Chapter 9 Morphological Image Processing

第九章　形态学图像处理

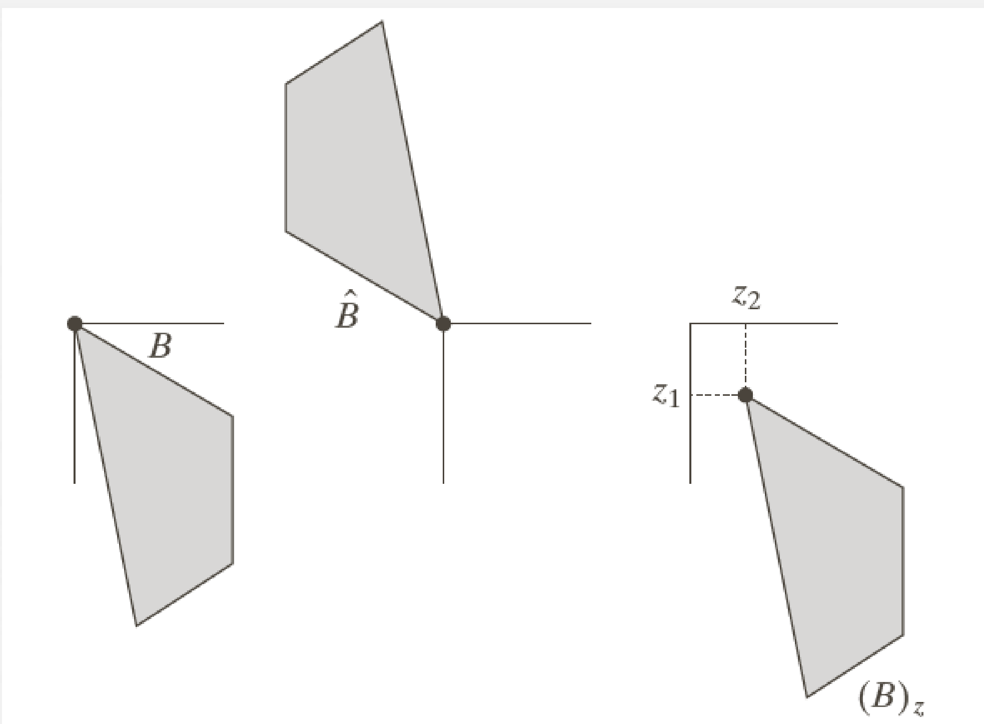
9.1 预备知识

集合B的反射，表示为$\hat{B}$ 定义为：

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

集合B平移到点$z = (z_1, z_2)$,表示为$(B)_z$，定义为：

$$(B)_z = \{c \mid c = b + z, b \in B\}$$



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

9.1 预备知识

结构元(SE, Structuring element)

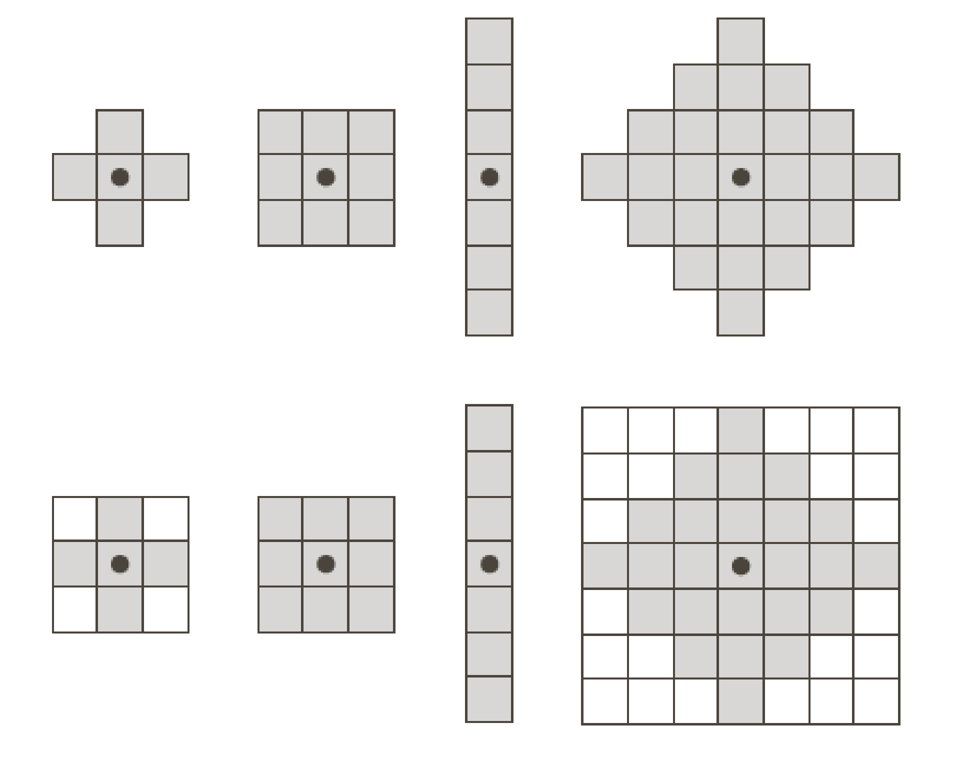
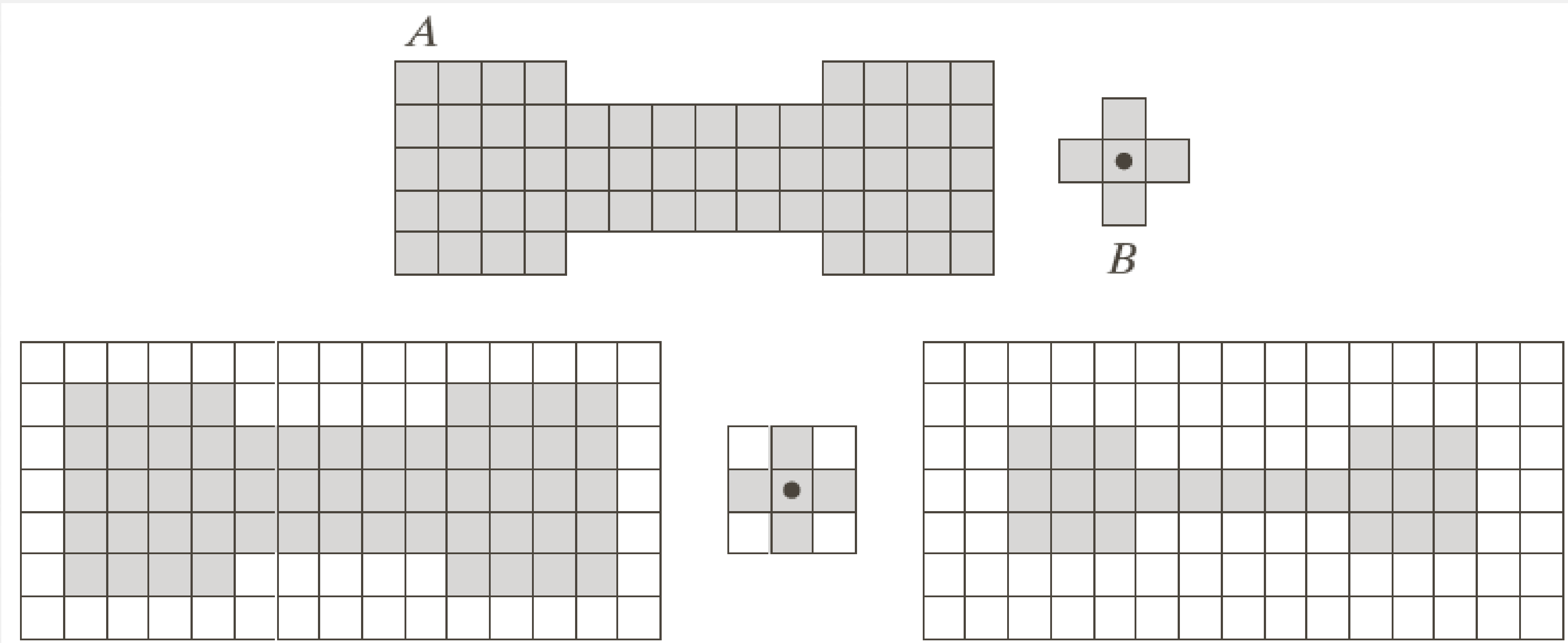


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

9.1 预备知识

腐蚀操作



a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

9.2 腐蚀和膨胀
9.2.1 腐蚀（Erosion）

A和B是Z^2中的集合，其中B为结构元。则B对A的腐蚀定义为：

$$A \ominus B = \{z|(B)_z \subseteq A\}$$

上述定义也可表示为如下等价形式：

$$A \ominus B = \{z|(B)_z \cap A^c = \emptyset\}$$

A^c是A的补集，∅是空集。

9.2 腐蚀和膨胀
9.2.1 腐蚀

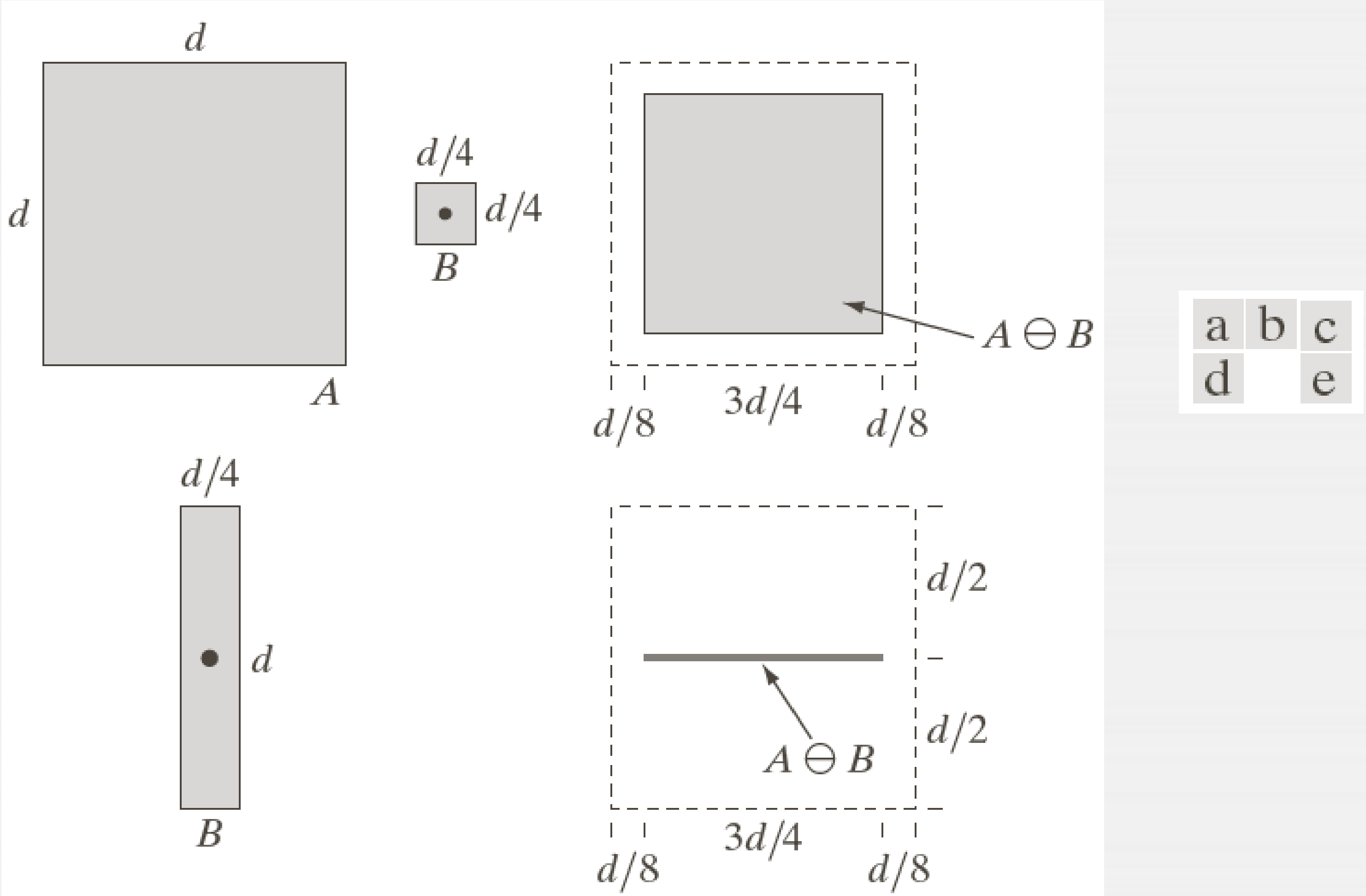
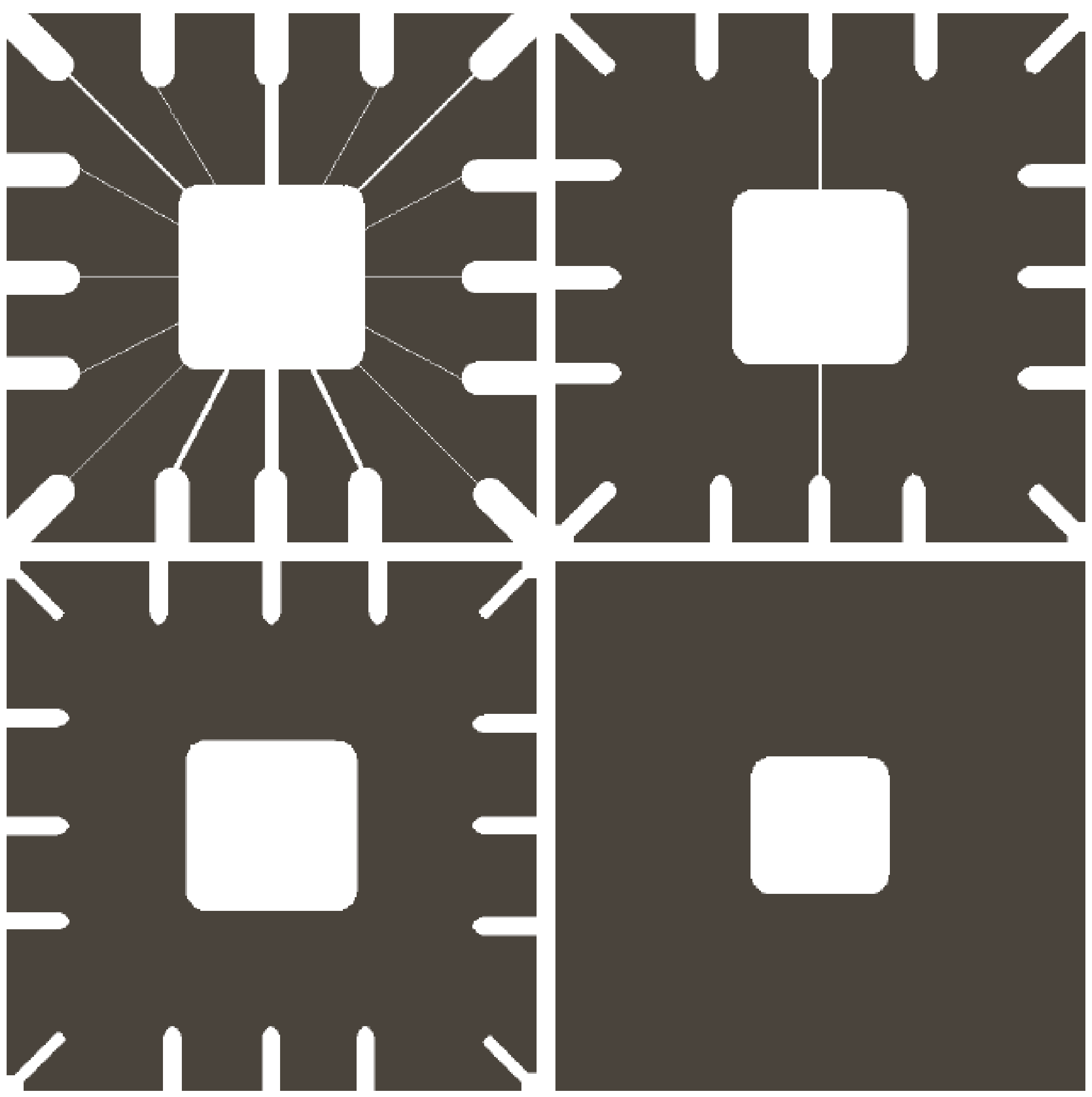


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

9.2.1 腐蚀



a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

9.2.2 膨胀(Dilation)

