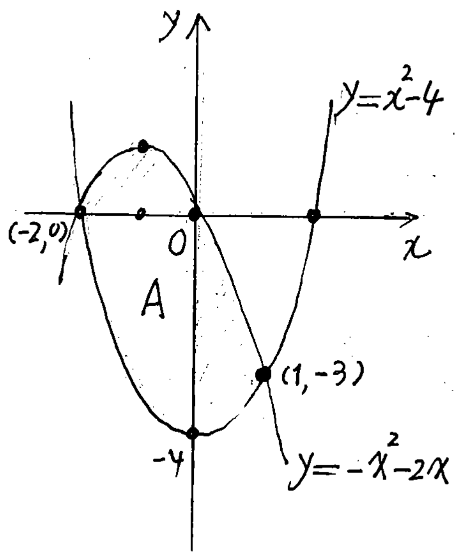
P.164.5. $y=x^2-4$ 与 $y=-x^2-2x=-(x+1)^2+1$

2011/7/16 —70.



求交点: $\begin{cases} y=x^2-4 \\ y=-x^2-2x \end{cases} \Longrightarrow x^2-4=-x^2-2x$

$x^2+x-2=0 \Rightarrow (x+2)\cdot(x-1)=0$

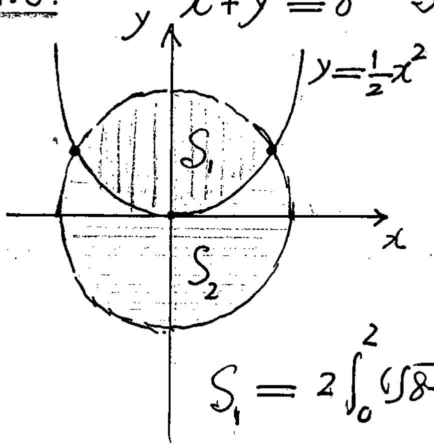
$\Longrightarrow \begin{cases} x=1 \\ y=3 \end{cases}, \quad \begin{cases} x=-2 \\ y=0 \end{cases}$

$$A=\int_{-2}^{1}\left[(-x^2-2x)-(x^2-4)\right]dx$$

$$=\int_{-2}^{1}(-2x^2-2x+4)\,dx=-\frac{2}{3}[x^3]_{-2}^{1}-[x^2]_{-2}^{1}+4[x]_{-2}^{1}$$

$$=-\frac{2}{3}(1+8)-(1-4)+4(1+2)=-6+3+12=9.$$

P.164.6. $x^2+y^2=8$ 与 $y=\frac{1}{2}x^2$.



求交点: $\begin{cases} x^2+y^2=8 \\ y=\frac{1}{2}x^2 \end{cases} \Longrightarrow y^2+2y-8=0 \Rightarrow (y+4)\cdot(y-2)=0$

$y=-4$(舍), $y=2$.

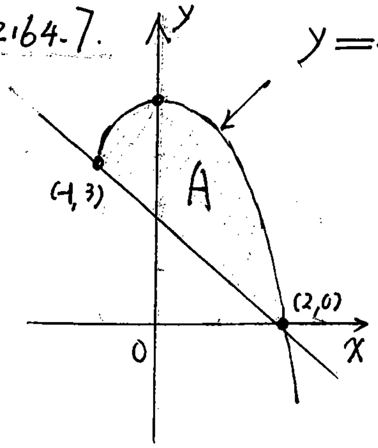
$\begin{cases} x=-2 \\ y=2 \end{cases} \quad \begin{cases} x=2 \\ y=2 \end{cases}$

$$S_1=2\int_0^2\left(\sqrt{8-x^2}-\frac{x^2}{2}\right)dx=2\int_0^2\sqrt{8-x^2}\,dx-\int_0^2 x^2\,dx=2(\pi+2)-\frac{8}{3}=2\pi+\frac{4}{3}.$$

$$S_2=\pi(\sqrt{8})^2-S_1=8\pi-\left(2\pi+\frac{4}{3}\right)=6\pi-\frac{4}{3}$$

注 $\int_0^2\sqrt{8-x^2}\,dx=\int_0^{\frac{\pi}{4}}\sqrt{8}\cdot\cos t\cdot\sqrt{8}\cdot\cos t\,dt=8\int_0^{\frac{\pi}{4}}\frac{1+\cos 2t}{2}dt=4\times\frac{\pi}{4}+2=\pi+2.$

P.164.7. $y=4-x^2$, 与 $y=-x+2$.



求交点: $\begin{cases} y=4-x^2 \\ y=-x+2 \end{cases}$ $\quad 4-x^2=-x+2$

$x^2-x-2=0, \quad (x-2)\cdot(x+1)=0$

$\begin{cases} x=-1 \\ y=3 \end{cases}, \quad \begin{cases} x=2 \\ y=0 \end{cases}$

$$A=\int_{-1}^{2}\left[(4-x^2)-(2-x)\right]dx=\int_{-1}^{2}(-x^2+x+2)\,dx$$

$$=-\left[\frac{x^3}{3}\right]_{-1}^{2}+\left[\frac{x^2}{2}\right]_{-1}^{2}+\left[2x\right]_{-1}^{2}$$

$$=-\left(\frac{8}{3}+\frac{1}{3}\right)+\frac{1}{2}(4-1)+2(2+1)=-3+\frac{3}{2}+6=\frac{9}{2}.$$