Warsaw University of Technology

FACULTY OF
POWER AND AERONAUTICAL ENGINEERING

Institute of Automation and Robotics

Homework 1

in the subject of Modelling and control of manipulator

Derivation of the kinematic model of the ABB IRb-7600 manipulator

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List of abbreviations

$cos(\theta_i) = c_i$

$sin(\theta_i) = s_i$

$cos(\theta_i + \theta_j) = c_{i+j}$

$sin(\theta_i + \theta_j) = s_{i+j}$

Homework

Derive the kinematic model of the ABB IRb-7600 manipulator, i.e. solve the direct and inverse kinematic problem for that manipulator.

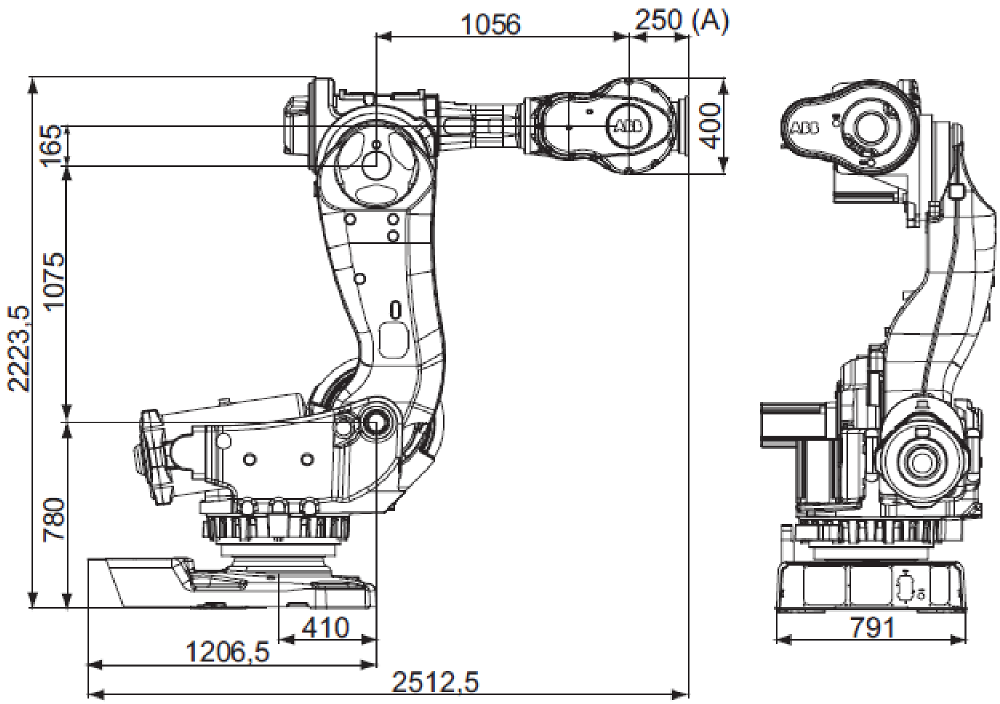

Figure 1 2D Drawing of the Model ABB IRb-7600 Manipulator

Step 1 – Putting the axes

From lecture notes Algorithm producing the kinematic model of a manipulator has steps as follows

- Draw the manipulator
- Identify the axes of rotation or translation (for revolute and prismatic joints)
- Assign the iz axis to the ith axis of rotation/translation
- Find the common perpendiculars between the $i-1z$ and iz axes (if those axes intersect the common perpendicular is defined along $i-1z \times iz$)
- Define the ix axes – the axis i–1x coincides with the common perpendicular between the $i-1z$ and iz axes and points from $i-1z$ to iz
- Determine the iy axes $iy = iz \times ix$
- The base coordinate frame 0 should be located in such a way that $\alpha 0$, $a0$ and $\theta 1$ or $d1$ are equal to 0) $0^z = 1^z$ and ${}^0_0P = {}^0_0P$

