P.98.1 求 n 阶导数.

(1) $y = x^n$; $\quad y' = n x^{n-1}$, $\quad y'' = n(n-1) \cdot x^{n-2}$, $\quad \cdots$, $\quad y^{(n)} = n!$

(2) $y = e^x$; $\quad y' = e^x$, $\quad y'' = e^x$, $\quad \cdots \cdots$, $\quad y^{(n)} = e^x$.

(3) $y = \dfrac{1}{1+x}$; $\quad y' = (-1) \cdot \dfrac{1}{(1+x)^2}$, $\quad y'' = (-1) \cdot (-2) \cdot \dfrac{1}{(1+x)^3}$, $\quad \cdots \cdots y^{(n)} = (-1)^n \cdot \dfrac{n!}{(1+x)^{n+1}}$.

P.98.2 设 $y = e^x \cdot \cos x$, 证明: $y'' - 2y' + 2y = 0$

证: $y' = e^x \cdot \cos x - e^x \sin x$

$y'' = (e^x \cos x - e^x \sin x) - (e^x \sin x + e^x \cos x) = -2 e^x \sin x$

$y'' - 2y' + 2y = -2 e^x \sin x - 2(e^x \cdot \cos x - e^x \sin x) + 2 e^x \cos x = 0$

P.98.3 设 $y = \dfrac{x-3}{x+4}$, $(x \neq -4)$ 证明: $2 y'^2 = (y-1) \cdot y''$.

证: $y' = \dfrac{(x-3)' \cdot (x+4) - (x-3) \cdot (x+4)'}{(x+4)^2} = \dfrac{x+4-x+3}{(x+4)^2} = \dfrac{7}{(x+4)^2}$

$y'' = \dfrac{-14}{(x+4)^3}$.

由 $2 y'^2 = 2 \times \dfrac{49}{(x+4)^4} = \dfrac{2 \times 49}{(x+4)^4}$

$(y-1) \cdot y'' = \left(\dfrac{x-3}{x+4} - 1\right) \cdot \dfrac{-14}{(x+4)^3} = \dfrac{-14(x-3-x-4)}{(x+4)^4} = \dfrac{(-14)(-7)}{(x+4)^4} = \dfrac{2 \times 49}{(x+4)^4}$

∴ $2 y'^2 = (y-1) \cdot y''$.

P.98.4 设 $y = (1-x) \cdot (2x+1)^2 \cdot (3x-1)^3$, 求 $y^{(6)}$, $y^{(7)}$

解 $y = -108 x^6 + a_1 x^5 + a_2 x^4 + a_3 x^3 + a_4 x^2 + a_5 x + a_6$

$y^{(6)} = [-108 \cdot x^6]^{(6)} + [a_1 x^5 + a_2 x^4 + \cdots \cdots + a_5 x + a_6]^{(6)}$

$= -108 \times 6! + 0$

$y^{(7)} = 0$