



Computer Graphics

Rendering Pipeline: Image Processing

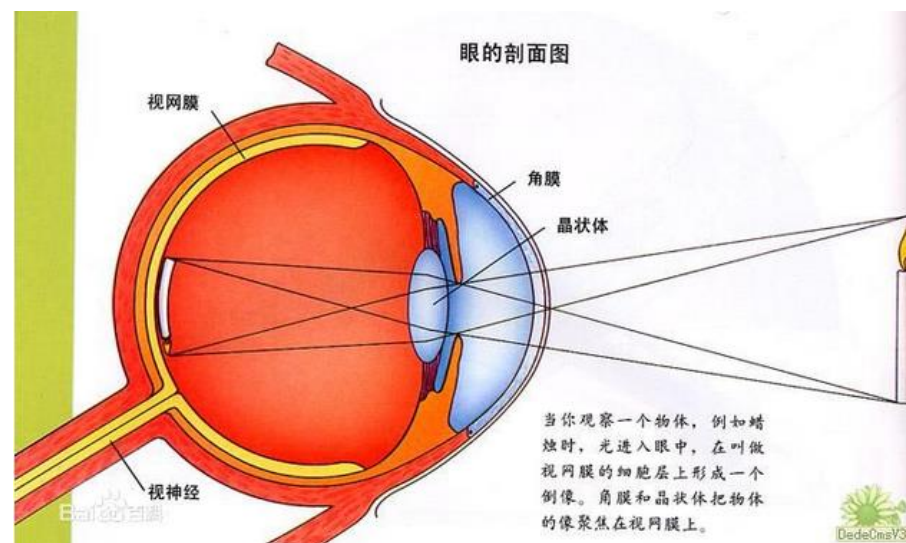
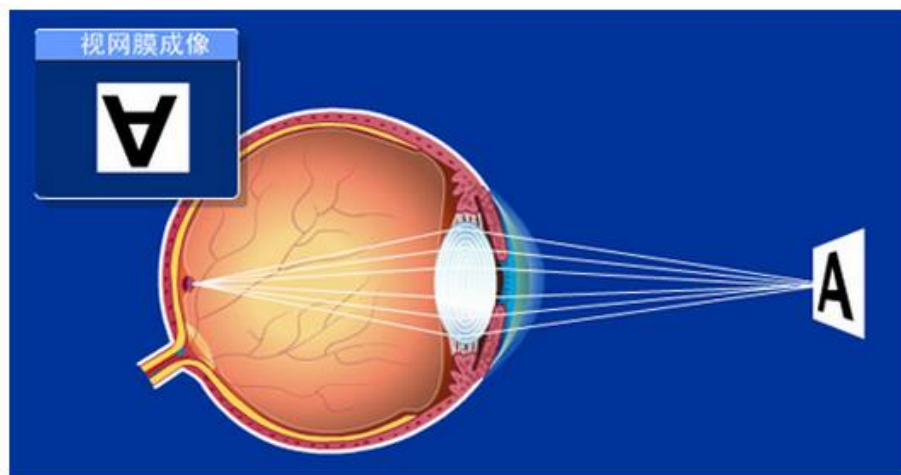
Teacher: Dr. Zhuo SU (苏卓)

E-mail: suzhuo3@mail.sysu.edu.cn

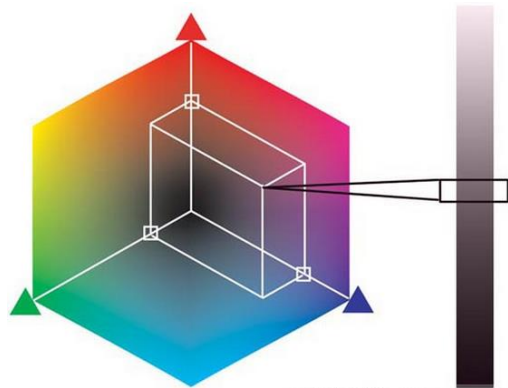
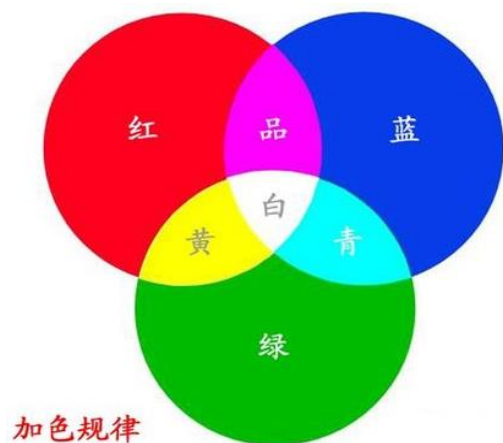
School of Data and Computer Science



