1. True of False. Whether following vectors could construct a subspace ($\mathrm{R^n}$ is the n- dimensional space).
   a) The n-dimensional vectors in which each elements is integer
   b) The solution of equation $x_1 + x_2 + \cdots + x_n = 0$.
   c) The solution of equation $x_1 + x_2 + \cdots + x_n = 1$.
   d) The n-dimensional vectors in which the first two elements are equal.
   e) The points in the first quadrant.

2. Let $a_1 = (0,1,1)^T, a_2 = (1,0,1)^T, a_3 = (1,1,0)^T$. Prove that $\mathbf{span}\{a_1, a_2, a_3\}$= R^3.

3. Let $a_1 = [1,2,-1,0]^T, a_2 = [1,1,0,2]^T, a_1 = [2,1,1,a]^T$. If dim $\mathbf{span}\{a_1, a_2, a_3\}$=2, what is the value of a?

4. Let W is a real number set in [0, 1]. Then we define the **addition** operator on W: if $f_1 + f_2 \in W$, then $f_1 + f_2$ is a function as $(f_1 + f_2)(x) = f_1(x) + f_2(x)$. Also we define the **multiplication function by scalars** as $(rf)(x) = r \cdot f(x)$ where r is a real number. Proof W is a vector space on R, and given the zero vector in W.

5. Computer the $\mathrm{Nul}\,A$

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$

6. Let $W = \left\{ \begin{bmatrix} s+3t \\ r+s-2t \\ 2r+s \\ 3r-s+t \end{bmatrix} : r,s,t \in R \right\}$. If $W = \mathrm{Col}\,A$, compute the matrix A.

7. Determine whether the vector v either in $\mathrm{Col}\,A$ or in $\mathrm{Nul}\,A$, or in both $\mathrm{Col}\,A$ and $\mathrm{Nul}\,A$.

$$v = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}, \ A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix}$$

8. Let$\{a_1, a_2, a_3\}$ be a basis for R^3, and $b_1=a_1+a_2-2a_3$, $b_2=a_1-a_2-a_3$, $b_3=a_1+a_3$, $\beta= 6a_1-a_2-a_3$. Proof that $\{b_1, b_2, b_3\}$is also a basis for R^3, and find the coordinate vector of β relative to $\{b_1, b_2, b_3\}$