A laboratory report on

ANALYSIS OF STRESS DISTRIBUTION IN A THICK CYLINDER

By

Idoko Job John
@00512412

Submitted to

Dr Babaie Meisam

Petroleum and Gas Engineering Division, School of Computing Science and Engineering, University of Salford, Manchester

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ABSTRACT

In this experiment, a thick cylinder was subjected to internal loading and the behaviour of the stress distribution within it was investigated. A thick cylinder has been identified as one with a thickness not less than one-twentieth of the internal diameter.

To achieve the above-specified aim, a thick cylinder was subjected to internal loading using a hydraulic ram and three objectives were carried out: (i) the hoop strains and radial strains distribution were examined against the radius of propagation of the strain; (ii) the hoop stresses and radial stresses distribution, in the cylinder, with respect to the strain location were also computed and; (iii) the parameters obtained from the experiment were compared with those obtained using established theorems, specifically, Lame's Equation.

The experiment revealed that, while the hoop strain produced in the cylinder increases as the depth of plug penetration increases, the radial strain decreases with increase in the depth of plug penetration. On the other hand, the circumferential stress decreases as the strain propagates away from the centre of the thick cylinder while the radial stress increases with increase in the radial position.

It was also discovered that the values of the stress parameters computed using the Lame's equation differ significantly from those estimated from the experiment.

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1. INTRODUCTION

1.1 Background of Study

1.1.1 Thick and Thin Cylinders

A cylinder is referred to as "a thin-walled cylinder if its thickness is smaller than one-twentieth of its internal diameter", otherwise, it is referred to as "a thick-walled cylinder" or simply, "thick cylinder" (Raju et al., 2015). Pressure vessels are designed to be either thick-walled or thin-walled depending on how much load they are likely to be subjected to on-the-job. It is, therefore, expected that high-pressure vessels such as gas transmission pipelines and crude oil separators are thick cylinders while canned drinks and gas distribution pipes are examples of thin cylinders (Thin and Thick Cylinders, n.d.).

Accurate knowledge of the stress distribution in pressure vessels is essential for design purposes, safety, and structural stability. This experiment investigates how the stress exerted within a thick cylinder is distributed as it is subjected to internal pressure.

1.2 Literature Review

When a cylinder is under pressure, it experiences three mutually perpendicular principal stresses within its walls. These forces stresses are (I) Hoop or circumferential stress, σ_θ; (II) Longitudinal or axial stress, σ_L and (III) Radial stress, σ_r (Advanced Structural Analysis EGF316, n.d.).

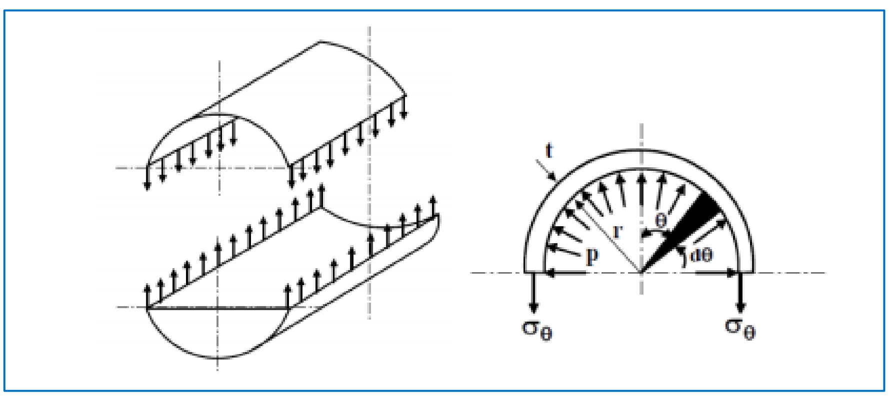

Figure 1-1: Illustration of Hoop's Stress (Elso, 2012)

