软 件 学 院 2009 级 软 件 工 程 专 业 (2009-2)

《离散数学》期末试题试卷(A)

参考答案

1. (10 points) A, B and C are sets, prove or disprove the following statements.

(1). if $A - B = A - C$, then $B = C$

(2). if $A \times B = A \times C$, $A \neq \varnothing$, then $B = C$

证明:

(1). 错误. (能说明问题的任何反例都可以)

例如, A={1,2}, B={1}, C={1,3}.

显然有 $A - \mathrm{B} = A - C = \{2\}$, 但 $B \neq C$. 所以, 该命题不成立.

(2). 正确. (能证明结论的任何证明方法都可以)

(2.1) $B \subseteq C$

$$\begin{aligned} & \forall y \in B \\ & \Rightarrow x \in A \wedge y \in B && (A \neq \varnothing) \\ & \Rightarrow (x, y) \in A \times B \\ & \Rightarrow (x, y) \in A \times C && (A \times B = A \times C) \\ & \Rightarrow y \in C && (\text{笛卡尔积的定义}) \end{aligned}$$

所以, 有: $B \subseteq C$.

(2.2) $C \subseteq B$

$$\begin{aligned} & \forall y \in C \\ & \Rightarrow x \in A \wedge y \in C && (A \neq \varnothing) \\ & \Rightarrow (x, y) \in A \times C \\ & \Rightarrow (x, y) \in A \times B && (A \times B = A \times C) \\ & \Rightarrow y \in B && (\text{笛卡尔积的定义}) \end{aligned}$$

所以, 有: $C \subseteq B$.

由(2.1)和(2.2)可知: $B = C$.

2. (10 points) Write each of the following statements in terms of propositional variables, predicates, quantifiers and logical connectives. You can choose any propositional variables and predicates freely.

(1). If I like the course or the teacher, I will attend the class. (Statement and its negation)

(2). For all students of our school, someone studies hard and has good score, someone studies hard and has not good score.

Note: The first question is expressed in propositional logic, the second is expressed in predicate logic.

解:

(1) Suppose: p – I like the course, q – I like the teacher, r – I will attend the class

Conditional statement: $(p \vee q) \Rightarrow r$

Negation statement: $\sim((p \vee q) \Rightarrow r) \equiv \sim(\sim(p \vee q) \vee r) \equiv (p \vee q) \wedge (\sim r)$

(2) Suppose: $P(x)$ – x studies hard, $Q(x)$ – x has good score

Predicate logical statement: $\exists x(P(x) \wedge Q(x)) \wedge \exists x(P(x) \wedge \sim Q(x))$, $x \in$ { all students of our school }

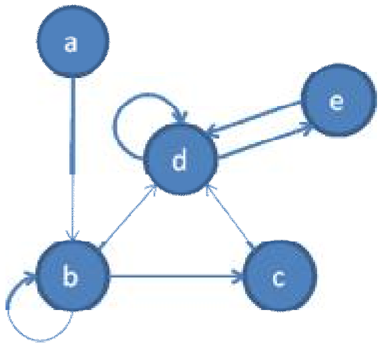
3. (15 points) Let $A=\{a,b,c,d,e\}$, a relation R on A is $\{(a,b), (b,b), (b,c), (b,d), (c,d), (d,d), (d,e), (e,d)\}$.

(1) Give the digraph and matrix of relation R;

(2) Compute R^2, reflexive closure $r(R)$ and symmetric closure $s(R)$.

