中 山 大 学 软 件 学 院 **2010** 软 件 工 程 专 业 (2010 学年春季学期)

《SE-106 离散数学》 期 末 试 题 试 卷(A)答案

(考 试 形 式： 闭 卷 考 试 时 间：2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：____________________ 姓名：__________ 学号：____________

注意：答案一定要写在答卷中，写在本试题卷中不给分。本试卷要和答卷一起交回。

1. **(10 points)** Let A and B be sets, Prove or disprove the following statements
   (a) $A \cap B = A \cap C$, then B=C
   (b) If $A \cup B = B$ for all any set B, then $A = \Phi$

Solution:

(a) Let $A=\Phi$, B={1}, C={2}, then $A \cap B = A \cap C = \Phi$, but $B \neq C$

(b) Let $B=\Phi$, then $A = A \cup \Phi = B = \Phi$

2. **(10 points)** Determine whether the following statements are tautology

   (a) $\sim P \Rightarrow (p \Rightarrow q)$

   (b) $(p \Rightarrow q) \wedge (p \vee q)$

Solution:

Make the corresponding truth tables as follows

(a) $\sim P \Rightarrow (p \Rightarrow q)$ is a tautology

p	q	~p	p⇒q	$\sim P \Rightarrow (p \Rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(b) $(p \Rightarrow q) \wedge (p \vee q)$ is not a tautology

p	q	$p \Rightarrow q$	$p \vee q$	$(p \Rightarrow q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

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They can also be proved by calculation.

Eg.

$(p \Rightarrow q) \wedge (p \vee q) = (\neg p \vee q) \wedge (p \vee q) = (\neg p \wedge p) \vee q = F \vee q = q$

Obviously q is not a tautology, and neither is $(p \Rightarrow q) \wedge (p \vee q)$.

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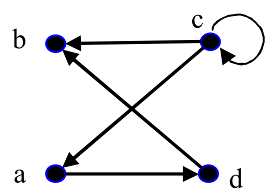
3. **(15 points)** Let A={a, b, c, d}, and R={(a, d), (c, a), (c, b), (c, c), (d, b)}
   (a) Construct the digraph of R
   (b) Show the corresponding matrix M_R and then compute $M_R{}^2$
   (c) Give the transitive closure of R

Solution:

(a)



(b)

$$M_R = \begin{bmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&0 \\ 0&1&0&0 \end{bmatrix}, \quad M_{R^2} = (M_R)^2_{\odot} = \begin{bmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&0 \\ 0&1&0&0 \end{bmatrix} \odot \begin{bmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&0 \\ 0&1&0&0 \end{bmatrix} = \begin{bmatrix} 0&1&0&0 \\ 0&0&0&0 \\ 1&1&1&1 \\ 0&0&0&0 \end{bmatrix}$$

(c)

$$M_{R^3} = (M_R)^3_{\odot} = (M_{R^3})^2_{\odot} \odot M_R = \begin{bmatrix} 0100 \\ 0000 \\ 1111 \\ 0000 \end{bmatrix} \odot \begin{bmatrix} 0001 \\ 0000 \\ 1110 \\ 0100 \end{bmatrix} = \begin{bmatrix} 0000 \\ 0000 \\ 1111 \\ 0000 \end{bmatrix}$$

$$M_{R^4} = (M_R)^4_{\odot} = (M_R)^3_{\odot} \odot M_R = \begin{bmatrix} 0000 \\ 0000 \\ 1111 \\ 0000 \end{bmatrix} \odot \begin{bmatrix} 0001 \\ 0000 \\ 1110 \\ 0100 \end{bmatrix} = \begin{bmatrix} 0000 \\ 0000 \\ 1111 \\ 0000 \end{bmatrix}$$

$$M_{R^\infty} = M_R \vee M_{R^2} \vee M_{R^3} \vee M_{R^4} = \begin{bmatrix} 0101 \\ 0000 \\ 1111 \\ 0100 \end{bmatrix}$$

Therefore, the transitive closure of R is

$$R^\infty = \{(a,b),(a,d),(c,a),(c,b),(c,c),(c,d),(d,b)\}$$

4. **(10 points)** Let S = {1, 2, 3, 4, 5} and A=S×S. Define the following relation R on A: (a, b) R (a', b') if and only if b=b'.
   (a) Show that R is an equivalence relation
   (b) Compute A/R

