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P.144.22 $\int \frac{dx}{\sin x+2\cos x}$. 令 $t=\tan\frac{x}{2}$, $dx=\frac{2dt}{1+t^2}$

$\sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$

$=\int \frac{1}{\frac{2t}{1+t^2}+2\cdot\frac{(1-t^2)}{1+t^2}}\cdot\frac{2dt}{1+t^2}$

$=+\int\frac{dt}{t+1-t^2}=-\int\frac{dt}{t^2-t-1}=-\int\frac{dt}{(t-\frac{1}{2})^2-\frac{5}{4}}=-\int\frac{du}{u^2-(\frac{\sqrt{5}}{2})^2}$, $u=t-\frac{1}{2}$

$=\frac{1}{\sqrt{5}}\left[\ln\left|u+\frac{\sqrt{5}}{2}\right|-\ln\left|u-\frac{\sqrt{5}}{2}\right|\right]+C$

$=\frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+2t-1}{\sqrt{5}+2t+1}\right|+C=\frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+2\tan\frac{x}{2}-1}{\sqrt{5}+2\tan\frac{x}{2}+1}\right|+C$

P.144.23 $\int\frac{\sin x\cdot\cos x}{\sin^2 x+\cos^2 x}dx=\int\frac{-\cos x\, d\cos x}{1-\cos^2 x+\cos^4 x}\overset{令\cos x=t}{=\!=\!=}\int\frac{-t\,dt}{1-t^2+t^4}=\int\frac{-t\,dt}{t^4-t^2+1}$

$=\int\frac{-\frac{1}{2}d(t^2-\frac{1}{2})}{\frac{3}{4}+(t^2-\frac{1}{2})^2}=-\frac{1}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\cdot\arctan\frac{t^2-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+C=-\frac{1}{\sqrt{3}}\arctan\frac{2\cos^2 x-1}{\sqrt{3}}+C.$

P.144.24 $\int\frac{dx}{\sin^4 x}=-\int\frac{1}{\sin^2 x}d\cot x=-\int\csc^2 x\, d\cot x=-\int(1+\cot^2 x)\,d\cot x$

$=-\cot x-\frac{\cot^3 x}{3}+C$

P.144.25. $\int\sqrt{\frac{1-x}{1+x}}dx=\int\frac{1-x}{\sqrt{1-x^2}}dx=\int\frac{dx}{\sqrt{1-x^2}}+\int\frac{1}{2\sqrt{1-x^2}}d(1-x^2)=\arcsin x+\sqrt{1-x^2}+C.$

P.144.26. $\int\frac{1-\sqrt{x-1}}{1+\sqrt[3]{x-1}}dx$ 令 $\sqrt[6]{x-1}=t$, 则 $\sqrt[3]{x-1}=t^2$, $\sqrt{x-1}=t^6$

$x-1=t^6$, $dx=6t^5dt$

$=\int\frac{1-t^3}{1+t^2}\cdot 6t^5dt$

$=-6\int\frac{t^8-t^5}{1+t^2}dt$

$=-6\int(t^6-t^4-t^3+t^2+t-1+\frac{1-t}{t^2+1})dt$

$=-\frac{6}{7}t^7+\frac{6}{5}t^5+\frac{3}{2}t^4-2t^3-3t^2+6t$

$-6\int\frac{dt}{1+t^2}+3\int\frac{1}{1+t^2}d(1+t^2)$

$=-\frac{6}{7}t^7+\frac{6}{5}t^5+\frac{3}{2}t^4-2t^3-3t^2+6t$

$-6\arctan t+3\ln(1+t^2)+C$

$t^6-t^4-t^3+t^2+t-1$

$t^2+1\,\overline{)\,t^8-t^5}$

t^8+t^6

$-t^6-t^5$

$-t^6-t^4$

$-t^5+t^4$

$-t^5-t^3$

t^4+t^3

t^4+t^2

t^3-t^2

t^3+t

$-t^2-t$

$-t^2-1$

$-t+1$

$\frac{t^8-t^5}{t^2+1}=t^6-t^4-t^3+t^2+t-1+\frac{1-t}{t^2+1}.$