

5. 计算函数  $z = x^2y + y^2$  的全微分

A-2

$$dz = 2xydx + (x^2 + 2y)dy$$

6. 求  $\int \frac{\sin x \cos x}{1 + \sin^4 x} dx$

$$\int \frac{\sin x \cos x}{1 + \sin^4 x} dx = \int \frac{\sin x}{1 + \sin^4 x} d \sin x = \frac{1}{2} \int \frac{1}{1 + \sin^4 x} d \sin^2 x = \frac{1}{2} \arctan(\sin^2 x) + C$$

7. 求  $I = \int_0^2 xe^x dx$

$$I = \int_0^2 xe^x dx = \int_0^2 x de^x = xe^x \Big|_0^2 - \int_0^2 e^x dx = 2e^2 - e^x \Big|_0^2 = e^2 + 1$$

8. 求曲线  $y = x^2$  与  $y = 2 - x^2$  所围成的图形的面积.

$y = x^2$  与  $y = 2 - x^2$  交点为  $(-1, 1), (1, 1)$

$$\therefore A = \int_{-1}^1 (2 - x^2 - x^2) dx = 4 \int_0^1 (1 - x^2) dx = \frac{8}{3}$$

9. 求  $f(x) = x^3 - 6x^2 + 9x + 3$  的极值,

$$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0$$

$$\therefore x=1, x=3$$

$$f''(x) = 6x - 12,$$

$$f''(1) = -6 < 0, \therefore \text{极大值为 } f(1) = 7$$

$$f''(3) = 6 > 0, \therefore \text{极小值为 } f(3) = 3$$

10 求曲线  $L: \begin{cases} x=t, \\ y=t^2 \\ z=t^3 \end{cases}$  在  $P_0(1,1,1)$  处的切线方程和法平面方程。

$$L: \text{在 } P_0 \text{ 点的切向量为: } \vec{\tau} = (x'_t, y'_t, z'_t) \Big|_{t=1} = (1, 2t, 3t^2) \Big|_{t=1} = (1, 2, 3)$$

$$\therefore \text{切线方程为: } \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$$

$$\therefore \text{法平面方程为: } (x-1) + 2(y-1) + 3(z-1) = 0$$

二. 计算题 (每小题 6 分, 共 18 分)

1. 求函数  $y = \frac{1-x}{1+x}$  在点  $x_0 = 0$  处的  $n$  阶泰勒公式.

$$y = \frac{1-x}{1+x} = -1 + \frac{2}{1+x}$$