Thus based on the steps the axes are as follows

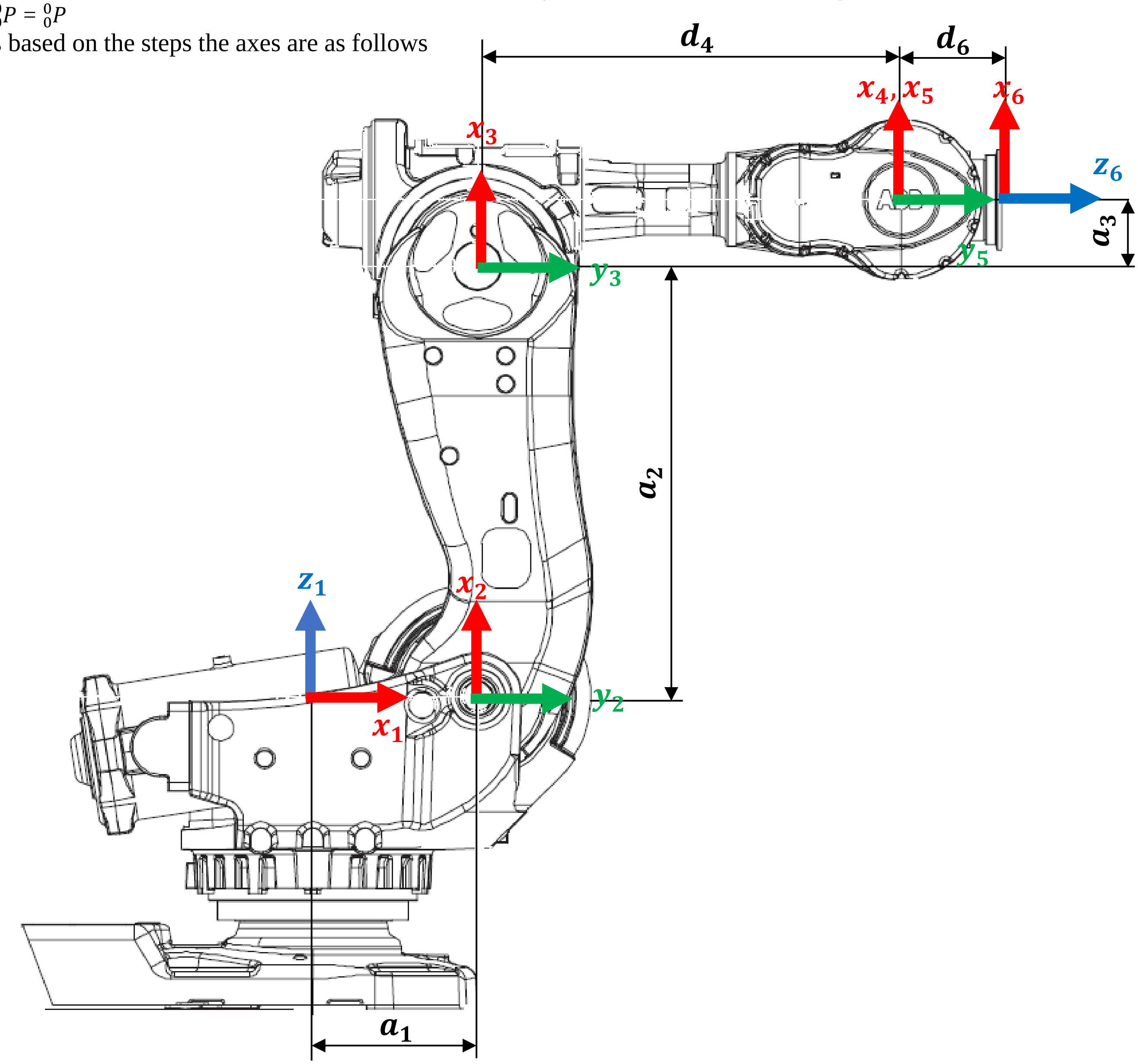

Figure 2 Model manipulator diagram with axes based on the modified Denavit-Hatenberg convention