视网膜成像：上亿个感光细胞



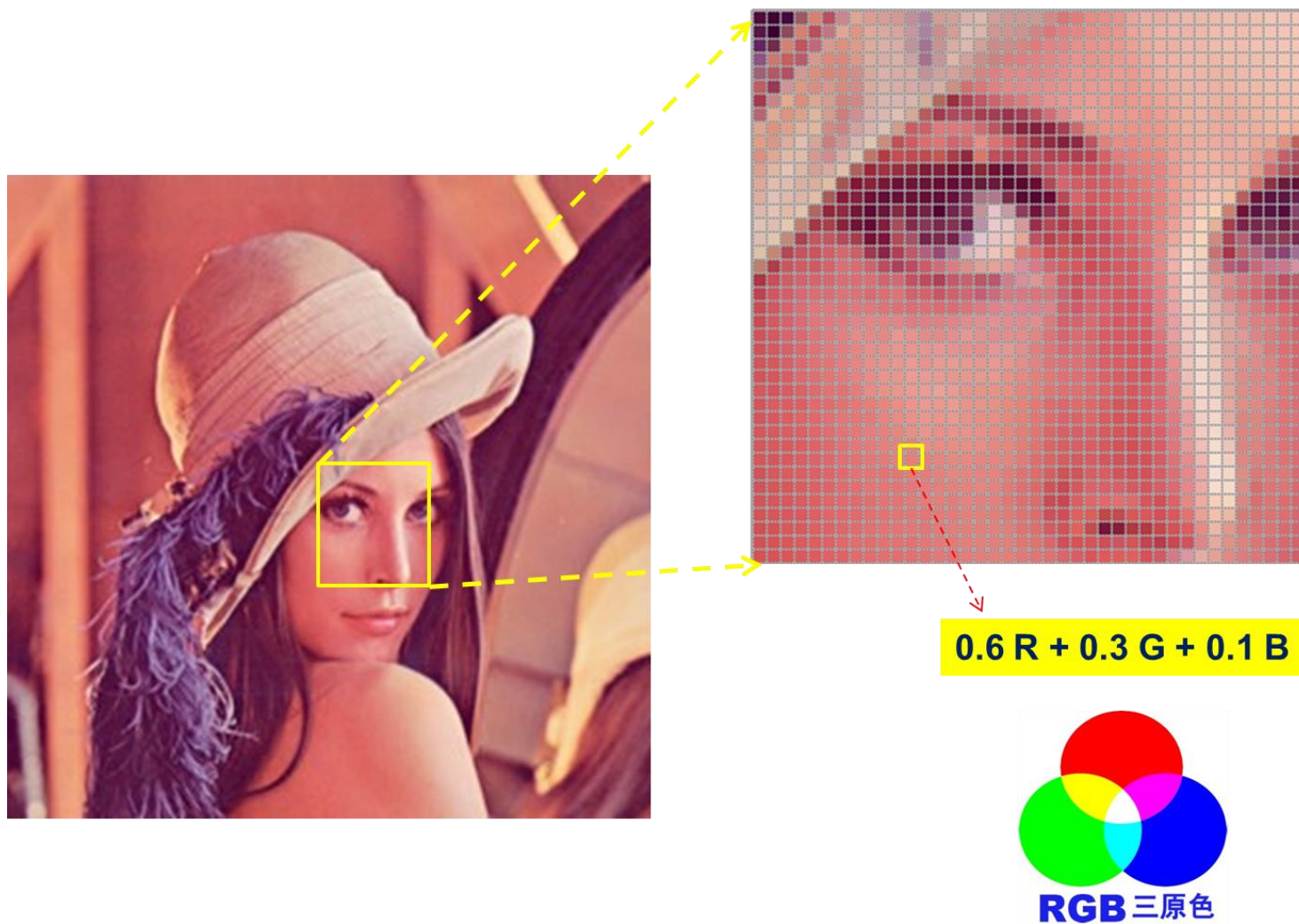
颜色三原色与RGB颜色空间



$$\text{Color} = a R + b G + c B$$

#FFFFCC	#CCFFFF	#FFCCCC	#99CCCC	#FFCC99	#FFCCCC	#FF9999	#996699	#FFCCCC
#CC9999	#FFFFCC	#CCCC99	#FFCCCC	#FFFF99	#CCCCFF	#0099CC	#CCCCCC	#FF6666
#FF9966	#FF6666	#FFCCCC	#CC9966	#666666	#CC9999	#FF6666	#FFFF66	#99CC66
#CC3333	#CCCCCC	#003366	#993333	#CCCC00	#663366	#CCCC99	#666666	#CC9999
#FF6666	#FFFF00	#0066CC	#CC0033	#333333	#CCCC00	#336633	#990033	#FFCC99
#993333	#CC9966	#003300	#FF0033	#333399	#CCCC00	#CC0033	#000000	#003399
#000000	#99CC00	#CC0033	#999933	#993333	#333300			

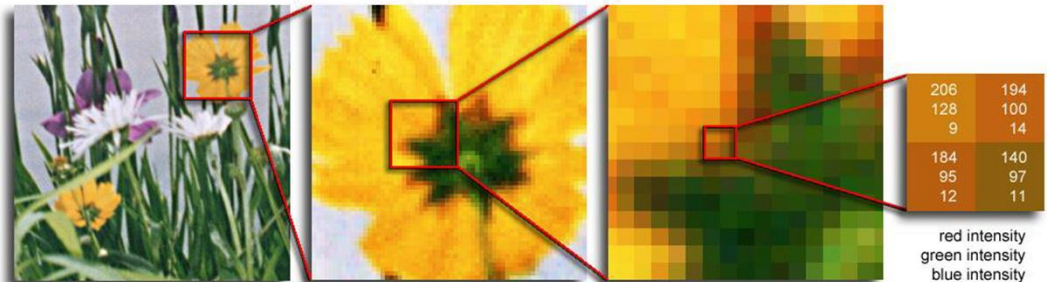
数字图像 (Digital Image)



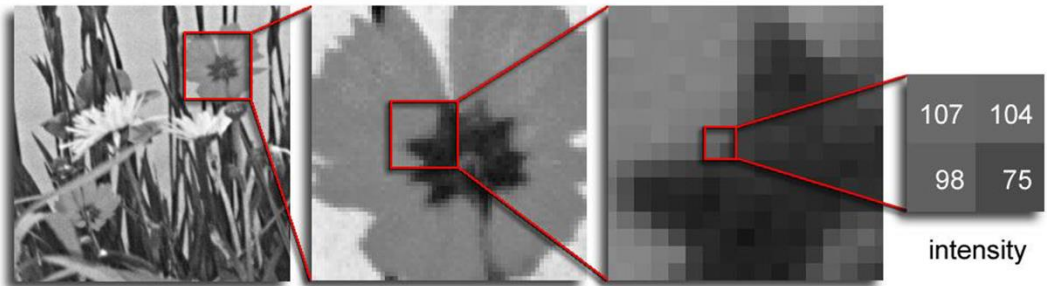
Digital Image

- Binary images (0 or 1)
- Gray images (0~255)
- Color images
 - indexed color images
 - full color images (24 bits per pixel, 8-red, 8-green, 8-blue))

**Color
Image**



**Grey
Image**



Color Components



Image Format

- BMP: bitmap
 - Raw data , 无压缩 , 文件大
- GIF
 - 文件小 , 能存储动画
- JPEG
 - 压缩率高
- JPEG 2000
 - 更高压缩率 , 支持渐近传输
- PSD, SWF...



Image Quality

- Image format and its compressive ratio.
- BMP , PNG , JPG



BMP
263 KB



PNG
112 KB



100% JPG 52 KB



10% JPG 6.38 KB

Video: Image Sequence



图像的连续数学模型：平面区域上的向量值函数

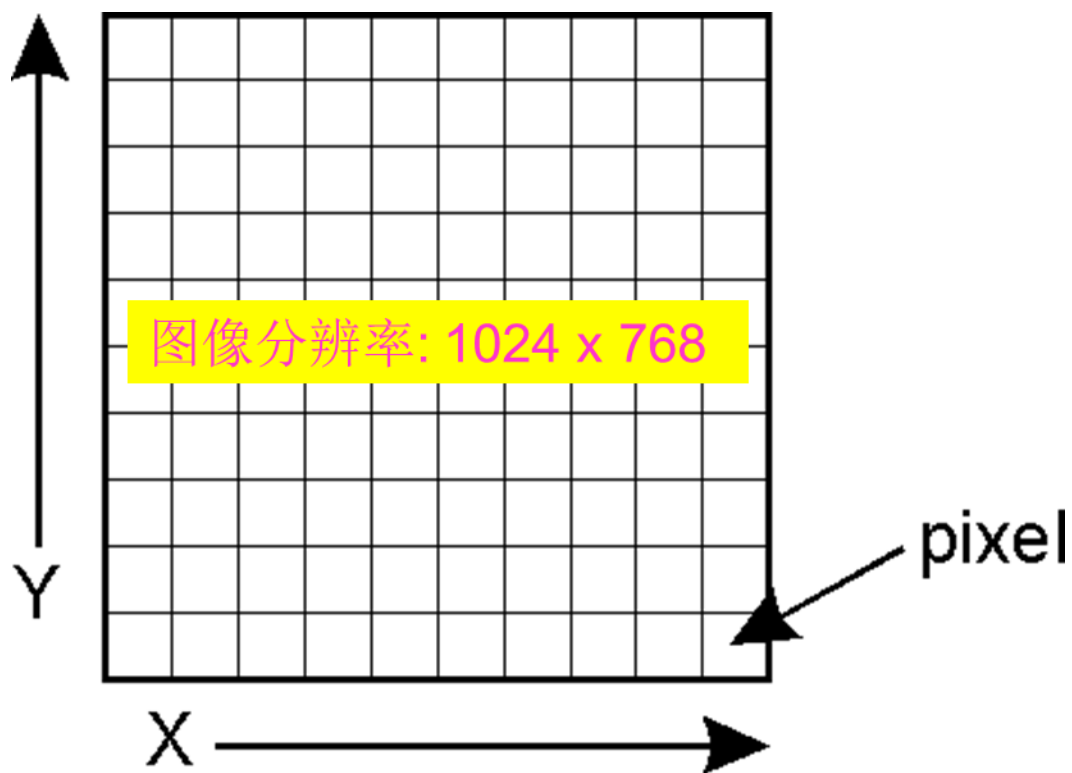
- 区域内每个点有个颜色值
 - 每种颜色值是红、绿、蓝三个成分的线性组合
 - 颜色空间是三维线性空间