解:

(1). The digraph of relation R is:



The matrix of relation R is:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(2) $R^2 = \{(a,b), (a,c), (a,d), (b,b), (b,c), (b,d), (b,e), (c,d), (c,e), (d,d), (d,e), (e,d), (e,e)\}$

$r(R) = \{(a,b), (b,b), (b,c), (b,d), (c,d), (d,d), (d,e), (e,d), \underline{(a,a), (c,c), (e,e)}\}$

$s(R) = \{(a,b), (b,b), (b,c), (b,d), (c,d), (d,d), (d,e), (e,d), \underline{(b,a), (c,b), (d,b), (d,c)}\}$

4. (15 points) Let $S\in Z^+$ and $A=S\times S$. Define the following relation R on A:

$$(a,b)\ R\ (a',b') \text{ if and only if } ab' = a'b$$

(1) Show that R is an equivalent relation;

(2) Let S=$\{1,2,3,4,5,6,7,8,9\}$, compute the equivalent class $[(2,4)]$.

解:

由关系 R 的定义可知: $(a,b)\ R\ (a',b')$ if and only if $a/b = a'/b'$

(1) 证明: 关系 R 是等价关系.

(1.1) 关系 R 是自反的

$\forall(a,b)\in A$

$\Rightarrow a\in Z^+ \wedge b\in Z^+$

$\Rightarrow a/b=a/b$

$\Rightarrow (a,b)\ R\ (a,b)$

∴ R is reflexive.

(1.2) 关系 R 是对称的

$\forall(a,b)\ R\ (a',b')$

$\Rightarrow a/b = a'/b'$

$\Rightarrow a'/b' = a/b$

$\Rightarrow (a',b')\ R\ (a,b)$

∴ R is symmetric.

(1.3) 关系 R 是对称的

$\forall(a,b)\ R\ (a',b'), (a',b')\ R\ (a'',b'')$

$\Rightarrow a/b = a'/b' \wedge a'/b' = a''/b''$

$\Rightarrow a/b = a''/b''$

$\Rightarrow (a,b)\ R\ (a'',b'')$

∴ R is transitive.

所以,由(1.1)~(1.3)可知: R is an equivalent relation on A.

(2) $[(2,4)] = \{(1,2), (2,4), (3,6), (4,8)\}$.

5. (10 points) Let function $f(x, y)=(x+3y, 2x+y)$, $(x, y)\in R\times R$, prove that f is bijection.

解:

(1) 证明: f 是单射.

假设: $f((x,y)) = f((u,v))$.

$(x+3y, 2x+y) = (u+3v, 2u+v)$

$\Rightarrow (x+3y = u+3v) \wedge (2x+y = 2u+v)$

$\Rightarrow (u=x) \wedge (v=y)$

$\Rightarrow (x,y) = (u,v)$

所以, f 是单射.

(2) 证明: f 是满射.

$\forall (u, v) \in R\times R$.

假设: $\exists (x, y) \in R\times R$, 使得: $f(x,y)=(u, v)$.

$(x+3y, 2x+y) = (u, v)$

$\Rightarrow (x+3y = u) \wedge (2x+y = v)$

$\Rightarrow x=(3v-u)/5, y=(2u-v)/5$

所以, $\forall (u, v) \in R\times R$, 都存在: $(x, y) \in R\times R$, 使得: $f(x, y) = (u, v)$, 即: f 是满射.

6. (15 points) Let $A=\{2, 4, 5, 6, 8, 10, 12, 20, 120\}$, R is the relation of divisibility on A.

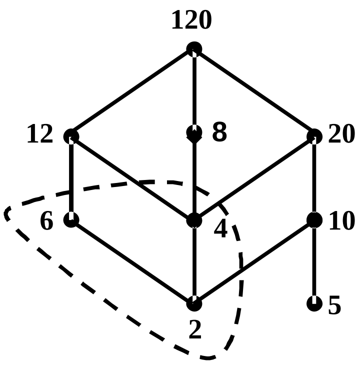
(1) Draw the Hasse diagram of the poset <A, R>;

(2) Find all the minimal elements, the maximal elements, the least element and the greatest element of the poset <A, R> if they exist;

(3) Let $B = \{2, 4, 6\}$, find the upper bound, the lower bound, the least upper bound and the greatest lower bound of B if they exist.

