人工智能
——人工神经网络 I

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Perceptron Learning Algorithm

- Perceptron Learning Algorithm (感知机学习算法)：单层前馈神经网络
- Dealing with all attributes jointly which are continuous variables

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- Dealing with all attributes jointly which are continuous variables
- For **x**=(x_1,x_2,…,x_d) with d features, compute a weighted 'score' and
  predict +1(good) if $\sum_{k=1}^{d} w_k x_k > threshold$
  predict -1(bad) if $\sum_{k=1}^{d} w_k x_k < threshold$
- **y**={+1(good), -1(bad)}

$$h(\mathbf{x}) = sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) - threshold\right)$$

Perceptron Learning Algorithm

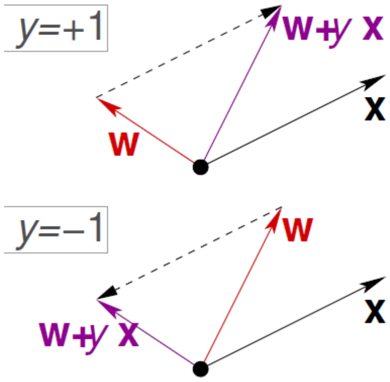
$$
\begin{aligned}
h(\mathbf{x}) &= sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) - threshold\right) \\
&= sign\left(\left(\sum_{k=1}^{d} w_k x_k\right) + \underbrace{(-threshold)}_{w_0} \cdot \underbrace{(+1)}_{x_0}\right) \\
&= sign\left(\sum_{j=0}^{d} w_j x_j\right) \\
&= sign\left(\tilde{\mathbf{W}}^{\mathrm{T}} \tilde{\mathbf{X}}\right)
\end{aligned}
$$

Perceptron Learning Algorithm

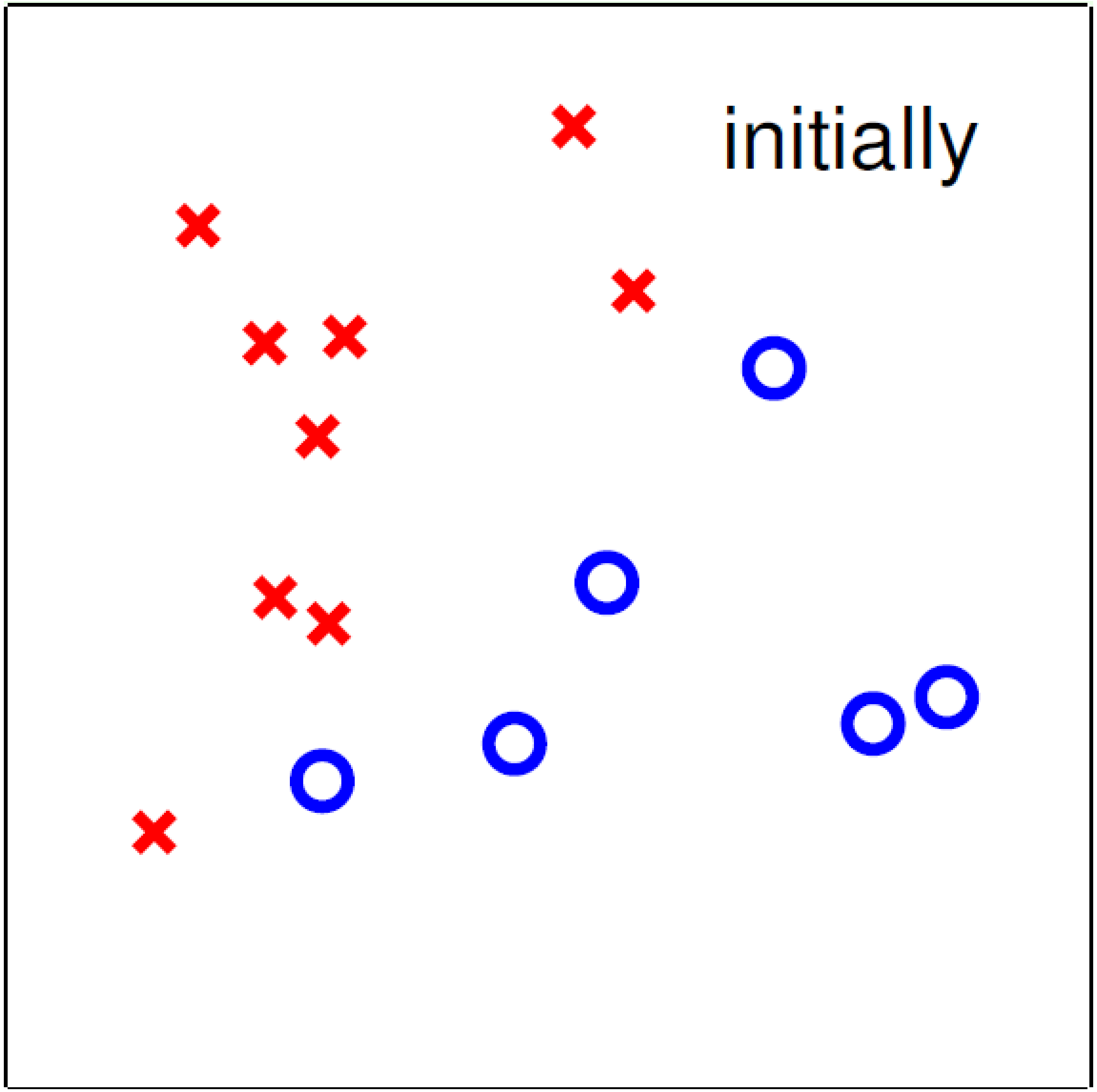
- Difficult: the set of $h(\mathbf{x})$ is of infinite size
  无限大
- Idea: start from some initial weight vector $\mathbf{w}_{(0)}$, and "correct" its mistakes on D
- For t = 0, 1,…
  - find a mistake of $\mathbf{w}_{(t)}$ called $(\mathbf{x}_{i(t)}, y_{i(t)})$

    $$sign\left(\tilde{\mathbf{w}}_{(t)}^{\mathrm{T}}\tilde{\mathbf{x}}_{i(t)}\right) \neq y_{i(t)}$$
  - (try to) correct the mistake by

    $$\tilde{\mathbf{w}}_{(t+1)} \leftarrow \tilde{\mathbf{w}}_{(t)} + y_{i(t)}\tilde{\mathbf{x}}_{i(t)}$$
  - until no more mistakes
- Return last $\mathbf{W}$ (called $\mathbf{W}_{\mathrm{PLA}}$)