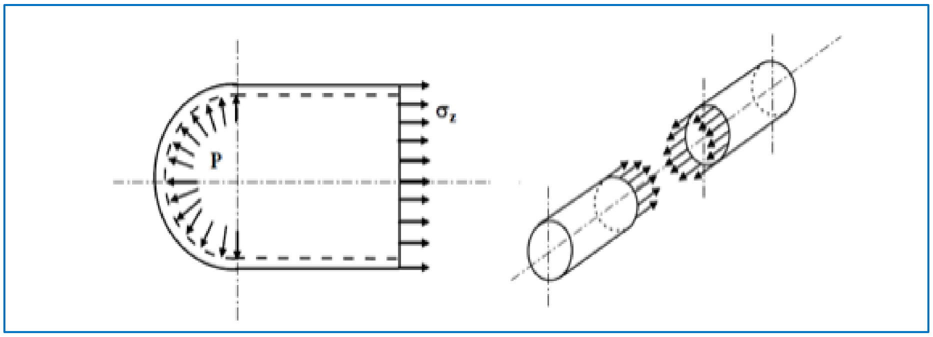

Figure 1-2: Illustration of Longitudinal or Axial Stress (Elso, 2012)

The **Hoop stress** is that stress that acts along the circumference of the cylinder and tends to pull the cylinder into two halves along its length. The **axial stress** acts along the pipe length and tends to cause failure across its circumference while the **radial stresses**, according to (Elso, 2012) are “normal to the curved plane of the isolated element” and “In thin-walled cylinder theory, they are normally not considered, because they are negligible compared to the other two stresses”. Figures 1 and 2 illustrate Hoop and Axial stresses.

1.3 Theorem

1.3.1 Lame’s Theorem

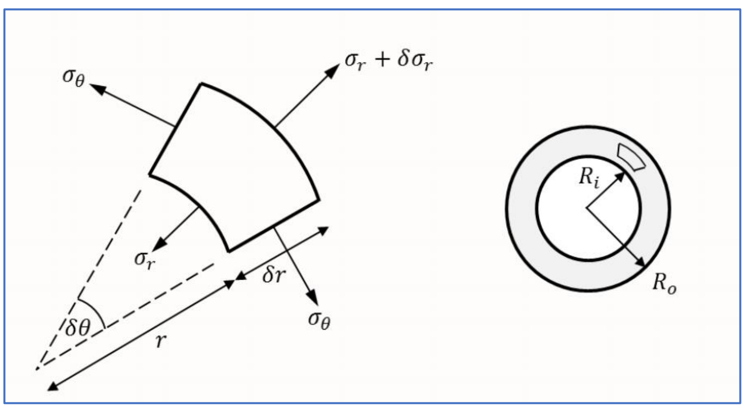

Figure 1-3: Lame's theorem (Advanced Structural Analysis, n.d.)

Thick cylinder stress analysis is governed by Lame’s theorem with the following assumptions (Lame’s Theorem - Strength of Materials, n.d.).

- The microstructure and components of the cylinder are isotropic and not heterogenous

- Plane sections of the cylinder perpendicular to the longitudinal axis remain plane under the pressure.

The second assumption implies that the longitudinal strain is the same at all points, i.e., the strain is independent of the radius.

Considering Figure 1-3, “Lame’s equations for a thick cylinder give”:

$$\sigma_\theta = A + \frac{B}{r^2} \quad \text{...} \quad \textbf{Equation 1.1}$$

$$\sigma_r = A - \frac{B}{r^2} \quad \text{...} \quad \textbf{Equation 1.2}$$

Where,

E = Young’s modulus of elasticity, μ = Poisson’s ratio and, A and B are constants.

“Applying boundary conditions: $\sigma_r = -p$ at R_i and $\sigma_r = 0$ at R_o, gives the following:

$$A = \frac{PR_i^2}{R_o^2 - R_i^2} \quad \text{...} \quad \textbf{Equation 1.3}$$

And

$$B = \frac{PR_o^2 R_i^2}{R_o^2 - R_i^2} \quad \text{...} \quad \textbf{Equation 1.4}$$

Where,

P = internal pressure, R_i = inner radius of the cylinder, R_o = outer radius of the cylinder”.

Alternatively, if the Young’s modulus and the Poisson ratio of the cylinder are known, the hoop and radial stresses can be obtained using the following equations (Roylance, 2001):

“Hoop stress, $\sigma_\theta = \frac{E}{1-\mu^2}(e_\theta + \mu e_r)$ **.. Equation 1.5**

Radial stress, $\sigma_r = \frac{E}{1-\mu^2}(e_r + \mu e_\theta)$ **..Equation 1.6”**

2. AIM AND OBJECTIVES OF THE EXPERIMENT

2.1 The Aim of the Experiment.

This experiment aims to investigate the distribution of the stress generated in a thick cylinder when it is under the effect of internal pressure.

