习题3-1 # 中山大學 本科生考试草稿纸 2011 10/1 -43

警示 《中山大学授予学士学位工作细则》第七条："考试作弊者不授予学士学位。"

P.129. 求下列定积分

1. $\int\sqrt{1+2x}\,dx = \frac{1}{2}\int\sqrt{1+2x}\,d(1+2x) = \frac{1}{3}(1+2x)^{\frac{3}{2}}+C.$

2. $\int\frac{3x\,dx}{(x^2+1)^2} = \frac{3}{2}\int\frac{1}{(x^2+1)^2}d(x^2+1) = -\frac{3}{2}\cdot\frac{1}{(1+x^2)}+C.$

3. $\int x\sqrt{7+2x^2}\,dx = \frac{1}{4}\int\sqrt{7+2x^2}\,d(7+2x^2) = \frac{1}{6}(7+2x^2)^{\frac{3}{2}}+C.$

4. $\int(2x^{\frac{3}{2}}+1)^{\frac{2}{3}}\cdot\sqrt{x}\,dx = \frac{2}{3}\int(2x^{\frac{3}{2}}+1)^{\frac{2}{3}}d(x^{\frac{3}{2}}) = \frac{1}{3}\int(2x^{\frac{3}{2}}+1)^{\frac{2}{3}}d(2x^{\frac{3}{2}}+1)$

$= \frac{1}{3}\cdot\frac{3}{5}(2x^{\frac{3}{2}}+1)^{\frac{5}{3}}+C = \frac{1}{5}(2x^{\frac{3}{2}}+1)^{\frac{5}{3}}+C.$

5. $\int\frac{e^{\frac{1}{x}}}{x^2}dx = -\int e^{\frac{1}{x}}d(\frac{1}{x}) = -e^{\frac{1}{x}}+C.$

6. $\int\frac{dx}{(2-x)^{100}} = -\int\frac{1}{(2-x)^{100}}d(2-x) = -\frac{1}{99}(2-x)^{-99}+C = \frac{1}{99}\cdot\frac{1}{(2-x)^{99}}+C.$

7. $\int\frac{dx}{3+5x^2} = \frac{1}{3}\int\frac{dx}{1+(\frac{\sqrt{5}}{\sqrt{3}}x)^2} = \frac{1}{\sqrt{15}}\int\frac{1}{1+(\sqrt{\frac{5}{3}}x)^2}d(\sqrt{\frac{5}{3}}x) = \frac{1}{\sqrt{15}}\arctan\sqrt{\frac{5}{3}}x+C.$

8. $\int\frac{dx}{\sqrt{7-3x^2}} = \frac{1}{\sqrt{3}}\int\frac{1}{\sqrt{7-3x^2}}d(\sqrt{3}x) = \frac{1}{\sqrt{3}}\int\frac{1}{\sqrt{1-(\frac{\sqrt{3}}{\sqrt{7}}x)^2}}d(\sqrt{\frac{3}{7}}x) = \frac{1}{\sqrt{3}}\arcsin\sqrt{\frac{3}{7}}x+C.$

9. $\int\frac{dx}{\sqrt{x}(1+x)} = 2\int\frac{1}{1+x}d\sqrt{x} = 2\arctan\sqrt{x}+C.$

10. $\int\frac{e^x}{2+e^{2x}}dx = \int\frac{1}{2+e^{2x}}de^x = \frac{1}{\sqrt{2}}\int\frac{1}{1+(\frac{e^x}{\sqrt{2}})^2}d(\frac{e^x}{\sqrt{2}}) = \frac{1}{\sqrt{2}}\arctan\frac{e^x}{\sqrt{2}}+C.$

11. $\int\frac{dx}{\sqrt{e^{-2x}-1}} = \int\frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \int\frac{de^x}{\sqrt{1-(e^x)^2}} = \arcsin e^x+C.$

12. $\int\frac{dx}{e^x-e^{-x}} = \int\frac{dx}{e^x-\frac{1}{e^x}} = \int\frac{e^x\,dx}{e^{2x}-1} = \int\frac{de^x}{(e^x-1)(e^x+1)} = \frac{1}{2}\int(\frac{1}{e^x-1}-\frac{1}{e^x+1})de^x = \frac{1}{2}\ln\left|\frac{1-e^x}{1+e^x}\right|+C.$

3. $\int\frac{\ln\ln x}{x\ln x}dx = \int\frac{\ln\ln x}{\ln x}d\ln x = \int\ln\ln x\,d\ln\ln x = \frac{(\ln\ln x)^2}{2}+C$

14. $\int\frac{dx}{1+\cos x} = \int\frac{dx}{2\cos^2\frac{x}{2}} = \int\sec^2\frac{x}{2}\,d(\frac{x}{2}) = \tan\frac{x}{2}+C.$