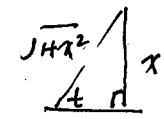
P.134.17. $\int \frac{x\cdot \operatorname{arctan} x}{(1+x^2)^{\frac{5}{2}}}dx = \frac{1}{2}\int \frac{\operatorname{arctan} x}{(1+x^2)^{\frac{5}{2}}}d(1+x^2) = \frac{1}{2}\left(-\frac{2}{3}\right)\int \operatorname{arctan} x\, d\frac{1}{(1+x^2)^{\frac{3}{2}}}$

$= -\frac{1}{3}\left[\operatorname{arctan} x\cdot \frac{1}{(1+x^2)^{\frac{3}{2}}} - \int \frac{1}{(1+x^2)^{\frac{3}{2}}}\cdot\frac{1}{1+x^2}dx\right]$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \frac{1}{3}\int \frac{dx}{(1+x^2)^{\frac{5}{2}}}$



令 $x=\tan t$, $dx=\sec^2 t\,dt$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \frac{1}{3}\int \frac{\sec^2 t}{\sec^3 t}dt$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \frac{1}{3}\int \cos^3 t\,dt$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \int (1-\sin^2 t)\,d\sin t$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \frac{\sin t}{3} - \frac{\sin^3 t}{9} + C$

$= -\frac{1}{3}\cdot\frac{\operatorname{arctan} x}{(1+x^2)^{\frac{3}{2}}} + \frac{1}{3}\cdot\frac{x}{\sqrt{1+x^2}} - \frac{1}{9}\cdot\frac{x^3}{(1+x^2)^{\frac{3}{2}}} + C.$

P.134.18. $\int \underbrace{x\cdot \ln(x+\sqrt{1+x^2})}_{u}dx = x^2\ln(x+\sqrt{1+x^2}) - \int x\, d[x\ln(x+\sqrt{1+x^2})]$

$= x^2\cdot\ln(x+\sqrt{1+x^2}) - \int x\cdot\left[\ln(x+\sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}}\right]dx$

$= x^2\cdot\ln(x+\sqrt{1+x^2}) - \int x\ln(x+\sqrt{1+x^2})\,dx - \int \frac{x^2}{\sqrt{1+x^2}}dx$

$2\int x\cdot\ln(x+\sqrt{1+x^2})\,dx = x^2\cdot\ln(x+\sqrt{1+x^2}) - \frac{1}{2}\int \frac{x}{\sqrt{1+x^2}}d(1+x^2)$

$\int x\cdot\ln(x+\sqrt{1+x^2})\,dx = \frac{x^2}{2}\ln(x+\sqrt{1+x^2}) - \frac{1}{2}\int x\,d\sqrt{1+x^2}$

$= \frac{x^2}{2}\ln(x+\sqrt{1+x^2}) - \frac{1}{2}\left[x\sqrt{1+x^2} - \int \sqrt{1+x^2}\,dx\right]$

$= \frac{x^2}{2}\ln(x+\sqrt{1+x^2}) - \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\int \sec t\cdot\sec^2 t\,dt$

$= \frac{x^2}{2}\ln(x+\sqrt{1+x^2}) - \frac{x}{2}\sqrt{1+x^2} + \frac{1}{4}(x\cdot\sqrt{1+x^2} + \ln(x+\sqrt{1+x^2})) + C$

$= \frac{x^2}{2}\ln(x+\sqrt{1+x^2}) - \frac{x}{4}\sqrt{1+x^2} + \frac{1}{4}\ln(x+\sqrt{1+x^2}) + C.$