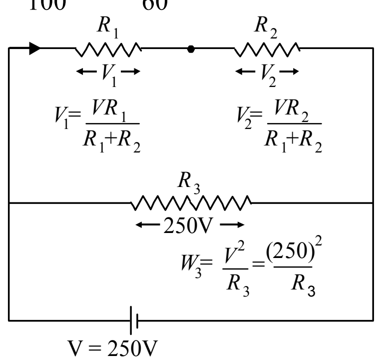


Power dissipation, $P_4 = I^2\left(\frac{3}{2}R\right) = 1.5\,I^2R$

$\therefore$ III < II < IV < I

16. **(d)** As we know, $P = \frac{V^2}{R}$ $\therefore R = \frac{V^2}{P}$

$\therefore \quad R_1 = \frac{V^2}{100}, R_2 = \frac{V^2}{60} = R_3;$



$W_1 = \frac{V_1^2}{R_1} = \frac{V^2R_1}{(R_1+R_2)^2}, W_2 = \frac{V_2^2}{R_2} = \frac{V^2R_2}{(R_1+R_2)^2}$

and $W_3 = \frac{V^2}{R_3}$

$W_3 : W_2 : W_1 = \frac{(250)^2}{R_3} : \frac{(250)^2 R_2}{(R_1+R_2)^2} \; R_2 : \frac{(250)^2}{(R_1+R_2)^2} R_1$

or $\quad W_3 : W_2 : W_1$

$= \frac{(250)^2}{V^2} \times 60 : \frac{(250)^2}{\left[\frac{1}{100}+\frac{1}{60}\right]^2 V^4} \times \frac{V^2}{60} : \frac{(250)^2 V^2}{\left[\frac{1}{100}+\frac{1}{60}\right]^2 V^4 \times 1000}$

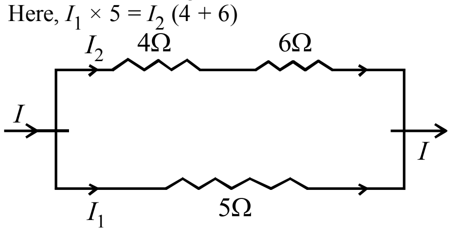
or $\quad W_3 : W_2 : W_1$

$= 60 : \frac{100 \times 100 \times 60 \times 60}{160 \times 160 \times 60} : \frac{100 \times 100 \times 60 \times 60}{160 \times 160 \times 100}$

= 64 : 25 : 15

Therefore $W_1 < W_2 < W_3$

17. **(c)** Here, $I_1 \times 5 = I_2\,(4 + 6)$



$\therefore I_1 = 2I_2 \Rightarrow I_2 = \dfrac{I_1}{2}$

$$\frac{\text{Heat generated in } 5\Omega \text{ resistance per second}}{\text{Heat generated in } 4\Omega \text{ resistance per second}} = \frac{I_1^2 \times 5}{\frac{I_1^2 \times 4}{4}}$$

$$\Rightarrow \frac{10}{H \text{in } 4\Omega} = 5 \Rightarrow H\text{in}4\Omega = \frac{10}{5} = 2 \text{ cal/s}$$

$\therefore$ Heat generated in the 4Ω resistor = 2 calories/s

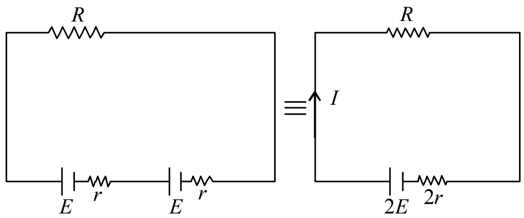
18. (b) Heat produced per second

$$H = \frac{V^2}{R} \text{ and } R = \int \frac{l}{A} = \frac{\rho\ell}{\pi r^2} \quad \therefore \quad H = V^2\left(\frac{\pi r^2}{\rho\ell}\right)$$

or $\quad H = \left(\dfrac{\pi V^2}{\rho}\right)\dfrac{r^2}{\ell}$ or, $H \propto \dfrac{r^2}{l}$

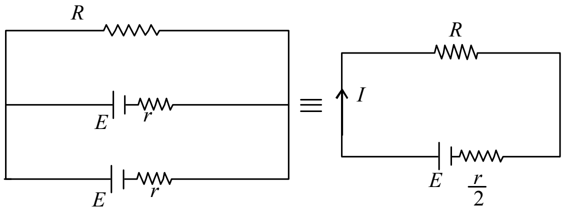
Thus heat developed (H) is doubled if both length (ℓ) and radius (r) are doubled.

19. (4) Cells connected in series



$$J_1 = I^2R = \left(\frac{2E}{2r+R}\right)^2 .R$$

Cells connected in parallel



$$J_2 = I^2R = \left(\frac{E}{R+\frac{r}{2}}\right)^2 \times R$$

$\therefore J_1 = 2.25\, J_2$ (Given)

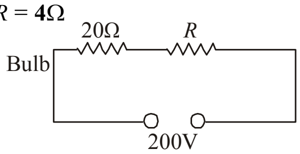
$$\frac{(2E)^2}{(2r+R)^2}.R = 2.25\frac{E^2}{(R+\frac{r}{2})^2}.R$$

$$\Rightarrow \frac{4}{(2r+R)^2} = \frac{2.25}{\left(R+\frac{r}{2}\right)^2}$$

$$\Rightarrow 4[R+0.5]^2 = 2.25[2+R]^2 \quad [\because r = 1\Omega]$$

$\Rightarrow 2\,(R + 0.5) = 1.5\,(2 + R) \therefore R = 4\Omega$

20. (20Ω) We know,



power $P = \dfrac{V^2}{R}$

$$\therefore \quad R_{bulb} = \frac{V^2}{P} = \frac{100 \times 100}{500} = 20\Omega$$

This would be possible only when $R = 20\ \Omega$ is in series with the bulb resistance ($R_{bulb} = 20\Omega$) because in that case both resistances will share equal p.d of 100V each.

21. (b, d) Heat produced, $H = \left(\dfrac{V^2}{R}\right)t = \dfrac{V^2}{R} \times 4$ ...(i)

where $R = \rho\dfrac{l}{A} = \dfrac{4\rho l}{\pi d^2}$

When resistances are connected in series

$$\text{Total resistance} = R_1 + R_2 = 2\left[\frac{4\rho l}{4\pi d^2}\right] = 2 \times \frac{R}{4} = \frac{R}{2}$$

$$\therefore H = \frac{V^2}{R/2} \times t_2 \quad ...(ii)$$

From eq. (i) and (ii) $t_2 = 2$ min.

When resistance are connected in parallel

$$\text{Total resistance} = \frac{R_1R_2}{R_1+R_2} = \frac{R_1^2}{2R_1} = \frac{R/4}{2} = \frac{R}{8}$$

$$\therefore H = \frac{V^2}{R/8} \times t_2 \quad ...(iii)$$

From eq. (i) and (iii) $t_2 = 0.5$ min

22. Resistance of the heater coil, $R = \dfrac{V^2}{P} = \dfrac{100 \times 100}{100} = 10\ \Omega$

Voltage across the coil of heater if it operates with a power $P' = 62.5$ W

$\therefore \quad V' = \sqrt{R \times P'} = \sqrt{10 \times 62.5} = \sqrt{625} = 25$

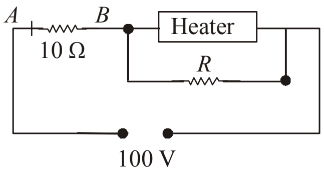
Since the voltage drop across the heater is $25V$ hence voltage drop across 10Ω resistor is $(100 - 25) = 75V$.

$\therefore \quad$ Current in the circuit, $= I = \dfrac{V}{R} = \dfrac{75}{10} = 7.5\ A$

This current divides into two parts. Let I_1 be the current that passes through the heater.

$\therefore V' = I_1 R \Rightarrow 25 = I_1 \times 10 \therefore I_1 = 2.5\ A$

Thus current through resistance 'R' $I_2 = 7.5 - 2.5 = 5$A.



From Ohm's law resistance across resistor R,

$V' = I_2 \times R \Rightarrow 25 = 5 \times R \quad \therefore R = 5\Omega$

Topic-5 : Wheatstone Bridge and Defferent Measuring Instruments

1. (d) The voltmeter of resistance $10k\Omega$ is parallel to the resistance of 400Ω. So, their equivalent resistance is

$$\frac{1}{R'} = \frac{1}{10\ k\Omega} + \frac{1}{400\Omega} = \frac{1}{10000} + \frac{1}{400}$$

$$\Rightarrow \frac{1}{R'} = \frac{1+25}{10000} = \frac{26}{10000} \Rightarrow R' = \frac{10000}{26}\Omega$$

Using Ohm's law, current in the circuit

$$I = \frac{\text{Voltage}}{\text{Net Resistance}} = \frac{6}{\frac{10000}{26}+800}$$

Potential difference measured by voltmeter

$$V = IR' = \frac{6}{\frac{10000}{26}+800} \times \frac{10000}{26} \Rightarrow V = \frac{150}{77} = 1.95 \text{ volt}$$

2. **(b)** Multimeter shows deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

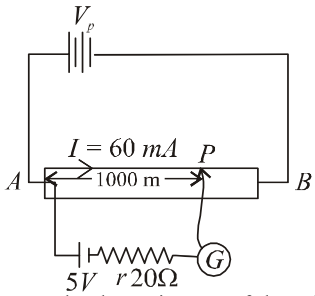
3. **(a)** Potential gradient, $x = \frac{\text{Potential drop}}{\text{length}}$

Here, Potential drop = 1.02

Balancing length from P = 100 – 49

$$\therefore x = \frac{1.02}{100-49} = 0.02 \text{ volt/cm}$$

4. **(d)**



Let R be the resistance of the whole wire

Potential gradient for the potentiometer wire

$$'AB' = -\frac{dV}{d\ell} = \frac{I \times R}{\ell} = \left[\frac{60 \times R}{\ell_{AB}}\right] mv/m$$

$$V_{AP} = \left(\frac{dV}{d\ell_{AB}}\right)\ell_{AP} = \frac{60 \times R}{1200} \times 1000 mV$$

$\Rightarrow V_{AP} = 50\text{ R } mV$

Also, $V_{AP} = 5\ V$ (for balance point at P)

$$\therefore R = \frac{V_{AP}}{50 \times 10^{-3}} = \frac{5}{50 \times 10^{-3}} = 100\Omega$$

5. **(c)** The resistance of potentiometer wire

R = 0.01 × 400 = 4 Ω

Current in the wire

$$i = \frac{V}{R_T} = \frac{3}{4+0.5+0.57+1} = \frac{1}{2} A$$

Now $V = iR_{AJ} = \frac{1}{2} \times (0.01 \times 50) = 0.25$ V.

6. **(d)**

7. **(c)** Let x be the length AJ at which galvanometer shows null deflection current,

$$\mathrm{i} = \frac{\varepsilon}{12\mathrm{r}+\mathrm{r}} = \frac{3}{13\mathrm{r}} \text{ or, } \mathrm{i}\left(\frac{\mathrm{x}}{\mathrm{L}}12\mathrm{r}\right) = \frac{\varepsilon}{2}$$

$$\Rightarrow \frac{\varepsilon}{13\mathrm{r}}\left[\frac{\mathrm{x}}{\mathrm{L}}\cdot 12\mathrm{r}\right] = \frac{\varepsilon}{2} \Rightarrow \frac{\varepsilon}{13\mathrm{r}}\left[\frac{\mathrm{x}}{\mathrm{L}}\cdot 12\mathrm{r}\right] = \frac{\varepsilon}{2} \text{ or, } \mathrm{x} = \frac{13\mathrm{L}}{24}$$

8. **(a)** Given, colour code of resistance,

R_1 = Orange, Red and Brown

$\therefore R_1 = 32 \times 10 = 320$

using balanced wheatstone bridge principle,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \Rightarrow \frac{320}{R_3} = \frac{80}{40}$$

$\therefore R_3 = 160$ i.e. colour code for R_3 Brown, Blue and Brown

9. **(b)** Using formula, internal resistance,

$$r = \left(\frac{l_1 - l_2}{l_2}\right)s = \left(\frac{52-40}{40}\right) \times 5 = 1.5\Omega$$

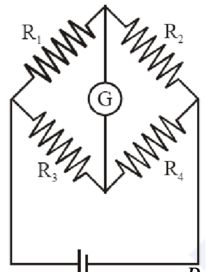
10. **(b)** For a balanced meter bridge,

$$\frac{X}{39.5} = \frac{Y}{(100-39.5)} \Rightarrow Y = 39.5 = X \times (100 - 39.5)$$

or, $X = \frac{12.5 \times 39.5}{60.5} = 8.16\Omega$

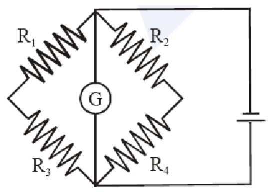
When X and Y are interchanged l_1 and $(100 - l_1)$ will also interchange so, $l_2 = 60.5$ cm

11. **(d)** There is no change in null point, if the cell and the galvanometer are exchanged in a balanced wheatstone bridge.



On balancing condition $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

After exchange



On balancing condition $\frac{R_1}{R_2} = \frac{R_3}{R}$

12. (c) As the two cells oppose each other hence, the effective emf in closed circuit is 15 – 10 = 5 V and net resistance is 1 + 0.6 = 1.6 Ω (because in the closed circuit the internal resistance of two cells are in series.

Current in the circuit,

$$I = \frac{\text{effective emf}}{\text{total resistance}} = \frac{5}{1.6} A$$

The potential difference across voltmeter will be same as the terminal voltage of either cell.

Since the current is drawn from the cell of 15 V

$\therefore \quad V_1 = E_1 - Ir_1$

$$= 15 - \frac{5}{1.6} \times 0.6 = 13.1 \text{ V}$$

13. **(c)** In case of balanced meter bridge

$\frac{R}{l} = \frac{X}{100-l}$ Given: $X = 90\ \Omega$, $l = 40.0$ cm

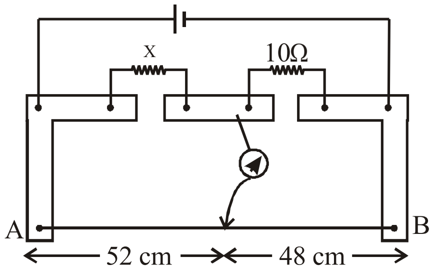
$$\therefore R = \frac{Xl}{100-l} = \frac{90 \times 40}{60} = 60\Omega$$

$$\therefore \frac{\Delta R}{R} = \frac{\Delta l}{l} + \frac{\Delta(100-l)}{100-l}$$

$$\Rightarrow \frac{\Delta R}{60} = \frac{0.1}{40} + \frac{0.1}{60} \therefore \Delta R = 0.25\Omega$$

Therefore, $R = (60 \pm 0.25)\Omega$

14. (b)



Applying balanced wheatstone bridge condition $\frac{P}{Q} = \frac{R}{S}$

$\frac{X}{\ell_1} = \frac{10}{\ell_2}$

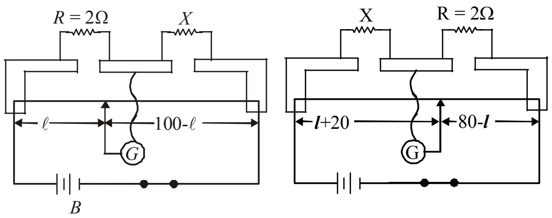
Here ℓ_1 = 52 + end correction = 52 + 1 = 53 cm

ℓ_2 = 48 + end correction = 48 + 2 = 50 cm

or $\frac{X}{53} = \frac{10}{50}$ $\therefore$ $X = \frac{53}{5} = 10.6\Omega$

15. (a) When the bridge is balanced $x > 2\Omega$

Applying balanced wheatstone bridge condition, $\frac{P}{Q} = \frac{R}{S}$



$\frac{R}{\ell} = \frac{X}{100 - \ell}$

or, $100R - R\ell = \ell X$ or, $200 - 2\ell = \ell X$ or, $\ell = \frac{200}{X+2}$

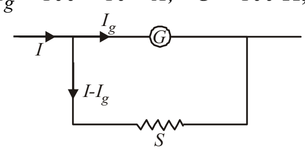
When the resistances are interchanged the jockey shifts 20 cm.

$\therefore$ $\frac{X}{\ell + 20} = \frac{2}{80 - \ell} \Rightarrow 80X - \ell X = 2\ell + 40$

or, $80X = \ell(X+2) + 40 = \left(\frac{200}{X+2}\right)(X+2) + 40$

or, $X = \frac{240}{80} = 3\Omega$.

16. (a) Here, $I_g = 100 \times 10^{-6} A$; $G = 100\ \Omega$; $S = 0.1\ \Omega$



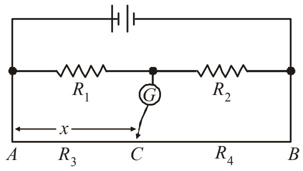
Using $I_g G = (I - I_g) S$

$I = I_g\left(\frac{G}{S} + 1\right) = 100 \times 10^{-6}\left(\frac{100}{0.1} + 1\right)$

or, $I = 100 \times 10^{-6} \times 1000.1 = 100.01$ mA

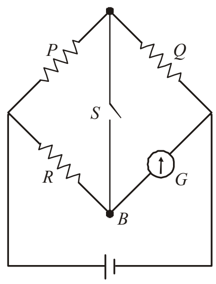
17. (a) In case of meter bridge at null point

$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{AC}{CB} = \frac{AC}{100 - CB}$



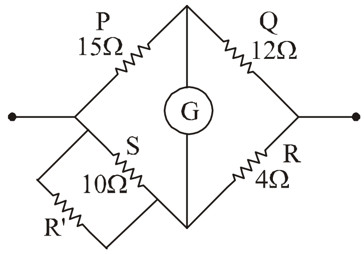
If radius of the wire is doubled, then the resistance of AC will change and also the resistance of CB will change. But since $\frac{R_1}{R_2}$ does not change so, $\frac{AC}{CB}$ should also not change at null point. Therefore the point C does not change.

18. (a) Since the opening or closing the switch 's' does not does not change the reading of gal vanometer it means that in both the cases there is no current passing through S. It is the case of balanced wheat stone bridge. Thus potential at A is equal to potential at B.



$\therefore I_P = I_Q$ and $I_R = I_G$

19. (10)



As per Wheatstone bridge balance condition $\frac{P}{Q} = \frac{S}{R}$

Let resistance R' is connected in parallel with resistance S of 10Ω

$\therefore \frac{15}{12} = \frac{10R'}{10 + R'} \Rightarrow 5 = \frac{10R'}{10 + R'} \Rightarrow 50 + 5R' = 10R'$

$\therefore R' = \frac{50\,4}{5} = 10\Omega$

20. (12) We know that

$E \propto \ell$ where l is the balancing length

$\therefore E = k(560)$ ....(i)

When the balancing length changes by 60 cm

$\frac{E}{r+10}10 = k(500)$ ...(ii)

Dividing (i) by (ii) we get

$\Rightarrow \frac{r+10}{10} = \frac{56}{50} \Rightarrow 50r + 500 = 560$

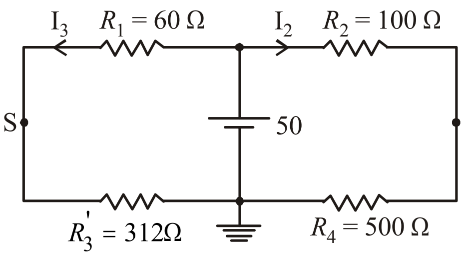
$\Rightarrow r = \frac{6}{5}\Omega = \frac{N}{10}\Omega \Rightarrow N = 12$

21. (0.27) According to question, resistance R_3 has temperature coefficient, $\alpha = 0.0004°C^{-1}$

And temperature is increased by 100° C i.e., $\Delta T = 100°$ C

$R'_3 = R_0(1 + \alpha\Delta T) = 300(1 + \alpha\Delta T)$

$\therefore R'_3 = 312\Omega$



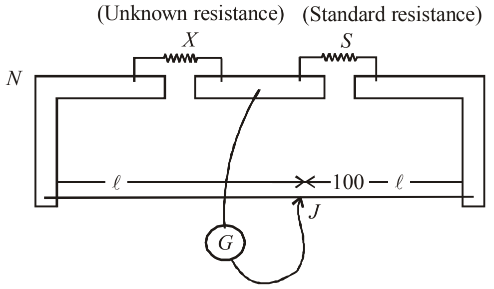
Here, $I_1 = \dfrac{V}{R_1 + R_3} = \dfrac{50}{372}$ and $I_2 = \dfrac{V}{R_2 + R_4} = \dfrac{50}{600}$

$V_S - V_T = 312 I_1 - 500 I_2$

$= 312 \times \dfrac{50}{372} - 50 \times \dfrac{500}{600} = 41.94 - 41.67 = 0.27V$

Hence voltage developed between S and T = 0.27 Volt

22. **(d)** When the temperature of metal increases; its resistance increases, $R_t = R_0(1 + \propto t)$



For a meter bridge at null point or at balanced condition

$\dfrac{X}{\ell} = \dfrac{S}{100 - \ell}$ or $\dfrac{X}{S} = \dfrac{l}{100 - l}$

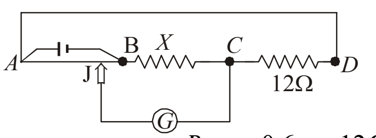
For null point 'J' at same point or, $\dfrac{\ell_1}{100 - \ell_i}$ to remain unchanged, $\dfrac{X}{S}$ should also remain unchanged. Hence if X is increasing then standard resistance 's' should also increase.

23. Position-B of null point gives most accurate reading. Null point at B is almost at the centre of resistance wire wheatstone bridge is most accurate when all the four resistances of the four arms are of the same order of magnitude.

In case of balanced wheatstone bridge condition, $\dfrac{P}{Q} = \dfrac{X}{R}$.

24. (a) No. There are no positive and negative terminals on the galvanometer. Whenever there is no current, the pointer of the galvanometer is at zero.

(b)



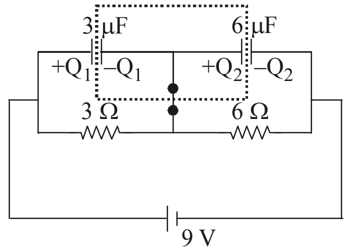
(c) $\because$ Bridge is balanced $\dfrac{R_{AJ}}{R_{JB}} = \dfrac{0.6\rho}{0.4\rho} = \dfrac{12\Omega}{X}$ $\quad \therefore X = 8\ \Omega$

where ρ is the resistance per unit length.

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** When the switch S is open total charge $Q_1 + Q_2 = 0\ CV_1 + C_2V_2 = 0\ 3 \times 6 + (-6 \times 3) = 0$. When the switch is closed and steady state is reached, the current I coming from the battery is

$9 = I(3 + 6) \Rightarrow I = 1A \qquad (\because V = IR)$



$\therefore$ Potential difference across 3Ω resistance = $3V$

Potential difference across 6Ω resistance = 6V

$\therefore$ p.d. across 3 μF capacitor = $3V$

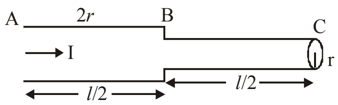
and p.d. across 6 μF capacitor = $6V$

$\therefore$ Charge on 3 μF capacitor $Q_1 = 3 \times 3 = 9\ \mu C$

Charge on 6 μF capacitor $Q_2 = 6 \times 6 = 36\ \mu C$

$\therefore$ Charge flows from y to $x = 36 - 9 = 27\ \mu C$

2. **(b)** As both wire of same material $\therefore I_{AB} = I_{BC}$



(a) $\dfrac{V_{AB}}{V_{BC}} = \dfrac{I_{AB}R_{AB}}{I_{BC}R_{BC}} = \dfrac{R_{AB}}{R_{BC}} = \dfrac{\rho\dfrac{\ell}{2[\pi \times 4r^2]}}{\rho\dfrac{\ell}{2[\pi r^2]}} = \dfrac{1}{4}$

$\therefore V_{AB} = \dfrac{V_{BC}}{4}$

(b) $\dfrac{P_{BC}}{P_{AB}} = \dfrac{I^2R_{BC}}{I^2R_{AB}} = \dfrac{\rho\dfrac{\ell}{2[\pi \times 4r^2]}}{\rho\dfrac{\ell}{2[\pi r^2]}} = \dfrac{1}{4}$

$\therefore$ $P_{AB} = 4P_{BC}$,

(c) $\dfrac{J_{AB}}{J_{BC}} = \dfrac{\dfrac{I}{\pi \times 4r^2}}{\dfrac{I}{\pi \times r^2}} = \dfrac{1}{4}$; $\therefore J_{BC} = 4J_{AB}$

(d) $\dfrac{E_{AB}}{E_{BC}} = \dfrac{\left[\dfrac{V_{AB}}{\ell/2}\right]}{\left[\dfrac{V_{BC}}{\ell/2}\right]} = \dfrac{1}{4}$; $\therefore E_{BC} = 4E_{AB}$

3. **(b)** Here, change in internal energy = heat supplied

i.e., $= \Delta U = Q = I^2Rt$

$= 1 \times 1 \times 100 \times 5 \times 60 = 30{,}000$ J = 30 kJ

4. **(a)** At any instant of time t during charging process, the transient current in the circuit

$I = \dfrac{V_0}{R}e^{-t/RC}$ $\quad\therefore$ Potential difference across resistor R

$V_R = \left[\dfrac{V_0}{R}e^{-t/RC}\right] \times R$

$= V_0 e^{-t/RC}$ ...(i)

$\therefore$ Potential difference across C

$V_c = V_0 - V_0 e^{-t/RC} = V_0(1 - e^{-t/RC})$ ...(ii)

$\because$ $V_c = 3V_R$

$\therefore \quad V_0\left(1-e^{-t/RC}\right) = 3V_0.e^{-t/RC}$

$\Rightarrow \quad 1-e^{-t/RC} = 3e^{-t/RC} \quad \Rightarrow \quad 1 = 4e^{-t/RC}$

Taking log on both sides

$\log_e 1 = 2\log_e 2 + \left(-\dfrac{t}{RC}\right)$

$\Rightarrow \quad 0 = 2 \times 2.303 \log_{10} 2 - \dfrac{t}{RC}$

$\Rightarrow \quad t = [2 \times 2.303 \log_{10} 2] \times 2.5 \times 10^6 \times 4 \times 10^{-6}$ or, $t = 13.86$ s

5. **(b)** Current in RC circuit, $I = I_0 e^{-t/RC}$

or $\quad lnI = lnI_0 - \dfrac{t}{RC}$ or $lnI = \left(\dfrac{-t}{RC}\right) + lnI_0$

$lnI = \left(\dfrac{-t}{RC}\right) + ln\left(\dfrac{E_0}{R}\right)$

Comparing with $y = mx + C$

Intercept $= ln\left(\dfrac{E_0}{R}\right)$ and slope $= -\dfrac{1}{RC}$

So, when x is changed to $2x$ then slope increases and current becomes less. Hence new graph is Q.

6. **(b)** Post office box is also works on the principle of balanced wheat stone bridge use to measure unknown resistance. Total external resistance will be the total resistance of whole length of box. It should be connected between A and D.

7. **(c)** As the current I is independent of R_6, it follows that the resistance R_1, R_2, R_3, R_4 and R_6 must form the balanced Wheatstone bridge. $\therefore R_1 R_4 = R_2 R_3$

8. **(b)** Let R be the resistance of wire.

Energy released in t second, $H = \dfrac{(3V)^2}{R} \times t \left(\because H = \dfrac{V^2}{R}t\right)$

Also, $H = mc\Delta T = \dfrac{9V^2}{R} \times t \quad (\because \text{But } Q = mc\Delta T) \quad .... \text{(i)}$

Let R' be the resistance of the wire of double length '$2l$'

$\therefore$ Now energy released in t-seconds H'

$= \dfrac{(NV)^2}{2R} \times t \ (\because R \propto l)$

Also $H' = m'c\Delta T = (2m)\, C\Delta T$

$2\, mc\Delta T = \dfrac{N^2V^2}{2R} \times t \quad .... \text{(ii)}$

Dividing eq. (i) by (ii)

$\dfrac{mc\Delta T}{2mc\Delta T} = \dfrac{9V^2 \times t/R}{N^2V^2\, t/2R}$ or, $\dfrac{1}{2} = \dfrac{9 \times 2}{N^2}$

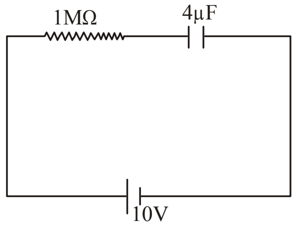
or, $\quad N^2 = 18 \times 2 \quad \therefore \quad N = 6$

9. **(a)** Intensity of light, $I \propto \dfrac{1}{r^2}; V \propto \dfrac{1}{r}; V \propto r^0$

10. **(5)** From $\left(\dfrac{I-I_g}{I_g}\right)S = \dfrac{V}{I_g} - R$

$\dfrac{1.5-0.006}{0.006} \times \dfrac{2n}{249} = \dfrac{30}{0.006} - 4990 \quad \therefore \ n = \dfrac{2490}{498} = 5$

11. **(2)** The equivalent circuit of the given circuit is as shown below.
$R = 1\text{M}\Omega$, $C = 4\mu\text{F}$



$\therefore$ The time constant
$\tau = RC = 4$ sec
Potential across 4μF capacitor at any time 't'

$V = V_0\left[1-e^{-\frac{t}{\tau}}\right]$

$\Rightarrow 4 = 10\left[1-e^{-\frac{t}{4}}\right]$

$\Rightarrow t = 2 \text{ s} \Rightarrow e^{-t/4} = 0.6 = \dfrac{3}{5}$

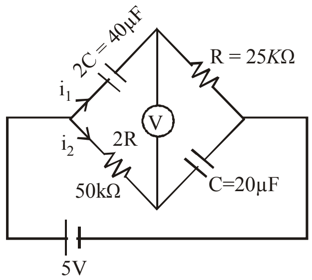
Taking log on both sides,

$-\dfrac{t}{4} = \ell n3 - \ell n5 \quad \Rightarrow t = 4(\ell n5 - \ell n3) = 2.5$

12. **(a, b, c, d)**
At t=0, i.e., as soon as the key is pressed, Capacitors act as short circuit and voltmeter display reading,–5V
At t = ∝, i.e., after a long time of key press, Capacitor acts as open circuit and no current flows through voltmeter (∵ very high resistance of voltmeter) so it display +5V.

$q_1 = \ 2CV\left(1-e^{-t/2CR}\right), \qquad i_1 = \dfrac{V}{R}e^{-\frac{t}{2CR}}$

$q_2 = \ CV\left(1-e^{-t/2CR}\right), \qquad i_2 = \dfrac{V}{2R}e^{-\frac{t}{2CR}}$



$\therefore \ \Delta V = -i_2 \times 2R + \dfrac{CV_1}{2C} = V\left[1-2e^{-t/2CR}\right] = 0$

i.e., At time t = ln 2s voltmeter will display reading OV.

At τ = 1sec, $i = \dfrac{i_o}{e} \qquad \left[\because i = i_o\, e^{-t/\tau}\right]$

i.e., After 1s current in the ammeter becomes $\dfrac{1}{e}$ of the initial value.
After a long time no current flows since both capcitor and voltmeter do not allow current to flow.

13. **(c, d)**
With the use of filament and the evaporation involved, the filament will become thinner thereby decreasing the area of cross-section and increasing the resistance. Therefore the filament will consume less power towards the end of life.

As power, $P = \dfrac{V^2}{R} or, P \propto \dfrac{1}{R} (\because V = \text{constant})$

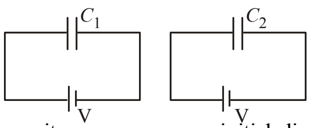
As the evaporation is non-uniform, the area of cross-section will be different at different cross-section. Therefore temperature distribution will be non-uniform. The filament will break at the point where the temperature is maximum.

When the filament temperature is higher $\left(\lambda_n \propto \dfrac{1}{T}\right)$, it emits light of lower wavelength or higher band of frequencies.

14. **(a,b,d)** Hence current passing-through an insulator and semiconductor at OK is zero when potential difference is applled.
At 0K an insulator does not permit any current to flow through it.
At 0K a semiconductor behaves as an insulator.

In reverse biasing at 300 K, a small finite current flows through a p-n junction diode.
In case of metal, the current flowing will be very-very high because a metal becomes super conductor at 0K.

15. **(b, d)** In R.C. circuit, discharging current
$I = I_0 e^{-t/RC}$



Initial current $I_0 = \dfrac{V}{R}$

As V and R for both capacitors are same so, initial discharging current will be same but non-zero.
Also, $q = q_0 e^{-t/RC}$

When $q = \dfrac{q_0}{2}$ then $\dfrac{q_0}{2} = q_0\, e^{-t/RC}$

or $e^{+t/RC} = 2. \Rightarrow t = RC \log_e 2 \therefore t \propto C.$

$\therefore \quad \dfrac{t_1}{t_2} = \dfrac{C_1}{C_2} = \dfrac{1}{2} = 0.5 \quad$ or, $t_1 = 0.5\, t_2$

Hence capacitor C_1 losses 50% of its initial charge sooner than C_2 losses 50% of its initial charge.

16. **A → *s***
Bimetallic strip is based on thermal expansion of solids i.e., $l = l_0 (1 + \propto t)$
B → *q*
Steam engine is based on energy conversion.
C → *p, q*
Incandescent lamp is based on energy conversion i.e., electrical energy into light energy and radiation from a hot body.
D → *q, r*
Electric fuse is based on melting of the fuse material which in turn depends on the heating effect of current.

17. Given $Q = Q_0[1 - e^{-\alpha t}]$
Here Q_0 = maximum charge and

$$\alpha = \frac{1}{\tau_c} = \frac{1}{C_{Req}}$$

Now the maximum charge
$Q_0 = C[V_0]$ where V_0 = max potential difference across C

$$= C\left[\frac{V}{R_1 + R_2} \times R_2\right]$$

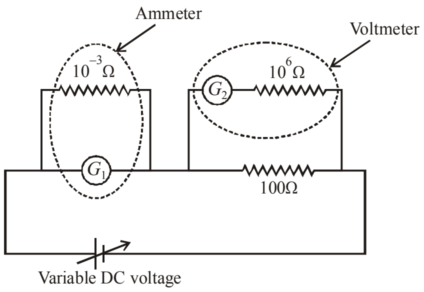
Here $\left(\dfrac{V}{R_1 + R_2}\right)$ is the steady state current through R_2.

And $\tau_c = C\, R_{eq} = C\left[\dfrac{R_1 R_2}{R_1 + R_2}\right]$

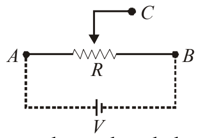
$(\because R_1$ and R_2 are in parallel combination$)$

$$\therefore\ \alpha = \frac{1}{\tau_c} = \frac{R_1 + R_2}{CR_1R_2}$$

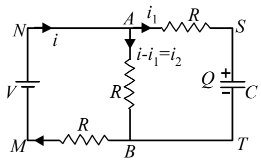
18. To verify ohm's law, the circuit diagram is as shown below.
An ammeter is a low resistance galvanometer which is connected on parallel with the resistance and voltmeter is a high resistance galvanometer which is connected in series with the resistance.



19. For rheostat to behave like a potential divider, battery should be connected across A and B. Output can be taken across the terminals A and C or B and C.



20. According to question, capacitor is initially uncharged and the switch s is closed at times $t = 0$
Let at any time t charge on capacitor C be Q. Let currents are as shown in fig. Since charge Q will increase with time 't'



$\therefore\ i_1 = \dfrac{dQ}{dt}$

(a) Applying KVL in the loop $MNABM$
$V = (i - i_1)\, R + iR$
or $\quad V = 2iR - i_1R \qquad$ ... (i)
Similarly, applying KVL in loop $MNSTM$,

$V = i_1R + \dfrac{Q}{C} + iR \qquad$ ...(ii)

Eliminating i from equation (i) and (ii), we get

$V = 3i_1R + \dfrac{2Q}{C}$ or $\quad 3i_1R = V - \dfrac{2Q}{C}$

or $\quad i_1 = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$ or $\quad \dfrac{dQ}{dt} = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$

or $\quad \dfrac{dQ}{\left(V - \dfrac{2Q}{C}\right)} = \dfrac{dt}{3R}$ or $\quad \displaystyle\int_0^Q \frac{dQ}{\left(V - \dfrac{2Q}{C}\right)} = \int_0^t \frac{dt}{3R}$

or, $Q = \dfrac{CV}{2}(1 - e^{-2t/3RC})$

(b) $\quad i_1 = \dfrac{dQ}{dt} = \dfrac{d}{dt}\left[\dfrac{CV}{2}\left(1 - e^{-2t/3RC}\right)\right]$

$= \dfrac{CV}{2} \times \dfrac{2}{3RC} \times e^{-2t/3RC} \quad = \dfrac{V}{3R} e^{-2t/3RC}$

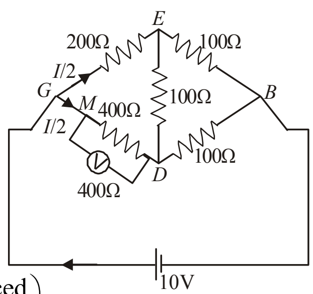
From equation (i)

$i = \dfrac{V + i_1R}{2R} = \dfrac{V + \dfrac{V}{3}e^{-2t/3RC}}{2R} \quad \therefore$ Current through AB

$i_2 = i - i_1 = \dfrac{V + \dfrac{V}{3}e^{-2t/3RC}}{2R} - \dfrac{V}{R}e^{-2t/3RC}$

$\Rightarrow i_2 = \dfrac{V}{2R} - \dfrac{V}{6R}e^{-2t/3RC}$ or, $i_2 = \dfrac{V}{2R}$ as $t \to \infty$

21. We can redraw the circuit as follows. This is a balanced wheat stone bridge.



Equivalent resistance between G and D

$$R_{GD} = \frac{400 \times 400}{400 + 400} = 200\,\Omega$$

$$\frac{R_{GE}}{R_{GD}} = \frac{R_{EB}}{R_{DB}}$$

$\left(\begin{array}{l}\text{condition of balanced} \\ \text{wheat stone bridge}\end{array}\right)$

Equivalent resistance across G and B

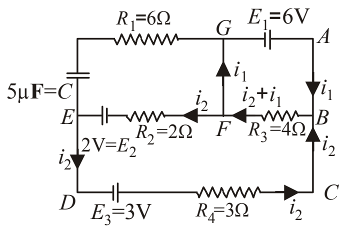
$R_{GB} = \dfrac{300 \times 300}{300 + 300} = 150\,\Omega$ $\therefore$ Current $I = \dfrac{V}{R_{GB}} = \dfrac{10}{150} = \dfrac{1}{15}$

Current $\dfrac{I}{2}$ further divides into two equal parts at M.

Hence potential difference across the voltmeter

$V_G = IR = \dfrac{I}{4} \times 400 = \dfrac{1}{15} \times \dfrac{400}{4} = \dfrac{20}{3} = 6.67$ V

22.



Applying Kirchhoff's law in loop $ABFGA$

$6 - (i_1 + i_2)\,4 = 0$ ... (i)

Applying Kirchhoff's law in loop $BCDEFB$

$i_2 \times 3 - 3 - 2 + 2i_2 + (i_2 + i_1)\,4 = 0$ ... (ii)

Putting the value of $4\,(i_1 + i_2) = 6$ in eq. (ii)

$3i_2 - 5 + 2i_2 + 6 = 0$

$\therefore \quad i_2 = -\dfrac{1}{5}A$

Substituting this value in eq. (i), we get

$i_1 = 1.5 - \left(-\dfrac{1}{5}\right) = 1.7\,A$

Hence, current in $R_3 = i_1 + i_2 = 1.7 - 0.2 = 1.5\,A$

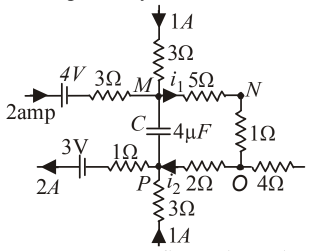
Across the capacitor potential difference,

$V_E - 2 - 0.2 \times 2 = V_G \quad \therefore V_E - V_G = 2.4\,V$

$\therefore$ Energy stored in capacitor, $U = \dfrac{1}{2}CV^2$

$= \dfrac{1}{2} \times 5 \times 10^{-6} \times (2.4)^2 = 1.44 \times 10^{-5}$ J

23. Applying Kirchhoff's first law, at junction M, and P current $i_1 = 3A$ and $i_2 = 1$A respectively.



At steady states, no current flows through capacitor

Moving the loop along MNO to P

$\therefore \quad V_M - 5 \times i_1 - 1 \times I_1 - 2 \times i_2 = V_P$

$\therefore \quad V_M - V_P = 6i_1 + 2i_2 = 6 \times 3 + 2 \times 1 = 20\,V$

$\therefore \quad V = 6 \times 3 + 2 \times 1 = 20\,V$

Energy stored in the capacitor

$U = \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 4 \times 10^{-6} \times 20 \times 20 = 8 \times 10^{-4}$ J

24. Resistance of 2Ω and 3Ω are in parallel

$(R_{eq})_{AB} = \dfrac{2 \times 3}{2 + 3} = 1.2\,\Omega$

Net current through the battery $= \dfrac{6}{1.2 + 2.8} = 1.5$ A

$\because P,d$ is same $\therefore 2I_1 = 3I_2 \Rightarrow I_2 = \dfrac{2}{3}$

$I_1 + I_2 = I_1 + 2I_1 = 1.5 \quad \therefore I_1 = \dfrac{3}{5} \times 1.5 = 0.9A$

25. Given P.d. across the 400 Ω resistance = 30 V.

$\therefore$ $P.d.$ across the 300 Ω resistance = 60 – 30V = 30 V.

As the voltmeter is in the parallel with the 400 Ω resistance, their combined resistance

$R' = \dfrac{400R}{(400 + R)}$ (Here R = resistance of voltmeter)

Here R' = 300Ω, as the potential difference of 60 V is equally shared between the 300 Ω and 400 Ω resistance.

$\therefore\ 300 = \dfrac{400R}{(400 + R)} \Rightarrow R = 1200\Omega,$

i.e., The resistance of the voitmeter, R = 1200Ω. When the voltmeter is connected across the 300Ω resistance, their combined resistance

$R'' = \dfrac{300R}{(300 + R)} = \dfrac{300 \times 1200}{(300 + 1200)} = 240\Omega$

$\therefore$ Total resistance in the ciruit = 400 + 240 = 640 Ω

$\therefore$ Current in the circuit

$I = \dfrac{V}{R} = \dfrac{60V}{640\Omega} = \dfrac{3}{32}A$ $\therefore$ Voltmeter reading

= Potential difference across 240 Ω resistance

$= V = IR'' = \dfrac{3}{32} \times 240 = 22.5V$

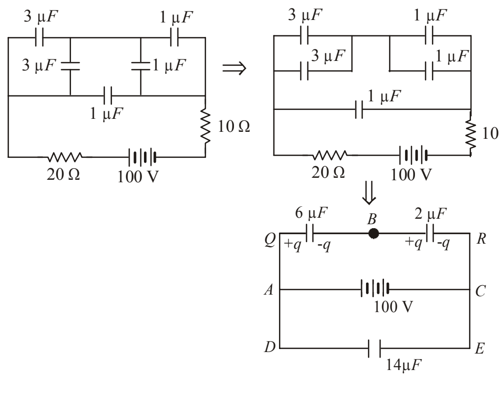
26. Inside the battery, current flows from the positive terminal to the negative terminal of the battery.

During charging, the potential difference between the two terminals of the battery.

$V = E + Ir = 2 + 5 \times 0.1 = 2.5\ V$

27. Applying Kirchh off's law KVL in loop $AQBRC$

$-\dfrac{q}{6} - \dfrac{q}{2} + 100 = 0 \Rightarrow q = 150\ \mu C$



$\therefore$ Potential difference between AB,

$V = \dfrac{q}{C} = \dfrac{150}{6} = 25V\ (V_{AB})$

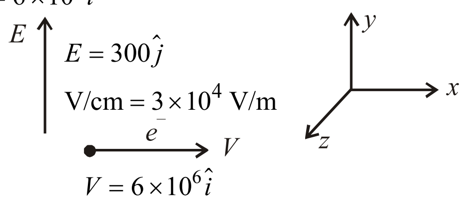
$\therefore$ Potential difference between BC

$V_{BC} = 100 - 25 = 75V$

Topic-1 : Motion of Charged Particle in Magnetic Field

1. **(c)** $\vec{E} = 300\hat{j}$ V/cm $= 3 \times 10^4$ V/m

$\vec{V} = 6 \times 10^6 \hat{i}$



$\vec{B}$ must be in $+z$ axis.

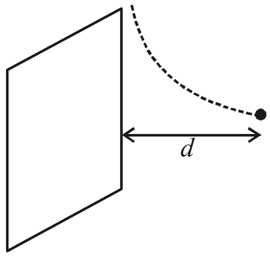
$q\vec{E} + q\vec{V} \times \vec{B} = 0$

$E = VB$

$\therefore\ B = \frac{E}{V} = \frac{3 \times 10^4}{6 \times 10^6} = 5 \times 10^{-3} T$

Hence, magnetic field $B = 5 \times 10^{-3}$ T along $+z$ direction.

2. **(c)** In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'r', particle will not hit the screen.



$r = \frac{mv}{qB_0}$ $\quad \left[\because \frac{mv^2}{r} = qvB_0\right]$

Hence, minimum value of v for which the particle will not hit the screen.

$v = \frac{qB_0 d}{m}$

3. **(a)** [Given: $q = 1\mu C = 1 \times 10^{-6} C$;

$\vec{V} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ m/s and

$\vec{B} = (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$ T]

$$\vec{F} = q(\vec{V} \times \vec{B}) = 10^{-6} \times 10^{-3} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{vmatrix}$$

$= (-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}$ N $\quad \therefore \vec{F} = (-30\hat{i} + 32\hat{j} - 9\hat{k})$

4. **(d)** Pitch $= (v\cos\theta)T$ and $T = \frac{2\pi m}{qB}$

$\therefore$ Pitch $= (V\cos\theta)\frac{2\pi m}{qB}$

$= (4 \times 10^5 \cos 60°)\frac{2\pi}{0.3}\left(\frac{1.67 \times 10^{-27}}{1.69 \times 10^{-19}}\right) = 4$ cm

5. **(c)** Time period of one revolution of proton, $T = \frac{2\pi m}{qB}$

Here, m = mass of proton

q = charge of proton

B = magnetic field.

Linear distance travelled in one revolution,

$p = T(v\cos\theta)$ (Here, v = velocity of proton)

$\therefore$ Length of region, $l = 10 \times (v\cos\theta)T$

$\Rightarrow l = 10 \times v\cos 60° \times \frac{2\pi m}{qB}$

$\Rightarrow l = \frac{20\pi mv}{qB} = \frac{20 \times 3.14 \times 1.67 \times 10^{-27} \times 4 \times 10^5}{1.6 \times 10^{-19} \times 0.3}$

$\Rightarrow l = 0.44$ m

6. **(d)** As particle is moving along a circular path

$\therefore R = \frac{mv}{qB}$ ...(i)

Path is straight line, then

$qE = qvB$

$E = vB \Rightarrow v = \frac{E}{B}$ ....(ii)

From equation (i) and (ii)

$m = \frac{qB^2R}{E} = \frac{1.6 \times 10^{-19} \times (0.5)^2 \times 0.5 \times 10^{-2}}{100}$

$\therefore$ m = 2.0 × 10^{-24} kg

7. **(b)** As mvr = qvB $\Rightarrow$ $r = \frac{mv}{qB} = \frac{\sqrt{2mK.E.}}{qB}$

[As : $\frac{1}{2}mv^2 = K.E.$

$\Rightarrow\ m^2v^2 = 2m K.E.$

$\Rightarrow\ mv = \sqrt{2m K.E.}$]

For proton, electron and α-particle,

$m_{He} = 4m_p$ and $m_p >> m_e$

Also $a_{He} = 2q_p$ and $q_p = q_e$

$\therefore$ As KE of all the particles is same then,

$r\ \alpha \frac{\sqrt{m}}{q}$

$\therefore r_{He} = r_p > r_e$

8. **(a)** Radius of the circular path will be $r = \frac{mv}{qB}$

$\Rightarrow\ r = \frac{\sqrt{2mKE}}{qB}$ $(\because\ p = mv = \sqrt{2mKE})$

$\because\ KE = q\Delta V$

$\therefore\ r = \frac{\sqrt{2mq\Delta V}}{qB} \Rightarrow r \propto \sqrt{\frac{m}{q}}$

$\therefore\ \frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$

9. **(d)** Radius of the path (r) is given by $r=\dfrac{mv}{qB}$

$$r=\frac{\sqrt{2mk}}{eB} \qquad \left(\because\ p=mv=\sqrt{2mk}\right)$$

$$=\frac{\sqrt{2meV}}{eB} \qquad (\because\ k=eV)$$

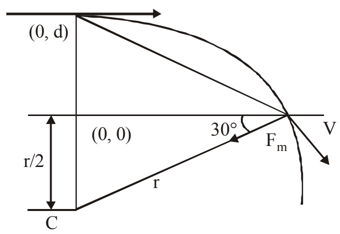
$$r=\frac{\sqrt{\dfrac{2m}{e}V}}{B}=\frac{\sqrt{\dfrac{2\times 9.1\times 10^{-31}}{1.6\times 10^{-19}}(500)}}{100\times 10^{-3}}$$

$$r=\frac{\sqrt{\dfrac{9.1}{0.16}\times 10^{-10}}}{10^{-1}}=\frac{3}{.4}\times 10^{-4}$$

$$=7.5\times 10^{-4}$$

10. **(BONUS)**
Assuming particle enters from (0, d)

$$r=\frac{mv}{qB},\quad d=\frac{r}{2}$$



$$a=\frac{qVB}{m}\left[\frac{-\sqrt{3}i-j}{2}\right]$$

this option is not given in the all above four choices.

11. **(b)** As we know, radius of circular path in magnetic field

$$r=\frac{\sqrt{2Km}}{qB}$$

For electron, $r_e=\dfrac{\sqrt{2Km_e}}{eB}$ ....(i)

For proton, $r_p=\dfrac{\sqrt{2Km_p}}{eB}$ ....(ii)

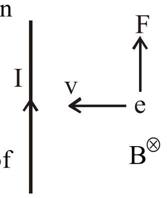
For α particle, $r_\alpha=\dfrac{\sqrt{2Km_a}}{q_\alpha B}=\dfrac{\sqrt{2K4m_p}}{2eB}=\dfrac{\sqrt{2Km_p}}{eB}$ ...(iii)

$\therefore\ r_e<r_p=r_\alpha \qquad (\because\ m_e<m_p)$

12. **(b)** The force is parallel to the direction of current in magnetic field,

hence $F=q(v\times \vec{B})$
According to Fleming's left hand rule,

we have, the direction of motion of charge is towards the wire.



13. **(b)** Q $F=qE$ and $F=qvB$
$\therefore E=vB$

And Gauss's law in Electrostatics $E=\dfrac{s}{e_0}$

$$E=\frac{s}{e_0}=vB \Rightarrow s=e_0vB$$

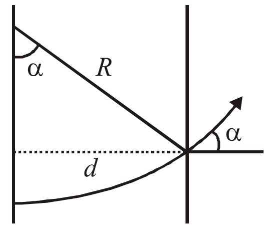
$$\sigma_1=-\sigma_2$$

14. (d) From figure, $\sin\alpha = d/R$

And we know, $\dfrac{mv^2}{R}=qvB$

$\Rightarrow\ R=\dfrac{mv}{qB}$

$\therefore\ \sin\alpha=\dfrac{dqB}{mv}$



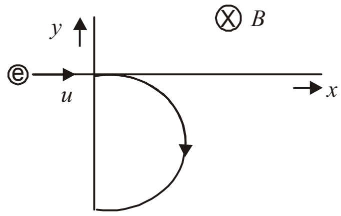
$$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}\left[\because qV=\frac{1}{2}mv^2\right]$$

15. **(c)** Since particle is moving undeflected
So, $q_E=qvB$

$$\Rightarrow B=\frac{E}{V}=\frac{10^4}{10}=10^3\ \text{wb/m}^2$$

16. **(a)** Use the vector form of B and v in the formulae $\vec{F}=q(\vec{v}\times\vec{B})$ to get the instantaneous direction of force at $x=a$ and $x=2a$.
In the region $a<x<2a$ force must be in the direction $\hat{i}\times\hat{j}$ *i.e.*, $+z$-direction so vertically upward. And in the region $2a<x<3a$ in $-z$-direction vertically downward.

17. **(b)** Magnetic force on the charged particle does not change its speed
$\therefore\ u=v$
The force acting on electron will be perpendicular to the direction of velocity till the electron remains in the magnetic field. So the electron will follow the path as given below.



Hence electron comes out with speed $v=u$ and at $y<0$.

18. **(b)** When $\vec{E}=0$ then path of the charged particle beyond P will be helix hence option (a) and (c) are incorrect. The velocity at P is in the X-direction (given).

Let $\vec{v}=k\hat{i}$.
After P, the positively charged particle gets deflected in the $x-y$ plane toward $-y$ direction and the path is non-circular.
Now, $\vec{F}=q(\vec{v}\times\vec{B})$

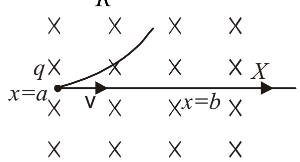
$\Rightarrow\ \vec{F}=q[k\hat{i}\times(c\hat{k}+a\hat{i})]$ for option (b)

$$=q[kc\hat{i}\times\hat{k}+\hat{k}a\hat{i}\times\hat{i}]=kcq(-\hat{j})$$

Since in option (b), electric field is also present $\vec{E}=a\hat{i}$, therefore it will also exert a force in the $+X$ direction. The net result of the two forces will be a non-circular path. Only option (b) fits for the above logic.

19. **(b)** When a charged particle is moving in a uniform megnetic field normal to its motion

$$qvB=\frac{mv^2}{R}$$



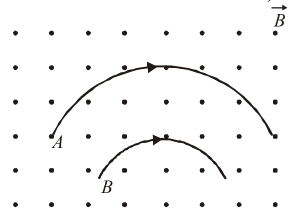
Width of the magnetic field region $(b-a) \le R$; where 'R' is its radius of curvature inside magnetic field,

$$\therefore \quad R = \frac{mv}{qB} \ge (b-a) \Rightarrow v_{\min} = \frac{(b-a)qB}{m}$$

20. **(b)** When a charged particle is moving normal to the magnetic field then a force acts on it which behaves as a centripetal force and moves the particle in circular motion.

$$\therefore \quad F = qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv^2}{qB}$$

If q and B are same for both then, $r \propto mv$



Since $r_A > r_B$

$\Rightarrow \quad m_A v_A > m_B v_B$

21. **(c)** Given electric field along $+x$ direction, $\vec{E} = E.\hat{i}$ and magnetic field along $+z$ direction $\vec{B} = B.\hat{k}$

Velocity of the ionised particle will be along direction determined by $q.\vec{E}$

or velocity $\vec{v} = AqE\hat{i}$ where A is a positive constant.

Here A, E and B are positive constants.

Charge on ions (q) may be positive or negative

Magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$

$\therefore \quad \vec{F} = q[(AqE\hat{i}) \times (B\hat{k})] = q^2 \; AEB(\hat{i} \times \hat{k})$

$= q^2 \; AEB(-\hat{j}) = (\pm q)^2 \, AFB$, along negative y-direction.

As magnetic force is along negative y-axis hence all ions, whether positive or negative will deflect towards negative y-axis.

22. **(c)** The angular momentum of the particle $L = mvr = mr^2\omega$ where $\omega = 2\pi n$. $[\because v = r\omega]$

$\therefore$ Frequency $n = \dfrac{\omega}{2\pi}$; Further $i = \dfrac{q}{t} = q \times n = \dfrac{\omega q}{2\pi}$

Magnetic moment, $M = iA = \dfrac{\omega q}{2\pi} \times \pi r^2 \Rightarrow M = \dfrac{\omega q r^2}{2}$

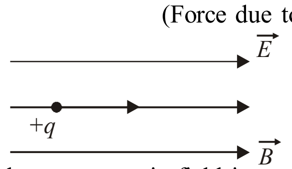
$$\therefore \quad \frac{M}{L} = \frac{\omega q r^2}{2mr^2\omega} = \frac{q}{2m}$$

23. **(a)** Uniform electric field $\vec{E}$ and magnetic field $\vec{B}$ are parallel to each other.

$F_E = qE$ (Force due to electric field)

$F_B = evB \sin \theta = qvB \sin 0 = 0$

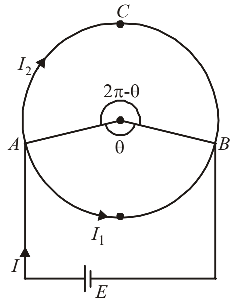
(Force due to magnetic field)



Since force due to magnetic field is zero. So force due to electric field will make the charged particle accelerate or decelerate parallel to itself in straight line.

24. **(d)** Magnetic field at the centre due to current in arc AB

$B_1 = \dfrac{\mu_0}{4\pi}\dfrac{I_1}{r}\theta$ (Upwards)



Magnetic field at the centre due to current in arc ACB

$B_2 = \dfrac{\mu_0}{4\pi}\dfrac{I_2}{r}(2\pi - \theta)$ (Downwards)

$\therefore$ Net magnetic field at the centre

$$B = \frac{\mu_0}{4\pi}\frac{I_1}{r}\frac{\theta}{\pi} - \frac{\mu_0}{4\pi}\frac{I_2}{r}(2\pi - \theta)$$

Also, $I_1 = \dfrac{E}{R_1} = \dfrac{E}{\rho\ell_1/A} = \dfrac{EA}{\rho r\theta} \quad \left[\because R = \rho\dfrac{\ell}{A}\right]$

and $I_2 = \dfrac{E}{R_2} = \dfrac{E}{\rho\ell_2/A} = \dfrac{EA}{\rho r(2\pi - \theta)}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\left[\frac{EA}{\rho r\theta} \times \frac{\theta}{r} - \frac{EA}{\rho r(2\pi - \theta)} \times \frac{(2\pi - \theta)}{r}\right] = 0$$

25. **(c)** After entering the magnetic field, a magnetic force acts on the charged particle which moves the charged particle in circular path

$$\therefore \quad \frac{mv^2}{R} = qvB \quad \Rightarrow \quad R = \frac{mv}{qB} \quad \text{or,} \quad R = \frac{\sqrt{2mK}}{qB}$$

Here, K, q, B are equal $\quad \therefore \quad R^2 \propto m$

Hence, $\dfrac{m_1}{m_2} = \dfrac{R_1^2}{R_2^2}$

26. Here undeviated path C is of chargeless particle neutron. According to Fleming's left hand rule, the force on electrons (negative charge) will be towards right *i.e.*, track D.

Also, by the same rule we find that the force on proton and α-particle *i.e.*, +(ve)ly charged particle is towards left. Now since the magnetic force will behave as centripetal force,

$\therefore \quad \dfrac{mv^2}{r} = qvB \qquad$ or $\quad r \propto \dfrac{m}{q}$

For proton $r \propto \dfrac{1}{1} = 1$; For α-particle $r \propto \dfrac{4}{2} = 2$

i.e., $(r)_\alpha > (r)_p \; \therefore \quad \alpha$-particle will take path B.

27. **False**

Kinetic energy, $K_{proton} = K_{electron} = K$

Here $\dfrac{mv^2}{r} = qvB \quad \Rightarrow \quad r = \dfrac{mv}{qB} \Rightarrow \quad r = \dfrac{\sqrt{2mK}}{qB}$

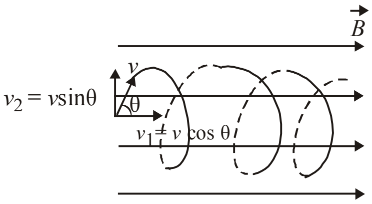
$[\because mv = P = \sqrt{2mk}\,]$

$\therefore \quad r \propto \sqrt{m}$ [For constant q, K and B]

$\therefore \quad r \propto \sqrt{m}$

Since mass of proton > mass of electron therefore radius of proton will be more.

28. False
The path of a particle is a circle when it enters normal to the magnetic field. The velocity component v_2 will be responsible in moving the charged particle in a circle.



The velocity component v_1 will be responsible in moving the charged particle in horizontal direction. Therefore the charged particle will travel in a helical path.

29. True
The magnetic force acting on a charged particle is in a direction perpendicular to the direction of velocity and hence it cannot change the speed of the charged particle.

Therefore, the kinetic energy $\left(=\frac{1}{2}\text{mv}^2\right)$ does not change.

As work done by this force is zero so the energy of moving changed particles remains unchanged.

30. (a, b) (a) For the charge +Q to return region 1.

$$\frac{mv^2}{(3R/2)} = QvB \Rightarrow \frac{2p}{3R} = QB \quad \left[\text{Here, radius } r = \frac{3}{2}R\right]$$

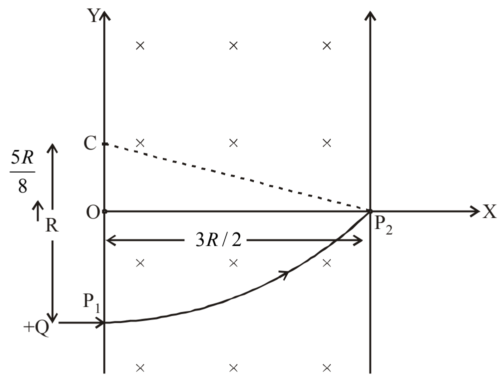
$$\therefore \quad B = \frac{2p}{3QR}$$

Therefore for $B \geq \frac{2p}{2QR}$, the particle will re-enter region 1.

(b) When $B = \frac{8p}{13QR}$

$$\frac{mv^2}{r} = Qv\left(\frac{8p}{13QR}\right) \qquad \therefore \; r = \frac{13R}{8}$$

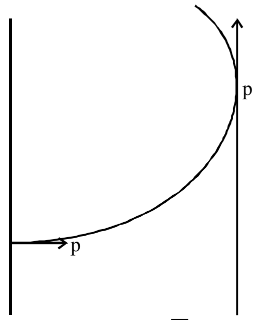
Thus 'C' is the of the centre of circular path of radius $\frac{13R}{8}$



Also $CP_2 = \sqrt{CO^2 + OP_2^{\,2}} = \sqrt{\left(\frac{5R}{8}\right)^2 + \left(\frac{3R}{2}\right)^2}$

$$\therefore \text{CP}_2 = \frac{13R}{8}$$

Thus the particle will enter region 3 through the point P_1 on X-axis



(c) Change in momentum = $\sqrt{2}p$

(d) Further $\frac{mv^2}{r} = qvB \;\therefore\; r = \frac{mv}{qB} \quad \therefore \quad r \propto m$

i.e., Distance is directly proportional to mass.

31. (a, b, c)
Magnetic force acting on a current carrying wire, placed in a uniform magnetic field,

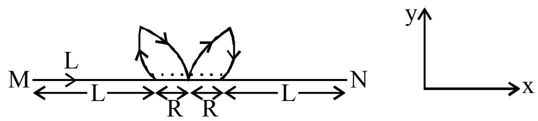
$\vec{F} = I(\vec{l} \times \vec{B})$

Here, $\vec{l}$ = displacement of the wire = $2(L + R)\,\hat{x}$

$\therefore \quad \vec{F} = 2I(L+R)(\hat{x} \times \vec{B})$

If $\vec{B} = B\hat{x}$ then

$\vec{F} = 2I(L+R)(\hat{x} \times \hat{x})\,B = 0$



If $\vec{B} = B\hat{y}$ then

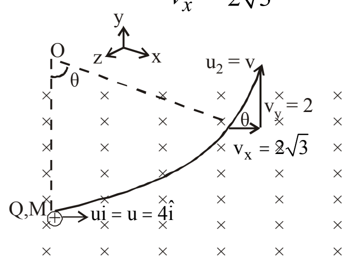
$\vec{F} = 2I(L+R)(\hat{x} \times \hat{y})B = 2IB(L+R)\,\hat{z}$
or $\quad F \propto (L+R)$

If $\vec{B} = B\hat{z}$ then

$\vec{F} = 2I(L+R)(\hat{x} \times \hat{z})B = -2IB(L+R)\hat{y}$
or, $\; F \propto (L+R)$.

32. (a, c)
According to Fleming's left hand rule, magnetic field should be in the –z direction.

From figure, $\tan\theta = \frac{v_y}{v_x} = \frac{2}{2\sqrt{3}}$



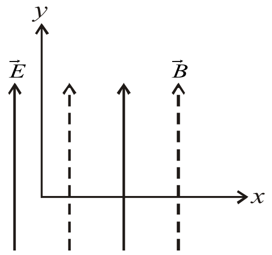
$\therefore \; \theta = \frac{\pi}{6}$

Angle rotated by the particle = $\frac{\text{arc}}{\text{radius}} = \frac{\text{speed} \times \text{time}}{\text{radius}}$

$$\frac{\pi}{6} = \frac{4 \times 10 \times 10^{-3}}{M \times 4/QB} \qquad \left[\because \text{radius} = \frac{\text{Mv}}{\text{QB}}\right]$$

$$\therefore \quad B = \frac{50\pi M}{3Q}$$

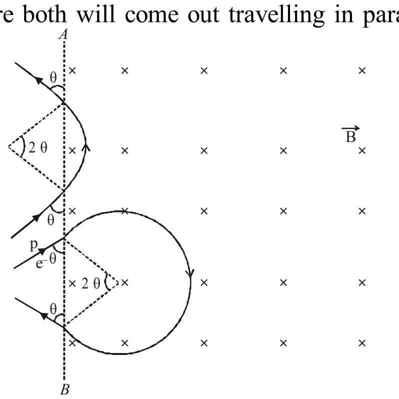
33. **(c, d)** If $\theta = 0^\circ$, the charged particle is projected along x-axis, due to magnetic field, $\vec{B}$ the charged particle will tend to move in a circular path in y-z plane but due to force of electric field $\vec{E}$, the particle will move in a helical path with increasing pitch. Hence options (A) and (B) are wrong.



If $\theta = 10^\circ$, we can resolve velocity into two rectangular components. One along x-axis ($v \cos 10^\circ$) and one along y-axis ($v \sin 10^\circ$). Due to $v \cos 10^\circ$, the particle will move in circular path and due to $v \sin 10^\circ$ plus the force due to electric field, the particle will undergo helical motion with its pitch increasing.

If $\theta = 90^\circ$, the charge is moving along the magnetic field. Therefore the force due to magnetic field is zero. But the force due to electric field will accelerate the particle along y-axis.

34. **(b,d)**
The entry and exit of electron & proton in the magnetic field makes the same angle with AB as shown.
Therefore both will come out travelling in parallel paths.



Also, $r = \dfrac{mv}{qB}$ or, $r \propto m$

$\because m_e < m_p \therefore r_e < r_p$

and, $T = \dfrac{2\pi m}{qB}$ or, $T \propto m$

$\therefore \quad T_e < T_p,\ t_e = \dfrac{T_e}{2}$ and $t_p = \dfrac{T_p}{2}$

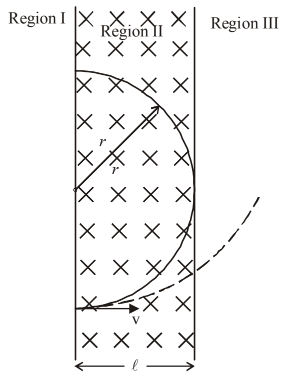
or, $t_e < t_p$

35. **(a, c, d)** As $V \perp B$ and B uniform, so path of the charged particle in region-II is circular and radius of circular path

$$r = \frac{mv}{qB}$$

- For the particle to enter region III, $r > \ell$ (path shown by daished line)

$$\frac{mv}{qB} > \ell \ \Rightarrow v > \frac{q\ell B}{m}$$



- **For maximum path length** in region II, $r = \ell$

$$\therefore \quad \ell = \frac{mv}{qB} \Rightarrow v = \frac{q\ell B}{m}$$

- The time taken by the particle to move in region II before coming back in region I

$t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$ which is independent of v *i.e.*, time spent is same in Region-II is same for any velocity.

36. **(a, c)** When the charged particles enter a magnetic field then a force acts on the particle which will act as a centripetal force.

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

or, $r = \dfrac{\sqrt{2mk}}{qB}$

$\therefore \quad r \propto \dfrac{\sqrt{m}}{q}$ [Kinetic energy, 'k' and 'B' are same]

$$r_{H^+} \propto \frac{\sqrt{1}}{1};\ r_{He^+} \propto \frac{\sqrt{4}}{1};\ r_{O^{++}} \propto \frac{\sqrt{16}}{2}$$

$$\Rightarrow \quad r_{H^+} \propto 1;\ r_{He^+} \propto 2;\ r_{O^{++}} \propto 2$$

Hence He^+ and O^{++} will be deflected equally.
H^+ will be deflected the most since its radius is smallest.

37. **(a,b,d)**
There is no change in velocity. It can be possible when
- Electric and magnetic fields are absent, *i.e.*, E = 0, B = 0
  So, F_e and F_m both are zero.
- Or when electric and magnetic fields are present but force due to electric field is equal and opposite to the magnetic force *i.e.*, $F_e + F_m = 0$ (*i.e.*, $E \neq 0, B \neq 0$).
- Or when $E = 0$. $B \neq 0$ provided
  $F = qvB \sin\theta = 0$

$\sin\theta = 0$, *i.e.*, $\theta = 0 \Rightarrow v$ and B are parallel or anti-parallel.

38. **(d)** For the particle to move in straight line with constant velocity

$$\vec{F}_e + \vec{F}_B = 0$$

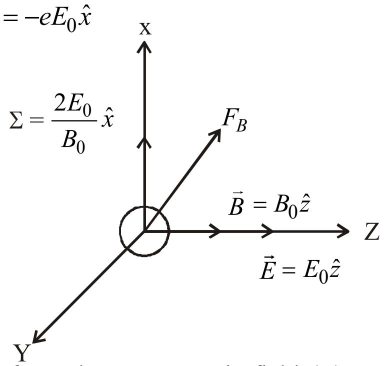
$$\vec{F}_E = -e\vec{E} = -e(-E_0\hat{x}) = eE_0\hat{x}$$

$$\vec{F}_B = q(v \times \vec{B}) = -e\left[\frac{E_0}{B_0}\hat{y} \times B_0\hat{z}\right]$$

$$\vec{F}_B = -eE_0\hat{x}$$

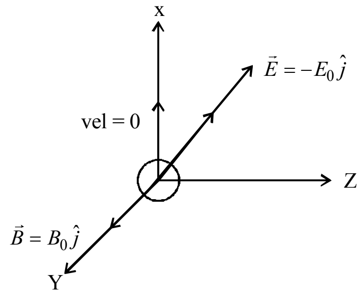
39. **(a)**

$$\Sigma = \frac{2E_0}{B_0}\hat{x}$$



The force due to magnetic field (B) F_B will provide the necessary centripetal force for circular motion which will be in X-Y plane. The force due to electric field (E) F_E will accelerate proton in Z-direction. Thus the path will be helical with increasing pitch.

40. **(b)** Particle will move in a straight line along- $\hat{y}$



If the electric field will apply a force on –Y axis thereby accelerating the charge along –Y axis.
And magnetic force, $F_B = qv B \sin\theta = 0$
Here $\theta = 180°$

41. Here, $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$ ...(i)

Potential energy = kinetic energy

$qV = \frac{1}{2}mv^2$ or, $V = \sqrt{\frac{2qv}{m}}$ ...(ii)

From eq. (i) and (ii)

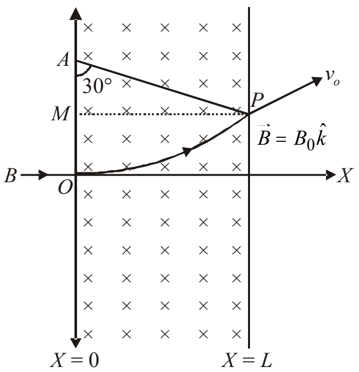
$r = \frac{m}{qB} \times \sqrt{\frac{2qV}{m}}$ or $r = \frac{\sqrt{2qVm}}{qB}$

or, $r \propto \sqrt{\frac{m}{q}}$

$\therefore \frac{r_p}{r_q} = \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}} = \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{1}{\sqrt{2}}$

42. Let the charged particle enter at 'O' and emerge out from the region of magnetic field at point P. Then the velocity vector $\vec{v}_0$ makes an angle 30° with x-axis. The perpendicular to circular path at P intersects the negative y-axis at point A.

(a)



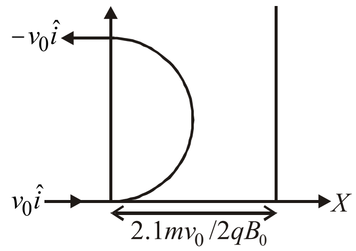
$\therefore AO = AP = R$ = radius of circular path.

$\frac{mv_0^2}{R} = B_0 q v_0 \Rightarrow R = \frac{mv_0}{qB_0}$ ... (i)

In ΔAPM, $\frac{L}{R} = \sin 30° \Rightarrow R \sin 30° = L \Rightarrow \frac{R}{2} = L$ ... (ii)

From eq. (i) and (ii), $L = \frac{mv_0}{2qB_0}$.

(b) Now the magnetic field extends upto 2.11



So new region of magnetic field

$= \frac{2.1R}{2} > R.$

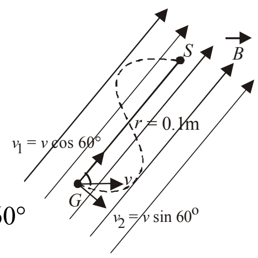
Thus, the required final velocity $= -v_0\hat{i}$.

From figure, the charged particle covers half the circle in the region of magnetic field.
Since the time period for complete revolution, $T = 2\pi m/qB_0$.
Therefore the time taken by the particle to cross the region of magnetic field *i.e.*, half the circle $= \frac{T}{2} = \pi m/qB_0$.

43. **(a)** The velocity of electron makes an angle 60° with magnetic field B. v_1 component of velocity move the charge particle in the direction of the magnetic field whereas v_2 component for revolving the charged particle in circular motion. Therefore overall path is helical. The particles will hit 's' with minimum value of B if pitch of helix.

$T \times v_1 = GS \Rightarrow \frac{2\pi m}{qB} \times v\cos 60° = 0.1$

$B = \frac{2\pi mv\cos 60°}{q \times 0.1}$



But $\frac{1}{2}mv^2 = E \Rightarrow v = \sqrt{\frac{2E}{m}}$

$\therefore B = \frac{2\pi m}{q \times 0.1} \times \sqrt{\frac{2E}{m}} \times \cos 60°$

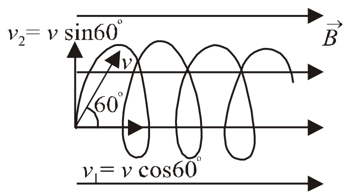
$= \frac{2\pi}{q \times 0.1} \times \sqrt{2mE} \times \cos 60° = \frac{2 \times 3.14}{1.6 \times 10^{-19} \times 0.1}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 10^3 \times 1.6 \times 10^{-19}} \times \frac{1}{2}$

$= \frac{149.8}{10^{-19}} \times 0.316 \times 10^{-23} = 47.37 \times 10^{-4}$

or, $B = 4.737 \times 10^{-3}$ T

44. A beam of protons enters a uniform magnetic field B = 0.3T at an angle $\theta = 60°$ with a velocity $V = 4 \times 10^5$ m/s



Here V_1 provides horizontal motion and V_2 circular motion to the beam of protons, so protons follow a helical path.

$\therefore \quad \dfrac{mv_2^2}{r} = qv_2 B$

$r = \dfrac{mv_2}{qB} = \dfrac{1.76\times10^{-27}\times4\times10^5\times\sqrt{3}}{1.6\times10^{-19}\times0.3\times2} = 0.012\text{ m}$

Pitch of helix = $v_1 \times T$

where $T = \dfrac{2\pi r}{v_2} = \dfrac{2\pi r}{v\sin\theta}$

$\therefore$ Pitch of helix = $v\cos\theta\times\dfrac{2\pi r}{v\sin\theta}$

$= 2\pi r\cot\theta = 2\times3.14\times0.012\times\cot 60° = 0.044$ m

45. $m = 1.6\times10^{-27}$ kg, $q = 1.6\times10^{-19}$ C

$B = 1$ T

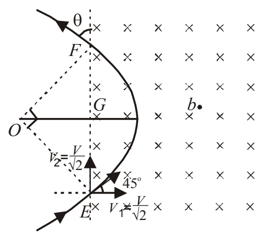
$v = 10^7$ m/s

$F = q\,.\,vB\sin\alpha$

(acting towards O by Fleming's left hand rule)

$\Rightarrow \quad F = qvB \qquad [\because \alpha = 90°]$

$qvB = ma \Rightarrow a = \dfrac{qvB}{m}$



$= \dfrac{1.6\times10^{-19}\times10^7\times1}{1.6\times10^{-27}}$

$= 10^{15}$ m/s^2

$\angle OEF = 45°$ ($\because OE$ act as a radius)

By symmetry $\angle OFE = 45°$

$\therefore \quad \angle EOF = 90°$ (by geometry)

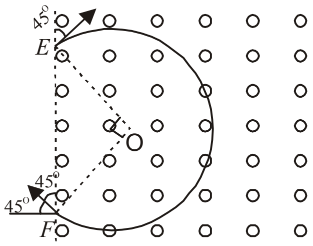
This is the centripetal acceleration

$\therefore \quad a_c = \dfrac{v^2}{r} = 10^{15} \quad \Rightarrow r = \dfrac{10^{14}}{10^{15}} = 0.1\text{m}.$

$\therefore$ EF $= 2r\cos45° = 2\times 0.1\times\dfrac{1}{\sqrt{2}} = 0.141$ m.

Angle, $\theta = 45$.

If the magnetic field is in the outward direction and the particle enters in the same way at E, then according to Fleming's left hand rule, the particle will turn towards clockwise direction and cover 3/4th of a circle as shown in the figure.



$\therefore$ Time required $= \dfrac{3}{4}T = \dfrac{3}{4}\left(\dfrac{2\pi}{\omega}\right) = \dfrac{3}{4}\times\left[\dfrac{2\pi r}{v}\right]$

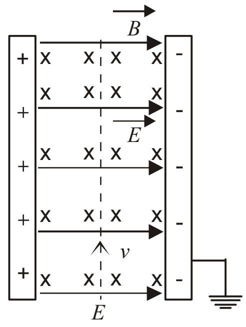
$= 4.71\times10^{-8}$ s.

46. As electron passes undeviated,

$\therefore$ Force due to electric field = force due to magnetic field.

$eE = evB$

$\therefore \quad B = \dfrac{E}{v} = \dfrac{V/d}{v} \qquad \left[\because E = \dfrac{V}{d}\right]$



(Given : $V = 600$ V, V $= 2\times10^6$ m/s and d = 3 mm $= 3\times10^{-3}$ m

$\therefore \quad B = \dfrac{600/3\times10^{-3}}{2\times10^6} = \dfrac{600}{3\times10^{-3}\times2\times10^6} = 0.1$ tesla

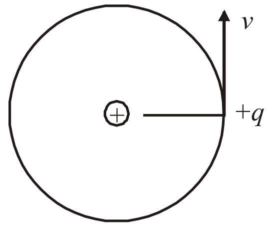
Here direction of force due to electric field (qE) is opposite to magnetic field q(v × B)

Here E is in positive direction

$\therefore$ V × B should be in negative x-direction or B should be in –(ve) z-direction ⊥ to paper inwards.

Topic-2 : Magnetic Field Lines, Biot-Savart'sLaw and Ampere's Circuital Law

1. **(d)**



Length of the circular path, $l = 2\pi r$

Current, $i = \dfrac{q}{T} = \dfrac{qv}{2\pi r}$

Magnetic moment M = Current × Area

$= i\times\pi r^2 = \dfrac{qv}{2\pi r}\times\pi r^2$

$M = \dfrac{1}{2}q\cdot v\cdot r$

Radius of circular path in magnetic field, $r = \dfrac{mv}{qB}$

$\therefore M = \dfrac{1}{2}qv\times\dfrac{mv}{qB} \Rightarrow M = \dfrac{mv^2}{2B}$

Direction of $\vec{M}$ is opposite of $\vec{B}$ therefore

$\vec{M} = \dfrac{-mv^2\vec{B}}{2B^2}$

(By multiplying both numerator and denominator by B).

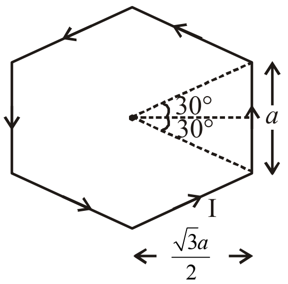
2. **(d)** Given : $I_A = 2$ A, $R_A = 2$ cm, $\theta_A = 2\pi - \dfrac{\pi}{2} = \dfrac{3\pi}{2}$

$I_B = 3$ A, $R_B = 4$ cm, $\theta_B = 2\pi - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$

Using, magnetic field, $B = \dfrac{\mu_0 I\theta}{4\pi R}$

$$\frac{B_A}{B_B}=\frac{I_A}{I_B}\times\frac{\theta_A R_B}{\theta_B R_A}=\frac{2\times\frac{3\pi}{2}\times 4}{3\times\frac{5\pi}{3}\times 2}=\frac{6}{5}$$

3. **(c)**



Magnetic field due to one side of hexagon

$$B=\frac{\mu_0 I}{4\pi\frac{\sqrt{3}a}{2}}(\sin 30°+\sin 30°)$$

$$\Rightarrow B=\frac{\mu_0 I}{2\sqrt{3}a}\left(\frac{1}{2}+\frac{1}{2}\right)=\frac{\mu_0 I}{2\sqrt{3}a\pi}$$

Now, magnetic field due to one hexagon coil

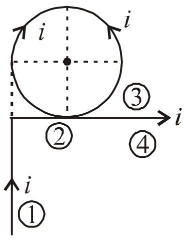
$$B=6\times\frac{\mu_0 I}{2\sqrt{3}a\pi}$$

Again magnetic field at the centre of hexagonal shape coil of 50 turns,

$$B=50\times 6\times\frac{\mu_0 I}{2\sqrt{3}a\pi} \qquad \left[\because a=\frac{10}{100}=0.1\text{ m}\right]$$

$$\text{or, } B=\frac{150\mu_0 I}{\sqrt{3}\times 0.1\times\pi}=500\sqrt{3}\,\frac{\mu_0 I}{\pi}$$

4. **(a)**



$$B_0=B_1+B_2+B_3+B_4$$

$$=\frac{\mu_0 I}{4\pi R}[\sin 90°-\sin 45°]+\frac{\mu_0 I}{2R}+\frac{\mu_0 I}{4\pi R}\ [\sin 45°+\sin 90°]$$

$$=-\frac{\mu_0 I}{4\pi R}\left(1-\frac{1}{\sqrt{2}}\right)+\frac{\mu_0 I}{2R}+\frac{\mu_0 I}{4\pi R}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\overrightarrow{B_0^{\odot}}=\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)^{\odot}$$

5. **(a)** $\vec{B}=\vec{B}_1+\vec{B}_2$

$$=\frac{\mu_0}{2\pi}\cdot\left(\frac{i^o}{d}\hat{k}+\frac{i^o}{d}\left(-\hat{k}\right)\right)=0$$

6. **(b)** $B=\frac{\mu_0}{4\pi}\cdot\frac{i}{r}(\sin\alpha+\sin\beta)$

Here $r=\sqrt{5^2-3^2}=4$ cm

$\alpha=\beta=37°$

$\therefore\ B=10^{-7}\times\frac{5}{4}2\sin 37°=1.5\times 10^{-5}$ T

7. **(c)** Let I be the current in each wire. (directed inwards) Magnetic field at 'O' due to LP and QM will be zero.

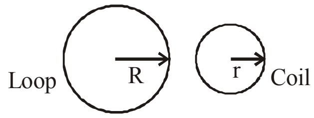
i.e., $B_0=B_{PS}+B_{QN}$

$\therefore$ Net magnetic field $B_0=\frac{\mu_0 i}{4\pi d}+\frac{\mu_0 i}{4\pi d}$

or $10^{-4}=\frac{\mu_0 i}{2\pi d}+\frac{2\times 10^{-7}\times i}{4\times 10^{-2}}$

$\therefore$ i = 20 A and the direction of magnetic field is perpendicular into the plane

8. **(d)**



$L=2\pi R$ $\qquad$ $L=N\times 2\pi r$

$R=Nr\Rightarrow r=\frac{R}{N}$

$$B_{Loop}=\frac{\mu_0 i}{2R}\quad B_{coil}=\frac{\mu_0 Ni}{2r}=\frac{\mu_0 Ni}{2\left(\frac{R}{N}\right)}=\frac{\mu_0 N^2 i}{2R}$$

$$\therefore\frac{B_L}{B_C}=\frac{1}{N^2}$$

9. **(c)** Magnetic field at the centre of loop, $B_1=\frac{\mu_0 I}{2R}$

Dipole moment of circular loop is m = IA

$m_1=I.A=I.\pi R^2$ {R = Radius of the loop}

If moment is doubled (keeping current constant) R becomes $\sqrt{2}R$

$$m_2=I.\pi\left(\sqrt{2}R\right)^2=2.I\pi R^2=2m_1$$

$$B_2=\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}$$

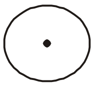
$$\therefore\frac{B_1}{B_2}=\frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}}=\sqrt{2}$$

10. **(b)** Point P is situated at the mid-point of the line joining the centres of the circular wires which have same radii (R).

The magnetic fields ($\vec{B}$) at P due to the currents in the wires are in same direction.

Magnitude of magnetic field at point, P

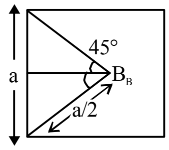
$$B=2\left\{\frac{\mu_0 NIR^2}{2\left(R^2+\frac{R^2}{4}\right)^{3/2}}\right\}=\frac{\mu_0 NIR^2}{\frac{5^{3/2}}{8}}=\frac{8\mu_0 NI}{5^{3/2}R}$$

11. **(b)** Case (a) : 

$$B_A = \frac{\mu_0}{4\pi}\frac{I}{R}\times 2\pi = \frac{\mu_0}{4\pi}\frac{I}{\ell/2\pi}\times 2\pi \ (\because 2\pi R = \ell)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{\ell}\times(2\pi)^2$$

Case (b) :



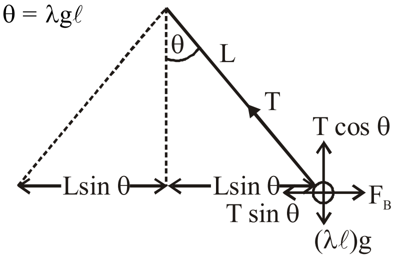
$$B_B = 4\times\frac{\mu_0}{4\pi}\frac{I}{a/2}[\sin 45° + \sin 45°]$$

$$= 4\times\frac{\mu_0}{4\pi}\times\frac{I}{\ell/8}\times\frac{2}{\sqrt{2}} = \frac{\mu_0}{4\pi}\frac{I}{\ell}\times\frac{64}{\sqrt{2}} = \frac{\mu_0 I}{4\pi\ell}32\sqrt{2} \qquad [4a = l]$$

$$\Rightarrow \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

12. **(d)** Let us consider 'ℓ' length of current carrying wire. At equilibrium

$T\cos\theta = \lambda g\ell$



and $T\sin\theta = \frac{\mu_0}{2\pi}\frac{I\times Il}{2L\sin\theta} \quad \left[\because \frac{F_B}{\ell} = \frac{\mu_0}{4\pi}\frac{2I\times I}{2\ell\sin\theta}\right]$

Therefore, $I = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{u_0\cos\theta}}$

13. **(b)** For loop $B = \frac{\mu_0 nI}{2a}$

where, a is the radius of loop.

