中山大學 本科生考试草稿纸 2011-10-17

警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.129. 33 $\int \frac{dx}{\sqrt{3+x-x^2}} = \int \frac{dx}{\sqrt{\frac{13}{4}-(x^2-x+\frac{1}{4})}} = \int \frac{d(x-\frac{1}{2})}{\sqrt{(\frac{\sqrt{13}}{2})^2-(x-\frac{1}{2})^2}} = \arcsin\frac{2x-1}{\sqrt{13}} + C.$

34. $\int \sqrt{7+x-x^2}\,dx = \int \sqrt{\frac{29}{4}-(x^2-x+\frac{1}{4})}\,dx = \int \sqrt{\frac{29}{4}-(x-\frac{1}{2})^2}\,d(x-\frac{1}{2})$

$= \frac{1}{2}\cdot\frac{29}{4}\arcsin\frac{x-\frac{1}{2}}{\frac{\sqrt{29}}{2}} + \frac{x-\frac{1}{2}}{2}\sqrt{7+x-x^2} + C$

$= \frac{29}{8}\arcsin\frac{2x-1}{\sqrt{29}} + \frac{2x-1}{4}\sqrt{7+x-x^2} + C.$

35. $\int \frac{dx}{1+\sqrt{x-1}}$，令 $\sqrt{x-1}=t$，则 $x-1=t^2$，$x=1+t^2$，$dx=2t\,dt$

$= \int \frac{1}{1+t}\,2t\,dt$

$= 2\int \frac{1+t-1}{1+t}\,dt = 2\int dt - \int \frac{1}{1+t}\,d(1+t)$

$= 2t - 2\ln(1+t) + C = 2\sqrt{x-1} - 2\ln(1+\sqrt{x-1}) + C.$

习题3—2

P.134. 1. $\int x\ln x\,dx = \frac{1}{2}\int \ln x\,dx^2 = \frac{1}{2}\left[x^2\ln x - \int x^2 d\ln x\right] = \frac{1}{2}\left[x^2\ln x - \int x\,dx\right]$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C.$

2. $\int x^2\cdot e^{ax}\,dx = \frac{1}{a}\int x^2\,de^{ax} = \frac{1}{a}\left[x^2e^{ax} - \int e^{ax}\,dx^2\right] = \frac{1}{a}\left[x^2e^{ax} - \int e^{ax}\cdot 2x\,dx\right]$

$= \frac{x^2}{a}e^{ax} - \frac{1}{a^2}\int x\,de^{ax} = \frac{x^2}{a}e^{ax} - \frac{1}{a^2}\left[xe^{ax} - \int e^{ax}\,dx\right]$

$= \frac{x^2}{a}e^{ax} - \frac{x}{a^2}e^{ax} + \frac{1}{a^3}e^{ax} + C = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) + C.$

3. $\int x\cdot\sin 2x\,dx = -\frac{1}{2}\int x\,d\cos 2x = -\frac{1}{2}\left[x\cdot\cos 2x - \int\cos 2x\,dx\right] = -\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + C.$

4. $\int \arcsin x\,dx = x\cdot\arcsin x - \int x\,d\arcsin x = x\cdot\arcsin x - \int \frac{x\,dx}{\sqrt{1-x^2}} = x\cdot\arcsin x + \int \frac{d(1-x^2)}{2\sqrt{1-x^2}}$

$= x\cdot\arcsin x + \sqrt{1-x^2} + C$