中 山 大 学 软 件 学 院 **2009** 级 软 件 工 程 专 业 (2009 秋季学期)

# 《线性代数》期末试题试卷(B)

**(考 试 形 式 ： 闭 卷　　考 试 时 间 ： 2 小 时)**

警 示

**《中山大学授予学士学位工作细则》第六条**

**考 试 作 弊 不 授 予 学 士 学 位**

方向：___________________　　姓名：__________　　学号：____________

## 1. Fill the blank (4 titles * 4 points/title = 16 points)

(1) The matrics A and B below are row equivalent.

$$A=\begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix};\quad B=\begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & -4 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for $Row\ A$ is ____________________________________________.

(2) If $A=\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{bmatrix}$, then $\det A=$___________.

(3) Find matrix $A=$________________ such that $ColA=\left\{\begin{bmatrix} 2s+3t \\ r+s-2t \\ 4r+s \\ 3r-s-t \end{bmatrix}: r,s,t\in R\right\}$.

(4) If $\alpha_1$ and $\alpha_2$ are orthonomal vectors，and $x=\alpha_1+5\alpha_2$， $y=4\alpha_1-3\alpha_2$， then

$x\cdot y=$___________。

## 2. Mark each statement True or False, and descript your reasons (3titles * 8 points/title = 24 points)

(1) If $AB=C$ and $C$ has 5 columns, then $A$ has 5 columns.

(2) All polynomials of degree at most 5 consists of a vector space.

(3) Let $A\in R^{n\times n}$ and $\det(A)=a$， then $\det(3A)=3a$.

(4) If $A^T = A$ and if vectors $u$ and $v$ satisfy $Au = 3u$ and $Av = 4v$, then $u \cdot v = 0$.

(5) Let $A \in R^{5\times 4}$ and $rankA = 3$, then $\dim NulA = 3$.

(6) $A \in R^{n\times n}$ is invertible if and only if all columns of A are linear independent.

(7) $Nul\ A = \{0\}$ if and only if the linear transformation $x \mapsto Ax$ is one to one.

(8) If $A \in R^{n\times n}$ is diagonalizable, then $A$ has $n$ distinct eigenvalues

**3. Calculation issues (40 points)**

(1) Make a change of variable, $x = Py$, that transforms the following quadratic form into one with no cross-product term. Give $P$ and the new quadratic form. (12 points).

$$3x_1^2 + 6x_2^2 + 3x_3^2 - 4x_1x_2 + 8x_1x_3 + 4x_2x_3$$

(2) Let the system equations be $\begin{cases} (2-\lambda)x_1 + 2x_2 - 2x_3 = 1 \\ 2x_1 + (5-\lambda)x_2 - 4x_3 = 2 \\ -2x_1 - 4x_2 + (5-\lambda)x_3 = -\lambda - 1 \end{cases}$. Find the approriate values for $\lambda$ to make the system have at most one solution, no solution, and infinite solutions, respectivly. When the system has infinite solutions, write them in parametric vector form. (12 points).

(3) Find a least-squares solution of Ax = b, and compute the least-squares error. (10 points).

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, \mathrm{b} = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$$

(4) Let $\mathfrak{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V, and let T: V--->V be a linear transformation with the property that $T(\mathbf{b}_1) = \mathbf{b}_1 + \mathbf{b}_3, T(\mathbf{b}_2) = 2\mathbf{b}_1 - \mathbf{b}_3$, and. $T(\mathbf{b}_3) = 3\mathbf{b}_1 + 4\mathbf{b}_2 + 5\mathbf{b}_3$. Find the matrix of T relative to the basis (6 points).

**4. Prove issues (20 points)**

(1) Let $\{\xi_1 \quad \xi_2 \quad \xi_3\}$ be a basis for $R^3$, and $\alpha_1 = \xi_1 + \xi_2 - 2\xi_3$, $\alpha_2 = \xi_1 - \xi_2 - \xi_3$, $\alpha_3 = \xi_1 + \xi_3$, $\beta = 6\xi_1 - \xi_2 - \xi_3$. Prove that $\{\alpha_1 \quad \alpha_2 \quad \alpha_3\}$ is also a basis for $R^3$, and find the coordinate vector of $\beta$ relative to $\{\alpha_1 \quad \alpha_2 \quad \alpha_3\}$. (10 points)

(2) Suppose $A$ is a $m \times n$ matrix such that the matrix $A^T A$ is invertible. Let $b$ be any vector in $R^n$. Show that the Linear system $Ax = b$ has at most one solution. (10 points)