1.True or False， and give your reasons

(1) $\forall \mathbf{b} \in \mathrm{R}^{\mathrm{n}}$, $\mathrm{A}\mathbf{x} = \mathbf{b}$ is consistent, then A is invertible.

(2) Given a vector $\mathbf{b} \in \mathrm{R}^{\mathrm{n}}$, if $\mathrm{A}\mathbf{x} = \mathbf{b}$ is consistent, then A is invertible.

(3) Given a vector $\mathbf{b} \in \mathrm{R}^{\mathrm{n}}$, if $\mathrm{A}\mathbf{x} = \mathbf{b}$ has a unique solution, then A is invertible.

2. Proof that if AB is invertible, then A and B are both invertible (*Hints: using determinant*).

3. Given three vectors $\alpha_1, \alpha_2, \alpha_3 \in \mathrm{R}^3$, we define matrix $A = [\alpha_1, \alpha_2, \alpha_3]$ and $B = [\alpha_3 - 2\alpha_1, 3\alpha_2, \alpha_1]$. If $|A| = 3$, compute $|B|$.

4. $\begin{vmatrix} b+c & c+a & a+b \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ = ____________

5. If the equation $\begin{cases} \lambda x_1 + x_2 + x_3 = 0 \\ x_1 + \mu x_2 + x_3 = 0 \\ x_1 + 2\mu x_2 + x_3 = 0 \end{cases}$ has non-zero solution, compute the λ and μ. (*Hints: using determinant*).

6. If $\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$ are linear independent vectors, $\{\alpha_1, \alpha_2, \cdots, \alpha_m, \beta\}$ are linear dependent. Proof that β is a combination of $\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$.

7. If $\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$ are linear independent vectors, β is not a combination of $\{\alpha_1, \alpha_2, \cdots, \alpha_m\}$. Proof that $\{\alpha_1, \alpha_2, \cdots, \alpha_m, \beta\}$ are independent dependent.