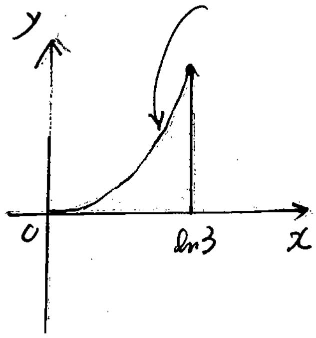
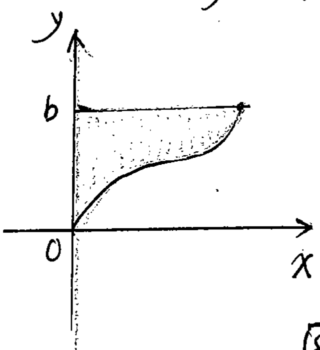
P.164.10 $y=e^x-1$，$x=\ln 3$，$y=0$，求 $V_x$. 2011 $\frac{17}{7}$ —72




$$V_x=\int_0^{\ln 3}\pi y^2 dx$$
$$=\int_0^{\ln 3}\pi(e^x-1)^2 dx=\int_0^{\ln 3}\pi(e^{2x}-2e^x+1)dx$$
$$=\pi\left(\frac{1}{2}e^{2x}\Big|_0^{\ln 3}-2e^x\Big|_0^{\ln 3}+x\Big|_0^{\ln 3}\right)$$
$$=\pi\left(\frac{9}{2}-\frac{1}{2}-2\times 3+2+\ln 3\right)=\pi\ln 3.$$

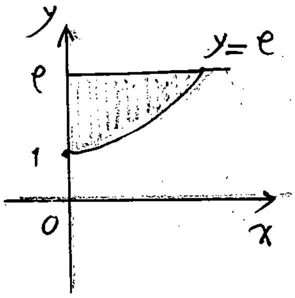
P.164.11

$ay^2=x^3$，$x=0$，$y=b$，$a>0$，求 $V_y$.



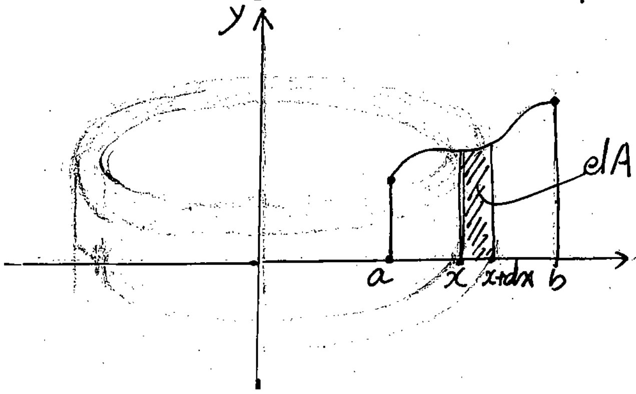

$$V_y=\int_0^b \pi x^2 dy=\pi\int_0^b(\sqrt[3]{a}\cdot\sqrt[3]{y^2})^2 dy$$
$$=\pi\cdot a^{\frac{2}{3}}\int_0^b y^{\frac{4}{3}}dy=\pi\cdot a^{\frac{2}{3}}\cdot\frac{3}{7}[y^{\frac{7}{3}}]_0^b=\frac{3\pi}{7}a^{\frac{2}{3}}b^{\frac{7}{3}}.$$

P.164.12 $x=\dfrac{\sqrt{8\ln y}}{y}$，$x=0$，$y=e$




$$V_y=\int_1^e \pi x^2 dy$$
$$=\int_1^e \pi\cdot\frac{8\ln y}{y^2}dy=-8\pi\int_1^e \ln y\, d\left(\frac{1}{y}\right)$$
$$=-8\pi\left[\frac{\ln y}{y}\Big|_1^e-\int_1^e\frac{1}{y}\cdot\frac{1}{y}dy\right]$$
$$=-8\pi\left(\frac{1}{e}-0+\frac{1}{y}\Big|_1^e\right)=-8\pi\left(\frac{2}{e}-1\right)=8\pi\left(1-\frac{2}{e}\right).$$

P.164.13 设 $y=f(x)$ 在 $[a,b]$，$(a>0)$ 上连续且不取负值。试用微分元法推导：由曲线 $y=f(x)$，直线 $x=a$, $x=b$ 及 $x$ 轴围成的平面图形绕 $y$ 轴旋转所成立体的体积为：$V=2\pi\int_a^b x f(x)dx$.




$$dA=f(x)dx$$
$$dV=2\pi x\,dA=2\pi x f(x)dx$$
$$V=\int_a^b dV=\int_a^b 2\pi x\cdot f(x)dx=2\pi\int_a^b x f(x)dx.$$