Step 2 - Denavit-Hartenberg(D-H) parameters

From lecture notes the modified D-H parameters are described as

$a_{i-1} = i-1^{z} \xrightarrow{i-1^{x}} i^{z}$, $\alpha_{i-1} = i-1^{z} \xrightarrow{i-1^{x}} i^{z}$, $d_i = i-1^{x} \xrightarrow{i^{z}} i^{x}$, and $\theta_i = i-1^{x} \xrightarrow{i^{z}} i^{x}$

Table 1 D-H parameters

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	a_1	$-\pi/2$	0	$\theta'_2 = \theta_2 - \pi/2$
3	a_2	0	0	θ_3
4	a_3	$-\pi/2$	d_4	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	d_6	θ_6

Step 3 - Direct Kinematics Problem

1. Joint/link description matrices

From class lecture notes we know that

$$ {}^{i-1}_{i}T = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & a_{i-1} \\ s_{\theta_i}c_{\alpha_{i-1}} & c_{\theta_i}c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & -d_i s_{\alpha_{i-1}} \\ s_{\theta_i}s_{\alpha_{i-1}} & c_{\theta_i}s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & d_i c_{\alpha_{i-1}} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Joint/link description matrices can be found by putting the DH parameters into the matrix above. Thus the matrices are as follows

$$ {}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{5}_{6}T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2. Composition of matrices

To solve the direct kinematics problem form lecture notes we have

$$ {}^{0}_{n}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \ldots {}^{n-1}_{n}T = \prod_{i=1}^{n} {}^{i-1}_{i}T $$

Where n is the end-effector frame,

Thus starting with ${}^0_1T$

$$ {}^0_1\boldsymbol{T} = \begin{bmatrix} \boldsymbol{c_1} & \boldsymbol{-s_1} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{s_1} & \boldsymbol{c_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} $$

- ${}^0_2T = {}^0_1T\,{}^1_2T$

$$ {}^0_2T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 & a_1c_1 \\ c_2s_1 & -s_1s_2 & c_1 & a_1s_1 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- ${}^0_3T = {}^0_2T\,{}^2_3T$

$$ {}^0_3T = \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 & a_1c_1 \\ c_2s_1 & -s_1s_2 & c_1 & a_1c_1 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3\boldsymbol{T} = \begin{bmatrix} \boldsymbol{c_{2+3}c_1} & \boldsymbol{-s_{2+3}c_1} & \boldsymbol{-s_1} & \boldsymbol{c_1(a_1+a_2c_2)} \\ \boldsymbol{c_{2+3}s_1} & \boldsymbol{-s_{2+3}s_1} & \boldsymbol{c_1} & \boldsymbol{s_1(a_1+a_2c_2)} \\ \boldsymbol{-s_{2+3}} & \boldsymbol{-c_{2+3}} & \mathbf{0} & \boldsymbol{-a_2s_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} $$

- ${}^0_4T = {}^0_3T\,{}^3_4T$

$$ {}^0_4T = \begin{bmatrix} c_{2+3}c_1 & -s_{2+3}c_1 & -s_1 & c_1(a_1+a_2c_2) \\ c_{2+3}s_1 & -s_{2+3}s_1 & c_1 & s_1(a_1+a_2c_2) \\ -s_{2+3} & -c_{2+3} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_4\boldsymbol{T} = \begin{bmatrix} \boldsymbol{s_1s_4 + c_{2+3}c_1c_4} & \boldsymbol{c_4s_1 - c_{2+3}c_1s_4} & \boldsymbol{-s_{2+3}c_1} & \boldsymbol{c_1(a_1 + a_3c_{2+3} - d_4s_{2+3} + a_2c_2)} \\ \boldsymbol{c_{2+3}c_4s_1 - c_1s_4} & \boldsymbol{-c_1c_4 - c_{2+3}s_1s_4} & \boldsymbol{-s_{2+3}s_1} & \boldsymbol{s_1(a_1 + a_3c_{2+3} - d_4s_{2+3} + a_2c_2)} \\ \boldsymbol{-s_{2+3}c_4} & \boldsymbol{s_{2+3}s_4} & \boldsymbol{-c_{2+3}} & \boldsymbol{-d_4c_{2+3} - a_3s_{2+3} - a_2s_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} $$

