软件学院　2010　级软件工程专业(2010-11)

《线性代数》期中试题试卷

(考试形式：　闭　卷　　考试时间：120 分钟)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：__________　　姓名：__________　　学号：__________成绩：__________

1. Fill the blank (4 titles * 5 points/title = 20 points)

(1) Given a linear system $\begin{pmatrix} -2 & h \\ 6 & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, and if the system is consistent, the constants h and k must satisfy________________.

(2) The reduced echelon form of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$ is _____________.

(3) Suppose T: $\mathbb{R}^2 \to \mathbb{R}^2$ is a transformation, which performs a horizontal shear that transforms $\vec{e}_2$ into $\vec{e}_2 - 2\vec{e}_1$ first (leave $\vec{e}_1$ unchanged), and then reflects points through the line $x_2 = -x_1$. So the standard matrix of T is _________.

(4) The determinant of $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix}$ is _______.

2. Mark each statement True or False, and descript your reasons (3titles * 5 points/title = 15 points)

(1) If $\vec{u}$ is a linear combination of $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^m$, then $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.

(2) If A is a 6 × 5 matrix, the linear transformation $\vec{x} \mapsto A\vec{x}$ can't map $\mathbb{R}^5$ onto $\mathbb{R}^6$.

(3) If A is 3 × 3 matrix, there exist element matrices $E_1, \ldots, E_p$ such that $E_1 \cdots E_p A = I$.

3. Problem issues (12 + 8 + 10 + 8 = 38 points)

(1) Determine the vectors below linear dependent or not. Describe your reasons. (2 titles * 6 points/title = 12 points)

a. $\begin{pmatrix}1\\7\\6\end{pmatrix}, \begin{pmatrix}2\\2\\1\end{pmatrix}, \begin{pmatrix}6\\1\\4\end{pmatrix}, \begin{pmatrix}2\\3\\3\end{pmatrix}$ b. $\begin{pmatrix}1\\0\\9\\6\end{pmatrix}, \begin{pmatrix}0\\1\\5\\3\end{pmatrix}, \begin{pmatrix}0\\0\\1\\3\end{pmatrix}, \begin{pmatrix}0\\0\\0\\3\end{pmatrix}$

(2) Calculate LU factorization of $\begin{pmatrix}2 & -4 & 4 & -2\\ 6 & -9 & 7 & -3\\ -1 & -4 & 8 & 0\end{pmatrix}$ matrix (8 points).

(3) Given the 4 × 4 matrix that translation by the vector $\vec{p}=(-6,4,5)$, using homogeneous coordinates (10 points).

(4) Suppose matrix $A=\begin{pmatrix}6 & 7 & 0 & 0\\ 4 & 5 & 0 & 0\\ 0 & 0 & 4 & 3\\ 0 & 0 & 9 & 8\end{pmatrix}$, find A^{-1} (8 points).

4. Prove issues (10 + 9 = 19 points)

(1) Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be the transformation. Show that if T maps two linear independent vectors onto a linear dependent set, then the equation $T(\vec{x})=0$ has a nontrivial solution (10 points).

(2) Show that if A is invertible, then $\operatorname{adj}A$ is also invertible.(9 points)

5. Synthesis (8 points)

Suppose A is an n × n invertible matrix, $\vec{a}$, $\vec{b} \in \mathbb{R}^n$ and $1+\vec{b}^T A^{-1}\vec{a} \neq 0$, prove $A+\vec{a}\vec{b}^T$ is invertible, and $(A+\vec{a}\vec{b}^T)^{-1} = A^{-1} - \dfrac{A^{-1}\vec{a}\vec{b}^T A^{-1}}{1+\vec{b}^T A^{-1}\vec{a}}$ (10 points).