

Chapter 8: Planning Representation And Algorithms

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Quick Review of Classical Planning

- Classical planning requires all eight of the restrictive assumptions:

A0: Finite

A1: Fully observable

A2: Deterministic

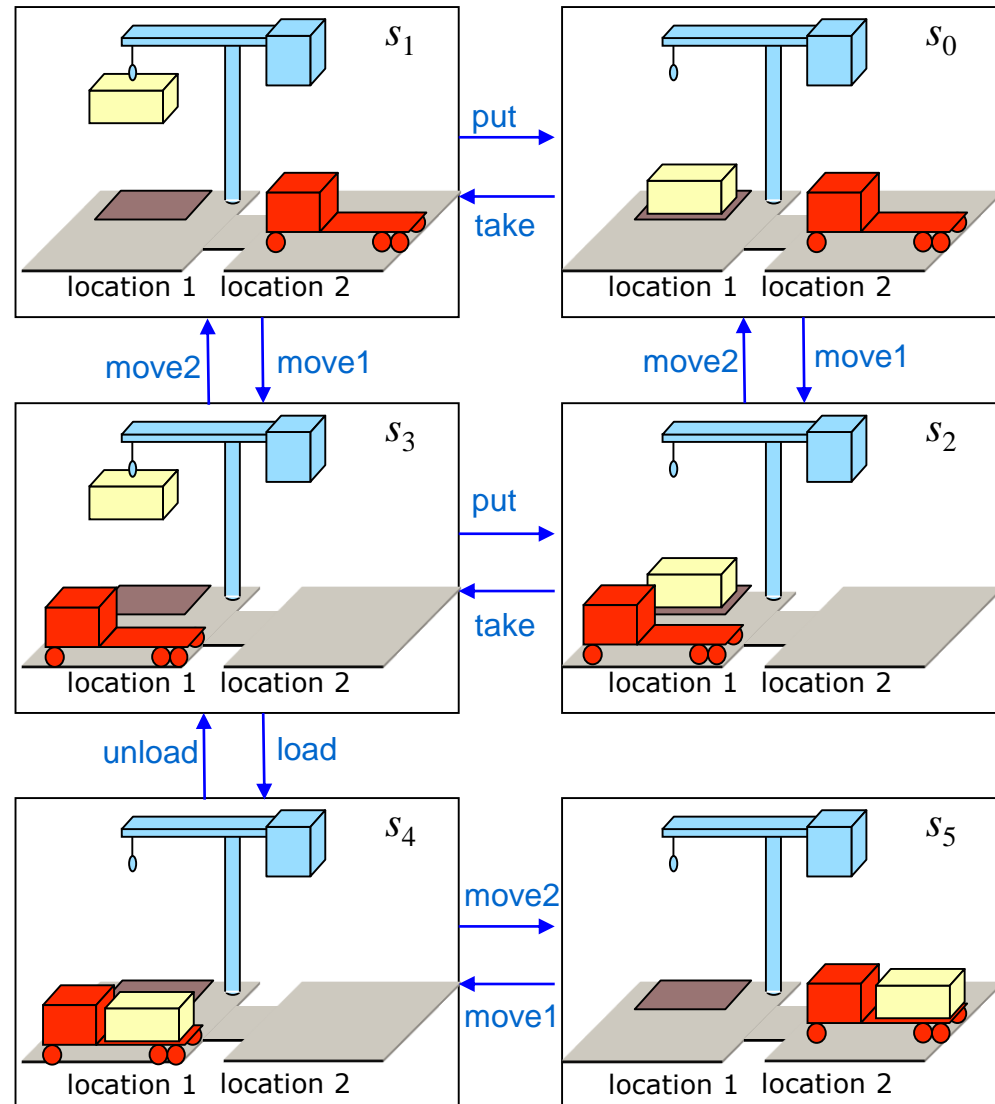
A3: Static

A4: Attainment goals

A5: Sequential plans

A6: Implicit time

A7: Offline planning



Representations: Motivation

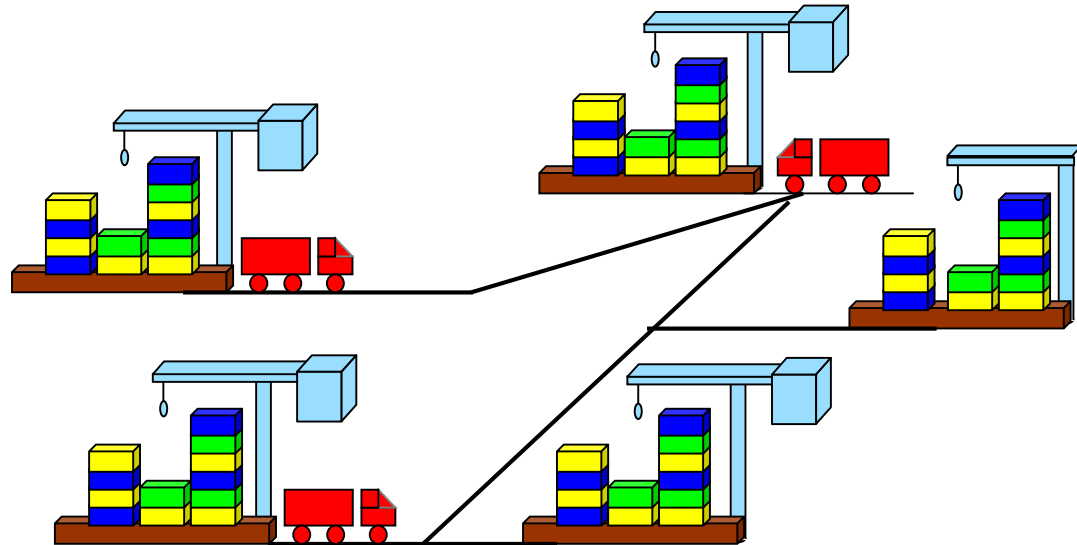
- In most problems, far too many states to try to represent all of them explicitly as s_0, s_1, s_2, \dots
- Represent each state as a set of features
 - ◆ e.g.,
 - » a vector of values for a set of variables
 - » a set of ground atoms in some first-order language L
- Define a set of *operators* that can be used to compute state-transitions
- Don't give all of the states explicitly
 - ◆ Just give the initial state
 - ◆ Use the operators to generate the other states as needed

Outline

- Representation schemes
 - ◆ Classical representation
 - ◆ Set-theoretic representation
 - ◆ State-variable representation
 - ◆ Examples: DWR and the Blocks World
 - ◆ Comparisons

Classical Representation

- Start with a *function-free* first-order language
 - ◆ Finitely many predicate symbols and constant symbols, but *no* function symbols
- Example: the DWR domain
 - ◆ Locations: l_1, l_2, \dots
 - ◆ Containers: c_1, c_2, \dots
 - ◆ Piles: p_1, p_2, \dots
 - ◆ Robot carts: r_1, r_2, \dots
 - ◆ Cranes: k_1, k_2, \dots



Classical Representation

- *Atom*: predicate symbol and args

◆ Use these to represent both fixed and dynamic relations

| | | |
|----------------------------------|---------------------------------|-------------------------------|
| adjacent(<i>l</i> , <i>l'</i>) | attached(<i>p</i> , <i>l</i>) | belong(<i>k</i> , <i>l</i>) |
| occupied(<i>l</i>) | at(<i>r</i> , <i>l</i>) | |
| loaded(<i>r</i> , <i>c</i>) | unloaded(<i>r</i>) | |
| holding(<i>k</i> , <i>c</i>) | empty(<i>k</i>) | |
| in(<i>c</i> , <i>p</i>) | on(<i>c</i> , <i>c'</i>) | |
| top(<i>c</i> , <i>p</i>) | top(pallet, <i>p</i>) | |

- *Ground* expression: contains no variable symbols - e.g., in(c1,p3)
- *Unground* expression: at least one variable symbol - e.g., in(c1,*x*)
- *Substitution*: $\theta = \{x_1 \leftarrow v_1, x_2 \leftarrow v_2, \dots, x_n \leftarrow v_n\}$
 - ◆ Each x_i is a variable symbol; each v_i is a term
- *Instance* of e : result of applying a substitution θ to e
 - ◆ Replace variables of e simultaneously, not sequentially

States

- *State*: a set s of ground atoms
 - ◆ The atoms represent the things that are true in one of Σ 's states
 - ◆ Only finitely many ground atoms, so only finitely many possible states

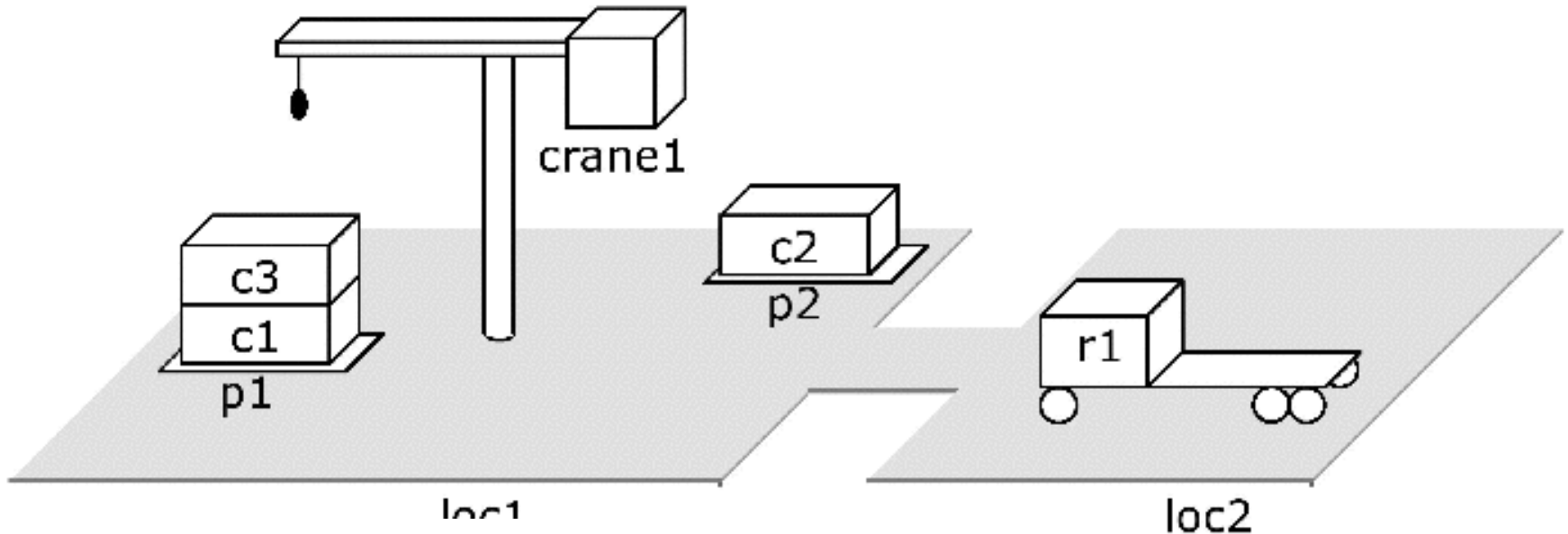


Figure 2.2: The DWR state $s_1 = \{\text{attached}(\text{p1}, \text{loc1}), \text{in}(\text{c1}, \text{p1}), \text{in}(\text{c3}, \text{p1}), \text{top}(\text{c3}, \text{p1}), \text{on}(\text{c3}, \text{c1}), \text{on}(\text{c1}, \text{pallet}), \text{attached}(\text{p2}, \text{loc1}), \text{in}(\text{c2}, \text{p2}), \text{top}(\text{c2}, \text{p2}), \text{on}(\text{c2}, \text{pallet}), \text{belong}(\text{crane1}, \text{loc1}), \text{empty}(\text{crane1}), \text{adjacent}(\text{loc1}, \text{loc2}), \text{adjacent}(\text{loc2}, \text{loc1}), \text{at}(\text{r1}, \text{loc2}), \text{occupied}(\text{loc2}), \text{unloaded}(\text{r1})\}$.

Operators

- *Operator*: a triple $o = (\text{name}(o), \text{precond}(o), \text{effects}(o))$
 - ◆ $\text{name}(o)$ is a syntactic expression of the form $n(x_1, \dots, x_k)$
 - » n : operator symbol - must be unique for each operator
 - » x_1, \dots, x_k : variable symbols (parameters)
 - must include every variable symbol in o
 - ◆ $\text{precond}(o)$: *preconditions*
 - » literals that must be true in order to use the operator
 - ◆ $\text{effects}(o)$: *effects*
 - » literals the operator will make true

$\text{take}(k, l, c, d, p)$

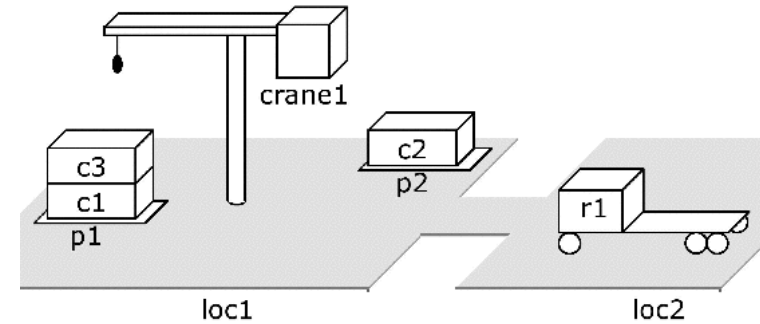
;; crane k at location l takes c off of d in pile p

precond: $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects: $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

Actions

- *Action*: ground instance (via substitution) of an operator



`take(k, l, c, d, p)`

`:: crane k at location l takes c off of d in pile p`

`precond: belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)`

`effects: holding(k, c), \neg empty(k), \neg in(c, p), \neg top(c, p), \neg on(c, d), top(d, p)`

`take(crane1,loc1,c3,c1,p1)`

`:: crane crane1 at location loc1 takes c3 off c1 in pile p1`

`precond: belong(crane1,loc1), attached(p1,loc1),
empty(crane1), top(c3,p1), on(c3,c1)`

`effects: holding(crane1,c3), \neg empty(crane1), \neg in(c3,p1),
 \neg top(c3,p1), \neg on(c3,c1), top(c1,p1)`

Notation

- Let S be a set of literals. Then
 - ◆ $S^+ = \{\text{atoms that appear positively in } S\}$
 - ◆ $S^- = \{\text{atoms that appear negatively in } S\}$
- More specifically, let a be an operator or action. Then
 - ◆ $\text{precond}^+(a) = \{\text{atoms that appear positively in } a\text{'s preconditions}\}$
 - ◆ $\text{precond}^-(a) = \{\text{atoms that appear negatively in } a\text{'s preconditions}\}$
 - ◆ $\text{effects}^+(a) = \{\text{atoms that appear positively in } a\text{'s effects}\}$
 - ◆ $\text{effects}^-(a) = \{\text{atoms that appear negatively in } a\text{'s effects}\}$

$\text{take}(k, l, c, d, p)$

;; crane k at location l takes c off of d in pile p

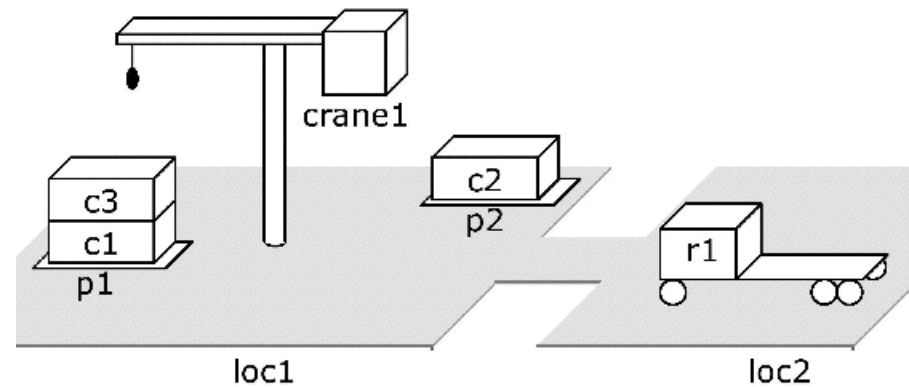
precond: $\text{belong}(k, l), \text{attached}(p, l), \text{empty}(k), \text{top}(c, p), \text{on}(c, d)$

effects: $\text{holding}(k, c), \neg \text{empty}(k), \neg \text{in}(c, p), \neg \text{top}(c, p), \neg \text{on}(c, d), \text{top}(d, p)$