- ${}^0_5T = {}^0_4T\,{}^4_5T$

$$ {}^0_5\boldsymbol{T} = \begin{bmatrix} \boldsymbol{c_5(s_1s_4 + c_{2+3}c_1c_4) - s_{2+3}c_1s_5} & \boldsymbol{-s_5(s_1s_4 + c_{2+3}c_1c_4) - s_{2+3}c_1c_5} & \boldsymbol{c_{2+3}c_1s_4 - c_4s_1} & \boldsymbol{c_1(a_1 + a_3c_{2+3} - d_4s_{2+3} + a_2c_2)} \\ \boldsymbol{-c_5(c_1s_4 - c_{2+3}c_4s_1) - s_{2+3}s_1s_5} & \boldsymbol{s_5(c_1s_4 - c_{2+3}c_4s_1) - s_{2+3}c_5s_1} & \boldsymbol{c_1c_4 + c_{2+3}s_1s_4} & \boldsymbol{s_1(a_1 + a_3c_{2+3} - d_4s_{2+3} + a_2c_2)} \\ \boldsymbol{-c_{2+3}s_5 - s_{2+3}c_4c_5} & \boldsymbol{s_{2+3}c_4s_5 - c_{2+3}c_5} & \boldsymbol{-s_{2+3}s_4} & \boldsymbol{-d_4c_{2+3} - a_3s_{2+3} - a_2s_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} $$

- ${}^{0}_{6}T = {}^{0}_{5}T\,{}^{5}_{6}T$

The output matrix can be written as

$$
{}^{0}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \text{where}
$$

$$\boldsymbol{r_{11} = s_1(c_4s_6 + c_5c_6s_4) - c_1(c_{2+3}(s_4s_6 - c_4c_5c_6) + s_{2+3}c_6s_5)}$$

$$\boldsymbol{r_{12} = s_1(c_4c_6 - c_5s_4s_6) - c_1(c_{2+3}(c_6s_4 + c_4c_5s_6) - s_{2+3}s_5s_6)}$$

$$\boldsymbol{r_{13} = -s_1s_4s_5 - c_1(s_{2+3}c_5 + c_{2+3}c_4s_5)}$$

$$\boldsymbol{p_x = c_1((a_1 + a_2c_2) + c_{2+3}(a_3 - d_6c_4s_5) - s_{2+3}(d_4 + d_6c_5)) - d_6s_1s_4s_5}$$

$$\boldsymbol{r_{21} = c_1(c_4s_6 + c_5c_6s_4) - s_1(c_{2+3}(s_4s_6 - c_4c_5c_6) + s_{2+3}c_6s_5)}$$

$$\boldsymbol{r_{22} = -c_1(c_4c_6 - c_5s_4s_6) - s_1(c_{2+3}(c_6s_4 + c_4c_5s_6) + s_{2+3}s_5s_6)}$$

$$\boldsymbol{r_{23} = -c_1s_4s_5 - s_1(s_{2+3}c_5 + c_{2+3}c_4s_5)}$$

$$\boldsymbol{p_y = s_1((a_1 + a_2c_2) + c_{2+3}(a_3 - d_6c_4s_5) - s_{2+3}(d_4 + d_6c_5)) + d_6c_1s_4s_5}$$

$$\boldsymbol{r_{31} = s_{2+3}(s_4s_6 - c_4c_5c_6) - c_{2+3}c_6s_5}$$

$$\boldsymbol{r_{32} = s_{2+3}(c_6s_4 - c_4c_5s_6) - c_{2+3}s_5s_6}$$

$$\boldsymbol{r_{33} = s_{2+3}c_4s_5 - c_{2+3}c_5}$$

$$\boldsymbol{p_z = -s_{2+3}(a_3 - d_6c_4s_5) - c_{2+3}(d_4 + d_6c_5) - a_2s_2}$$

