15. $\int \frac{dx}{1-\sin x} = \int \frac{(1+\sin x)dx}{(1+\sin x)(1-\sin x)} = \int \frac{dx}{\cos^2 x} - \int \frac{1}{\cos^2 x} d\cos x = \tan x + \frac{1}{\cos x} + C.$

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16. $\int \frac{x^{14}}{(x^5+1)^4} dx = \frac{1}{5}\int \frac{x^{10}}{(x^5+1)^4} dx^5$

$= \frac{1}{5}\int \frac{u^2}{(u+1)^4} du$

$= \frac{1}{5}\int [\frac{1}{(u+1)^2} - \frac{2}{(u+1)^3} + \frac{1}{(u+1)^4}] du$

$= \frac{1}{5}(-\frac{1}{u+1} + \frac{1}{(u+1)^2} - \frac{1}{3}\cdot\frac{1}{(u+1)^3}) + C$

$= \frac{1}{5}(-\frac{1}{x^5+1} + \frac{1}{(x^5+1)^2} - \frac{1}{3(x^5+1)^3}) + C = -\frac{1}{5}(x^5+1)^{-1} + \frac{1}{5}(x^5+1)^{-2} + \frac{1}{15}(x^5+1)^{-3} + C$

注: $\frac{x^2}{(x+1)^4} = \frac{x^2-1+1}{(x+1)^4} = \frac{x-1}{(x+1)^3} + \frac{1}{(x+1)^4}$

$= \frac{x+1-2}{(x+1)^3} + \frac{1}{(x+1)^4}$

$= \frac{1}{(x+1)^2} - \frac{-2}{(x+1)^3} + \frac{1}{(x+1)^4}.$

16. 方法二.

由于 $(\frac{x^5}{x^5+1})' = \frac{5x^4(x^5+1) - x^5\cdot 5x^4}{(x^5+1)^2} = \frac{5x^9+5x^4-5x^9}{(x^5+1)^2} = \frac{5x^4}{(x^5+1)^2}$

所以 $\int \frac{x^{14}}{(x^5+1)^4} dx = \frac{1}{5}\int \frac{x^{10}}{(x^5+1)^2}[\frac{5x^4}{(x^5+1)^2}] dx = \frac{1}{5}\int (\frac{x^5}{x^5+1})^2 d(\frac{x^5}{x^5+1})$

$= \frac{1}{15}\cdot(\frac{x^5}{x^5+1})^3 + C \quad = \frac{1}{15}\cdot\frac{x^{15}}{(x^5+1)^3} + C.$

16. 方法三. $\int \frac{x^{14}}{(x^5+1)^4} dx = -\frac{1}{5}\int \frac{x^{20}}{(x^5+1)^4} d(\frac{1}{x^5}) = -\frac{1}{5}\int \frac{x^{20}}{x^{20}(1+\frac{1}{x^5})^4} d(\frac{1}{x^5})$

$= -\frac{1}{5}\cdot\frac{1}{-4+1}(1+x^{-5})^{-4+1} + C = \frac{1}{15}(1+\frac{1}{x^5})^{-3} + C.$

17. $\int \frac{x^{2n-1}}{x^n-1} dx = \int \frac{x^{2n}}{x(x^n-1)} dx = \int \frac{x^{2n}-1+1}{x(x^n-1)} dx = \int \frac{x^n+1}{x} dx + \int \frac{dx}{x(x^n-1)}$

$= \int x^{n-1}dx + \int \frac{1}{x}dx + \int (\frac{x^{n-1}}{x^n-1} - \frac{1}{x})dx$

$= \frac{x^n}{n} + \ln|x| + \frac{1}{n}\int \frac{1}{x^n-1} d(x^n-1) - \ln|x| + C = \frac{x^n}{n} + \frac{1}{n}\ln|x^n-1| + C.$

18. $\int \frac{dx}{x(x^5+2)} = \int \frac{dx}{x^6(1+\frac{2}{x^5})} = -\frac{1}{10}\int \frac{1}{1+\frac{2}{x^5}} d(1+\frac{2}{x^5}) = -\frac{1}{10}\ln|1+\frac{2}{x^5}| + C = \frac{1}{10}\ln|\frac{x^5}{x^5+2}| + C.$

19. $\int \frac{\ln(x+2)-\ln x}{x(x+2)} dx = \int \frac{\ln(1+\frac{2}{x})}{x^2(1+\frac{2}{x})} = -\frac{1}{2}\int \frac{\ln(1+\frac{2}{x})}{(1+\frac{2}{x})} d(1+\frac{2}{x}) = -\frac{1}{2}\int \ln(1+\frac{2}{x}) d\ln(1+\frac{2}{x})$

$= -\frac{1}{2}\ln^2(1+\frac{2}{x}) + C.$

20. $\int \frac{e^{\arctan x} + x\cdot\ln(1+x^2)}{1+x^2} dx = \int e^{\arctan x} d\arctan x + \frac{1}{2}\int \ln(1+x^2) d\ln(1+x^2)$

$= e^{\arctan x} + \frac{1}{4}\ln^2(1+x^2) + C.$