中山大学软件学院 **2011** 级软件工程专业(2011-11)

《线性代数》期中试题答案

1. Fill in the blank（5×4=20 Pts）

(1) $\det(2A)$ =320

(2) a，b

(3) $X = B^{-1}$，$Y = -B^{-1}AC$

(4) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(5) 3, $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 13 \\ 10 \end{bmatrix} \right\}$

2. Mark each statement True or False, and descript your reasons. （6×4=24 Pts）

(1) F (2) F (3) T (4) F (5) T (6) F

3. Calculation issues （5×8=40 Pts）

(1) Solution

$$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & 1 & 5 \\ -1 & -2 & -2 & 1 & -4 \end{bmatrix} \sim \ldots\ldots \sim \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \text{ (with } x_2, x_4 \text{ free)}$$

(2) Solution

$$AB + I = A^2 + B \Rightarrow AB - B = A^2 - I \Rightarrow (A - I)\ B = A^2 - I \Rightarrow B = (A - I)^{-1}(A^2 - I)$$

thus $B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

(3) Solution

$A_1 = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$,

$$A=\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix},\ A^{-1}=\begin{bmatrix} A_1^{-1} & 0 \\ 0 & A_2^{-1} \end{bmatrix}=\begin{bmatrix} 3 & -5 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 4/3 \\ 0 & 0 & 0 & 1/2 & -2/3 \\ 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

(4) Solution

Take $U=\begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & 3 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, further row reduction of *U* is not needed. For *L*, copy the pivot columns that were found during the row reduction to *U*: $\begin{bmatrix} 2 \\ -1 \\ -2 \\ 3 \end{bmatrix},\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix},\begin{bmatrix} 3 \\ -5 \end{bmatrix}$. Divide each column by the pivot at the top and place into the lower triangular part of *L*, then fill in the remaining column of *L* with the last column of the 4×4 identity matrix:

$$L=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ -1 & 1/3 & 1 & 0 \\ 3/2 & 1/3 & -5/3 & 1 \end{bmatrix}$$

(5) Solution

a. Standard matrix of *T* is $A=\left[T(e_1)\quad T(e_2)\quad T(e_3)\right]=\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -7 \\ 4 & 5 & 5 \end{bmatrix}\sim\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -7 \\ 0 & 0 & -2 \end{bmatrix}$. Since A has three pivot positions, the IMT implies that A is invertible and the mapping is one-to-one.

b. $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$

4. Prove issues （2×8=16 Pts）

(1) Proof

Since $\{v_1,v_2,v_3\}$ is a linearly independent set, $v_1\neq 0$. Also, Theorem 7 shows that v_2 cannot be a multiple of v_1, and v_3 cannot be a linear combination of v_1 and v_2. By hypothesis, v_4 is not a linear combination of v_1, v_2 and v_3. Thus, by Theorem 7, $\{v_1,v_2,v_3,v_4\}$ cannot be a linearly dependent set and so must be linearly independent.

(2) Proof

Let A be a 5×3 matrix and B a 3×5 matrix. Since B has more columns than rows, its five columns are linearly dependent and there is a nonzero x such that $Bx = 0$. Thus $ABx = A0 = 0$. This shows that the matrix AB is not invertible, by the IMT.