- ◆ $\text{effects}^+(\text{take}(k, l, c, d, p)) = \{\text{holding}(k, c), \text{top}(d, p)\}$
- ◆ $\text{effects}^-(\text{take}(k, l, c, d, p)) = \{\text{empty}(k), \text{in}(c, p), \text{top}(c, p), \text{on}(c, d)\}$

Applicability

- An action a is *applicable* to a state s if s satisfies $\text{precond}(a)$,
 - ◆ i.e., if $\text{precond}^+(a) \subseteq s$ and $\text{precond}^-(a) \cap s = \emptyset$
- Here are an action and a state that it's applicable to:



`take(crane1,loc1,c3,c1,p1)`

`:: crane crane1 at location loc1 takes c3 off c1 in pile p1`

`precond: belong(crane1,loc1), attached(p1,loc1),
empty(crane1), top(c3,p1), on(c3,c1)`

`effects: holding(crane1,c3), \neg empty(crane1), \neg in(c3,p1),
 \neg top(c3,p1), \neg on(c3,c1), top(c1,p1)`

Result of Performing an Action

- If a is applicable to s , the result of performing it is

$$\gamma(s,a) = (s - \text{effects}^-(a)) \cup \text{effects}^+(a)$$

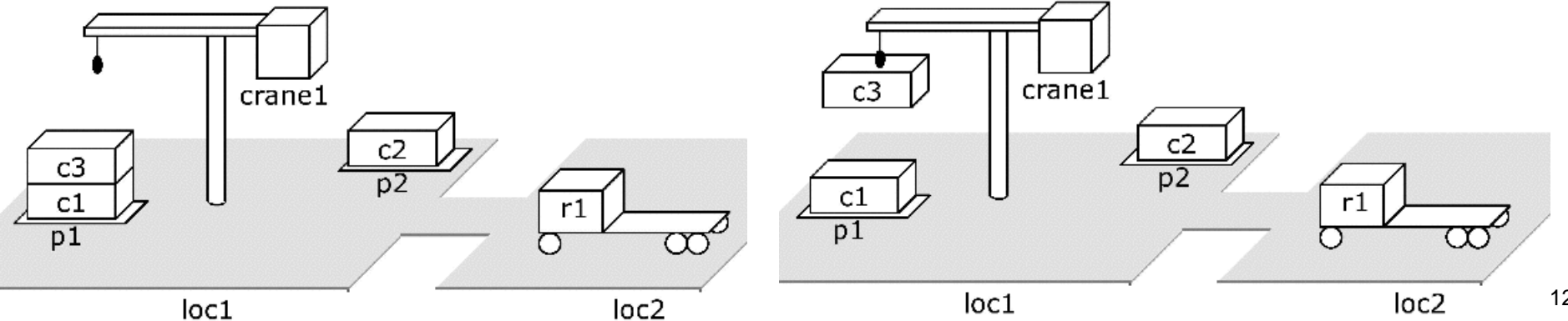
- ◆ Delete the negative effects, and add the positive ones

`take(crane1,loc1,c3,c1,p1)`

`:: crane crane1 at location loc1 takes c3 off c1 in pile p1`

`precond: belong(crane1,loc1), attached(p1,loc1),
empty(crane1), top(c3,p1), on(c3,c1)`

`effects: holding(crane1,c3), \neg empty(crane1), \neg in(c3,p1),
 \neg top(c3,p1), \neg on(c3,c1), top(c1,p1)`



`move(r, l, m)`

:: robot r moves from location l to location m

precond: `adjacent(l, m), at(r, l), \neg occupied(m)`

effects: `at(r, m), occupied(m), \neg occupied(l), \neg at(r, l)`

`load(k, l, c, r)`

:: crane k at location l loads container c onto robot r

precond: `belong(k, l), holding(k, c), at(r, l), unloaded(r)`

effects: `empty(k), \neg holding(k, c), loaded(r, c), \neg unloaded(r)`

`unload(k, l, c, r)`

:: crane k at location l takes container c from robot r

precond: `belong(k, l), at(r, l), loaded(r, c), empty(k)`

effects: `\neg empty(k), holding(k, c), unloaded(r), \neg loaded`

`put(k, l, c, d, p)`

:: crane k at location l puts c onto d in pile p

precond: `belong(k, l), attached(p, l), holding(k, c), top(d, p)`

effects: `\neg holding(k, c), empty(k), in(c, p), top(c, p), on(c, d), \neg top(d, p)`

`take(k, l, c, d, p)`

:: crane k at location l takes c off of d in pile p

precond: `belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)`

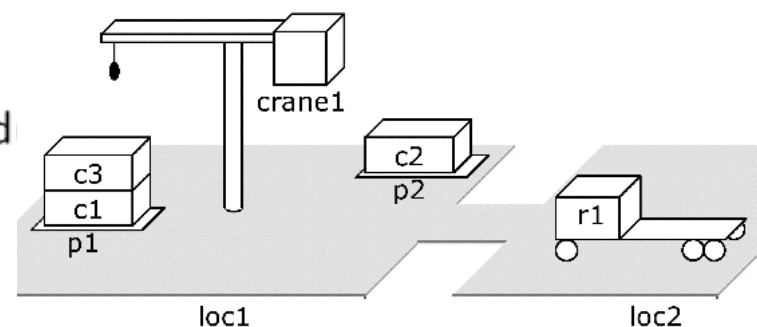
effects: `holding(k, c), \neg empty(k), \neg in(c, p), \neg top(c, p), \neg on(c, d), top(d, p)`

- Planning domain:

language plus operators

- ◆ Corresponds to a set of state-transition systems

- ◆ Example: operators for the DWR domain



Planning Problems

- Given a planning domain (language L , operators O)
 - ◆ *Statement of a planning problem: a triple $P=(O,s_0,g)$*
 - » O is the collection of operators
 - » s_0 is a state (the initial state)
 - » g is a set of literals (the goal formula)
 - ◆ *The actual *planning problem*: $P = (\Sigma,s_0,S_g)$*
 - » s_0 and S_g are as above
 - » $\Sigma = (S,A,\gamma)$ is a state-transition system
 - » $S = \{\text{all sets of ground atoms in } L\}$
 - » $A = \{\text{all ground instances of operators in } O\}$
 - » $\gamma = \text{the state-transition function determined by the operators}$
- I'll often say “planning problem” when I mean the statement of the problem

Plans and Solutions

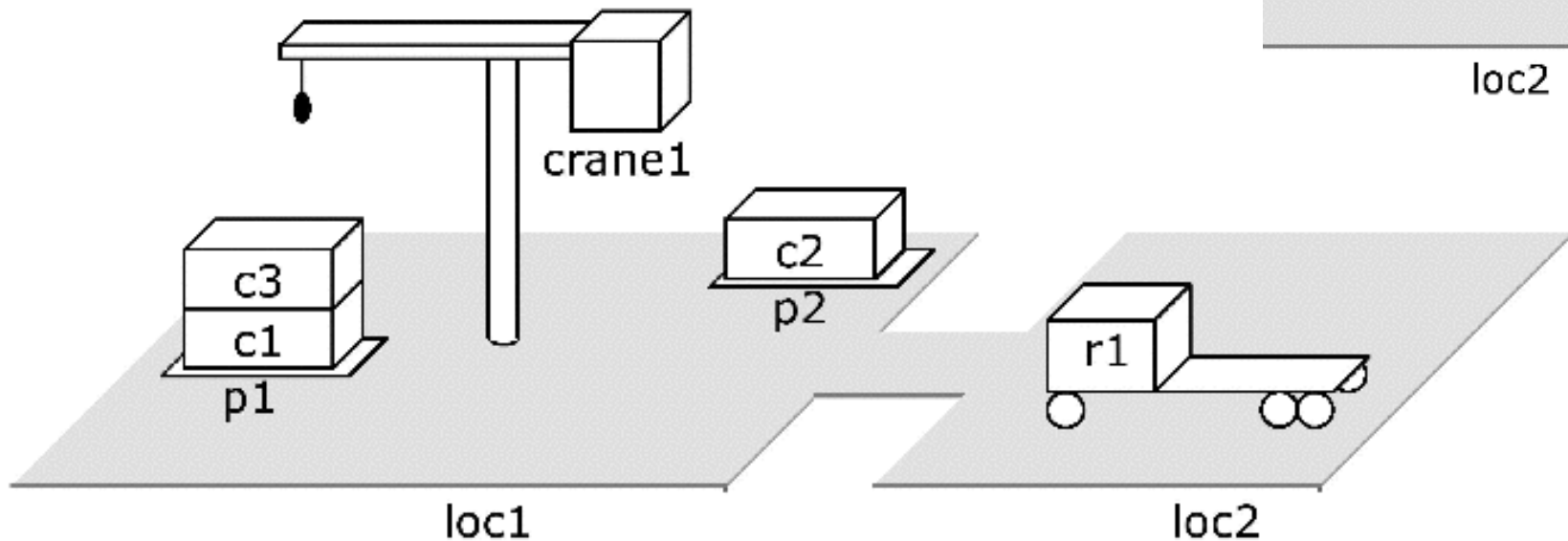
- *Plan*: any sequence of actions $\sigma = \langle a_1, a_2, \dots, a_n \rangle$ such that each a_i is a ground instance of an operator in O
- The plan is a *solution* for $P=(O, s_0, g)$ if it is executable and achieves g
 - ◆ i.e., if there are states s_0, s_1, \dots, s_n such that
 - » $\gamma(s_0, a_1) = s_1$
 - » $\gamma(s_1, a_2) = s_2$
 - » ...
 - » $\gamma(s_{n-1}, a_n) = s_n$
 - » s_n satisfies g

Example

- Let $P_1 = (O, s_1, g_1)$, where

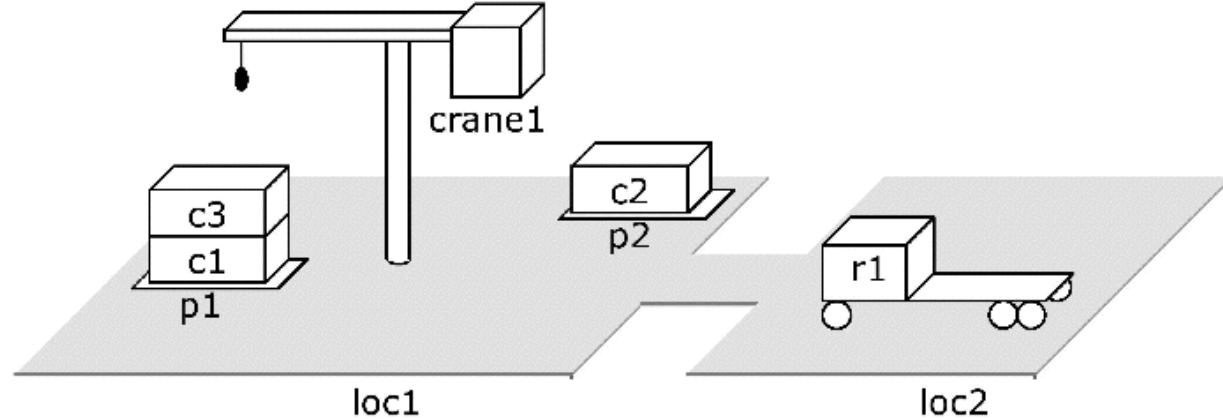
◆ O is the set of operators given earlier

◆ $g_1 = \{\text{loaded}(r1, c3), \text{at}(r1, \text{loc2})\}$



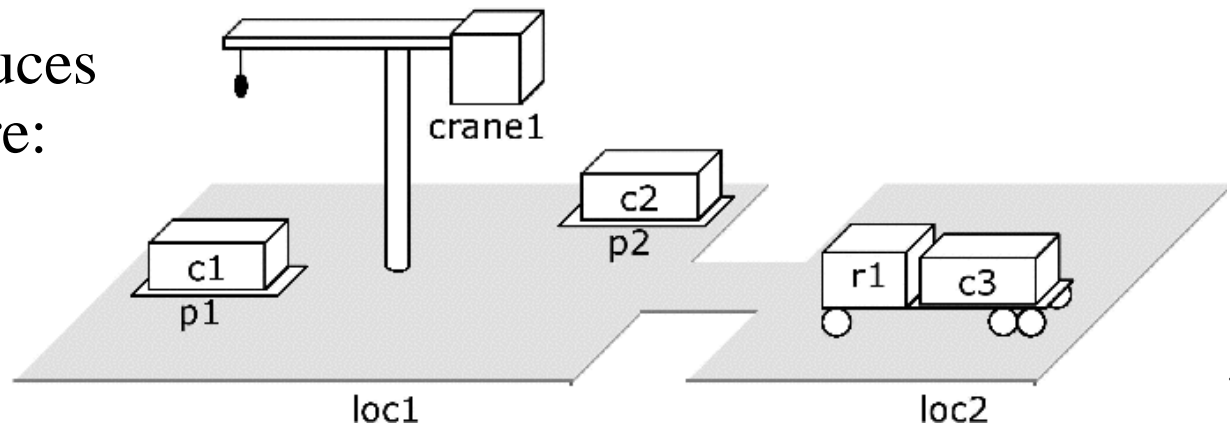
◆ $s_1 = \{\text{attached}(p1, \text{loc1}), \text{in}(c1, p1), \text{in}(c3, p1), \text{top}(c3, p1), \text{on}(c3, c1), \text{on}(c1, \text{pallet}), \text{attached}(p2, \text{loc1}), \text{in}(c2, p2), \text{top}(c2, p2), \text{on}(c2, \text{pallet}), \text{belong}(\text{crane1}, \text{loc1}), \text{empty}(\text{crane1}), \text{adjacent}(\text{loc1}, \text{loc2}), \text{adjacent}(\text{loc2}, \text{loc1}), \text{at}(r1, \text{loc2}), \text{occupied}(\text{loc2}), \text{unloaded}(r1)\}$.

