1

The different-level quadratic minimization (DLQM) is a new and special optimization problem, which can be described as

$$\text{minimize} \qquad ax^2(t)/2 + cx(t) + p\dot{x}^2(t)/2 + q\dot{x}(t) \tag{1}$$

where time variable parameter $t \in [0, t_\mathrm{f}] \subseteq [0, \infty)$ and $a, c, p, q, x(t), \dot{x}(t) \in \mathbb{R}$. Besides, for simplicity here, $a > 0$, c, $p > 0$ and q are given constant. Note that $x(t)$ and $\dot{x}(t) = \mathrm{d}x(t)/\mathrm{d}t$ vary with time variable parameter t.

For example, $a = 1$, $c = 1$, $p = 1$, $q = -1$, initial value $x(0) = 1$, and initial value $\dot{x}(0) = 1/2$. Note that you can choose other suitable values for investigations.

2

In addition to the above constant-coefficient DLQM, now let us consider different-level time-varying quadratic minimization (DLTVQM), of which the coefficients vary with time variable parameter $t \in [0, t_\mathrm{f}] \subseteq [0, \infty)$.

$$\text{minimize} \qquad a(t)x^2(t)/2 + c(t)x(t) + p(t)\dot{x}^2(t)/2 + q(t)\dot{x}(t). \tag{2}$$

For example, $a(t) = \sin(t) + 2$, $c(t) = -\cos(t)/2$, $p(t) = \cos(2t) + 3$, $q(t) = \sin(3t)/3$, initial value $x(0) = 1$, and initial value $\dot{x}(0) = 1/2$. Note that you can choose other suitable analytical forms or numerical values of them for investigations.