中山大学软件学院 2013 级软件工程专业(2013 学年秋季学期)

# 《SE-103 线性代数》期末试题(A 卷)

(考试形式：开/闭 卷　考试时间：2 小时)

警示　《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：__________　姓名：________　学号：________

**1. Fill in the blanks（6×4=24 Pts）**

(1) Let $A=\begin{bmatrix}1&0&1\\0&2&0\\-2&0&1\end{bmatrix}$ and $A^2B-A-B=I$, where $A$ and $B$ are 3×3 matrices, then $|B|=$________.

(2) Given a subspace $H=\left\{\begin{bmatrix}a-3b+6c\\5a\\b-2c-d\\0\end{bmatrix}: a,b,c,d \text{ in } R\right\}$, a basis is ________________,

and the dimension of $H$ is ____________.

(3) Let $A=\begin{bmatrix}0&1\\-8&4\end{bmatrix}$ act on $C^2$. Then an eigenvalues of $A$ is $\lambda=$____________.

And a basis for the eigenspace corresponding to $\lambda$ is ____________________.

(4) Let $y=\begin{bmatrix}5\\-9\\5\end{bmatrix}$ and $W=\text{span}\{u_1,u_2\}$, where $u_1=\begin{bmatrix}-3\\-5\\1\end{bmatrix}$ and $u_2=\begin{bmatrix}-3\\2\\1\end{bmatrix}$, then

$\text{proj}_W y$ = ______________, and the distance from $y$ to $W$ is ____________.

(5) Let $A=\begin{bmatrix}1 & 5\\ 3 & 1\\ -2 & 4\end{bmatrix}$ and $b=\begin{bmatrix}4\\ -2\\ -3\end{bmatrix}$, then a least-squares solution of $Ax=b$ is ___________, and the associated least-squares error is ___________________.

(6) Let $A$ be the matrix of the quadratic form $(x_1+x_2)^2+(x_2-x_3)^2+(x_3+x_1)^2$, then $A$= ___________________________, and rank $A$ is ______________.

**2. Make each statement True or False, and descript your reasons. ($6\times 3=18$ Pts)**

(1) If $A$ is a positive definite symmetric $n\times n$ matrix, then $A^{-1}$ is also positive definite.

(2) Suppose a 3×5 matrix $A$ has dim Row $A$ =3. Then the equation $Ax=b$ always has a unique solution.

(3) If $V$ is a vector space having dimension $n$, and if $S$ is a subset of $V$ with $n$ vectors, then $S$ is linearly independent if and only if $S$ spans V.

(4) Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.

(5) If $A$ is produced by multiplying row 2 of $B$ by 3, then det $A$ =3det $B$.

(6) If a matrix $U$ has orthonormal columns, then $UU^T=I$, where $I$ is the $n\times n$ identity matrix.

**3. Calculation (5 ×8 = 40 Pts)**

(1) Let $A=\begin{bmatrix}1 & 0\\ -1 & 2\\ 0 & 1\end{bmatrix}$, $B=A^TA$

a. Find the eigenvalues and the corresponding eigenvectors of $B$.

b. Computer $B^k$, where k represents an arbitrary positive integer.

(2) Find a QR factorization of $A=\begin{bmatrix}5 & 9\\ 1 & 7\\ -3 & -5\\ 1 & 5\end{bmatrix}$.

(3) If $B=\begin{bmatrix}2 & 3 & 0\\ 3 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$, and if $A$ and $B$ are similar.

a. Find $|A-2I|$.

b. Let $\vec{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$. Computer $x^TBx$ for the matrix $B$.

(4) Suppose $A=\begin{bmatrix}1 & 0 & -2 & 1\\ 0 & 1 & a & 0\\ 2 & 1 & 0 & a\\ 0 & 0 & 0 & a-1\end{bmatrix}$. If the columns of the matrix $A$ are linearly dependent and $a\neq 1$.

a. Find $a$.

b. Find bases for the null space, the column space, and the row space of the matrix $A$.

(5) If $T$ is the linear transformation from $P_2$ to $P_2$ .

a. Suppose the set $B=\{1+t, 1+t^2, t+t^2\}$ is a basis for $P_2$. Find the coordinate vector of $p(t)=1+3t-2t^2$ relative to $B$.

b. IF $T(a_0+a_1t+a_2t^2)=2a_1+4a_2t^2$, find the $C$-matrix for T, when $C$ is the basis $\{1, t, t^2\}$.

**4. Prove issue（2 ×9 = 18 Pts）**

(1) Suppose $A=I-3uu^T$, where $u$ is a unit vector in $R^n$ and $I$ is the $n\times n$ identity matrix.

a. Let $u$ be an eigenvector of $A$, find the corresponding eigenvalue.

b. If $v$ is any vector orthogonal to $u$, show that $v$ is an eigenvector of $A$ and find the eigenvalue.

(2) Suppose $A$ and $B$ are both $n\times n$ symmetric matrices. Show that $AB$ is an symmetric matrix if and only if $AB=BA$.