2.2 Objectives of the Experiment

To achieve the above aim, the following objectives were outlined:

i. To investigate the effect of plug penetration on the hoop strain and radial strain in a thick cylinder subjected to internal pressure.
ii. To examine how the hoop stress and radial stress behave with respect to the strain position when a thick cylinder is subjected to internal pressure.
iii. Compare the experimental analysis of the stress distribution in a thick cylinder with a theoretical analysis based on established theorems.

3. DESCRIPTION OF APPARATUS

3.1Equipment description

Figure 3.1 shows the set of the apparatus for the experiment. The apparatus consists of a supportive frame, designed to provide structural support and withstand the maximum pressure that the thick-walled cylinder will be subjected to at the course of the experiment, a thick walled-cylinder of an outer radius (R_o) of $152mm$ and an inner radius (R_t) of $77.5mm$ containing 10 strain gauges, a pressure gauge which measures the pressure inside the cylinder, a hydraulic hand pump for pressurising the system, a hydraulic ram lubricated with oil, tapered plug and a data logger or Versatile Data Acquisition system (VDAS). The mode of operation of the apparatus is described in the next section along with the experimental procedures.

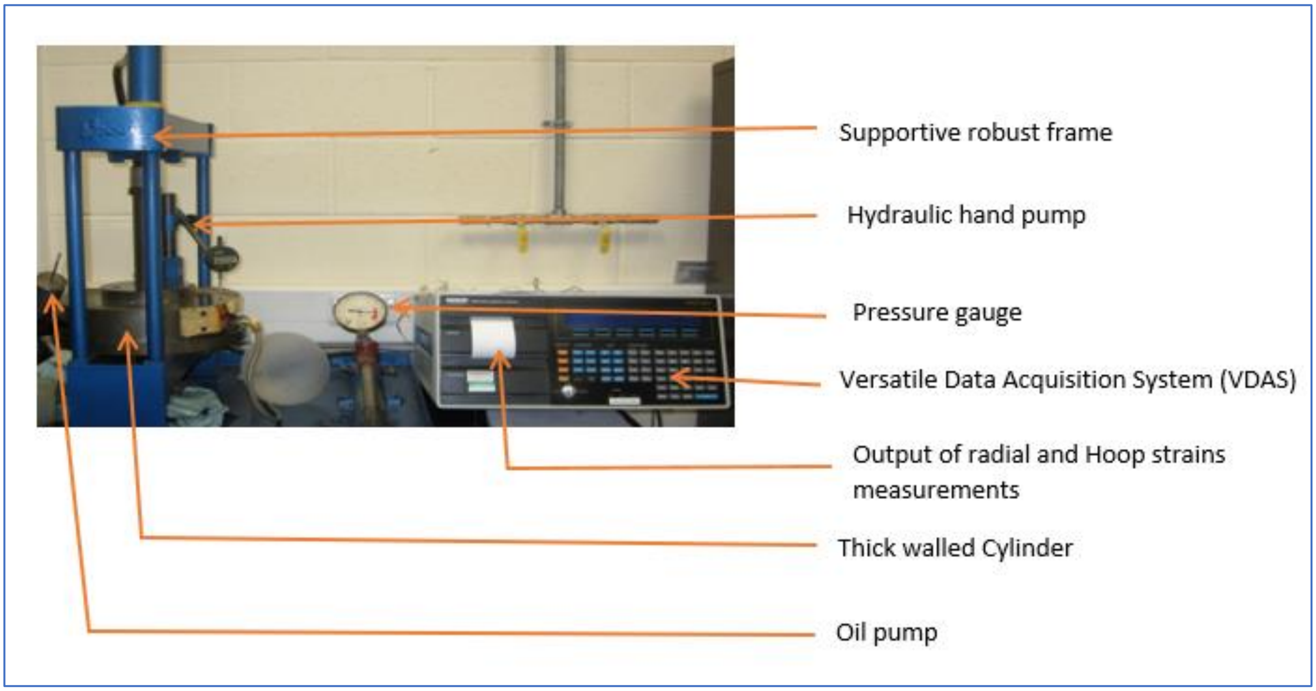

Figure 3-1: Setup of the Experimental Apparatus

4. EXPERIMENTAL METHODOLOGY

i. Datum was initiated by setting gauges to zero.