Perceptron Learning Algorithm



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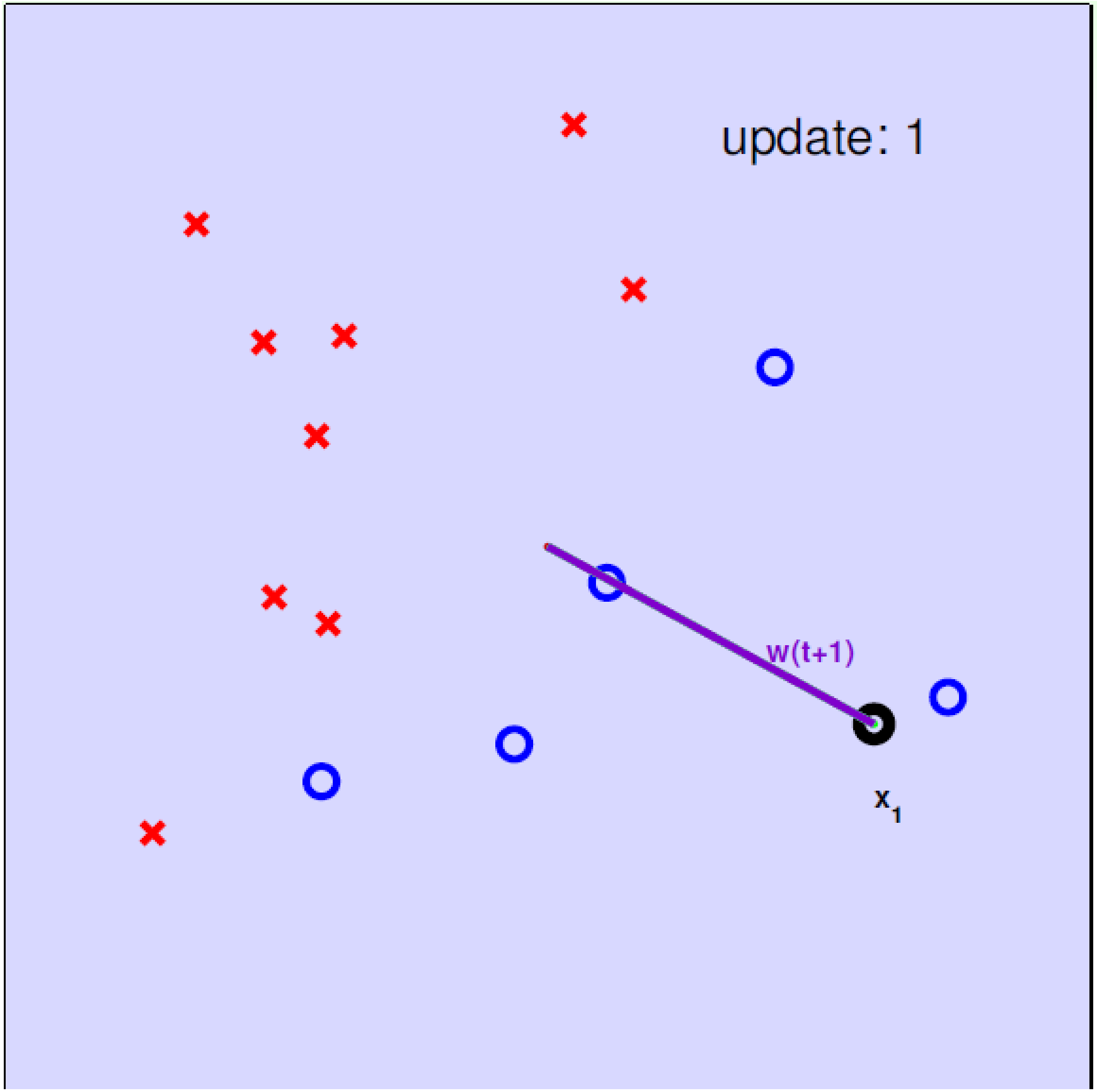

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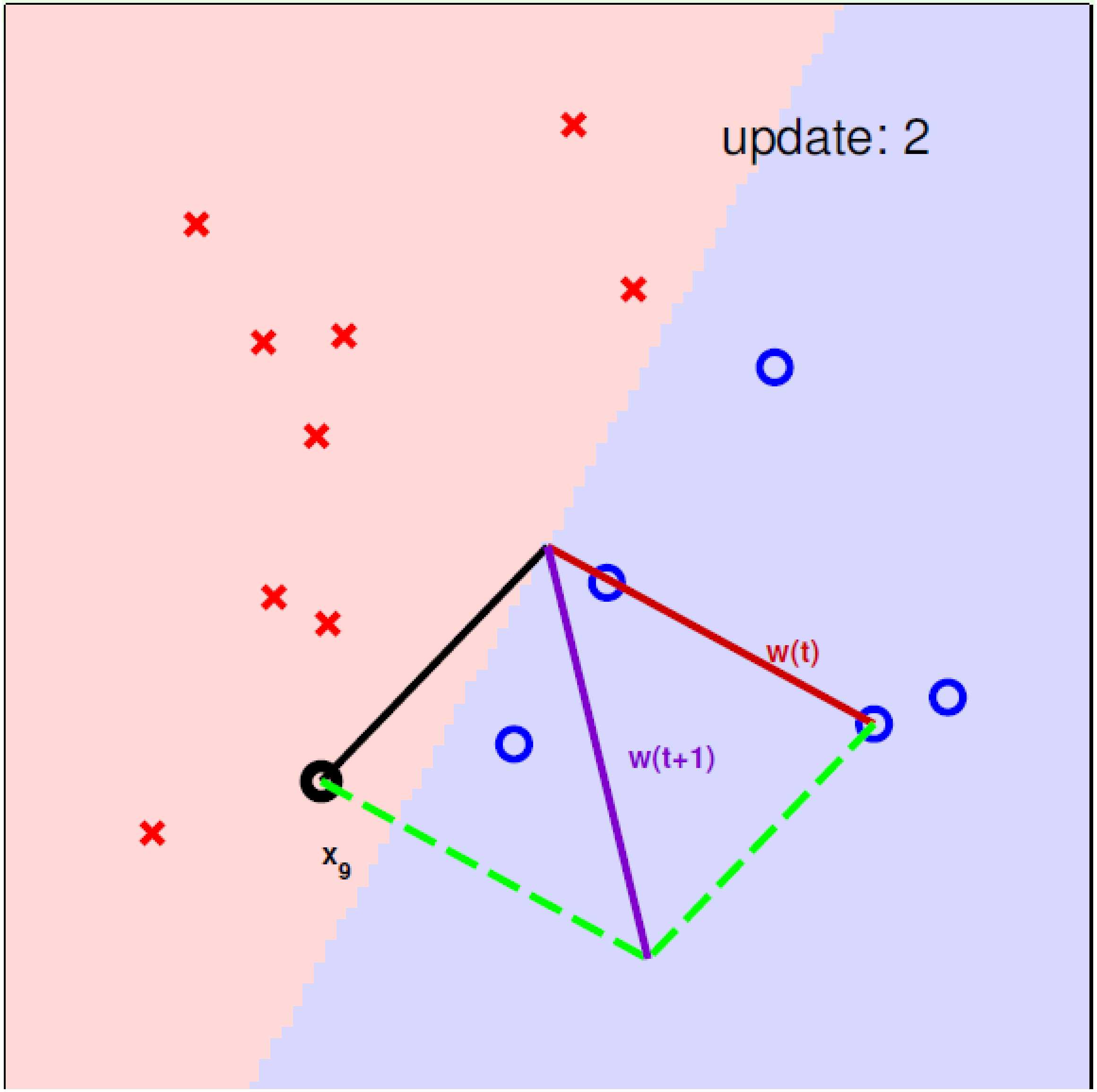