$$0.6 R + 0.3 G + 0.1 B$$

图像的离散数学模型：矩阵

- 分辨率



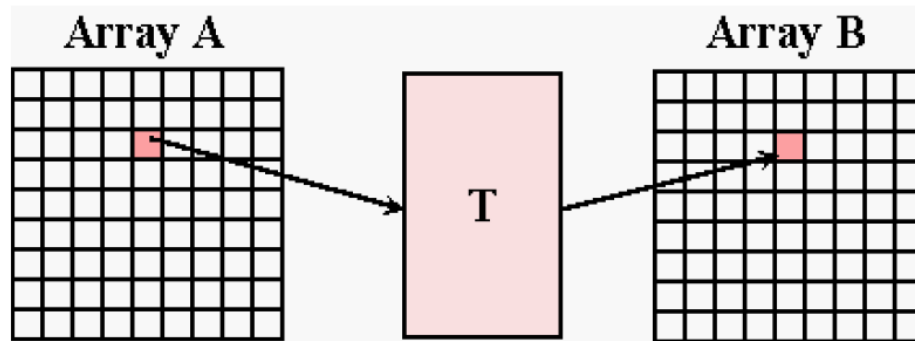
- 图像的抽象
- 我们需要哪些信息可以决定一幅图像？
 - 宽、高
 - 每个元素的颜色

- 像素的矩阵！

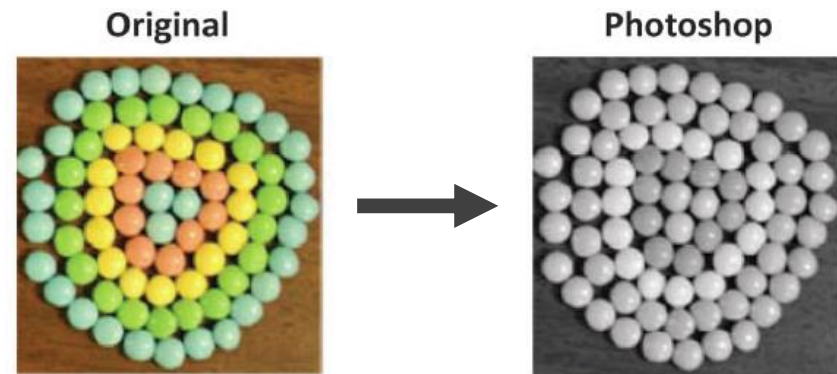
```
class CImage
{
    public:
        int GetWidth();
        int Get Height();
        CColor GetPixelAt(int x, int y);
        void SetPixelAt(int x, int y, CColor c);
};
```

Image Processing (Image Transformation)

$$B[x, y] = T[A[x, y]]$$

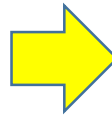


- Examples : Reverse color
- R, G, B channels
- Color2Gray
 - $\text{Gray} = a R + b G + c B$
 - $\text{Gray} = R \cdot 0.299 + G \cdot 0.587 + B \cdot 0.114$
- Add noise
- ...



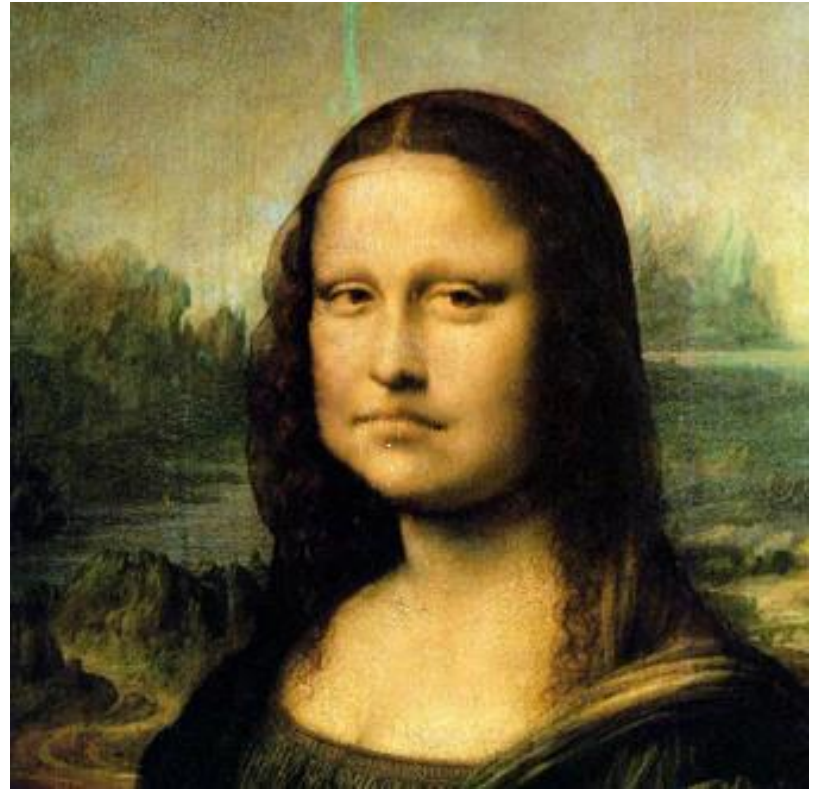
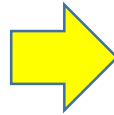
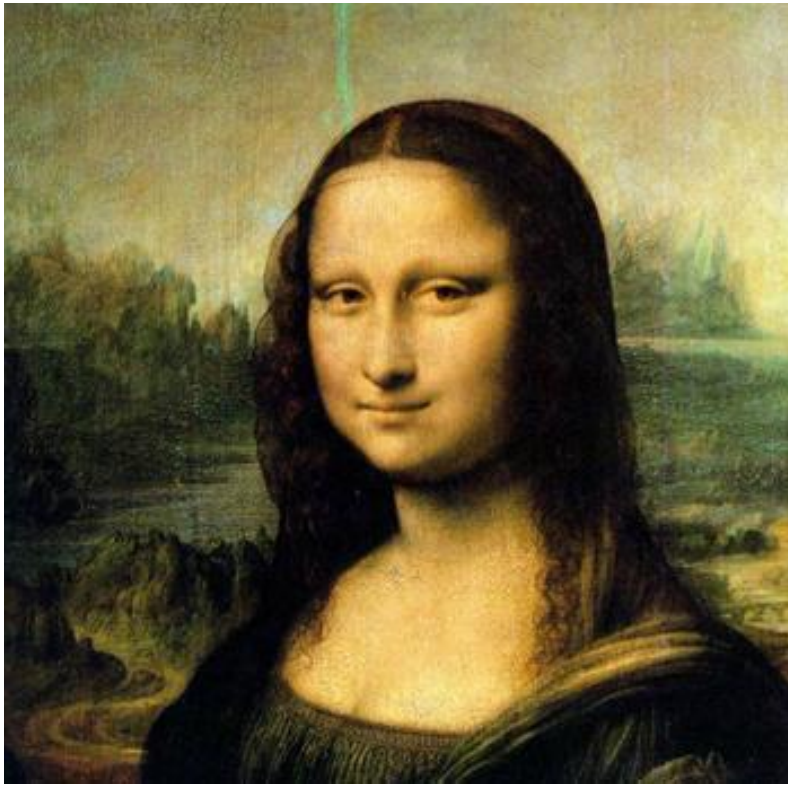
Case 1

