Discrete Mathematical Structures

A General Review

Outline

- Chapter 1. Fundamentals
- Chapter 2. Logic
  - Topic.1.Review of Logic
- Chapter 4. Relations and Digraph
- Chapter 5. Function
  - Topic.2.Counting
  - Topic.3.Solving Recurrence Relations
- Chapter 6. Ordered Relations and Structures
- Chapter 7. Trees
- Chapter 8. Topics in Graph Theory
  - Topic.4.Transport Networks
  - Topic.5.An Introduction to Discrete Probability

Basic Concepts

Chapter 1. Fundamentals

- Intuitive definition of sets
- The description of a set
- Set elements: the "$\in$" notation
- Venn diagram
- Empty set, Universal set, Subset and "$\subseteq$" notation, Power set
- Equal sets: the axioms of extensionality
- Cardinality
- Set operations: Union, Intersection, Difference, Complement
- The characteristic function
- Mathematical structures
- Properties of set operations: Commutative, Associative, Distributive, De Morgan's Laws
- Identity and Inverse in a mathematical structure

Basic Concepts

Chapter 2. Logic

- Propositions, True value, Compound propositions. Truth tables
- Logical operators: Negation, Conjunction, Disjunction, Complement
- Predicates: Universal and Existential quantifiers
- Bound and Free variables, Scope of quantifiers
- Nested quantifiers
- Tautology, Contradiction (absurdity) & Contingency
- Logical equivalence in propositional logic
- Logical equivalences involving quantifiers
- Properties of logical operations
- Valid argument, Inference and Proof
- Mathematical Induction

Basic Concepts

Chapter 4. Relations and Digraphs

- Ordered pair, Cartesian product, Partitions
- Relations: Domain, Range, R-relative sets and properties
- Relation matrices
- The Digraph of a relation: Degree, Path, Connectivity
- Reachability relation and Reachability matrix of a relation
- R^n-relation of a relation R
- Properties of relations: Reflexive, Irreflexive, Symmetric, Antisymmetric, Transitive
- Relation properties with relation matrices & digraph
- Equivalence relations, Classes & Partitions on set A
- Operations on relations and their combinations: Relations as sets, Inverse, Closures, Composition
- Transitive closure and paths in digraph, *Warshall*'s algorithm

Basic Concepts

Chapter 5. Functions

- Functions: Argument, Value, Mapping, Image
- Functions defines on a graph: Labeled digraph
- Properties: Everywhere defined, Onto, One to one, One-to-one correspondence, Invertible
- Function operations: composition and inverse
- Functions for computer science: Ceiling and floor functions
- Big-*O* notation
- Common Complexity Functions
- Some Important Big-*O* Results
- Big-Θ notation and some Important Big-Θ Results
- Permutation, Inverse of permutations, Product of permutations, Cyclic permutation, Disjoint cycles, Transposition, Even & Odd transposition

Basic Concepts

Chapter 6. Ordered Relations and Structures

- Partial order, Partially ordered set, comparable elements, linearly ordered set , well-ordered set, Product partial order, Lexicographic order
- *Hasse* diagram
- Topological sorting
- Isomorphism of posets
- Maximal elements and minimal elements of a poset
- The greatest and the least element of a poset
- Upper bound, lower bound, LUB, and GLB of a subset in a poset
- Lattice, Sublattice, Properties of lattices
- Lattices of Bounded, Distributive, Complemented
- Finite Boolean algebra B_n, $(\mathcal{P}(S),\subseteq)$, and D_n for some n's

Basic Concepts

Chapter 7. Trees

- Trees: Definitions and Terminology
- Subtree
- Binary tree
- Properties of Trees
- Functions on trees: Labeled tree
- Prefix codes and *Huffman* tree
- Tree traversal: Preorder, Inorder, and Postorder
- Spanning tree
- Minimum-cost spanning tree (MST)
- *Prim*'s Algorithm and *Kruskal*'s Algorithm

Basic Concepts

Chapter 8. Graphs

- Undirected graph and Directed graph: Terminology
- Simple graph
- Connected, Disconnected, Strongly connected, Weakly connected, and Strongly connected components
- The Handshaking Theorem
- Some Special Graphs: K_n, C_n, W_n, Bipartite Graphs, $K_{m,n}$,
- Subgraph and quotient graph
- Adjacency Matrices
- *Euler* Paths and Circuits, *Euler* graph, and *Euler*'s Theorem
- *Hamilton* Paths and Circuits, *Hamilton* graph, and *Ore*'s Theorem