ii. The plug was then forced into the ring approximately $0.3mm$.

iii. Five penetration depths or plug insertions were obtained at roughly $0.75mm$ increments.

iv. To produce the internal pressure(P), a tapered plug was forced into the bore or ring with the hydraulic ram and the hydraulic hand pump was used to increase the pressure inside the cylinder.

v. Results from the strain gauges were processed by the Versatile Data Acquisition System (VDAS) or data logger and hard readings of radial and Hoop strains were produced.

vi. The experiment was repeated, and strain readings were taken for each penetration depth obtained from step (iii) and recorded.

vii. Data were tabulated and presented in Table 5.1.

5. RESULTS AND DISCUSSION

5.1Experimental Data

Table 5.1 presents the raw data obtained from the experiment.

Table 5-1: Strain Gauge Readings Obtained from the Experiment with the VDAS

Penetration Depths, D_p (mm)	Channels or Gauge Numbers									
	1	3	5	7	9	11	13	15	17	19
	Radial Strains (e_r)					Hoop Strains (e_θ)				
0.73	-11	-16	-26	-29	-37	43	51	59	70	85
1.57	-26	-38	-55	-73	-110	100	117	137	164	206
2.6	-47	-66	-93	-129	-200	165	192	224	269	338
3.38	-63	-88	-118	-171	-264	215	248	289	346	435
4.16	-78	-109	-145	-212	-324	266	307	358	427	535

5.2Data Analysis

5.2.1 Empirical Analysis of Thick Cylinder Stress Distribution under Internal Pressure

5.2.1.1 Effect of Plug Penetration on the Hoop and Radial Strains.

To investigate the effect of the depth of the plug penetration on the strain produced in the thick cylinder, the gauge readings of Table 5.1 are plotted against penetration depths as shown in Figure 5.1.

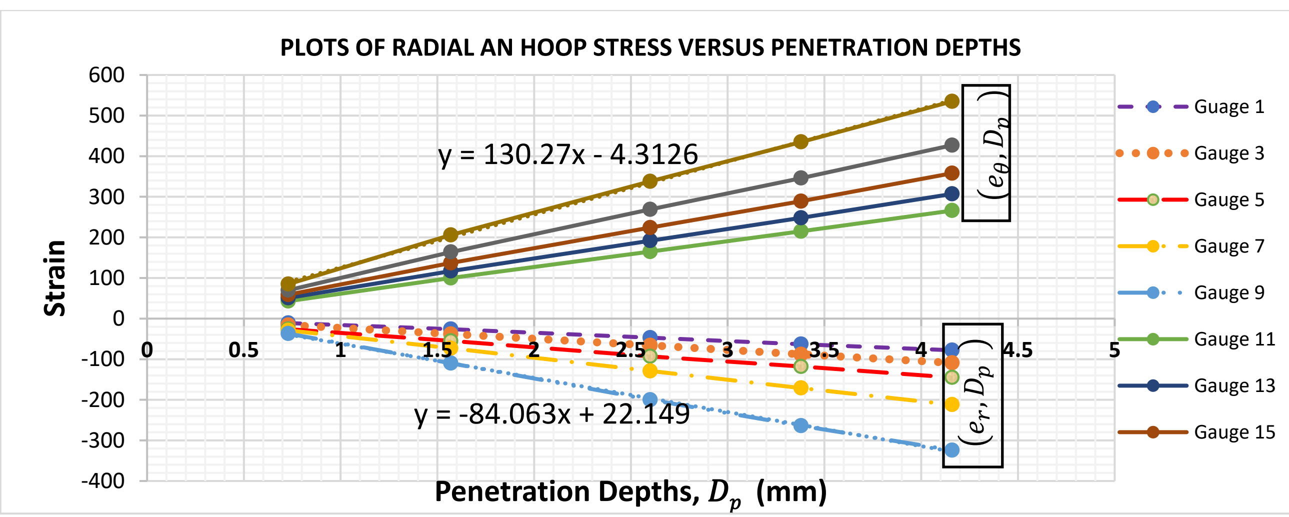

Figure 5-1: Graph Showing the Effect of Plug Penetration Depth Hoop and Radial Strains produced in the Thick Cylinder

