1. If the matrix B is the echelon form of matrix A, compute the basis of $\mathrm{ColA}, \mathrm{RowA}, \mathrm{NulA}$

A=$\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$, B=$\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2. If T is a one-to-one transformation. Prove that if the set of images $\{T(v_1), \cdots, T(v_p)\}$ is linearly dependent, then the vector set $\{v_1, \cdots, v_p\}$ is also linearly dependent.

3. Given the polynomials $\mathbf{p}_1(t) = 1 + t^2$, $\mathbf{p}_2(t) = 1 - t^2$, prove that $\{\mathbf{p}_1, \mathbf{p}_2\}$ is a linearly independent set in $\mathbf{P_3}$.

4. If a subset $\{\mathbf{u}_1, \cdots, \mathbf{u}_p\} \subset \mathbf{V}$ is linearly dependent, then the set of coordinate vectors $\left\{[\mathbf{u_1}]_\beta, \cdots, [\mathbf{u_p}]_\beta\right\}$ is also linearly dependent.

5. Let H be a nonzero subspace of V, and let $T(H)$ be the set of images of vectors in H. Prove that $\dim T(H) \leq \dim H$

6. Let $A = \begin{bmatrix} 1 & -2 & 3k \\ -1 & 2k & -3 \\ k & -2 & 3 \end{bmatrix}$, compute the value of k make that (1) rankA=1; (2) rankA=2; (3) rankA=3

7. A and B are $n \times n$ matrics. Prove $\mathrm{Rank}(A * B) \leq \min\{\mathrm{Rank}(A), \mathrm{Rank}(B)\}$.

8. Find the eigenvalues and eigenvectors of A and A^2 and A^{-1} and $A + 4I$: $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$