P.95.3 设 $\alpha(x)=O(x)$, $(x\to 0)$; $\beta(x)=O(x)$ $(x\to 0)$ 2011 3/4-30.

证明: $\alpha(x)+\beta(x)=O(x)$.

证: 由条件可知: $\alpha(x)=\eta_1(x)\cdot x$, 而 $\lim\limits_{x\to 0}\eta_1(x)=0$

$\beta(x)=\eta_2(x)\cdot x$, $\lim\limits_{x\to 0}\eta_2(x)=0$

则 $[\alpha(x)+\beta(x)]=[\eta_1(x)+\eta_2(x)]\cdot x$, 且 $\lim\limits_{x\to 0}[\eta_1(x)+\eta_2(x)]=0$

从而 $\alpha(x)+\beta(x)=O(x)$

方法2. 由条件 $\lim\limits_{x\to 0}\dfrac{\alpha(x)}{x}=0$, $\lim\limits_{x\to 0}\dfrac{\beta(x)}{x}=0$

从而 $\lim\limits_{x\to 0}\dfrac{\alpha(x)+\beta(x)}{x}=\lim\limits_{x\to 0}\dfrac{\alpha(x)}{x}+\lim\limits_{x\to 0}\dfrac{\beta(x)}{x}=0+0=0$

即 $\alpha(x)+\beta(x)=O(x)$.

P.95.4 计算下列函数在指定点 x_0 处的微分:

(1) $y=x\cdot\sin x$, $x_0=\dfrac{\pi}{4}$.

$dy=d(x\sin x)=\sin x\,dx+x\,d\sin x=(\sin x+x\cdot\cos x)dx$

$dy\big|_{x=\frac{\pi}{4}}=(\sin\frac{\pi}{4}+\frac{\pi}{4}\cos\frac{\pi}{4})dx=\frac{1}{\sqrt{2}}(1+\frac{\pi}{4})dx$.

(2) $y=(1+x)^{\alpha}$, $x_0=0$ $(\alpha>0)$

$dy=\alpha(1+x)^{\alpha-1}dx$

$dy\big|_{x=0}=\alpha(1+0)^{\alpha-1}dx=\underline{\alpha\,dx}$

P.95.5 求下列各函数的微分。

(1) $y=\dfrac{1-x}{1+x}$ $(x\neq -1)$

$dy=d(\dfrac{1-x}{1+x})=\dfrac{(1+x)d(1-x)-(1-x)d(1+x)}{(1+x)^2}=\dfrac{-(1+x)dx-(1-x)dx}{(1+x)^2}=\dfrac{-2dx}{(1+x)^2}$

(2) $y=x\cdot e^{x}$, $dy=d(xe^{x})=x\,de^{x}+e^{x}dx$

$=xe^{x}dx+e^{x}dx=(x+1)\cdot e^{x}dx$.