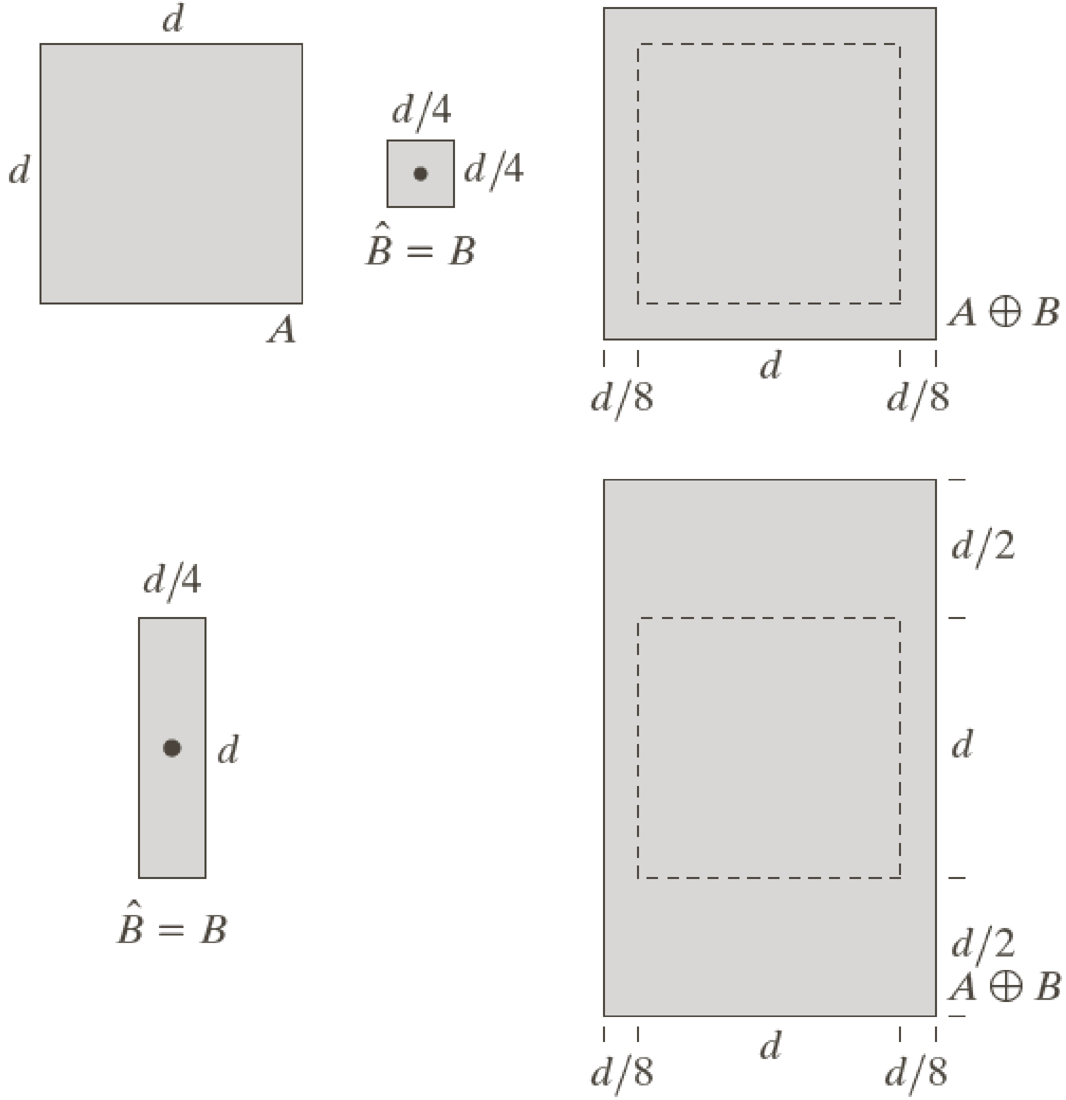
A和B是Z^2中的集合，其中B为结构元。则B对A的膨胀定义为：

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

B对A的膨胀是所有位移z的集合，这样， $\hat{B}$和A至少有一个元素是重叠的。上述定义也可表示为如下等价形式：

$$A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$$

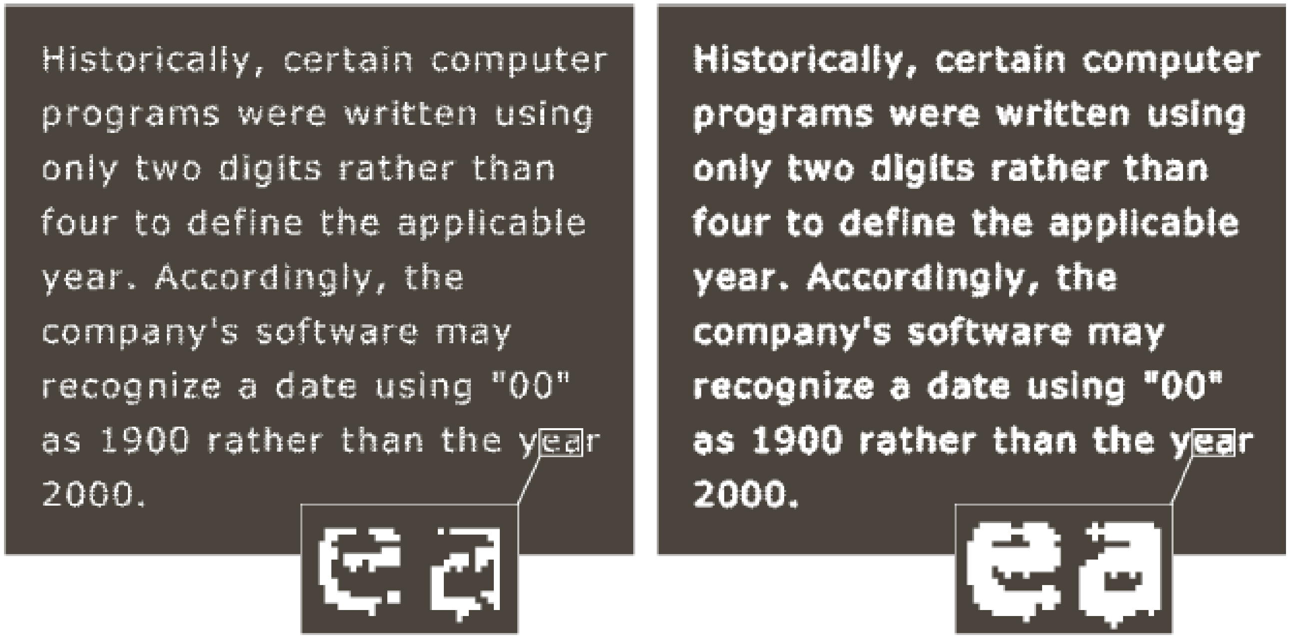
9. 2. 2 膨胀



a	b	c
d		e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

9.2.2 膨胀



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

9.2.3 对偶性

膨胀与腐蚀的关系

$$(A \ominus B)^c = A^c \oplus \hat{B} \text{或} (A \oplus B)^c = A^c \ominus \hat{B}$$

证明:

$$(A \ominus B)^c = \{z | (B)_z \subseteq A\}^c$$

如果集合 $(B)_z$ 包含于A, 则等价于 $(B)_z \bigcap A^c = \phi$

于是: $(A \ominus B)^c = \{z | (B)_z \cap A^c = \emptyset\}^c = \{z | (B)_z \cap A^c \neq \emptyset\}$

依定义: $A \oplus B = \{z \,|\, (\hat{B})_z \bigcap A \neq \phi\}$

类似可定义: $A^c \oplus \hat{B} = \{z \,|\, (B)_z \bigcap A^c \neq \phi\}$

故 $(A \ominus B)^c = A^c \oplus \hat{B}$

9.3 开操作与闭操作

- 开操作一般使对象的轮廓变得光滑，断开狭窄的间断和消除细的突出物。
- 闭操作使对象的更为连通，它能消除小的孔洞，并填补轮廓线中的断裂。

使用结构元素B对集合A进行开操作的定义为:

$$A \circ B = (A \ominus B) \oplus B$$

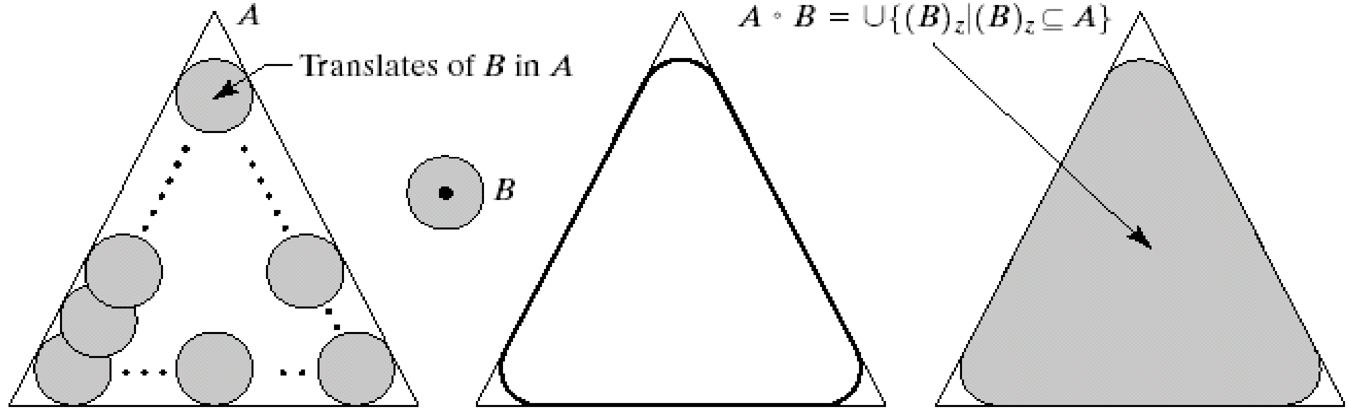
即，使用结构元素B对集合A进行先腐蚀，然后膨胀。

使用结构元素B对集合A进行闭操作的定义为:

$$A \bullet B = (A \oplus B) \ominus B$$

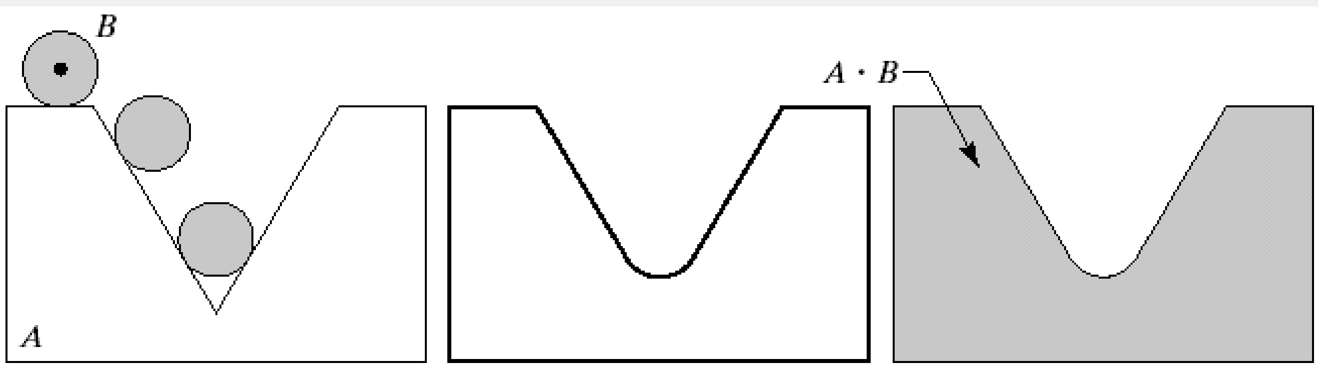
即，使用结构元素B对集合A进行先膨胀，然后腐蚀。

9.3 开操作与闭操作



a b c d

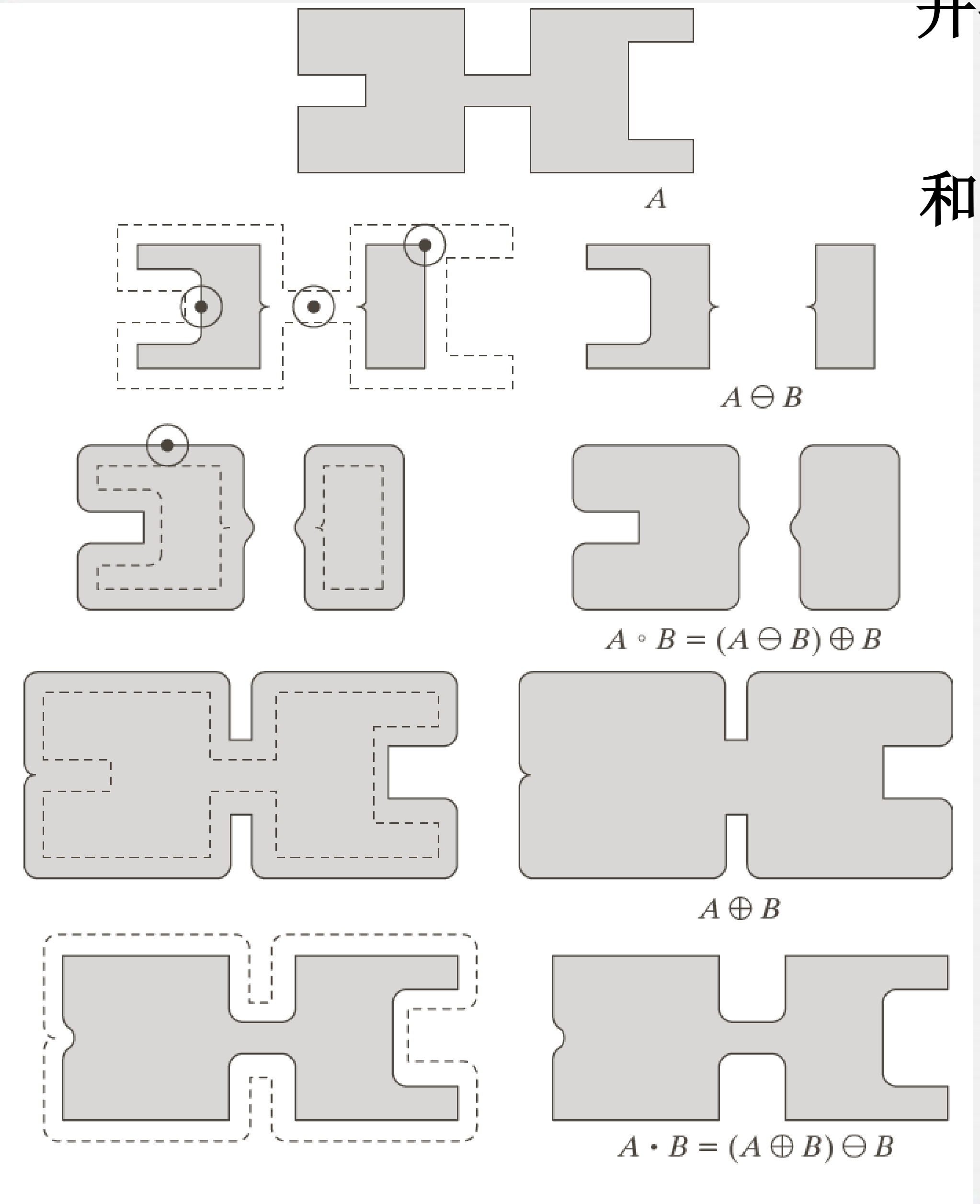
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

9.3 开操作与闭操作



开操作与闭操作的关系：

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

和：

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

9.3 开操作与闭操作

开操作与闭操作的性质：

开操作满足下列性质：

(i) $A \circ B$是A的子集合（子图）。

(ii) 如果C是D的子集合，则$C \circ B$是$D \circ B$的子集。

(iii) $(A \circ B) \circ B = A \circ B$

同样，闭操作也满足下列性质：

(i) A是$A \bullet B$的子集（子图）。

(ii) 如果C是D的子集，则$C \bullet B$是$D \bullet B$的子集。

(iii) $(A \bullet B) \bullet B = A \bullet B$

注意，由两个情况下的条件（iii）可知，算子应用一次后，一个集合进行多少次开操作或闭操作或闭操作都不会有变化。

9.3 开操作与闭操作

a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

9.4 击中或击不中(Hit-or-Miss)变换

- 击中或击不中变换是形状检测的基本工具.
- 令每种形状的重心为它的原点.

设形状D包含在一个小窗口W中，D的背景定义为W-D. 令B是由D和W-D组成的集合，使用结构元素B对集合A进行匹配操作的定义为:

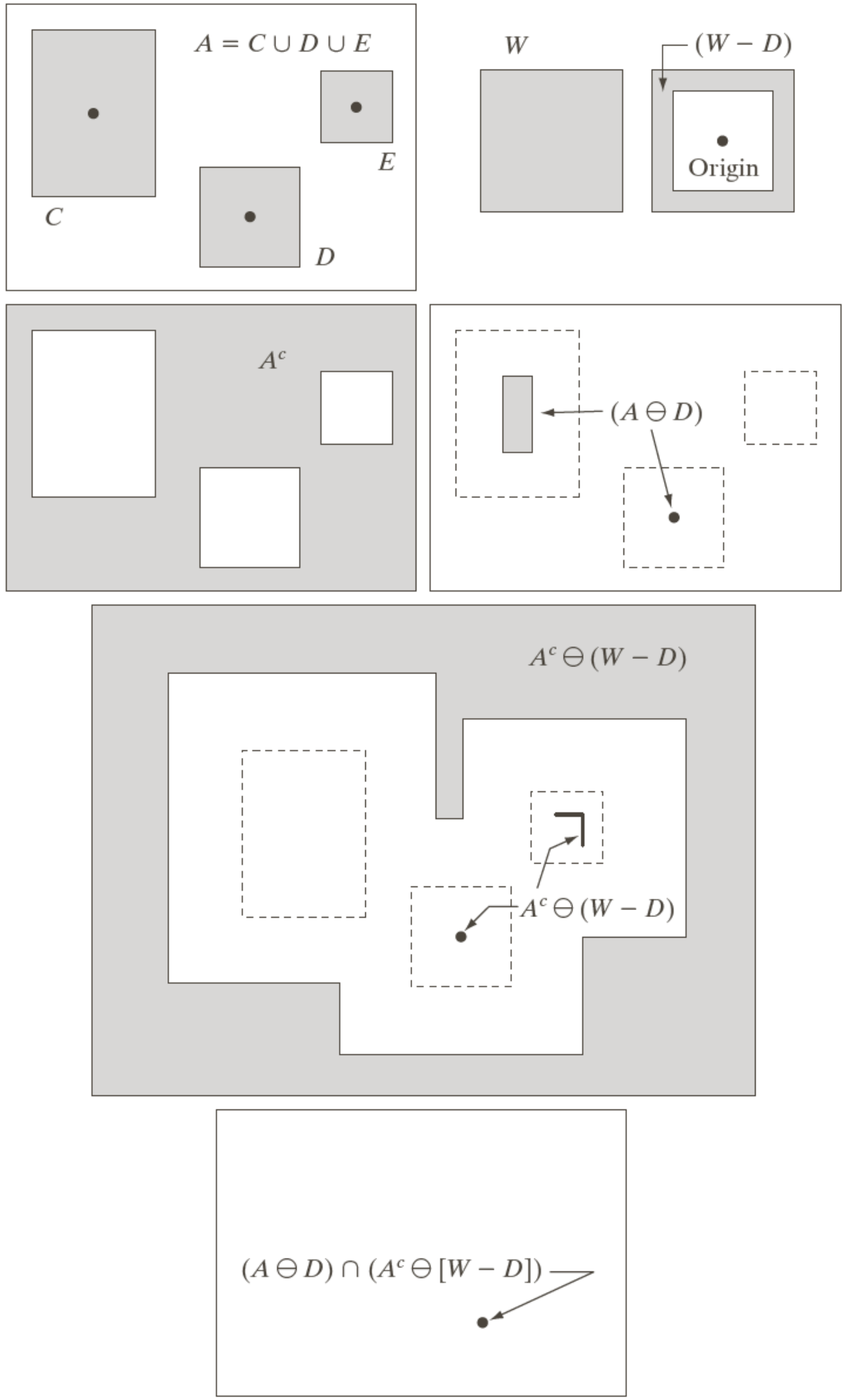
$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

更一般地,令 $B = (B_1, B_2)$ ，使用结构元素B对集合A进行匹配操作的定义为:

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

根据$A^c \oplus \hat{B} = (A \ominus B)^c$(式9.2.5)和$A \cap B^c = A - B$(式2.6.19)

