中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2010 学年秋季学期)

《线性代数》 期 末 试 题 答 案 (A/B)

1. Fill in the blank（10*3=30 Pts）

(1) x= $\begin{bmatrix} 11 \\ 2 \end{bmatrix}$

(2) $\det A$ **=6**

(3) a **=8,** b **= -7**

(4) $\hat{y} = \frac{2}{3}\begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$

(5) $A = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$

(6) A basis for $Row\ A$ **is {（2，-3，6，2，5)，（0，0，3，-1，1)，（0，0，0，1，3）},**

$\dim Nul\ A$ **is 2，** $rank\ A^T$ **is 3**

(7) $v_1 = \begin{bmatrix} 1+3i \\ 2 \end{bmatrix}$

2. Mark each statement True or False, and descript your reasons. （5*4=20 Pts）

(1) T

(2) T

(3) F

(4) T

(5) F

3. Calculation issues (5*6=30 Pts）

(1)

Solution

a. Determine if the equation $c_1v_1+c_2v_2+c_3v_3=x$ is consistent.

$$\begin{bmatrix} 5 & 3 & 1 & -2 \\ 0 & 2 & -1 & 3 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 1 & -3 \end{bmatrix} \sim \begin{bmatrix} 5 & 3 & 1 & -2 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$ **Thus x isn't in W.**

b. Since $y\cdot v_1=0$, $y\cdot v_2=0$, $y\cdot v_3=0$, so y is in $W^{\perp}$.

(2)

Solution

$$AB+I=A^2+B \Rightarrow AB-B=A^2-I \Rightarrow (A-I)\ B=A^2-I \Rightarrow B=(A-I)^{-1}(A^2-I)$$

thus $B=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

(3)

Solution

a. Computer and find $Av_1=-5v_1, Av_2=4v_2, Av_3=4v_3$**, and** $\{v_2, v_3\}$ **must be replace by an orthogonal basis.**

$$\hat{v}_3=\left(\frac{v_3\cdot v_2}{v_2\cdot v_2}\right)v_2=\frac{1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix},\ v_3-\hat{v}_3=\begin{bmatrix}1\\0\\2\end{bmatrix}-\frac{1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}1/2\\-1/2\\2\end{bmatrix},$$

And normalize the eigenvectors.

Place eigenvectors into $P=\begin{bmatrix} -2/3 & 1/\sqrt{2} & 1/\sqrt{18} \\ 2/3 & 1/\sqrt{2} & -1/\sqrt{18} \\ 1/3 & 0 & 4/\sqrt{18} \end{bmatrix}_1$ **, and set** $D=\begin{bmatrix} -5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}_1$

Equation $A=PDP^{-1}$

b. $3x_3^2+8x_1x_2+4x_1x_3-4x_2x_3$**. The quadratic form is indefinite.**

(4)

Solution

$$A^TA=\begin{bmatrix}1 & 2 & 3\\ -1 & 5 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 5\\ 3 & 4\end{bmatrix}=\begin{bmatrix}14 & 21\\ 21 & 42\end{bmatrix},\ A^Tb=\begin{bmatrix}1 & 2 & 3\\ -1 & 5 & 4\end{bmatrix}\begin{bmatrix}-1\\ 10\\ 3\end{bmatrix}=\begin{bmatrix}28\\ 63\end{bmatrix}.$$

Solving $A^TAx=A^Tb$, $\begin{bmatrix}14 & 21 & 28\\ 21 & 42 & 63\end{bmatrix}\sim\ldots\sim\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\end{bmatrix}$**, so the solution is** $\hat{x}=\begin{bmatrix}-1\\ 2\end{bmatrix}$**.**

Or $\hat{x}=(A^TA)^{-1}A^Tb=\frac{1}{147}\begin{bmatrix}42 & -21\\ -21 & 14\end{bmatrix}\begin{bmatrix}28\\ 63\end{bmatrix}=\frac{1}{147}\begin{bmatrix}-147\\ 294\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$

And $b-A\hat{x}=\begin{bmatrix}-1\\ 10\\ 3\end{bmatrix}-\begin{bmatrix}1 & -1\\ 2 & 5\\ 3 & 4\end{bmatrix}\begin{bmatrix}-1\\ 2\end{bmatrix}=\begin{bmatrix}2\\ 2\\ -2\end{bmatrix}$**, thus** $\|b-A\hat{x}\|=\sqrt{2^2+2^2+(-2)^2}=\sqrt{12}$

(5)

Solution

A typical element of H can be written as

$$\begin{bmatrix} 3a+7b-c \\ -5b+8c-2d \\ 3d-4e \\ 5b-8c-d+4e \end{bmatrix} = a\begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 7 \\ -5 \\ 0 \\ 5 \end{bmatrix} + c\begin{bmatrix} -1 \\ 8 \\ 0 \\ -8 \end{bmatrix} + d\begin{bmatrix} 0 \\ -2 \\ 3 \\ -1 \end{bmatrix} + e\begin{bmatrix} 0 \\ 0 \\ -4 \\ 4 \end{bmatrix}$$

$\mathbf{u}_1$ $\mathbf{u}_2$ $\mathbf{u}_3$ $\mathbf{u}_4$ $\mathbf{u}_5$

a. H is a vector space because it is the set of all linear combinations of a set of vectors.

b. Row reduce:

$$\begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 5 & -8 & -1 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & -3 & 4 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & -1 & 0 & 0 \\ 0 & -5 & 8 & -2 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivot columns are 1,2, and 4. So { $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_4$} is a basis for H .

4. Prove issues （2*10=20 Pts）

(1)

Proof Since $A^T = (I_n - \frac{2}{\|x\|^2}xx^T)^T = I_n - \frac{2}{\| x \|^2}xx^T = A$, A **is symmetric.**

And because $A^T A = A^2 = (I_n - \frac{2}{\|x\|^2}xx^T)(I_n - \frac{2}{\|x\|^2}xx^T)$

$$= I_n - \frac{4}{\| x \|^2}xx^T + \frac{4}{\| x \|^4}x(x^T x)x^T$$

$$= I_n - \frac{4}{\| x \|^2}xx^T + \frac{4}{\| x \|^4}x(\| x \|^2)x^T$$

$$= I_n - \frac{4}{\| x \|^2}xx^T + \frac{4}{\| x \|^2}xx^T$$

$$= I_n,$$

thus A is a orthogonal matrix.

Hence A is both orthogonal and symmetric.

(2)

Proof If there is an orthogonal matrix *T* such that $T^{-1}AT = B$, then A is similar to B, thus they have the same characteristic polynomial.

Conversely, if they have the same characteristic polynomial, then they have the same eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, thus there is orthogonal matrix T_1 and T_2 such that

$T_1^{-1}AT_1=[\lambda_1,\lambda_2,...,\lambda_n]$

$T_2^{-1}BT_2=[\lambda_1,\lambda_2,...,\lambda_n]$

Then

$T_1^{-1}AT_1=T_2^{-1}BT_2$

$(T_1T_2^{-1})^{-1}AT_1T_2^{-1}=B$

Let $T=T_1T_2^{-1}$ **, then** T **is an orthogonal matrix such that** $T^{-1}AT=B$