习题5-2

P.231.1 写出点 (x, y, z) 分别到 x 轴、y 轴、z 轴、xOy 面、yOz 面及原点的距离。
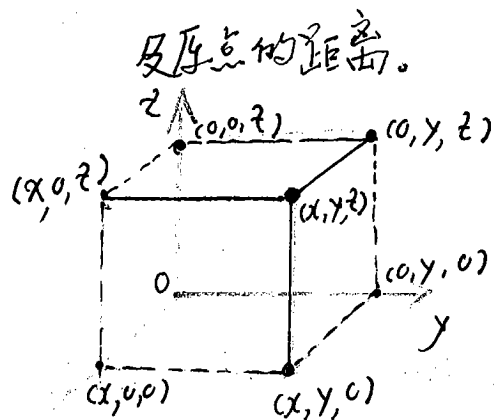
$$d_x = \sqrt{(x-x)^2+(y-0)^2+(z-0)^2} = \sqrt{y^2+z^2}$$

$$d_y = \sqrt{(x-0)^2+(y-y)^2+(z-0)^2} = \sqrt{x^2+z^2}$$

$$d_z = \sqrt{(x-0)^2+(y-0)^2+(z-z)^2} = \sqrt{x^2+y^2}.$$

$$d_{Oxy} = |z|, \quad d_{Oyz} = |x|, \quad d_{(0,0,0)} = \sqrt{x^2+y^2+z^2}.$$

P.229.2. 已知三点. $A(-1,2,1)$、$B(3,0,1)$、$C(2,1,2)$

求 $\vec{AB}, \vec{BA}, \vec{AC}, \vec{BC}$ 的坐标与模。

解: $\vec{AB} = (4, -2, 0)$, $\vec{AC} = (3, -1, 1)$, $\vec{BA} = (-4, 2, 0)$, $\vec{BC} = (-1, 1, 1)$

$$|\vec{AB}| = |\vec{BA}| = \sqrt{4^2+(-2)^2+0} = \sqrt{20} = 2\sqrt{5}.$$

$$|\vec{AC}| = \sqrt{3^2+(-1)^2+1^2} = \sqrt{11}, \quad |\vec{BC}| = \sqrt{(-1)^2+1^2+1} = \sqrt{3}.$$

P.229.3 设 $\vec{a} = (3,-2,2)$, $\vec{b} = (1,3,2)$, $\vec{c} = (8,6,-2)$

求 $3\vec{a} - 2\vec{b} + \frac{1}{2}\vec{c} = (9,-6,6) + (-2,-6,-4) + (4,3,-1) = (11,-9,1)$.

P.229.4. 设 $\vec{a} = (2,5,1)$, $\vec{b} = (1,-2,7)$ 分别求 $\vec{a}$ 与 $\vec{b}$ 同方向的单位向量，并求常数 k, 使向量 $k\vec{a}+\vec{b}$ 与 xOy 平面平行。

解: $\vec{a}^0 = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{(2,5,1)}{\sqrt{2^2+5^2+1}} = \dfrac{1}{\sqrt{30}}(2,5,1)$

$\vec{b}^0 = \dfrac{\vec{b}}{|\vec{b}|} = \dfrac{(1,-2,7)}{\sqrt{1^2+(-2)^2+7^2}} = \dfrac{1}{3\sqrt{6}}(1,-2,7)$

$k\vec{a} + \vec{b} = (2k, 5k, k) + (1,-2,7) = (2k+1, 5k-2, k+7)$

要使 $k\vec{a}+\vec{b} \parallel Oxy$ 面, 必 $k+7=0$, $k=-7$.