Step 4 - Inverse kinematic problem solution

Given the following parameters solve the inverse kinematic problem

$$ {}^{0}_{6}T_d = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

1. **Solve $\boldsymbol{\theta_1}$ as a function of given parameters**

$$ {}^{1}_{4}T_d = {}^{0}_{1}T^{-1}\,{}^{0}_{6}T_d\,{}^{4}_{6}T^{-1} $$

$$ {}^{1}_{4}T_d = \begin{bmatrix} c_5c_6(r_{11}c_1 + r_{21}s_1) - s_5(r_{13}c_1 + r_{23}s_1) - c_5s_6(r_{12}c_1 + r_{22}s_1) & \dots & \dots & p_xc_1 - d_6(r_{13}c_1 + r_{23}s_1) + p_ys_1 \\ c_5c_6(r_{21}c_1 - r_{11}s_1) - s_5(r_{23}c_1 - r_{13}s_1) - c_5s_6(r_{22}c_1 - r_{12}s_1) & \dots & \dots & p_yc_1 - d_6(r_{23}c_1 - r_{13}s_1) - p_xs_1 \\ r_{31}c_5c_6 - r_{33}s_5 - r_{32}c_5s_6 & \dots & \dots & p_z - d_6r_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{4}T = \begin{bmatrix} c_{2+3}c_4 & -c_{2+3}s_4 & -s_{2+3} & a_1 + a_3c_{2+3} - d_4s_{2+3} + a_2c_2 \\ -s_4 & -c_4 & 0 & 0 \\ -s_{2+3}c_4 & s_{2+3}s_4 & -c_{2+3} & -d_4c_{2+3} - a_3s_{2+3} - a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

From ${}^{1}_{4}T_{d_{24}} = {}^{1}_{4}T_{24}$

$$ p_yc_1 - d_6(r_{23}c_1 - r_{13}s_1) - p_xs_1 = 0 $$

Thus solving for θ_1

$$ \boldsymbol{\theta_1 = arctan\left(\frac{p_y - d_6r_{23}}{p_x - d_6r_{13}}\right)} $$

2. **Solve $\boldsymbol{\theta_3}$ as a function of $\boldsymbol{\theta_1}$ and given parameters**

$$ {}^{2}_{5}T_d = {}^{0}_{2}T^{-1}\,{}^{0}_{6}T_d\,{}^{5}_{6}T^{-1} $$

$$ {}^{2}_{5}T_d = \begin{bmatrix} c_6(r_{11}c_1c_2 - r_{31}s_2 + r_{21}c_2s_1) - s_6(r_{12}c_1c_2 - r_{32}s_2 + r_{22}c_2s_1) & \dots & \dots & p_xc_1c_2 - a_1c_2 - p_zs_2 - d_6(r_{13}c_1c_2 - r_{33}s_2 + r_{23}c_2s_1) + p_yc_2s_1 \\ s_6(r_{32}c_2 + r_{12}c_1s_2 + r_{22}s_1s_2) - c_6(r_{31}c_2 + r_{11}c_1s_2 + r_{21}s_1s_2) & \dots & \dots & d_6(r_{33}c_2 + r_{13}c_1s_2 + r_{23}s_1s_2) - p_zc_2 + a_1s_2 - p_xc_1s_2 - p_ys_1s_2 \\ c_6(r_{21}c_1 - r_{11}s_1) - s_6(r_{22}c_1 - r_{12}s_1) & \dots & \dots & p_yc_1 - d_6(r_{23}c_1 - r_{13}s_1) - p_xs_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{5}T = \begin{bmatrix} c_3c_4c_5 - s_3s_5 & -c_5s_3 - c_3c_4s_5 & c_3s_4 & a_2 + a_3c_3 - d_4s_3 \\ c_3s_5 + c_4c_5s_3 & c_3c_5 - c_4s_3s_5 & s_3s_4 & d_4c_3 + a_3s_3 \\ -c_5s_4 & s_4s_5 & c_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