解:

(1) the Hasse diagram of the poset <A, R>



(2) the minimal element: 2, 5;

the maximal element: 120;

the least element: None;

the greatest element: 120.

(3) for set $B = \{2, 4, 6\}$,

the upper bound: 12, 120;

the lower bound: 2;

the least upper bound: 12;

the greatest lower bound: 2.

7. (15 points) Use the labeling algorithm (Ford-Fulkerson's) to find a maximum flow for the following transport network in Fig. 1. Use of figures is required to show the variety of flows during the procedure.

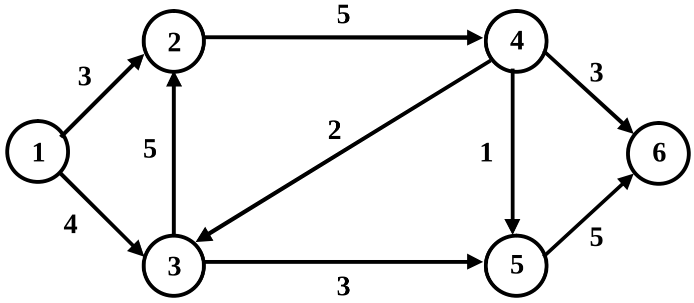
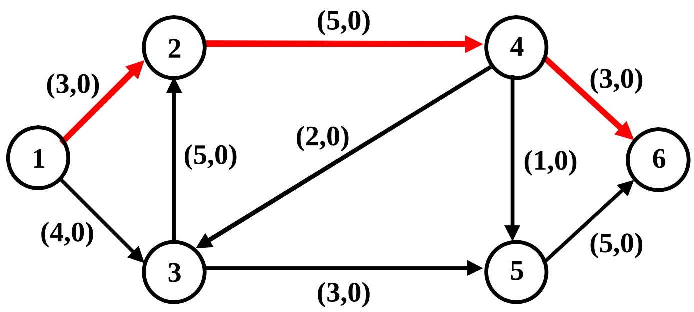
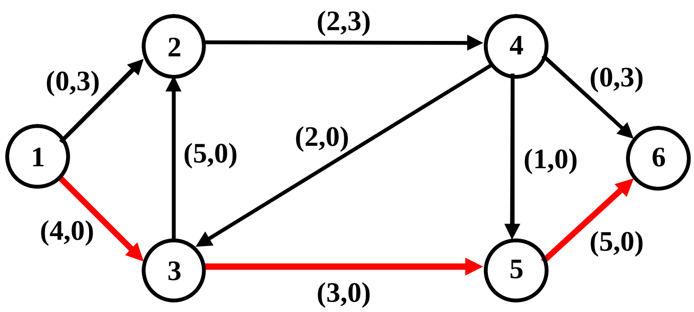


Fig. 1. transport network for question 7

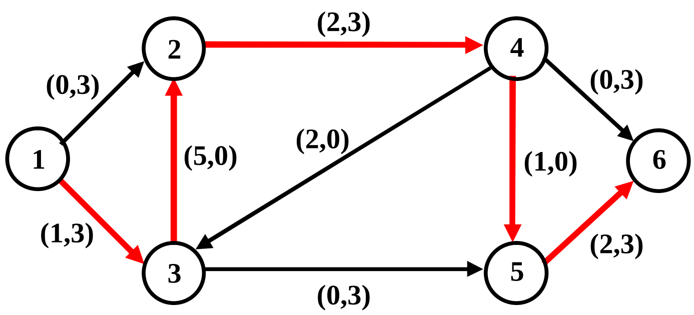
解：按标定算法把传输网络中边的流标定如下： 标识的图例:(e_{ij}, e_{ji})



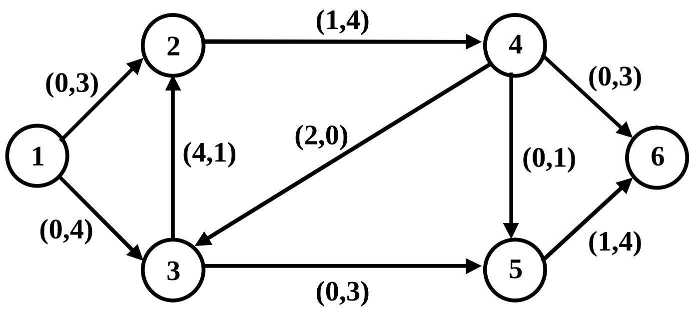
对上图，由标定算法得路径 1→2→4→6，该路径的流量为 3. 逆向修改路径上边的标识，得下图.