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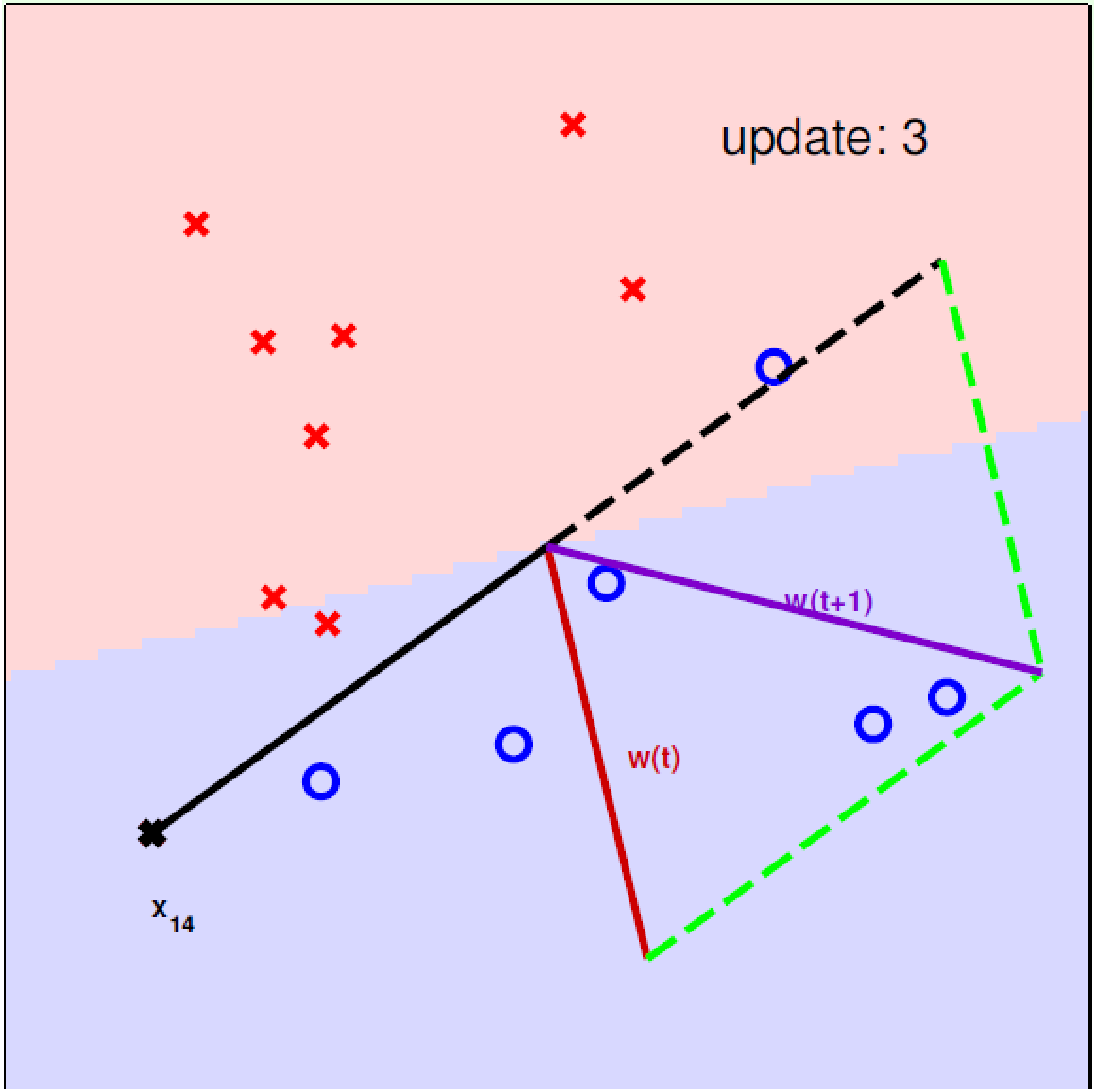

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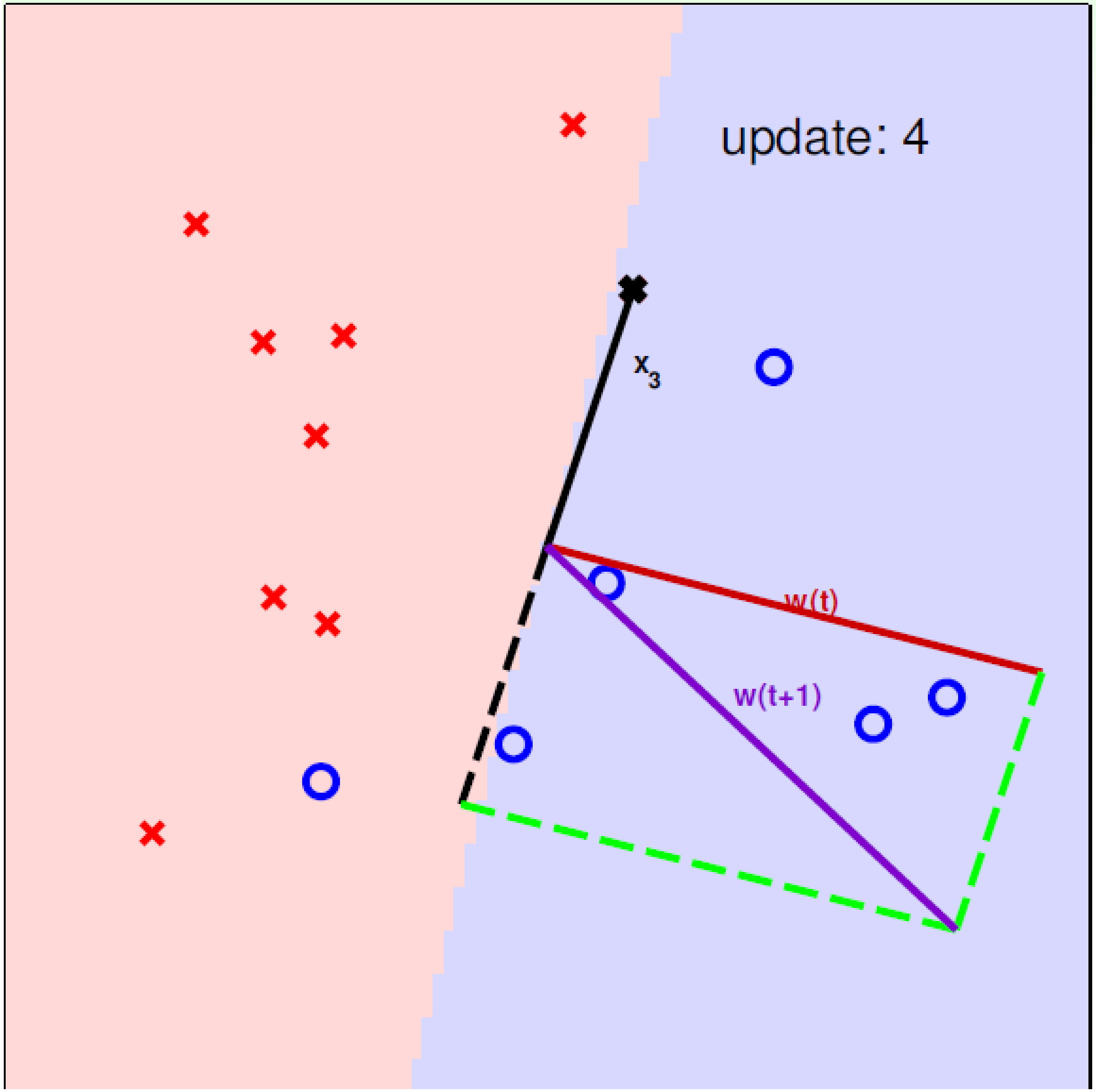