因此, $A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$



a	b
c	d
e	
f	

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

9.5 一些基本的形态学算法

1.介绍提取图像的边界、连通分量、凸壳和区域骨架的形态学算法；
2. 介绍区域填充、细化、粗化和修剪等预处理算法和后处理算法。

9.5.1 边界提取

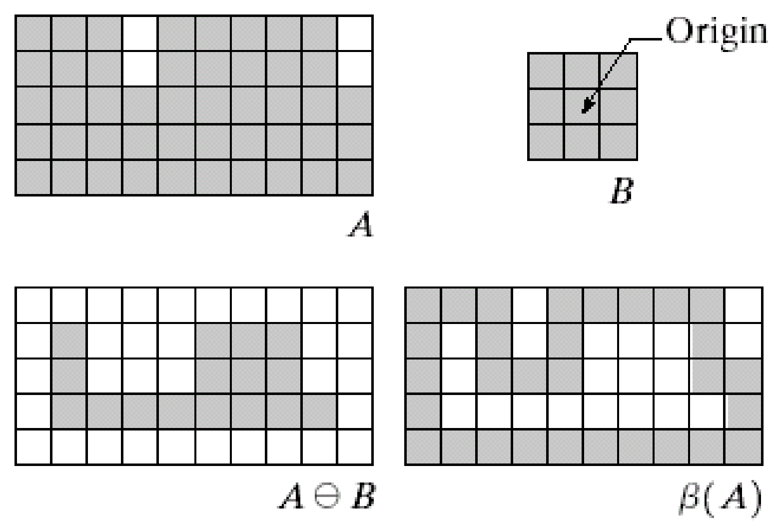
设集合A的边界表示为$\rho(A)$, 定义：

$$\rho(A) = A - (A \ominus B)$$

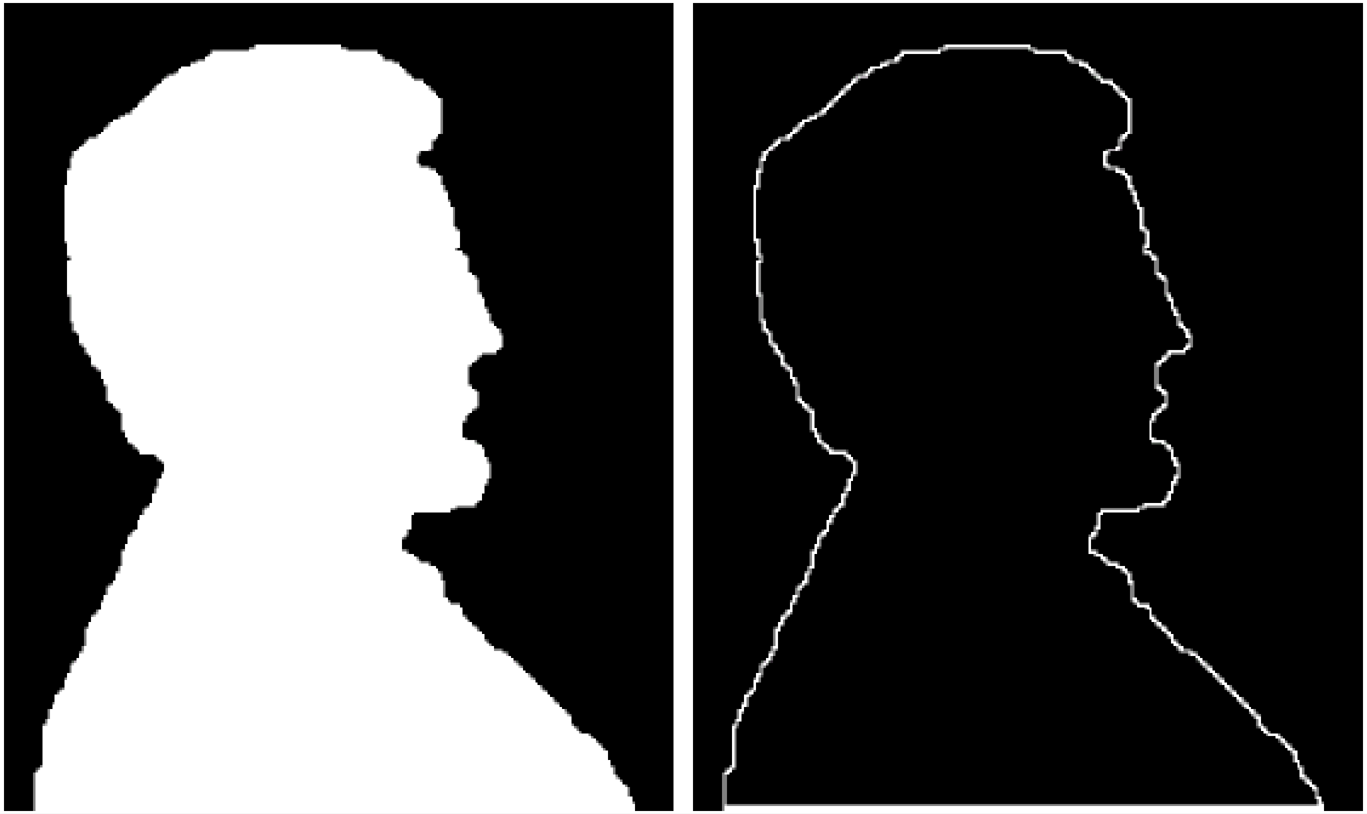
其中B为结构元素。

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



注意B的原点位于集合的边线的处理方法.



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.5.2. 区域(空洞)填充

设A为包含子集的集合，区域填充算法是一个迭代算法：

1）取一个边界内的初始点*p*，将1赋给*p*，令 $X_0 = p$

2） $X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,\ldots$

3）如果$X_k = X_{k-1}$ ，则 $X_k \cup A$为所求，结束，否则转到2).

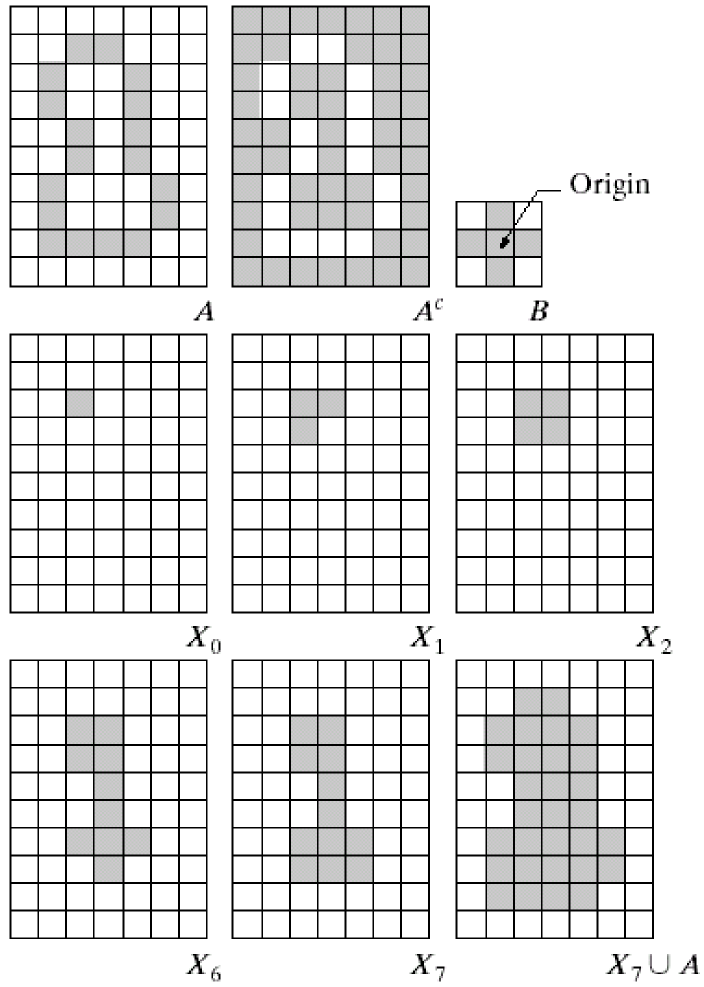
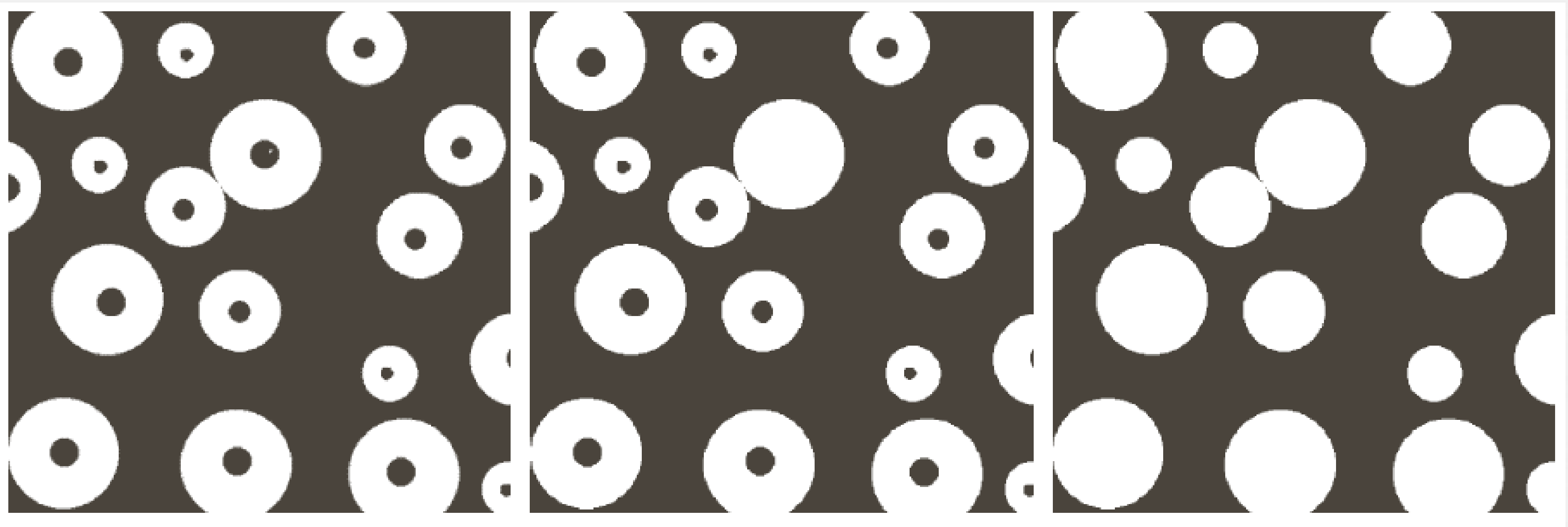


FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

9.5.2. 区域(空洞)填充



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

9.5.3 连通分量的提取

邻接性、连通性、区域和边界：

1）位于（x,y）的像素p有4个水平和垂直相邻像素：

$$(x+1,y),(x-1,y),(x,y+1),(x,y-1)$$

记为 $N_4(p)$

2）p有4个对角相邻像素：

$$(x+1,y+1),(x+1,y-1),(x-1,y+1),(x-1,y-1)$$

记为 $N_D(p)$

3）p的8相邻像素：

$$N_8(p)=N_4(p)\cup N_D(p)$$

4）令V用于定义邻接性的灰度集合，例如在二值图像中，V={1}。

邻接性有两个要素：一个是灰度值的邻接性（值域V）、一个是物理位置的邻接性（邻域，如$N_4(P)$等）。例如，二值图象中，像素值都为1（或都为0）的像素才有可能被称为是邻接的。在一般图像中，可定义一个值域V，V是0到255中取值的一个子集。

一般我们考虑三种邻接性：

（a）4邻接：如果点q在$N_4(P)$中，并具有V中的数值，则q和p是4邻接的；

（b）8邻接：如果点q在$N_8(P)$中，并具有V中的数值，则q和p是8邻接的；

（c）m邻接（混合邻接）：满足下列条件的任一个，则具有V中数值的p和q是m连接的。

（i）q在中$N_4(P)$

（ii）q在$N_D(P)$中，且集合$N_4(P) \cap N_4(q)$中没有V值的像素。

注意：混合邻接是8邻接的改进，为了消除8邻接的二义性。例如图2.26。

两个集合邻接的概念：如果集合S1中的某些像素和S2中的某些像素邻接，则称这两个集合是邻接的。这里说的邻接指的是4、8或者m邻接。

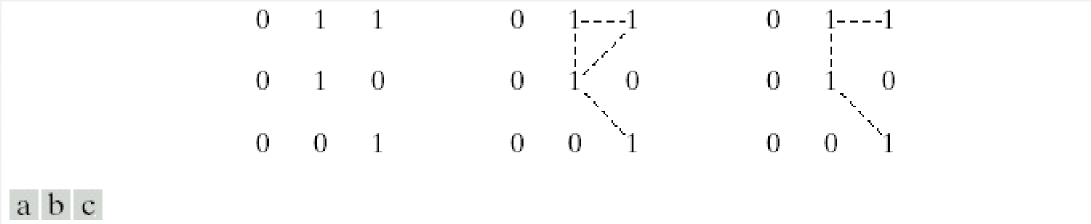


FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

从具有坐标(x, y)的p到具有坐标(s, t)的q的4(或8)通路，如果存在 $(x_0,y_0),(x_1,y_1),\ldots,(x_n,y_n)$

其中 $(x_0,y_0)=(x,y),(x_n,y_n)=(s,t)$ 并且(x_i,y_i)和 (x_{i-1},y_{i-1})是4(8)邻接的。

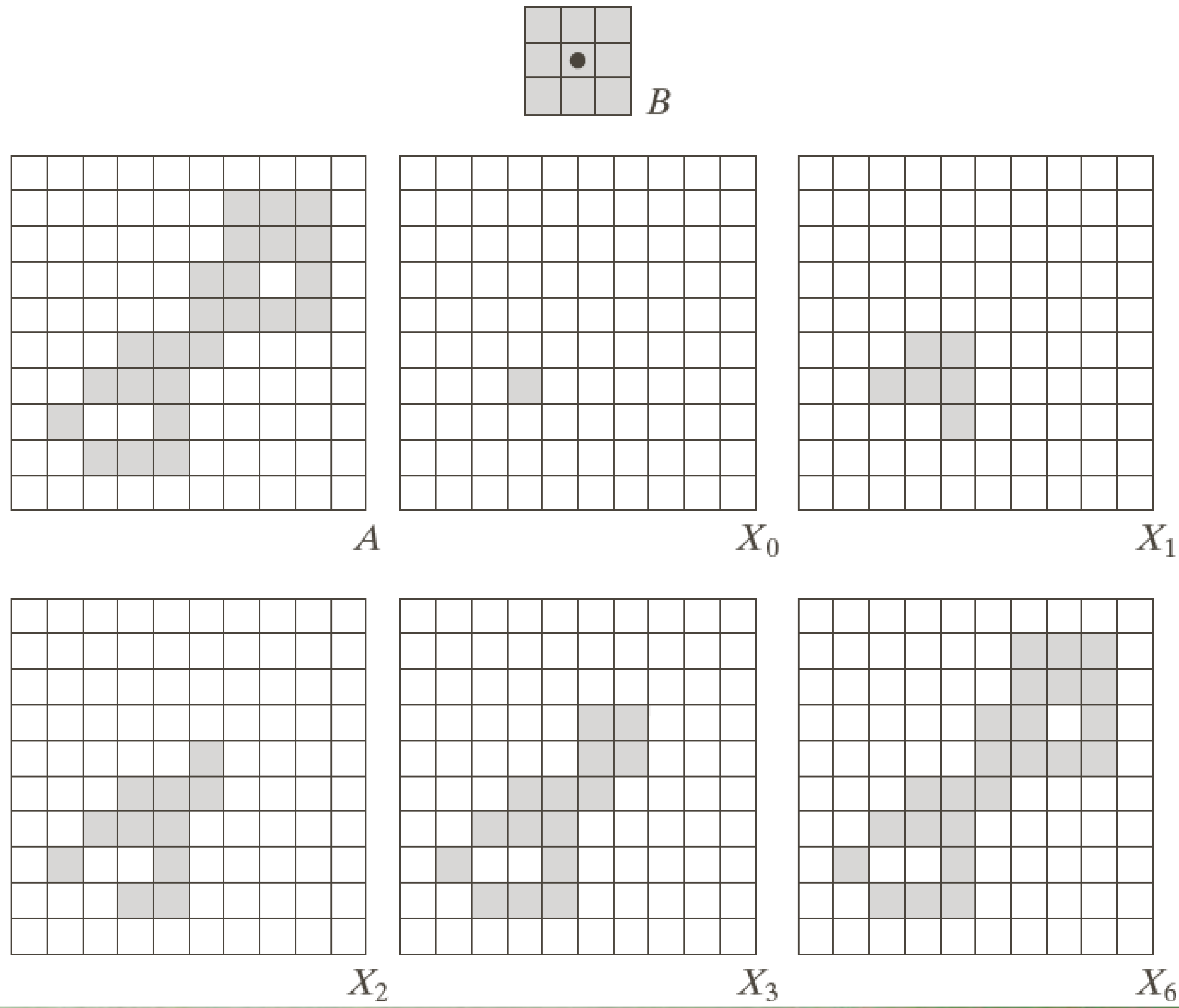
4(或8)连通子集：令S代表图像中像素的子集，如果S中全部像素之间存在一个4(或8)通路，那么称S是4(或8)连通子集。

连通分量的提取算法是一个迭代算法：

1）取子集A中的一个的点p为初始点，令 $X_0 = p$

2） $X_k = (X_{k-1} \oplus B) \cap A, k = 1,2,3,...$

3）如果 $X_k = X_{k-1}$ ，则 $Y = X_k$ 结束；否则转到2).



ents. (a) Structuring element. (b) Array
ent. (c) Initial array containing a 1 in the
Various steps in the iteration of Eq. (9.5-3).

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

9.5.4 凸壳

◆ **凸集的定义:**

如果连接集合A内任意两点的直线段都在A的内部,就称A是凸集.

◆ **凸壳的定义:**

任意集合S的凸壳H是包含S的最小凸集. 集合差H-S称为S的凸缺.

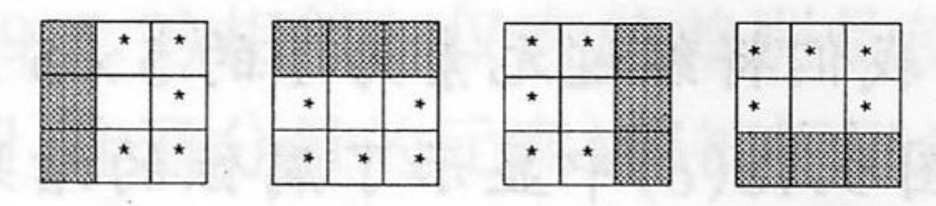
◆ **集合A的凸壳C(A)求取算法:**

令$B^i, i = 1,2,3,4$表示图9.19(a)中的4个结构元素。这个过程由计算公示组成:

$$X_k^i = (X_{k-1} \circledast B^i) \cup A, i = 1,2,3,4, k = 1,2,3,\ldots$$

其中 $X_0^i = A$。现令 $D^i = X_{conv}^i$ ，这里下标“conv”表示在 $X_k^i = X_{k-1}^i$ 时收敛。

A的凸壳为: $C(A) = \bigcup_{i=1}^{4} D^i$



上述计算的凸壳不能确保凸性所需的最小尺寸.

