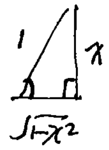
2011 $\frac{11}{7}$ —50.

13. $$\int \frac{x\cdot \arccos x}{(1-x^2)^2}dx = -\frac{1}{2}\int \frac{\arccos x}{(1-x^2)^2}\,d(1-x^2) = \frac{1}{2}\int \arccos x \, d\frac{1}{1-x^2}$$

$$= \frac{\arccos x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{1-x^2}\,d\arccos x = \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\int \frac{1}{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$$

令 $x=\sin t$,



$$= \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\int \frac{\cos t\,dt}{\cos^3 t} = \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\tan t + C$$

$$= \frac{\arccos x}{2(1-x^2)} + \frac{1}{2}\cdot\frac{x}{\sqrt{1-x^2}} + C = \frac{1}{2}\left(\frac{\arccos x}{1-x^2} + \frac{x}{\sqrt{1-x^2}} + C\right)$$

14. $$\int \arctan\sqrt{x}\,dx = x\cdot\arctan\sqrt{x} - \int x\,d\arctan\sqrt{x}$$

$$= x\cdot\arctan\sqrt{x} - \int x\cdot\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}\,dx$$

$$= x\cdot\arctan\sqrt{x} - \int \frac{1+x-1}{1+x}\cdot\frac{dx}{2\sqrt{x}}$$

$$= x\cdot\arctan\sqrt{x} - \int\frac{dx}{2\sqrt{x}} + \int\frac{1}{1+x}\,d\sqrt{x}$$

$$= x\cdot\arctan\sqrt{x} - \sqrt{x} + \arctan\sqrt{x} + C$$

$$= (x+1)\cdot\arctan\sqrt{x} - \sqrt{x} + C.$$

15. $$\int \frac{\arctan x}{x^2}dx = -\int \arctan x\,d(\frac{1}{x}) = -\left[\frac{\arctan x}{x} - \int\frac{1}{x}\,d\arctan x\right]$$

$$= -\frac{\arctan x}{x} + \int\frac{1}{x(1+x^2)}\,dx$$

$$= -\frac{\arctan x}{x} + \int\frac{1}{x^3(1+\frac{1}{x^2})}\,dx$$

$$= -\frac{\arctan x}{x} - \frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}\,d(1+\frac{1}{x^2})$$

$$= -\frac{\arctan x}{x} - \frac{1}{2}\ln(1+\frac{1}{x^2}) + C.$$

16. $$\int x^3\cdot(\ln x)^3dx = \frac{1}{4}\int(\ln x)^3\,dx^4 = \frac{1}{4}\left[x^4(\ln x)^3 - \int x^4\cdot 3\ln^2 x\cdot\frac{1}{x}\,dx\right]$$

$$= \frac{1}{4}\left[x^4\cdot(\ln x)^3 - 3\int x^3\ln^2 x\,dx\right] = \frac{x^4}{4}(\ln x)^3 - \frac{3}{4}\int x^3\ln^2 x\,dx$$

$$= \frac{x^4}{4}(\ln x)^3 - \frac{3}{16}\int \ln^2 x\,dx^4 = \frac{x^4}{4}(\ln x)^3 - \frac{3}{16}(x^4\ln^2 x - \int x^4\cdot 2\ln x\cdot\frac{1}{x}\,dx)$$

$$= \frac{x^4}{4}(\ln x)^3 - \frac{3x^4}{16}(\ln x)^2 - \frac{6}{16}\int x^3\cdot\ln x\,dx = \frac{x^4}{4}(\ln x)^3 - \frac{3x^4}{16}(\ln x)^2 - \frac{3}{16\times 2}\int \ln x\,dx^4$$

$$= \frac{x^4}{4}(\ln x)^3 - \frac{3x^4}{16}(\ln x)^2 - \frac{3}{16\times 2}(x^4\ln x - \int x^4\cdot\frac{1}{x})dx = \frac{x^4}{4}(\ln^3 x - \frac{3}{4}\ln^2 x - \frac{3}{8}\ln x - \frac{3}{16}) + C.$$