



设焦点 F_1, F_2 , $F_1(c, 0), F_2(-c, 0)$, $c = \sqrt{a^2 - b^2}$

只需证明: $\gamma_1 = \gamma_2$ 或 $\tan \gamma_1 = \tan \gamma_2$.

$$\tan \alpha = \frac{dy}{dx} = -\frac{b^2}{a^2} \cdot \frac{x}{y}$$

$$a^2 - b^2 = c^2$$

$$\tan \beta_1 = \frac{y}{c+x}, \quad \tan \beta_2 = \frac{y}{c-x}$$

$$a^2 y^2 + b^2 x^2 = a^2 b^2$$

$$\tan \alpha_1 = \tan(x - \alpha) = -\tan \alpha = \frac{b^2}{a^2} \cdot \frac{x}{y}$$

$$\begin{aligned} \tan \gamma_1 = \tan(\alpha_1 + \beta_1) &= \frac{\tan \alpha_1 + \tan \beta_1}{1 - \tan \alpha_1 \cdot \tan \beta_1} = \frac{\frac{b^2}{a^2} \cdot \frac{x}{y} + \frac{y}{c+x}}{1 - \frac{b^2}{a^2} \cdot \frac{x}{y} \cdot \frac{y}{c+x}} = \frac{\frac{b^2 x(c+x) + a^2 y^2}{a^2 y(c+x)}}{\frac{a^2(c+x) - b^2 x}{a^2(c+x)}} \\ &= \frac{a^2 b^2 + b^2 c x}{y(a^2 c + a^2 x - b^2 x)} = \frac{b^2(a^2 + c x)}{y[a^2 c + c^2 x]} = \frac{b^2 \cdot (a^2 + c x)}{c y (a^2 + c x)} = \frac{b^2}{c y} \end{aligned}$$

$$\begin{aligned} \tan \gamma_2 = \tan(\beta_2 - \alpha_1) &= \frac{\tan \beta_2 - \tan \alpha_1}{1 + \tan \beta_2 \cdot \tan \alpha_1} \\ &= \frac{\frac{y}{c-x} - \frac{b^2}{a^2} \cdot \frac{x}{y}}{1 + \frac{y}{c-x} \cdot \frac{b^2}{a^2} \cdot \frac{x}{y}} = \frac{\frac{a^2 y^2 - b^2 x(c-x)}{a^2 y(c-x)}}{\frac{a^2(c-x) + b^2 x}{a^2(c-x)}} = \frac{a^2 y^2 + b^2 x^2 - b^2 c x}{y(a^2 c - a^2 x + b^2 x)} = \frac{a^2 b^2 - b^2 c x}{y(a^2 c - c^2 x)} = \frac{b^2}{c y} \end{aligned}$$

$$\therefore \tan \gamma_1 = \tan \gamma_2 = \frac{b^2}{c y},$$

从而 $\gamma_1 = \gamma_2$, 入射线 = 反射线.

入射线经过 F_1 , 反射线过 F_2 . F_1, F_2 为焦点.