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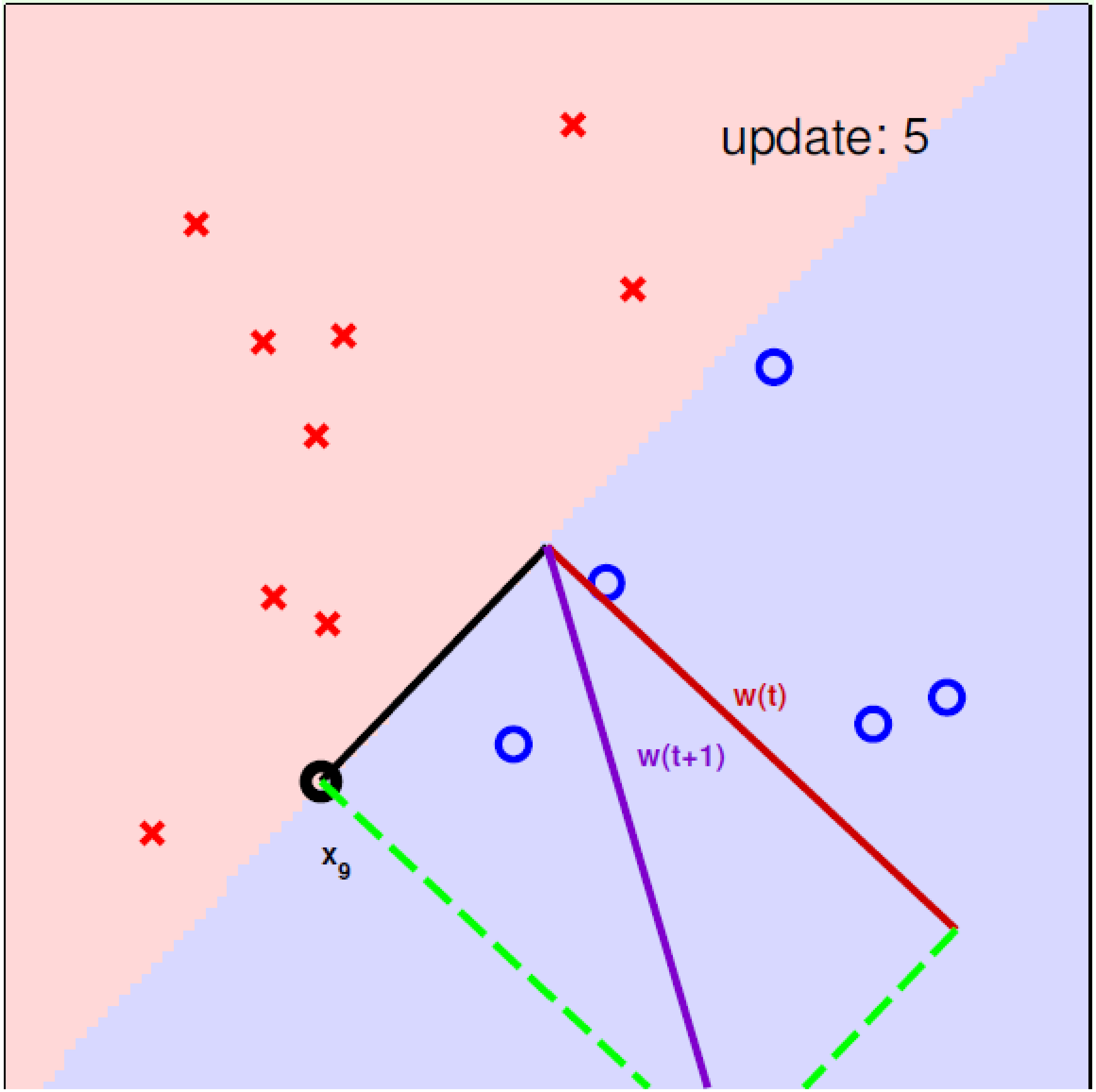

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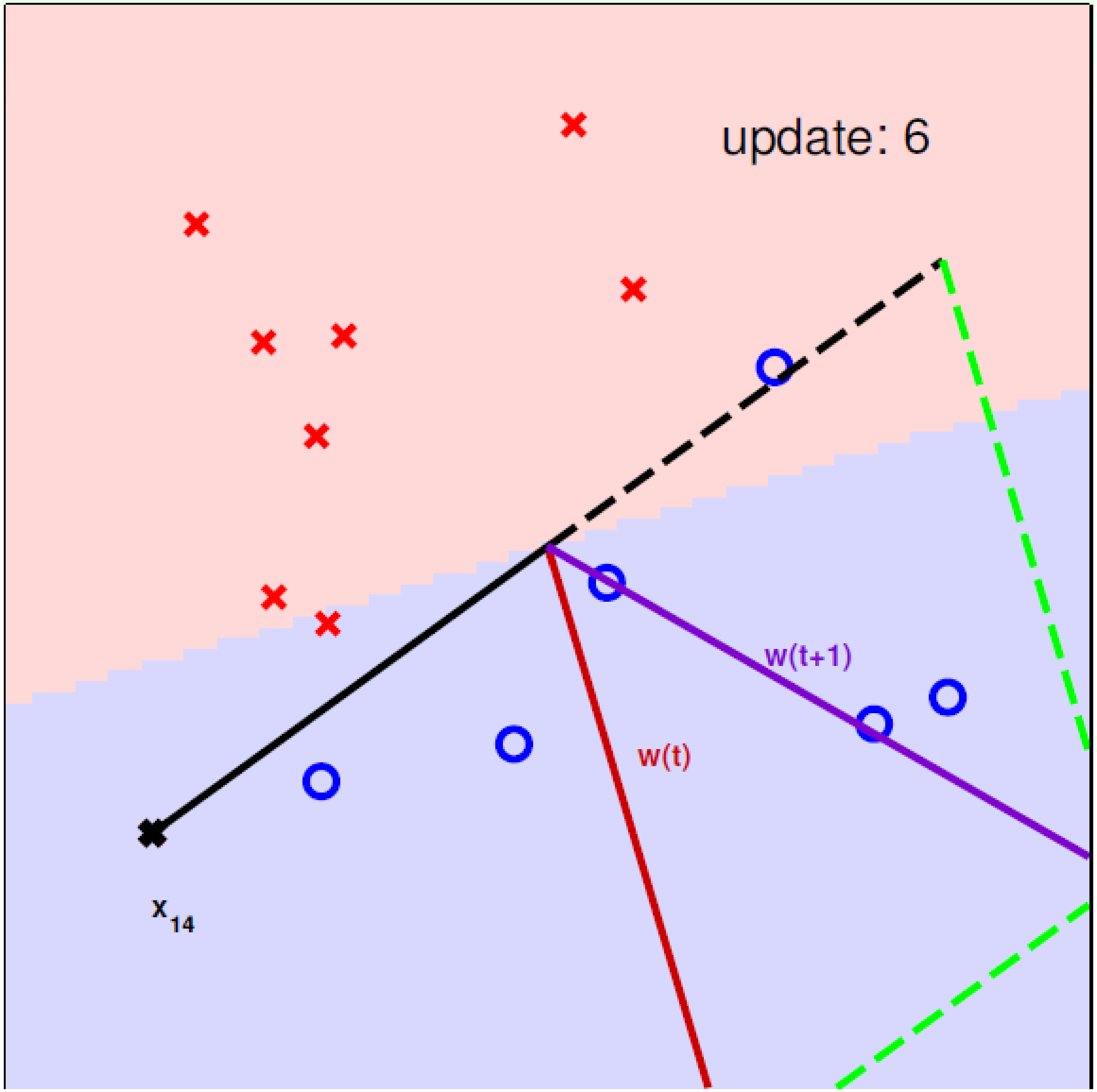

