P.129. 21. $\int \sin 2x \cdot \cos 2x \, dx = \frac{1}{2} \int \sin 4x \, dx = -\frac{1}{8} \int \sin 4x \, d(4x) = -\frac{1}{8} \cos 4x + C.$

22. $\int \sin^2 \frac{x}{2} \cos \frac{x}{2} \, dx = 2 \int \sin^2 \frac{x}{2} \cdot \cos \frac{x}{2} \, d\frac{x}{2} = 2 \int \sin^2 \frac{x}{2} \, d\sin\frac{x}{2} = \frac{2}{3} \sin^3 \frac{x}{2} + C.$

23. $\int \sin 5x \cdot \sin 6x \, dx = -\frac{1}{2} \int [\cos(6+5)x - \cos(6-5)x] \, dx$

$= -\frac{1}{2} [\int \cos 11x \, dx - \int \cos x \, dx]$

$= -\frac{1}{2} [\frac{\sin 11x}{11} - \sin x] + C = \frac{1}{2} \sin x - \frac{1}{22} \sin 11x + C.$

24. $\int \frac{2x-1}{\sqrt{1-x^2}} dx = \int \frac{-1}{\sqrt{1-x^2}} d(1-x^2) - \int \frac{dx}{\sqrt{1-x^2}} = -2\sqrt{1-x^2} - \arcsin x + C.$

25. $\int \frac{x^3+x}{\sqrt{1-x^2}} dx = \int \frac{x^3}{\sqrt{1-x^2}} dx - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} d(1-x^2)$

$= \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx^2 - \sqrt{1-x^2} + C$

$= -\frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx^2 - \sqrt{1-x^2} + C$

$= -\frac{1}{2} \int \sqrt{1-x^2} \, d(1-x^2) - \int \frac{1}{2\sqrt{1-x^2}} d(1-x^2) - \sqrt{1-x^2} + C$

$= \frac{1}{3} (1-x^2)^{\frac{3}{2}} - 2\sqrt{1-x^2} + C.$

26. $\int \frac{dx}{(a^2-x^2)^{\frac{3}{2}}}$, 设 $x = a\sin t$, $dx = a \cdot \cos t \, dt$

$= \int \frac{a \cdot \cos t \, dt}{a^3 \cdot \cos^3 t} = \frac{1}{a^2} \int \sec^2 t \, dt = \frac{1}{a^2} \tan t + C = \frac{x}{a^2 \cdot \sqrt{a^2-x^2}} + C$

27. $\int \frac{\sqrt{x^2-a^2}}{x} dx$, 设 $x = a \cdot \sec t$, $\sqrt{x^2-a^2} = \sqrt{a^2\sec^2 t - a^2} = a\tan t$ $x > a$

         $\frac{x}{a} = \sec t = \frac{1}{\cos t}$, $\cos t = \frac{a}{x}$, $dx = a \cdot \sec t \cdot \tan t \, dt$.

$= \int \frac{a \cdot \tan t}{a \cdot \sec t} \cdot a \sec t \cdot \tan t \, dt$

$= a \int \tan^2 t \, dt = a \int (\sec^2 t - 1) \, dt = a(\tan t - t) + C = \sqrt{x^2-a^2} - a \cdot \arccos \frac{a}{x} + C. \quad x > a$

$= \sqrt{x^2-a^2} + a \cdot \arccos \frac{a}{x} + C \quad x < -a$