1. If the vectors $\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4} \in V$ are linear independent,
   a) Whether $\boldsymbol{a_1} + \boldsymbol{a_2}, \boldsymbol{a_2} + \boldsymbol{a_3}, \boldsymbol{a_3}+\boldsymbol{a_4}, \boldsymbol{a_3}+\boldsymbol{a_4}$ are linear independent? Explain.
   b) Find a basis of a subspace $\mathrm{W} = \{\boldsymbol{a_1} + \boldsymbol{a_2}, \boldsymbol{a_2} + \boldsymbol{a_3}, \boldsymbol{a_3}+\boldsymbol{a_4}, \boldsymbol{a_3}+\boldsymbol{a_4}\}$, and compute the dimension of W.

2. True or False
   a) If the vectors $\boldsymbol{a_1}, \boldsymbol{a_2}, \cdots, \boldsymbol{a_m}$ are linear dependent, then $\boldsymbol{a_1}$ is a linear combination of $\boldsymbol{a_2}, \cdots, \boldsymbol{a_m}$.
   b) If the numbers $c_1, c_2, \ldots, c_m$ are not all zero and $\mathrm{c}_1\boldsymbol{a_1} + \cdots + \mathrm{c}_\mathrm{m}\boldsymbol{a_m} + \mathrm{c}_1\boldsymbol{b1} + \cdots + \mathrm{c}_\mathrm{m}\,\boldsymbol{b_m} = \boldsymbol{0}$, then the vectors $\boldsymbol{a_1}, \boldsymbol{a_2}, \cdots, \boldsymbol{a_m}$ are linear dependent and $\boldsymbol{b_1}, \boldsymbol{b_2}, \cdots, \boldsymbol{b_m}$ are linear dependent also.
   c) If there exists a linearly dependent set $(\mathrm{v}_1, \ldots, \mathrm{v}_\mathrm{p})$ in $\boldsymbol{V}$, then $\mathbf{dim\,V} \le p$.
   d) If every set of p elements in $\boldsymbol{V}$ fails to $\mathbf{span}\boldsymbol{V}$, then $\mathbf{dim\,V} > p$.
   e) If $p \ge 2$ and $\mathbf{dim}V = p$, then every set of $p-1$ nonzero vectors is linearly independent.

3. The set β={1+t^2, t+t^2, 1+2t+t^2} is a basis for P^2, find the coordinate vector of P(t) = 1+4t+7t^2 relative to β.

4. Find the basis of the subspace spanned by vectors $\begin{bmatrix}1\\0\\2\\1\end{bmatrix}\begin{bmatrix}1\\2\\0\\1\end{bmatrix}\begin{bmatrix}2\\1\\3\\0\end{bmatrix}\begin{bmatrix}2\\5\\-1\\4\end{bmatrix}\begin{bmatrix}1\\-1\\3\\-1\end{bmatrix}$

5. Prove the Theorem 8. (*Onto and One-to-one map*)

6. (a)Proof:If β_1={$\mathbf{a_1}$, $\mathbf{a_2}$, …, $\mathbf{a_n}$} is a basis of space V in R^n, then β_2={$\mathbf{a_1}$, $\mathbf{a_1+a_2}$, …, $\mathbf{a_1+a_2+\ldots+a_n}$} is also a basis of space V.
   (b)Even more if $[v]_{\beta_1} = (\mathrm{n}, \mathrm{n}-1, \ldots, 2, 1)$, compute $[v]_{\beta_2}$

7. Let T: $\mathrm{R^n} \rightarrow \mathrm{R^m}$ be a linear transformation.
   a) What is the dimension of the range of T if T is a one-to-one mapping? Explain.
   b) What is the dimension of the kernel of T (see Section 4.2) if T maps Rn onto $\mathrm{R^m}$? Explain.

8. Consider the polynomials $\mathrm{p_1(t) = 1 + t}$, $\mathrm{p_2(t) = 1 - t}$, $\mathrm{p_3(t) = 4}$, $\mathrm{p_4(t) = 1 + t^2}$, and $\mathrm{p_5(t) = 1 + 2t + t^2}$, and let *H* be the subspace of P5 spanned by the set *S*={$\mathrm{p_1, p_2, p_3, p_4, p_5}$}. Produce a basis for *H*. (Explain how to select appropriate members of *S*.)