   Solution:

   (a) Proof:
   1. R is reflexive
      For any (a, b) in A, (a, b) R (a, b) since b=b.
   2. R is symmetric
      If (a, b) R (a', b'), then b=b'. Therefore, we have (a', b') R (a, b), since b'=b.
   3. R is transitive
      If (a, b) R (a', b'), and (a', b') R (a'', b''), then we have b=b' and b'=b'', and thus b=b'', therefore (a, b) R (a'', b'')

   (b) A/R= { { (1, 1), (2, 1), (3, 1), (4, 1), (5, 1) }; { (1, 2), (2, 2), (3, 2), (4, 2), (5, 2) }; (1, 3), (2, 3), (3, 3), (4, 3), (5, 3) }; { (1, 4), (2, 4), (3, 4), (4, 4), (5, 4) }; (1, 5), (2, 5), (3, 5), (4, 5), (5, 5) } }

5. **(10 points)** Let A={1, 2, 3, 4, 5, 6} and

$$p = \begin{pmatrix} 1\ 2\ 3\ 4\ 5\ 6 \\ 3\ 1\ 2\ 5\ 4\ 6 \end{pmatrix}$$

be a permutation of A

(a) Write p as a product of disjoint cycles;

(b) Compute p^{-1} and p^2.

Solution:

(a)

$$p=\begin{pmatrix}1&2&3&4&5&6\\3&1&2&5&4&6\end{pmatrix}=(1,2,3)\circ(4,5)$$

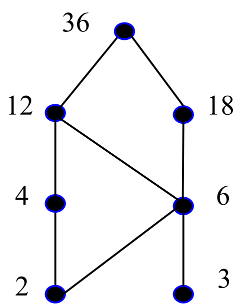
(b)

$$p^{-1}=\begin{pmatrix}1&2&3&4&5&6\\2&3&1&5&4&6\end{pmatrix},\quad p^2=\begin{pmatrix}1&2&3&4&5&6\\3&1&2&5&4&6\end{pmatrix}\circ\begin{pmatrix}1&2&3&4&5&6\\3&1&2&5&4&6\end{pmatrix}=\begin{pmatrix}1&2&3&4&5&6\\2&3&1&4&5&6\end{pmatrix}$$

6. **(10 points)** A={2, 3, 4, 6, 12, 18, 36} with the partial order of divisibility
   (a) Draw the corresponding Hasse diagram;
   (b) Determine the greatest, least, maximal and minimal elements, if they exist, of the poset.

Solution:

(a)

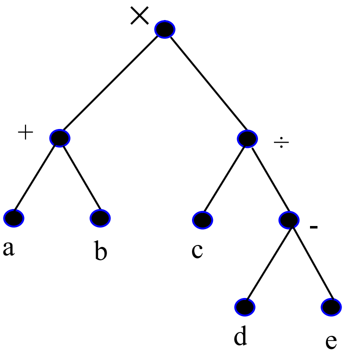


(b) The greatest and maximal element are 36, the minimal elements are 2 and 3. There is no least element.

7. **(15 points)** Consider the completely parenthesized expression (a + b) × (c÷(d-e))
   (a) Show a tree representation of the expression;
   (b) Travel the tree in (a) using POSTORDER algorithm;
   (c) Let a=1, b=2, c=3, d=4, e=3, calculate the expression (a + b) × (c ÷ (d-e)) according to the string obtained in (b) step by step.

Solution:

(a)

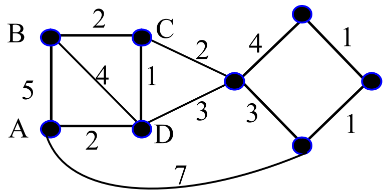


(b) The postfix or reverse polish: a b + c d e - ÷ ×

(c) If a=1, b=2, c=3, d=4 and e=3, the expression is evaluated in the following sequence of steps.

