习题3—3

P.143.1 $\int \frac{x-1}{x^2+6x+8}dx = \int \frac{(x+2)-3}{(x+2)(x+4)}dx = \int \frac{dx}{x+4} - 3\int \frac{dx}{(x+2)(x+4)}$

$= \int \frac{dx}{x+4} - \frac{3}{2}\int (\frac{1}{x+2} - \frac{1}{x+4})dx$

$= \frac{5}{2}\int \frac{dx}{x+4} - \frac{3}{2}\int \frac{dx}{x+2} = \frac{5}{2}\ln|x+4| - \frac{3}{2}\ln|x+2| + C.$

2. $\int \frac{3x^4+x^2+1}{x^2+x-6}dx$

$= x^3 - \frac{3}{2}x^2 + 22x - 20\int \frac{2x+1}{x^2+x-6}dx + \frac{153}{5}\int (\frac{1}{x-2} - \frac{1}{x+3})dx$

$= x^3 - \frac{3}{2}x^2 + 22x - 20\ln|x^2+x-6| + \frac{153}{5}\ln|x-2| - \frac{153}{5}\ln|x+3| + C$

$= x^3 - \frac{3}{2}x^2 + 22x - 20\ln|x-2| - 20\ln|x+3| + \frac{153}{5}\ln|x-2| - \frac{153}{5}\ln|x+3| + C$

$= x^3 - \frac{3}{2}x^2 + 22x + \frac{57}{5}\ln|x-2| - \frac{153}{5}\ln|x+3| + C.$

$3x^2 - 3x + 22$

$x^2+x-6 \overline{)\,3x^4+x^2+1}$

$3x^4+3x^3-18x^2$

$-3x^3+19x^2+1$

$-3x^3-3x^2+18x$

$22x^2-18x+1$

$22x^2+22x-132$

$-40x+133$

$\frac{3x^4+x^2+1}{x^2+x-6} = 3x^2-3x+22 + \frac{-40x+133}{x^2+x-6}$

$= 3x^2-3x+22 + \frac{-20(2x+1)+153}{x^2+x-6}$

$= 3x^2-3x+22 - \frac{20(2x+1)}{x^2+x-6} + \frac{153}{5}(\frac{1}{x-2} - \frac{1}{x+3})$

3. $\int \frac{2x^2-5}{x^4-5x^2+6}dx = \int \frac{2x^2-4-1}{(x^2-2)(x^2-3)}dx$

$= \int \frac{2(x^2-2)-1}{(x^2-2)(x^2-3)}dx = \int \frac{2}{x^2-3}dx - \int \frac{dx}{(x^2-2)(x^2-3)}$

$= \int \frac{2}{x^2-3}dx - [\int \frac{1}{x^2-3}dx - \int \frac{1}{x^2-2}dx] = \int \frac{dx}{x^2-3} + \int \frac{dx}{x^2-2}$

$= \frac{1}{2\sqrt{3}}\int (\frac{1}{x-\sqrt{3}} - \frac{1}{x+\sqrt{3}})dx + \frac{1}{2\sqrt{2}}\int (\frac{1}{x-\sqrt{2}} - \frac{1}{x+\sqrt{2}})dx$

$= \frac{1}{2\sqrt{3}}\ln|\frac{x-\sqrt{3}}{x+\sqrt{3}}| + \frac{1}{2\sqrt{3}}\ln|\frac{x-\sqrt{2}}{x+\sqrt{2}}| + C.$

4. $\int \frac{dx}{(x-1)^2(x-2)}$

$\frac{1}{(x-1)^2\cdot(x-2)} = \frac{A}{x-2} + \frac{Bx+C}{x^2-2x+1} = \frac{(A+B)x^2+(-2A-2B+C)x+(A-2C)}{(x-2)\cdot(x-1)^2}$

$\begin{cases} A+B=0 \\ -2A-2B+C=0 \\ A-2C=1 \end{cases} \Rightarrow \begin{cases} C=0 \\ A=1 \\ B=-1 \end{cases}$

$= \int \frac{dx}{x-2} + \int \frac{-x}{x^2-x+1}dx$

$= \ln|x-2| - \frac{1}{2}\int \frac{1}{x^2-x+1}d(x^2-x+1) - \int \frac{dx}{x^2-2x+1}$

$= \ln|x-2| - \frac{1}{2}\ln(x-1)^2 + \frac{1}{x-1} + C.$