Example (continued)

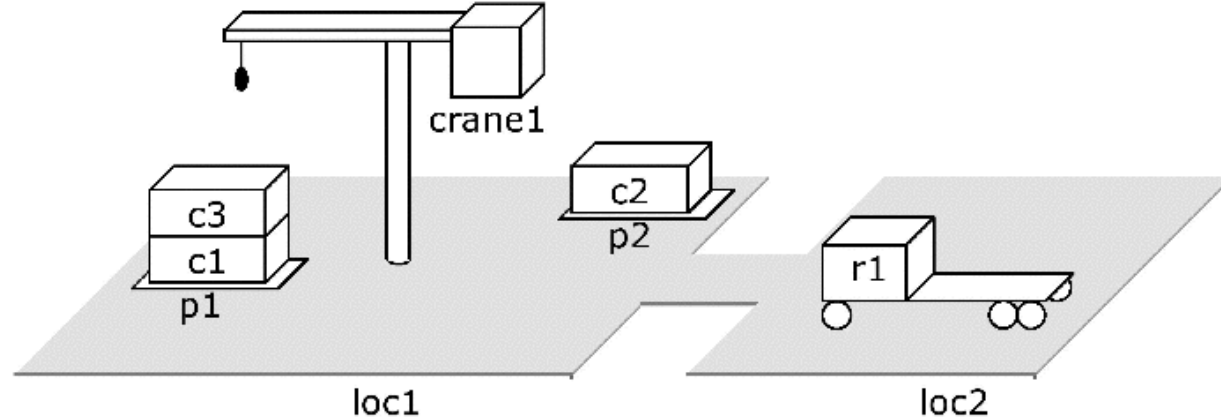


- Here are three solutions for P_1 :
 - ◆ $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$
 - ◆ $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$
 - ◆ $\langle \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$

- Each of them produces the state shown here:

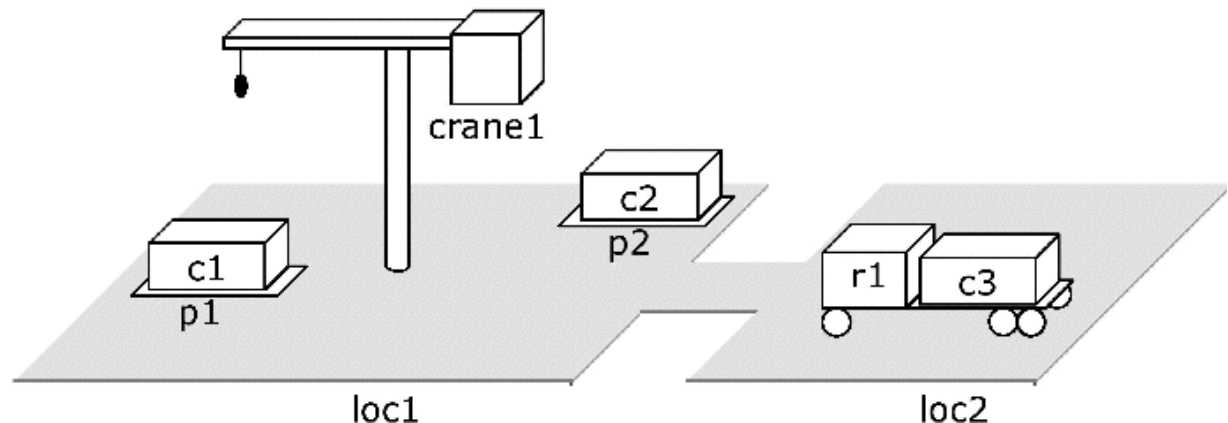


Example (continued)



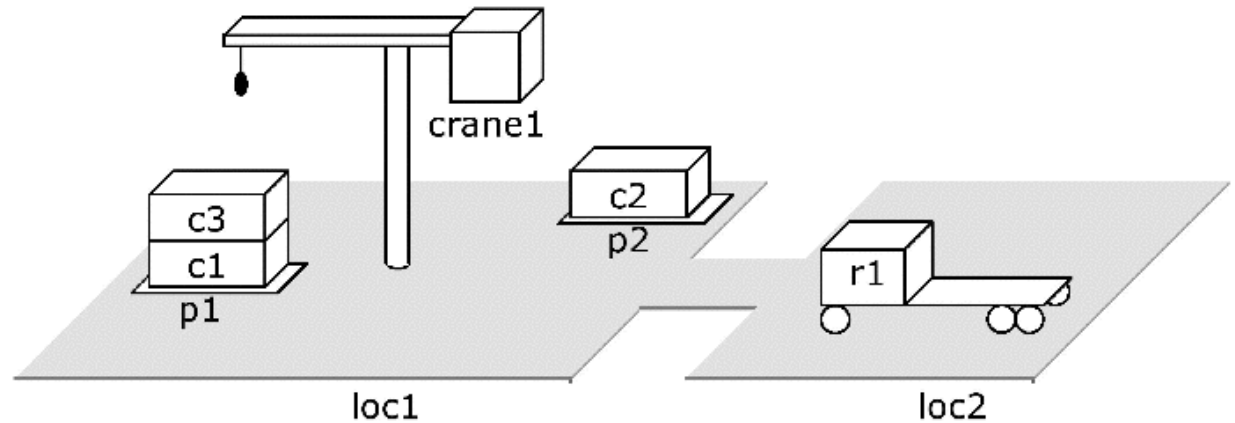
- The first is *redundant*: can remove actions and still have a solution
 - ◆ $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$
 - ◆ $\langle \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$
 - ◆ $\langle \text{move}(\text{r1}, \text{loc2}, \text{loc1}), \text{take}(\text{crane1}, \text{loc1}, \text{c3}, \text{c1}, \text{p1}), \text{load}(\text{crane1}, \text{loc1}, \text{c3}, \text{r1}), \text{move}(\text{r1}, \text{loc1}, \text{loc2}) \rangle$

- The 2nd and 3rd are *irredundant* and *shortest*



Set-Theoretic Representation

- Like classical representation, but restricted to propositional logic



- States:

- ◆ Instead of a collection of ground atoms ...

$\{\text{on}(\text{c1}, \text{pallet}), \text{on}(\text{c1}, \text{r1}), \text{on}(\text{c1}, \text{c2}), \dots, \text{at}(\text{r1}, \text{l1}), \text{at}(\text{r1}, \text{l2}), \dots\}$

... use a collection of propositions (boolean variables):

$\{\text{on-c1-pallet}, \text{on-c1-r1}, \text{on-c1-c2}, \dots, \text{at-r1-l1}, \text{at-r1-l2}, \dots\}$

- Instead of operators like this one,

```
take(k, l, c, d, p)
```

```
;; crane k at location l takes c off of d in pile p
```

```
precond: belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)
```

```
effects:  holding(k, c),  $\neg$ empty(k),  $\neg$ in(c, p),  $\neg$ top(c, p),  $\neg$ on(c, d), top(d, p)
```

take all of
the operator
instances,

e.g., this one,

```
take(crane1,loc1,c3,c1,p1)
```

```
;; crane crane1 at location loc1 takes c3 off c1 in pile p1
```

```
precond: belong(crane1,loc1), attached(p1,loc1),  
         empty(crane1), top(c3,p1), on(c3,c1)
```

```
effects:  holding(crane1,c3),  $\neg$ empty(crane1),  $\neg$ in(c3,p1),  
          $\neg$ top(c3,p1),  $\neg$ on(c3,c1), top(c1,p1)
```

and rewrite
ground atoms
as propositions

```
take-crane1-loc1-c3-c1-p1
```

```
precond: belong-crane1-loc1, attached-p1-loc1,  
         empty-crane1, top-c3-p1, on-c3-c1
```

```
delete:  empty-crane1, in-c3-p1, top-c3-p1, on-c3-p1
```

```
add:     holding-crane1-c3, top-c1-p1
```

Comparison

- A set-theoretic representation is equivalent to a classical representation in which all of the atoms are ground
- Exponential blowup
 - ◆ If a classical operator contains n atoms and each atom has arity k , then it corresponds to c^{nk} actions where $c = |\{\text{constant symbols}\}|$

State-Variable Representation

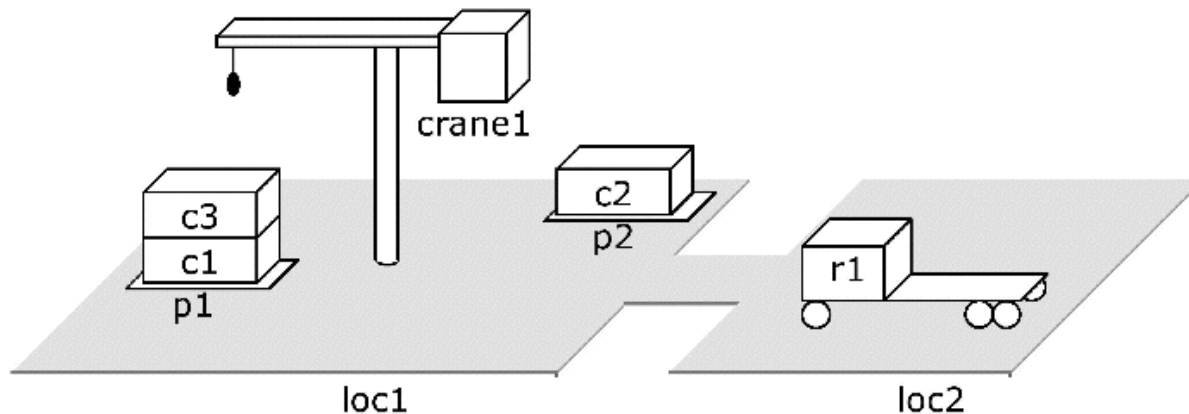
- Use ground atoms for properties that do not change, e.g., `adjacent(loc1, loc2)`
- For properties that can change, assign values to *state variables*
 - ◆ Like fields in a record structure
- Classical and state-variable representations take similar amounts of space
 - ◆ Each can be translated into the other in low-order polynomial time

`move(r, l, m)`

:: robot *r* at location *l* moves to an adjacent location *m*

precond: `rloc(r) = l, adjacent(l, m)`

effects: `rloc(r) \leftarrow m`



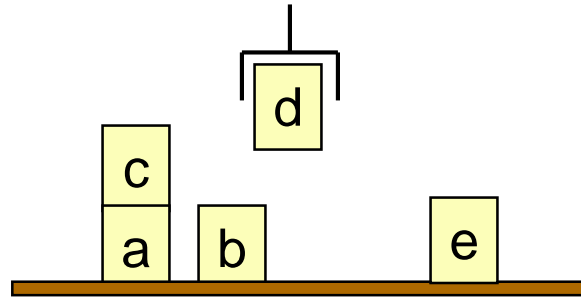
{ `top(p1)=c3,`
 `cpos(c3)=c1,`
 `cpos(c1)=pallet,`
 `holding(crane1)=nil,`
 `rloc(r1)=loc2,`
 `loaded(r1)=nil, ...` }

Example: The Blocks World

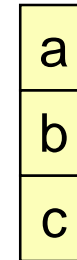
- Infinitely wide table, finite number of blocks
- Ignore where a block is located on the table
- A block can sit on the table or on another block
- Want to move blocks from one configuration to another

◆ e.g.,

initial state



goal



- Can be expressed as a special case of DWR
 - ◆ But the usual formulation is simpler
- I'll give classical, set-theoretic, and state-variable formulations
 - ◆ For the case where there are five blocks

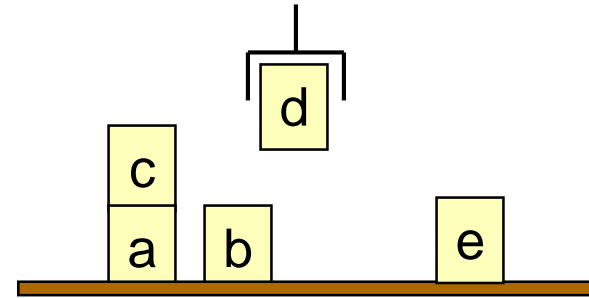
Classical Representation: Symbols

- Constant symbols:

- ◆ The blocks: a, b, c, d, e

- Predicates:

- ◆ `ontable(x)` - block x is on the table
 - ◆ `on(x,y)` - block x is on block y
 - ◆ `clear(x)` - block x has nothing on it
 - ◆ `holding(x)` - the robot hand is holding block x
 - ◆ `handempty` - the robot hand isn't holding anything



Classical Operators

unstack(x,y)

Precond: $\text{on}(x,y)$, $\text{clear}(x)$, handempty

Effects: $\sim\text{on}(x,y)$, $\sim\text{clear}(x)$, $\sim\text{handempty}$,
 $\text{holding}(x)$, $\text{clear}(y)$

stack(x,y)

Precond: $\text{holding}(x)$, $\text{clear}(y)$

Effects: $\sim\text{holding}(x)$, $\sim\text{clear}(y)$,
 $\text{on}(x,y)$, $\text{clear}(x)$, handempty

pickup(x)

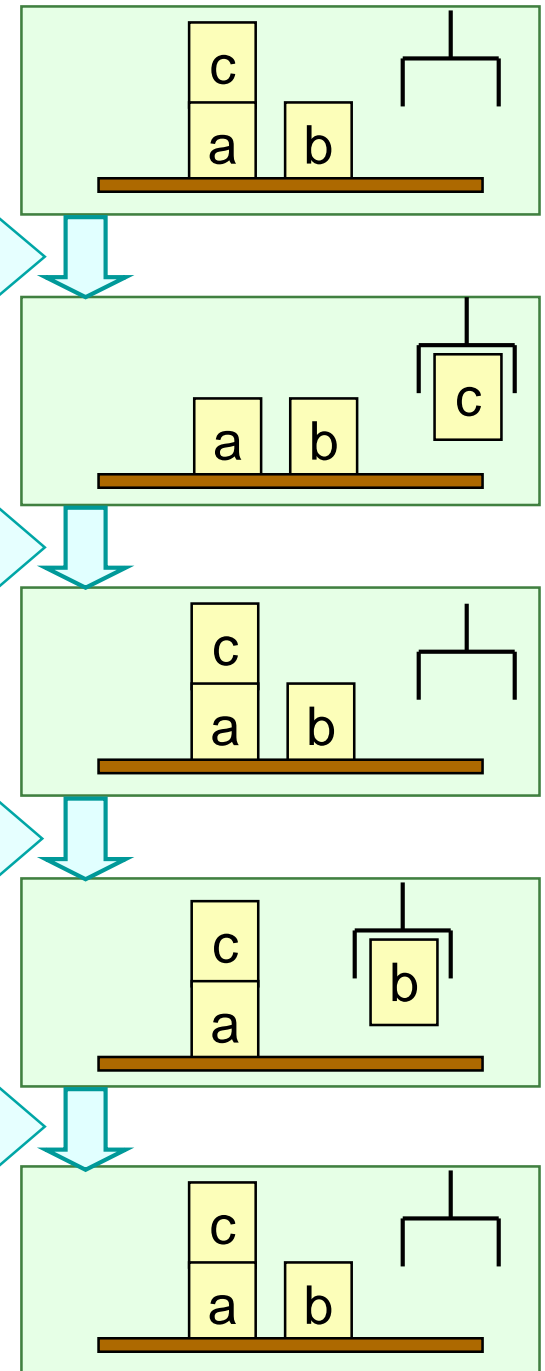
Precond: $\text{ontable}(x)$, $\text{clear}(x)$, handempty

Effects: $\sim\text{ontable}(x)$, $\sim\text{clear}(x)$,
 $\sim\text{handempty}$, $\text{holding}(x)$

putdown(x)

Precond: $\text{holding}(x)$

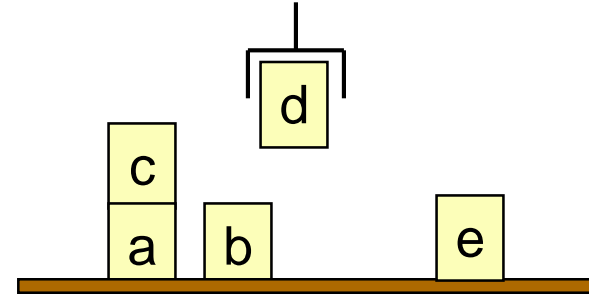
Effects: $\sim\text{holding}(x)$, $\text{ontable}(x)$,
 $\text{clear}(x)$, handempty



Set-Theoretic Representation: Symbols

- For five blocks, there are 36 propositions
- Here are 5 of them:

| | |
|-----------|---|
| ontable-a | - block a is on the table |
| on-c-a | - block c is on block a |
| clear-c | - block c has nothing on it |
| holding-d | - the robot hand is holding block d |
| handempty | - the robot hand isn't holding anything |



Set-Theoretic Actions

Fifty
different
actions

Here are
four of
them:

unstack-c-a

Pre: on-c,a, clear-c, handempty

Del: on-c,a, clear-c, handempty

Add: holding-c, clear-a

stack-c-a

Pre: holding-c, clear-a

Del: holding-c, clear-a

Add: on-c-a, clear-c, handempty

pickup-c

Pre: ontable-c, clear-c, handempty

Del: ontable-c, clear-c, handempty

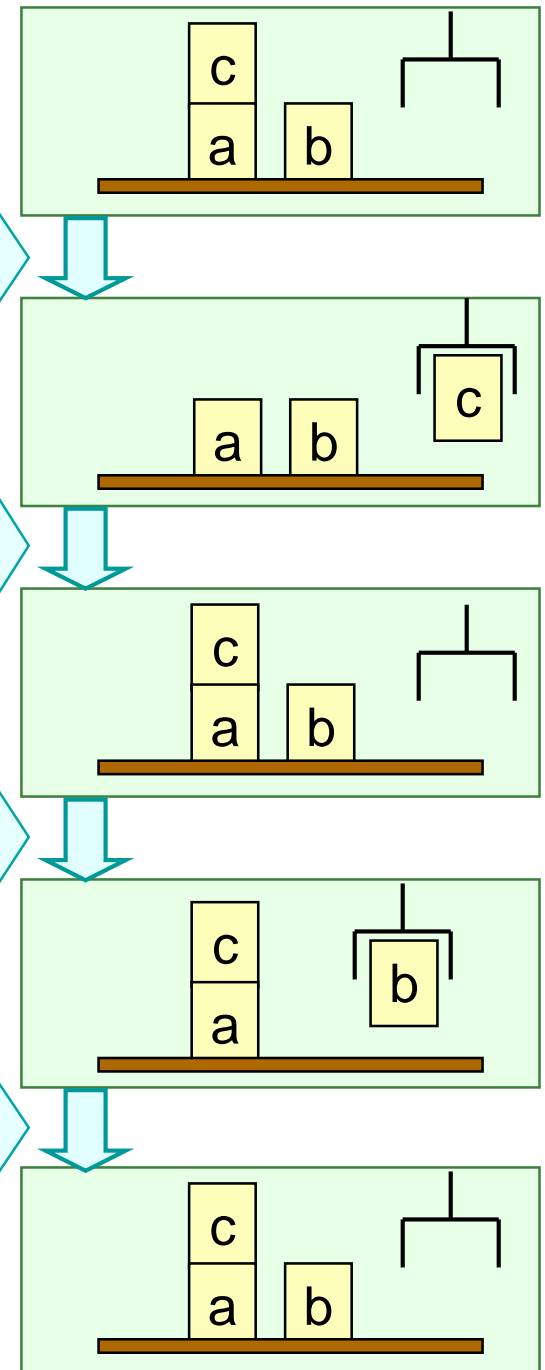
Add: holding-c

putdown-c

Pre: holding-c

Del: holding-c

Add: ontable-c, clear-c, handempty



State-Variable Representation: Symbols

- Constant symbols:

a, b, c, d, e of type block

$0, 1, \text{table}, \text{nil}$ of type other

- State variables:

$\text{pos}(x) = y$ if block x is on block y

$\text{pos}(x) = \text{table}$ if block x is on the table

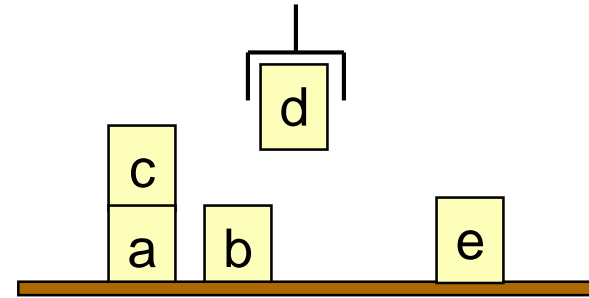
$\text{pos}(x) = \text{nil}$ if block x is being held

$\text{clear}(x) = 1$ if block x has nothing on it

$\text{clear}(x) = 0$ if block x is being held or has another block on it

$\text{holding} = x$ if the robot hand is holding block x

$\text{holding} = \text{nil}$ if the robot hand is holding nothing

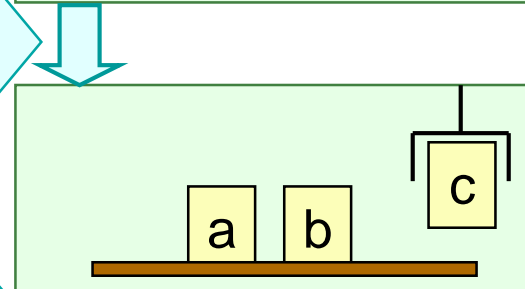
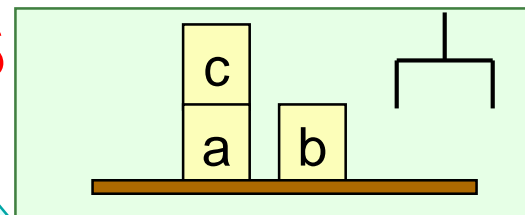


State-Variable Operators

unstack(x : block, y : block)

Precond: $\text{pos}(x)=y$, $\text{clear}(y)=0$, $\text{clear}(x)=1$, $\text{holding}=\text{nil}$

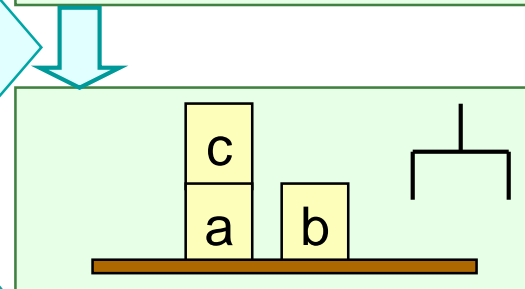
Effects: $\text{pos}(x)=\text{nil}$, $\text{clear}(x)=0$, $\text{holding}=x$, $\text{clear}(y)=1$



stack(x : block, y : block)

Precond: $\text{holding}=x$, $\text{clear}(x)=0$, $\text{clear}(y)=1$

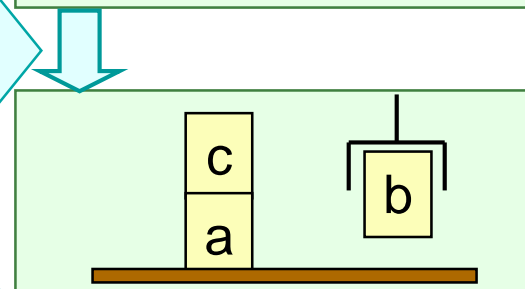
Effects: $\text{holding}=\text{nil}$, $\text{clear}(y)=0$, $\text{pos}(x)=y$, $\text{clear}(x)=1$



pickup(x : block)

Precond: $\text{pos}(x)=\text{table}$, $\text{clear}(x)=1$, $\text{holding}=\text{nil}$

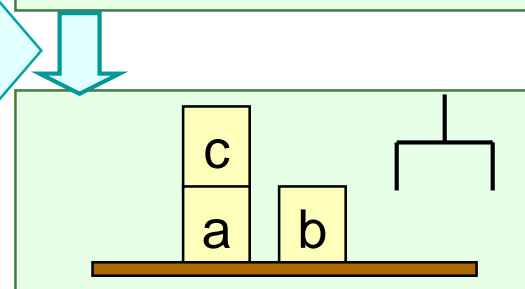
Effects: $\text{pos}(x)=\text{nil}$, $\text{clear}(x)=0$, $\text{holding}=x$



putdown(x : block)

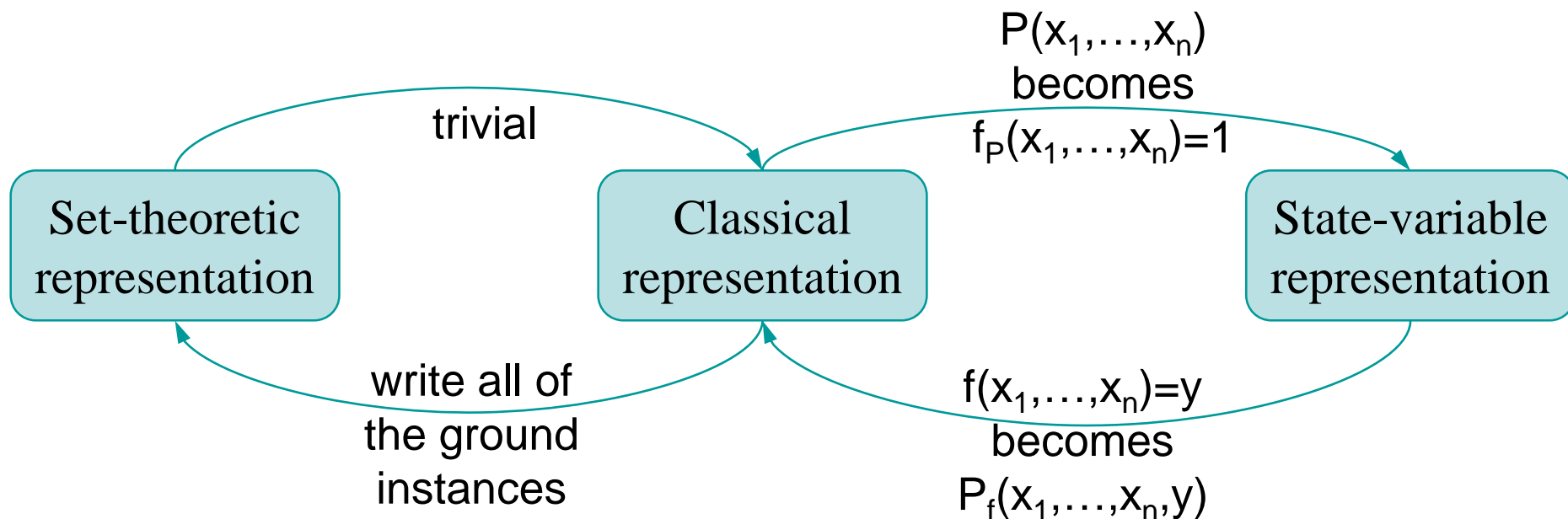
Precond: $\text{holding}=x$

Effects: $\text{holding}=\text{nil}$, $\text{pos}(x)=\text{table}$, $\text{clear}(x)=1$



Expressive Power

- Any problem that can be represented in one representation can also be represented in the other two
- Can convert in linear time and space, except when converting to set-theoretic (where we get an exponential blowup)



Comparison

- Classical representation
 - ◆ The most popular for classical planning, partly for historical reasons
- Set-theoretic representation
 - ◆ Can take much more space than classical representation
 - ◆ Useful in algorithms that manipulate ground atoms directly
 - » e.g., planning graphs (Chapter 6), satisfiability (Chapters 7)
 - ◆ Useful for certain kinds of theoretical studies
- State-variable representation
 - ◆ Equivalent to classical representation in expressive power
 - ◆ Less natural for logicians, more natural for engineers
 - ◆ Useful in non-classical planning problems as a way to handle numbers, functions, time

Planning Algorithms

Motivation

- Nearly all planning procedures are search procedures
- Different planning procedures have different search spaces
- *State-space planning*
 - ◆ Each node represents a state of the world
 - » A plan is a path through the space
- *Plan-space planning*
 - ◆ Each node is a set of partially-instantiated operators, plus some constraints
 - » Impose more and more constraints, until we get a plan

Outline

- State-space planning
 - ◆ Forward search
 - ◆ Backward search
 - ◆ Lifting
 - ◆ STRIPS
 - ◆ Block-stacking

Forward-search(O, s_0, g)

$s \leftarrow s_0$

$\pi \leftarrow$ the empty plan

loop

if s satisfies g then return π

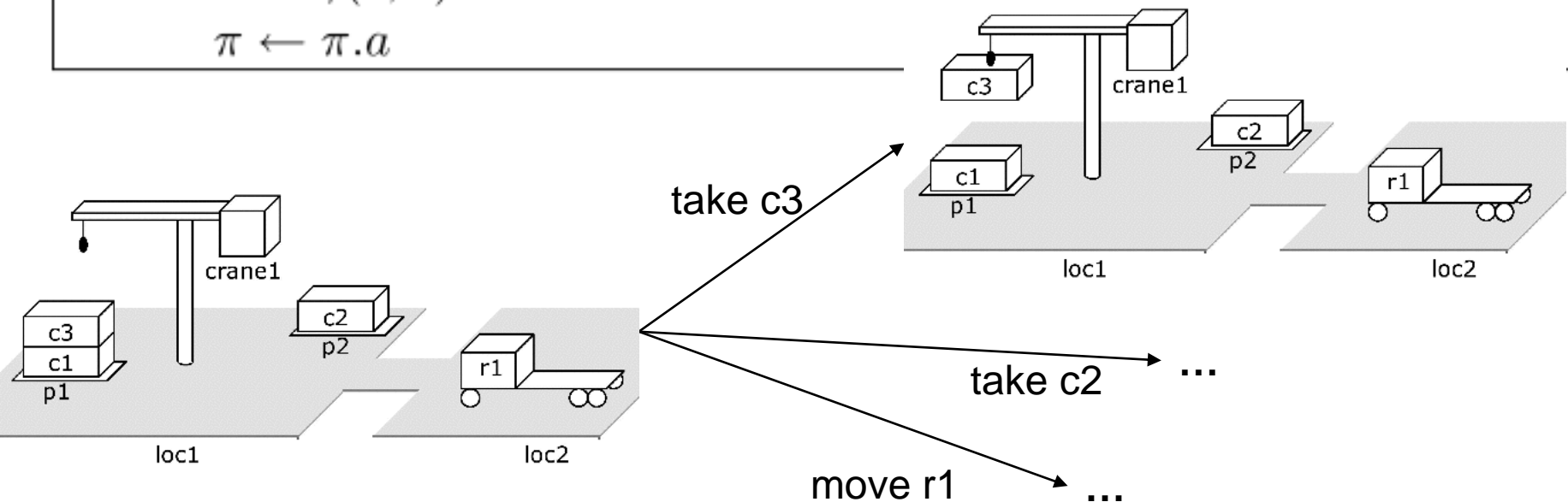
$E \leftarrow \{a \mid a \text{ is a ground instance an operator in } O,$
and $\text{precond}(a)$ is true in $s\}$

if $E = \emptyset$ then return failure

nondeterministically choose an action $a \in E$

$s \leftarrow \gamma(s, a)$

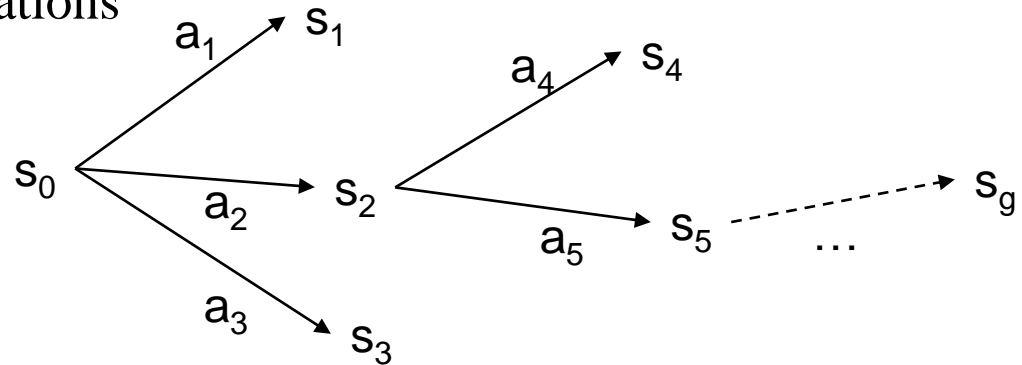
$\pi \leftarrow \pi.a$



Deterministic Implementations

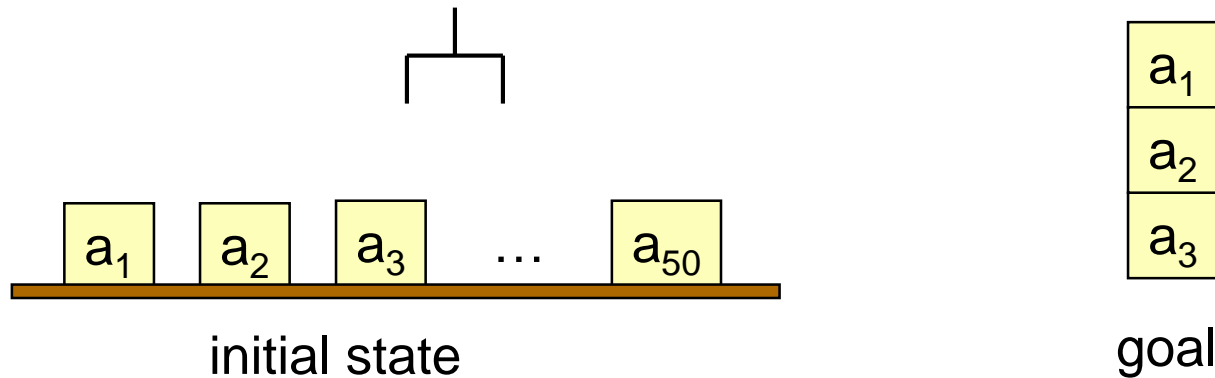
- Some deterministic implementations of forward search:

- ◆ breadth-first search
- ◆ depth-first search
- ◆ best-first search (e.g., A^*)
- ◆ greedy search



- Breadth-first and best-first search are sound and complete
 - ◆ But they usually aren't practical because they require too much memory
 - ◆ Memory requirement is exponential in the length of the solution
- In practice, more likely to use depth-first search or greedy search
 - ◆ In general, sound but not complete
 - » But classical planning has only finitely many states
 - » Thus, can make depth-first search complete by doing loop-checking

Branching Factor of Forward Search



- Forward search can have a very large branching factor
 - ◆ E.g., many applicable actions that don't progress toward goal
- Why this is bad:
 - ◆ Deterministic implementations can waste time trying lots of irrelevant actions
- Need a good heuristic function and/or pruning procedure

Backward Search

- For forward search, we started at the initial state and computed state transitions
 - ◆ new state = $\gamma(s, a)$
- For backward search, we start at the goal and compute inverse state transitions
 - ◆ new set of subgoals = $\gamma^{-1}(g, a)$
- To define $\gamma^{-1}(g, a)$, must first define *relevance*:
 - ◆ An action a is relevant for a goal g if
 - » a makes at least one of g 's literals true
 - $g \cap \text{effects}(a) \neq \emptyset$
 - » a does not make any of g 's literals false
 - $g^+ \cap \text{effects}^-(a) = \emptyset$ and $g^- \cap \text{effects}^+(a) = \emptyset$

Inverse State Transitions

- If a is relevant for g , then
 - ◆ $\gamma^{-1}(g,a) = (g - \text{effects}(a)) \cup \text{precond}(a)$
- Otherwise $\gamma^{-1}(g,a)$ is undefined
- Example: suppose that
 - ◆ $g = \{\text{on}(b1,b2), \text{on}(b2,b3)\}$
 - ◆ $a = \text{stack}(b1,b2)$
- What is $\gamma^{-1}(g,a)$?