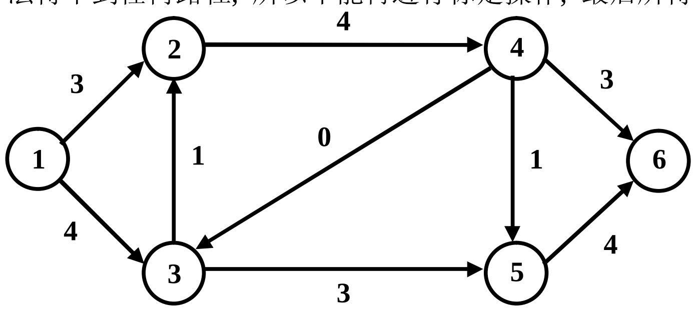
对上图，由标定算法得路径 1→3→5→6，该路径的流量为 3. 逆向修改路径上边的标识，得下图.



对上图，由标定算法得路径 1→3→2→4→5→6，该路径的流量为 1. 逆向修改路径上边的标识，得下图.



对上图，根据标定算法得不到任何路径，所以不能再进行标定操作，最后所得到的流量如下图所示.



综上所述，该网络的最大流量为 3+3+1=7.

8. (10 points) Use Kruskal’s algorithm to find a minimal spanning tree of graph in Fig. 2. The sequence of edges-selecting is ordered to be shown up.

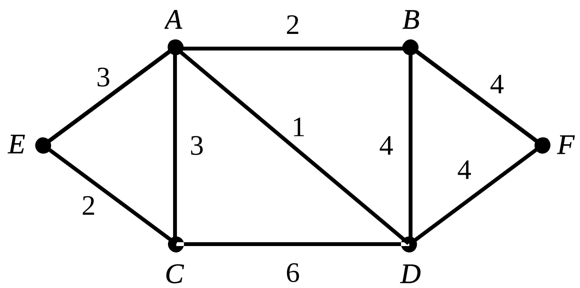


Fig. 2. The graph for question 8

解:

(1) 选择当前最小权的边(A, D);

(2) 选择当前最小权的边(A, B)和(C, E)，因为它和已选用的边不会构成圈;

(3) 选择当前最小权的边(A, C)和(A, E)中的一个，因为选择两个将构成圈．不妨选择边(A, E);

(4) 不能选择边(B, D)，因为选择它将构成圈;

(5) 选择当前最小权的边(B, F)和(D, F)中的一个，因为选择两个将构成圈．不妨选择边(B, F);

这时已选择了 5 条边，算法结束.

最小生成树的权为: 1+2+2+3+4=12.

本题所得到的最小生成树如图 2.1 所示．本题答案不唯一，权为 12 的生成树都是正确的.

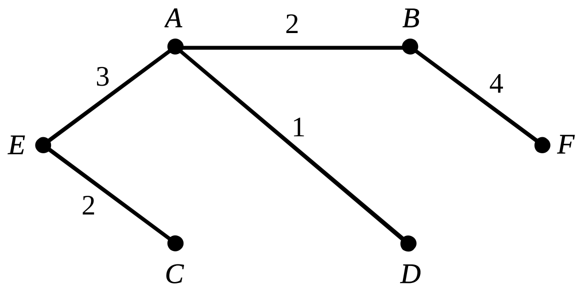


Fig. 2.1. The minimal spanning tree for question 8