P.154.26 求定积分 $I=\int_0^{2n\pi}\frac{dx}{\sin^4x+\cos^4x}$ （n为正整数） Jun 16/7 — 68.

解：由于 $\frac{1}{\sin^4x+\cos^4x}$ 以π为周期

从而
$$I=\int_0^{2n\pi}\frac{dx}{\sin^4x+\cos^4x}=2n\int_0^{\pi}\frac{dx}{\sin^4x+\cos^4x}$$
$$=2n\left[\int_0^{\frac{\pi}{2}}\frac{dx}{\sin^4x+\cos^4x}+\int_{\frac{\pi}{2}}^{\pi}\frac{dx}{\sin^4x+\cos^4x}\right]$$
$$=4n\int_0^{\frac{\pi}{2}}\frac{dx}{\sin^4x+\cos^4x}$$
$$=4n\cdot\int_0^{\frac{\pi}{2}}\frac{dx}{(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x}$$
$$=4n\cdot\int_0^{\frac{\pi}{2}}\frac{dx}{1-\frac{1}{2}\sin^2 2x}$$
$$=8n\int_0^{\frac{\pi}{2}}\frac{1}{2-\sin^2 2x}\,dx$$
$$=8n\cdot\int_0^{\frac{\pi}{2}}\frac{dx}{2\cos^2 2x+\sin^2 2x}$$
$$=8n\left[\int_0^{\frac{\pi}{4}}+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{dx}{2\cos^2 2x+\sin^2 2x}\right]$$
$$=16n\int_0^{\frac{\pi}{4}}\frac{dx}{2\cos^2 2x+\sin^2 2x}$$
$$=8n\int_0^{\frac{\pi}{4}}\frac{d(2x)}{2\cos^2 2x+\sin^2 2x}=8n\int_0^{\frac{\pi}{4}}\frac{1}{2+\tan^2 2x}\,d\tan 2x$$
$$=8n\cdot\frac{1}{\sqrt{2}}\arctan\left[\frac{\tan 2x}{\sqrt{2}}\right]_0^{\frac{\pi}{4}}$$
$$=\frac{8n}{\sqrt{2}}\left(\frac{\pi}{2}-0\right)=2\sqrt{2}n\pi.$$

注①：
$$\int_{\frac{\pi}{2}}^{\pi}\frac{dx}{\sin^4x+\cos^4x}\xlongequal{t=\pi-x}\int_{\frac{\pi}{2}}^{0}\frac{-dt}{\sin^4t+\cos^4t}=\int_0^{\frac{\pi}{2}}\frac{dx}{\sin^4x+\cos^4x}.$$

注②
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{dx}{2\cos^2 2x+\sin^2 2x}\xlongequal{令t=\frac{\pi}{2}-x}\int_{\frac{\pi}{4}}^{0}\frac{-dt}{2\cos^2 2x+\sin^2 2x}=\int_0^{\frac{\pi}{4}}\frac{dx}{2\cos^2 2x+\sin^2 2x}$$