

$$\begin{aligned}
 28. \int \frac{x^2}{\sqrt{a^2-x^2}} dx &= - \int \frac{a^2-x^2+a^2}{\sqrt{a^2-x^2}} dx = - \int \sqrt{a^2-x^2} dx + a^2 \int \frac{dx}{\sqrt{a^2-x^2}} \\
 &= -\frac{a^2}{2} \arcsin \frac{x}{a} - \frac{x}{2} \sqrt{a^2-x^2} + a^2 \cdot \arcsin \frac{x}{a} + C \\
 &= \frac{a^2}{2} \arcsin \frac{x}{a} - \frac{x}{2} \sqrt{a^2-x^2} + C.
 \end{aligned}$$

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$$\begin{aligned}
 29. \int \frac{dx}{\sqrt{1+e^{3x}}} & \quad \left\{ \begin{array}{l} \sqrt{1+e^{3x}} = t \\ 1+e^{3x} = t^2, \quad e^{3x} = t^2-1 \\ 3x = \ln(t^2-1), \quad x = \frac{1}{3} \ln(t^2-1) \\ dx = \frac{1}{3} \cdot \frac{2t}{t^2-1} dt = \frac{2t}{3(t^2-1)} dt \end{array} \right. \\
 &= \int \frac{1}{t} \cdot \frac{2t}{3(t^2-1)} dt = \frac{2}{3} \int \frac{dt}{(t-1)(t+1)} \\
 &= -\frac{1}{3} \int \left(\frac{1}{t+1} - \frac{1}{t-1} \right) dt \\
 &= -\frac{1}{3} \left[\ln|t+1| - \ln|t-1| \right] + C \\
 &= -\frac{1}{3} \left[\ln|\sqrt{1+e^{3x}}+1| - \ln|\sqrt{1+e^{3x}}-1| \right] + C \\
 &= \frac{1}{3} \cdot \ln \frac{\sqrt{1+e^{3x}}-1}{\sqrt{1+e^{3x}}+1} + C = \frac{1}{3} \ln \frac{(\sqrt{1+e^{3x}}-1)^2}{e^{3x}} + C = \frac{2}{3} \ln(\sqrt{1+e^{3x}}-1) - x + C.
 \end{aligned}$$

$$30. \int \frac{x^3}{\sqrt{1+x^8}} dx = \frac{1}{4} \int \frac{dx^4}{\sqrt{1+(x^4)^2}} = \frac{1}{4} \ln(x^4 + \sqrt{1+x^8}) + C.$$

$$\begin{aligned}
 31. \int \frac{dx}{x^6 \sqrt{1+x^2}} & \quad \left\{ \begin{array}{l} x = \tan t \\ \sec^2 t dt \\ \tan^6 t \cdot \sec t \\ \frac{dt}{\tan^6 t \cdot \sec t} = \int \frac{\cos^5 t}{\sin^6 t} dt = \int \frac{\cos^4 t}{\sin^6 t} d\sin t \\ = \int \frac{(1-\sin^2 t)^2}{\sin^6 t} d\sin t = \int \frac{1-2\sin^2 t + \sin^4 t}{\sin^6 t} d\sin t \\ = \int \left(\frac{1}{\sin^6 t} - \frac{2}{\sin^4 t} + \frac{1}{\sin^2 t} \right) d\sin t \\ = -\frac{1}{5} \cdot \frac{1}{\sin^5 t} + \frac{2}{3} \cdot \frac{1}{\sin^3 t} - \frac{1}{\sin t} + C \\ = -\frac{1}{5} \cdot \frac{(\sqrt{1+x^2})^5}{x^5} + \frac{2}{3} \cdot \frac{(\sqrt{1+x^2})^3}{x^3} - \frac{\sqrt{1+x^2}}{x} + C \\ = -\frac{(1+x^2)^{\frac{5}{2}}}{5x^5} + \frac{2(1+x^2)^{\frac{3}{2}}}{3x^3} - \frac{\sqrt{1+x^2}}{x} + C \end{array} \right. \\
 & \quad \begin{array}{c} \sqrt{1+x^2} \\ \triangle x \\ 1 \end{array} \quad x, \quad \sin t = \frac{x}{\sqrt{1+x^2}}
 \end{aligned}$$

$$\begin{aligned}
 32. \int \frac{e^{2x}}{\sqrt[3]{1+e^x}} dx &= \int \frac{e^x}{\sqrt[3]{1+e^x}} de^x = \int \frac{1+e^x-1}{\sqrt[3]{1+e^x}} de^x = \int (1+e^x)^{\frac{2}{3}} d(1+e^x) - \int \frac{d(1+e^x)}{\sqrt[3]{1+e^x}} \\
 &= \frac{3}{5} (1+e^x)^{\frac{5}{3}} - \frac{3}{2} (1+e^x)^{\frac{2}{3}} + C.
 \end{aligned}$$