Then, $B_1 = \frac{\mu_0 I}{2a}$

Now, for coil $B = \frac{\mu_0 I}{4\pi}\cdot\frac{2nA}{x^3}$

at the centre x = radius of loop

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\times3\times(I/3)\times\pi(a/3)^2}{(a/3)^3} = \frac{\mu_0.3I}{2a}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{\mu_0 I/2a}{\mu_0.3I/2a}$$

$B_1 : B_2 = 1:3$

14. **(b)** Given : Radius = R

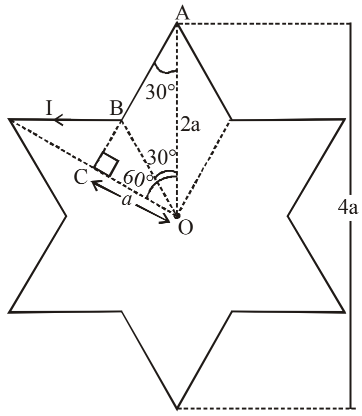
Distance $x = 2\sqrt{2}R$

$$\frac{B_{centre}}{B_{axis}} = \left(1+\frac{x^2}{R^2}\right)^{3/2} = \left(1+\frac{(2\sqrt{2}R)^2}{R^2}\right)^{3/2}$$

$$= (9)^{3/2} = 27$$

15. **(a)** From figure,

In Δ OAC, $\quad \cos 60° = \frac{OC}{OA} = \frac{OC}{2a}$



$\therefore$ OC $= 2a\times\frac{1}{2} = a$

Magnetic field at 'O' due to element AB

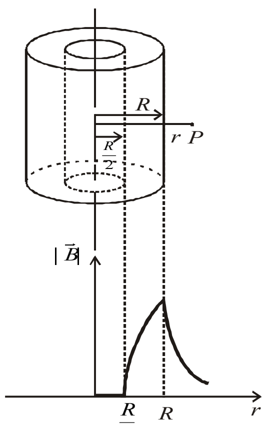
$$= \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 60° - \sin 30°]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{\sqrt{3}}{2}-\frac{1}{2}\right] = \frac{\mu_0 I}{4\pi a}\times\frac{1}{2}(\sqrt{3}-1)$$

$\therefore$ Magnetic field at the centre, due to complete loop

$$= \left[\frac{\mu_0}{4\pi}\frac{I}{a}\times\frac{1}{2}(\sqrt{3}-1)\right]\times 12 = \frac{\mu_0}{4\pi}\frac{I}{a}\times 6(\sqrt{3}-1)$$

16. **(d)** For $r < \frac{R}{2}, \quad B = 0$



For $\frac{R}{2} \le r < R,$

$$B = \frac{\mu_0}{2}\left[r - \frac{R^2}{2r}\right]J$$

For $r > R, \quad B = \frac{\mu_0 i}{2\pi r}$

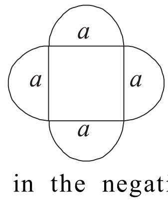
i.e., $B \propto \frac{1}{r}$

Hence graph (*d*) correctly depicts $|\vec{B}|$ versus r graph.

17. **(b)** Magnetic moment of a current carrying loop $\vec{M} = NI\vec{A}$

Here $N = 1, A = a^2 + 2\pi\left(\frac{a}{2}\right)^2 = a^2\left[1+\frac{\pi}{2}\right]$

From Screw law, direction of m is outward or in +ve z-direction.



$$\therefore \quad \vec{M} = Ia^2\left[1+\frac{\pi}{2}\right]\hat{k}$$

18. **(a)**

19. **(a)** The wire carries a current I in the negative z-direction. We have to consider the magnetic vector field $\vec{B}$ at (x, y) in the $z = 0$ plane.

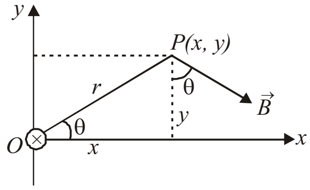
Magnetic field $\vec{B}$ is perpendicular to OP.

$\therefore\ \vec{B} = B\sin\theta\hat{i} - B\cos\theta\hat{j}$

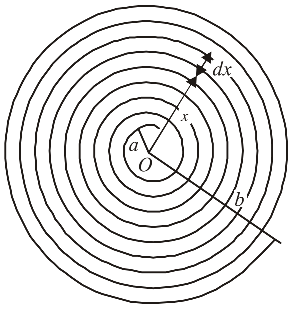
$\sin\theta = \dfrac{y}{r},\ \cos\theta = \dfrac{x}{r}\quad B = \dfrac{\mu_0 I}{2\pi r},\ x^2 + y^2 = r^2$

$\therefore\quad \vec{B} = \dfrac{\mu_0 I}{2\pi r^2}\left(y\hat{i} - x\hat{j}\right)$

or $\quad \vec{B} = \dfrac{\mu_0 I\left(y\hat{i} - x\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}.$



20. **(c)** Let us consider an element of thickness dx of wire. Let it be at a distance x from the centre O.



Number of turns per unit length $= \dfrac{N}{b-a}$

$\therefore$ Number of turns in thickness $dx = \dfrac{N}{b-a}\,\text{dx}$

Magnetic field due to this small element at O

$$dB = \frac{\mu_0}{2}\frac{NI}{(b-a)}\frac{dx}{x}$$

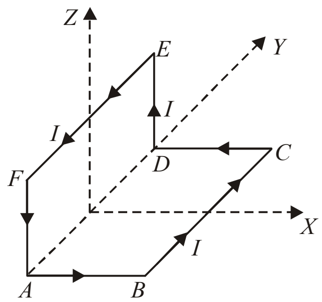
$$B = \int_a^b \frac{\mu_0}{2}\frac{NI}{b-a}\frac{dx}{x} = \frac{\mu_0}{2}\frac{NI}{(b-a)}$$

$$\int_a^b \frac{dx}{x} = \frac{\mu_0}{2}\frac{NI}{(b-a)}[\log_e x]_a^b$$

$$\therefore\quad B = \frac{\mu_0}{2}\frac{NI}{(b-a)}\ln\frac{b}{a}$$

21. **(d)** Here loop $ADEFA$ in y-z plane and loop $ABCDA$ in the $x-y$ plane.

By choosing the loops we find that in one loop we have to take current from A to D and in the other one from D to A. Effectively there is no current in AD. Hence these two cancel out the effect of each other as far as creating magnetic field at the point P is considered.



The point (a, 0, a) is in the X-Z plane.

The magnetic field due to current in $ABCDA$ will be in + ve Z-direction.

Due to symmetry the y-components and x-components will cancel out each other.

Similarly the magnetic field due to current in $ADEFA$ will be in x-direction.

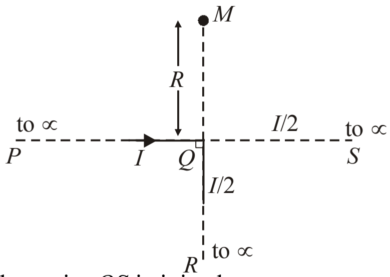
$\therefore$ The direction of resultant magnetic field at $P(a, 0, a)$

$$\vec{B} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{k}).$$

22. **(c)** Magnetic field at M due to PQ and QR

$$H_1 = \frac{1}{2}\left[\frac{\mu_0 I}{2\pi R}\right] + 0 = \frac{\mu_0 I}{4\pi R}$$

[$\because$ Magnetic field $B = 0$ at any point on the current carrying straight conductor]



Now when wire QS is joined.

H_2 = (Magnetic field at M due to PQ) + (magnetic field at M due to QR) + (Magnetic field at M due to QS)

$$= \frac{1}{2}\left[\frac{\mu_0 I}{2\pi R}\right] + 0 + \frac{1}{2}\left[\frac{\mu_0 I/2}{2\pi R}\right] = \frac{3\mu_0 I}{8\pi R}$$

$$\therefore\quad \frac{H_1}{H_2} = \frac{\mu_0 I}{4\pi R}\Big/\frac{3\mu_0 I}{8\pi R} = \frac{2}{3}$$

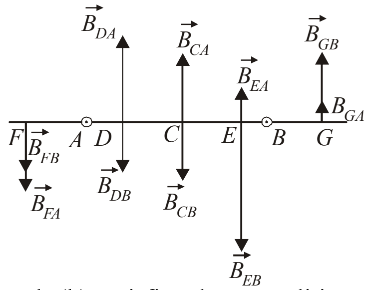
23. **(b)** Here current flowing out of the plane of paper so magnetic field at points to the right of the wire will be upwards and to the left will be downwards. Let us consider certain points.

Point C (mid point between A and B) : The magnetic field at C due to A ($\vec{B}_{CA}$) is in upward direction but magnetic field at C due to B is in downward direction. Net field is zero.

Point E : Magnetic field due to A is upward and magnetic field due to B is downward but $|\vec{B}_{EA}| < |\vec{B}_{EB}|$.

$\therefore$ Net magnetic field is in downward direction.

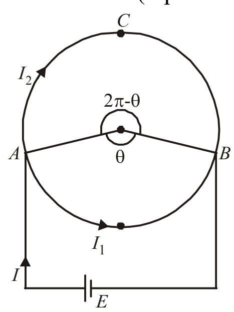
Point D : $|\vec{B}_{DA}| > |\vec{B}_{DB}|$. Net field upwards. Similarly, other points can be considered.



Hence graph (b) satisfies these condition.

24. **(d)** Magnetic field at the centre due to current in arc AB

$B_1 = \dfrac{\mu_0}{4\pi}\dfrac{I_1}{r}\theta\qquad$ (Upwards)



Magnetic field at the centre due to current in arc ACB

$$B_2 = \frac{\mu_0}{4\pi}\frac{I_2}{r}(2\pi-\theta) \quad \text{(Downwards)}$$

$\therefore$ Net magnetic field at the centre

$$B = \frac{\mu_0}{4\pi}\frac{I_1}{r}\frac{\theta}{\pi} - \frac{\mu_0}{4\pi}\frac{I_2}{r}(2\pi-\theta)$$

Also, $I_1 = \frac{E}{R_1} = \frac{E}{\rho\ell_1/A} = \frac{EA}{\rho r\theta} \quad \left[\because R = \rho\frac{\ell}{A}\right]$

and $I_2 = \frac{E}{R_2} = \frac{E}{\rho\ell_2/A} = \frac{EA}{\rho r(2\pi-\theta)}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\left[\frac{EA}{\rho r\theta}\times\frac{\theta}{r} - \frac{EA}{\rho r(2\pi-\theta)}\times\frac{(2\pi-\theta)}{r}\right] = 0$$

25. **(5)** Current density $J = \frac{\text{current}}{\text{area}} = \frac{I}{A} \Rightarrow I = \text{JA}$

Magnetic field B_R after removing cavity (C)

$B_R = B_{total} - B_{cavity}$

$$\frac{\mu_0 I_t}{2\pi a} - \frac{\mu_0 I_c}{2\pi\left(\frac{3}{2}a\right)}$$

$$= \frac{\mu_0}{\pi a}\left[\frac{I_t}{2} - \frac{I_c}{3}\right] \quad \left(\text{here } I_t = J(\pi a^2)\ I_c = J\left(\frac{\pi a^2}{4}\right)\right)$$

$$= \frac{\mu_0}{\pi a}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right]$$

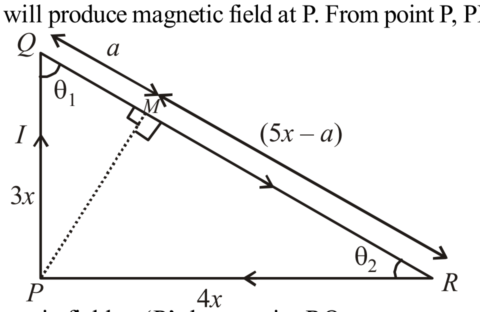
or, $B_R = \frac{5\mu_0 aJ}{12}$

Comparing it with $\frac{N}{12}\mu_0 aJ$

We get $N = 5$

26. **(7)**

The magnetic field B due to wires PR and PQ = 0. Only wire QR will produce magnetic field at P. From point P, PM $\perp$ QR



Magnetic field at 'P' due to wire RQ

$$B = \frac{\mu_0}{4\pi}\frac{I}{PM}(\cos\theta_1 + \cos\theta_2) \quad \ldots\text{(i)}$$

In ΔPQM, $9x^2 = PM^2 + a^2$ … (ii)

In ΔPRM,

$16x^2 = PM^2 + (5x - a)^2$ … (iii)

$\Rightarrow \quad 7x^2 = 25x^2 - 10xa \Rightarrow 10xa = 18x^2$

$\Rightarrow \quad a = 1.8\,x$ … (iv)

From eq. (ii) & (iv),

$$9x^2 = PM^2 + (1.8x)^2$$

$\therefore \quad PM = \sqrt{9x^2 - 3.24x^2} = \sqrt{5.76x^2} = 2.4x$ … (v)

Also $\cos\theta_1 = \frac{a}{3x} = \frac{1.8x}{3x} = 0.6$ … (vi)

$\cos\theta_2 = \frac{5x-a}{4x} = \frac{5x-1.8x}{4x} = \frac{3.2}{4} = 0.8$ …(vii)

Therefore, from eq. (i), (v), (vi) and (vii),

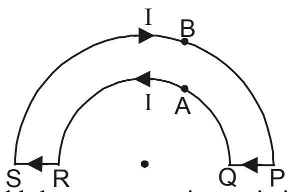
$$B = \frac{\mu_0}{4\pi}\times\frac{I}{2.4x}[0.6+0.8] = \frac{\mu_0}{4\pi}\times\frac{I}{2.4x}\times 1.4 = 7\left[\frac{\mu_0 I}{48\pi x}\right]$$

Comparing it with B = $k\left[\frac{\mu_0 I}{48\pi x}\right]$, we get, k = 7.

27. The magnetic field at centre C due to current carrying wires PQ and RS is zero.

Magnetic field due to current in semi-circular arc QAR

$B_1 = \frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_1}\right]$ [$\perp$ to paper outwards]



Magnetic field due to current in semi-circular arc SBP

$B_2 = \frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_2}\right]$ [$\perp$ to paper inwards]

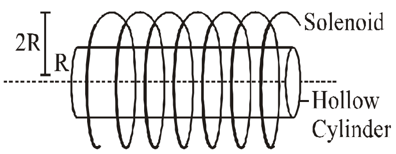
$\therefore$ Net Magnetic field B = $B_1 - B_2$

$$= \frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_1}\right] - \frac{1}{2}\left[\frac{\mu_0}{2}\frac{I}{R_2}\right]$$

or, $B = \frac{\mu_0 I}{4}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$.

Directed towards the reader perpendicular to plane of paper.

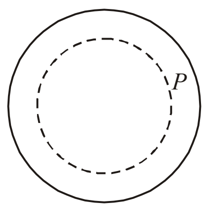
28. **(a, d)** In the region 0 < r < R, the net magnetic field is due to current in solenoid.



In the region r > 2 R, the magnetic field is present due to the current in the cylinder.

For the region R < r < 2R, the magnetic field is neither along the common axis, nor tangential to the circle of radius r.

29. **(b)** The magnetic field at every point inside the loop is zero. Consider any point P inside the thin walled pipe. Let us consider a circular loop and using Ampere's circuital law,

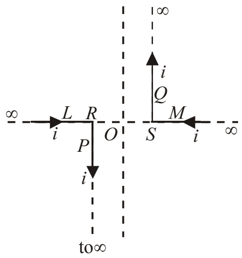


$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$$

Since current inside the loop is zero.

$\therefore$ Magnetic field, $\vec{B} = 0$

30.



$\therefore$ Magnetic field due to current carrying conductor P at point O

$B_1 = \dfrac{\mu_0}{4\pi}\dfrac{i}{(OR)}$ [$\perp$ to paper outwards]

Magnetic field due to current carrying conductor Q at point O

$B_2 = \dfrac{\mu_0}{4\pi}\dfrac{i}{(OS)}$ [$\perp$ to paper outwards]

Magnetic field due to current carrying conductors L and M at O is zero.

$\therefore$ Resultant magnetic field at O

$B = B_1 + B_2$

(As B_1 and B_2 are in the same direction)

$$B = \frac{\mu_0}{4\pi}\frac{i}{OR} + \frac{\mu_0}{4\pi}\frac{i}{OS} = \frac{\mu_0}{4\pi} i\left[\frac{1}{OR} + \frac{1}{OS}\right]$$

$$= 10^{-7} \times 10 \times \left[\frac{1}{0.02} + \frac{1}{0.02}\right] = 10^{-4}\,\text{T}.$$

Topic-3 : Force and Torque on Current Carrying Conductor

1. **(c)** Torque on the loop,

$$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin\theta = MB\sin 90°$$

Magnetic field, $B = \dfrac{\mu_0 I}{2\pi d}$

$$\therefore \tau = I_1(2a)^2\left(\frac{\mu_0 I_2}{2\pi d}\right)\sin 90°$$

$$= \frac{2\mu_0 I_1 I_2}{\pi d} \times a^2 = \frac{2\mu_0 I^2 a^2}{\pi d}$$

2. **(b)** Magnetic moment of loop $ABCD$,

M_1 = area of loop × current

$\vec{M}_1 = (abI)(\hat{j})$ (Here, ab = area of rectangle)

Magnetic moment of loop $DEFA$,

$\vec{M}_2 = (abI)(\hat{i})$

Net magnetic moment,

$$\vec{M} = \vec{M}_1 + \vec{M}_2 \Rightarrow \vec{M} = abI(\hat{i} + \hat{j})$$

$$\Rightarrow |\vec{M}| = \sqrt{2}abI\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$$

3. **(c)** Torque on circular loop, $\tau = MB\sin\theta$

where, M = magnetic moment

B = magnetic field

Now, using $\tau = I\alpha$

$\therefore \tau = MB\sin\theta = I\alpha$

$$\Rightarrow \pi R^2 IB\theta = \frac{mR^2\alpha}{2}$$

($\because m = IA$ and moment of inertia of circular loop, $I = \dfrac{mR^2}{2}$)

$$\Rightarrow \pi R^2 IB\,\theta = \frac{mR^2}{2}\omega\theta$$

$$\Rightarrow \omega = \sqrt{\frac{2\pi IB}{m}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{2\pi IB}{m}}$$

$$\Rightarrow T = \sqrt{\frac{2\pi m}{IB}}$$

4. **(b)** Torque experienced by circular loop, $\tau = \vec{M} \times \vec{B} = MB\sin\theta$ [Here, $\theta = \pi/2$]

where magnetic moment, $\vec{M} = iA = i\pi R^2$ [R = radius of circular coil]

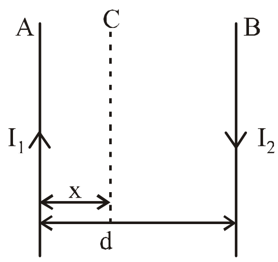
and magnetic field, $B = B_0$

$\therefore \tau = MB\sin\theta = i\pi R^2 N B_0$ [N = no. of turns]

Also, torque, $\tau = \dfrac{\Delta L}{\Delta t} \Rightarrow \Delta L$ (= magnitude of angular) momentum gained, $= \tau\Delta t = i\pi R^2 N B_0 . \Delta t = \pi NQB_0R^2$

[$\because i\,\Delta t = Q$]

5. **(d)**



As net force on the third wire C is zero.

$$\Rightarrow \vec{F} = \frac{\mu_0 I_1}{2\pi x} + \frac{\mu_0 I_2}{2\pi(d-x)} = 0$$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x-d)}$$

$I_1x - I_1d = I_2x$

$$x = \frac{I_1 d}{I_1 - I_2}$$

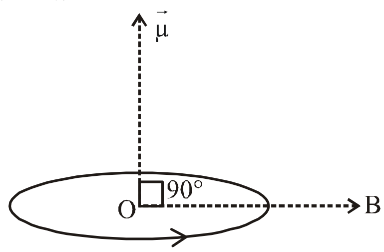
Two cases may be possible if $I_1 > I_2$ or $I_2 > I_1$

6. **(c)** $F = \dfrac{\mu_0}{2\pi}\left(\dfrac{i_i i_2}{a} - \dfrac{i_1 i_2}{2a}\right) \times a = \dfrac{\mu_0 i_1 i_2}{4\pi}$

7. **(b)** $|\vec{\tau}| = |\vec{\mu} \times \vec{B}|$ [$\mu = NIA$]

$= NIA \times B\sin 90°$ [$A = \pi r^2$]

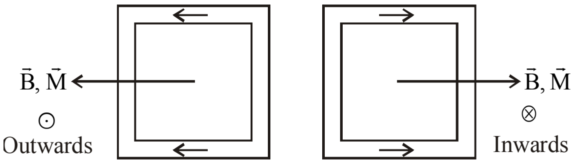
$\Rightarrow \tau = NI\pi r^2 B$



8. **(a)** Magnetic moment, $\mu = IA = \dfrac{qv}{2\pi r}(\pi r^2)$

or, $\mu = \dfrac{qr\omega}{2\pi r}(\pi r^2) = \dfrac{1}{2}qr^2\omega$

9. **(c)** Magnetic moment of current carrying rectangular loop of area A is given by M = NIA
magnetic moment of current carrying coil is a vector and its direction is given by right hand thumb rule, for rectangular loop, $\vec{B}$ at centre due to current in loop and $\vec{M}$ are always parallel.



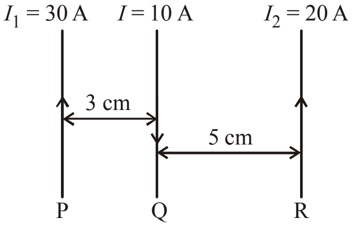
Hence, (c) corresponds to stable equilibrium.

10. **(c)** $\vec{F}_1 = \vec{F}_2 = 0$
because of action and reaction pair

11. **(a)** For stable equilibrium $\vec{M} \parallel \vec{B}$
For unstable equilibrium $\vec{M} \parallel (-\vec{B})$

12. **(a)** $I_1 I_2$ = Positive
(attract) F = Negative
$I_1 I_2$= Negative
(repell) F = Positive
Hence, option (a) is the correct answer.

13. **(a)**



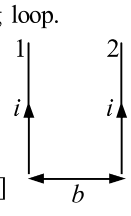
Also given; length of wire Q
= 25 cm = 0.25 m
Force on wire Q due to wire R

$F_{QR} = 10^{-7} \times \dfrac{2 \times 20 \times 10}{0.05} \times 0.25$
$= 20 \times 10^{-5}$ N (Towards left)
Force on wire Q due to wire P

$F_{QP} = 10^{-7} \times \dfrac{2 \times 30 \times 10}{0.03} \times 0.25$
$= 50 \times 10^{-5}$ N (Towards right)
Hence, $F_{net} = F_{QP} - F_{QR}$
$= 50 \times 10^{-5}$ N $- 20 \times 10^{-5}$ N
$= 3 \times 10^{-4}$ N towards right

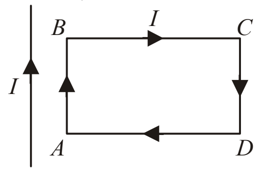
14. **(b)** In uniform magnetic field, net force on a current carrying loop is zero. Hence loop cannot move. Fleming's left hand rule, we find that a force is acting in the radially outward direction throughout the circumference of the conducting loop.

15. **(b)** As we know, the force per unit length f/l between two wires carrying currents I_1 and I_2 separated by a distance r.



$\dfrac{P}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi r} = \dfrac{\mu_0 i^2}{2\pi b}$ $[\because I_1 = I_2 = i \text{ and } r = b]$

16. **(c)** The loop will move towards the wire AB part of the rectangular loop will get attracted to the long straight wire as the currents are parallel and in the same direction whereas CD part will be repelled because currents are in opposite direction. But since this force $F \propto \dfrac{1}{r}$ where r is the distance between the wires. Therefore, there will be a net attractive force on the rectangular loop. Force on BC is equal and opposite to that on AD, so these forces will cancel each other.



17. **(3)** $\dfrac{mv^2}{R} = qvB \Rightarrow R = \dfrac{mv}{qB}$

or $R \propto \dfrac{1}{B}$ $[\because m, q, v$ are the same$]$

$\therefore \dfrac{R_1}{R_2} = \dfrac{B_2}{B_1}$

or, $\dfrac{R_1}{R_2} = \dfrac{\dfrac{\mu_0}{4\pi} \times 2I\left[\dfrac{1}{X_1} + \dfrac{1}{X_0 - X_1}\right]}{\dfrac{\mu_0}{4\pi} \times 2I\left[\dfrac{1}{X_1} - \dfrac{1}{X_0 - X_1}\right]}$

$= \dfrac{X_0 - X_1 + X_1}{X_0 - X_1 - X_1} = \dfrac{X_0}{X_0 - 2X_1}$ $\left[\text{Given } \dfrac{X_0}{X_1} = 3\right]$

$\therefore \dfrac{R_1}{R_2} = \dfrac{\dfrac{X_0}{X_1}}{\dfrac{X_0}{X_1} - 2} = \dfrac{3}{3-2} = 3$

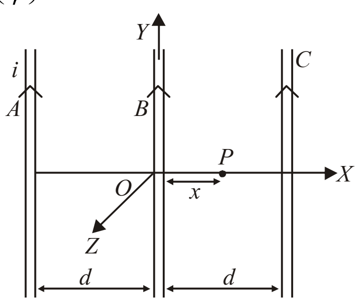
18. Here, net force on the loop $EDCBE$ will be zero. Also force due to segment FE and BA will be zero. Force due to segment EB

$\vec{F} = I[L\hat{i} \times B\hat{j}] = ILB\hat{k}$ in the positive z direction.

19. **True**
A current carrying coil is a magnetic dipole. The net magnetic force on a magnetic dipole placed in a uniform magnetic field is zero.

20. (i) Magnetic field due to an infinitely long current carrying wire at distance r is given by

$B = \dfrac{\mu_0}{4\pi}\left(\dfrac{2i}{r}\right)$



In case of three identical wires, resultant field can be zero only if the point P is between the two wires, otherwise field B due to all the wires will be in the same direction and so resultant B cannot be zero. Hence, if point P is at a distance x from the central wire as shown in figure, then,

$$\vec{B}_P = \vec{B}_{PA} + \vec{B}_{PB} + \vec{B}_{PC}$$

where $\vec{B}_{PA}$ = magnetic field at P due to A

$\vec{B}_{PB}$ = Magnetic field at P due to B

$\vec{B}_{PC}$ = Magnetic field at P due to C.

Direction of B is given by right hand palm rule.

$$\vec{B}_P = \frac{\mu_0}{4\pi}2i\left[\frac{1}{d+x}+\frac{1}{x}-\frac{1}{d-x}\right](-\hat{k}).$$

For $\vec{B}_P = 0$,

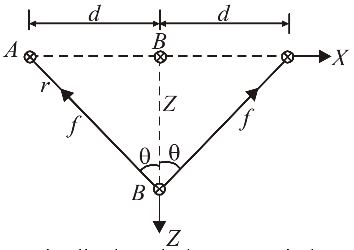
$$\frac{1}{d+x}+\frac{1}{x} = \frac{1}{d-x}$$

Solving we get, $x = \pm\frac{d}{\sqrt{3}}$

(ii) The force per unit length between two parallel current carrying wires

$$\frac{\mu_0}{4\pi}\frac{2i_1 i_2}{r} = f \text{ (say)}$$

Attractive if currents are in the same direction.



If the wire B is displaced along Z-axis by a small distance Z, the restoring force per unit length $\frac{F}{\ell}$ on the wire B due to wires A and C will be

$$\frac{F}{\ell} = 2f\cos\theta = 2\frac{\mu_0}{4\pi}\frac{2i_1 i_2}{r}\times\frac{z}{r}\quad\left[\text{as}\cos\theta = \frac{z}{r}\right]$$

or $$\frac{F}{\ell} = \frac{\mu_0}{4\pi}\cdot\frac{4i^2 z}{(d^2+z^2)}\quad [\text{as } I_1 = I_2 \text{ and } r^2 = d^2+z^2]$$

or $$\frac{F}{\ell} = -\frac{\mu_0}{4\pi}\left(\frac{2i}{d}\right)^2 z\ [\text{as } d >> z \text{ and } F \text{ is opposite to } z]\quad ...(i)$$

The components of force along x-axis will be cancelled. Here $F \propto -z$, the motion is simple harmonic.

Comparing eq. (i) with the standard equation of S.H.M.

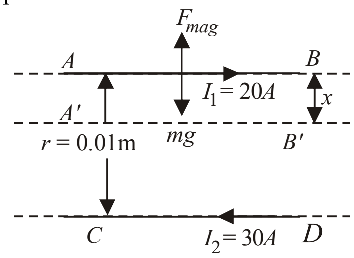
$F = -m\omega^2 z$ i.e., $\frac{F}{\ell} = -\frac{m}{\ell}\omega^2 z$

$= -\lambda\,\omega^2 z$, we get

$$\lambda\,\omega^2 = \frac{\mu_0}{4\pi}\times\frac{4i^2}{d^2} \Rightarrow \omega = \sqrt{\frac{\mu_0 i^2}{\pi d^2\lambda}}$$

$$\Rightarrow 2\pi f = \frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} \Rightarrow f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

21. When wire AB is slightly depressed, force of repulsion due to current. In it opposite to the wire CD will push it upwards and AB may vibrate. If force of attraction exists, CD will attraction AB and vibration of AB will not be possible.
When the rod is depressed by a distance x, then the force acting on the upper wire increases and behaves as a restoring force.



Restoring force/length $= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r-x} - \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

$$= \frac{\mu_0}{4\pi}2I_1I_2\left[\frac{1}{r-x}-\frac{1}{r}\right] = \frac{\mu_0}{4\pi}\frac{2I_1I_2 x}{r(r-x)}$$

When x is small i.e., $x << r$ then $r = x \approx r$

Restoring force/length $F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2}x = \frac{mg}{r}$

Since, $F \propto x$ and directed to equilibrium position. Therefore the motion is simple harmonic

$$\therefore\ \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2} = \text{(mass per unit length)}\ \omega^2 (\because a = \ell\omega^2)\ ...(i)$$

At equilibrium, $\frac{mg}{r} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

Mass per unit length $= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{rg}$ ... (ii)

From eq. (i) and (ii)

$$\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r^2} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{rg}\times\omega^2 \Rightarrow \omega = \sqrt{\frac{g}{r}}$$

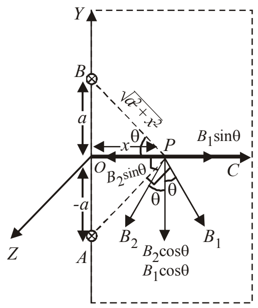
$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{r}}$$

$$\therefore\ T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}} = 0.2\text{ s}$$

22. Let the magnetic field due to currents in A and B at P be B_1 and B_2 respectively in the directions perpendicular to AP and BP as shown in figure.

Let $\angle BPO = \angle APO = \theta$

$$|\vec{B}_1| = \frac{\mu_0}{4\pi}\frac{2I}{\sqrt{a^2+x^2}} = |\vec{B}_2|$$



On resolving B_1 and B_2 we get that the $\sin\theta$ components cancel out and the $\cos\theta$ components add up.

$\therefore B = 2B_1\cos\theta$

$$= \frac{2\mu_0}{4\pi}\frac{2I}{\sqrt{a^2+x^2}}\times\frac{x}{\sqrt{a^2+x^2}} = \frac{\mu_0}{4\pi}\frac{4Ix}{(a^2+x^2)}$$

(towards $-Y$ direction)

Let us consider a small portion of wire OC at P of length dx. (Because magnetic field produced at different points on OC will be different). The small amount of force acting on that small portion

$$\vec{d}F = I(\vec{d}x \times \vec{B}) \quad \therefore\ dF = I\,dx\,B \sin 90^\circ$$

$$\Rightarrow\ dF = I\,dx \times \frac{\mu_0}{4\pi} \times \frac{4Ix}{(a^2+x^2)}$$

$$\Rightarrow\ dF = \frac{\mu_0}{4\pi} 4I^2 \frac{xdx}{(a^2+x^2)}$$

$$\therefore\ \text{Total force, } F = \frac{\mu_0}{4\pi} \times 4I^2 \int_0^L \frac{xdx}{(a^2+x^2)}$$

$$= \frac{\mu_0}{4\pi} \times 4I^2 \left[\frac{1}{2}\log_e(a^2+x^2)\right]_0^L$$

$$\Rightarrow\ F = \frac{\mu_0}{4\pi} \times 2I^2 \left[\log_e \frac{a^2+L^2}{a^2}\right]$$

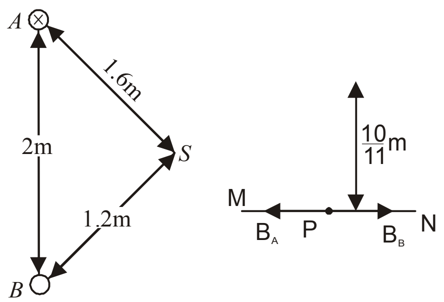
The direction of $\vec{F}$ is towards $-Z$ direction as per Fleming's left hand rule.

When direction of current in B is reversed, net magnetic field is along the current, hence force.

$F = I\ell B \sin\theta$ and $\theta = 180^\circ$ is zero.

23. The two parallel wires are perpendicular to the plane of the paper. The wire A carries current directed into the plane of paper.

The current in wire B should be in upward direction so as to cancel the magnetic field due to A at P. (By right hand Thumb rule)



(i) The magnetic field at P due to current in wire A.

$$B_A = \frac{\mu_0}{4\pi}\frac{2I_A}{r_{AP}} = \frac{\mu_0}{4\pi} \times \frac{2\times 9.6}{\left(2+\frac{10}{11}\right)} \text{ (Direction } P \text{ to } M) \quad ...(i)$$

The magnetic field at P due to current in wire B

$$B_B = \frac{\mu_0}{4\pi} \times \frac{2I_B}{\left(\frac{10}{11}\right)} \quad ...(ii)$$

From eq. (i) and (ii)

$$\frac{\mu_0}{4\pi} \times \frac{2\times 9.6}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{4\pi} \times \frac{2I_B}{\left(\frac{10}{11}\right)}$$

$$\Rightarrow\ \frac{9.6\times 11}{32} = \frac{I_B \times 11}{10} \Rightarrow I_B = \frac{96}{32} = 3\text{A}$$

(ii) The dimensions given shows that $SA^2 + SB^2 = AB^2\ \therefore\ \angle ASB = 90^\circ$ (By pythagorous theorem)

Magnetic field due to A at S

$$B_{SA} = \frac{\mu_0}{4\pi}\cdot\frac{2I_A}{r_{SA}} = \frac{\mu_0}{4\pi} \times \frac{2\times 9.6}{1.6} \text{ (Directed } S \text{ to } B)$$

Magnetic field due to B at S

$$B_{SB} = \frac{\mu_0}{4\pi}\cdot\frac{2I_B}{r_{SB}} = \frac{\mu_0}{4\pi}\frac{2\times 3}{1.2} \text{ (Directed } S \text{ to } A)$$

Since B_{SA} and B_{SB} are mutually perpendicular

$\therefore$ net magnetic field,

$$B = \sqrt{B_{SA}^2 + B_{SB}^2} = \frac{\mu_0}{4\pi}\sqrt{\left(\frac{9.6}{0.8}\right)^2 + \left(\frac{3}{0.6}\right)^2}$$

$$= 10^{-7} \times 13 = 1.3 \times 10^{-6}\,\text{T}$$

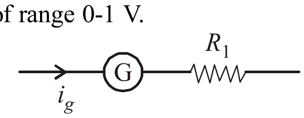
(iii) Force per unit length on wire B

$$= \frac{\mu_0}{4\pi}\frac{2I_AI_B}{r_{AB}}$$

$$= \frac{10^{-7} \times 2 \times 9.6 \times 3}{2} = 28.8\times 10^{-7}\,\text{N/m}$$

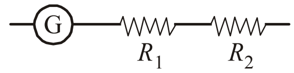
Topic-4 : Galvanometer and its Conversion into Ammeter and Voltmeter

1. **(d)** Galvanometer of resistance (G) converted into a voltmeter of range 0-1 V.



$$V = 1 = i_g(G + R_1) \quad ...(i)$$

To increase the range of voltmeter 0-2 V



$$2 = i_g(R_1 + R_2 + G) \quad ...(ii)$$

Dividing eq. (i) by (ii),

$$\Rightarrow \frac{1}{2} = \frac{G+R_1}{G+R_1+R_2}$$

$$\Rightarrow G + R_1 + R_2 = 2G + 2R_1$$

$$\therefore R_2 = G + R_1$$

2. **(d)** Given,

Current passing through galvanometer, $I = 6$ mA

Deflection, $\theta = 2^\circ$

Figure of merit of galvanometer

$$= \frac{I}{\theta} = \frac{6\times 10^{-3}}{2} = 3\times 10^{-3} \text{ A/div}$$

3. **(Bonus)** $v = i_g(R + G)$

$\Rightarrow 5 = 10^{-4}(2 \times 10^6 + x)$

$x = -195 \times 10^4\Omega$

4. **(c)** $V = i_g(G + R) = 4 \times 10^{-3}(50 + 5000) = 20V$

5. **(d)** $C\theta = NBiA \sin 90^\circ$

$$\text{or } 10^{-6}\left(\frac{\pi}{180}\right) = 175B(10^{-3}) \times 10^{-4}$$

$\therefore B = 10^{-3}$ T

6. **(b)** When key K_1 is closed and key K_2 is open

$$i_g = \frac{E}{220 + R_g} = C\theta_0 \qquad \text{... (i)}$$

When both the keys are closed

$$i_g = \left(\frac{E}{220 + \frac{5R_g}{5 + R_g}}\right) \times \frac{5}{(R_g + 5)} = \frac{C\theta_0}{5}$$

$$\Rightarrow \frac{5E}{225R_g + 1100} = \frac{C\theta_0}{5} \qquad \text{... (ii)}$$

$$\frac{E}{220 + R_g} = C\theta_0 \qquad \text{... (i)}$$

Dividing (i) by (ii), we get

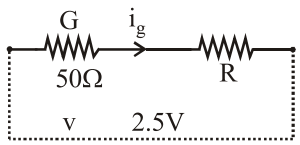
$$\Rightarrow \frac{225R_g + 1100}{1100 + 5R_g} = 5$$

$\Rightarrow 5500 + 25R_g = 225R_g + 1100$

$200R_g = 4400$

$R_g = 22\Omega$

7. **(b)** Galvanometer has 25 divisions $I_g = 4 \times 10^{-4} \times 25 = 10^{-2}$ A



$V = I_g R_{net}$

$v = I_g (G + R)$

$2.5 = (50 + R)\, 10^{-2} \therefore R = 200\Omega$

8. **(c)** Deflection current

$= Ig_{max} = nxk = 0.005 \times 30$

Where, n = Number of divisions = 30 and k = 0.005 amp/division

$= 15 \times 10^{-2} = 0.15$

$v = I_g[20 + R]$

$15 = 0.15\,[20 + R]$

$100 = 20 + R$

$R = 80\ \Omega$

9. **(d)** Given,

Resistance of galvanometer, $G = 100\Omega$

Current, $i_g = 1\ mA$

A galvanometer can be converted into voltmeter by connecting a large resistance R in series with it.

Total resistance of the combination $= G + R$

According to Ohm's law, $V = i_g (G + R)$

$\therefore 10 = 1 \times 10^{-3} (100 + R_0)$

$\Rightarrow 10000 - 100 = 9900\ \Omega = R_0$

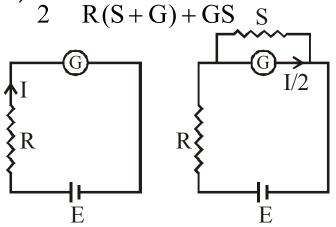
$\Rightarrow R_0 = 9.9\ k\Omega$

10. **(b)** Figure of merit of a galvanometer is the correct required to produce a deflection of one division in the galvanometer

i.e., figure of merit $= \dfrac{I}{\theta}$

$$I = \frac{\varepsilon}{R + G} \quad G = \frac{1}{9} K\Omega$$

$$\frac{1}{2} = \frac{\varepsilon}{R + \frac{GS}{G + S}} \times \frac{S}{S + G} \Rightarrow \frac{1}{2} = \frac{\varepsilon S}{R(S + G) + GS}$$



$$S = \frac{RG \times \frac{1}{2}}{\varepsilon - \frac{(R + G)I}{2}}$$

$$S = \frac{11 \times 10^3 \times \frac{1}{2} \times 10^2 \times 270 \times 10^{-6}}{6 - \left(\frac{6}{2}\right)} = 110\Omega$$

11. **(c)** Given : Current through the galvanometer,

$i_g = 5 \times 10^{-3} A$

Galvanometer resistance, $G = 15\Omega$

Let resistance R to be put in series with the galvanometer to convert it into a voltmeter.

$V = i_g (R + G)$

$10 = 5 \times 10^{-3} (R + 15)$

$\therefore \quad R = 2000 - 15 = 1985 = 1.985 \times 10^3\ \Omega$

12. **(c)** $Ig\,G = (I - Ig)s$

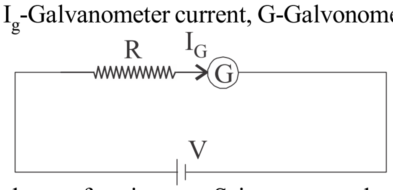
$\therefore 10^{-3} \times 100 = (10 - 10^{-3}) \times S$

$\therefore \ S \approx 0.01\Omega$

13. **(a)** According to Ohm's Law, $I = \dfrac{V}{R}$

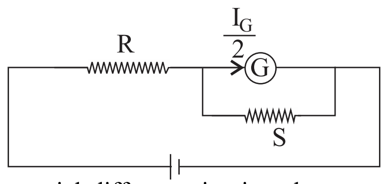
$$I_g = \frac{V}{R + G}$$

where, I_g-Galvanometer current, G-Galvonometer resistance



When shunt of resistance S is connected parallel to the Galvanometer then $G = \dfrac{GS}{G + S}$

$$\therefore \ I = \frac{V}{R + \frac{GS}{G + S}}$$



Equal potential difference is given by

$I'_g G = (I - I'_g)S$

$I'_g (G + S) = IS$

$$\Rightarrow \frac{I_g}{2} = \frac{IS}{G + S}$$

$$\Rightarrow \frac{V}{2(R + G)} = \frac{V}{R + \frac{GS}{G + S}} \times \frac{S}{G + S}$$

$$\Rightarrow \frac{1}{2(R + G)} = \frac{S}{R(G + S) + GS}$$

$\Rightarrow R(G + S) + GS = 2S(R + G)$

$\Rightarrow RG + RS + GS = 2S(R + G)$

$\Rightarrow RG = 2S(R + G) - S(R + G)$

$\therefore RG = S(R + G)$

14. **(5.56)** [Given: $B = 0.02T$, $C = 10^{-4}$ Nm rad^{-1}

$\theta = 0.2$ rad

$N = 50$ and

$A = 2 \times 10^{-4}\, m^2]$

We know, $C\theta = NBA\, I_g$

$\therefore \quad I_g = \dfrac{C\theta}{NBA} = \dfrac{10^{-4} \times 0.2}{50 \times 2 \times 10^{-4} \times 0.02} = 0.1A$

To convert a galvanometer to ammeter, a shunts is used in parallel to the galvanometer.

$I_g \times G = (I - I_g)\, S$

$\therefore \quad S = \dfrac{I_g G}{I - I_g} = \dfrac{0.1 \times 50}{1 - 0.1} = \dfrac{50}{9} = 5.56\,\Omega$

15. **(b, d)** Here, $G = 10\Omega$; $I_g = 2 \times 10^{-6}$ A, V = 100 m $V = 0.1V, I = 10^{-3}A$

Using, $V = I_g\,(G + R)$ [R = resistance connected in series with galvanometer]

$\Rightarrow 0.1 = 2 \times 10^{-6}\, R_v$

$\therefore\ R_v = 5 \times 10^4\,\Omega$ (Resistance of voltmeter)

Also $I_g\, G = (I - I_g)s$

$2 \times 10^{-6} \times 10 = (10^{-3} - 2 \times 10^{-6})s$

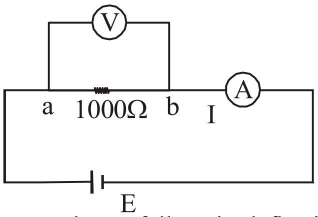
$\therefore\ S = 2 \times 10^{-2}\,\Omega$ (Resistance of ammeter)

$$R_A = \frac{GS}{G+S} = \frac{10 \times 0.02}{10 + 0.02} \approx 0.02\Omega$$

$$I = \frac{E}{\dfrac{50,000 \times 1000}{51000} + 0.02} = \frac{E}{980.41}$$

$$V_{ab} = \frac{E}{980.41} \times \frac{50,000 \times 1000}{51000} = \frac{E}{980.41} \times 980.39$$

$$\therefore\ R_{measured} = \frac{V_{ab}}{I} = \frac{E}{980.41} \times \frac{980.39}{E/980.41} = 980.39\Omega$$



If the voltmeter shows full scale deflection, then

$$0.1 = \frac{E}{980} \times \left(\frac{1000}{51000}\right) \times 5 \times 10^4$$

$\therefore\ E = 999.6$ mv.

Since $i_A = 10^{-3}A$

$$\therefore \quad \text{Maximum reading of } R = \frac{999.6 \times 10^{-3}}{1 \times 10^{-3}} = 999.6\,\Omega$$

16. **(a, c)** The range of voltmeter.

$V = I_g(R_{eq} + G)$

$\therefore$ Maximum voltage can be obtained if equivalent resistance of components is maximum, *i.e.* when all the components are connected in series.

The range of ammeter

$$I = I_g\left(1 + \frac{G}{S_{eq}}\right)$$

$\therefore$ Maximum current range can be obtained if equivalent shunt resistance is minimum, *i.e.*, when all the components are connected in parallel.

17. **(b, c)** To convert ammeter into a volt meter a high resistance is connected in series and alow resistance is connected in parallel to convert a galvanometer into an ammeter.

For $V = I_g\,(G + R) = 5 \times 10^{-5}\,[100 + 200{,}000] = 10V$

For $I = I_g\left(\dfrac{G}{S} + 1\right) = 5 \times 10^{-5}\left[\dfrac{100}{1} + 1\right] = 5mA.$

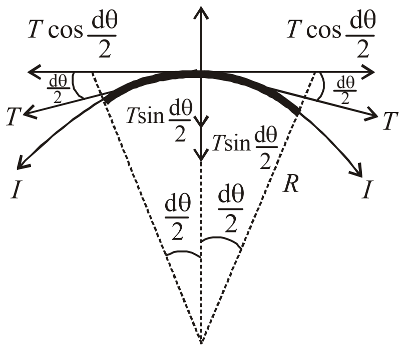
18. **(c)** Sensitivity of a moving coil galvanometer, $= \dfrac{\theta}{I} = \dfrac{NBA}{C}$.

If B increases, by using iron core $\dfrac{\theta}{I}$ *i.e.*; sensitivity increases.

Soft iron can be easily magnetised and de magnetized.

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** Let us consider an elemental length dl subtending an angle $d\theta$ at the centre of the circle 'O'. Let F_B be the magnetic force acting on this length.



$\therefore \quad F_B = BI\,(dl)$ (upwards)

$= BI\,(Rd\theta) \qquad \left[\because \text{angle}\,(d\theta) = \dfrac{\text{arc}(dl)}{\text{radius } R}\right]$

$= BI\left(\dfrac{L}{2\pi}\right)d\theta \qquad \left[\because 2\pi R = L \Rightarrow R = \dfrac{L}{2\pi}\right]$

Let T be the tension in the wire acting along both ends of the elemental length.

At equilibrium $2T\sin\left(\dfrac{d\theta}{2}\right) = BI\dfrac{L}{2\pi}d\theta$

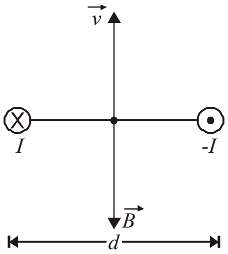
$\Rightarrow 2T\dfrac{d\theta}{2} = BI\dfrac{L}{2\pi}d\theta \qquad \left[\because \dfrac{d\theta}{2}\text{ very-very small}\right]$

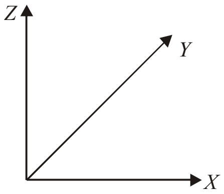
$\therefore\ T = \dfrac{BIL}{2\pi}$

2. **(d)** The magnetic lines of force created due to current will be in such a way that on $x - y$ plane these lines will be perpendicular. These lines will be in circular loops. The number of lines moving downwards in $x - y$ plane will be same in number to that coming upwards of the $x - y$ plane. Therefore, total magnetic flux through $x - y$ plane will be zero.

3. **(d)** The net magnetic field due to both the wires will be downward as shown below in the figure. Since angle between $\vec{v}$ and $\vec{B}$ is 180°,

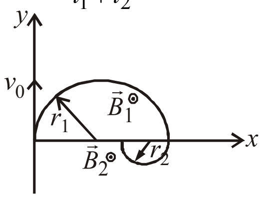
Therefore, magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$





Wires are in X-Y plane and velocity in Z-direction

4. **(2)** Average speed along x-axis, $v_x = \dfrac{d_1 + d_2}{t_1 + t_2}$



Here, $r_1 = \dfrac{mv_0}{qB_1}$ and $r_2 = \dfrac{mv_0}{qB_2}$

$\because B_1 = \dfrac{B_2}{4} \qquad \therefore \quad r_1 = 4r_2$

Time spent by charged particle in B_1, $t_1 = \dfrac{\pi m}{qB_1}$

Time spent by charged particle in B_2, $t_2 = \dfrac{\pi m}{qB_2}$

Total distance along x-axis

$d_1 + d_2 = 2r_1 + 2r_2 = 2(r_1 + r_2) = 2(5r_2) = 10r_2$

Average speed $= \dfrac{10r_2}{5t_2} = 2\dfrac{mv_0}{qB_2} \times \dfrac{qB_2}{\pi m}$

$= 2\text{ms}^{-1} \qquad (\because v_0 = \pi \text{ms}^{-1})$

5. **(6)** Consider the circular tube as a long solenoid. The wires are closely wound. Magnetic field B inside the solenoid $B = \mu_0 ni$

$\therefore \quad B = \dfrac{\mu_0 I}{L} \qquad \left[\because ni = \dfrac{I}{L}\right]$

Flux passing through the circular coil

$\phi = BA = \left(\dfrac{\mu_0 I}{L}\right)(\pi r^2)$

Induced emf $e = -\dfrac{d\phi}{dt} = -\left(\dfrac{\mu_0 \pi r^2}{L}\right) \cdot \dfrac{dI}{dt}$

Induced current, $i = \dfrac{e}{R} = -\left(\dfrac{\mu_0 \pi r^2}{LR}\right) \cdot \dfrac{dI}{dt}$

Magnetic moment, $M = iA = i\pi r^2$

or $\quad M = -\left(\dfrac{\mu_0 \pi^2 r^4}{LR}\right) \cdot \dfrac{dI}{dt} \quad$ ...(i)

Given, $I = I_0 \cos(300t)$

$\therefore \quad \dfrac{dI}{dt} = -300 I_0 \sin(300t)$

$M = \left(\dfrac{300\,\pi^2 r^4}{LR}\right) \mu_0 I_0 \sin(300t)$

$N = \dfrac{300\,\pi^2 r^4}{LR}$

$\dfrac{300\,(22/7)^2 (0.1)^4}{(10)\,(0.005)} = 5.926$ or $N \simeq 6$

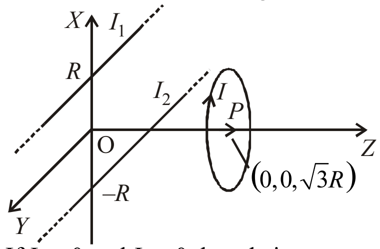
6. The force experience by moving charge. $\vec{F} = q(\vec{v} \times \vec{B})$

$= (-e)(-v\hat{i} \times B\hat{j}) = evB\,\hat{k}$

The direction of flow of electrons is opposite to that of current. Due to this force, the lowering of the potential of the face ABCD as electron collected at this face.

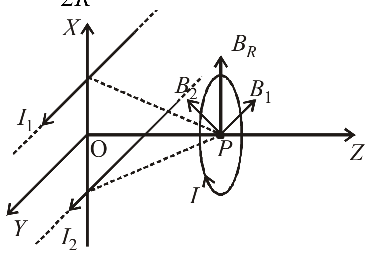
7. **(a, b, d)** (a) If $I_1 = I_2$, then the magnetic fields due to I_1 and I_2 at origin 'O' will cancel out each other. But the magnetic field at 'O' due to the ring will be present. Therefore B cannot be zero at origin.

(b) If $I_1 > 0$ and $I_2 < 0$, then the magnetic field due to both current will be in $+Z$ direction and add-up. The magnetic field due to current I will be in $-Z$ direction and if its magnitude is equal to the combined magnitudes of I_1 and I_2 then B can be zero at the origin.



(c) If $I_1 < 0$ and $I_2 > 0$ then their net magnetic field at origin will be in $-Z$ direction and the magnetic field due to I at origin will also be in $-Z$ direction. Therefore $\vec{B}$ at origin cannot be zero.

(d) If $I_1 = I_2$ then the resultant of the magnetic field B_R at P is along $+X$ direction. Therefore the magnetic field at P is only due to the current I which is in $-Z$ direction and is equal to $\vec{B} = \dfrac{\mu_0 I}{2R}(-\hat{k})$.



8. **(b, d)** Induced emf $e = -\dfrac{d\phi}{dt}$. For identical rings induced emf will be same. But currents will be different. Given $h_A > h_g$.

Hence, $V_A > V_B$ as $\left(h = \dfrac{v^2}{2g}\right)$.

If $\rho_A > \rho_B$, then, $I_A < I_B$. In this case given condition can be fulfilled if $m_A < m_B$.

If $\rho_A < \rho_B$, then $I_A > I_B$. In this case given condition can be fulfilled if $m_A \le m_B$.

9. **A : q, r**

B at P due to upper wire in downward direction and due to lower wire in upward direction.

Hence, q is correct.

As P is the mid point, the two magnetic fields, cancel out each other. Therefore, r is correct.

B : p

B at P, due to current in loop A is along the axial line towards right and due to current in loop B is also along the axial line towards right.

Hence B and P due to the currents in the wires are in the same direction.

C : q, r

The magnetic field due to current in loop A at P is equal and opposite to the magnetic field due to current in loop B at P.

D : q, s

'B' at P due to current in inner loop is perpendicular to the plane of paper directed vertically upwards.

'B' at P due to current in outer loop is perpendicular to the plane of paper directed vertically downward.

As the current are in opposite direction the wires repel each other. But net force on each wire is zero.

10. **A : q**

Thermal energy is generated in the wire, when a charged capacitor is connected to the ends of the wire, a variable current (decreasing in magnitude with time) passes through the wire (shown as resistor). The potential difference across the wire also decreases with time. The charge on the capacitor plate also decreases with time.

B : r, s

The wire is moved perpendicular to its length (ℓ) with a constant velocity (v) in a uniform magnetic field (B) ⊥ to the plane of motion $e = B\ell v$

When B, ℓ, v are constant, e is constant

A constant potential difference develops across the ends of the wire and charges of constant magnitude appear at the ends of the wire.

C : s

When wire is placed ma constant electric field that has a direction along the length of the wire. The free electrons move under the influence of electric field opposite to the direction of electric field. This movement of e^- continues till the electric field inside the wire is zero. Charges of constant magnitude appear at the ends of the wire.

D : p, q, r

Since *emf* of the battery E, R are constant, a constant current flows in the wire. Due to heating effect of current, thermal energy is generated in the wire. Also a constant potential difference develops between the ends of the wire.

11. A – P; B – q, s; C – q, s; D – q, r, s

(A) Charge on dielectric ring will create electrostatic field which is time independent.

(B) The rotating charge is like a current. This will create a magnetic field and a magnetic moment.

(C) Constant current in ring, so net charge is zero there will be no time independent electric field. The current produces magnetic field and magnetic moment.

(D) $i = i_0 \cos \omega t$; A changing magnetic field will be produced. This will create a induced electric field. Also a changing magnetic moment will be produced.

12. **(a, d)** When megnetic force balances electric force

$F_B = F_E \Rightarrow q\, v_d\, B = q\, E$

$$\therefore\ v_d B = \frac{V}{w} \qquad [\because V = E \times w]$$

$$\therefore\ V = w v_d B = w\left[\frac{I}{newd}\right] \times B \qquad \left[v_d = \frac{I}{neA} = \frac{I}{newd}\right]$$

$$\therefore\ V = \frac{I}{ned} \times B$$

$$\text{or, } V \propto \frac{1}{d} \Rightarrow V_1 d_1 = V_2 d_2$$

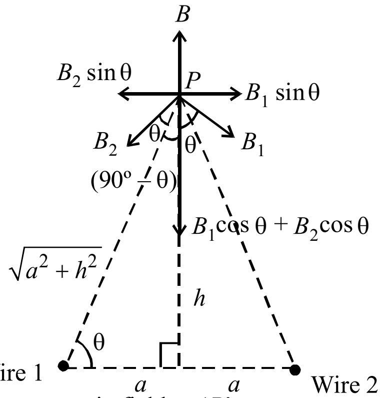
If $d_1 = 2d_2$, $V_2 = 2V_1$
and if $d_1 = d_2$, $V_2 = V_1$

13. **(a, c)** $\because V = \dfrac{I}{ne_d} \times B$

$$\therefore\ V \propto \frac{B}{n} \Rightarrow \frac{V_1 n_1}{B_1} = \frac{V_2 n_2}{B_2}$$

If $B_1 = B_2$ and $n_1 = 2n_2 \Rightarrow V_2 = 2V_1$
and vf $B_1 = 2B_2$ and $n_1 = n_2 \Rightarrow V_2 = 0.5V_1$

14. **(c)** Here $B_1 \sin\theta$ and $B_2 \sin\theta$ cancelled each other.



For zero magnetic field at 'P'

Magnetic field due to current carrying circular loop = Magnetic field due to straight wires

$B = B_1 \cos\theta + B_2 \cos\theta = 2\, B_1 \cos\theta$

$$\frac{\mu_0 I a^2}{2\left(a^2+h^2\right)^{3/2}} = 2\left[\frac{\mu_0 I}{2\pi\sqrt{a^2+h^2}}\right] \times \frac{a}{\sqrt{a^2+h^2}}$$

Solving we get,

$h \approx 1.2a$

The current is from P to Q and R to S in wire 1 and wire 2 respectively.

15. **(b)** We know torque

$$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin\theta$$

$$= \left(I \times \pi a^2\right) \times \left[2 \times \frac{\mu_0 I}{2\pi d}\right] \sin 30°$$

$$\therefore\ \tau = \frac{\mu_0 I^2 a^2}{2d}$$

16. **(d)** The levitation of the train is due to magnetic repulsion. The magnetised coils running along the track repel large magnets on the train's under carriage.

17. **(d)** High initial cost.

18. **(b)** Maglev is the abbreviation of magnetic levitation. The magnetic force will pull the maglev trains.

19. **(a)** Torque acting on a rectangular coil placed in a uniform magnetic field

$\vec{\tau} = \vec{M} \times \vec{B} \Rightarrow \tau = MB\sin\theta$

$M = N\, i\, A$ and $\theta = 90°$ (for moving coil galvanometer)

$\therefore\ \tau = N\, i\, A\, B \sin 90° = N\, i\, A\, B$

$\tau = k\, i$ (given)

$\therefore\ k\, i = NiAB \Rightarrow k = NAB$

(b) Torsion constant, $C = \dfrac{\tau}{\theta} = \dfrac{NiAB}{\theta}$

Given, $i = i_0$, $\theta = \pi/2$

$\therefore \quad C = \dfrac{2N i_0 AB}{\pi}$ ... (i)

(c) $\int \tau dt = \int NiAB\, dt = NAB \int i\, dt \quad (\because \tau = NiAB)$
$= NABQ$

$\int \tau\, dt = I\omega$

$\therefore \quad I\omega = NABQ \Rightarrow \omega = \dfrac{NABQ}{I}$

Let us apply the law of energy conservation to find the angle of rotation.

Rotational kinetic energy of coil = potential energy at maximum deflection

$\dfrac{1}{2} I\omega^2 = \dfrac{1}{\lambda} C\theta^2$ max

$\dfrac{1}{2} I \left(\dfrac{NABQ}{I}\right)^2 = \dfrac{1}{2}\left(\dfrac{2NIAB}{\pi}\right)\theta^2_{max}$

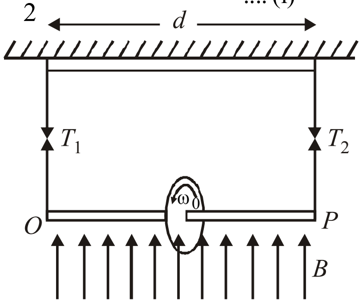
$\therefore \quad \theta_{max} = Q\sqrt{\dfrac{\pi BNA}{2I}}$

20. When the wheel is not rotating

Wt. of wheel = tension in string

$mg = 2T_0$

$\therefore \quad T_0 = \dfrac{mg}{2}$ .... (i)



When the wheel is rotating and magnetic field is applied magnetic moment

$M = iA = \dfrac{Q}{T} \times \pi r^2 = \dfrac{Q}{2\pi}\omega \times \pi R^2$

Now, the tensions in the strings will become unequal. Let the tensions in the strings be T_1 and T_2.

For translational equilibrium

$T_1 + T_2 = mg$ ... (ii)

Torque acting on the ring about the centre of ring

$\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = M \times B \times \sin 90°$

$\tau = \dfrac{Q}{2\pi}\omega \times \pi R^2 \times B = \dfrac{Q\omega BR^2}{2}$

For rotational equilibrium, the torque about the centre of ring should be zero.

$\therefore \quad T_1 \times \dfrac{d}{2} - T_2 \times \dfrac{d}{2} = \dfrac{Q\omega BR^2}{2}$

$\Rightarrow \quad T_1 - T_2 = \dfrac{Q\omega BR^2}{d}$ ... (iii)

From eq. (ii) and (iii),

$T_1 = \dfrac{mg}{2} + \dfrac{Q\omega BR^2}{2d}$

But the maximum tension or breaking tension $= \dfrac{3T_0}{2}$

$\therefore \quad \dfrac{3T_0}{2} = T_0 + \dfrac{Q\omega_{max} BR^2}{2d} \quad \left[\because T_0 = \dfrac{mg}{2}\right]$

$\therefore \quad \omega_{max} = \dfrac{dT_0}{BQR^2}$

21. Magnetic field produced by the circuit at its centre,

$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$

as straight part of the circuit will produce zero magnetic field at the centre.

$\therefore \quad B = \dfrac{\mu_0 i}{4r_1} + \dfrac{\mu_0 i}{4r_2}$

[Given $i = 10_A$, $r_1 = 0.08$ m and $r_2 = 0.12$ m]

or, $B = \dfrac{\mu_0}{4} \times 10 \times \left(\dfrac{1}{0.08} + \dfrac{1}{0.12}\right)$

$\therefore \quad B = (6.54 \times 10^{-5})\, T$ [$\perp$ to the paper outwards.] (from right hand thumb rule)

(b) Force acting on a current carrying conductor placed in a magnetic field

$\vec{F} = I(\vec{\ell} \times \vec{B}) = I\ell B \sin\theta$

For force acting on the wire at the centre

In this case $\theta = 180°$

$\therefore \quad F = I\ell B \sin 180° = 0$

On arc AC due to current at the centre

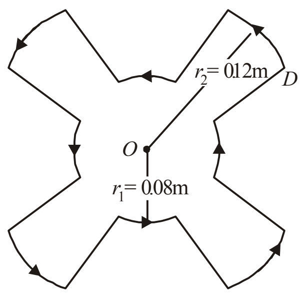
$|\vec{B}|$ on AC, $B = \dfrac{\mu_0 I}{2\pi r_1}$

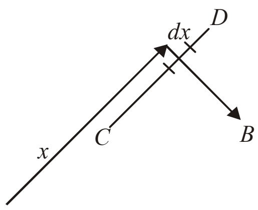
The direction of this magnetic field on any small segment of AC will be tangential

$\therefore \quad \theta = 180° \Rightarrow F = 0$

On segment CD.

Magnetic field at a distance x due to central wire $B = \dfrac{\mu_0 I}{2\pi x}$





Force on a small segment dx distant r from O.

$dF = I\, dxB$

$= 10 \times dx \times \dfrac{\mu_0 I}{2\pi x} = \dfrac{5\mu_0 I}{\pi}\dfrac{dx}{x}$

On integrating

$\therefore \quad F = \dfrac{5\mu_0 I}{\pi}\int_{r_1}^{r_2}\dfrac{dx}{x} \qquad \therefore \quad F = \dfrac{5\mu_0 I}{\pi}[\log_e x]_{r_1}^{r_2}$

$\therefore \quad F = \dfrac{5\mu_0 I}{\pi}\log_e \dfrac{r_2}{r_1} = \dfrac{5\mu_0 \times 10}{\pi}\log_e\left(\dfrac{0.12}{0.08}\right)$

$= 8.1 \times 10^{-6}$ N (inwards)

(By Fleming Left Hand rule).

22. (a) Magnetic field $(\vec{B})$ at the origin = magnetic field due to semicircle KLM + magnetic field due to other semicircle KNM.

$\therefore \ \vec{B} = \dfrac{\mu_0 I}{4R}(-\hat{i}) + \dfrac{\mu_0 I}{4R}(\hat{j})$

[$\because$ KLM lies in the y-z plane and KNM in the x-z plane]

or, $\vec{B} = -\dfrac{\mu_0 I}{4R}\hat{i} + \dfrac{\mu_0 I}{4R}\hat{j} = \dfrac{\mu_0 I}{4R}(-\hat{i}+\hat{j})$

$\therefore$ Magnetic force acting on the particle

$\vec{F} = q(\vec{v}\times\vec{B}) = q\left\{(-v_0\hat{i})\times(-\hat{i}+\hat{j})\times\dfrac{\mu_0 I}{4R}\right\}$

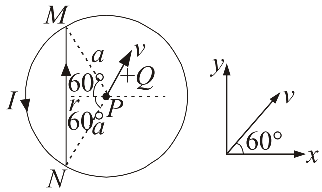
or, $F = \dfrac{-\mu_0 q v_0 I}{4R}\hat{k}$

(b) $\vec{F}_{KLM} = \vec{F}_{KNM} = \vec{F}_{KM}$

and $\vec{F}_{KM} = BI(2R)\hat{i} = 2BIR\hat{i}$

Therefore, $\vec{F}_1 = \vec{F}_2 = 2BIR\hat{i}$

$\therefore$ Total force on the loop, $\vec{F} = \vec{F}_1 + \vec{F}_2 \Rightarrow \vec{F} = 4BIR\hat{i}$

23. **(a)** 

(i) Current I through the arc MN and straight wire NM setup a magnetic field at the centre P.

The charged particle is acted upon by the magnetic force due to these fields. The magnetic force produces an instantaneous acceleration in the charged particle. Field due to arc MN.

$B_1 = \dfrac{1}{3}\dfrac{\mu_0 I}{2a}$ outwards

or $B_1 = \dfrac{\mu_0 I}{6a}$ outwards $= \dfrac{0.16\,\mu_0 I}{a}$ outwards

or $\vec{B}_1 = \dfrac{0.16\,\mu_0 I}{a}\hat{k}$

Field due to straight wire NM

$B_2 = \dfrac{\mu_0 I}{4\pi r}(\sin 60° + \sin 60°)$, where $r = a\cos 60°$

or $B_2 = \dfrac{\mu_0}{4\pi}\dfrac{I}{a\cos 60°}(2\sin 60°)$

or $B_2 = \dfrac{\mu_0 I}{2\pi a}\times\sqrt{3}$ (inwards)

or $\vec{B}_2 = -\dfrac{0.27\,\mu_0 I}{a}\hat{k}$

$\therefore \ \vec{B}_{net} = \vec{B}_1 + \vec{B}_2$

or $\vec{B}_{net} = -\dfrac{0.11\,\mu_0 I}{a}\hat{k}$

$\vec{v} = v\cos 60°\ \hat{i} + v\sin 60°\ \hat{j}$

or $\vec{v} = \dfrac{v}{2}\hat{i} + \dfrac{\sqrt{3}v}{2}\hat{j}$

$\therefore$ Magnetic force

$\therefore \ \vec{F}_m = Q(\vec{v}\times\vec{B}_{net}) = \dfrac{0.11\mu_0 IQv}{2a}(\hat{j}-\sqrt{3}\hat{i})$

$\therefore$ Acceleration mass $\vec{a} = \dfrac{\vec{F}_m}{m}$

or $\vec{a} = \dfrac{0.11\mu_0 IQv}{2ma}\left(\hat{j}-\sqrt{3}\hat{i}\right)$

(ii) Force and torque on the loop:

In uniform magnetic field $\vec{B} = B\hat{i}$, force on current loop is zero.

Torque on loop $= \vec{M}\times\vec{B}$

$\vec{M} = (IA)\hat{k}$

or $A = \dfrac{\pi a^2}{3} - \dfrac{a^2}{2}\sin 120°$ or $A = 0.61\,a^2$

$\therefore \ \vec{M} = (0.61\,Ia^2)\hat{k} \qquad \vec{B} = B\hat{i}$

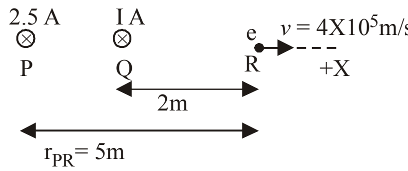
$\therefore \ \vec{\tau} = \vec{M}\times\vec{B}$ or $\vec{\tau} = (0.61\,Ia^2 B)(\hat{k}\times\hat{i})$

or $\vec{\tau} = (0.61 Ia^2 B)\hat{j}$

24. (i) Magnetic field due to current in wire P at R

$B_1 = \dfrac{\mu_0}{4\pi}\times\dfrac{2I_P}{r_{PR}} = \dfrac{\mu_0}{4\pi}\times\dfrac{2\times 2.5}{5} = \dfrac{\mu_0}{4\pi}$

[in the plane of paper downwards]



Similarly, the magnetic field due to current in wire Q at R

$B_2 = \dfrac{\mu_0}{4\pi}\times\dfrac{2\times I}{2} = \dfrac{\mu_0}{4\pi}I$ [in the plane of paper downwards]

Total magnetic field at R [due to P and Q]

$B = B_1 + B_2 = \dfrac{\mu_0}{4\pi} + \dfrac{\mu_0}{4\pi}I = \dfrac{\mu_0}{4\pi}(1+I)$

[in the plane of paper downwards]

Force experienced by the electron

$F = qvB\sin\theta$

$= \pi vB\sin 90° = 1.6\times 10^{-19}\times 4\times 10^5\times\dfrac{\mu_0}{4\pi}(1+I)$

$\therefore \ 3.2\times 10^{-20} = 1.6\times 10^{-19}\times 4\times 10^5\times 10^{-7}(1+I)$

or, $I = 4$ A

(ii) For net field at R to be zero, magnetic field due to third wire carrying a current of 2.5 A should be

$B = 10^{-7}(I+1) = 5\times 10^{-7}$ T

Let r be the distance of this wire from R

$\therefore \ 5\times 10^{-7} = \dfrac{\mu_0}{4\pi}\times\dfrac{2\times 2.5}{r}$ or, $r = 1$m

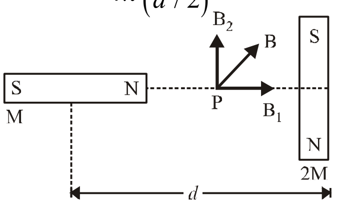
i.e., either left or right 1m from R direction of current outwards and inwards respectively.

Topic-1 : Magnetism, Gaus's Law, Magnetic Moment, Properties of Magnet

1. **(c)** Here, $\theta = 30°$, $\tau = 0.018$ N-m, $B = 0.06$ T
Torque on a bar magnet :
$\tau = MB\sin\theta$
$0.018 = M \times 0.06 \times \sin 30°$
$\Rightarrow 0.018 = M \times 0.06 \times \frac{1}{2} \Rightarrow M = 0.6 \text{ A-m}^2$
Position of stable equilibrium ($\theta = 0°$)
Position of unstable equilibrium ($\theta = 180°$)
Minimum work required to rotate bar magnet from stable to unstable equilibrium
$\Delta U = U_f - U_i = -MB\cos 180° - (-MB\cos 0°)$
$W = 2MB = 2 \times 0.6 \times 0.06$
$\therefore\ W = 7.2 \times 10^{-2}$ J

2. **(a)** Given,
Moment of inertia of circular coil, $I = 0.8$ kg m^2
Magnetic moment of circular coil, $M = 20$ Am^2
Rotational kinetic energy of circular coil,
$\text{KE} = \frac{1}{2} I\omega^2$
Here, ω = angular speed of coil
Potential energy of bar magnet $= -MB\cos\phi$
From energy conservation
$\frac{1}{2} I\omega^2 = U_{\text{in}} - U_f = -MB\cos 60° - (-MB)$
$\Rightarrow \frac{MB}{2} = \frac{1}{2} I\omega^2$
$\Rightarrow \frac{20 \times 4}{2} = \frac{1}{2}(0.8)\omega^2$
$\Rightarrow 100 = \omega^2 \Rightarrow \omega = 10$ rad

3. **(b)** $B_1 = \frac{\mu_0}{4\pi}\frac{2K}{(d/2)^3}$



and $B_2 = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$

$\therefore\ \tan\theta = \frac{B_2}{B_1} = \frac{\frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}}{\frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}} = 1$

or $\theta = 45°$

The resultant field is 45° from B_1. The angle between $\vec{B}$ and $\vec{v}$ zero, so force on the particle is zero.

4. **(c)**
Work done, $W = 2\,m{\cdot}B$
$= 2 \times 10^{-2} \times 1\cos(0.125) = 0.02$ J

5. **(b)** We know that, magnetic dipole moment
$M = NiA\cos\theta$ i.e., $M \propto \cos\theta$
When two magnetic fields are inclined at an angle of 75° the equilibrium will be at 30°, so
$\cos\theta = \cos(75° - 30°) = \cos 45° = \frac{1}{\sqrt{2}}$
$\frac{x}{\sqrt{2}} = \frac{15}{2}$ \ x » 11

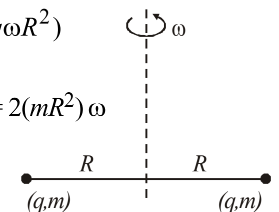
6. **(c)** $\vec{M}$ (mag. moment/volume) $= \frac{NiA}{A\ell}$
$= \frac{Ni}{\ell} = \frac{(500)15}{25 \times 10^{-2}} = 30000 \text{ Am}^{-1}$

7. **(a)** Potential energy, $U = -\vec{M}.\vec{B} = -MB\cos\theta$
In case I, $\theta = 180°$, $U = +MB$
In case II, $\theta = 90°$, $U = 0$
In case III, θ = acute, U = + ve (less than $+MB$)
In case IV, θ = obtuse, U = – ve
$\therefore$ I > III > II > IV.

8. **(d)** Magnetic lines of force form closed loops. Inside a magnet, these are directed from south to north pole and outside the magnet from north to south.

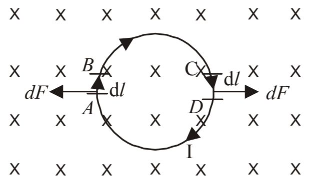
9. **(c)** The angular momentum of the particle
$L = mvr = mr^2\omega$ where $\omega = 2\pi n$. [$\because v = r\omega$]
$\therefore$ Frequency $n = \frac{\omega}{2\pi}$; Further $i = \frac{q}{t} = q \times n = \frac{\omega q}{2\pi}$
Magnetic moment, $M = iA = \frac{\omega q}{2\pi} \times \pi r^2 \Rightarrow M = \frac{\omega q r^2}{2}$
$\therefore\ \frac{M}{L} = \frac{\omega q r^2}{2mr^2\omega} = \frac{q}{2m}$

10. **(a)** Current, $i = \frac{q}{t}$ = (frequency)(charge) $= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$
Magnetic moment,
$M = (i)(A) = \left(\frac{q\omega}{\pi}\right)(\pi R^2) = (q\omega R^2)$



Angular momentum, $L = 2I\omega = 2(mR^2)\omega$
$\therefore\ \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$

11. **(c)** In a uniform magnetic field, magnetic force on a current carrying loop is zero. The magnetic field B_0 is perpendicular to the plane of the paper. Let us consider two diametrically opposite elements. By Fleming's left hand rule, on element AB force
$dF = I(d\ell)B\sin 90° = I(d\ell)B$ (leftwards)



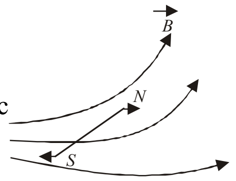
On element CD, force

$dF = I(d\ell)B$ [Right on the plane of paper]

These two forces will cancel out.

Similarly, all forces acting on the diametrically opposite elements will cancel out in pair. Hence the net force acting on the loop will be zero.

12. **(a)** The force on north pole $= m\vec{B}_1$



The force on south pole $= m\vec{B}_2$

Since the two poles of the magnetic needle will be acted upon by different forces in different directions therefore, the magnetic needle experiences a force as well as a torque.

13. $i = \frac{q}{t} = \frac{ne}{t} = ve = 10^{16} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-3}$ A.

Magnetic moment, $M = i \times A = i \times \pi r^2$

$= 1.6 \times 10^{-3} \times 3.14 \times 0.5 \times 10^{-10} \times 0.5 \times 10^{-10}$

$= 1.25 \times 10^{-23}$ Am^2

14. Wire of length L is bent in the form of a circle,

$\therefore$ Perimeter of the circle = length of wire

$$\Rightarrow 2\pi r = L \quad \Rightarrow r = \frac{L}{2\pi}$$

$\therefore$ Area of the circle $= \pi r^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$

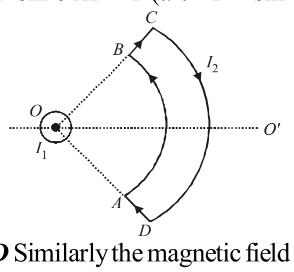
Magnetic moment of a loop in which current i flows

or, $M = iA = \frac{iL^2}{4\pi}$.

15. **(a, c)**

Force on *AB* The magnetic field due to current I_1 is along AB.

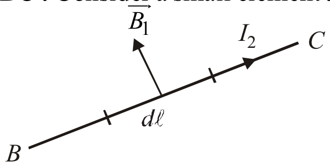
$\therefore$ $F = Id\ell B \sin\theta\, AB = I\,(d\,\ell \times B \times \sin 0°) = 0$



Force on *CD* Similarly the magnetic field due to current I_1 is along DC. Because $\theta = 180°$ here, therefore force on DC is zero.

$F_{CD} = Id\ell B \sin\theta = Id\ell \sin 180° = 0$

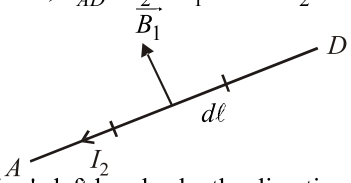
Force on *BC* : Consider a small element $d\ell$.



$F_{BC} = I_2 d\ell B_1 \sin 90° \Rightarrow dF = I_2 d\ell B_1$

By Fleming's left hand rule, the direction of this force is perpendicular to the plane of the paper directed outwards.

Force on *AD*, $F_{AD} = I_2 d\ell B_1 \sin 90°\ I_2 d\ell B_1$



By Fleming's left hand rule, the direction of this force is perpendicular to the plane of paper directed inwards. Since the current elements are located symmetrical to current I_1, therefore force on BC will cancel out the effect of force on AD.

Hence net force on loop $ABCD$, $F_{net} = 0$

The force on BC and AD will create a torque on $ABCD$ in clockwise direction about OO' axis as seen by the observer at O.

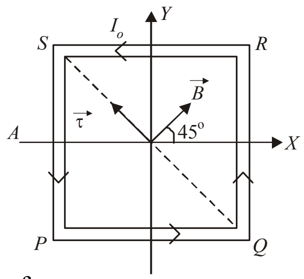
16. **(c, d)**

Only induced electric field and magnetostatic field form closed loops of field lines.

17. (a) As the magnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis, in $x-y$ plane components of the magnetic field

$B_x = B\cos 45° = B/\sqrt{2}$ and

$B_y = B\sin 45° = B/\sqrt{2}$



So, in vector from

$$\vec{B} = \hat{i}\left(\frac{B}{\sqrt{2}}\right) + \hat{j}\left(\frac{B}{\sqrt{2}}\right) \text{ and } \vec{M} = (iA)\hat{k} = I_0 L^2 \hat{k}$$

Therefore torque acting on the loop.

$$\vec{\tau} = \vec{M} \times \vec{B} = I_0 L^2 \hat{k} \times \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right) = \frac{I_0 L^2 B}{\sqrt{2}}(\hat{j} - \hat{i})$$

$I_0^2 L^2 B$ and is directed along line QS from Q to S.

(b) Here the torque is in $\hat{j} - \hat{i}$ direction *i.e.*, parallel to QS. Hence the loop will rotate about an axis passing through Q and S. According to the theorem of perpendicular axes, moment of inertia of the frame about QS.

$$I_{QS} = \frac{1}{2}I_z = \frac{1}{2}\left(\frac{4}{3}ML^2\right) = \frac{2}{3}ML^2$$

Also, torque $\tau = I\alpha$,

$$\therefore \quad \alpha = \frac{\tau}{I} = \frac{I_0 L^2 B \times 3}{2ML^2} = \frac{3}{2}\frac{I_0 B}{M}$$

Here α is constant, therefore from

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \text{ here } \omega_0 = 0,$$

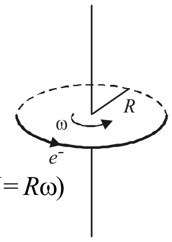
$$\therefore \quad \theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{3I_0 B}{2M}\right)(\Delta t)^2$$

or $\theta = \frac{3}{4}\frac{I_0 B}{M}(\Delta t)^2$

18. (i) Magnetic moment $M = IA$
where I current due to orbital motion of electron and A = area of loop made by electron.

$$M = IA = \frac{e}{T} \times \pi R^2 \Rightarrow M = \frac{e\omega}{2\pi} \times \pi R^2 \left(\because 1 = \frac{e}{\sigma}\right)$$

$$\therefore \quad M = \frac{1}{2} e\omega R^2$$



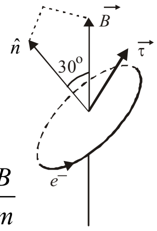
But according to Bohr's theory
$mvx = nh/2\pi$

$$\Rightarrow \; mR\omega^2 = \frac{nh}{2\pi} \Rightarrow R\omega^2 = \frac{nh}{2\pi m} \quad (\because V = R\omega)$$

$$\therefore M = \frac{e}{2} \times \frac{nh}{2\pi m} = \frac{nhe}{4\pi m} = \frac{eh}{4\pi m} \; (\because n = 1 \text{ for ground state})$$

Direction of magnetic momentum m perpendicular to the plane of orbit.

(ii) We know that torque



$\vec{\tau} = \vec{M} \times \vec{B} \;\; \Rightarrow \tau = MB \sin\theta$

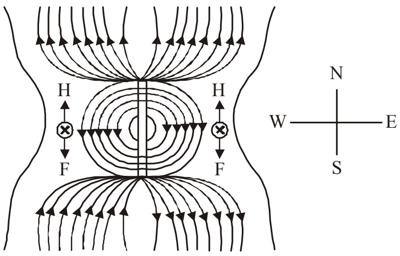
where θ is the angle between M and B

$\theta = 30°$

$$\therefore \quad \tau = MB \sin\theta = \frac{he}{4\pi m} \times B \sin 30° = \frac{heB}{8\pi m}$$

Direction of torque can be found by right hand thumb rule. The direction of torque is perpendicular to both M and B.

19.



20. Here, $P = m \times 2\ell = 14.4 \quad \Rightarrow \quad m = \dfrac{14.4}{0.25} = 57.6 \text{ A-m}^2$

(Given: $2\ell = 25$ cm)

Torque due to magnetic field

$$= p_m \times B \times \sin 60° = 14.4 \times 0.25 \times \frac{\sqrt{3}}{2}$$

Torque due to the force $= F \times 0.12$

For equilibrium $F \times 0.12 = 14.4 \times 0.25 \times \dfrac{\sqrt{3}}{2}$

$\Rightarrow F = 25.98$ N

If the force F is removed, then the torque due to magnetic field will move the bar magnet. It will start oscillating about the mean position where the angle between $\vec{p}_m$ and $\vec{B}$ is zero.

Topic-2 : The Earth Magnetism, Magnetic Materials and their Properties

1. **(b)** Given,

Volume of iron rod, $V = 10^{-3}$ m^3

Relative permeability, $\mu_r = 1000$

Number of turns per unit length, $n = 10$

Magnetic moment of an iron core solenoid,

$$M = (\mu_r - 1) \times NiA$$

$$\Rightarrow M = (\mu_r - 1) \times Ni\frac{V}{l} \Rightarrow M = (\mu_r - 1) \times \frac{N}{l} iV$$

$$\Rightarrow M = 999 \times \frac{10}{10^{-2}} \times 0.5 \times 10^{-3} = 499.5 \approx 500.$$

2. **(d)** For paramagnetic material. According to curies law

$$\chi \propto \frac{1}{T}$$

For two temperatures T_1 and T_2

$$\chi_1 T_1 = \chi_2 T_2$$

But $\chi = \dfrac{I}{B}$

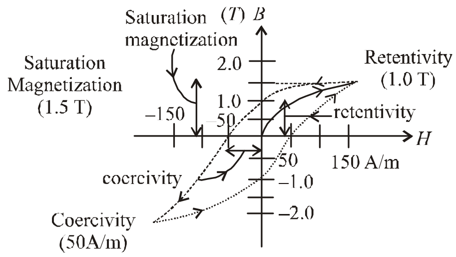
$$\therefore \frac{I_1}{B_1} T_1 = \frac{I_2}{B_2} T_2$$

$$\Rightarrow \frac{6}{0.4} \times 4 = \frac{I_2}{0.3} \times 24 \Rightarrow I_2 = \frac{0.3}{0.4} = 0.75 \text{ A/m}$$

3. **(b)** When magnetic field is applied to a diamagnetic substance, it produces magnetic field in opposite direction so net magnetic field inside the cavity of sphere will be zero. So, field inside the paramagnetic substance kept inside the cavity is zero.

4. **(d)** Permanent magnets (P) are made of materials with large retentivity and large coercivity. Transformer cores (T) are made of materials with low retentivity and low coercivity.

5. **(d)**



6. **(a)** According to Curie law for paramagnetic substance,

$$\chi \propto \frac{1}{T_C} \Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8 \times 10^{-4}}{\chi_2} = \frac{300}{350}$$

$$\chi_2 = \frac{2.8 \times 350 \times 10^{-4}}{300} = 3.266 \times 10^{-4}$$

7. **(d)** Magnetic susceptibility,

$$\chi = \frac{I}{H}$$

where, $I = \dfrac{\text{Magnetic moment}}{\text{Volume}} = \dfrac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ N/m}^2$

Now, $\chi = \dfrac{20}{60 \times 10^3} = \dfrac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$

8. **(b)** Corecivity, $H = \dfrac{B}{\mu_0}$ and $B = \mu_0 ni \left(n = \dfrac{N}{\ell}\right)$

or, $H = \dfrac{N}{\ell} i = \dfrac{100}{0.2} \times 5.2 = 2600$ A/m

9. **(b)** Given Number of turns,
n = 1000 turns/cm = 1000 × 100 turns/m
Coercivity of ferromagnet, H = 100 A/m
Current to demagnetise the ferromagnet, I = ?
Using, H = nI
or, $100 = 10^5 \times I$

$\therefore \quad I = \frac{100}{10^5} = 1\ \text{mA}$

10. **(b)** Graph [A] is for material used for making permanent magnets (high coercivity)
Graph [B] is for making electromagnets and transformers.

11. **(d)** $V_B = VB_H l = 240 \times 5 \times 10^{-5} \cos(\theta) \times 5 = 44.7$ mv
By right hand rule, the charge moves to the left of pilot.

