Homework 1

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1.Chap 1.2 Ex 10 (8 point) Are these system specifications consistent? Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system,then they can save new files. If users cannot save new files, then the system software is not being upgraded.

Solution :We write these symbolically: $u \to \neg a, a \to s, \neg s \to \neg u$. Note that we can make all the conclusion true by making a false, s true, and u false. Therefore if the users cannot access the file system, they can save new files, and the system is not being upgraded, then all the conditional statements are true. Thus the system is consistent

2.Chap 1.2 Ex 34 (8 point) Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

Solution :This information is enough to determine the entire system. Let each letter stand for the statement that the person whose name begins with that letter is chatting. Then the given information can be expressed symbolically as follows: $\neg K \to H, R \to \neg V, \neg R \to V, A \to R, V \to K, K \to V, H \to A, H \to K$. Note that we were able to convert all of these statements into conditional statements. In what follows we will sometimes make use of the contrapositives of these conditional statements as well. First suppose that H is true. Then it follows that A and K are true, whence it follows that R and V are true. But R implies that V is false, so we get a contradiction. Therefore H must be false. From this it follows that K is true; whence V is true, and therefore R is false, as is A. We can now check that this assignment leads to a true value for each conditional statement. So we conclude that Kevin and Vijay are chatting but Heather, Randy, and Abby are not.

3.Chap 1.3 Ex 10 (8 point) Show that each of these conditional statements is a tautology by using truth tables.
a)$[\neg p \wedge (p \vee q)\ ] \to q$
b)$[(p \to q) \wedge (q \to r)] \to (p \to r)$
c)$[p \wedge (p \to q)] \to q$
d)$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Solution : omit

4,Chap 1.3 Ex 24,26,30 (10 point)

Solution : omit

5,Chap 1.4 Ex 44 (8 point) Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

Solution : We want propositional functions P and Q that are sometimes, but not always, true (so that the second biconditional is $F \leftrightarrow F$ and hence true), but such that there is an x making one true and the other false. For example, we can take $P(x)$ to mean that x is an even number (a multiple of 2) and $Q(x)$ to mean that $x$is a multiple of 3. Then an example like x = 4 or x = 9 shows that $\forall x(P(x) \leftrightarrow Q(x))$ is false

6,Chap 1.5 Ex 12 (12 point)
a) Jerry does not have an Internet connection.
b) Rachel has not chatted over the Internet with Chelsea.
c) Jan and Sharon have never chatted over the Internet.
d) No one in the class has chatted with Bob.
e) Sanjay has chatted with everyone except Joseph.
f) Someone in your class does not have an Internet connection.
g) Not everyone in your class has an Internet connection.
h) Exactly one student in your class has an Internet connection.
i) Everyone except one student in your class has an Internet connection.
j) Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.
k) Someone in your class has an Internet connection but has not chatted with anyone else in your class.
l) There are two students in your class who have not chatted with each other over the Internet.
m) There is a student in your class who has chatted with everyone in your class over the Internet.
n) There are at least two students in your class who have not chatted with the same person in your class.
o) There are two students in the class who between them have chatted with everyone else in the class.

Solution :

The answers to this exercise are not unique; there are many ways of expressing the same propositions symbolically. Note that $C(x, y)$ and $C(y, x)$ say the same thing.

a) $\neg I(\text{Jerry})$ **b)** $\neg C(\text{Rachel}, \text{Chelsea})$ **c)** $\neg C(\text{Jan}, \text{Sharon})$ **d)** $\neg \exists x\, C(x, \text{Bob})$
e) $\forall x(x \neq \text{Joseph} \leftrightarrow C(x, \text{Sanjay}))$ **f)** $\exists x \neg I(x)$ **g)** $\neg \forall x\, I(x)$ (same as **(f)**)
h) $\exists x \forall y(x = y \leftrightarrow I(y))$ **i)** $\exists x \forall y(x \neq y \leftrightarrow I(y))$ **j)** $\forall x(I(x) \rightarrow \exists y(x \neq y \wedge C(x, y)))$
k) $\exists x(I(x) \wedge \forall y(x \neq y \rightarrow \neg C(x, y)))$ **l)** $\exists x \exists y(x \neq y \wedge \neg C(x, y))$ **m)** $\exists x \forall y\, C(x, y)$
n) $\exists x \exists y(x \neq y \wedge \forall z \neg(C(x, z) \wedge C(y, z)))$ **o)** $\exists x \exists y(x \neq y \wedge \forall z(C(x, z) \vee C(y, z)))$

7,Chap 1.5 Ex 30 (8 point) Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

Solution :

a) $\forall y \forall x \neg P(x,y)$ **b)** $\exists x \forall y \neg P(x,y)$ **c)** $\forall y(\neg Q(y) \vee \exists x\, R(x,y))$
d) $\forall y(\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))$ **e)** $\forall y(\exists x \forall z \neg T(x,y,z) \wedge \forall x \exists z \neg U(x,y,z))$

8,Chap 1.6 Ex 14 (12 point) For each of these arguments, explain which rules of inference are used for each step.

a) Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.
b) Each of five roommates, Melissa, Aaron, Ralph,Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.
c) All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.
d) There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.

Solution :

a) Let $c(x)$ be "x is in this class," let $r(x)$ be "x owns a red convertible," and let $t(x)$ be "x has gotten a speeding ticket." We are given premises $c(\text{Linda})$, $r(\text{Linda})$, $\forall x(r(x) \to t(x))$, and we want to conclude $\exists x(c(x) \wedge t(x))$.

