P.181.9. 证明下列等式：　　　　　　　　　　　　　　2011 18/7 —84.

(1) $\arcsin x + \arccos x = \frac{\pi}{2}$, $-1 \le x \le 1$

证：设 $f(x) = \arcsin x + \arccos x$，则 $f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$，$f(x) = C$.

又 $f(1) = \arcsin 1 + \arccos 1 = \frac{\pi}{2} + 0 = \frac{\pi}{2}$，所以 $f(x) = f(1) = C = \frac{\pi}{2}$.

即 $\arcsin x + \arccos x = \frac{\pi}{2}$.

(2) $\arctan x = \arcsin \frac{x}{\sqrt{1+x^2}}$, $-\infty < x < +\infty$.

证：设 $f(x) = \arctan x - \arcsin \frac{x}{\sqrt{1+x^2}}$

则 $f'(x) = \frac{1}{1+x^2} - \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \cdot \frac{\sqrt{1+x^2} - x \cdot \frac{2x}{2\sqrt{1+x^2}}}{(1+x^2)} = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$

所以 $f(x) = C$，而 $f(1) = \arctan 1 - \arcsin \frac{1}{\sqrt{2}} = \frac{\pi}{4} - \frac{\pi}{4} = 0$

即 $f(x) = 0$，即 $\arctan x - \arcsin \frac{x}{\sqrt{1+x^2}} = 0$

P.181.10 证明不等式：$\frac{2x}{\pi} < \sin x < x$, $(0 < x < \frac{\pi}{2})$

证：① $\sin x - \sin 0 = \cos\xi \cdot (x-0)$　　$0 < \xi < x$

$\sin x = \cos\xi \cdot x < 1 \cdot x \Rightarrow \sin x < x$

② 设 $f(x) = \frac{\sin x}{x}$，$f'(x) = \frac{x\cdot\cos x - \sin x}{x^2} = \frac{\cos x \cdot (x - \tan x)}{x^2}$

令 $g(x) = x - \tan x$，则 $g'(x) = 1 - \frac{1}{\cos^2 x} \le 0$，$g(x)$ 递减，$g(x) \le g(0) = 0$

所以 $x - \tan x < 0$，$0 < x < \frac{\pi}{2}$

所以 $f'(x) < 0$，　$0 < x < \frac{\pi}{2}$.

$f(\frac{\pi}{2}) - f(x) = f'(\xi)\cdot(\frac{\pi}{2} - x) < 0$，　$\frac{\pi}{2} - x > 0$

$\frac{2}{\pi} - \frac{\sin x}{x} < 0 \Rightarrow \frac{2x}{\pi} < \sin x$

由①、②可得：$\frac{2x}{\pi} < \sin x < x$.