- Image Deformation



Case 1

- Image Deformation

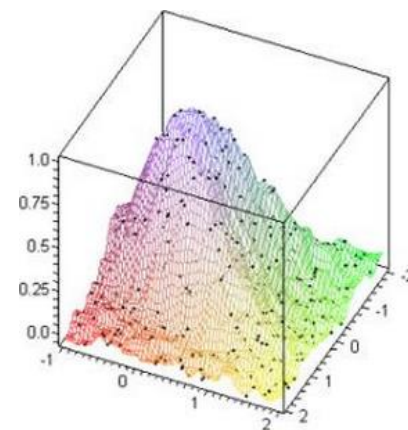


Case 1



Case 1

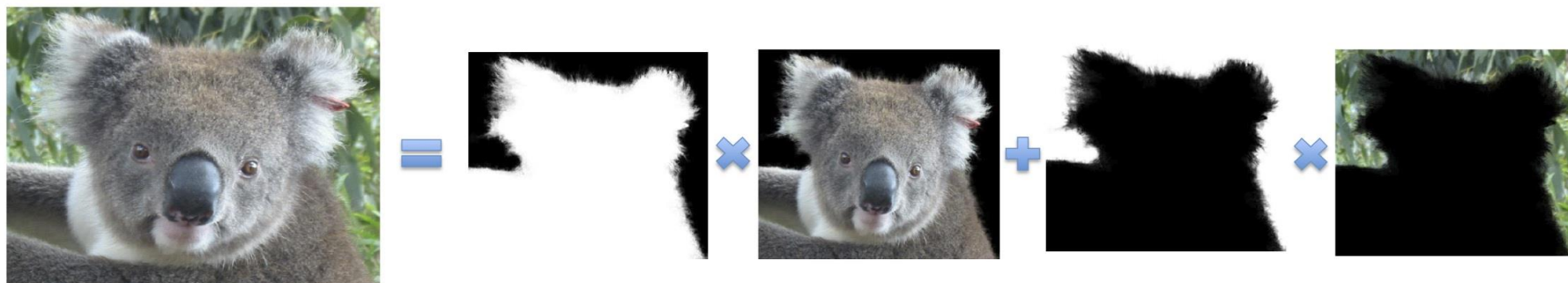
- How to do?



Scattered Point Interpolation

Case 2

- Image Matting
 - Cut out the object from a given image



Case 2

- How to do
 - Photoshop Magic Wand Tool



Original Image

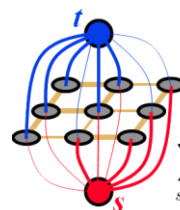
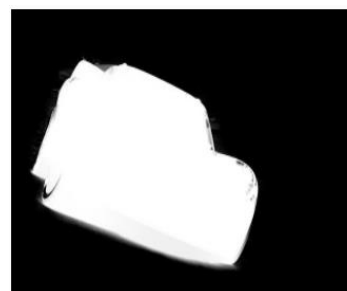


Select an area with the Wand tool



Add additional areas

- Interactive sketch



图论：最大流最小割算法

$$\sum_{s \in V} \bar{\theta}_s(x_s) + \sum_{(s,t) \in E} \bar{\theta}_{st}(x_s, x_t)$$

各种图像处理软件

- Photoshop (PS)

