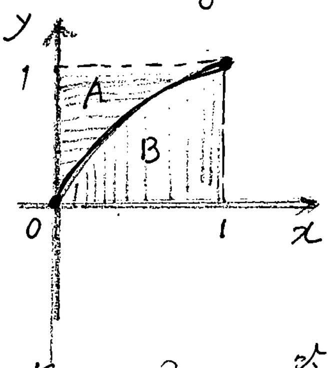
Don 8/5—28.

P.110.3 $\int_0^1 x^2 dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(\xi_i)\cdot\Delta x_i$, $f(\xi_i)=\xi_i^2=(\frac{i}{n})^2$, $\Delta x_i=\frac{1}{n}$.

$$=\lim_{n\to\infty}\sum_{i=1}^{n}(\frac{i}{n})^2\cdot\frac{1}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n^3}=\lim_{n\to\infty}\frac{1^2+2^2+\cdots+n^2}{n^3}$$

$$=\lim_{n\to\infty}\frac{\frac{1}{6}n\cdot(n+1)(2n+1)}{n^3}=\frac{1}{6}\lim_{n\to\infty}(1+\frac{1}{n})\cdot(2+\frac{1}{n})=\frac{2}{6}=\frac{1}{3}.$$

P.110.4 求 $\int_0^1\sqrt{x}\,dx$ （利用2,3题结果）



$$A+B=\int_0^1 y^2 dy+\int_0^1\sqrt{x}\,dx=1\times1=1$$

$$\int_0^1\sqrt{x}\,dx=1-\int_0^1 x^2 dy=1-\frac{1}{3}=\frac{2}{3}.$$

P.111-5. 证明下列不等式

(1) $\frac{\pi}{2}<\int_0^{\frac{\pi}{2}}(1+\sin x)dx<\pi$;

证: $\frac{\pi}{2}=\int_0^{\frac{\pi}{2}}(1+0)dx<\int_0^{\frac{\pi}{2}}(1+\sin x)dx<\int_0^{\frac{\pi}{2}}(1+1)dx=2\cdot\int_0^{\frac{\pi}{2}}dx=2\cdot\frac{\pi}{2}=\pi$

(2) $\sqrt{2}<\int_0^1\sqrt{2+x-x^2}\,dx<\frac{3}{2}$.

证: $x\in[0,1]$ 时, $\sqrt{2}\le\sqrt{2+x-x^2}=\sqrt{\frac{9}{4}-(x-\frac{1}{2})^2}<\sqrt{\frac{9}{4}-0}=\frac{3}{2}$

$x^2\le x$

从而, $\int_0^1\sqrt{2}\,dx<\int_0^1\sqrt{2+x-x^2}\,dx<\int_0^1\frac{3}{2}dx$

$$\sqrt{2}<\int_0^1\sqrt{2+x-x^2}\,dx<\frac{3}{2}.$$

P.111-6 比较定积分的大小。

(1) $\int_0^1 e^x dx>\int_0^1 e^{x^2}dx$; 因为 $x\in(0,1)$ 时 $x^2<x$. $e^{x^2}<e^x$.

(2) $\int_0^{\frac{\pi}{2}}x^2 dx>\int_0^{\frac{\pi}{2}}(\sin x)^2 dx$. 因为, $x\in[0,\frac{\pi}{2}]$, $0<\sin x<x$, $0<\sin^2 x<x^2$.

(3) $\int_0^1 x\,dx<\int_0^1\sqrt{1+x^2}\,dx$, 因为, $x\in[0,1]$, $0<x=\sqrt{x^2}<\sqrt{1+x^2}$.