1. Find the characteristic polynomial and the eigenvalues of the matrix $A$.

$$A=\begin{bmatrix} 3 & -4 \\ 4 & 8 \end{bmatrix}$$

2. It can be shown that the algebraic multiplicity of an eigenvalue $\lambda$ is always greater than or equal to the dimension of the eigenspace corresponding to $\lambda$. Find h in the matrix A below such that the eigenspace for $\lambda = 5$ is two-dimensional:

$$A=\begin{bmatrix} 5 & -2 & 6 & 1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. True or false

a. The determinant of $A$ is the product of the diagonal entries in $A$.

b. An elementary row operation on $A$ does not change the determinant.

c. $(\det A)(\det B) = \det AB$

d. If $\lambda + 5$ is a factor of the characteristic polynomial of $A$, then $5$ is an eigenvalue of $A$.

4. Given two matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}, B=\begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, if $A$ is similar to $B$ (given your reason).

5. Suppose the matrix $A=\begin{bmatrix} 1 & a & -3 \\ -1 & 4 & -3 \\ 1 & -2 & 5 \end{bmatrix}$ has *multiplicity* eigenvalue. If the matrix $A$ could be **Diagonalizable** (given your reason).

6. Suppose $A=\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$, compute $\varphi(A) = A^{10} - 5A^{9}$.

7. Suppose matrix $B=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and matrix $A$ is similar to $B$. Compute the $\mathrm{Rank}(A-2I)+\mathrm{Rank}(A-I)$, where $I$ is the identity matrix.

8. Suppose the vector space $V=\mathbb{R}^3$, $\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$ is a basis of $V$. Given a linear transformation $T$: $V\longrightarrow V$:

$$T(\varepsilon_1)=\varepsilon_1, T(\varepsilon_2)=\varepsilon_1+\varepsilon_2, T(\varepsilon_3)=\varepsilon_1+\varepsilon_2+\varepsilon_3$$

(1) Find β-matrix for $T$, when β is the basis $\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$.

(2) Find β-matrix for $T^{-1}$, when β is the basis $\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$.

(3) Find β-matrix for $T^{-1}$, when β is the basis $\{T(\varepsilon_1),T(\varepsilon_2),T(\varepsilon_3)\}$.

9. Define $T:P_3\to R^4$ by $T(p)=\begin{bmatrix} p(-3) \\ p(-1) \\ p(1) \\ p(3) \end{bmatrix}$.

a. Show that T is a linear transformation.

b. Find the matrix for T relative to the basis $\{1,t,t^2,t^3\}$ for $P_3$ and the standard basis for $R^4$.

10. 课本 P341: 第一题