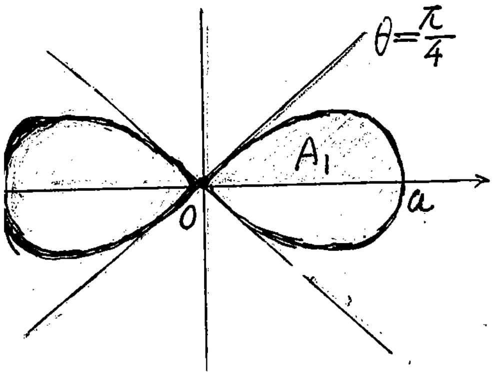
中山大學 本科生考试草稿纸 2015-7-17

警示 《中山大学授予学士学位工作细则》第七条:"考试作弊者不授予学士学位。"

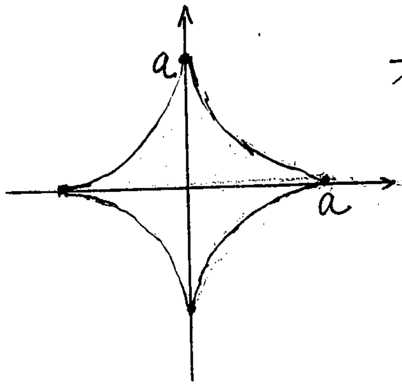
P.164.8. 求双纽线: $r^2 = a^2\cdot\cos2\varphi$, $(a>0)$ 所围面积A.



$$A_1 = \frac{1}{2}\int_0^{\frac{\pi}{4}} r^2 d\varphi$$
$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} a^2\cdot\cos2\varphi\, d\varphi$$
$$= \frac{a^2}{4}[\sin2\varphi]_0^{\frac{\pi}{4}} = \frac{a^2}{4}(1-0) = \frac{a^2}{4}$$
$$A = 4A_1 = 4\times\frac{a^2}{4} = a^2.$$

P.164.9 求下列图形绕x轴旋转一周所形成的旋转体体积。

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, $a>0$, 求V_x.



方法1. $y^{\frac{2}{3}} = a^{\frac{2}{3}} - x^{\frac{2}{3}}$

$$y^2 = (a^{\frac{2}{3}} - y^{\frac{2}{3}})^3 = a^2 - 3a^{\frac{4}{3}}\cdot x^{\frac{2}{3}} + 3a^{\frac{2}{3}}\cdot x^{\frac{4}{3}} - x^2$$
$$V_x = 2\int_0^a \pi y^2 dx = 2\pi\int_0^a (a^2 - 3a^{\frac{4}{3}}\cdot x^{\frac{2}{3}} + 3a^{\frac{2}{3}}\cdot x^{\frac{4}{3}} - x^2)dx$$
$$= 2\pi(a^3 - 3a^{\frac{4}{3}}\cdot\frac{3}{5}a^{\frac{5}{3}} + 3a^{\frac{2}{3}}\cdot\frac{3}{7}a^{\frac{7}{3}} - \frac{a^3}{3})$$
$$= 2\pi a^3(1 - \frac{9}{5} + \frac{9}{7} - \frac{1}{3})$$
$$= 2\pi a^3(\frac{2}{3} - \frac{63}{35} + \frac{45}{35}) = 2\pi a^3(\frac{2}{3} - \frac{18}{35})$$
$$= 2\pi a^3\cdot\frac{70-54}{105} = \frac{32}{105}\pi a^3$$

方法2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ 的参数方程:

$$\begin{cases} x = a\cdot\cos^3 t \\ y = a\cdot\sin^3 t \end{cases}$$

$$V_x = 2\int_0^a \pi y^2 dx = 2\pi\int_{\frac{\pi}{2}}^0 a^2\sin^6 t\, d\, a\cdot\cos^3 t$$
$$= 2\pi a^3\int_0^{\frac{\pi}{2}} \sin^6 t\cdot 3\cos^2 t\sin t\, dt$$
$$= 6\pi a^3\int_0^{\frac{\pi}{2}} \sin^7 t\cdot(1-\sin^2 t)dt$$
$$= 6\pi a^3[\int_0^{\frac{\pi}{2}} \sin^7 t\, dt - \int_0^{\frac{\pi}{2}} \sin^9 t\, dt]$$
$$= 6\pi a^3(\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3} - \frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}) = 6\pi a^3\frac{1}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3} = \frac{32\pi a^3}{105}$$