微积分公式

1、导数公式：

$(x^{\mu})' = \mu x^{\mu-1}$

$(\sin x)' = \cos x;\qquad (\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x;\qquad (\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \cdot \tan x;\quad (\csc x)' = -\csc x \cdot \cot x$

$(a^x)' = a^x \ln a;\qquad (e^x)' = e^x$

$(\log_a x)' = \dfrac{1}{x\ln a};\qquad (\ln|x|)' = \dfrac{1}{x}$

$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \dfrac{1}{1+x^2}$

$(arc\cot x)' = -\dfrac{1}{1+x^2}$

2、积分表：

·基本积分公式：

$\int x^{\mu}dx = \dfrac{1}{\mu+1}x^{\mu+1} + C$

$\int \dfrac{1}{\sqrt{1-x^2}}dx = arc\sin x + C$

$\int \dfrac{1}{1+x^2}dx = \arctan x + C$

$\int a^x dx = \dfrac{a^x}{\ln a} + C; \int e^x dx = e^x + C$

$\int \dfrac{1}{x}dx = \ln|x| + C$

$\int \cos x dx = \sin x + C$

$\int \sin x dx = -\cos x + C$

$\int \sec^2 x dx = \tan x + C$

$\int cs\,\mathrm{c}^2 x dx = -\cot x + C$

$\int \sec x \cdot \tan x dx = \sec x + C$

$\int \csc x \cdot \cot x dx = -\csc x + C$

·补充积分公式：

$\int \tan x dx = -\ln|\cos x| + C$

$\int \cot x dx = \ln|\sin x| + C$

$\int \sec x dx = \ln|\sec x + \tan x| + C$

$\int \csc x dx = \ln|\csc x - \cot x| + C$

$\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\arctan\dfrac{x}{a} + C$

$\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right| + C$

$\int \dfrac{dx}{\sqrt{a^2-x^2}} = \arcsin\dfrac{x}{a} + C$

上述公式方法：第一换元法

$\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$ (第二换元法)

$\int \sqrt{a^2-x^2}dx = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\arcsin\dfrac{x}{a} + C$ (第二换元法或分部积分法)

$\int \sqrt{x^2 \pm a^2}dx = \dfrac{x}{2}\sqrt{x^2 \pm a^2} \pm \dfrac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$ (第二换元法或分部积分法)

上述公式方法：第二换元法

$\sqrt{a^2-x^2}$换元$x = a\sin t$; $\sqrt{x^2+a^2}$换元$x = a\tan t$; $\sqrt{x^2-a^2}$换元$x = a\sec t$

分部积分：$\int udv = uv - \int vdu$

$\int x^n (\ln x)^m dx$; $\int x^n$ 反三角函数 dx

$\int x^n \sin bxdx$; $\int x^n \cos bxdx$; $\int x^n e^{bx} dx$

$\int e^{ax} \sin bxdx$; $\int e^{ax} \cos bxdx$

附三角函数公式：

· 诱导公式：

角 \ 函数	sin	cos	tan	cot
-α	-sinα	cosα	-tanα	-cotα
$\pi/2$-α	cosα	sinα	cotα	tanα
$\pi/2$+α	cosα	-sinα	-cotα	-tanα
π-α	sinα	-cosα	-tanα	-cotα
π+α	-sinα	-cosα	tanα	cotα
2π-α	-sinα	cosα	-tanα	-cotα
2π+α	sinα	cosα	tanα	cotα

· 和差角公式：

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$$

$$\cot(\alpha \pm \beta) = \frac{\cot\alpha \cdot \cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$

· 和差化积公式：

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

· **积化和差公式：**

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta=-\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$$

· **倍角公式：**

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$\cot 2\alpha=\frac{\cot^2\alpha-1}{2\cot\alpha},\qquad \tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin 3\alpha=3\sin\alpha-4\sin^3\alpha$$

$$\cos 3\alpha=4\cos^3\alpha-3\cos\alpha$$

$$\tan 3\alpha=\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$$

· **半角公式：**

$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}\qquad \cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}\qquad \cot\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{1+\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1-\cos\alpha}$$

· **万能公式：**

$$\sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},\quad \cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}},\quad \tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

· **反三角函数性质：** $\arcsin x=\dfrac{\pi}{2}-\arccos x\qquad \arctan x=\dfrac{\pi}{2}-arc\cot x$