中 山 大 学 软 件 学 院 **2011** 级 软 件 工 程 专 业 (2011 学年秋季学期)

《SE-103+线性代数》 期末试题答案 (B)

1. Fill in the blank（5×4=20 Pts）

(1) $(2a_0+a_1)+(3a_1-a_2)t+(a_0-2a_1+4a_2)t^2$

(2) **3, Yes**

(3) $A=PDP^{-1}=\begin{bmatrix}2 & 1 & -1\\ 1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

(4) $a\neq 0$ **and** $a\neq 3$

(5) $[x_1\ \ x_2\ \ x_3]\begin{bmatrix}3 & -1/2 & 0\\ -1/2 & 5 & 4\\ 0 & 4 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$

2. Make each statement True or False, and descript your reasons.（5×4=20 Pts）

(1) F **(2) T** **(3) T** **(4) F** **(5) F**

3. Calculation (5×8=40 Pts）

(1) Solution **Since** $B=[b_1+b_2+b_3\quad b_1+2b_2+4b_3\quad b_1+3b_2+9b_3]$

$$=[b_1\ \ b_2\ \ b_3]\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix},$$

Then $\det B=\det\left([b_1\ \ b_2\ \ b_3]\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}\right)$

By the multiplicative property of determinants, $\det B = \det A \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = 1 \times 2 = 2$

(2) Solution $\det(A - \lambda I) = (\lambda - 2)(\lambda - 1)$ **. The eigenvalues are 2 and 1, and the corresponding eigenvectors are** $v_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ **and** $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **. Next, form** $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ **,**

$D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ **,and** $P^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

Since $A = PDP^{-1}$, $A^6 = PD^6P^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2^6 & 0 \\ 0 & 1^6 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 190 & -189 \\ 126 & -125 \end{bmatrix}$

(3) Solution A typical element of H **can be written as**

$$\begin{bmatrix} 3a + 7b - c \\ -5b + 8c - 2d \\ 3d - 4e \\ 5b - 8c - d + 4e \end{bmatrix} = a \underbrace{\begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{u}_1} + b \underbrace{\begin{bmatrix} 7 \\ -5 \\ 0 \\ 5 \end{bmatrix}}_{\mathbf{u}_2} + c \underbrace{\begin{bmatrix} -1 \\ 8 \\ 0 \\ -8 \end{bmatrix}}_{\mathbf{u}_3} + d \underbrace{\begin{bmatrix} 0 \\ -2 \\ 3 \\ -1 \end{bmatrix}}_{\mathbf{u}_4} + e \underbrace{\begin{bmatrix} 0 \\ 0 \\ -4 \\ 4 \end{bmatrix}}_{\mathbf{u}_5}$$

a. H **is a vector space because it is the set of all linear combinations of a set of vectors.**

b. Row reduce:

$$\begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 5 & -8 & -1 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & -3 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivot columns are 1,2, and 4. So { $\mathbf{u}_1$ **,** $\mathbf{u}_2$ **,** $\mathbf{u}_4$ **} is a basis for** H **.**

(4) Solution (a) Because the columns a_1 **and** a_2 **of** A **are orthogonal, the orthogonal projection of** b **onto** $\operatorname{Col} A$ **is given by**

$$\hat{b} = \frac{b \cdot a_1}{a_1 \cdot a_1} a_1 + \frac{b \cdot a_2}{a_2 \cdot a_2} a_2 = \frac{2}{7} a_1 + \frac{1}{7} a_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(b) $\hat{x} = \begin{bmatrix} 2/7 \\ 1/7 \end{bmatrix}$ **, since we already know what weights to place on the columns of** A **to produce** $\hat{b}$**.**

(c) $\| b - \hat{b} \| = \sqrt{(4-1)^2 + (-2-1)^2 + (-3)^2} = 3\sqrt{3}$

(5) Solution **Let** $v_1 = x_1$ **and** $v_2 = x_2 - \dfrac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 = x_2 - \dfrac{1}{2} v_1 = \begin{bmatrix} 3 \\ 3/2 \\ 3/2 \end{bmatrix}$**. Thus** $\{v_1, v_2\}$ **is an orthogonal basis for** W**.**

Since $\| v_1 \| = \sqrt{30}$ **and** $\| v_2 \| = \sqrt{27/2} = 3\sqrt{6}/2$ **, an orthonormal basis for** W **is**

$$\left\{ \frac{v_1}{\| v_1 \|}, \frac{v_2}{\| v_2 \|} \right\} = \left\{ \begin{bmatrix} 2/\sqrt{30} \\ -5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \right\}.$$

4. Prove issues（2×6=12 Pts）

(1) Solution **If** u **and** v **in** R^n**, then**

$$T(u+v) = [(u+v)^T w_1]w_1 + [(u+v)^T w_2]w_2 + \cdots + [(u+v)^T w_p]w_p$$

$$= [(u^T + v^T)w_1]w_1 + [(u^T + v^T)w_2]w_2 + \cdots + [(u^T + v^T)w_p]w_p$$

$$= (u^T w_1)w_1 + (u^T w_2)w_2 + \cdots + (u^T w_p)w_p$$

$$+ (v^T w_1)w_1 + (v^T w_2)w_2 + \cdots + (v^T w_p)w_p$$

$$= T(u) + T(v)$$

It can be shown similarly that $T(cu) = cT(u)$ **for each scalar** c**, so** T **is a linear transformation.**

(2) Solution **Since** A **is similar to** B**, there is an invertible matrix** P **such that** $A = PBP^{-1}$**, then** $A^3 - 3A + I = (PBP^{-1})^3 - 3(PBP^{-1}) + I$

$$= PB^3P^{-1} - 3(PBP^{-1}) + I$$

$$= P(B^3 - 3B + I)P^{-1}$$

$$= 0$$

Thus $B^3 - 3B + I = 0$ **because** P **is an invertible matrix.**

5. Synthesis (8 points)

Solution **Since** $x^T x = 1$**, thus** $Ax = (I - xx^T)x = x - xx^T x = x - 1x = 0$ **.**

Next, we know $x \neq 0$ **because** $x^T x = 1$**. Hence the homogeneous equation** $Ax = 0$ **has a nontrivial solution. This shows that the equation has at least one free variable, and** $rank(A) < n$