

P.144.8 $\int \frac{x^3+x^2+2}{(x^2+2)^2} dx = \int \frac{x^3}{(x^2+2)^2} dx + \int \frac{x^2+2}{(x^2+2)^2} dx$ Lesson 13 / -54.

$$= \frac{1}{4} \int \frac{4x^3+8x-8x}{x^4+4x^2+4} dx + \frac{1}{\sqrt{2}} \int \frac{1}{1+(\frac{x}{\sqrt{2}})^2} d(\frac{x}{\sqrt{2}})$$

$$= \frac{1}{4} \int \frac{1}{x^4+4x^2+4} d(x^4+4x^2+4) - \int \frac{2x}{(x^2+2)^2} dx + \frac{1}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}}$$

$$= \frac{1}{4} \ln(x^2+2)^2 + \frac{1}{x^2+2} + \frac{1}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} + C.$$

P.144.9 $\int \frac{e^x dx}{e^{2x}+3e^x+2} = \int \frac{1}{(e^x+2)(e^x+1)} de^x = \int \frac{1}{e^x+1} d(e^x+1) - \int \frac{1}{e^x+2} d(e^x+2)$

$$= \ln(1+e^x) - \ln(2+e^x) + C.$$

P.144.10 $\int \frac{\cos x dx}{\sin^2 x + \sin x - 6} = \int \frac{d\sin x}{(\sin x+3)(\sin x-2)} = \frac{1}{5} \int (\frac{1}{\sin x-2} - \frac{1}{\sin x+3}) d\sin x$

$$= \frac{1}{5} \ln|\sin x-2| - \frac{1}{5} \ln|\sin x+3| + C.$$

P.144.11 $\int \frac{x^3}{x^4+x^2+2} dx = \frac{1}{4} \int \frac{4x^3+2x-2x}{x^4+x^2+2} dx = \frac{1}{4} \int \frac{d(x^4+x^2+2)}{x^4+x^2+2} - \frac{1}{2} \int \frac{x dx}{x^4+x^2+2}$

$$= \frac{1}{4} \ln(x^4+x^2+2) - \frac{1}{4} \int \frac{d(x^2+\frac{1}{2})}{(x^2+\frac{1}{2})^2+\frac{7}{4}} = \frac{1}{4} \ln(x^4+x^2+2) - \frac{1}{2\sqrt{7}} \arctan \frac{2x^2+1}{\sqrt{7}} + C.$$

P.144.12 $\int \frac{dx}{(x+2)(x^2-2x+2)}$ $\frac{1}{(x+2)(x^2-2x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+2} = \frac{(A+B)x^2+(-2A+2B+C)x+2A+2C}{(x+2)(x^2-2x+2)}$

$$= \frac{1}{10} \int (\frac{1}{x+2} + \frac{-x+4}{x^2-2x+2}) dx$$

$$= \frac{1}{10} \int \frac{1}{x+2} d(x+2) - \frac{1}{20} \int \frac{2x-8}{x^2-2x+2} dx$$

$$= \frac{1}{10} \ln|x+2| - \frac{1}{20} \int \frac{1}{x^2-2x+2} d(x^2-2x+2) + \frac{6}{20} \int \frac{dx}{x^2-2x+2}$$

$$= \frac{1}{10} \ln|x+2| - \frac{1}{20} \ln|x^2-2x+2| + \frac{3}{10} \int \frac{d(x-1)}{(x-1)^2+1}$$

$$= \frac{1}{10} \ln|x+2| - \frac{1}{20} \ln|x^2-2x+2| + \frac{3}{10} \arctan(x-1) + C.$$

$$\begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 2A+2C=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{10} \\ B=\frac{1}{10} \\ C=\frac{4}{10} \end{cases}$$