Basic Concepts

Chapter 8. Graphs

- Weighted graphs
- Shortest paths and their properties, *Dijkstra*'s Algorithm, TSP
- Transport networks: Capacity, Flow, Maximum flows
- The max-flow min-cut theorem
- *Ford-Fulkerson*'s Algorithm
- Matching problems: Matching set, M-saturated, M-alternating chain, M-augmenting path
- *Hall*'s Marriage Theorem
- Coloring, Dual graph, Chromatic number, k-chromatic graph
- $\chi(G)$ of some special graphs
- Chromatic polynomials, $P_G(k)$ of some special graphs

Problems & Solutions

Chapter 1. Fundamentals

- Use Venn diagrams to represent set operations, or to prove set identities
- Prove or disprove some statements by making use of properties of set operations
- Applications of Addition Principle, Inclusive-Exclusive Principle

Problems & Solutions

Chapter 2. Logic

- Determine the value of compound propositions by making use of truth tables
- Translate English to logical expressions in propositional logic
- Use some important equivalences to construct new logical equivalences
- Translate English to logical expressions in predicate logic
- Negate nested quantifiers
- Construct proofs using rules of inference
- Construct proofs using mathematical induction

Problems & Solutions

Chapter 4. Relations and Digraphs

- Representation of a relation: Ordered pairs, matrix and digraph
- Show some properties of a given relation
- Determine whether a relation is an equivalence relation
- Find the equivalence classes of an equivalence relation
- Construct an equivalence relation from a partition
- Find R^n-relation of a relation R
- Find reflexive closure/symmetric closure of a relation
- Use *Warshall*'s algorithm to find the transitive closure of a relation

Problems & Solutions

Chapter 5. Functions

- Determine whether a function is onto, one to one or one-to-one correspondence
- Find the composition and inverse of functions
- Determine the big-O and big-Θ of a function.
- Find the composition and inverse of permutations
- Determine the Inverse of a permutation, the Product of permutations
- Write a permutation as a product of disjoint cycles

Problems & Solutions

Chapter 6. Ordered Relations and Structures

- Determine whether a relation is partially ordered
- Draw the *Hasse* diagram of a poset, find its maximal and minimal elements, and its greatest and least element if they exist. Find Upper bound, lower bound, LUB, and GLB of some subset of a poset if they exist
- Find the result of a topological sorting for a poset with its *Hasse* diagram
- Show that two posets are isomorphism
- Determine whether a poset is a lattice
- Understand some special Boolean algebras like $(\mathcal{P}(S),\subseteq)$, D_n for some n's,

Problems & Solutions

Chapter 7. Trees

- Make use of properties of trees
- Use *Huffman's* Coding algorithm to construct a prefix codes
- Do a tree traversal by: Preorder, Inorder, and Postorder
- Use *Prim*'s Alg. and *Kruskal*'s Alg. to construct a MST

Chapter 8. Graphs

- Determine whether a graph is an *Euler* graph, or a half *Euler* graph (using *Fleury*'s algorithm)
- Determine whether a graph is a *Hamilton* graph, or a half *Hamilton* graph
- Use *Dijkstra*'s Algorithm to find shortest paths from one vertex to all the others in the graph
- Use *Ford-Fulkerson*'s algorithm to find a maximum flow for a network

Problems & Solutions

Chapter 8. Graphs

- Use *Hall*'s Theorem to determine whether a bi-graph has a complete matching
- Gain the chromatic number of a graph by an approach of reduction
- Gain the chromatic polynomial of a graph by an approach of reduction

Algorithms

Chapter 3. Relations and Digraphs

- *Warshall*'s algorithm finding the transitive closure of a relation

Chapter 6. Ordered Relations and Structures

- A topological sorting algorithm for a poset with its *Hasse* diagram

Chapter 7. Trees

- Pre-order, In-order and Post-order traversal algorithms
- The *Huffman's* Coding algorithm
- *Prim*'s Algorithm and *Kruskal*'s Algorithm

Chapter 8. Graphs

- *Fleury's* Algorithm
- **Dijkstra*'s Algorithm
- *Ford-Fulkerson*'s Algorithm

Office Hours

- Office hours:
  - 17, Jun. (Monday), 3:00 pm - 5:00 pm, A304,
  - 20, Jun. (Thursday), 3:00 pm - 5:00 pm, A304.

Thank You

- Thank you for your attention!
- I will see you in the **Web Security Course** next, next year.