因此要求:

限制生长以便凸壳不会超过初始集合在水平和垂直方向上的尺寸大小.

a
b c d
e f g
h

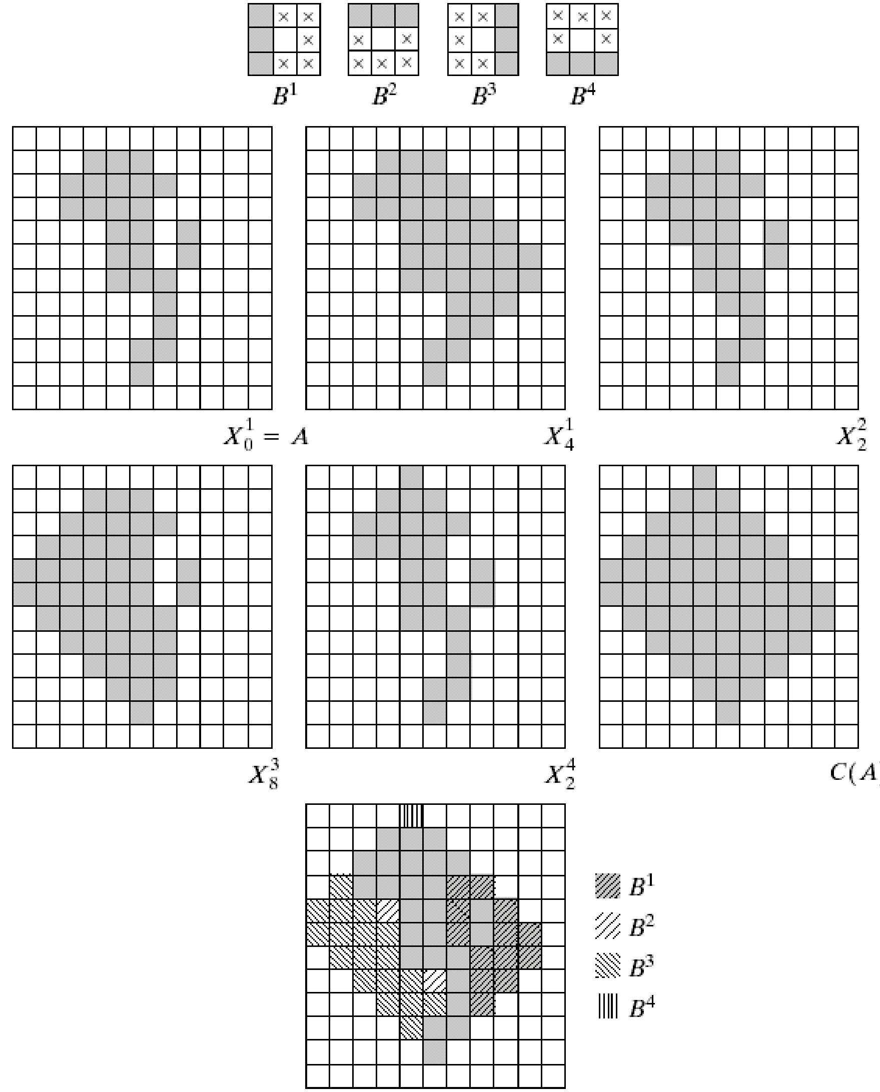


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

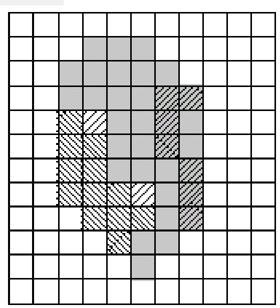


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

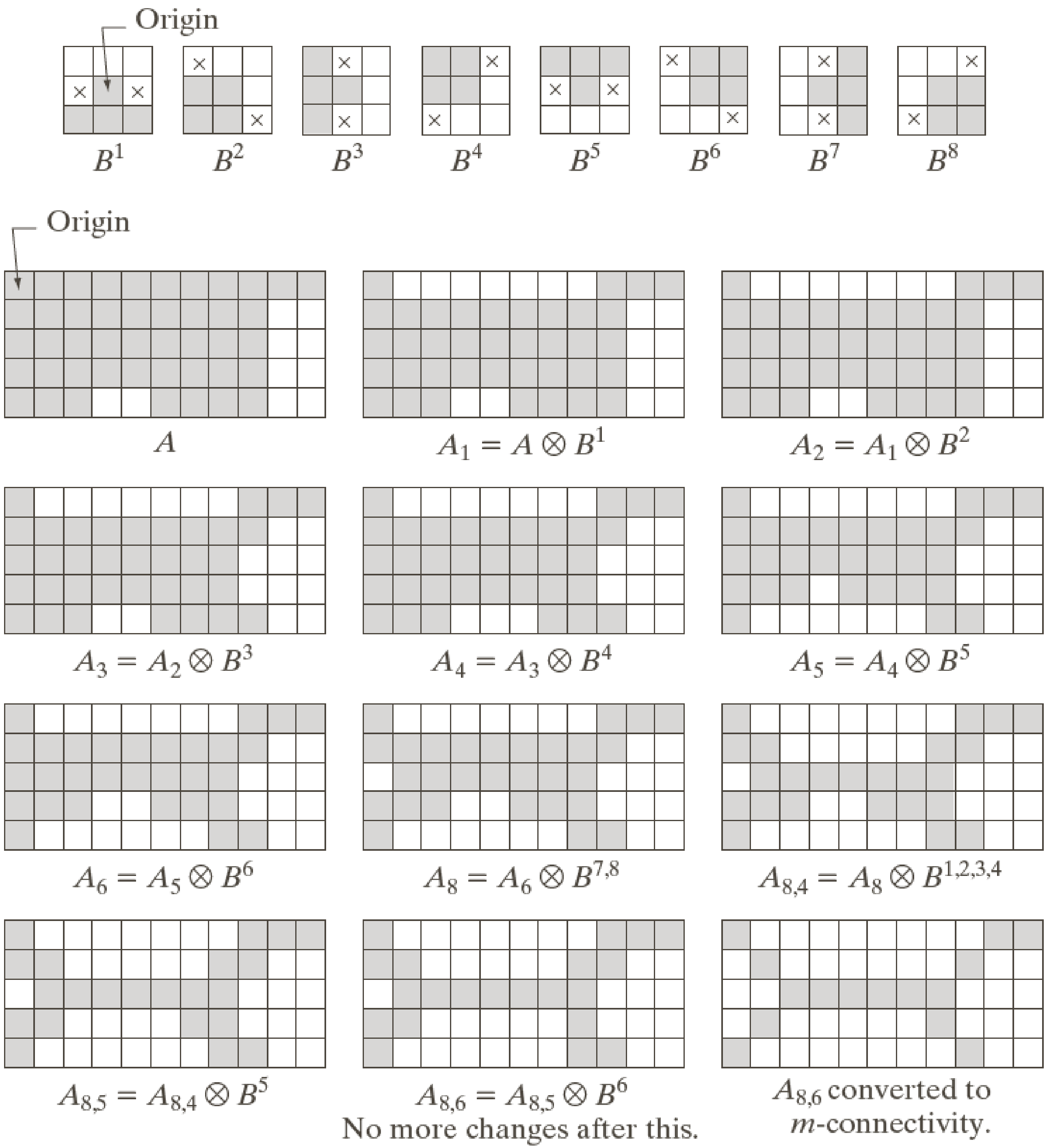
9.5.5 细化

- 用结构元素B对集合A的细化定义为:

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

- 用结构元素序列$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$对集合A的细化定义为:

$$A \otimes \{B\} = ((\cdots((A \otimes B^1) \otimes B^2) \cdots) \otimes B^n)$$



	a	
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.21 (a
(c) Result of th
seven element
(j) Result of u
Conversion to

9.5.5 粗化

- 用结构元素B对集合A的粗化定义为:

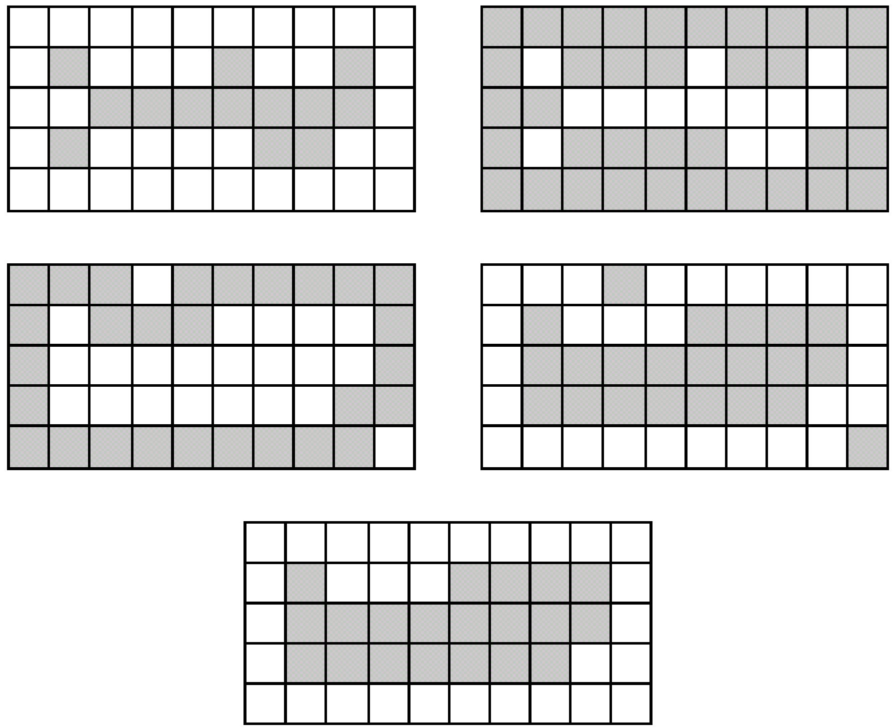
$$A \odot B = A \cup (A \circledast B)$$

- 用结构元素序列$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$对集合A的粗化定义为:

$$A \odot \{B\} = ((\cdots((A \odot B^1) \odot B^2)\cdots) \odot B^n)$$

- 注意:

1) 粗化和细化的结构元素类似,但1和0要互换.
2) 通常用对背景的细化代替粗化,但需要消除断点.



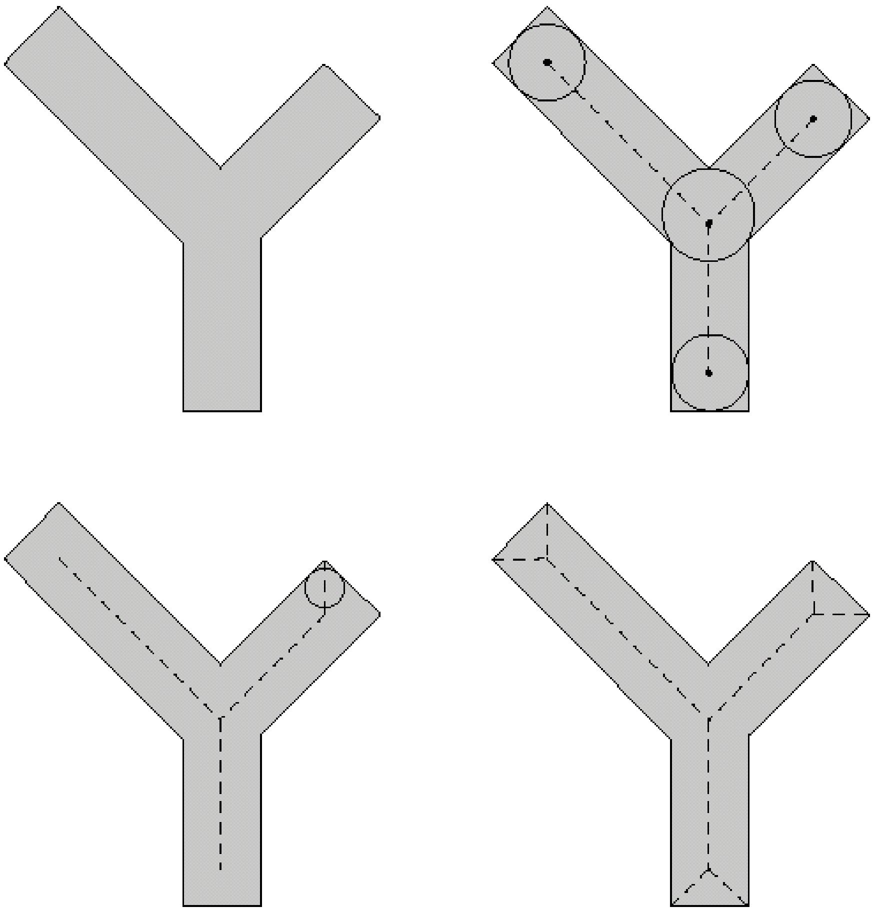
a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

9.5.5 骨架

- 集合A的的骨架S(A)定义为:

$$S(A)=\{z\,|\,A\text{内最大园盘}(D)_z\text{的圆心}\}$$



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

A的骨架可用腐蚀和开操作表达。即骨架可以表达为如下式所示：

$$S(A)=\bigcup_{k=0}^{K} S_k(A)$$

其中 $S_k(A)=(A\ominus kB)-(A\ominus kB)\circ B$

上式中，B是一个结构元素，而$A\ominus kB$表示对A的连续k次腐蚀：

$$(A\ominus kB)=\left((\cdots(A\ominus B)\ominus B)\ominus\cdots\right)\ominus B)$$

第$\boldsymbol{K}$次是A被腐蚀为空集合前进行的最后一次迭代。就是说：

$$K=\max\{k|(A\ominus kB)\neq\emptyset\}$$

可以证明：

$$A=\bigcup_{k=0}^{K}(S_k(A)\oplus kB)$$

这里$(S_k(A)\oplus kB)$表示对$S_k(A)$的k次连续的膨胀，即，

$$(S_k(A)\oplus kB)=\left((\cdots(S_k(A)\oplus B)\oplus B)\oplus\cdots\right)\oplus B)$$

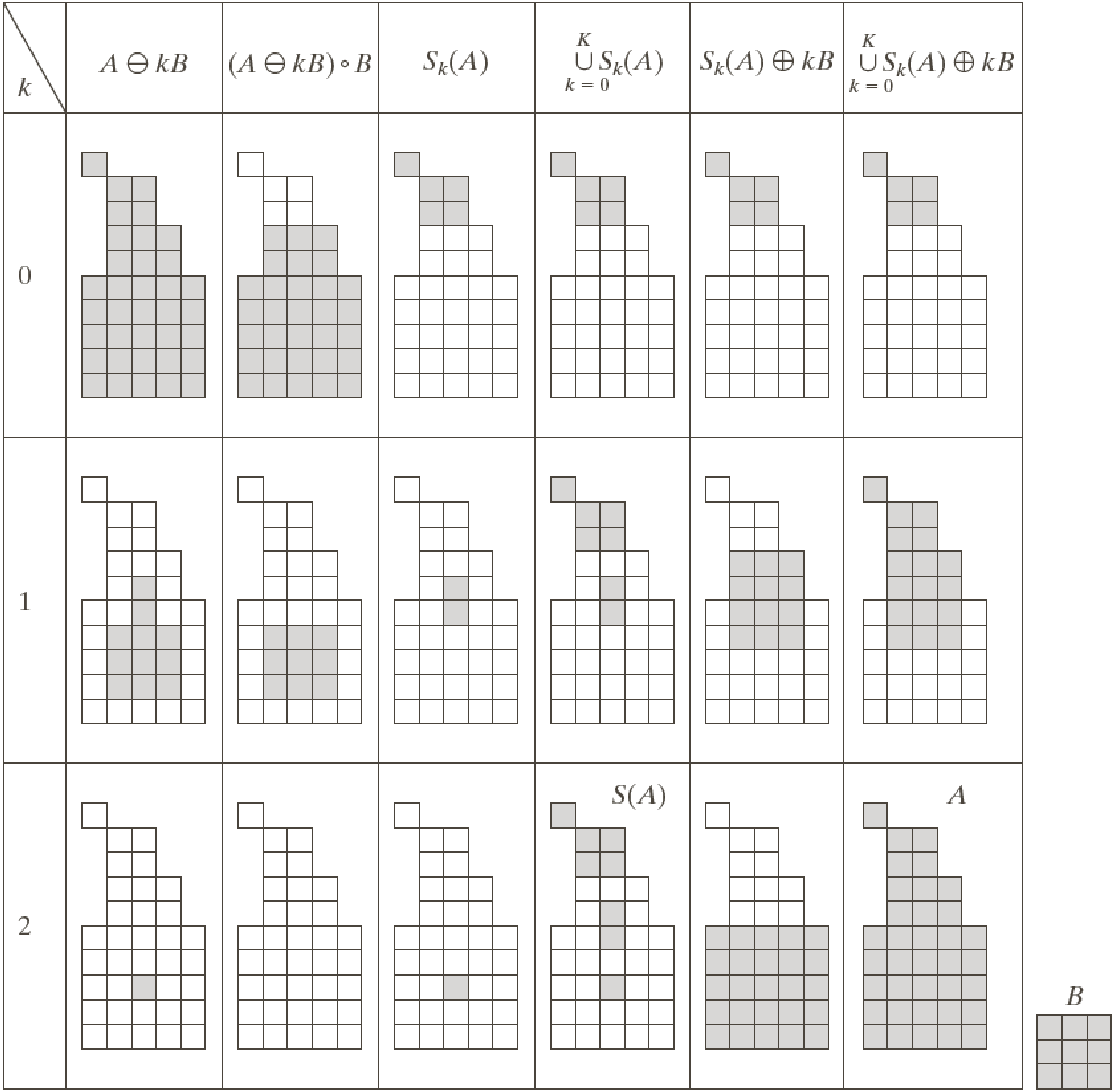


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

9.5.8 剪裁

- 剪裁通常用于细化和骨架绘制算法的后处理.
- 用于消除”毛刺”—比较短的像素端点，比如说小于等于3个像素长度.

剪裁算法（例子）：

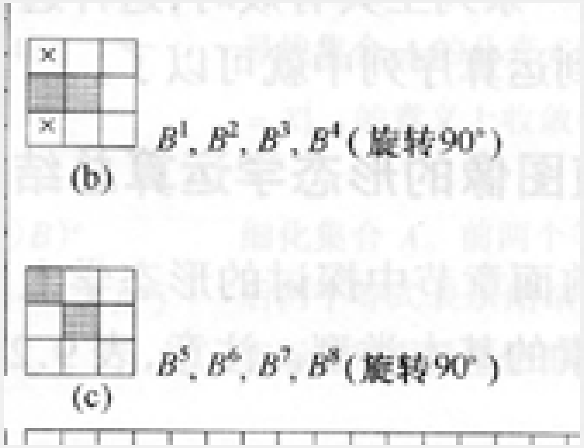
1) 三次细化$X_1 = A \otimes \{B\}$

2) 提取终点$X_2 = \cup_{k=1}^{8}(X_1 \circledast B^k)$

3) 三次限制膨胀$X_3 = (X_2 \oplus H) \cap A$

其中H是元素值为1的3x3的结构元素.

