软 件 学 院　２ ０ ０ 8　级 软 件 工 程 专 业 (2008-2)

《线性代数》期中试题试卷

(考 试 形 式：　闭　卷　　考 试 时 间：120 分 钟)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：____________　姓名：___________　学号：___________成绩：_____________

1. Fill the blank (4 titles * 5 points/title = 20 points)

(1) Given a linear system $\begin{cases} x_1 + x_2 = -a_1 \\ x_2 + x_3 = a_2 \\ x_3 + x_4 = -a_3 \\ x_1 + x_4 = a_4 \end{cases}$, and if the system is consistent, the constants a_1, a_2, a_3, a_4 must satisfy   .

(2) The reduced echelon form of $A = \begin{bmatrix} 2 & -1 & -1 & 1 \\ 1 & 1 & -2 & 1 \\ 4 & -6 & 2 & -2 \end{bmatrix}$ is ________________.

(3) Suppose T: $R^2 \to R^2$ is a transformation, which reflection through the x_1-axis first, and then reflection through the line $x_2=x_1$. So the standard matrix of T is ___________.

(4) Given matrix $A = \begin{bmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{bmatrix}$, the basis of Col A is _______________;

Nul A is a ________ dimensional subspace of Rk, and k is ________, the rank of A is ________.

2. Mark each statement True or False, and descript your reasons (3titles * 5 points/title = 15 points)

(1) Suppose vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ span a plane in R^3, and $\mathbf{v}_3$ is not in this plane. Then

$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linear independent.

(2) If 3*3 matrices A and B each have three pivot positions, then A can be transformed into B by elementary row operations.

(3) If A is 3*3 matrix, there exist element matrices $E_1,\dots,E_p$ such that $E_1,\dots,E_pA=I$

3. Problem issues (12 + 8 + 10 + 8 = 38 points)

(1) Determine the vectors below linear dependent or not. Describe your reasons. (2 titles * 6 points/title = 12 points)

a. $\begin{bmatrix} -4 & 12 \\ 1 & -3 \\ -3 & 8 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & 7 & 0 \\ -4 & -6 & 5 \\ 6 & 13 & -3 \end{bmatrix}$

(2) Calculate LU factorization of $\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & 9 \\ 15 & 1 & 2 \end{bmatrix}$ matrix. (8 points).

(3) Let $e_1=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $e_2=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $y_1=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$,and $y_2=\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$,Suppose $T: R^2 \mapsto R^3$ is a linear transformation which mapping e_1 into y_1, and mapping e_2 into y_2. Find the image of $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$. (10 points).

(4) Suppose matrix $A=\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, find A^{-1}.(8 points).

4. Prove issues (10 + 7 = 17 points)

(1) Suppose the vectors $\alpha_1,\dots,\alpha_t$ are solutions of homogeneous linear system Ax=0 and the vectors is linear independent. β is not the solution of Ax=0. Please prove

that $\beta+\alpha_1,\ldots,\beta+\alpha_t$ are linear independent. (10 points).

(2) Suppose A is 6*4 matrix, B is 4*6 matrix, prove 6*6 matrix AB is not invertible. (7 points).

5. Synthesis (10 points)

Suppose A is an nxn matrix, and satisfy $A^2-A+I_n=0$, show A is invertible, and find A^{-1}. (10 points).