Backward-search(O, s_0, g)

$\pi \leftarrow$ the empty plan

loop

if s_0 satisfies g then return π

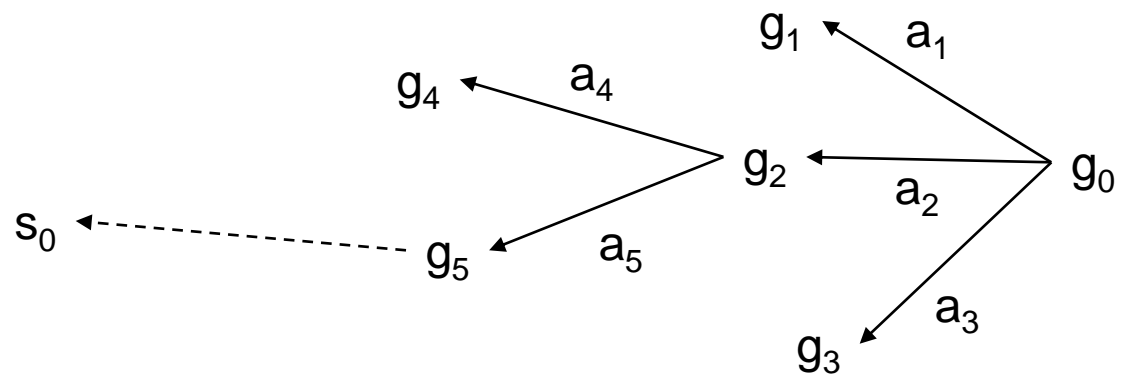
$A \leftarrow \{a \mid a \text{ is a ground instance of an operator in } O$
and $\gamma^{-1}(g, a)$ is defined}

if $A = \emptyset$ then return failure

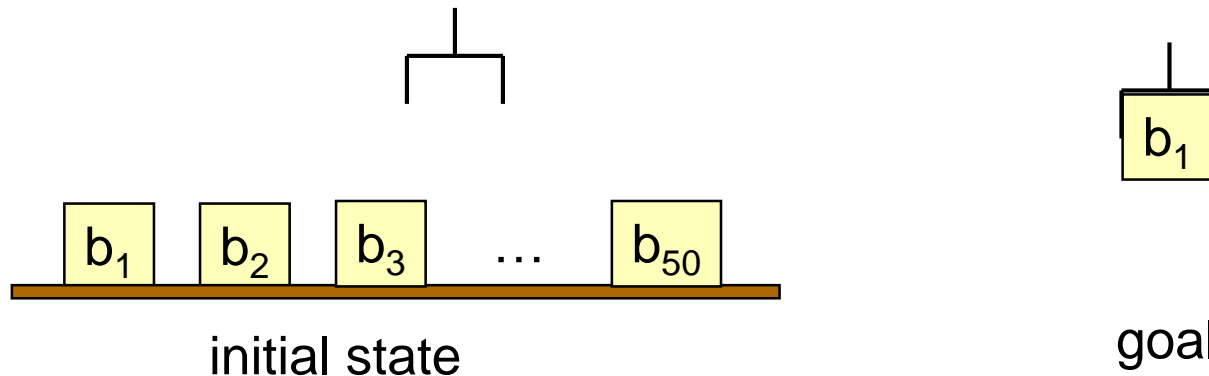
nondeterministically choose an action $a \in A$

$\pi \leftarrow a.\pi$

$g \leftarrow \gamma^{-1}(g, a)$

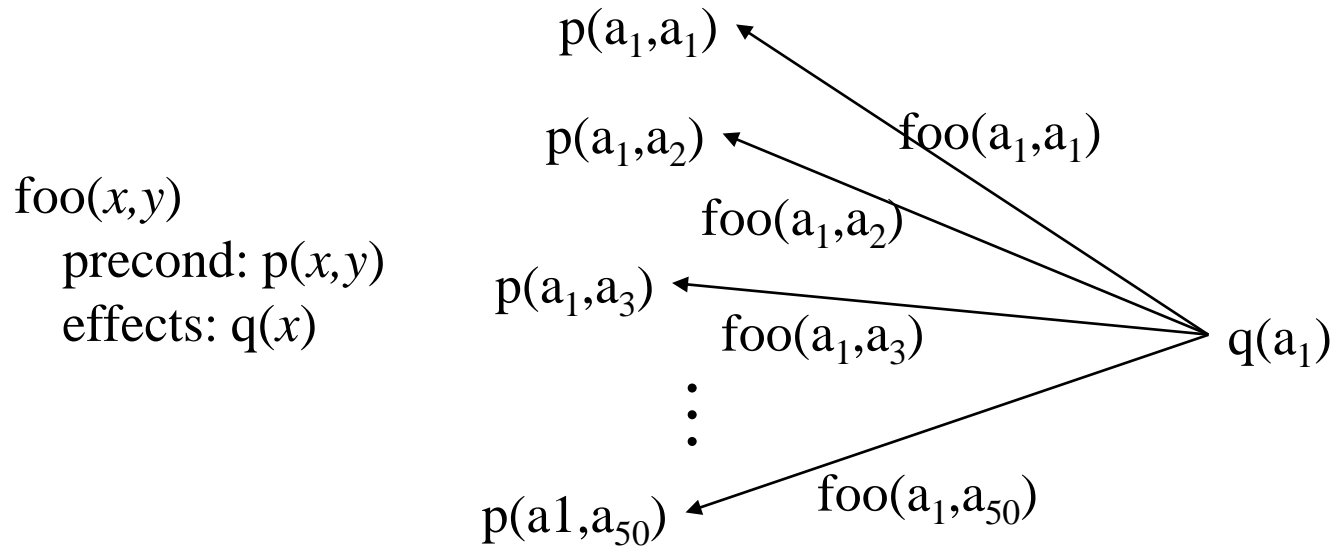


Efficiency of Backward Search



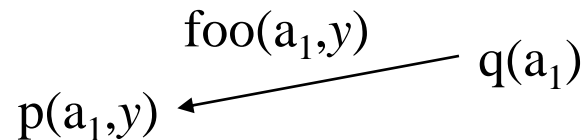
- Backward search can *also* have a very large branching factor
 - ◆ E.g., an operator o that is relevant for g may have many ground instances a_1, a_2, \dots, a_n such that each a_i 's input state might be unreachable from the initial state
- As before, deterministic implementations can waste lots of time trying all of them

Lifting



- Can reduce the branching factor of backward search if we *partially* instantiate the operators

◆ this is called *lifting*



Lifted Backward Search

- More complicated than Backward-search
 - ◆ Have to keep track of what substitutions were performed
- But it has a much smaller branching factor

Lifted-backward-search(O, s_0, g)

$\pi \leftarrow$ the empty plan

loop

if s_0 satisfies g then return π

$A \leftarrow \{(o, \theta) \mid o \text{ is a standardization of an operator in } O,$
 $\theta \text{ is an mgu for an atom of } g \text{ and an atom of } \text{effects}^+(o),$
 $\text{and } \gamma^{-1}(\theta(g), \theta(o)) \text{ is defined}\}$

if $A = \emptyset$ then return failure

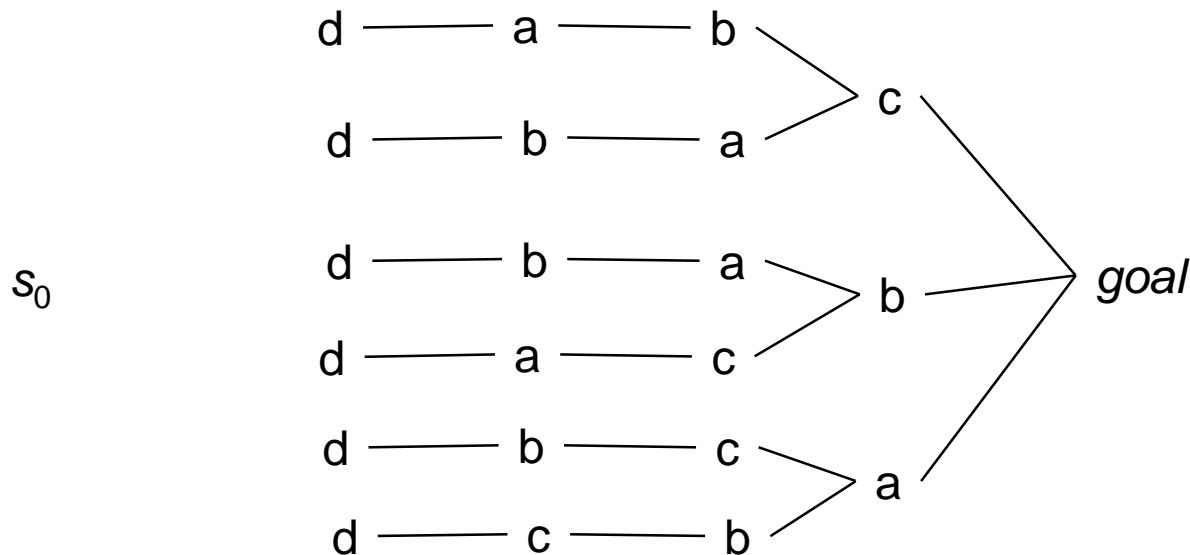
nondeterministically choose a pair $(o, \theta) \in A$

$\pi \leftarrow$ the concatenation of $\theta(o)$ and $\theta(\pi)$

$g \leftarrow \gamma^{-1}(\theta(g), \theta(o))$

The Search Space is Still Too Large

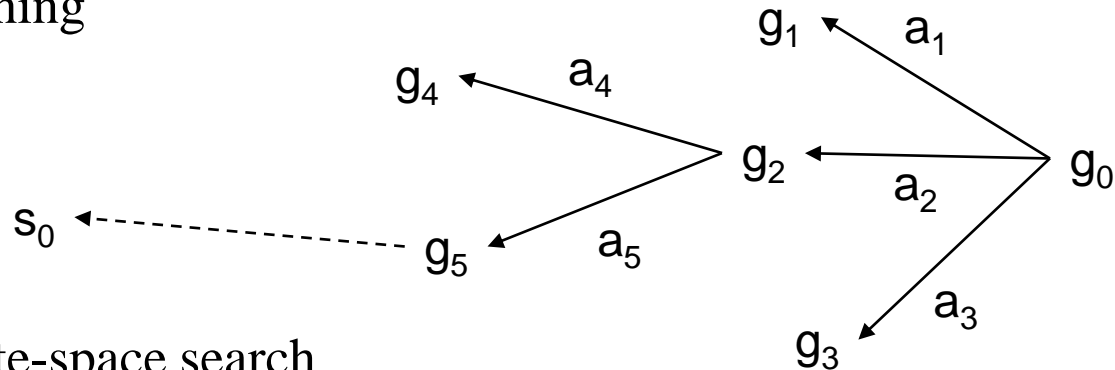
- Lifted-backward-search generates a smaller search space than Backward-search, but it still can be quite large
 - ◆ Suppose actions a , b , and c are independent, action d must precede all of them, and there's no path from s_0 to d 's input state
 - ◆ We'll try all possible orderings of a , b , and c before realizing there is no solution



Plan-space planning

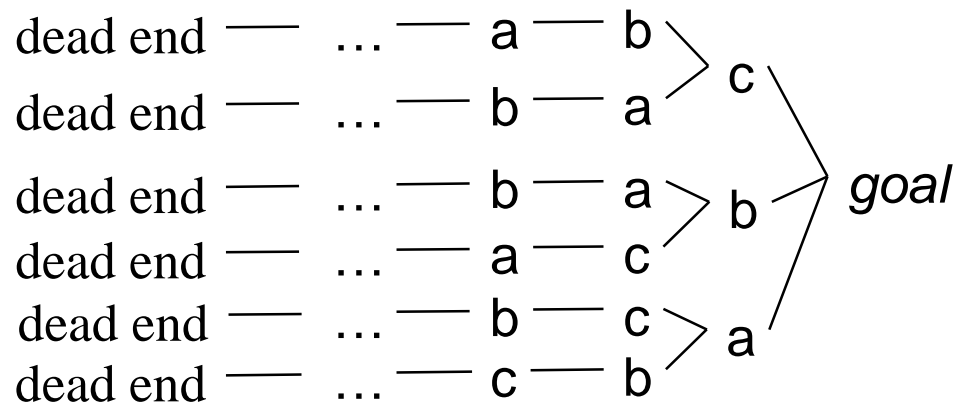
Motivation

- State-Space Planning



- Problem with state-space search

◆ In some cases we may try many different orderings of the same actions before realizing there is no solution



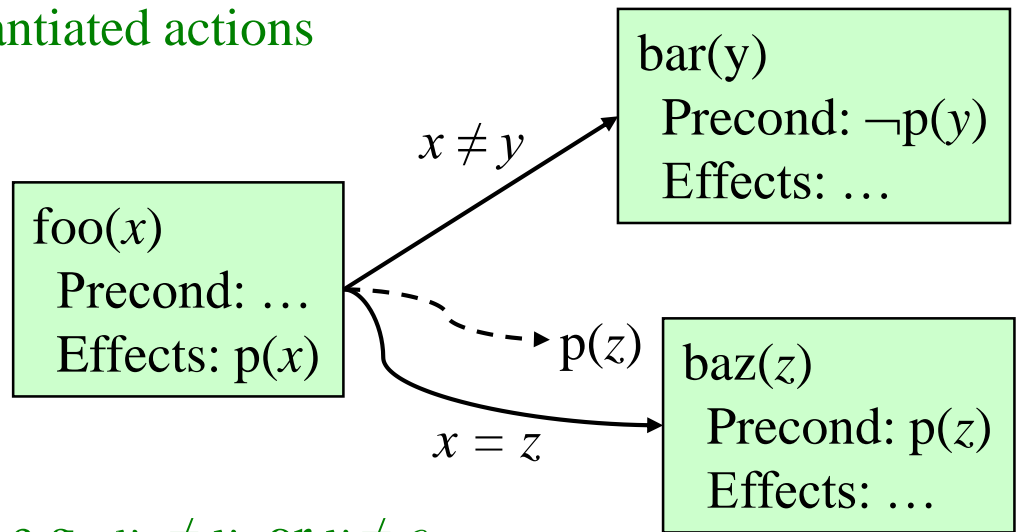
- *Least-commitment strategy*: don't commit to orderings, instantiations, etc., until necessary

Outline

- Basic idea
- Open goals
- Threats
- The PSP algorithm
- Long example
- Comments

Plan-Space Planning - Basic Idea

- Backward search from the goal
- Each node of the search space is a *partial plan*
 - » A set of partially-instantiated actions
 - » A set of constraints



- Types of constraints:

