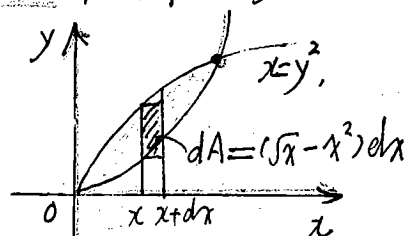


# 题3-5 中山大学 本科生考试草稿纸 2011/4-69.



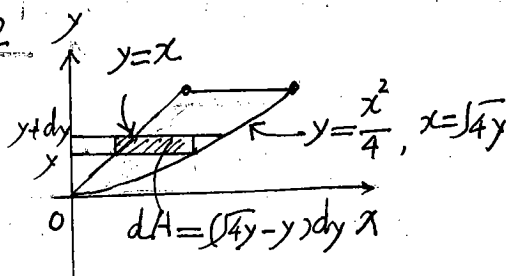
《中山大学授予学士学位工作细则》第七条：“考试作弊者不授予学士学位。”

P.164.1 求面积.  $y = x^2$  与  $x = y^2$



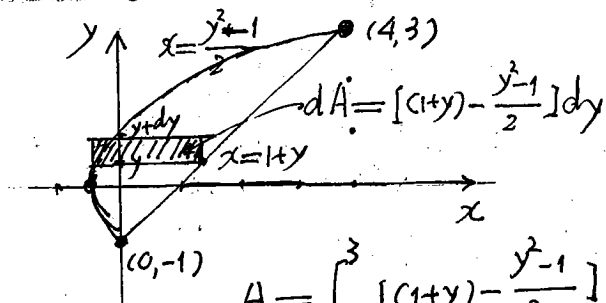
$$A = \int_0^1 (\sqrt{x} - x^2) dx = \frac{2}{3} [x^{\frac{3}{2}}]_0^1 - [\frac{x^3}{3}]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

P.164.2



$$A = \int_0^1 (\sqrt{4y} - y) dy = 2 \cdot \frac{2}{3} [y^{\frac{3}{2}}]_0^1 - [\frac{y^2}{2}]_0^1 = \frac{4}{3} - \frac{1}{2} = \frac{8-3}{6} = \frac{5}{6}$$

P.164.3  $y^2 = 2x+1$  与  $x-y=1$ .



$$\text{交点: } (y-1)^2 = 2x+1 \Rightarrow y^2 - 2y + 1 = 2x+1 \Rightarrow y^2 - 2y = 0 \Rightarrow y(y-2) = 0$$

$$\begin{cases} x=0 \\ y=-1 \end{cases}, \begin{cases} x=4 \\ y=3 \end{cases}$$

$$A = \int_{-1}^3 [(1+y) - \frac{y^2-1}{2}] dy = \int_{-1}^3 (-\frac{y^2}{2} + y + \frac{3}{2}) dy$$

$$= [-\frac{y^3}{6}]_{-1}^3 + [\frac{y^2}{2}]_{-1}^3 + \frac{3}{2} [y]_{-1}^3$$

$$= -\frac{1}{6}(27+1) + \frac{1}{2}(9-1) + \frac{3}{2}(3+1) = -\frac{28}{6} + \frac{8}{2} + \frac{12}{2} = -\frac{28}{6} + \frac{20}{2} = \frac{-28+60}{6} = \frac{16}{3}$$

P.164.4

$$y=0 \text{ 与 } \begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$$

$0 \leq x \leq 2\pi, a > 0$ . 摆线拱.

$$A = \int_0^{2\pi} y dx = \int_0^{2\pi} a(1 - \cos t) \cdot a(1 - \cos t) dt$$

$$= a^2 \int_0^{2\pi} (1 - 2\cos t + \cos^2 t) dt$$

$$= a^2 (2\pi - 0 + \int_0^{2\pi} \frac{1 + \cos 2t}{2} dt) = a^2 (2\pi + \frac{1}{2} \cdot 2\pi) = 3\pi a^2$$