4) 剪裁的结果:$X_3 = X_1 \cup X_3$



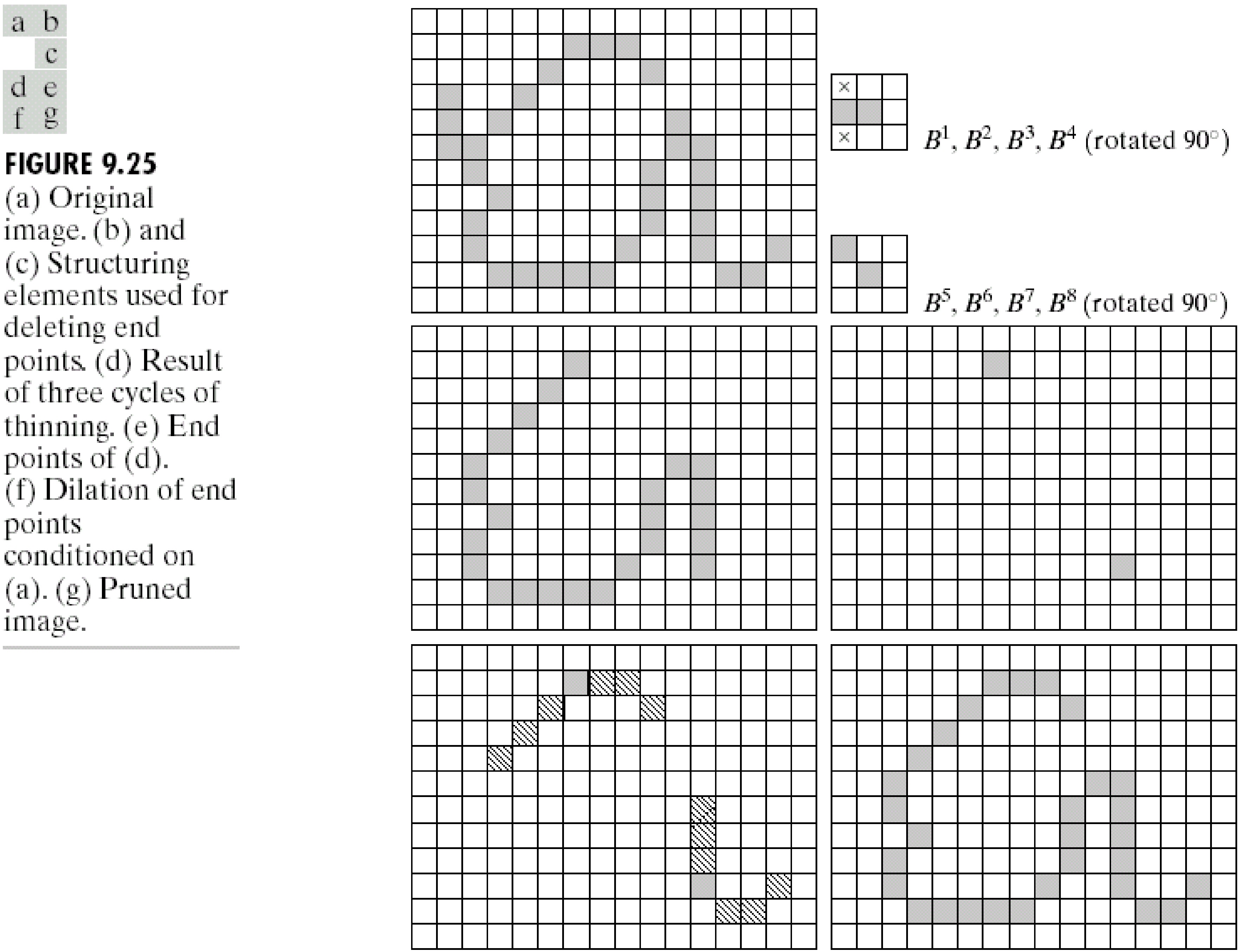


FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

9.5.9 形态学重建

◆ **测地膨胀**

令F表示标记图像（包含变换的起始点），G表示模板图像（约束该变换）。假定两者均为二值图像，且 $F \subseteq G$。标记图像关于模板的大小为1的测试膨胀定义为：

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

F关于G的大小为n的测地膨胀定义为：

$$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)]$$

其中$D_G^{(0)}(F) = F$。

9.5.9 形态学重建

◆ 测地膨胀

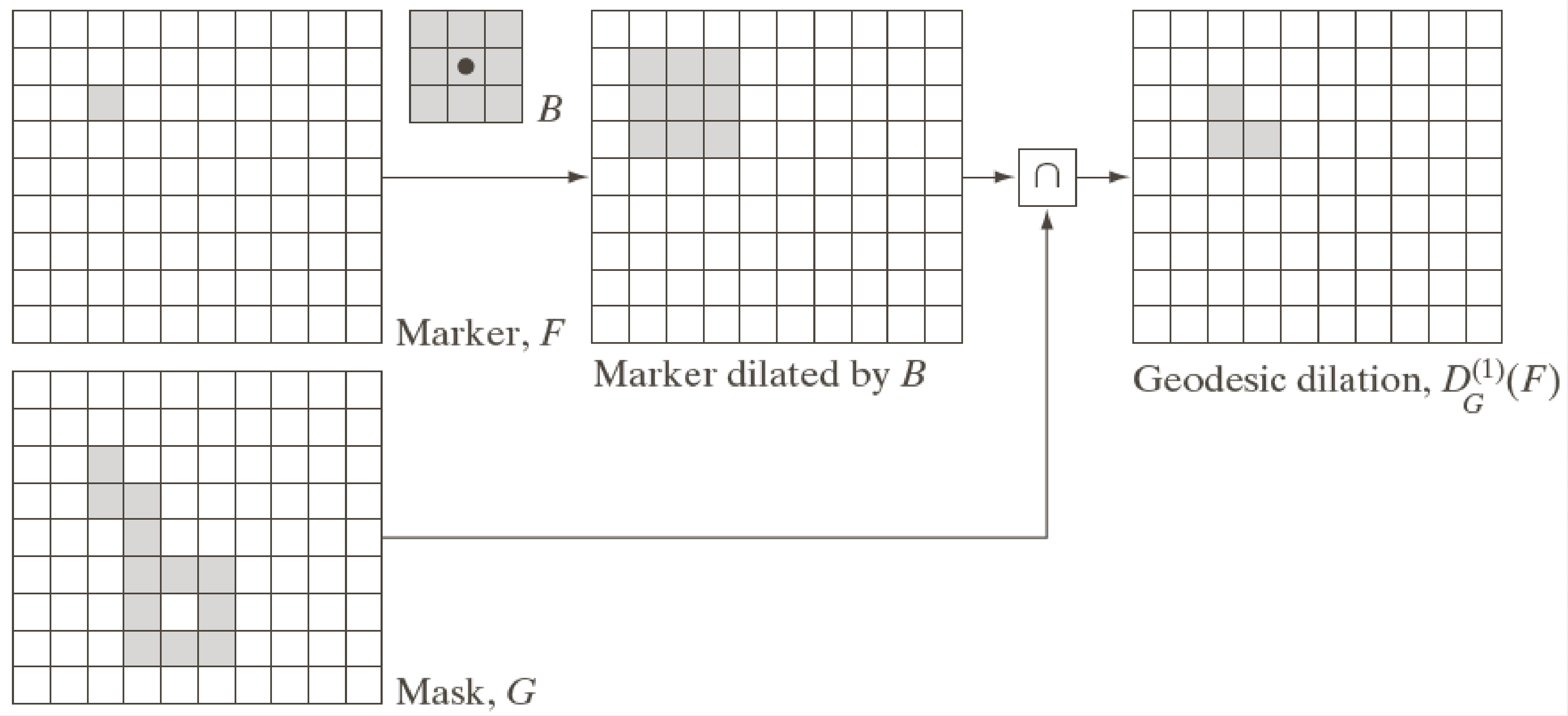


FIGURE 9.26
Illustration of geodesic dilation.

9.5.9 形态学重建

◆ **测地腐蚀**

类似的，标记图像关于模板的大小为1的测试腐蚀定义为：

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

F关于***G***的大小为***n***的测地腐蚀定义为：

$$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)]$$

其中$E_G^{(0)}(F) = F$。

9.5.9 形态学重建

◆ **测地腐蚀**

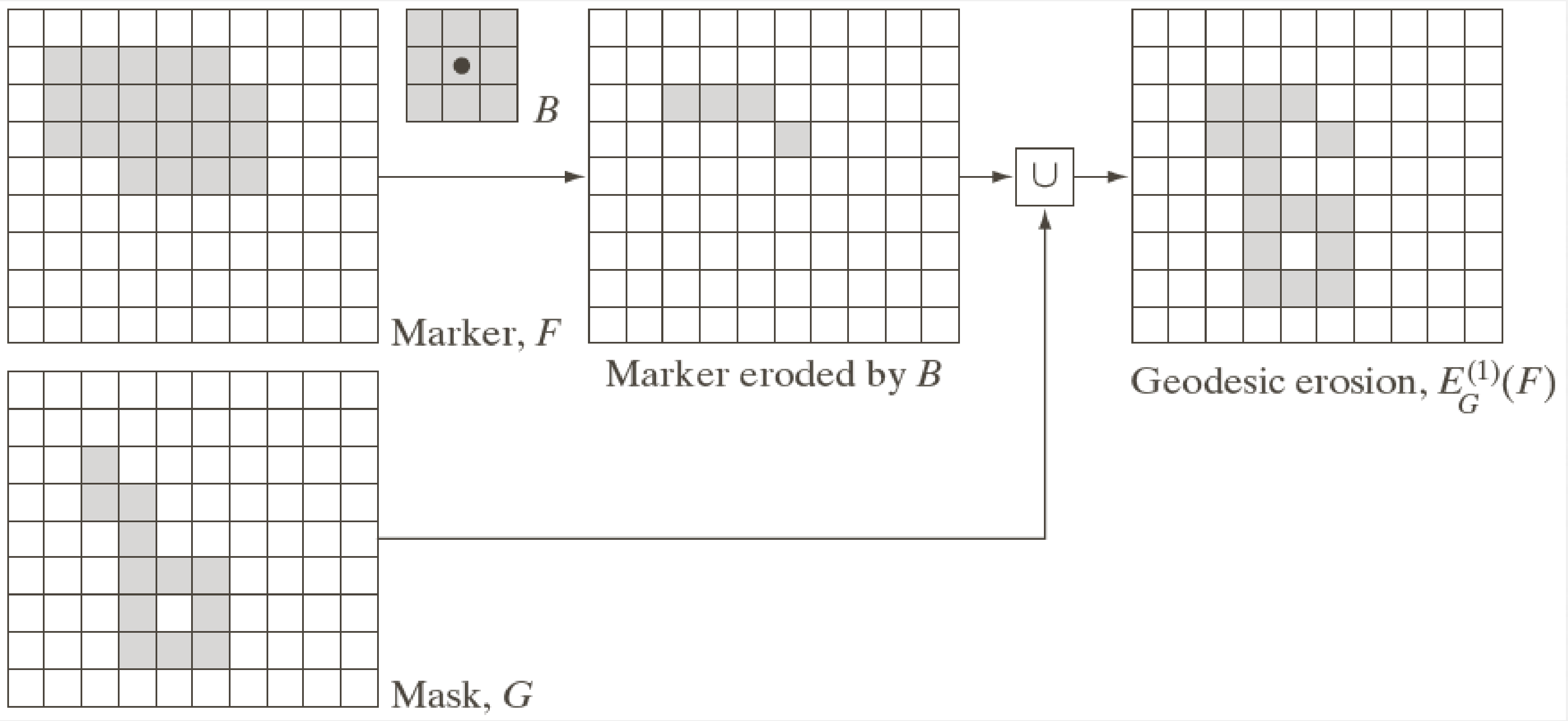


FIGURE 9.27
Illustration of geodesic erosion.

9.5.9 形态学重建

◆ 基于膨胀的形态学重建

标记图像**F**对模板**G**的基于膨胀的形态学重建记为$R_G^D(F)$,它被定义为**F**关于**G**的反复迭代***k***次的测地膨胀

$$R_G^D(F) = D_G^{(k)}(F)$$

直至到稳定状态$D_G^{(k)}(F) = D_G^{(k+1)}(F)$。

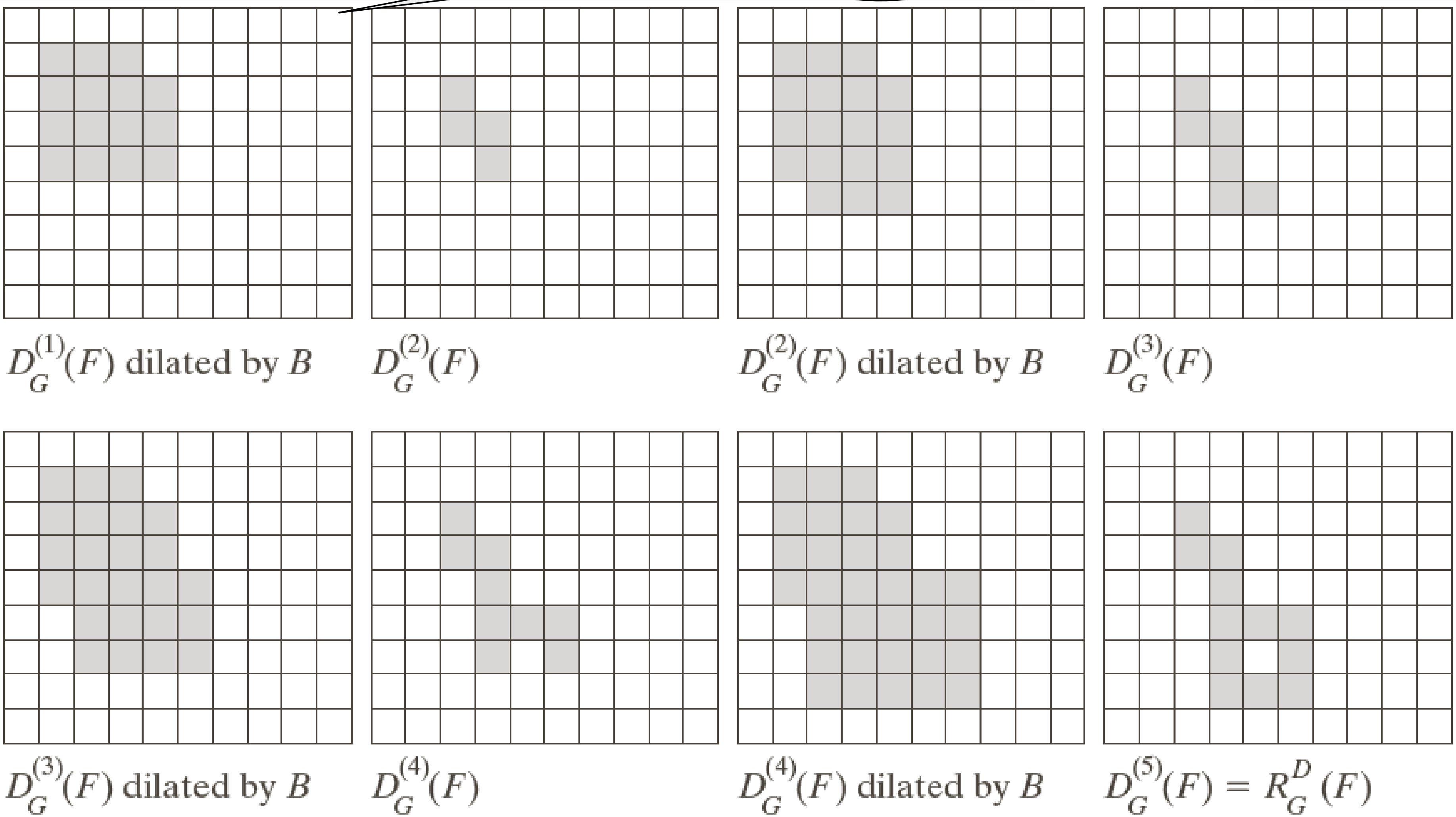
9.5.9 形态学重建

◆ **基于膨胀的形态学重建**

参见图9.26结果

a b c d
e f g h

FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.