1. **1 2 +** 3 4 3 - ÷ ×
2. 3 3 **4 3 -** ÷ × replacing 1 2 + by 3 since 1 +2=3
3. 3 **3 1 ÷** × replacing 4 3 – by 1 since 4-3=1
4. **3 3 ×** replacing 3 1 ÷ by 3 since 3÷1=3
5. 9 replacing 3 3 × by 9 since 3×3=9

8. **(10 points)** Consider the following weight graph

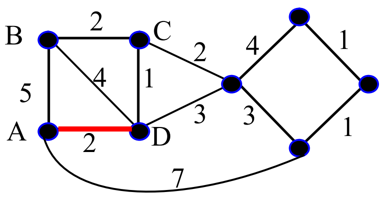


(a) Use Prim's Algorithm to find a minimal spanning tree (start at vertex A)

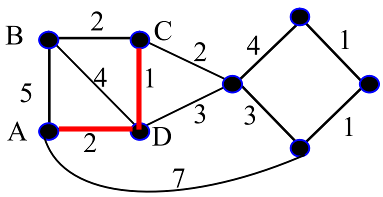
(b) Use Kruskal's Algorithm to find a minimal spanning tree

Solution:

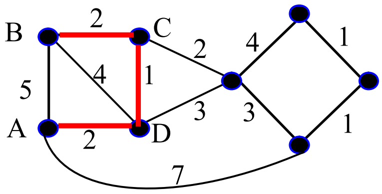
(a) Note that the solution is not unique.



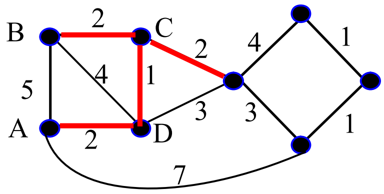
Step 1



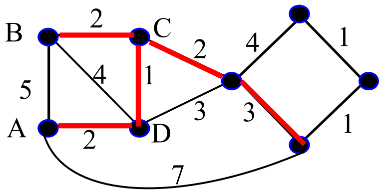
Step 2



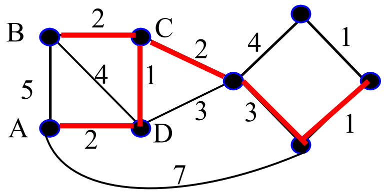
Step 3



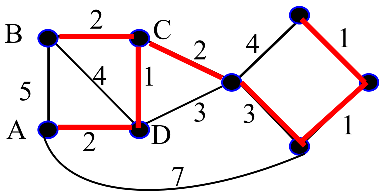
Step 4



Step 5



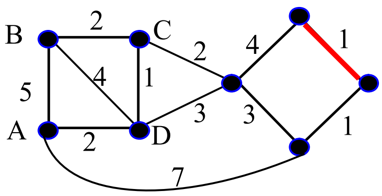
Step 6



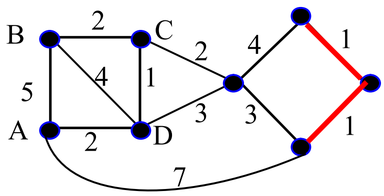
Step 7

The total cost is 12.

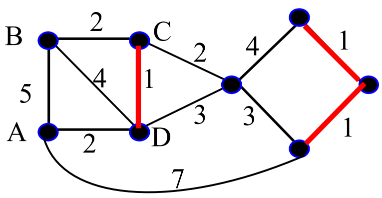
(b) Note that the solution is not unique.



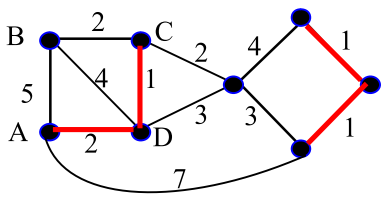
Step 1



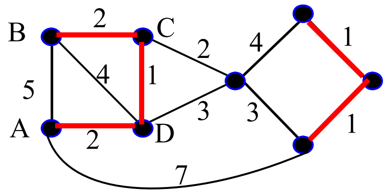
Step 2



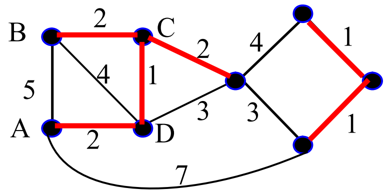
Step 3



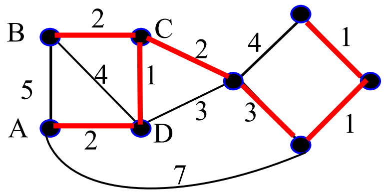
Step 4



Step 5



Step 6



Step 6

The total cost is 12.

