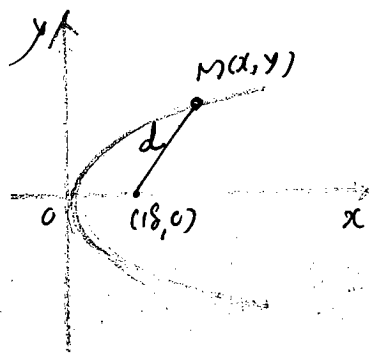


P.209.9. 在曲线 $y^2 = 4x$ 上, 求到点 $(18, 0)$ 的距离最短的点。2011/4-98.

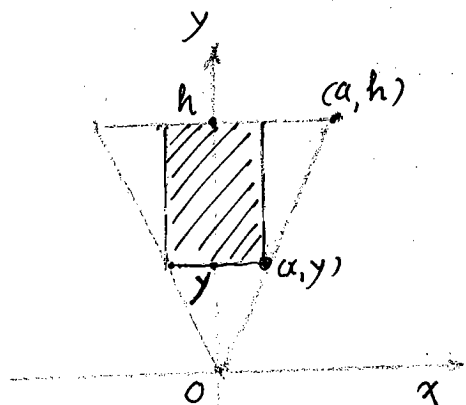


$$d^2 = (x-18)^2 + y^2 = (x-18)^2 + 4x$$

$$(d^2)'_x = 2(x-18) + 4 = 2(x-16)$$

$$\text{令 } (d^2)'_x = 0, \text{ 得 } x=16 \quad \text{则 } \begin{cases} x=16 \\ y=8 \end{cases} \quad \begin{cases} x=16 \\ y=-8 \end{cases}$$

P.207.10 求内接于一定圆锥体, 且有最大体积的圆柱体的高度。



解: 如图, $V_{\text{圆锥}} = \frac{1}{3}\pi a^2 h$

$$y = \frac{h}{a}x$$

$$V_{\text{圆柱}} = \pi x^2(h-y) = \pi x^2(h - \frac{h}{a}x) = \pi h x^2(1 - \frac{x}{a})$$

$$V'(x) = \pi h [2x(1 - \frac{x}{a}) + x^2 \cdot (-\frac{1}{a})]$$

$$= \pi h x (2 - \frac{2x}{a} - \frac{x}{a}) = \pi h x (2 - \frac{3x}{a})$$

$$\text{令 } V'(x) = 0, \text{ 得 } x=0 \text{ (舍)}$$

$$2 - \frac{3x}{a} = 0, \quad x = \frac{2a}{3}$$

$$\text{圆柱体的高 } H = h - y = h - \frac{h}{a} \cdot \frac{2a}{3} = h - \frac{2h}{3} = \frac{h}{3}$$