

中山大学 本科生考试草稿纸 2011/7-49

警示

《中山大学授予学士学位工作细则》第七条：“考试作弊者不授予学士学位。”

$$\begin{aligned} \text{P.134.9} \quad \int \sqrt{1+9x^2} dx &= x \cdot \sqrt{1+9x^2} - \int x d\sqrt{1+9x^2} = x \cdot \sqrt{1+9x^2} - \int x \cdot \frac{18x}{2\sqrt{1+9x^2}} dx \\ &= x \cdot \sqrt{1+9x^2} - \int \frac{9x^2}{\sqrt{1+9x^2}} dx = x \cdot \sqrt{1+9x^2} - \int \frac{1+9x^2-1}{\sqrt{1+9x^2}} dx \end{aligned}$$

$$= x \cdot \sqrt{1+9x^2} - \int \sqrt{1+9x^2} dx + \int \frac{dx}{\sqrt{1+9x^2}}$$

$$2 \int \sqrt{1+9x^2} dx = x \cdot \sqrt{1+9x^2} + \frac{1}{3} \int \frac{1}{\sqrt{1+(3x)^2}} d(3x)$$

$$\int \sqrt{1+9x^2} dx = \frac{x}{2} \sqrt{1+9x^2} + \frac{1}{6} \arctan 3x + C.$$

$$11. \int \ln(x + \sqrt{1+x^2}) dx = x \cdot \ln(x + \sqrt{1+x^2}) - \int x d \ln(x + \sqrt{1+x^2})$$

$$= x \ln(x + \sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}} dx$$

$$= x \ln(x + \sqrt{1+x^2}) - \int \frac{1}{2\sqrt{1+x^2}} d(1+x^2)$$

$$= x \cdot \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C.$$

$$12. \int (\operatorname{arccos} x)^2 dx = x (\operatorname{arccos} x)^2 - \int x d(\operatorname{arccos} x)^2$$

$$= x \cdot (\operatorname{arccos} x)^2 - \int x \cdot 2 \operatorname{arccos} x \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) dx$$

$$= x \cdot (\operatorname{arccos} x)^2 - \int \operatorname{arccos} x \cdot \frac{1}{\sqrt{1-x^2}} d(1-x^2)$$

$$= x \cdot (\operatorname{arccos} x)^2 - 2 \int \operatorname{arccos} x d\sqrt{1-x^2}$$

$$= x \cdot (\operatorname{arccos} x)^2 - 2 \sqrt{1-x^2} \operatorname{arccos} x + 2 \int \sqrt{1-x^2} d \operatorname{arccos} x$$

$$= x \cdot (\operatorname{arccos} x)^2 - 2 \sqrt{1-x^2} \cdot \operatorname{arccos} x + 2 \int \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= x \cdot (\operatorname{arccos} x)^2 - 2 \sqrt{1-x^2} \cdot \operatorname{arccos} x - 2x + C.$$