中山大學 本科生考试草稿纸

警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.144.5 $\int \frac{x^2dx}{1-x^4} = \int \frac{x^2+1-1}{(1-x^2)(1+x^2)}dx = \int \frac{dx}{1-x^2} - \int \frac{dx}{(1-x^2)(1+x^2)} = \int \frac{dx}{1-x^2} - \frac{1}{2}\int (\frac{1}{1-x^2} + \frac{1}{1+x^2})dx$

$= \frac{1}{2}\int \frac{dx}{1-x^2} - \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{4}\int (\frac{1}{1-x} + \frac{1}{1+x})dx - \frac{1}{2}\int \frac{dx}{1+x^2}$

$= \frac{1}{4}\ln\left|\frac{1+x}{1-x}\right| - \frac{1}{2}\arctan x + C.$

P.144.6 $\int \frac{dx}{x^3+1}$ $\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} = \frac{(A+B)x^2+(-A+B+C)x+(A+C)}{(x+1)(x^2-x+1)}$

$\begin{cases} A+B=0 \\ -A+B+C=0 \\ A+C=1 \end{cases} \Rightarrow \begin{cases} 2B+C=0 \\ A+C=1 \\ A+B=0 \end{cases} \Rightarrow \begin{cases} 2B+C=0 \\ C-B=1 \end{cases} \Rightarrow \begin{cases} 3B=-1, B=-\frac{1}{3} \\ A=\frac{1}{3}, C=\frac{2}{3}. \end{cases}$

$\frac{1}{x^3+1} = \frac{\frac{1}{3}}{x+1} + \frac{-\frac{1}{3}x+\frac{2}{3}}{x^2-x+1}$

$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{x-2}{x^2-x+1}dx$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{2x-1-3}{x^2-x+1}dx$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\int \frac{1}{x^2-x+1}d(x^2-x+1)$

$+ \frac{1}{2}\int \frac{dx}{x^2-x+1}$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{1}{2}\int \frac{dx}{\frac{3}{4}+(x-\frac{1}{2})^2}$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{(x-\frac{1}{2})}{\frac{\sqrt{3}}{2}} + C$

$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}} + C$

P.144.7 $\int \frac{dx}{x^4+1} = \frac{1}{2}\int \frac{x^2+1}{x^4+1}dx - \frac{1}{2}\int \frac{x^2-1}{x^4+1}dx = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$

$= \frac{1}{2}\int \frac{d(x-\frac{1}{x})}{(x-\frac{1}{x})^2+2} - \frac{1}{2}\int \frac{d(x+\frac{1}{x})}{(x+\frac{1}{x})-2}$

$= \frac{1}{2\sqrt{2}}\arctan\frac{x-\frac{1}{x}}{\sqrt{2}} - \frac{1}{2\sqrt{2}}\int \frac{d(\frac{x+\frac{1}{x}}{\sqrt{2}})}{(\frac{x+\frac{1}{x}}{\sqrt{2}})^2-1}$

$= \frac{1}{2\sqrt{2}}\arctan\frac{x-\frac{1}{x}}{\sqrt{2}} - \frac{1}{4\sqrt{2}}\int (\frac{1}{\frac{x+\frac{1}{x}}{\sqrt{2}}-1} - \frac{1}{\frac{x+\frac{1}{x}}{\sqrt{2}}+1})d(\frac{x+\frac{1}{x}}{\sqrt{2}})$

$= \frac{1}{2\sqrt{2}}\arctan\frac{x-\frac{1}{x}}{\sqrt{2}} - \frac{1}{4\sqrt{2}}\ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C.$