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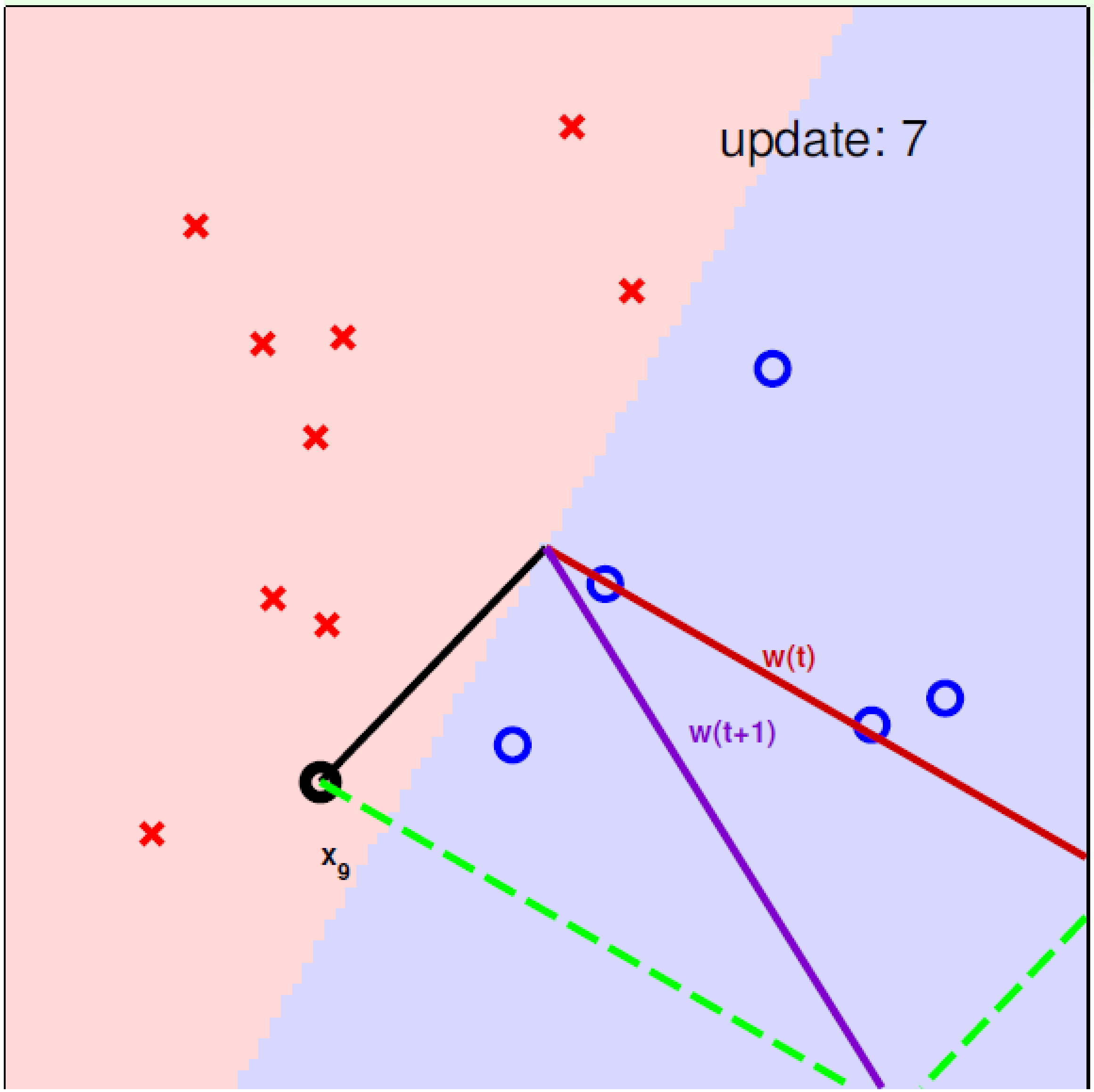

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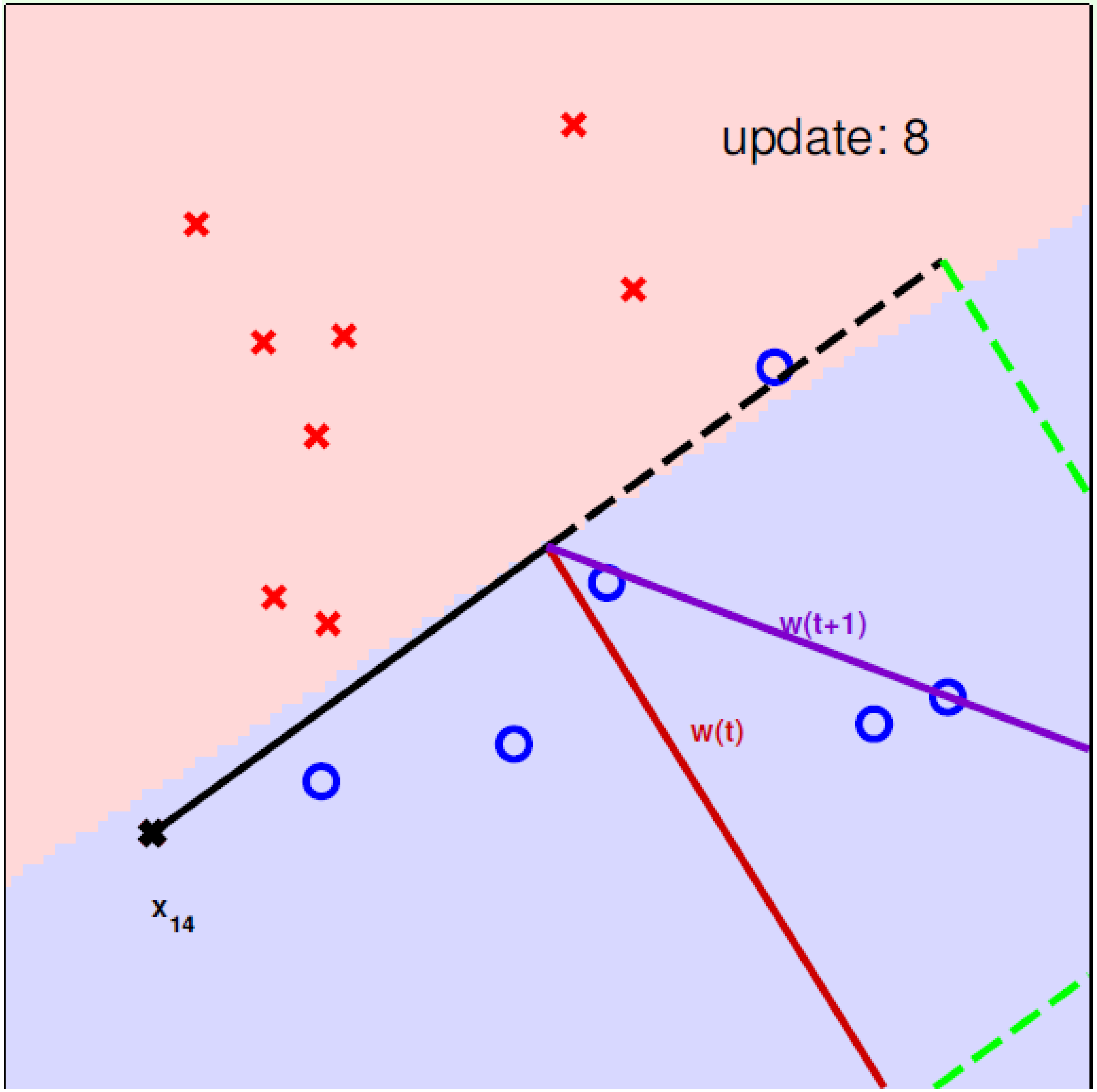