12. **(c)** Magnetic field in solenoid $B = \mu_0 ni$

$\Rightarrow \frac{B}{\mu_0} = ni$ (Where n = number of turns per unit length)

$\Rightarrow \frac{B}{\mu_0} = \frac{Ni}{L} \Rightarrow 3 \times 10^3 = \frac{100i}{10 \times 10^{-2}}$

$\Rightarrow i = 3\text{A}$

13. **(b)** Diamagnetic materials are repelled in an external magnetic field.
Bar B represents diamagnetic materials.

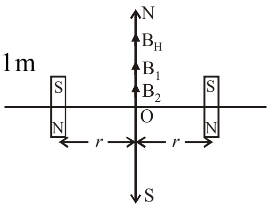
14. **(d)** In magnetic dipole

Force $\propto \frac{1}{r^4}$

In the given question,
Force $\propto x^{-n}$
Hence, n = 4

15. **(b)** **Given :** $M_1 = 1.20\ Am^2$

$M_2 = 1.00\ Am^2;\ r = \frac{20}{2} cm = 0.1\text{m}$



$B_{net} = B_1 + B_2 + B_H$

$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$

$= \frac{10^{-7}(1.2+1)}{(0.1)^3} + 3.6 \times 10^{-5} = 2.56 \times 10^{-4}\ \text{wb/m}^2$

16. **(a)** Given $M = 8 \times 10^{22}\ \text{Am}^2$
$d = R_e = 6.4 \times 10^6$m

Earth's magnetic field, $B = \frac{\mu_0}{4\pi}.\frac{2M}{d^3}$

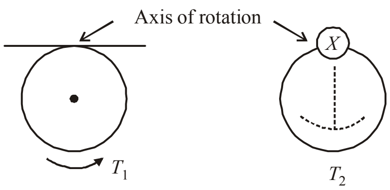
$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 8 \times 10^{22}}{(6.4 \times 10^6)^3} \cong 0.6$ Gauss

Topic-3 : Magnetic Equipment

1. **(a)** Let I_1 and I_2 be the moment of inertia in first and second case respectively.

$I_1 = 2MR^2$

$I_2 = MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$



Time period, $T = 2\pi\sqrt{\frac{I}{mgd}}$

$T \propto I$

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$

2. **(Bonus)** We have, $T = 2\pi\sqrt{\frac{I}{MB_x}}$

$\therefore \frac{T_1^2}{T_2^2} = \frac{Bx_2}{Bx_1}$

or $\left(\frac{2}{1.5}\right)^2 = \frac{B_2 \cos 45°}{B_1 \cos 30°} = \frac{B_2 \times 2}{\sqrt{2} \times B_1 \times \sqrt{3}}$

$\left(\frac{4}{3}\right)^2 = \frac{B_2}{B_1} \times \frac{2}{\sqrt{6}}$

$\therefore \frac{B_1}{B_2} = \frac{9}{8\sqrt{6}} = 0.46$

3. **(a)** Using, time /oscillation period,

$T = 2\pi\sqrt{\frac{I}{MB}}$

Where, M = magnetic moment, I moment of inertia and B = magnetic field

$T_h = 2\pi\sqrt{\frac{mR^2}{(2MB)}}$

$T_c = 2\pi\sqrt{\frac{1/2mR^2}{MB}}$

Clearly, $T_h = T_c$

4. **(c)** Given : Magnetic moment, $M = 6.7 \times 10^{-2}\ Am^2$
Magnetic field, B = 0.01 T
Moment of inertia, $I = 7.5 \times 10^{-6}$ Kgm^2

Using, $T = 2\pi\sqrt{\frac{I}{MB}}$

$= 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = \frac{2\pi}{10} \times 1.06\,s$

Time taken for 10 complete oscillations
$t = 10T = 2\pi \times 1.06$
$= 6.6568 \approx 6.65$ s

Topic-4 : Miscellaneous Problems

1. **(a)** It is clear from the given graph that with increase of the magnitude of magnetic field (B), the critical temperature $T_C(B)$ decreases.
If $B_2 > B_1$. then for B_2, the temperature at which the resistance becomes zero should be less.

2. **(b)** Given at B = 0, T_C = 100 K
and at B = 7.5T, T_C = 75 K
With increase in B T_C decreases
$\therefore$ At B = 5T, T_C should be between 75 K and 100 K.

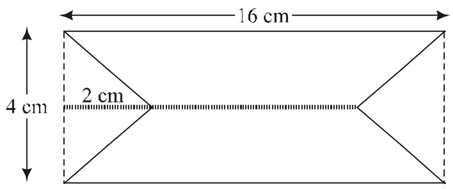
20 Electromagnetic Induction

Topic-1 : Magnetic Flux, Faraday's and Lenz's Law

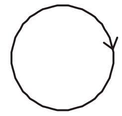
1. **(b)** **Case (a) :** When bar magnet is entering with constant speed, flux (ϕ) will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.
Case (b) : When magnet is completely inside, flux (ϕ) will not change, so galvanometer will show null deflection.
Case (c) : When bar magnet is making on exit, again flux (ϕ) will change and an e.m.f. is induced in opposite direction so galvanometer will deflect in negative direction i.e. reverse direction.

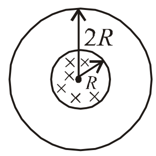
2. **(a)** As we know, emf $\varepsilon = NAB\omega \cos \omega t$, Here $N = 1$
Average power,
$$<P> = <\frac{\varepsilon^2}{R}> = <\frac{A^2B^2\omega^2\cos^2\omega t}{R}> = \frac{A^2B^2\omega^2}{R}\left(\frac{1}{2}\right)$$
Therefore average power loss in the loop due to Joule heating
$$<P> = \frac{\pi^2a^2b^2B^2}{2R}(\omega^2)$$

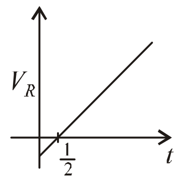
3. **(b)** Given,
Length of wire, $l = 30$ cm
Radius of wire, $r = 2$ mm $= 2 \times 10^{-3}$ m
Resistivity of metal wire, $\rho = 1.23 \times 10^{-8}\,\Omega$m
Emf generated, $|e| = \frac{d\phi}{dt} = \frac{dB}{dt}(A)$ $\quad (\because \phi = B.A.)$
Current, $i = \frac{e}{R}$
But, resistance of wire, $R = \rho\frac{l}{A}$
$$\therefore i = \left|\frac{dB}{dt}\right|\frac{(A)^2}{\rho l} = \frac{0.032 \times \{\pi \times 2 \times 10^{-3}\}^2}{1.23 \times 10^{-8} \times 0.3} = 0.61 \text{ A.}$$

4. **(a)** According to question, $dB = 1000 - 500 = 500$ gauss $= 500 \times 10^{-4}T$
Time $dt = 5$ s
Using faraday law
Induced EMF, $e = \left|-\frac{d\phi}{dt}\right| = \left|A\frac{dB}{dt}\right|$
$$\frac{dB}{dt} = \frac{1000-500}{5} \times 10^{-4} = 10^{-2} \text{ T/sec}$$


Area, A = ar of ▭ -2 ar of $\Delta = (16 \times 4 - 2 \times \text{Area of triangle})\text{ cm}^2$
$$= \left(64 - 2 \times \frac{1}{2} \times 2 \times 4\right)\text{cm}^2 = 56 \times 10^{-4} \text{ m}^2$$
$$\therefore \varepsilon_{induced} = \left|A\frac{dB}{dt}\right| = 56 \times 10^{-4} \times 10^{-2} = 56 \times 10^{-6}V = 56\mu V$$

5. **(d)** As magnetic field lines form close loop, hence every magnetic field line creating magnetic flux through the inner region (ϕ_i) must be passing through the outer region. Since flux in two regions are in opposite region.

$\therefore \quad \phi_i = -\phi_0$

6. **(d)** According to question,


$I(t) = I_0t(1-t)$
$\therefore I = I_0t - I_0t^2$
$\phi = B.A$
$\phi = (\mu_0\, nI) \times (\pi R^2)$
$(\because\ B = \mu_0 \text{ nI } and\ A = \pi R^2)$
$$V_R = \frac{-d\phi}{dt}$$
$$V_R = \mu_0 n\pi R^2(I_0 - 2I_0t)$$
$$\Rightarrow V_R = 0 \text{ at } t = \frac{1}{2}s$$


7. **(b)** We have given, time period, $T = 10$s
$\therefore$ Angular velocity, $\omega = \frac{2\pi}{10} = \frac{\pi}{5}$
Magnetic flux, $\phi(t) = BA \cos \omega t$
Emf induced, $E = \frac{-d\phi}{dt} = BA\omega \sin \omega t = BA\omega\sin(\omega t)$
Induced emf, $|\varepsilon|$ is maximum when $\omega t = \frac{\pi}{2}$
$$\Rightarrow \quad t = \frac{\frac{\pi}{2}}{\frac{\pi}{5}} = 2.5 \text{ s}$$
For induced emf to be minimum i.e zero
$$\omega t = \pi \quad \Rightarrow \quad t = \frac{\pi}{\frac{\pi}{5}} = 5\ s$$
$\therefore$ Induced emf is zero at t = 5 s

8. **(a)** $Q = BA$
$= (\mu_0\, n_i)A$
$= \mu_0\, n\,(kt\, e^{-\alpha t})A$
$$e = -\frac{dQ}{dt} = -\mu_0 nAk\frac{d}{dt}(te^{-\alpha t})$$
$$= -\mu_0 nAk[t(-1)e^{-\alpha t} + e^{-\alpha t} \times 1]$$
$$= -\mu_0 nAk[e^{-\alpha t}(1-t)]$$
$$i = \frac{e}{R} = \frac{-\mu_0 nAk}{R}[e^{-\alpha t}(1-t)]$$
At $t = 0$, $i \Rightarrow -$ve

9. **(b)** According to faraday's law of electromagnetic induction, $e = \frac{-d\phi}{dt}$
$$L \times \frac{di}{dt} = 25 \Rightarrow L \times \frac{15}{1} = 25 \text{ or } L = \frac{5}{3}\text{H}$$

Change in the energy of the inductance,

$$\Delta E = \frac{1}{2}L\left(i_1^2 - i_2^2\right) = \frac{1}{2}\times\frac{5}{3}\times(25^2 - 10^2)$$

$$= \frac{5}{6}\times 525 = 437.5\,J$$

10. **[Bonus]**

Net charge $Q = \frac{\Delta\phi}{R} = \frac{1}{10}A(B_f - B_i) = \frac{1}{10}\times 3.5\times 10^{-3}$

$$\left(0.4\sin\frac{\pi}{2} - 0\right)$$

$$= \frac{1}{10}(3.5\times 10^{-3})(0.4 - 0) = 1.4\times 10^{-4}$$

No option matches, So it should be a bonus.

11. **(a)** According to Faraday's law of electromagnetic induction, $\varepsilon = \frac{d\phi}{dt}$

Also, $\varepsilon = iR$

$$\therefore\quad iR = \frac{d\phi}{dt} \quad\Rightarrow\quad \int d\phi = R\int i\,dt$$

Magnitude of change in flux ($d\phi$) = R × area under current vs time graph

or, $d\phi = 100\times\frac{1}{2}\times\frac{1}{2}\times 10 = 250$ Wb

12. **(b)** Electric flux is given by

$\phi = B.A$

$$\phi = B_0\pi r^2 e^{-t/\tau} \qquad \left(Q\ B = B_0 e^{-t/t}\right)$$

Induced E.m.f. $\varepsilon = \frac{d\phi}{dt} = \frac{B_0\pi r^2}{\tau^2}e^{-t/\tau}$

$$\text{Heat} = \int_0^\infty \frac{\varepsilon^2}{R} = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

13. **(d)** According to Faraday's law of electromagnetic induction,

Induced emf, $e = \frac{Ldi}{dt}$

$$50 = L\left(\frac{5-2}{0.1\text{sec}}\right) \Rightarrow L = \frac{50\times 0.1}{3} = \frac{5}{3} = 1.67\ H$$

14. **(a)** Inside the sphere field varies linearly i.e., $E \propto r$ with distance and outside varies according to $E \propto \frac{1}{r^2}$

Hence the variation is shown by curve (a)

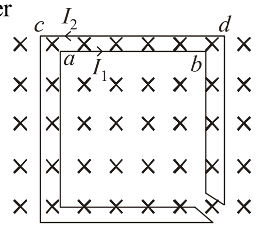
15. **(a)** As we know, Magnetic flux, $\phi = B.A$

$$\frac{\mu_0(2)(20\times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]}\times\pi(0.3\times 10^{-2})^2$$

On solving

$= 9.216\times 10^{-11} = 9.2\times 10^{-11}$ weber

16. **(d)** The magnetic field is increasing in the downward direction. Therefore, according to Lenz's law the current I_1 will flow in the direction ab and I_2 in the direction dc.



17. **(c)** Polarity of emf will be opposite when the magnet enters and leaves the coil.

Only graph (c) shows these characteristic.

18. **(b)** Current induced $i = \frac{|e|}{R} \Rightarrow i = \frac{1}{R}\frac{d\phi}{dt}$

But $i = \frac{dq}{dt}$

$$\therefore\quad \frac{dq}{dt} = \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \int dq = \frac{1}{R}\int d\phi \ \therefore q = \frac{BA}{R}$$

19. **(4)** Time constant, T = RC

Impedance $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Given $Z = R\sqrt{1.25}$

$$\therefore\quad R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\therefore RC = \frac{2}{\omega} = \frac{2}{500}\times 1000\ \text{ms} \quad \therefore RC = 4\ \text{ms}$$

20. **(5)**

For coil C_1, No. of turns $N_1 = 500$ and radius, $r = 1$ cm.

For coil C_2, No. of turns $N_2 = 200$ and radius, $R = 20$ cm

$$I = (5t^2 - 2t + 3) \Rightarrow \frac{dI}{dt} = (10t - 2)$$

$$\phi_{small} = BA = \left(\frac{\mu_0 I N_2}{2R}\right)(\pi r^2)$$

Induced emf in small coil,

$$e = \frac{d\phi}{dt} = \left(\frac{\mu_0 N_2}{2r}\right)\pi r^2 N_1\frac{di}{dt} = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right)(10t - 2)$$

At $t = 1$ s

$$e = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right)8 = 4\frac{\mu_0 N_1 N_2 \pi r^2}{R}$$

$$= \frac{4(4\pi)10^{-7}\times 200}{20}\times 500\times\frac{10^{-4}}{10^{-2}}\pi$$

$$= 80\times\pi^2\times 10^{-7}\times 10\times 10^2\times 10^{-2}$$

$$= 8\times 10^{-4}\ \text{volt} = 0.8\ \text{mV} = \frac{4}{x} \Rightarrow x = 5.$$

21. **(15)**

Here, $B = 3.0\times 10^{-5}$ T, $R = 10$ cm $= 0.1$ m

$$\omega = \frac{2\pi}{2T} = \frac{\pi}{0.2}$$

Flux as a function of time $\phi = \vec{B}\cdot\vec{A} = AB\cos(\omega t)$

Emf induced, $e = \frac{-d\phi}{dt} = AB\omega\sin(\omega t)$

Max. value of Emf $= AB\omega = \pi R^2 B\omega$

$$= 3.14\times 0.1\times 0.1\times 3\times 10^{-5}\times\frac{\pi}{0.2}$$

$$= 15\times 10^{-6}\ V = 15\ \mu V$$

22. **(10)** Given $dI = 0.25 - 0 = 0.25$ A

$dt = 0.025$ ms

Induced voltage

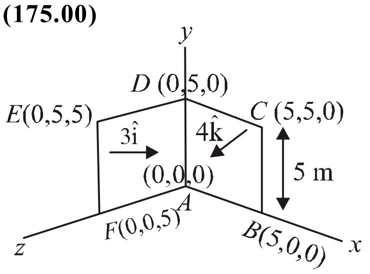
$E_{ind} = 100$ v

Self-inductance, L = ?

Using, $E_{\text{ind}} = \dfrac{\Delta\phi}{\Delta t} \quad \Rightarrow 100 = \dfrac{L(0.25-0)}{.025\times10^{-3}}$

$\Rightarrow L = 10^{-3}$ H = 10 mH

23. **(175.00)**



Flux through the loop ABCDEFA,

$\phi = \vec{B}.\vec{A} = (3\hat{i}+4\hat{k}).(25\hat{i}+25\hat{k})$

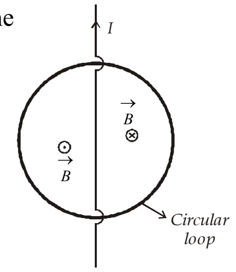
$\Rightarrow \phi = (3\times 25) + (4\times 25) = 175$ weber

24. When the source is switched off, the current left to right decreases to zero. The induced current opposes the cause or change as per Lenz's law. Therefore, the induced current will be from left to right.

25. **False.** The coil of metal wire is kept stationary in a non-uniform magnetic field. And for induced emf to develop in a coil, the magnetic flux through the coil must change. But in this case magnetic flux linked with the coil is not changing.

26. **True.** when the wire is in motion, the electrons have a net velocity in the direction of motion.
A charged particle moving in a magnetic field experiences a force $\vec{F} = q(\vec{v}\times\vec{B})$.
Here also each electron experiences a force and therefore, electrons will move towards one end creating an emf between the two ends of a straight copper wire.

27. **(a, c)** If the current is constant, the emf induced in the loop zero. Emf will be induced in the circular wire loop when flux through it changes with time.



$e = -\dfrac{\Delta\phi}{\Delta t}$

when the current is constant, the flux changing through it will be zero.
Also, if the current decreases at steady rate, the emf. induced in the loop is zero.
When the current is decreasing at a steady rate then the change in the flux (decreasing inwards) on the right half of the wire is equal to the change in flux (decreasing outwards) on the left half of the wire such that $\Delta\phi$ through the circular loop is zero.

28. **(a)** To find the relation among $i, \dfrac{di}{dt}$ and $\dfrac{d\phi}{dt}$
Applying Kirchhoff's second law,

$$\frac{d\phi}{dt} - iR - L\frac{di}{dt} = 0$$

or $\dfrac{d\phi}{dt} = iR + L\dfrac{di}{dt}$ ...(i)

(b) Expression for the net charge through R from $t = 0$ to $t = T$.

$$d\phi = iRdt + Ldi \left(\text{from eq.}(i)\right)$$

Integrating, we get

$$\Delta\phi = R\Delta q + Li_1$$

$\Delta q = \dfrac{\Delta\phi}{R} - \dfrac{Li_1}{R}$ ...(ii)

Here, $\Delta\phi = \phi_f - \phi_i = \displaystyle\int_{x=2x_0}^{x=x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x} l dx = \frac{\mu_0 I_0 l}{2\pi}\text{In}(2)$

So, from Eq. (ii) charge flown through the resistance upto time t = T, when current i_1, is

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\text{In}(2) - Li_1\right]$$

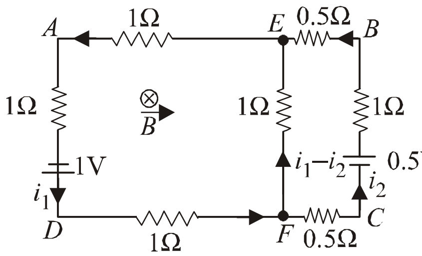
$i = i_0 e^{-t/\tau L}$

Here, $i = \dfrac{i_1}{4}, i_0 = i_1, t = (2T - T) = T$ and $\tau_L = \dfrac{L}{R}$

Substituting these values $i = i_0 e^{-t/\tau L}$

$\dfrac{i_0}{4} = i_0 e^{-T/(L/R)} \Rightarrow \tau_L = \dfrac{L}{R} = \dfrac{T}{In 4}$

29.



The entire circuit is placed in steadily increasing uniform magnetic field directed into the plane of paper and normal to it, an induced emf will be produced in the $AEFD$ as well as in the circuit $EBCF$ such that the current flowing in the loop creates magnetic lines of force in the upward direction (to the plane of paper).
Therefore, the current should flow in the anticlockwise direction in both the loops.
Induced emf in loop $AEFD$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA = -A\frac{dB}{dt} = -1\times1 = -1\text{V}$$

Induced emf in loop $EBCF$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA' = -A'\frac{dB}{dt} = -0.5\times1 = -0.5\text{ V}$$

Applying junction law at F, we get current in branch FE $(i_1 - i_2)$
Applying Kirchhoff 's law in loop $EADFE$

$-1\times i_1 - 1\times i_1 + 1 - 1\times i_1 - 1(i_1 - i_2) = 0$

$\Rightarrow \; 4i_1 - i_2 = 1$ ... (i)

Applying Kirchhoff 's law in loop $EBCFE$

$+0.5i_1 - 0.5 + 1i_2 + 0.5i_2 - 1(i_1 - i_2) = 0$

$-i_1 + 3i_2 = 0.5$ ... (ii)

Solving eq. (i) and (ii)

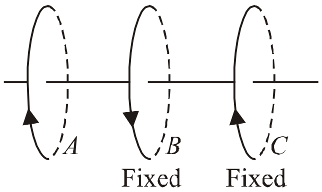
$11 i_1 = 3.5 \Rightarrow i_1 = 3.5/11 = \dfrac{7}{22}A$

Also $11 i_2 = 3 \Rightarrow i_2 = 3/11\,A = \dfrac{6}{22}A$

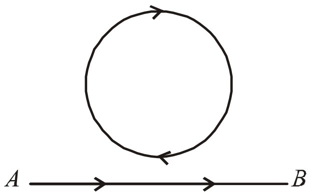
$\therefore$ Current in segment, $AE = i_1 = \dfrac{7}{22}A$; $BE = i_2 = \dfrac{6}{22}A$

and $EF = (i_1 - i_2) = \dfrac{1}{22}A$

30. Yes, When the coil A moves towards B, the number of magnetic lines of force passing through B changes. Therefore, an induced emf and hence induced current is produced in B.
The direction of current in B will be such as to oppose the field change in B and therefore, will be in the opposite direction of A.



31. The direction of induced current in the loop as shown below.
As the current increases, the number of magnetic lines of force passing through the loop increases in the outward direction. To oppose this change, the current will flow in the clockwise direction as per Lenz's law.



Topic-2 : Motional and Static EMI and Application of EMI

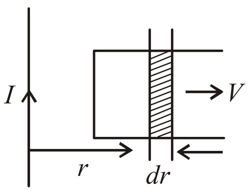
1. **(d)** Magnetic field at a distance r from the wire

$$B=\frac{\mu_0 I}{2\pi r}$$

Magnetic flux for small displacement dr,

$\phi = B\cdot A = Bldr$ [$\because$ $A = l\,dr$ and $B.A = BA\cos 0°$]

$$\Rightarrow \phi=\frac{\mu_0 I}{2\pi r}l\,dr$$



Emf, $e=\dfrac{d\phi}{dt}=\dfrac{\mu_0 Il}{2\pi r}\cdot\dfrac{dr}{dt}$

$$\Rightarrow e=\frac{\mu_0}{2\pi}\cdot\frac{Ivl}{r}$$

Induce current in the loop, $i=\dfrac{e}{R}=\dfrac{\mu_0}{2\pi}\cdot\dfrac{Ivl}{Rr}$

2. **(b)** Due to motion of magnet above the disc, the plate moves through the magnetic flux, due to which an EMF is generated in the plate and eddy currents are induced. These currents are such that it opposes the relative motion so disc will rotate in the same direction as the direction of magnet's motion.
This apparatus is called Arago's disk and the effect was discovered in 1824 by Arago.

3. **(b)** Induced emf,
$e = Bv\,\ell = 1\times 10^{-2}\times 0.05 = 5\times 10^{-4}$ V
Equivalent resistance,

$$R=\frac{4\times 2}{4+2}+1.7=\frac{4}{3}+1.7\simeq 3\,\Omega$$

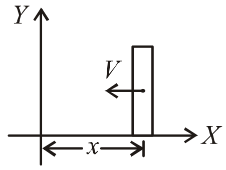
Current, $i=\dfrac{e}{R}=\dfrac{5\times 10^{-4}}{3}\simeq 170\,\mu$A

4. **(d)** Inductance $=\dfrac{\mu_0 N^2 A}{L}$

5. **(c)** Force on the strip when it is at stretched position x from mean position is

$$F=-kx-iIB=-kx-\frac{BIv}{R}\times IB$$

$$F=-kx-\frac{B^2I^2}{R}\times v$$



Above expression shows that it is case of damped oscillation, so its amplitude can be given by

$$\Rightarrow A=A_0e^{-\frac{bt}{2m}}$$

$$\Rightarrow \frac{A_0}{e}=A_0e^{-\frac{bt}{2m}}\quad \left[\text{as per question } A=\frac{A_0}{e}\right]$$

$$\Rightarrow t=\frac{2m}{\left(\frac{B^2I^2}{R}\right)}=\frac{2\times 50\times 10^{-3}\times 10}{0.01\times 0.01}$$

Given, $m = 50\times 10^{-3}$ kg
B = 0.1 T
l = 0.1 m
R = 10 Ω
k = 0.5 N

Time period, $T=2\pi\sqrt{\dfrac{m}{k}}\simeq 2$ s

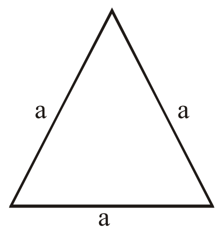
so, required number of oscillations,

$$N=\frac{10000}{2}=5000$$

6. **(a)** Induced emf, $\varepsilon = Bv\ell$
$= 0.3\times 10^{-4}\times 5\times 10 = 1.5\times 10^{-3}$ V

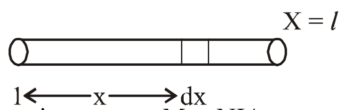
7. **(c)** As total length L of the wire will remain constant
L = (3a) N (N = total turns)
and length of winding = (d) N
(d = diameter of wire)
self inductance $=\mu_0 n^2 A\ell$

$$=\mu_0 n^2\left(\frac{\sqrt{3}a^2}{4}\right)dN$$



$\propto a^2 N \propto a$ [as $N = L/3a \Rightarrow N\propto \dfrac{1}{a}$]
Now 'a' increased to '3a'
So self inductance will become 3 times

8. **(c)**



Magnetic moment, M = NIA
$dQ = \rho\, dx$

$$dI=\frac{dQ}{2\pi}.\omega$$

dM = dI × A

$$=\frac{\omega}{2\pi}.\frac{\rho_0}{\ell}.x\pi x^2\,dx\qquad \Rightarrow M=\frac{\rho_0}{\ell}n\pi\int_0^{\ell}x^3\,dx$$

$$=\frac{\pi}{4}.n\rho\ell^3$$

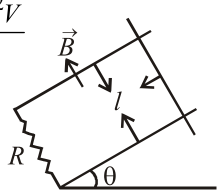
9. **(b)** From Faraday's law of electomagnetic induction,

$$e=\frac{d\phi}{dt}=\frac{d(BA)}{lt}=\frac{d(Bll)}{dt}=\frac{Bdl\times l}{dt}=BVl$$

Also, $F = ilB = \left(\frac{BV}{R}\right)(l^2B) = \frac{B^2l^2V}{R}$

At equilibrium

$mg\sin\theta = \frac{B^2lV}{R} \Rightarrow V = \frac{mgR\sin\theta}{B^2l^2}$

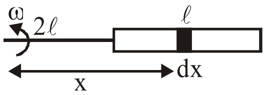


10. **(b)** In the given question,
Current flowing through the wire, I = 1A
Speed of the frame, v = 10 ms^{-1}
Side of square loop, l = 10 cm
Distance of square frame from current carrying wires x = 10 cm.
We have to find, e.m.f induced e = ?
According to Biot-Savart's law

$$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{x^2} = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{1\times10^{-1}}{\left(10^{-1}\right)^2} = 10^{-6}$$

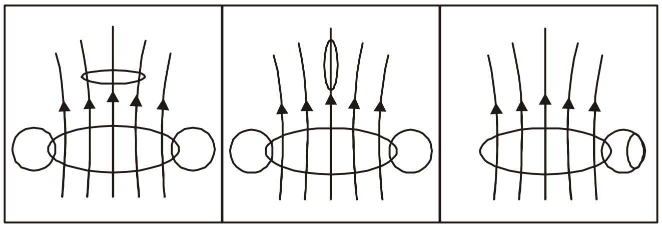
Induced e.m.f. e = Blv
$= 10^{-6} \times 10^{-1} \times 10 = 1\ \mu v$

11. **(d)** Here, induced e.m.f.

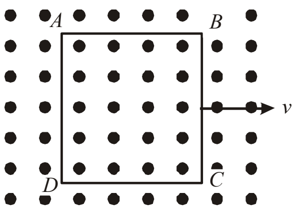


$$e = \int_{2\ell}^{3\ell}(\omega x)Bdx = B\omega\frac{[(3\ell)^2-(2\ell)^2]}{2} = \frac{5B\ell^2\omega}{2}$$

12. **(a)** When current flows in any of the coils, the flux linked with the other coil is maximum when surface area to receive flux is maximum. Clearly the flux linkage is maximum in case (a) due to the spatial arrangement of the two circular coils.



13. **(d)** Electric field will be induced, as *ABCD* moves, in both *AD* and *BC*. The metallic square loop moves in its own plane with velocity *v* in a uniform magnetic field perpendicular to the plane of the square loop. *AD* and *BC* are perpendicular to the velocity as well as perpendicular to field applied.



14. **(d)** Using current-time equation in L–R circuit,
$I = I_0\,(1 - e^{-t/\tau})$

But $I_0 = \frac{V}{R}$ and $\tau = \frac{L}{R}$

$\therefore\quad I = \frac{V}{R}(1-e^{-Rt/L}) = \frac{12}{6}\left[1-e^{-6t/8.4\times10^{-3}}\right] = 1$ (given)

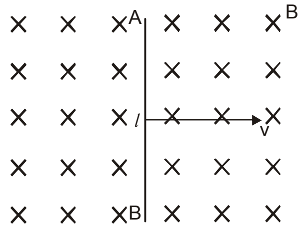
$\therefore\quad t = 0.97\times10^{-3}$ s ≈ 1ms

15. **(d)** Induced emf produced across *MNQ* will be same as the induced emf produced in straight wire *MQ*.

$\therefore\ e_{MNQ} = e_{MQ} = Bv\ell = Bv\times 2R$ with Q at higher potential.

16. **(b)** In this case there is an electric field in the rod.

A motional emf, e = Blv is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



17. **(b)** We know magnetic field due to a current flowing in a wire of finite length magnetic

$$B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$

Applying the above formula for *AB* for finding the magnetic field at centre *O*, of the loop

$$B = \frac{\mu_0 I}{4\pi(L/2)}(\sin 45° + \sin 45°) = \frac{\mu_0 I}{\sqrt{2}\pi L}$$

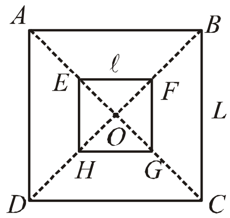
or $B \propto \frac{I}{L}$ or, $B = K\frac{I}{L}$ (K = constant)

Magnetic flux linked with the smaller loop, $\phi = B.A = \frac{KI}{L}l^2$

And, $\phi = ME$

where M = Mutual inductance

$\therefore MI = \frac{KI}{L}l^2\ \therefore M \propto \frac{l^2}{L}.$



18. **(d)** Since the rate of change of magnetic flux $\frac{d\phi}{dt}$ is zero, hence induced emf or current in the loop is zero.

19. The coil is broken into two identical coils and connected in parallel.

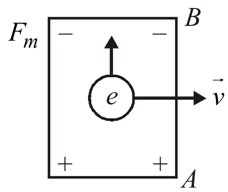
$$\therefore L_{eq} = \frac{L/2\times L/2}{L/2+L/2} = \frac{L}{4} = \frac{1.8\times10^{-4}}{4} = 0.45\times10^{-4}\text{ H},$$

$$\therefore R_{eq} = \frac{R/2\times R/2}{R/2+R/2} = \frac{R}{4} = \frac{6}{4} = 1.5\,\Omega$$

Time constant, $\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.45\times10^{-4}}{1.5} = 0.3\times10^{-4}$s

Steady state current through the battery, $I = \frac{E}{R_{eq}} = \frac{15}{1.5} = 10$ A.

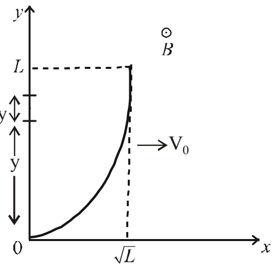
20. **True.** According to fleming's left hand rule, when conduction rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive z-direction, then the electrons will experience a force towards B. Hence, the end A will become positive due to deficiency of electrons at A.



21. **(a, b, d)** Given: $B = B_0\left[1+\left(\frac{y}{L}\right)^f\right]\hat{K}\ \vec{V} = V_0\hat{i}$

We now consider a infinite small length of wire dy at a distance y from the origin.

Emf induced across the length dy $|\Delta\phi| = B(dy)V_0$



$$|\Delta\phi| = B_0\left[1+\left(\frac{y}{L}\right)^B\right]V_0 dy$$

$\therefore$ Induced emf across the complete projection

$$|\Delta\phi| = B_0V_0\int_0^L\left[1+\left(\frac{y}{L}\right)^\beta\right]V_0 dy \;= B_0V_0L\left[1+\frac{1}{\beta+1}\right]$$

For $\beta = 0, |\Delta\phi| = 2B_0V_0L$. Clearly, $|\Delta\phi| \propto L$

For $\beta = 2$ $|\Delta\phi| = \frac{4}{3}B_0V_0L$

For a straight wire of length $\sqrt{2}L$ placed along y = x then the value of $|\Delta\phi|$ will remain the same as its projection of y-axis is same L as that of previous.

22. **(a, b)** $i = \frac{e}{R} = \frac{BLv}{R}$ – (i) [Counter–clockwise direction while entering, Zero when completely inside and clockwise while exiting]

$F = iLB = \frac{B^2L^2v}{R}$ – (ii) [Toward left while entering and exiting and zero when completely inside]

$$\therefore \quad -mV\frac{dv}{dx} = \frac{B^2L^2v}{R}$$

$$\therefore \quad \int_{v_0}^{v} dV = -\frac{B^2L^2}{mR}\int_0^x dx \Rightarrow V - V_0 = -\frac{B^2L^2}{mR}x$$

$$\therefore \quad V = V_0 - \frac{B^2L^2x}{mR} \qquad \text{... (iii)}$$

[V decreases from x = 0 to x = L, remains constant for x = L to x= 3L again decreases from x=3L to x=4L hence graph (a) is correct]

From (i) and (iii)

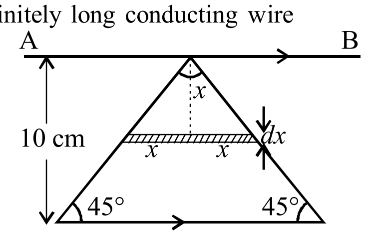
$$i = \frac{BL}{R}\left[V_0 - \frac{B^2L^2x}{mR}\right]$$

[i decreases from x=0 to x=L i becomes zero from x = L to x = 3Li changes direction and decreases from x = 3L to x = 4L]

Hence graph (b) is correct.

23. **(a, d)**

The flux passing through the triangular wire if i current flows through the inifinitely long conducting wire



$$\left(\frac{dI}{dt} = 10AS^{-1} \text{ given}\right)$$

$$d\phi = \int_0^{0.1}\frac{\mu_0 i}{2\pi x}\times 2\pi dx$$

$$\phi = \frac{\mu_0 i}{10\pi} = Mi$$

$$\therefore \; M = \frac{\mu_0}{10\pi}$$

Induced emf in the wire, $e = M\frac{di}{dt} = \frac{\mu_0}{10\pi}\times 10 = \frac{\mu_0}{\pi}V$

There will be no extra induced emf in the wire because there is no change in the magnetic.

Flux due to rotation of loop.

As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

24. **(a, b, c, d)**

(a) Inductance, $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

(b) Induced *emf*, $e = -L\left(\frac{di}{dt}\right)$

$\therefore \; L = -\frac{e}{(di/dt)}$ or henry $= \frac{\text{volt} - \text{second}}{\text{ampere}}$

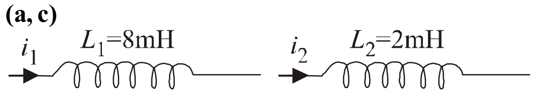
(c) Energy, $U = \frac{1}{2}Li^2$

$\therefore \; L = \frac{2U}{i^2}$ or henry $= \frac{\text{joule}}{(\text{ampere})^2}$

(d) $U = \frac{1}{2}Li^2 = i^2Rt$

$\therefore \; L$ = R.t or henry = ohm-second.

25. **(a, c)**



According to question, rate of change of current $\frac{di_1}{dt}$ = constant = $m(say)$

And according to faraday's law,

Induced emf $V_1 = -L_1\frac{di_1}{dt} = -8\times 10^{-3}\times m$

$$\therefore \; \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2\times 10^{-3}}{8\times 10^{-3}} = \frac{1}{4}$$

Since Power given to the two coils are equal

$\therefore \; V_1 i_1 = V_2 i_2$

$$\Rightarrow \; \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4} \qquad \text{...(i)}$$

Energy $W = \frac{1}{2}Li^2$

$$\therefore \; \frac{W_2}{W_1} = \frac{\frac{1}{2}L_2i_2^2}{\frac{1}{2}L_1i_1^2} = \frac{1}{4}\times 4\times 4 = 4$$

26. Given: $\beta = B_0\left(\frac{y}{a}\right)\hat{k}$ and side of square loop = a

Suppose at $t = 0$, $y = 0$ and $t = t$, $y = y$

(a) Total magnetic flux $\phi = \vec{B}.\vec{A}$

$$\therefore \; \phi = \frac{B_0 y}{a}.a^2 = B_0 ya$$

Net emf., $e = -\frac{d\phi}{dt} = -B_0 a\frac{dy}{dt} = -B_0 av(t)$

$\therefore |i| = \frac{|e|}{R} = \frac{B_0 a v(t)}{R}$ [Anticlock wise]

As loop goes down, magnetic flux linked with it increases, hence induced current flows in such a direction so as to reduce the magnetic flux linked with it. Therefore, induced current flows in anticlockwise direction.

(b) Each side will experience a force as shown in figure (A current carrying segment in a magnetic field experiences a force).

$\vec{F}_1 = i(\vec{\ell} \times \vec{B}) = i\left(-a\hat{i} \times \frac{B_0 y}{a}\hat{k}\right) = B_0 y(\hat{i} \times \hat{j});$

$\vec{F}_3 = i\left(+a\hat{i} \times \frac{B_0(y+a)}{a}\hat{k}\right) = iB_0(y+a)\hat{j}$

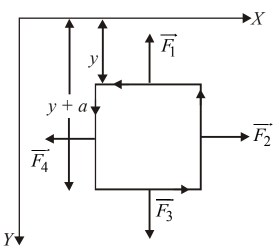
$\vec{F}_2 = -\vec{F}_4$ and hence will cancel out each other.

Net force,

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= -iB_0 a\hat{j} = -\frac{B_0^2 a^2 v(t)}{R}\hat{j}$

in upward direction.



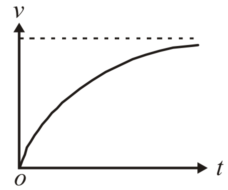
(c) Net force on the loop,

$$F = mg\,\hat{j} + \vec{F} = \left[mg - \frac{B_0^2 a^2 v(t)}{R}\right]\hat{j};$$

$$\therefore \quad m\frac{dv}{dt} = mg - \frac{B_0^2 a^2 v(t)}{R}$$

Integrating it, we get, $\int_0^v \frac{dv}{g - \frac{B_0^2 a^2 v(t)}{mR}} = \int_0^t dt$

$$\frac{\log\left[g - \frac{B_0^2 a^2 v(t)}{mR}\right]_0^{(v)t}}{\frac{-B_0^2 a^2}{mR}} = t$$



$$\text{or } \log\left[\frac{g - \frac{B_0^2 a^2 v(t)}{mR}}{g}\right] = -\frac{B_0^2 a^2 t}{mR}$$

$$\text{or } \quad 1 - \frac{B_0^2 a^2 v(t)}{mgR} = e^{-\left(B_0^2 a^2 t\right)/mR}$$

$$\text{or } \quad 1 - e^{-\left(B_0^2 a^2 t\right)/mR} = \frac{B_0^2 a^2}{mgR} v(t);$$

$$\therefore \quad v(t) = \frac{mgR}{B_0^2 a^2}\left[1 - e^{-\left(B_0^2 a^2 t\right)/mR}\right]$$

When terminal velocity is attained, $v(t)$ does not depend on t

$$\therefore \quad v(t) = \frac{mgR}{B_0^2 a^2}$$

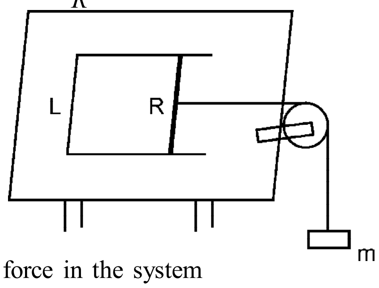
27. (i) Let v be the velocity of the rod at any time t,

$\therefore$ Induced emf $e = BvL$ and so induced current in the rod

$$I = \frac{\text{Induced e.m.f.}}{R} = \frac{BvL}{R}$$

Due to this current, the rod in the field B will experience a force

$F = BIL = \frac{B^2 L^2 v}{R}$ (opposite to its motion) ... (i)



So, net force in the system

$T - F = 0 \times a$, i.e., $T = F$ [$\because$ rod is massless]

$\Rightarrow \quad mg - T = ma \Rightarrow a = g - \frac{T}{m} = g - \frac{B^2 L^2 v}{mR}$ ... (ii)

So rod will acquire terminal velocity when its acceleration is zero i.e.,

$g - \frac{B^2 L^2 v_T}{mR} = 0$ i.e. $v_T = \frac{mgR}{B^2 L^2};$

(ii) When the velocity of the rod is half of its terminal velocity

$v = \frac{v_T}{2} = \frac{mgR}{2B^2 L^2}$

Substituting this value of velocity in eq. (ii)

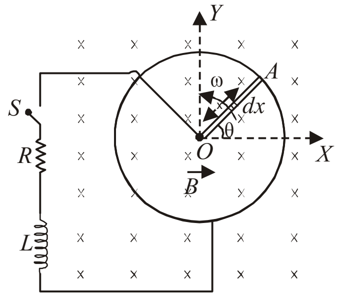
Acceleration, $a = g - \frac{B^2 L^2}{mR} \times \frac{1}{2}\frac{mgR}{B^2 L^2} = g - \frac{1}{2}g = \frac{g}{2}$

28. (a) (i) Let us consider a small element of length dx of metal rod OA at a distance x from the origin. Small amount of emf induced in this small length due to uniform magnetic field B

$de = B\,(dx)\,v$ ... (i)

where v is the velocity of small length dx

$v = x\omega$ ... (ii)



$\therefore$ The total emf acoss the whole metallic rod OA

$$e = \int_0^r Bx\omega\,dx = B\omega\left[\frac{x^2}{2}\right]_0^r = \frac{Br^2\omega}{2}$$

(b) (i) The above diagram can be reconstructed as follows. Switch s is a closed at time $t = 0$. Hence it is a case of growth of current in L-R circuit. Therefore current at any time 't'

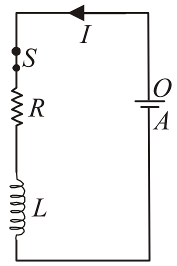
$I = I_0\,(1 - e^{t/\tau})$

Here,

$I_0 = \frac{e}{R} = \frac{B\omega r^2}{2R}$

and $\tau = \frac{L}{R}$

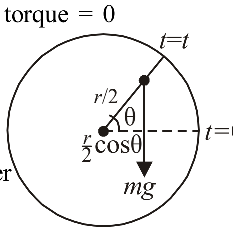
$\therefore \quad I = \frac{B\omega r^2}{2R}[1 - e^{-\left(\frac{R}{L}\right)t}]$



(ii) In steady state,

$$I = \frac{B\omega r^2}{2R} \quad [\because\ t \text{ has a large value and } e^{-\left(\frac{R}{L}\right)t} \to 0]$$

Since the rod is rotating in a vertical plane, work needs to be done to keep it at constant angular speed.
At constant angular speed, net torque = 0
Power loss due to current I



$$P = I^2R = \left(\frac{Br^2\omega}{2R}\right)^2 R$$

If torque required for this power is τ_1 then

$$P = \tau_1\omega \;\Rightarrow\; \tau_1 = \frac{B^2r^4\omega}{4R}$$

Torque of weight (mg) about centre

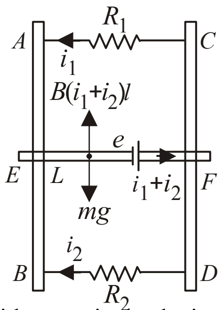
$$\tau_2 = mg \times \frac{r}{2}\cos\theta$$

The total torque
$\tau = \tau_1 + \tau_2$ (Clockwise)

$$\tau = \frac{B^2r^4\omega}{4R} + \frac{mgr}{2}\cos\omega t$$

The required torque will be of same magnitude and in anticlockwise direction. The net torque will then be zero and the angular speed of the rod will be maintained constant.

29. The direction of induced current can be found with the help of Lenz's law.
Potential difference across parallel combinations remains the same
Also, $P_1 = ei_1 = 0.76$ W
and $P_2 = ei_2 = 1.2$ W



$$\therefore\ \frac{i_1}{i_2} = \frac{1.76}{1.2} \Rightarrow i_1 = \frac{1.76}{1.2}i_2 \ \ldots\text{(ii)}$$

The horizontal metallic bar L moves with a terminal velocity. i.e., the net force on the bar is zero.

$\therefore\ B(i_1 + i_2) = mg$

$$\Rightarrow\ i_1 + i_2 = \frac{mg}{B\ell} = \frac{0.2 \times 9.8}{0.6 \times 1} = \frac{49}{15}\text{amp. ......(iii)}$$

From eq. (ii) and (iii)

$$\frac{1.76}{1.2}i_2 + i_2 = \frac{49}{15}$$

$$\Rightarrow\ i_2 = 2\text{ A.} \Rightarrow\ i_1 = \frac{19}{15}\text{A.} \Rightarrow\ e = \frac{0.76}{19/15} = 0.6V$$

Induced emf across L due to the movement of bar L in a magnetic field

$$e = Bv_TL \;\Rightarrow\; v_T = \frac{e}{BL} = \frac{0.6}{0.6 \times 1} = 1\text{m/s}$$

Also from eq. (i),

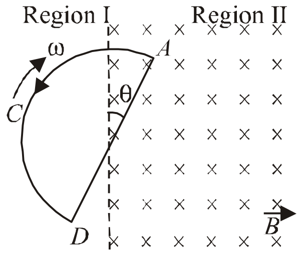
$$R_1 = \frac{e}{i_1} = \frac{0.6}{19/15} = 0.47\Omega \text{ and } R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\Omega$$

30. (i) Induced emf

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(B \times A)$$

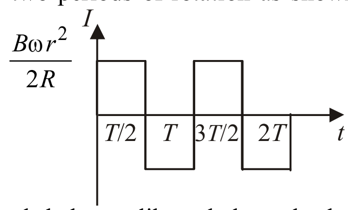
$$= -\frac{d}{dt}\left[B\left(\frac{1}{2}r^2\theta\right)\right] = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega\ [\because\ \theta = \omega t]$$

$$\therefore\quad I = \frac{E}{R} = -\frac{1}{2}\frac{Br^2\omega}{R} \Rightarrow |I| = \frac{1}{2}\frac{Br^2\omega}{R}$$

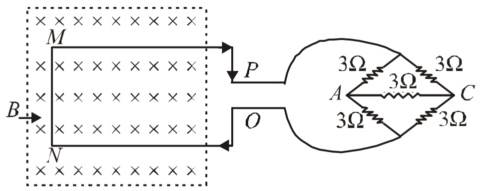


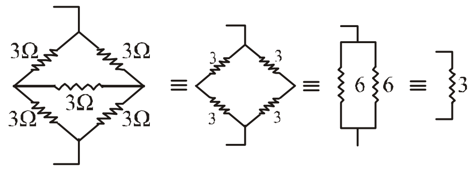
(ii) When the loop is entering in the magnetic field i.e., in region–II, magnetic lines of force passing through the loop is increasing in the downward direction. Therefore, as per Lenz's law current will flow in the loop in such a direction which will oppose the change. Hence the current will flow in the anti-clockwise direction.

(iii) Graph between induced emf and period of rotation:
For first half rotation, ($t = T/2$), when the loop enters the field, the current is in anticlockwise direction.
Magnitude of current remains constant at $I = B\omega r^2/2R$.
For next half rotation, when the loop comes out of the field, current of the same magnitude is set up clockwise.
Anticlockwise current is supposed to be positive. The I-t graph for two periods of rotation as shown below.



31. The network behaves like a balanced wheatstone bridge. So no current flows through AC.





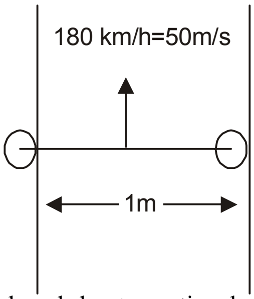
$\therefore$ Equivalent resistance of the circuit $R = 3 + 1 = 4\Omega$
The emf induced, $e = BV_0l$

$$e = IR = BV_0l \Rightarrow V_0 = \frac{IR}{Bl}$$

$$\text{or, } V_0 = \frac{1 \times 10^{-3} \times 4}{2 \times 0.1} = 0.02\text{m/s}$$

The free electrons in the portion MN of the rod have a velocity v in the right direction. Applying Fleming's left hand rule, we find that the force on electron will be towards N. Hence, M will be + ve and N will be negative. Current will flow in clockwise direction.

32. Here, $B = 0.2 \times 10^{-4}\ wb/m^2$, $l = 1$ m



Here emf developed due to motional emf.

$e = vB\ell = 50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3} = 1$ mv

The reading of milli voltmeter is $1mv$.

Topic-3 : Miscellaneous Problems

1. **(a)** As cylinder is kept parallel to an uniform magnetic field, so no change in magnetic flux and hence induced current will be zero.

2. **(b)** Power, $P = \frac{E^2}{R} = \frac{\pi r^2}{\rho\ell}\left(\frac{d\phi}{dt}\right)^2 = \frac{\pi r^2}{\rho\ell}\left[\frac{d}{dt}(NBA)^2\right]$

[Here, E = induced emf and $R = \rho\frac{l}{A}$]

or, $P = \frac{\pi r^2}{\rho\ell}N^2A^2\left(\frac{dB}{dt}\right)^2 \Rightarrow P \propto \frac{N^2r^2}{\ell}$

When number of turns quadrupled and the wire radius haved Power $\rho' \propto \frac{(4N)^2(r/2)^2}{4\ell}$

$\therefore \frac{P}{\rho'} = \frac{1}{1}$ $\therefore$ Power remains the same.

3. **(d)** When switch S is closed, a magnetic field is set-up in the space around P. The field lines threading Q increases in the direction from right to left. According to Lenz's law, I_{Q_1} will flow so as to oppose the cause or chnage and flow in anticlockwise direction as seen by E. Opposite is the case when S is opened. I_{Q_2} will be clockwise.

4. **(b)** A conducting ring is placed coaxially within a coil and the coil has inductance as well as resistance. The magnetic field at the centre of the coil

$B(t) = \mu_0 nI_1$.

As the current increases, B will also increase with time till it reaches a maximum value (when the current becomes steady).

Induced emf in the ring

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\vec{B}.\vec{A}) = -A\frac{d}{dt}(\mu_0 nI_1)$$

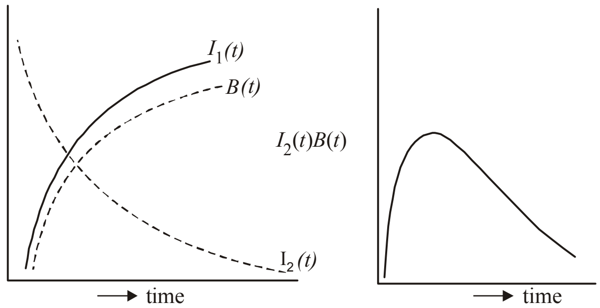
$\therefore$ Induced current in the ring

$$I_2(t) = \frac{|e|}{R} = \frac{\mu_0 nA}{R}\frac{dI_1}{dt}$$

[$\frac{dI_1}{dt}$ decreases with time and hence I_2 also decreases with time.]

Where $I_1 = I_{max}(1 - e^{-t/\tau})$

The relevant graphs are as follows.



Hence as a function of time (t>0) the product $I_2(t)$ B(t) decreases with time.

5. **(b)** $\oint \vec{E}.\vec{d\ell} = \frac{d\phi}{dt} = \frac{d}{dt}(\vec{B}.\vec{A}) = \frac{d}{dt}(BA\cos 0°) = A\frac{dB}{dt}$

$\Rightarrow E(2\pi r) = \pi a^2\frac{dB}{dt}$ for $r \geq a$

$\Rightarrow E = \frac{a^2}{2r}\frac{dB}{dt} \Rightarrow E \propto \frac{1}{r}$

Hence magnitude of the induced electric field at a distance r from centre of circular region decreases as $\frac{1}{r}$.

6. **(c)** Loop-A carries a current. And hen the current in the loop A increases with time the magnetic lines of force in loop B also increases as loop A is placed near loop B. This induces an emf in B in such a direction that current flows opposite in B as compared to A hence the loop B is repelled by loop A.

7. **(a, b, c)** At t = 0 inductors L_1 and L_2 will offer infinite resistance hence current through circuit is zero.

After a long time the current through the resistor is constant I will divide into two parts L_1 and L_2 which are in parallel

$\therefore I_1L_1 = I_2L_2$ $[I = I_1 + I_2]$

$$I_1 = \frac{V}{R}\left[\frac{L_2}{L_1 + L}\right]$$

and $I_2 = \frac{V}{R}\left[\frac{L_1}{L_1 + L_2}\right]$

Also the ratio of currents through L_1 and L_2 is fixed at all times At t = 0, $I \approx 0$

8. **(b, d)** The net magnetic flux through the loops at time t

$\phi = B.2A\cos\omega t - BA\cos\omega t = B\ (2A - A)\cos\omega t = BA\cos\omega t$

$\therefore \left|\frac{d\phi}{dt}\right| = B\omega A\sin\omega t$

So, $\left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \pi/2$

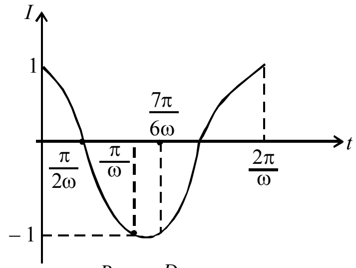
The emf induced in the smaller loop,

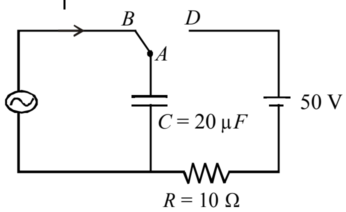
$$\varepsilon_{smaller} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A\sin\omega t$$

$\therefore$ Amplitude of maximum net emf induced in both the loops = Amplitude of maxium emf induced in the smaller loop alone.

9. **(c, d)** For maximum charge on the capacitor, $\frac{dQ}{dt} = I = 0$

$I = I_0\cos\omega t = \cos 500\ t$





Till $t = \frac{7\pi}{6\omega}$, the charge will be maximum at $\frac{\pi}{2\omega}$

$$Q' = \int_0^{\pi/2\omega} \cos 500t \, dt = \left[\frac{\sin 500t}{500}\right]_0^{\pi/2\omega}$$

$$= \frac{1}{500}\sin\left(500 \times \frac{\pi}{2 \times 500}\right) = \frac{1}{500}\text{C}$$

i.e., $Q_{max} = \frac{1}{500}c = 2 \times 10^{-3} C$

From the graph it is clear that just before $t = \frac{7\pi}{6\omega}$, the current is in anticlockwise direction.

Immediately after A is connected to D.

At $t = \frac{7\pi}{6\omega}$, the charge on the upper plate of capacitor

$$\int_0^{\frac{7\pi}{6\omega}} \cos 500t \, dt = \frac{1}{500}\sin\left(500 \times \frac{7\pi}{6 \times 500}\right)$$

$$= -\frac{1}{500} \times \frac{1}{2} = -10^{-3}\text{C}$$

Now applying KVL

$$50 + \frac{10^{-3}}{20 \times 10^{-6}} - i \times 10 = 0 \Rightarrow i = 10 \text{ A}$$

The maximum charge on C, $Q = CV = 20 \times 10^{-6} \times 50 = 10^{-3}$C

Therefore, the total charge flown from the battery $= 2 \times 10^{-3}$ C

10. **(a,b,c)** $\frac{1}{RC}$, R/L and $1/\sqrt{LC}$ have the dimensions of frequency i.e., $[M^{\circ} L^{\circ} T^{-1}]$

11. **(b)** $\int \vec{E} . \vec{dl} = \frac{-d\phi}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$

$$= -\pi R^2 B$$

$\therefore$ E × 2πR = – πR²B

$\therefore$ $E = \frac{-BR}{2}$

12. **(b)** Given M = γ L

$\therefore$ M = γ mωR²

$\therefore$ M = γ m (Δω) R² ...(1)

But $\Delta\omega = \frac{Q \times B}{2m}$ ...(2)

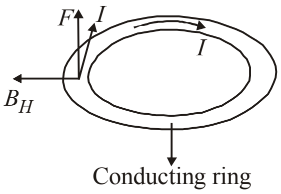
From eq. (i) and (ii)

$$\Delta M = -\gamma \, m \left(\frac{QB}{2m}\right) R^2 = \frac{-\gamma BQR^2}{2}$$

Here –(ve) sign shows that change is opposite to the direction of magnetic field, B.

(b) is the correct option.

13. **(a)** The horizontal component of the magnetic field B_H interacts with the induced current produced in the conducting ring which produces an average force in the upward direction. This is an accordance with Fleming's left hand rule.



14. A cylindrical shell of length L, thickness d and radius R surrounds a coaxial solenoid of radius a. The coil of the solenoid carries a variable current $i = i_0 \sin \omega t$.

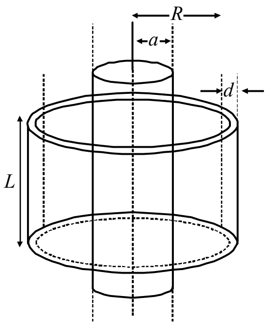
Outside the solenoid, the magnetic field is zero.

Magnetic field inside the solenoid, $B = \mu_0 \, n \, i_0 \sin \omega t$.

$\therefore$ Magnetic flux linked with solenoid

$\phi = BA$

or $\phi = (\mu_0 \, n \, i_0 \sin \omega t)(\pi a^2)$



$\therefore \frac{d\phi}{dt} = \pi\mu_0 n a^2 i_0 \omega co \; \omega t$

or $\varepsilon = -\pi\mu_0 n \, a^2 i_0 \; \omega \cos \omega t$

$$\left[\because e = -\frac{d\phi}{dt}\right]$$

Resistance of shell, $R = \frac{pl}{A}$

$\therefore$ Resistance, $R = \frac{\rho \times 2\pi R}{L \times d}$ or Resistance $= \frac{2\pi R\rho}{Ld}$

$\therefore$ Induced current in shell, I

$$= \frac{|\varepsilon|}{\text{resistance}} = \frac{\pi\mu_0 n a^2 i\omega \cos \omega t}{\frac{2\pi Rp}{Ld}}$$

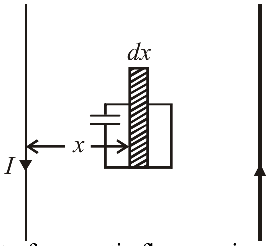
$\therefore$ Induced current $= \frac{\mu_0 n a^2 Ld \; i_0 \omega \cos \omega t}{2\rho R}$.

15. **(a)** Let us consider a small strip of thickness dx at a distance x from the left wire. As shown in the figure. The magnetic field at this strip due to both wires

$B = B_A + B_B$

(Perpendicular to the plane of paper directed upwards)

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{(3a - x)} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a - x}\right]$$



Small amount of magnetic flux passing through the strip of thickness dx

$$d\phi = B \times a\,dx = \frac{\mu_0 Ia \times 3a\,dx}{2\pi x(3a-x)}$$

Total flux through the square loop

$$\phi = \int_a^{2a} \frac{\mu_0 I \times 3a^2}{2\pi}\frac{dx}{x(3a-x)} = \frac{\mu_0 Ia}{\pi}\ln 2$$

or, $\phi = \dfrac{\mu_0 a \ln(2)}{\pi}\ (I_0 \sin\omega t)$

The emf produced $e = \left|-\dfrac{d\phi}{dt}\right| = \dfrac{\mu_0 a I_0 \omega}{\pi}\ln(2)\cos\omega t$

Charge stored in the capacitor

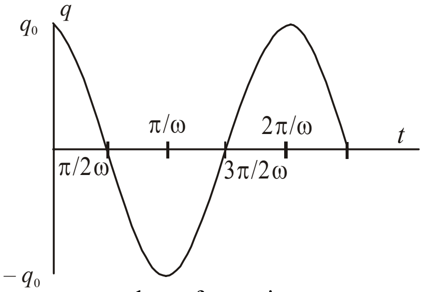
$$q = C \times e = C \times \frac{\mu_0 a I_0 \omega}{\pi}\ln(2)\cos\omega t \qquad \text{... (i)}$$

$\therefore$ Current in the loop

$$i = \frac{dq}{dt} = \frac{C \times \mu_0 a I_0 \omega^2}{\pi}\ln(2)\sin\omega t$$

$$\therefore\ i_{\max} = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(b) From eq. (i), the graph between charge and time $(q-t)$ is as shown below



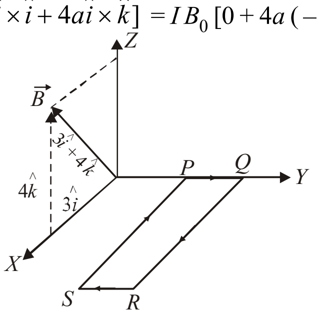
Charge on upper plate of capacitor, $q = q_0 \cos\omega t$

Here, $q_0 = \dfrac{C \times \mu_0 a I_0 \omega \ln(2)}{\pi}$

16. (a) Let us consider the current in the clockwise direction in loop $PQRS$ i.e., P to Q in the wire PQ. Force on wire QR,

$$\vec{F}_{QR} = I(\vec{\ell} \times \vec{B}) = I[(a\hat{i}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= I B_0[3a\hat{i} \times \hat{i} + 4a\hat{i} \times \hat{k}] = IB_0[0 + 4a(-\hat{j})] = -4aB_0 I\hat{j}$$



Force on wire PS

$$\vec{F}_{PS} = I(\vec{\ell} \times \vec{B}) = I[a(-\hat{i}) \times (3\hat{i} + 4\hat{k})B_0] = 4aB_0 I\hat{j}$$

i.e., force on QR is equal and opposite to that on PS and balance each other.

Force on RS

$$\vec{F}_{RS} = I(\vec{\ell} \times \vec{B}) = I[b(-\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= IbB_0[3\hat{k} - 4\hat{i}] \qquad \text{... (i)}$$

Torque about PQ by this force

$$\vec{\tau}_{RS} = \vec{r} \times \vec{F} = (\hat{i}a) \times (3\hat{k} - 4\hat{i})IbB_0$$

$$= -IabB_0(3\hat{j}) \qquad \text{... (ii)}$$

Torque about PQ due to weight of the wire $PQRS$

$$\tau = mg\left(\frac{a}{2}\right) \qquad \text{... (iii)}$$

(b) Magnetic force on Rs from eq (i), $F_{RS} = \text{Ib } B_0\left[3\hat{k} - 4\hat{i}\right]$

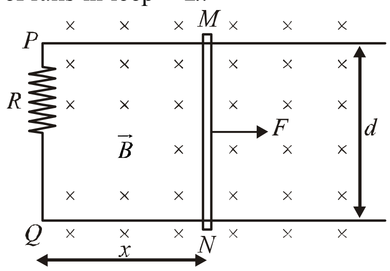
(c) For the wire loop to be horizontal,

$$3IabB_0 = mg\frac{a}{2} \Rightarrow I = \frac{mg}{6bB_0} \qquad \text{... (iv)}$$

As because torque due to mg and current are in opposite directions. Therefore, current is from P to Q.

17. (i) As shown in figure, a variable force F is applied to the rod MN such that as the rod moves in the uniform magnetic field. A constant current flows through R. Consider the loop $MPQN$. Let MN be at a distance x from PQ.

Length of rails in loop = $2x$



$\therefore$ Resistance of rails in loop = $2x\lambda$

$\therefore$ Total resistance of loop $R_{net} = R + 2\lambda x$

Let V be the velocity of rod at this instant, then due to the motion of the rod emf induced, $e = Bvd$

$$\therefore\ \text{Induced current } (I) = \frac{e}{R_{net}} = \frac{Bvd}{R + 2\lambda x}$$

So for constant I, $\quad v = \dfrac{(R + 2\lambda x)}{Bd}I \qquad \text{...(i)}$

And due to induced current I the wire will experience a force $F_M = BId$ opposite to its motion, the equation of motion of the wire

$F - F_M = ma \quad \Rightarrow \quad F = F_M + ma$

But as here $F_M = BId$ and from equation (i)

$$a = \frac{dv}{dt} = \frac{2\lambda I}{Bd}\frac{dx}{dt} = \frac{2\lambda Iv}{Bd} = \frac{2\lambda I^2}{(Bd)^2}(R + 2\lambda x)$$

$$\therefore\ F = BId + \frac{2\lambda m I^2}{(Bd)^2}(R + 2\lambda x)$$

(ii) As the work done by force F per second.

$$\frac{dW}{dt} = P = Fv = \left[BId + \frac{2\lambda m I^2}{(Bd)^2}(R + 2\lambda x)\right]\left[\frac{R + 2\lambda x}{Bd}.I\right]$$

$$\text{i.e., } P = \left[I^2(R + 2\lambda x) + \frac{2\lambda m I^3}{B^3 d^3}(R + 2\lambda x)^2\right]$$

and heat produced per second,

$$H = I^2(R + 2\lambda x)$$

$$\therefore\ f = \frac{H}{P} = \left[1 + \frac{2\lambda m I(R + 2\lambda x)}{B^3 d^3}\right]^{-1}$$

Topic-1 : Alternating Current, Voltage and Power

1. **(d)** As $V(t) = 220 \sin 100\,\pi t$

so, $I(t) = \dfrac{220}{50} \sin 100\,\pi t$

i.e., $I = I_m = \sin(100\,\pi t)$

For $I = I_m$

$$t_1 = \frac{\pi}{2} \times \frac{1}{100\pi} = \frac{1}{200} \text{ sec.}$$

and for $I = \dfrac{I_m}{2}$

$$\Rightarrow \frac{I_m}{2} = I_m \sin(100\,\pi t_2) \Rightarrow \frac{\pi}{6} = 100\,\pi t_2 \Rightarrow t_2 = \frac{1}{600} s$$

$$\therefore\ t_{req} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300} s = 3.3 \text{ ms}$$

2. **(a)** For two concentric circular coil,

Mutual Inductance M $= \dfrac{\mu_0 \pi N_1 N_2 a^2}{2b}$

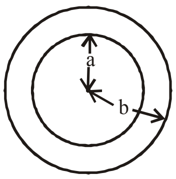
here, $N_1 = N_2 = 1$

Hence, M $= \dfrac{\mu_0 \pi a^2}{2b}$ ..... (i)

and given $I = I_0 \cos \omega t$ ..... (ii)

Now according to Faraday's second law induced emf

$$e = -M\frac{dI}{dt}$$



From eq. (ii),

$$e = \frac{-\mu_0 \pi a^2}{2b} \frac{d}{dt}(I_0 \cos \omega t)$$

$$e = \frac{\mu_0 \pi a^2}{2b} I_0 \sin \omega t\,(\omega)$$

$$e = \frac{\pi \mu_0 I_0}{2} . \frac{a^2}{b} \omega \sin \omega t$$

3. **(b)** In a pure inductive circuit current always lags behind the emf by $\dfrac{\pi}{2}$.

If $v(t) = v_0 \sin \omega t$

then $I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$

Now, given v(t) = 100 sin (500 t)

and $I_0 = \dfrac{E_0}{\omega L} = \dfrac{100}{500 \times 0.02}$ $[\because L = 0.02H]$

$$I_0 = 10 \sin\left(500t - \frac{\pi}{2}\right)$$

$$I_0 = -10 \cos(500t)$$

Topic-2 : AC Circuit, LCR Circuit, Quality and Power Factor

1. **(a)** Quality factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$= \frac{1}{100}\sqrt{40 \times 10^3} = \frac{200}{100} = 2$$

2. **(a)** Potential energy stored in the inductor

$$U = \frac{1}{2} L I^2$$

During growth of current,

$$i = I_{max}(1 - e^{-Rt/L})$$

For U to be $\dfrac{U_{max}}{n}$; i has to be $\dfrac{I_{max}}{\sqrt{n}}$

$$\therefore \quad \frac{I_{max}}{\sqrt{n}} = I_{max}(1 - e^{-Rt/L})$$

$$\Rightarrow e^{-Rt/L} = 1 - \frac{1}{\sqrt{n}} = \frac{\sqrt{n} - 1}{\sqrt{n}}$$

$$\Rightarrow -\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right) \quad \Rightarrow t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

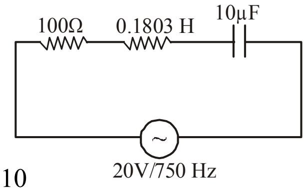
3. **(d)** Here, $R = 100$, $X_L = L\omega = 0.1803 \times 750 \times 2\pi = 850\Omega$,

$$X_C = \frac{1}{C\omega} = \frac{1}{10^{-5} \times 2\pi \times 750} = 21.23\Omega$$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (850 - 21.23)^2} = 834.77 \simeq 835$$

$$H = i_{rms}^2 Rt = \left(\frac{V_{rms}}{|Z|}\right)^2 RT$$



$= (ms)\Delta t$

$$\Rightarrow \frac{20}{835} \times \frac{20}{835} \times 100t = (2) \times 10$$

$\because V_{rms} = 20$ V and $\Delta t = 10°C$

$\therefore$ Time, $t = 348.61$ s.

4. **(a)** Given,

Reactance of inductance coil, $Z = 100\Omega$

Frequency of AC signal, $v = 1000$ Hz

Phase angle, $\phi = 45°$

$$\tan \phi = \frac{X_L}{R} = \tan 45° = 1 \Rightarrow X_L = R$$

Reactance, $Z = 100 = \sqrt{X_L^2 + R^2}$

$$\Rightarrow 100 = \sqrt{R^2 + R^2}$$

$$\Rightarrow \sqrt{2}R = 100 \Rightarrow R = 50\sqrt{2}$$

$$\therefore X_L = 50\sqrt{2}$$

$\Rightarrow L\omega = 50\sqrt{2}$ $\quad (\because X_L = \omega L)$

$\Rightarrow L = \dfrac{50\sqrt{2}}{2\pi \times 1000}$ $\quad (\because \omega = 2\pi v)$

$= \dfrac{25\sqrt{2}}{\pi}$ mH $= 1.1 \times 10^{-2}$ H

5. **(b)** We have, $i = i_0(1 - e^{-t/c}) = \dfrac{\varepsilon}{R}(1 - e^{-t/c})$

Charge, $q = \int_0^{\tau} i\,dt$

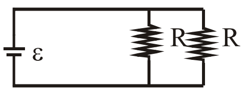
$= \dfrac{\varepsilon}{R}\int_0^{\tau}(1 - e^{-t/\tau})dt = \dfrac{E}{R}\dfrac{\tau}{e} = \dfrac{E}{R} \times \dfrac{(L/R)}{e} = \dfrac{EL}{2.7R^2}$

6. **(c)** $i^2R = \left(\tau\dfrac{di}{dt}\right)i \quad \Rightarrow \dfrac{di}{dt} = \dfrac{i}{\tau}$

$\Rightarrow \mathrm{t} = \tau\, ln2 = 2ln2 \quad \left[\text{as}\ \tau = \dfrac{L}{R} = \dfrac{20}{10} = 2\right]$

7. **(d)** Long time after switch is closed, the inductor will be idle so, the equivalent diagram will be as below

$\mathrm{I} = \dfrac{\varepsilon}{\left(\dfrac{\mathrm{R} \times \mathrm{R}}{\mathrm{R} + \mathrm{R}}\right)} = \dfrac{2\varepsilon}{\mathrm{R}} = \dfrac{2 \times 15}{5} = 6\,\mathrm{A}$

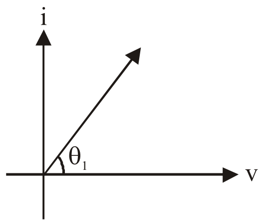


8. **(Bonus)**
Capacitive reactance,

$X_c = \dfrac{1}{\omega C} = \dfrac{4}{10^{-6} \times \sqrt{3} \times 100} = \dfrac{2 \times 10^4}{\sqrt{3}}$

$\tan\theta_1 = \dfrac{X_C}{R_2} = \dfrac{10^3}{\sqrt{3}}$

θ_1 is close to $90°$
For L-R circuit



$X_L = \omega L = 100 \times \dfrac{\sqrt{3}}{10} = 10\sqrt{3}$

$R_1 = 10$

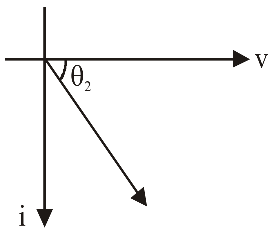
$\tan\theta_2 = \dfrac{X_L}{R_1}$

$\tan\theta_2 = \sqrt{3} \Rightarrow \theta_2 = \tan^{-1}(\sqrt{3})$

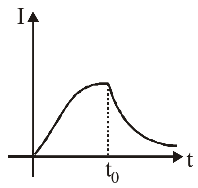
$\theta_2 = 60°$

So, phase difference comes out $90° + 60° = 150°$

If R_2 is 20 KΩ then phase difference comes out to be 60 + 30 = 90°.



9. **(b)**



The current will grow for the time t = 0 to t = t_0 and after that decay of current takes place.

10. **(a)** Given, Inductance, L= 40 mH
Capacitance, C = 100 μF
Impedance, $Z = X_C - X_L$

$\Rightarrow Z = \dfrac{1}{\omega C} - \omega L \quad \left(\because X_c = \dfrac{1}{\omega C}\ \text{and}\ X_L = \omega L\right)$

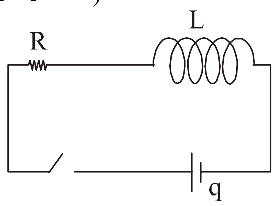
$= \dfrac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3} = 19.28\Omega$

Current, $i = \dfrac{V_0}{Z}\sin(\omega t + \pi/2)$

$\Rightarrow i = \dfrac{10}{19.28}\cos\omega t = 0.52\cos(314\,\mathrm{t})$

11. **(a)** For series connection of a resistor and inductor, time variation of current is $I = I_0(1 - e^{-t/T_c})$

Here, $T_C = \dfrac{L}{R}$



$q = \int_0^{T_c} i\,dt$

$\Rightarrow \int dq = \int \dfrac{E}{R}\left(1 - e^{-t/t_c}\right)dt$

$\Rightarrow q = \dfrac{\in}{R}\left[t + t_C e^{-t/t_c}\right]_0^{t_c}$

$\Rightarrow q = \dfrac{\in}{R}\left[t_C + \dfrac{t_C}{e} - t_C\right] \Rightarrow q = \dfrac{\in}{R}\dfrac{L}{\mathrm{Re}}$

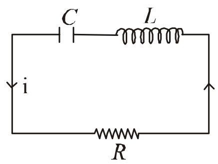
$\therefore q = \dfrac{\in L}{R^2 e}$

12. **(d)** In damped harmonic oscillation,

$\dfrac{md^2x}{dt^2} = -kx - bv$

$\Rightarrow \dfrac{md^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0 \quad$ ....(i)

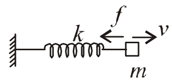
In LCR circuit, $\dfrac{-q}{C} - iR - \dfrac{Ldi}{dt} = 0$



$L\dfrac{d^2}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = 0 \quad$ ...(ii)

Comparing equations (i) & (ii)

$L \leftrightarrow m,\ C \leftrightarrow \dfrac{1}{k},\ R \leftrightarrow b$



13. **(a)** The current (I) in LR series circuit is given by

$I = \dfrac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$

At $t = \infty$,

$I_\infty = \dfrac{20}{5}\left(I - e^{\frac{-\infty}{L/R}}\right) = 4 \quad$ ...(i)

At t = 40s,

$\left(1 - e^{\frac{-40 \times 5}{10 \times 10^{-3}}}\right) = 4(1 - e^{-20,000}) \quad$ ...(ii)

Dividing (i) by (ii) we get

$\Rightarrow \dfrac{I_\infty}{I_{40}} = \dfrac{1}{1 - e^{-20,000}}$,

14. **(b)** As we know, average power $P_{avg} = V_{rms} I_{rms} \cos\theta$

$= \left(\dfrac{V_0}{\sqrt{2}}\right)\left(\dfrac{I_0}{\sqrt{2}}\right)\cos\theta = \left(\dfrac{100}{\sqrt{2}}\right)\left(\dfrac{20}{\sqrt{2}}\right)\cos 45° \quad (\because \theta = 45°)$

$P_{avg} = \dfrac{1000}{\sqrt{2}}$ watt

Wattless current I = $I_{rms} \sin\theta$

$= \dfrac{I_0}{\sqrt{2}} \sin\theta = \dfrac{20}{\sqrt{2}} \sin 45° = 10A$

15. **(a)** Quality factor $Q = \dfrac{\omega_0}{2\Delta\omega} = \dfrac{\omega_0 L}{R}$

16. **(b)** Given,
$V_0 = 283$ volt, $\omega = 320$, $R = 5\ \Omega$, $L = 25$ mH, $C = 1000\ \mu F$
$x_L = \omega L = 320 \times 25 \times 10^{-3} = 8\ \Omega$

$x_C = \dfrac{1}{\omega C} = \dfrac{1}{320 \times 1000 \times 10^{-6}} = 3.1\,\Omega$

Total impedance of the circuit :

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{25 + (4.9)^2} = 7\ \Omega$

Phase difference between the voltage and current

$\tan\phi = \dfrac{X_L - X_C}{R}$

$\tan\phi = \dfrac{4.9}{5} \approx 1 \Rightarrow \phi = 45°$

17. **(b)** Here

$i = \dfrac{e}{\sqrt{R^2 + X_L^2}} = \dfrac{e}{\sqrt{R^2 + \omega^2 L^2}} = \dfrac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$

$10 = \dfrac{220}{\sqrt{64 + 4\pi^2 (50)^2 L}}$ $\quad [\because R = \dfrac{V}{I} = \dfrac{80}{10} = 8]$

On solving we get
L = 0.065 H

18. **(b)** $I(0) = \dfrac{15 \times 100}{0.15 \times 10^3} = 0.1A$

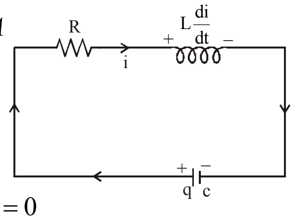
$I(\infty) = 0$

$I(t) = [I(0) - I(\infty)]\ e^{\frac{-t}{L/R}} + i(\infty)$

$I(t) = 0.1\ e^{\frac{-t}{L/R}} = 0.1\, e^{\frac{R}{L}}$

$I(t) = 0.1\ e^{\frac{0.15 \times 1000}{0.03}} = 0.67mA$

19. **(c)** From KVL at any time t



$\dfrac{q}{c} - iR - L\dfrac{di}{dt} = 0$

$i = -\dfrac{dq}{dt} \Rightarrow \dfrac{q}{c} + \dfrac{dq}{dt}R + \dfrac{Ld^2q}{dt^2} = 0$

$\dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{Lc} = 0$

From damped harmonic oscillator, the amplitude is given by A $= A_o e - \dfrac{dt}{2m}$

Double differential equation

$\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{k}{m}x = 0$

$Q_{max} = Q_o e^{-\frac{Rt}{2L}} \Rightarrow Q_{max}^2 = Q_o^2 e^{-\frac{Rt}{L}}$

Hence damping will be faster for lesser self inductance.

20. **(c)** Power factor

$\cos\phi = \dfrac{R}{\sqrt{R^2 + \left[\omega L - \dfrac{1}{\omega(C + C')}\right]^2}} = 1$

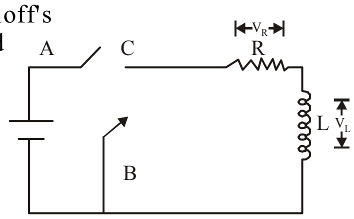
On solving we get,

$\omega L = \dfrac{1}{\omega(C + C')}$

$C' = \dfrac{1 - \omega^2 LC}{\omega^2 L}$

Hence option (c) is the correct answer.

21. **(c)** Applying Kirchhoff's law of voltage in closed loop $-V_R - V_C = 0$



$\Rightarrow \quad \dfrac{V_R}{V_C} = -1$

22. **(c)** Given, $V_L : V_C : V_R = 1 : 2 : 3$
V = 100 V
$V_R = ?$
As we know,

$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Solving we get, $V_R \simeq 90V$

23. **(c)** Charge on he capacitor at any time t is given by $q = CV\ (1 - e^{t/\tau})$
at $t = 2\tau$
$q = CV\ (1 - e^{-2})$

24. **(d)** Power factor $_{(old)}$

$= \dfrac{R}{\sqrt{R^2 + X_L^2}} = \dfrac{R}{\sqrt{R^2 + (2R)^2}} = \dfrac{R}{\sqrt{5}R}$

Power factor$_{(new)}$

$= \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{R}{\sqrt{R^2 + (2R - R)^2}} = \dfrac{R}{\sqrt{2}R}$

$\therefore \dfrac{\text{New power factor}}{\text{Old power factor}} = \dfrac{\dfrac{R}{\sqrt{2}R}}{\dfrac{R}{\sqrt{5}R}} = \sqrt{\dfrac{5}{2}}$

25. **(d)**

26. **(b)** $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{V_{rms}}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$

As ω increases, I_{rms} through the bulb increases. Hence the bulb glows brighter.

27. **(b)** Time constant of $R - C$ circuit, $\tau = R_{eq} C_{eq}$
(i) R_1 & R_2 in series and C_1 & C_2 in parallel.
$\tau_1 = (2 + 1)(2 + 4) = 18\ \mu s$.
(ii) R_1 & R_2 in parallel and C_1 & C_2 in series.

$\tau_2 = \left(\dfrac{2\times1}{2+1}\right)\left(\dfrac{2\times4}{2+4}\right) = \dfrac{8}{9}\mu s$

(iii) R_1 & R_2 in parallel and C_1 & C_2 in parallel.

$\tau_3 = \left(\dfrac{2\times1}{2+1}\right) \times (4 + 2) = 4\mu s$

28. **(a)** From $e = E_0 \sin(100t)$ $\therefore$ $\omega = 100$
From given graph, current leads emf,
Hence this is an $R - C$ circuit.

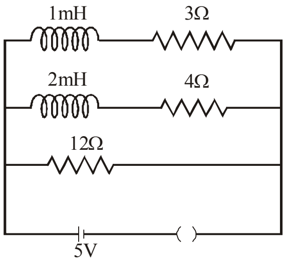
$$\tan\phi = \frac{X_C}{R}$$

Here $\phi = 45° = \frac{\pi}{4}$ $\quad\therefore\quad X_C = R$

$$\frac{1}{\omega C} = R \quad \Rightarrow RC\omega = 1$$

or $\quad RC = \frac{1}{100} s^{-1}$ $\therefore R = 1k\,\Omega, C = 10\mu F$

29. **(8)** At t = 0 $I_{min} = \frac{5}{12}$



At t = ∞

$$I_{max} = \frac{5}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3}$$

$$\left[\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}\right]$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{10}{3} \times \frac{12}{5} = 8$$

30. **(33)**
Here, $L = 50$ mH $= 50 \times 10^{-3}$ H; $I = 1$ A, $R = 2\Omega$

$$V_P - L\frac{dI}{dt} - 30 + RI = V_Q$$

$$\Rightarrow V_P - V_Q = 50 \times 10^{-3} \times 10^2 + 30 - 1 \times 2$$

$$= 5 + 30 - 2 = 33 \text{ V.}$$

31. **(400)**
Given: Power $P = 400$ W, Voltage $V = 250$ V

$P = V_m \cdot I_{rms} \cdot \cos\phi$

$\Rightarrow 400 = 250 \times I_{rms} \times 0.8 \Rightarrow I_{rms} = 2$ A

Using $P = I_{rms}^2 R$

$(I_{rms})^2 \cdot R = P \Rightarrow 4 \times R = 400$

$\Rightarrow R = 100\Omega$

Power factor is,

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow 0.8 = \frac{100}{\sqrt{100^2 + X_L^2}} \Rightarrow 100^2 + X_L^2 = \left(\frac{100}{0.8}\right)^2$$

$$\Rightarrow X_L = \sqrt{-100^2 + \left(\frac{100}{0.8}\right)^2} \Rightarrow X_L = 75\Omega$$

When power factor is unity,

$$X_C = X_L = 75 \Rightarrow \frac{1}{\omega C} = 75$$

$$\Rightarrow C = \frac{1}{75 \times 2\pi \times 50} = \frac{1}{7500\pi} F$$

$$= \left(\frac{10^6}{2500} \times \frac{1}{3\pi}\right) \mu F = \frac{400}{3\pi} \mu F$$

$N = 400$

32. **(55.00)**
Given: Mutual inductance, M = 5mH
$L_1 = 10$ mH, $V_1 = 5$V, $L_2 = 20$ mH & $V_2 = 20$V

$$I_1 = \frac{V_1}{R_1} = \frac{5}{5} = 1A; I_2 = \frac{V_2}{R_2} = \frac{20}{10} = 2A$$

After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF = increase in magnetic energy

$$\therefore W = \Delta U = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + MI_1I_2$$

$$= \frac{1}{2} \times (10 \times 10^{-3}) 1^2 + \frac{1}{2} \times (20 \times 10^{-3}) \times 2^2$$

$+ (5 \times 10^{-3}) \times 1 \times 2$

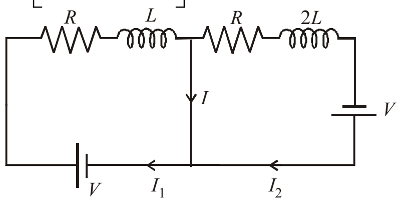
$= (5 + 40 + 10) \times 10^{-3}$J

$\therefore$ W = 55 mJ

33. **(b, d)** Here $I + I_2 = I_1$ $\quad\therefore\quad I = I_1 - I_2$

$$\therefore \quad I = \frac{V}{R}\left[1 - e^{\frac{-Rt}{2L}}\right] - \frac{V}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$$

$$\Rightarrow I = \frac{V}{R}\left[e^{\frac{-Rt}{L}} - e^{\frac{-Rt}{2L}}\right] \quad \text{...(i)}$$



For I_{max}, $\frac{dI}{dt} = 0$

$$\therefore \quad \frac{V}{R}\left[\frac{-R}{L}e^{\frac{-Rt}{L}} - \left(\frac{-R}{2L}\right)e^{\frac{-Rt}{2L}}\right] = 0$$

$$\therefore \quad e^{\frac{-Rt}{2L}} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2 \qquad \therefore \quad t = \frac{2L}{R}\ln 2$$

This is the time when I is maximum
Putting this value of time in eq.(i)

$$\text{Further } I_{max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ell n 2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ell n 2\right)}\right]$$

$$\Rightarrow I_{max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right] = \frac{V}{4R}$$

34. **(b, c)** The frequency at which the current is in phase with the voltage is resonance frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{(10^{-6} \times 10^{-6})^{1/2}} = 10^6 \text{ rad s}^{-1}$$

This frequency is independent of 'R'

At $\omega \approx 0$, the current $i = \dfrac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

i.e, current through the circuit nearly becomes zero.
If $\omega >> \omega$, $X_L > X_C$ so circuit behaves like an inductor.

