总复习

考试范围:

Chapter 1 -7

考题类型:

填空题、判断题、计算题、证明题、综合题

复习要求:

- 基本概念要清晰、章节之间知识点能融会贯通。
- 重要的定理要掌握，相关证明要理解。
- 基本方法和基本运算是重点，必须掌握。
- 书上例题、作业要多思考。
- 全面复习，归纳总结。把书读薄，把书读厚

Chapter1　　Linear equations in linear algebra

Review:

§ 1.1 Systems of linear equations

§ 1.2 Row reduction and echelon forms

§ 1.3 Vector equations

§ 1.4 The matrix equation Ax=b

§ 1.5 Solutions sets of linear systems

§ 1.7 linear independence

§ 1.8 Introduction to Linear Transformations

§ 1.9 The Matrix of a Linear Transformation

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Chapter1 Linear equations in linear algebra

Definitions

- A system of linear equations.
- Row equivalent, echelon form, reduced echelon form.
- Span{v}, Span{u,v} and geometric interpretation in R^2 or R^3, Span{v_1,…, v_p}.
- Linearly independent, linearly dependent.
- Linear transformation, Standard matrix of a linear transformation.
- One-to-one and onto (for linear transformation).

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Chapter1 Linear equations in linear algebra

Theorems

- **Theorem 2 (Existence and Uniqueness Theorem)**
- **Theorem 3 (Matrix equation, vector equation, system of linear equations)**
- **Theorem 4 (when do the columns of A span R^m?)**
- **Theorem 7, 8, 9 (properties of linearly dependent sets)**
- **Theorem 10 (standard matrix for the linear transformation)**
- **Theorem 11 and 12 (one-to-one and onto linear transformations).**

Chapter1 Linear equations in linear algebra

Important skills

•Determine when a system is consistent. Write the general solution in parametric vector form.

•Describe existence or uniqueness of solution in terms of pivot positions.

•Determine when a homogeneous system has a nontrivial solution.

•Determine when a vector is in a subset spanned by specified vectors.

•Exhibit a vector as a linear combination of specified vectors.

Chapter1 Linear equations in linear algebra

Important skills

- **Determine whether the columns of an $m \times n$ matrix span R^m**
- **Determine whether a set of vector is linearly independent.**
- **Find the standard matrix of a linear transformation.**
- **Determine whether a linear transformation is one-to-one and/or onto.**
- **Determine whether a specified vector is in the range of a linear transformation.**

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Chapter 2 Matrix Algebra

Review:

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Chapter 2 Matrix Algebra

Definitions

The definition of a matrix product AB, A^T, A^{-1}, A^k, singular/nonsingular matrix, elementary matrix.

Theorems

Theorem 3, 4, 5, 6, and 7,

Theorem 8 (The Invertible Matrix Theorem) , including new statements in section 4.6 and 5.2

Theorem 9

Important skills

- **Matrix operations.**
- **Use an inverse matrix to solve a system of linear equations.**
- **Use matrix algebra to solve equation involving matrices.**

Chapter 3 Determinants

Review:

§ 3.1 Introduction to Determinants

§ 3.2 Properties of Determinants

§ 3.3 Cramer's Rule, Volume, and Linear Transformations

Definitions

det A, Cij(余因子), adj A(伴随矩阵)

Theorems

Theorem 1, 2, 3, 4, 5, 6 (properties of determinants).

Theorem 8, 9, 10 (determinants as area or volume). Theorem 7 (Cramer's Rule)

Important skills

- **Compute the determinant of a 4×4 matrix.**
- **Computer the area of the parallelogram (or its image).**

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Chapter 4 Vector Spaces

Review:

Chapter 4 Vector Spaces

Definitions

Subspace, null space, column space, row space,
basis, B-coordinate vector of x,
kernel/range of a linear transformation,
dimension of V, rank of A.

