中山大学软件学院 **2010** 级软件工程专业(2010 学年秋季学期)

《线性代数》 期末试题 (A/B 卷)

(考试形式：闭 卷 考试时间：2 小时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：________________ 姓名：__________ 学号：____________

出卷：________________________ 复核：__________________

1. Fill in the blank（10*3=30 Pts）

(1) If the vector x determined by the coordinate vector $[x]_B=\begin{bmatrix}-1\\3\end{bmatrix}$ **and the basis** $B=\left\{\begin{bmatrix}-2\\1\end{bmatrix},\begin{bmatrix}3\\1\end{bmatrix}\right\}$**, then x=____________.**

(2) Let A **be a 4×4 matrix, and suppose the eigenvalues of** A **are 3，1，1，and 2, then** $\det A$ =__________

(3) The matrix $A=\frac{1}{9}\begin{bmatrix}-1&4&a\\a&4&-1\\4&b&4\end{bmatrix}$ **is an orthogonal matrix, then** a =__________, b =_________________.

(4) If $v_1=\begin{bmatrix}1\\3\\-2\end{bmatrix},v_2=\begin{bmatrix}5\\1\\4\end{bmatrix},y=\begin{bmatrix}1\\3\\5\end{bmatrix}$**, and** $W=Span\{v_1,v_2\}$**, then the closest point in** W **to** y **is ______________________.**

(5) If the transformation $x\mapsto Ax$ **maps** $\begin{bmatrix}1\\3\end{bmatrix}$ **and** $\begin{bmatrix}2\\7\end{bmatrix}$ **into** $\begin{bmatrix}1\\1\end{bmatrix}$ **and** $\begin{bmatrix}3\\1\end{bmatrix}$**,respectively, then the matrix** A=_______________.

(6) Assume the matrix A **is row equivalent to** B.

$$A=\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for $Row\ A$ **is ____, and** $\dim Nul\ A$ **is___________, and** $rank\ A^T$ **is ________________________.**

(7) $A=\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ **act on** C^n**, the eigenvalues of** A **is** $\lambda = 2\pm 3i$**, and a basis for the eigenspace corresponding to** $\lambda_1 = 2-3i$ **is _________________________.**

2. Mark each statement True or False, and descript your reasons. （5*4=20 Pts）

(1) Let $T: R^n \mapsto R^n$ **be a linear transformation defined by** $T(x)=Ax$**. Then** T **is onto if and only if** $\det(A)\neq 0$**.**

(2) If A **is an** $n\times n$ **matrix, then** $rank\ A < n$ **if and only if some eigenvalue of** A **is zero.**

(3) The set of all solution to the linear system $Ax=b$**, where** A **is** $m\times n$ **and** $b\neq 0$**, is a subspace of** R^n**.**

(4) Every set of five orthonormal vectors is a basis for R^5**.**

(5) The linear transformation $T: P_2 \mapsto P_2$ **defined by** $T(at^2+bt+c)=2at+b$ **is one-to-one.**

3. Calculation issues (5*6=30 Pts）

(1) Let $v_1=\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2=\begin{bmatrix} 3 \\ 2 \\ -2 \\ -2 \end{bmatrix}, v_3=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix}, x=\begin{bmatrix} -2 \\ 3 \\ 0 \\ -3 \end{bmatrix}, y=\begin{bmatrix} 0 \\ 3 \\ 6 \\ -3 \end{bmatrix}$**, and** $W=Span\{v_1,v_2,v_3\}$

a. Determine whether x is in W

b. Determine whether y is in the orthogonal of W**.**

(2) If A **and** B **are 3×3 matrices,** I **is the identity matrix, and** $AB+I=A^2+B$**, where** $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$**. Find** B**.**

(3) Let $A=\begin{bmatrix}0 & 4 & 2\\ 4 & 0 & -2\\ 2 & -2 & 3\end{bmatrix}$, $v_1=\begin{bmatrix}-2\\ 2\\ 1\end{bmatrix}$, $v_2=\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}$, $v_3=\begin{bmatrix}1\\ 0\\ 2\end{bmatrix}$. **The** v_1, v_2, **and** v_3 **are eigenvectors for** A.

a. Find matrices P **and** D **that orthogonally diagonalize** A, **and write the equation that relates** A **to** P **and** D.

b. Write the quadratic form associated with A **using variables** x_1, x_2, **and** x_3. **And classify the quadratic form as positive (or negative) definite, or indefinite.**

(4) Let $A=\begin{bmatrix}1 & -1\\ 2 & 5\\ 3 & 4\end{bmatrix}$, $b=\begin{bmatrix}-1\\ 10\\ 3\end{bmatrix}$. **Find the least-squares solution of** $Ax=b$, **and determine the least-squares error in the least-squares solution of** $Ax=b$.

(5) Let $H=\left\{\begin{bmatrix}3a+7b-c\\ -5b+8c-2d\\ 3d-4e\\ 5b-8c-d+4e\end{bmatrix}: a,b,c,d,e \text{ any real numbers}\right\}$,

a. Show H **is a subspace of** R^4.

b. Find a basis for H **.**

4. Prove issues （2*10=20 Pts）

(1) Show that if x is a nonzero vector in R^n, **then the** $n\times n$ **matrix**

$$A=I_n-\frac{2}{\|x\|^2}xx^T$$

is both orthogonal and symmetric.

(2) Suppose A **and** B **are both real symmetric matrix, then they have the same characteristic polynomial if and only if there is an orthogonal matrix** T **such that** $T^{-1}AT=B$ **.**