35. **(a,c)** Impedance across AB, RC part of the circuit

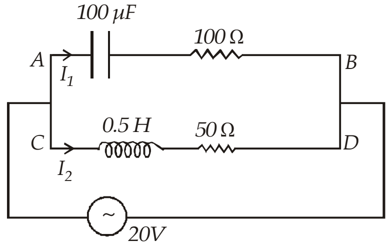
$$Z_1 = \sqrt{X_c^2 + R_1^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R_1^2}$$

$$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$$

$$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}}$$

[leads emf by ϕ_1]

where $\cos\phi_1 = \dfrac{R}{Z_1} = \dfrac{100}{100\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$



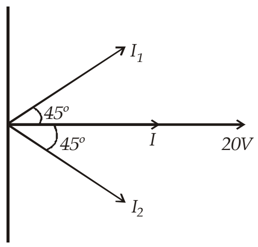
Impedance across CD, LR part of the circuit.

$$Z_2 = \sqrt{X_L^2 + R_2^2} = \sqrt{(\omega L)^2 + R_2^2}$$

$$= \sqrt{(0.5 \times 100)^2 + (50)^2} = 50\sqrt{2}\,\Omega$$

$$\therefore \quad I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}}$$

[leads emf by ϕ_2]



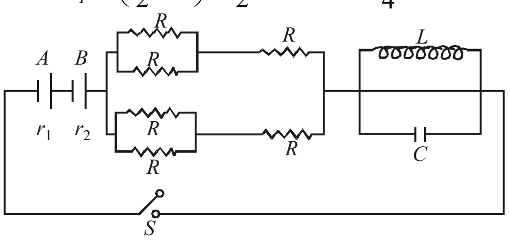
where $\cos\phi_2 = \dfrac{R}{Z_2} = \dfrac{50}{50\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45^\circ$

$\therefore$ Current I from the circuit

$$I = \frac{20}{100\sqrt{2}} + \frac{20}{50\sqrt{2}} = I_1 + I_2 \simeq 0.3 \text{ A}$$

36. After a long time capacitor will be fully charged, hence no current will flow through capacitor and all the current will flow from inductor. Thus resistance across capacitor becomes infinite and across an inductor becomes zero.

$$\therefore \quad R_{eq} = \left(\frac{R}{2} + R\right) \times \frac{1}{2} + r_1 + r_2 = \frac{3R}{4} + r_1 + r_2$$



$$I = \frac{\varepsilon + \varepsilon}{R_{eq}} \Rightarrow I = \frac{2\varepsilon}{R_{eq}} = \frac{2\varepsilon}{(3R/4) + r_1 + r_2}$$

Resistance 'R' for which potential difference across battery A is zero

$$\varepsilon - I r_1 = 0 \quad \Rightarrow \quad \varepsilon = \frac{2\varepsilon}{(3R/4) + r_1 + r_2} r_1$$

$$\Rightarrow \quad r_1 = r_2 + 3R/4 \quad \text{or} \quad R = \frac{4}{3}(r_1 - r_2)$$

37. Given, $V_{rms} = 220$ V, $\nu = 50$ Hz, $L = 35$ mH, $R = 11\Omega$

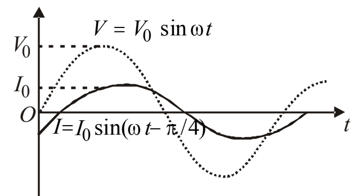
Impedance $Z = \sqrt{(\omega L)^2 + R^2} = 11\sqrt{2}\,\Omega$

Also, current amplitude, $I_0 = \dfrac{V_0}{Z}$

$V_0 = V_{rms}\sqrt{2} \qquad \therefore \quad I_0 = \dfrac{V_{rms}\sqrt{2}}{Z} = 20A$

$\cos\phi = \dfrac{R}{Z} = \dfrac{11}{11\sqrt{2}} = \dfrac{1}{\sqrt{2}} \qquad \therefore \quad \phi = \dfrac{\pi}{4}$ phase

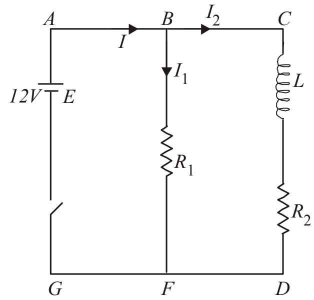
In L-R circuit, voltage leads the current. $I_{instantaneous} = 20 \sin\left(\omega t - \pi/4\right)$, the current time graph as shown below.



38. Given: $R_1 = R_2 = 2\Omega$, $E = 12V$ and $L = 400$mH
The circuit given is an R-L circuit in which the current grows as soon as current is switched on.
Applying Kirchhoff's law in the loop ABCDFGA
we get, starting from G moving clockwise

$$E - L\frac{dI_2}{dt} - I_2 R_2 = 0$$

or $I_2 = \dfrac{E}{R_2}\left[1 - e^{-\frac{R}{L}t}\right]$



Also we know that the emf (V) produced across the inductor

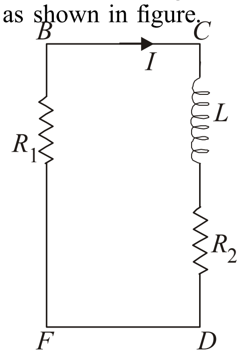
$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}[LI_2] = -L\frac{dI_2}{dt}$$

$$= -L\frac{d}{dt}\left[\frac{E}{R_2}\left(1 - e^{\frac{-R_2}{L}t}\right)\right]$$

$V = -E\, e^{-\frac{R_2}{L}t}$. Here the negative sign shows the opposition to the growth of current.

$$\therefore \quad V = 12\, e^{-\frac{2}{400\times10^{-3}}t} = 12\, e^{-5t}\text{ V}$$

When the switch is open, the current flows through the circuit BCDF only in the direction as shown in figure.
Applying Kirchhoff 's law



$$I(R_1 + R_2) - \left(-\frac{L\,dI}{dt}\right) = 0$$

$$\therefore \quad \frac{dI}{I} = -\left(\frac{R_1 + R_2}{L}\right)dt$$

$\therefore$ On integrating,

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{(R_1 + R_2)}{L}\int_0^t dt$$

$$\therefore \quad I = I_0\, e^{-\frac{(R_1+R_2)t}{L}}$$

Here, $\frac{R_1 + R_2}{L} = \frac{2+2}{400 \times 10^{-3}} = 10$

and $I_0 = \frac{E}{R_1 + R_2} = \frac{12}{4} = 3$ A

$\therefore$ $I = 3e^{-10t}$ A, clockwise.

39. As we know, energy $U = \frac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\frac{1}{4}$th of its maximum value when current is reached half of its maximum value.

In L-R circuit, equation of current growth

$i = i_0\,(1 - e^{-t/\tau L})$

Here, i_0 = Maximum value of current

τ_L = Time constant = L/R $= \frac{10H}{2\Omega} = 5s$

$\therefore$ $i = i_0/2 = i_0\,(1 - e^{-t/5})$

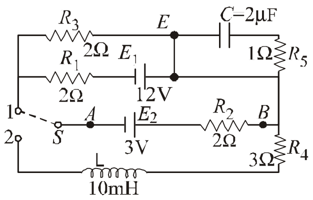
or $\frac{1}{2} = 1 - e^{-t/5} \Rightarrow e^{-t/5} = \frac{1}{2}$

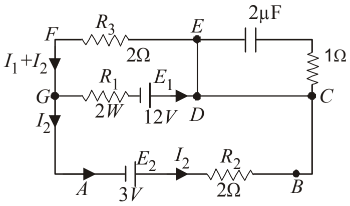
or $-t/5 = \text{In}\left(\frac{1}{2}\right) \Rightarrow t/5 = \text{In}(2) = 0.693$

$\therefore$ $t = (5)(0.693)$ *or* $t = 3.465s$

40. **(a)** (i) When the switch S is in position '1'

In steady state, current through the capacitor is zero.





Using Kirchhoff's law in loop ABCDGA

$+3 - I_2 \times 2 - 12 + I_1 \times 2 = 0$

$\Rightarrow$ $2I_1 - 2I_2 = 9$ ...(i)

Applying Kirchhoff 's law in loop DEFGD

$-2I_1 + 12 - (I_1 + I_2)\,2 = 0$

$\Rightarrow$ $2I_1 + I_2 = 6$ ...(ii)

From eq. (i) and (ii) $I_1 = \frac{21}{6}A.$

$\therefore$ From (ii) $I_2 = -1\,A.$

Potential difference between A and B

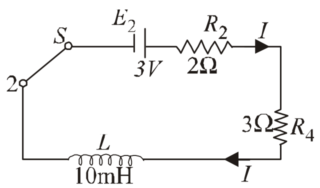
$V_A + 3 - (-1) \times 2 = V_B$ $\Rightarrow$ $V_A - V_B = -5V$

Rate of production of heat in R_1

$PR_1 = I_1^2 R_1 = \left(\frac{21}{6}\right)^2 \times 2 = 24.5W$

(b) (i) When the switch is put in position 2 then the circuit is as shown below.

When the steady state current is reached then the inductor plays no role in the circuit



$E_2 = I\,(R_2 + R_4)$

$\Rightarrow$ $I = \frac{E_2}{R_2 + R_4} = \frac{3}{5} = 0.6$ A.

(ii) The growth of current in L–R circuit

$I = I_0\left[1 - e^{-\frac{R}{L}t}\right]$

When $I = \frac{I_0}{2}$ half the steady value, then

$$\frac{I_0}{2} = I_0\left[1 - e^{-\frac{R}{L}t}\right]$$

$\Rightarrow$ $\frac{1}{2} = 1 - e^{\frac{-R}{L}t}$ $\Rightarrow$ $e^{-\frac{R}{L}t} = \frac{1}{2}$

Taking log on both sides

$$\log_e e^{-\frac{R}{L}t} = \log_e \frac{1}{2}$$

$\Rightarrow$ $\frac{R}{L}t = 0.693 \Rightarrow t = 0.693\frac{L}{R} = \frac{0.6930 \times 10 \times 10^{-3}}{(2+3)}$

when $R = R_2 + R_4$

$\therefore$ $t = 1.386 \times 10^{-3}$ s

Energy stored by the inductor

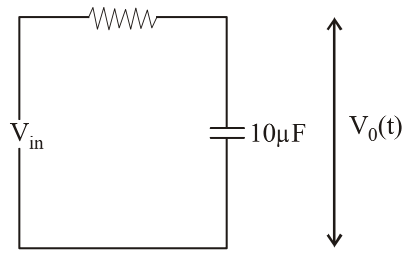
$E = \frac{1}{2}LI^2 = \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{0.6}{2}\right)^2 = 4.5 \times 10^{-4}$ J

Topic-3 : Transformers and LC Oscillations

1. **(a)** When first pulse is applied, the potential across capacitor

$V_0(t) = V_{\text{in}}\left(1 - e^{\frac{1}{RC}}\right)$

At $t = 5\mu s = 5 \times 10^{-6}\,s$



$V_0(t) = 5\left(1 - e^{\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right) = 5(1 - e^{-0.5}) = 2V$

When no pulse is applied, capacitor will discharge.

Now, $V_{\text{in}} = 0$ means discharging.

$V_0(t) = 2e^{\frac{1}{RC}} = 2e^{-0.5} = 1.21$ V

Now for next 5 μs

$V_0(t) = 5 - 3.79e^{\frac{1}{RC}}$

After 5 μs again, $V_0(t) = 2.79$ Volt ≈ 3 V

Hence, graph (a) correctly depicts.

2. **(c)** Power output $(V_2I_2) = 2.2$ kW

$\therefore\ V_2 = \dfrac{2.2\text{kW}}{(10\text{A})} = 220$ volts

$\therefore$ Input voltage for step-down transformer

$\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} = 2$

$V_{input} = 2 \times V_{output} = 2 \times 220$
$= 440$ V

Also $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$

$\therefore\ I_1 = \dfrac{1}{2} \times 10 = 5\text{A}$

3. **(b)** Efficiency, $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{V_sI_s}{V_pI_p}$

$\Rightarrow 0.9 = \dfrac{230 \times I_s}{2300 \times 5} \quad \Rightarrow I_s = 0.9 \times 50 = 45\text{A}$

Output current = 45A

4. **(c)** Given : $V_P = 2300$ V, $V_s = 230$ V, $I_P = 5$ A, $n = 90\% = 0.9$

Efficiency $n = 0.9 = \dfrac{P_s}{P_P} \Rightarrow P_s = 0.9\,P_p$

$V_sI_s = 0.9 \times V_PI_P \quad (\because\ P = VI)$

$I_s = \dfrac{0.9 \times 2300 \times 5}{230} = 45\text{A}$

Topic-4 : Miscellaneous Problems

1. **(a,c)** In RC - circuit impedance, $Z = \sqrt{R^2 + \left(\dfrac{1}{WC}\right)^2}$

The capacitance in case B is four times the capacitance in case A

$\therefore$ Impedance in case B is less then that of case A $(Z_B < Z_A)$

Now $I = \dfrac{V}{Z}\ \therefore I_R^A < I_R^B$.

and $V_R^A < V_R^B$. $\Rightarrow\ V_C^A > V_C^B$

[$\because$ If V is the applied potential difference access series R-C circuit then $V = \sqrt{V_R^2 + V_C^2}$]

2. **A-r,s,t; B-q,r,s,t; C-p,q; D-q,r,s,t**

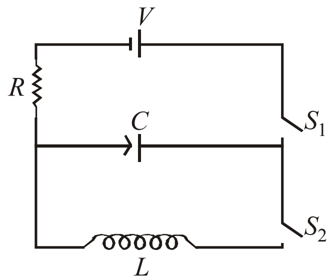
For DC circuit, in steady state, the current I through the capacitor (c) is zero. In case of L-C circuit, the potential difference (v) across the inductor (L) is zero and that across the capacitor = applied potential difference. In case of L-R circuit, = (V) across inductor (L) = across (R) = applied voltage.

For AC circuit in steady state, I_{rms} current flows through the capacitor (c), inductor (R) and (L) and resistor (R). The potential difference across resistor, inductor and capacitor I. And for changing current, the potential difference across (V) inductor (L), capacitor (c) or resistor (R) $\propto$ Current (I).

3. **(b)** When charging is complete, the potential difference between the capacitor plates will be V and the charge stored in this case will be maximum.

$\therefore Q_0 = CV$.

When $t = 2\tau$, $Q = CV\left[1 = e^{\frac{-2\tau}{\tau}}\right]$
$= CV[1 - e^{-2}]$

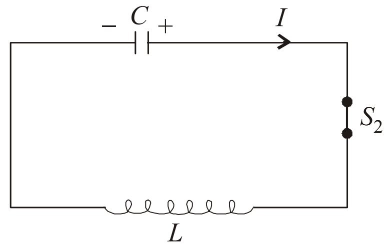


4. **(d)** Instantaneous charge on plates at any time t during discharging

$Q_{inst} = Q_0 \cos \omega t$

$\therefore$ Instantaneous current,

$I_{instan} = \dfrac{dQ}{dt} = Q_0\,\omega \sin \omega t$



$\therefore\ I_{max} = Q_0\omega$

Here $Q_0 = CV$ and $\omega = \dfrac{1}{\sqrt{LC}}$

$\therefore\ I_{max} = CV \times \dfrac{1}{\sqrt{LC}} = V\sqrt{\dfrac{C}{L}}$

5. **(c)** Applying Kirchhoff 's law

$\dfrac{Q}{C} - L\dfrac{dI}{dt} = 0 \Rightarrow \dfrac{Q}{C} = L\dfrac{dI}{dt}$

$\Rightarrow\ Q = LC\dfrac{d}{dt}\left(-\dfrac{dQ}{dt}\right) = -LC\dfrac{d^2Q}{dt^2}$

6. **(a)** Step up transformer $\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p} \Rightarrow \dfrac{10}{1} = \dfrac{V_s}{4000}$

$\therefore\ V_s = 40{,}000$ V

Step down transformer $\dfrac{N_p}{N_s} = \dfrac{V_p}{V_s} = \dfrac{40{,}000}{200} = \dfrac{200}{1}$

7. **(b)** Power $P = V \times I$

$\Rightarrow\ I = \dfrac{P}{V} = \dfrac{600 \times 1000}{4000} = 150$ A

Total resistance = $0.4 \times 20 = 8\ \Omega$

$\therefore$ Power dissipated as heat = $I^2R = (150)^2 \times 8 = 180{,}000\text{W} = 180$ kW

$\therefore\ \%$ loss $= \dfrac{180}{600} \times 100 = 30\%$

22 Electromagnetic Waves

Topic-1 : Electromagnetic Waves, Conduction and Displacement Current

1. **(a)** Relation between electric field E_0 and magnetic field B_0 of an electromagnetic wave is given by

$c = \frac{E_0}{B_0}$ (Here, c = Speed of light)

$\Rightarrow E_0 = B_0 \times c = 1.2 \times 10^{-7} \times 3 \times 10^8 = 36$

As the wave is propagating along x-direction, magnetic field is along z-direction

and $(\hat{E} \times \hat{B}) \parallel \hat{C}$

$\therefore \vec{E}$ should be along y-direction.

So, electric field $\vec{E} = E_0 \sin \vec{E} \cdot (x, t)$

$= [-36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

2. **(c)** In electromagnetic wave, $\frac{E_0}{B_0} = C$

$\therefore$ Maximum value of magnetic field, $B_0 = \frac{E_0}{C}$

$F_{\max} = qVB_{\max} \sin 90° = \frac{qV_0E_0}{C}$

(Given $V_0 = 0.1$ C and $E_0 = 30$)

$= \frac{1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 30}{3 \times 10^8} = 4.8 \times 10^{-19}$N

3. **(a)** $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$

Direction of propagation of em wave $= +\hat{k}$

Unit vector in the direction of electric field, $\hat{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

The direction of electromagnetic wave is perpendicular to both electric and magnetic field.

$\therefore \hat{k} = \hat{E} \times \hat{B}$

$\Rightarrow \hat{k} = \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) \times \hat{B} \Rightarrow \hat{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$

$\therefore \vec{B} = \frac{E_0}{c}(-\hat{x} + \hat{y})\sin(kz - \omega t)$

4. **(d)** Given : $\overline{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{i}T$

$\therefore B_0 = 3 \times 10^{-8}$

$E_0 = CB_0 \Rightarrow E_0 = 3 \times 10^8 \times 3 \times 10^{-8} = 9$ V/m

Directiono f wave propagation

$(\overline{E} \times \overline{B}) \parallel \overline{C}$

$\hat{B} = \hat{i}$ and $\hat{C} = -\hat{j}$ $\therefore \hat{E} = -\hat{k}$

$\therefore \overline{E} = E_0 \sin[200\pi(y + ct)](-\hat{k})$ V/m

or, $\overline{E} = -9\sin[200\pi(y + ct)]\hat{k}$ V/m

5. **(c)** Relation between electric field and magnetic field for an electromagnetic wave in vacuum is $B_0 = \frac{E_0}{c}$.

In free space, its speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Here, μ_0 = absolute permeability, ε_0 = absolute permittivity

$\therefore B_0 = \frac{E_0}{c} = \frac{E_0}{1/\sqrt{\mu_0\varepsilon_0}} = E_0\sqrt{\mu_0\varepsilon_0}$

As the electromagnetic wave is propagating along x direction and electric field is along y direction.

$\therefore \hat{E} \times \hat{B} \parallel \hat{C}$ (Here, $\hat{C}$ = direction of propagation of wave)

$\therefore \vec{B}$ should be in $\hat{k}$ direction.

$\therefore$ B = $E_0\sqrt{\mu_0\varepsilon_0}$ cos (ωt – kx) $\hat{k}$

At t = 0

B = $E_0\sqrt{\mu_0\varepsilon_0}$ cos (kx) $\hat{k}$

6. **(b)** Energy density $= \frac{1}{2}\frac{B^2}{\mu_0} \Rightarrow B = \sqrt{2 \times \mu_0 \times \text{Energy density}}$

$\mu_0 = \frac{1}{C^2\varepsilon_0} = 4\pi \times 10^{-7}$

$\therefore B = \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}} = 160 \times 10^{-9} = 160$ nT

7. **(a)** Electromagnetic wave will propagate perpendicular to the direction of Electric and Magnetic fields

$\hat{C} = \hat{E} \times \hat{B}$

Here unit vector $\hat{C}$ is perpendicular to both $\hat{E}$ and $\hat{B}$

Given, $\vec{E} = \hat{k}, \vec{B} = 2\hat{i} - 2\hat{j}$

$\therefore \hat{C} = \hat{E} \times \hat{B} = \frac{1}{\sqrt{2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} \Rightarrow \hat{C} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

8. **(a)** Direction of polarisation $= \hat{E} = \hat{k}$

Direction of propagation $= \hat{E} \times \hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

But $\vec{E}.\vec{B} = 0 \therefore \hat{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$

9. **(d)** Amplitude of electric field (E) and Magnetic field (B) of an electromagnetic wave are related by the relation

$\frac{E}{B} = c \Rightarrow E = Bc$

$\Rightarrow E = 5 \times 10^{-8} \times 3 \times 10^8 = 15$ N/C

$\Rightarrow \vec{E} = 15\hat{i}\ V/m$

10. **(b)** Given, $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10}$ t)

Using, $E_0 = B_0 \times C = 3 \times 10^{-8} \times 3 \times 10^8 = 9$ V/m

$\therefore$ Electric field,

$\vec{E} = 9\sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\hat{k}\ V/m$

11. **(c)** At $t = 0$, $z = \frac{\pi}{k}$

$\therefore \vec{E} = \frac{E_0}{\sqrt{2}}(\hat{i} + \hat{j})\cos[\pi] = -\frac{E_0}{\sqrt{2}}(\hat{i} + \hat{j})$

$\vec{F}_E = q\vec{E}$

Force due to electric field will be in the direction $\frac{-(\hat{i} + \hat{j})}{\sqrt{2}}$

Force due to magnetic field is in direction

$q(\vec{v} \times \vec{B})$ and $\vec{v} \parallel \hat{k}$. Therefore, it is parallel to $\vec{E}$.

$\Rightarrow \vec{F}_{\text{net}} = \vec{F}_E + \vec{F}_B$ is antiparallel to $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

12. **(c)** $\hat{S} = \frac{6\hat{j} + 8\hat{k}}{\sqrt{6^2 + 8^2}} = \frac{-3\hat{j} + 4\hat{k}}{5}$

13. **(b)** $B_0 = \frac{E_0}{C} = \frac{60}{3\times10^8}$
$= 20 \times 10^{-8}\ T = 2 \times 10^{-7}\ T$
$K = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi\times23.9\times10^9}{3\times10^8} = 500$
Therefore, $\vec{B} = B_0 \sin(kz - \omega t)$
$= 2 \times 10^{-7}\ \sin(0.5\times10^3 z - 1.5\times10^{11} t)i$

14. **(a)** $\frac{E_0}{B_0} = C \Rightarrow B_0 = \frac{E_0}{C}$
Given that $\vec{E} = E_0\cos(kz)\cos(\omega t)\hat{i}$
$\vec{E} = \frac{E_0}{2}\left[\cos(kz-\omega t)\hat{i} - \cos(kz+\omega t)\hat{i}\right]$
Correspondingly
$\vec{B} = \frac{B_0}{2}\left[\cos(kz-\omega t)\hat{j} - \cos(kz+\omega t)\hat{j}\right]$
$\vec{B} = \frac{B_0}{2}\times 2\sin kz \sin\omega t$
$\vec{B} = \left(\frac{E_0}{C}\sin kz\sin\omega t\right)\hat{j}$

15. **(c)** Pressure, $P = \frac{I}{C}$
$\Rightarrow \frac{F}{A} = \frac{I}{C} \quad \Rightarrow F = \frac{IA}{C} = \frac{\Delta p}{\Delta t} \quad \Rightarrow \Delta p = \frac{I}{C}A\Delta t$
$= \frac{(25\times25)\times10^4\times10^{-4}\times40\times60}{3\times10^8}$ N-s
$= 5 \times 10^{-3}$ N-s

16. **(b)** As we know,
$|\vec{B}| = \frac{|\vec{E}|}{C} = \frac{6.3}{3\times10^8} = 2.1\times10^{-8}\ T$ and $\hat{E}\times\hat{B} = \hat{C}$
$\hat{j}\times\hat{B} = \hat{i}$ [$\because$ EM wave travels along +(ve) x-direction.]
$\therefore\ \hat{B} = \hat{k}$ or $\vec{B} = 2.1\times10^{-8}\hat{k}T$

17. **(b)** $F = (1+r)\frac{IA}{C}$
$= \frac{(1+0.25)\times50\times1}{3\times10^8} \simeq 20\times10^{-8}$ N

18. **(a)** The relation between amplitudes of electric and magnetic field in free space is given by
$B_0 = \frac{E_0}{c} = \frac{6}{3\times10^8} = 2\times10^{-8}T$
Propagation direction $= \hat{E}\times\hat{B}$
$\hat{i} = \hat{j}\times\hat{B} \Rightarrow \hat{B} = \hat{k}$
$\therefore$ The magnetic field component will be along z direction.

19. **(c)** $E_0 = cB_0 = 3 \times 10^8 \times 1.6 \times 10^{-6} = 4.8 \times 10^2$ V/m
Also $\vec{S} \Rightarrow \vec{E}\times\vec{B}$
or $-\vec{K} \Rightarrow \vec{E}\times(2\hat{i}+\hat{j})$
Therefore direction of $\vec{E} \rightarrow (-\hat{i}+2\hat{j})$

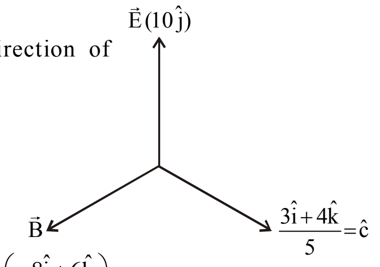
20. **(d)** $I = \frac{B_0^2}{2\mu_0}\cdot C \Rightarrow \frac{B_0^2}{2} = \frac{I\mu_0}{C}$
$\Rightarrow B_{rms} = \sqrt{\frac{I\mu_0}{C}} = \sqrt{\frac{10^8\times4\pi\times10^{-7}}{3\times10^8}} \simeq 6\times10^{-4}$ T
Which is closest to 10^{-4}.

21. **(c)** The speed of electromagnetic wave in free space is given by
$C = \frac{1}{\sqrt{\mu_0\in_0}}$ ...(i)
In medium, $v = \frac{1}{\sqrt{k\in_0\mu_0}}$ ...(ii)
Dividing equation (i) by (ii), we get
$\therefore\ \frac{C}{V} = \sqrt{k} = n$
$\frac{1}{2}\in_0 E_0^2 C = \text{intensity} = \frac{1}{2}\in_0 kE^2 v$
$\therefore\ E_0^2C = kE^2v$
$\Rightarrow \frac{E_0^2}{E^2} = \frac{kV}{C} = \frac{n^2}{n} \Rightarrow \frac{E_0}{E} = \sqrt{n}$
similarly
$\frac{B_0^2C}{2\mu_0} = \frac{B^2v}{2\mu_0} \Rightarrow \frac{B_0}{B} = \frac{1}{\sqrt{n}}$

22. **(d)** EM wave intensity
$\Rightarrow I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0E_0^2c$
[where E_0= maximum electric field]
$\Rightarrow \frac{27\times10^{-3}}{10\times10^{-6}} = \frac{1}{2}\times9\times10^{-12}\times E_0^2\times3\times10^8$
$\Rightarrow E_0 = \sqrt{2}\times10^3$ kV / m=1.4kV / m

23. **(b)** Using, formula $E_0 = B_0 \times C$
$= 100 \times 10^{-6} \times 3 \times 10^8$
$= 3 \times 10^4$ N/C
Here we assumed that
$B_0 = 100 \times 10^{-6}$ is in tesla (T) units

24. **(b)** $\vec{E} = 10\hat{j}\cos\left[(6\hat{i}+8\hat{k}).(x\hat{i}+z\hat{k})\right]$
$= 10\hat{j}\cos\left[\vec{K}.\vec{r}\right]$
$\therefore \vec{K} = 6\hat{i}+8\hat{K}$; direction of waves travel
i. e. direction of 'c'.
$\hat{C}\times\hat{E} = \frac{-4\hat{i}+3\hat{k}}{5}$
$\vec{B} = \frac{E}{C} = \frac{10}{C}$
$\therefore \vec{B}\frac{10}{C}\left(\frac{-4\hat{i}+3\hat{k}}{5}\right) = \left(\frac{-8\hat{i}+6\hat{k}}{C}\right)$



or, magnetic field $\vec{B}(x,z,t) = \frac{1}{C}$
$(6\hat{k}-8\hat{i})\cos(6x+8z-10ct)$

25. **(c)** Velocity of EM wave is given by $v = \frac{1}{\sqrt{\mu\in}}$
Velocity in air $= \frac{\omega}{k} = C$
Velocity in medium $= \frac{C}{2}$
Here, $\mu_1 = \mu_2 = 1$ as medium is non-magnetic
$\therefore\ \frac{\frac{1}{\sqrt{\in_{r_1}}}}{\frac{1}{\sqrt{\in_{r_2}}}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \Rightarrow \frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$

26. **(d)** Average energy density of magnetic field,
$u_B = \frac{B_0^2}{4\mu_0}$.

Average energy density of electric field,

$u_E = \dfrac{\varepsilon_0 E_0^2}{4}$

Now, $E_0 = CB_0$ and $C^2 = \dfrac{1}{\mu_0 \in_0}$

$u_E = \dfrac{\varepsilon_0}{4} \times C^2 B_0^2 = \dfrac{\varepsilon_0}{4} \times \dfrac{1}{\mu_0 \varepsilon_0} \times B_0^2 = \dfrac{B_0^2}{4\mu_0} = u_B$

$\therefore u_E = u_B$

Since energy density of electric and magnetic field is same, so energy associated with equal volume will be equal i.e., $u_E = u_B$

27. **(c)** $\hat{E} \times \hat{B}$ should give the direction of wave propagation

$\Rightarrow \hat{K} \times \hat{B} \parallel \dfrac{\hat{i} \times \hat{j}}{\sqrt{2}} \Rightarrow \hat{K} \times \left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \dfrac{\hat{j}-(-\hat{i})}{\sqrt{2}} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}} \parallel \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

Option (a), option (b) and option (d) does not satisfy.

Wave propagation vector $\hat{K}$ should along $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$.

28. **(c)** Given, Electric field component of monochromatic radiation,

$(\vec{E}) = 2E_0 \hat{i} \cos kz \cos \omega t$

We know that, $\dfrac{dE}{dz} = -\dfrac{dB}{dt}$

$\dfrac{dE}{dz} = -2E_0 k \sin kz \cos \omega t = -\dfrac{dB}{dt}$

$dB = +2E_0 k \sin kz \cos \omega t \, dt$ ..... (i)

Integrating eq^n (i), we have

$B = +2E_0 k \sin kz \int \cos \omega t \, dt$

Magnetic field is given by,

$= +2E_0 \dfrac{k}{\omega} \sin kz \sin \omega t$

We also know that,

$\dfrac{E_0}{B_0} = \dfrac{\omega}{k} = c$

Magnetic field vector,

$\vec{B} = \dfrac{2E_0}{c} \hat{j} \sin kz \sin \omega t$

29. **(d)** Wave in X-direction means E and B should be function of x and t.

$\hat{y} - \hat{z} \perp \hat{y} + \hat{z}$

30. **(b)** As we know, $\vec{E}.\vec{B} = 0 \quad \because [\vec{E} \perp \vec{B}]$

and $\vec{E} \times \vec{B}$ should be along Z direction

As $(-2\hat{i} - 3\hat{j}) \times (3\hat{i} - 2\hat{j}) = 5\hat{k}$

Hence option (b) is the correct answer.

31. **(c)** $E_0 = CB_0$ and $C = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Electric energy density $= \dfrac{1}{2}\varepsilon_0 E_0{}^2 = \mu_E$

Magnetic energy density $= \dfrac{1}{2}\dfrac{Bo^2}{\mu_0} = \mu_B$

Thus, $\mu_E = \mu_B$

Energy is equally divided between electric and magnetic field.

32. **(b)** Wavelength of monochromatic green light

$= 5.5 \times 10^{-5}$ cm

Intensity $I = \dfrac{\text{Power}}{\text{Area}} = \dfrac{100 \times (3/100)}{4\pi(5)^2} = \dfrac{3}{100\pi} \text{Wm}^{-2}$

Now, half of this intensity (I) belongs to electric field and half of that to magnetic field, therefore,

$\dfrac{I}{2} = \dfrac{1}{4}\varepsilon_0 E_0^2 C$

or $E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 C}}$

$= \sqrt{\dfrac{2 \times \left(\dfrac{3}{100}\pi\right)}{\left(\dfrac{1}{4\pi \times 9 \times 10^9}\right) \times (3 \times 10^8)}} = \sqrt{\dfrac{6}{25} \times 30} = \sqrt{7.2}$

$\therefore \quad E_0 = 2.68$ V/m

33. **(b)** From question,

$B_0 = 20\ nT = 20 \times 10^{-9} T$

($\because$ velocity of light in vacuum $C = 3 \times 10^8$ ms^{-1})

$\vec{E}_0 = \vec{B}_0 \times \vec{C}$

$|\vec{E}_0| = |\vec{B}| \cdot |\vec{C}| = 20 \times 10^{-9} \times 3 \times 10^8 = 6$ V/m.

34. **(b)**

Topic-2 : Electromagnetic Spectrum

1. **(b)** The orderly arrangement of different parts of EM wave in decreasing order of wavelength is as follows:

$\lambda_{\text{radiowaves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X-rays}}$

2. **(c)** $\xrightarrow{\text{E, Decreases}}$

γ-rays X-rays uv-rays Visible rays VIBGYOR IR rays Microwaves Radio waves

Radio wave < yellow light < blue light < X-rays

(Increasing order of energy)

3. **(c)** Microwave oven acts on the principle of giving vibrational energy to water molecules.

4. **(c)** Gamma rays < X-rays < Ultra violet < Visible rays < Infrared rays < Microwaves < Radio waves.

5. **(a)** $E = \dfrac{hc}{\lambda} \Rightarrow \lambda = \dfrac{hc}{E}$

$\Rightarrow \lambda = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{11 \times 1000 \times 1.6 \times 10^{-19}}$

$= 12.4$ Å

Increasing order of frequency →

x-rays u-v rays visible Infrared

wavelength range of visible region is 4000Å to 7800Å.

6. **(d)** Energy sequence of radiations is

$E_{\gamma\text{-Rays}} > E_{\text{X-Rays}} > E_{\text{microwave}} > E_{\text{AM Radiowaves}}$

$\therefore \lambda_{\gamma\text{-Rays}} < \lambda_{\text{X-Rays}} < \lambda_{\text{microwave}} < \lambda_{\text{AM Radiowaves}}$

From the above sequence, we have

(a) Microwave $\to 10^{-3}$ m (iv)

(b) Gamma Rays $\to 10^{-15}$ m (ii)

(c) AM Radio wave $\to 100$ m (i)

(d) X-Rays $\to 10^{-10}$ m (iii)

7. **(c)** Optical Fibre Communication – Infrared Light

Radar – Radio Waves

Sonar – Ultrasound

Mobile Phones – Microwaves

8. **(d)**

(1) Infrared rays are used to treat muscular strain because these are heat rays.

(2) Radio waves are used for broadcasting because these waves have very long wavelength ranging from few centimeters to few hundred kilometers.

(3) X-rays are used to detect fracture of bones because they have high penetrating power but they can't penetrate through denser medium like dones.

(4) Ultraviolet rays are absorbed by ozone of the atmosphere.

9. **(d)** Wavelength emitted by atomic hydrogen in interstellar space - Part of short radio wave of electromagnetic spectrum.

Doublet of sodium - visible radiation.

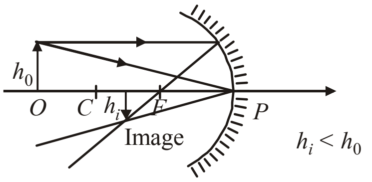
10. **(d)** Vibration of atoms and molecules 700 nm to 1 mm

Radioactive decay of the nucleus < 10^{-3} nm

Magnetron valve 1 mm to 0.1 m

Topic-1 : Plane Mirror, Spherical Mirror and Reflection of Light

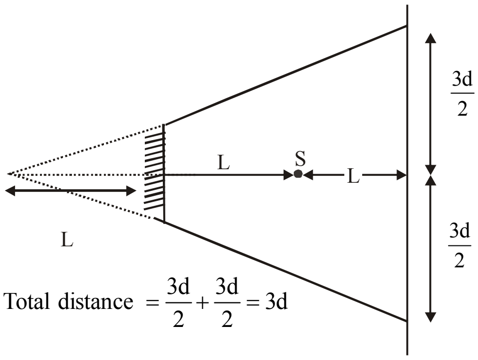
1. **(d)** Object is placed beyond radius of curvature (R) of concave mirror hence image formed is real, inverted and diminished or unmagnified.



2. **(c)** $+5 = -\frac{v}{u} \Rightarrow v = -5u$

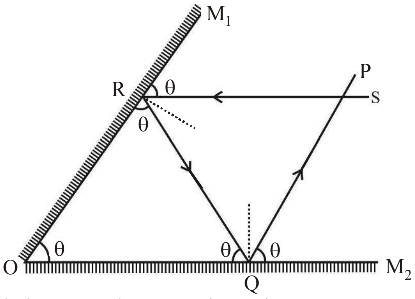
Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{-5u}+\frac{1}{u}=\frac{1}{0.4}$ $\therefore$ u = 0.32 m

3. **(c)**



Total distance $= \frac{3d}{2}+\frac{3d}{2} = 3d$

4. **(b)**



Let angle between the two mirrors be θ.

Ray PQ || mirror M_1 and Rs || mirror M_2

$\therefore$ $M_1Rs = \angle ORQ = \angle M_1OM_2 = \theta$

Similarly, $\angle M_2QP = \angle OQR = \angle M_2OM_1 = \theta$

$\therefore$ In ΔORQ, $3\theta = 180° \Rightarrow \theta = \frac{180°}{3} = 60°$

5. **(c)** Using mirror formula, magnification is given by

$$m = \frac{f}{u-f} = \frac{-1}{1-\frac{u}{f}}$$

At focus magnification is ∞

And at $u = 2f$, magnification is 1.

Hence graph (d) correctly depicts 'm' versus distance of object 'x' graph.

6. **(c)** When object is at 8 cm

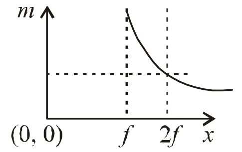


Image $V_1 = \frac{f \times u}{u-f} = \frac{5\times 8}{8-5} = -\frac{40}{3}$ cm

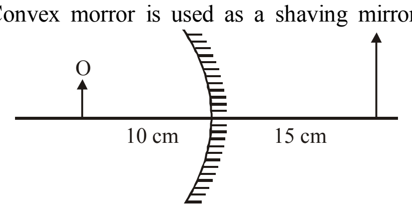
When object is at 12 cm

Image $V_2 = \frac{f \times u}{u-f} = \frac{5\times 12}{12-5} = -\frac{60}{7}$ cm

Separation $= |V_1 - V_2| = \frac{40}{3} - \frac{60}{7} = \frac{100}{21}$ cm

So A, C and D are correct statements.

7. **(c)** Convex morror is used as a shaving mirror.



From question : $v = 15$ cm, $u = -10$ cm

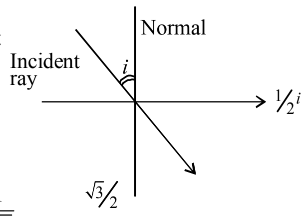
Radius of curvature, $R = 2f = ?$

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\frac{1}{15}+\frac{1}{(-10)}=\frac{1}{f} \Rightarrow f = -30$ cm

Therefore radius of curvature, $R = 2f = -60$ cm

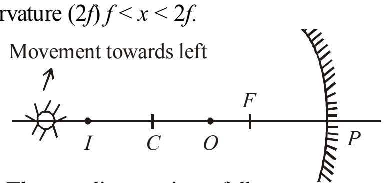
8. **(a)** According to question, ray of light is travelling in $\frac{1}{2}(i+\sqrt{3}j)$ direction incident on a plane mirror.



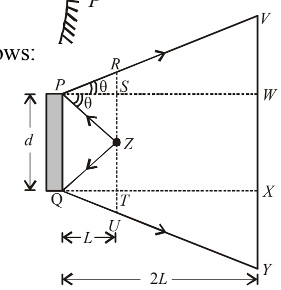
From figure, $\tan i = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\therefore$ $i = 30°$ (Angle of incidence)

9. **(b)** As shown in the figure, when the object (O) is placed between F and C, the image (I) is formed beyond C. It is in this condition that when the student shifts his eyes towards left, the image appears to the right of the object pin. Object O lies between focus (f) and centre of curvature ($2f$) $f < x < 2f$.



10. **(d)** The ray diagram is as follows:



From figure, $WX = PQ = d$

$ZS = RS = \frac{d}{2}$

$\because$ $PW = 2PS$

$\therefore$ $VW = 2RS = 2\frac{d}{2} = d$

Similarly, $XY = d$

$\therefore$ $VY = VW + WX + XY = d + d + d = 3d$

11. **(d)** From mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\therefore$ $\frac{dv}{v^2}\frac{du}{u^2}=0$

$\therefore$ $\frac{dv}{du}=\frac{-v^2}{u^2}=-\left(\frac{f}{u-f}\right)^2 \Rightarrow dv=-\left(\frac{f}{4-f}\right)^2 dx$

$\therefore$ Image size $=\left(\frac{f}{u-f}\right)^2 \times b$

12. **(3)** **Using mirror formula for first position**

$u_1=?, v_1=\frac{25}{3}\text{cm}, f=+10\text{cm}\left(=\frac{R}{2}\right)$

$\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{f}, \frac{3}{25}+\frac{1}{u_1}=\frac{1}{10}$ $\therefore u_1=-50$ m

Using mirror formula for the second position

$u_2=?, v_2=\frac{50}{7}$ and $f=10$ cm

$\frac{1}{v_2}+\frac{1}{u_2}=\frac{1}{f} \Rightarrow \frac{7}{50}+\frac{1}{u_2}=\frac{1}{10} \Rightarrow \frac{1}{u_2}=\frac{1}{10}-\frac{7}{50}$

$u_2=-25$m

Speed of object $=\frac{u_1-u_2}{\text{time}}=\frac{25}{30}\times\frac{18}{5}=3\,\text{km h}^{-1}$

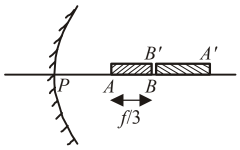
13. **(1)** Distance of object, $u=-30$ cm

Distance of image, $v=10$ cm

Magnification, $m=\frac{-v}{u}=\frac{(-10)}{-30}=\frac{1}{3}$

Speed of image $=m^2\times$ speed of object $=\frac{1}{9}\times 9=1$ cm s^{-1}

14. Since the image formed is real and elongated, the object lies between focus F and centre of curvature C the situation is as shown in the figure. Since the image of B is formed at B' itself



$\therefore$ B is situated at the centre of curvature that is at a distance of $2f$ from the pole.

$\therefore$ $PA=2f-\frac{f}{3}=\frac{5f}{3}$

Let us find the image of A. For point A, $u=-\frac{5f}{3}, v=?$

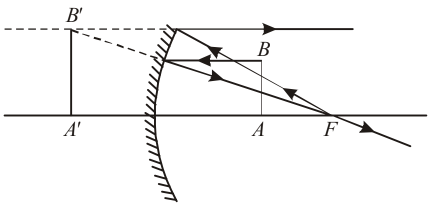
From formula, $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{\frac{-5f}{3}}+\frac{1}{v}=\frac{1}{-f}$

$\Rightarrow$ $\frac{1}{v}=-\frac{1}{f}+\frac{3}{5f}$ $\therefore$ $v=-2.5f$

Image length of rod $=2.5f-2f=0.5f$

$\therefore$ Magnification $M=\frac{v}{u}=\frac{0.5f}{f/3}=1.5\left[\text{Given } u=\frac{f}{3}\right]$

15. **(d)** Distance of point A from the mirror is $f/2$.



From mirror formula,

$\frac{1}{v}+\frac{1}{f/2}=\frac{1}{-f} \Rightarrow \frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}$ $\therefore$ $v=f$

Image $A'B'$ of line AB should be I principle axis. Image of F will be formed at infinity.

Also light ray from infinity or towards infinity seems parallel to the principle axis of the mirror.

16. **(c, d)** Given $f=-24$ cm

Applying mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$u-v$ values of options (a) and (b) match with mirror formula. Whereas option (c) and (d) do not match with mirror formula.

For (66, 33)

$\Rightarrow$ $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-24}+\frac{1}{66}=\frac{-66+24}{24\times 66}=\frac{-42}{24\times 66}$

$\Rightarrow$ $v=-\frac{24\times 66}{42}=-37.7$

But the value of $v=33$. The absolute error is $37.7-33=4.7$ cm which is greater than 0.2 cm. Therefore a wrong reading.

For (78, 39) when $u=78$ then

$\frac{1}{v}+\frac{1}{-78}=\frac{1}{-24}$ $\Rightarrow v=-34.67$

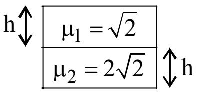
The absoluate error is $39-34.67=4.33$ which is greater than 0.2 cm.

17. **(c)** The formula connecting u, v and f for a spherical mirror $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ is valid only for mirrors of small apertures where the size of aperture is very small as compared to the radius of curvature of the mirror.

Laws of mirror are valid for plane as well as large spherical surfaces. The laws of reflection are valid when ever the light is reflected.

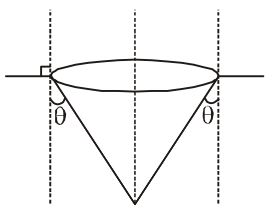
Topic-2 : Refraction of Light at Plane Surface and Total Internal Reflection

1. **(d)** Apparent depth,



$D=\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}=\frac{h}{\sqrt{2}}+\frac{h}{2\sqrt{2}}=\frac{3h}{2\sqrt{2}}=\frac{3h\sqrt{2}}{4}$

2. **(c)** Given,



Refractive index, $\mu=\frac{4}{3}$

$\frac{4}{3}\sin\theta=1\sin 90°$

$\Rightarrow \sin\theta=\frac{3}{4}$; $\cos\theta=\frac{\sqrt{7}}{4}$

Solid angle, $\Omega=2\pi(1-\cos\theta)=2\pi(1-\sqrt{7}/4)$

Fraction of energy transmitted

$=\frac{2\pi(1-\cos\theta)}{4\pi}=\frac{1-\sqrt{7}/4}{2}=0.17$

Percentage of light emerges out of surface $=0.17\times 100=17\%$

3. **(b)** Here, from question, relative permittivity

$\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}=3 \Rightarrow \varepsilon=3\varepsilon_0$

Relative permeability $\mu_r = \frac{\mu}{\mu_0} = \frac{4}{3} \Rightarrow \mu = \frac{4}{3}\mu_0$

$\therefore \mu\varepsilon = 4\mu_0\varepsilon_0$

$$\sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}} = \frac{v}{c} = \frac{1}{2} \left(\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$n = \sqrt{\mu_r\varepsilon_r} = \sqrt{\frac{4}{3}\times 3} = 2$$

And $n = \frac{1}{\sin\theta_c} \Rightarrow \sin\theta_c = \frac{1}{n} = \frac{1}{2}$

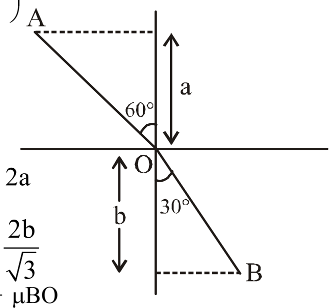
$\therefore$ Critical angle, $\theta_c = 30°$

4. **(c)** Using, $\sin\theta_{max} = \mu_1\sqrt{\mu_2^2 - \mu_1^2} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

or $\theta_{max} = \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right)$

For T_1R, $\theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right)$

5. **(d)** From the given figure



As $\sin 60° = \mu \sin 30°$

$\Rightarrow \mu = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

$\frac{a}{AO} = \cos 60° \Rightarrow AO = 2a$

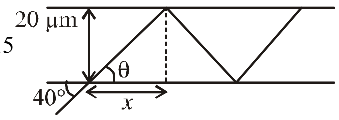
$\frac{b}{BO} = \cos 30° \Rightarrow BO = \frac{2b}{\sqrt{3}}$

Optical path length = AO + μBO

$= 2a + (\sqrt{3})\frac{2b}{\sqrt{3}} = 2a + 2b$

6. **(d)** Using Snell's law of refraction,

$1 \times \sin 40° = 1.31 \sin\theta$



$\Rightarrow \sin\theta = \frac{0.64}{1.31} = 0.49 \approx 0.5$

$\Rightarrow \theta = 30°$

$x = 20\ \mu m \times \cot\theta$

$\therefore$ Number of reflections $= \frac{2}{20\times 10^{-6}\times\cot\theta}$

$= \frac{2\times 10^6}{20\times\sqrt{3}} = 57735 \approx 57000$

7. **(c)** As 4% of light gets reflected, so only (100 – 4 = 96%) of light comes after refraction so,

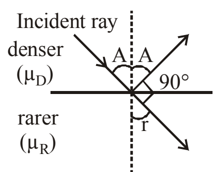
$P_{refracted} = \frac{96}{100}P_I \Rightarrow K_2A_t^2 = \frac{96}{100}K_1A_i^2$

$\Rightarrow r_2A_t^2 = \frac{96}{100}r_1A_i^2 \Rightarrow A_t^2 = \frac{96}{100}\times\frac{1}{\frac{3}{2}}\times(30)^2$

$A_t\sqrt{\frac{64}{100}\times(30)^2} = 24$

8. **(d)**

From Snell's law, $\frac{\mu_R}{\mu_D} = \frac{\sin i}{\sin r}$ ..... (i)



$\because \angle i = A$ and $\angle r = (90° - A)$

We also know that, $\sin\theta_C = \frac{\mu_R}{\mu_D}$

From eq^n (i), $\sin\theta_C = \frac{\sin A}{\sin(90° - A)}$

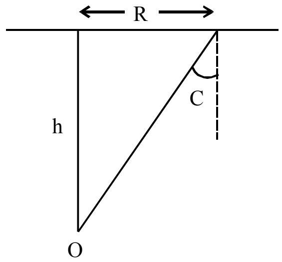
$\sin\theta_C = \frac{\sin A}{\cos A}$

$\sin\theta_C = \tan A$ or $A = \tan^{-1}(\sin\theta_C)$

9. **(d)** Given, $\mu = \frac{4}{3}$

h = 15 cm

R = ?



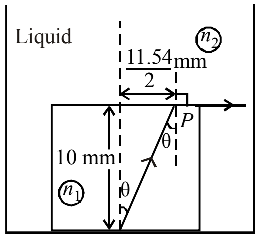
$\frac{\sin 90°}{\sin C} = \mu$

$\Rightarrow \sin C = \frac{1}{\mu} = \frac{R}{\sqrt{R^2 + h^2}} = \frac{3}{4}$

$\Rightarrow 16R^2 = 9R^2 + 9h^2$

or, $7R^2 = 9h^2$ or, $R = \frac{3}{\sqrt{7}}h = \frac{3}{\sqrt{7}}\times 15\,cm$

10. **(c)**



Applying Snell's law at point P

$n_1 \sin\theta = n_2 \sin 90°$

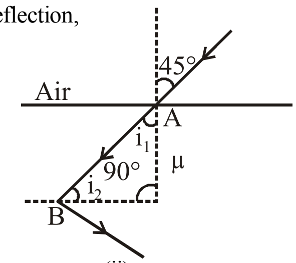
$\sin\theta = \frac{11.54/4}{\sqrt{10^2 + (11.54/2)^2}}$ and n_2 = refractive index of liquid

$\therefore n_2 = 2.72\times\frac{11.54/2}{\sqrt{(10)^2 + \left(\frac{11.54}{2}\right)^2}}$ $\therefore n_2 = 1.36$

11. **(d)** At point A by Snell's law

$\mu = \frac{\sin 45°}{\sin r} \Rightarrow \sin r = \frac{1}{\mu\sqrt{2}}$ ... (i)

At point B, for total internal reflection,



$\sin i_1 = \frac{1}{\mu}$

From figure, $i_1 = 90° - r$

$\therefore (\sin 90° - r) = \frac{1}{\mu}$

$\Rightarrow \cos r = \frac{1}{\mu}$ ... (ii)

Now $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{1}{2\mu^2}} = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}}$ ... (iii)

From eqs (ii) and (iii) $\frac{1}{\mu} = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}}$

Squaring both sides and then solving, we get $\mu = \sqrt{\frac{3}{2}}$

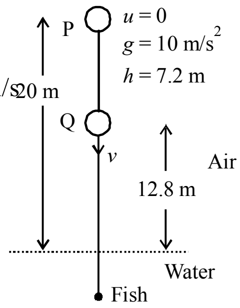
12. **(c)** Ball falling P to Q

Applying $v^2 - u^2 = 2gh$

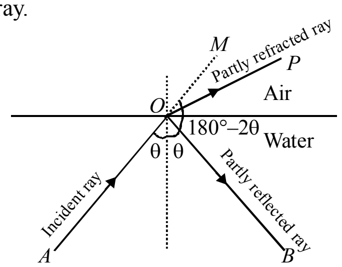
$v^2 - 0^2 = 2 \times 10 \times 7.2 \Rightarrow v = 12 \text{ m/s}$

Velocity of ball as perceived by fish

$$v' = {}_w\mu \times v = \frac{4}{3} \times 12 = 16 \text{ m/s}$$



13. **(c)** The incident ray is partly reflected and partly refracted. $\angle MOB = 180° - 2\theta > \angle POB$ the angle between refracted and reflected ray.



14. **(a)** Distance of virtual image from surface = 6 cm

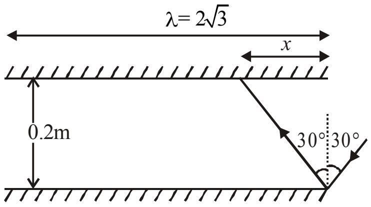
The rays coming from the point object fall on the glass-air interface normally and hence pass undeviated. Therefore if we retrace the path of the refracted rays backwards, the image will be formed at the centre only.

15. **(b)** From snell's law, $\mu_g \sin i = \mu_{air} \sin 90°$

$\therefore \quad \mu_g = \dfrac{1}{\sin i}$

16. **(b)** Maximum number of reflection $= \dfrac{2\sqrt{3}}{x}$

where $x = 0.2 \tan 30° = 0.2/\sqrt{3}$.



$\therefore$ Maximum number of reflection $= \dfrac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$

17. **(b)** For the image of point P of the rod PQ to be seen by the observer, it should be formed at point Q.

In ΔQNS,

$NS = QS = 2h$

$\therefore \quad \angle NQS = 45°$

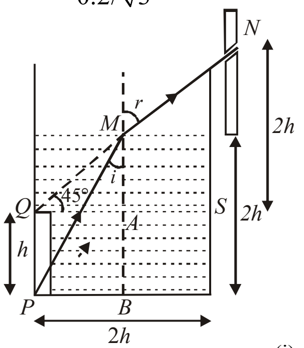
$\therefore \quad r = 45°$

Now in ΔQMA,

$\angle MQA = 45°$

$\therefore \quad MA = QA = h$

$$ {}^1_2\mu = \frac{\sin r}{\sin i} = \frac{\sin 45°}{\sin i} \quad \text{...(i)}$$



In ΔPMB,

$PM^2 = 4h^2 + h^2 = 5h^2$

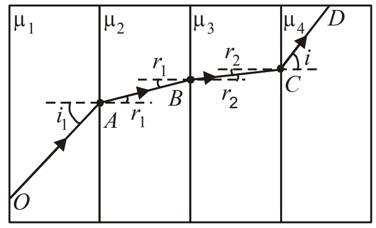
$$\therefore \quad \sin i = \frac{h}{\sqrt{5}h} = \frac{1}{\sqrt{5}} \quad \text{...(ii)}$$

From eq. (i) and (ii)

$$ {}^1_2\mu = \frac{\sin 45°}{\sin i} = \sqrt{\frac{5}{2}}$$

18. **(d)** Applying Snell's law at A,

$$ {}^1\mu_2 = \frac{\sin i}{\sin r_1} = \frac{\mu_2}{\mu_1} \quad \text{...(i)}$$



Applying Snell's law at B,

$$ {}^2\mu_3 = \frac{\sin r_1}{\sin r_2} = \frac{\mu_3}{\mu_2} \quad \text{...(ii)}$$

Again applying Snell's law at C

$$ {}^3\mu_4 = \frac{\sin r_2}{\sin i} = \frac{\mu_4}{\mu_3} \quad \text{...(iii)}$$

Multiplying eq. (i), (ii) and (iii)

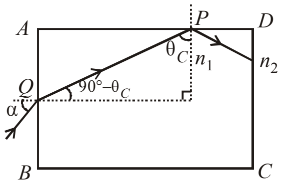
$$\frac{\mu_2}{\mu_1} \times \frac{\mu_3}{\mu_2} \times \frac{\mu_4}{\mu_3} = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i}$$

$\therefore \quad \dfrac{\mu_4}{\mu_1} = 1 \quad \therefore \quad \mu_1 = \mu_4$

If the emergent ray is parallel to incident ray after travelling a number of parallel interfaces then the refractive index of the first and the last medium is always same.

Here incident ray is medium-1 OA || CD emergent ray in medium-4 $\quad \therefore \quad \mu_1 = \mu_4$

19. **(a)** From the diagram, the ray will come out from CD if it suffers total internal reflection at surface AD, i.e., it strikes the surface AD at critical angle θ_C (the limiting case).



Applying Snell's law at P face AD of slab

$n_1 \sin \theta_C = n_2$ or $\sin \theta_C = \dfrac{n_2}{n_1}$

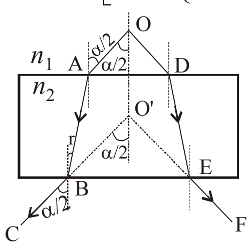
Applying Snell's law at Q

$n_2 \sin \alpha = n_1 \cos \theta_C$

$$\Rightarrow \quad \sin \alpha = \frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$$

$$\text{or} \quad \alpha = \sin^{-1}\left[\frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$$

20. **(b)**



The incident and emergent ray OA ∥ BC & OD ∥ EF of a glass slab are parallel therefore, the divergence angle remains the same.

21. **(a)** Applying Snell's law

$${}_{2}^{1}\mu = \frac{\sin r'}{\sin i} = \frac{\sin(90-r)}{\sin r} \qquad [\because i = r;\ r' + r = 90°]$$

$$\therefore\ {}_{2}^{1}\mu = \frac{\cos r}{\sin r}$$

Critical angle $\theta_c = \sin^{-1}\left(\frac{1}{14}\right) = \sin^{-1}\left(\frac{\sin r}{\cos r}\right)$

$\therefore\ \theta_c = \sin^{-1}(\tan r)$

22. **(a)** **Frequency (v)** does not change with medium
Glass slab is an optically denser medium, the velocity of light decreases and therefore the wavelength in glass decreases.

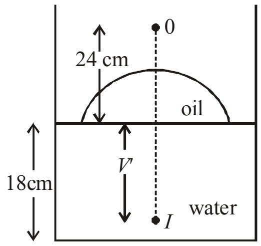
23. **(8)** Here, $n \sin\theta = (n - m\Delta n) \sin 90°$

$\Rightarrow$ $n \times \sin 30° = [n - m \times 0.1] \sin 90°$

$\therefore$ $1.6 \times \sin 30° = 1.6 - m \times 0.1$ $\therefore$ $m = 8$

24. **(2)** For the convex spherical refracting surface i.e., air-oil interface

$u = -24$ cm, $v = ?$, $\mu_1 = 1$, $\mu_2 = \frac{7}{4}$ and $R = 6$ cm



$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore\ \frac{-1}{(-24)} + \frac{7/4}{v} = \frac{\frac{7}{4}-1}{6}$$

$\therefore$ $v = 21$ cm

This image will not as object for the water-oil interface

$u = 21$ cm, $v = v'$, $\mu_1 = \frac{7}{4}$, $\mu_2 = \frac{4}{3}$ and $R = \infty$

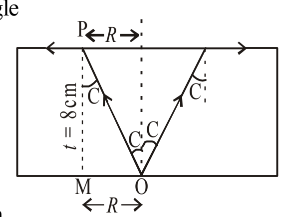
$$\frac{\frac{-7}{4}}{+21} + \frac{\frac{4}{3}}{V'} = 0$$

$\therefore$ $V' = 16$ cm.

Therefore the distance of the image from the bottom of the tank $= 18 - 16 = 2$ cm.

25. **(6)** In the figure, C = critical angle

In ΔPOM, $\tan C = \frac{\text{OM}}{\text{PM}} = \frac{R}{8}$



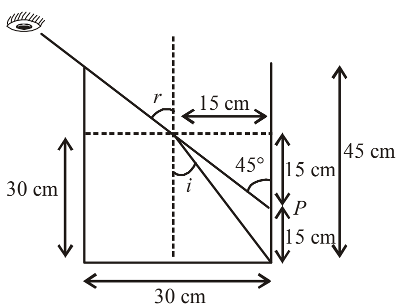
$\because$ $\sin C = \frac{1}{\mu} = \frac{3}{5}$ $\therefore$ $\tan C = \frac{3}{4}$

$\therefore$ $\frac{R}{8} = \frac{3}{4}$ $\therefore$ $R = \frac{3}{4} \times 8 = 6\,\text{cm}$

26. **(158)** From figure, $\sin i = \frac{15}{\sqrt{15^2 + 30^2}}$ and $\sin r = \sin 45°$

From Snell's law, $\mu \times \sin i = 1 \times \sin r$

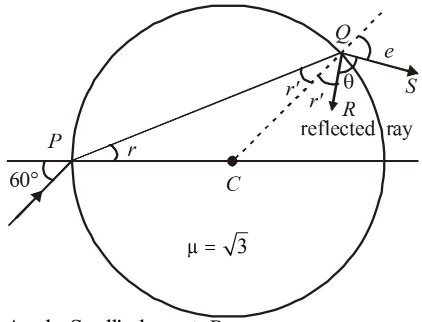
$$\Rightarrow \mu \times \frac{15}{\sqrt{15^2 + 30^2}} = 1 \times \sin 45° = \frac{1}{\sqrt{2}}$$



$$\therefore\ \mu = \frac{\frac{1}{\sqrt{2}}}{\frac{15}{\sqrt{1125}}} = 158 \times 10^{-2} = \frac{N}{100}$$

Hence, value of $N \simeq 158$.

27. **(90.00)**
In the figure, QR is the reflected ray and QS is refracted ray. CQ is normal.



Apply Snell's law at P

$1 \sin 60° = \sqrt{3} \sin r \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30°$

From geometry, $CP = CQ$

$\therefore r' = 30°$

Again apply snell's law at Q,

$\sqrt{3} \sin r' = 1 \sin e \Rightarrow \frac{\sqrt{3}}{2} = \sin e \Rightarrow e = 60°$

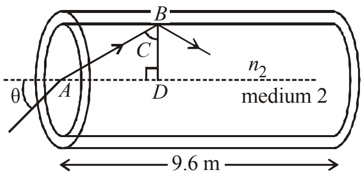
From geometry

$r' + \theta + e = 180°$ (As angles lies on a straight line)

$\Rightarrow 30° + \theta + 60° = 180° \Rightarrow \theta = 90°.$

28. **(50)** Let 'C' be the critical angle.
1.5 sin C = 1.44 sin 90°

$\Rightarrow$ $\sin C = \frac{1.44}{1.50}$



But $\sin C = \frac{AD}{AB}$

$\therefore$ $\frac{AD}{AB} = \frac{1.44}{1.50}$ $\therefore$ $AB = \frac{1.50}{1.44} AD$

If we replace AD by 9.6 then total length travelled by light $=$

$\frac{1.50}{1.44} \times 9.6 = 10\text{m}$

Maximum, time taken by a ray to exit the plane

$$t = \frac{d}{v_2} = \frac{d}{C/n_2}$$

$$\therefore\ t = \frac{10}{3 \times 10^8 / 1.5} = 5 \times 10^8\,\text{s} = 50 \times 10^{-9}\,\text{s}$$

Comparing it with $t \times 10^{-9}$ s we get, t = 50.00

29. Frequency does not change with medium

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14}\,\text{Hz}$$

Wavelength in medium, $\lambda^1 = \frac{\lambda}{\mu} = \frac{6000\text{Å}}{1.5} = 4000\text{Å}$

30. $\therefore \quad \lambda_{medium} = \dfrac{\lambda_{air}}{\mu} = \dfrac{6000}{1.5} = 4000\text{Å}$

Frequency does not change with the medium

$$f_{\text{medium}} = \frac{V_{medium}}{\lambda_{medium}} = \frac{V_{air}}{\lambda_{air}} = \frac{3\times10^8}{6\times10^{-7}} = 5\times10^{14}\,\text{Hz}$$

31. Velocity of light in medium, $V = \dfrac{c}{\mu}$

$$= \frac{3\times10^8}{1.5} = 2\times10^8\,\text{m/s};$$

Wavelength, $\lambda = \dfrac{v}{f} = \dfrac{2\times10^8}{5\times10^{14}} = 4\times10^{-7}\,m$

32. **(a, c, d)**

From Snell's law, $n_1 \sin\theta_i = n_2 \sin\theta_f$ [$\because$ 1 and 2 interfaces are parallel]

l depends on the refractive index of transparent slab $n(z)$ but not on n_2. In fact θ_f depends, on n_2.

Because lateral displacement (l) is possible due μ of slab and angle of incidence θi.

33. **(c, d)** For the ray undergoes total internal reflection

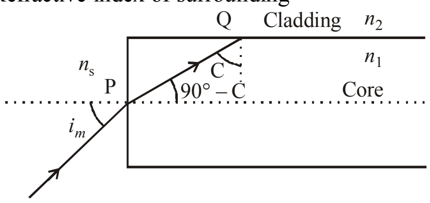
Angle of incidence, $i >$ critical angle, θ_c

or $\sin 45° > \dfrac{1}{n}$ or $\dfrac{1}{\sqrt{2}} > \dfrac{1}{n}$ or $n > \sqrt{2}$ or $n > 1.414$.

34. **(a, c)**

Using Snell's law at P; $n_s \sin i_m = n_1 \sin(90° - C)$ ...(i)

n_s = Refractive index of surrounding



Also $\sin C = \dfrac{n_2}{n_1}$ $\quad\therefore\quad$ $\cos C = \sqrt{1-\left(\dfrac{n_2}{n_1}\right)^2}$

Now from eq. (i)

Numerical aperture,

$$NA = \sin i_m = \frac{n_1}{n_s}\cos C = \frac{n_1}{n_s}\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\therefore \quad NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_s}$$

For S_1 (in air)

$NA = \sqrt{\dfrac{45}{16} - \dfrac{9}{4}} = \dfrac{3}{4}$

For S_1 (in water)

$NA = \dfrac{3}{4}\sqrt{\dfrac{45}{16} - \dfrac{9}{4}} = \dfrac{9}{16}$

For s_1 $\left(\text{in } n_s = \dfrac{6}{\sqrt{15}}\right)$

$NA = \dfrac{\sqrt{15}}{6}\sqrt{\dfrac{45}{16} - \dfrac{9}{4}}$

$= \dfrac{3\sqrt{15}}{24}$

For S_2 (in water)

$NA = \dfrac{3}{4}\sqrt{\dfrac{64}{25} - \dfrac{49}{25}}$

$= \dfrac{3}{4}\dfrac{\sqrt{15}}{5}$

For S_2 (in air)

$NA = \sqrt{\dfrac{64}{25} - \dfrac{49}{25}} = \dfrac{\sqrt{15}}{5}$

For S_2 (in $n_s = \dfrac{4}{\sqrt{15}}$)

$NA = \dfrac{\sqrt{15}}{4}\sqrt{\dfrac{64}{25} - \dfrac{49}{25}} = \dfrac{3}{4}$

For S_2 $\left(\text{in } n_s = \dfrac{16}{3\sqrt{15}}\right)$

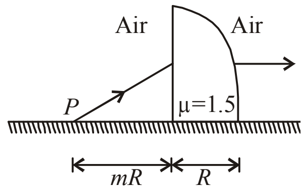
$NA = \dfrac{3\sqrt{15}}{16}\sqrt{\dfrac{64}{25} - \dfrac{49}{25}} = \dfrac{9}{16}$

35. **(d)** Numerical aperture, $NA = \dfrac{1}{n_s}\sqrt{n_1^2 - n_2^2}$

Here, $NA_2 < NA_1$

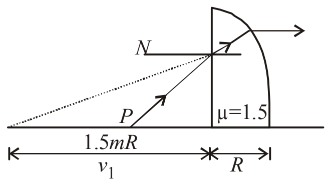
$\therefore$ Numerical aperture, of combined structure is equal to the smaller value of the two numerical apertures.

36. Apply formula for refraction at a spherical surface when light travels from air ($\mu_1 = 1$) to medium (μ_2).



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$$

First refraction occurs on plane surface for which $r = \infty$.



$$\therefore \quad \frac{1.5}{v_1} - \frac{1}{-mR} = \frac{1.5-1}{\infty}$$

or $\quad v_1 = -(1.5mR)$

Then, on curved surface

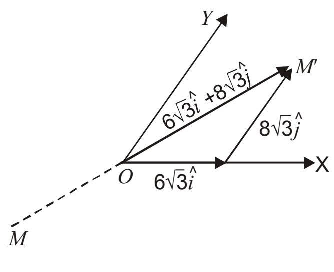
$\dfrac{\mu_1}{v_2} - \dfrac{\mu_2}{u_2} = \dfrac{\mu_1 - \mu_2}{r}$, where $u_2 = v_1 + R$, $v_2 = \infty$

or $\quad \dfrac{1}{\infty} - \dfrac{1.5}{-(1.5mR + R)} = \dfrac{1-1.5}{-R}$

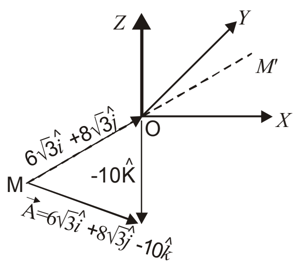
or $\quad \dfrac{1.5}{R(1.5m+1)} = \dfrac{0.5}{R}$ $\quad$ or $\quad 3 = 1.5m + 1$

or $\quad 1.5m = 2$ $\quad$ or $\quad m = \dfrac{4}{3}$

37. The x - y plane is the boundary between medium-1 and medium-2



(Fig. a)



(Fig. b)

Figure (a) shows vector $OM' = 6\sqrt{3}\text{i} + 8\sqrt{3}\hat{\text{j}}$

Figure (b) shows vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

The perpendicular to line MOM' is Z-axis which has a unit vector of $\hat{k}$.

Angle between vector $\overrightarrow{MP}$ and $\overrightarrow{OP}$ can be found by dot product.

$\overrightarrow{MP} \cdot \overrightarrow{OP} = (MP)(OP)\cos i$

$$\frac{\left(6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}-10\hat{k}\right).(-\hat{k})}{\sqrt{(6\sqrt{3})^2+(8\sqrt{3})^2+(-10)^2}+\sqrt{(-1)^2}} = \cos i$$

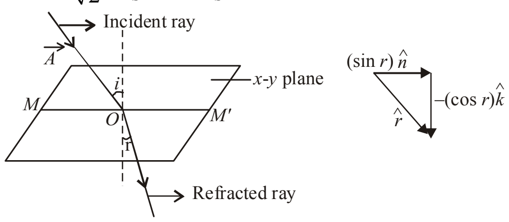
$\Rightarrow \quad i = 60°$

Unit vector in the direction of MOM' from fig. (a)

$$\hat{n} = \frac{6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}}{[(6\sqrt{3})^2+(8\sqrt{3})^2]^{1/2}}, \quad \hat{n} = \frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}$$

To find the angle of refraction, we use Snell's law

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin r} \Rightarrow r = 45°$$



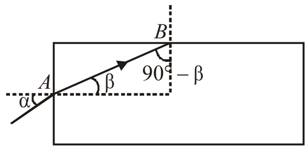
Now, $\hat{r} = (\sin r)\hat{n} - (\cos r)\hat{k}$

$$= (\sin 45°)\left[\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}\right] - (\cos 45°)\hat{k}$$

$$\therefore \quad r = \frac{1}{5\sqrt{2}}[3\hat{i}+4\hat{j}-5\hat{k}]$$

38. The angle of incidence at the curved surface of the cylindrical rod is given by $(90° - \beta)$.

The light entering the rod does not emerge from the curved surface of the rod, if the angle $(90° - \beta)$ is greater than the critical angle C.



i.e., $\mu \le \frac{1}{\sin C}$ where C is the critical angle.

Here, $C = 90° - \beta$

$$\Rightarrow \quad \mu \le \frac{1}{\sin(90°-\beta)} \Rightarrow \mu \le \frac{1}{\cos\beta}$$

As a limiting case, $\mu = \frac{1}{\cos\beta}$ ... (i)

Applying Snell's law at A

$$\mu = \frac{\sin\alpha}{\sin\beta} \Rightarrow \sin\beta = \frac{\sin\alpha}{\mu} \quad \text{... (ii)}$$

If $\alpha = \pi/2$ as a limiting case on planar end of rod

$$\therefore \quad \sin\beta = \frac{\sin\alpha}{\mu} = \frac{\sin\pi/2}{\mu} \Rightarrow \mu = \frac{1}{\sin\beta} \quad \text{... (iii)}$$

From eq. (i) and (iii), $\sin\beta = \cos\beta$

or $\quad \beta = 45°$

$$\therefore \quad \mu = \frac{1}{\cos 45°} = \frac{1}{1/\sqrt{2}} \quad \text{or} \quad \mu = \sqrt{2}$$

$\therefore$ The least value of the refractive index of rod for light entering the rod and not leaving it from the curved surface is $\sqrt{2}$.

39. (i) According to question, a parallel beam of light is travelling in water by a spherical bubble of radius 2 nm.

Initially the object is in denser medium and $u = \infty$ using the formula of refraction at a spherical surface for AB see diagram below.

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v} = \frac{\mu_1-\mu_2}{R} \Rightarrow \frac{-4/3}{-\infty}+\frac{1}{v} = \frac{1-4/3}{2}$$

$\therefore \quad v = -6$ mm

This is the position of the image due to refraction at the first surface i.e., at 6 mm left of first surface. This image will behave as a virtual object for the refraction at the second surface.

$u = -6 - 4 = -10$ mm

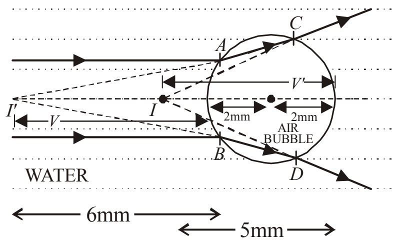
Again using the formula of refraction at a spherical surface for CD

$$-\frac{\mu_1}{u'}+\frac{\mu_2}{v'} = \frac{+\mu_2-\mu_1}{R}, \quad -\frac{1}{10}+\frac{4/3}{v'} = \frac{\frac{4}{3}-1}{-2}$$

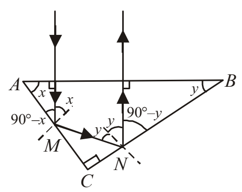
$\therefore \quad v' = -5$ mm.

i.e., Final image, is formed 5 mm to the left of second surface.

(ii) Ray diagram showing the positions of both the images.



40. (i) Let x is the incident angle for reflection at face AC of the prism. For total internal reflection $x > i_C$ (critical angle)



Let y be the incident angle of the ray on face CB of the prism. For total internal reflection

$y > i_C$

$\therefore \quad x + y > 2i_C$

But $\quad x = \angle A$ and $y = \angle B$ (from geometry)

$\therefore \quad x + y = 90°$

$\Rightarrow \quad 90 > 2i_C \Rightarrow i_C < 45°$

$\therefore$ Minimum value of refractive index n of the medium for this to be possible.

$$n = \frac{1}{\sin i_C} = \frac{1}{\sin 45°} = \sqrt{2}$$

(ii) For $n = \frac{5}{3}$

$$\sin i_C' = \frac{1}{\mu} = \frac{1}{5/3} = \frac{3}{5} \Rightarrow i_C' = 37°$$

$y = 30°$ (Given) $\therefore \; x = 60°$

$x > i_C'$ but $y < i_C'$

$\therefore$ Total internal reflection will take place on face AC but not on CB.

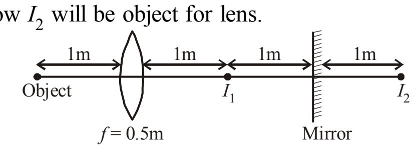
Topic-3 : Refraction at Curved Surface Lenses and Power of Lens

1. **(d)** Focal length of the convex lens, $f = 0.5$ m

Object is at $2f$ so, image (I_1) will also be at $2f$.

Image of I_1 i.e., I_2 will be 1 m behind mirror.

Now I_2 will be object for lens.



$\therefore u = (-1) + (-1) + (-1) = -3$ m

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+0.5} + \frac{1}{-3}$ or $v = \frac{3}{5} = 0.6$ m

Hence, distance of image from mirror
= 2 + 0.6 = 2.6 m and real.

2. **(d)** Given, using lens maker's formula

$$\frac{1}{f} = (k-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $R_1 = R_2 = R$ (For double convex lens)

$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\Rightarrow P = \frac{1}{f} = (\mu - 1)\frac{2}{R} \qquad \text{...(i)}$$

For plano convex lens,

$R_1 = R'$, $R_2 = \infty$

Using lens maker's formula again, we have

$$1.5P = (\mu - 1)\left(\frac{1}{R'} - \frac{1}{\infty}\right) \qquad \text{...(ii)}$$

$$\Rightarrow \frac{3}{2}P = \frac{\mu - 1}{R'}$$

From (i) and (ii),

$$\frac{3}{2} = \frac{R'}{2R} \Rightarrow R' = \frac{R}{3}$$

3. **(d)** From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f}$$

Case-I : If $v = u \Rightarrow f + u = f \Rightarrow u = 0$

Case-II : If $u = \infty$ then $v = f$.

Hence, correct u versus v graph, that satisfies this condition is (a).

4. **(b)** Using lens maker's formula

$$\frac{1}{f} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, μ_g and μ_a are the refractive index of glass and air respectively

$$\Rightarrow \frac{1}{f} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(i)}$$

When immersed in liquid

$$\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

[Here, μ_l = refractive index of liquid]

$$\Rightarrow \frac{1}{f_l} = \left(\frac{1.5}{1.42} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(ii)}$$

Dividing (i) by (ii)

$$\Rightarrow \frac{f_l}{f} = \frac{(1.5-1)1.42}{0.08} = \frac{1.42}{0.16} = \frac{142}{16} \approx 9$$

5. **(a)** Focal length of plano-convex lens-

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R} \Rightarrow f_1 = \frac{R}{(\mu_1 - 1)}$$

Focal length of plano-concave lens -

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{\mu_2 - 1}{-R} \Rightarrow f_2 = \frac{-R}{(\mu_2 - 1)}$$

For the combination of two lens-

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R} \Rightarrow f_{eq} = \frac{R}{\mu_1 - \mu_2}$$

6. **(d)** From the equation of line

$m = k_1 v + k_2$ $\quad (\because y = mx + c)$

$$\Rightarrow \frac{v}{u} = k_1 v + k_2 \qquad \left(\because m = \frac{v}{u}\right)$$

$$\Rightarrow \frac{1}{u} = k_1 + \frac{k_2}{v} \text{ (Dividing both sides by v)} \Rightarrow \frac{k_2}{v} - \frac{1}{u} - k_1$$

Comparing with lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$k_1 = \frac{1}{-f} \text{ and } k_2 = 1 \qquad \therefore f = \frac{1}{\text{slope of } m - v \text{ graph}} = -\frac{b}{c}$$

7. **(a)** $\frac{1}{f_1} = \frac{2}{f_\ell}$

Here $2f_1 = 18$ cm or $f_1 = 9$ cm

So, $\frac{1}{9} = \frac{2}{f_\ell}$ or $f_\ell = 18$ cm

Using, $\frac{1}{f_\ell} = (\mu - 1)\left(\frac{2}{R}\right)$ or $\frac{1}{18} = (1.5 - 1)\left(\frac{2}{R}\right)$

$\therefore R = 18$ cm

when liquid is put between, then

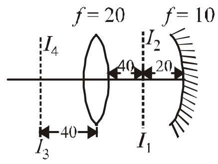
$\frac{1}{f_2} = \frac{2}{f_\ell} + \frac{2}{f}$ or $\frac{1}{(27/2)} = \frac{2}{18} + \frac{2}{f}$

or $f = -54$ cm

Now $-\frac{1}{54} = (\mu_1 - 1) \times \frac{1}{R} = (\mu_1 - 1) \times \left(\frac{1}{-18}\right)$

$\therefore \mu_1 = \frac{1}{3} + 1 = \frac{4}{3}$

8. **(Bouns)** $v_1 = \frac{40 \times 20}{(40 - 20)} = 40\,cm$

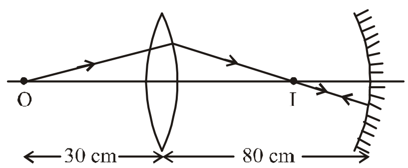


$u_2 = 60 - 40 = 20$ cm

$\therefore v_2 = \frac{20 \times 10}{(20 - 10)} = 20\,cm$

$\therefore$ Image traces back to object itself as image formed by lens is a centre of curvature of mirror.

9. **(c)** For lens



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} - \frac{1}{-30} = \frac{1}{20}$

$\therefore \quad v = +60$ cm

According to the condition, image formed by lens should be the centre of curvature of the mirror, and so
$2f' = 20$ or $f' = 10$ cm

10. (d) By lens's formula, $\frac{1}{V}-\frac{1}{u}=\frac{1}{f}$

For first lens, $[u_1 = -20]$

$$\frac{1}{V_1}-\frac{1}{-20}=\frac{1}{5}\Rightarrow V_1=\frac{20}{3}$$

Image formed by first lense will behave as an object for second lens

so, $u_2=\frac{20}{3}-2=\frac{14}{3}$

$$\frac{1}{V_2}-\frac{1}{\frac{14}{3}}=\frac{1}{-5}\Rightarrow V_2=70\,\text{cm}$$

11. (a) According to lens maker's formula,

$$\frac{1}{f}=(\mu_{rel}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Focal length of lens will change due to change in refrective index μ_{rel}. So, image will be formed at new position. Hence image disappears

12. (d) By lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};\quad \frac{1}{v}-\frac{1}{(-20)}=\frac{10}{3}$$

$$\frac{1}{v}=\frac{10}{3}-\frac{1}{20};\quad \frac{1}{v}=\frac{197}{60};\; v=\frac{60}{197}$$

Magnification of lens (m) is given by

$$m=\left(\frac{v}{u}\right)=\frac{\left(\frac{60}{197}\right)}{20}$$

velocity of image wrt. to lens is given by
$v_{I/L} = m^2 v_{O/L}$
direction of velocity of image is same as that of object
$v_{O/L} = 5$ m/s

$$v_{I/L}=\left(\frac{60\times 1}{197\times 20}\right)^2(5)$$

$= 1.16 \times 10^{-3}$ m/s towards the lens

13. (b) From lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right);\quad \frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{1}{2f_2}$$

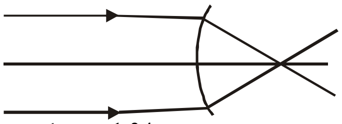
Similarly, for plano-concave lens

$$\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$$

Dividing $\frac{1}{f_1}$ by $\frac{1}{f_2}$ we get,

$$\frac{(\mu_1-1)}{R}=\frac{(\mu_2-1)}{2R}\quad \text{or, } 2\mu_1-\mu_2=1$$

14. (d) using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $\quad R = 7.8$ mm

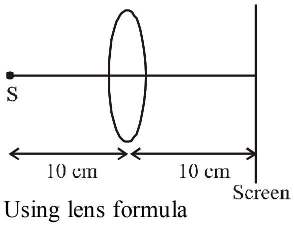


$\mu_1 = 1$ $\mu_2 = 1.34$

$$\Rightarrow\frac{1.34}{V}-\frac{1}{\infty}=\frac{1.34-1}{7.8}[\because u=\infty]$$

$\therefore$ V = 30.7 mm = 3.07 cm $\simeq$ 3.1 cm

15. (d)



Using lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{10}-\frac{1}{-10}=\frac{1}{f}\Rightarrow f=5\text{cm}$$

Shift due to slab, $=t\left(1-\frac{1}{\mu}\right)$ in the direction of incident ray

or, $d=1.5\left(1-\frac{2}{3}\right)=0.5$

Now, u = – 9.5

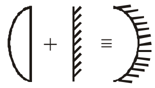
Again using lens formulas $\frac{1}{v}-\frac{1}{-9.5}=\frac{1}{5}$

$$\Rightarrow\frac{1}{v}=\frac{1}{5}-\frac{2}{19}=\frac{9}{95}$$

or, $v=\frac{95}{9}=10.55\,\text{cm}$

Thus, screen is shifted by a distance
d = 10.55 – 10 = 0.55 cm away from the lens.

16. (b) Case-1

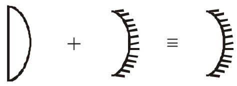


$$\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)\quad f=-28$$

$$P=2P_1+P_2\Rightarrow\frac{1}{28}=2\left(\frac{\mu-1}{R}\right)$$

$\left(\because \text{Power, } P=\frac{1}{f}\;\&\; f_{\text{plane mirror}}=\infty\right)$

Case-2



$$\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)\quad f_2=-\frac{R}{2}\quad f=-10\text{ cm}$$

$$P=2P_1+P_2\Rightarrow\frac{1}{10}=2\left(\frac{\mu-1}{2}\right)+\frac{2}{R}$$

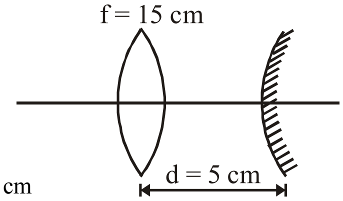
or, $\frac{1}{10}=\frac{1}{28}+\frac{2}{R}\Rightarrow\frac{2}{R}=\frac{1}{10}-\frac{2}{28}=\frac{18}{280}$

or, $R=\frac{280}{9}$ cm

or, $\frac{1}{28}=2\left(\frac{\mu-1}{280}\right)9\Rightarrow\mu-1=\frac{5}{9}$

$\therefore\quad \mu=1+\frac{5}{9}=\frac{14}{9}=1.55$

17. (a) Given, focal length of lens (f) = 15 cm
object is placed at a distance (u) = – 20 cm
By lens formula,



$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{15}-\frac{1}{20}$$

$$\frac{1}{v}=\frac{4-3}{60}\quad\text{or}\quad v=60\text{ cm}$$

The image I gets formed at 60 cm to the right of the lens and it will be inverted.
The rays from the image (I) formed further falls on the convex mirror forms another image. This image should formed in such a way that it coincide with object at the same point due to reflection takes place by convex mirror.
Distance between lens and mirror will be
d = image distance (v) – radius of curvature of convex mirror.
$5 = 60 - 2f$ or $2f = 60 - 5$

$f = \frac{55}{2} = 27.5$ cm (convex mirror)

18. **(b)** Given, radius of hemispherical glass R = 10 cm

$\therefore$ Focal length $f = \frac{10}{2} = -5$ cm
$u = (10 - 6) = -4$ cm.
By using mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-4}=\frac{1}{-5} \Rightarrow v = 20 \text{ cm.}$$

Apparent height, $h_a = h_r \frac{\mu_1}{\mu_2} = 30 \times \frac{1}{1.5} = 20$ cm below flat surface.

