中山大学数据科学与计算机学院2015级软件工程专业（2016.11）

《线性代数》期中考试题

（考试形式：闭卷　考试时间：2小时）

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

教学班：________ 姓名：________ 学号：________ 成绩：________

1. Fill in the blank（6×4=24 Pts）

(1) Let $A=\begin{bmatrix}1&2&3\\2&3&4\\3&4&5\end{bmatrix}$, then det A=________, adj$A$=

(2) Let $A=\begin{pmatrix}1&0\\k&1\end{pmatrix}$, Then A^2=________, A^n=________.(n is a positive integer, n>0)

(3) Let $A=\begin{bmatrix}1&2&4\\2&1&3\\1&0&2\end{bmatrix}$, $B=\begin{bmatrix}1&2&4\\2&1&3\\2&2&6\end{bmatrix}$, $C=\begin{bmatrix}1&2&4\\0&-1&-3\\2&2&6\end{bmatrix}$. Find an elementary matrix E such that $EA=B$, E=________.

Find an elementary matrix F such that $FB=C$, F=________.

(4) Consider a linear system $A=\begin{bmatrix}1&-2&5&-2\\-1&2&-3&4\\1&-2&9&2\\-1&2&1&8\end{bmatrix}$, which has been reduced by row operations to a reduced echelon matrix U, then U=________.

(5) Find a= ________ and b= ________ such that the subset $\left\{\begin{bmatrix}2\\a-b\\1\end{bmatrix},\begin{bmatrix}a\\b\\3\end{bmatrix}\right\}$ is linear dependent.

(6)Let T: $R^2\to R^3$ be a linear transformation such that $T\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}x_1-2x_2\\-x_1+3x_2\\3x_1-2x_2\end{pmatrix}$. Find a matrix A such that $T(\mathbf{x})=A\mathbf{x}$, the matrix A=____________. Find $\mathbf{x}$ such that $T(x)=\begin{pmatrix}-1\\4\\9\end{pmatrix}$, $\mathbf{x}$=____________.

2. Make each statement True or False, and descript your reasons.（8×3=24 Pts）

(1) Let A be an n×n matrix and the equation Ax=b has a unique solution for some b in R^n, then A is invertible.

(2) If A is an $m\times n$ matrix with m pivot columns, then the linear transformation $\mathbf{x}\to A\mathbf{x}$ maps R^n onto R^m.

(3) If the columns of A span R^4, then $\det(-A)=-\det A$

(4) Let $T:R^n\to R^n$ be a linear transformation defined by $T(x)=Ax$. Then T is invertible if and only if T is one-to-one.

(5) If $\{u, v, w\}$ is linearly dependent, then the vector w is in Span$\{u, v\}$.

(6) The (i, j)-cofactor C_{ij} of a matrix A is the matrix A_{ij} obtained by deleting from A its ith row and jth column.

(7) If A is invertible and if $AB=BA$, then $(A^T)^{-1}B^T=B^T(A^T)^{-1}$.

(8) If none of the vectors in the set $V=\{v_1, v_2, v_3\}$ in R^n is a multiple of one of the other vectors, then V is linearly independent.

3. Calculation（4×8=32 Pts）

(1) Determine the values of a and b such that linear system

$$\begin{cases}x_1+x_2+x_3+x_4=0,\\ x_2+2x_3+2x_4=1,\\ -x_2+(a-3)x_3-2x_4=b,\\ 3x_1+2x_2+x_3+ax_4=-1\end{cases}$$

(a) has only a solution;

(b) has no solution;

(c) has indefinitely many solutions. In this case compute the solution of linear system.

(2) Prove that $\begin{bmatrix} 3I & A \\ 0 & 2I \end{bmatrix}$ is invertible and find its inverse. (Note: I is the identity matrix)

(3) Give a matrix $A = \begin{bmatrix} 2 & 4 & 2 & -2 \\ -4 & -5 & -8 & 2 \\ 2 & -5 & 1 & 8 \\ -6 & 0 & -3 & 1 \end{bmatrix}$,

(a) Find an LU factorization of A.

(b) Compute determinants of A.

(4) Let $T: R^3 \to R^3$ be a linear transformation such that

$T(e_1) = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} -2 \\ -2 \\ 21 \end{bmatrix}$, where e_1, e_2, e_3 are the columns of I_3

(a) Determine if T is one-to-one map. Explain.

(b) Write the 4×4 matrix that represents T when homogeneous coordinates are used for vectors in R^3.

4. Prove issues（2×10=20 Pts）

(1). Let A be a 5×4 matrix and B a 4×5 matrix. Show that the det $AB = 0$.(Hint: utilize the equation $AB\mathbf{x}$=0)

(2) Square matrix A is invertible if and only if detA≠0