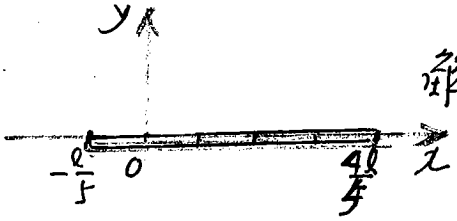
P.165.27. 有一均匀细棒，长为 l，质量 M。计算细棒距离一端 $\frac{l}{5}$ 的转动惯量。



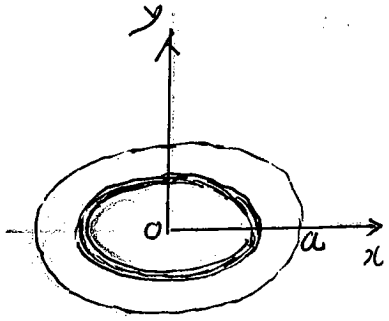
解：$\rho_0 = \frac{M}{l}$，$ds = dx$

$$J_y = \int_{-\frac{l}{5}}^{\frac{4l}{5}} x^2 \cdot \rho_0 \, ds = \frac{M}{l}\int_{-\frac{l}{5}}^{\frac{4l}{5}} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-\frac{l}{5}}^{\frac{4l}{5}}$$

$$= \frac{M}{l}\cdot\frac{1}{3}\left[\frac{4^3 l^3}{5^3} + \frac{l^3}{5^3}\right] = \frac{M}{l}\cdot\frac{64+1}{3\times 5^3} = \frac{13Ml^2}{75}.$$

P.165.28. 设有一均匀圆盘，半径为 a，质量 M。

求它对于通过圆心且与盘面垂直的轴之转动惯量 I。



解：$\rho_0 = \frac{M}{\pi a^2}$

$$dI = r^2 \cdot \rho_0 \cdot 2\pi r \, dr = \frac{M}{\pi a^2} 2\pi r^3 dr = \frac{2M}{a^2} r^3 dr$$

$$I = \int_0^a \frac{2M}{a^2} r^3 dr = \frac{2M}{a^2}\left[\frac{r^4}{4}\right]_0^a = \frac{1}{2}Ma^2.$$