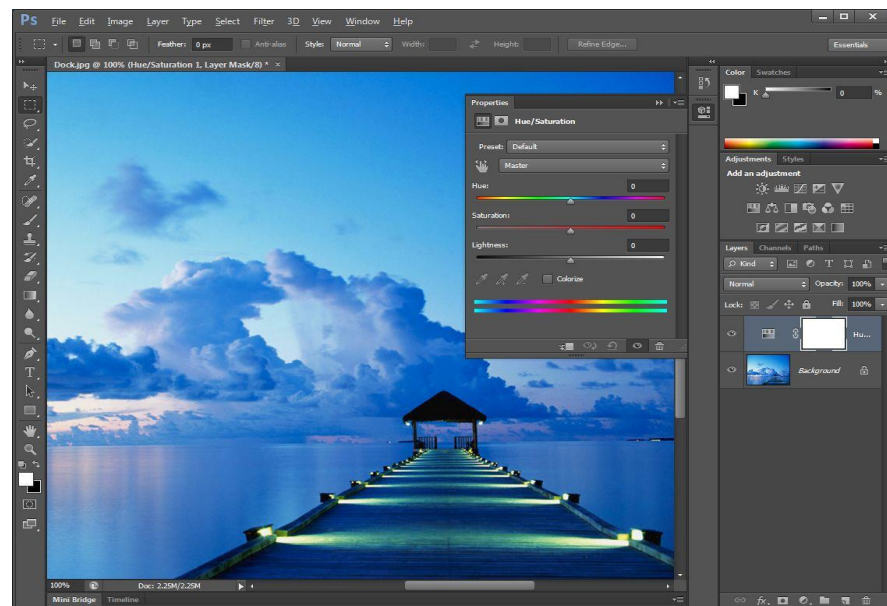
- CorelDraw

- Picasa

- 美图秀秀

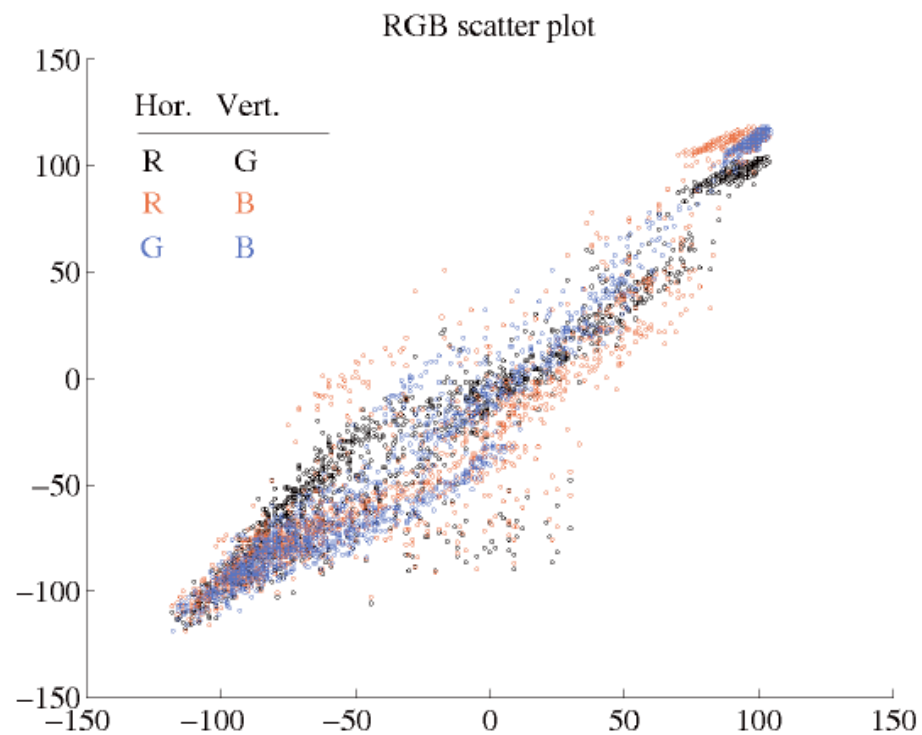
- 光影魔术手

- ...



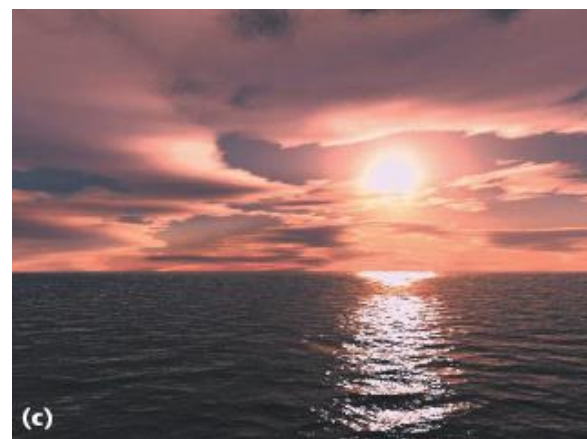
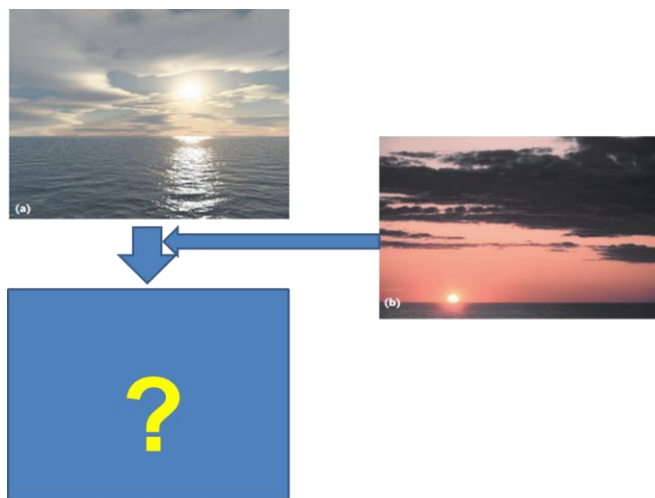
从数学的角度看图像处理

1、图像是线性空间的数据点集



从数学的角度看图像处理

- Example 1: Color Transfer



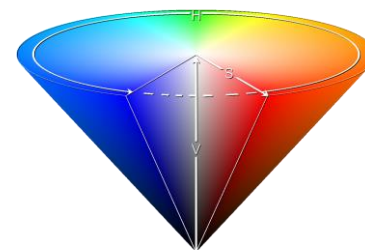
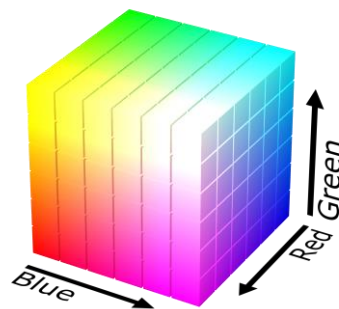
- 使点云位置、形状相似（均值和方差变换）

$$\begin{aligned}l^* &= l - \langle l \rangle \\ \alpha^* &= \alpha - \langle \alpha \rangle \\ \beta^* &= \beta - \langle \beta \rangle \\ l' &= \frac{\sigma_l^l}{\sigma_s^l} l^* \\ \alpha' &= \frac{\sigma_l^\alpha}{\sigma_s^\alpha} \alpha^* \\ \beta' &= \frac{\sigma_l^\beta}{\sigma_s^\beta} \beta^*\end{aligned}$$

从数学的角度看图像处理

- Color Spaces: Different Basis

- RGB
- CMY
- HSV
- $l\alpha\beta$



$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.3811 & 0.5783 & 0.0402 \\ 0.1967 & 0.7244 & 0.0782 \\ 0.0241 & 0.1288 & 0.8444 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$L = \log L$$

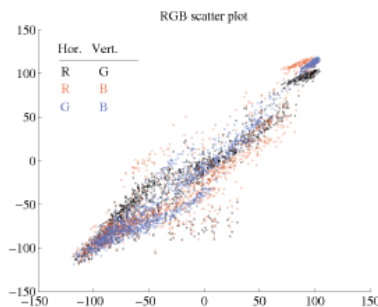
$$M = \log M$$

$$S = \log S$$

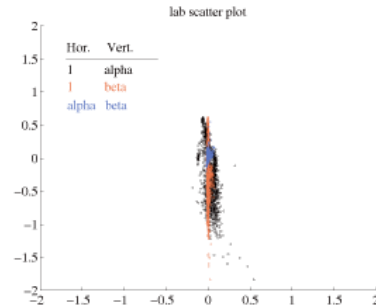
$$\begin{bmatrix} l \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$