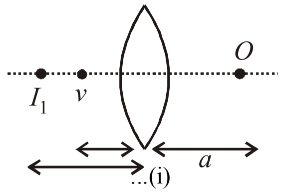
19. **(a)** Len's formula is given by $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

For convex lens, $\frac{1}{30}=\frac{1}{v}+\frac{1}{60} \Rightarrow \frac{1}{60}=\frac{1}{v}$

Similarly for concave lens $\frac{1}{-120}=\frac{1}{v}-\frac{1}{40} \Rightarrow \frac{1}{v}=\frac{1}{60}$
Virtual object 10 cm behind plane mirror.
Hence real image 10 cm infront of mirror or, 60 cm from convex lens.

20. (d) When object is kept at a distance 'a' from thin covex lens

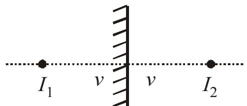
By lens formula : $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



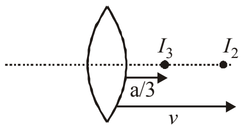
$$\frac{1}{V}-\frac{1}{(-a)}=\frac{1}{f}$$

or, $\frac{1}{v}=\frac{1}{f}-\frac{1}{a}$

Mirror forms image at equal distance from mirror



Now, again from lens formula



$$\frac{3}{a}-\frac{1}{V}=\frac{1}{f} \quad \text{or} \quad \frac{3}{a}-\frac{1}{f}+\frac{1}{a}=\frac{1}{f} \qquad \text{[From eqn. (i)]}$$

Hence, $a = 2f$

21. **(b)** By Lens maker's formula for convex lens

$$\frac{1}{f}=\left(\frac{\mu}{\mu_L}-1\right)\left(\frac{2}{R}\right)$$

for, $\mu_{L_1}=\frac{4}{3}, f_1 = 4R$

for $\mu_{L_2}=\frac{5}{3}, f_2 = -5R \Rightarrow f_2 = (-)$ ve

22. **(a)** If a lens of refractive index μ is immersed in a medium of refractive index μ_1, then its focal length in medium is given by

$$\frac{1}{f_m}=({}_m\mu_l - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

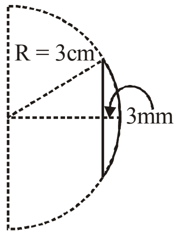
If f_a is the focal length of lens in air, then

$$\frac{1}{f_a}=({}_a\mu_l - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{f_m}{f_a}=\frac{({}_a\mu_l - 1)}{({}_m\mu_l - 1)}$$

If $\mu_1 > \mu$, then f_m and f_a have opposite signs and the nature of lens changes i.e. a convex lens diverges the light rays and concave lens converges the light rays. Thus given option (a) is correct.

23. **(c)** $\therefore n = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$

$\therefore n = \frac{3}{2}$



$3^2 + (R - 3\text{mm})^2 = R^2$
$\Rightarrow 3^2 + R^2 - 2R(3\text{mm}) + (3\text{mm})^2 = R^2$
$\Rightarrow R \approx 15$ cm

$$\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{15}\right) \Rightarrow f = 30 \text{ cm}$$

24. **(d)** If side of object square = ℓ
and side of image square = ℓ'

From question, $\frac{\ell'^2}{\ell}=9$ or $\frac{\ell'}{\ell}=3$
i.e., magnification m = 3
u = – 40 cm
v = 3 × 40 = 120 cm
f = ?

From formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{120}-\frac{1}{-40}=\frac{1}{f} \text{ or, } \frac{1}{f}=\frac{1}{120}+\frac{1}{40}=\frac{1+3}{120}$$

$\therefore$ f = 30 cm

25. **(c)** Here $\mu = \frac{\lambda_{air}}{\lambda_{medium}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} = 1.5$

Also $m = \frac{v}{u} = -\frac{1}{3}$ ($\because v = 8m$)

$\therefore u = -24$ cm.

For a plano-convex lens

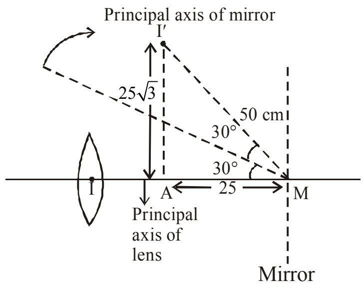
$$\frac{1}{f}=\frac{(\mu-1)}{R}=\frac{1}{v}-\frac{1}{u} \text{ or } \frac{1.5-1}{R}=\frac{1}{8}-\left(\frac{1}{-24}\right)=\frac{1}{8}+\frac{1}{24}=\frac{1}{6}$$

$\therefore$ R = 3m

26. **(c)** *For convex lens* $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{v}-\frac{1}{-50}=\frac{1}{30}$ ($\because v = -50$ cm, $f = 30$ cm)

$\therefore$ v = 75 cm



The image formed by convex lens acts as an object for mirror.

For minor $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\therefore$ $\frac{1}{v}+\frac{1}{25}=\frac{1}{50}$ $\therefore$ v = – 50 cm

The image I would have formed as shown had the mirror been straight. But here the mirror is tilted by 30°. Therefore the image will be tilted by 60° and will be formed at A.

Here MA = 50 cos 60° = 25 cm (x-coordinate of the image)

and I′A = 50 sin 60° = $25\sqrt{3}$ cm (y-coordinate of the image)

Hence, coordinate of the point at which image is formed (25, $25\sqrt{3}$).

27. **(b)** The focal length (f_1) of the plano-convex lens with n = 1.5 using lens-maker formula

$$\frac{1}{f_1}=(n_1-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.5-1)\left[\frac{1}{14}-\frac{1}{\infty}\right]=\frac{1}{28}$$

The focal length (f_2) of the plano-convex lens with n = 1.2

$$\frac{1}{f_2}=(n_2-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.2-1)\left[\frac{1}{\infty}-\frac{1}{-14}\right]=\frac{1}{70}$$

Focal length F of the combination

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{20}$$

Now, applying lens formula for the combination of lens

$$\frac{1}{V}-\frac{1}{U}=\frac{1}{F} \Rightarrow \frac{1}{V}-\frac{1}{-40}=\frac{1}{20} \qquad \text{[Given } \mu = 40 \text{ cm]}$$

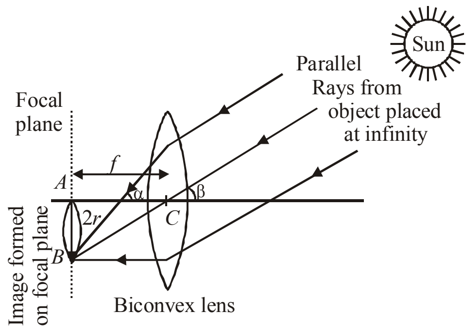
$\therefore$ V = 40 cm

28. **(b)** From the figure in ΔABC, $\tan\beta=\frac{AB}{AC}$

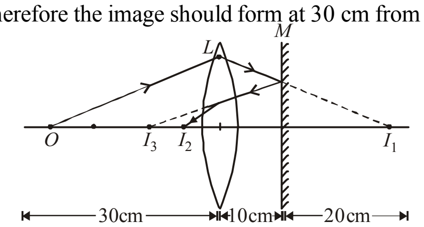
$\Rightarrow$ $AB = AC \tan\beta$

$\Rightarrow$ $2r = f\tan\beta$ $\therefore$ $r=\frac{f}{2}\tan\beta$

$\therefore$ Area of image formed by sun = $\pi r^2=\pi\frac{\tan^2\beta}{2}f^2\propto f^2$



29. **(b)** Focal length of the biconvex lens L is 15 cm. A small object is placed at a distance of 30 cm from the lens i.e. at a distance of 2f. Therefore the image should form at 30 cm from the lens at I_1.



The image I_1 acts as a virtual object for the mirror. The mirror forms an image I_2 at a distance of 20 cm in front of it.

The image I_2 acts as an object for the lens.

Here, u = +10 cm, f = +15cm

Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

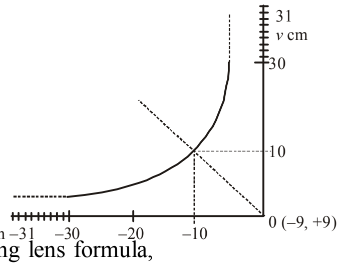
$$\Rightarrow \frac{1}{v}-\frac{1}{10}=\frac{1}{15} \Rightarrow \frac{1}{v}=\frac{1}{15}+\frac{1}{10}=\frac{25}{150} \qquad \therefore\ v = 6 \text{ cm.}$$

Therefore final real image is formed at a distance of 16 cm from the plane mirror.

30. **(c)** From the graph u = –10 cm; v = 10 cm $\Delta u = \Delta v$ = 0.1

From lens formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{10}-\frac{1}{-10}$

$\therefore$ f = 5 cm



Differentiating lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{\Delta f}{f^2}=\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2} \qquad \text{(for maximum error in f)}$$

$$\Rightarrow \frac{\Delta f}{25}=\frac{0.1}{(10)^2}+\frac{0.1}{(10)^2} \Rightarrow \Delta f = 25 \times 0.1 \times 2 \times 0.01 = 0.05$$

$\therefore$ Focal length, $f \pm \Delta f$ = (5.00 ± 0.05) cm.

31. **(b)** The focal length f of the equivalent mirror when lens is silvered,

$$\frac{1}{f}=\frac{2}{f_1}+\frac{1}{f_m}=\frac{2}{15}+\frac{1}{\infty} \Rightarrow f=\frac{15}{2}\text{cm}$$

Since f has a positive value, the combination behaves as a converging mirror.

Here u = – 20cm, $f=-\frac{15}{2}$cm , v = ?

According to mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \frac{1}{v}-\frac{1}{-20}=\frac{1}{-15/2} \Rightarrow v = -12 \text{ cm}$$

Negative sign indicates that the image is 12 cm in front or left of mirror.

32. **(a)** Focal length of concave lens, $f_2=-\frac{3}{2}f_1$

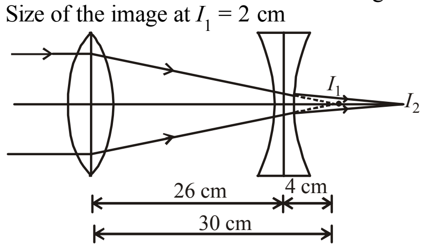
f_1 = focal length of convex lens.

$$\frac{1}{f}=\frac{1}{f_1}-\frac{1}{f_2} \Rightarrow \frac{1}{30}=\frac{1}{f_1}-\frac{2}{3f_1}$$

$\therefore$ f_1 = 10 cm and $f_2=-\frac{3}{2}\times 10=-15$ cm

33. **(b)** The image formed by convex lens, I_1 acts as virtual object for concave lens. Concave lens forms the image of I_1 at I_2.

Size of the image at I_1 = 2 cm



For concave lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

or, $\frac{1}{v}-\frac{1}{4}=-\frac{1}{20}$ or $\frac{1}{v}=-\frac{1}{20}+\frac{1}{4}=\frac{4}{20}=\frac{1}{5}$

or v = 5 cm = Distance of I_2 from concave lens.

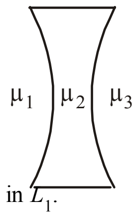
$\therefore$ Magnification, $m = \frac{v}{u} = \frac{\text{size of image}}{\text{size of object}} = \frac{5}{4}$

or $\frac{\text{size of image}}{2} = 1.25$

$\therefore$ Size of image due to concave lens = 2.5 cm

34. (d) The Lens maker formula,

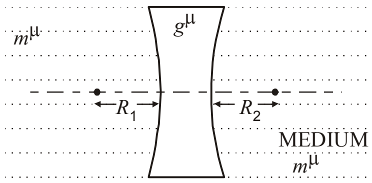
$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



If $\mu_2 > \mu_1$, the concave lens maintains its nature otherwise the nature of the lens will be reversed.

So, the lens should be filled with L_2 and immerse in L_1.

35. (a) For concave lens as shown in figure, in this case $R_1 = -R$ and $R_2 = +R$



From lens maker formula $\frac{1}{f} = \left({}_g^m\mu - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$${}_g^m\mu = \frac{{}_g\mu}{{}_m\mu} = \frac{1.5}{1.75}$$

$$\therefore \quad \frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = +\frac{0.25 \times 2}{1.75\,R}$$

or, $f = +3.5\,R$

Hence lens behaves as convergent lens of focal length $f = 3.5$R

36. (c) A convex mirror and a concave lens always produce virtual image. Which cannot be taken on the screen.

Therefore, option (b) and (d) are not correct. The image formed by a convex lens is diminished when the object is placed beyond $2f$.

Let $u = 2f + x$

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-(2f+x)} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{2f+x} = \frac{2f+x-f}{f(2f+x)} = \frac{(f+x)}{f(2f+x)}$$

But $u + v = 1$ (given)

$$(2f+x) + \frac{f(2f+x)}{f+x} \le 1$$

$$2f + x\left[1 + \frac{f}{f+x}\right] \le 1 \Rightarrow \frac{(2f+x)^2}{f+x} \le 1$$

$\therefore$ $(2f+x)^2 \le f + x$. This is true for $f < 0.25$ m.

37. (c) Spherical aberration occurs due to the inability of a lens to converge marginal rays of the same wavelength to the focus as it converges the paraxial rays. i.e. , the marginal and paraxial says are focussed at different places on the axis of the lens and therefore the image so formed is blurred. This aberration can be reduced by using a circular annular mask over the lens.

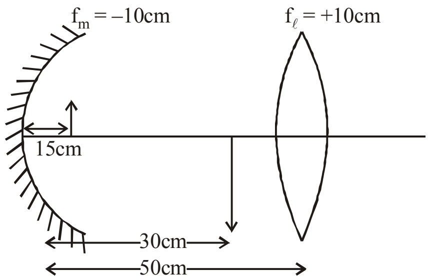
38. (7) Applying mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{15}$$

$$\therefore \quad \frac{1}{v} = \frac{-15+10}{150} = \frac{-5}{150} = \frac{-1}{30}$$

$\therefore$ $v = -30$cm

And magnification, $m_1 = -\frac{v}{u} = -\frac{-30}{-15} = -2$



Now for refraction from lens, $u = -(50 - 30) = -20$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \quad \therefore \ v = 20 \text{ cm}$$

Magnification, $m_2 = \frac{v}{u} = \frac{20}{-20} = -1$

Magnification produced by the combination,

$M_1 = m_1 \times m_2 = (-2) \times (-1) = 2$

Again, when system is kept in a medium of refractive index 7/6.

There is no change for mirror in this case,

For lens, $\frac{1}{f_l'} = \left(\frac{\mu_l}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_l'} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_l'} = \frac{2}{7}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Also, when lens was in air

$$\frac{1}{f_l} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10} \quad \therefore \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{5}$$

Using this result in eqn. (i), we get

$$\frac{1}{f_l'} = \frac{2}{7} \times \frac{1}{5} \quad \therefore \ f_l' = \frac{35}{2} \text{ cm}$$

Again using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_l'}$

$$\frac{1}{v} - \frac{1}{-20} = \frac{2}{35} \Rightarrow \frac{1}{v} = \frac{2}{35} - \frac{1}{20} = \frac{1}{140} \quad \therefore \ v = 140 \text{ cm}$$

Magnification, $m_2' = \frac{v}{u} = \frac{140}{-20} = -7$

Magnification produced by the combination,

$M_2 = m_1 \times m_2' = (-2) \times (-7) = 14$

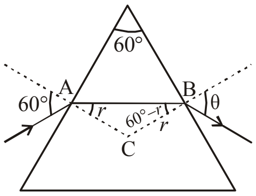
$$\therefore \quad \left|\frac{M_2}{M_1}\right| = \frac{14'}{2} = 7$$

39. (2) Applying Snell's law at A

$\sin 60° = n \sin r$ ...(i)

Differentiating w.r.t 'n' we get

$0 = \sin r + n \cos r \times \frac{dr}{dn}$ ...(ii)



Again, applying Snell's law at B

$\sin\theta = n\sin(60° - r)$ ...(iii)

Differentiating w.r.t 'n' we get

$$\cos\theta\,\frac{d\theta}{dn} = \sin(60° - r) + n\cos(60° - r)\left[-\frac{dr}{dn}\right]$$

$$\therefore\quad \cos\theta\,\frac{d\theta}{dn} = \sin(60° - r) - n\cos(60° - r)\left[-\frac{\tan r}{n}\right]$$

[from (ii)]

$$\therefore\quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}[\sin(60° - r) + \cos(60° - r)\tan r] \quad ...(iv)$$

From eq. (i), for $n = \sqrt{3}$ we get $r = 30°$

From eq (iii), for $n = \sqrt{3}$, $r = 30°$ we get $\theta = 60°$

Substituting the values of r and θ in eq (iv) we get

$$\frac{d\theta}{dn} = \frac{1}{\cos 60°}[\sin 30° + \cos 30°\tan 30°] = 2\left(\frac{1}{2}+\frac{1}{2}\right) = 2$$

40. **(476.19)**

Given,

Distance between an object and screen, $D = 100$ cm

Distance between the two position of lens, $d = 40$ cm

Focal length of lens,

$$f = \frac{D^2 - d^2}{4D} = \frac{100^2 - 40^2}{4(100)} = \frac{(100+40)(100-40)}{4(100)} = 21 \text{ cm}$$

Power, $P = \frac{1}{f} = \frac{100}{21} = \frac{N}{100}$

$\therefore N = 476.19.$

41. **(60)** Given : $\mu = 1.5$; $R_{curved} = 30$ cm

Using, Lens-maker formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For plano-convex lens

$R_1 \to \infty$ then $R_2 = -R$

$$\therefore f = \frac{R}{\mu - 1} = \frac{30}{1.5 - 1} = 60 \text{ cm}$$

42. When two lenses are in contact, $P = P_1 + P_2 = 10$

When two lenses are separated by a distance d(d = 0.25m) $P = P_1 + P_2 - (0.25)P_1P_2 = 6$

From these two expressions, we get

$P_1P_2 = 16\text{m}^{-2}$

$$P_1 - P_2 = \sqrt{(P_1+P_2)^2 - 4P_1P_2}$$

$$= \sqrt{(10^{-1})^2 - 4(16^{-1})} = 6\text{m}^{-1}$$

$\therefore\quad P_1 = 8\text{m}^{-1}$ and $P_2 = 2\text{ m}^{-1}$

$$\therefore\quad f_1 = \frac{1}{P_1} = \frac{1}{8}\text{m} = 0.125\text{m and } f_2 = \frac{2}{P_2} = \frac{1}{2}\text{m} = 0.5\text{m}$$

43. $${}_g^m\mu = \frac{{}_g\mu}{{}_m\mu} = \frac{1.5}{4/3} = 1.125$$

using $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ [Lens-maker's formula]

$$\frac{1}{15} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ....(i)$$

and $$\frac{1}{f'} = (1.125 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ....(ii)$$

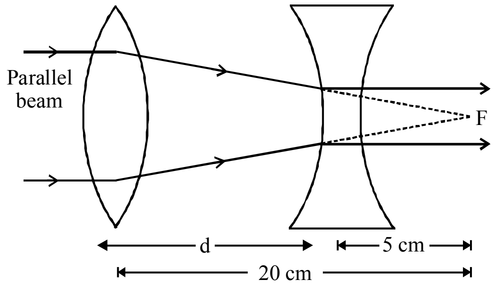
Dividing eq. (i) by (ii)

$$\frac{f'}{15} = \frac{1.5-1}{1.125-1} = \frac{0.5}{0.125} = 4$$

$\therefore\quad f' = 60$ cm

44. From the diagram it is clear that the focus of both the lenses should coincide at F

$\therefore d = 20 - 5 = 15$ cm.



45. **True;**

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{F} = \frac{1}{-15} + \frac{1}{30} = \frac{-2+1}{30}$$

$\therefore\quad F = -30$ cm.

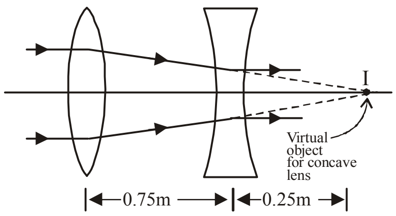
Hence combination of lenses behaves as a concave lens of focal length 30 cm.

Since $F_v < F_r$, violet colour is deviated more than red colour when white light passes through a lens or prism

$\therefore$ One sees coloured pattern with violet colour at the outer edge.

46. **False;**

The image formed by the convex lens at the focus of the concave lens. Therefore I will act as a virtual object for concave lens and image will be formed at infinity. Hence after refraction from both the lenses, rays will become parallel to the optic axis.



47. **(a, d)** From figure, in ΔOAB

$R^2 = (R - h)^2 + r^2$

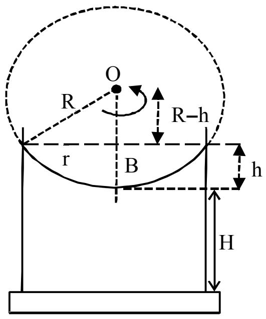
$R^2 = R^2 - 2hR + h^2 + r^2$

$\Rightarrow 2hR = h^2 + r^2$

$$\therefore R = \frac{h^2 + r^2}{2h}$$

Hence option (a) is correct.

Now from equation of surface



$$y = y_0 + \frac{\omega^2 r^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$

Now using formula for refraction at curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{V} - \frac{\frac{4}{3}}{-(H-h)} = \frac{1-\frac{4}{3}}{R} \Rightarrow \frac{1}{V} - \frac{1}{3R} - \frac{4}{3H}$$

$[\because h << H]$

$$\Rightarrow \frac{1}{V} = \frac{2h}{3r^2} - \frac{4}{3H}$$

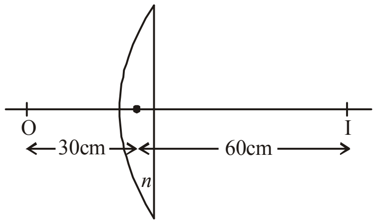
$$\Rightarrow \frac{1}{V} = -\frac{4}{3H}\left[1 - \frac{\omega^2 H}{4g}\right]$$

$$\therefore v = \frac{3H}{4}\left[1 + \frac{\omega^2 H}{4g}\right]^{-1} \quad \text{(Apparent depth)}$$

Hence option (d) is correct.

48. **(a, d)**

For lens, applying lens formula



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{f}$$

$$\frac{1}{60} - \frac{1}{-30} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{60} + \frac{1}{30}$$

$\therefore \quad f = 20$ cm

$$\because \quad m = \frac{v}{u} \Rightarrow -2 = \frac{v}{-30} \Rightarrow v = 60 \text{ cm}$$

For reflection, applying mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R} \Rightarrow \frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow R = 30\text{cm}$$

The image formed by convex side is faint erect and virtual.

By lens maker formula

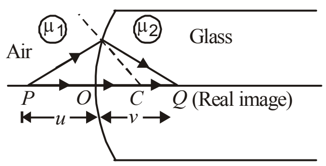
$$\frac{1}{f} = \left(\frac{n_l}{n_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{20} = \left(\frac{n_l}{1} - 1\right)\left(\frac{1}{30}\right) \quad \therefore n_l = 2.5$$

(Refractive index of lens)

49. **(a)** Using formula for spherical refracting surface

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



$PO = OQ$

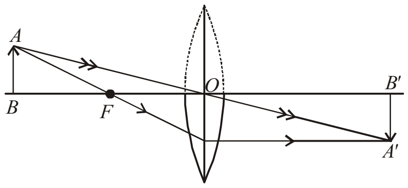
Here $u = -x$, $v = +x$, $R = +R$, $\mu_1 = 1$, $\mu_2 = 1.5$

$$\frac{-1}{-x} + \frac{1.5}{x} = \frac{1.5-1}{R} \quad \therefore \ x = 5R = PO$$

50. **(b)** Spherical aberation is smaller when the curved surface is facing the object because the total deviation is shared between the curved and the plane surfaces.

51. **(b, c)** Image formed by concave lens and convex mirror is virtual and erect. Concave lens and convex mirror are diverging in nature. Therefore the refracted/reflected rays do not meet. These rays are produced backwards to make them meet.

52. **(b, d)** The image formed will be complete because light rays from all parts of the object will strike on the lower half.



But since the upper half light rays are cut off, the intensity will reduce.

53. **(a)** Power of lens combination,

$$P = \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.4} + \frac{1}{-0.25} = -1.5 \text{ D}$$

54. **(b)** For double convex lens, (P) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\Rightarrow (1.5-1)\left(\frac{1}{r} - \frac{1}{r}\right) = (1.5-1)\left[\frac{2}{r}\right] = \frac{1}{r} \Rightarrow f = r$$

$$\frac{1}{F_{eq.}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}$$

$$\therefore \quad F_{eq} = \frac{r}{2}$$

For (Q) plano-convex lens $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$= (1.5-1)\left[\frac{1}{\infty} - \frac{1}{-r}\right] = \frac{0.5}{r} = \frac{1}{2r} \quad \therefore f = 2r$$

$$\frac{1}{F_{eq.}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{2r} + \frac{1}{2r} = \frac{2}{2r} = \frac{1}{r} \quad \therefore F_{eq.} = r$$

For (R) plano-concave lens

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-r} - \frac{1}{\infty}\right) \Rightarrow f = -2r$$

$$\frac{1}{F_{eq.}} = \frac{1}{f} + \frac{1}{f} = \frac{1}{-2r} + \frac{1}{-2r} \Rightarrow F_{eq.} = -r$$

For (S) combination of one double convex and one plano-concave lens

$$\frac{1}{F_{eq.}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow F_{eq.} = 2r$$

55. **A-p, r; B-q,s,t; C-p,r,t, D-q,s**

When a ray of light enters from rarer medium to denser medium it bends towards the normal and its opposite when the ray of light travels.

From denser to rarer, it bends away from the normal.

(a) As $\mu_1 < \mu_2$, the ray of light entering the lens will bend towards the normal.

(B) As $\mu_1 > \mu_2$, the ray of light entering the lens will bend away from the normal.
(C) As $\mu_2 = \mu_3$, the ray of light coming out from the lens without any deviation.
(D) As $\mu_2 > \mu_3$, the ray of light coming out of the lens deviates away from the normal.

56. A-p, q, r, s; B-q; C-p, q, r, s; D-p, q, r, s
Image formed by concave mirror (A) convex lens (C) and concavo-convex lens (D)
Can be real, virtual, magnified, diminished. And image is at infinity when object is at focus.
Image formed by convex mirror (B) is always virtual.

39. Given: $f = 0.3$ m, $u = -0.4$ m

Using lens formula, $\frac{1}{v} - \frac{1}{-0.4} = \frac{1}{0.3} \Rightarrow v = 1.2$ m

Now, differentiating lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ w.r.t. time t

$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0 \text{ given } \frac{du}{dt} = 0.01 \text{ m/s}$$

$$\therefore \left(\frac{dv}{dt}\right) = \frac{(1.20)^2}{(0.4)^2} \times 0.01 = 0.09 \text{ m/s}$$

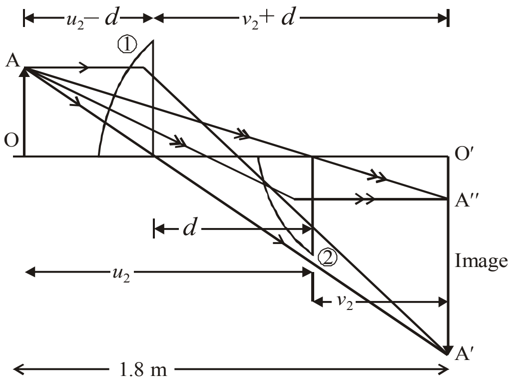
Therefore, magnitude of rate of change of position of the image (w.r.t. the lens) = 0.09 m/s

Lateral magnification, $m = \frac{v}{u}$ $\therefore \frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2}$

$$= \frac{-(0.4)(0.09) - (1.2)(0.01)}{(0.4)^2} = -0.35\,\text{s}^{-1}$$

i.e., Magnitude of rate of change of lateral magnification $= 0.35\ \text{s}^{-1}$.

58. The first half lens (1) forms image of OA at O′A′ and the second half lens (2) forms image of OA at O′A′.



Here, $u_2 + v_2 = 1.8$ m ... (i)

The magnification of lens (1) is 2 (given)

$$\therefore \quad 2 = \frac{v_2 + d}{u_2 - d} \qquad \text{... (ii)}$$

From eq. (i) and (ii)

$u_2 = 0.6 + d$, $v_2 = 1.2 - d$

Applying lens formula for lens (1)

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{f} \qquad \text{.... (iii)}$$

Again applying lens formula for lens (2)

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f} \qquad \text{... (iv)}$$

From eq. (iii) and (iv)

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{v_2} + \frac{1}{u_2}$$

$$\Rightarrow \frac{1}{1.2 - d + d} + \frac{1}{0.6 + d - d} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

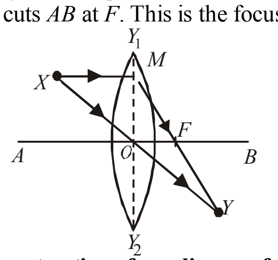
$\therefore$ $d = 0.6$ m

Substituting this value in eq. (iv) $\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$

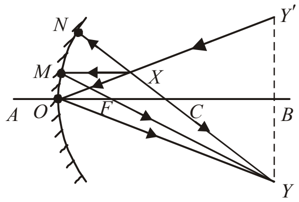
$$\Rightarrow \frac{1}{1.2 - d} + \frac{1}{0.6 + d} = \frac{1}{f}$$

$$\frac{1}{1.2 - 0.6} + \frac{1}{0.6 + 0.6} = \frac{1}{f} \quad \therefore f = 0.4 \text{ m}$$

59. (i) Since Y is below of optic axis AB and X above, therefore the image is real and inverted and lens should be a convex lens.
Steps of construction of ray diagram for a lens:
(1) Join XY. This represents the ray originating from the source X and meeting the image Y. Since the ray is undeviated after passing through the lens, therefore O is the optical centre of the lens. Draw Y_1OY_2 perpendicular to AB.
(2) Draw a ray from X, parallel to AB, XM. It strikes Y_1OY_2 at M. Join MY. It cuts AB at F. This is the focus of the convex lens.

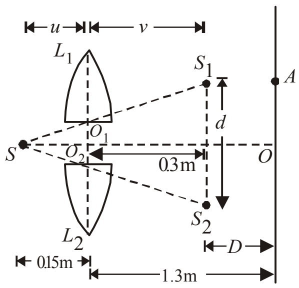


(ii) Steps of construction of ray diagram for concave mirror
(1) Draw a line YY' perpendicular to AB such that BY = BY'



(2) Join $Y'X$ and extend the line it cuts the line AB at point O. The point O is the pole of mirror.
(3) Join X and Y. It cuts AB at the point C. This point C is the centre of curvature of the mirror.
(4) Draw XM parallel to the principal axis. Join M to Y. Let it cut AD at F. Therefore, F is the focus of concave mirror.

60. (i) Here, the two identical halves of convex lens L_1 and L_2 will form two seperate images S_1 and S_2 of the source S at equal distance. These images S_1 and S_2 will behave as two coherent sources.



For lens L_1 and L_2
$u = -0.15$ m, $v = ?$, $f = +0.1$ m

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{0.1}+\frac{1}{-0.15}$

$\therefore \quad v = 0.3$ m

Linear magnification, $m = \frac{v}{u}$

$\therefore \quad m = \frac{0.3}{-0.15} = -2$

$\therefore$ Two images S_1 and S_2 of S will be formed at 0.3 m from the lens.

S_1 will be 0.5 mm above its optic axis as $m = -2$.

S_2 will be 0.5 mm below its optic axis as $m = -2$.

$\therefore \quad d = S_1S_2 = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

$D = 1.30 - 0.30 = 1.0$ m

$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m} = 5 \times 10^{-7}$ m.

$\therefore$ Fringe width $\beta = \frac{D\lambda}{d}$

or $\beta = \frac{(1.0)(5\times10^{-7})}{1.5\times10^{-3}}$ or $\beta = \frac{1}{3} \times 10^{-3}$ m

Since point A is at third maxima,

$OA = 3\beta = 3 \times \left(\frac{1}{3}\times10^{-3}\right) = 10^{-3}$ m

$\therefore \quad OA = 10^{-3}$ m

(ii) Variation of OA with gap between L_1 and L_2.

If the gap between L_1 and L_2 is reduced, d will decrease.

$\because \quad \beta = \frac{D\lambda}{d} \quad \therefore$ β will increase.

$\therefore$ Distance OA will increase.

61. Ray of light first enters from medium II to medium III, i.e., from denser to rarer medium.

Case (i) : When $n_3 < n_1$

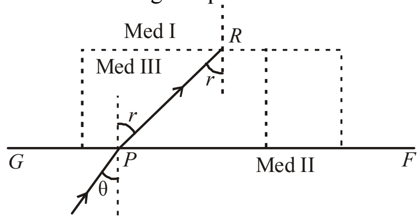
i.e., $n_3 < n_1 < n_2$

or, $\frac{n_3}{n_2} < \frac{n_1}{n_2}$ or, $\sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_1}{n_2}\right)$

Obviously $n_3 < n_2$ and the angle θ is greater than the critical angle required for the ray passing from medium II to medium III. Therefore total internal reflection will also take place when a ray strikes with the same angle at the interface of medium II and medium III.

Case (ii) : $n_3 > n_1$ but $n_3 < n_2$

The ray will get refracted in medium III as the angle θ will now be less than the critical angle required for medium II and medium III pair.



Applying Snell's law at P

$\frac{\sin\theta}{\sin r} = \frac{n_3}{n_2} \quad \therefore \sin r = \frac{n_2}{n_3}\sin\theta$

$\because \quad n_2 > n_3 \quad \therefore \quad r > \theta$

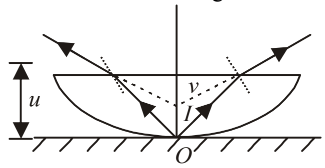
When the refracted ray PR meets the boundary DE, it is travelling from a denser medium to a rarer medium. Hence the ray will be totally internally reflected at DE if its angle of incidence r is more than the critical angle for med III and I.

$\sin i'' = \frac{n_1}{n_3}$

$\because \quad \sin r > \frac{n_1}{n_3} \Rightarrow \sin r > \sin i'' \Rightarrow r > i''$

$\therefore$ Ray PR will be totally internal reflected along RQ. On reaching Q, the ray will be refracted in med II. Hence, the ray will ultimately be reflected back in medium II.

62. Here plane surface is the refracting surface $\therefore R = \infty$



$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \Rightarrow -\frac{\mu_1}{-4}+\frac{\mu_2}{-3}=0$

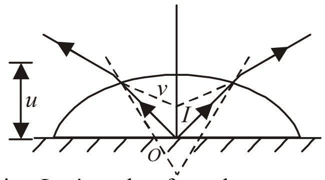
$\therefore \quad \frac{\mu_2}{\mu_1}=\frac{3}{4}$ ...(i)

Again refraction takes place from spherical surface,

$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

$\Rightarrow -\frac{1}{u}+\frac{\mu_2/\mu_1}{v}=\frac{(\mu_2/\mu_1)-1}{R}$

$\Rightarrow -\frac{1}{-4}+\frac{3/4}{-25/8}=\frac{3/4-1}{R} \Rightarrow R = 25$ cm



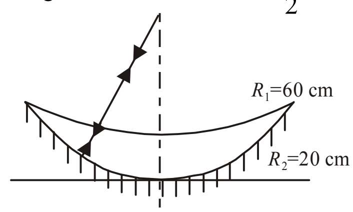
Now, applying Len's maker formula,

$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{4}{3}-1\right)\left(\frac{1}{25}-\frac{1}{\infty}\right) \therefore f = 75$cm

63. (i) Here the silvered concavo-convex lens behaves like a mirror whose focal length $\frac{1}{f}=\frac{2}{f_1}+\frac{1}{f_2}$

f_1 = focal length of concave surface = $\frac{60}{2}$ = 30 cm

f_2 = focal length of concave mirror = $\frac{20}{2}$ = 10 cm



$\therefore \quad \frac{1}{f}=\frac{2}{-30}+\frac{1}{-10}=-\frac{4}{30} \Rightarrow f = -7.5$ cm

Let P in be placed at a distance x on the optic axis such that its image is formed at the same place i.e., $u = v = x$

Using mirror formula

$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{1}{-7.5}=\frac{1}{-x}+\frac{1}{-x}$

$\therefore \quad x = 15$ cm

(ii) When concave part is filled with water of $\mu = \frac{4}{3}$.

Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let y be the distance of pin, then the plane surface will form its image at a distance $\frac{4}{3}y$ ($h_{app.} = \mu h$) from it.

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

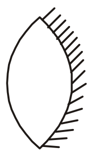
$$\frac{1.5}{-20} - \frac{4/3}{-\frac{4x}{3}} = \frac{1.5 - 4/3}{-60}$$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$ or $x = 13.84$ cm

$\therefore$ $\Delta x = x_1 - x_2 = 15$ cm – 13.84 cm = 1.16 cm

64. From figure (i), there is no refraction. Therefore $\mu_1 = \mu$
From figure (ii), here the convex lens behaves as a diverging lens.
Therefore , $\mu < \mu_2 \therefore \mu_1 < \mu_2$

65. Focal length of the equivalent mirror



$$\frac{1}{F} = \frac{1}{f_\ell} + \frac{1}{f_\ell} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

$$= \frac{2}{20} + \frac{2}{22} = \frac{1}{10} + \frac{1}{11} = \frac{21}{110}$$

$$\Rightarrow F = \frac{110}{21}$$

As the focal length is positive it is behaves as concave mirror
Applying mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{-10} + \frac{1}{v} = \frac{1}{-110/21}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{21}{110}$$

$v = -11$cm

The negative sign shows that the image formed infront of mirror hence the image is real.

Topic-4 : Prism and Dispersion of Light

1. **(a)** When angle of prism is small, then angle of deviation is given by $D_m = (\mu - 1)A$
So, if wavelength of incident light is increased, μ decreases and hence D_m decreases.

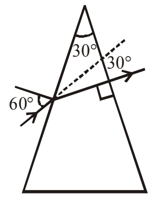
2. **(c)** For minimum deviation:

$$r_1 = r_2 = \frac{A}{2} = 30^o$$

by Snell's law $\mu_1 \sin i = \mu_2 \sin r$

$$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \Rightarrow i = 60$$

3. **(c)** Angle of prism, A = 30°, i = 60°,
angle of deviation, $\delta = 30°$



Using formula, $\delta = i + e - A$

$\Rightarrow e = \delta + A - i$

$= 30° + 30° - 60° = 0°$

$\therefore$ Emergent ray will be perpendicular to the face
So it will make angle 90° with the force through which it emerges.

4. **(c)** We know that $i + e - A = \delta$
$35° + 79° - A = 40°$ $\quad\therefore\quad A = 74°$

$$\text{But } \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin\left(\frac{74+\delta_m}{2}\right)}{\sin\frac{74}{2}}$$

$$= \frac{5}{3}\sin\left(37° + \frac{\delta_m}{2}\right)$$

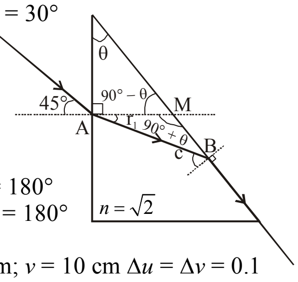
μ_{max} can be $\frac{5}{3}$. That is μ_{max} is less than $\frac{5}{3} = 1.67$

But δ_m will be less than 40° so

$$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \Rightarrow \mu = 1.5$$

5. **(a)** Applying Snell's law at A

$1 \times \sin 45° = \sqrt{2} \times \sin r_1 \quad \therefore r_1 = 30°$



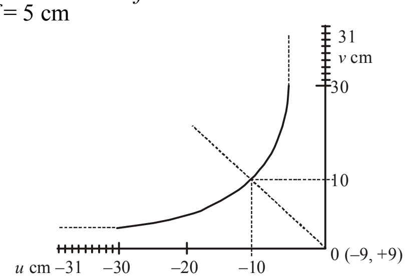
$\sin C = \frac{1}{n} = \frac{1}{\sqrt{2}}$

$\therefore$ C = 45°
In ΔAMB,
$90° + \theta + r_1 + (90° - C) = 180°$
$\Rightarrow$ $90° + \theta + 30° + 90° - 45° = 180°$
$\therefore$ $\theta = 15°$

6. **(c)** From the graph $u = -10$ cm; $v = 10$ cm $\Delta u = \Delta v = 0.1$

From lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \frac{1}{-10}$

$\therefore$ $f = 5$ cm



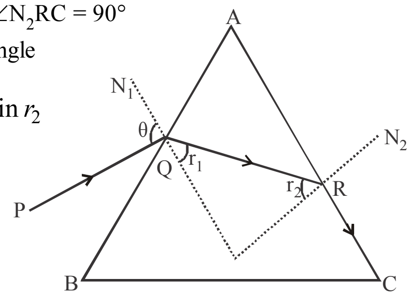
Differentiating lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} \quad \text{(for maximum error in f)}$$

$$\Rightarrow \frac{\Delta f}{25} = \frac{0.1}{(10)^2} + \frac{0.1}{(10)^2}$$

$\Rightarrow$ $\Delta f = 25 \times 0.1 \times 2 \times 0.01 = 0.05$
$\therefore$ Focal length, $f \pm \Delta f = (5.00 \pm 0.05)$ cm.

7. **(c)** When $r_2 = C$, $\angle N_2RC = 90°$
Where C = critical angle



As $\sin C = \frac{1}{\mu} = \sin r_2$

Applying snell's law at 'R'
$\mu \sin r_2 = 1 \sin 90°$ ...(i)
Applying snell's law at 'Q'
$1 \times \sin\theta = \mu \sin r_1$ ...(ii)
But $r_1 = A - r_2$

So, $\sin\theta = \mu \sin(A - r_2)$

$\sin\theta = \mu \sin A \cos r_2 - \cos A$ ...(iii) [using (i)]

From (1)

$$\cos r_2 = \sqrt{1-\sin^2 r_2} = \sqrt{1-\frac{1}{\mu^2}} \quad \text{...(iv)}$$

By eq. (iii) and (iv)

$$\sin\theta = \mu \sin A\sqrt{1-\frac{1}{\mu^2}} - \cos A$$

on further solving we can show for ray not to transmitted through face AC

$$\theta = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

So, for transmission through face AC

$$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

8. **(c)** For the prism as the angle of incidence (i) increases, the angle of deviation (δ) first decreases goes to minimum value and then increases.

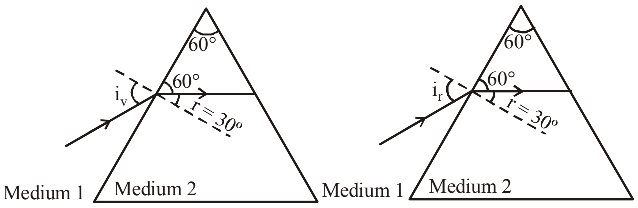
9. **(d)** For light to come out through face 'AC', total internal reflection must not take place.

i.e., $\theta < c \Rightarrow \sin\theta < \sin c$

$$\Rightarrow \sin\theta < \frac{1}{\mu} \quad \text{or} \quad \mu < \frac{1}{\sin\theta} \Rightarrow \mu < \frac{1}{\sin 45°}$$

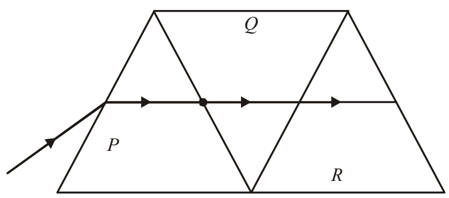
$$\Rightarrow \mu < \sqrt{2} \quad \Rightarrow \quad \mu < 1.414$$

10. **(a)** For minimum deviation the ray in the prism is parallel to the base of the prism. This condition does not depend on the colour (or wave length) of incident radiation. So in both the cases, for both the colours by geometry, angle of refraction, r = 30º.

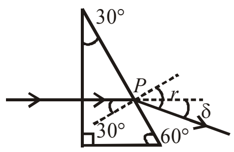


11. **(b)** For minimum deviation, incident angle is equal to emerging angle. And ray QR inside the equilateral prism is parallel to base.

12. **(c)** There will be no refraction from P to Q and then from Q to R all being identical made of same material. Hence the ray will now have the same deviation.



13. **(c)** Applying Snell's law at P,



$$\mu = \frac{\sin r}{\sin 30°} \quad \because \angle i = 30°$$

$$\Rightarrow \sin r = \frac{1.44}{2} = 0.72$$

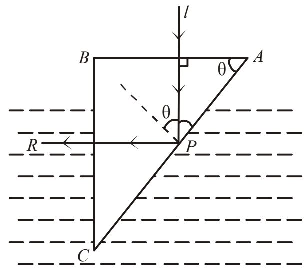
$\therefore \quad \delta = r - 30° = \sin^{-1}(0.72) - 30°$

$\therefore$ The rays make an angle of

$2\delta = 2[\sin^{-1}(0.72) - 30°]$ with each other.

14. **(a)** The phenomenon of total internal reflection takes place during reflection at P to reach at BC

$$\therefore \quad \sin\theta_{critical} = \frac{1}{{}_g^w\mu} = \frac{M_w}{M_g} = \frac{4/3}{3/2} = \frac{8}{9}$$



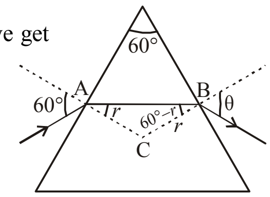
$\therefore$ $\sin\theta$ should be greater than critical angle $\theta_c = \frac{8}{9}$.

15. **(2)** Applying Snell's law at A

$\sin 60° = n \sin r$ ...(i)

Differentiating w.r.t 'n' we get

$$0 = \sin r + n \cos r \times \frac{dr}{dn} \quad \text{...(ii)}$$



Again, applying Snell's law at B

$\sin\theta = n \sin(60° - r)$ ...(iii)

Differentiating w.r.t 'n' we get

$$\cos\theta \frac{d\theta}{dn} = \sin(60° - r) + n\cos(60° - r)\left[-\frac{dr}{dn}\right]$$

$$\therefore \quad \cos\theta \frac{d\theta}{dn} = \sin(60° - r) - n\cos(60° - r)\left[-\frac{\tan r}{n}\right]$$

[from (ii)]

$$\therefore \quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}[\sin(60° - r) + \cos(60° - r)\tan r] \quad \text{...(iv)}$$

From eq. (i), for $n = \sqrt{3}$ we get $r = 30°$

From eq (iii), for $n = \sqrt{3}$, $r = 30°$ we get $\theta = 60°$

Substituting the values of r and θ in eq (iv) we get

$$\frac{d\theta}{dn} = \frac{1}{\cos 60°}[\sin 30° + \cos 30° \tan 30°] = 2\left(\frac{1}{2}+\frac{1}{2}\right) = 2$$

16. **(1.50)**

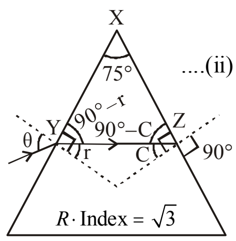
In ΔXYZ, $90° \times r + 90° - C + 75° = 180°$

$\therefore \quad r + C = 75 \Rightarrow r = 75° - C$ ....(i)

Applying Snell's law at Z

$\sqrt{3} \sin C = n \sin 90°$

$\sqrt{3} \sin C = n$ ....(ii)



Applying snell's law at Y

$1 \times \sin\theta = \sqrt{3} \sin r$

$= \sqrt{3} \sin(75° - C)$

From eq. (i)

For $\theta = 60°$

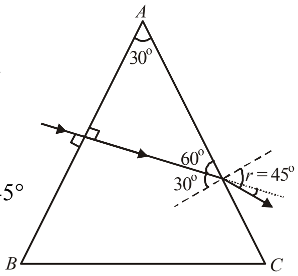
$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75° - C) \quad \therefore \quad C = 45°$$

From eq. (ii) $n = \sqrt{3}\sin 45° = \frac{\sqrt{3}}{\sqrt{2}}$

$\therefore \quad n^2 = 1.50$

17. At face AB of prism ray falls normally using Snell's law for the refraction at AC,

$\mu \sin i = (1) \sin r$



$\sqrt{2}\sin 30° = \sin r \Rightarrow r = 45°$

Angle of deviation at face AC

$= 45° - 30° = 15°$

18. $\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2}$, $\therefore \sqrt{2} = \frac{\sin\left(\frac{60+m}{2}\right)}{\sin 60/2}$

($\angle A = 60°$ for an equilateral triangle)

$\Rightarrow \frac{60+fm}{2} = 45° \Rightarrow fm = 30°$

The condition is for minium deviation. In this case the ray inside the prism becomes parallel to base. Therefore the angle made by the ray inside the prism with the base of the prism is 0°.

19. **True;**

For the light to split, the material through which the light passes should have refractive index greater than 1.

Since the prism is hollow, we get no spectrum as no refraction and no dispersion occur. The thickness of glass slabs through which the prism is made can be neglected.

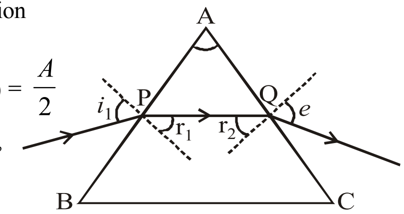
20. **(a, c, d)**

For minimum deviation

$i_1 = e$

$r_1 = r_2 = r \text{ (say)} = \frac{A}{2}$

$\delta_m = 2i_1 - A$,



Here $\delta_m = A$

$\therefore \quad i_1 = A = e = A$

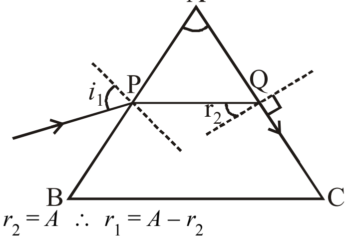
$\therefore \quad r_1 = \frac{i_1}{2}$

Relation between μ and A

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin A/2} = \frac{2\sin A/2 \cos A/2}{\sin A/2} = 2\cos A/2$$

When emergent ray tangential to the surface

$$\mu = \frac{\sin 90°}{\sin r_2} = \frac{1}{\sin r_2} \Rightarrow r_2 = \sin^{-1}\left(\frac{1}{\mu}\right)$$



But $r_1 + r_2 = A \quad \therefore \quad r_1 = A - r_2$

$\therefore \quad r_1 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$

Applying Snell's law at 'P'

$$\mu = \frac{\sin i_1}{\sin r_1} \quad \therefore \quad i_1 = \sin^{-1}\left[\mu\sin(A - \sin^{-1}\frac{1}{\mu})\right]$$

For minimum deviation through isoscletes prism if $\angle B = \angle C$, PQ || BC.

21. **(c)** Angle of deviation for the first prism P_1

$\delta_1 = (\mu_1 - 1)A_1$

Angle of deviation for the second prism P_2

$\delta_2 = (\mu_2 - 1)A_2$

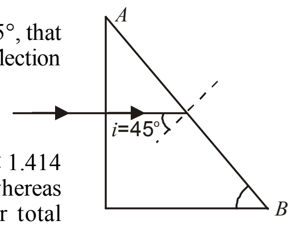
For dispersion without deviation, $\delta_{net} = 0$

$\therefore \quad (\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$ or, $A_2 = \frac{(1.54-1)}{(1.72-1)}4° = 3°$

22. **(a)** The colour beam of light for which $i > c$ will get total internal reflection.

$$\mu = \frac{1}{\sin C} = \frac{1}{\sin 45°} = 1.414$$

i.e. For an angle of incidence of 45°, that colour will suffer total internal reflection for which $\mu > 1.414$.



Therefore, red light for which $\mu < 1.414$ will be refracted at interface AB whereas blue and green light will suffer total internal reflection.

23. **(d)** $e \to f$. When the ray enters from the rectangular block to prism then angle of incidence > angle of refraction, so $\mu_2 > \mu_1$. The ray then moves away from the normal when it emerges out of the rectangular block. Therefore $\mu_2 > \mu_3$.

$e \to g$. As there is no deviation of the ray as it emerges out of the prism, $\therefore \mu_2 = \mu_1$.

$e \to h$. As the ray emerges out of prism, it moves away from the normal. $\therefore \mu_2 < \mu_1$. And the ray moves away from the normal as it emerges out of the rectangular block, $\therefore \mu_2 > \mu_3$.

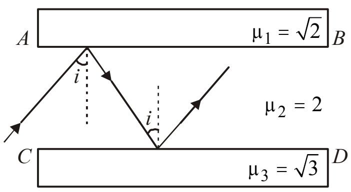
$e \to i$. At the prism surface, total internal reflection has taken place.

$\therefore$ Critical angle $45° > C \Rightarrow \sin 45° > \sin C$

$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \quad \therefore \mu_1 > \sqrt{2}\mu_2$$

24. **For total internal reflection on interface AB**

$$\sin i = \sin C_1 = \frac{1}{{}_2^1\mu} = \frac{{}_1\mu}{{}_2\mu} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow i = C_1 = 45°$$



For total internal reflection on interface CD.

$$\sin i = \sin C_2 = \frac{1}{{}_2^3\mu} = \frac{{}_3\mu}{{}_2\mu} = \frac{\sqrt{3}}{2} \Rightarrow i = C_2 = 60°$$

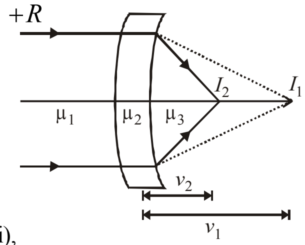
Hence the minimum angle of incidence such that the total internal reflection occurs on both the surfaces is 60°.

25. For refraction at first surface, if an object placed at infinity the image after first refraction will be formed at a distance v_1

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \quad \text{...(i)}$$

For refraction at second surface

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \quad \text{...(ii)}$$



Adding eq. (i) and (ii),

$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R} \Rightarrow v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

When incident rays are parallel, image formed at focus

Therefore, focal length of the given lens system $= \dfrac{\mu_3 R}{\mu_3 - \mu_1}$

26. (a) The rays incident at any angle on the interface BC pass through without bending at this surface if $n_1 = n_2$

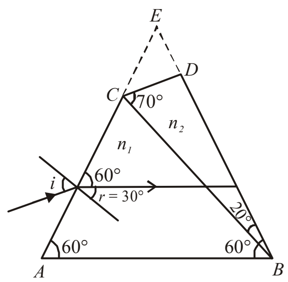
$$1.20 + \frac{10.8 \times 10^{-4}}{\lambda_0^2} = 1.45 + \frac{1.80 \times 10^{-4}}{\lambda_0^2}$$

(where λ_0 is in nm)

or $\lambda_0 = \left(\dfrac{9.0 \times 10^4}{0.25}\right)^{1/2} = 600$ nm

(b) For light of wavelength $\lambda_0 = 600$ nm, the combination of prism acts as a single prism shaped like an isosceles triangle ABE as shown in figure. At the minimum deviation, the ray inside the prism will be parallel to the base.

$\therefore \quad r = \dfrac{A}{2} = \dfrac{60°}{2} = 30°$



$n_1 = \dfrac{\sin i}{\sin r} \Rightarrow \sin i = n_1 \sin r$

$\Rightarrow \quad \sin i = \left[1.20 + \dfrac{10.8 \times 10^4}{(600)^2}\right] \times \dfrac{1}{2} \quad (\because \lambda_0 = 600 \text{ nm})$

or, $\sin i = \left(\dfrac{3}{4}\right)$

$\therefore$ Angle of incidence, $i = \sin^{-1}\left(\dfrac{3}{4}\right)$

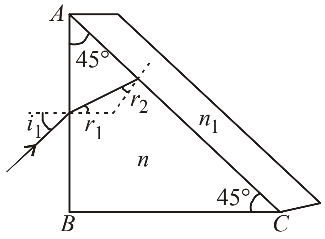
27. (i) At face AC

Critical angle, $\theta_c = \sin^{-1}\left(\dfrac{n_1}{n}\right)$ or $\sin \theta_c = \dfrac{n_1}{n}$

Now, it is given that $r_2 = \theta_c$

$\therefore r_1 = A - r_2 = (45° - \theta_c)$

Applying Snell's law at fact AB,



$n = \dfrac{\sin i_1}{\sin r_1}$

or $\quad \sin i_1 = n \sin r_1$

$\therefore \quad i_1 = \sin^{-1}(n \sin r_1)$

Substituting value of r_1, we get

$i_1 = \sin^{-1}\{n \sin (45° - \theta_c)\}$

$= \sin^{-1}\{n(\sin 45° \cos \theta_c - \cos 45° \sin \theta_c)\}$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}\left(\sqrt{1 - \sin^2 \theta_c} - \sin \theta_c\right)\right\}$$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}\left(\sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n}\right)\right\}$$

This is the required angle of incidence (i_1) at face AB for which the ray strikes at the diagonal face AC at critical angle is

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2} - n_1\right)\right\}$$

(ii) Here $n_1 \neq n$ (because $n_1 < n$ is given) so for ray to pass undeviated through AC

$r_2 = 0°$ (Ray $\perp$ on face AC)

or $\quad r_1 = A - r_2 = 45° - 0° = 45°$

Now applying Snell's law at face AB, we have $n = \dfrac{\sin i_1}{\sin r_1}$

or $\quad 1.352 = \dfrac{\sin i_1}{\sin 45°}$

$\therefore \quad \sin i_1 = (1.352)\left(\dfrac{1}{\sqrt{2}}\right) \qquad \sin i_1 = 0.956$

$\therefore \quad i_1 = \sin^{-1}(0.956) \approx 73°$

i.e., the required angle of incidence $i = 73°$

28. The situation can be shown as in the figure.

Here, $i = 60°$, $A = 30°$,

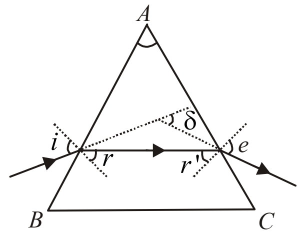
$\delta = 30°$, $e = ?$

And, $A + \delta = i + e$

$\Rightarrow \quad e = A + \delta - i$

$= 30° + 30° - 60° = 0$

Hence the emergent ray is normal to the face from which it emerges.



When $e = 0$, $r' = 0$

$r' + r = A = r = 30°$.

From Snell's law, refractive index of prism,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.732.$$

Topic-5 : Optical Instruments

1. **(d)** For telescope

Tube length (L) $= f_o + f_e = 60$

and magnification $(m) = \dfrac{f_o}{f_e} = 5 \Rightarrow f_0 = 5 f_e$

$\therefore \quad f_0 = 50$ cm and $f_e = 10$ cm
Hence focal length of eye-piece, $f_e = 10$ cm

2. **(a)** According question, $M = 375$
$L = 150$ mm, $f_0 = 5$ mm and $f_e = ?$

Using, magnification, $M \simeq \frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$

$$\Rightarrow 375 = \frac{150}{5}\left(1+\frac{250}{f_e}\right) \quad (\because D = 25 \text{ cm} = 250 \text{ mm})$$

$$\Rightarrow 12.5 = 1+\frac{250}{f_e} \quad \Rightarrow f_e = \frac{250}{11.5} = 21.7 \approx 22\,mm$$

3. **(b)** A telescope magnifies by making the object appearing closer.

4. **(c)** Reading one Þ without slab
Reading two Þ with slab
Reading three Þ with saw dust
Minimum three readings are required to determine refractive index of glass slab using a travelling microscope.

5. **(c)** Magnifying power of telescope,

$$MP = \frac{\beta \text{ (angle subtended by image at eye piece)}}{\alpha \text{ (angle subtended by object on objective)}}$$

Also, $MP = \frac{f_o}{f_e} = \frac{150}{5} = 30$

$$\alpha = \frac{50}{1000} = \frac{1}{20}\text{rad}$$

$$\therefore \quad \beta = \theta = \text{MP} \times \alpha = 30\times\frac{1}{20} = \frac{3}{2} = 1.5\,\text{rad}$$

or, $\beta = 1.5\times\frac{180°}{\pi} \simeq 84°$

6. **(a)** **Given :** $f_0 = 1.2$ cm; $f_e = 3.0$ cm
$u_0 = 1.25$ cm; $M_\infty = ?$

From $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0} \Rightarrow \frac{1}{1.2} = \frac{1}{v_0} - \frac{1}{(-1.25)}$

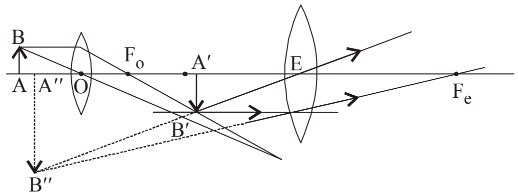
$$\Rightarrow \frac{1}{v_0} = \frac{1}{1.2} - \frac{1}{1.25} \Rightarrow v_0 = 30 \text{ cm}$$

Magnification at infinity,

$$M_\infty = -\frac{v_0}{u_0}\times\frac{D}{f_e} = \frac{30}{1.25}\times\frac{25}{3}$$

($\because D = 25$ cm least distance of distinct vision) = 200
Hence the magnifying power of the compound microscope is 200

7. **(c)** The intermediate image in compound microscope is real, inverted and magnified.



8. **(a)** Here $f_0 = 2$ cm and $f_e = 3$ cm.
Using lens formula for eye piece

$$\Rightarrow \frac{-1}{u_e} + \frac{1}{\infty} = \frac{1}{3} \Rightarrow u_e = -3 \text{ cm}$$

[$\because$ final image formed by the eyepiece at ∞]
Given distance between objective and eye piece = 15 cm
$\therefore$ Distance of image formed by the objective
$V_0 = 15 - 3 = 12$ cm.
Let u_0 be the object distance from objective, then for objective lens

$$-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0} \text{ or } \frac{-1}{u_o} + \frac{1}{12} = \frac{1}{2}$$

$$\Rightarrow \frac{-1}{u_o} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \quad \therefore u_o = -\frac{12}{5} = -2.4 \text{ cm}$$

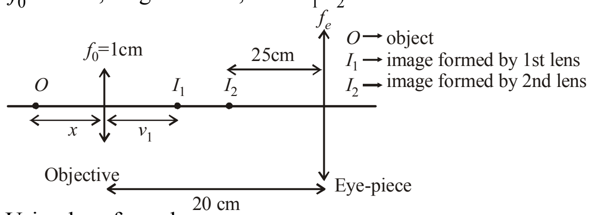
9. **(50)**
Given : Length of compound microscope, $L = 10$ cm
Focal length of objective $f_0 = 1$ cm and of eye-piece, $f_e = 5$ cm
$u_0 = f_e = 5$ cm
Final image formed at infinity (∞), $v_e = \infty$
$v_0 = 10 - 5 = 5$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{5} - \frac{1}{u_0} = \frac{1}{1} \Rightarrow u_0 = -\frac{5}{4} \text{ cm}$$

or, $\frac{5}{4} = \frac{N}{40} \qquad \therefore N = \frac{200}{4} = 50$ cm.

10. **(4.48)**
According to question, final image i.e., $v_2 = 25$ cm, $f_0 = 1$ cm, magnification, $m = m_1 m_2 = 100$



Using lens formula,

For first lens or objective $= \frac{1}{v_1} - \frac{1}{-x} = \frac{1}{1} \Rightarrow v_1 = \frac{x}{x-1}$

Also magnification $|m_1| = \left|\frac{v_1}{u_1}\right| = \frac{1}{x-1}$

For 2nd lens or eye-piece, this is acting as object

$$\therefore u_2 = -(20-v_1) = -\left(20-\frac{x}{x-1}\right) \text{ and } v_2 = -25 \text{ cm}$$

Angular magnification $|m_A| = \left|\frac{D}{u_2}\right| = \frac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$

$$\left(\frac{1}{x-1}\right)\left(\frac{25}{20-\frac{x}{x-1}}\right) = 100$$

$$\Rightarrow \frac{25}{20(x-1)-x} = 100 \Rightarrow 1 = 80(x-1) - 4x$$

$$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76}$$

$$\Rightarrow u_2 = -\left(20 - \frac{\frac{81}{76}}{\frac{81}{76} - 1}\right) = \frac{-19}{5}$$

Again using lens formula for eye-piece

$$\frac{1}{-25} - \frac{1}{-\frac{19}{5}} = \frac{1}{f_e} \Rightarrow f_e = \frac{25\times19}{106} \approx 4.48\,\text{cm}$$

11. The resolving power of a microscope

$$\propto \frac{1}{\lambda\ used}$$

The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is smaller than the wavelength of visible light.

12. **(a, b, c, d)** In an astronomical telescope the distance between the objective lens and eyepiece lens

$L = f_0 + f_e = 16 + 0.02 = 16.02$ m

Angular magnification $= -\dfrac{f_{\text{objective}}}{f_{\text{eye piece}}} = \dfrac{-16}{0.02} = -800$

The image seen by the astronomical telescope is inverted. Also the objective is larger than eye piece.

13. **(d)** In an astronomical telescope when the final image is formed at infinity:

Angular magnification

$$\therefore \quad M = \frac{f_o}{f_e} = 5 \Rightarrow f_o = 5f_e$$

And the separation between the objectives and eye-piece.

$L = f_o + f_e = 36 \Rightarrow 5f_e + f_e = 36$ or $f_e = 6$ cm

$\therefore \quad f_o = 5f_e = 5 \times 6 = 30$ cm

Topic-6 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** When the light is incident on glass - an interface travelling from glass at an angle less than critical angle a small part of light will be reflected and most part will be transmitted.

When the light is incident greater than the critical angle, it gets completed reflected (total internal reflection) so 0% transmission and 100% reflection.

These characteristics are depicted in option (c).

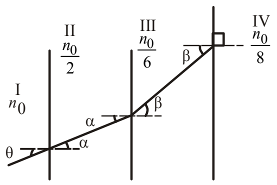
2. **(b)** For refraction at parallel interfaces

From Snell's law,

$$n_0 \sin\theta = \frac{n_0}{2}\sin\alpha = \frac{n_0}{6}\sin\beta = \frac{n_0}{8}\sin 90^\circ$$

The angle of refraction in the region IV must be 90° as the beam just misses entering the region IV.

$\therefore \quad \sin\theta = \dfrac{1}{8}$



or, $\theta = \sin^{-1}\left(\dfrac{1}{8}\right)$

3. **(c)** The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of

$$= \frac{33.25}{1.33} = 25\,\text{cm} \quad \left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object. Distance of object from mirror,

$u = -(15 + 25) = -40$ cm

and $v = -\left[15 + \dfrac{25}{1.33}\right]$

Where $\dfrac{25}{1.33}$ is the real depth of the image.

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-33.8} + \frac{1}{-40}$$

$\therefore \quad f = -18.31$ cm

4. **(a)** Distance of virtual image from surface = 6 cm

The rays coming from the point object fall on the glass-air interface normally and hence pass undeviated. Therefore if we retrace the path of the refracted rays backwards, the image will be formed at the centre only.

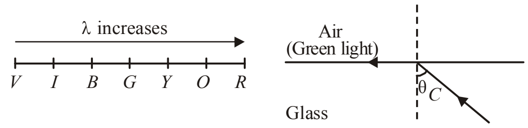
5. **(c)** Frequency does not change with change of medium.

$\therefore$ Frequency of sound in water = Frequency heard in air = 600 Hz

6. **(c)** $\sin\theta_C = \dfrac{1}{\mu}$ and $\mu \propto \dfrac{1}{\lambda}$

$\therefore \quad \sin\theta_C \propto \lambda$

For higher value of λ, the critical angle θ_C also increases



Hence yellow, orange and red colours of light for which $\theta_C >$ incidence angle will come out to air.

7. **(c)** Since both surfaces have same radius of curvature $R_1 = R_2 = R$ on the same side, no dispersion will occur.

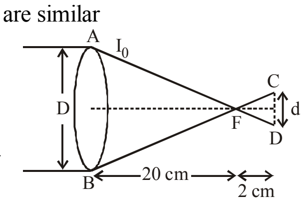
For no dispersim $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = 0$

8. **(130.00)**

Let D be the initial area covered by light and d be the final area covered by light at 22 cm.

From figure, ΔAFB and ΔCFD are similar

$\therefore \quad \dfrac{d}{D} = \dfrac{2}{20} = \dfrac{1}{10}$



$\therefore$ Ratio of area $= \dfrac{d^2}{D^2} = \dfrac{1}{100}$

As there is no energy loss $\therefore I_0 D^2 = I d^2$

$\therefore$ Average intensity of light at a distance 22 cm

$$I = \frac{D^2}{d^2} I_0 = 1.3 \times 100 = 130.00\ \text{kWm}^{-2}$$

9. Velocity of light in vacuum $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Velocity of light in a medium $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$

Refractive index, $\mu = \dfrac{c}{v} = \dfrac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \dfrac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$

10. For refraction at APB

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}$$

$$\Rightarrow \quad \frac{-2}{-15}+\frac{1}{v}=\frac{1-2}{-10} \quad \Rightarrow v=-30 \text{ cm}$$

Therefor the image of O will be formed at 30 cm to the right of P.

11. **True;**

Intensity of a line source of power (P) at a distance r from the source

$$I=\frac{p}{A}=\frac{P}{2\pi rl} \quad \text{or,} \quad I\propto\frac{1}{r}$$

12. **True;**

This is due to atmospheric refraction. The light coming from sun bends towards the normal. As we go higher in the atmosphere density by decreases. Therefore, sun appears higher.

13. **(b, d)** As we know, $\mu = \dfrac{\text{real depth}}{\text{apparent depth}}$

Case - I $\quad H_1=\dfrac{H}{1.5}=\dfrac{30}{1.5}=20\text{cm}$

Case - II $\quad \dfrac{-n_1}{u}+\dfrac{n_2}{v}=\dfrac{n_2-n_1}{R}$

$$\therefore \frac{-1.5}{-30}+\frac{1}{-H_2}=\frac{1-1.5}{-300} \quad \therefore H_2=20.68 \text{ cm}$$

Case - III $\quad \therefore \dfrac{-1.5}{-30}+\dfrac{1}{-H_3}=\dfrac{1-1.5}{300}$

$$\therefore \frac{1}{-H_3}=-\frac{1}{600}-\frac{1}{20} \quad \therefore H_3=19.35 \text{ cm}$$

14. **(a, b, c)** $\dfrac{1}{f}=(n-1)\dfrac{2}{R}\Rightarrow f=\dfrac{R}{2(n-1)}$

$$\frac{1}{f+\Delta f}=\frac{(n-1)}{R}+\frac{(n+\Delta n-1)}{R}=\frac{2(n-1)+\Delta n}{R}$$

$$\therefore \quad f+\Delta f=\frac{R}{2(n-1)+\Delta n}$$

$$\therefore \quad \frac{f+\Delta f}{f}=\frac{R}{2(n-1)+\Delta n}\times\frac{[2(n-1)]}{R}$$

$$=\frac{[2(n-1)]}{[2(n-1)+\Delta n]}$$

$$\therefore \quad \frac{\Delta f}{f}=\frac{2n-2-2n+2-\Delta n}{[2(n-1)+\Delta n]}$$

$$=\frac{-\Delta n}{2n-2+\Delta n}=\frac{-\Delta n}{2(n-1)} \qquad \text{...(i)}$$

$[\because \ \Delta n << (n-1)]$

From equation (i) if $\dfrac{\Delta n}{n}<0$, then $\dfrac{\Delta f}{f}>0.$

Also, $\left|\dfrac{\Delta f}{f}\right|>\left|\dfrac{\Delta n}{n}\right|$

The relation between $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ remains unchanged, if convex surface are replaced by concave surface of the same radius of curvature.

For $n=1.5$, $\Delta n=10^{-3}$ and $f=20$ cm then eq. (i)

$$\frac{\Delta f}{20}=-\frac{10^{-3}}{2(1.5-1)}\Rightarrow \Delta f=-0.02 \text{ cm}$$

or $\quad |\Delta f|=0.02$ cm

15. **(b)** For refraction in S_1

$$-\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}\Rightarrow -\frac{1.5}{-50}+\frac{1}{V}=\frac{1-1.5}{-10}$$

$\Rightarrow v=50$ cm.

Again for refraction in S_2

$$-\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$$

$$-\frac{1}{-(d-50)}+\frac{1.5}{\infty}=\frac{1.5-1}{10}$$

$$\therefore \quad \frac{1}{d-50}=\frac{1}{20}\Rightarrow d-50=20 \ \therefore \ d=70 \text{ cm.}$$

16. **(a, b, c)**

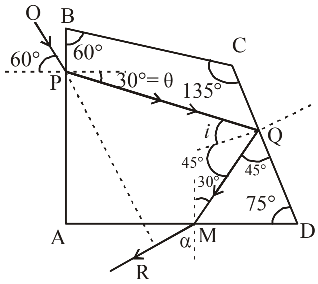
(a) Applying Snell's law at P

$n_1 \sin i = n_2 \sin r$

$\Rightarrow \quad n_1 \sin 60° = n_2 \sin\theta$

$\Rightarrow \quad \sin 60° = \sqrt{3}\sin\theta \Rightarrow \theta = 30° = r$

In quadrilateral BCQP,



$60° + (90° + 30°) + 135° + \angle PQC = 360°$

$\Rightarrow \angle PQC = 45° \quad \Rightarrow i = 45°$

The critical angle for prism - air pair of media

$C = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ which is less than 45°.

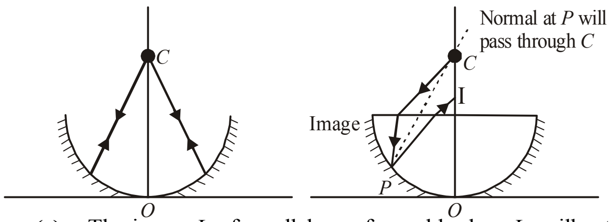
Therefore total internal reflection takes place at face CD.

(b) In ΔQDM, $\angle$QMD = 180° – (45° + 75°) = 60°

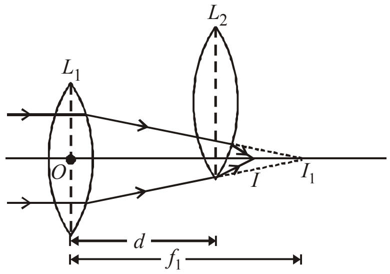
Therefore the angle of incidence of ray QM on AD is 30°. This angle is less than the critical angle. Hence the ray emerges out of face AD.

(c) From the figure, angle between the incident ray OP and the emergent ray MR is 90°.

17. **(d)** The ray diagram is shown in figure. Therefore, the image will be real and between C and O. When the mirror is filled with water.



18. **(c)** The image I_1 of parallel rays formed by lens L_1 will act as virtual object for second lens L_2.



Applying lens formula for lens L_2

$$\Rightarrow \quad \frac{1}{v_2} - \frac{1}{f_1 - d} = \frac{1}{f_2} \Rightarrow v_2 = \frac{f_2(f_1 - d)}{f_2 + f_1 - d}$$

$\therefore$ Horizontal distance of the image I from O

$$x = d + \frac{f_2(f_1 - d)}{f_2 + f_1 - d} = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

To find the y-coordinate, we use magnification formula for lens L_2

$$m = \frac{v_2}{u_2} = \frac{\frac{f_2(f_1 - d)}{f_1 + f_2 - d}}{f_1 - d} = \frac{f_2}{f_1 + f_2 - d}$$

Also $m = \frac{h_2}{\Delta} \Rightarrow h_2 = \frac{\Delta \times f_2}{f_1 + f_2 - d}$

$\therefore$ y-coordinate $y = \Delta - h_2$

$$= \Delta - \frac{\Delta f_2}{f_1 + f_2 - d} = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$$

19. **(c)** Let n_1 = refractive index of air and
n_2 = refractive index of meta material which is negative

From Snell's law, $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$

$\because$ n_2 negative $\therefore$ θ_2 is also negative hence graph (c) is correct.

20. **(b)** Speed of light in a medium, $V = \frac{C}{n}$

n for meta-material = $|n|$

$\therefore$ Speed of light in the meta-material $V = \frac{C}{|n|}$.

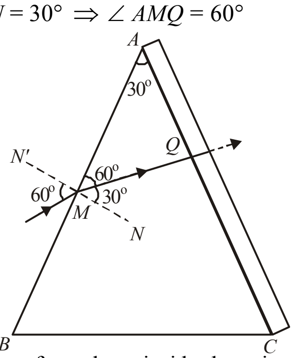
21. **(a)** Applying Snell's law at surface AB
$\mu_1 \sin i = \mu_2 \sin r$

$\Rightarrow$ $\mu_{air} \sin 60° = \mu_p \sin r_1 \Rightarrow \frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1 \Rightarrow r_1 = 30°$

$\because$ $r_1 + r_2 = A$

$\therefore$ $r_2 = A - r_1 = 0°$

But $\angle QMN = 30° \Rightarrow \angle AMQ = 60°$



Therefore, the refracted ray inside the prism hits the other face at 90° hence angle of emergence $e = 0$.

(b) Multiple reflections occur in the film for minimum thickness. When the ray from surface AC of the prism enters the film multiple reflections occur in the film.

The intensity of emergent ray will be maximum if transmitted waves undergo constructive interference.

$\therefore$ For minimum thickness, $\Delta x = \lambda$

$\Rightarrow$ $\Delta x = 2\mu t = \lambda$ (where t = thickness of film)

$\therefore$ $t = \frac{\lambda}{2\mu} = 125\,\text{nm}$

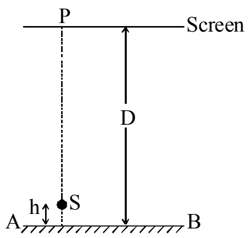
22. (a) Because S is a point source, so interference fringes will be circular.

(b) If intensity of light falling on P directly from S, $I_1 = I$, then the intensity of light falling at P after reflection from the mirror AB, $I_2 = 36\%$ of $I = 0.36\,I$

$$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I} - \sqrt{0.36\,I}}{\sqrt{I} + \sqrt{0.36\,I}}\right)^2 = \left(\frac{0.4}{1.6}\right)^2 = \frac{1}{16}$$

(c) For maximum intensity at P, path difference = $n\lambda$

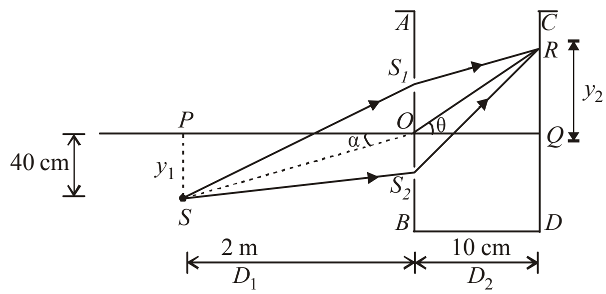
If AB is shifted by a distance x, it will cause an additional path difference of $2x$.



For minimum value of x, $n = 1$

$\therefore$ $2x = \lambda \Rightarrow x = \frac{\lambda}{2} = \frac{600}{2} = 300\,\text{nm}$

23. (i) Given: $D_1 = 2$ m, $D_2 = 10$ cm, $y_1 = 40$ cm.
O is the middle point of two slits S_1 and S_2.
$S_1S_2 = d = 0.8$ mm



$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$\therefore$ $\sin\alpha = \frac{1}{\sqrt{26}} = \frac{1}{5.1} \approx \frac{1}{1.5} \approx \tan\alpha$

Path difference $\Delta X_1 = SS_1 - SS_2$

or $\Delta X_1 = d\sin\alpha = (0.8\text{ mm})\left(\frac{1}{5}\right) = 0.16$ mm ... (i)

Let R denotes the position of central bright fringe. Net path difference = 0

Now $\Delta X_2 = S_2R - S_1R$ or $\Delta X_2 = d\sin\theta$ ... (ii)

For central bright fringe

$\Delta X_2 - \Delta X_1 = 0$ or $d\sin\theta - \Delta X_1 = 0$

or $\quad d\sin\theta = \Delta X_1 = 0.16\,\text{mm}$

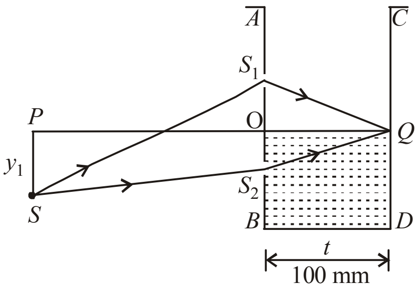
or $\quad (0.8)\sin\theta = 0.16$ or $\sin\theta = \dfrac{0.16}{0.8} = \dfrac{1}{5}$

$\therefore \quad \tan\theta = \dfrac{1}{\sqrt{24}} = \dfrac{1}{4.9} \approx \dfrac{1}{5} = \sin\theta \quad \therefore \quad \tan\theta = \dfrac{y_2}{D_2}$

or $\quad \dfrac{1}{5} = \dfrac{y_2}{D_2}$ or $y_2 = \dfrac{D_2}{5} = \dfrac{10}{5} = 2$ cm

$\therefore$ Position of central bright fringe is 2 cm above point Q on side CD.

(ii)



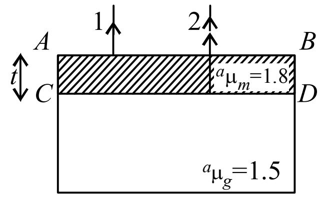
Let a liquid of refractive index μ is poured into the vessel. The central bright fringe is found at Q if net path difference = 0.

$\therefore \quad (\mu - 1)t = \Delta X_1$ or $(\mu - 1)(100) = 0.16$

or $\quad \mu - 1 = 0.0016$ or $\mu = 1.0016$

24. (b) The path difference between the two rays ray-1 reflected from the upper surface AB and ray-2 reflected from lower surface CD

$\Delta x = {}^a\mu_m \times 2t + \dfrac{\lambda}{2}$



Here $\dfrac{\lambda}{2}$ is the path difference as the ray 1 suffer reflection from a denser medium on surface AB

For constructive interference

Path difference = $n\lambda$ where n is 1, 2,....

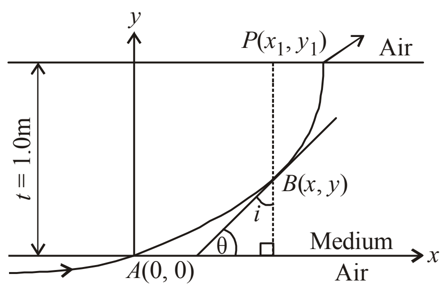
$\therefore \quad {}^a\mu_m \times 2t + \dfrac{\lambda}{2} = n\lambda \;\Rightarrow\; 2\,{}^a\mu_m t = \left(n - \dfrac{1}{2}\right)\lambda$

For least value of thickness t, $n = 1$

$\therefore \quad t = \dfrac{\lambda}{4\,{}^a\mu_m} = \dfrac{648}{4 \times 1.8} = 90\,\text{nm}.$

25. **(a)**

(a) A ray of light travelling in air is incident at grazing angle (incident angle $\cong 90°$) on a slab. The point of incidence is the origin $A(0, 0)$.



Refractive index of medium = μ

$\mu(y) = [ky^{3/2} + 1]^{1/2}$ ...(i)

where $k = 1.0$ (metre)$^{-3/2}$. The refractive index of air is 1.

The ray travels from air to denser medium. It continuously bends towards normal.

Slope at B is $\tan\theta$. Angle of incidence = i at B.

$\therefore \quad \dfrac{dy}{dx} = \tan\theta \qquad$ According to geometry, $i + \theta = 90°$

or $\quad \theta = \dfrac{\pi}{2} - i \quad \therefore \quad$ slope$\left(\dfrac{dy}{dx}\right) = \tan\theta$

or $\quad \dfrac{dy}{dx} = \tan\left(\dfrac{\pi}{2} - i\right)$ or $\dfrac{dy}{dx} = \cot i$ ...(ii)

(b) Equation for trajectory $y(x)$ of ray in medium:

Applying Snell's law at points A and B.

$\mu_A \sin(i_A) = \mu_B \sin(i_B)$ ...(iii)

$\mu_A = 1$ from (i) because $y = 0$

$i_A = 90°$, because of grazing incidence. $i_B = i$

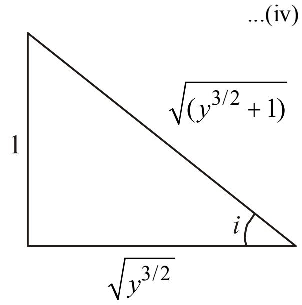
$\mu_B = [ky^{3/2} + 1]$ where $k = 1.0$ m$^{-3/2}$

or $\quad \mu_B = [y^{3/2} + 1]$ ...(iv)

From (iii) and (iv), we get

$(1)(1) = [y^{3/2} + 1]\sin(i_B)$

or $\quad \sin i = \dfrac{1}{(y^{3/2} + 1)}$



or $\quad \cot i = \sqrt{y^{3/2}}$

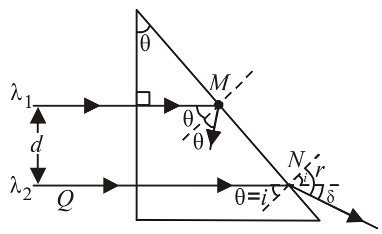
or $\quad \cot i = y^{3/4}$

or $\quad \dfrac{dy}{dx} = y^{3/4}$ or $y^{-3/4}\,dy = dx$

or $\quad \int_0^y y^{3/4} dy = \int_0^x dx$ or $4y^{1/4} = x \Rightarrow y = (x/4)^4$

$\therefore$ Equation of trajectory of ray in medium is $y = (x/4)^4$.

26. (a) For TIR to take place, θ should be greater than C. For smaller values of C, the values of μ should be high or in other words the value of λ should be small.



From the given expression of

$$\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$$

Total internal reflection will be given by $\lambda_1 = 4000$ Å

Here, $\sin\theta = 0.8$ (given) $\Rightarrow \theta = 53.1°$

$\therefore \quad \mu = \dfrac{1}{\sin\theta} = \dfrac{1}{0.8} = 1.25$

$\therefore \quad \mu = 1.2 + \dfrac{b}{(4000 \times 10^{-10})^2} = 1.25$

$\therefore \quad b = 0.8 \times 10^{-14}$ m^2

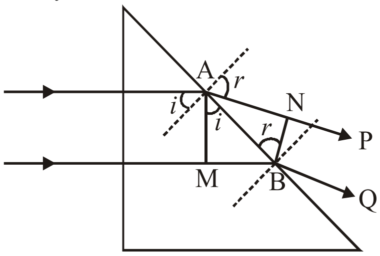
(b) Applying Snell's law at N for wavelength $\lambda_2 = 5000$Å

$\mu = \dfrac{\sin r}{\sin i}$ where $\mu = 1.2 + \dfrac{0.8 \times 10^{-14}}{(5000 \times 10^{-10})^2} = 1.232$

$\Rightarrow \quad 1.232 = \dfrac{\sin r}{0.8} \Rightarrow r = 80.3°$

From the figure it is clear that the deviation,
$\delta = r - i = 80.3° - 53.1° = 27.2°$

(c) The intensities of transmitted beams are $4I$ and I respectively.



Path difference, $\Delta x = \mu(MB) - AN$

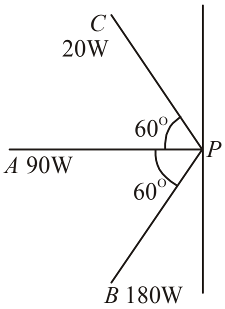
$$= \frac{\sin r}{\sin i}(AB\sin i) - AB\sin\theta = 0$$

Phase difference between rays P and Q is zero. Hence maximum intensity will be obtained from their interference.

$\therefore$ Resultant Intensity

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{4I} + \sqrt{I}\right)^2 = (3\sqrt{I})^2 = 9I.$$

27. Intensity at point P, $I_P = I_A + I_B + I_C$



$$I_A = \frac{(\text{Illumination power}) \times \cos\theta}{4\pi r^2}$$

$$= \frac{90 \times \cos 0°}{4\pi \times 3^2} = \frac{10}{4\pi}\text{ watt/m}^2$$

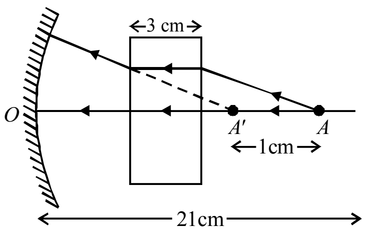
Similarly,

$$I_B = \frac{180 \times \cos 60°}{4\pi \times (1.5)^2} = \frac{10}{\pi}\text{ watt/m}^2$$

$I_C = 20 \cos 60° = 10$

$$\therefore \quad I_p = \frac{10}{4\pi} + \frac{10}{\pi} + 10 = 13.9\text{ W/m}^2$$

28. The rays originating from the point object at A suffers refraction due to glass slab before striking the concave mirror.
For the mirror the rays are coming from A'



Apparent shift $AA' = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1$ cm

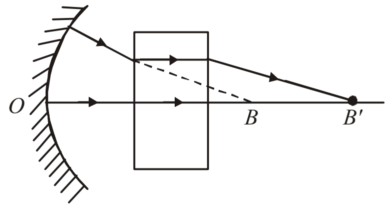
Therefore the object distance
$u = OA' = OA - AA' = 21 - 1 = 20$ cm

The apparent object at A' as a virtual object for concave mirror.

