

中山大学 本科生考试草稿纸 ¹³/₇₋₅₅



《中山大学授予学士学位工作细则》第七条：“考试作弊者不授予学士学位。”

P.144.13 $\int \frac{1}{2+\sin x} dx$, $\frac{1}{2} \tan \frac{x}{2} = u$, $\sin x = 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}$
 $= \int \frac{1}{2 + \frac{2u}{1+u^2}} \cdot \frac{2du}{1+u^2}$
 $= \int \frac{du}{u^2 + u + 1} = \int \frac{d(u + \frac{1}{2})}{(u + \frac{1}{2})^2 + \frac{3}{4}}$
 $= \frac{2}{\sqrt{3}} \arctan \frac{2u+1}{\sqrt{3}} + C$
 $= \frac{2}{\sqrt{3}} \arctan \frac{2 \tan \frac{x}{2} + 1}{\sqrt{3}} + C$

$\sin x = 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}$
 $= 2 \tan \frac{x}{2} \cdot \frac{1}{\sin^2 \frac{x}{2}}$
 $= \frac{2 \tan u}{1 + \tan^2 u} = \frac{2u}{1+u^2}$
 $x = 2 \arctan u, dx = \frac{2du}{1+u^2}$

P.144.14 $\int \frac{dx}{1 + \sin x + \cos x}$, $\frac{1}{2} \tan \frac{x}{2} = u$, $dx = \frac{2du}{1+u^2}$
 $= \int \frac{1}{1 + \frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}} \cdot \frac{2du}{1+u^2}$
 $= \int \frac{du}{1+u} = \ln|1+u| + C = \ln|1 + \tan \frac{x}{2}| + C$

$\sin x = \frac{2u}{1+u^2}$, $\cos x = \frac{1-u^2}{1+u^2}$

P.144.15 $\int \cot^4 x dx = \int (\csc^2 x - 1)^2 dx = \int (\csc^4 x - 2 \csc^2 x + 1) dx$
 $= \int \csc^4 x dx - 2 \int \csc^2 x dx + \int dx$
 $= -\int (1 + \cot^2 x) d \cot x + 2 \cot x + x + C$
 $= -\cot x - \frac{\cot^3 x}{3} + 2 \cot x + x + C$
 $= \cot x - \frac{\cot^3 x}{3} + x + C$

P.144.16 $\int \sec^4 x dx = \int \sec^2 x d \tan x$
 $= \int (1 + \tan^2 x) d \tan x$
 $= \tan x + \frac{\tan^3 x}{3} + C$