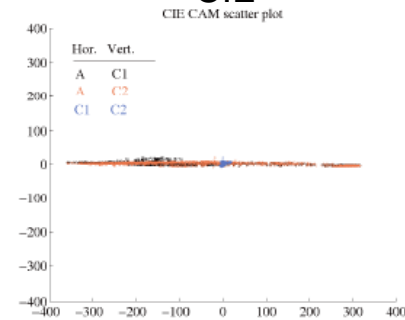
RGB



$l\alpha\beta$



CIE



从数学的角度看图像处理

- Example 2: Color2Gray

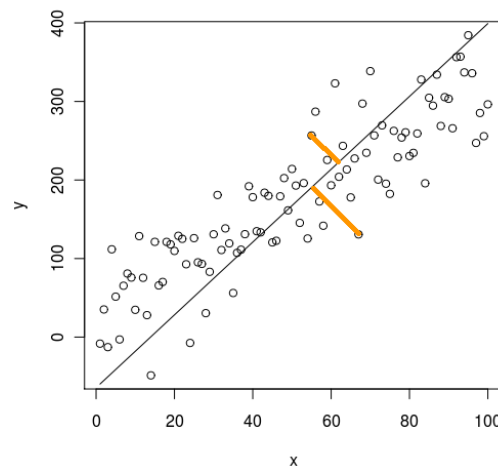


Color



Grayscale

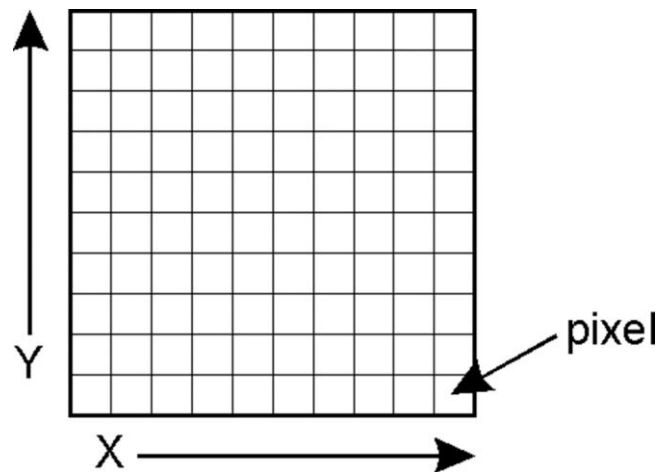
- Problem: Dimension reduction
 - 3D (Color) -> 1D (Gray)



从数学的角度看图像处理

- 2、图像是一个矩阵

This image is 600×465 pixels



从数学的角度看图像处理

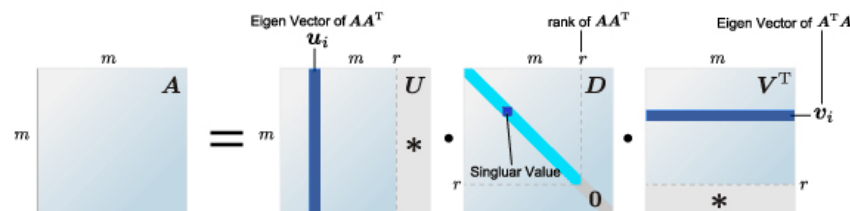
- Matrix Decomposition

- LU Decomposition
- QR Decomposition
- Cholesky Decomposition
- Jordan Decomposition
- Spectral Decomposition
- Singular Value Decomposition (SVD)
- Low-Rank Decomposition

$$A = L U$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ 0 & l_{12} & l_{23} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$A = U D V^T$$



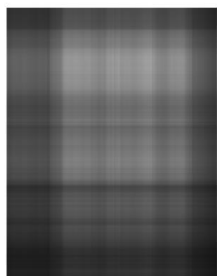
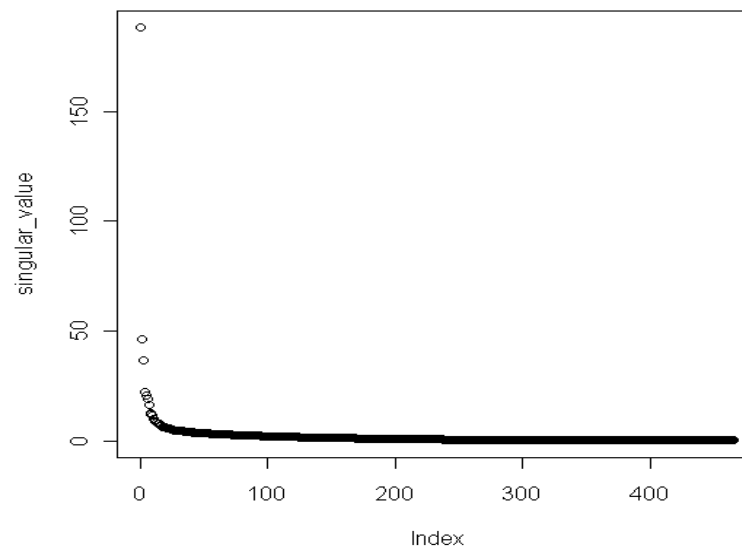
where $k \ll \min\{m, n\}$

$$\begin{matrix} A \\ m \times n \end{matrix} \approx \begin{matrix} B \\ m \times k \end{matrix} \times \begin{matrix} C \\ k \times n \end{matrix}$$

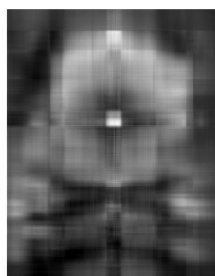


从数学的角度看图像处理

- Singular values of flowers image



Rank-1



Rank-5



Rank-20



Rank-80



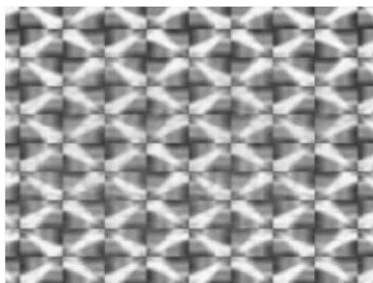
Rank-150



True Image

从数学的角度看图像处理

- Low Rank Matrix
 - 高维数据往往具有低维结构



71) which turns out in the end to be mathematically equivalent to maximum entropy, the problem is interesting also in that we can see a continuous gradation from decision problems so simple that common sense tells us the answer instantly, with no need for any mathematical theory, through problems more and more involved so that common sense moves and more difficulty in making a decision, until finally we reach a point where only has yet claimed to be able to see the right decision intuitively, and we require the mathematics to tell us what to do.

Initially, the widget problem turns out to be very close to an important real problem faced by all prospectors. The details of the real problem are shrouded in proprietary caution; but not giving away any secrets to report that, a few years ago, the writer spent a week at research laboratories of one of our large oil companies, lecturing for over 20 hours on the widget problem. We went through every part of the calculation in excruciating detail in a room full of engineers armed with calculators, checking up on every stage of the mathematical work.