9. **(10 points) consider the following graphs**

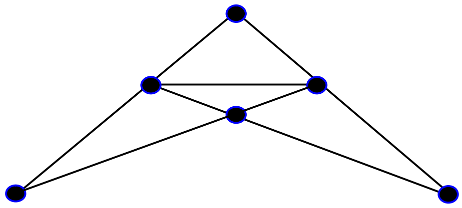


Fig. 1

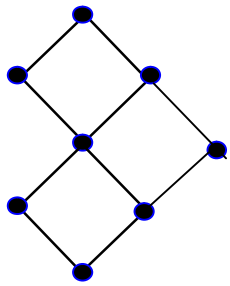


Fig. 2

(a) Determine which of the graphs has a Euler circuit, and give reasons for your choice;

(b) Use Fleury's algorithm to produce an Euler circuit for the graph in (a)

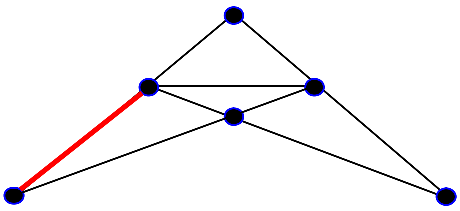
Solution:

(a)

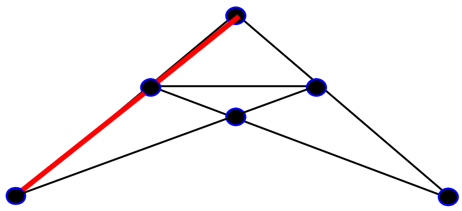
In Fig.1, each vertex has even degree. Therefore, there is an Euler circuit.

In Fig.2, there are two vertices of odd degree. Therefore, there is no Euler circuit.

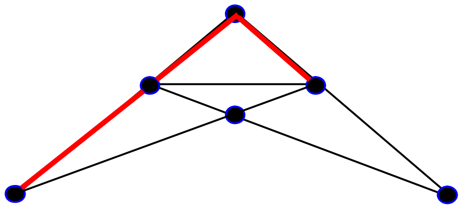
(b) Note the solution is not unique.



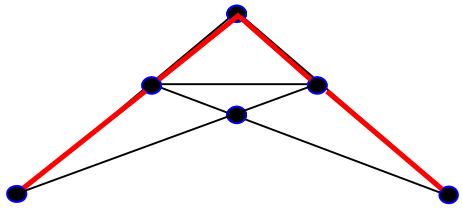
Step 1



Step 2



Step 3



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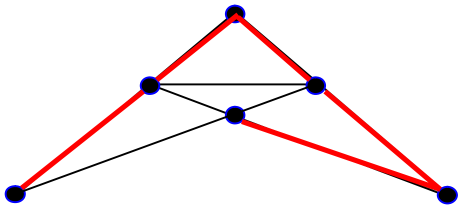
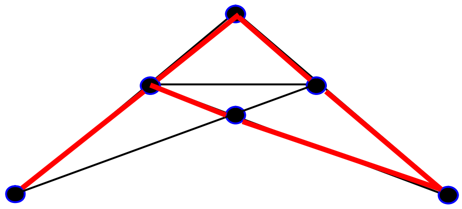
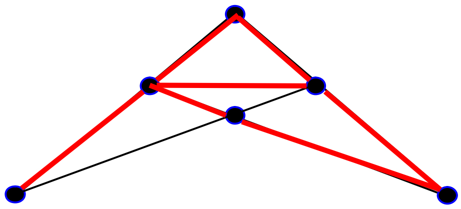


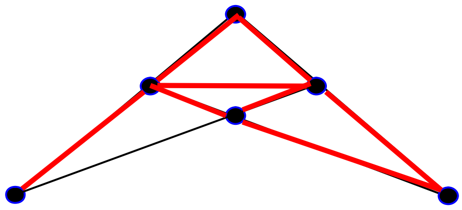
Step 5



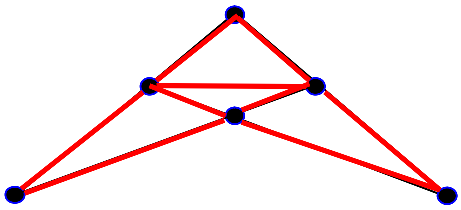
Step 6



Step 7



Step 8



Step 9