P.153.12 $\int_0^{\frac{\pi}{2}} \sin^{11}x\,dx = I_{11} = \frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}\times 1 = \frac{2\times8\times2\times4\times2}{11\times9\times7} = \frac{256}{693}$. 2011 15/7 —64.

P.153.13 $\int_0^{\pi} \sin^6\frac{x}{2}\,dx = 2\int_0^{\frac{\pi}{2}} \sin^6 t\,dt = 2I_6 = 2\times\frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2} = \frac{5\pi}{16}$.

P.153.14 $\int_0^{\pi}(x\cdot\sin x)^2dx = \int_0^{\pi} x^2\cdot\frac{1-\cos 2x}{2}dx = \frac{1}{2}[\int_0^{\pi}x^2dx - \int_0^{\pi}x^2\cos 2x\,dx]$

$= \frac{1}{2}[\frac{x^3}{3}\Big|_0^{\pi} - \frac{1}{2}\int_0^{\pi}x^2\,d\sin 2x] = \frac{\pi^3}{6} - \frac{1}{4}[x^2\sin 2x\Big|_0^{\pi} - \int_0^{\pi}\sin 2x\cdot 2x\,dx]$

$= \frac{\pi^3}{6} - \frac{1}{4}\int_0^{\pi}x\,d\cos 2x = \frac{\pi^3}{6} - \frac{1}{4}[x\cdot\cos 2x\Big|_0^{\pi} - \int_0^{\pi}\cos 2x\,dx]$

$= \frac{\pi^3}{6} - \frac{1}{4}(\pi - 0) = \frac{\pi^3}{6} - \frac{\pi}{4}$

P.153.15. $\int_0^{\frac{\pi}{4}}\tan^4x\,dx = \int_0^{\frac{\pi}{4}}\frac{\sin^4x}{\cos^4x}dx = \int_0^{\frac{\pi}{4}}\frac{(1-\cos^2x)^2}{\cos^4x}dx = \int_0^{\frac{\pi}{4}}\frac{1-2\cos^2x+\cos^4x}{\cos^4x}dx$

$= \int_0^{\frac{\pi}{4}}(1-2\sec^2x+\frac{1}{\cos^4x})dx$

$= [x]_0^{\frac{\pi}{4}} - [2\tan x]_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}}(1+\tan^2x)\,d\tan x$

$= \frac{\pi}{4} - 2 + [\tan x]_0^{\frac{\pi}{4}} + [\frac{\tan^3x}{3}]_0^{\frac{\pi}{4}} = \frac{\pi}{4} - 2 + 1 + \frac{1}{3} = \frac{\pi}{4} - \frac{2}{3}$.

P.153.16 $\int_0^1 \arcsin x\,dx = x\cdot\arcsin x\Big|_0^1 - \int_0^1 x\,d\arcsin x$

$= 1\cdot\arcsin 1 - 0 - \int_0^1 x\cdot\frac{dx}{\sqrt{1-x^2}}$

$= \frac{\pi}{2} + \int_0^1\frac{1}{2\sqrt{1-x^2}}d(1-x^2) = \frac{\pi}{2} + \sqrt{1-x^2}\Big|_0^1 = \frac{\pi}{2} - 1$.

P.153.17 $\int_0^{\pi}\ln(x+\sqrt{x^2+a^2})dx$

$= x\cdot\ln(x+\sqrt{x^2+a^2})\Big|_0^{\pi} - \int_0^{\pi}x\,d\ln(x+\sqrt{x^2+a^2})$

$= \pi\cdot\ln(\pi+\sqrt{\pi^2+a^2}) - 0 - \int_0^{\pi}x\cdot\frac{dx}{\sqrt{x^2+a^2}}$

$= \pi\cdot\ln(\pi+\sqrt{\pi^2+a^2}) - \int_0^{\pi}\frac{1}{2\sqrt{x^2+a^2}}d(x^2+a^2)$

$= \pi\cdot\ln(\pi+\sqrt{\pi^2+a^2}) - \sqrt{x^2+a^2}\Big|_0^{\pi} = \pi\cdot\ln(\pi+\sqrt{\pi^2+a^2}) + |a| - \sqrt{\pi^2+a^2}$.