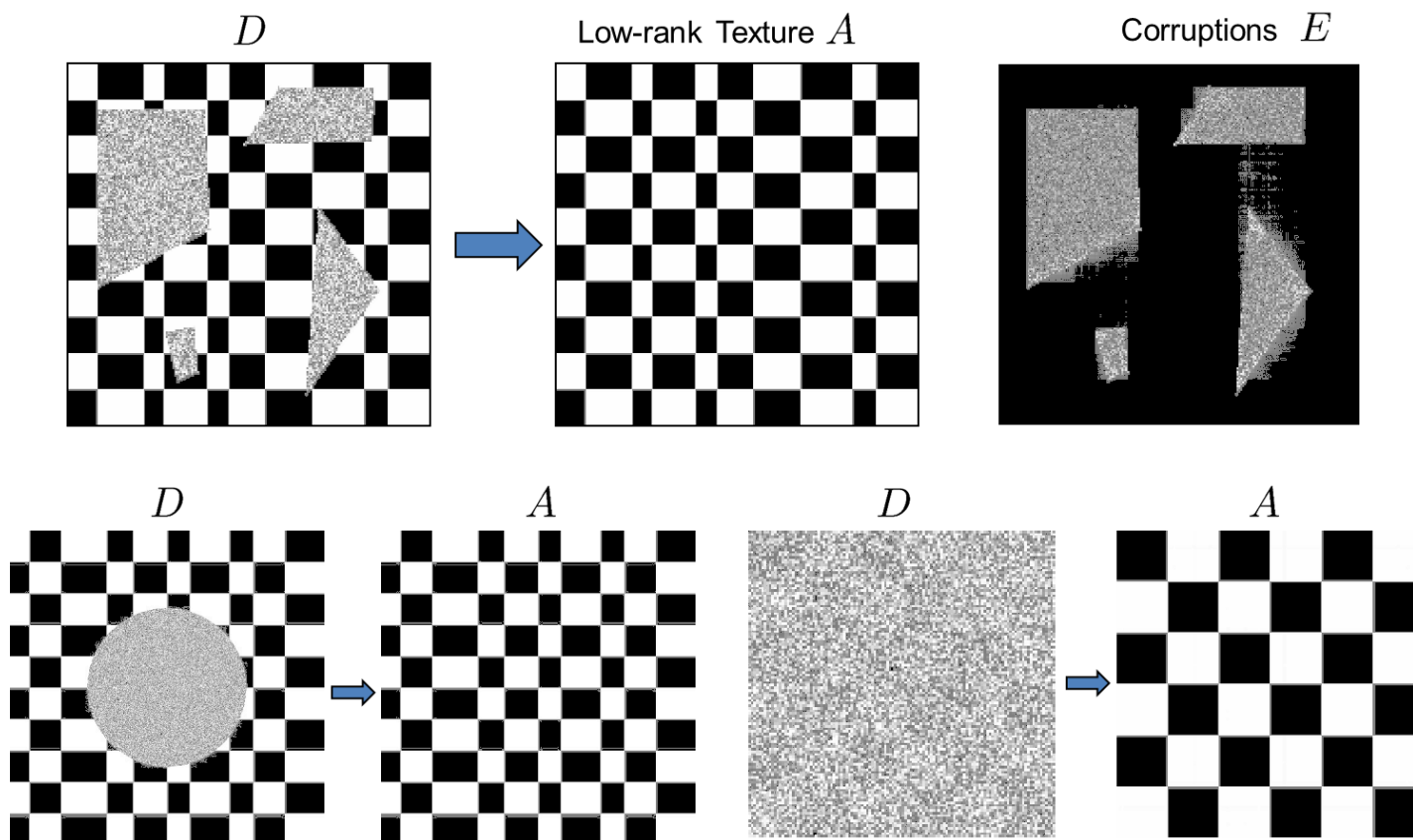
Here is the problem: Mr A is in charge of a widget factory, which proudly advertises that it can make delivery in 24 hours on any size order. This, of course, is not really true, and Mr A is anxious to protect, as best he can, the advertising manager's reputation for veracity. This means that each morning he must decide whether the day's run of 200 widgets will be painted red, blue, or green. (For complex technological reasons, not relevant to the present problem, only one color can be produced per day.) We follow his problem of decision through several



- Visual data exhibit low-dimensional structures due to rich local regularities, global symmetries, repetitive patterns, or redundant sampling.

从数学的角度看图像处理

- Example 1: Image Completion



从数学的角度看图像处理

- Example 1: Image Completion



从数学的角度看图像处理

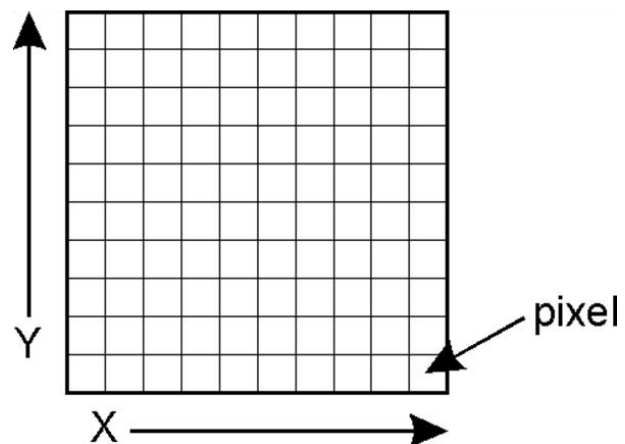
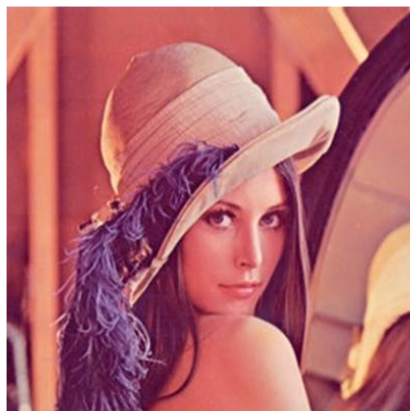
3、图像是一个函数

- 2D区域上的向量值函数

$$I = I(x, y)$$

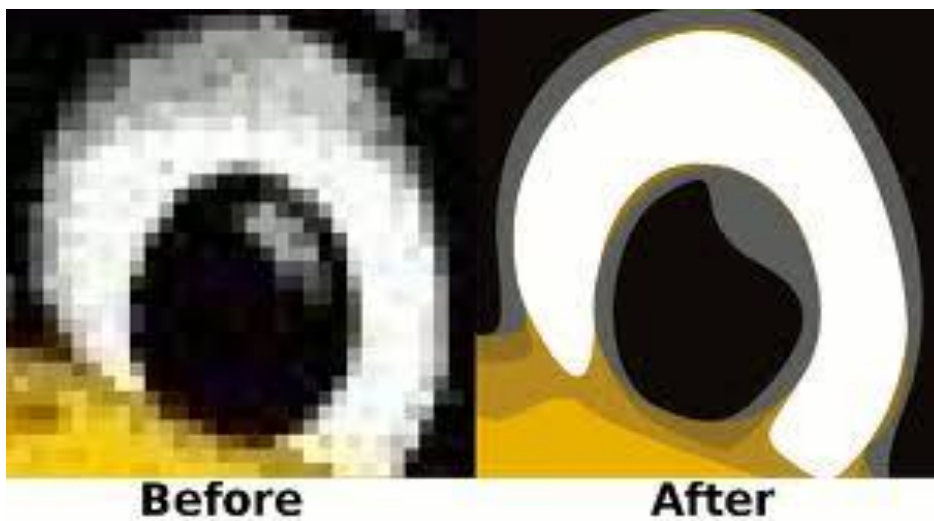
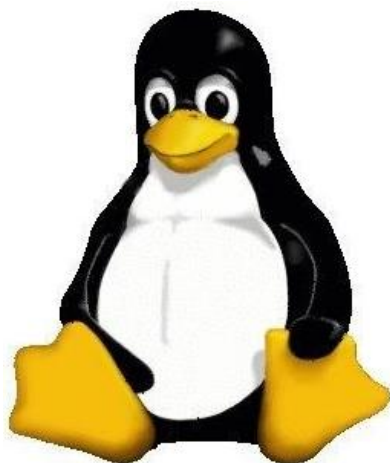
Discrete sampling

$$I = f(i, j)$$



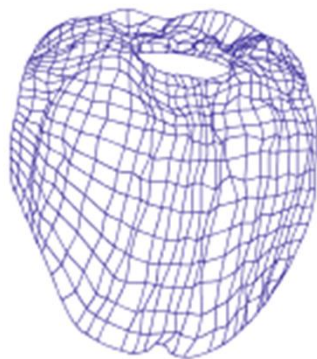
从数学的角度看图像处理

- Example 1: Vectorization

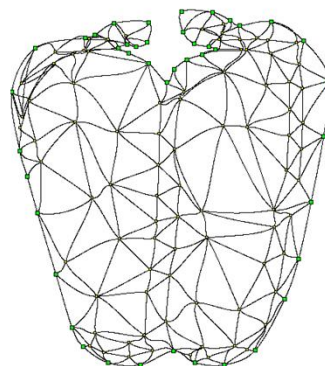
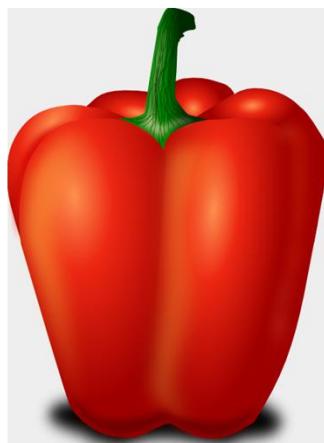


