

HW 6.3-6.5

1. Let $y = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$, $u_1 = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}$, $W = \text{Span}\{u_1\}$

a. Let U be the 2×1 matrix whose only column is u_1 . Compute $U^T U$ and $U U^T$.

b. Compute $\text{Proj}_W y$ and $(U U^T)y$.

2. True or false

- A. If W is a subspace of \mathbb{R}^n and if vector v is in both W and W^\perp , then v must be the zero vector.
- B. In the Orthogonal Decomposition Theorem, each term in formula (2) for \hat{y} is itself an orthogonal projection of y onto a subspace of W .
- C. If $y = z_1 + z_2$, where z_1 is in a subspace W and z_2 is in W^\perp , then z_1 must be the orthogonal projection of y onto W .
- D. The best approximation to y by elements of a subspace W is given by the vector $y - \text{Proj}_W y$.
- E. If an $n \times p$ matrix U has orthonormal columns, then $U U^T x = x$ for all x in \mathbb{R}^n .

3. Find the QR factorization for matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

4. Suppose $A = QR$, where R is an invertible matrix. Show that A and Q have the same column space.

5. Find the orthogonal projection of b onto $\text{Col } A$ and a least-squares solution of $Ax = b$.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

6. Prove that the matrix $A^T A$ is invertible if and only if the columns of A are linearly independent.

课本 6.3 节习题： 第 23, 24 题

课本 6.4 节习题： 第 16 题

课本 6.4 节习题： 第 22 题