9.5.9 形态学重建

◈ **基于腐蚀的形态学重建**

标记图像***F***对模板***G***的基于腐蚀的形态学重建计为$R_G^E(F)$**,**它被定义为***F***关于***G***的反复迭代***k***次的测地腐蚀

$$R_G^E(F) = E_G^{(k)}(F)$$

直至到稳定状态$E_G^{(k)}(F) = E_G^{(k+1)}(F)$。

9.5.9 形态学重建

◆ **应用实例**

1）基于重建的开操作

在开操作中，腐蚀会删除小的物体，而后续的膨胀试图恢复遗留物体的形状。这种恢复的准确性高度依赖于物体的形状和所用结构元的相似性。基于重建的开操作可正确恢复腐蚀后所保留物体的形状。图像$\boldsymbol{F}$的大小为$\boldsymbol{n}$的基于重建的开操作定义如下：

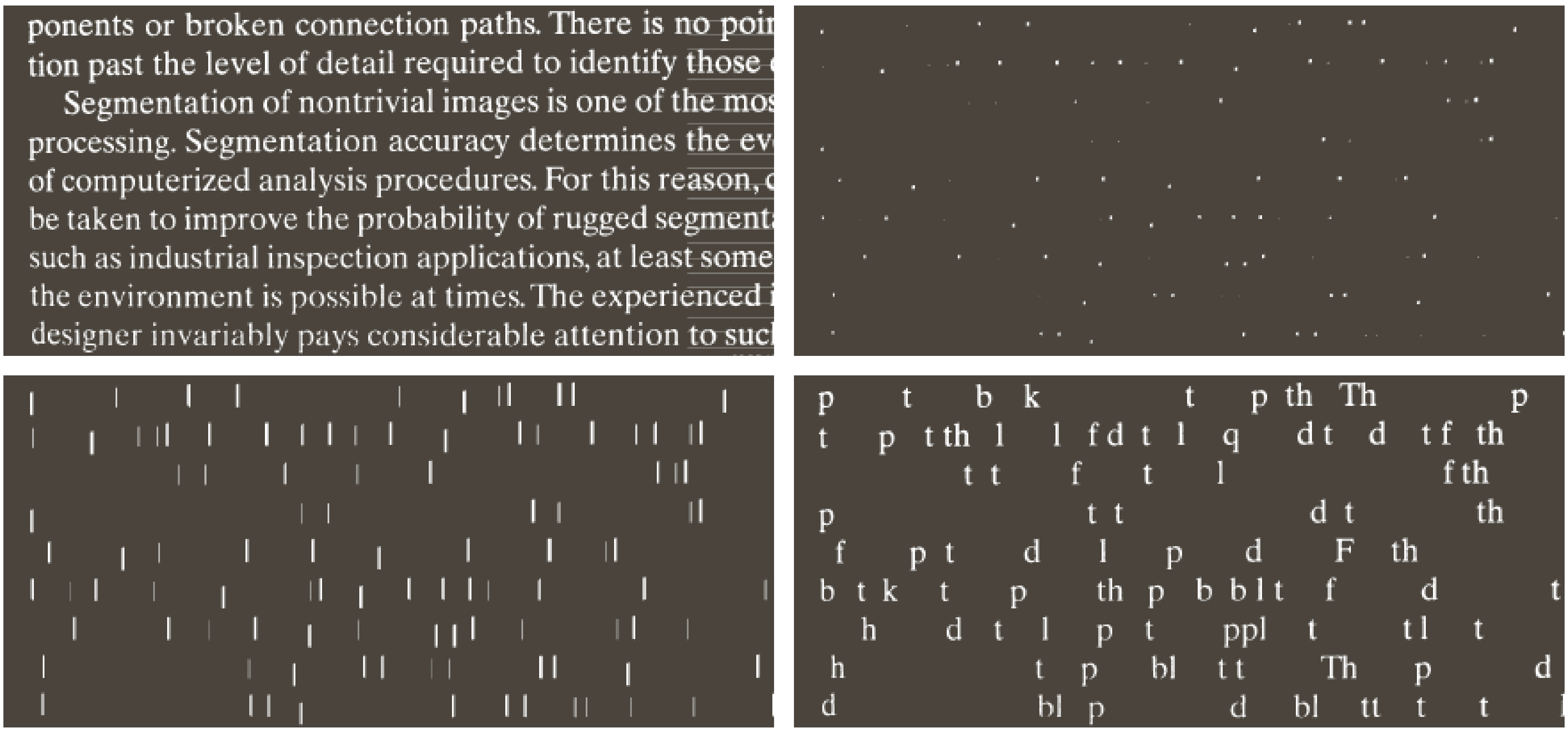
$$O_R^{(n)}(F) = R_F^D[(F \ominus nB)] = D_F^{(k)}((F \ominus nB))$$

$(F \ominus nB)$表示$\boldsymbol{B}$对$\boldsymbol{F}$的$\boldsymbol{n}$次腐蚀。注意这个应用中$\boldsymbol{F}$是模板图像。

9.5.9 形态学重建

◆ 应用实例

1）基于重建的开操作



a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

9.5.9 形态学重建

◈ 应用实例

2）孔洞填充

在9.5.2节，我们开发了一种填充孔洞的算法，该算法在每个孔洞中需要一个已知起点。这里，我们给出一个基于形态学重建的全自动过程。令$I(x, y)$代表一幅二值图像，形成如下标记图像F，该标记图像除了在边界位置为1−I之外，在其它位置均为0，即

$$F(x,y)=\begin{cases}1-I(x,y), & (x,y)\text{在}I\text{的边界上}\\ 0, & \text{其它}\end{cases}$$

则

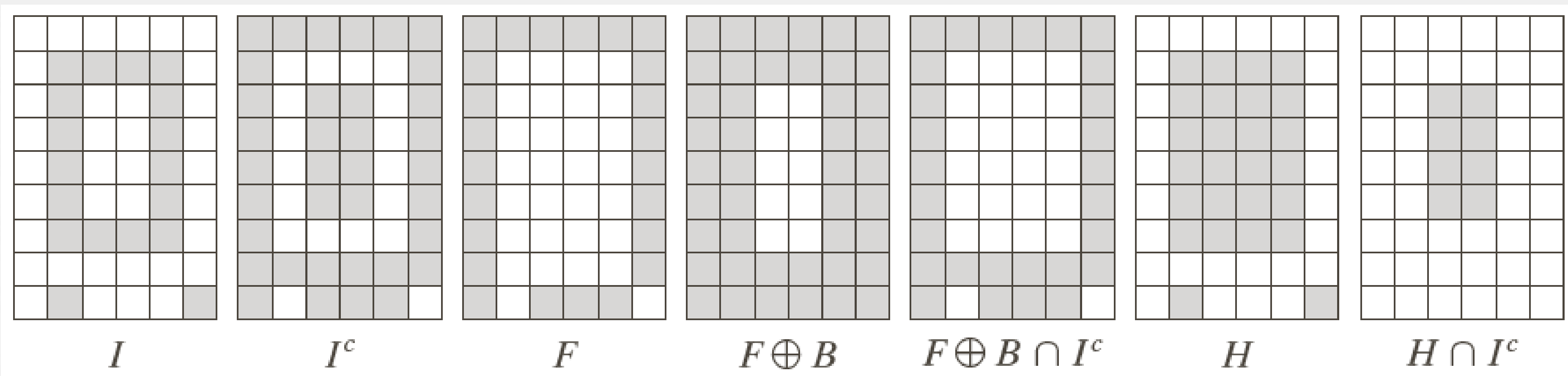
$$H=\left[R_{I^c}^{D}(F)\right]^c$$

是一幅等于所有I的孔洞被填充的图像。

9.5.9 形态学重建

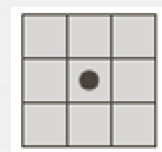
◈ 应用实例

2）孔洞填充



a b c d e f g

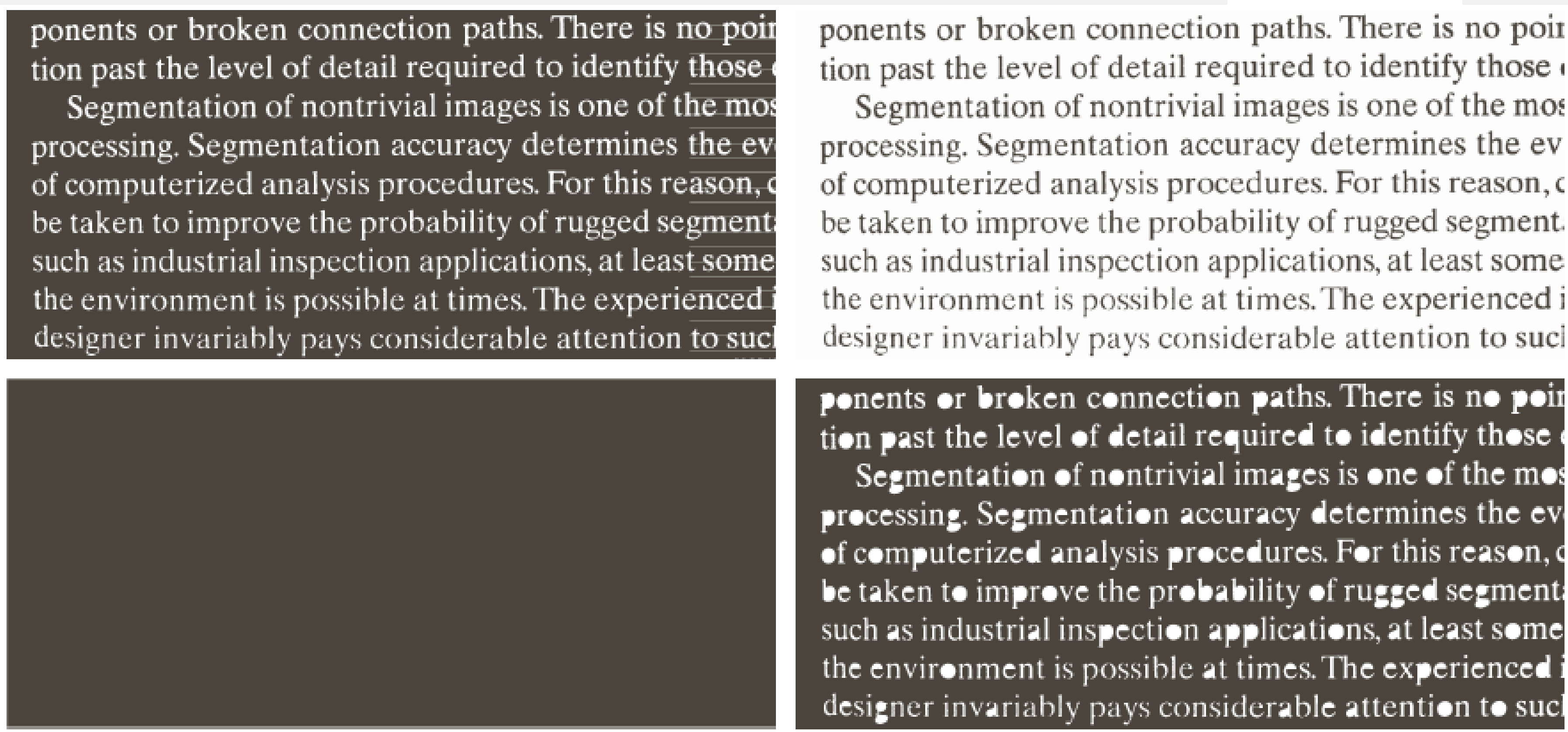
FIGURE 9.30 Illustration of hole filling on a simple image.



9.5.9 形态学重建

◆ 应用实例

2）孔洞填充



a b
c d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

9.5.9 形态学重建

◆ 应用实例

3）边界清除

在自动图像处理中，提取目标以用于后续的分析是一个基本任务。删除那些接触到边界的物体是一个很有用的工具，因为：（1）可以屏蔽图像，以便于后续只处理保留完整的目标；（2）它可用作部分对象存在视野中的一个信号。以原始图像I作为模板图像，标记图像如下

$$F(x,y)=\begin{cases} I(x,y), (x,y)\text{在}I\text{的边界上} \\ 0, \quad \text{其它} \end{cases}$$

则边界清除算法首先计算$R_I^D(F)$，然后计算差

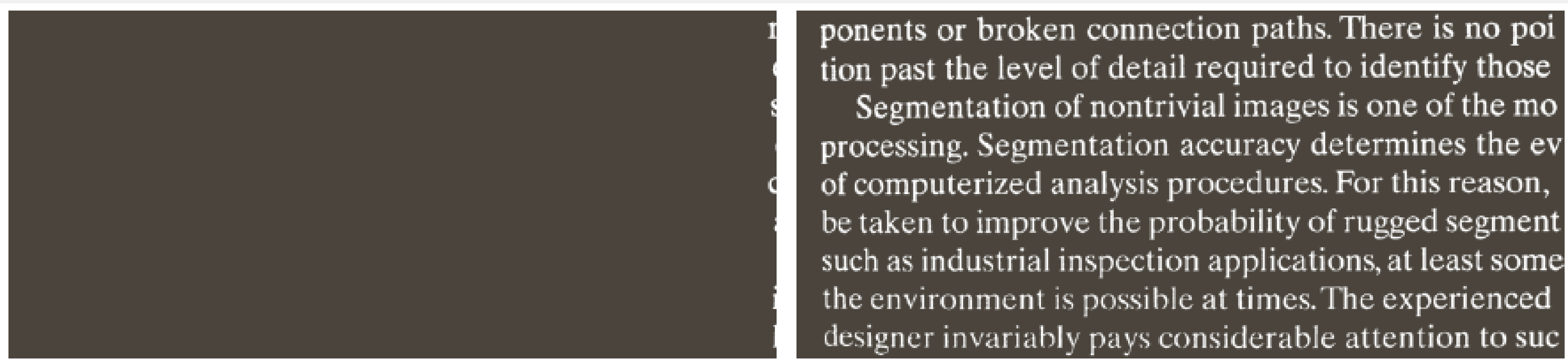
$$X = I - R_I^D(F)$$

得到一幅不接触边界的图像X。

9.5.9 形态学重建

◆ 应用实例

3）边界清除



a b

FIGURE 9.32
Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

9.5.10 关于二值图像的形态学运算总结

各种形态学处理的结构元基本类型

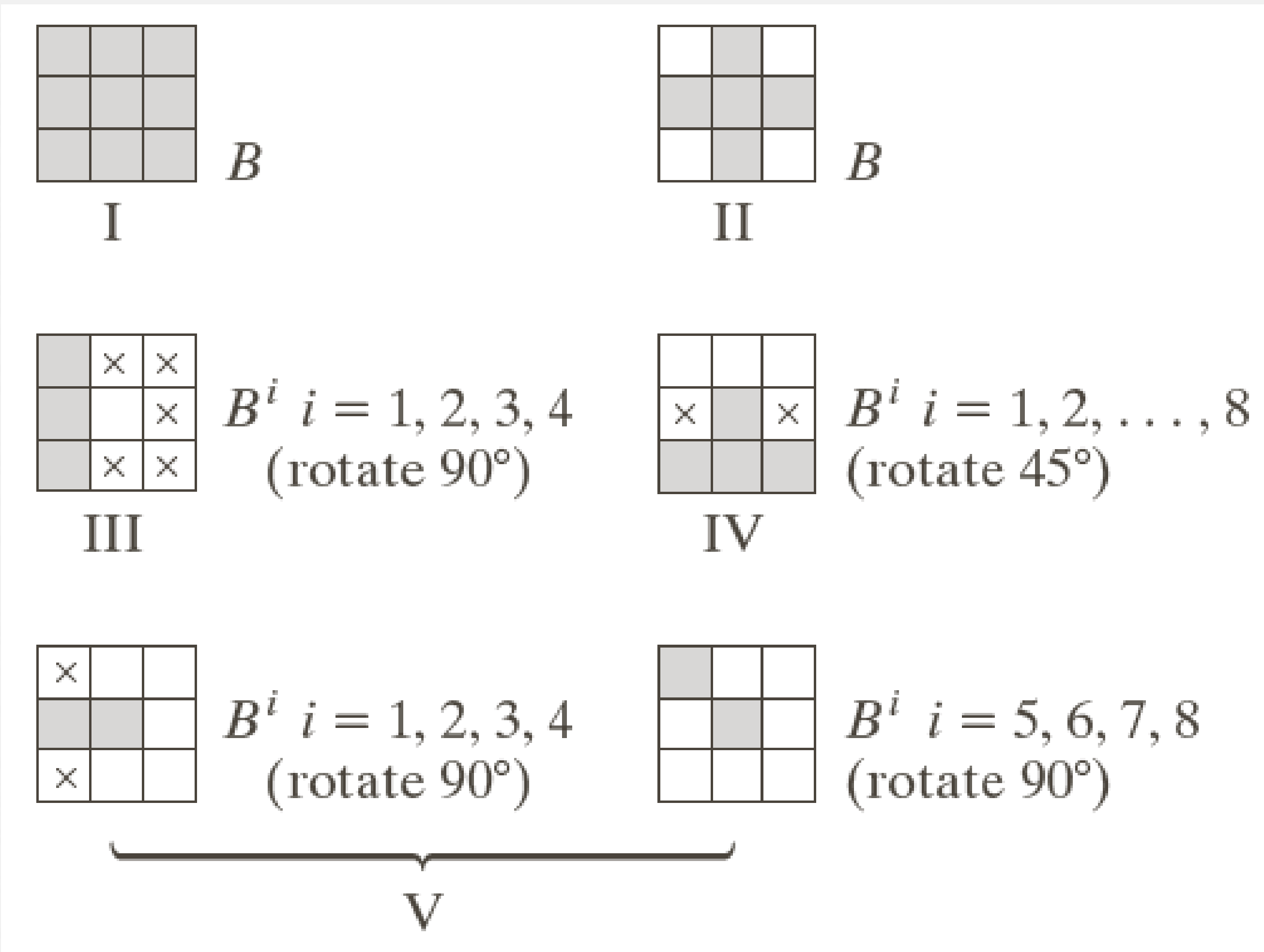


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

表9.2 形态学操作及其性质的总结

TABLE 9.1 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; $X_0 =$ array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; $X_0 =$ array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(Continued)

TABLE 9.1 *(Continued)*

TABLE 9.1 (Continued)

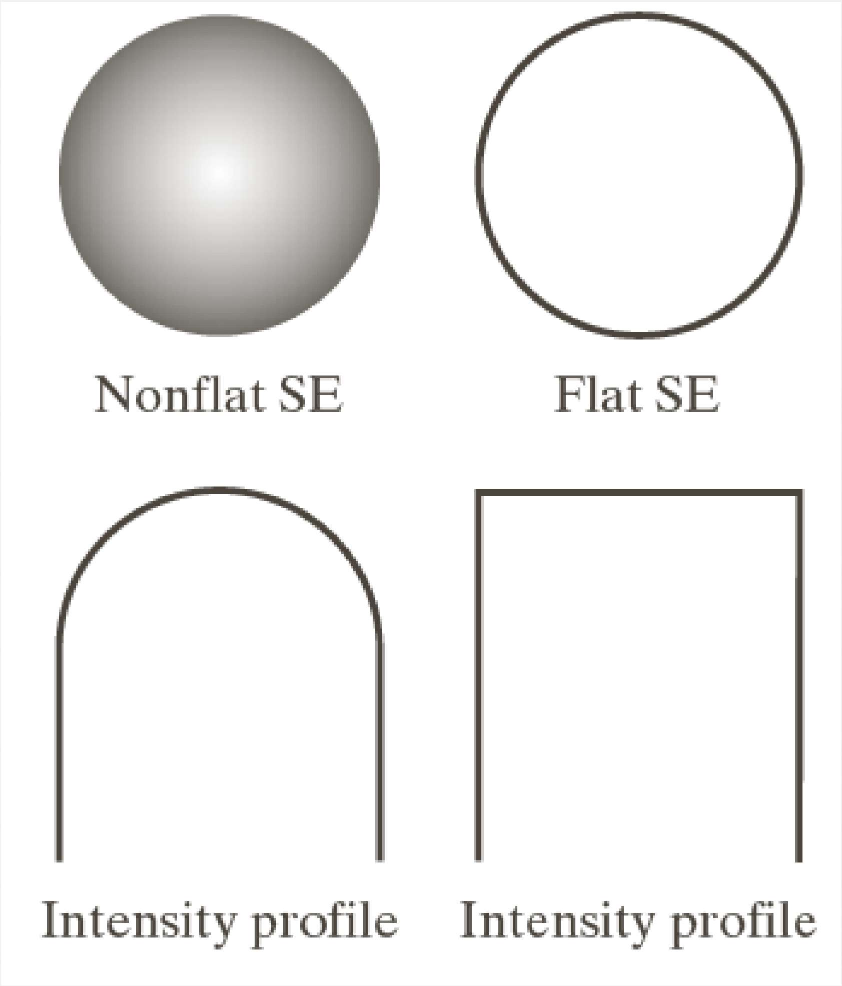
Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