5.2.1.2 Effect of the Strain Radius on Hoop Stress and Radial Stress.

To find out the effect of the Strain Radius, r, on the Hoop Stress, σ_θ and Radial Stress, σ_r produced in the thick cylinder, the following values will be used for the indicated parameters.

Young's Modulus, $(E) = 208KN/mm^2$

Poisson's ratio $(\mu) = 0.3$

Penetration depth$D_p = 4.16mm$.

Using Equations 1.5 and 1.6 shown below, and the corresponding e_θ and e_r of the chosen penetration depth (4.16mm), obtained from the graph of Figure 5.1, Table 5.2 is generated for five values of Strain Radius, r, using Microsoft Excel.

Hoop stress, $\sigma_\theta = \frac{E}{1-\mu^2}(e_\theta + \mu e_r)$ **………………………………….……….. Equation 5.5**

Radial stress,

Radial stress, $\sigma_r = \frac{E}{1-\mu^2}(e_r + \mu e_\theta)$ ……………………..................…………Equation 5.6

Table 5-2: Hoop Stress and Radial Stress at Different Strain Radii estimated from the experiment.

r (mm)	**90.5**	**103**	**115.5**	**128**	**140.5**
$1/r^2$ (10^{-5}mm-^{2})	12.21	9.43	7.50	6.10	5.07
e_θ **(microstrain)**	535	427	358	307	266
e_r **(microstrain)**	-324	-212	-145	-109	-78
σ_θ **(N/mm^2)**	100.07	83.06	71.89	62.70	55.45
σ_r **(N/mm^2)**	-37.37	-19.18	-8.59	-3.86	0.41

Figure 5.2 shows the relationship between Hoop Stress, Radial Stress and the Strain Radii while Figure 5.3 shows how the stresses vary with the inverse of the square of the strain radii.

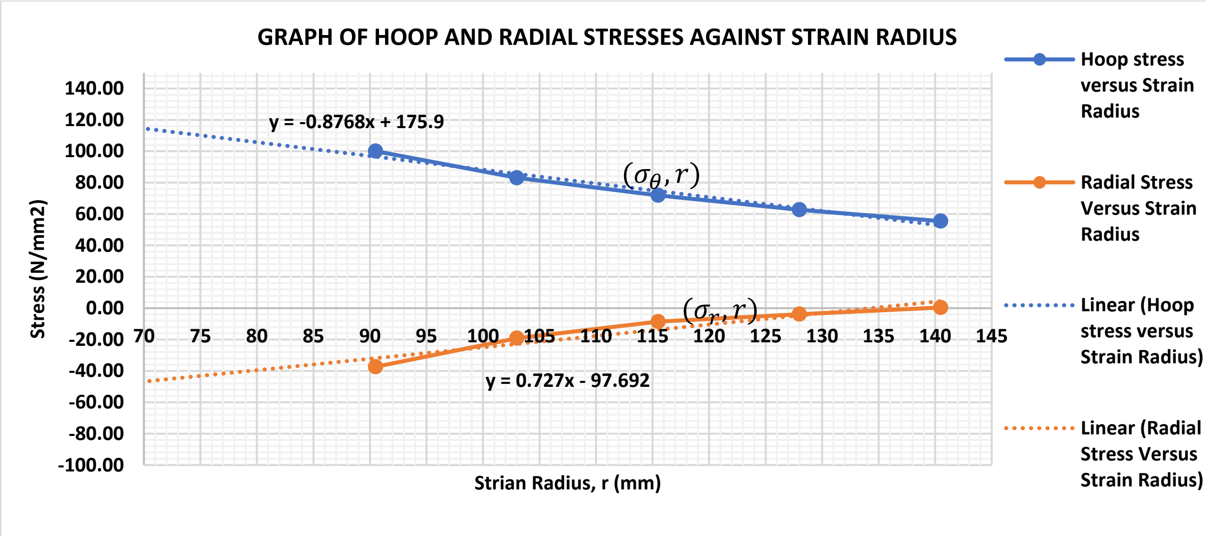

Figure 5-2: Graph Showing the Effect of Strain Radius on Hoop Stress and Radial Stress in a Thick Cylinder.

