

《线性代数》期中试题试卷(A)

(考试形式：闭卷 考试时间：100 分钟)



《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：_____ 姓名：_____ 学号：_____ 成绩：_____

注意：答案一定要写在答卷中，写在本试题卷中不给分。答题时注明各题题号，并在答题纸上写上姓名和学号。本试卷要和答卷一起交回。

一、填空题（每小题 4 分，共 16 分）

1. Let $A = \begin{bmatrix} -1 & X \\ 2y & -3 \end{bmatrix}$ and $B = \begin{bmatrix} a & -4 \\ 4 & a-b \end{bmatrix}$ if $A = B$, then $a =$ _____, $b =$ _____, $x =$ _____, $y =$ _____.

2. Find the inverses of matrices $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

3. Find, if possible, an LU factorization of each of the following matrices:

$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & 5 & 21 \\ 1 & 7 & -5 \end{bmatrix}$$

4. Let $\alpha_1 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $\alpha_2 = \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}$, $\alpha_3 = \begin{bmatrix} -1 \\ 5 \\ \lambda \end{bmatrix}$, when $\lambda =$ _____, $\{\alpha_1, \alpha_2, \alpha_3\}$ is linearly

dependent set.

二、判断题（每小题 3 分，共 24 分）

1. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -0 & 1 \end{bmatrix}$ is elementary.
2. If A and B are invertible matrices, so is A + B.
3. The function $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-y \\ y-z \\ z-x+2 \\ x+y+z \end{bmatrix}$ is a linear transformation
4. if $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation such that $T\left(\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then T is not one to one
5. If a matrix A has row echelon form u, the pivot columns of A are the pivot columns of u
6. If a 4*4 matrix A has row echelon form $U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, the system $Ax = 0$ has only the trivial solution.
7. If A and B are invertible matrices and $XA = B$, the $X = A^{-1}B$.
8. If x_0 is a solution to the system $Ax = b$ with $b \neq 0$, then $2x_0$ is also a solution.

三、计算题（40 分）

1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \\ -6 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 4 \\ 6 & 2 \\ -8 & 2 \end{bmatrix}$ solve the equation $2A + 4X = 3B$.

2. Let $A = \begin{bmatrix} 1 & -7 & 1 \\ 2 & -9 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 0 & 1 \\ 2 & 4 \end{bmatrix}$

(a). Compute AB and BA

(b). Is A invertible? Explain.

3. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$, $X = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$

(a). Find Ax

(b). Find $x^T A^T$ by two different methods

4. Let $V_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, $V_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $V_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, and $V_4 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

Determine whether every vector in R^3 is a Linear combination of the given vectors. write the solution x_0 the following Linear systems in the form $x = x_p + x_h$ where x_p is a particular solution and x_h is a solution of the corresponding homogeneous system

$$\begin{cases} x_1 + 5x_2 + 7x_3 = -2 \\ 9x_3 = 3 \end{cases}$$

四、证明题（每题 10 分， 20 分）

1. Suppose A and B are $m \times n$ matrices such that $Ae_i = Be_i$ for each $e_i \in R^n$, prove that

$$A = B$$

2. Show that if A is an invertible matrix, then so is A^T , and $(A^T)^{-1} = (A^{-1})^T$