9.6 灰度级形态学

1. 将形态学处理扩展到灰度图像的基本操作,即膨胀、腐蚀、开操作和闭操作;
2. 介绍基于形态学梯度运算的边界提取算法、基于纹理内容的区域分割算法、形态学图像平滑处理、Top-hat变换等。

处理形如$f(x, y)$和$b(x, y)$的图像，$f(x, y)$是输入图像、$b(x, y)$是结构元素。

9.6 灰度级形态学



a	b
c	d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

9.6.1 腐蚀和膨胀

平滑结构元b在任意位置(x, y)对图像f进行的腐蚀定义如下：

$$[f \ominus b](x, y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

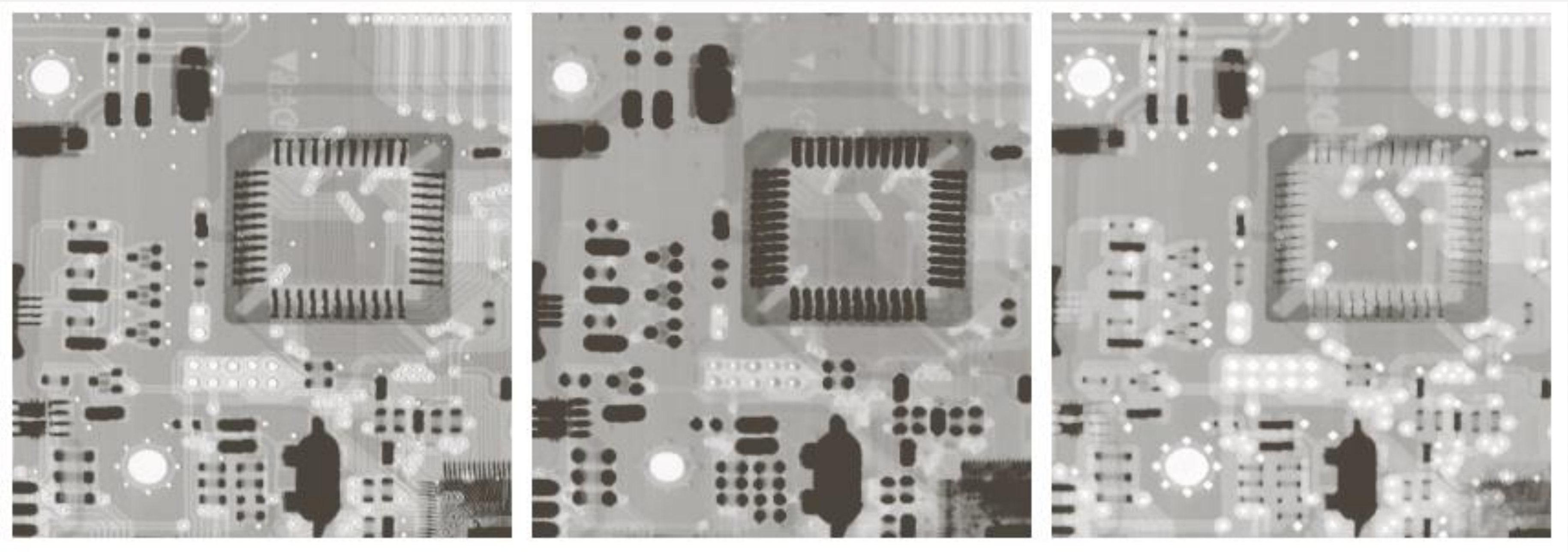
即，结构元b的原点和图像任意位置(x, y)重合，取结构元所覆盖范围内的最小值。

平滑结构元b在任意位置(x, y)对图像f进行的膨胀定义为其反射$\hat{b}$的原点位于(x, y)时，$\hat{b}$所覆盖范围内的最大值。即：

$$[f \oplus b](x, y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$

注意，上式用到了$\hat{b} = b(-x, -y)$。

9. 6. 1 腐蚀和膨胀



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

9.6.1 腐蚀和膨胀

非平滑结构元b_N在任意位置(x,y)对图像f进行的腐蚀定义如下：

$$[f \ominus b_N](x,y) = \min_{(s,t)\in b_N} \{f(x+s,y+t) - b_N(s,t)\}$$

即，结构元b的原点和图像任意位置(x,y)重合，取结构元所覆盖范围内像素与b_N之差的最小值（有意义的b_N比较难选择）。

同样，非平滑结构元b在任意位置(x,y)对图像f进行的膨胀定义为：

$$[f \oplus b_N](x,y) = \max_{(s,t)\in b_N} \{f(x-s,y-t) + b_N(s,t)\}$$

b_N的值为常量时，退化为平滑结构元。

9.6.2 开操作和闭操作

灰度图像的开操作和闭操作与二值图像的对应操作具有相同的形式。

使用结构元素b对图像f进行开操作的定义为：

$$f \circ b = (f \ominus b) \oplus b$$

即，使用结构元素b对图像f进行先腐蚀，然后膨胀。

使用结构元素b对图像f进行闭操作的定义为：

$$f \bullet b = (f \oplus b) \ominus b$$

即，使用结构元素b对图像f进行先膨胀，然后腐蚀。

灰度开操作与闭操作的关系：

$$(f \bullet b)^c = f^c \circ \hat{b}$$

和

$$(f \circ b)^c = f^c \bullet \hat{b}$$

由于$f^c = -f(x,y)$，$(f \bullet b)^c = f^c \circ \hat{b}$亦可写成$-(f \bullet b) = (-f \circ \hat{b})$。

9.6.2 开操作和闭操作

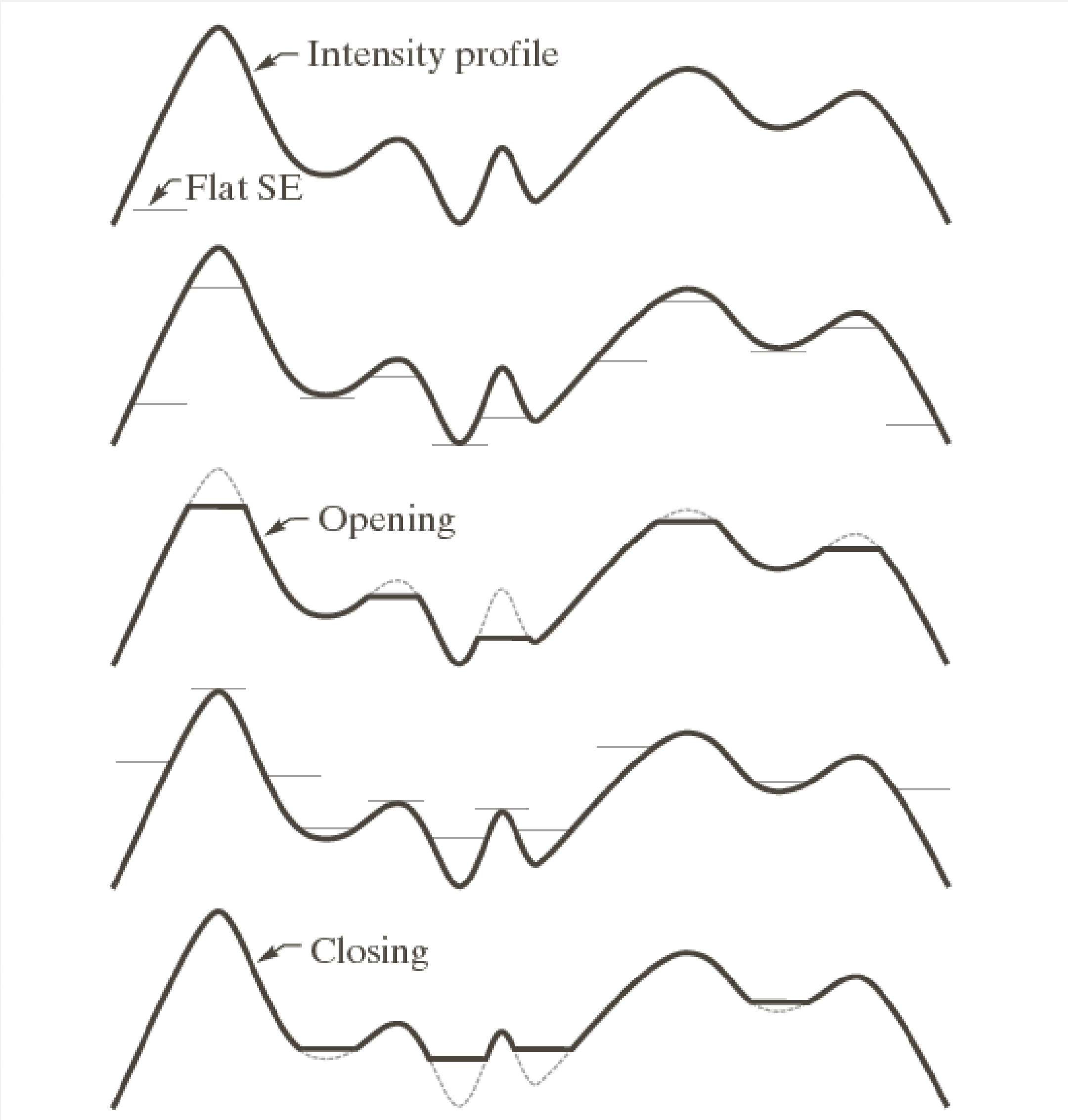


FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

- 开操作经常用于去除小的明亮的细节；
- 闭操作经常用于去除小的黑暗的细节；

- 开操作经常用于去除小的明亮的细节；
- 闭操作经常用于去除小的黑暗的细节；

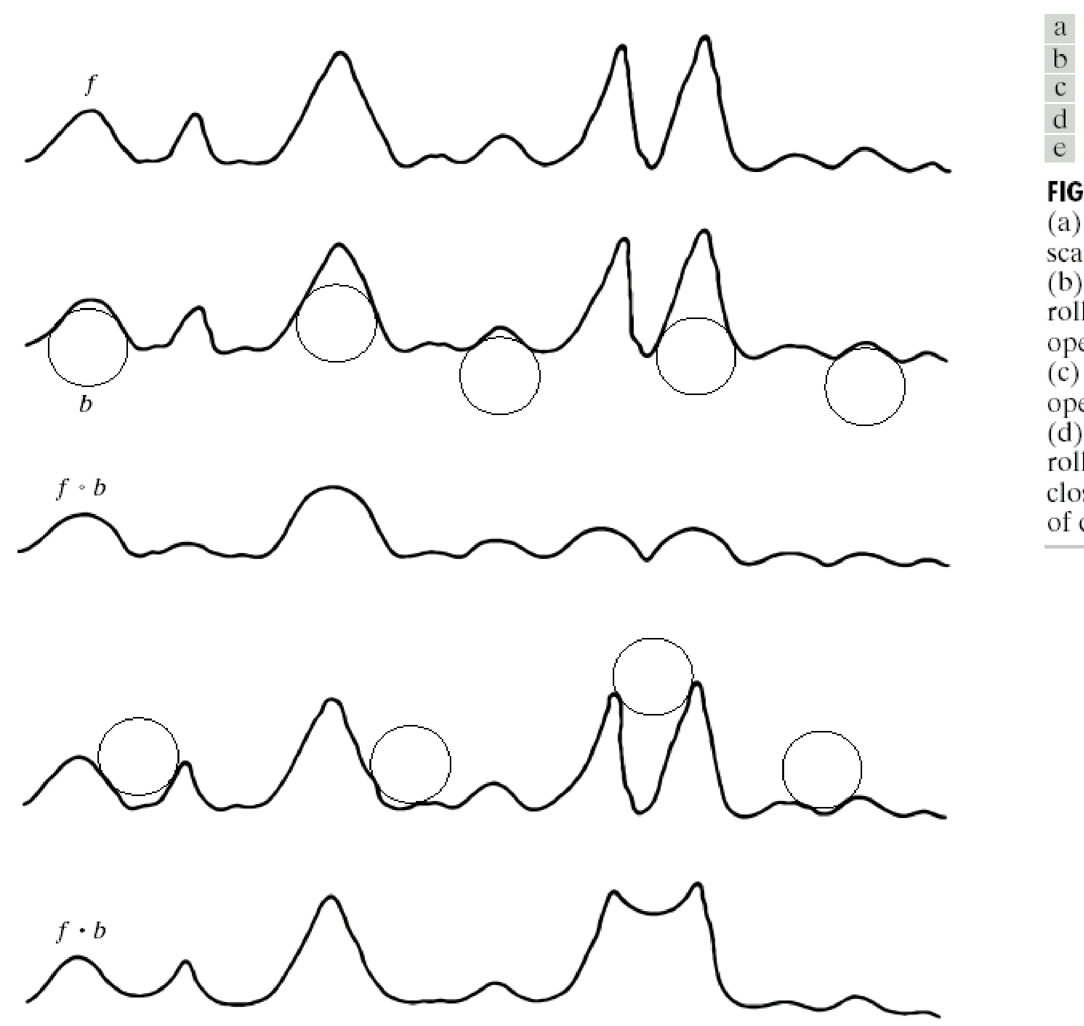
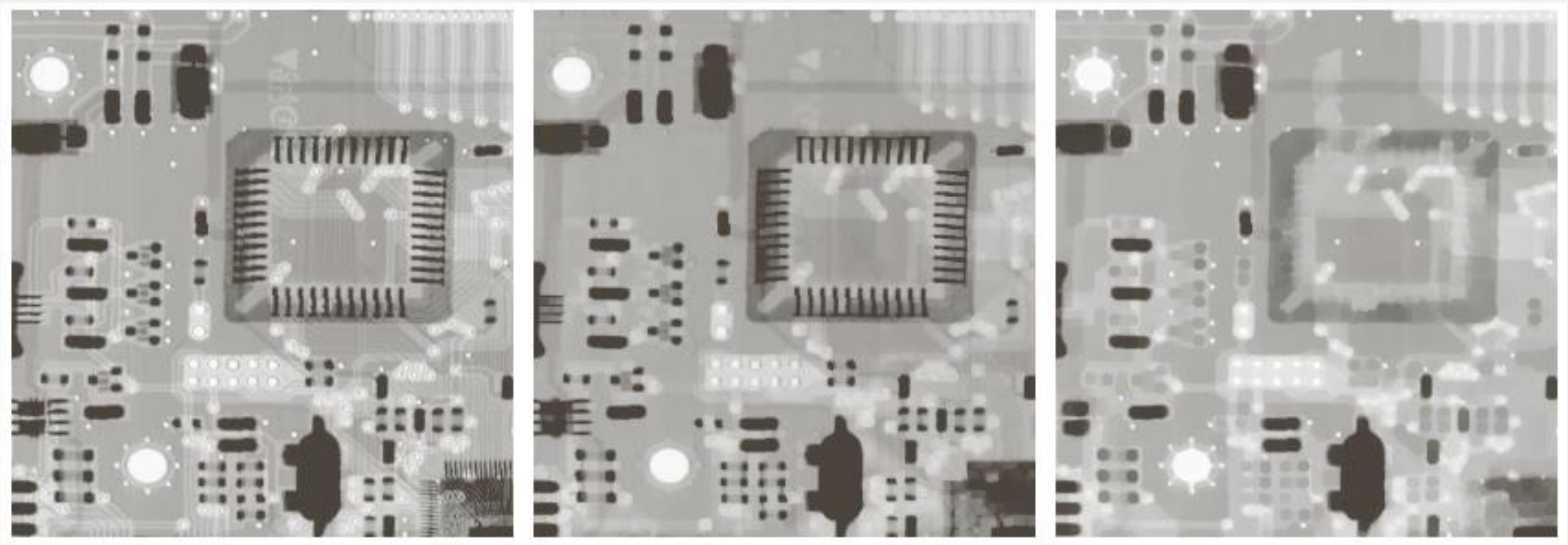


FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

9.6.2 开操作和闭操作



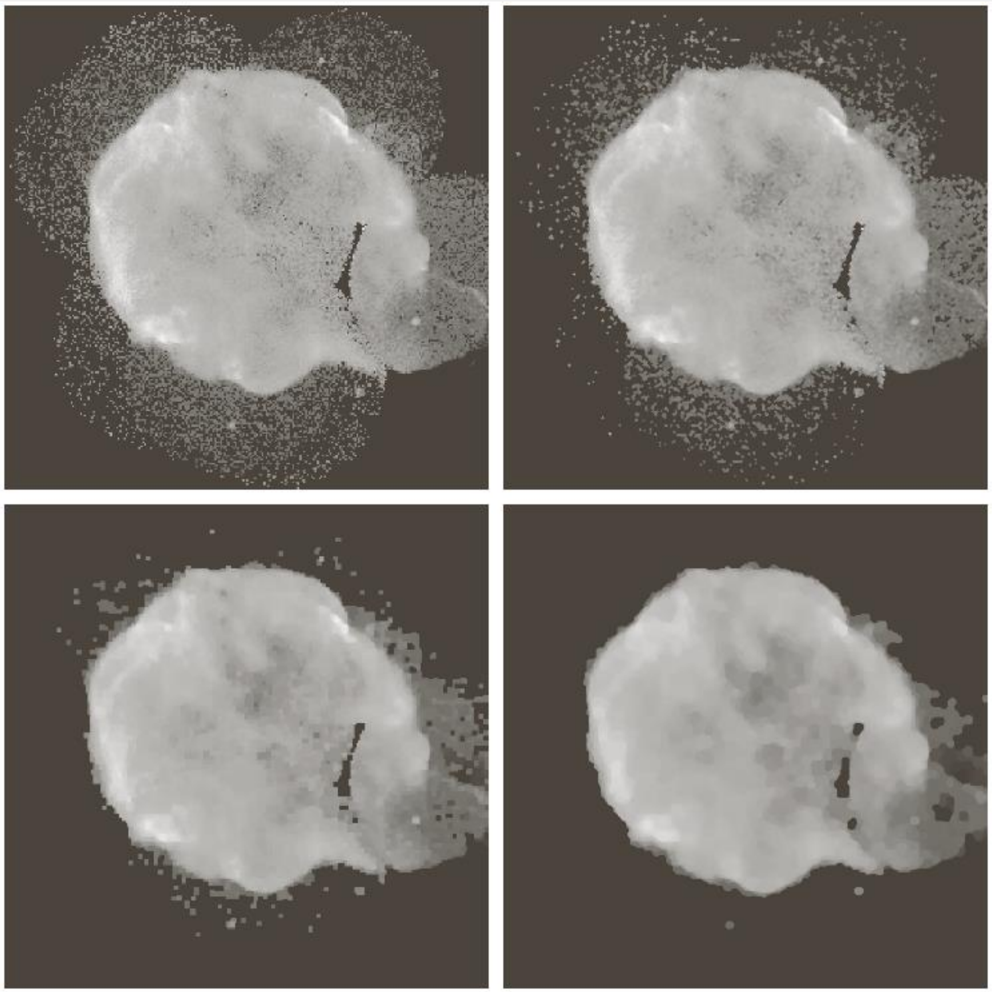
a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