Now from mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

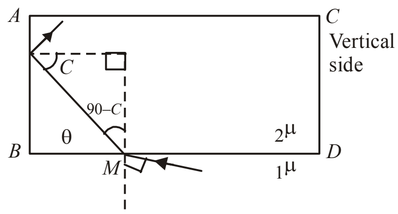
$$v = \frac{uf}{u - f} = \frac{20 \times 5}{20 - 5} = \frac{20}{3}\text{cm} = 6.67\text{cm}$$

The reflected rays again pass through the glass slab suffering a further shift of 1 cm. The image should have formed at B in the absence of glass slab. But due to its presence the image is formed at B'.



Therefore image distance, $OB = OB + BB'$
$= 6.67 + 1 = 7.67$ cm

29. For a grazing incident ray at BD for which i ≈ 90° the angle of refraction (90° – C) is maximum. For this C is least. Let C is greater than the critical angle.



Applying Snell's law at M

$${}_2^1\mu = \frac{\sin 90°}{\sin(90° - C)} \Rightarrow {}_2^1\mu = \frac{1}{\cos C} \qquad \text{...(i)}$$

Also $${}_2^1\mu = \frac{1}{\sin C} \qquad \text{...(ii)}$$

When C is the critical angle.

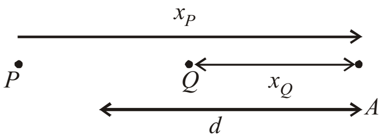
From (i) and (ii), $\frac{1}{\cos C} = \frac{1}{\sin C} \Rightarrow C = 45°$

$$\therefore \quad {}_2^1\mu = \frac{1}{\sin 45°} = \sqrt{2} = 1.41$$

Therefore, the minimum value of $\mu_{glassslab} = 1.41$ for which letters on the page are not visible from any of the vertical faces of the block.

Topic-1 : Wavefront, Interference of Light, Coherent and Incoherent Sources

1. **(b)** For (A)



$x_P - x_Q = (d + 2.5) - (d - 2.5) = 5$ m

Phase difference $\Delta\phi$ due to path difference

$= \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5) = \frac{\pi}{2}$.

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

$\therefore$ Total phase difference at A $\frac{\pi}{2} - \frac{\pi}{2} = 0$

(due to P being ahead of Q by 90°)

$I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(0) = 4I$

For C,

Path difference, $x_Q - x_P = 5$ m

Phase difference $\Delta\phi$ due to path difference

$= \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5) = \frac{\pi}{2}$

Total phase difference at C $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\phi)$

$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$

For B, Path difference, $x_P - x_Q = 0$

Phase difference, $\Delta\phi = \frac{\pi}{2}$ (due to P being ahead of Q by 90°)

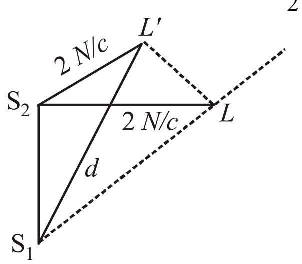
$I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$

Therefore intensities of radiation at A, B and C will be in the ratio $I_A : I_B : I_C = 4I : 2I : 0 = 2 : 1 : 0$.

2. **(d)** Initially, $S_2L = 2$ m

$S_1L = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2} = 2.5$ m

Path difference, $\Delta x = S_1L - S_2L = 0.5 \text{ m} = \frac{\lambda}{2}$



When the listner move from L, first maxima will appear if path difference is integral multiple of wavelength.

For example

$\Delta x = n\lambda = 1\lambda$ (n = 1 for first maxima)

$\therefore \Delta x = \lambda = S_1L' - S_2L \Rightarrow 1 = d - 2 \Rightarrow d = 3$ m

3. **(c)** The distance traversed by light in a medium of refractive index m in time t is given by

$d = vt$ ...(i)

where v is velocity of light in the medium. The distance traversed by light in a vacuum in this time,

$\Delta = ct = c \times \frac{d}{v}$ [from equation (i)]

$= d\frac{c}{v} = \mu d$ ...(ii) $(\because \mu = \frac{c}{v})$

This distance is the equvalent distance in vacuum and is called optical path.

Optical path for first ray which travels a path L_1 through a medium of refractive index $n_1 = n_1L_1$

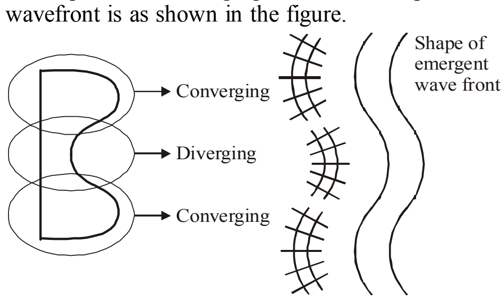
Optical path for second ray which travels a path L_2 through a medium of refractive index $n_2 = n_2L_2$

Path difference $= n_1L_1 - n_2L_2$

Now, phase difference

$= \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times (n_1L_1 - n_2L_2)$

4. **(a)** Clearly middle part of glass is diverging and upper and lower part are converging so correct shape of the emergent wavefront is as shown in the figure.



5. **(c)** Given amplitude ratio of waves is $\frac{a_1}{a_2} = \frac{3}{1}$

so, $\frac{I_{max}}{I_{min}} - \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2$

$= \left(\frac{\frac{a_2}{a_1} + 1}{\frac{a_2}{a_1} - 1}\right)^2 = \left(\frac{3+1}{3-1}\right)^2 = \left(\frac{4}{2}\right)^2 = \frac{4}{1} = 4$

6. **(b)** As we know, $\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$

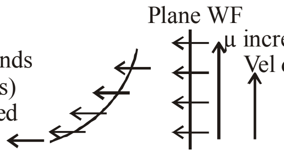
and $\sqrt{\frac{I_1}{I_2}} = \frac{A_1}{A_2}$

$\frac{I_{max}}{I_{min}} = 16 \Rightarrow \frac{A_{max}}{A_{min}} = 4 \Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$

Using componendo and dividendo.

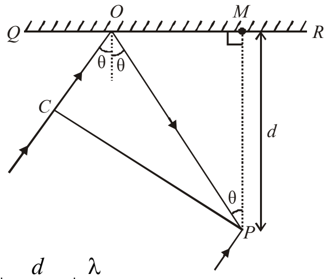
$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

7. **(b)**



8. **(c)**

9. **(b)** In ΔOPM,



$OP = \frac{d}{\cos\theta}$

In ΔCOP,

$OC = \frac{d\cos 2\theta}{\cos\theta}$

Path difference between the two rays reaching P

$= CO + OP + \frac{\lambda}{2} = \frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$

$= \frac{d}{\cos\theta}(\cos 2\theta + 1) + \frac{\lambda}{2} = 2d\cos\theta + \frac{\lambda}{2}$

For constructive interference at P, path difference = $n\lambda$

$\therefore \quad 2d\cos\theta + \frac{\lambda}{2} = n\lambda \quad \Rightarrow \cos\theta = \frac{(2n-1)}{4}\frac{\lambda}{d}$

For $n = 1$, $\cos\theta = \frac{\lambda}{4d}$

10. **(b)** As we know, $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

When phase difference is $\pi/2$

$I_{\pi/2} = I + 4I \quad \Rightarrow \quad I_{\pi/2} = 5I$

Again when d phase difference is π

$I_\pi = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\pi = I$

$\therefore \quad I_{\pi/2} - I_\pi = 5I - I = 4I$

11. **(a)** Locus of equal path difference are lines running parallel to axis of the cylinder. Hence straight interference fringes will be observed.

12. **(c)** Let $I_1 = I$ and $I_2 = 4I$

$\therefore \quad I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I} + \sqrt{4I})^2 = (3\sqrt{I})^2 = 9I$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{I} - \sqrt{4I})^2 = I$

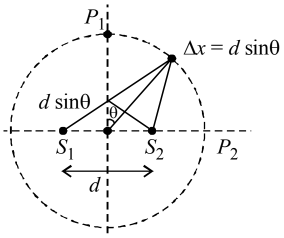
13. **(b, c)**

The path difference at any point P on the circumference $\Delta x = d\sin\theta$

Here $d = 1.8$ mm

At P_1 $\quad \Delta x = d\sin\theta \quad (\because \theta = 0°)$

At P_2 $\quad \Delta x = d\sin\theta \quad (\because \theta = 90°)$



Number of maxima, $n = \frac{\Delta x}{\lambda} = \frac{1.8 \text{ mm}}{600 \text{ nm}} = 3000$

At P_2 $\quad \Delta x = 3000\,\lambda$ $\quad$ [$\therefore$ Bringt fringe]

At P_2, 3000th maxima is formed.

For (a) option

$\Delta x = d\sin\theta \Rightarrow d\Delta x = d\cos d\theta$

$R_1 = d\cos\theta\, Rd\theta \Rightarrow Rd\theta = \frac{R\lambda}{d\cos\theta}\,\theta \therefore Rd\theta \propto \frac{1}{\cos\theta}$

As we move from P_1 to P_2, θ increases, cos θ decreases.

14. When the space between the lens and the mirror is filled with water, a system of two lenses.

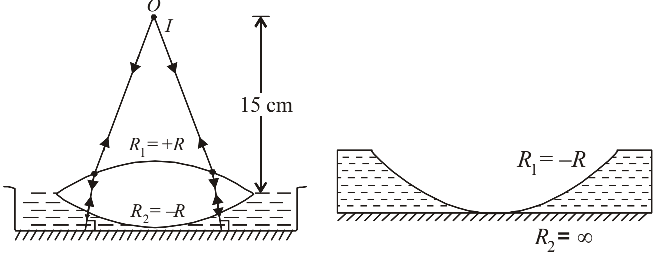
(i) equi-convex lens of $\mu_1 = \frac{3}{2} = 1.5$ and

(ii) plano-concave water lens of $\mu_2 = \frac{4}{3} = 1.33$

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$= \left(\frac{3}{2} - 1\right)\frac{2}{R} + \left(\frac{4}{3} - 1\right)\left(\frac{-1}{R}\right) = \frac{1}{R} - \frac{1}{3R} = \frac{2}{3R}$

$\therefore \quad F = \frac{3}{2}R$



A convex lens placed on a plane mirror behaves like a concave mirror. The image is formed at the object itself if the object is placed at centre of curvature of concave mirror.

After refraction through lens, the rays fall on the plane mirror normally and retrace their path to form image at the object itself.

$\therefore$ Focal length of system (F) = 15 cm

$\therefore \quad 15 = \frac{3}{2}R \Rightarrow R = 10$ cm

Now let μ be the refractive index of the liquid betweenlens and mirror. The image coincides with object when it is placed at 25 cm $\therefore F' = 25$ cm

In this case

$\frac{1}{F'} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$\Rightarrow \quad \frac{1}{F'} = \left(\frac{3}{2} - 1\right) \times \frac{2}{R} - \frac{(\mu - 1)}{R}$

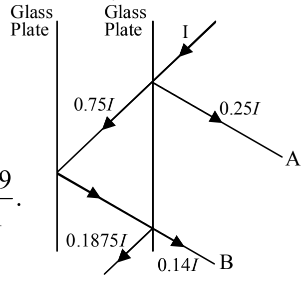
$\Rightarrow \quad \frac{1}{F'} = \frac{1}{R} - \frac{(\mu - 1)}{R} = \frac{2 - \mu}{R}$ or, $\frac{1}{25} = \frac{2 - \mu}{10} \quad \therefore \mu = 1.6$

15. As shown in the figure, the interference will be between $0.25\,I = I_1$ and $0.14\,I = I_2$ as each plate reflects 25% and transmits 75% of light.

Interference pattern takes place between rays A and B.

$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$

$= \frac{[\sqrt{0.25I} + \sqrt{0.14\,I}]^2}{[\sqrt{0.25I} - \sqrt{0.14\,I}]^2} = \frac{49}{1}.$



Topic-2 : Young's Double Slit Experiment

1. **(b)** Given : Wavelength of light, $\lambda = 500$ nm
Distance between the slits, $d = 0.05$ mm
Angular width of the fringe formed,
$$\theta = \frac{\lambda}{d} = \frac{500\times10^{-9}}{0.05\times10^{-3}} = 0.01 \text{ rad} = 0.57°.$$

2. **(a)** Path difference, $\Delta P = d\sin\theta = d\theta$
d = distance between slits = 1 mm = 10^{-3} mm
D = distance between the slits and screen = 100 cm = 1 m
y = distance between central bright fringe and observed fringe = 1.27 mm
$$\therefore \Delta P = \frac{dy}{D} = \frac{10^{-3}\times1.270\,\text{mm}}{1\text{ m}} = 1.27\ \mu\text{m}$$

3. **(d)** Let n_1 fringes are visible with light of wavelength λ_1 and n_2 with light of wavelength λ_2. Then
$$\beta = \frac{n_1D\lambda_1}{d} = \frac{n_2D\lambda_2}{d} \qquad \left(\because \beta = \frac{n\lambda D}{d}\right)$$
$$\Rightarrow \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow n_2 = \frac{700}{400}\times16 = 28$$

4. **(a)** Given, Path difference, $\Delta x = \frac{\lambda}{8}$
Phase differences, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \frac{\pi}{4}$
$$I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$$
$$\Rightarrow \frac{I}{I_0} = \cos^2\left(\frac{\frac{\pi}{4}}{2}\right) = \cos^2\left(\frac{\pi}{8}\right) \Rightarrow \frac{I}{I_0} = 0.853$$

5. **(c)** Given, distance between screen and slits, $D = 1.5$ m
Seperation between slits, d = 0.15 mm
Wavelength of source of light, $\lambda = 589$ nm
Fringe-width,
$$w = \frac{D}{d}\lambda = \frac{1.5}{0.15\times10^{-3}}\times589\times10^{-9}\,m$$
$= 589\times10^{-2}$ mm = 5.89 mm ≈ 5.9 mm

6. **(c)** Given, $\Delta = \beta$
or $\frac{D(\mu-1)t}{d} = \frac{D\lambda}{d}$ $\qquad \therefore\ t = \frac{\lambda}{(\mu-1)}$

7. **(Bonus)** Shift = $n\beta$ (given)
$$\therefore\ D\frac{(\mu-1)t}{a} = \frac{n\lambda D}{a} \quad \left[\because \text{Shift} = \frac{D(\mu-1)t}{a}\right]$$
or $t = \frac{n\lambda}{(\mu-1)}$

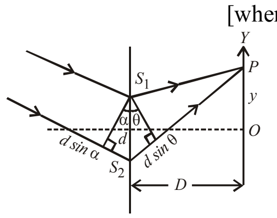
8. **(b)** Given, path difference, $\Delta x = \frac{\lambda}{8}$
Phase difference ($\Delta\phi$) is given by
$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) \qquad \Delta\phi = \frac{(2\pi)}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$
For two sources in different phases,
$$I = I_0\cos^2\left(\frac{\pi}{8}\right) \quad \text{or} \quad \frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$
$$= \frac{1+\cos\frac{\pi}{4}}{2} = \frac{1+\frac{1}{\sqrt{2}}}{2} = 0.85$$

9. **(a)** Path difference = $d\sin\theta \approx d\theta = 0.1\times\frac{1}{40}$ mm = 2500 nm
For bright fringe, path difference must be integral multiple of λ.
$\therefore 2500 = n\lambda_1 = m\lambda_2$
$\therefore \lambda_1 = 625$ (for n = 4), $\lambda_2 = 500$ (for m = 5)

10. **(a)** Here, $x_1 = 2d$ and $x_2 = \sqrt{5d}$
For, first minima, $\Delta x = \frac{\lambda}{2}$
$$\therefore \Delta x = x_2 - x_1 = \sqrt{5}d - 2d = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$$

11. **(d)** For 'n' number of maximas
$d\sin\theta = n\lambda$
$0.32\times10^{-3}\sin30° = n\times500\times10^{-9}$
$$\therefore\quad n = \frac{0.32\times10^{-3}}{500\times10^{-9}}\times\frac{1}{2} = 320$$
Hence total no. of maximas observed in angular range $-30° \le \theta \le 30°$
= 320 + 1 + 320 = 641

12. **(c)** Path difference, $\Delta x = d\sin\alpha + d\sin\theta = d\alpha + \frac{yd}{D}$
[when α and θ are small]



(a) For $\alpha = 0$, path difference $\Delta x = \frac{yd}{D}$
$$= \frac{0.3\times11}{1000} = 33\times10^{-4}\text{ mm}$$
$$\text{Now } \frac{\Delta x}{\lambda} = \frac{33\times10^{-4}}{600\times10^{-6}} = \frac{11}{2}$$
$$\therefore\quad \Delta x = \frac{11}{2}\lambda \Rightarrow \Delta x = (2n-1)\frac{\lambda}{2}$$
Hence interference at P is destructive.

(b) Fringe width $\beta = \frac{\lambda D}{d}$ is independent of $\propto$

(c) For $\alpha = \frac{0.36}{\pi}$ degree (at point P)
$$\Delta x = d\left[\alpha + \frac{y}{D}\right] = 0.3\times10^{-3}\left[\frac{0.36}{180} + \frac{11\times10^{-3}}{1}\right]\text{m} = 3900\text{ nm}$$
$$\text{Now } \frac{\Delta x}{\lambda} = \frac{3900}{600} = \frac{13}{2}\lambda$$
Hence destructive interference at P.

(d) For $\alpha = \frac{0.36}{\pi}$ degree (at point O)
$$\Delta x = d\alpha = 0.3\times10^{-3}\times\frac{0.36}{180}$$
$= 600\times10^{-9}$m = 600 nm
Now $\frac{\Delta x}{\lambda} = 1 \Rightarrow \Delta x = 1\lambda$
Hence constructive interference at O.

13. **(d)** For common maxima, $n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{520\times10^{-9}}{650\times10^{-9}} = \dfrac{4}{5}$

For λ_1

$y = \dfrac{n_1\lambda_1 D}{d}$, $\lambda_1 = 650$ nm

$y = \dfrac{4\times650\times10^{-9}\times1.5}{0.5\times10^{-3}}$ or, $y = 7.8$ mm

14. **(c)** $2I_0 = 4I_0 \cos^2\left(\dfrac{\Delta\phi}{2}\right)$ here, $\Delta\phi = \dfrac{\pi}{2}$

But, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$ so, $\Delta x = \dfrac{\lambda}{4}$

$\dfrac{dy}{D} = \dfrac{\lambda}{4}$ ....(i) $\dfrac{\lambda D}{d} = \beta$ ....(ii)

Multiplying equation (i) and (ii) we get,

$y = \dfrac{\beta}{4}$

15. **(c)**

16. **(d)** It will be concentric circles.

17. **(c)**

18. **(b)** Intensity $I = I_o \cos^2\dfrac{f}{2}$ where I_o is the peak intensity

Here $I = \dfrac{I_o}{2}, \therefore \dfrac{I_o}{2} = I_o\cos^2\dfrac{\phi}{2}, \therefore \phi = \dfrac{\pi}{2}(2n+1)$

$\therefore \phi = \dfrac{\pi}{2}, \dfrac{3}{2}\pi, \dfrac{5}{2}\pi....$

And path difference,

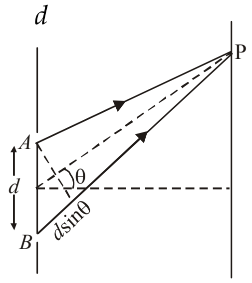
$\Delta x = \left(\dfrac{\lambda}{2\pi}\right)\phi \quad \therefore \Delta x = \dfrac{\lambda}{4}, \dfrac{3}{4}\lambda ..... \dfrac{(2n+1)}{4}\lambda$

19. **(d)** We know that fringe width, $\beta = \dfrac{\lambda D}{d}$

$\because \lambda_R > \lambda_G > \lambda_B \quad \therefore \beta_R > \beta_G > \beta_B$

20. **(c)** $I = I_{max}\cos^2$

$= I_{max}\cos^2\dfrac{\pi d\sin\theta}{\lambda}$



$4 = \dfrac{2\pi}{\lambda}d\sin\theta$

$\Rightarrow \dfrac{I_{max}}{4} = I_{max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$

or, $\cos\dfrac{\pi d\sin\theta}{\lambda} = \dfrac{1}{2} \Rightarrow \dfrac{\pi d\sin\theta}{\lambda} = \dfrac{\pi}{3} \quad \therefore \theta = \sin^{-1}\left(\dfrac{\lambda}{3d}\right)$

21. **(d)** At the area of total darkness minima will occur for both the wavelengths incident simultaneously and normally.

$\therefore \dfrac{(2n+1)}{2}\lambda_1 = \dfrac{(2m+1)}{2}\lambda_2 \Rightarrow (2n+1)\lambda_1 = (2m+1)\lambda_2$

or $\dfrac{(2n+1)}{(2m+1)} = \dfrac{560}{400} = \dfrac{7}{5}$ or $10n = 14m + 2$

By inspection for $m = 2$, $n = 3$ and for $m = 7$, $n = 10$, the distance between them will be the distance between such points.

i.e., $\Delta s = \dfrac{D\lambda_1}{d}\left\{\dfrac{(2n_2+1)-(2n_1+1)}{2}\right\}$

Put $n_2 = 10, n_1 = 3$

$\Delta s = \dfrac{1\times(400\times10^{-9})}{0.1\times10^{-3}}\left[\dfrac{21-7}{2}\right] = 28\text{mm}$

22. **(a)** Path difference = $(\mu - 1)t = n\lambda$;

For minimum t, $n = 1$; $\therefore t = \dfrac{n\lambda}{(\mu-1)} = \dfrac{\lambda}{(1.5-1)} = 2\lambda$

23. **(b)** Fringe width, $\omega = \dfrac{12\lambda_1 D}{d} = \dfrac{k\lambda_2 D}{d}$

$\Rightarrow k = \dfrac{12\times600}{400} = 18$

Hence the number of fringes observed in the same segment of the screen = 18.

24. **(a)** When slits are of equal width.

$I_{max} \propto (a+a)^2 \ (= 4a^2)$

$I_{min} \propto (a-a)^2 \ (= 0)$

When one slit's width is twice that of other

$\dfrac{I_1}{I_2} = \dfrac{W_1}{W_2} = \dfrac{a^2}{b^2} \Rightarrow \dfrac{W}{2W} = \dfrac{a^2}{b^2} \Rightarrow b = \sqrt{2}a$

$\therefore I_{max} \propto (a + \sqrt{2}a)^2 = (5.8\,a^2)$

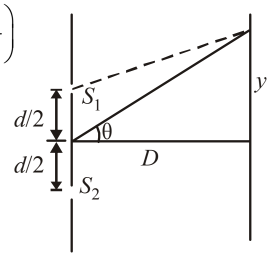
$I_{min} \propto (\sqrt{2}a - a)^2 = (= 0.17\,a^2)$

Clearly, the intensities of both the maxima and minima increase.

25. **(c)** We know that intensity of light

$I(\theta) = I_0\cos^2\dfrac{\delta}{2}$ where $\delta = \dfrac{2\pi d\tan\theta}{\lambda}$

$\therefore I(\theta) = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$



$\lambda = \dfrac{v}{h} = \dfrac{3\times10^8}{106} = 300m$

and $d = 150$ m

For $\theta = 30°$

$\delta = \left(\dfrac{2\pi}{300}\right)(150)\left(\dfrac{1}{2}\right) = \dfrac{\pi}{2}$

$\therefore \dfrac{\delta}{2} = \dfrac{\pi}{4} \qquad \therefore I(\theta) = I_0\cos^2\left(\dfrac{\pi}{4}\right) = \dfrac{I_o}{2}$

For $\theta = 90°$

$\delta = \left(\dfrac{2\pi}{300}\right)(150)(1) = \pi$ or $\dfrac{\delta}{2} = \dfrac{\pi}{2}$ and $I(\theta) = 0$

For $\theta = 0°$, $\delta = 0$ or $\dfrac{\delta}{2} = 0$

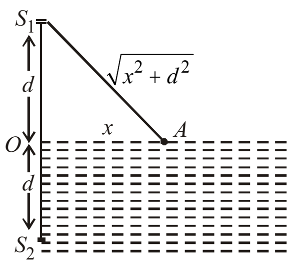
$\therefore I(\theta) = I_0$

26. **(d)** Here $\beta = \dfrac{\lambda D}{d}$ and $\beta' = \dfrac{\lambda(2D)}{d/2} = 4\dfrac{\lambda D}{d} = 4\beta$

The fringe width is quadrupled.

27. **(3)** For maxima, Path defference = $m\lambda$

$\therefore S_2A - S_1A = m\lambda$



$\therefore \left[(n-1)\sqrt{d^2+x^2}+\sqrt{d^2+x^2}\right]-\sqrt{d^2-x^2} = m\lambda$

$\therefore \quad (n-1)\sqrt{(d^2+x^2)} = m\lambda$

$\therefore \quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2} = m\lambda \;\therefore\; \sqrt{d^2+x^2} = 3m\lambda$

$\therefore \quad d^2+x^2 = 9m^2\lambda^2 \;\therefore\; x^2 = 9m^2\lambda^2 - d^2$

Comparing this equation with the given equation $x^2 = p^2m^2\lambda^2 - d^2$, we get

$p^2 = 9 \;\therefore\; p = 3$

28. **(9)**

In young's double slit experiment, intensity at a point is given by

$$I = I_0\cos^2\frac{\phi}{2} \qquad \text{...(i)}$$

where, ϕ = phase difference,

Using phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path difference

For path difference λ, phase difference $\phi_1 = 2\pi$

For path difference, $\frac{\lambda}{6}$, phase difference $\phi_2 = \frac{\pi}{3}$

Using equation (i),

$$\frac{I_1}{I_2} = \frac{\cos^2\left(\frac{\phi_1}{2}\right)}{\cos^2\left(\frac{\phi_2}{2}\right)} = \frac{\cos^2\left(\frac{2\pi}{2}\right)}{\cos^2\left(\frac{\pi}{3}\right)}$$

$$\Rightarrow \frac{K}{I_2} = \frac{1}{\frac{3}{4}} = \frac{4}{3} \Rightarrow I_2 = \frac{3K}{4} = \frac{9K}{12}$$

$\therefore n = 9.$

29. **(750)** Fringe width, $\beta = \frac{\lambda D}{d}$ where, λ = wavelength, D = distance of screen from slits, d = distance between slits

ATQ

$$15 \times \frac{\lambda_1 D}{d} = 10 \times \frac{\lambda_2 D}{d}$$

$\Rightarrow \quad 15\lambda_1 = 10\lambda_2$

$\Rightarrow \quad \lambda_2 = 1.5\lambda_1\ 15\lambda_1 = 1.5 \times 500$ nm

$\Rightarrow \quad \lambda_2 = 750$ nm

30. For coherent sources, for constructive interference

The amplitude at the mid point = $A + A = 2A$

$I_{max} \propto (2A)^2 \quad \Rightarrow \quad I_{max} = 4I_0 = I_1$

D_0 = Intensity due to one slit

For incoherent sources, the intensity add up normally (no interference).

Therefore, the total intensity $I_2 = I_0 + I_0 = 2I_0$ ... (ii)

From eqs. (i) and (ii)

$$\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2$$

31. **False**

In Young's double slit experiment if source is of white light than the central fringe is white with coloured fringes on either side.

32. **True**

When the two slits of Young's double slit experiment are illuminated by two different sodium lamps, then the sources are not coherent and to get interference sources must be coherent.

33. **(b,d)**

Path difference at O = d = 0.6003mm

For $n\frac{\lambda}{2} = d$ we get n = 2001

As n is a whole number, the condition for minima is satisfied. Therefore 'O' will be dark. i.e., minima is formed at 'O'.

Also, as the screen is perpendicular to the plane containing the slits S_1, S_2, therefore fringes obtained will be semi-circular (only top half of the screen is available)

34. **(a, b, c)** We know that fring width, $\beta = \frac{\lambda D}{d}$

$\because \quad \lambda_2 > \lambda_1 \therefore \beta_2 > \beta_1$

Number of fringes in a given width $m \propto \frac{1}{\beta} \;\therefore m_1 > m_2$

$$3 \times \frac{\lambda_2 D}{d} = \frac{(2\times 5-1)\lambda_1}{2}\frac{D}{d}$$

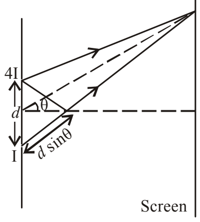
$3 \times 600 = 4.5 \times 400$

Angular separation $\frac{\lambda}{d} \propto \lambda$

So it is greater for λ_2.

35. **(a, b)** Condition to obtain maxima in young's double slit experiment is

$d \sin\theta = n\lambda$ where n is an integer



(a) When $d = \lambda$

$\lambda \sin\theta = n\lambda \Rightarrow \sin\theta = n$

When $n = 0$, $\theta = 0$

When $n = 1$, $\theta = 90°$

(This will be a point on the screen which will be at infinity and therefore not practical)

Other values of n are invalid as $-1 \le \sin\theta \le 1$.

Hence, only one maxima when $d = \lambda$.

(b) When $\lambda < d < 2\lambda$

$$\Rightarrow \quad \lambda < \frac{n\lambda}{\sin\theta} < 2\lambda \qquad \left[\because d = \frac{n\lambda}{\sin\theta}\right]$$

$$\Rightarrow \quad 1 < \frac{n}{\sin\theta} < 2$$

$\therefore$ Possible values of n are 0, + 1, –1.

Hence, there is at least one more maxima besides the central maxima.

As we know,

$I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ and $I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2$

Initially $I_1 = 4$ I and $I_2 = $ I

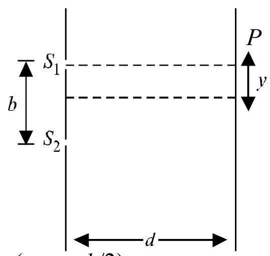
$\therefore \quad I_{max} = 9$ I and $I_{min} = $ I

When $I_1 = I_2 = $ I then $I_{max} = 4$ I and $I_{min} = 0$

i.e., When the intensities become equal, I_{min} reduces to zero.

36. **(a, c)** Those wavelengths will be missing for which, path difference

$\Delta x = (2n-1)\lambda/2$



Here $y = (2n-1)\frac{\lambda}{2}\frac{D}{d}$

$= (2n-1)\frac{\lambda}{2}\frac{d}{b}$

$(\because d = b$ and $D = d)$

$\therefore \quad \frac{b}{2} = (2n-1)\frac{\lambda}{2}\frac{d}{b} \qquad (\because y = b/2)$

$$\Rightarrow \quad \lambda = \frac{b^2}{(2n-1)d} \quad \text{when } n = 1, 2$$

$$\lambda = \frac{b^2}{d}, \frac{b^2}{3d}, \ldots$$

37. **(b, d)** $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \dfrac{9}{1}$

$$\therefore \quad \frac{I_1}{I_2} = 4$$

Amplitude ratio, $\dfrac{A_1}{A_2} = \sqrt{\dfrac{I_1}{I_2}} = \sqrt{4} = 2$

38. **A-p, s; B-q; C-t; D-r, s, t**

For path difference $\lambda/4$, phase difference is $\pi/2$.
For path difference $\lambda/3$, phase difference is $2\pi/3$.

Here, $S_1P_0 - S_2P_0 = 0$

$\therefore \quad \delta(P_0) = 0$

The path difference for P_1 and P_2 will not be zero. The intensities at P_0 is maximum.

Intensity continuously decreases from P_0 towards P_2.

$\therefore \quad I(P_0) > I(P_1)$

(B) At P_1, path difference is zero. Hence P_1 is the brightest central fringe and $\delta P_1 = 0$.

$$\delta P_0 = \frac{\lambda}{4}, \ \delta P_1 = 0, \ \delta P_2 = \frac{\lambda}{12}$$

(C) Here $\delta(P_0) = -\lambda/2;\ \delta(P_1) = -\lambda/4,\ \delta(P_2) = -\lambda/6$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$
$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2I_0 = 0$$
$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi/2)$$
$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$
$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(-\frac{\pi}{3}\right)$$
$$= I_1 + I_2 + \sqrt{I_1}\sqrt{I_2} = I_0 + I_0 + I_0 = 3I_0$$

$\therefore \quad I(P_2) > I(P_1) > I(P_0)$

(D) Here $\delta P_0 = 3\lambda/4; \delta P_1 = -\lambda/2; \delta P_2 = -5\lambda/12$

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\left(\frac{-3\pi}{2}\right)$$
$$= I_1 + I_2 = I_0 + I_0 = 2I_0$$
$$I(P_1) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(-\pi)$$
$$= I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} = I_0 + I_0 - 2\sqrt{I_0}\sqrt{I_0} = 0$$
$$I(P_2) = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos[-5\pi/6]$$
$$= I_1 + I_2 - \sqrt{3}\sqrt{I_1}\sqrt{I_2} = \left(2-\sqrt{3}\right)I_0$$

Clearly, $I(P_1) = 0$

$I(P_0) > I(P_1)$ and $I(P_2) > I(P_1)$

39. Here, $n_1 = 1$, $n_2 = \sqrt{2}$, $i = 45°$, $r = ?$

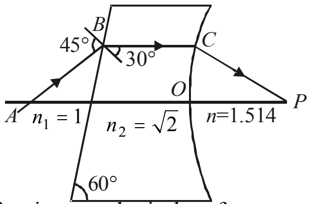
Use Snell's law at B

$n_1 \sin i = n_2 \sin r$

$$\therefore \quad \sin r = \frac{1 \times \sin 45°}{\sqrt{2}} = \frac{1}{2} \quad \Rightarrow r = 30°$$

$\therefore$ Angle made by the first surface with refracted ray BC is $60°$.

Hence the refracted ray at $B \parallel$ to horizontal.



Now for refraction at spherical surface,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad (\because u = \infty)$$

$$\Rightarrow \quad \frac{1.514}{OP} = \frac{1.514 - 1.414}{0.4} \quad \therefore \ OP = 6.056 \text{ m}$$

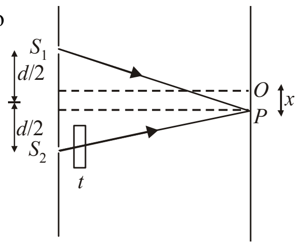
40. (a) Let the central maxima is obtained at a distance x below O.

$$\therefore \quad \Delta x_1 = S_1P - S_2P = \frac{xd}{D}$$

and $\Delta x_2 = \left(\dfrac{\mu_g}{\mu_m} - 1\right)$ + (Due to glass sheet)

Here, net path difference is zero

$\therefore \quad \Delta x_1 = \Delta x_2$



$$\therefore \quad \frac{xd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow x = \left(\frac{\mu_g}{\mu_m} - 1\right)t \times \frac{D}{d}$$

$$= \left(\frac{1.5}{4/3} - 1\right) \times \frac{(10.4 \times 10^{-6})(1.5)}{0.45 \times 10^{-3}} = 4.33 \times 10^{-3} \text{ m}$$

(b) **For O,** path difference $= \left(\dfrac{\mu_g}{\mu_m} - 1\right)t \quad (\because \Delta x_1 = 0)$

$\therefore$ Phase difference

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$= \frac{2 \times 3.14}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6}) = 6.8 \text{ rad}$$

$$\because \quad I = I_0 \cos^2\frac{\phi}{2} \quad \therefore \quad \frac{I}{I_0} = \cos^2(6.8) = 0.75$$

or, $I = \dfrac{3}{4}I_0$

(c) **For maximum intensity at O**

Again path difference $= \left(\dfrac{\mu_g}{\mu_m} - 1\right)t$

For maxima, path difference $= n\lambda$

$$\therefore \quad n\lambda = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow \quad \lambda = \left(\frac{\mu_g}{\mu_m} - 1\right)\frac{t}{n} = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}}{n}$$

$$= \frac{1.3 \times 10^{-6}\,\text{m}}{n} = \frac{1300}{n}\,\text{nm}$$

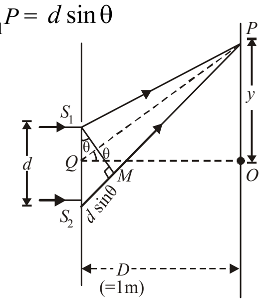
Putting $n = 1, 2, 3$ ..... to find the wavelength in the range of 0.4×10^{-6} m to 0.7×10^{-6} m we get

$\lambda = 6.5 \times 10^{-7}$ m and 4.33×10^{-7} m

41. (a) If the incident beam falls normally,

Path difference (Δx) from the ray starting from S_1 and S_2 and reaching a point P

$\Delta x = S_2P - S_1P = d\sin\theta$



And path difference for minimum intensity = $(2n - 1)\dfrac{\lambda}{2}$ where $n = 1, 2, 3...$

$$\therefore \quad d\sin\theta = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)0.5}{2\times 1.0} = \frac{2n-1}{4}$$

Also $-1 \le \sin\theta \le 1$ $\therefore$ possible values of m are ± 1, $\pm$ 2, 0

From Δ POQ position of minima

$$y = D\tan\theta = \frac{D\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \qquad (\because D = 1\text{m})$$

Therefore the y-coordinates of all the interference minima on the screen as follows

For $m = +1$, $\sin\theta = \dfrac{1}{4}$ and $y = 0.26$

$m = -1$, $\sin\theta = -\dfrac{3}{4}$ and $y = -1.13$ m

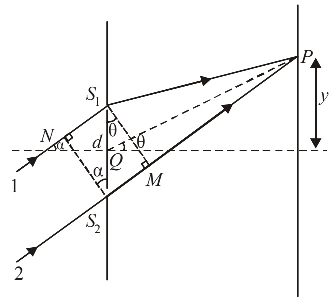
$m = +2$, $\sin\theta = \dfrac{3}{4}$ $\therefore y = +1.13$ m

$m = 0$, $\sin\theta = -\dfrac{1}{4}$ $\therefore y = -0.26$ m

(b) If the incident beam makes anangle of 30° with x-axis

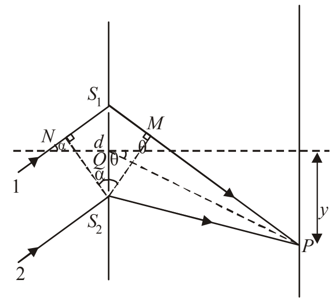
Path difference between ray 1 and 2 reaching $P = S_2M - NS_1$

$$\therefore \quad \Delta x_1 = d\sin\theta - d\sin\alpha$$



Path difference between ray 1 and 2 reaching $P = NS_1 + S_1M$

$\Delta x_2 = d\sin\alpha + d\sin\theta$



For central maxima, path difference should be zero.

$\therefore \quad \Delta x_1 = 0$ or $\Delta x_2 = 0$

$\therefore \quad d\sin\alpha = d\sin\theta$

$\therefore \quad \propto = \theta = 30°$

$$\because \quad y = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = 0.58\,\text{m}$$

For first minima; $d\sin\theta - d\sin\alpha = \dfrac{\lambda}{2}$

$$\Rightarrow \quad d\sin\theta = \frac{\lambda}{2} + d\sin\alpha$$

$$\therefore \quad \sin\theta = \frac{\lambda}{2d} + \sin\alpha = \frac{0.5}{2\times 1} + \sin 30° = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\therefore \quad y = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = 1.13\text{ m}$$

For first minima on the other side

$d\sin\theta = \dfrac{\lambda}{2}$ or

$\sin\theta = \dfrac{\lambda}{2d}$ or, $\sin\theta = \dfrac{0.5}{2\times 0.1} = \dfrac{1}{4}$

$$\therefore \quad y = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = 0.26\text{ m}$$

Hence y-coordinates of the first minima on either side of the central maximum are 0.26 m and 1.13 m.

42. The phase difference $\phi = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi}{\lambda}(5\lambda + \Delta)$

Where $\Delta < \lambda/2$

We know that $I(\phi) = I_{\max}\cos^2\left(\dfrac{\phi}{2}\right)$

Intensity of P = $\frac{3}{4} I_{max}$

$$\frac{3}{4} I_{max} = I_{max} \cos^2 \frac{\phi}{2} \Rightarrow \frac{\phi}{2} = 30° = \frac{\pi}{6}$$

$$\Rightarrow \frac{2\pi}{6} = \frac{2\pi}{\lambda}(5\lambda + \Delta) \Rightarrow \Delta x \; 5\lambda + \frac{\lambda}{6} = 0.3\ t$$

$\therefore \quad t = 9.3 \times 10^{-6}$ m

43. (i) $\,_m^a\mu = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{\,_m^a\mu}$

(Wave length of light in the given liquid)

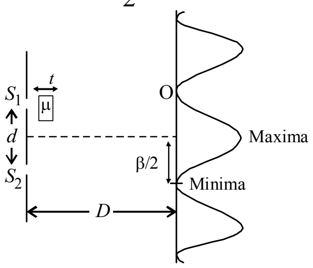
$\therefore$ Fringe width B = $\frac{D}{d}\lambda = \frac{\lambda_a D}{\,_m^a\mu d} = \frac{6300 \times 10^{-10} \times 1.33}{1.33 \times 10^{-3}}$

$= 6.3 \times 10^{-4}$m

(ii) Let 't' be the thickness of glass hect which covers one of the slits. The shift of fringes when one slit is covered with thin glass sheet

$$= \frac{Dt}{d}\left[\frac{_g\mu}{_m\mu} - 1\right]$$

The shift has to be such that the minima shifts to the axis i.e., shifting of the fringes = $\frac{\beta}{2}$



$$\therefore \quad \frac{Dt}{d}\left[\frac{_g\mu - _m\mu}{_m\mu}\right] = \frac{\beta}{2}$$

$$\Rightarrow \text{t} = \frac{\beta d_m\mu}{2(_g\mu - _m\mu) \times D} = \frac{6.3 \times 10^{-4} \times 10^{-3} \times 1.33}{2(1.53 - 1.33) \times 1.33}$$

$= 15.75 \times 10^{-7}$ m $= 1.575 \times 10^{-6}$ m

44. Here power transmitted by A
$P_1 = 10\%$ of $I(\pi r_A^2)$

$$= \left[10\% \text{ of}\left(\frac{10}{\pi}\right)\right] \times \pi(0.001)^2 = 10^{-6} \text{ W}$$

Similarly, power transmitted by B

$$P_2 = \left[10\% \text{ of}\left(\frac{10}{\pi}\right)\right] \times \pi \times (0.002)^2 = 4 \times 10^{-6} \text{ W}$$

Let $\Delta\phi$ be the phase difference introduced by film

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}$ (path difference introduced by the film)

$$= \frac{2\pi}{\lambda} \times (\mu - 1)t = \frac{2\pi}{6000 \times 10^{-10}}[1.5 - 1] \times 2000 \times 10^{-10} = \frac{\pi}{3}$$

$\therefore$ Power received at focal spot, F

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \Delta\phi$$

$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{10^{-6} \times 4 \times 10^{-6}} \cos\frac{\pi}{3} = 7 \times 10^{-6} \text{ W.}$$

45. (i) As we know, distance of the nth bright fringe from the central maxima

$y_n = \frac{n\lambda D}{d}$, For 3rd bright fringe $n = 3$

$$\therefore \quad y_3 = \frac{3 \times 6500 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 10^{-3}} = 1.17 \times 10^{-3} \text{ m}$$

(ii) Let nth bright fringe of wavelength 6500 Å coincide with mth bright fringe of wavelength 5200Å. Here distance will be same from the central bright.

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d} \quad \therefore \quad \frac{n}{m} = \frac{5200}{6500} = \frac{4}{5}$$

The least distance from the central maximum thus corresponds to 4th bright fringe of $\lambda_1 = 6500$ Å of 5th bright fringe of $\lambda_2 = 5200$ Å. Its distance from the central maxima

$$y_n = \frac{4 \times 6500 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 10^3} = 1.56 \times 10^{-3} \text{m}$$

Topic-3 : Diffraction, Polarisation of Light and Resolving Power

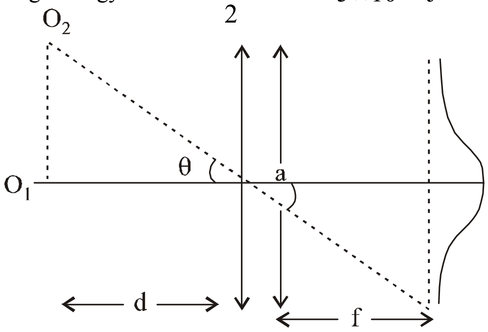
1. **(d)** Given :
Intensity, $I_0 = 3.3$ Wm^{-2}
Area, $A = 3 \times 10^{-4}$ m^2
Angular speed, $\omega = 31.4$ rad/s

Average energy $= I_0 A \langle \cos^2\theta \rangle$

$\because \langle \cos^2\theta \rangle = \frac{1}{2}$ per revolution

$\therefore$ Average energy $= \frac{(3.3)(3 \times 10^{-4})}{2} \simeq 5 \times 10^{-4}$ J

2. **(a)**



Smallest angular separation between two distant objects here moon and earth,

$$\theta = 1.22\frac{\lambda}{a}$$

a = aperture diameter of telescope
Distance $O_1O_2 = (\theta)d$
Minimum separation between objects on the surface of moon,

$$= \left(1.22\frac{\lambda}{a}\right)d$$

$$= \frac{(1.22)(5500 \times 10^{-10}) \times 4 \times 10^5 \times 10^3}{5}$$

$= 5368 \times 10^{-2}$ m $= 53.68$ m ≈ 60 m

3. **(b)** According to question, the intensity of light coming out of the analyser is just 10% of the original intensity (I_0)
Using, $I = I_0\cos^2\theta$

$\Rightarrow \frac{I_0}{10} = I_0 \cos^2\theta \Rightarrow \frac{1}{10} = \cos^2\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} = 0.316 \Rightarrow \theta \approx 71.6°$

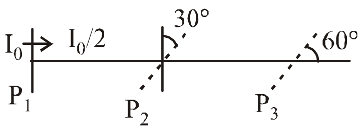
Therefore, the angle by which the analyser need to be rotated further to reduced the output intensity to be zero

$\phi = 90° - \theta = 90° - 71.6° = 18.4°$

4. **(a)** $x = \frac{1.22\lambda}{2\mu\sin\theta}$

$= \frac{1.22 \times 5000}{2 \times 1.25} = 0.24\ \mu m$

5. **(c)** $I = \left(\frac{I_0}{2}\right) \cos^2 30° \cos^2 60°$



$= \frac{I_0}{2} \times \frac{3}{4} \times \frac{1}{4}$

$\therefore \quad \frac{I_0}{I} = \frac{32}{3} = 10.67$

6. **(c)** $\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$

$= 3.0 \times 10^{-7}$ rad

7. **(a)** $\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 500 \times 10^{-9}}{2} = 305 \times 10^{-9}$ rad.

8. **(b)**

9. **(b)** According to Brewster's law, refractive index of material (μ) is equal to tangent of polarising angle

$\because \tan i_b = \mu = \frac{1.5}{\mu}$

$\frac{1}{\mu} < \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}} \ (\because \sin i_c < \sin i_b)$

$\therefore \sin i_b = \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$

or, $\sqrt{\mu^2 + (1.5)^2} < 1.5 \times \mu$

$\Rightarrow \mu^2 + (1.5)^2 < (\mu \times 1.5)^2$

$\Rightarrow \mu < \frac{3}{\sqrt{5}}$ i.e. minimum value of μ should be $\frac{3}{\sqrt{5}}$

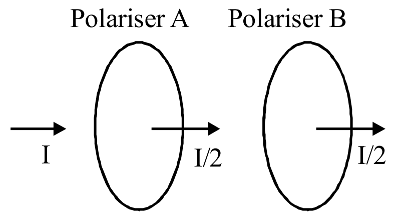
10. **(a)** Angular width of central maxima $= \frac{2\lambda}{d}$

or, $\lambda = \frac{d}{2}$; Fringe width, $\beta = \frac{\lambda \times D}{d'}$

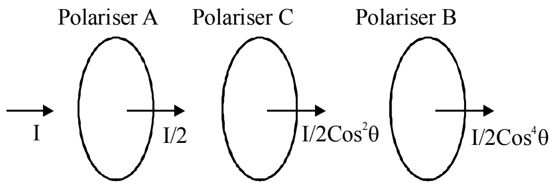
$10^{-2} = \frac{d}{2} \times \frac{50 \times 10^{-2}}{d'} = \frac{10^{-6} \times 50 \times 10^{-2}}{2 \times d'}$

Therefore, slit separation distance, d' = 25μm

11. **(c)** Axis of transmission of A & B are parallel.



After introducing polariser C between A and B,



$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ or, $\theta = 45°$

12. **(a)** If angular position of 2nd maxima from central maxima is θ then

$\sin\theta = \frac{(2n-1)\lambda}{2a} = \frac{3\lambda}{20} = \frac{3 \times 550 \times 10^{-9}}{2 \times 22 \times 10^{-7}}$

$\therefore \quad \theta \simeq \frac{\pi}{8}$ rad

13. **(a)** Polariser A and B have same alignment of transmission axis.

Lets assume polariser c is introduced at θ angle

$\frac{1}{2}\cos^2\theta \times \cos^2\theta = \frac{1}{3}$

or, $\cos^4\theta = \frac{2}{3} \Rightarrow \cos\theta = \left(\frac{2}{3}\right)^{1/4}$

14. **(a)** Use relativistic doppler's effect as velocity of observer is not small as compared to light

$f = f_0\sqrt{\frac{c+v}{c+v}}$; V = relative speed of approach

f_0 = 10 GHz

$f = 10\sqrt{\frac{c+\frac{c}{2}}{c-\frac{c}{2}}} = 10\sqrt{3} = 17.3$ GHz

15. **(b)** For secondary minima,

b sin θ = nλ ⇒ sin θ $= \frac{n\lambda}{b}$

Distance of nth secondary minima x = D sin θ or $\sin\theta_1 = \frac{x_1}{D}$

$\sin\theta_1 = \frac{2\lambda}{b}$

n = 4

$\sin\theta_2 = \frac{4\lambda}{b} = \frac{x_2}{D}$

$x_2 - x_1 = \frac{4\lambda}{b} - \frac{2\lambda}{b} = \frac{2\lambda}{b}$

$3 = \frac{2\lambda}{b} \Rightarrow b = \frac{2\lambda}{3}$ ..... (i)

Width of central maxima $= \frac{2\lambda}{b}$

$= \frac{2\lambda}{\frac{2\lambda}{3}} = 3$ cm.

.. from eq. (i)

16. **(a)** Given geometrical spread = a

Diffraction spread $= \frac{\lambda}{a} \times L = \frac{\lambda L}{a}$

The sum $b = a + \frac{\lambda L}{a}$

For b to be minimum

$\frac{db}{da} = 0 \qquad \frac{d}{da}\left(a + \frac{\lambda L}{a}\right) = 0$

$a = \sqrt{\lambda L}$

b min $= \sqrt{\lambda L} + \sqrt{\lambda L} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}$

17. **(a)** We know that $\Delta\theta = \frac{0.61\lambda}{4} = \frac{l}{R}$

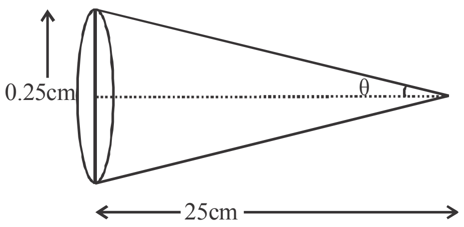
The minimum distance between them

$l = \frac{R}{9} 0.61' \ l = \frac{9.46' \ 10^{15}{}' \ 10' \ 0.61' \ 600' \ 10^{-9}}{0.3}$

$= 1.15 \times 10^{11}$ m

$\Rightarrow 1.115 \times 10^{8}$ km.

18. **(d)** $\sin\theta = \frac{0.25}{25} = \frac{1}{100}$



Resolving power $= \frac{1.22\lambda}{2\mu\sin\theta} = 30$ µm.

19. **(a)** When unpolarised light is incident at Brewster's angle then reflected light is completely polarized and the intensity of the reflected light is less than half of the incident light.

20. **(d)** According to malus law, intensity of emerging beam is given by,

$I = I_0\cos^2\theta$

Now, $I_{A'} = I_A \cos^2 30°$

$I_{B'} = I_B \cos^2 60°$

As $I_{A'} = I_{B'}$

$\Rightarrow \quad I_A \times \frac{3}{4} = I_B \times \frac{1}{4}; \frac{I_A}{I_B} = \frac{1}{3}$

21. **(c)** Here, $\sin\theta_{ic} / \sin\theta_{iB} = 1.28$

As we know,

$\mu = \frac{\sin\theta_{iB}}{\sin\left(\frac{\pi}{2} - \theta_{iB}\right)}$

where, θ_{iB} is Brewster's angle of incidence,

And, $\mu = \frac{1}{\sin\theta_{ic}}$

On solving we get, relative refractive index of the two media.

22. **(b)** In a single slit experiment,

For diffraction maxima,

$a\sin\theta = (2n+1)\frac{\lambda}{2}$

and for diffraction minima,

$a\sin\theta = n\lambda$

According to question,

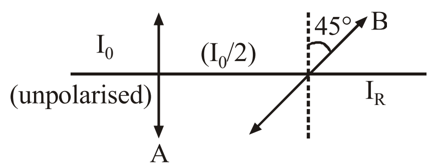
$(2\times1+1)\frac{\lambda}{2} = 1\times 6600$

$(\because \lambda_R = 6600Å)$

$\lambda = \frac{6600\times 2}{3}$

$\lambda = 4400Å$

23. **(c)** Relation between intensities



$I_r = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$

24. **(b)**

25. **(d)** For diffraction pattern to be observed, the width of slit should be comparable to the wave length of light used. The wavelength of X-rays (1 – 100 Å) << slit width 0.6 mm.

26. **(c)** At the angular position of the first diffraction minimum, the phase difference (θ) between the wavelets from the opposite edges of the slit = 2π radian.

27. **(d)** The distance between the first dark fringe on either side of the central maximum = width of central maximum. Hence distance between two dark fringes on either side

$= \frac{2D\lambda}{a} = \frac{2\times2\times600\times10^{-9}}{10^{-3}} = 2.4\times10^{-3}\text{ m} = 2.4\text{ mm}$

28. **(d)** For first minima the path difference between the rays coming from the two edges should be λ therefore corresponding phase difference

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \lambda = 2\pi.$$

29. **(198)**
For obtaining secondary minima at a point path difference should be integral multiple of wavelength

$\therefore d \sin\theta = n\lambda$

$\therefore \sin\theta = \frac{n\lambda}{d}$

For n to be maximum $\sin\theta = 1$

$$n = \frac{d}{\lambda} = \frac{6\times10^{-5}}{6\times10^{-7}} = 100$$

Total number of minima on one side = 99
Total number of minima = 198.

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. $I \propto \frac{1}{r^2}$ and $I \propto A^2$

$\therefore \quad A \propto \frac{1}{r}$

or, $\frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{9}$.

2. **(A) → (p); (B) → (r); (C) → (r); (D) → (p), (q), (r);**
(A) More the radius of aperture more is the amount of light entering the telescope.

(B) Angular magnification, $M = \frac{f_0}{f_e}$

(C) Length of telescope, $L = f_0 + f_e$
Sharpness of image depends on dispersion of lens, spherical aberration and radius of aperture.

3. **(a)** For plane wavefronts, the beam of light is parallel. Hence light travels as a parallel beam in each medium.

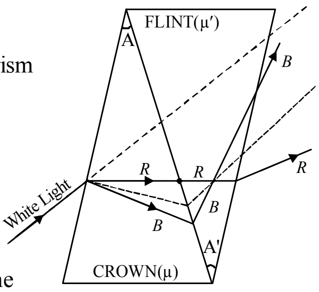
4. **(c)** Since points c and d are on the same wavefront,
$\therefore \quad \phi_d = \phi_c$
Similarly, $\phi_e = \phi_f \quad \therefore \phi_d - \phi_f = \phi_c - \phi_e$

5. **(b)** From figure, the gap between consecutive wavefronts in medium 2 is less than that in medium 1. Hence, wavelength of light in medium 2 is less than that in medium 1. Therefore, speed of light ($v = f\lambda$) in medium 1 greater than in medium 2.

6. (a) For no deviation
$(\mu - 1)A = (\mu' - 1)A'$
A' = angle of the flint glass prism

$\Rightarrow \quad A' = \left[\frac{\mu - 1}{\mu' - 1}\right]A$

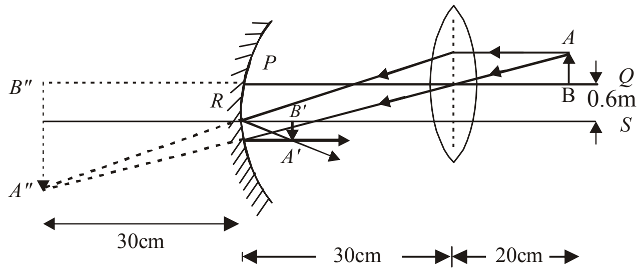
$\therefore \quad A' = \frac{1.5-1}{1.75-1} \times 6° = 4°$



(b) Net dispersion due to the two prisms

$= (\mu_{b_1} - \mu_{r_1})A + (\mu_{b_2} - \mu_{r_2})A'$
$= (1.51 - 1.49) \times 6° + (1.77 - 1.73) \times -4° = -0.04°$

7. **(a)** Image formed by lens



Applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_\ell} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{20} = \frac{1}{60} \Rightarrow v = 60 \text{ cm}$$

Magnification, $m = \frac{v}{u} = \frac{60}{-20} = -3$

The image formed by lens is real inverted and magnified $A'B'$ size 3×1.2 cm = 3.6 cm
For the mirror, applying mirror formula,

$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f_m} \Rightarrow \frac{1}{v'} = \frac{1}{-30} - \frac{1}{30} = -\frac{2}{30}$$

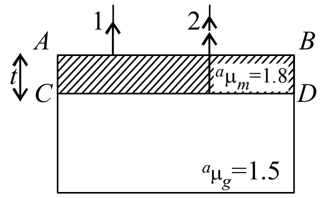
$\therefore \quad v' = -15$ cm

Magnification, $m = -\frac{v'}{u'} = -\frac{-15}{30} = \frac{1}{2}$

Size of image $= \frac{1}{2} \times 3.6 = 1.8$ cm.

i.e., The final image $A''B''$ is inverted w.r.t. the original image AB and its position will be 0.3 cm above RS and 1.5 cm below RS. The position of the image is 15 cm to the right of the mirror.
(b) The path difference between the two rays ray-1 reflected from the upper surface AB and ray-2 reflected from lower surface CD

$\Delta x = {}^a\mu_m \times 2t + \frac{\lambda}{2}$



Here $\frac{\lambda}{2}$ is the path difference as the ray 1 suffer reflection from a denser medium on surface AB
For constructive interference
Path difference = $n\lambda$ where n is 1, 2,....

$$\therefore \quad {}^a\mu_m \times 2t + \frac{\lambda}{2} = n\lambda \Rightarrow 2\,{}^a\mu_m t = \left(n - \frac{1}{2}\right)\lambda$$

For least value of thickness t, $n = 1$

$$\therefore \quad t = \frac{\lambda}{4\,{}^a\mu_m} = \frac{648}{4 \times 1.8} = 90 \text{ nm}.$$

25 Dual Nature of Radiation and Matter

Topic-1 : Matter Waves, Cathode and Positive Rays

1. **(c)** de-Broglie wavelength, $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(\text{KE})}}$

$\therefore \lambda \propto \frac{1}{\sqrt{m}}$ As $m_{\text{He}^{++}} > m_P > m_e$

$\lambda_{\text{He}^{++}} > \lambda_P > \lambda_e$ or $\lambda_e > \lambda_P > \lambda_{\text{He}^{++}}$

2. **(a)** Rms speed of gas molecule, $V_{rms} = \sqrt{\frac{3kT}{m}}$

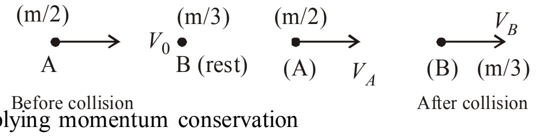
de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$

$$\therefore \lambda = \frac{h}{\sqrt{2m \times \frac{1}{2} m V_{rms}^2}} = \frac{h}{\sqrt{m \times \frac{3}{2} kT}} = \frac{h}{\sqrt{3mkT}}$$

Substituting the respective values we get

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 4.64 \times 10^{-26} \times 1.38 \times 10^{-13} \times 400}} = 0.24 \text{Å}$$

3. **(d)**



Applying momentum conservation

$$\frac{m}{2} \times V_0 + \frac{m}{3} \times (0) = \frac{m}{2} V_A + \frac{m}{3} V_B$$

$$= \frac{V_0}{2} = \frac{V_A}{2} + \frac{V_B}{3} \qquad \text{...(i)}$$

Since, collision is elastic

$$e = 1 = \frac{V_B - V_A}{V_0} \Rightarrow V_0 = V_B - V_A \qquad \text{...(ii)}$$

On solving equations (i) and (ii) : $V_A = \frac{V_0}{5}$

Now, de-Broglie wavelength of A before collision :

$$\lambda_0 = \frac{h}{m_A V_0} = \frac{h}{\left(\frac{m}{2}\right) V_0} \Rightarrow \lambda_0 = \frac{2h}{mV_0}$$

Final de-Broglie wavelength :

$$\lambda_f = \frac{h}{m_A V_0} = \frac{h}{\frac{m}{2} \times \frac{V_0}{5}} \Rightarrow \lambda_f = \frac{10h}{mV_0}$$

$$\therefore \Delta\lambda = \lambda_f - \lambda_0 = \frac{10h}{mV_0} - \frac{2h}{mV_0}$$

$$\Rightarrow \Delta\lambda = \frac{8h}{mv_0} \Rightarrow \Delta\lambda = 4 \times \frac{2h}{mv_0}$$

$$\therefore \Delta\lambda = 4\lambda_0$$

4. **(d)** de Broglie wavelength

$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v}$ Clearly, $m \propto \frac{1}{\lambda v}$

If λ and v be the wavelength and velocity of electron and λ' and v' be the wavelength and velocity of the particle then

$$\Rightarrow \frac{m'}{m} = \frac{v\lambda}{v'\lambda'} = \frac{1}{5} \times \frac{1}{1.878} \times 10^{-4} \Rightarrow m = 9.7 \times 10^{-28} \text{ kg}$$

5. **(c)** As per question, when KE of particle E, wavelength λ and when KE becomes $E + \Delta E$ wavelength becomes $\lambda/2$

Using, $\lambda = \frac{h}{\sqrt{2mKE}}$

$$\frac{\lambda}{2} = \frac{h}{\sqrt{2m(KE + \Delta E)}} \Rightarrow \frac{\lambda}{\lambda/2} = \sqrt{\frac{KE + \Delta E}{KE}}$$

$$\Rightarrow 4 = \frac{KE + \Delta E}{KE} \Rightarrow 4KE - KE = \Delta E$$

$\therefore$ $\Delta E = 3\ KE = 3\ E$

6. **(d)** Acceleration of electron in electric field, $a = \frac{eE}{m}$

Using equation

$$v = u + at \Rightarrow v = 0 + \frac{eE}{m} t$$

$$\Rightarrow v = \frac{eEt}{m} \qquad \text{..... (i)}$$

De-broglie wavelength λ is given by

$$\lambda = \frac{h}{mv} = \frac{h}{m\left(\frac{eEt}{m}\right)} \qquad \text{[using (i)]}$$

$$\Rightarrow \lambda = \frac{h}{eEt}$$

Differentiating w.r.t. t

$$\frac{d\lambda}{dt} = \frac{d\left(\frac{h}{eEt}\right)}{dt} \Rightarrow \frac{d\lambda}{dt} = \frac{-h}{eEt^2}$$

7. **(c)** Given, Initial velocity, $u = v_0\hat{i} + v_0\hat{j}$

Acceleration, $a = \frac{qE_0}{m} = \frac{eE_0}{m}$

Using $v = u + at$

$$v = v_0\hat{i} + v_0\hat{j} + \frac{eE_0}{m} t\hat{k} \qquad \therefore |\vec{v}| = \sqrt{2v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}$$

de-Broglie wavelength, $\lambda = \frac{h}{p} \Rightarrow \lambda = \frac{h}{mv}$ ($\because p = mv$)

Initial wavelength, $\lambda_0 = \frac{h}{mv_0\sqrt{2}}$

Final wavelength,

$$\lambda = \frac{h}{m\sqrt{2v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}}$$

$$\frac{\lambda}{\lambda_0}=\frac{1}{\sqrt{1+\left(\frac{eE_0t}{\sqrt{2}mv_0}\right)^2}}\Rightarrow\lambda=\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

8. **(b)** $P_1-P_2=(P_1+P_2)=P$

As $P\propto\frac{1}{\lambda}$ or $\frac{1}{\lambda_x}-\frac{1}{\lambda_y}=\frac{1}{\lambda}$ or $\frac{\lambda_y-\lambda_x}{\lambda_x\lambda_y}=\frac{1}{\lambda}$

9. **(c)** de Broglie wavelength (λ) is given by
K = qV

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}=\frac{h}{\sqrt{2mqV}}\left(\because p=\sqrt{2mK}\right)$$

Substituting the values we get

$$\therefore\ \frac{\lambda_A}{\lambda_B}=\frac{\sqrt{2m_Bq_BV_B}}{\sqrt{2m_Aq_AV_A}}=\sqrt{\frac{4m.q.2500}{m.q.50}}$$

$$=2\sqrt{50}=2\times7.07=14.14$$

10. **(d)** de-Broglie wavelength,

$$\lambda=\frac{h}{mv}=10^{-3}\left(\frac{3\times10^8}{6\times10^{14}}\right)\quad\left[\because\ \lambda=\frac{c}{v}\right]$$

$$v=\frac{6.63\times10^{-34}\times6\times10^{14}}{9.1\times10^{-31}\times3\times10^5}$$

$v = 1.45\times10^6$ m/s

11. **(c)** Momentum (p) of each electron $\frac{h}{\lambda_1}\hat{i}$ and $\frac{h}{\lambda_2}\hat{j}$

Velocity of centre of mass

$$V_{cm}=\frac{h}{2m\lambda_1}\hat{i}+\frac{h}{2m\lambda_2}\hat{j}\quad(\because\ p=mv)$$

Velocity of 1st particle about centre of mass

$$V_{1cm}=\frac{h}{2m\lambda_1}\hat{i}-\frac{h}{2m\lambda_2}\hat{j}$$

$$\lambda_{cm}=\frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2}+\frac{h^2}{4\lambda_2^2}}}=\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}\quad\left(\because\ \lambda=\frac{h}{p}\right)$$

12. **(d)** From question, $m_A=M;\ m_B=\frac{m}{2}$

$u_A=V\quad u_B=0$
Let after collision velocity of $A=V_1$ and velocity of $B=V_2$
Applying law of conservation of momentum,

$$mu=mv_1+\left(\frac{m}{2}\right)v_2$$

or, $24=2v_1+v_2$ ....(*i*)

By law of collision $e=\frac{v_2-v_1}{u-0}$

or, $u=v_2-v_1$ ....(*ii*)

[$\because$ collision is elastic, $e=1$]

using eqns (*i*) and (*ii*)

$$v_1=\frac{4}{3}\text{ and }v_2=\frac{4}{3}u$$

de-Broglie wavelength $\lambda=\frac{h}{p}$

$$\therefore\ \frac{\lambda_A}{\lambda_B}=\frac{P_B}{P_A}=\frac{\frac{m}{2}\times\frac{4}{3}u}{m\times\frac{4}{3}}=2$$

13. **(a)** From Bragg's equation $d\sin\theta=\lambda$

$\sin\theta=\frac{\lambda}{d}<1$ $\quad\therefore\ \lambda<d$

$\frac{h}{|p_y|}<d$ $\quad\left[\because\lambda=\frac{h}{|p_y|}\right]$

$\therefore\ h<|p_y|\,d$

14. **(d)** Among the given particles most difficult to experimentally verify the de-broglie relationship is for a dust particle.

15. **(d)** Energy in joule (E)
= charge × potential diff. in volt
$E_{electron}=q_eV$ and $E_{proton}=q_p4V$

de-Broglie wavelength $\lambda=\frac{h}{P}=\frac{h}{\sqrt{2mE}}$

$\lambda_e=\frac{h}{\sqrt{2m_eeV}}$ and $\lambda_P=\frac{h}{\sqrt{2m_Pe4V}}$ $\quad(\because\ q_e=q_p)$

$$\therefore\ \frac{\lambda_e}{\lambda_P}=\frac{\frac{h}{\sqrt{2m_eeV}}}{\frac{h}{\sqrt{2m_Pe4V}}}=\sqrt{\frac{2m_Pe4V}{2m_eeV}}=2\sqrt{\frac{m_P}{m_e}}$$

16. **(d)** For photon, $\lambda_2=\frac{hc}{E}$ ...(i)

For proton, $\lambda_1=\frac{h}{p}=\frac{h}{\sqrt{2mE}}\ \because\ p=\sqrt{2mE}$ ...(ii)

$$\therefore\ \frac{\lambda_2}{\lambda_1}=\frac{hc}{E\times\frac{h}{\sqrt{2mE}}}\propto E^{-1/2}$$

17. **(c)** Applying conservation of linear momentum, $P_i=P_f$
$0=m_1v_1-m_2v_2\Rightarrow m_1v_1=m_2v_2$

de-Broglie wavelength $\lambda=\frac{h}{p}$

$$\therefore\ \frac{\lambda_1}{\lambda_2}=\frac{h/m_1v_1}{h/m_2v_2}=1$$

$\because\ |M_1V_1|=|M_2V_2|$

18. **(3)** We know that, $\lambda=\frac{h}{\sqrt{2mqV}}$ or, $\lambda\propto\frac{1}{\sqrt{qm}}$

$$\therefore\ \frac{\lambda_p}{\lambda_\alpha}=\sqrt{\frac{m_\alpha q_\alpha}{m_pq_p}}=\sqrt{\frac{4}{1}\times\frac{2}{1}}=\sqrt{8}\approx3$$

19. de-Broglie wavelength $\lambda=\frac{h}{mv}\Rightarrow\lambda=\frac{h}{\sqrt{2mK}}$

when $0\le x\le1$
Potential energy E_0
and total energy $=2E_0$ (given)
$\therefore$ Kinetic energy K.E. $=2E_0-E_0=E_0$

$\lambda_1=\frac{h}{\sqrt{2mE_0}}$ ...(i)

when $x>1$

Potential energy = 0
and total energy = $2E_0$ (given)
$\therefore$ Kinetic energy = $2E_0$

$$\therefore \lambda_2 = \frac{h}{\sqrt{2m(2E_0)}} \quad ....(ii)$$

Dividing eq. (i) by (ii) $\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{2E_0}{E_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$

Topic-2: Photon, Photoelectric Effect X-rays and Davisson-Germer Experiment

1. **(a)** Graph of V_s and f given at B (5.5, 0)
Minimum energy for ejection of electron
= Work function (ϕ).

$$\phi = hV \text{ joule or } \phi = \frac{hV}{e} \text{ eV (for } V = 0)$$

$$\therefore \phi = \frac{6.62\times10^{-34}\times5.5\times10^{14}}{1.6\times10^{-19}} \text{eV} = 2.27 \text{eV}$$

2. **(d)** According to Einstein's photoelectric equation

$$K_{max} = h\nu - \phi_0 \Rightarrow eV_s = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow V_s = \frac{hc}{\lambda e} - \frac{\phi_0}{e}$$ where λ = wavelength of incident light

ϕ_0 = work function
V_s = stopping potential

Comparing the above equation with $y = mx + c$, we get slope $= \frac{hc}{e}$

Increasing the frequency of incident radiation has no effect on work function and frequency. So, graph will not change.

3. **(d)** Using equation, $= \frac{hc}{\lambda} - \phi$

$$KE_{max} = \frac{hc}{\lambda} - \phi = \frac{hc}{500} - \phi \quad ...(1)$$

$$\text{Again, } 3KE_{max} = \frac{hc}{200} - \phi \quad ...(2)$$

Dividing equation (2) by (1), $\frac{3KE_{max}}{KE_{max}} = \frac{3}{1} = \frac{\frac{hc}{200}-\phi}{\frac{hc}{500}-\phi}$

Putting the value of $hc = 1237.5$ and solving we get, work function, $\phi = 0.61$ eV.

4. **(a)** Given,
Wavelength of X-rays, $\lambda_1 = 1 \text{ nm} = 1\times10^{-9}\text{m}$
Wavelength of visible light, $\lambda_2 = 500\times10^{-9}\text{m}$
The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as

$$n = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc} \quad (\because E = h\nu \text{ and } \nu = \frac{c}{\lambda})$$

$$\Rightarrow \text{Clearly } n \propto \lambda \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}$$

5. **(a)** Using Einstein's photoelectric equation,
$E = \omega_0 + KE_{max} \Rightarrow \omega_0 = KE_{max} - E$

$$p = \sqrt{2mKE} \Rightarrow KE = \frac{p^2}{2m}$$

$$r = \frac{p}{eB} \Rightarrow p = reB$$

$$K_{max} = \frac{r^2e^2B^2}{2m} \quad KE_{max} = \frac{12420}{\lambda} - \omega_0$$

$$\Rightarrow \omega_0 = \frac{12420}{6561} - \frac{r^2eB^2}{2m}(In\,\text{eV})$$

$$= 1.89(eV) - \frac{(10^{-4})(1.6\times10^{-19})9\times10^5}{2\times9.07\times10^{-31}}$$

$$= 1.89(eV) - \frac{(10^{-4})(1.6\times10^{-19})9\times10^5}{2\times9.07\times10^{-31}}$$

= (1.89 – 0.79) eV= 1.1 eV

6. **(a)** de-Broglie wavelength (λ),

$$\text{Momentum, } mv = \frac{h}{\lambda} = p = \sqrt{2m(KE)}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mKE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{K_B}{K_A}} = \sqrt{\frac{T_A-1.5}{T_A}} \quad \text{(as given)}$$

$$\text{Also, } \frac{\lambda_A}{\lambda_B} = \frac{1}{2}$$

On solving we get, $T_A = 2\ eV$
$\therefore KE_B = T_A - 1.5 = 2 - 1.5 = 0.5\ eV$
$\therefore$ Work function of metal B is
$\phi_B = E_B - KE_B = 4.5 - 0.5 = 4\ eV$

7. **(b)** $f_0 = 4\times10^{14}$ Hz
$W_0 = hf_0 = 6.63\times10^{-34}\times(4\times10^{14})$ J

$$= \frac{(6.63\times10^{-34})\times(4\times10^{14})}{1.6\times10^{-19}} = 1.66 \text{ eV}$$

8. **(a)** Energy of photon (E) is given by

$$E = \frac{hc}{\lambda}$$

Number of photons of wavelength λ emitted in t second from laser of power P is given by

$$n = \frac{Pt\lambda}{hc}$$

$$\Rightarrow n = \frac{2\times\lambda}{hc} = \frac{2\times10^{-3}\times5\times10^{-7}}{2\times10^{-25}} \quad (\because t = 1S)$$

$\Rightarrow n = 5\times10^{15}$

9. **(c)** Here w = $2\pi\times6\times10^{14}$ or f = 6×10^{14} Hz

$$\text{Wavelength } \lambda = \frac{C}{f} = \frac{3\times10^8}{6\times10^{14}} = 0.5\times10^{-6}m = 5000\text{A}^\circ$$

$$\text{Now } E = \frac{12374}{5000} = 2.48eV$$

Using E = w + eV_s
2.48 = 2 + eV_s or V_s = 0.48 V

10. **(BONUS)**

11. **(c)** Let ϕ = work function of the metal,

$$\frac{hc}{\lambda_1} = \phi + eV_1 \quad ....(i)$$

$$\frac{hc}{\lambda_2} = \phi + eV_2 \quad ....(ii)$$

Sutracting (ii) from (i) we get

$$hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)=e(V_1-V_2)$$

$$\Rightarrow V_1-V_2=\frac{hc}{e}\left(\frac{\lambda_2-\lambda_1}{\lambda_1\cdot\lambda_2}\right)\quad\left[\begin{array}{l}\lambda_1=300\,\text{nm}\\ \lambda_2=400\,\text{nm}\\ \frac{hc}{e}=1240\,\text{nm}-\text{V}\end{array}\right]$$

$$=(1240\,\text{nm}-\text{v})\left(\frac{100\text{nm}}{300\text{ nm}\times 400\text{nm}}\right)=1.03\text{ V}\approx 1\text{V}$$

12. **(c)** using, intensity $I=\frac{nE}{At}$

n = no. of photoelectrons

$$\Rightarrow 16\times10^{-3}=\left(\frac{n}{t}\right)\times\frac{10\times1.6\times10^{-19}}{10^{-4}}\quad\text{or,}\quad\frac{n}{t}=10^{12}$$

So, effective number of photoelectrons ejected per unit time = $10^{12}\times 10/100=10^{11}$

13. **(a)** From Einstein's photoelectric equation,

$$\frac{hc}{\lambda_1}=\phi+\frac{1}{2}m(2v)^2\quad\text{....(i)}$$

$$\text{and}\quad\frac{hc}{\lambda_2}=\phi+\frac{1}{2}mv^2\quad\text{....(ii)}$$

As per question, maximum speed of photoelectrons in two cases differ by a factor 2

From eqn. (i) & (ii)

$$\Rightarrow\frac{\frac{hc}{\lambda_1}-\phi}{\frac{hc}{\lambda_2}-\phi}=4\Rightarrow\frac{hc}{\lambda_1}-\phi=\frac{4hc}{\lambda_2}-4\phi$$

$$\Rightarrow\frac{4hc}{\lambda_2}-\frac{hc}{\lambda_1}=3\phi\Rightarrow\phi=\frac{1}{3}hc\left(\frac{4}{\lambda_2}-\frac{1}{\lambda_1}\right)$$

$$=\frac{1}{3}\times1240\left(\frac{4\times350-540}{350\times540}\right)=1.8\text{ eV}$$

14. **(d)** According to question, there are two EM waves with different frequency,

$B_1=B_0\sin(\pi\times10^7c)t$

and $B_2=B_0\sin(2\pi\times10^7c)t$

To get maximum kinetic energy we take the photon with higher frequency

using, $B=B_0\sin\omega t$ and $\omega=2\pi v\Rightarrow v=\frac{\omega}{2\pi}$

$B_1=B_0\sin(\pi\times10^7c)t\Rightarrow v_1=\frac{10^7}{2}\times c$

$B_2=B_0\sin(2\pi\times10^7c)t\Rightarrow v_2=10^7c$

where c is speed of light $c=3\times10^8$ m/s

Clearly, $v_2>v_1$

so KE of photoelectron will be maximum for photon of higher energy.

$v_2=10^7c$ Hz

$hv=\phi+KE_{max}$

energy of photon

$E_{ph}=hv=6.6\times10^{-34}\times10^7\times3\times10^9$

$E_{ph}=6.6\times3\times10^{-19}$J

$$=\frac{6.6\times3\times10^{-19}}{1.6\times10^{-19}}\text{eV}=12.375\text{eV}$$

$KE_{max}=E_{ph}-\phi$

$=12.375-4.7=7.675\text{ eV}\approx7.7\text{ eV}$

15. **(c)** In X-ray tube, $\lambda_{min}=\frac{hc}{eV}$

$$\ln\lambda_{min}=\ln\left(\frac{hc}{e}\right)-\ln V$$

Clearly, log λ_{min} versus log V graph slope is negative hence option (c) correctly depicts.

16. **(c)** As the metal surface is same, work function (ϕ) is same for both the case.

Initially $KE_{max}=nh-\phi$ ..... (i)

After increase

$KE'_{max}=3\,nh-\phi$ .... (ii)

For work function ϕ – not to be –ve or zero, $v'>\sqrt{3}v$

17. **(d)** From Einsteins photoelectric equation,

$$\frac{hC}{\lambda}-\phi_0=K.E_{max}=\frac{p^2}{2m_e}=\frac{h^2}{2m_e\lambda_d^{\,2}}\left(\because\lambda=\frac{h}{p}\right)$$

Differentiating on both sides

$$\frac{-hC}{\lambda^2}d\lambda=\frac{h^2}{2m_e}\left(\frac{-2}{\lambda_d^{\,3}}\times d\lambda_d\right)$$

$$\therefore\frac{d\lambda_d}{d\lambda}\propto\frac{\lambda_d^{\,3}}{\lambda^2}$$

18. **(c)** $h\frac{c}{\lambda}-hv_0=\frac{1}{2}mv^2$

$$\therefore\frac{4}{3}\frac{hc}{\lambda}-hv_0=\frac{1}{2}mv'^2$$

$$\therefore\frac{v'^2}{v^2}=\frac{\frac{4}{3}v-v_0}{v-v_0}\quad\therefore\quad v'=v\sqrt{\frac{\frac{4}{3}v-v_0}{v-v_0}}$$

$$\therefore v'>v\sqrt{\frac{4}{3}}$$

19. **(a)** From Einstein's photoelectric equation

$$K.E._\lambda=\frac{hc}{\lambda}-\phi\quad\text{...(i)}$$

(for monochromatic light of wavelength λ)

where ϕ is work function

$$K.E._{\lambda/2}=\frac{hc}{\lambda/2}-\phi\quad\text{...(ii)}$$

(for monochromatic light of wavelength $\lambda/2$)

From question,

$$K.E._{\lambda/2}=3(K.E._\lambda)\Rightarrow\frac{hc}{\lambda/2}-\phi=3\left(\frac{hc}{\lambda}-\phi\right)$$

$$\frac{2hc}{\lambda}-\phi=3\frac{hc}{\lambda}-3\phi\Rightarrow 2\phi=\frac{hc}{\lambda}\therefore\phi=\frac{hc}{2\lambda}$$

20. **(b)** $\frac{hc}{e\lambda_1}-\frac{\phi}{e}=V_{0_1}$ and $\frac{hc}{e\lambda_2}-\frac{\phi}{e}=V_{0_2}\left(\because\frac{he}{\lambda}-\phi=eV_0\right)$

$$\therefore\frac{hc}{e}\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right]=V_{0_1}-V_{0_2}$$

$$\therefore h=\frac{e(V_{0_1}-V_{0_2})\lambda_1\lambda_2}{(\lambda_2-\lambda_1)c}$$

From the first two values given in data

$$h=\frac{1.6\times10^{-19}[2-1]\times0.4\times0.3\times10^{-6}}{0.1\times3\times10^8}$$

$$h=0.64\times10^{-33}=6.4\times10^{-34}\text{ J-s}$$

Similarly if we calculate h for the last two values of data we get same values of $h=6.4\times10^{-34}$J-s

21. **(a)** Frank-Hertz experiment - Discrete energy levels of atom, Photoelectric effect - Particle nature of light.
Davison - Germer experiment - wave nature of electron.

22. **(b)** Given, $\lambda_1 = 4972\text{Å}$ and $\lambda_2 = 6216\text{Å}$
and $I = 3.6\times10^{-3}\,\text{Wm}^{-2}$
Intensity associated with each wavelength

$= \frac{3.6\times10^{-3}}{2} = 1.8\times10^{-3}\ \text{Wm}^{-2}$

work function $\phi = h\nu$

$= \frac{hc}{\lambda} = \frac{(6.62\times10^{-34})(3\times10^{8})}{\lambda}$

$= \frac{12.4\times10^{3}}{\lambda}\text{ev}$

for different wavelengths

$\phi_1 = \frac{12.4\times10^3}{\lambda_1} = \frac{12.4\times10^3}{4972} = 2.493\text{ eV} = 3.984\times10^{-19}\text{ J}$

$\phi_2 = \frac{12.4\times10^3}{\lambda_2} = \frac{12.4\times10^3}{6216} = 1.994\text{ eV} = 3.184\times10^{-19}\text{ J}$

Work function for metallic surface $\phi = 2.3$ eV (given) $\phi_2 < \phi$
Therefore, ϕ_2 will not contribute in this process.
Now, no. of electrons per m^2-s = no. of photons per m^2-s

no. of electrons per m^2-s $= \frac{1.8\times10^{-3}}{3.984\times10^{-19}}\times10^{-4}$

$(\because 1\,\text{cm}^2 = 10^{-4}\,\text{m}^2) = 0.45\times10^{12}$

So, the number of photo electrons liberated in 2 sec.
$= 0.45\times10^{12}\times2$
$= 9\times10^{11}$

23. **(a)** Here, $\frac{hC}{\lambda_1} - \phi = \frac{1}{2}mu_1^2$ ...(i)

and $\frac{hC}{\lambda_2} - \phi = \frac{1}{2}mu_2^2$ ...(ii)

Dividing equations, (i) by (ii)

$$\frac{\frac{hC}{\lambda_1}-\phi}{\frac{hC}{\lambda_2}-\phi} = \frac{u_1^2}{u_2^2} \quad \therefore \frac{\frac{1240}{248}-\phi}{\frac{1240}{310}-\phi} = \frac{4}{1}$$

$$\therefore \frac{1240}{248} - \phi = \frac{4\times1240}{310} - 4\phi \quad \therefore \phi = 3.7\text{ eV}$$

Hence the work function of the metal is nearly, $\phi = 3.7$ *eV*

24. **(b)** $\because \frac{1}{\lambda} = R(z-b^2)\left(\frac{1}{n_i^2}-\frac{1}{n_i^2}\right)$ For K-series, $b = 1$ and for k_∞ transition $n_f = 2$ to $n_i = 1$ $\therefore \frac{1}{\lambda} \propto (z-1)^2$

$$\because \frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2} = \left(\frac{42-1}{29-1}\right)^2 = \left(\frac{41}{28}\right)^2 = 2.14$$

25. **(d)** As λ is increased, there will be a value of λ above which photoelectrons will be cease to come out so photocurrent will become zero. Hence (d) is correct answer.

26. **(d)** $\frac{hc}{\lambda} - \phi = eV_0$

$V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

For metal A: $\frac{\phi A}{hc} = \frac{1}{\lambda}$

For metal B: $\frac{\phi B}{hc} = \frac{1}{\lambda}$

As the value of $\frac{1}{\lambda}$ (increasing and decreasing) is not specified hence we cannot say that which metal has comparatively greater or lesser work function (ϕ).

27. **(b)** Final momentum,

$p = \frac{E}{c} = \frac{P\times t}{c} = \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^8}$

$= 1.0\times10^{-17}\text{ kg ms}^{-1}$

28. **(a)** Energy of photon $E = \frac{hc}{\lambda} = \frac{1240}{\lambda}$ The energy possessed by photons of wavelength 550 nm

$E_1 = \frac{1240}{550} = 2.25\text{eV}$

The energy possessed by photons of wavelength 450 nm $E_2 = \frac{1240}{450} = 2.76\text{eV}$

The energy possessed by photons of wavelength 350 nm $E_3 = \frac{1240}{350} = 3.54\text{eV}$

Work function of $\phi_p = 2.0$ *eV* is least among *p*, *q*, and *r* so it can emit photoelectrons by E_1, E_2 and E_3 or from all three wavelengths. Hence magnitude of *V* and *I* both will be maximum for metal plate *P*.

29. **(b)** The wavelength of continuous X - rays is independent of the atomic number of target material. Wave length of ckaracteristic X-ray depends on atomic number of target material.

