

2、

**解**  $E(Y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y f(x, y) dx dy = \int_1^{+\infty} dx \int_{1/x}^x \frac{1}{2x^3 y} dy = \frac{3}{4}$

$$E\left(\frac{1}{XY}\right) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{xy} f(x, y) dx dy = \int_1^{+\infty} dx \int_{1/x}^x \frac{1}{2x^4 y^3} dy = \frac{3}{5}$$

3、

$$E(X) = \int_{-1}^0 x(1+x) dx + \int_0^1 x(1-x) dx = 0,$$

$$E(X^2) = \int_{-1}^0 x^2(1+x) dx + \int_0^1 x^2(1-x) dx = \frac{1}{6},$$

于是  $D(X) = E(X^2) - [E(X)]^2 = \frac{1}{6} - 0^2 = \frac{1}{6}.$

4、

$$D(2X^3 + 5) = D(2X^3) + D(5)$$

$$= 4D(X^3)$$

$$= 4[E(X^6) - (E(X^3))^2]$$

$$E(X^6) = (-2)^6 \times \frac{1}{3} + 0^6 \times \frac{1}{2} + 1^6 \times \frac{1}{12} + 3^6 \times \frac{1}{12} = \frac{493}{6},$$

$$[E(X^3)]^2 = \left[ (-2)^3 \times \frac{1}{3} + 0^3 \times \frac{1}{2} + 1^3 \times \frac{1}{12} + 3^3 \times \frac{1}{12} \right]^2$$

$$= \frac{1}{9},$$

$$\text{故 } D(2X^3 + 5) = 4[E(X^6) - (E(X^3))^2]$$

$$= \frac{2954}{9}.$$

5、

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xf(x, y) dx dy = \int_0^1 dx \int_x^1 x \cdot 8xy dy = \frac{8}{15}$$

$$E(Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} yf(x, y) dx dy = \int_0^1 dx \int_x^1 y \cdot 8xy dy = \frac{4}{5}$$

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x, y) dx dy = \int_0^1 dx \int_x^1 xy \cdot 8xy dy = \frac{4}{9}$$

$$Cov(X, Y) = \frac{4}{9} - \frac{4}{5} \cdot \frac{8}{15} = \frac{4}{225}$$

6、

解：由 $X \sim N(1, 3^2)$ 有 $EX=1, DX=9$ ,  
由 $Y \sim N(0, 4^2)$ 有 $EY=0, DY=16$ , 又由  
 $\rho_{XY} = -\frac{1}{2}$ 有 $\text{Cov}(X, Y) = \rho_{XY} \sqrt{DXDY} = -6$ 。

$$(1) \quad EZ = \frac{1}{3}EX - \frac{1}{2}EY = \frac{1}{3},$$
$$DZ = \frac{1}{9}DX + \frac{1}{4}DY - 2\frac{1}{3}\frac{1}{2}\text{Cov}(X, Y) = 7。$$

$$(2) \quad \text{Cov}(X, Z) = \text{Cov}\left(X, \frac{1}{3}X - \frac{1}{2}Y\right)$$
$$= \frac{1}{3}DX - \frac{1}{2}\text{Cov}(X, Y) = 6,$$

$$\rho_{XZ} = \frac{\text{Cov}(X, Z)}{\sqrt{DXDZ}} = \frac{2\sqrt{7}}{7}.$$

(3)  $X$ 与 $Z$ 不独立。因为  $\rho_{XZ} \neq 0$ 。