中山大學 本科生考试草稿纸

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警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.153.6. $\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2dx}{\sqrt{1-x^2}} = \int_0^{\frac{\pi}{3}} \frac{\sin^2 t}{\cos t} \cdot \cos t\, dt = \int_0^{\frac{\pi}{3}} \frac{1-\cos 2x}{2} dx = \frac{1}{2}\left[x\Big|_0^{\frac{\pi}{3}} - \frac{\sin 2x}{2}\Big|_0^{\frac{\pi}{3}}\right]$

$$= \frac{1}{2}\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) = \frac{1}{2}\left(\frac{\pi}{3} - \frac{1}{2}\cdot\sin\frac{\pi}{3}\right) = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{8}.$$

P.153.7. $\int_0^1 \sqrt{4-x^2}\, dx$ ，令 $x = 2\sin t$，则 $dx = 2\cos t\, dt$

$x=0, t=0$；$x=1, t=\frac{\pi}{6}$.

$$= \int_0^{\frac{\pi}{6}} 2\cdot\cos t \cdot 2\cos t\, dt = 4\int_0^{\frac{\pi}{6}} \frac{1+\cos 2t}{2} dt$$

$$= 2\left[t\Big|_0^{\frac{\pi}{6}} + \frac{1}{2}\sin 2t\Big|_0^{\frac{\pi}{6}}\right] = 2\left(\frac{\pi}{6} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}.$$

P.153.8. $\int_0^3 x\cdot\sqrt[3]{1-x^2}\, dx = -\frac{1}{2}\int_0^3 \sqrt[3]{1-x^2}\, d(1-x^2) = -\frac{1}{2}\cdot\frac{3}{4}(1-x^2)^{\frac{4}{3}}\Big|_0^3 = -\frac{3}{8}\left[8^{\frac{4}{3}}-1\right] = \frac{-3}{8}(16-1)$

$$= -\frac{45}{8}.$$

P.153.9. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x}\, dx = 2\int_0^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x}\, dx = 2\cdot\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\cdot\sin x\, dx$

$$= -2\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, d\cos x = -2\cdot\frac{2}{3}\left[\cos^{\frac{3}{2}} x\right]_0^{\frac{\pi}{2}} = -\frac{4}{3}(0-1) = \frac{4}{3}.$$

P.153.10. $\int_0^{\frac{\pi}{2}} \cos^n 2x\, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^n 2x\, d(2x) = \frac{1}{2}\int_0^{\pi} \cos^n u\, du$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \cos^n x\, dx + \int_{\frac{\pi}{2}}^{\pi} \cos^n x\, dx\right]$$

$$= \frac{1}{2}\left[1+(-1)^n\right]\cdot\int_0^{\frac{\pi}{2}} \cos^n x\, dx$$

$$= \begin{cases} 0, & n\text{为奇数。} \\ \frac{(n-1)!!}{n!!}\cdot\frac{\pi}{2}, & n\text{为偶数。} \end{cases}$$

注：$\int_{\frac{\pi}{2}}^{\pi} \cos^n u\, du \overset{\text{令}x=\pi-u}{=\!=} \int_{\frac{\pi}{2}}^{0} (-\cos x)^n(-dx)$

$$= (-1)^n\cdot\int_0^{\frac{\pi}{2}} \cos^n x\, dx$$

P.153.11 $\int_0^a (a^2-x^2)^{\frac{n}{2}} dx$ ，令 $x = a\sin t$

$$= \int_0^{\frac{\pi}{2}} a^n\cos^n t\cdot a\cdot\cos t\, dt = a^{n+1}\cdot\int_0^{\frac{\pi}{2}} \cos^{n+1} t\, dt = a^{n+1} I_{n+1}$$

$= a^{n+1}\cdot\frac{n!!}{\ldots}\begin{cases}1 \\ \frac{\pi}{\ldots}\end{cases} = \begin{cases} a^{n+1}\cdot\frac{n!!}{(n+1)!!}\cdot 1, & n\text{为偶.} \\ \ldots & \text{奇} \ldots \end{cases}$