9.6.3 一些基本的灰度形态学算法

(1)形态学图像平滑

先采用开操作，然后采用闭操作以去除亮和暗的噪声。



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

9.6.3 一些基本的灰度形态学算法

(2) 形态学图像梯度

$$g = (f \oplus b) - (f \ominus b)$$

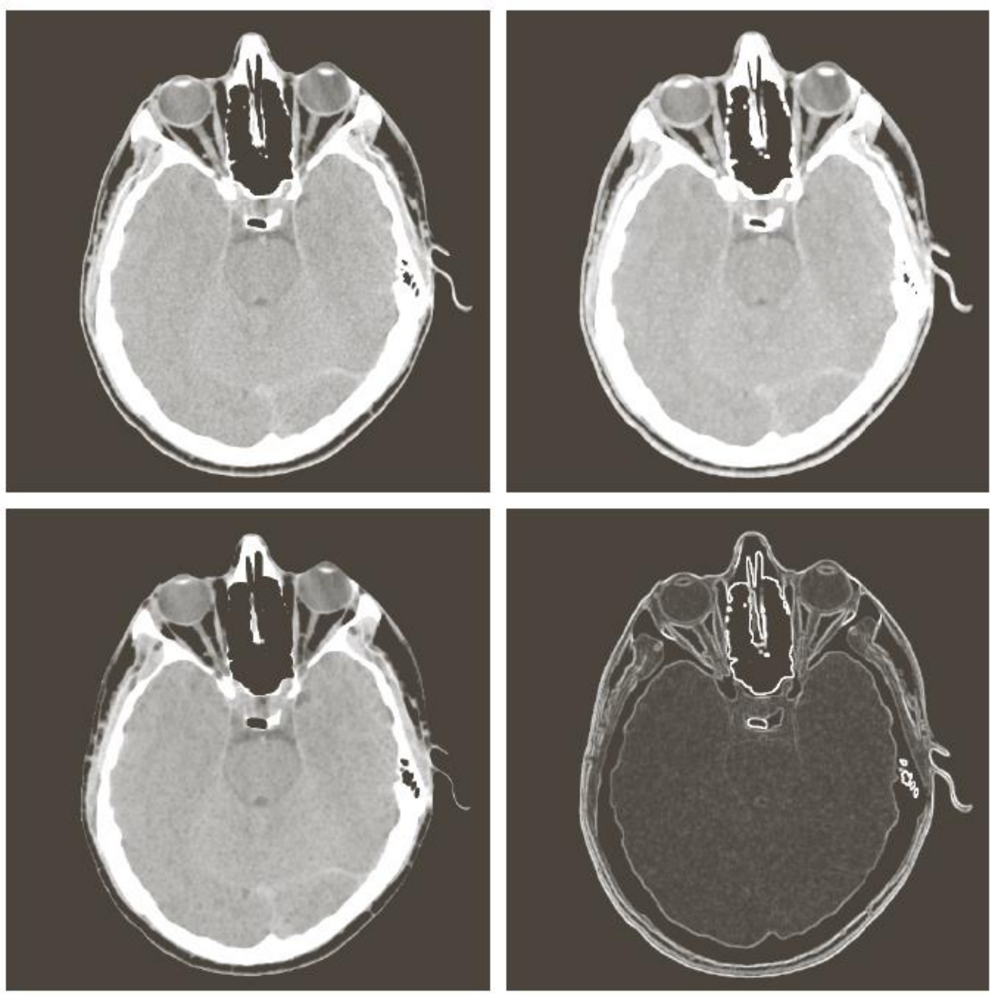


FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

9.6.3 一些基本的灰度形态学算法

(3)形态学top-hat变换

将图像相减与开操作和闭操作结合起来，可得到所谓的顶帽变换和底帽变换。

顶帽变换定义为：

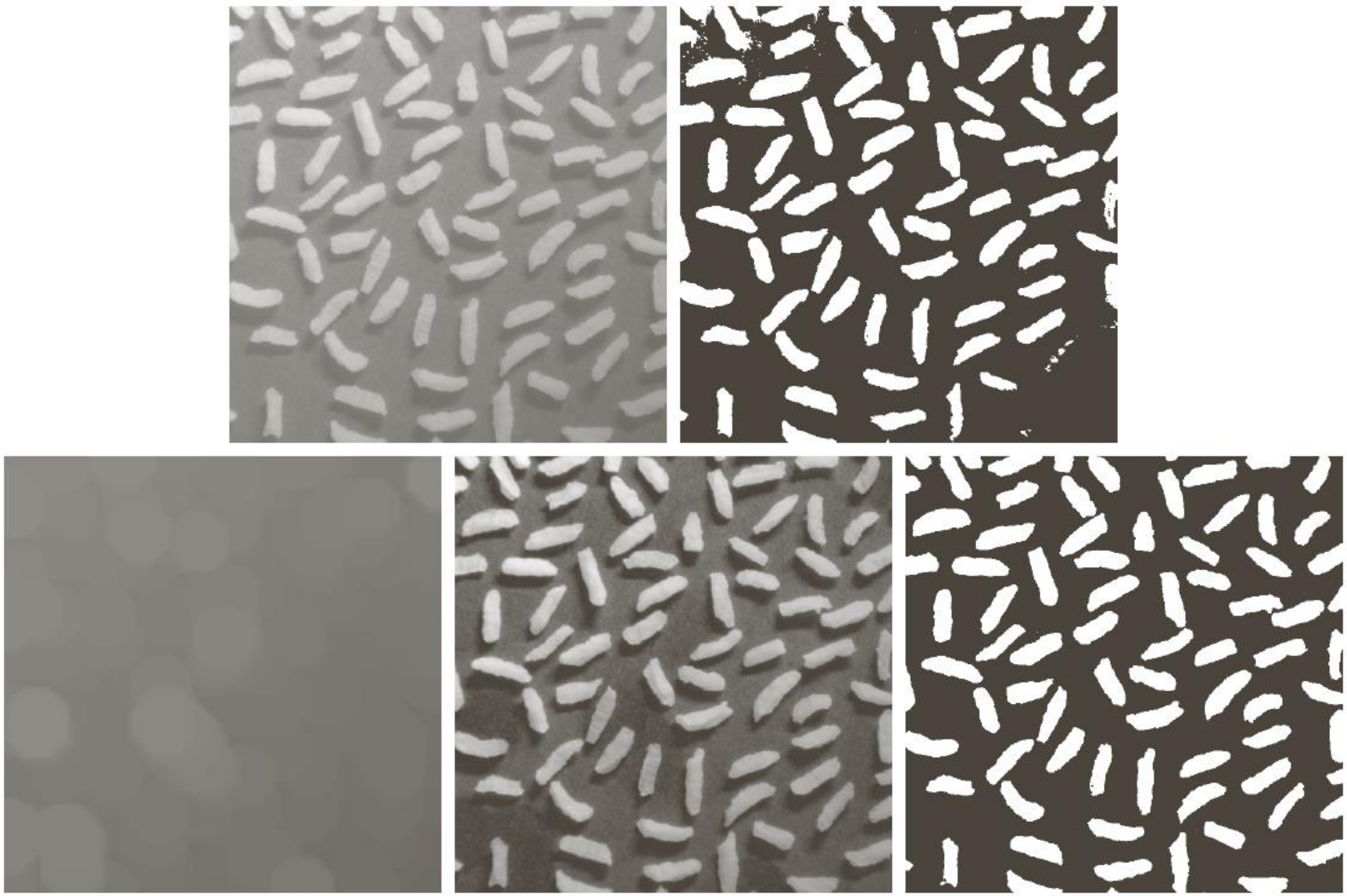
$$T_{hat}(f) = f - (f \circ b)$$

底帽变换的定义为：

$$B_{hat}(f) = (f \bullet b) - f$$

9.6.3 一些基本的灰度形态学算法

(3)形态学top-hat变换

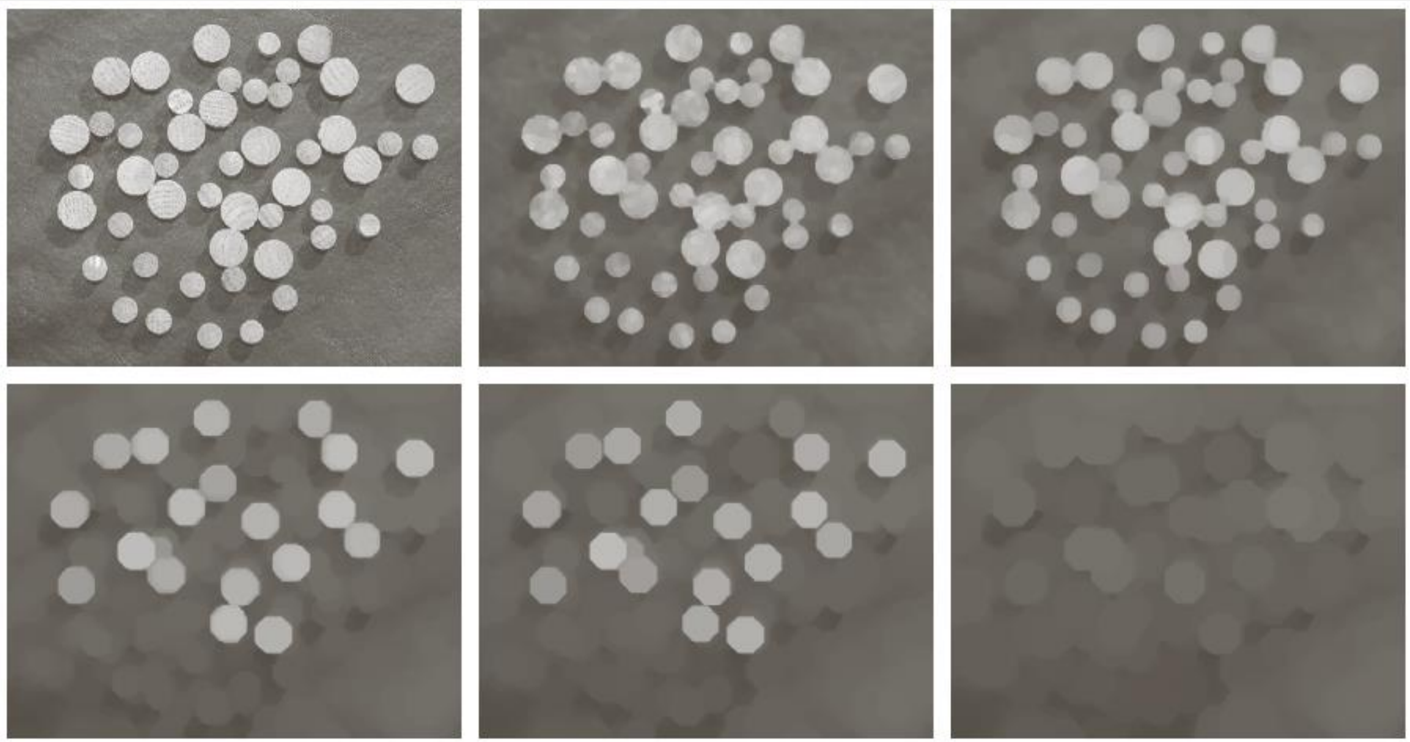


a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

9.6.3 一些基本的灰度形态学算法

(4) 粒度测定



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

原理：以某一特定的尺度对含有相近尺度颗粒的图像区域进行开操作，然后通过计算输入图像和输出图像之间的差异可以对相近尺寸颗粒的相对数量进行测算。

9.6.3 一些基本的灰度形态学算法

（4）粒度测定

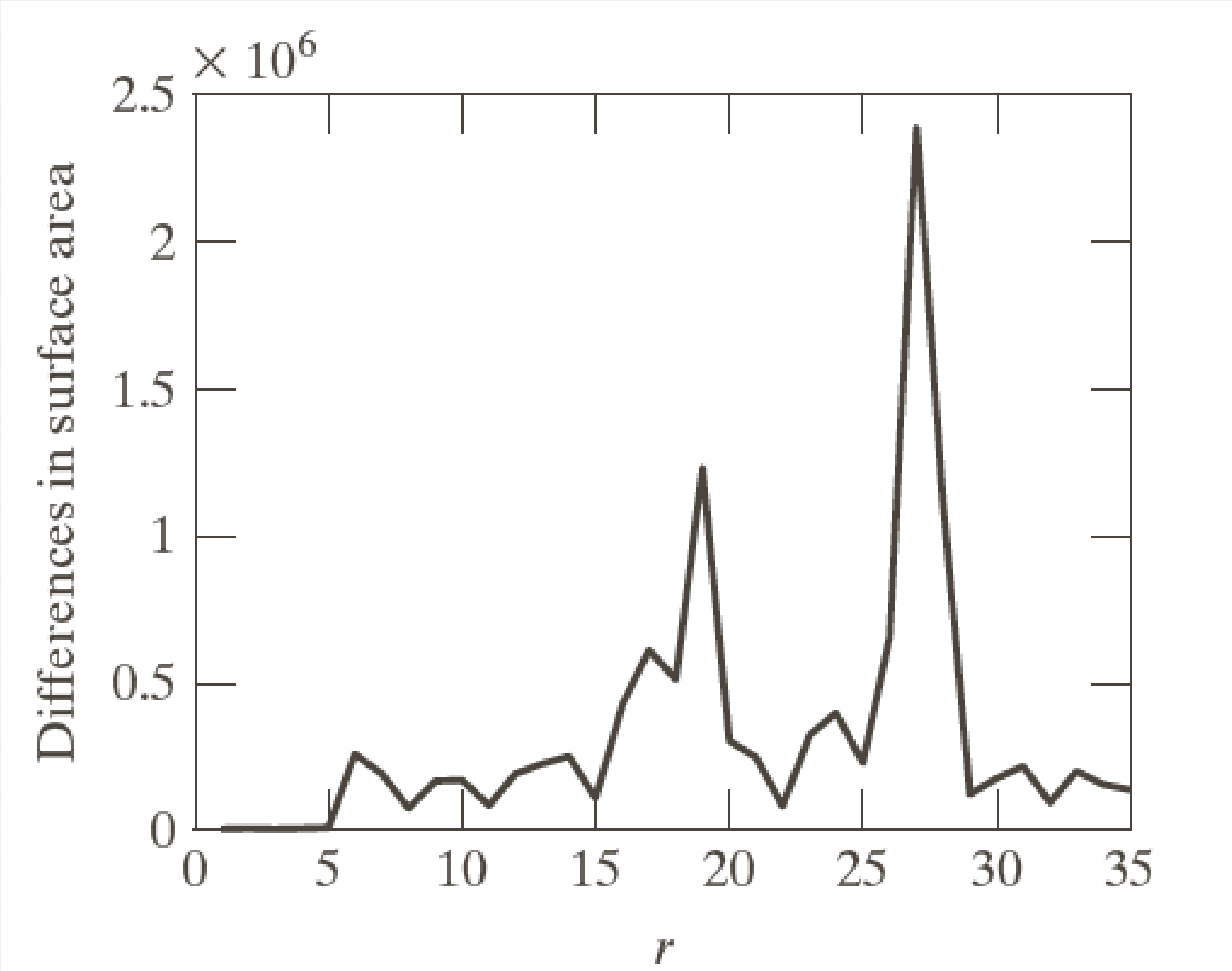


FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

9.6.3 一些基本的灰度形态学算法

(5) 纹理分割

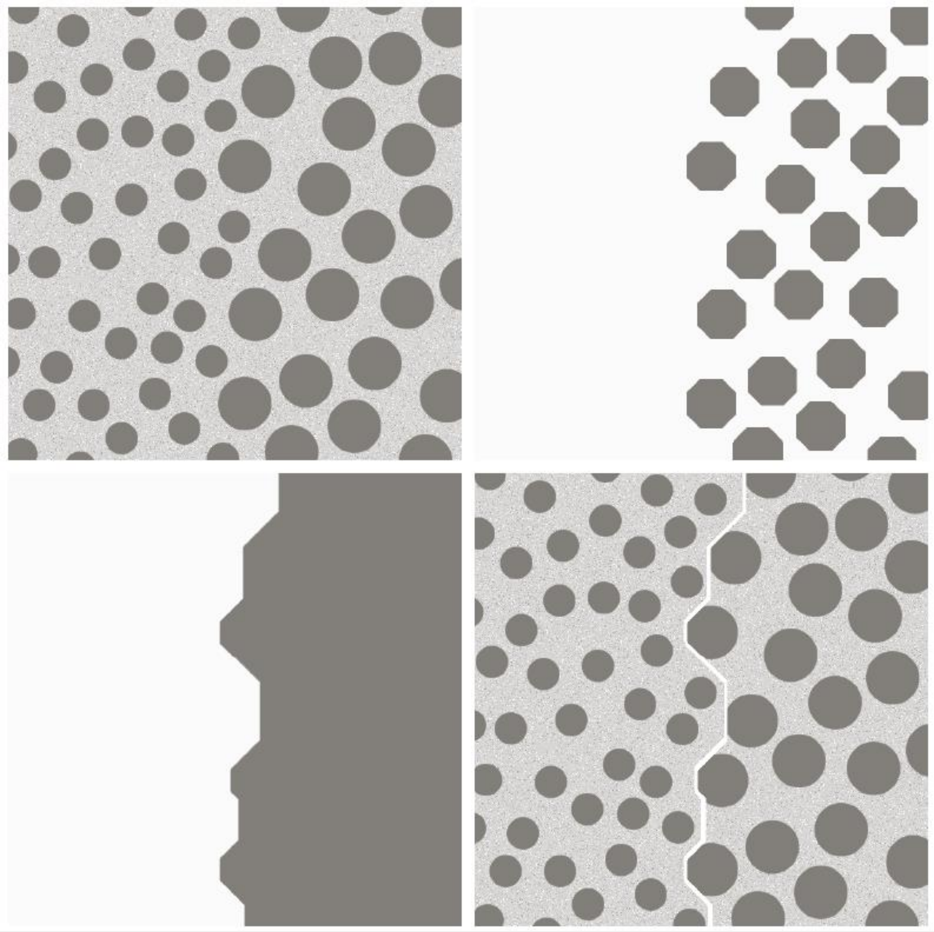


FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

右边区域的圆点直径比左边大。目的是以纹理为基础找到区域的边界。算法如下：

(i) 取尺寸与小斑点同大小（或稍大）的结构元素做闭运算；

(ii) 取比大斑点间隙大的结构元素做开操作；

(iii)做二值化.

作业：9.6、9.18