中山大学 考试草稿纸

P.244.1) 求椭球面 $2x^2 + 3y^2 + 4z^2 - 4x - 6y + 16z + 16 = 0$

的中心坐标及三个半轴的长度。

解： $2(x^2 - 2x + 1) + 3(y^2 - 2y + 1) + 4(z^2 + 4z + 4) - 5 = 0$

$2(x-1)^2 + 3(y-1)^2 + 4(z+2)^2 = 5$

$$\frac{(x-1)^2}{\frac{5}{2}} + \frac{(y-1)^2}{\frac{5}{3}} + \frac{(z+2)^2}{\frac{5}{4}} = 1.$$
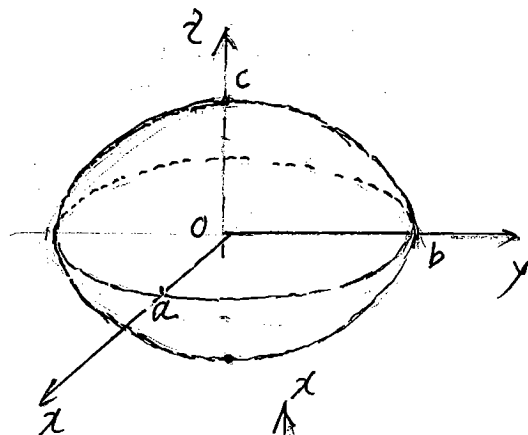
中心 $(1, 1, -2)$

半轴. $a = \sqrt{\frac{5}{2}}$, $b = \sqrt{\frac{5}{3}}$, $c = \frac{\sqrt{5}}{2}$
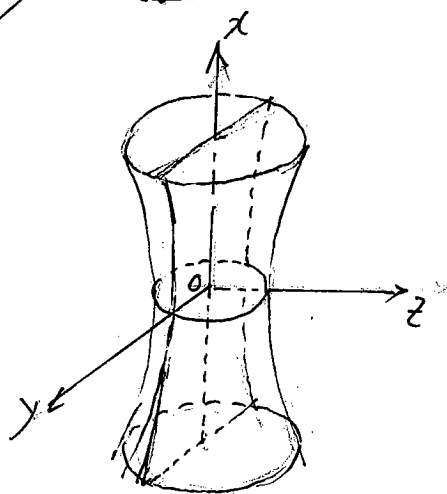
P.244.2) 说出下列曲面的名称，并画出略图。

(1) $8x^2 + 11y^2 + 24z^2 = 1$

$$\frac{x^2}{(\frac{1}{\sqrt{8}})^2} + \frac{y^2}{(\frac{1}{\sqrt{11}})^2} + \frac{z^2}{(\frac{1}{\sqrt{24}})^2} = 1.$$ 椭球.

$a = \frac{1}{\sqrt{8}}$, $b = \frac{1}{\sqrt{11}}$, $c = \frac{1}{\sqrt{24}}$.

(2) $4x^2 - 9y^2 - 14z^2 = -25$.

$$-\frac{x^2}{(\frac{5}{2})^2} + \frac{y^2}{(\frac{5}{3})^2} + \frac{z^2}{(\frac{5}{\sqrt{14}})^2} = 1.$$ 单叶双曲面.

(3) $2x^2 + 9y^2 - 16z^2 = -9$

$$\frac{x^2}{(\frac{3}{\sqrt{2}})^2} + \frac{y^2}{1} + \frac{z^2}{(\frac{3}{4})^2} = (-1)$$

双叶双曲面.