警 示

中山大学软件学院 2012 级软件工程专业（2012-11）

《线性代数》期中考试题

（考试形式：闭卷　　考试时间：2 小时）

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

教学班：________________　姓名：_______________　学号：______________　成绩：___________

1. Fill in the blank（5×4=20 Pts）

(1)Let $u=\begin{bmatrix}1\\0\\-2\\1\\0\end{bmatrix}$, $v=\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix}$, $w=\begin{bmatrix}-3\\0\\2\\0\\2\end{bmatrix}$, is the set $\{u, v, w\}$ linearly dependent? __________.

(2) Suppose $T:R^2\to R^2$ first performs a horizontal shear that transforms e_2 into e_2-2e_1 (leaving e_1 unchanged) and then reflects points through the line x_1-axis. So the standard matrix of T is ____________________________.

(3) If $A=\begin{bmatrix}-2&0&0&-3\\0&1&0&-2\\3&-1&2&1\\-1&0&3&0\end{bmatrix}$, then $\det(-2A)=$___________________.

(4) The matrices A and B below are row equivalent,

$$A=\begin{bmatrix}2&-6&14&4&18\\-1&6&-19&4&-6\\-2&7&-18&1&-11\\3&-8&17&3&18\end{bmatrix},\quad B=\begin{bmatrix}2&0&-10&0&-8\\0&1&-4&0&-3\\0&0&0&1&2\\0&0&0&0&0\end{bmatrix}$$

then dim Nul A =_____________, and a basis for the Col A is ___________________.

（5）Suppose matrix $A=\begin{bmatrix} -7 & -5 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & -3 & -7 \end{bmatrix}$, find A^{-1} =________________.

2. Make each statement True or False, and descript your reasons.（6×4=24 Pts）

(1) A set of five vectors in R^4 is linear independent.

(2) If A is a 4×4 matrix, then $\det(-A)=\det A$.

(3) Suppose a 3×5 matrix A has three pivot columns, is $Nul\ A = R^2$?

(4) If A and B are row equivalent m×n matrices and if the columns of A span R^m, then so do the columns of B.

(5) The equation $Ax=0$ has the trivial solution if and only if there are no free variables.

(6) Any system of n linear equation in n variables can be solved by Cramer’s rule.

3. Calculation（5×8=40 Pts）

(1) Find the general solution of the following system of equation. Write your answer in parametric vector form.

$$\begin{aligned} 2x_2 + 6x_3 - 8x_4 &= 4 \\ x_1 - 5x_2 - 9x_3 + 8x_4 &= -7 \\ x_2 + 3x_3 - 4x_4 &= 2 \end{aligned}$$

(2) If A and B are 3×3 matrices, I is the identity matrix, and $A^T B = A^2 + B - I$, where $A=\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. Find B.

(3) Let $T: R^3 \to R^3$ be a linear transformation such that

$$T(e_1)=\begin{bmatrix}1\\3\\-5\end{bmatrix},\ T(e_2)=\begin{bmatrix}0\\1\\-1\end{bmatrix},\ T(e_3)=\begin{bmatrix}-2\\-2\\9\end{bmatrix}$$, where e_1, e_2, e_3 are the columns of I_3

a. Determine if *T* maps R^3 onto R^3. Explain.

b. Write the 4×4 matrix that represents *T* when homogeneous coordinates are used for vectors in R^3.

(4) Find an LU factorization of $A=\begin{bmatrix}1&-2&-4&-3\\2&-7&-7&-6\\-1&2&6&4\\-4&-1&9&8\end{bmatrix}$

(5) Determine the value(s) of a such that $\{\begin{bmatrix}1\\2a\end{bmatrix},\begin{bmatrix}1-a\\3a\end{bmatrix}\}$ is linearly independent.

4. Prove issues（2×8=16 Pts）

(1) Let $T:R^3\to R^3$ be a linear transformation and its standard matrix is invertible, let $\{v_1,v_2\}$ be a linear independent set in R^n. Show that the set $\{T(v_1),T(v_1+v_2)\}$ is also linear independent.

（2）Suppose A is an $n\times n$ matrix and $rank\ A=n$. Explain why A^TA is invertible, then show that $A^{-1}=(A^TA)^{-1}A^T$.