30. **(a)** The cut off wavelength of the emitted X-ray,

$\lambda_0 = \frac{hc}{eV}$ ...(i)

According to de Broglie equation

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

or $\lambda^2 = \frac{h^2}{2meV} \Rightarrow V = \frac{h^2}{2me\lambda^2}$ ..(ii)

From eq. (i) and (ii),

$\lambda_0 = \frac{hc\times2me\lambda^2}{eh^2} = \frac{2mc\lambda^2}{h}$

31. **(a)** For K_α, $\frac{1}{\lambda} \propto (Z-1)^2$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{(Z_1-1)^2}{(Z_2-1)^2} \Rightarrow \frac{4\lambda}{\lambda} = \frac{(11-1)^2}{(Z_2-1)^2}$

$\Rightarrow Z_2 - 1 = \frac{10}{2} \quad \therefore Z_2 = 6$

32. **(d)** Saturation current $\propto$ (intensity) and stopping potential increases with increase in frequency. From the graph it is clear that *A* and *B* have the same stopping potential and therefore, the same frequency. Also, *B* and *C* have the same intensity.

33. **(a)** As we know, $I = \frac{q}{t} = \frac{ne}{t}$

$\therefore$ No. of electrons striking the target per second

$\frac{n}{t} = \frac{I}{e} = 2 \times 10^{16}$

34. **(a)** In case of Coolidge tube $\lambda_{\min} = \frac{hc}{eV} = \lambda$

Thus the minimum wavelength is inversely proportional to accelerating voltage. As V increases, λ_c decreases. λ_k is the wavelength of K_∞ line which is independent of accelerating voltage of bombarding electron. Since λ_k always refers to a photon wavelength of transition of e^- from the target element from $2 \to 1$. Therefore $\lambda_k - \lambda_c$ increases as accelerating voltage is increased.

35. **(d)** Minimum wave length of continuous X-ray spectrum

$$\lambda_{\min} = \frac{hc}{E}$$

$$\therefore \lambda_{\min} = \frac{12375}{80 \times 10^3} \text{Å} = 0.155\text{Å} \qquad [\because E = 80 \text{ kev given}]$$

Energy of incident electrons is greater than the ionization energy of electrons in K-shell, the K-shell electrons will be knocked off. Hence, characteristic X-ray spectrum will be obtained.

36. **(c)** $\lambda_{\min} = \frac{hc}{\phi} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4(1.6 \times 10^{-19})} = 310 \times 10^{-9}\text{m}.$

$= 310 \text{ nm}$

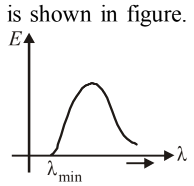
37. **(b)** The continuous X-ray spectrum is shown in figure.

All wavelengths $> \lambda_{\min}$ are found

i.e., $\gamma_{\min}$ to ∞; $\gamma_{\min} > 0$

where $\lambda_{\min} = \frac{12400}{\text{V (in volt)}}$ Å

Here V is the applied voltage.



38. **(b)** Stopping potential is the negative potential applied to stop the electrons having maximum kinetic energy.

$\therefore$ Stopping potential = 4 volt.

39. **(d)** According to question, 24.6 ev is required to remove one of the electrons from a neutral hellum atom.

For helium ion, $Z = 2$ and for first orbit $n = 1$.

$$\therefore \quad E = \frac{-13.6 \times z^2}{n^2} = \frac{-13.6}{(1)^2} \times 2^2 = -54.4 \text{ eV}$$

$\therefore$ Energy required to remove this $e^- = +54.4$ eV

$\therefore$ Total energy required = 54.4 + 24.6 = 79 eV

40. **(c)** Cut-off wave length, $\lambda_{\min} = \frac{hc}{eV}$

41. **(b)** The wavelength of characteristic X-rays depends on the type of atoms of which the target material is made. It does not depend on the accelerating potential. Cut-off wavelength depends on the acceterating voltage.

When an electric beam strikes the target in an X-ray tube, part of the kinetic energy approximately (2%) is converted into X-ray energy.

42. **(1)** From conservation of momentum principle, change in momentum of photon = change in momentum of mirror

$2(\text{NP}) = \text{MV}_{\max}$

$$\Rightarrow 2\left[\text{N}\left(\frac{\text{h}}{\lambda}\right)\right] = \text{MV}_{\max}$$

$$\therefore 2\frac{\text{Nh}}{\lambda} = \text{M}(\text{A}\Omega) \qquad [\because \text{V}_{\max} = \text{A}\Omega]$$

$$\text{N} = \left(\frac{\text{M}\Omega}{\text{h}}\right)\frac{\text{A}\lambda}{2} = \frac{10^{24}}{4\pi} \times \frac{10^{-6} \times 8\pi \times 10^{-6}}{2}$$

$$\left[\because \frac{m\Omega}{h} = \frac{10^{24}}{4\pi}; A = 1hm; \lambda = 8\pi \times 10^{-6}\right]$$

$\therefore$ N = $1 \times 10^{12} = x \times 10^{12}$ $\therefore$ $x = 1$

43. **(1)** For photoelectric effect

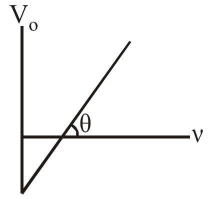
$h\nu - \phi = \text{eV}_0$

$\Rightarrow \frac{h\nu}{e} - \frac{\phi_0}{e} = \text{V}_0$

The slope of V_0 versus ν graph is a straight line with slope

$\tan\theta = \frac{\text{h}}{\text{e}}$ = constant

$\therefore$ The ratio of two slopes will be 1.



44. **(7)** From $\frac{hc}{\lambda} - \phi = -eV_0$

Stopping potential $V_0 = \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right]$ where

hc = 1240eV –nm

$$= \frac{1}{e}\left[\frac{1240}{200} - 4.7\right] = \frac{1}{e}[6.2 - 4.7] = \frac{1}{e} \times 1.5eV = 1.5\text{V}$$

But $\text{V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{ne}{r}$ $\left(\begin{array}{l} n = \text{no. of photo-} \\ \text{electrons emitted} \end{array}\right)$

$$\Rightarrow \text{n} = \frac{Vr(4\pi\varepsilon_0)}{e} = \frac{1.5 \times 10^{-2}}{9 \times 10^9 \times 1.6 \times 10^{-19}}$$

$\therefore$ n = 1.04×10^7

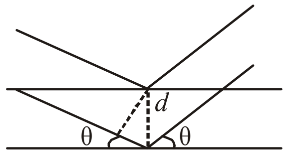
Comparing it with A × 10^z we get, z = 7

45. **(50)**

From Bragg's equation $2d\sin\theta = \lambda$ and de-Broglie wavelength,

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}}$$

$$2d\sin\theta = \lambda = \frac{h}{\sqrt{2mE}}$$



$$\Rightarrow 2 \times 10^{-10} \times \frac{\sqrt{3}}{2} = \frac{6.6 \times 10^{-34}}{\sqrt{2mE}}$$

$[\because \theta = 60°$ and $d = 1\text{Å} = 1 \times 10^{-10}\text{m}]$

$$\therefore E = \frac{1}{2} \times \frac{6.64^2 \times 10^{-48}}{9.1 \times 10^{-31} \times 3 \times 1.6 \times 10^{-19}} \simeq 50 \text{ eV}$$

46. **(2)**

From the Einstein's photoelectric equation

Energy of photon

= Kinetic energy of photoelectrons + Work function

$\Rightarrow$ Kinetic energy = Energy of Photon – Work Function

Let ϕ_0 be the work function of metal and v_1 and v_2 be the velocity of photoelectrons. Using Einstein's photoelectric equation we have

$$\frac{1}{2}mv_1^2 = 4 - \phi_0 \quad ...(i) \qquad \frac{1}{2}mv_2^2 = 2.5 - \phi_0 \quad ...(ii)$$

$$\Rightarrow \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{4 - \phi_0}{2.5 - \phi_0} \Rightarrow (2)^2 = \frac{4 - \phi_0}{2.5 - \phi_0} \Rightarrow 10 - 4\phi_0 = 4 - \phi_0$$

$\phi_0 = 2eV$

47. **(9)**

When radiation of wavelength A, λ_A is used to illuminate, stopping potential $V_A = V$

$$\frac{hc}{\lambda} = \phi + eV \quad \text{...(i)}$$

When radiation of wavelength B, λ_B is used to illuminate, stopping potential, $V_B = \dfrac{V}{4}$

$$\frac{hc}{3\lambda} = \phi + \frac{eV}{4} \quad \text{...(ii)}$$

From eq. (i) – (ii),

$$\frac{hc}{\lambda}\left(1-\frac{1}{3}\right) = \frac{3}{4}eV \Rightarrow \frac{hc}{\lambda}\frac{2}{3} = \frac{3}{4}eV \Rightarrow eV = \frac{8}{9}\frac{hc}{\lambda}$$

$$\frac{hc}{\lambda} = \phi + \frac{8}{9}\frac{hc}{\lambda}$$

$\therefore \phi = \dfrac{hc}{9\lambda} = \dfrac{hc}{n\lambda}$, so, $n = 9$.

48. **(11.00)** Energy of proton

$E = \dfrac{hc}{\lambda} = \dfrac{1240}{310} = 4eV > 2eV$ [= ϕ]

(so emission of photoelectron will take place)

$= 4 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19}$ joule

$$N = \frac{6.4\times10^{-5}\times1}{4\times6.4\times10^{-19}} = 10^{14}$$

No. of photoelectrons emitted per second

$= \dfrac{10^{14}}{10^{3}} = 10^{11}$ ($\because$ 1 in 10^3 photons ejects an electron)

$\therefore$ Value of $X = 11.00$

49. **(24.00)** Number of electrons emitted/s

$$= \frac{200\ W}{6.25\times1.6\times10^{-19} J}$$

Force, F = Rate of change of linear momentum = $N\sqrt{2mV}$

$$= \frac{200}{6.25\times1.6\times10^{-19}} \times \sqrt{2\times9\times10^{-31}\times1.6\times10^{-19}\times500}$$

$\Rightarrow$ $F = 24 \times 10^{-4}$ N [$\because$ K = eV : e = 1.6×10^{-19} = V = 500]

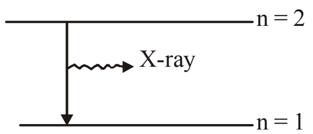
$\therefore$ $n = 24.00$

50. $\dfrac{1}{\lambda} = R(Z-1)^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

For K_α, $n_2 = 2$ and $n_1 = 1$

$$\therefore \frac{1}{0.76\times10^{-10}} = 1.097\,(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$$

$\Rightarrow z - 1 = 40 \quad \Rightarrow Z = 41$



The atomic number of the anode material of the tube is 41.

51. $\lambda_{min} = \dfrac{hc}{eV} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times20\times10^{3}} = 0.62\text{Å}$

52. All electromagnetic waves propagate at 3×10^8 m/s in vacuum.

$\therefore$ Speed of X-rays is 3×10^8m/s in vacuum.

53. We know that

K_α corresponds to n = 2 to $n = 1$, $\dfrac{1}{\lambda} = C\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$, where C is a constant

$$\Rightarrow \frac{1}{0.32\text{Å}} = C\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = \frac{3C}{4}$$

K_β, corresponds to n = 3 to n = 1, $\dfrac{1}{\lambda} = C\left[\dfrac{1}{1^2}-\dfrac{1}{3^2}\right] = \dfrac{8C}{9}$

Dividing, we get $\lambda = 0.27$ Å.

54. More the number of electrons striking the anode, more number of photons of X-rays emitted. Hence more is the intensity of X-rays.

When the speed of the striking electrons on anode is increased, the emitted X-rays have greater energy. And energy, $E = \dfrac{hc}{\lambda}$.

$\therefore$ when E increases then λ decreases.

55. According to Einstein's photoelectric equation

$(K.E.)_{max} = h\nu - h\nu_0$

$\therefore$ Maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the frequency of incident radiation.

56. For characteristic X-rays

$$\frac{1}{\lambda} = R_\alpha(Z-1)^2\left[1-\frac{1}{n^2}\right] = \frac{E}{hc}$$

$$\therefore \frac{E_1}{hc} = R_\alpha(Z-1)^2\left[1-\frac{1}{2^2}\right] \quad \text{......(i)}$$

As for minimum accelerating voltage, electron jumps from n = 1 to n = 2.

The binding energy of innermost electron in tungsten, $E_2 = 40$ keV

$$\therefore \frac{E_2}{hc} = R_\alpha(Z-1)^2\left[1-\frac{1}{\infty^2}\right] \quad \text{.....(ii)}$$

Dividing eq. (i) by (ii)

$$\frac{E_1}{E_2} = \frac{\left[1-\frac{1}{2^2}\right]}{\left[1-\frac{1}{\infty^2}\right]} \Rightarrow E_1 = \frac{3}{4}E_2 = \frac{3}{4}\times40{,}000\text{ eV}$$

$\therefore V_{min} = 30{,}000\text{ eV} = 30\times10^3\text{V}$

57. **(False)**

According to Einstein's photoelectric equation, $(K.E.)_{max} = h\nu - h\nu_0 \Rightarrow (K.E.)_{max} \propto \nu$

Thus maximum kinetic energy is proportional to frequency and not intensity.

58. **(False)**

The kinetic energy of photo electrons emitted depends on frequency (ν) of incident radiation. For photoelectric effect

$h\nu - h\nu_0 = (K.E.)_{max}$

where h = Planck's constt.

ν_0 = Threshold frequency

$\Rightarrow (K.E.)_{max} \propto \nu$

K.E. does not depend on the intensity of incident radiation.

59. **(a, c)** Cut-off wavelength, $\lambda = \dfrac{hc}{eV} \Rightarrow \alpha\dfrac{1}{V}$

$\Rightarrow$ (Cut-off wavelength)$_{new} = \dfrac{\lambda}{2}$

As potential difference V is increased V to 2V so cut-off wavelength will reduce to half. But wavelength of characteristic x-ray will not be affected.

$\because$ Intensity, $I = \dfrac{dN}{dt}\times\dfrac{hc}{\lambda}$

$\therefore \dfrac{dN}{dt}$ decreases

Hence intensity of all the x-ray will decrease.

60. (c) de-Broglie wavelength passing through the anode,

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2m(K.E)}}$$

When ϕ increases, K.E. decreases and therefore λ_e increases

When λ_{ph} increases, N_{ph} decreases, K.E decreases and therefore λe increases.

λe is independent of the distance d.

Also after reaching anode, $\frac{hc}{\lambda_{ph}} + eV - \phi = \frac{h^2}{2m\lambda_e^2}$

$$\left[\lambda e = \frac{h}{\sqrt{2mk.E}}\right] \quad \therefore \frac{hc}{e\lambda_{ph}} + V - \frac{\phi}{e} = \frac{h^2}{2me\lambda_e^2} \qquad ...(i)$$

For $V >> \frac{\phi}{e}$, $\phi << eV$

Also $\frac{hc}{e\lambda_{Ph}} << V$. $\therefore$ from eq (i). $\lambda_e \propto \frac{1}{\sqrt{V}}$

Hence, if V is made our times, λ_e is approximately half.

61. (a, c) We know that

$$\frac{hC}{\lambda} - \phi = eV_0 \Rightarrow \frac{hc}{e\lambda} - \frac{\phi}{e} = V_0 \Rightarrow \frac{he}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e} = V_0$$

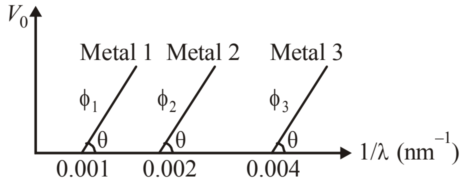
Therefore V_0 versus $\frac{1}{\lambda}$ graph is a straight line with negative slope $\left(\frac{hc}{e}\right)$ and positive intercept $\left(\frac{\phi}{e}\right)$.

For V_0 versus λ, we will get a hyperbola as λ decreases V_0 increases. And V_0 becomes zero when $\left(\frac{hc}{\lambda}\right) = \phi$ i.e., when $\lambda = \lambda_0$

62. (a, c) From $eV = \frac{hc}{\lambda} - \phi$ $V = \frac{hc}{e\lambda} - \frac{\phi}{e}$ At $V = 0$, $\phi_1 : \phi_2 : \phi_3$.

$$= \frac{hc}{\lambda_1} : \frac{hc}{\lambda_2} : \frac{hc}{\lambda_3}$$

$$= 0.001 : 0.002 : 0.004 = 1 : 2 : 4$$



By Einstein's photoelectric equation, $\frac{hc}{\lambda} - \phi = eV$

$$\Rightarrow V = \frac{hc}{e\lambda} - \frac{\phi}{e} \qquad ...(i)$$

Comparing equation (i) by $y = mx + c$, we get the slope of the line

$$m = \frac{hc}{e} = \tan\theta$$

From the graph V_0 versus $\frac{1}{\lambda}$ it is clear that,

$$\frac{1}{\lambda_{0_1}} = 0.001(nm)^{-1} \therefore \lambda_{0_1} = \frac{1}{0.001} = 1000mn$$

$$\frac{1}{\lambda_{0_2}} = 0.002(nm)^{-1} \therefore \lambda_{0_2} = 500nm \text{ and } \lambda_{0_3} = 250nm$$

Violet colour light has wavelength 400 nm.

Therefore, this light will be unable to show photoelectric effect on plate 3 can eject photoelectrons from metal–1 and metal–2.

63. (a,b,c) From Einstein's photoelectric equation, $K_{max} = E - W$

For metal A

$4.25 = W_A + T_A$ ...(i)

Also $T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{m^2v_A^2}{m} = \frac{p_A^2}{2m} = \frac{h^2}{2m\lambda_A^2}$ ...(ii)

$$\left[\because \lambda = \frac{h}{p} \text{and } P = mV\right]$$

For metal *B*

$4.7 = (T_A - 1.5) + W_B$ ...(iii)

And $T_B = \frac{h^2}{2m\lambda_B^2}$ ...(iv)

Dividing equation (iv) by (ii), $\frac{T_B}{T_A} = \frac{h^2}{2m\lambda_B^2} \times \frac{2m\lambda_A^2}{h^2} = \frac{\lambda_A^2}{\lambda_B^2}$

$$\Rightarrow \frac{T_A - 1.5}{T_A} = \frac{\lambda_A^2}{(2\lambda_A)^2} = \frac{\lambda_A^2}{4\lambda_A^2} = \frac{1}{4} \quad [\because \lambda_B = 2\lambda_A \text{ given}]$$

$\Rightarrow 4T_A - 6 = T_A \therefore T_A = 2$ eV

Putting this value of T_A in equation (i) and (ii) we get

$W_A = 2.25$ eV and $W_B = 4.2$ eV

Also $T_B = T_A - 1.5 \Rightarrow T_B = 0.5$eV

64. (b, d) There is no change in stopping potential by increasing the distance of source from the cell as neither the energy of incident light nor the work function of metal wil change. The saturation current depends on the intensity (D) of incident light on the cathode of the photocell.

Intensity, $I \propto \frac{1}{r^2}$ and saturation current $\propto I$

$$\therefore \text{Saturation current} \propto \frac{1}{r^2} \therefore \frac{(\text{Saturation current})_{\text{final}}}{(\text{Saturation current})_{\text{initial}}} = \frac{r_{\text{initial}}^2}{r_{\text{final}}^2}$$

$$\therefore (\text{Saturation Current})_{\text{final}} = \frac{0.2 \times 0.2}{0.6 \times 0.6} \times 18 = 2\text{mA}$$

Hence the saturation current will be 2.0 mA

65. (c,d) When potential difference (V) applied to an X-ray tube is increased, the minimum wavelength $\lambda_{\min} = \frac{12375}{V}$ will decrease.

Potential difference or voltage (V) does not change the intensity of emitted radiation.

66. (a,b,c)

67. (c,d) For ejection of electrons, the wavelength of the light should be less than threshold wavelength, 5200 Å, U.V light has less wavelength than 5200 Å.

68. (b,d) Note : Shortest wavelength means highest frequency So highest energy.

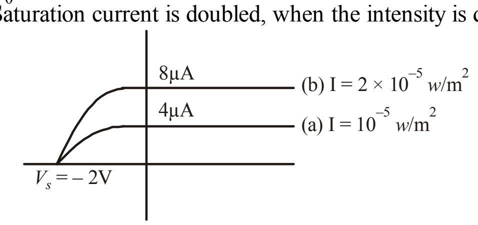
Shortest or cut-off wave length of X-rays emitted from an X-ray tube depends on the voltage applied to the tube.

Also, according to Moseley's law $\sqrt{\nu} = a(Z - b)$. Thus the frequency also depends on the atomic number.

69. (a) $eV_0 = h\nu - h\nu_0 = 5 - 3 = 2$ eV

$\therefore$ $V_0 = 2$ volt

(b) Saturation current is doubled, when the intensity is doubled



70. Energy released during transition from n_2 to n_1

$$\Delta E = h\nu = Rhc(Z-b)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

For transition L shell $\rightarrow$ K shell

$b = 1, n_2 = 2, n_1 = 1 \therefore (Z-1)^2 Rhc\left[\frac{1}{1}-\frac{1}{4}\right] = h\nu$

Putting the value of $R = 1.1 \times 10^7\ m^{-1}$ and $c = 3 \times 10^8$ m/s, and solving we get $Z = 42$

71. **(a)** Number of electrons incident on the metal plate A $= 10^{16} \times (5 \times 10^{-4})$

$\therefore$ Number of photoelectrons emitted from plate A in 10 s

$$n_e = \frac{(5\times10^4)\times10^{16}}{10^6}\times10 = 5\times10^7$$

(b) At $t = 10$ sec charge on plate B

$Q_b = 33.7 \times 10^{-12} - 5 \times 10^7 \times 1.6 \times 10^{-19} = 25.7 \times 10^{-12}\ C$

and $Q_a = 8 \times 10^{-12}\ C$

$$E = \frac{\sigma_B}{2\varepsilon_0}-\frac{\sigma_A}{2\varepsilon_0} = \frac{1}{2A\varepsilon_0}(Q_B - Q_A)$$

$$= \frac{17.7\times10^{-12}}{5\times10^{-4}\times8.85\times10^{-12}} = 2000\ N/C$$

(c) K.E. of most energetic particles = energy of photoelectrons due to light + due to work done by photoelectrons between the plates $= (hn - f) + e\,(Ed) = 23$ eV

72. Given: Work function, $\phi = 1.9$ ev and $\lambda = 400$nm $= 400 \times 10^{-9}$ m The energy of the incident photon is

$$E_1 = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\,\text{eVs})(3\times10^8\,\text{m/s})}{(400\times10^{-9}\,\text{m})} = 3.1\text{eV}$$

The maximum kinetic energy of the emitted photelectrons

$E_{max} = E_1 - W = 3.1\text{ eV} - 1.9\text{ eV} = 1.2\text{eV}$

As per question,

$$\begin{pmatrix}\text{Emitted electrons of}\\ \text{maximum kinetic energy}\end{pmatrix} + {}_2He^{2+} \longrightarrow \underset{\text{in 4th excited state}}{He^+} + 1 \text{ photon}$$

The fourth excited state implies that the electron enters in the $n = 5$ state.

In this state its energy

$$E_5 = -13.6\frac{z^2}{n^2}ev = -\frac{(13.6\text{eV})Z^2}{n^2} = -\frac{(13.6\text{eV})(2)^2}{5^2}$$

$= -2.18$ eV.

The energy of the emitted photon in the above combination

$E = E_{max} + (-E_5) = 1.2\text{ eV} + 2.18\text{ eV} = 3.4$ eV

Similarly, energies in other states of He^+

$$E_4 = \frac{(-13.6\text{eV})(2^2)}{4^2} = -3.4\text{eV} \quad E_3 = \frac{(-13.6\text{eV})(2^2)}{3^2} = -6.04\text{eV}$$

$$E_2 = \frac{(-13.6\text{eV})(2^2)}{2^2} = -13.6\text{eV}$$

The possible transitions are

$n = 5 \rightarrow n = 4$

$\Delta E = E_5 - E_4 = [-2.18 - (-3.4)]\text{ eV} = 1.28$ eV

$n = 5 \rightarrow n = 3$

$\Delta E = E_5 - E_3 = [-2.18 - (-6.04)]\text{ eV} = 3.84$ eV

$n = 5 \rightarrow n = 2$

$\Delta E = E_5 - E_2 = [-2.18 - (-13.6)]\text{ eV} = 11.4$ eV

$n = 4 \rightarrow n = 3$

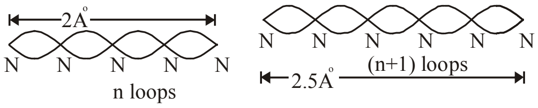
$\Delta E = E_4 - E_3 = [-3.4 - (-6.04)]\text{ eV} = 2.64$ eV

Hence, the photons that are likely to be emitted in the range of 2 eV to 4 eV are 3.4 eV during combination, 3.84 eV and 2.64 eV after combination.

73. Nodes are formed at each of the atomic sites,

$$\therefore 2\text{Å} = n\left(\frac{\lambda}{2}\right) \quad ...(i)$$

[$\because$ Distance between successive nodes = $\lambda/2$]



And $2.5\text{ Å} = (n+1)\dfrac{\lambda}{2}$ ...(ii)

From eq (ii) – (i) $0.5 = \dfrac{\lambda}{2} \therefore = 1\text{Å}$

Now, de broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mK}} \text{ or } K = \frac{h^2}{\lambda^2.2m}$$

$$\therefore \quad K = \frac{(6.63\times10^{-34})^2}{(1\times10^{-10})^2\times2\times9.1\times10^{-31}\times1.6\times10^{-19}}\text{ eV}$$

$$= \frac{(6.63)^2}{8\times9.1\times1.6}\times10^2\text{ eV} = 151\text{ eV}$$

d will be minimum, when only one loop is formed $\therefore n = 1$

$$\therefore d_{\min} = \frac{\lambda}{2} = \frac{1\text{Å}}{2} = 0.5\text{Å}$$

Topic-3 : Miscellaneous (Mixed Concepts) Problems

1. **(7)** From energy conservation Loss in K.E. of proton = gain in potential energy of the proton – nucleus system

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \qquad \therefore \frac{p^2}{2m} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$\therefore \frac{1}{2m}\left(\frac{h^2}{\lambda^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \qquad \therefore \lambda = \sqrt{\frac{4\pi\epsilon_0\, r\,.\,h^2}{q_1 q_2\,(2\,m)}}$$

Putting the values of $4\pi\varepsilon_0$, r, h, q_1, q_2 and m we get, de-Broglie wavelength of proton, $\lambda = 7$fm

2. **(b, c, d)** According to question,

surface area of filament of light bulb, A = 64 mm^2

Temperature of filament, T = 2500 K

distance of bulb or source from observer, d = 100m

Radius of the pupil of the eyes of the observer, Re = 3 mm = 3 $\times 10^{-3}$ m

(a) Power radiated by the filament $P = \sigma AeT^4$

$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4 = 141.75$ w

($\because$ e = 1 for black body)

Hence, option (a) is incorrect.

(b) Radiated power entering into one eye of the observer,

$$I = \frac{P}{4\pi d^2}\times(\pi R_e^2)$$

$$= \frac{141.75}{4\pi\times(100)^2}\times\pi\times(3\times10^{-3})^2 = 3.189375 \times 10^{-8}\text{ W}$$

Hence, option (b) is correct

(c) From wein's displacement law, $\lambda_m T = b$

or, $\lambda_m \times 2500 = 2.9 \times 10^{-3}$

or, $\lambda_m = 1.16 \times 10^{-6} = 1160$ nm

Hence, option (c) is correct

(d) Total no. of photons entering per second into one eye of the observer $= \left(\dfrac{hc}{\lambda}\right)\times N_{photons} = I$

$$3.189375 \times 10^{-8} = \frac{6.63\times10^{-34}\times3\times10^8}{1740\times10^{-9}}\times N_{photons}$$

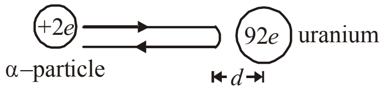
$\therefore N_{photons} = 2.79 \times 10^{11}$

Hence, option (D) is correct.

Topic-1 : Atomic Structure and Rutherford's Nuclear Model

1. **(c)** Graph (c) correctly depicts (Y) versus (θ).
2. **(c)** Let 'r' be the distance of closest approach. One point charge is $\left({}^{235}_{92}\text{U}\right)$ uranium nucleus $\therefore\ q_1 = 92e$

Another point charge is α particle $\therefore\ q_2 = +2e$



Here loss in K.E. = gain in P.E. Energy conservation.

$$\therefore \frac{1}{2}mv^2 = k\frac{q_1q_2}{r} \Rightarrow r = k\frac{2q_1q_2}{\frac{1}{2}mv^2}$$

$$\therefore\quad r = \frac{9\times10^9\times2\times1.6\times10^{-19}\times92\times1.6\times10^{-19}}{5\times1.6\times10^{-13}}$$

$= 529.92 \times 10^{-16}$ m

Hence the distance of closest approach is of the order of 10^{-12} cm.

Topic-2 : Bohr's Model and the Spectra of the Hydrogen Atom

1. **(b)** Total energy of electron in n^{th} orbit of hydrogen atom

$$E_n = -\frac{Rhc}{n^2}$$

Total energy of electron in $(n + 1)^{th}$ level of hydrogen atom

$$E_{n+1} = -\frac{Rhc}{(n+1)^2}$$

When electron makes a transition from $(n + 1)^{th}$ level to n^{th} level Change in energy,

$$\Delta E = E_{n+1} - E_n$$

$$h\nu = Rhc\cdot\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \qquad (\because E = h\nu)$$

$$\nu = R\cdot c\left[\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right] \Rightarrow \nu = R\cdot c\left[\frac{1+2n}{n^2(n+1)^2}\right]$$

For $n >> 1$

$$\Rightarrow \nu = R\cdot c\left[\frac{2n}{n^2\times n^2}\right] = \frac{2RC}{n^3} \Rightarrow \nu \propto \frac{1}{n^3}$$

2. **(b)** According to Bohr's Theory the wavelength of the radiation emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\because\quad Z = 3 \qquad \therefore\quad \frac{1}{\lambda} = 9R\left(1 - \frac{1}{9}\right)$$

$$\Rightarrow\ \lambda = \frac{1}{8R} = \frac{1}{8\times10973731.6} \quad (R = 10973731.6\ \text{m}-1)$$

$\Rightarrow\ \lambda = 11.39$ nm

3. **(b)** For first excited state n' = 3

Time period $T \propto \dfrac{n^3}{z^2}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{n'^3}{n^3}$$

∴ T2 = 8T1 = 8 × 1.6 × 10–16s

$$\therefore \text{ Frequency, } \nu = \frac{1}{T_2} = \frac{1}{8\times1.6\times10^{-16}}$$

≈ 7.8 × 1014 Hz

4. **(a)** $\dfrac{1}{\lambda_1} = R\left(\dfrac{1}{3^2} - \dfrac{1}{4^2}\right) = \dfrac{7R}{16\times9}$

And $\dfrac{1}{\lambda_2} = R\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{5R}{36}$

Now $\dfrac{\lambda_1}{\lambda_2} = \dfrac{(5R/36)}{7R/(16\times9)} = \dfrac{20}{7}$

5. **(d)** Spectral lines obtained on account of transition from nth orbit to various lower orbits is $\dfrac{n(n-1)}{2}$

$$\Rightarrow 6 = \frac{n(n-1)}{2} \Rightarrow n = 4$$

$$\Delta E = \frac{hc}{\lambda} = \frac{-Z^2}{n^2}(13.6eV)$$

$$\Rightarrow \frac{1}{\lambda} = Z^2\left(\frac{13.6eV}{hc}\right)\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = (13.4)(3)^2\left[1 - \frac{1}{16}\right]eV$$

$$\Rightarrow \lambda = \frac{1242\times16}{(13.4)\times(9)(15)}\text{nm} \simeq 10.8\text{nm}$$

6. **(b)** $\dfrac{1}{\lambda_1} = -R\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{5R}{36}$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16} \qquad \therefore \frac{\lambda_2}{\lambda_1} = \frac{80}{108}$$

$$\lambda_2 = \frac{80}{108}\lambda_1 = \frac{80}{108}\times660 = 488.9\text{ nm.}$$

7. **(d)** Energy released by hydrogen atom for transition n = 2 to n = 1

$$\therefore\ \Delta E_1 = 13.6\times\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\times13.6\text{ eV} = 10.2\text{ eV}$$

This energy is absorbed by He+ ion in transition from n = 2 to n = n1 (say)

$$\therefore\ \Delta E_2 = 13.6\times4\times\left(\frac{1}{4} - \frac{1}{n_1^2}\right) = 10.2\text{ eV} \Rightarrow\ n1 = 4$$

So, possible transition is n = 2 → n = 4

8. **(c)** Energy of photon $= \dfrac{hc}{\lambda} = \dfrac{12500}{980} = 12.75\text{eV}$

Energy of electron in n^{th} orbit is given by

$$\text{En} = \frac{-13.6}{n^2} \Rightarrow E_n - E_1 = -13.6\left[\frac{1}{n^2} \frac{-1}{1^2}\right]$$

$\Rightarrow 12.75 = 13.6\left[\frac{1}{1^2} \frac{-1}{n^2}\right] \Rightarrow n = 4$

$\therefore$ Electron will excite to n = 4

We know that 'R' $\propto n^2$

$\therefore$ Radius of atom will be $16a_0$

9. **(d)** When electron jumps from M $\rightarrow$ L shell

$$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{K \times 5}{36} \quad ..... \text{(i)}$$

When eletron jumps from N $\rightarrow$ L shell

$$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{K \times 3}{16} \quad ....\text{(ii)}$$

solving equation (i) and (ii) we get

$$\lambda' = \frac{20}{27}\lambda$$

10. **(a)** Wavelength of emitted photon from n^{th} state to the ground state,

$$\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

Since n is very large, using binomial theorem

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

As we know, $\lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

11. **(d)** $h\nu_L = E_\infty - E_1$ ...(i)

$h\nu_f = E_\infty - E_5$ ...(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

Eqn (i) / (ii) $\Rightarrow \frac{h\nu_L}{h\nu_f} = \frac{E_1}{E_5} \Rightarrow \frac{\nu_L}{\nu_f} = \frac{25}{1} \Rightarrow \nu_f = \frac{\nu_L}{25}$

12. **(b)** Energy required to remove e^- from singly ionized helium atom $= \frac{(13.6)Z^2}{1^2} = 54.4$ eV $(\because Z = 2)$

Energy required to remove e^- from helium atom = x eV

According to question, 54.4 eV = 2.2x $\Rightarrow$ x = 24.73 eV

Therefore, energy required to ionize helium atom

= (54.4 + 24.73) eV = 79.12 eV

13. **(b)** From energy level diagram, using $\Delta E = \frac{hc}{\lambda}$

For wavelength λ_1 $\Delta E = -E - (-2E) = \frac{hc}{\lambda_1}$

$\therefore \quad \lambda_1 = \frac{hc}{E}$

For wavelength λ_2 $\Delta E = -E - \left(-\frac{4E}{3}\right) = \frac{hc}{\lambda_2}$

$\therefore \quad \lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)} \qquad \therefore r = \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$

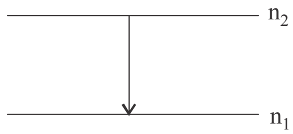
14. **(d)** Magnetic field at the centre of nucleus of H-atom,

$$B = \frac{\mu_0 I}{2r} \quad ..... \text{(i)}$$

According to Bohr's model, radius of orbit r $\propto n^2$

from eq. (i) we can also write as B $\propto n^{-2}$

15. **(b)** A hydrogen atom makes a transition from n = 2 to n = 1



Then wavelength $= Rcz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = Rc(1)^2\left[1 - \frac{1}{4}\right]$

$$\lambda = Rc\left[\frac{3}{4}\right] \quad ...(1)$$

For ionized lithium

$$\lambda = Rc(3)^2\left[\frac{1}{n^2}\right] = Rc9\left[\frac{1}{n^2}\right] \quad ...(2)$$

$$Rc\left[\frac{3}{4}\right] = Rc9\left[\frac{1}{n^2}\right] \Rightarrow \frac{3}{4} = \frac{9}{n^2} \Rightarrow n = \sqrt{12} = 2\sqrt{3}$$

$\therefore$ The least quantum number must be 4.

16. **(c)** Kinetic energy of electron is

$$K.E. \propto \left(\frac{Z}{N}\right)^2$$

When the electron makes transition from excited state to ground state, then n increases and kinetic energy increases.

Total energy = – KE

$\therefore$ Total energy also decreases.

Potential energy is lowest for ground state.

17. **(c)** $qVB = \frac{mv^2}{r}$ ....(i)

$\frac{nh}{2\pi} = mvr$ ....(ii)

Multiplying equation (i) and (ii),

$$\frac{qBnh}{2\pi} = m^2v^2$$

Now multiplying both sides by $\frac{1}{2m}$,

$$n\frac{qBh}{4\pi m} = \frac{1}{2}mv^2$$

i.e. KE $= n\left[\frac{qBh}{4\pi m}\right]$

18. **(b)** Radius of circular path followed by electron is given by,

$$r = \frac{m\upsilon}{qB} = \frac{\sqrt{2meV}}{eB} = \frac{1}{B}\sqrt{\frac{2m}{e}V}$$

$$\Rightarrow \quad V = \frac{B^2r^2e}{2m} = 0.8V$$

For transition between 3 to 2.

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{13.6 \times 5}{36} = 1.88eV$$

Work function = 1.88 eV – 0.8 eV = 1.08 eV $\approx$ 1.1eV

19. **(c)** Wave number $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right] \Rightarrow \lambda \propto \frac{1}{Z^2}$

$\therefore \lambda Z^2$ = constant

By question n = 1 and n_1 = 2

Then, $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

20. **(b)** For first excited state, n = 2 and for Li^{++} Z = 3

$$E_n = \frac{13.6}{n^2}\times Z^2 = \frac{13.6}{4}\times 9 = 30.6 \text{ eV}$$

21. **(d)** $\Delta E = h\nu$ $\quad \nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k(2n-1)}{n^2(n-1)^2}$

$$\approx \frac{2k}{n^3} \quad \text{or} \quad \nu \propto \frac{1}{n^3}$$

22. **(c)** As $F = \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r^2}+\frac{\beta}{r^3}\right)$

and $mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$

$$\therefore m\left(\frac{nh}{2\pi mr}\right)^2\times\frac{1}{r} = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r^2}+\frac{\beta}{r^3}\right)$$

or, $\frac{1}{r^2}+\frac{\beta}{r^3} = \frac{mn^2h^2 4\pi\epsilon_0}{4\pi^2 m^2 e^2 r^3}$

or, $\frac{a_0 n^2}{r^3} = \frac{1}{r^2}+\frac{\beta}{r^3}$ $\quad \left(\because a_0 = \frac{\epsilon_0 h^2}{m\pi e^2} \text{ Given}\right)$

For n^{th} atom

$\therefore r_n = a_0 n^2 - \beta$

23. **(a)** We know for hydrogen or hydrogen like atom,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

For first spectral line in the Balmer series of hyrogen atom n_1 = 2 and n_2 = 3. Here $z = 1$

$$\therefore \frac{1}{6561} = R(1)^2\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5R}{36} \quad \text{...(i)}$$

For the second spectral line in the Balmer series of singly ionised helium ion $n_2 = 4$ and $n_1 = 2$; Z = 2

$$\therefore \frac{1}{\lambda} = R(2)^2\left[\frac{1}{4}-\frac{1}{16}\right] = \frac{3R}{4} \quad \text{...(ii)}$$

Dividing eq. (i) by (ii)

$$\frac{\lambda}{6561} = \frac{5R}{36}\times\frac{4}{3R} = \frac{5}{27} \qquad \therefore \lambda = 1215 \text{ Å}$$

24. **(b)** The smallest frequency and longest wavelength in ultraviolet region will be for transition of electron from $n = 2$ to $n = 1$ i.e., Lyman series.

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{122\times10^{-9}m} = R\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = R\left[1-\frac{1}{4}\right] = \frac{3R}{4}$$

$$\therefore R = \frac{4}{3\times122\times10^{-9}}m^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron $n = \infty$, $n = 3$ corresponds to paschem series.

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{4}{3\times122\times10^{-9}}\left(\frac{1}{3^2}-\frac{1}{\infty}\right)$$

$$\therefore \lambda = \frac{3\times122\times9\times10^{-9}}{4} = 823.5\text{nm}$$

25. **(a)** Initially a photon of energy 10.2eV collides inelastically with a hydrogen atom in ground state. For hydrogen atom,

$E_1 = -13.6$ eV; $E_2 = -\frac{13.6}{4}$ eV $= -3.4$eV

$\therefore E_2 - E_1 = 10.2$ eV

The electron of hydrogen atom will jump to second orbit after absorbing the photon of energy 10.2 eV. Another photon of energy 15 eV strikes the hydrogen atom inelastically. This energy is sufficient to knock out the electron from the atom as ionisation energy is 13.6 eV. The remaining energy 15 – 13.6 = 1.4 eV is left which is released by the second photon.

26. **(d)** For an atom following Bohr's model, the radius of the fifth orbit.

$r_m = \frac{r_0 m^2}{Z}$ where r_0 = Bohr's radius

For ${}_{100}Fm^{257}$, $m = 5$ (Fifth orbit in which the outermost electron is present) and $z = 100$

$$\therefore r_m = \frac{r_0\times5^2}{100} = nr_0 \text{ (given)} \quad \therefore n = \frac{1}{4}$$

27. **(a)** Given potential energy (U) between electron and proton

$= eV_0 \ln\frac{r}{r_0}$ $\qquad [\because |U| = eV]$

$$\therefore |F| = \left|\frac{-du}{dr}\right| = \frac{d}{dr}\left[eV_0\log_e\frac{r}{r_0}\right] = \frac{eV_0}{r_0}\times\frac{1}{r}$$

This force will provide the necessary centripetal force

$$\therefore \frac{mv^2}{r} = \frac{eV_0}{rr_0} \Rightarrow mv^2 = \frac{eV_0}{r_0} \quad \text{...(i)}$$

As per Bohr's postulate, $mvr = \frac{nh}{2\pi}$ ....(ii)

From eq. (i) and (ii),

$$\frac{m^2v^2r^2}{mv^2} = \frac{n^2h^2r_0}{4\pi^2\times V_0 e} \Rightarrow r^2 = \frac{n^2h^2r_0}{4\pi V_0 me} \Rightarrow r\propto n$$

28. **(b)** $l = \frac{nh}{2\pi}, |E| \propto Z^2/n^2; n = 3$

$\Rightarrow l_H = l_{Li}$ and $|E_H| < |E_{Li}|$

29. **(d)** For state transition, 2 to 1, 3 to 2 and 4 to 2 we get energy that $n = 4$ to $n = 3$,
Infrared radiation has less energy and greater than ultraviolet radiation.
Infrared radiation will be obtained in the transition 5 to 4.

30. **(a)** According to question, in a hydrogen atom makes atransition from an excited state to the ground state i.e., electron comes nearer to the nucleus so r decreases.

$\therefore$ Potential energy P.E. $= \frac{-Kze^2}{r}$ decreases

Kinetic energy K.E. will increase $\because$ K.E $= \frac{1}{2}\frac{kZe^2}{r}$

$\therefore$ Total energy decreases

$$\because \text{T.E.} = \text{P.E.} + \text{K.E.} = -\frac{1}{2}\frac{kZe^2}{r}$$

31. **(c)** For ordinary hydrogen atom, longest wavelength

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5R}{36} \text{ or } \lambda = \frac{36}{5R}$$

With hypothetical particle, required wavelength

$$\lambda' = \frac{1}{2}\times\frac{36}{5R} = \frac{18}{5R} \quad \because \lambda \propto \frac{1}{m}$$

32. **(b)** According to Doppler's effect of light, the wavelength shift

$$\Delta\lambda = \frac{v}{c} \times \lambda \Rightarrow v = \frac{\Delta\lambda \times c}{\lambda}$$

$$\therefore V = \frac{(706-656)}{656} \times 3 \times 10^8 \approx 2 \times 10^7 \text{ m/s}$$

33. **(d)** Ground state energy of doubly ionized lithium atom ($Z = 3$, $n = 1$)

$$E_1 = -13.6\frac{(Z^2)}{(n^2)}\text{eV} = (-13.6)\frac{(3)^2}{(1)^2} = -122.4 \text{ eV}$$

$\therefore$ Ionization energy or the minimum energy of an electron in ground state of doubly ionized lithium atom will be 122.4eV.

34. **(d)** As we know, $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \Rightarrow \frac{1}{\lambda} \propto Z^2$

λ is shortest when Z is highest. Z is highest for doubly ionised lithium.

35. **(3)** According to question, the photon emitted $n = 2 \to n = 1$ transition has energy 74.8 eV higher than the photon emitted $n = 3 \to n = 2$

$\therefore \Delta E_{2-1} = 74.8 + \Delta E_{3-2}$

$$13.6z^2\left[1 - \frac{1}{4}\right] = 74.8 + 13.6z^2\left[\frac{1}{4} - \frac{1}{9}\right] \qquad \therefore z = 3$$

36. **(5)** Here $\frac{V_i}{V_f} = \frac{\frac{-27.2}{n_f^2}}{\frac{-27.2}{n_i^2}} = \frac{n_f^2}{n_i^2} = 6.25$

$$\therefore \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

$\therefore$ Smallest possible $n_f = 5$

37. **(6)** Energy of incident light,

$$E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}} eV = 12.75 eV$$

$\therefore$ The energy of electron after absorbing this photon $= -13.6 + 12.75 = -0.85$eV

Let the electron tumps to nth state after excitation

$$\therefore \frac{-13.6}{n^2} = -0.85 \quad \therefore n = 4$$

$$\therefore \text{Total number of spectral line} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

38. **(2)** Angular momentum, $L = mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi} \Rightarrow n = 3$

$$\because \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{hr}{mvr} \Rightarrow mvr = \frac{hr}{\lambda}$$

$$\therefore \frac{hr}{\lambda} = \frac{3h}{2\pi}$$

$$\therefore \lambda = \frac{2\pi r}{3} = \frac{2}{3}\pi\left[a_0\frac{n^2}{z}\right] \qquad \left[\because r = a_0\frac{n^2}{z}\right]$$

$$\therefore \lambda = \frac{2}{3}\pi a_0\left[\frac{3 \times 3}{3}\right] = 2\pi a_0 \; [\because n = 3; z = 3] \quad \therefore p = 2$$

39. **(2)** Using energy conservation principle,

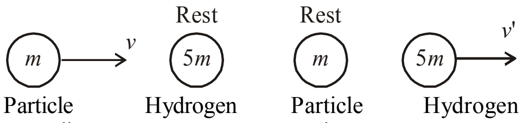
$E_{\text{photon}} = E_{\text{ionize}} + E_k$

$$\Rightarrow \frac{hc}{\lambda} = \frac{13.6}{n^2} + E_k$$

$$\therefore \frac{1242}{90} = \frac{13.6}{n^2} + 10.2 \Rightarrow n^2 = 4 \qquad \therefore n = 2$$

40. **(51)**

Before collision **After collision**



From linear momentum conservation, $L_i = L_f$

$$mV + 0 = 0 + 5mV' \Rightarrow V' = \frac{v}{5}$$

$$\text{Loss of KE} = KE_i - KE_f = \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2$$

$$= \frac{1}{2}mv^2\left(1 - \frac{1}{5}\right) = \frac{4}{5}\left(\frac{mv^2}{2}\right)$$

$$= \frac{4}{5}KE_i = 10.2 \text{ eV}$$

[$\because$ Energy in first excited state of atom = 10.2 eV]

$$KE_i = 12.75\text{eV} = \frac{N}{4} \Rightarrow N = 51$$

The value of $N = 51$.

41. **(10553.14)**

From Bohr's formula for hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

For Lyman series :

$$\frac{1}{\lambda_{\text{min.}}} = R(1) = R \qquad \because n_2 = \infty \text{ and } n_1 = 1$$

$$\frac{1}{\lambda_{\text{max.}}} = R\left\{1 - \frac{1}{4}\right\} = \frac{3R}{4} \qquad \because n_1 = 2,\ n_1 = 1$$

$$\therefore \lambda_{\text{max.}} - \lambda_{\text{min.}} = \frac{4}{3R} - \frac{1}{R} = \frac{1}{3R} = 304 \text{ (Given)}$$

For Paschen series :

$$\lambda'_{\text{min.}} = R\left(\frac{1}{9}\right) \text{ and } \lambda'_{\text{max.}} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{16 \times 9}$$

$$\lambda'_{\text{max.}} - \lambda'_{\text{min.}} = \frac{16 \times 9}{7R} - \frac{9}{R} = \frac{81}{7R}$$

$$\text{or, } \lambda'_{\text{max.}} - \lambda'_{\text{min.}} = \frac{81}{7R} = \frac{81 \times 3}{7 \times 3R} = \frac{81 \times 3}{7} \times 304$$

$$\left(\because \frac{1}{3R} = 304\text{Å}\right)$$

$\therefore$ For Pachen series, $\lambda'_{\text{max.}} - \lambda'_{\text{min.}} = 10553.14$

42. **(486.00)**

The wavelength of the spectral line of hydrogen spectrum is given by formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Where, R = Rydberg constant

For the first member of Balmer series nF = 2, ni = 3

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad \text{...(i)}$$

For last member of Balmer series, nf = 2, ni = 4

$$\text{So, } \frac{1}{\lambda'} = R\left[\frac{1}{4} - \frac{1}{16}\right] \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{5\times16}{9\times4\times3} \qquad \Rightarrow \lambda' = \frac{5\times4\times656.1}{9\times3}(nm) = 486\,nm$$

43. In the bohr model of hydrogen atom, kinetic energy, K.E. $= \frac{kZe^2}{2r}$ and

Total energy, $\text{T.E.} = \frac{-kZe^2}{2r}$ $\quad \therefore \frac{\text{K.E.}}{\text{T.E.}} = -1$

44. The fifth valence electron of phosphorous is in its third shell, i.e., $n = 3$. For phosphorous, $Z = 15$.
$\therefore$ Bohr's radius for nth orbit

$$r_n = \left(\frac{n^2}{Z}\varepsilon_r\right) r_0 = \frac{3^2}{15}\times 12\times 0.529\text{ Å} = 3.81\text{Å}$$

45. **(b, c)** Given: Potential energy of the particle of mass 'm' moving in circular orbit, V(r) = Fr

$$\Rightarrow F = \frac{V(r)}{r}$$

Also centripetal force

$$F = \frac{mv^2}{R} \qquad \text{...(1)}$$

According to Bohr's second postulate,

$$mvR = \frac{nh}{2\pi} \Rightarrow V = \frac{nh}{2\pi mr}$$

Putting this value of v in eqn (1)

$$F = \frac{m}{R}\times\frac{n^2h^2}{2\pi^2}\times\frac{1}{m^2R^2} \quad \Rightarrow R = \left(\frac{n^2h^2}{4\pi^2 mR}\right)^{1/3} \quad \therefore R \propto n^{2/3}$$

Now putting this value of R in $v = \frac{nh}{2\pi mR}$

$$v = \frac{nh}{2\pi m}\left(\frac{4\pi^2 mF}{n^2h^2}\right)^{1/3} \quad \therefore v \propto n^{1/3}$$

Hence, option (b) is correct.
Total energy

$$E = \frac{1}{2}mv^2 + v(r) = \frac{1}{2}mv^2 + FR$$

$$\Rightarrow E = \frac{1}{2}m\left(\frac{n^{2/3}h^{2/3}F^{2/3}}{2^{2/3}\pi^{2/3}m^{4/3}}\right) + F\times\left(\frac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$$

$$\Rightarrow E = \left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}\left[\frac{1}{2}+1\right] \text{ or, } E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$$

Hence, option (c) is correct.

46. **(b, c)**
(a) Change in linear momentum due to absorption

$$\Delta P_a = \frac{h}{\lambda a} \quad \left(\because \lambda = \frac{h}{P}\right)$$

Change in linear momentum due to emission

$$\Delta P_e = \frac{h}{\lambda e} \quad \because \frac{\Delta P_a}{\Delta P_e} = \frac{\lambda_e}{\lambda_a} = 5\left(\because \frac{\lambda_a}{\lambda_e} = \frac{1}{5}\right) \text{ given}$$

(b) Kinetic energy $K_n \propto \frac{z^2}{n^2} \therefore \frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$

(c) For absorption of energy, $n = 1$ to $n = 4$

$$E_4 - E_1 = \frac{hc}{\lambda_a} = 13.6\left(\frac{1}{1} - \frac{1}{4^2}\right) \qquad \text{...(i)}$$

Per emission of energy, $n = m$ to $n = 4$

$$E_4 - E_m = \frac{h_c}{\lambda e} = 13.6\left(\frac{1}{m^2} - \frac{1}{4^2}\right) \qquad \text{...(ii)}$$

Dividing eq (ii) by (i)

$$\frac{\lambda a}{\lambda e} = \frac{\frac{1}{m^2} - \frac{1}{16}}{1 - \frac{1}{16}} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{1}{5}\times\frac{15}{16} \Rightarrow \frac{1}{m^2} = \frac{3}{16} + \frac{1}{16} = \frac{4}{16}$$

$\therefore m = 2$

(d) Now from eq. (ii)

$$\frac{hc}{\lambda e} = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \lambda e = \frac{1242\times16}{13.6\times3} \approx 486.2nm$$

47. **(a, b, d)**
We know radius, $rn = r_0\frac{n^2}{z}$, Energy $E_n = -\frac{13.6Z^2}{n^2}$,

angular momentum, $L_n = \frac{nh}{2\pi}$

Relative change in the radii of two consecutive orbitals

$$\frac{\Delta r}{r} = \frac{r_n - r_{n-1}}{r_n} = 1 - \frac{r_{n-1}}{r_n} = 1 - \frac{(n-1)^2}{n^2}$$

$$= \frac{2n-1}{n^2} \approx \frac{2}{n} \quad (\because n >> 1)$$

Relative change in the energy of two consecutive orbitals

$$\frac{\Delta E}{E} = \frac{E_n - E_{n-1}}{E_n} = 1 - \frac{E_{n-1}}{E_n} = 1 - \frac{n^2}{(n-1)^2} = \frac{-2n+1}{(n-1)^2} \approx \frac{-2}{n}$$

$$\frac{\Delta L}{L} = \frac{L_n - L_{n-1}}{L_n} = 1 - \frac{L_{n-1}}{L_n} = 1 - \frac{(n-1)}{n} = \frac{1}{n}$$

48. **(a, c)** According to Bohr's quantisation, angular momentum

$$L = \frac{nh}{2\pi} = \frac{3h}{2\pi}. \therefore n = 3.$$

Also $r_n = \frac{a_o n^2}{z} = 4.5a_o$

$$\therefore \quad \frac{n^2}{z} = 4.5 \qquad \Rightarrow \frac{9}{z} = 4.5 \Rightarrow z = 2$$

According to Rydberg formula

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{1}{\lambda} = 4R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For $n_2 = 3, \rightarrow n_1 = 1$ $\lambda = \frac{9}{8\times4R} = \frac{9}{32R}$

For $n_2 = 3, \rightarrow n_1 = 2$ $\lambda = \frac{36}{5\times4R} = \frac{9}{5R}$

For $n_2 = 2, \rightarrow n_1 = 1$ $\lambda = \frac{4}{3\times4R} = \frac{1}{3R}$

49. **(a, d)** The time period of the electron in a Bohr orbit in nth state

$$T_n = \frac{2\pi rn}{V_n} \text{ or, } T_n \propto \frac{r_n}{V_n} \qquad \text{...(i)}$$

Bohr radius of a hydrogen atom

$r_n = n^2\left(\dfrac{h^2\varepsilon_0}{\pi m e^2}\right)$ or , $r_n \propto n_2$ ...(ii)

And, $V_n \propto \dfrac{1}{n}$ ...(iii)

From eq. (i), (ii) and (iii) $T_n \propto n^3$.
And according to question, $Tn_1 = 8T_2 \therefore n_1 = 2n_2$

50. **(a, c, d)** Radius of nth orbit, $r_n \propto n^2$

$$E_n = \frac{-13.6Z^2}{n^2}\text{eV}$$

Angular momentum, $L_n = mvr = \dfrac{nh}{2\pi}$

| P.E. | = 2 × | K.E. |

51. **A → p, r**
The lines in the hydrogen spectrum is obtained due to transition of electrons from one energy level to another.
Characteristic X-ray are produced due to transition of electrons from one energy level to another.
B → q, s
In photoelectric effect electrons from the metal surface are emitted when light of appropriate frequency incident on it.
In β-decay, electrons are emitted from the nucleus of an atom.
C → p
According to Moseley law frequency of emitted X-ray is related with the atomic number (z) of the target material as $\sqrt{\nu} = a(Z-b)$
D → q
According to Einstein's photoelectric effect, energy of photons of incident ray gets converted into kinetic energy of emitted electrons K. $E_{\max} = h\nu - \phi$

52. **(d)** Rotational kinetic energy $K{\cdot}E_{\text{rot}} = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}I\left[\dfrac{L}{I}\right]^2$

$[\because L = I\omega]$

And according to Bohr's quantisation principle $L = \dfrac{nh}{2\pi}$

$$\therefore\ K{\cdot}E_{\text{rot}} = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2I}\times\frac{n^2h^2}{4\pi^2} = n^2\left[\frac{h^2}{8\pi^2 I}\right] \quad ...(i)$$

53. **(b)** From ground (n = 1) to first excited state (n = 2)
Energy given = change in kinetic energy

$$h\nu = K_f - K_i = \frac{h^2}{8\pi^2 I}[2^2 - 1^2] \quad \text{[From eq. (i)]}$$

$$h\nu = \frac{3h^2}{8\pi^2 I} \Rightarrow I = \frac{3h}{8\pi^2\nu} = \frac{3\times 2\pi\times 10^{-34}}{8\pi^2\times\frac{4}{\pi}\times 10^{11}} = \frac{3}{16}\times 10^{-45}$$

$= 1.87 \times 10^{-46}$ kg m^2

54. **(c)** Moment of inertia of CO molecule,

$I = \mu r^2 \Rightarrow r^2 = \dfrac{I}{\mu}$

where, μ = reduced mass of the CO molecule, r = distance between C and O
Reduced mass of the CO molecule

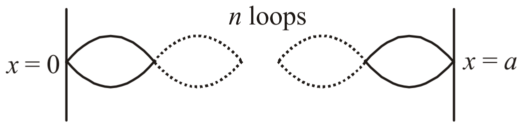
$$\mu = \frac{m_1m_2}{m_1+m_2} = \left[\frac{(12)(16)}{12+16}\right]\times\frac{5}{3}\times 10^{-27}\text{kg}$$

But $I = 1.87 \times 10^{-46}$ kg m^2 (from the above question)

$$\therefore\ r^2 = \left[\frac{1.87\times 10^{-46}}{\frac{12\times 16}{28}\times\frac{5}{3}\times 10^{-27}}\right] = \frac{1.87\times 10^{-46}\times 28\times 3}{12\times 16\times 5\times 10^{-27}}$$

$\therefore r = 1.3 \times 10^{-10}$ m

55. **(a)**



Energy, $E = \dfrac{h^2}{2m\lambda^2}$ $\left(\because E = \dfrac{P^2}{2m} \text{ and } P = \dfrac{h}{\lambda}\right)$

The length in which the particle is restricted to move is

$a = n\dfrac{\lambda}{2} \Rightarrow \lambda = \dfrac{2a}{n}$

Putting this value of λ we get

$$E = \frac{h^2n^2}{2m\times 4a^2} = \frac{n^2h^2}{8m\,a^2} \qquad \therefore\ E \propto a^{-2}$$

56. **(b)** For ground state n = 1,
Given $m = 1.0 \times 10^{-30}$ kg, $a = 6.6 \times 10^{-9}$ m

$$\therefore\ E = \frac{n^2h^2}{8ma^2} = \frac{1^2\times(6.6\times 10^{-34})^2}{8\times 1\times 10^{-30}\times(6.6\times 10^{-9})^2}\text{J} = 8\text{ meV}$$

57. **(d)** $\lambda = \dfrac{h}{p} \Rightarrow \lambda = \dfrac{h}{mv}$ But $\dfrac{n\lambda}{2} = a \Rightarrow \lambda = \dfrac{2a}{n}$

$$\therefore \frac{h}{mv} = \frac{2a}{n} \therefore\ mv = \frac{nh}{2a} \Rightarrow v = \frac{nh}{2am} \therefore\ v \propto n$$

58. **(c)** For hydrogen or hydrogen like atoms

$$E_n = \frac{-13.6\,Z^2}{n^2}\text{eV/atom}$$

For hydrogen atom (Z = 1) $E_1 = -13.6$eV (for n = 1)
$E_2 = -3.4$eV (for n = 2)

$\therefore \Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

i.e., When hydrogen comes to ground state from its first excited state it will release 10.2 eV of energy.

For He$^+$ ion (Z = 2) $E_1 = -13.6 \times 4$ eV $= -54.4$ eV (for n = 1)
$E_2 = -13.6$ eV (for n = 2)
$E_3 = -6.04$ eV (for n = 3)
$E_4 = -3.4$eV (for n = 4)

Here He$^+$ ion is in the first excited state i.e., possessing energy –13.6 eV. After receiving energy of + 10.2 eV from excited hydrogen atom on collision, the energy of electron will be (–13.6 + 10.2) eV = –3.4 eV. Hence the quantum number of the state finally populated in He$^+$ ions, n = 4.

59. **(c)** Wavelength of visible light lies in the range,
$\lambda_1 = 4000$Å to $\lambda_2 = 7000$Å.
Therefore

$$E_1 = \frac{12375}{\lambda_1} = \frac{12375}{4000} = 3.09\text{ eV}$$

$$E_2 = \frac{12375}{\lambda_2} = \frac{12375}{7000} = 1.77\text{ eV}$$

For He$^+$ atom in transition from n = 4 to n = 3, energy of photon released will lie between E_1 and E_2.

$$E_4 - E_3 = -3.4 - (-6.04) = 2.64\text{ eV}$$

∴ Wavelength of photon corresponding to this energy, (2.64 eV)

$$\lambda = \frac{12375}{2.64}\text{Å} = 4687.5\text{ Å} = 4.68\times 10^{-7}\text{ m}$$

60. **(a)** Kinetic energy for hydrogen or hydrogen like atom K

$$K = \frac{-13.6Z^2}{n^2} \Rightarrow K \propto Z^2$$

$$\frac{K_H}{K_{He^+}} = \left(\frac{Z_H}{Z_{He^+}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

61. For an electron to revolve in $(n + 1)$th orbit.
$2\pi r = (n + 1)\lambda$

$$\Rightarrow \lambda = \frac{2\pi}{(n+1)} \times r = \frac{2\pi}{(n+1)}\left[0.529 \times 10^{-10}\right]\frac{(n+1)^2}{Z}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{Z}{2\pi\left[0.529 \times 10^{-10}\right](n+1)} \quad ..(i)$$

And when electron jumps from $(n + 1)$th to 1st orbit.

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{1^2} - \frac{1}{(n+1)^2}\right] = 1.09 \times 10^7 Z^2\left[1 - \frac{1}{(n+1)^2}\right] \quad ...(ii)$$

From eq. (i) and (ii)

$$\frac{Z}{2\pi(0.529 \times 10^{-10})(n+1)} = 1.09 \times 10^7 Z^2\left[1 - \frac{1}{(n+1)^2}\right]$$

Putting the value of $z = 11$ and solving, we get $n = 24$

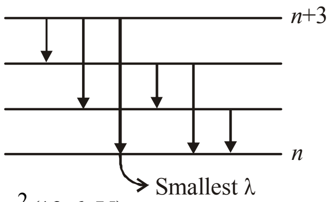
62. For different lines of Balmer series

$$\frac{hc}{\lambda} = 13.6\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ eV, where } n = 3, 4, 5$$

Solving, we get $\lambda = 657$nm, 487 nm between 450 nm and 700 nm. Wavelength 487 nm < 657 nm, electron of max. K.E. will be emitted for photon corresponding to wavelength 487 nm with

$$(\text{K.E.}) = \frac{hc}{\lambda} - W = \left(\frac{1242}{487} - 2\right) = 0.55 \text{ eV}$$

63. **(a)** Let



We have $\dfrac{-z^2(13.6\text{eV})}{n^2} = -0.85\text{eV}$ ...(i)

and $\dfrac{-z^2(13.6\text{eV})}{(n+3)^2} = -0.544$ eV ...(ii)

From eq. (i) and (ii) n = 12 and z = 3

(b) Smallest wavelength λ

$$\frac{hc}{\lambda} = (0.85 - 0.544) \text{ eV}$$

Putting the value of h, and c and solving, we get
$\lambda = 4052$ nm.

64. Energy for an orbit of hydrogen or hydrogen like atom

$$E_n = -\frac{13.6Z^2}{n^2}$$

For transition from $n = 2n$ orbit to $n = 1$ orbit

Maximum energy, $E_{\max} = 13.6Z^2\left(\dfrac{1}{1} - \dfrac{1}{(2n)^2}\right) = 204$

Also for transition $n = 2n$ to $n = n$.

$$40.8 = 13.6Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) \Rightarrow 40.8 = 13.6Z^2\left(\frac{3}{4n^2}\right)$$

$\Rightarrow 40.8 = 40.8\dfrac{Z^2}{4n^2} \Rightarrow 4n^2 = Z^2$ or $2n = Z$ ....(ii)

From eq. (i) and (ii)

$$204 = 13.6Z^2\left(1 - \frac{1}{Z^2}\right) = 13.6Z^2 - 13.6$$

$13.6Z^2 = 204 + 13.6 = 217.6 \Rightarrow Z^2 = \dfrac{217.6}{13.6} = 16, \therefore Z = 4$

$$n = \frac{Z}{2} = \frac{4}{2} = 2$$

For minimum energy = transition from 4 to 3.

$$E_{\min} = 13.6 \times 4^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 13.6 \times 4^2\left(\frac{7}{9 \times 16}\right) = 10.5 \text{ eV.}$$

65. (i) Kinetic energy = total energy
$\therefore$ Kinetic energy, K.E. = 3.4 eV
(ii) The de Broglie wavelength of electron

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.64 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} \text{ eV}$$

$= 0.66 \times 10^{-9}$ m = 6.6Å

66. For hydrogen or hydrogen like atoms

$$E_n = -\frac{13.6}{n^2}Z^2 \text{ eV/atom}$$

From the condition given,
$E_n - E_2 = 10.2 + 17 = 27.2$ eV ..(i)
$E_n - E_3 = 4.24 + 5.95 = 10.2$ eV
$\therefore E_3 - E_2 = 17$

But $E_3 - E_2 = -\dfrac{13.6}{9}Z^2 - \left(-\dfrac{13.6}{4}Z^2\right)$

$$= -13.6Z^2\left[\frac{1}{9} - \frac{1}{4}\right] = -13.6Z^2\left[\frac{4-9}{36}\right] = \frac{13.6 \times 5}{36}Z^2$$

$$\therefore \frac{13.6 \times 5}{36}Z^2 = 17 \Rightarrow Z = 3$$

$$E_n - E_2 = -\frac{13.6}{n_2} \times 3^2 - \left[-\frac{13.6}{2^2} \times 3^2\right]$$

$$= -13.6\left[\frac{9}{n^2} - \frac{9}{4}\right] = -13.6 \times 9\left[\frac{4-n^2}{4n^2}\right] \quad ...(ii)$$

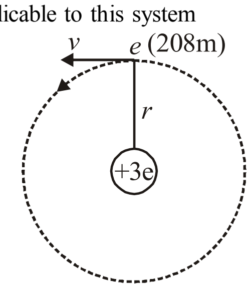
From eq. (i) and (ii),

$$-13.6 \times 9\left[\frac{4-n^2}{4n^2}\right] = 27.2 \Rightarrow -122.4\,(4 - n^2) = 108.8n^2$$

$$\Rightarrow n^2 = \frac{489.6}{13.6} = 36 \;\therefore n = 6$$

67. **(i)** Let mass of electron = m
$\therefore$ Mass of mu-meson = $(208)m$
Here Bohr model of the atom is applicable to this system

$\therefore$ Angular momentum, mur $= \dfrac{nh}{2\pi}$



$$\therefore \quad (208\text{m})\, vr = \frac{nh}{2\pi}$$

$\therefore \quad v = \dfrac{nh}{2\pi \times 208mr} = \dfrac{nh}{416\pi mr}$ ...(i)

Since mu-meson is revolving in a circular path, around the nucleus which is of infinite mass and at rest therefore, it needs centripetal force which is provided by the electrostatic force between the nucleus and mu-meson.

$\therefore \quad \dfrac{(208m)v^2}{r} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{3e \times e}{r^2}$

$\therefore \quad r = \dfrac{3e^2}{4\pi\varepsilon_0 \times 208mv^2}$

Putting the value of v from eq. (i), we get

$r = \dfrac{3e^2 \times 416\pi mr \times 416\pi mr}{4\pi\varepsilon_0 \times 208mn^2h^2}$

Therefore, the radius of the nth Bohr orbit.

$r = \dfrac{n^2h^2\varepsilon_0}{624\pi me^2}$ ....(ii)

(ii) To find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for hydrogen atom,

The radius of the first Bohr orbit of the hydrogen atom

$= \dfrac{\varepsilon_0 h^2}{\pi me^2} \quad \therefore \quad \dfrac{n^2h^2\varepsilon_0}{624\pi me^2} = \dfrac{\varepsilon_0 h^2}{\pi me^2} \quad \therefore \; n = \sqrt{624} \approx 25$

(iii) The wavelength of the radiation emitted when mu-meson jumps from the third or bit ($n = 3$) to first orbit ($n = 1$).

$\dfrac{1}{\lambda} = 208R \times Z^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$\Rightarrow \dfrac{1}{\lambda} = 208 \times 1.097 \times 10^7 \times 3^2\left[\dfrac{1}{1^2} - \dfrac{1}{3^2}\right]$

$\Rightarrow \lambda = 5.478 \times 10^{-11}$m

68. (i) Wavelength of the radiation required to excite the electron in $Li++$ from $n_1 = 1$ to $n_2 = 3$ $\quad E_n = -\dfrac{13.6}{n^2}Z^2$ eV/atom

For Li^{2+}, $Z = 3 \quad \therefore \; E_n = \dfrac{-13.6 \times 9}{n^2}$ eV/atom

$\therefore \; E_1 = -\dfrac{13.6 \times 9}{1}$ and $E_3 = -\dfrac{13.6 \times 9}{9} = -13.6$

$\Delta E = E_3 - E_1 = -13.6 - (-13.6 \times 9)$
$= 13.6 \times 8 = 108.8$ eV/atom

$\lambda = \dfrac{12400}{E \text{ (in eV)}}$ Å $= \dfrac{12400}{108.8} = 114$Å

(ii) Number of spectral lines observed in the emission spectrum

$= \dfrac{n(n-1)}{2} = \dfrac{3(3-1)}{2} = 3$

69. (i) $E_n = -\dfrac{\text{I.E.}}{n^2}$ for Bohr's hydrogen atom.

I.E. = Ionisation energy

Here, I.E. = $4R$ (R = 1 Rydberg = $Rhc = 2.2 \times 10^{-18}$ J)

$\therefore \; E_n = \dfrac{-4R}{n^2}$

$\therefore E_2 - E_1 = \dfrac{-4R}{2^2} - \left(-\dfrac{4R}{1^2}\right) = 3R$ ..(i)

$E_2 - E_1 = h\nu = \dfrac{hc}{\lambda}$ ...(ii)

From eq. (i) and (ii)

$\dfrac{hc}{\lambda} = 3R \; \therefore \; \lambda = \dfrac{hc}{3R}$

or, $\lambda = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.2 \times 10^{-18} \times 3} = 300$Å

(ii) **The radius of the first orbit**

$|E_n| = +0.22 \times 10^{-17}Z^2 = 4R = 4 \times 2.2 \times 10^{-18} \; \therefore \; Z = 2$

$\therefore \; r_1 = \dfrac{r_0}{Z} = \dfrac{5 \times 10^{-11}}{Z} = \dfrac{5 \times 10^{-11}}{2} = 2.5 \times 10^{-11}$m

70. Energy $E = \dfrac{h_c}{\lambda} \Rightarrow E = \dfrac{12400}{\lambda(\text{inÅ})}$ eV $= \dfrac{12400}{975} = 12.75$ eV ...(i)

Let the electron excites from $n_1 = 1$ to n_2 state

$13.6\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right] = 12.75 \Rightarrow \left[\dfrac{1}{1} - \dfrac{1}{n_2^2}\right] = \dfrac{12.75}{13.6} \Rightarrow n_2 \approx 4$

Total number of lines in emission spectrum $= \dfrac{n(n-1)}{2}$

$= \dfrac{4(4-1)}{2} = 6$

For longest wavelength, the frequency should be smallest.

This corresponds to the transition from $n = 4$ to $n = 3$,

$E_4 = -\dfrac{13.6}{4^2}; E_3 = -\dfrac{13.6}{3^2}$

$\therefore E_4 - E_3 = \dfrac{-13.6}{4^2} - \left(\dfrac{-13.6}{3^2}\right) = 13.6\left[\dfrac{1}{9} - \dfrac{1}{16}\right] = 0.66$ eV

Now, $\because \; E = \dfrac{12400}{\lambda(\text{inÅ})}$ eV $\therefore \; \lambda = \dfrac{12400}{E} = \dfrac{12400}{0.66}$ Å

$= 1.875 \times 10^{-6}$ m

Topic-3 : Miscellaneous (Mixed Concepts) Problems

1. **(a)** From Einstein's photoelectric equation, energy of photon causing photoelectric emission (E)

= Work function of sodium metal + KE of the fastest photoelectron K_{max}.

= 1.82 + 0.73 = 2.55 eV

(b) For hydrogen of hydrogen like atom

$E_n = \dfrac{-13.6}{n^2}\dfrac{\text{eV}}{\text{atom}}$

For $n = 1$, $E_1 = -13.6$ ev
For $n = 2$, $E_2 = -3.4$ ev
For $n = 3$, $E_3 = -1.5$ ev
For $n = 4$, $E_4 = -0.85$ ev
Here, $E_4 - E_2 = 2.55$ ev

Hence the quantum numbers of the two levels involved in the emission of these photons are from $n = 4$ to $n = 2$

(c) Change in angular momentum in transition from $n = 4$ to $n = 2$

$\Delta L = \dfrac{n_1 h}{2\pi} - \dfrac{n_2 h}{2\pi} = \dfrac{h}{2\pi}(2-4) = \dfrac{h}{2\pi} \times (-2) = -\dfrac{h}{\pi}$

(d) The momentum of emitted photon can be found by de Broglie relationship

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h\nu}{c} = \frac{E}{c} \quad \therefore\ p = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

According to conservation of linear momentum, momentum of emitted photon = momentum of hydrogen atom

$$\therefore\ m \times v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$$\text{or, } v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8 \times 1.67 \times 10^{-27}} = 0.814 \text{ m/s}$$

2. For hydrogen like atom energy of the nth Bohr orbit

$$E_n = -\frac{13.6}{n^2} Z^2 \text{ eV/atom}$$

For transition from $n = 5$ to $n = 4$,

$$\frac{h_c}{\lambda} = 13.6 \times 9\left[\frac{1}{16} - \frac{1}{25}\right] = \frac{13.6 \times 9 \times 9}{16 \times 25} = 2.754 \text{ eV}$$

For transition from $n = 4$ to $n = 3$,

$$\frac{hc}{\lambda} = 13.6 \times 9\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{13.6 \times 9 \times 7}{9 \times 16} = 5.95\text{eV}$$

Clearly longer wave length will correspond to transition from $n = 5$ to $n = 4$

For photoelectric effect, $h\nu' - \phi = eV_0$, where ϕ = work function

$5.95 \times 1.6 \times 10^{-19} - W = 1.6 \times 10^{-19} \times 3.95$

$\therefore\ \phi = 2 \times 1.6 \times 10^{-19} = 2$ eV

Again applying $h\nu - W = eV'_0$

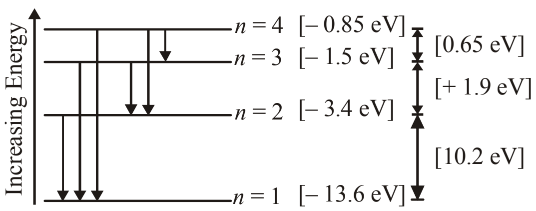
$2.754 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19}\ V'_0$

$\therefore\ V'_0 = 0.754$ V (stopping potential for longer wavelength)

3. **(i)** Since the atoms emitradiation of only six different photon energies

$$\therefore n_f\left(\frac{n_f - 1}{2}\right) = 6 \text{ or, } n_f = 4 \text{ (Final quantum number)}$$

The transition state of six different photon energies as shown in figure.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicates that the excited level B is $n = 2$ if $n = 3$ is the excited level then energy less than 2.7 eV is not possible.

(ii) For hydrogen like atoms

$$E_n = \frac{-13.6}{n^2} Z^2 \text{ eV/atom}$$

$$E_4 - E_2 = \frac{-13.6}{16} Z^2 - \left(\frac{-13.6}{4}\right) Z^2 = 2.7$$

$$\Rightarrow Z^2 \times 13.6\left[\frac{1}{4} - \frac{1}{16}\right] = 2.7 \Rightarrow Z^2 = \frac{2.7}{13.6} \times \frac{4 \times 16}{12}$$

$$\text{Ionisation enrgy, I.E.} = 13.6Z^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} = 14.46 \text{ eV}$$

(iii) Maximum energy of the emitted photons

Corresponds to transition from $n = 4$ to $n = 1$

$$E_4 - E_3 = -13.6Z^2\left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{15}{16} = 13.5 \text{ eV}$$

And minimum energy of the emitted photons-corresponds to transition from n = 4 to $n = 3$

$$E_4 - E_3 = -13.6Z^2\left(\frac{1}{4^2} - \frac{1}{3^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{7}{9 \times 16} = 0.7\text{eV}$$

4. (i) According to Bohr for hydrogen/hydrogen like atom,

$$E_n = \frac{-13.6z^2}{n^2} eV$$

$$E_2 = -\frac{13.6}{n^2} Z^2, \quad E_3 = -\frac{13.6}{9} Z^2$$

$$\therefore E_3 - E_2 = -13.6Z^2\left(\frac{1}{9} - \frac{1}{4}\right) = +\frac{13.6 \times 5}{36} Z^2$$

But $E_3 - E_2 = 47.2$ eV (Given)

$$\therefore \quad \frac{13.6 \times 5}{36} Z^2 = 47.2 \qquad \therefore\ Z = \frac{\sqrt{47.2 \times 36}}{13.6 \times 5} = 5$$

(ii) Energy required to excite the electron from the Bohr orbit

$$n = 3 \text{ to } n = 4\ E_4 = \frac{-13.6}{16} Z^2$$

$$\therefore \quad E_4 - E_3 = -13.6Z^2\left[\frac{1}{16} - \frac{1}{9}\right] = -13.6Z^2\left[\frac{9 - 16}{9 \times 16}\right]$$

$$= \frac{+13.6 \times 25 \times 7}{9 \times 16} = 16.53\text{eV}$$

(iii) The wavelength of radiation required to remove the electron

$$\text{from } n = 1 \text{ to } n = \infty\ E_1 = -\frac{13.6}{1} \times 25 = -340\text{eV}$$

$\therefore\ E = E_\infty - E_1 = 340$ eV $= 340 \times 1.6 \times 10^{-19}$ J $[E_\infty = 0 \text{ eV}]$

$$\text{But } E = \frac{hc}{\lambda}$$

$$\therefore\ \lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{340 \times 10^{-19} \times 1.6} = 3.65 \times 10^{-19} \text{m}$$

(iv) Total Energy of 1st orbit

$$= \frac{-13.6z^2}{n^2} = \frac{13.6 \times 5^2}{1} = -340 \text{ eV}$$

We know that – (T.E). = K.E. [in case of electron revolving around nucleus]

and 2T.E. = P.E.

$\therefore$ K.E. = 340 eV and P.E. = $2 \times -340 = -680$ eV

Angular momentum in 1st orbit : According to Bohr's postulate,

$$\text{Angular moment } L = mvr = \frac{nh}{2\pi}$$

For $n = 1$,

$$L = \frac{nh}{2\pi} = \frac{6.6 \times 10^{-34}}{2\pi} = 1.05 \times 10^{-34}\text{J-s.}$$

(v) Radius of first Bohr orbit

$$r_1 = \frac{5.3 \times 10^{-11}}{Z} = \frac{5.3 \times 10^{-11}}{5} = 1.06 \times 10^{-11} \text{ m}$$

27 Nuclei

Topic-1 : Composition and Size of the Nuclei

1. **(d)** Density of nucleus, $\rho = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{mA}{\frac{4}{3}\pi R^3}$

$\Rightarrow \rho = \dfrac{mA}{\frac{4}{3}\pi(R_0 A^{1/3})^3}$ $(\because R = R_0 A^{1/3})$

Here m = mass of a nucleon

$\therefore \rho = \dfrac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.3 \times 10^{-15})^3}$ (Given, $R_0 = 1.3 \times 10^{-15}$)

$\Rightarrow \rho = 2.38 \times 10^{17}$ kg/m^3

2. **(a)** Nuclear density is independent of atomic number.
3. **(c)** Let heavy nucleus breaks into two nuclei of mass m_1 and m_2 and move away with velocities V_1 and V_2 respectively.

According to question, $\dfrac{V_1}{V_2} = \dfrac{8}{27}$

$m_1 V_1 = m_2 V_2$ (Law of momentum conservation)

$\Rightarrow \dfrac{m_1}{m_2} = \dfrac{V_2}{V_1} = \dfrac{27}{8}$

$\dfrac{\rho \times \frac{4}{3}\pi R_1^3}{\rho \times \frac{4}{3}\pi R_2^3}$ $\left(\because \text{ density } \rho = \dfrac{\text{mass}}{\text{volume}}\right)$

$\Rightarrow \left(\dfrac{R_1}{R_2}\right) = \left(\dfrac{27}{8}\right)^{\frac{1}{3}} = \left(\dfrac{3}{2}\right)^{3 \times \frac{1}{3}}$ $\therefore \dfrac{R_1}{R_2} = \dfrac{3}{2}$

4. **(a)** We know that radius of the nucleus $R = R_0 A^{1/3}$, where A is the mass number.

$\therefore R^3 = R_0^3 A$

Volume, $V = \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi R_0^3 A$ $\therefore$ Mass $\propto$ volume

5. **(b)** Nuclear density of an atom

$d = \dfrac{\text{mass}}{\text{volume}} = \dfrac{A(1.67 \times 10^{-27})}{\frac{4}{3}\pi[1.25 \times 10^{-15} A^{1/3}]^3}$ $(A = \text{mass number})$

$\left[\because V = \dfrac{4}{3}\pi R^3, R = R_0 A^{1/3}, R_0 = 1.25 \times 10^{-15}\right]$

$\therefore d = 2 \times 10^{17}$ kg/m^3.

6. **False;** The order of nuclear density is 10^{17} kg/m^3. Density

$= \dfrac{m}{V} = \dfrac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi\left[R_0 A^{1/3}\right]^3} = \dfrac{1.67 \times 10^{-27}}{1.33 \times 3.14 \times (1.1 \times 10^{-15})}$

$= 3 \times 10^{17}$ kg/m^3

7. **(c, d)** In the case of hydrogen, atomic number = mass number (= 1) In the other atoms, atomic number (z) < mass number (A = Z + n).

Topic-2 : Mass-Energy Equivalence and Nuclear Reactions

1. **(d)** ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \longrightarrow 2\left({}^{4}_{2}\text{He}\right)$

$\Delta m \rightarrow [m_{\text{Li}} + m_{\text{H}}] - 2[M_{\text{He}}]$

Energy released $= \Delta mc^2$

In use of 1 g Li energy released $= \dfrac{\Delta mc^2}{m_{\text{Li}}}$

In use of 20 g energy released $= \dfrac{\Delta mc^2}{m_{\text{Li}}} \times 20$ g

$= \dfrac{[(7.016 + 1.0079) - 2 \times 4.0026]u \times c^2}{7.016 \times 1.6 \times 10^{-24}} \times 20$ g

$= \left(\dfrac{0.0187 \times 1.6 \times 10^{-19} \times 10^9}{7.016 \times 1.6 \times 10^{-24}} \times 20\right) = 480 \times 10^{10}$ J

$\because$ 1 J $= 2.778 \times 10^{-7}$ kWh

$\therefore$ Energy released $= 480 \times 10^{10} \times 2.778 \times 10^{-7}$
$= 1.33 \times 10^6$ kWh

2. **(c)** For the momentum and energy conservation, mass defect (Δm) should be positive. Since some energy is lost in every process.

$(m_p + m_n) > m_d$

3. **(d)** Mass defect,

$\Delta m = (50 m_p + 70 m_n) - (m_{sn})$
$= (50 \times 1.00783 + 70 \times 1.008) - (119.902199)$
$= 1.096$

Binding energy $= (\Delta m)C^2 = (\Delta m) \times 931 = 1020.56$

$\dfrac{\text{Binding energy}}{\text{Nucleon}} = \dfrac{1020.5631}{120} = 8.5$ MeV

4. **(b)** Power output of the reactor,

$P = \dfrac{\text{energy}}{\text{time}}$

$= \dfrac{2}{235} \times \dfrac{6.023 \times 10^{26} \times 200 \times 1.6 \times 10^{-19}}{30 \times 24 \times 60 \times 60} \simeq 60$ MW

5. **(d)**
6. **(c)** Power level of reactor, $P = \dfrac{E}{\Delta t} = \dfrac{\Delta mc^2}{\Delta t}$

mass of the fuel consumed per hour in the reactor,

$\dfrac{\Delta m}{\Delta t} = \dfrac{P}{c^2} = \dfrac{10^9}{(3 \times 10^8)^2} = 4 \times 10^{-2}$ gm

7. **(c)** Nuclear fission equation

${}_{92}\text{U}^{135} + {}_0\text{n}^1 \longrightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3\,{}_0\text{n}^1 + \text{Q(energy)}$

Hence particle x is neutron.

8. **(a)** The rest mass of parent nucleus i.e., ${}^{236}_{92}U$ should be greater than the rest mass of daughter nuclei.

9. **(c)** $4\,{}^4_2He \longrightarrow {}^{16}_{8}O$
B.E. $= \Delta m \times 931.5$ MeV
$= (4 \times 4.0026 - 15.9994) \times 931.5 = 10.24$ MeV

10. **(c)** Energy is released when stability increases. This will happen when binding energy per nucleon increases i.e.,
$\left(\frac{B.E.}{A}\right)_{\text{Product}} > \left(\frac{B.E.}{A}\right)_{\text{Reactant}}.$
$\because \quad |M_1V_1| = |M_2V_2|$

Reactant	Product
Reaction (a) $60 \times 8.5\text{MeV} = 510\text{MeV}$	$2 \times 30 \times 5 = 300$ MeV
Reaction (b) $120 \times 7.5 = 900$ MeV	$(90 \times 8 + 30 \times 5) = 870$ MeV
Reaction (c) $120 \times 7.5 = 900$ MeV	$2 \times 60 \times 8.5 = 1020$ MeV
Reaction (d) $90 \times 8 = 720$ MeV	$(60 \times 8.5 + 30 \times 5) = 600$ MeV

11. **(b)** Fast neutrons can be easily slowed down by passing them through water. In nuclear reactors heavy water in used as moderator.

12. This is a nuclear fusion reaction. In a nuclear fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.
Energy released $Q = (\Delta m)\,[931.5\ MeV/u]$
$= [2 \times 2.0141 - 4.0024] \times 931.5\ MeV \approx 24\ MeV$

13. ${}^{11}_{6}C \to {}^{11}_{5}B + \beta^+ + X \Rightarrow {}^{11}_{6}C \to {}^{11}_{5}B + {}^{0}_{+1}e + \nu$ (neutrino)
When balancing atomic number and mass number both sides of arrow then, X stands for neutrino.

