中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2010 学年秋季学期)

《SE-103　线性代数》期末试题参考答案(A)

1.　Fill in the blank（10*3=30 Pts）

(1) x= $\begin{bmatrix} 11 \\ 2 \end{bmatrix}$

(2) $\det A$ **=6**

(3) a **=8,** b **= -7**

(4) $\hat{y} = \frac{2}{3}\begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$

(5) $A = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$

(6) A basis for $Row\ A$ **is {（2，-3，6，2，5)，（0，0，3，-1，1)，（0，0，0，1，3）}, (注：原矩阵 A 的前三行线性相关，故不能取 A 的前三行)；** $\dim Nul\ A$ **is 2，** $rank\ A^T$ **is 3**

(7) $v_1 = \begin{bmatrix} 1+3i \\ 2 \end{bmatrix}$

2.　Mark each statement True or False, and descript your reasons.（5*4=20 Pts）

(1)　T

(2)　T

(3)　F

(4)　T

(5)　F

3.　Calculation issues (5*6=30 Pts）

(1)

Solution

a. Determine if the equation $c_1\mathbf{v}_1+c_2\mathbf{v}_2+c_3\mathbf{v}_3=\mathbf{x}$ **is consistent.**

$$\begin{bmatrix} 5 & 3 & 1 & -2 \\ 0 & 2 & -1 & 3 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 5 & 3 & 1 & -2 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$ **Thus x is in W.**

b. Since $\mathbf{y}\cdot\mathbf{v}_1=0$, $\mathbf{y}\cdot\mathbf{v}_2=0$, $\mathbf{y}\cdot\mathbf{v}_3=0$, **so y is in** $W^{\perp}$.

(2)

Solution

$$AB+I=A^2+B \Rightarrow AB-B=A^2-I \Rightarrow (A-I)\ B=A^2-I \Rightarrow B=(A-I)^{-1}(A^2-I)$$

thus $B=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

(3)

Solution

a. Computer and find $Av_1=-5v_1, Av_2=4v_2, Av_3=4v_3$**, so take**

$\boldsymbol{P}=\begin{bmatrix} \mathbf{-2} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{2} \end{bmatrix}$ **and** $D=\begin{bmatrix} -5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}_1$**, we have** $\boldsymbol{A=PDP^{-1}}$

b. $\{v_2, v_3\}$ **must be replace by an orthogonal basis.**

$$\hat{v}_3=(\frac{v_3\cdot v_2}{v_2\cdot v_2})v_2=\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ v_3-\hat{v}_3=\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}-\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 1/2 \\ -1/2 \\ 2 \end{bmatrix},$$

And normalize the eigenvectors.

Place eigenvectors into $P=\begin{bmatrix} -2/3 & 1/\sqrt{2} & 1/\sqrt{18} \\ 2/3 & 1/\sqrt{2} & -1/\sqrt{18} \\ 1/3 & 0 & 4/\sqrt{18} \end{bmatrix}_1$**, and set** $D=\begin{bmatrix} -5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}_1$

Equation $\boldsymbol{A=QDQ^T}$**.**

(4)

Solution

$$A^TA=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} 14 & 21 \\ 21 & 42 \end{bmatrix},\ A^Tb=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 10 \\ 3 \end{bmatrix}=\begin{bmatrix} 28 \\ 63 \end{bmatrix}.$$

Solving $A^TAx=A^Tb$, $\begin{bmatrix} 14 & 21 & 28 \\ 21 & 42 & 63 \end{bmatrix}\sim\ldots\sim\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$**, so the solution is** $\hat{x}=\begin{bmatrix} -1 \\ 2 \end{bmatrix}$**.**

Or $\hat{x}=(A^TA)^{-1}A^Tb=\frac{1}{147}\begin{bmatrix} 42 & -21 \\ -21 & 14 \end{bmatrix}\begin{bmatrix} 28 \\ 63 \end{bmatrix}=\frac{1}{147}\begin{bmatrix} -147 \\ 294 \end{bmatrix}=\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

And $b-A\hat{x}=\begin{bmatrix} -1 \\ 10 \\ 3 \end{bmatrix}-\begin{bmatrix} 1 & -1 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}=\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$**, thus** $\|b-A\hat{x}\|=\sqrt{2^2+2^2+(-2)^2}=\sqrt{12}$

(5)

Solution

A typical element of H **can be written as**

$$\begin{bmatrix} 3a+7b-c \\ -5b+8c-2d \\ 3d-4e \\ 5b-8c-d+4e \end{bmatrix} = a\underset{\mathbf{u}_1}{\begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}} + b\underset{\mathbf{u}_2}{\begin{bmatrix} 7 \\ -5 \\ 0 \\ 5 \end{bmatrix}} + c\underset{\mathbf{u}_3}{\begin{bmatrix} -1 \\ 8 \\ 0 \\ -8 \end{bmatrix}} + d\underset{\mathbf{u}_4}{\begin{bmatrix} 0 \\ -2 \\ 3 \\ -1 \end{bmatrix}} + e\underset{\mathbf{u}_5}{\begin{bmatrix} 0 \\ 0 \\ -4 \\ 4 \end{bmatrix}}$$

a. H **is a vector space because it is the set of all linear combinations of a set of vectors.**

b. Row reduce:

$$\begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 5 & -8 & -1 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & -3 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivot columns are 1,2, and 4. So { $\mathbf{u}_1$ **,** $\mathbf{u}_2$ **,** $\mathbf{u}_4$**} is a basis for** H **.**

4. Prove issues （2*10=20 Pts）

(1)

Proof Since $A^T = (I_n - \frac{2}{\|x\|^2}xx^T)^T = I_n - \frac{2}{\|x\|^2}xx^T = A$**,** A **is symmetric.**

And because $A^TA = A^2 = (I_n - \frac{2}{\|x\|^2}xx^T)(I_n - \frac{2}{\|x\|^2}xx^T)$

$$= I_n - \frac{4}{\|x\|^2}xx^T + \frac{4}{\|x\|^4}x(x^Tx)x^T$$

$$= I_n - \frac{4}{\|x\|^2}xx^T + \frac{4}{\|x\|^4}x(\|x\|^2)x^T$$

$$= I_n - \frac{4}{\|x\|^2}xx^T + \frac{4}{\|x\|^2}xx^T$$

$$= I_n,$$

thus A **is a orthogonal matrix.**

Hence A **is both orthogonal and symmetric.**

(2)

Proof $Ax=(I-2xx^T)x=x-2x(x^Tx)=-x$**, so –1 is an eigenvalue, the eigenspace is span{x}.**

Take a nonzero vector v in (span{ x **})**$^{\perp}$**,** $Av=(I-2xx^T)v=v-2x(x^Tv)=v$**, so 1 is the other**

eigenvalue, its eigenspace is (span{ x })$^{\perp}$.