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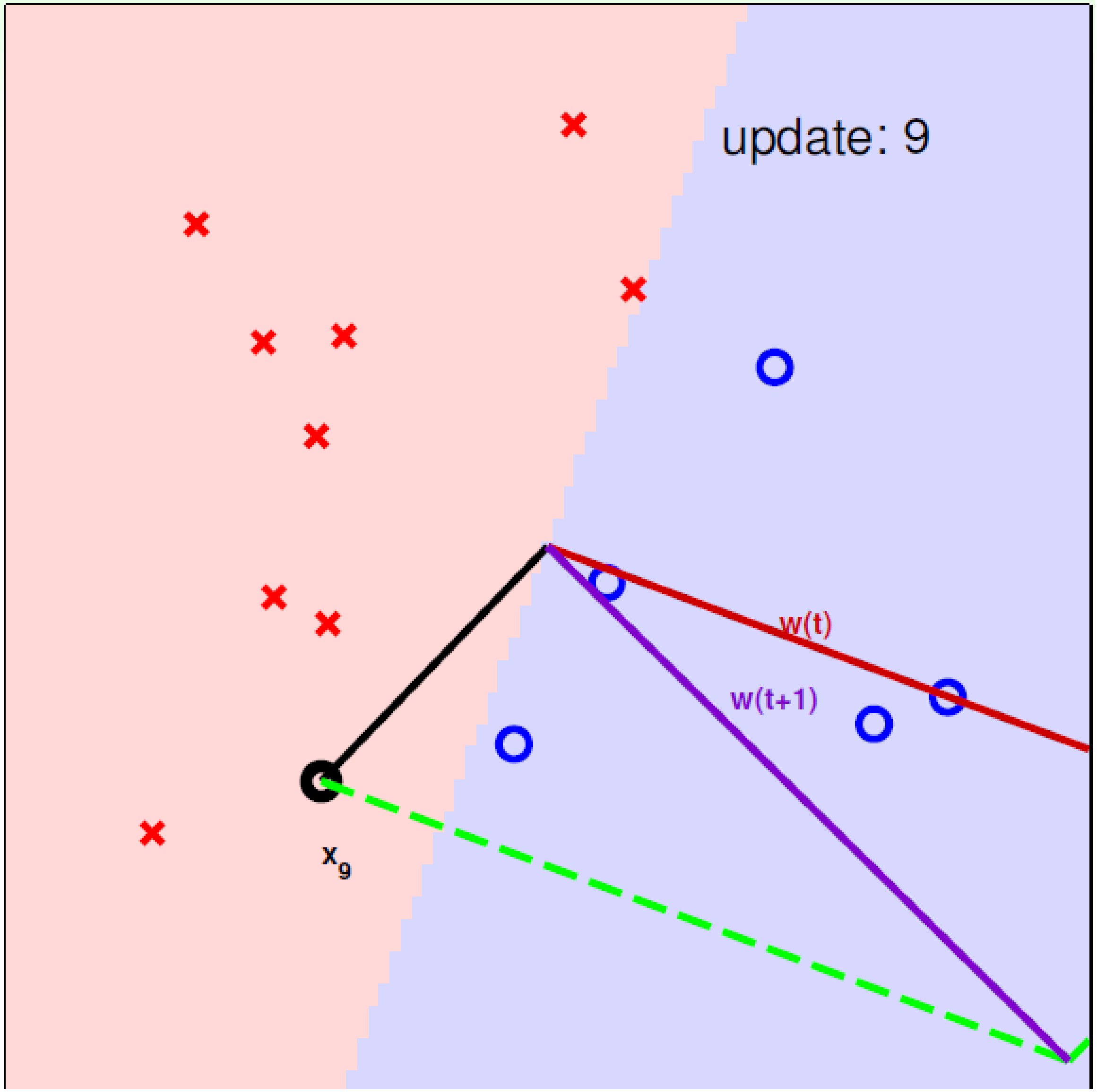

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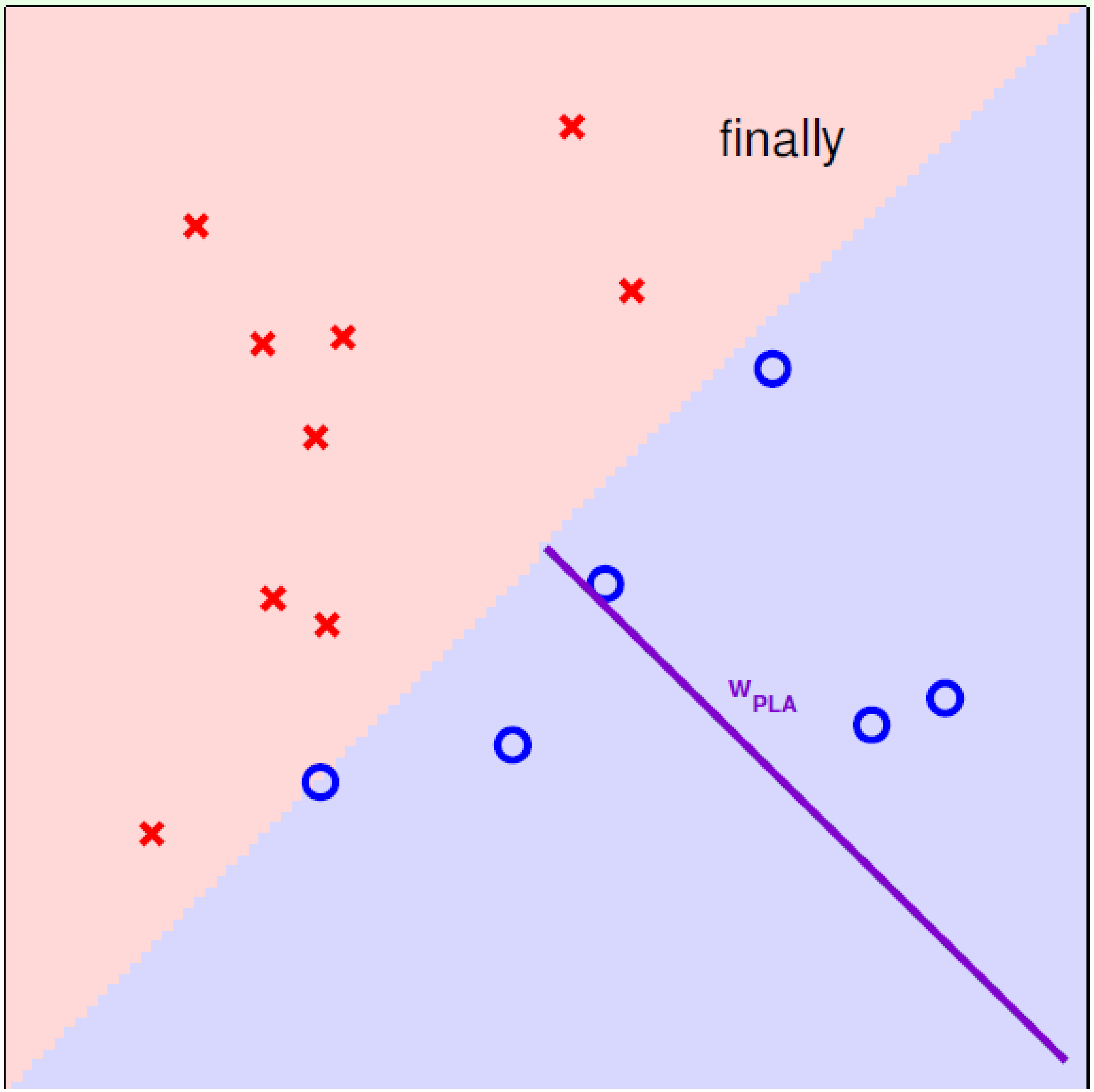

