

# 中山大学 本科生考试草稿纸 2011/18-77

**警示**

《中山大学授予学士学位工作细则》第七条：“考试作弊者不授予学士学位。”

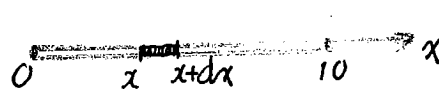
P.165.24 求心脏线:  $r = a(1 + \cos\theta)$  绕极轴旋转并成的旋转体的侧面积。

解:  $r'(\theta) = -a\sin\theta$

$$\begin{aligned} F &= 2\pi \int_0^\pi r(\theta) \cdot \sin\theta \cdot \sqrt{r^2(\theta) + r'^2(\theta)} d\theta \\ &= 2\pi \int_0^\pi a(1 + \cos\theta) \cdot \sin\theta \cdot \sqrt{a^2(1 + \cos\theta)^2 + a^2\sin^2\theta} d\theta \\ &= 2\pi \int_0^\pi a(1 + \cos\theta) \cdot \sin\theta \cdot \sqrt{2} \cdot \sqrt{1 + \cos\theta} d\theta \\ &= -2\sqrt{2}\pi a^2 \int_0^\pi (1 + \cos\theta)^{\frac{3}{2}} d(1 + \cos\theta) \\ &= -2\sqrt{2}\pi a^2 \cdot \frac{2}{5} [(1 + \cos\theta)^{\frac{5}{2}}]_0^\pi \\ &= -\frac{4}{5}\sqrt{2}\pi a^2 [0 - (\frac{1}{2})^{\frac{5}{2}}] = \frac{4}{5}\sqrt{2}\pi a^2 \cdot 4\sqrt{2} = \frac{32}{5}\pi a^2 \end{aligned}$$

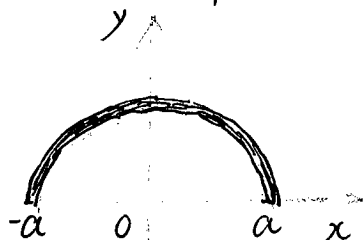
P.165.25 有一细棒长 10 m, 设距左端点  $x$  处的线密度是  $\rho(x)$ ,

$\rho(x) = (7 + 0.2x) \text{ kg/m}$ , 求细棒的质量。



$$\begin{aligned} dm &= \rho(x) dx = (7 + 0.2x) dx \\ m &= \int_0^{10} dm = \int_0^{10} (7 + 0.2x) dx = 7x \Big|_0^{10} + 0.1x^2 \Big|_0^{10} \\ &= 70 + 0.1 \times 100 = 70 + 10 = 80 \text{ (kg)}. \end{aligned}$$

P.165.27. 求半径为  $a$  的均匀半圆周的质心坐标。



解:  $m = \pi a \rho_0$ ,  $y = \sqrt{a^2 - x^2}$ ,  $y' = \frac{-x}{\sqrt{a^2 - x^2}}$

$$\bar{x} = \frac{\int_S x \rho_0 ds}{\pi a \rho_0} = \frac{\rho_0 \int_{-a}^a x \cdot \frac{a}{\sqrt{a^2 - x^2}} dx}{\pi a \rho_0} = 0$$

$$\bar{y} = \frac{\int_S y \rho_0 ds}{\pi a \rho_0} = \frac{\int_{-a}^a \sqrt{a^2 - x^2} \cdot \frac{a dx}{\sqrt{a^2 - x^2}}}{\pi a} = \frac{2a}{\pi}, \quad \text{质心}(\bar{x}, \bar{y}) = (0, \frac{2a}{\pi})$$