P. 144. 17 $\int \frac{\cos x\,dx}{5-3\cos x}$, 令 $u=\tan\frac{x}{2}$, $dx=\frac{2du}{1+u^2}$ 2011/3/13 一次

$$=\int \frac{\frac{1-u^2}{1+u^2}}{5-3\cdot\frac{1-u^2}{1+u^2}}\cdot\frac{2du}{1+u^2} \qquad \cos x=\frac{1-u^2}{1+u^2}$$

$$=\int \frac{1-u^2}{5+5u^2-3+3u^2}\frac{2du}{1+u^2}=\int \frac{1-u^2}{2+8u^2}\cdot\frac{2du}{1+u^2}=-\int \frac{(u^2+1-2)\,du}{(1+4u^2)\cdot(1+u^2)}$$

$$=-\int \frac{du}{1+4u^2}+\frac{2}{3}\int\left(\frac{4}{1+4u^2}-\frac{1}{1+u^2}\right)du$$

$$=\frac{5}{3}\int\frac{du}{1+4u^2}-\frac{2}{3}\int\frac{du}{1+u^2}$$

$$=\frac{5}{6}\operatorname{arctan}2u-\frac{2}{3}\operatorname{arctan}u+C$$

$$=\frac{5}{6}\operatorname{arctan}\left(2\tan\frac{x}{2}\right)-\frac{2}{3}\cdot\frac{x}{2}+C=\frac{5}{6}\operatorname{arctan}\left(2\tan\frac{x}{2}\right)-\frac{x}{3}+C.$$

P. 144. 18 $\int\frac{\cos^3x\,dx}{\sin x+\cos x}=\int\frac{\cos^2x\,d\sin x}{\sin x+\cos x}=\int\frac{\cos^2x-\sin^2x}{\sin x+\cos x}d\sin x+\int\frac{\sin^2x}{\sin x+\cos x}d\sin x$

$$I=\int(\cos x-\sin x)d\sin x+\int\frac{1-\cos^2x}{\sin x+\cos x}\cdot\cos x\,dx$$

$$=\int\cos^2x\,dx-\frac{\sin^2x}{2}+\int\frac{\cos x\,dx}{\sin x+\cos x}-I$$

$$2I=\int\frac{1+\cos 2x}{2}dx-\frac{\sin^2x}{2}+\int\frac{\sin x+\cos x-\sin x}{\sin x+\cos x}dx$$

$$=\frac{x}{2}+\frac{\sin 2x}{4}-\frac{\sin^2x}{2}+x-\int\frac{\sin x}{\sin x+\cos x}dx$$

令 $\int\frac{\sin x}{\sin x+\cos x}dx=\int\frac{\frac{2u}{1+u^2}}{\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}\cdot\frac{2du}{1+u^2}=\int\frac{u}{2u+1-u^2}\cdot\frac{2du}{1+u^2}$

$$=\int\frac{-2u\,du}{(u^2-2u+1)(1+u^2)}=\int\left(\frac{1}{1+u^2}-\frac{1}{u^2-2u+1}\right)du$$

$$=\operatorname{arctan}u-\int\frac{1}{1+(u-1)^2}d(u-1)$$

$$=\frac{x}{2}-\operatorname{arctan}(u-1)+C$$

$$=\frac{x}{2}-\operatorname{arctan}\left[\tan\frac{x}{2}-1\right]+C$$

以下 $\int\frac{\cos^3x\,dx}{\sin x+\cos x}=\frac{x}{2}+\frac{\sin 2x}{8}-\frac{\sin^2x}{4}+\operatorname{arctan}\left[\tan\frac{x}{2}-1\right]+C.$