Theorems

Theorem 1, 2, 3,
Contrast between Nul A and Col A.(p 232)
Theorem 4, 5 (spanning set theorem).
Theorem 6, 7(unique representation theorem).
Theorem 8, 9, 10, 12(basis theorem), 13, 14(rank theorem)

Chapter 4 Vector Spaces

Important skills

- Determine if a set of vectors spans (or is a basis for) R^n.
- Determine if a set is a subspace (using theorem 1,2,or 3 in chapter 4).
- Determine if a vector is in Nul A or in Col A.
- Determine if a set is a basis for a subspace.
- Find a basis for Nul A or in Col A, or other subspace.
- Find the coordinate vector of a vector relative to a basis.
- Use coordinate vectors to check if a set is linearly independent.
- Find the dimension of Nul A ,Col A, Row A, or other subspace.
- Determine the rank of a matrix.
- Use the Rank Theorem to determine facts about a system of linear equations.

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Chapter 5 Eigenvalues and Eigenvectors

Review:

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Chapter 5 Eigenvalues and Eigenvectors

Definitions

Eigenvalue, eigenvector, eigenspace, diagonalizable.
Similar matrix.
Matrix of a linear transformation relative to a basis.

Theorems

Theorem 1, 2, Theorem 3(Properties of Determinants (continued))

Theorem 4,5(Diagonalization theorem)

Theorem 6, 7, and 8

Important skills

• Determine if a number (vector) is an eignenvalue (eigenvector) of a matrix.

• Find the characteristic equation and eigenvalues of a 3×3 matrix.

•Find an basis for an eigenspace.

Chapter 5 Eigenvalues and Eigenvectors

Important skills

• If A is diagonalizable, find P and D such that $A=PDP^{-1}$.

• Show how to compute high powers of a diagonalizable matrix.

• Find the B-matrix $[T]_B$ of a linear transformation $T:V\rightarrow V$ relative to a basis B of V.

• Find complex eigenvalues and corresponding eigenvectors.

• Find a factorization of a 2×2 matrix with a complex eigenvalue, $A=PDP^{-1}$, where the transformation $x \rightarrow Cx$ is a composition of a rotation and possibly a scaling transformation. Determine the angle of the rotation and scale factor.

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CHAPTER 6 Orthogonality and Least Squares

Review:

CHAPTER 6 Orthogonality and Least Squares

Definitions

Length of a vector, unit vector, orthogonal set, Orthogonal vector, orthonormal basis, orthogonal complements, orthogonal matrix.

QR factorization.

General least-squares problem.

Least-squares solution of Ax=b, normal equations.

Theorems

Theorem 3, 5, 6

Theorem 8 (orthogonal decomposition), theorem 9 (best approximation)

Theorem 11 (Gram-Schmidt),theorem 12 (QR factorization)

Theorem 13,14,15

CHAPTER 6 Orthogonality and Least Squares

Important skills

• Compute length of a vector, distance between two vectors.

• Normalize a vector.

• Check a set for orthogonality.

• Compute the orthogonal projection onto a line (through 0) or other subspace.

• Decompose a vector into a component in the direction of u and a component orthogonal to u.

• Decompose a vector into the sum of a vector in W and a vector in $W^{\perp}$.

CHAPTER 6 Orthogonality and Least Squares

Important skills

• Determine if a set is orthogonal, normalize a vector, construct an orthonormal set from an orthogonal set. Know $||x||^2=x^Tx=x\bullet x$.

• Compute orthogonal projection of a vector onto a subspace, find the closest point in a subspace, find the distance from a vector to a subspace, decompose a vector as in the orthogonal decomposition theorem.

• Perform the Gram-Schmidt process on a linearly independent set of vectors.

• Construct a QR factorization of a matrix.

• Find a least-squares solution to Ax=b, find the least-squares error associated with this solution, know the normal equations.

CHAPTER 7 Symmetric Matrices and Quadratic Forms

Review:

§ 7.1 Diagonalization of Symmetric Matrices

§ 7.2 Quadratic Forms

Definitions

Symmetric matrix, orthogonally diagonalizable, quadratic form, matrix of the quadratic form

Theorems

Theorem 1, 2, theorem 3 (The Spectral Theorem for Symmetric Matrices).
Theorem 5 (Quadratic forms and eigenvalues)

Important skills

- orthogonally diagonalize a symmetric matrix.
- Find the matrix of the quadratic form.

Other material

1、本书网站 http://www.laylinalgebra.com

2、试题（放课件邮箱上）

3、Solutions （放课件邮箱上）

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