从数学的角度看图像处理

- Example 1: Vectorization



四边形域的函数

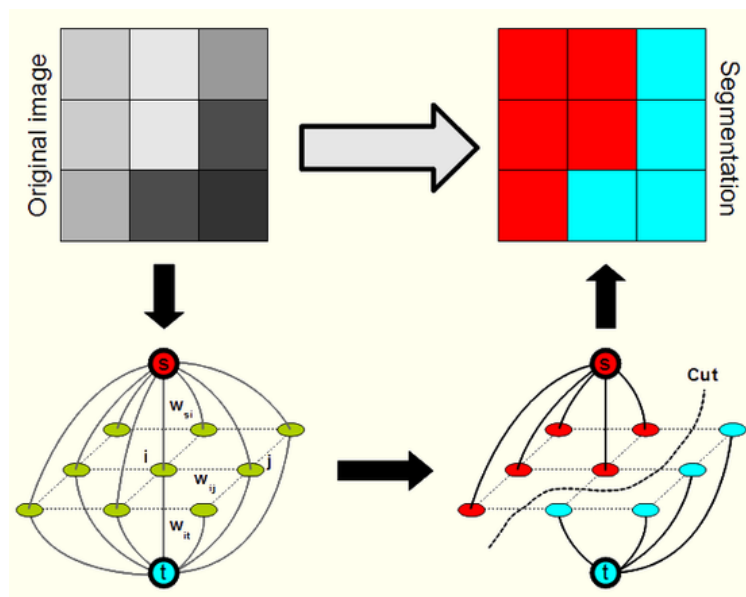


三角形域的函数

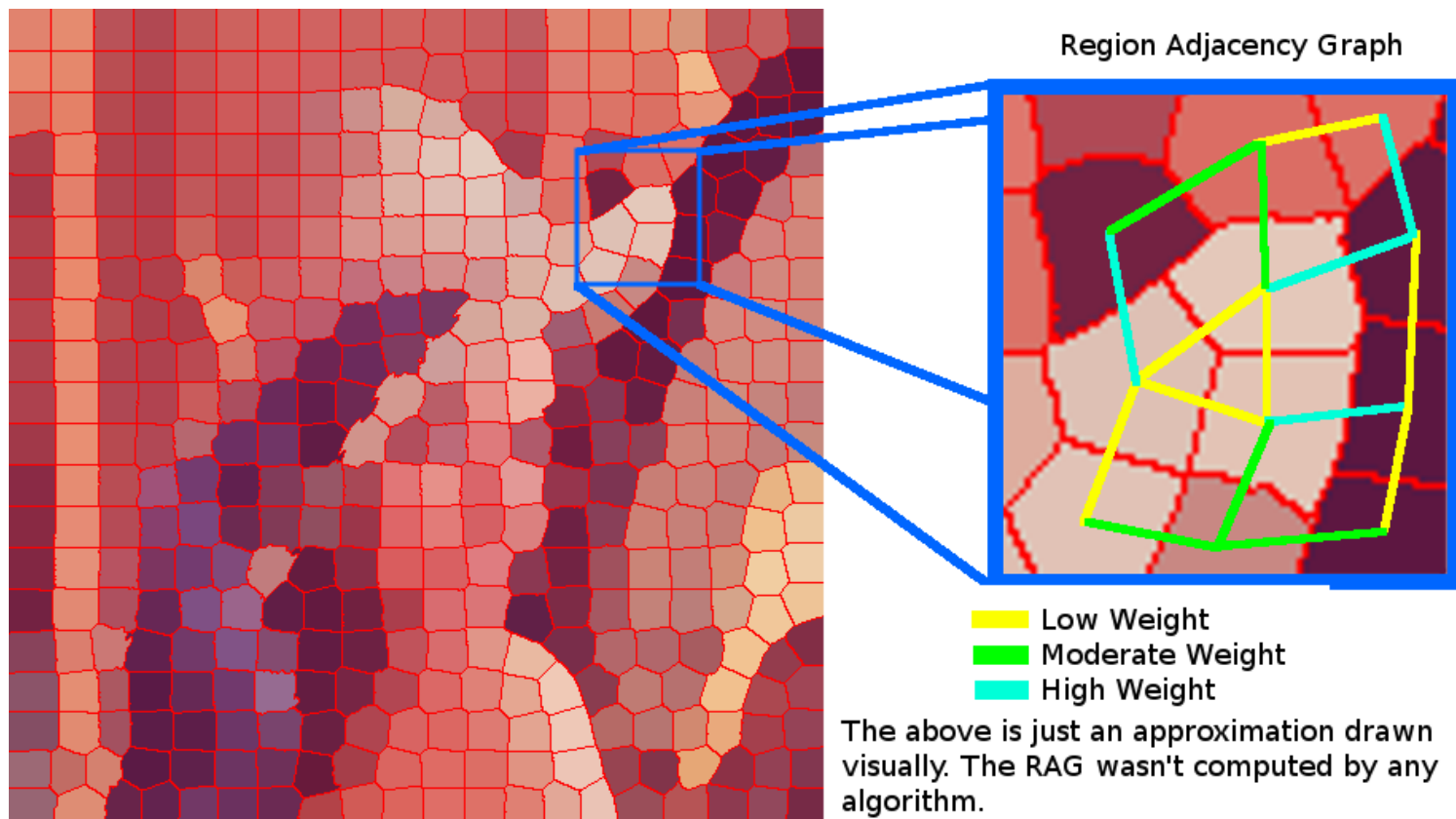


从数学的角度看图像处理

4、图像是一个图 (Graph)



从数学的角度看图像处理



Acknowledgement

- USTC Computer Graphics (Spring 2018), Prof. Ligang Liu
 - http://staff.ustc.edu.cn/~lgliu/Courses/ComputerGraphics_2018_spring-summer/default.htm
- ZJU CAD Computer Graphics 2017, Dr. Hongxin Zhang
 - <http://www.cad.zju.edu.cn/home/zhx/CG/2017/doku.php>
- XMU Digital Geometry Processing, Dr. Zhonggui Chen
 - <http://graphics.xmu.edu.cn/courses/dgp/index.html>
- Tsinghua Computer Graphics, Prof. Shimin Hu
 - <http://cg.cs.tsinghua.edu.cn/course/>