Step	**Reason**
1. $\forall x(r(x) \to t(x))$	Hypothesis
2. $r(\text{Linda}) \to t(\text{Linda})$	Universal instantiation using (1)
3. $r(\text{Linda})$	Hypothesis
4. $t(\text{Linda})$	Modus ponens using (2) and (3)
5. $c(\text{Linda})$	Hypothesis
6. $c(\text{Linda}) \wedge t(\text{Linda})$	Conjunction using (4) and (5)
7. $\exists x(c(x) \wedge t(x))$	Existential generalization using (6)

b) Let $r(x)$ be "r is one of the five roommates listed," let $d(x)$ be "x has taken a course in discrete mathematics," and let $a(x)$ be "x can take a course in algorithms." We are given premises $\forall x(r(x) \to d(x))$ and $\forall x(d(x) \to a(x))$, and we want to conclude $\forall x(r(x) \to a(x))$. In what follows y represents an arbitrary person.

Step	Reason
1. $\forall x(r(x) \to d(x))$	Hypothesis
2. $r(y) \to d(y)$	Universal instantiation using (1)
3. $\forall x(d(x) \to a(x))$	Hypothesis
4. $d(y) \to a(y)$	Universal instantiation using (3)
5. $r(y) \to a(y)$	Hypothetical syllogism using (2) and (4)
6. $\forall x(r(x) \to a(x))$	Universal generalization using (5)

c) Let $s(x)$ be "x is a movie produced by Sayles," let $c(x)$ be "x is a movie about coal miners," and let $w(x)$ be "movie x is wonderful." We are given premises $\forall x(s(x) \to w(x))$ and $\exists x(s(x) \wedge c(x))$, and we want to conclude $\exists x(c(x) \wedge w(x))$. In our proof, y represents an unspecified particular movie.

Step	Reason
1. $\exists x(s(x) \wedge c(x))$	Hypothesis
2. $s(y) \wedge c(y)$	Existential instantiation using (1)
3. $s(y)$	Simplification using (2)
4. $\forall x(s(x) \to w(x))$	Hypothesis
5. $s(y) \to w(y)$	Universal instantiation using (4)
6. $w(y)$	Modus ponens using (3) and (5)
7. $c(y)$	Simplification using (2)
8. $w(y) \wedge c(y)$	Conjunction using (6) and (7)
9. $\exists x(c(x) \wedge w(x))$	Existential generalization using (8)

d) Let $c(x)$ be "x is in this class," let $f(x)$ be "x has been to France," and let $l(x)$ be "x has visited the Louvre." We are given premises $\exists x(c(x) \wedge f(x))$, $\forall x(f(x) \to l(x))$, and we want to conclude $\exists x(c(x) \wedge l(x))$.

In our proof, y represents an unspecified particular person.

Step	Reason
1. $\exists x(c(x) \wedge f(x))$	Hypothesis
2. $c(y) \wedge f(y)$	Existential instantiation using (1)
3. $f(y)$	Simplification using (2)
4. $c(y)$	Simplification using (2)
5. $\forall x(f(x) \to l(x))$	Hypothesis
6. $f(y) \to l(y)$	Universal instantiation using (5)
7. $l(y)$	Modus ponens using (3) and (6)
8. $c(y) \wedge l(y)$	Conjunction using (4) and (7)
9. $\exists x(c(x) \wedge l(x))$	Existential generalization using (8)

9,Chap 1.6 Ex 28 (8 point) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \to R(x))$ are true, then $\forall x(\neg R(x) \to P(x))$ is also true, where the domains of all quantifiers are the same.

solution : We want to show that the conditional statement $\neg R(a) \to P(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $\neg R(a)$ is true for a particular a, then $P(a)$ is also true. For such an a, universal modus tollens applied to the second premise gives us $\neg(\neg P(a) \wedge Q(a))$. By rules from propositional logic, this gives us $P(a) \vee \neg Q(a)$. By universal generalization from the first premise, we have $P(a) \vee Q(a)$. Now by resolution we can conclude $P(a) \vee P(a)$, which is logically equivalent to $P(a)$, as desired.

10,Chap 1.8 Ex 26 (8 point) Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [Hint: Work backward, assuming that you did end up with nine zeros.]

solution : If we were to end up with nine 0s, then in the step before this we must have had either nine 0s or nine 1s, since each adjacent pair of bits must have been equal and therefore all the bits must have been the same. Thus if we are to start with something other than nine 0s and yet end up with nine 0s, we must have had nine 1s at some point. But in the step before that each adjacent pair of bits must have been different; in other words, they must have alternated 0, 1, 0, 1, and so on. This is impossible with an odd number of bits. This contradiction shows that we can never get nine 0s.

11,Chap 1.8 Ex 48 (10 point) Find all squares, if they exist, on an 8 8 checkerboard such that the board obtained by removing one of these square can be tiled using straight triominoes. [Hint: First use arguments based on coloring and rotations to eliminate as many squares as possible from consideration.]

solution : If we study Figure 7, we see that by rotating or reflecting the board, we can make any square we wish nonwhite, with the exception of the squares with coordinates (3, 3), (3, 6), (6, 3), and (6, 6). Therefore the same argument as was used in Example 22 shows that we cannot tile the board using straight triominoes if any one of those other 60 squares is removed. The following drawing (rotated as necessary) shows that we can tile the board using straight triominoes if one of those four squares is removed.