14. ${}^2_1H + {}^2_1H \longrightarrow {}^4_2He$
Binding energy of two deuterons
$= 2\,[1.1 \times 2] = 4.4$ MeV
Binding energy of helium nucleus $= 4 \times 7.0 = 28\ MeV$
$\therefore$ Energy released $= 28 - 4.4 = 23.6\ MeV$

15. **(b, d)** In fusion two or more lighter nuclei combine to form a comparatively heavier nucleus. When binding energy per nucleon increases for a nuclear process, energy is released. In fission, a heavy nucleus breaks into two or more lighter nuclei.
(a) For $1 < A < 50$, on fusion mass number of the resulting nucleus will be less than 100.
(b) For $51 < A < 100$, on fusion mass number the resulting nucleus is between 100 and 200. B/A increases, energy will be released.
(c) On fission for $100 < A < 200$, the mass number for fission nuclei will be between 50 to 100. B/A decreases, no energy will be released.
(d) On fision for $200 < A < 260$, the mass number for fission nuclei will be between 100 to 130, B/A will increase, energy will be released.

16. **(c, d)** Due to mass defect which is finally responsible for the binding energy of the nucleus, mass of a nucleus is always less than the sum of masses of its constituent particles. i.e., protons and neutrons.
${}^{20}_{10}Ne$ is made up of 10 protons and 10 neutrons.
$\therefore$ Mass of ${}^{20}_{10}Ne$ nucleus
$M_1 < 10\,(m_p + m_n)$
Heavier the nucleus, more is the mass defect
$20\,(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$
or, $10\,(m_n + m_p) > M_2 - M_1$
or, $M_2 < M_1 + 10\,(m_p + m_n)$
Now, since $M_1 < 10\,(m_p + m_n)$
$\therefore \quad M_2 < 2M_1$

17. **(a, d)** In nuclear fission, a heavy nucleus breaks into two nuclei and more products and released energy.

18. **(d)** In nuclear fusion reaction two light nuclei combine to give a heavier nucleus and possibly other products and huge amount of energy.

19. **(b, c)** Nuclear fusion occurs when two or more lighter nuclei combine to form a comparatively heavier nucleus with release of a huge amount of energy.

20. Efficiencys $\eta = \frac{P_{out}}{P_{in}}$
$\therefore \quad P_{in} = \frac{P_{out}}{\eta} = \frac{1000 \times 10^6}{0.1} = 10^{10}$ W
Energy $E = P \times t$
$= 10^{10} \times 86{,}400 \times 365 \times 10$
$= 3.1536 \times 10^{18}$ J
$200 \times 1.6 \times 10^{-13}$ J of energy is released by 1 fission
$\therefore$ 3.1536×10^{18} J of energy is released by
$\frac{3.1536 \times 10^{18}}{200 \times 1.6 \times 10^{-13}}$ fission $= 0.9855 \times 10^{29}$ fission
$= 0.985 \times 10^{29}$ of U^{235} atoms.
6.023×10^{23} atoms of Uranium has mass 235g
$\therefore$ 0.9855×10^{29} atoms of Uranium has mass
$\frac{235 \times 0.9855 \times 10^{29}}{6.023 \times 10^{23}}$ g $= 38451$ kg

21. Energy required per day to run the reactor
$E = P \times t = 200 \times 10^6 \times 24 \times 60 \times 60$
$= 1.728 \times 10^{13}$J
Energy released in the given nuclear fusion reaction
$=$ (mass defect) $\times$ (931.3) mev
$= [2\,(2.0141) - 4.0026] \times 931.5$ MeV
$= 23.85$ MeV $= 23.85 \times 106 \times 1.6 \times 10^{-19} = 38.15 \times 10^{-13}$J
$\therefore$ No. of fusion reactions required
$= \frac{1.728 \times 10^{13}}{38.15 \times 10^{-13}} = 0.045 \times 10^{26}$
$\therefore$ No. of deuterium required $= 2 \times 0.045 \times 10^{26} = 0.09 \times 10^{26}$
Number of moles of deuterium $= \frac{0.09 \times 10^{26}}{6.02 \times 10^{23}} = 14.95$
$\therefore$ Mass in gram of deuterium $= 14.95 \times 2 = 29.9$ g
Since the energy from the reaction is used with a 25% efficiency in the reactor
$\therefore = (4 \times 29.9)$g $= 119.6$ g

Topic-3 : Radioactivity

1. **(c)** Since, $R = R_0 e^{-\lambda t}$
$\ln R = \ln R_0 + (-\lambda \ln t)$
$\lambda = \frac{\ln 2}{t_{1/2}} = \text{Slope}$

$\lambda_A = \frac{6}{10} \Rightarrow T_A = \frac{10}{6}\ln 2 \quad \lambda_B = \frac{6}{5} \Rightarrow T_B = \frac{5\ln 2}{6}$

$\lambda_C = \frac{2}{5} \Rightarrow T_C = \frac{5\ln 2}{6}$

$\therefore T_{\frac{1}{2}A} : T_{\frac{1}{2}B} : T_{\frac{1}{2}C} = \frac{10}{6} : \frac{5}{6} : \frac{15}{6} = 2:1:3$

2. **(a)** Let λ_1 and λ_2 be the decay constants of two process. N be the number of nuclei left undecayed after two process. From the law of radioactive decay we have

$-\frac{dN}{dt} = \lambda_1 N + \lambda_2 N \qquad \left[\because -\frac{dN}{dt} = \lambda N\right]$

$\Rightarrow -\frac{dN}{dt} = (\lambda_1 + \lambda_2)N \qquad \Rightarrow \lambda_{eq.} = (\lambda_1 + \lambda_2)$

$\Rightarrow \frac{\ln 2}{T} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2} \qquad \left(\because \lambda = \frac{\ln 2}{T}\right)$

$\Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

$\Rightarrow \frac{1}{T} = \frac{1}{10} + \frac{1}{100} = \frac{11}{100}$ [Given: $T_1 = 10$ s & $T_2 = 100$ s]

$\Rightarrow T = \frac{100}{11} = 9$ sec.

3. **(c)** As we know, for first order decay, $N(t) = N_0 e^{-\lambda t}$

According to question,

$\frac{N(t)}{N_0} = \frac{9}{16} = e^{-\lambda t}$

After time, $t/2$;

$N(t/2) = N_0 e^{-\lambda(t/2)}$

$\frac{N(t/2)}{N_0} = \sqrt{e^{-\lambda t}} = \sqrt{\frac{9}{16}}$

$\therefore N(t/2) = \frac{3}{4} N_0$

4. **(b)** We know that

Activity, $A = A_0 e^{-\lambda t}$

$A = A_0 e^{-tIn2/T_{1/2}} \left(\because \lambda = \frac{In_2}{T_{1/2}}\right)$

$\Rightarrow \quad 500 = 700\, e^{-tIn2/T_{1/2}}$

$\Rightarrow \quad In\frac{7}{5} = \frac{30 In2}{T_{1/2}} \qquad (\because\ t = 30 \text{ minute})$

$\Rightarrow \quad T_{1/2} = 30\frac{In\ 2}{In\ 1.4} = 61.8$ minute

($\because$ ln 2 = 0.693 and ln.1.4 = 0.336)

$\Rightarrow \quad T_{1/2} \approx 62$ minute

5. **(a)** Let N_1 and N_2 be the number of radioactive nuclei of substance at anytime t.

$N_1(\text{at } t) = N_0 e^{-5\lambda t}$ (i)

$N_2(\text{at } t) = N_0 e^{-\lambda t}$ (ii)

Dividing equation (i) by (ii), we get

$\frac{N_1}{N_2} = \frac{1}{e^2} = e^{-4\lambda t} \Rightarrow 4\lambda t = 2$

$\Rightarrow t = \frac{2}{4\lambda} = \left(\frac{1}{2\lambda}\right)$

6. **(c)** When one α- particle emitted then danghter nuclei has 4 unit less mass number (A) and 2 unit less atomic (z) number (z).

${}^{232}_{90}Th \rightarrow {}^{208}_{78}Y + 6\,{}^{4}_{2}He$

${}^{208}_{78}Y \rightarrow {}^{208}_{82}X + 4\beta$ praticle

7. **(a)** According to question, at t = 0, A0 = $\frac{dN}{dt}$ = 1600 C/s

and at t = 8s, A = 100 C/s

$\therefore \frac{A}{A_0} = \frac{1}{16}$ in 8s

Therefore half life period, t1/2 = 2s

$\therefore$ Activity at t = 6s = $1600\left(\frac{1}{2}\right)^3 = 200$C/s

8. **(c)** Half life of A = ℓn2

$(t_{1/2})A = \frac{\ell n2}{\lambda}$

$\therefore \quad \lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$

$N_A = N_B$ at t = 0

At t = t $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

$e^{-(\lambda_B - \lambda_A)t} = e^{-3t} \Rightarrow \lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$(t_{1/2})_B = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

9. **(d)** Let initial activity = No = 0.8 μ ci

$0.8 \times 3.7 \times 10^4$ dps

Activity in 1 cm^3 of blood at t = 10 hr,

$n = \frac{300}{60}$ dps = 5 dps

N = Activity of whole blood at time t = 10 hr.

Total volume of the blood in the person, V = $\frac{N}{n}$

$= \frac{N_0 e - \lambda t}{n} = \frac{0.8 \times 3.7 \times 10^4 \times 0.7927}{5} \cong 5$ litres

10. **(d)** Let initially there are total N_0 number of nuclei At time t

$$\frac{N_B}{N_A}=0.3 \text{ (given)} \Rightarrow N_B=0.3N_A$$

$$N_0=N_A+N_B=N_A+0.3N_A$$

$$\therefore N_A=\frac{N_0}{1.3}$$

As we know $N_t=N_0e^{-\lambda t}$

or, $\frac{N_0}{1.3}=N_0e^{-\lambda t}$

$$\frac{1}{1.3}=e^{-\lambda t} \qquad \Rightarrow ln(1.3)=\lambda t$$

$$\text{or, } t=\frac{ln(1.3)}{\lambda} \qquad \Rightarrow t=\frac{ln(1.3)}{\frac{ln(2)}{T}}=\frac{ln(1.3)}{ln(2)}T$$

11. **(b)** For $A_{t_{1/2}}=20$ min, $t=80$ min, number of half lifes $n=4$

$\therefore$ Nuclei remaining $=\frac{N_0}{2^4}$. Therefore nuclei decayed

$=N_0-\frac{N_0}{2^4}$

For $B_{t_{1/2}}=40$ min., $t=80$ min, number of half lifes $n=2$

$\therefore$ Nuclei remaining $=\frac{N_0}{2^2}$. Therefore nuclei decayed

$=N_0-\frac{N_0}{2^2}$

$$\therefore \text{Required ratio}=\frac{N_0-\frac{N_0}{2^4}}{N_0-\frac{N_0}{2^2}}=\frac{1-\frac{1}{16}}{1-\frac{1}{4}}=\frac{15}{16}\times\frac{4}{3}=\frac{5}{4}$$

12. **(b)** We know that $N_\beta=N_0\,(1-e^{-\lambda t})$

$$N_\beta=\frac{6.023\times10^{23}}{24}\left[1-e^{\frac{\ell n2}{15}\times7.5}\right]$$

on solving we get,

$N_\beta=7.4\times10^{21}$

13. **(c)** **Given:** $\frac{dN_0}{dt}=20$ decays/min

$\frac{dN}{dt}=2$ decays/min

$T_{1/2}=5730$ years

As we know,

$N=N_0e^{-\lambda t}$

$\text{Log}\frac{N_0}{N}=\lambda t$

$$\therefore t=\frac{1}{\lambda}\text{Log}\frac{N_0}{N}=\frac{2.303\times T_{1/2}}{0.693}\times\text{Log}_{10}\frac{N_0}{N}$$

$$\text{But } \frac{\frac{dN_0}{dt}}{\frac{dN}{dt}}=\frac{N_0}{N}=\frac{20}{2}=10$$

$$\therefore t=\frac{2.303\times5730}{0.693}\times1=19039 \text{ years}$$

14. **(b)** Let N be the number of nuclei at any time t then,

$$\frac{dN}{dt}=100-\lambda N \quad \text{or} \quad \int_0^N\frac{dN}{(100-\lambda N)}=\int_0^t dt$$

$$-\frac{1}{\lambda}[\log(100-\lambda N)]_0^N=t$$

$$\log(100-\lambda N)-\log 100=-\lambda t$$

$$\log\frac{100-\lambda N}{100}=-\lambda t$$

$$\frac{100-\lambda N}{100}=e^{-\lambda t} \quad 1-\frac{\lambda N}{100}=e^{-\lambda t}$$

$$N=\frac{100}{\lambda}(1-e^{-\lambda}t)$$

As, $N=50$ and $\lambda=0.5$/sec

$$\therefore \quad 50=\frac{100}{0.5}(1-e^{-0.5t})$$

Solving we get,

$$t=2\ln\left(\frac{4}{3}\right)\text{sec}$$

15. **(a)** Here $-\frac{dN}{dt}=\lambda_1N+\lambda_2N$

In tegrating on both sides

$$t=\frac{2.303}{\lambda_1+\lambda_2}\log_{10}\frac{N_0}{N}$$

$$\Rightarrow t=\frac{2.303}{5\times10^{-10}}\log_{10}\frac{100}{1}$$

$$\left[\begin{array}{l}\because \lambda_1+\lambda_2=4.5 10^{-10}+0.5\times10^{-10}\\ \text{and } N_0/N=100\end{array}\right]$$

$\therefore \quad t=9.2\times10^9$ year

16. **(a)** Let λ_1 and λ_2 be the decay constants and N_1 and N_2 be the number of active nuclei present for the two samples S_1 and S_2 respectively.

i.e., $\lambda_1N_1=5\mu Ci$ and $\lambda_2N_2=10\mu Ci\Rightarrow\lambda_2N_2=2\lambda_1N_1$

Also, $N_1=2N_2$

$\therefore \lambda_2N_2=2\lambda_1(2N_2)$ or $\lambda_2=4\lambda_1\Rightarrow\frac{\lambda_2}{\lambda_1}=4$ Also $T_{1/2}=0.693$

$\therefore (T_{1/2})=4(T_{1/2})_2$.

17. **(c)** For a nucleus to disintegrate in two half life, $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ i.e., The probability is $\frac{3}{4}$ as 75% of the nuclei will disintegrate in this time.

18. **(b)** In two half lives, the activity will remain $\frac{1}{4}$ of its initial activity. $\therefore$ Initial activity = 4 × 6000 = 24000 dps.

19. **(b)** By conservation of momentum, $p_1 = p_2$

$$\sqrt{2K_1m_1} = \sqrt{2K_2m_2} \qquad \left[\because P = \sqrt{2\,km}\right]$$

$$\Rightarrow \sqrt{2K_1(216)} = \sqrt{2K_2(4)}$$

$\therefore$ $K_2 = 54K_1$ ...(i)

And given $K_1 + K_2 = 5.5$ MeV ...(ii)

Solving equation (i) and (ii) we get $K_\alpha = K_2 = 5.4$ MeV

20. **(c)** In γ-decay, the atomic number (Z) and mass number (A) do not change.

21. **(a)** $A = A_0\,(1/2)^n$; n = number of half lives.

$$\frac{A_0}{16} = A_0\left(\frac{1}{2}\right)^n \quad \therefore \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \quad \therefore n = 4$$

Therefore time taken to decay $\frac{1}{16^{th}}$ of its initial value.

$$\therefore\ t = n \times t_{1/2} = (4 \times 100)\mu s = 400\ \mu s$$

22. **(d)** $N_1 = N_0 e^{-\lambda_1 t} = N_0 e^{-\frac{t}{\tau}}$ ....(i)

Mean life time $\tau = \frac{1}{\lambda_1}$

Similarly, $N_2 = N_0 e^{-\lambda_2 t} = N_0 e^{-\frac{t}{5\tau}}$ ....(ii) as $5\tau = \frac{1}{\lambda_2}$

Adding eq. (i) and (ii) we get

$$N = N_1 + N_2 = N_0\left(e^{-t/\tau} + e^{-t/5\tau}\right)$$

The total number of radioactive nuclei (N) as a function of time only decreases exponentially hence graph (d) correctly depicts.

23. **(c)** In a nucleus a neutron converts into a proton as follows

$n \to p^+ + e^{-1}$

Therefore, decay of neutron is responsible for β-radiation origination

24. **(d)** Number of nuclei of X_1, $N_1 = N_0 e^{-10\lambda t}$ and number of nuclei of X_2 $N_2 = N_0 e^{-\lambda t}$.

$$\therefore\ \frac{N_1}{N_2} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = \frac{1}{e^{9\lambda t}}$$

$$\text{Given } \frac{N_1}{N_2} = \frac{1}{e};\quad \therefore\ \frac{1}{e^{9\lambda t}} = \frac{1}{e}\left(\because \frac{N_1}{N_2} = \frac{1}{e}\text{given}\right)$$

$$\Rightarrow 9\lambda t = 1 \quad \therefore\ t = \left(\frac{1}{9\lambda}\right)$$

i.e., After time $t = \frac{1}{9\lambda}$ the ratio of the number of nuclei of X_1 to that of X_2 will be $\frac{1}{e}$.

25. **(a)** Beta rays are same as cathode rays as both are stream of electrons.

26. **(a)** Let after 't' years $\frac{1}{4}th$ of the material remains.

$$\frac{-dN}{dt} = \lambda_1 N + \lambda_2 N \quad \Rightarrow \log_e \frac{N}{N_o} = -(\lambda_1 + \lambda_2)t$$

when N_o is initial number of atoms

$$\text{Decay constant } \lambda = \frac{0.693}{t_{1/2}}$$

$$\therefore\ \lambda_1 = \frac{0.693}{1620} \text{ and } \lambda_2 = \frac{0.693}{810};$$

$$\frac{N}{N_o} = \frac{1}{4} \Rightarrow \log_e \frac{1}{4} = -\left(\frac{0.693}{1620} + \frac{0.693}{810}\right)t$$

$\Rightarrow$ $t = 1080$ years

27. **(a)** The penetrating power is dependent on velocity. For a given energy, the velocity of γ radiation is highest and α-particle is least.

28. **(c)** $4\,{}_1^1H^+ \to {}_2^4He^{2+} + 2e^- + 26\,\text{MeV}$

represent a fusion reaction.

In a nuclear fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus and releases energy.

29. **(c)** β-particles (${}_{-1}B^0$) are charged particles emitted by the nucleus.

30. **(b)** Half life, $t_{1/2} = 3.8$ day

$$\therefore\ \text{Decay constant, } \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.8} = 0.182$$

If initial number of atom $N = A_0$ then after time t the number of atoms is $N/20 = A$.

$$\therefore\ t = \frac{2.303}{\lambda}\log\frac{A_0}{A} = \frac{2.303}{0.182}\log\frac{N}{N/20} \simeq 16.5 \text{ days}$$

31. **(5)** According to question,

$$I^{131} \xrightarrow[T_{1/2} = 8\text{ days}]{} Xe^{131} + \beta$$

$A_0 = 2.4 \times 10^5\ \text{Bq} = \lambda N_0$

Let the volume is V

Given: At $t = 0$, $A_0 = \lambda N_0 = 2.4 \times 10^5$ Bq

$t = 11.5$ hrs, $A = \lambda N$

After $t = 11.5\ h$, 2.5 ml of blood is drawn from person's body and gives an activity of 115 Bq.

$$\therefore 115 = \lambda\left(\frac{N}{V} \times 2.5\right); 115 = \frac{\lambda}{V} \times 2.5 \times N_0 e^{\lambda t}$$

$$\Rightarrow 115 = \frac{\lambda N_0}{V} \times 2.5 \times e^{-\frac{\text{In }2}{8\text{ days}}(11.5\text{ hrs})}$$

$$\Rightarrow V = \frac{2.4 \times 10^5}{115} \times 2.5\left[1 - \frac{1}{24}\right]$$

[from approximation $e^x \approx 1 + x$]

$$\Rightarrow V = \frac{2.4 \times 10^5}{115} \times 2.5 \times \frac{23}{24} = 5 \times 10^3\ \text{ml} = 5 \text{ litres.}$$

32. **(2)** Activity,

$$R=-\frac{dA}{dt}=-\frac{d}{dt}\left[-\frac{dN}{dt}\right]=\frac{d^2N}{dt^2}=\frac{d^2\left(N_o e^{-\lambda t}\right)}{dt^2}$$

$$\therefore\ R=N_o\lambda^2 e^{-\lambda t}=(N_o\lambda)\lambda e^{-\lambda t}=A_o\lambda e^{-\lambda t}\quad[\because A_o=N_o\lambda]$$

$$\therefore\ \frac{R_P}{R_Q}=\frac{\lambda_P e^{-\lambda_P t}}{\lambda_Q e^{-\lambda_Q t}}=\frac{\lambda P}{\lambda Q}\times\frac{e^{\lambda_Q t}}{e^{\lambda_P t}}=\frac{2\tau}{\tau}\,\frac{e^{\frac{2\tau}{2\tau}}}{e^{\frac{2\tau}{\tau}}}=\frac{2}{e}=\frac{n}{e}$$

$\therefore\ n=2$

33. **(4)** For a radioactive decay

$N=N_0(1-e^{-\lambda t})$

$\therefore \dfrac{N}{N_0}=e^{-\lambda t}$ $\qquad \therefore 1-\dfrac{N}{N_0}=1-e^{-\lambda t}$

$$\therefore \frac{N_0-N}{N}=1-e^{-\frac{0.693}{t_{1/2}}\times t}=1-e^{-0.04}=1-(1-0.04)$$

$[\because e^{-x}=1-x\ x<<1]$

% decayed $\approx 0.04\times 100=4\%$

34. **(1)** We know that, $\left|\dfrac{dN}{dt}\right|=\lambda N=\dfrac{1}{T_{mean}}N\left[\because \lambda=\dfrac{1}{T_{mean}}\right]$

$\therefore\ 10^{10}=\dfrac{1}{10^9}\times N\ \therefore\ N=10^{19}$

i.e. 10^{19} radioactive atoms are present in the freshly prepared sample.

$\therefore$ Mass of the sample = N × mass of me atom

$=10^{19}\times 10^{-25}\,kg=10^{-6}kg=1\,mg$

35. **(8)** We know that $N=N_0e^{-\lambda t}$

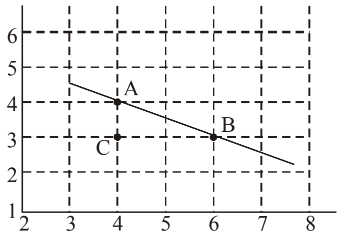
$\therefore \dfrac{dN}{dt}=N_0e^{-\lambda t}(-\lambda)=-N_0\lambda e^{-\lambda t}$

Taking log on both sides

$\log_e\dfrac{dN}{dt}=\log_e(\lambda N_0)-\lambda t$

Comparing it with the graph line,

Decay constant, $\lambda=\dfrac{1}{2}yr^{-1}$ $\qquad\left[\dfrac{AC}{BC}=\dfrac{1}{2}\right]$



$\therefore T_{1/2}=\dfrac{0.693}{\lambda}=0.693\times 2=1.386$ years

$n\,(t_{1/2})=4.16\ \therefore n=\dfrac{4.16}{1.386}\approx 3$

$\therefore\ N=N_0\left(\dfrac{1}{2}\right)^3$

$\therefore\ \dfrac{1}{P}=\dfrac{1}{8}$ $\qquad \therefore\ P=8]$

36. When boron nucleus ($^{10}_5B$) is bombarded by nentron $\left(^1_0 n\right)$

$^{10}_5B+{}^1_0n\longrightarrow {}^4_2He+{}^7_3Li$

the resulting nucleus is of element lithium and mass number A = 7.

37. $^{238}_{92}U\to{}^{206}_{82}Pb+x\,{}^4_2He+y\,{}^0_{-1}e$

No. of ∝ – particles emitted = 8 and no. of β-particles emitted = 6 ∝ – particle is $_2He^4$ so by emission of 1 ∝ – particle mass number (A) decreases by 4 units and a tomic number (Z) decreases by 2 units. β - particle is electron –1C° there is no change in mass number (A) but atomic number increases by 1 unit by emission of 1 beta particle.

38. Using $A=A_0\left(\dfrac{1}{2}\right)^n$ where A_0 = initial activity = 1000 dps (given)

A = Activity after n number of half lives

At $t=1, n=1\ \therefore A=1000\left(\dfrac{1}{2}\right)^1=500$ dps

At $t=3, n=3\ \therefore A=1000\left(\dfrac{1}{2}\right)^3=125$ dps

39. **(a, c)** No. of α-particles emitted, $N\propto=\dfrac{232-212}{4}=\dfrac{20}{4}=5$

$\therefore\ {}^{232}_{90}Th\longrightarrow{}^{212}_{82}Pb+5\,{}^4_2He+2\,{}^0_{-1}\beta$

Considering the laws of conservation of mass number (A) and atomic number (Z) number of β–particles emitted $N_\beta=2$

40. **(c)** According to question, $(t_{1/2})_x=(t_{mean})Y\Rightarrow$

$\dfrac{0.693}{\lambda_x}=\dfrac{1}{\lambda_Y}$ $\qquad \therefore \lambda_x=0.693\lambda_Y$ i.e., $\lambda_x<\lambda_Y$.

Now, rate of decay $\left(\dfrac{-dN}{dt}\right)=\lambda N$

Initially, number of atoms (N) of both x and y are equal but since $\lambda_Y<\lambda_x$, therefore element Y will decay at a faster rate than element x.

41. **(d)** The result follows from the formula based on laws of radioactive decay $N=N_0e^{-\lambda t}$ And rate of decay $\left(\dfrac{-dN}{dt}\right)=\lambda N$

The nucleus start decaying after time $t=0$

42. **(b)** Half-life $T_{1/2}=\dfrac{\ell n\,2}{\lambda}$ and Mean life, $\tau=\dfrac{1}{\lambda}$

43. **(b)** The intensity of radiation emitted is proportional to the rate of decay of radioactive meterial.

$t=\dfrac{2.303}{\lambda}\log_{10}\dfrac{N_0}{N}$ And $\lambda=\dfrac{0.693}{t_{1/2}}$

$\therefore\ t=\dfrac{2.303}{0.693/2}\log_{10}\dfrac{N_0}{N_0/64}$ or, $t=12$ hours.

44. **(c)** For negative beta decay

$$_0n^1 \rightarrow {}_1H^1 + \left({}_{-1}B^0\right)$$
(Neutron) $\quad$ (*Electron*)

45. For A → r, t; B → p, s; C → p, q, r, t; D → p, q, r, t

46. **(c)** In B^+-decay mass number (Z) decreases by 1 and mass number (A) remains unchanged.

$$^{15}_{8}O \longrightarrow {}^{15}_{7}N + \underset{\beta^+ \text{ particle}}{^{0}_{1}\beta}$$

In α-decay mass number (A) decreases by 4 unit and atomic number (Z) by 2 unit.

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + \underset{\alpha-\text{particle}}{^{4}_{2}He}$$

In proton $\left({}^1_1H\right)$ emission both (A) and (Z) decreases by 1.

$$^{185}_{83}Bi \longrightarrow {}^{184}_{82}Pb + \underset{\text{proton}}{^{1}_{1}H}$$

In fission process heavier nucleus breaks into two fragments.

$$^{239}_{94}Pu \longrightarrow {}^{140}_{57}La + {}^{99}_{37}X$$

47. **A → p, q; B → p, r; C → p, s; D → p, q, r**

In a nuclear fusion reaction two or more lighter nuclei combine to give a comparatively heavier nucleus and some matter is converted into energy.

In a nuclear fission reaction a heavy nucleus breaks into two or more lighter nuclei and some matter is converted into energy.

β-decay essentially proceeds by weak nuclear forces and converts some matter into energy.

Exothermic nuclear reaction possible for both nuclei with low and high atomic number and releases energy.

48. **(a)** $^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$

Mass defect

$\Delta m = [209.982876 - (205.974455 + 4.002603)] = 0.005818\mu$

$= 5.422$ MeV $= 5422$ keV

$\therefore Q = (\Delta m) \times 932 = (0.005818) \times 932$

$$K_\alpha = \frac{(A-4)}{A}Q = \left(\frac{210-4}{210}\right) \times 5422$$

$\therefore K_\alpha = 5319$ keV

49. **(c)** In case of (c) only, mass defect (Δm) is positive in all other cases (a) (b) and (d) Δm is negative.

Hence Deuteron and α-particle can undergo complete fusion.

50. Let initial Uranium atom $= a$

$\therefore$ atoms left

$$(a-x) = a\left(\frac{1}{2}\right)^n \text{ and } n = \frac{t}{t_{1/2}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3}$$

$$\therefore a - x = a\left(\frac{1}{2}\right)^{1/3}$$

$$\Rightarrow \frac{a}{a-x} = \frac{1}{(1/2)^{1/3}} = \frac{2^{1/3}}{1} = 1.26$$

$$\Rightarrow \frac{x}{a-x} = 1.259 - 1 = 0.259 \Rightarrow \frac{a-x}{x} = \frac{1}{0.259} = 3.86$$

51. Using, $\lambda = \dfrac{\log_e \frac{A_0}{A}}{t} = \dfrac{1}{2}\log_e \dfrac{n}{0.75n}$

Mean Life $= \dfrac{1}{\lambda} = \dfrac{2}{\log_e 4/3} = 6.94$ s

52. $X \xrightarrow[\lambda_x = 0.1\text{s}^{-1}]{T_{1/2} = 10\text{sec}} Y \xrightarrow[\lambda_y = \frac{1}{30}\text{s}^{-1}]{T_{1/2} = 30\text{sec}} Z$

$$\text{Here } \frac{dN_x}{dt} = -\lambda_x N_x \quad \text{...(i)}$$

$$\frac{dN_y}{dt} = -\lambda_y N_y + \lambda_x N_x \quad \text{...(ii)}$$

$$\frac{dN_z}{dt} = -\lambda_y N_y \quad \text{...(iii)}$$

Integrating, we get

$$N_x = N_0 e^{-\lambda_x t} \quad \text{...(iv)}$$

Given $N_y = \dfrac{\lambda_x N_0}{\lambda_x - \lambda_Y}\left[e^{-\lambda_y t} - e^{\lambda_x t}\right]$

To determine the maximum N_Y, $\dfrac{dN_Y}{dt} = 0$

From eq. (ii)

$$-\lambda_y N_y + \lambda_x N_x = 0$$

$$\Rightarrow \lambda_x N_x = \lambda_y N_y \quad \text{.......(v)}$$

$$\Rightarrow \lambda_x(N_0\, e^{-\lambda_x t}) = \lambda_y\left[\frac{\lambda_x N_0}{\lambda_x - \lambda_y}\left(e^{-\lambda_y t} - e^{\lambda_x t}\right)\right]$$

$$\Rightarrow \frac{\lambda_x - \lambda_y}{\lambda_y} = \frac{e^{-\lambda_y t} - e^{-\lambda_x t}}{e^{-\lambda_x t}} \Rightarrow \frac{\lambda x}{\lambda_y} = e^{(\lambda_x - \lambda_y)t}$$

$$\Rightarrow \log_e \frac{\lambda_x}{\lambda_y} = \left(\lambda_x - \lambda_y\right)t$$

$$\Rightarrow t = \frac{\log_e\left(\lambda_x / \lambda\gamma\right)}{\lambda_x - \lambda_\gamma} = \frac{\log_e\left[0.1/\left(\frac{1}{30}\right)\right]}{0.1 - \frac{1}{30}} = 15\log_e 3$$

$$\therefore N_x = N_0 e^{-0.1(15\log_e 3)} = N_0 e^{\log_e(3^{-1.5})}$$

$$\Rightarrow N_x = N_0 3^{-1.5} = \frac{10^{20}}{3\sqrt{3}} = 1.9 \times 10^{19}$$

Since, $\dfrac{dN_y}{dt} = 0$ at $t = 15\log_e 3$, $\therefore N_y = \dfrac{\lambda_x N_x}{\lambda_y} = \dfrac{10^{20}}{\sqrt{3}}$

$= 5.77 \times 10^{19}$

and $N_z = N_0 - N_x - N_y$

$$= 10^{20} - \left(\frac{10^{20}}{3\sqrt{3}}\right) - \frac{10^{20}}{\sqrt{3}} = 10^{20}\left(\frac{3\sqrt{3}-4}{3\sqrt{3}}\right) = 2.32 \times 10^{19}$$

53. (a) Let at time 't' number of radioactive nuclei $= N$.

Net rate of formation of nuclei of A.

$$\frac{dN}{dt} = \alpha - \lambda N \text{ or } \frac{dN}{\alpha - \lambda N} = dt \quad [\propto = \text{rate of production}]$$

$$\text{or } \int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving, we get

$$N=\frac{1}{\lambda}\left[\alpha-(\alpha-\lambda N_0)e^{-\lambda t}\right] \quad ...(i)$$

(b) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \frac{\ln(2)}{\lambda}$ in (i),

$$N=\frac{3}{2}N_0$$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in equation (i), we get

$$N=\frac{\alpha}{\lambda}=\frac{2\lambda N_0}{\lambda}=2N_0.$$

54. **(i)** From the given information, number of nuclei reduced to half 25% to 12.50% m 10s $\therefore$ half life $T_{1/2} = 10$s

Mean life $\tau=\frac{1}{\lambda}=\frac{1}{0.693/t_{1/2}}=\frac{t_{1/2}}{0.693}=\frac{10}{0.693}=14.43$ sec.

(ii) $N = N_0e^{-\lambda t} \Rightarrow \frac{N}{N_0}=\frac{6.25}{100}$

$$\frac{6.25}{100}=e^{-0.0693t} \Rightarrow e^{+0.0693t}=\frac{100}{6.25}=16$$

$$\left(\because \lambda = 0.0693\,s^{-1}\right)$$

$0.0693t = \ln 16 = 2.773$ or $t=\frac{2.733}{0.0693}=40$ sec.

55. Given: Half-life, $t_{1/2} = 15$ hours
Activity initially $A_0 = 10^{-6}$ Curie $= 3.7 \times 10^4$dps
After 5 hours, $A = 296$ dpm $= 296/60$ dps
The initial activity can be found by the formula

$$t=\frac{2.303}{\lambda}\log_{10}\frac{A_0}{A} \Rightarrow 5=\frac{2.303}{0.693/15}\times\log_{10}\frac{A_0}{296}$$

$$\Rightarrow \log_{10}\frac{A_0}{296}=\frac{5\times0.693}{2.303\times15}=\frac{0.3010}{3}=0.10033$$

$$\Rightarrow \frac{A_0}{296}=1.26 \Rightarrow A_0=373\text{ dpm}=\frac{373}{60}\text{ dps}$$

This is the activity level in blood volume 1 cm^3.
$\therefore$ Activity 3.7×10^4 dps in blood volume

$$V=\frac{3.7\times10^4}{373/60}=5951.7\,cm^3=5.95l\text{ litre}$$

56. **(a)** ${}^{A}_{92}X \to {}^{228}_{X}Y + {}^{4}_{2}He \; \left(\because \alpha = {}^4_2He\right)$

$\therefore A = 228 + 4 = 232$ and $92 = Z + 2 \therefore Z = 90$

(b) Let v be the velocity with which α - particle is emitted.

The magnetic force qvB provides centripetal force $\frac{mv^2}{r}$ to α-particle for its circular motion.

$$\therefore \frac{mv^2}{r}=qvB \Rightarrow v=\frac{qrB}{m}=\frac{2\times1.6\times10^{-19}\times0.11\times3}{4.003\times10^{-27}}$$

$\therefore v = 1.59 \times 10^7$ms^{-1}.
Applying law of conservation of linear momentum during α-decay

$m_Yv_Y = m_\alpha v_\alpha$ ...(i)
Total kinetic energy,

$$E=K.E._\alpha+K.E._Y=\frac{1}{2}m_\alpha {v_\alpha}^2+\frac{1}{2}m_Y{v_Y}^2$$

$$=\frac{1}{2}m_\alpha v_\alpha^2+\frac{1}{2}m_Y\left[\frac{m_\alpha v_\alpha}{m_Y}\right]^2=\frac{1}{2}m_\alpha {v_\alpha}^2+m_\alpha v_\alpha^2+\frac{m_\alpha^2v_\alpha^2}{2m_Y}$$

$$=\frac{1}{2}m_\alpha v_\alpha^2\left[1+\frac{m_\alpha}{m_Y}\right]$$

$$=\frac{1}{2}\times4.033\times1.6\times10^{-27}\times(1.59\times10^7)^2\left[1+\frac{4.003}{228.03}\right]J$$

$$=8.55\times10^{-13}J=\frac{8.55\times10^{-13}}{1.6\times10^{-19}}=5.34\text{ MeV}$$

$\therefore$ Mass equivalent of this energy $=\frac{5.34}{931.5}=0.0051$ a.m.u.

Also, $m_x = m_Y + m_\alpha$ + mass equivalent
$= 228.03 + 4.003 + 0.0057 = 232.0387$ u.
Number of nucleons = 92 protons + 140 neutrons.
$\therefore$ Binding energy of nucleus X
$= [92 \times 1.008 + 140 \times 1.009] - 232.0387] \times 931.5$ Mev
$= 1.9571 \times 931.5 = 1823$ MeV.

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. **(c)** Using, $\frac{A}{A_0}=\frac{1}{2^n}$

n = number of half lives

$$\therefore 2^n=\frac{A_0}{A}=\frac{64}{1}=2^6 \Rightarrow n=6$$

$\therefore$ Time $= n(t_{1/2}) = 6 \times t_{1/2} = 6 \times 18 = 108$ days
Hence after 108 days the laboratory can be considered safe for use.

2. **(c)** Binding energy of nitrogen atom
$[8M_n + 7M_p - M_N] \times 931$
$= [8 \times 1.008665 + 7 \times 1.007825 - 15.000109] \times 931$
Binding energy of oxygen atom
$[8M_n + 8M_p - M_o] \times 931$
$= [7 \times 1.008665 + 8 \times 1.007825 - 15.003065] \times 931$
$\therefore$ Difference $= 0.0037960 \times 931$ MeV ...(i)

$$\text{Also } E_O=\frac{3}{5}\times\frac{8\times7}{R}\times\frac{e^2}{4\pi\epsilon_0}=\frac{3}{5}\times\frac{56}{R}\times1.44\text{MeV}$$

$$E_N=\frac{3}{5}\times\frac{7\times6}{R}\times\frac{e^2}{4\pi\epsilon_0}=\frac{3}{5}\times\frac{42}{R}\times1.44\text{MeV}$$

$$\therefore E_O-E_N=\frac{3}{5}\times\frac{14}{R}\times1.44\text{MeV} \quad ...(ii)$$

From eq. (i) & (ii)

$$\frac{3}{5}\times\frac{14}{R}\times1.44=0.0037960\times931 \;\therefore R=3.42\text{ fm}$$

3. **(b)** ${}^{22}_{10}Ne \to {}^4_2He + {}^4_2He + {}^{14}_{6}X$
Atomic number of neon Ne is 10 and α-particle is helium ${}^4_2He$.

The new element X has atomic number 6. Therefore, it is carbon atom A = 14 and C = 6.

4. **(9)** Here, ${}^{12}_{5}\beta \longrightarrow {}^{13}_{6}C + {}^{0}_{-1}e + \bar{\nu}$
Maximum kinetic energy of β-particle
$= [\text{mass of } {}^{12}_{5}\beta - \text{mass of } {}^{12}_{6}C] \times 931.5 - 4.041$
$= [12.014 - 12] \times 931.5 - 4.041] = 9\text{MeV}$

5. **(3)** $N_o \xrightarrow{T} \frac{N_o}{2} \xrightarrow{T} \frac{N_o}{4} \xrightarrow{T} \frac{N_o}{8}$
100% 50% 25% 12.5%
So, three half lives are required.
$\therefore t = n(T) = 3T \therefore n = 3$

6. Atomic number, mass number

7. **(a)** For the given fission reaction,
${}^{236}_{92}\text{U} \to {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$
From conservation laws of mss number (A) and atomic number (Z).
$x = n, y = n$ i.e., $x = {}^{1}_{0}n$ and $y = {}^{1}_{0}n$
From conservation of momentum, $|P_{xe}| = |P_{st}|$
From $K = \frac{P^2}{2m} \Rightarrow K \propto \frac{1}{M} \therefore \frac{K^{st}}{K_{xe}} = \frac{m^{xe}}{m_{st}}$
$\therefore K_{st} = 129$ MeV and $K_{xe} = 86$ MeV

8. **(c)** ${}_1H^2 + {}_1H^2 \to {}_1H^3 + p$
${}_1H^2 + {}_1H^3 \to {}_2He^4 + n$
By adding given two equation $3\,{}_1H^2 \to {}_2He^4 + p + n$
$\Delta m = 3\,(2.014) - [4.001 + 1.007 + 1.008] = 0.026$
3 deuterons release 3.87×10^{-12} J
$\therefore 10^{40}$ deuterons release $= \frac{3.87 \times 10^{-12} \times 10^{40}}{3}$
$= 1.29 \times 10^{28}$J
Power, $P = \frac{E}{t} \Rightarrow t = \frac{E}{P} = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12}$

9. **(a)** At room temperature, thermal energy of air molecule = 0.02 eV
photon energy of visible light (λ = 4000Å to 700Å) = $2eV$.

10. **A → p, q, t; B → q; C → s; D → s**
(p) When an uncharged capacitor is connected to a battery, it becomes charged and energy stored $E = \frac{1}{2}QV$ in the capacitor.
(q) When a gas in an adiabatic container fitted with an adiabatic piston is compressed by pushing the piston
(i) the internal energy of the system increases
$\Delta U = Q - W = 0 - (-PdV) = +PdV$
(ii) Mechanical energy is proceeded to the piston which is converted into kinetic energy of the gas molecules.
(r) When the gas in a rigid container gets cooled, the internal energy of the system will decrease. Due to its conversion into mechanical energy.
(s) When a heavy nucleus initially at rest splits into two nuclei of nearly equal masses and some neutrons are emitted as in case of nuclear fission internal energy of the system is converted into mechanical energy and converts some matter into energy.
(t) When a resistive wire loops is placed in a time varying magnetic field perpendicular to its palne then energy of system is increased due to induced current.

11. **A → p; B → t; C → u; D → r**

(A)	Energy of thermal neutrons	(p)	0.025 eV
(B)	Energy of X-rays	(t)	10 k eV
(C)	Binding energy per nucleon	(u)	8 M eV
(D)	Photoelectric threshold of a metal	(r)	3 eV

12. **(d)** Kinetic energy (K) of electron will be minimum or zero when total energy is shared by proton and anti-neutrino
$\therefore 0 \le K < 0.8 \times 10^6$ eV

13. **(c)** $\because K_p^- + K_e^- + K_\nu^- = 0.8 \times 10^6$ eV
When $K_e^- = 0$ then $K_p^- + K_\nu^- = 0.8 \times 10^6$ eV
Mass of $\bar{\nu} << P \therefore$ maximum energy of anti-neutrino is nearly 0.8×10^6 eV.

14. **(d)** In the core of nuclear fusion reactor the gas becomes plasma a collection of ${}^{2}_{1}\text{H}$ nuclei and electron which is formed due to high temperature main tained inside for the reactor core. High temperature is required for nuclear fusion.

15. **(a)** From conservation of mechanical energy
Loss of kinetic energy of two deuteron nuclei
= gain in their potential energy.
$$2 \times 1.5kT = \frac{1}{4\pi\varepsilon_0}\frac{e \times e}{r} = \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{r}$$
$$\Rightarrow 2 \times 1.5 \times \left(8.6 \times 10^{-5}\frac{eV}{k}\right) \times T = \frac{(1.44 \times 10^{-9}\,eVm)}{4 \times 10^{-15}\,m}$$
$$\therefore \quad T = \frac{1.44 \times 10^{-9}}{2 \times 1.5 \times 8.6 \times 10^{-5} \times 4 \times 10^{-15}} = 1.4 \times 10^9\text{K}$$

16. **(b)** As given in the passage, the product of the deutron density (n) and confinement time (t_0) $nt_o > 5 \times 10^{14}$ which is the Lawson criterion for a reactor to work successfully.
$\therefore$ Here $n = 8.0 \times 10^{-14}$ cm^{-3} and $t_0 = 9.0 \times 10^{-1}$s

17. Radius of nucleus $r = r_0 A^{1/3}$
where r_0 = constt, and A = mass number.
Here $r_1 = r_0 4^{1/3}$
and unknown nucleus $r_2 = r_0(A)^{1/3}$
$$\therefore \frac{r_2}{r_1} = \left(\frac{A}{4}\right)^{1/3}, \; (14)^{1/3} = \left(\frac{A}{4}\right)^{1/3} \Rightarrow A = 56$$
$\therefore$ No of proton = mass number, A – no. of neutrons = 56 – 30 = 26
$\therefore$ Atomic number, $Z = 26$
(b) Using $\nu = Rc(Z - b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
$R = 1.1 \times 10^7, c = 3 \times 10^8, Z = 26$
$b = 1$ (for K_α), $n_1 = 1, n_2 = 2$
$\therefore \nu = 1.1 \times 10^7 \times 3 \times 10^8 [26 - 1]^2 \left[\frac{1}{1} - \frac{1}{4}\right]$
$= 3.3 \times 10^{15} \times 25 \times 25 \times \frac{3}{4} = 1.546 \times 10^{18}$ Hz

18. Here reaction ${}^{A}_{Z}\text{X} \to {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$
$m_y = 223.61$ amu and $m_\alpha = 4.002$ amu
Momentum, $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}} = 1.15 \times 10^{-19}$ kg m/s
This will be the momentum of α-particle as well as y (law of conservation of linear momentum

$\therefore$ K.E. $= \dfrac{p^2}{2m_y} + \dfrac{p^2}{2m_\alpha} \Rightarrow$ K.E. $= \dfrac{p^2}{2}\left[\dfrac{1}{m_y} + \dfrac{1}{m_\alpha}\right]$

$\therefore$ K.E. $= \dfrac{(1.15\times10^{-19})^2}{2\times1.66\times10^{-27}}\left[\dfrac{1}{223.61} + \dfrac{1}{4.002}\right] = 10^{-12}$ J

From $E = \Delta mc^2$

$\therefore \Delta m = \dfrac{E}{c^2} = \dfrac{10^{-12}}{(3\times10^8)^2}$ kg $= \dfrac{10^{-28}}{3^2}\times\dfrac{1}{1.67\times10^{-27}}$ amu

$= 0.00665\, u$

Mass of the parent nucleus X

$m_x = m_y + m_\alpha + \Delta m$

$= 223.61 + 4.002 + 0.00665 = 227.62$ amu

19. In α-decay ${}^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$

Mass defect

Δm = Mass of ${}^{248}_{96}$Cm – Mass of ${}^{244}_{94}$Pu – Mass of ${}^{4}_{2}$He

$= (248.072220 - 244.064100 - 4.002603)$ u $= 0.005517$u

$\therefore$ Energy released in α-decay

$E_\alpha = (0.005517 \times 931)$ MeV $= 5.136$ MeV

Similarly, $E_{\text{fission}} = 200$ MeV (given)

Mean life is given as $t_{\text{mean}} = 10^{13}\text{s} = \dfrac{1}{\lambda}$

$\therefore$ Disintegration constant $\lambda = 10^{-13}\ \text{s}^{-1}$

Rate of decay at the moment when number of nuclei are 10^{20}

$$\frac{dN}{dt} = \lambda N = (10^{-13})(10^{20}) = 10^7 \text{ dps}$$

8% disintegrations are in fission and 92% are in α-decay.

$\therefore$ Energy released per second

$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV

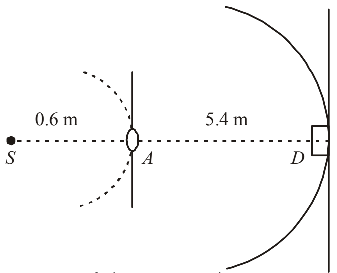
$= 2.074 \times 10^8$ MeV

$\therefore$ Power output (in watt) = energy released per second (J/s) $= (2.074 \times 10^8)(1.6 \times 10^{-13})$

$\therefore$ Power output, $P_{\text{output}} = 3.32 \times 10^{-5}$ watt.

20. Energy of one photon, $E = \dfrac{hc}{\lambda} = \dfrac{(6.6\times10^{-34})(3.0\times10^8)}{6000\times10^{-10}}$

$= 3.3 \times 10^{-19}$ J



Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$n_1 = \frac{2}{3.3\times10^{-19}} = 6.06\times10^{18}\ /\text{s}$$

At distance 0.6 m, number of photons incident per unit area per unit time :

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34\times10^{18}\ /\text{m}^2/\text{s}$$

Area of aperture $S_1 = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\ \text{m}^2$

$\therefore$ Total number of photons incident per unit time on the aperture,

$n_3 = n_2 s_1 = (1.34\times10^{18})(7.85\times10^{-3})/\text{s} = 1.052\times10^{16}/\text{s}$

The aperture will become new source of light.

Now these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area per unit time :

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times10^{16}}{4\pi(5.4)^2} = 2.87\times10^{13}\ \text{s}^{-1}\ \text{m}^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm^2 or 0.5×10^{-4} m^2.

Therefore, total number of photons incident on the detector per unit time :

$$n_5 = (0.5\times10^{-4})(2.87\times10^{13}\, d) = 1.435\times10^9\ \text{s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time

$$n_6 = 0.9 n_5 = 1.2915\times10^9\ \text{s}^{-1}$$

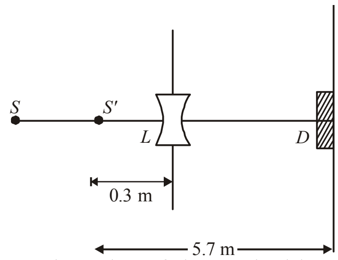
or, photocurrent in the detector

$$i = (e)n_6 = (1.6\times10^{-19})(1.2915\times10^9) = 2.07\times10^{-10}\ \text{A}$$

(b) Using the lens formula : $\dfrac{1}{V} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{v} - \dfrac{1}{-0.6} = \dfrac{1}{-0.6}$ or $v = -0.3$ m

i.e., Image of source (say S') is formed at 0.3 m from the lens.



Total number of photons incident per unit time on the lens are still n_3 or 1.052×10^{16}/s. 80% of it transmits to second medium. Therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time

$$n_7 = \frac{(80/100)(1.05\times10^{16})}{(4\pi)(5.7)^2} = 2.06\times10^{13}\ \text{s}^{-1}\text{m}^{-2}$$

This is the photon flux at the detector.

New value of photocurrent

$i = (2.06\times10^{13})(0.5\times10^{-4})(0.9)(1.6\times10^{-19})$

$= 1.483 \times 10^{-10}$ A

(c) The stopping potential depends on incident frequency, therefore it remains same with or without lens

$$\frac{hc}{\lambda} = (E_K)_{\max} + \phi = eV_0 + \phi$$

$$\therefore eV_0 = \frac{hc}{\lambda} - \phi = \frac{3.315\times10^{-19}}{1.6\times10^{-19}} - 1 = 1.07\text{eV}$$

($\because \phi = 1$ ev given)

or, stopping potential, $V_0 = 1.07$ Volt

21. (i) In a nucleus, number of electrons = 0 ($\because$ electrons don't reside in the nucleus of atom, electron revolves round the nucleus in its permissible orbit).

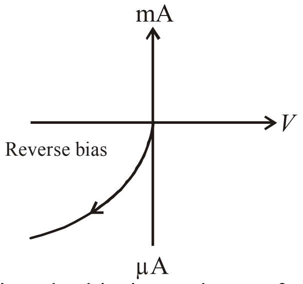
(ii) number of protons = atomic number = 11

(iii) number of neutrons = mass number – atomic number $= 24 - 11 = 13$

28 Semiconductor Electronics: Materials, Devices and simple circuit

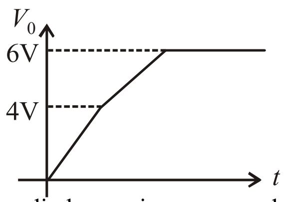
Topic-1 : Solid Semiconductors and P-N Junction Diode

1. **(d)** I-V characteristic of a photodiode is as follows :



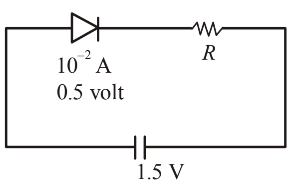
On increasing the biasing voltage of a photodiode, the magnitude of photocurrent first increases and then attains a saturation.

2. **(c)** Till input voltage reaches 4 V. No zener is in breakdown region so $V_0 = V_i$. Then now when V_i changes between 4 V to 6 V one zener with 4 V will breakdown and P.D. across this zener will become constant and remaining potential will dorp across resistance in series with 4 V zener.
Now current in circuit increases abruptly and source must have an internal resistance due to which some potential will get drop across the source also so correct graph between V_0 and t will be



3. **(c)** Here two zener diodes are in reverse polarity so if one is in forward bias the other will be in reverse bias and above 6V the reverse bias will too be in conduction mode. Hence when V > 6V the output will be constant. And when V < 6V it will follow the inut voltage.

4. **(c)** According to question, when diode is forward biased, $V_{diode} = 0.5$ V
Safe limit of current, $I = 10$ mA $= 10^{-2}$ A
$R_{min} = ?$
Voltage through resistance



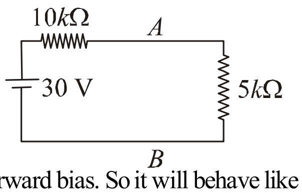
$V_R = 1.5 - 0.5 = 1$ volt
$iR = 1\ (=V_R)$
$\therefore R_{min} = \frac{1}{i} = \frac{1}{10^{-2}} = 100\ \Omega$

5. **(d)** Given,
Wavelength of photon, $\lambda = 400$ nm
A photodiode can detect a wavelength corresponding to the energy of band gap. If the signal is having wavelength greater than this value, photodiode cannot detect it.
$\therefore$ Band gap $E_g = \frac{hc}{\lambda} = \frac{1237.5}{400} = 3.09$ eV

6. **(c)** Both the diodes are reverse biased, so, there is no flow of current through 5Ω and 20Ω resistances.
Now, two resistors of 10Ω and two resistors of 5Ω are in series.
Hence current I through the network = 0.3 A

7. **(c)** In case I diode is reverse biased, so no current flows
$\therefore \quad Q_A = CV$
In case II, current will flow as diode is forward biased. So, it offers negligible resistance to the flow of current and thus be replaced by short circuit. Now, the charge of capacitor will leak through the resistance and decay exponentially with time.
During discharging of capacitor
Potential difference across the capacitor at any instant
$V' = Ve^{-\frac{t}{CR}}$
But $t = CR$
$V' = Ve^{-1} = \frac{V}{e}$
$\therefore \quad$ Charge $Q_B = CV' = \frac{CV}{e}$

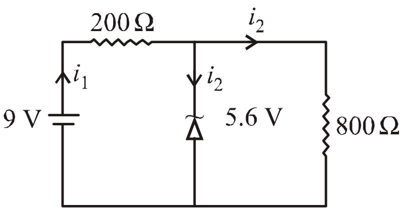
8. **(a)** The given circuit has two 10$k\Omega$ resistances in parallel, so we can reduce this parallel combination to a single equivalent resistance of 5$k\Omega$.



Diode is in forward bias. So it will behave like a conducting wire.
$$V_A - V_B = \frac{30}{5+10} \times 5 = 10\ V$$

9. **(d)** Current in load resistance,
$$i_1 = \frac{6}{4\times10^3} = 1.5 \times 10^{-3}\ \text{A} = 1.5\ \text{mA}$$
For $V = 16$ volt,
$$i_s = \frac{(16-6)}{2\times10^3} = 5\ \text{mA}$$
$\therefore \quad i_2 = i_s - i_1 = 5 - 1.5 = 3.5$ mA

10. **(a)**

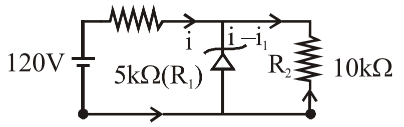


P.D. across 800Ω resistors = 5.6 V
so, $I_{800\Omega} = \frac{5.6}{800} A = 7$ mA
Now, P.D. across 200Ω resistors = 9 – 5.6 V = 3.4 V
so, $I_{200\Omega} = \frac{9-5.6}{200} = 17$ mA
so, current through zener diode = $I_2 = 17 - 7 = 10$ mA

11. **(a)** Since voltage across zener diode does not reach to breakdown voltage therefore its resistance will be infinite & current through it is 0.

12. **(b)** As D_2 is reversed biased, so no current through 75Ω resistor.
now $R_{eq} = 150 + 50 + 100 = 300\ \Omega$
So, required current $I = \dfrac{\text{Battery Voltage}}{300}$
$I = \dfrac{6}{300} = 0.02$

13. **(a)** The voltage across zener diode is constant



$i_{(R_2)} = \dfrac{V}{R} = \dfrac{50}{10\times10^3} = 5\times10^{-3}\,\text{A}$

$i_{(R_1)} = \dfrac{V}{R} = \dfrac{120-50}{5\times10^3} = \dfrac{70}{5\times10^3} 14\times10^{-3}\,\text{A}$

$\therefore\ i_{zenerdiode} = 14\times10^{-3} - 5\times10^{-3} = 9\times10^{-3}\,\text{A} = 9\ \text{mA}$

14. **(c)** As we know, current density,
$j = \sigma E = nev_d$

$\sigma = ne\dfrac{v_d}{E} = ne\mu$

$\dfrac{1}{\sigma} = \rho = \dfrac{1}{n_e e\mu_e}$ = Resistivity

$= \dfrac{1}{10^{19}\times1.6\times10^{19} - 19\times1.6}$

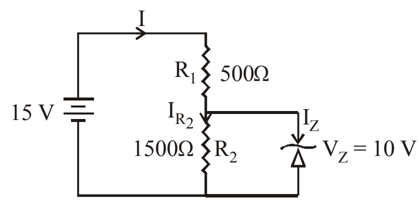
or $P = 0.4\ \Omega\text{m}$

15. **(d)** Initially Ge and Si are both forward biased so current will effectivily pass through Ge diode $\therefore\ V_\circ = 12 - 0.3 = 11.7$ V
And if "Ge" is revesed then current will flow through "Si" diode
$\therefore\ V_\circ = 12 - 0.7 = 11.3$ V
Clearly, $V_\circ$ changes by $11.7 - 11.3 = 0.4$ V

16. **(c)** Clearly from fig. given in question, Silicon diode is in forward bias.
$\therefore$ Potential barrier across diode
$\Delta V = 0.7$ volts

Current, $I = \dfrac{V-\Delta V}{R} = \dfrac{3-0.7}{200} = \dfrac{2.3}{200} = 11.5\text{mA}$

17. **(b)**



The voltage drop across R_2 is $V_{R_2} = V_Z = 10\,\text{V}$
The current through R_2 is

$I_{R_2} = \dfrac{V_{R_2}}{R_2} = \dfrac{10\,\text{V}}{1500\Omega} = 0.667\times10^{-2}\,\text{A}$
$= 6.67\times10^{-3}\,\text{A} = 6.67\ \text{mA}$

The voltage drop across R_1 is
$V_{R_1} = 15\text{V} - V_{R_2} = 15\,\text{V} - 10\,\text{V} = 5\,\text{V}$
The current through R_1 is

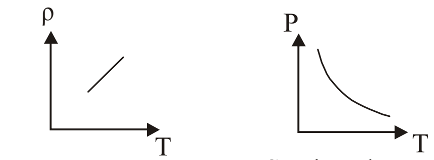
$I_{R_1} = \dfrac{V_{R_1}}{R_1} = \dfrac{5\,\text{V}}{500\Omega} = 10^{-2}\,\text{A} = 10\times10^{-3}\,\text{A} = 10\,\text{mA}$

The current through the zener diode is
$I_Z = I_{R_1} - I_{R_2} = (10 - 6.67)\text{mA} = 3.3\,\text{mA}$

18. **(a)** The conductivity of semiconductor
$\sigma = e\,(\eta_e\mu_e + \eta_h\mu_h)$
$= 1.6\times10^{-19}(5\times10^{18}\times2 + 5\times10^{19}\times0.01)$
$= 1.6\times1.05 = 1.68$

19. **(c)** Graph (i) is for a simple diode.
Graph (ii) is showing the V Break down used for zener diode.
Graph (iii) is for solar cell which shows cut-off voltage and open circuit current.
Graph (iv) shows the variation of resistance h and hence current with intensity of light.

20. **(a)**



Metal (for limited range of temperature)   Semiconductor $\rho = \rho_o\, e^{\frac{-Eg}{K_BT}}$

21. **(c)** The minimum voltage range of DC source is given by
$V^2 = PR$ Q $P = 1$ watt, $R = 100\Omega$
$= 1\times100$
$\therefore\ V = 10$ volt.

22. **(d)** Using $U_{av} = \dfrac{1}{2}\varepsilon_0 E_0^{\,2}$

But $U_{av} = \dfrac{P}{4\pi r^2\times c}$

$\therefore\quad \dfrac{P}{4\pi r^2} = \dfrac{1}{2}\varepsilon_0 E_0^{\,2}\times c$

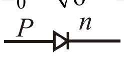
$E_0^2 = \dfrac{2P}{4\pi r^2\varepsilon_0 c} = \dfrac{2\times0.1\times9\times10^9}{1\times3\times10^8}$

$\therefore\quad E_0 = \sqrt{6} = 2.45\text{V/m}$

23. **(a)** When positive terminal connected to A then diode D_1 is forward biased, current, $I = \dfrac{2}{5} = 0.4A$
When positive terminal connected to B then diode D_2 is forward biased, current, $I = \dfrac{2}{10} = 0.2A$

$E_0^2 = \dfrac{2P}{4\pi r^2\varepsilon_0 c} = \dfrac{2\times0.1\times9\times10^9}{1\times3\times10^8}$

$\therefore\quad E_0 = \sqrt{6} = 2.45\text{V/m}$

24. **(a)** 

For forward bias, p-side must be at higher potential than n-side. $\Delta V = (+)Ve$

25. **(d)** Here, $R = 4\ \text{k}\Omega = 4\times10^3\ \Omega$
$V_i = 60$ V
Zener voltage $V_z = 10$ V
$R_L = 2\ \text{k}\Omega = 2\times10^3\ \Omega$

Load current, $I_L = \dfrac{V_Z}{R_L} = \dfrac{10}{2\times10^3} = 5\ \text{mA}$

Current through R, $I = \dfrac{V_i - V_Z}{R} = \dfrac{60-10}{4\times10^3} = \dfrac{50}{4\times10^3}$

$= 12.5$ mA

Fom circuit diagram,

$I = I_Z + I_L \Rightarrow \quad 12.5 = I_Z + 5 \Rightarrow I_Z = 12.5 - 5 = 7.5$ mA

26. **(a)** For same value of current higher value of voltage is required for higher frequency hence (a) is correct answer.

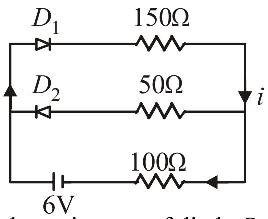
27. **(a)** Here, diodes D_1 and D_2 are forward biased and D_3 is reverse biased.
Therefore current through R_3

$$i = \frac{V}{R'} = \frac{6}{120} = \frac{1}{20} A = 50\,mA$$

28. **(c)** In n-type semiconductors, electrons are the majority charge carriers and in P type semiconductor holes are the majority charge carriers.

29. **(b)** In the circuit, diode D_1 is forward biased, while D_2 is reverse biased. Therefore, current i (through D_1 and 100Ω resistance)

$$i = \frac{V}{R} = \frac{6}{50+100+150} = 0.02A$$



Here, 50Ω is the resistance of diode D_1 in forward biasing not that connected with diode D_2.

30. **(b)** The contribution from the diode (1) is A,C. As shown in the fig. (a) during one half cycle the polarity of primary P and secondary S are opposite such that diode (1) is reversed biased and hence non conducting.

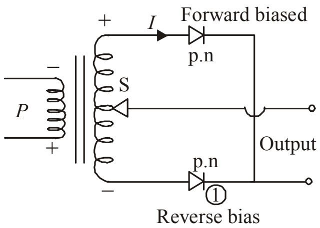


fig. (a)

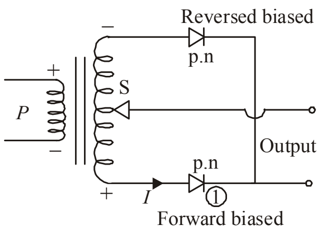


fig. (b)

During the other half cycle, diode (1) gets forward biased and is conducting. Thus diode (1) conducts in one half cycle and does not conduct in the other.

31. **(b)** For a semi conductor $n = n_0 e^{-Eg/kT}$
where n_0 = no. of free electrons at absolute zero, n = no. of free electrons at temperature T kelvin, E_g = energy gap, k = Boltzmann constant.
When E_g increases, n decreases exponentially.

32. **(12)** Right hand diode is reversed biased and left hand diode is forward biased.
Hence Voltage at 'A'
$V_A = 12.7 - 0.7 = 12$ volt

33. **(40)** Current in the circuit, $I = \dfrac{12-8}{400} = 10^{-2} A$

Power dissipited in each diode, $P = VI$

$\Rightarrow \quad P = 4\times10^{-2} = 40\,mW$

34. Reverse

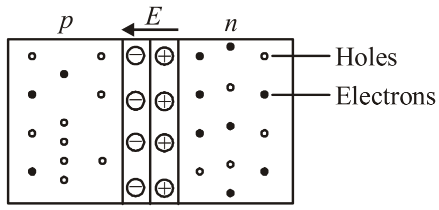
35. To act as full wave rectifier, *B and D is a.c. input* and *A and C is the d.c. output.*
When B is –ve and D is +ve
Current follows path *DA CB*
When B is + ve and D is – ve
Current follows path *VA CB*

36. Reverse, negative terminal.

37. Positive, p-side, n-side

38. **True;** When the cathode temperature is higher, then more number of electrons will be emitted which in turn will increase the anode current. There fore $T_2 > T_1$.

39. **(c)** At junction in a p-n junction diode, a potential barrier/ depletion layer is formed as shown, with n-side at higher potential and p-side at lower potential.



Therefore, there is an electric field at the junction directed from the n-side to p-side

40. **(a,c)** Holes are electron vacancies which participate in electrical conductivity. These are produced in semiconductors. In n-type semiconductor electrons are the charge carriers.

41. **(b)** When the resistances will be equal, the potential drops will be equal. In circuit –2 both the diodes are forward biased whereas in circuit –3 both, the diodes are reversed biased.

42. **(b, d)** To make a P-type semiconductor, tri-valent impurity atoms like boron, aluminium ete should be dopped with pure silicon.

43. **(a)** A diode can used as a rectifier.

Topic-2 : Junction Transistor

1. **(Bonus)** $\beta = \dfrac{\Delta i_c}{\Delta i_b} = \dfrac{200-100}{10-5} = 20$

Voltage gain $= \beta \dfrac{R_2}{R_1} = \dfrac{20\times100\times10^3}{100} = 20\times10^3$

Power gain $= \beta^2 \dfrac{R_2}{R_1} = 20^2\left(\dfrac{100\times10^3}{100}\right) = 4\times10^5$

2. **(a)** Power gain $= 60 = 10\log\left(\dfrac{P_0}{P_i}\right)$

$\Rightarrow 6 = \log\left(\dfrac{P_0}{P_i}\right) \Rightarrow \dfrac{P_0}{P_i} = 10^6 = \beta^2\left(\dfrac{R_{out}}{R_{in}}\right)$

$\Rightarrow 10^6 = \beta^2\left(\dfrac{10000}{100}\right)$ [as $R_{out} = 10,000\Omega$ $R_{in} = 100\Omega$]

$\Rightarrow \beta = 100$

3. **(b)** $\beta = \dfrac{\Delta Ic}{\Delta Ib} = \dfrac{3\times10^{-3}}{15\times10^{-6}} = 200$

We have $\frac{V_0}{V_i} = \beta \frac{R^2}{R_1}$ or $\frac{V_0}{V_i} = 200\left(\frac{1000}{R_1}\right)$

If $R_1 = 0.67\text{k}\Omega \Rightarrow \frac{V_0}{V_i} = 300$

4. **(a)** Given, $\beta = 250$

Voltage gain, $\frac{V_{CC}}{V_B} = \beta \frac{R_C}{R_B}$

$$\frac{10}{V_B} = 250 \times \frac{10^3}{R_B}$$

$$\therefore \frac{V_B}{R_B} = \frac{1}{25 \times 10^3} = 40\mu\text{A}$$

5. **(a)** At saturation, $V_{CE} = 0$

$V_{CE} = V_{CC} - I_C R_C \Rightarrow I_C = \frac{V_{CC}}{R_c} = 5 \times 10^{-3}$ A

Current gain,

$$\beta_{dc} = \frac{I_C}{I_B}$$

$$I_B = \frac{5 \times 10^{-3}}{200} = 25\ \mu\text{A}$$

At input side

$V_{BB} = I_B R_B + V_{BE} = (25\text{ mA})(100\text{ k}\Omega) + 1\text{V}$

$V_{BB} = 3.5$ V

6. **(d)** Curent gain $\beta = \frac{\Delta I_C}{I_B}$

Voltage gain A_v = Current gain × Resistance gain = $\beta \frac{R_L}{R_{BE}}$

Power gain A_p = (Current gain)^2 × Resistance gain

$= \beta^2 \frac{R_L}{R_{BE}}$

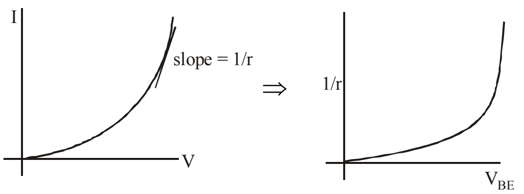
7. **(b)** In common emitter configuration for *n-p-n* transistor input and output signals are 180° out of phase *i.e.,* phase difference between output and input voltage is 180°.

8. **(b)** We know that $\alpha = \frac{I_c}{I_e}$ and $\beta = \frac{I_c}{I_b}$

Also $I_e = I_b + I_c$

$$\therefore \alpha = \frac{I_c}{I_b + I_c} = \frac{\frac{I_c}{I_b}}{1 + \frac{I_c}{I_b}} = \frac{\beta}{1+\beta}$$

9. **(c)**



10. **(c)** In the given question, A, B and C refer base, collector and emitter respectively.

11. **(d)** When the potential of both grid and plate is positive then the, plate current in a triode is maximum for a given plate voltage. The electrons emitted by emitter are collected to the maximum by the plate in this case.

12. **(b)** Amplification factor, $\mu = g_m \times r_0$

Given: g_m = mutual conductance = 1.5×10^{-3} A/V

r_0 = plate resistance = $3 \times 10^3\ \Omega$

$\therefore\ \mu = 3 \times 10^3 \times 1.5 \times 10^{-3} = 4.5$

13. **(150)**

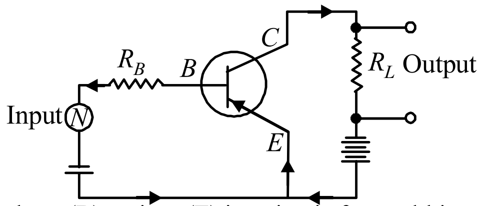
At $V_{CE} = 10$ V and $I_C = 4$ mA

Change in base current, $\Delta I_B = (30 - 20) = 10\ \mu$A

Change in collector current, $\Delta I_C = (4.5 - 3) = 1.5$ mA

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right) = \frac{1.5\ mA}{10\mu A} = 150$$

14. **(a,c)** The circuit for a *p-n-p* transistor used in the common emitter mode as an amplifier is shown in figure.



The base (B) emitter (E) junction is forward-biased and the input signal is connected in series with the voltage applied to bias the base emitter junction.

15. **(b,c)** Given: $I_c = 10$ mA

And I_c = 90% of I_e

$$\therefore I_e = \frac{I_e}{90\% I_e} = \frac{10}{0.9} \simeq 11mA\ I_e = I_b + I_c\ 11 = I_b + 10$$

$$\therefore I_b = 11 - 10 \text{ or, } I_b = 1mA$$

16. Given: Anode current, $I = 0.125\ V - 7.5$

$\Rightarrow dI = 0.125\ dV$ or $\frac{dV}{dI} = \frac{1}{0.125} = 8$ V/A

$\therefore$ Plate resistance, $r_p = \frac{dV}{dI} = 8\text{m}\Omega$

Transconductance, $g_m = \left[\frac{dI}{dV_g}\right]_{V=\text{constt}}$

At $V_g = -1$ volt, $V = 300$ volt, the plate current

$I = [0.125 \times 300 - 7.5]$ mA = 30 mA

Also it is given that $V_g = -3V$, $V = 300\ V$ and $I = 5$mA

$$\therefore g_m = \left[\frac{30-5}{-1-(-3)}\right] = \frac{25}{2} \times 10^{-3} = 12.5 \times 10^{-3} \text{ A/V}$$

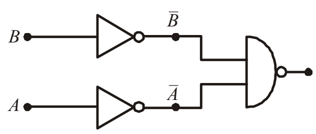
$\therefore$ Amplification factor, μ

$= r_p \times g_m = 8 \times 10^3 \times 12.5 \times 10^{-3} = 100$

Topic-3 : Digital Electronics and Logic Gates

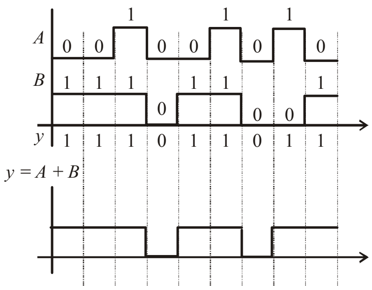
1. **(a)** Boolean expression,

$$y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$



Truth table :

A	B	Y
0	1	1
1	0	1
0	0	0
1	1	1



2. **(c)** When two inputs of NAND gate is shorted, it behaves like a NOT gate so boolen equation will be

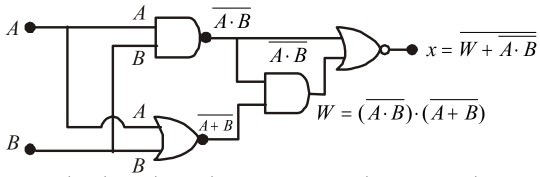
$y = \overline{\overline{A} + \overline{B} + \overline{C}}$

$y = A \cdot B \cdot C$

A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

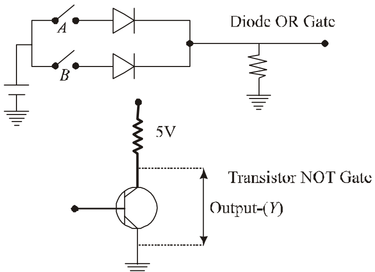
Thus, whole arrangement behaves like a AND gate.

3. **(a)**



A	B	$\overline{A \cdot B}$	$\overline{A + B}$	$W = (\overline{A \cdot B}) \cdot (\overline{A + B})$	$Q = W + \overline{A \cdot B}$	$\overline{Q} = x$
1	0	1	0	0	1	0
0	1	1	0	0	1	0
1	1	0	0	0	0	1
0	0	1	1	1	0	0

4. **(d)**

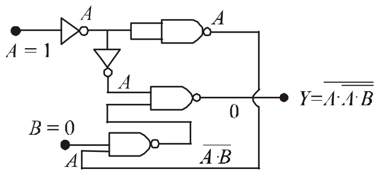


OR + NOT $\rightarrow$ NOR Gate

Hence Boolean relation at the output stage – Y for the circuit,

$Y = \overline{A + B} = \overline{A}.\overline{B}$

5. **(a)**



$Y = \overline{\overline{AB}.A} = \overline{\overline{AB}} + \overline{A} = AB + \overline{A}$

For $A = 1, B = 0$

$Y = (1) \times 0 + 0$

$\Rightarrow Y = 0 + 0 = 0$

6. **(d)** A logic gate is reversible if we can recover input data from the output. Hence NOT gate.

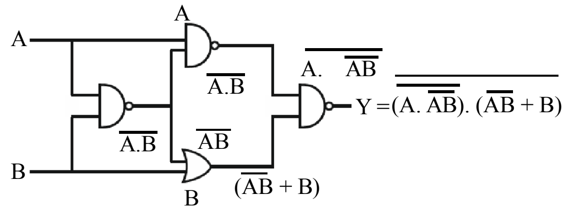
7. **(c)**

A	B	(A + B)	(A + B). A	$\overline{(A + B).A}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

8. **(b)** Truth table $\rightarrow$
The output is of OR-gate

A	B	$\overline{A}$	$\overline{B}$	$\overline{\overline{A}.\overline{B}}$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

9. **(c)**



$$Y = \left(\overline{\overline{A \cdot \overline{AB}}}\right) + \left(\overline{\overline{\overline{AB} + B}}\right)$$

$$= A \cdot \overline{AB} + AB \cdot \overline{B} = A \cdot \left(\overline{A} + \overline{B}\right) + AB \cdot \overline{B}$$

$$= A\overline{B}$$

10. **(c)** From the given logic circuit,

$p = \overline{x} + y \qquad Q = \overline{\overline{y}.x} = y + \overline{x}$

Output, R = $\overline{P + Q}$

To make output 1

P + Q must be '0'

So, x = 1, y = 0

11. **(c)** Truth table of the circuit is as follows

x	y	$\overline{x}$	$a = x.y$	$b = \overline{x}.y$	$z = \overline{a.b}$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	0	1	0	1

12. **(a)** In case of an 'OR' gate the input is zero when all inputs are zero. If any one input is ' 1', then the output is '1'.

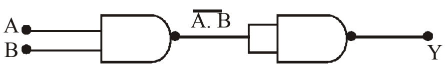
13. **(c)** Truth table for given logical circuit

a	b	(a + b)	c	Y = (a + b) .c
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

Output of OR gate must be 1 and c = 1

So, a = 1, b = 0 or a = 0, b = 1.

14. **(a)**



$Y = \overline{\overline{AB}.\overline{AB}} = \overline{\overline{AB}} + \overline{\overline{AB}} = AB + AB = AB$

In this case output Y is equivalent to AND gate.

Topic-1 : Communication Systems

1. **(b)** From the given expression,
$V_m = 5\,(1 + 0.6 \cos 6280t) \sin(211 \times 10^4 t)$
Modulation index, $\mu = 0.6$

$\because A_m = \mu A_c$

$$\frac{A_{\max} + A_{\min}}{2} = A_c = 5 \quad \text{...(i)}$$

$$\frac{A_{\max} - A_{\min}}{2} = A_m = 3 \quad \text{...(ii)}$$

From equation (i) + (ii),
Maximum amplitude, $A_{\max} = 8$.
From equation (i) – (ii),
Minimum amplitude $A_{\min} = 2$.

2. **(d)** Modulation index, $\mu = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$

Given, $fe = \frac{20000\pi}{2\pi} = 10000$ Hz. and $fm = \frac{2000\pi}{2\pi} = 1000$ Hz.
$\therefore$ LSB $= f_e - f_m = 10000 - 1000 = 9000$ Hz.

3. **(c)** Range of frequency $= (f_c - f_m)$ to $(f_c + f_m)$
$\therefore$ Band width $= 2f_m = 2 \times 100 \times 10^6$ Hz $= 2 \times 10^8$ Hz

and Modulation index $= \frac{A_m}{A_c} = \frac{100}{400} = 0.25$

4. **(a)**

5. **(b)** Size of antenna,

$l = \frac{\lambda}{4}$. As $\lambda = \frac{C}{f}$ $\therefore l \propto \frac{1}{f}$

6. **(b)** Carrier waves of wavelength 1500 nm is used in modern optical fiber communication.

7. **(c)** LOS $= \sqrt{2h_T R} + \sqrt{2h_R R}$

or $50 \times 103 = \sqrt{2h_T \times 6.4 \times 10^6} + \sqrt{2 \times 70 \times 6.4 \times 40^6}$

On solving, $h_T = 32$ m

8. **(c)** Maximum amplitude $= E_m + E_c = 160$
$E_m + 100 = 160$
$E_m = 160 - 100 = 60$
Modulation index,

$\mu = \frac{E_m}{E_c} = \frac{60}{100}$ or $\mu = 0.6$

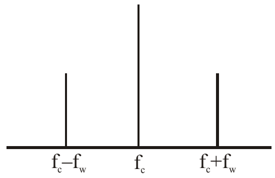
9. **(c)** As we know, Range $= \sqrt{2hR}$
therefore to double the range height 'h' should be 4 times.

10. **(c)** Equation given
$V(t) = 10\,[1 + 0.3 \cos (2.2 \times 10^4)]$
$\sin (5.5 \times 10^5 t)$
$= 10 + 1.5\,[\sin (57.2 \times 10^4 t) + \sin (52.8 \times 10^4 t)]$
$\omega_c + \omega_w = 57.2 \times 10^4 = 2\pi f_1$

$$f_1 = \frac{57.2 \times 10^4}{2 \times \left(\frac{22}{7}\right)} = 9.1 \times 10^4 \simeq 91\,\text{KHz}$$

$\omega_c - \omega_w = 52.8 \times 10^4$

$$f_2 = \frac{52.8 \times 10^4}{2 \times \left(\frac{22}{7}\right)} \simeq 84\,\text{KHz}$$



Upper side band frequency (f_1) is

$$f_1 = f_c - f_w = \frac{52.8 \times 10^4}{2\pi} \approx 85.00 \text{ kHz}$$

Lower side band frequency (f_2) is

$$f_2 = f_c + f_w = \frac{57.2 \times 10^4}{2\pi} \approx 90.00 \text{ kHz}$$

11. **(c)** After analysing the graph we may conclude that
(i) Amplitude varies as 8–10 V or 9 ± 1
(ii) Two time period as
100 μs (signal wave) & 8 μs (carrier wave)
So, equation of AM signal is

$$\left[9 \pm 1 \sin\left(\frac{2\pi t}{T_1}\right) \sin\left(\frac{2\pi t}{T_2}\right)\right]$$

$= [9 \pm \sin (2\pi \times 10^4 t)] \sin (2.5\pi \times 10^5 t)V]$

12. **(a)** Maximum distance upto which signal can be broadcasted

$d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

where h_T and h_R are heights of transmission tower and receiving antenna respectively.
Putting values R, hT and hR

$d_{\max} = \sqrt{2 \times 6.4 \times 106}\left[\sqrt{140} + \sqrt{40}\right]$ or, $d_{\max} \simeq 65$ km

13. **(d)** According to question, modulation frequency, 250 Hz is 10% of carrier wave

$f_{carrier} = \frac{250}{0.1} = 2500\,\text{KHZ}$

$\therefore$ Range of signal 2500 ± 250 KHz $= 2250$ Hz to 2750 Hz For 2000 KHZ

$f_{mod} = 200$ Hz

$\therefore$ Range = 1800 KHZ to 2200 KHZ

14. **(c)** Frequency, $f = \frac{V}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-7}} = \frac{30}{8} \times 10^{14}$ Hz

$= 3.75 \times 10^{14}$ Hz
1% of $f = 0.0375 \times 10^{14}$ Hz
$= 3.75 \times 10^{12}$ Hz $= 3.75 \times 10^6$ MHz
As we know, number of channels accomodated for transmission =

$$\frac{\text{total bandwidth of Channel}}{\text{bandwidth needed per channel}} = \frac{3.75 \times 10^6}{6} = 6.25 \times 10^5$$

15. **(c)** If n = no. of channels
10% of 10 GHz $= n \times 5$ KHz or,
$\Rightarrow n = 2 \times 10^5$

16. **(a)** Given : modulation index $m = 80\% = 0.8$
$E_c = 14$ V, $E_m = ?$

using, $m = \frac{E_m}{E_c} \Rightarrow E_m = m \times E_c = 0.8 \times 14 = 11.2$V

17. **(b)** Given, modulating frequency f_m = 15 KHz

$\therefore$ Bandwidth of one channel = $2f_m$ = 30 kHz

$\therefore$ No of channels accommodate = $\frac{300\text{kHz}}{30\text{kHz}} = 10$

18. **(c)** Given : Inductance, L = 49 μH = 49 × 10^{-6} H, capacitance C = 2.5 nF = 2.5 × 10^{-9} F

Using $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{49\times10^{-6}\times\frac{2.5}{10}\times10^{-9}}} = \frac{1}{7\times5\times10^{-8}} = \frac{10^8}{7\times5}$$

or, $\frac{10^8}{7\times5} = 2\pi\times f = 2\times\frac{22}{7}\times f \quad (\because\ \omega = 2\pi f)$

or, $f = \frac{10^7}{22} = \frac{10^4}{22}\text{kHz} = 454.54\text{kHz}$

Therefore frequency range 454.54 ± 12kHz

i.e., 442 kHz – 466 kHz

19. **(c)** Modulated carrier wave contains frequency ω_c and $\omega_c \pm \omega_m$

20. **(a)** Length of antenna = comparable to λ

Power radiated by linear antenna inversely depends on the square of wavelength and directly on the length of the antenna. Hence,

Power $P = \mu\left(\frac{1}{\lambda}\right)^2$

here μ = K

21. **(d)** Modulation index (m) $= \frac{V_m}{V_0} = \frac{5}{25} = 0.2$

Given, frequency of carrier wave (f_c) = 1.2 × 10^6 Hz
= 1200 kHz.

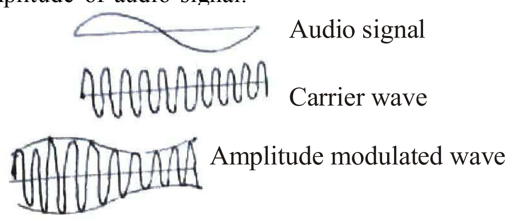
Frequency of signal (f_0) = 20 kHz.

Side frequency bands = $f_c \pm f_0$

f_1 = 1200 – 20 = 1180 kHz

f_2 = 1200 + 20 = 1220 kHz

22. **(c)** In amplitude modulation, the amplitude of the high frequency carrier wave made to vary in proportional to the amplitude of audio signal.



23. **(b)** Comparing the given equation with standard modulated signal wave equation, m = $A_c \sin \omega_c t + \frac{\mu A_c}{2}$

$\cos(\omega_c - \omega_s)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_s)t$

$m\frac{A_c}{2} = 10$ Þ $m = \frac{2}{3}$ (modulation index)

$A_c = 30$

$\omega_c - \omega_s = 200\pi$

$\omega_c + \omega_s = 400\pi$

$\Rightarrow f_c = 150,\ f_s = 50$ Hz.

24. **(c)** Ratio of AM signal Bandwidths

$= \frac{15200 - 200}{2700 - 200} = \frac{15000}{2500} = 6.$

25. **(a)** Amplitude modulated wave consists of three frequencies are $\omega_c + \omega_m$, ω, $\omega_c - \omega_m$

i.e. 2005 kHz, 2000kHz, 1995 kHz

26. **(b)** Frequency of radio waves for sky wave propagation is 2 MHZ to 30 MHZ.

27. **(b)** Sky wave propagation is suitable for frequency range 5 MHz to 25 MHz.

28. **(b)** Given : h_R = 32 m

h_T = 50 m

Maximum distance, d_M = ?

Applying, $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$= \sqrt{2\times6.4\times10^6\times50} + \sqrt{2\times6.4\times10^6\times32} = 45.5$ km

29. **(b)** **Given :** Resistance R = 100 kilo ohm

= 100 × 10^3 Ω

Capacitance C = 250 picofarad

= 250 × 10^{-12}F

τ = RC = 100 × 10^3 × 250 × 10^{-12} sec

= 2.5 × 10^7 × 10^{-12} sec

= 2.5 × 10^{-5} sec

The higher frequency whcih can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} = \frac{1}{2\pi\times0.6\times2.5\times10^{-5}}Hz$$

$$= \frac{100\times10^4}{25\times1.2\pi}Hz = \frac{4}{1.2\pi}\times10^4\,Hz$$

= 10.61 KHz

This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated singnal voltage for proper detection of mdoulated signal.

30. **(b)** Short-wave has the best noise tolerance.

31. **(c)** Above critical frequency (f_c), an electromagnetic wave penetrates the ionosphere and is not reflected by it.

32. **(a)** Effective refractive index of the ionosphere

$$n_{eff} = n_0\left[1 - \frac{80.5N}{f^2}\right]^{1/2}$$

where f is the frequency of em waves

33. **(c)** Modulation index

$$m_a = \frac{E_m}{E_c} = \frac{A}{A} = 1$$

Equation of modulated signal [$C_m(t)$]

$= E_{(C)} + m_a E_{(C)} \sin \omega_m t$

$= A(1 + \sin w_c t) \sin \omega_m t$

(As $E_{(C)} = A \sin\omega_c t$)