中山大學 本科生考试草稿纸

14/5—61

警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.144.32 $\int \frac{2x+3}{\sqrt{x^2+x}}dx = \int \frac{2x+1+2}{\sqrt{x^2+x}}dx = \int \frac{1}{\sqrt{x^2+x}}d(x^2+x) + 2\int \frac{dx}{\sqrt{x(x+1)}}$

$= 2\sqrt{x^2+x} + 4\int \frac{1}{\sqrt{1+x}}d\sqrt{x} = 2\sqrt{x^2+4} + 4\arctan\sqrt{x} + C.$

P.144.33. $\int \frac{2+x}{\sqrt{4x^2-4x+5}}dx = \frac{1}{8}\int \frac{1}{\sqrt{4x^2-4x+5}}d(4x^2-4x+5) + \frac{5}{2}\int \frac{dx}{\sqrt{4x^2-4x+5}}$

而 $\frac{1}{8}\int \frac{1}{\sqrt{4x^2-4x+5}}d(4x^2-4x+5) = \frac{1}{4}\sqrt{4x^2-4x+5} + C_1$

$\frac{5}{2}\int \frac{dx}{\sqrt{4x^2-4x+5}} = \frac{5}{2}\int \frac{d(x-\frac{1}{2})}{\sqrt{(x-\frac{1}{2})^2+1}} = \frac{5}{4}\int \frac{d(2x-1)}{\sqrt{1+(2x-1)^2}}$, $u=2x-1$

$\underline{\underline{令 u=\tan t}}\ \frac{5}{4}\cdot\int \frac{\sec^2 t}{\sec t}dt = \frac{5}{4}\int \sec t\, dt = \frac{5}{4}\ln|\sec t+\tan t| + C_2$

$= \frac{5}{4}\ln|\sqrt{1+u^2}+u| + C_2 = \frac{5}{4}\ln|2x-1+\sqrt{4x^2-4x+5}| + C_2$

从而 $\int \frac{2+x}{\sqrt{4x^2-4x+5}}dx = \frac{1}{4}\cdot\sqrt{4x^2-4x+5} + \frac{5}{4}\ln|2x-1+\sqrt{4x^2-4x+5}| + C$

P.144.34 $\int \sqrt{5-2x+x^2}\,dx = \int \sqrt{(x-1)^2+2^2}\,dx$, 令 $x-1=u$

$= \int \sqrt{u^2+2^2}\,du = \int 2\sec t\cdot 2\sec^2 t\,dt$

再令 $u=2\tan t$

则 $du = 2\sec^2 t\,dt$

$\sqrt{u^2+2^2} = 2\sec t$

$= 4\int \sec^3 t\,dt$

$= 4\cdot\frac{1}{2}[\sec t\cdot\tan t + \ln|\sec t+\tan t|] + C$

$= 2\cdot\frac{x-1}{2}\cdot\frac{1}{2}\sqrt{5-2x+x^2} + 2\ln\left|\frac{x-1}{2} + \frac{\sqrt{5-2x+x^2}}{2}\right| + C$

$= \frac{(x-1)}{2}\cdot\sqrt{5-2x+x^2} + 2\ln|x-1+\sqrt{5-2x+x^2}| + C.$