中山大學 本科生考试草稿纸 Jan 14/5-59.

警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.144.27 $\int \frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}dx = \int \frac{x+1+2\sqrt{x^2-1}+(x-1)}{(x+1)-(x+1)}dx = \int x\,dx + \int\sqrt{x^2-1}\,dx$

令 $x=\sec t$，则 $dx=\sec t\cdot\tan t\,dt$

$$\int\sqrt{x^2-1}\,dx = \int \tan t\cdot \sec t\cdot\tan t\,dt = \underline{\int \tan^2 t\cdot\sec t\,dt}$$

$$=\int(\sec^2 t-1)\cdot\sec t\,dt$$

$$=\int\sec^3 t\,dt - \int\sec t\,dt$$

$$=\int \sec t\,d\tan t - \int\sec t\,dt$$

$$=\sec t\cdot\tan t - \int\tan t\,d\sec t - \int \sec t\,dt$$

$$=\sec t\cdot\tan t - \int\sec t\,dt - \int\tan^2 t\sec t\,dt$$

$$2\int\tan^2t\cdot\sec t\,dt = \sec t\cdot\tan t - \int\sec t\,dt$$

$$\int\sqrt{x^2-1}\,dx = \frac12\sec t\cdot\tan t - \frac12\int\sec t\,dt = \frac{x}{2}\sqrt{x^2-1}-\frac12\ln|x+\sqrt{x^2-1}|+C.$$

所以原式 $=\frac{x^2}{2}+\frac{x}{2}\sqrt{x^2-1}-\frac12\ln|x+\sqrt{x^2-1}|+C.$

P.144.28 $\int\frac{dx}{\sqrt[3]{(x+1)^2\cdot(x-1)^4}} = \int\frac{1}{x^2-1}\cdot\sqrt[3]{\frac{x+1}{x-1}}\,dx$

令 $\sqrt[3]{\frac{x+1}{x-1}}=u$

则 $\frac{x+1}{x-1}=u^3=\frac{(x-1)+2}{x-1}=1+\frac{2}{x-1}$

$\frac{2}{x-1}=u^3-1$, $\frac{x-1}{2}=\frac{1}{u^3-1}$

$x-1=\frac{2}{u^3-1}$, $x=1+\frac{2}{u^3-1}$

$$dx=\frac{3u^2(u^3-1)-(u^3+1)\cdot3u^2}{(u^3-1)^2}du$$

$$=\frac{-6u^2}{(u^3-1)^2}du$$

$$=\int\frac{u}{\left(\frac{u^3+1}{u^3-1}\right)^2-1}\cdot\frac{-6u^2\,du}{(u^3-1)^2}$$

$$=\int\frac{-6u^3}{u^6+2u^3+1-(u^6-2u^3+1)}du$$

$$=\int\frac{-6u^3}{4u^3}du=-\frac32u+C$$

$$=-\frac32\cdot\sqrt[3]{\frac{x+1}{x-1}}+C$$