From Figure 5-3, the following has been obtained from the experiment:

$$\boldsymbol{A = 26.638\, {}^{N}\!/_{mm^2}; \quad B = 5.738 * 10^5 N.}$$

Also, the internal pressure, (P) corresponds to the absolute value of the radial stress (σ_r) when the strain radius or radius of analysis (r) is equal to the inner radius of the thick cylinder (R_i).

At $r = \boldsymbol{R_i = 77.5mm}$, ${}^{1}\!/_{r^2} = 16.6 \times 10^{-5} mm$ and $\sigma_r = -60\, {}^{N}\!/_{mm^2}$.

Therefore, $\boldsymbol{P = 60\, {}^{N}\!/_{mm^2}}$.

5.2.2 Comparison of the Empirical Stress Distribution Analysis Versus the Theoretical Stress analysis using Lame's Theorem

5.2.2.1 Estimation of A and B from Lame's Equation.

From Equations 1.3 and 1.4, $A = \dfrac{PR_i^2}{R_o^2 - R_i^2}$ and $B = \dfrac{PR_o^2 R_i^2}{R_o^2 - R_i^2}$

Where:

$\boldsymbol{P = 60\, {}^{N}\!/_{mm^2}}$ **(Internal pressure obtained from Figure 5-3 in Section 5.2.1.2)**

$R_i = 77.5mm$ (Inner radius given in Section 3.1)

$R_o = 152mm$ (Outer radius given in Section 3.1)

Therefore,

$$A = \frac{60 \times 77.5^2}{152^2 - 77.5^2} = \mathbf{21.08}\ {}^{N}/_{mm^2} \quad \textbf{And} \qquad B = \frac{60 \times 77.5^2 \times 152^2}{152^2 - 77.5^2} = \mathbf{4.87 * 10^5 N}$$

Table 5-3 compares the values of A and B obtained from the experiment as in Figure 5-3, with the values of A and B estimated in Section 5.2.2.1, above, using Lame's theorem and the internal pressure derived Figure 5-3.