- ◆ *precedence constraint:*
a must precede b

- ◆ *binding constraints:*

- » inequality constraints, e.g., $v_1 \neq v_2$ or $v \neq c$

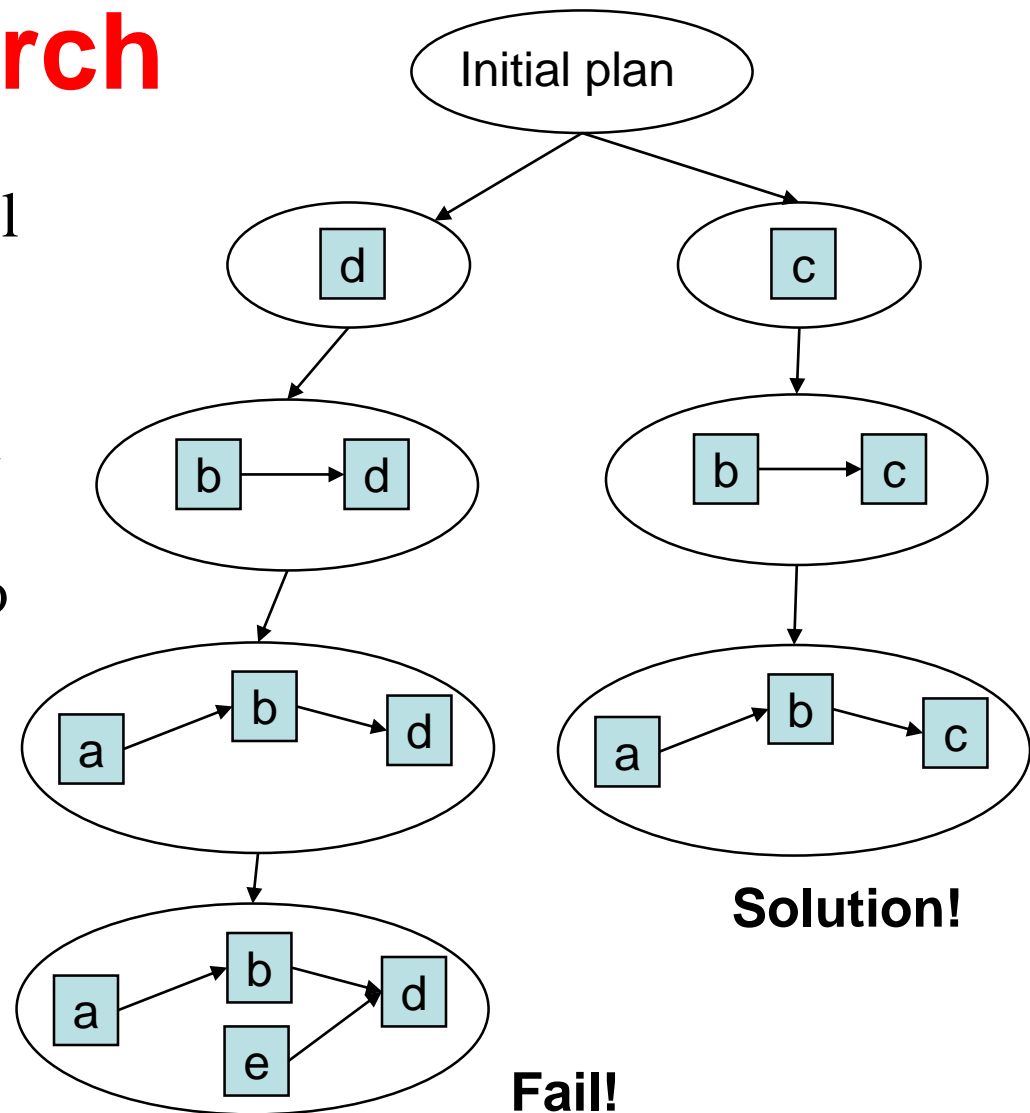
- » equality constraints (e.g., $v_1 = v_2$ or $v = c$) and/or substitutions

- ◆ *causal link:*

- » use action *a* to establish the precondition *p* needed by action *b*

Plan-Space Search

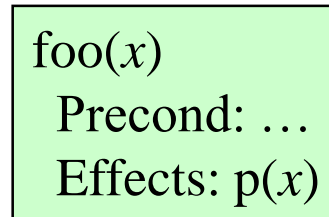
- Start with an initial plan (will explain this later)
- Make more and more refinements to the plan, until a solution is found
- If there are no refinements to a plan during search, then backtrack
- Questions:
 - ◆ How to tell we have a solution?
 - » No more *flaws* in the plan
 - ◆ How to refine a plan?
 - » Resolve the existing *flaws*



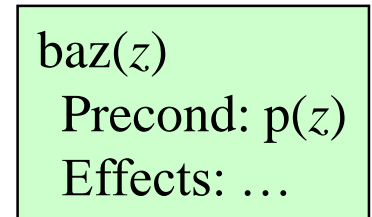
Flaws: 1. Open Goals

- Open goal:

- ◆ An action a has a precondition p that we haven't decided how to establish

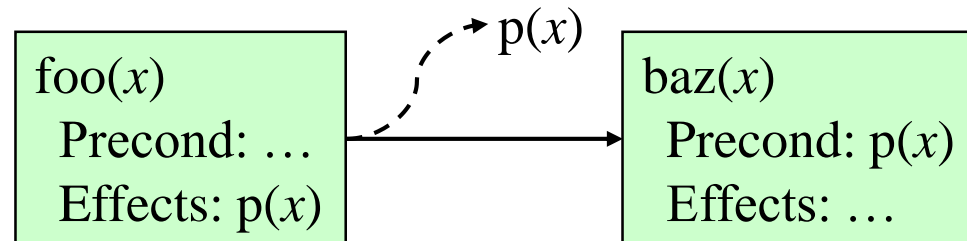


p(z)



- Resolving the flaw:

- ◆ Find an action b
 - (either already in the plan, or insert it)
- ◆ that can be used to establish p
 - can precede a and produce p
- ◆ Instantiate variables and/or constrain variable bindings
- ◆ Create a causal link

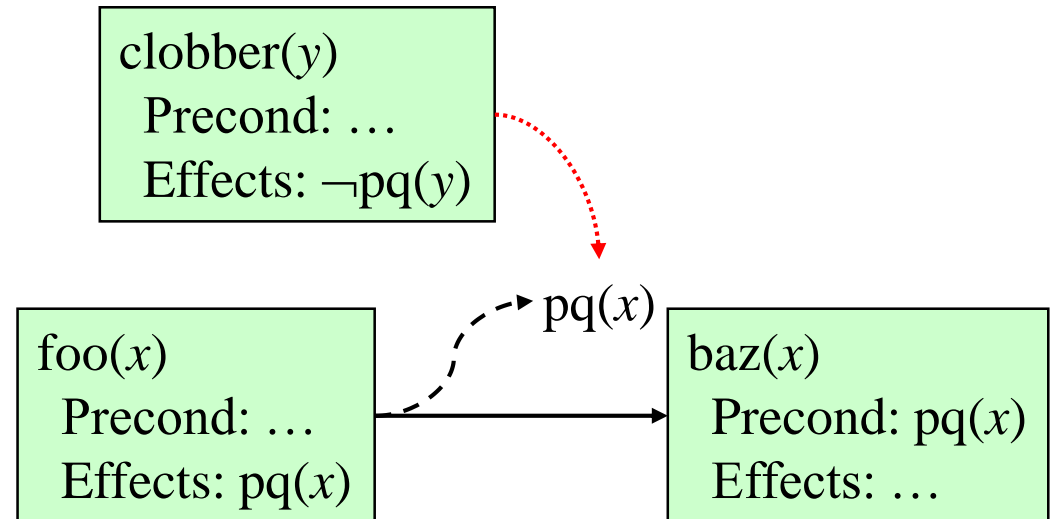


Flaws: 2. Threats

- Threat: a deleted-condition interaction
 - ◆ Action a establishes a precondition (e.g., $pq(x)$) of action b
 - ◆ Another action c is capable of deleting pq

- Resolving the flaw:
 - ◆ impose a constraint to prevent c from deleting pq

- Three possibilities:
 - ◆ Make b precede c
 - ◆ Make c precede a
 - ◆ Constrain variable(s) to prevent c from deleting pq



The PSP Procedure

PSP(π)

$flaws \leftarrow \text{OpenGoals}(\pi) \cup \text{Threats}(\pi)$

if $flaws = \emptyset$ then return(π)

select any flaw $\phi \in flaws$

$resolvers \leftarrow \text{Resolve}(\phi, \pi)$

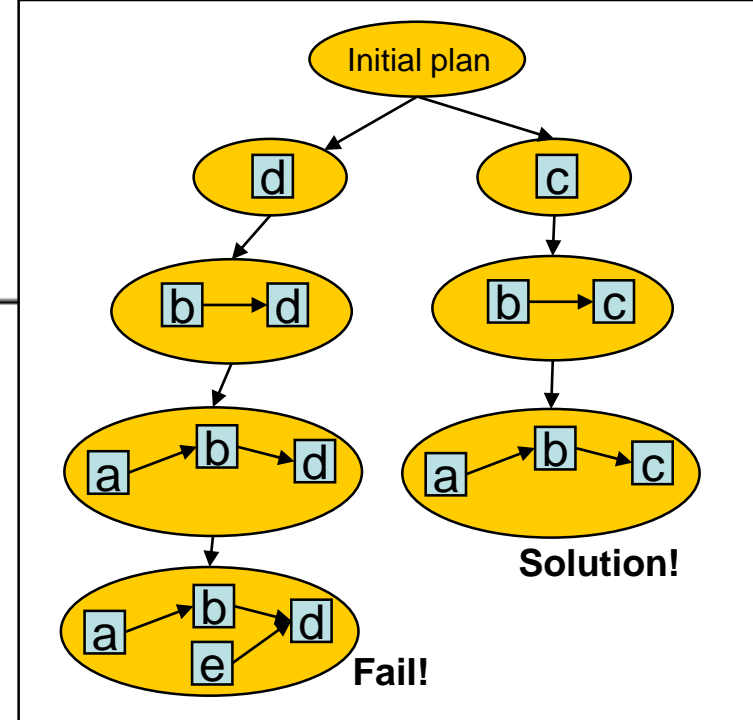
if $resolvers = \emptyset$ then return(failure)

nondeterministically choose a resolver $\rho \in resolvers$

$\pi' \leftarrow \text{Refine}(\rho, \pi)$

return(PSP(π'))

end



- PSP is both sound and complete
- It returns a partially ordered solution plan
 - ◆ Any total ordering of this plan will achieve the goals
 - ◆ Or could execute actions in parallel if the environment permits it

Example

- Similar (but not identical) to an example in Russell and Norvig's *Artificial Intelligence: A Modern Approach* (1st edition)

- Operators:

- ◆ **Start**

Precond: none

Effects: At(Home), sells(HWS,Drill), Sells(SM,Milk), Sells(SM,Banana)

- ◆ **Finish**

Precond: Have(Drill), Have(Milk), Have(Banana), At(Home)

Effects: none

- ◆ **Go(*l*,*m*)**

Precond: At(*l*)

Effects: At(*m*), \neg At(*l*)

- ◆ **Buy(*p*,*s*)**

Precond: At(*s*), Sells(*s*,*p*)

Effects: Have(*p*)

Start and **Finish** are dummy actions that we'll use instead of the initial state and goal

Example (continued)

- Need to give PSP a plan π as its argument

◆ Initial plan: **Start**, **Finish**, and
an ordering constraint

PSP(π)

$flaws \leftarrow \text{OpenGoals}(\pi) \cup \text{Threats}(\pi)$

if $flaws = \emptyset$ then return(π)

select any flaw $\phi \in flaws$

$resolvers \leftarrow \text{Resolve}(\phi, \pi)$

if $resolvers = \emptyset$ then return(failure)

nondeterministically choose a resolver $\rho \in resolvers$

$\pi' \leftarrow \text{Refine}(\rho, \pi)$

return(PSP(π'))

end

Start

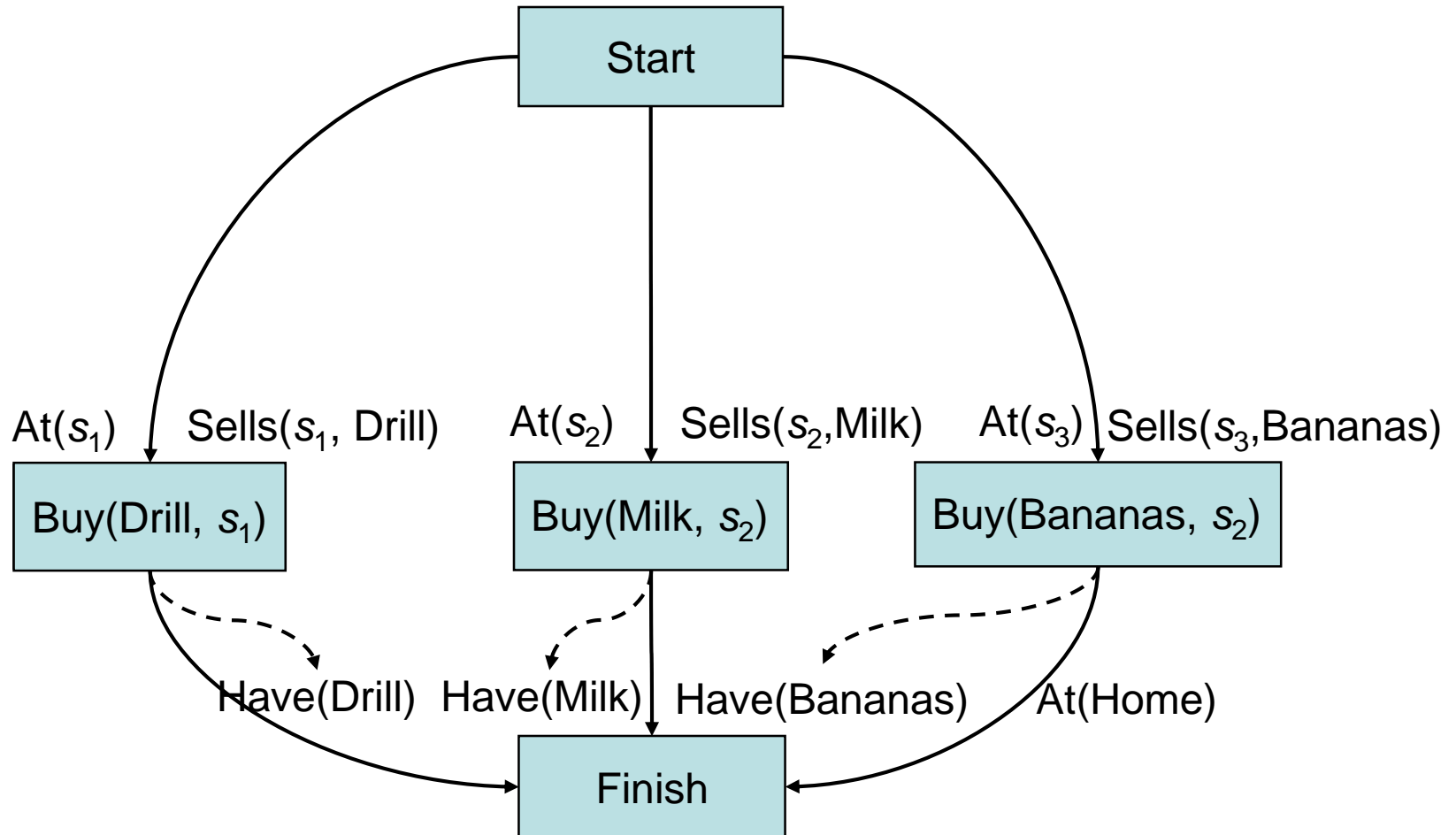
Effects: At(Home), Sells(HWS,Drill), Sells(SM,Milk), Sells(SM,Bananas)

Precond: Have(Drill) Have(Milk) Have(Bananas) At(Home)

