警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

计算机科学系 /（电子系 2+2） 2013 第 2 学期

《 高等代数/线性代数 》期中考试试题

年级： 班别： 专业： 姓名：________ 学号：__________成绩

温馨提示：(1) 计算可以慢点，细心点，每部分最后一题有点难度;(2) 卷中向量均有→在符号的顶部。
(3) 2+2 班同学需要用全英作答，普通班同学可以中英互用

PART 1: Elementary Questions 基础题 (无难度，只考会还是不会，这里理应基本全对; 1 小时)

1. (10%) For a matrix equation $A\vec{x}=\vec{b}$, where $\vec{x}$ is the solution vector, please tell (1) when there is no solution, (2) when there is a unique solution, and (3) when there are infinitely many solutions.

2. (5%) If A is a square matrix and A is invertible, please show me at least three different ways to solve the matrix equation $A\vec{x}=\vec{b}$ using linear algebra.

3. (20%) Compute the reduced echelon form of the following matrix

$$\begin{pmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}$$

Please point out the pivot positions. Is the above matrix invertible? If it is invertible, please compute its inverse.

4. (10%) Compute the determinant of the following matrices:

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} x & +1 & 0 & 0 \\ 0 & x & -1 & 0 \\ 0 & 0 & x & +1 \\ a_4 & a_3 & a_2 & x+a_1 \end{pmatrix}$$

5. (10%) Are the following vectors linearly independent? Why?

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -1 \\ 4 \\ -8 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} -1 \\ 3 \\ 9 \\ 2 \end{pmatrix}, \vec{a}_3 = \begin{pmatrix} 1 \\ 4 \\ 16 \\ 0 \end{pmatrix}, \vec{a}_4 = \begin{pmatrix} 3 \\ -3 \\ 3 \\ -3 \end{pmatrix}$$

6. (20%) Given the following matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & -1 & 0 & -1 \\ 3 & -5 & 1 & 0 & 0 & 2 \\ -1 & 5 & -2 & 1 & 1 & 0 \\ 3 & -7 & 0 & 1 & -1 & 0 \end{pmatrix}$$

Please

(a) Describe its column space and a basis for it.

(b) Compute the rank of matrix *A*

(c) Find the null space of matrix *A*.

7. (10%) Please perform LU factorisation for the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 5 & 3 \end{bmatrix}$$

PART 2：Proof Questions 证明题 (能做多少，做多少。目的：90 分以上 or not; 30 分钟)

8. (10%) Let $\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \vec{a}_3 = \begin{pmatrix} 1 \\ 2 \\ a-3 \\ 1 \end{pmatrix}, \vec{a}_4 = \begin{pmatrix} 1 \\ 2 \\ -2 \\ a \end{pmatrix}, \vec{\beta} = \begin{pmatrix} 0 \\ 1 \\ b \\ -1 \end{pmatrix}$

Then, what are the values of *a* and *b* if $\vec{\beta}$ cannot be linearly represented by $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$? And what are the values of *a* and *b* if $\vec{\beta}$ can be linearly represented by $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$.

9. (5%) Let $\vec{\eta}$ be a solution of matrix equation $A\vec{x} = \vec{b}$, where *A* is a matrix. Let $\{\vec{\xi}_1, \vec{\xi}_2, \cdots, \vec{\xi}_r\}$ be a basis of the null space of matrix *A*, and $\vec{b}$ is not a zero vector. Please prove: $\vec{\eta}, \vec{\eta} - \vec{\xi}_1, \vec{\eta} - \vec{\xi}_2, \cdots, \vec{\eta} - \vec{\xi}_r$ are linearly independent