From ${}^{2}_{5}T_{d_{14}} = {}^{2}_{5}T_{14}$,

$$ a_2 + a_3c_3 - d_4s_3 = p_xc_1c_2 - a_1c_2 - p_zs_2 - d_6(r_{13}c_1c_2 - r_{33}s_2 + r_{23}c_2s_1) + p_yc_2s_1 $$

From ${}^{2}_{5}T_{d_{24}} = {}^{2}_{5}T_{24}$,

$$ d_4c_3 + a_3s_3 = d_6(r_{33}c_2 + r_{13}c_1s_2 + r_{23}s_1s_2) - p_zc_2 + a_1s_2 - p_xc_1s_2 - p_ys_1s_2 $$

Now rearranging the equation

$$a_2 + a_3c_3 - d_4s_3 = s_2(\boldsymbol{d_6r_{33} - p_z}) - c_2(\boldsymbol{a_1 + c_1(-p_x + d_6r_{13}) - p_ys_1 + d_6r_{23}s_1})$$

$$d_4c_3 + a_3s_3 = c_2(\boldsymbol{d_6r_{33} - p_z}) + s_2(\boldsymbol{a_1 + c_1(-p_x + d_6r_{13}) - p_ys_1 + d_6r_{23}s_1})$$

Introducing the following substitutions

$$\mathrm{E} = d_6r_{33} - p_z$$

$$\mathrm{F} = a_1 + c_1(-p_x + d_6r_{13}) - p_ys_1 + d_6r_{23}s_1$$

Substituting and squaring both sides the following standard system of equations is obtained:

$$(a_2 + a_3c_3 - d_4s_3)^2 = (s_2E - c_2F)^2$$

$$(d_4c_3 + a_3s_3)^2 = (c_2E + s_2F)^2$$

Combining the two equations after squaring

$$a_3c_3 + \frac{-E^2 - F^2 + a_2^2 + a_3^2 + d_4^2}{2a_2} = d_4s_3$$

Introducing the following substitutions

$$G = \frac{-E^2 - F^2 + a_2^2 + a_3^2 + d_4^2}{2a_2}$$

Thus it will be

$$a_3c_3 + G = d_4s_3$$

From this solving for θ_3

$$\boldsymbol{\theta_3} = \mathbf{arctan}\left(\frac{Gd_4 - \sqrt{a_3{}^2(-G^2 + a_3{}^2 + d_4{}^2)}}{(a_3{}^2 + d_4{}^2)\sqrt{1 - \frac{\left(-Gd_4 + \sqrt{a_3{}^2(-G^2 + a_3{}^2 + d_4{}^2)}\right)^2}{(a_3{}^2 + d_4{}^2)^2}}}\right), \text{ or } \boldsymbol{\theta_3} = \mathbf{arctan}\left(\frac{Gd_4 + \sqrt{a_3{}^2(-G^2 + a_3{}^2 + d_4{}^2)}}{(a_3{}^2 + d_4{}^2)\sqrt{1 - \frac{\left(Gd_4 + \sqrt{a_3{}^2(-G^2 + a_3{}^2 + d_4{}^2)}\right)^2}{(a_3{}^2 + d_4{}^2)^2}}}\right)$$

3. Solve $\boldsymbol{\theta_2}$ as a function of $\boldsymbol{\theta_3}$ and given parameters

From ${}^1_4T_{d_{34}} = {}^1_4T_{34}$,

$$p_z - d_6r_{33} = -d_4c_{2+3} - a_3s_{2+3} - a_2s_2$$

From this solving for $\theta_2{}'$

$$\theta_2' = \mathbf{2arctan}\left(\frac{a_2 + a_3c_3 - d_4s_3 \pm \sqrt{a_3^2c_3^2 + d_4^2c_3^2 + a_3^2s_3^2 + d_4^2s_3^2 + a_2^2 - p_z^2 - d_6^2r_{33}^2 - 2a_2d_4s_3 + 2d_6r_{33} + 2a_2a_3c_3}}{d_6r_{33} - p_z + d_4c_3 + a_3s_3}\right)$$

