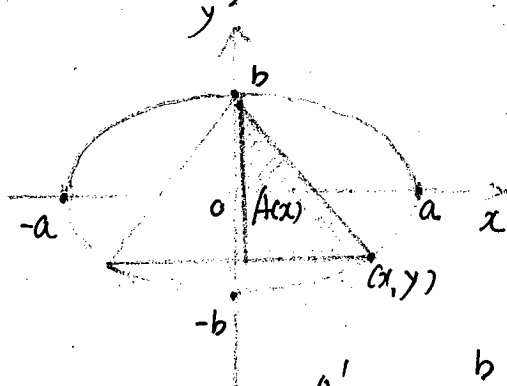


中山大學 本科生考試草稿紙 2011/5-99

警告

《中山大學授予學士學位工作細則》第七條：“考試作弊者不授予學士學位。”

P.207-11. 試求內接于橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, 且底邊平行于 x 軸的最大的等腰三角形的面積。



$$\begin{aligned} \text{解: } A(x) &= \frac{1}{2} |x| \cdot (|x| + b) \\ &= \frac{x}{2} \cdot \left(\frac{b}{a} \sqrt{a^2 - x^2} + b \right) \\ &= \frac{b}{2a} x (\sqrt{a^2 - x^2} + a) \end{aligned}$$

$$\begin{aligned} A'(x) &= \frac{b}{2a} \left[\sqrt{a^2 - x^2} + a + x \cdot \frac{-2x}{2\sqrt{a^2 - x^2}} \right] \\ &= \frac{b}{2a} \left[\sqrt{a^2 - x^2} - \frac{x^2}{\sqrt{a^2 - x^2}} + a \right] \\ &= \frac{b}{2a} \cdot \frac{a^2 - x^2 - x^2 + a\sqrt{a^2 - x^2}}{\sqrt{a^2 - x^2}} = \frac{b}{2a} \cdot \frac{a^2 - 2x^2 + a\sqrt{a^2 - x^2}}{\sqrt{a^2 - x^2}} \end{aligned}$$

$$\begin{aligned} \text{令 } A'(x) &= 0, \text{ 得 } a\sqrt{a^2 - x^2} = a^2 - 2x^2 \\ a^2(a^2 - x^2) &= a^4 - 4a^2x^2 + 4x^4 \\ 4x^4 &= 3a^2x^2 \\ x^2 &= \frac{3a^2}{4}, \quad x = \frac{\sqrt{3}}{2}a. \end{aligned}$$

$$|x| = \frac{b}{a} \sqrt{a^2 - \frac{3}{4}a^2} = \frac{b}{a} \cdot \frac{a}{2} = \frac{b}{2}.$$

$$A_{\text{最大}} = 2 \times \frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \left(b + \frac{b}{2} \right) = \frac{3\sqrt{3}ab}{4}.$$