Finish

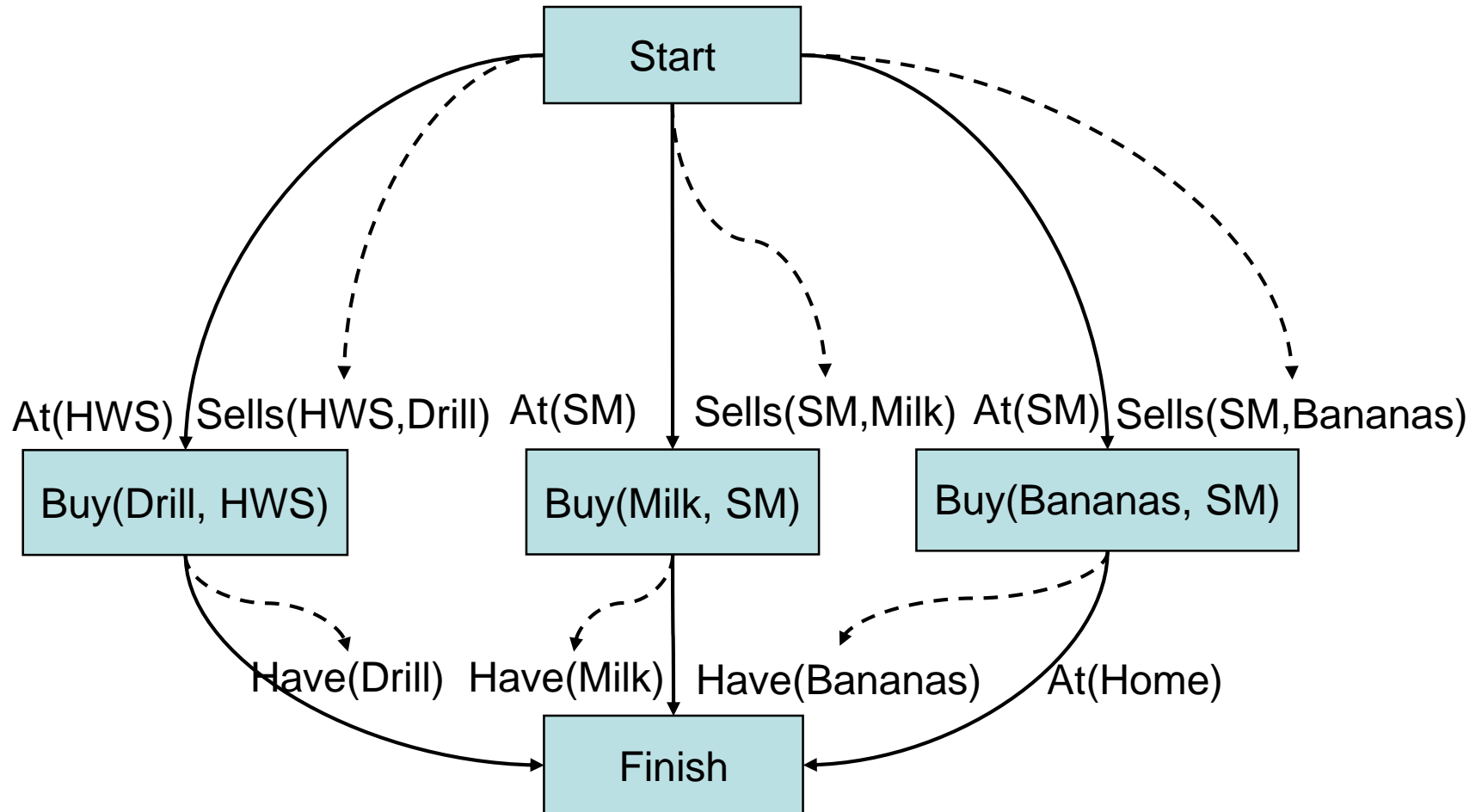
Example (continued)

- The first three refinement steps
 - ◆ These are the only possible ways to establish the Have preconditions



Example (continued)

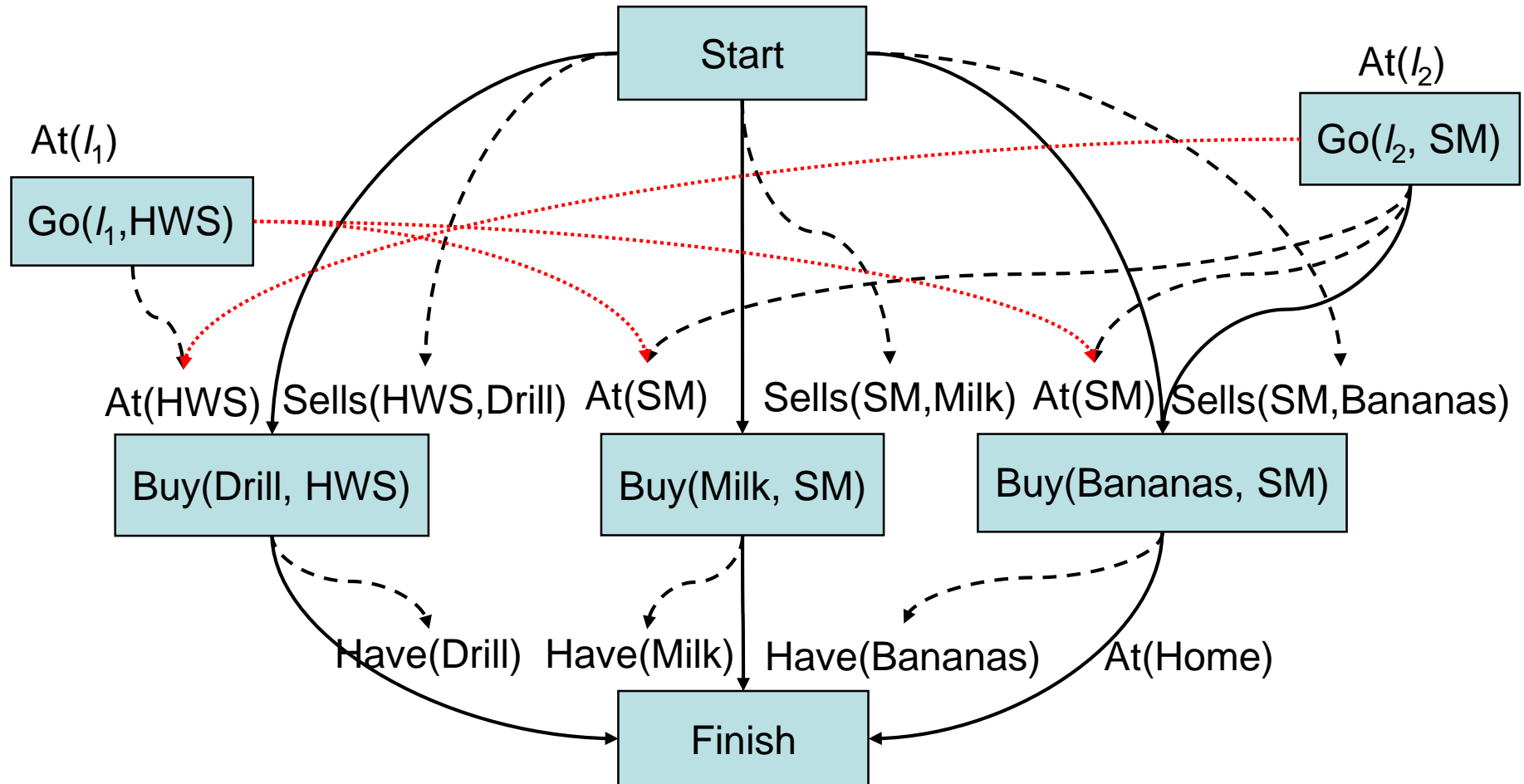
- Three more refinement steps
 - ◆ The only possible ways to establish the Sells preconditions



Example (continued)

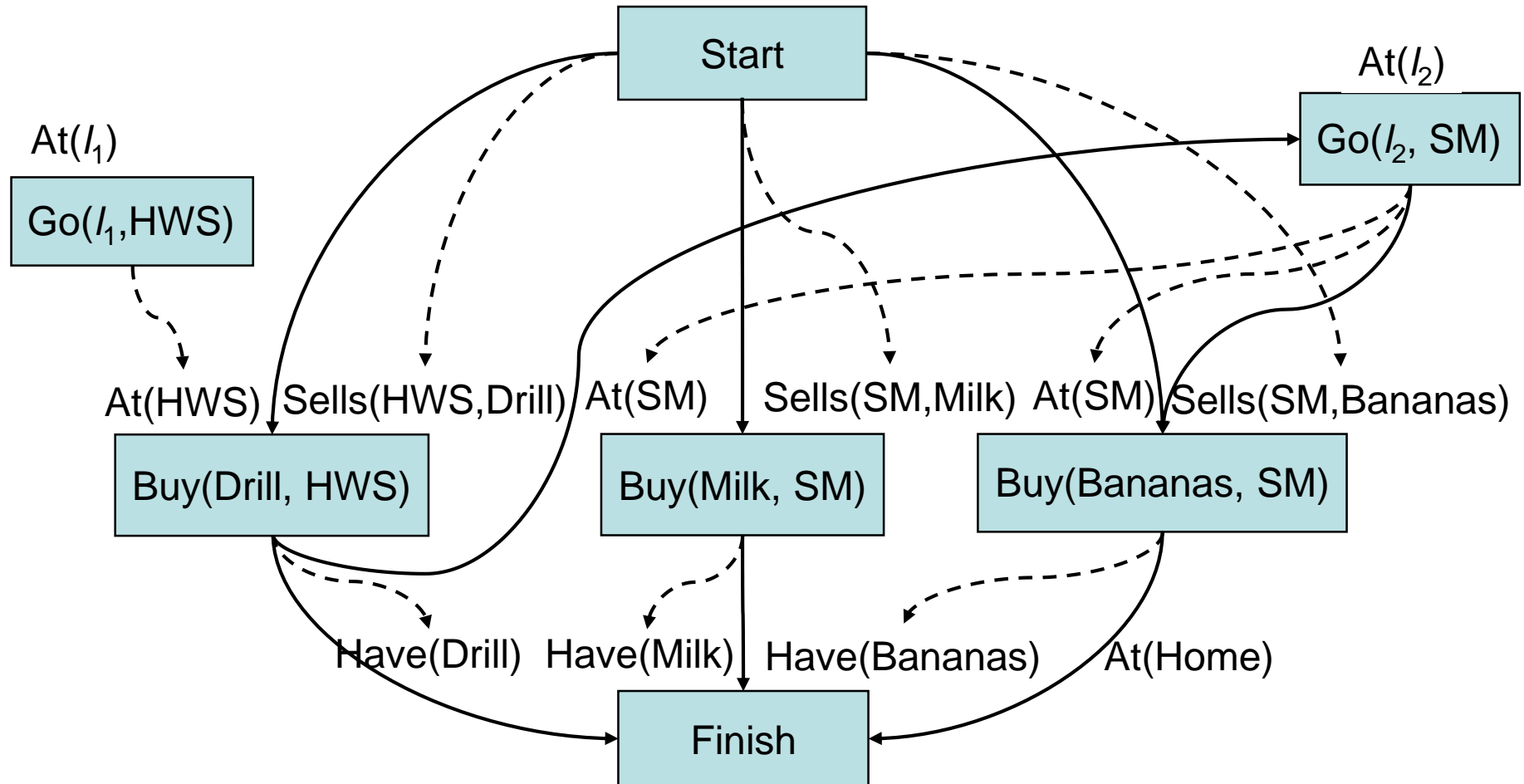
- Two more refinements: the only ways to establish $At(HWS)$ and $At(SM)$

◆ This time, several threats occur



Example (continued)

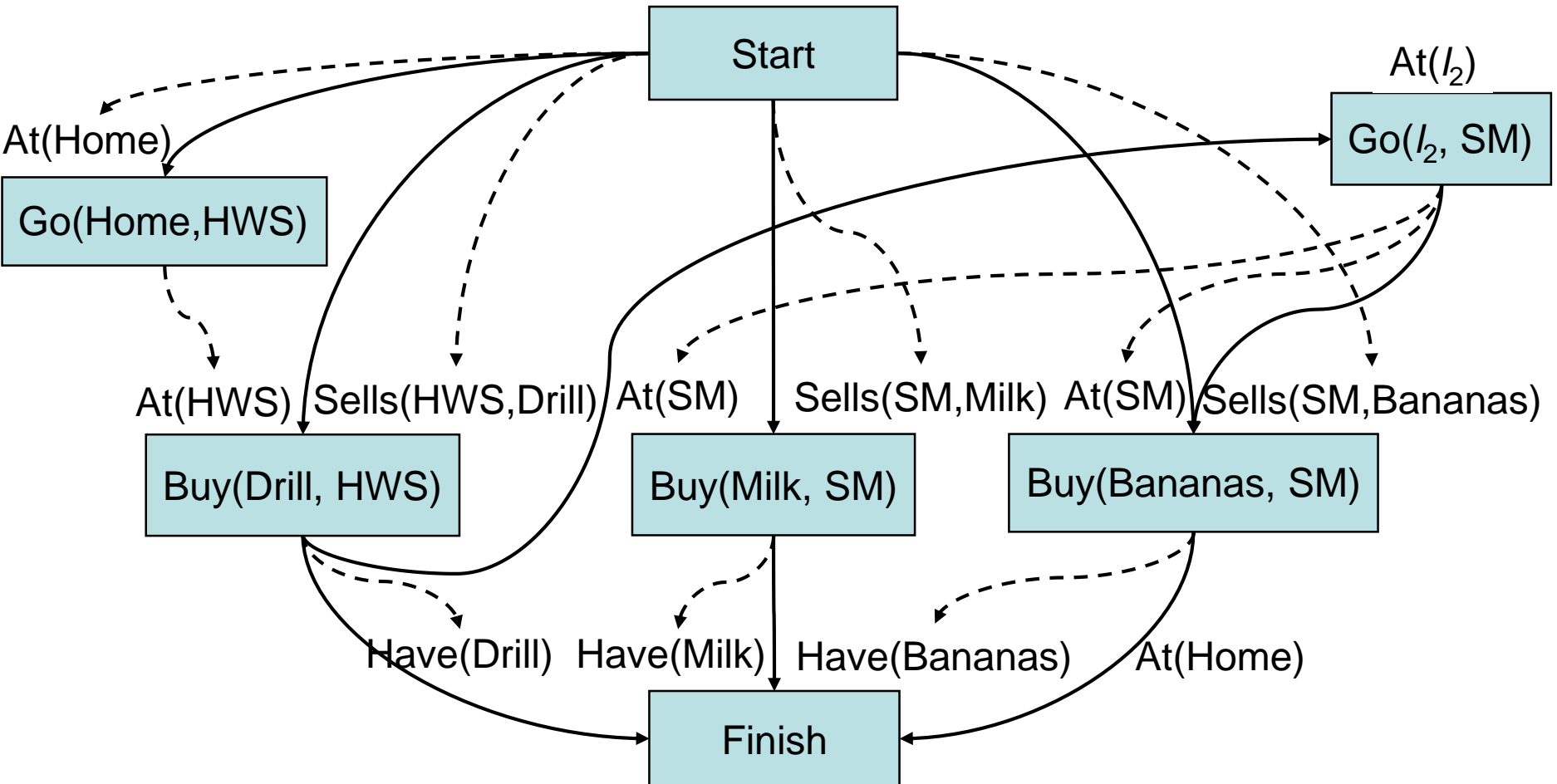
- Finally, a nondeterministic choice: how to resolve the threat to $At(s_1)$?
 - ◆ Our choice: make $Buy(Drill)$ precede $Go(SM)$
 - ◆ This also resolves the other two threats



Example (continued)

- Nondeterministic choice: how to establish $At(l_1)$?

