中山大學 考试草稿纸

P.244.3. 求下列曲面的参数方程.

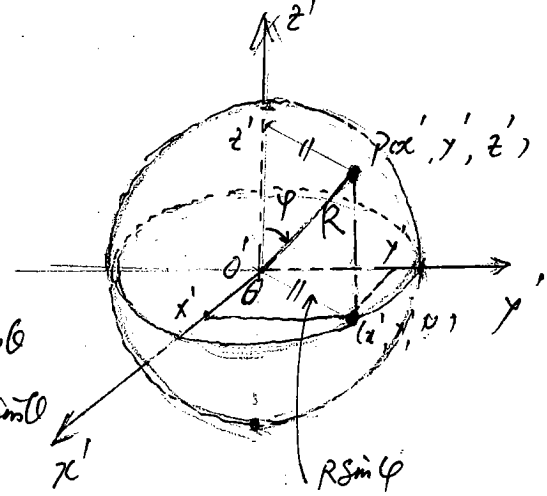
(1) $(x-1)^2+(y+1)^2+(z-3)^2=R^2$

$x'^2+y'^2+z'^2=R^2$

$\begin{cases} x'=x-1=R\sin\varphi\cos\theta \\ y'=y+1=R\sin\varphi\sin\theta \\ z'=z-3=R\cos\varphi \end{cases}$

$\begin{cases} x=1+R\sin\varphi\cos\theta \\ y=-1+R\sin\varphi\sin\theta \\ z=3+R\cos\varphi \end{cases}$

$0\le\theta\le 2\pi,\ 0\le\varphi\le\pi.$



$\begin{cases} x'=R\sin\varphi\cos\theta \\ y'=R\sin\varphi\sin\theta \\ z'=R\cos\varphi \end{cases}$

(2) $\frac{x^2}{1}+\frac{y^2}{9}+\frac{z^2}{4}=1$

令 $\frac{x}{1}=x',\ \frac{y}{3}=y',\ \frac{z}{2}=z'$

则 $x'^2+y'^2+z'^2=1.$

$\begin{cases} x'=\sin\varphi\cos\theta & 0\le\theta\le 2\pi \\ y'=3\sin\varphi\sin\theta & 0\le\varphi\le\pi \\ z'=2\cos\varphi \end{cases}$

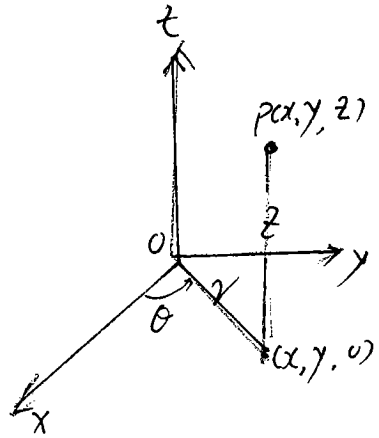
参数方程: $\begin{cases} x=\sin\varphi\cdot\cos\theta \\ y=3\sin\varphi\sin\theta \\ z=2\cos\varphi \end{cases}$

(3) $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1.$

$\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1+\frac{z^2}{16}.$

$\frac{x^2}{\left(2\sqrt{1+\frac{z^2}{16}}\right)^2}+\frac{y^2}{\left(3\sqrt{1+\frac{z^2}{16}}\right)^2}=1.$

令 $\begin{cases} x=2\sqrt{1+\frac{z^2}{16}}\cos\theta \\ y=3\sqrt{1+\frac{z^2}{16}}\sin\theta \\ z=z \end{cases}$



$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases}$ 柱坐标.