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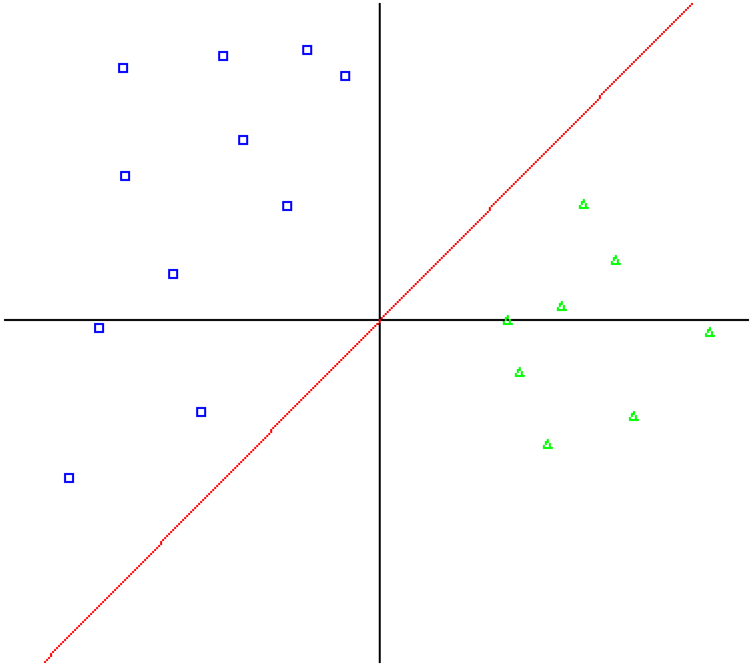
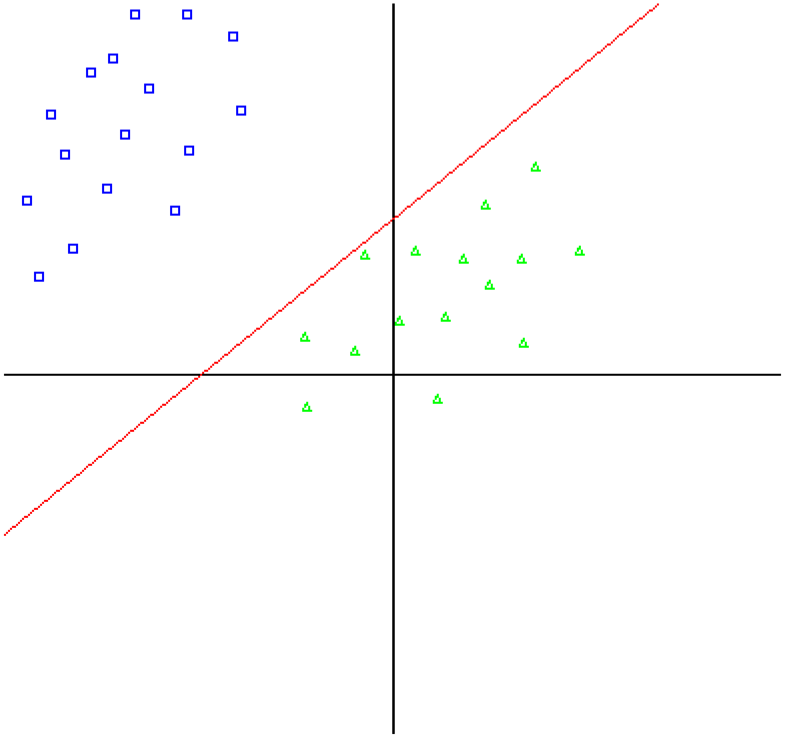


