P.153.1. $\int_{-1}^{1}\frac{x\,dx}{\sqrt{5-4x}}$ 　令 $\sqrt{5-4x}=t$, $5-4x=t^2$, $x=\frac{5-t^2}{4}$ 　2011 14/5 — 62.

$dx=-\frac{t}{2}dt$

$x=-1,\ t=3$

$x=1,\ t=1$

$$=\int_3^1\frac{1}{t}\cdot\frac{5-t^2}{4}\cdot(-\frac{t}{2}dt)$$

$$=\frac{1}{8}\int_1^3(5-t^2)\cdot t\,dt$$

$$=\frac{1}{8}[5t\Big|_1^3-\frac{t^3}{3}\Big|_1^3]=\frac{1}{8}(15-5-\frac{27}{3}+\frac{1}{3})=\frac{1}{8}(10-\frac{26}{3})=\frac{1}{8}\cdot\frac{4}{3}=\frac{1}{6}.$$

P.153.2 $$\int_0^{\ln 2}xe^{-x}dx=-\int_0^{\ln 2}x\,de^{-x}=-[xe^{-x}\Big|_0^{\ln 2}-\int_0^{\ln 2}e^{-x}dx]=-[\ln 2\cdot e^{\ln\frac{1}{2}}-0+e^{-x}\Big|_0^{\ln 2}]$$

$$=-\frac{1}{2}\ln 2-e^{\ln\frac{1}{2}}+e^0=-\frac{1}{2}\ln 2-\frac{1}{2}+1=\frac{1}{2}(1-\ln 2)$$

P.152.3 $$\int_0^1x^2\cdot\sqrt{1-x^2}\,dx=\int_0^{\frac{\pi}{2}}\sin^2t\cdot\cos t\cdot\cos t\,dt=\int_0^{\frac{\pi}{2}}\sin^2t\,dt-\int_0^{\frac{\pi}{2}}\sin^4t\,dt$$

$$=\frac{1}{2}\cdot\frac{\pi}{2}-\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}=\frac{1}{4}\cdot\frac{\pi}{4}=\frac{\pi}{16}.$$

P.152.4 $$\int_0^{\pi}x\cdot\sin x\,dx=-\int_0^{\pi}x\,d\cos x=-[x\cdot\cos x\Big|_0^{\pi}-\int_0^{\pi}\cos x\,dx]$$

$$=-[\pi\cdot\cos\pi-0-\sin x\Big|_0^{\pi}]=-\pi\cdot(-1)=\pi.$$

P.152.5 $$\int\sqrt{x^2+9}\,dx=\int 3\sec t\cdot 3\sec^2t\,dt=9\int\sec^3t\,dt$$

$$=\frac{9}{2}(\sec t\cdot\tan t+\ln|\sec t+\tan t|)+C$$

$$=\frac{9}{2}\cdot[\frac{\sqrt{x^2+9}}{3}\cdot\frac{x}{3}+\ln|\frac{\sqrt{x^2+9}}{3}+\frac{x}{3}|]+C$$

$$=\frac{9}{2}[\frac{x\sqrt{x^2+9}}{9}+\ln|x+\sqrt{x^2+9}|]+C$$

$$\int_0^4\sqrt{x^2+9}\,dx=\frac{9}{2}[\frac{x\sqrt{x^2+9}}{9}+\ln|x+\sqrt{x^2+9}|]_0^4$$

$$=\frac{9}{2}(\frac{4\times 5}{9}+\ln(4+5)-\ln 3)=10+\frac{9}{2}\ln 3.$$