Remember this is solution for $\theta_2{}'$ to solve for $\boldsymbol{\theta_2}$ use the equation

$$\theta'_2 = \theta_2 - \pi/2$$

4. **Solve $\boldsymbol{\theta_5}$ as a function of $\boldsymbol{\theta_1, \theta'_2, \theta_3}$ and given parameters**

$$ {}^3_6T_d = {}^0_3T^{-1}\,{}^0_6T_d $$

$$ {}^3_6T_d = \begin{bmatrix} r_{11}c_{2+3}c_1 - r_{31}s_{2+3} + r_{21}c_{2+3}s_1 & \dots & \dots & p_xc_{2+3}c_1 - p_zs_{2+3} - a_2c_3 - a_1c_{2+3} + p_yc_{2+3}s_1 \\ -r_{31}c_{2+3} - r_{11}s_{2+3}c_1 - r_{21}s_{2+3}s_1 & \dots & \dots & a_1s_{2+3} - p_zc_{2+3} + a_2s_3 - p_xs_{2+3}c_1 - p_ys_{2+3}s_1 \\ r_{21}c_1 - r_{11}s_1 & \dots & \dots & p_yc_1 - p_xs_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_6s_4 - c_4c_5s_6 & -c_4s_5 & a_3 - d_6c_4s_5 \\ c_6s_5 & -s_5s_6 & c_5 & d_4 + d_6c_5 \\ -c_4s_6 - c_5c_6s_4 & c_5s_4s_6 - c_4c_6 & s_4s_5 & d_6s_4s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

From ${}^3_6T_{d_{23}} = {}^3_6T_{23}$

$$ c_5 = -r_{33}c_{2+3} - r_{13}s_{2+3}c_1 - r_{23}s_{2+3}s_1 $$

Thus θ_5 will be

$$ \boldsymbol{\theta_5 = \arctan\left(\frac{\sqrt{1 - (-r_{33}c_{2+3} - r_{13}s_{2+3}c_1 - r_{23}s_{2+3}s_1)^2}}{-r_{33}c_{2+3} - r_{13}s_{2+3}c_1 - r_{23}s_{2+3}s_1}\right)} $$

5. **Solve $\boldsymbol{\theta_6}$ as a function of $\boldsymbol{\theta'_2, \theta_3, \theta_5}$ and given parameters**

From ${}^3_6T_{d_{21}} = {}^3_6T_{21}$

$$ c_6s_5 = -r_{31}c_{2+3} - r_{11}s_{2+3}c_1 - r_{21}s_{2+3}s_1 $$

Thus θ_6 will be

$$ \boldsymbol{\theta_6 = \arctan\left(\frac{\sqrt{1 - \left(\frac{-r_{31}c_{2+3} - r_{11}s_{2+3}c_1 - r_{21}s_{2+3}s_1}{s_5}\right)^2}}{\frac{-r_{31}c_{2+3} - r_{11}s_{2+3}c_1 - r_{21}s_{2+3}s_1}{s_5}}\right)} $$

Where $s_5 \neq 0$

6. **Solve $\boldsymbol{\theta_4}$ a function of $\boldsymbol{\theta_6}$, and given parameters**

From ${}^1_4T_{d_{22}} = {}^1_4T_{22}$,

$$ -c_4 = c_6(r_{22}c_1 - r_{12}s_1) + s_6(r_{21}c_1 - r_{11}s_1) $$

Thus θ_4 will be

$$ \boldsymbol{\theta_4 = \arctan\left(\frac{\sqrt{1 - \left(-c_6(r_{22}c_1 - r_{12}s_1) - s_6(r_{21}c_1 - r_{11}s_1)\right)^2}}{-c_6(r_{22}c_1 - r_{12}s_1) - s_6(r_{21}c_1 - r_{11}s_1)}\right)} $$

Conclusion

In this homework the derivation of the kinematic model of the ABB IRb-7600 manipulator is done, i.e. solving the direct and inverse kinematic problem for that manipulator. During the derivation of the equations MATLAB 2021b has been used to solve the matrix problems.