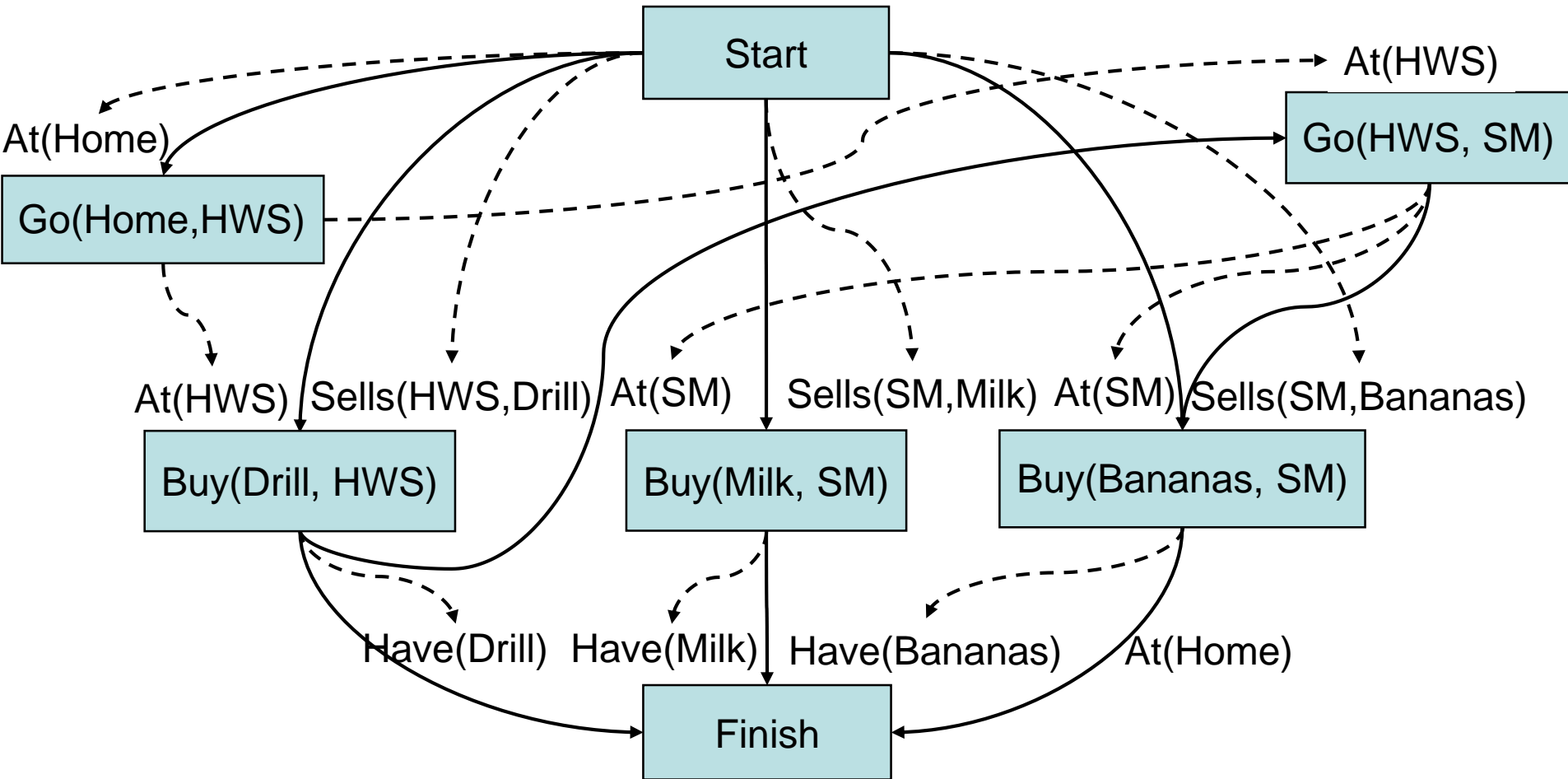
◆ We'll do it from Start, with $l_1=Home$



Example (continued)

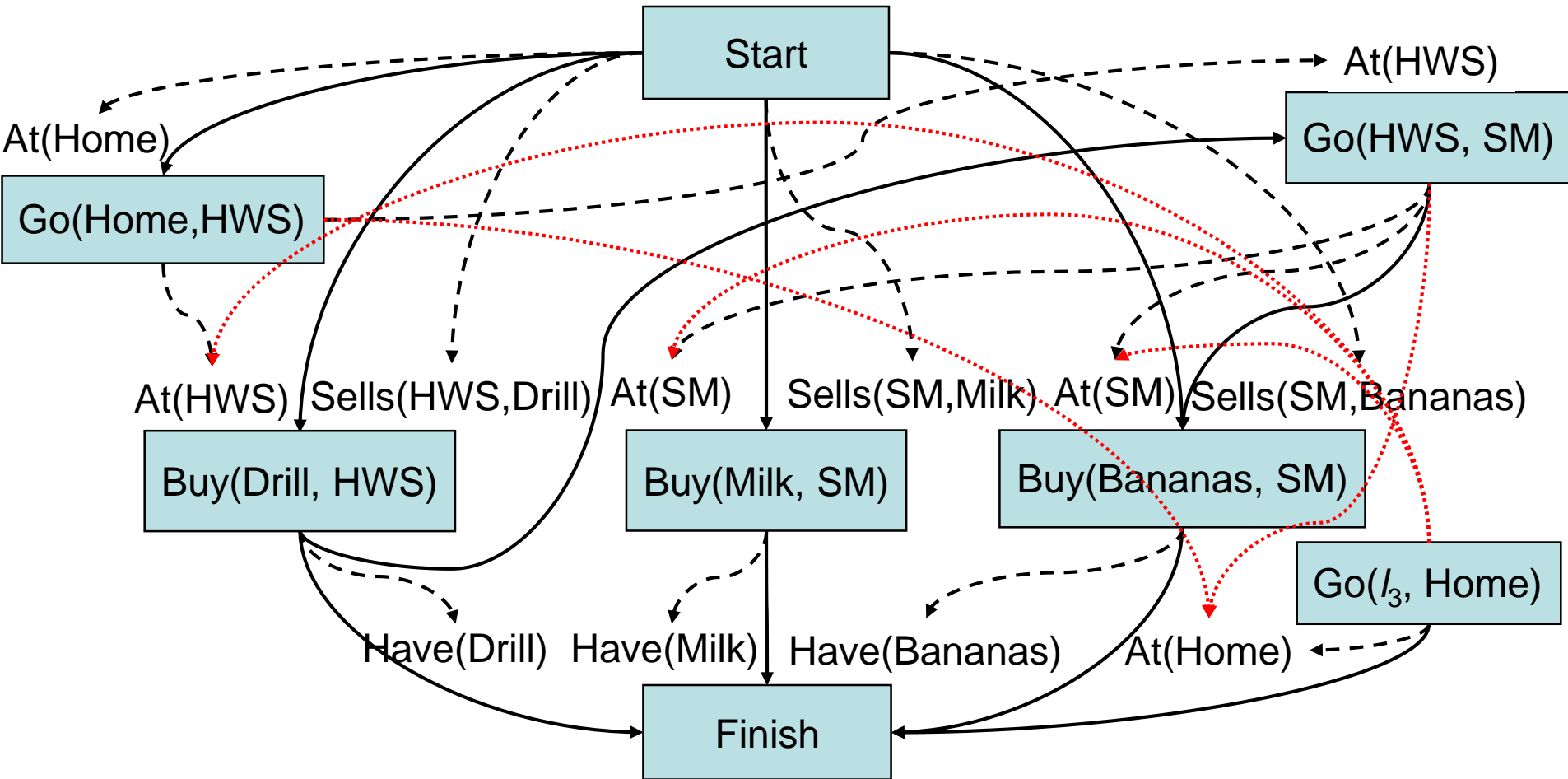
- Nondeterministic choice: how to establish $At(l_2)$?

◆ We'll do it from $Go(Home, HWS)$, with $l_2 = HWS$



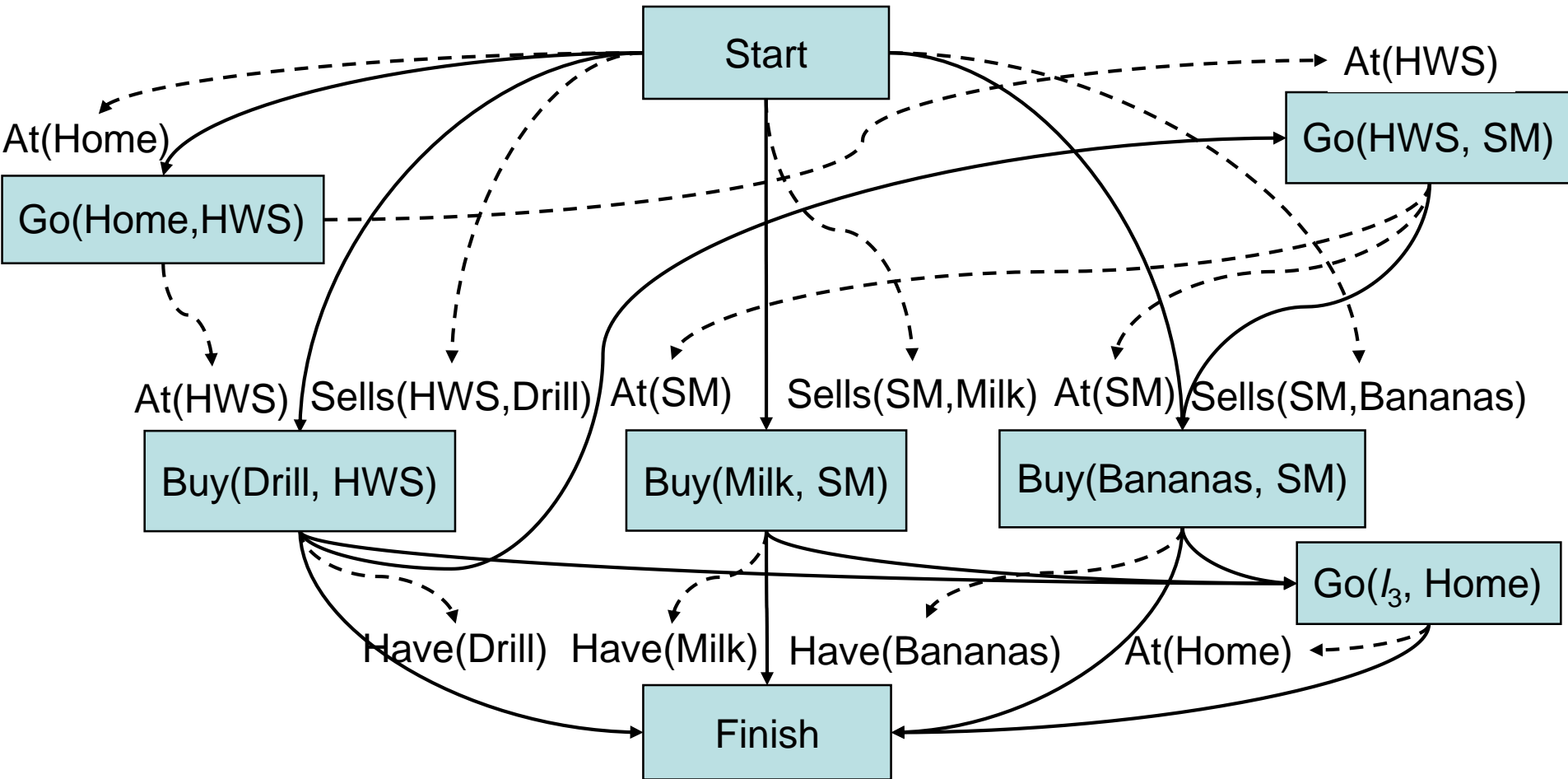
Example (continued)

- The only possible way to establish $\text{At}(\text{Home})$ for Finish
 - ◆ This creates a bunch of threats



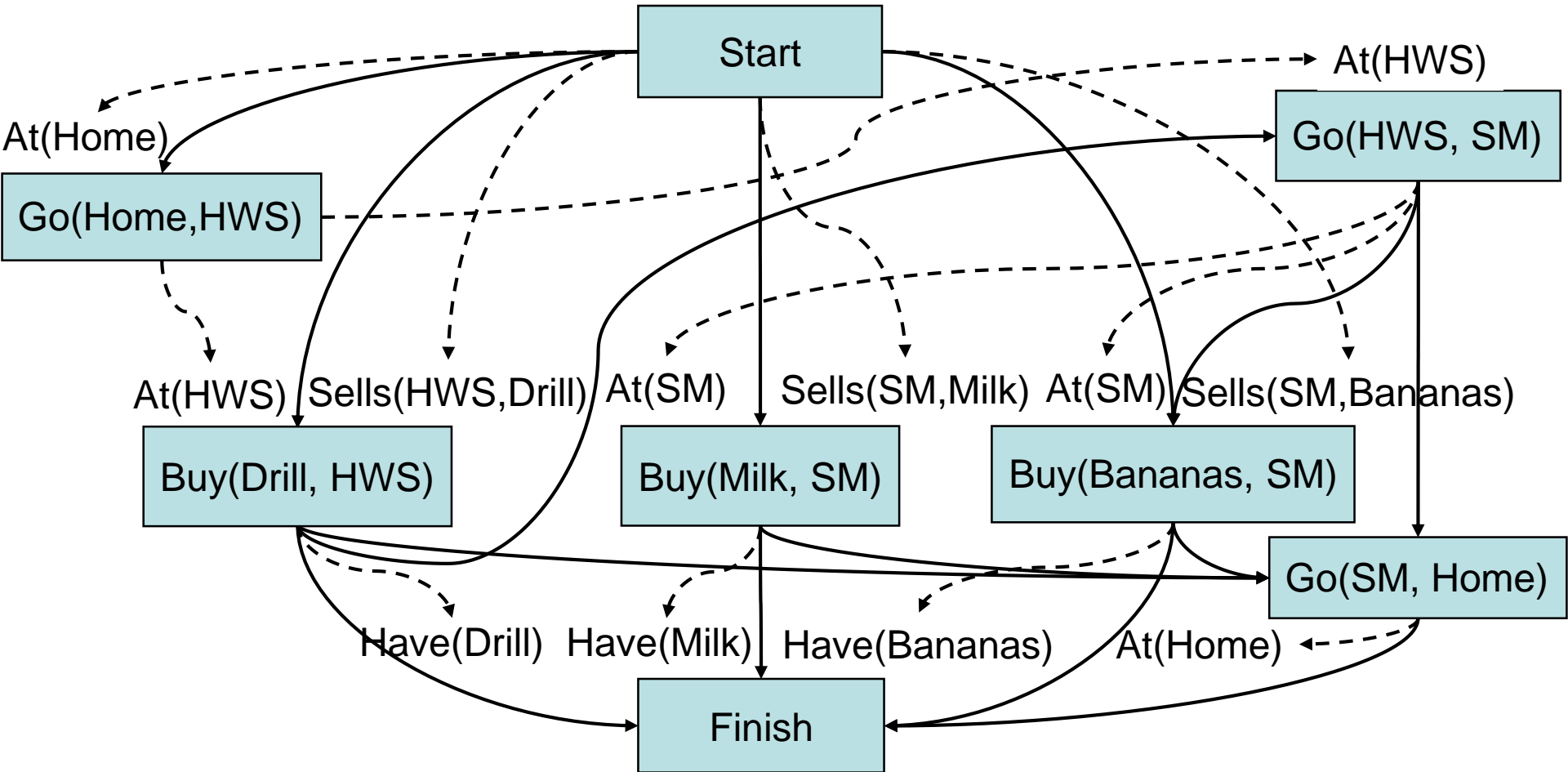
Example (continued)

- To remove the threats to $At(SM)$ and $At(HWS)$, make them precede $Go(l_3, Home)$
 - ◆ This also removes the other threats



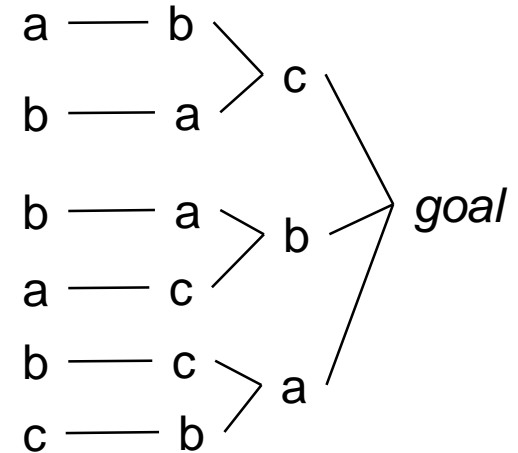
Final Plan

- Establish $At(l_3)$ with $l_3=SM$



Discussion

- How to choose which flaw to resolve first and how to resolve it?
 - ◆ There are lots of things!!!
- PSP doesn't commit to orderings and instantiations until necessary
 - ◆ Avoids generating search trees like this one:
- Problem: how to prune infinitely long paths?
 - ◆ In state-space planning, loop detection is based on recognizing states we've seen before
 - ◆ In a partially ordered plan, we don't know the states
- Can we prune if we see the same *action* more than once?
 - ... — go(b,a) — go(a,b) — go(b,a) — at(a)
 - ◆ No. Sometimes we might need the same action several times in different states of the world (see next slide)



Example

- 3-digit binary counter starts at 000, want to get to 110

$$s_0 = \{d_3=0, d_2=0, d_1=0\}$$

$$g = \{d_3=1, d_2=1, d_1=0\}$$

- Operators to increment the counter by 1:

incr0

Precond: $d_1=0$

Effects: $d_1=1$

incr01

Precond: $d_2=0, d_1=1$

Effects: $d_2=1, d_1=0$

incr011

Precond: $d_3=0, d_2=1, d_1=1$

Effects: $d_3=1, d_2=0, d_1=0$

A Weak Pruning Technique

- Can prune all partial plans of n or more actions, where $n = |\{\text{all possible states}\}|$
 - ◆ This doesn't help very much
- There's no good pruning technique for plan-space planning
 - ◆ Russell and Norvig's *Artificial Intelligence: A Modern Approach* describes a preliminary approach
 - ◆ In early 90's, researchers have looked at learning pruning rules during PSP planning

Thanks and Questions!