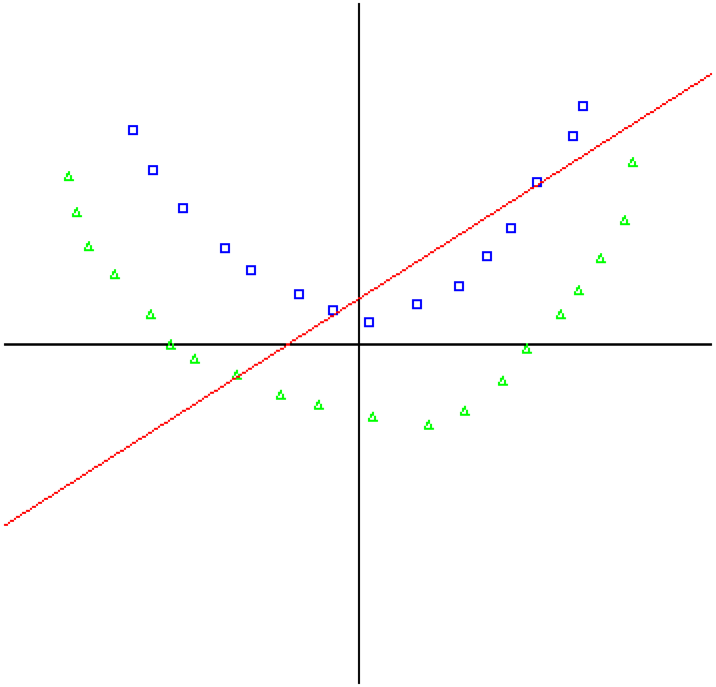


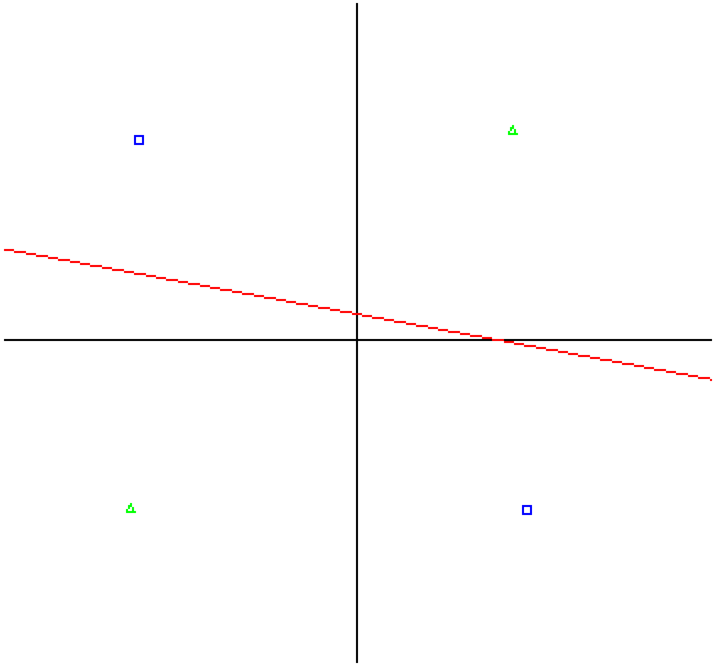
Perceptron Learning Algorithm

- Only if there exists an hyperplane that correctly classifies the data, the Perceptron procedure is guaranteed to converge; furthermore, the algorithm may give different results depending on the order in which the elements are processed, indeed several different solutions exist.

只有存在正确分类数据的超平面时，感知机才会收敛
该算法可以根据处理元素的顺序给出不同的结果
实际上存在几种不同的解

Perceptron Learning Algorithm





Artificial Neuron

可用于非线性可分

神经元

- The unit of computation in neural networks is the artificial neuron. 神经网络的计算单位是人工神经元
- An artificial neuron consists of
  - Input signals x_i. These signals represent data from the environment or activation of other neurons.

    激活
  - A set of real-valued weights w_i. The values of these weights represent connection strengths.

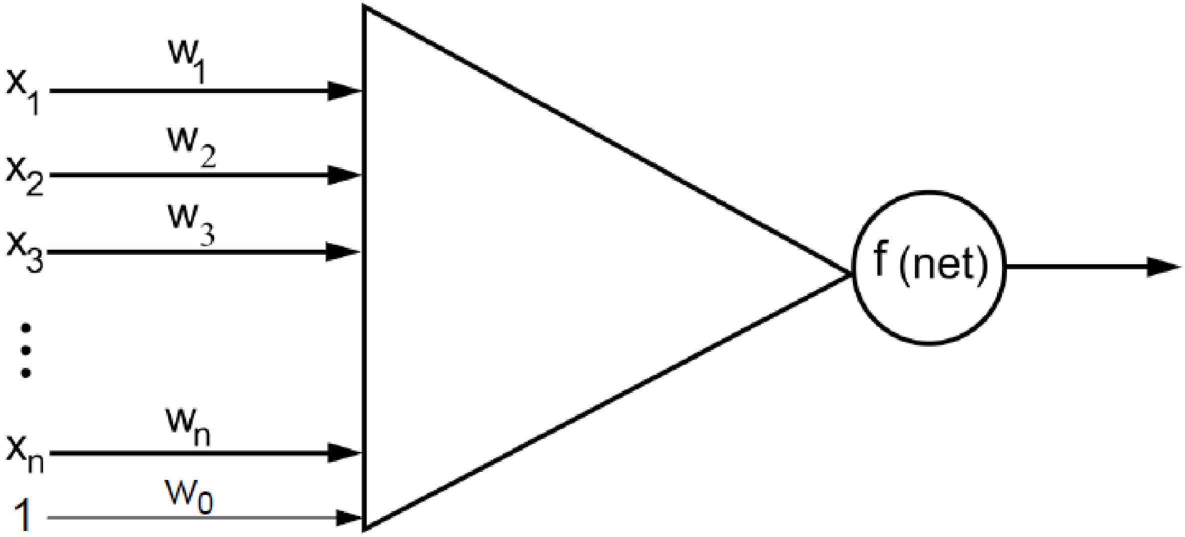
    实值
  - An activation level $\sum_i w_i x_i$. The neuron's activation level is determined by the sum of the weighted inputs.
  - A threshold function f. This function computes the final output by determining if the activation is below or above a threshold.

    阈值

Artificial Neuron

- Given the activation value $net = \sum_i w_i x_i$, the output of the neuron is given by

$$f(net) = \begin{cases} +1 & if \sum_i w_i x_i \geq 0 \\ -1 & if \sum_i w_i x_i < 0 \end{cases}$$



Example

- An artificial neuron can be used to compute the logic AND function.
  - The neuron has three inputs
  - x_1 and x_2 are the original inputs.
  - The third is the bias 偏移量 input which has a constant value of +1.
  - The input data and bias have weights of +1, +1, and –2 respectively.
- What about the logic OR function?

Artificial Neuron

- The perceptron learning algorithm (PLA) can be used to adjust 调整 the weights of an artificial neuron.
- The weights are adjusted until the outputs of the neuron become consistent 一致 with the true outputs of training examples.
- The following rule is used

$$\mathbf{w}_{(t+1)} \leftarrow \mathbf{w}_{(t)} + y_{n(t)}\mathbf{x}_{n(t)}$$

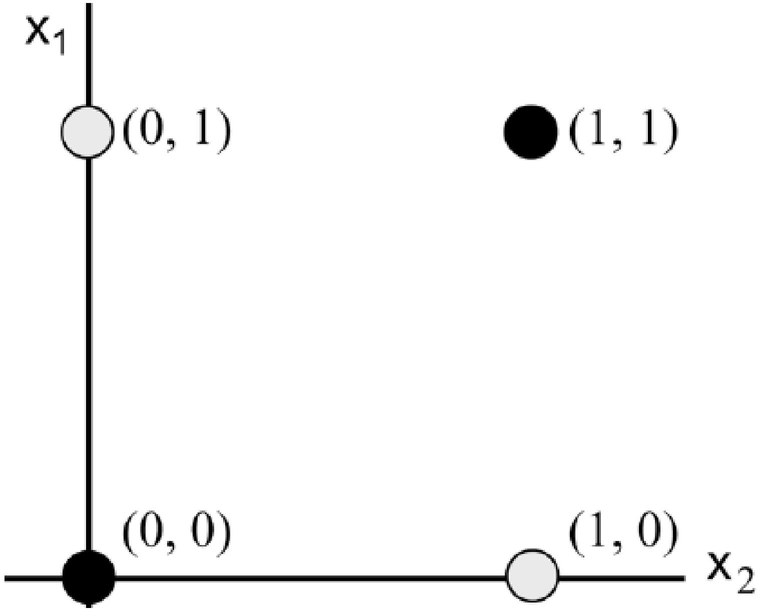
Artificial Neuron

PLA不能解决不可线性分离的问题

- Perceptron learning algorithm can not solve those problems where the patterns are not linearly separable.
- An example of this is the exclusive-OR problem.
- Multilayer networks are required for solving such kinds of problems.

Example

x_1	x_2	Output
1	1	-1
1	0	1
0	1	1
0	0	-1


x_1
(0, 1)
(1, 1)
(0, 0)
(1, 0)
x_2