P.25.2 求下列函数的值域 $f(X)$，其中 X 为题中指定的定义域。 2011 27/6 -6

(1) $f(x) = x^2+1$, $X=(0,3)$

解: $f(0)=1$, $f(3)=10$, $f(X) = (1, 10)$

(2) $f(x) = \ln(1+\sin x)$, $X = (-\frac{\pi}{2}, \pi]$

解: $f(\pi) = \ln(1+\sin\pi) = \ln 1 = 0$, $f(\frac{\pi}{2}) = \ln(1+\sin\frac{\pi}{2}) = \ln 2$.

$$\lim_{x\to -\frac{\pi}{2}+0} f(x) = \lim_{x\to-\frac{\pi}{2}+0} \ln(1+\sin x) = \lim_{u\to 0+0} \ln u = -\infty$$

$f(X) = (-\infty, \ln 2]$

(3) $f(x) = \sqrt{3+2x-x^2}$, $X = [-1, 3]$

解: $f(-1) = \sqrt{3-2-1} = 0$, $f(0)=\sqrt{3}$, $f(3) = \sqrt{3+6-9} = 0$, $f(1)=2$, $f(2)=\sqrt{3}$

$f(X) = [0, 2]$

(4) $f(x) = \sin x + \cos x$, $X = (-\infty, +\infty)$

解: $f(x) = \sin x + \cos x = \sqrt{2}(\sin x\cdot\cos\frac{\pi}{4} + \cos x\cdot\sin\frac{\pi}{4}) = \sqrt{2}\cdot\sin(x+\frac{\pi}{4})$

$f(X) = [-\sqrt{2}, \sqrt{2}]$

P.25.3 求函数值.

(1) 设 $f(x) = \frac{\ln x^2}{\ln 10}+1$; 求 $f(-1)$, $f(-0.001)$, $f(100)$.

解: $f(-1) = \frac{\ln(-1)^2}{\ln 10}+1 = 0+1 = 1$

$f(-0.001) = \frac{\ln(0.001)^2}{\ln 10}+1 = \frac{2\ln\frac{1}{1000}}{\ln 10}+1 = \frac{-2\times 3\ln 10}{\ln 10}+1 = -5$

$f(100) = \frac{\ln 100^2}{\ln 10}+1 = 4+1 = \ln 5$.

(2) 设 $f(x) = \arcsin\frac{x}{1+x^2}$, 求 $f(0)$, $f(1)$, $f(-1) = \arcsin\frac{-1}{1+(-1)^2}$

解: $f(0) = \arcsin 0 = 0$, $f(1) = \arcsin\frac{1}{2} = \frac{\pi}{6}$, $f(-1) = \arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$

(3) $f(x) = \begin{cases} \ln(1-x), & -\infty < x \le 0 \\ -x & 0 < x < +\infty \end{cases}$, 求 $f(-3)$, $f(0)$, $f(5)$.

解: $f(-3) = \ln 4 = 2\ln 2$, $f(0) = \ln(1-0) = \ln 1 = 0$, $f(5) = -5$.

(4) 设 $f(x) = \begin{cases} \cos x, & 0 \le x < 1 \\ \frac{1}{2}, & x=1 \\ 2^x & 1 < x \le 3 \end{cases}$ 求 $f(0)$, $f(1)$, $f(\frac{3}{2})$, $f(2)$.

解: $f(0) = \cos 0 = 1$, $f(1) = \frac{1}{2}$, $f(\frac{3}{2}) = 2^{\frac{3}{2}} = \sqrt{8} = 2\sqrt{2}$, $f(2) = 4$.