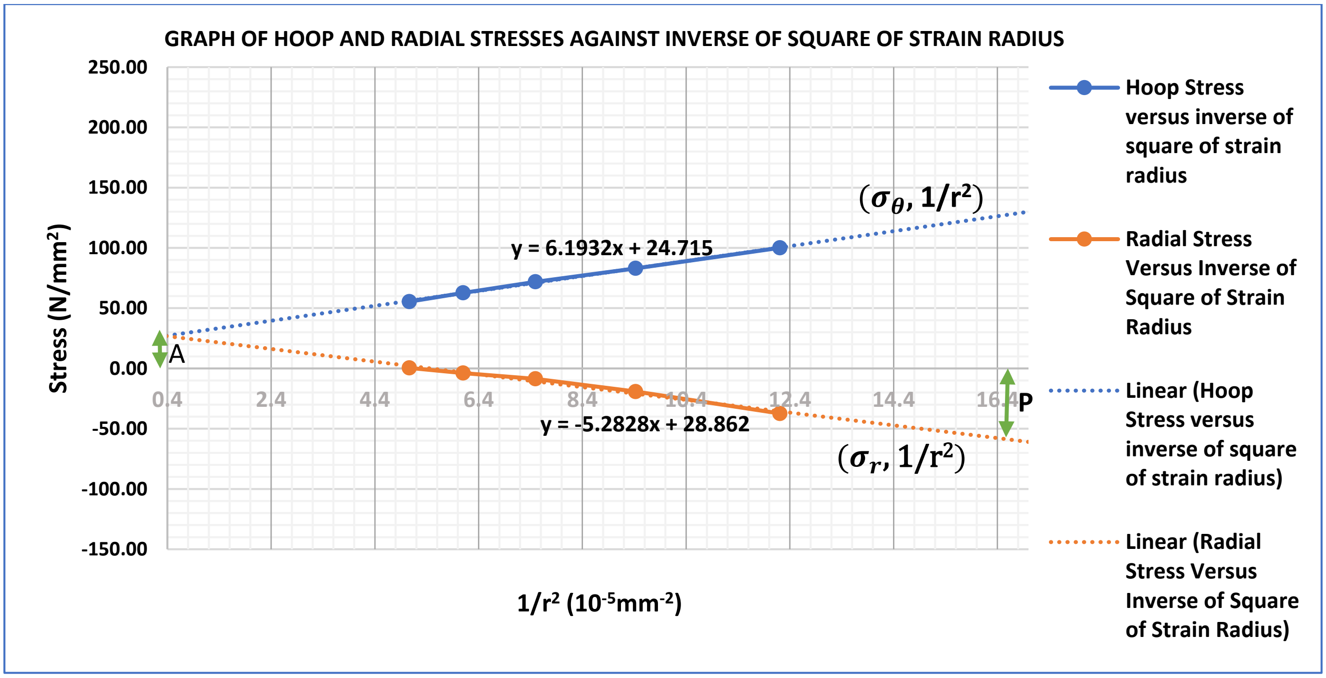

Figure 5-3: Graph of Hoop and Radial Stresses against Inverse of Square of Strain Radius

Table 5-3: Comparison of Empirical Stress Analysis with Theoretical Stress Analysis

Parameter	Empirical value	Theoretical value	Percentage difference (%)
$A\ \left({}^{N}/_{mm^2}\right)$	26.638	21.08	21.09768
$B\ (N)$	$5.738 * 10^5$	$4.87 * 10^5$	15.12722

5.3Precautions Taken to Reduce Errors

i. We ensured that the plug was not loaded while taking gauge readings.

ii. The ram was properly lubricated to reduce friction.

5.4Discussion of Results

Figure 5-1 shows that the hoop or circumferential strains have direct proportionate relationships with the depth of plug penetration while the radial strains have inverse proportionate relationships with the penetration depth. This is shown by the positive slopes of the $\left(e_{\theta},\ D_p\right)$ plots and the negative slopes of the $\left(e_r,\ D_p\right)$ series.

Hoop stress decreases proportionately with an increase in the strain radius while the Radial stress increases proportionately to the strain radius. This is also shown by the negative slope of the $(\sigma_{\theta},\ r)$ curve and the positive slope of the $(\sigma_r,\ r)$ curve of Figure 5.2.

Point A (used in Equation 1.1) is supposed the common intercept for the two graphs while point B (to be used in Equation 1.2) is meant to be the absolute value of their slopes (Al-Badri, 2011, 2011; Elso, 2012; Lame’s Theorem - Strength of Materials, n.d.). However, from the graphs of Figure 5-3, there are slight variations on the absolute values of the slopes of the two graphs and their intercepts. This is attributable to errors in the experiment. To enhance accuracy and obtain single values of $A\ and\ B$, the average value of the two slopes and two intercepts have been computed as $\boldsymbol{26.638\,{}^{N}\!/_{mm^2}\ for\ A}$ and $\boldsymbol{5.738 * 10^5 N\ for\ B.}$

The above-mentioned values of A and B were found to differ significantly from those calculated using Lame’s theorems. This is shown in Table 5-3 where the Theoretical and Empirical A values differ by over 20% while the values of B differ by over 15%.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

From the analysis of the experimental results and the foregoing discussions, the following conclusions have been reached.

- The hoop strain produced in the cylinder increases as the depth of plug penetration increases (direct proportionality) while the radial strain decreases with increase in the depth of plug penetration (Inverse proportionality).
- The hoop stress also decreases as the radial position increases or as the strain propagates away from the centre of the thick cylinder while the radial stress increases as the radial position increases.
- The values of the Lame's equations constants, $A \text{ and } B$ computed using Lame's equation and the internal pressure obtained from the graph were found to differ significantly from those estimated from the experiment.

6.2 Recommendations

- The experiment must not be performed without the guidance of a trained instructor.
- High safety and good housekeeping ethics must be observed during the conduct of the experiment.

7. REFERENCES

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