警 示

中山大学软件学院 2011 级软件工程专业（2011-11）

# 《线性代数》期中考试题

（考试形式：闭卷　　考试时间：2 小时）

**《中山大学授予学士学位工作细则》第六条**

**考 试 作 弊 不 授 予 学 士 学 位**

教学班：________________　姓名：_______________　学号：______________　成绩：___________

## 1. Fill in the blank（5×4=20 Pts）

(1) If $A=\begin{bmatrix}5 & -7 & 2 & 2\\ 0 & 3 & 0 & -4\\ -5 & -8 & 0 & 3\\ 0 & 5 & 0 & -6\end{bmatrix}$, then $\det(2A)=$__________________.

(2) For each matrix below, determine whether its columns form a linearly independent set. ____________.

a. $\begin{bmatrix}-4 & 12\\ 1 & -3\\ -3 & 8\end{bmatrix}$ b. $\begin{bmatrix}2 & 7 & 0\\ -4 & -6 & 5\\ 6 & 13 & -3\end{bmatrix}$ c. $\begin{bmatrix}1 & 5 & -3 & 2\\ 0 & 4 & -9 & 18\\ 0 & 0 & 0 & 1\end{bmatrix}$

(3) Suppose matrices $A$, $B$, and $C$ are square. Find matrices $X$ and $Y$ such that

$$\begin{bmatrix}I & 0\\ A & B\end{bmatrix}\begin{bmatrix}0 & C\\ X & Y\end{bmatrix}=\begin{bmatrix}0 & C\\ I & 0\end{bmatrix}$$

$X=$___________, $Y=$________________.

(4) Suppose $T$: $R^2 \to R^2$ is a transformation, which reflection through the $x_1$-axis first, and then reflection through the line $x_2=x_1$. So the standard matrix of $T$ is __________________________.

(5) The matrices $A$ and $B$ below are row equivalent,

$$A=\begin{bmatrix}2 & -4 & -1 & -3 & 5 & 2\\ -1 & 2 & -4 & -3 & -7 & -7\\ 3 & -6 & 6 & -7 & 15 & 13\\ 5 & -10 & 5 & -10 & 20 & 10\end{bmatrix},\quad B=\begin{bmatrix}2 & -4 & -1 & -3 & 5 & 2\\ 0 & 0 & 3 & -1 & 3 & 4\\ 0 & 0 & 0 & 0 & 0 & 5\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

then dim Nul $A=$____________, and a basis for the Col $A$ is __________________.

## 2. Make each statement True or False, and descript your reasons.（6×4=24 Pts）

(1) The linear transformation $x \mapsto Ax$ is one-to-one when $A$ is a 3×4 matrix.

(2) If matrices $A$ and $B$ are row equivalent, they have the same echelon form..

(3) If {u, v, w} is linear independent, then u, v, and w are not in $R^2$ .

(4) The set of all solutions to the linear system $Ax = b$, where $A$ is $m \times n$ and $b \neq 0$, is a subspace of $R^n$.

(5) Let $T: R^n \to R^n$ be a linear transformation defined by $T(x) = Ax$. Then $T$ is onto if and only if $\det(A) \neq 0$.

(6) If $A$ is a 4×4 matrix, then $\det(-A) = -\det A$.

## 3. Calculation（5×8=40 Pts）

(1) Solve the following system of linear equations and write the general solution in parametric vector form..

$$\begin{aligned} x_1 + 2x_2 - x_3 + 2x_4 &= 1 \\ 2x_1 + 4x_2 + x_3 + x_4 &= 5 \\ -x_1 - 2x_2 - 2x_3 + x_4 &= -4 \end{aligned}$$

(2) If $A$ and $B$ are 3×3 matrices, $I$ is the identity matrix, and $AB + I = A^2 + B$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ . Find $B$.

(3) Suppose matrix $A = \begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$, find $A^{-1}$.

(4) suppose that a matrix A has been reduced to echelon form as shown below. Construct an LU factorization of $A$.

$$A = \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ -1 & 6 & -19 & 4 & -6 \\ -2 & 7 & -18 & 1 & -11 \\ 3 & -8 & 17 & 3 & 18 \end{bmatrix} \sim \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & 3 \\ 0 & 1 & -4 & 5 & 7 \\ 0 & 1 & -4 & -3 & -9 \end{bmatrix} \sim \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & 3 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix}$$

$$\sim\begin{bmatrix}2 & -6 & 14 & 4 & 18\\ 0 & 3 & -12 & 6 & 3\\ 0 & 0 & 0 & 3 & 6\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\sim\begin{bmatrix}2 & -6 & 14 & 4 & 18\\ 0 & 3 & -12 & 6 & 3\\ 0 & 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\sim\begin{bmatrix}2 & -6 & 14 & 0 & 10\\ 0 & 3 & -12 & 0 & -9\\ 0 & 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\sim\begin{bmatrix}2 & -6 & 14 & 0 & 10\\ 0 & 1 & -4 & 0 & -3\\ 0 & 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\sim\begin{bmatrix}2 & 0 & -10 & 0 & -8\\ 0 & 1 & -4 & 0 & -3\\ 0 & 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

(5) Let $T: R^3 \to R^3$ be a linear transformation such that

$$T(e_1)=\begin{bmatrix}1\\0\\4\end{bmatrix},\ T(e_2)=\begin{bmatrix}2\\3\\5\end{bmatrix},\ T(e_3)=\begin{bmatrix}0\\-7\\5\end{bmatrix}$$, where $e_1, e_2, e_3$ are the columns of $I_3$

a. Determine if *T* is a one-to-one transformation.

b. Write the 4×4 matrix that represents *T* when homogeneous coordinates are used for vectors in $R^3$.

## 4. Prove issues（2×8=16 Pts）

(1) Explain why a set $\{v_1, v_2, v_3, v_4\}$ in $R^5$ must be linearly independent when $\{v_1, v_2, v_3\}$

is linearly independent and $v_4$ is not in Span$\{v_1, v_2, v_3\}$.

(2) Let $A$ be a 5×3 matrix and $B$ a 3×5 matrix. Show that the 5×5 matrix $AB$ cannot be invertible.