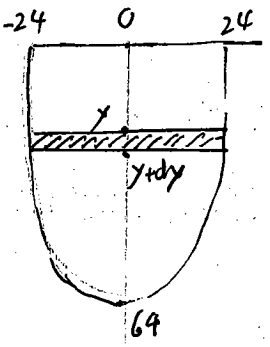
P.166.32. 一水闸门的边界线为一抛物线，水面宽度为48m，最低处在水下64m。求水对闸门的压力。 2011年4月—80



解：设此抛物线为：$y-64=ax^2$

由 $y=0,\ x=\pm 24$

则 $0-64=a\times 24^2$

$$a=-\frac{64}{24^2}=-\frac{1}{9}$$

$$y-64=-\frac{x^2}{9},\quad x^2=9(64-y)$$

$$x=\pm 3\sqrt{64-y}$$

$$dp=\rho g\cdot y\cdot 6\sqrt{64-y}\,dy=\rho g\cdot 6y\sqrt{64-y}\,dy$$

$$P=\int_0^{64}\rho g\cdot 6y\cdot\sqrt{64-y}\,dy$$

$$=-6\rho g\int_0^{64}y\sqrt{64-y}\,d(64-y)$$

$$=-6\rho g\cdot\frac{2}{3}\int_0^{64}y\,d(64-y)^{\frac{3}{2}}$$

$$=-4\rho g\left[y(64-y)^{\frac{3}{2}}\Big|_0^{64}-\int_0^{64}(64-y)^{\frac{3}{2}}dy\right]$$

$$=-4\rho g\left[0+\int_0^{64}(64-y)^{\frac{3}{2}}d(64-y)\right]$$

$$=-\frac{8}{5}\left[(64-y)^{\frac{5}{2}}\right]_0^{64}\rho g$$

$$=-\frac{8}{5}\rho g\cdot[0-8^5]=\rho g\cdot\frac{8^6}{5}$$

$$=\rho g\cdot\frac{262144}{5}\approx 52428.8\rho g.$$