中山大学软件学院 **2010** 级软件工程专业(2010 学年秋季学期)

《SE-103 线性代数》期末试题(A 卷)

(考试形式：闭 卷 考试时间：2 小时)

警 示

《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：____________ 姓名：________ 学号：________

出卷：______伍丽华______ 复核：______何伟弘______

1. Fill in the blank（10*3=30 Pts）

(1) If the vector x **determined by the coordinate vector** $[x]_B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ **and the basis** $B = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$**, then** x**=____________.**

(2) Let A **be a 4×4 matrix, and suppose the eigenvalues of** A **are 3，1，1，and 2, then** $\det A$ =__________

(3) The matrix $A = \frac{1}{9}\begin{bmatrix} -1 & 4 & a \\ a & 4 & -1 \\ 4 & b & 4 \end{bmatrix}$ **is an orthogonal matrix, then** a =__________, b =________________.

(4) If $v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$**, and** $W = Span\{v_1, v_2\}$**, then the closest point in** W **to** y **is ______________________.**

(5) If the transformation $x \mapsto Ax$ **maps** $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ **and** $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$ **into** $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **and** $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$**,respectively, then the matrix** A=_______________**.**

(6) Assume the matrix A **is row equivalent to** B**.**

$$A=\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for $Row\ A$ **is ____, and** $\dim Nul\ A$ **is____________, and** $rank\ A^T$ **is _________________________.**

(7) $A=\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ **act on** C^n**, the eigenvalues of** A **is** $\lambda=2\pm 3i$**, and a basis for the eigenspace corresponding to** $\lambda_1=2-3i$ **is __________________________.**

2. Mark each statement True or False, and descript your reasons. （5*4=20 Pts）

(1) Let $\boldsymbol{T}:\boldsymbol{R}^n\to\boldsymbol{R}^n$ **be a linear transformation defined by** $T(x)=Ax$**. Then** T **is onto if and only if** $\det(A)\neq 0$**.**

(2) If A **is an** $\boldsymbol{n\times n}$ **matrix, then** $rank\ A<n$ **if and only if zero is an eigenvalue of** A**.**

(3) The set of all solutions to the linear system $Ax=b$**, where** A **is** $\boldsymbol{m\times n}$ **and** $b\neq 0$**, is a subspace of** R^n**.**

(4) Every set of five orthonormal vectors is a basis for R^5**.**

(5) The linear transformation $\boldsymbol{T}:\boldsymbol{P}_2\to\boldsymbol{P}_2$ **defined by** $T(at^2+bt+c)=2at+b$ **is one-to-one.**

3. Calculation issues (5*6=30 Pts）

(1) Let $v_1=\begin{bmatrix}5\\0\\0\\0\end{bmatrix}, v_2=\begin{bmatrix}3\\2\\-2\\-2\end{bmatrix}, v_3=\begin{bmatrix}2\\-1\\1\\1\end{bmatrix}, x=\begin{bmatrix}-2\\3\\0\\-3\end{bmatrix}, y=\begin{bmatrix}0\\3\\6\\-3\end{bmatrix}$**, and** $W=Span\{v_1,v_2,v_3\}$

a. Determine whether ***x*** **is in** W

b. Determine whether ***y*** **is in the orthogonal complement of** W**.**

(2) If A **and** B **are 3×3 matrices,** I **is the identity matrix, and** $AB+I=A^2+B$**, where** $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$**. Find** B**.**

(3) Let $A=\begin{bmatrix}0&4&2\\4&0&-2\\2&-2&3\end{bmatrix}$, $v_1=\begin{bmatrix}-2\\2\\1\end{bmatrix}$, $v_2=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $v_3=\begin{bmatrix}1\\0\\2\end{bmatrix}$**, and** v_1**,** v_2**, and** v_3 **are eigenvectors for** A**.**

a. Diagonalize *A*. (Find an invertible matix *P* and a diagonal matrix *D* such that $A=PDP^{-1}$**)**

b. Use orthogonal matrix to Diagonalize *A*. (Find an orthogonal matrix Q such that $A=QDQ^{T}$**)**

(4) Let $A=\begin{bmatrix}1&-1\\2&5\\3&4\end{bmatrix}$, $b=\begin{bmatrix}-1\\10\\3\end{bmatrix}$**. Find the least-squares solution of** $Ax=b$**, and determine the least-squares error in the least-squares solution of** $Ax=b$**.**

(5) Let $H=\left\{\begin{bmatrix}3a+7b-c\\-5b+8c-2d\\3d-4e\\5b-8c-d+4e\end{bmatrix}: a,b,c,d,e \text{ any real numbers}\right\}$,

a. Show that H **is a subspace of** R^4**.**

b. Find a basis for H **.**

4. Prove issues （2*10=20 Pts）

(1) Let *x* be a unit vector in R^n**, prove that the** $n\times n$ **matrix** $\boldsymbol{A}=\boldsymbol{I}_n-2\boldsymbol{x}\boldsymbol{x}^T$ **is both orthogonal and symmetric.**

(2) Let *x* be a unit vector in R^3**, prove that** $\boldsymbol{A}=\boldsymbol{I}_3-2\boldsymbol{x}\boldsymbol{x}^T$ **has eigenvalues** $\lambda=1$ **and** $\lambda=-1$ **, furthermore, descript the corresponding eigenspaces. (Hint: the transformatiom** $\boldsymbol{v}\mapsto \boldsymbol{A}\boldsymbol{v}$ **reflects points across the plane (span{** x **})**$^{\perp}$**)**