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P.144.29 $\displaystyle\int \frac{x\,dx}{\sqrt{x^2-x+3}} = \frac{1}{2}\int \frac{2x-1+1}{\sqrt{x^2-x+3}}\,dx$

$\displaystyle = \frac{1}{2}\int \frac{1}{\sqrt{x^2-x+3}}\,d(x^2-x+3) + \frac{1}{2}\int \frac{dx}{\sqrt{x^2-x+3}}$

$\displaystyle = \sqrt{x^2-x+3} + \frac{1}{2}\int \frac{1}{\sqrt{(x-\frac{1}{2})^2+(\frac{\sqrt{11}}{2})^2}}\,d(x-\tfrac{1}{2})$

$\displaystyle = \sqrt{x^2-x+3} + \frac{1}{2}\int \frac{du}{\sqrt{u^2+a^2}} = \sqrt{x^2-x+3} + \frac{1}{2}\ln(u+\sqrt{u^2+a^2}) + C$

$\displaystyle = \sqrt{x^2-x+3} + \frac{1}{2}\ln(x-\tfrac{1}{2}+\sqrt{x^2-x+3}) + C$

P.144.30 $\displaystyle\int \frac{x\,dx}{(1+x^{\frac{1}{3}})^{\frac{1}{2}}}$, 令 $\sqrt{1+x^{\frac{1}{3}}}=t$, 则 $1+x^{\frac{1}{3}}=t^2$, $x^{\frac{1}{3}}=t^2-1$

$x=(t^2-1)^3$, $dx = 3(t^2-1)^2\cdot 2t\,dt$

$= 6t(t^2-1)^2dt$

$\displaystyle = \int \frac{(t^2-1)^3}{t}\cdot 6t\cdot(t^2-1)^2\,dt$

$\displaystyle = 6\int (t^2-1)^5\,dt = -6\int (1-t^2)^5\,dt$

$\displaystyle = -6\int \left[1+5\cdot(-t^2)+\frac{5\cdot4}{2!}(-t^2)^2+\frac{5\cdot4\cdot3}{3!}(-t^2)^3+\frac{5\cdot4\cdot3\cdot2}{4!}(-t^2)^4+(-t^2)^5\right]dt$

$\displaystyle = -6\int (1-5t^2+10t^4-10t^6+5t^8-t^{10})\,dt$

$\displaystyle = -6\left(t-\frac{5}{3}t^3+2t^5-\frac{10}{7}t^7+\frac{5}{9}t^9-\frac{t^{11}}{11}\right)+C.$

$\displaystyle = \frac{6}{11}t^{11}-\frac{10}{3}t^9+\frac{60}{7}t^7-12t^5-10t^3-6t+C$, $t=\sqrt{1+x^{\frac{1}{3}}}$.

P.144.31. $\displaystyle\int \frac{\sqrt{x}}{\sqrt[4]{x^3}+1}\,dx$ 令 $\sqrt[4]{x}=t$, 则 $\sqrt[4]{x^3}=t^3$, $x=t^4$, $dx=4t^3dt$

$\sqrt{x}=t^2$

$\displaystyle = \int \frac{t^2}{t^3+1}\cdot 4t^3\,dt = \frac{4}{3}\int \frac{t^3}{t^3+1}\,dt^3$

$\displaystyle = \frac{4}{3}\int \frac{t^3+1-1}{t^3+1}\,dt^3 = \frac{4}{3}\left[\int dt^3 - \int \frac{1}{t^3+1}\,d(t^3+1)\right]$

$\displaystyle = \frac{4}{3}(t^3-\ln|t^3+1|)+C$

$\displaystyle = \frac{4}{3}(\sqrt[4]{x^3}-\ln|\sqrt[4]{x^3}+1|)+C.$