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P.189.10 $\lim\limits_{y\to0}\dfrac{y-\arcsin y}{\sin^3 y}=\lim\limits_{y\to0}\dfrac{y-\arcsin y}{y^3}=\lim\limits_{y\to0}\dfrac{1-\frac{1}{\sqrt{1-y^2}}}{3y^2}$

$=\lim\limits_{y\to0}\dfrac{0-(-\frac12)\cdot\frac{-2y}{(\sqrt{1-y^2})^3}}{6y}=\lim\limits_{y\to0}(-\frac16)\cdot\dfrac{1}{(\sqrt{1-y^2})^3}=-\dfrac16$

P.189.11 $\lim\limits_{y\to1}(\dfrac{y}{y-1}-\dfrac{1}{\ln y})\overset{\infty-\infty}{=}\lim\limits_{y\to1}\dfrac{y\ln y-(y-1)}{(y-1)\cdot\ln y}=\lim\limits_{y\to1}\dfrac{y\cdot\frac1y+\ln y-1}{\ln y+1-\frac1y}=\lim\limits_{y\to1}\dfrac{\frac1y}{\frac1y+\frac1{y^2}}=\dfrac12.$

P.189.12 $\lim\limits_{x\to0}\dfrac{1-x^2-e^{-x^2}}{x\cdot\sin^3x}\overset{\frac00}{=}\lim\limits_{x\to0}\dfrac{1-x^2-e^{-x^2}}{x^4}=\lim\limits_{x\to0}\dfrac{-2x+2xe^{-x^2}}{4x^3}=\lim\limits_{x\to0}\dfrac{-1+e^{-x^2}}{2x^2}=\lim\limits_{x\to0}\dfrac{-2xe^{-x^2}}{4x}=-\dfrac12.$

P.189.13 $\lim\limits_{x\to0}(\dfrac{\arctan x}{x})^{\frac{1}{x^2}}=e^{\lim\limits_{x\to0}\frac{1}{x^2}\cdot\ln\frac{\arctan x}{x}}=e^{-\frac13}$

因为: $\lim\limits_{x\to0}\dfrac{1}{x^2}\cdot\ln\dfrac{\arctan x}{x}=\lim\limits_{x\to0}\dfrac{\ln\frac{\arctan x}{x}}{x^2}=\lim\limits_{x\to0}\dfrac{\ln(\arctan x)-\ln x}{x^2}=\lim\limits_{x\to0}\dfrac{\frac{1}{\arctan x}\cdot\frac{1}{1+x^2}-\frac1x}{2x}$

$=\lim\limits_{x\to0}\dfrac{1}{2x}\cdot\dfrac{x-(1+x^2)\arctan x}{x(1+x^2)\cdot\arctan x}=\dfrac12\lim\limits_{x\to0}\dfrac{x}{\arctan x}\cdot\lim\limits_{x\to0}\dfrac{x-(1+x^2)\cdot\arctan x}{x^3\cdot(1+x^2)}$

$=\dfrac12\times1\times\lim\limits_{x\to0}\dfrac{1-1-2x\arctan x}{3x^2+5x^4}=\dfrac12\cdot\lim\limits_{x\to0}\dfrac{-2\arctan x-2\cdot\frac{x}{1+x^2}}{6x+20x^3}=\lim\limits_{x\to0}\dfrac{\frac{-2}{1+x^2}-2\cdot\frac{(1+x^2)-2x^2}{(1+x^2)^2}}{2(6+60x^2)}$

$=\dfrac{-4}{2\times6}=-\dfrac13$

P.189.14. $\lim\limits_{x\to+\infty}(\frac{\pi}{2}-\arctan x)^{\frac{1}{\ln x}}$

$\overset{0^0}{=}e^{\lim\limits_{x\to+\infty}\frac{\ln(\frac{\pi}{2}-\arctan x)}{\ln x}}=e^{-1}.$

因为: $\lim\limits_{x\to+\infty}\dfrac{\ln(\frac{\pi}{2}-\arctan x)}{\ln x}=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{\frac{\pi}{2}-\arctan x}\cdot\frac{-1}{1+x^2}}{\frac1x}=\lim\limits_{x\to+\infty}\dfrac{-\frac{x}{1+x^2}}{\frac{\pi}{2}-\arctan x}=\lim\limits_{x\to+\infty}\dfrac{-\frac{(1+x^2)-x\cdot2x}{(1+x^2)^2}}{-\frac{1}{1+x^2}}$

$=\lim\limits_{x\to+\infty}\dfrac{1-x^2}{1+x^2}=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{x^2}-1}{\frac{1}{x^2}+1}=\dfrac{0-1}{0+1}=-1.$

P.189.15 $\lim\limits_{x\to0}\dfrac{\tan x-x}{x-\sin x}\overset{\frac00}{=}\lim\limits_{x\to0}\dfrac{\sec^2x-1}{1-\cos x}=\lim\limits_{x\to0}\dfrac{2\sec x\cdot\sec x\cdot\tan x}{\sin x}=\lim\limits_{x\to0}\dfrac{2}{\cos^3x}=2.$

P.189.16 $\lim\limits_{x\to0}\dfrac{\mathrm{ch}x-\cos x}{x^2}=\lim\limits_{x\to0}\dfrac{\frac12(e^x+e^{-x})-\cos x}{x^2}=\lim\limits_{x\to0}\dfrac{\frac12(e^x-e^{-x})+\sin x}{2x}=\lim\limits_{x\to0}[\dfrac{e^x+e^{-x}}{4}+\dfrac12\dfrac{\sin x}{x}]=1.$

P.189.17 $\lim\limits_{x\to1}\dfrac{x^x-x}{\ln x-x+1}\overset{\frac00}{=}\lim\limits_{x\to1}\dfrac{x^x(1+\ln x)-1}{\frac1x-1}=\lim\limits_{x\to1}\dfrac{x^x(1+\ln x)^2+x^x\cdot\frac1x}{-\frac{1}{x^2}}=\lim\limits_{x\to1}\dfrac{x^{x+2}(1+\ln x)^2+x^{x+1}}{-1}=-2.$

P.189.18 $\lim\limits_{x\to+\infty}(\frac{2}{\pi}\arctan x)^x\overset{1^\infty}{=}\lim\limits_{x\to+\infty}e^{x\ln\frac{2}{\pi}\arctan x}=e^{\lim\limits_{x\to+\infty}\frac{\ln\frac{2}{\pi}\arctan x}{\frac1x}}=e^{-\frac{2}{\pi}}.$

而 $\lim\limits_{x\to+\infty}\dfrac{\ln\frac{2}{\pi}\arctan x}{\frac1x}=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{\frac{2}{\pi}\arctan x}\cdot\frac{2}{\pi}\cdot\frac{1}{1+x^2}}{-\frac{1}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1}{\arctan x}\cdot(-1)\cdot\dfrac{x^2}{1+x^2}$

$=\lim\limits_{x\to+\infty}\dfrac{-1}{\arctan x}\cdot\dfrac{1}{\frac{1}{x^2}+1}=(-1)\cdot\dfrac{1}{\frac{\pi}{2}}=-\dfrac{2}{\pi}.$